Local mapping of transport through quantum nano-structures from semiconductors, to superconductors, to graphene

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Local mapping of transport through quantum nano-structures: From semiconductors, to superconductors, to graphene

A dissertation submitted to
ETH ZURICH
for the degree of
Doctor of Sciences

presented by

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2010
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Abstract

We present transport and scanning gate measurements on nano-structures of diverse shapes and materials carried out in a home built low-temperature scanning gate microscopy setup. The measurements are supported by electrostatic simulations performed using the program Comsol Multiphysics.

We start by discussing the construction and operation of a scanning force microscope that was built into a $^3$He refrigerator with a base temperature of 280 mK. Stick-slip motors in various configurations have been used to achieve the coarse positioning of the sample at 4.2 K. A five-electrode scan piezo is used for fine movement. The setup is completed by special cabling, thermal anchoring, heaters and thermometers. Magnetic fields up to 9 T can be applied. We employ a metallic tip mounted onto a tuning fork sensor as a flying nano-gate that induces a small potential perturbation into the sample under investigation.

Using this measurement setup we have carried out transport measurements and scanning probe experiments on a variety of nano-structures. We start by presenting measurements performed on a graphene nano-ring. In the transport measurements we focus mainly on the amplitude, phase and visibility of the Aharonov–Bohm oscillations. The scanning gate measurements show that local manipulation of the charge carrier density is possible, allowing to diminish or enhance the charge carrier density locally. We show signs of a complex potential landscape in the sample near the Dirac point and investigate the fine structure in the current maps.

Scanning gate measurements on a superconducting Aluminum single electron transistor allow us to translate features from the charge stability diagrams into real space. We investigate the shape of the tip induced potential acting on the nano-structure supporting the theory that the tip potential is made up of two independent parts [1]. Spatially resolved mapping of transport properties such as the capacitances, the superconducting gap and the charging energy was carried out.

Furthermore we show transport and scanning probe experiments performed on a double quantum dot structure fabricated via local anodic oxidation into a two-dimensional electron gas. We show how distorted hexagons can be observed in real space. We investigate the apparent relative position of both quantum dots and its
dependence on gate voltage settings. In extremis we are able to form a single dot within the double dot geometry and determine its real space position.

Finally we show scanning gate experiments localizing leakage currents occurring in a structure fabricated via local anodic oxidation. We show that leakage currents flow at distinct points across the oxide barriers, not homogenously along their whole length. We show evidence that leakage currents are most likely to occur at positions where two oxide lines intersect.
Zusammenfassung


Zusammenfassung


Schlussendlich zeigen wir SGM Experimente, welche Leckströme in einer Struktur die mittels lokaler aniodischer Oxidation hergestellt wurde untersuchen. Wir zeigen dass die Leckströme an definierten Punkten die Oxidbarriere überqueren und nicht homogen entlang der ganzen Länge der Oxidbarriere auftreten. Wir zeigen Hinweise dafür, dass Leckströme am wahrscheinlichsten an Positionen auftreten, an denen zwei Oxidlinien einander kreuzen.
List of constants, variables, and acronyms

**Variables**

<table>
<thead>
<tr>
<th>variable</th>
<th>explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>area of a quantum dot</td>
</tr>
<tr>
<td>α&lt;sub&gt;i&lt;/sub&gt;</td>
<td>lever arm of gate i acting on a quantum dot</td>
</tr>
<tr>
<td>B</td>
<td>magnetic field</td>
</tr>
<tr>
<td>C&lt;sub&gt;Σ&lt;/sub&gt;</td>
<td>total capacitance of a quantum dot</td>
</tr>
<tr>
<td>D(E) = 2E/(πc&lt;sup&gt;2&lt;/sup&gt;ℏ)&lt;sup&gt;2&lt;/sup&gt;</td>
<td>density of states of graphene</td>
</tr>
<tr>
<td>Δ&lt;sub&gt;N&lt;/sub&gt;</td>
<td>single level spacing</td>
</tr>
<tr>
<td>Δ</td>
<td>superconducting gap</td>
</tr>
<tr>
<td>E&lt;sub&gt;C&lt;/sub&gt; = e&lt;sup&gt;2&lt;/sup&gt;/C&lt;sub&gt;Σ&lt;/sub&gt;Δ&lt;sub&gt;N&lt;/sub&gt;</td>
<td>charging energy</td>
</tr>
<tr>
<td>E&lt;sub&gt;F&lt;/sub&gt;</td>
<td>Fermi energy</td>
</tr>
<tr>
<td>E&lt;sub&gt;J&lt;/sub&gt;</td>
<td>Josephson energy</td>
</tr>
<tr>
<td>E&lt;sub&gt;T&lt;/sub&gt;</td>
<td>thermal energy</td>
</tr>
<tr>
<td>G</td>
<td>conductance</td>
</tr>
<tr>
<td>I</td>
<td>current</td>
</tr>
<tr>
<td>I&lt;sub&gt;tc&lt;/sub&gt;</td>
<td>tunnel current in STM-mode</td>
</tr>
<tr>
<td>l&lt;sub&gt;e&lt;/sub&gt;</td>
<td>mean free path</td>
</tr>
<tr>
<td>λ&lt;sub&gt;F&lt;/sub&gt;</td>
<td>Fermi wave length</td>
</tr>
<tr>
<td>μ&lt;sub&gt;N&lt;/sub&gt;</td>
<td>electrochemical potential of a quantum dot with N charge carriers</td>
</tr>
<tr>
<td>N</td>
<td>number of charge carriers trapped on a quantum dot</td>
</tr>
<tr>
<td>n&lt;sub&gt;2D&lt;/sub&gt;</td>
<td>electron sheet density</td>
</tr>
<tr>
<td>g</td>
<td>density of states</td>
</tr>
<tr>
<td>σ</td>
<td>conductivity</td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
</tr>
<tr>
<td>τ</td>
<td>momentum relaxation time</td>
</tr>
<tr>
<td>V&lt;sub&gt;bias&lt;/sub&gt;</td>
<td>bias voltage across a device, e. g. a quantum dot</td>
</tr>
<tr>
<td>v&lt;sub&gt;F&lt;/sub&gt;</td>
<td>Fermi velocity</td>
</tr>
</tbody>
</table>
### Constants

<table>
<thead>
<tr>
<th>constant</th>
<th>value</th>
<th>explanation</th>
</tr>
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<tbody>
<tr>
<td>$\alpha_* = e^2 / (4\pi\varepsilon_0 \hbar c_*)$</td>
<td>$\sim 1$ (for $\varepsilon = (\varepsilon_{\text{vac}} + \varepsilon_{\text{SiO}_2}) / 2$)</td>
<td>effective fine-structure constant for graphene</td>
</tr>
<tr>
<td>$c_*$</td>
<td>$\sim 10^6$ m/s</td>
<td>effective velocity of charge carriers in graphene</td>
</tr>
<tr>
<td>$e =</td>
<td>e</td>
<td>$</td>
</tr>
<tr>
<td>$\varepsilon_{\text{vac}}$</td>
<td>1</td>
<td>relative dielectric constant of vacuum</td>
</tr>
<tr>
<td>$\varepsilon_{\text{SiO}_2}$</td>
<td>3.9</td>
<td>relative dielectric constant of SiO$_2$</td>
</tr>
<tr>
<td>$\hbar = \hbar / 2\pi$</td>
<td>$1.0546 \times 10^{-34}$ Js</td>
<td>Planck constant</td>
</tr>
<tr>
<td>$k_B$</td>
<td>$1.3807 \times 10^{-23}$ J/K</td>
<td>Boltzmann constant</td>
</tr>
<tr>
<td>$m_e$</td>
<td>$9.1094 \times 10^{-31}$ kg</td>
<td>(free) electron mass</td>
</tr>
<tr>
<td>$m^*$</td>
<td>$0.067 m_e$</td>
<td>effective mass of an electron in GaAs</td>
</tr>
</tbody>
</table>

### Acronyms

<table>
<thead>
<tr>
<th>acronym</th>
<th>explanation</th>
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<tbody>
<tr>
<td>2DEG</td>
<td>two-dimensional electron gas</td>
</tr>
<tr>
<td>AFM</td>
<td>atomic-force microscope/microscopy</td>
</tr>
<tr>
<td>CB</td>
<td>Coulomb blockade</td>
</tr>
<tr>
<td>DD</td>
<td>double dot</td>
</tr>
<tr>
<td>EBL</td>
<td>electron beam litography</td>
</tr>
<tr>
<td>MBE</td>
<td>molecular beam epitaxy</td>
</tr>
<tr>
<td>PG</td>
<td>plunger gate</td>
</tr>
<tr>
<td>QD</td>
<td>quantum dot</td>
</tr>
<tr>
<td>QPC</td>
<td>quantum point contact</td>
</tr>
<tr>
<td>SET</td>
<td>single-electron transistor</td>
</tr>
<tr>
<td>SGM</td>
<td>scanning-gate microscope/microscopy</td>
</tr>
<tr>
<td>STM</td>
<td>scanning-tunneling microscope/microscopy</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

We have a tendency to only believe what we can see. Therefore it is no surprise that our everyday life is still ruled by the power of graphical images. We believe what we see. Children believe in Santa Claus not only because their parents tell them of his existence, but because every year thousands of guys dress up in red costumes to convince children that Santa Claus is real by appearing on their doorsteps. And in most cases this visual proof really works for an extended period of time.

But children are not the only ones easily convinced by what they see. The fact that watching the news on TV has become more popular than simply listening to the facts on the radio is evidence, that the concept of ’visual proof’ is also popular among adults. We like to be able to ’put a face on a name’, even if knowing this face does not change a single thing about the actually delivered facts. Strikingly present is the visual proof in many sports: Road bike races for example still rely on the photo finish to visually examine who crossed the finish line first. This is done despite the electronic transponders every biker carries on his bike frame.

Sometimes it seems that, despite all the phenomenal technical measuring devices, our eyes really still are the most accurate probing tool and we have good reason to only trust the things we can see for ourselves. This is also the reason why when publishing a paper we cannot simply write that we observe a peak of certain height, shape and position, we rather have to show the measurement curve as a graphic to convince the sceptical reader.

So all scepticism aside why should we suddenly become credible when it comes to microscopic explanations of electric transport through a nano-structure? In a conventional transport experiment we can measure the current through a nano-structure in dependence on a slew of external parameters such as gate voltages, temperature, magnetic field and so on. However we are blind to anything that happens in the nano-structure. We as well as most measurement equipment are located outside the cryostat, far away from the sample. Therefore we do not see what really happens at the sample, because we have no probing or imaging
tool directly at the sample. And despite this problem we put together theories that state where the current flows through the nano-structure, what the electron probability density is and how the charge carriers behave in order to explain the transport data. Obviously we have ample evidence to support theories such as the existence of edge states in the transport data alone. But nevertheless, we have never really seen inside the sample.

So in order to really be able to prove or disprove those theories we need to be able to look at transport in real space and time while it is happening. Along the lines of “among the blind the one-eyed is king”, we use the principle of scanning gate microscopy to spatially resolve transport. This method allows us to get a look into the transport processes that are on the base of all electronic transport properties we can measure on the outside of our cryostat.

In scanning gate microscopy (SGM) we use a metallic tip as a flying nano-gate to locally probe the transport properties of a nano-structure. This allows us to image transport in a spatially resolved way and therefore draw conclusions about the underlying transport properties.

The basic idea is that the metallic tip induces a small potential perturbation in the sample underneath. We can freely move this potential to any position in the sample and therefore see directly how different real space regions of the sample contribute to the overall transport properties.

The beauty of this method is that it is not limited to a specific type of nano-structure. In principle we can image any nano-structure that can be approached by a SGM tip and whose transport properties can be influenced by a local potential (see chap. 2 for a review on the technique and existing measurements in literature). Following this we will investigate very diverse nano-structures in this thesis.

Graphene has been hyped to be the new “IT” material among solid-state physicists. A special advantage of graphene for scanning probe experiments is its direct accessibility as it is located on the surface without any protection layer. Therefore it seemed an obvious choice to perform SGM on a graphene nano-structure. In chapter 5 we will show transport and scanning gate microscopy measurements on a graphene nano-ring. We first give an introduction to graphene and the Aharonov–Bohm effect in general. Subsequently we focus on evaluating the dependence of the Aharonov–Bohm oscillations on different external parameters, as well as the in plane gate voltages, showing phase jumps of $\pi$ in the oscillations. We then show how the charge carrier density can be changed locally by the metallic tip and further investigate locally resolved transport properties. Based on earlier scanning probe measurements by Hackens [2] we show that short range oscillations can also be seen in our measurements. The measurements will then be backed up by electrostatic simulations.
Chapter 6 deals with a completely different nano-structure, namely a superconducting single electron transistor (SET). In order to understand the scanning gate measurements performed on this nano-structure we will first give a short introduction to superconductivity and explain the basic principles of transport through a SET first in the normal conducting and then in the superconducting state on the basis of the transport measurements performed on this structure. Scanning gate measurements then focus on the tip potential as well as the spatial resolution of transport parameters such as the superconducting gap and the charging energy $E_C$. Electrostatic simulations are used to back up the experimental findings.

Chapters 7 and 8 focus on measurements on a more traditional structure based on a two-dimensional electron gas (2DEG). In chapter 7 we present measurements on a double quantum dot fabricated via local anodic oxidation. We start by giving an introduction to 2DEGs in general. We then use the transport measurements to explain the transport properties of a double quantum dot structure. The scanning gate measurements show a so called distorted hexagon pattern which allows us to determine the apparent relative position of the two quantum dots in real space. We will evaluate the shift of these apparent positions due to the gate voltages. Electrostatic simulations will be used to support the measurements.

Chapter 8 focuses on spatially resolved investigation of leakage currents. We use scanning gate microscopy to locate leakage currents within a sample that was defined via local anodic oxidation. We show that current flows across the oxide barriers not along their complete extend but rather at a single well defined position. We find that leakage currents occur with higher probabilities at positions where two oxide barriers cross.

For all of the investigated structures the common denominator is that in order to observe processes based on energy scales such as the charging energy, they have to be performed at low enough temperatures to make the thermal energy $E_T$ small compared to the other energy scales in the system. Therefore a scanning force microscope was built inside a $^3$He refrigerator as part of this thesis and used for all measurements presented here. As such a low temperature AFM comes with many technical complications and challenges we will describe the setup in chapter 4.

No Ph.D. thesis no matter with how much commitment and dedication it is carried out is able to answer all questions and cover all experiments the Ph.D. candidate aims to perform. Therefore we will give a short summary and outlook over further directions in which scanning gate microscopy could be taken in the last chapter.
Chapter 2

Scanning gate technique

In this chapter we motivate [sec. 2.1] and introduce [sec. 2.2] the method of scanning gate microscopy. Subsequently we give an overview [sec. 2.3] of the most important experiments that have used this technique in different groups.

2.1 Motivation

Since the basic concepts of current and voltage have been understood about 200 years ago, physicists have been making enormous efforts to understand every aspect of it. New impulses came from miniaturization, where electronic devices are rapidly decreased in size. A well known embodiment of this trend is Moore’s Law, which states that the number of transistors that can be placed on an integrated circuit doubles every two years [3]. However already Moore himself stated, that this trend will reach fundamental limits at some point as “we’re approaching the size of atoms which is a fundamental barrier”[4]. This fundamental barrier, however, has lead to new questions such as: What happens when current flows through a constriction that only consists of few atoms? How does current flow through (artificial) atoms? Where does the current flow?

The field of mesoscopic physics is concerned with investigating structures, that are larger than single atoms but still much smaller than bulk material, so quantum mechanical effects are important. The field of investigated nano-structures is very diverse and ranges from measuring transport through comparably simple nano-structures such as constrictions and nano-ribbons [5, 6, 7, 8, 9] to complex coupled multiterminal structures [10, 11, 12]. Many groups have gained amazing mastery over their nano-structures enabling them to even trap single electrons onto a quantum dot [13, 14]. However most transport experiments on nano-structures rely on the concept of applying a voltage across the sample and measuring the current through the sample versus an external controlled quantity such as a gate
voltage or a magnetic field. In such measurements the current or conductance that is recorded remains a macroscopic quantity that has little to no spatial information about how the current flows through the sample.

On the other hand, models that are used to explain the phenomena we observe in such measurements through nano-structures usually rely heavily on explaining the flow of charge carriers in space. Already basic ideas such as the scattering of an electron at an impurity evoke the need of spatially resolved current measurements. More complex concepts, such as the formation of edge channels [15] in the quantum Hall effect do so even more. Hence there is a strong necessity to spatially investigate transport through nano-structures. One promising option that allows us to acquire data with a spatial resolution is scanning gate microscopy where a conductive tip is scanned across a nano-structure, inducing a local potential into the sample. The principle of this method, as well as the experimental results this technique has lead to in the past will be discussed below.

2.2 Principle

2.2.1 The method of scanning gate microscopy

In scanning gate microscopy a metallic tip is used to locally induce a potential into the investigated sample. This local potential can be moved to any desired point in the sample, by moving the tip as schematically shown in Fig. 2.1. Its strength can be altered either by increasing the distance between the structure and the tip, or by applying a different voltage to the tip. No current flows from the tip to the sample.

We now record the change of the resistance of the sample, while positioning the tip, i.e. a small electrostatic perturbation, at different points. Usually so called current maps $I(x_{\text{tip}}, y_{\text{tip}})$ are recorded by scanning the tip at a constant height over the sample, while measuring the current flow through the sample in dependence on the tip position $x_{\text{tip}}, y_{\text{tip}}$. Depending on the precise measurement techniques conductance maps or resistance maps might be more suitable, relying on the same principle. However it is possible to record all measurable quantities with a spatial resolution, as will be discussed in section 6 using examples of capacitances, charging energy and the superconducting gap.

2.2.2 Time line of a scanning gate cool down

Preparing sample and setup for a new cool down cycle has to take place under ambient conditions. Upon mounting the sample into the sample holder, the microscope is assembled around it. An optical microscope is used to position the tip
Figure 2.1: (a) Schematic of the local potential induced by the SFM tip. (b) Size comparison. In order to locate the nano-structure (brown) on the sample (gray) at low temperatures topography scans have to be carried out. The red square below the word ‘scan frame’ is the size of the scan frame available at low temperatures.

of the scanning probe sensor [see sec. 4.8] as close as possible to the nano-structure that is going to be investigated. Then the tip is retracted in z-direction by several μm to avoid tip crashes during the final assembly steps, evacuation and cool down process.

Upon carefully fixing all loose parts and cables of the insert to avoid mechanical vibrations as well as mechanical contact to the inner vacuum chamber (IVC) which will be at 4.2 K upon cooldown, the IVC is slid over the setup and evacuated. Directly before cooling down the insert, exchange gas is filled in the IVC reaching a pressure of around $10^{-4}$ mbar. The insert is then carefully lowered into the cryostat and cooled down to 4.2 K [see sec. A and sec. B for detailed cool down instructions]. However careful one can be when lowering the insert into the cryostat, mechanical vibrations cannot be avoided. These mechanical vibrations alone already shift the position of the tip of the sensor relative to the position of the nano-structure by up to several hundred micrometers. This mechanically induced shift is not reproducible or predictable in direction or size.

In case the complete cryostat is at room temperature when starting the experiment it is therefore highly advisable to proceed in the following order:

- assemble and evacuate setup
- lower the insert into the cryostat
- reposition the sample with respect to the sensor
Chapter 2. Scanning gate technique

- fill in exchange gas
- cool down the complete setup

By proceeding in this order, the shift due to mechanical vibrations can be minimized. It is favorable to carry out the repositioning while still at room temperature, as the scan frame of the scan piezo is about a factor of ten bigger at room temperature than at low temperatures. A larger scan frame allows for easier positioning. Additionally the coarse positioning motors usually work best at room temperature.

However in addition to the mechanically induced shift of the sample with respect to the sensor, the temperature difference will also lead to an additional shift. The complete setup is constructed of various materials. They are chosen in a way to minimize this thermally induced shift. But it cannot be eliminated completely. On the positive side: This shift is systematic. That means, the approximate direction and size of the thermal shift is known, as long as all parts of the setup stay the same. Whenever a single part of the microscope is replaced the thermal shift will change.

No matter how careful one goes about assembly and cooldown, one will almost certainly encounter that the sample has shifted by up to a few hundred micrometers from the position adjusted at room temperature. To carry out scanning gate measurements it is therefore unavoidable to carry out topography scans in order to reposition the tip above the nano-structure at measurement temperature.

2.2.3 A necessary prerequisite: Scanning topography

When performing a cooldown with the scanning probe setup the usual goal is to perform a scanning probe experiment. To administer a scanning probe experiment, we would only need a tip, that can be moved to different positions using a scan piezo with a range of several micrometers. We have seen in the last section that mechanical and thermal shifts in the order of several hundred \( \mu \)m prior to performing scanning probe experiments are unavoidable. Therefore we will need to be able to image the topography in order to assert the position of the tip on the sample. And additionally we need coarse motors with a range of roughly a millimeter to position the tip above the nano-structure. As we are often working with samples that display a mostly isolating surface, the most convenient method to image a surface at low temperature and vacuum is atomic force microscopy (AFM).

Using atomic force microscopy we can image the topography of the sample once the setup has stabilized at 4.2 K. For scanning gate measurements we want to be able to use a tip that is as sharp as possible and free of any undesired particles.
2.2. Principle

Unfortunately recording topography scans tend to flatten the tip and make the adhesion of particles onto the tip very likely. It is therefore advisable to minimize topography scanning as much as possible.

Figure 2.1(b) shows the typical situation anybody working with scanning probe setups might encounter. The sample is designed in a very symmetric way (as visible in the grey mesa). In total few lines are visible and many of those lines have a similar or even parallel slope. The scan frame at 4.2 K that can be used for an extended period of time without depolarizing the scan piezo is $7 \times 7 \mu m^2$. This corresponds to the small red square in Fig. 2.1(b). The nano-structure itself is located in the geometric center of this image as shown by the brown inset. From the different length scales it is intuitively obvious that it is a very challenging task to position the tip right above the nano-structure, when only being able to record tiny pieces of topography within the scan frame.

It would be therefore highly desirable for future scanning gate samples to feature markers, that allow for a faster orientation on the sample. Ideally a single, unique marker should be placed within each scan frame on the sample. Therefore upon recording a single topography scan one would be able to directly position the tip in the close vicinity of the nano-structure. This would minimize the topography scanning, therefore maximizing our tip quality and optimize the recorded scanning gate measurements in terms of lateral resolution and interpretability.

2.2.4 Fine positioning: "Scanning feel"

Some nano-structures are very sensitive to mechanical intrusions. It is therefore a good measure of caution not to record the topography of the sample before all measurements have been concluded. This also minimizes wear on the tip.

Upon positioning within a few micro meters of the sample, the fine positioning can be carried out as follows. With the tip positioned on one corner of the scan frame we record a sweep of the resistance of the sample versus the tip voltage $R(V_{tip})$. We proceed to do this for all four corners of the scan frame.

We then evaluate the change in $R$ for each tip position. We then choose the direction into which the tip has to be moved by comparing these four $R(V_{tip})$ sweeps. For a quantum dot we see for example a change in lever arm, i.e. the spacing of the Coulomb peaks changes.

This way we can prevent damage to the tip and the sample at once, while still enabling us to conclude the fine positioning.

2.2.5 A helpful tool: The STM modus

The setup needed to perform atomic force microscopy and the setup needed to perform scanning tunneling microscopy (STM) are very similar. Basically the
only change needed is the ability to change the feedback parameter and record a tunnel current from the tip. The feedback parameter for AFM is usually the deflection of a cantilever, or in our case the frequency shift of a tuning fork sensor. The feedback parameter to perform STM is the tunnel current from the tip to the sample.

The setup built as part of this thesis incorporates both options, allowing for STM measurements when desired. The additional advantage of being able to perform STM is the fact, that it allows for additional option to alter the tip in situ by driving big tunnel currents from the tip to a conductive part of the sample.

### 2.2.6 Reducing thermal effects: The necessity of low temperature measurements

A goal of measurements on nano-structures is the observation of quantum effects. The energy scales involved are usually in the order of magnitude of meV [see sec. 6.3.4]. In order to be able to resolve these energy scales the thermal energy \( E_t = k_B T \) has to be lower than the other energy scales of interest. To achieve this the sample and scanning probe setup are built and operated in a \(^3\)He refrigerator with a base temperature of 280 mK. Obviously building a complete AFM that is functional at high vacuum and 280 mK brings with it a whole range of unique challenges. Therefore the setup will be discussed in chapter 4.

### 2.3 History/Literature overview

Scanning probe microscopy is a method that is very involved in terms of technical equipment, money and manpower. Nevertheless this method has proven to be a powerful tool to locally investigate transport properties. It helps us to look 'inside' the transport mechanisms, rather than being 'blind' when it comes to spatial resolution as in conventional transport measurements. Due to its rather high input-to-output ratio only a limited number of groups have tackled the task of performing scanning probe experiments.

A review of the scanning gate technique with related experiments (before 2004) is given in [17]. Excellent short reviews are given in previous PhD thesis' in this group, for example [18, 19]

The best known results of scanning probe microscopy measurements are those on quantum point contacts. The standard work here is the publication by Topinka [16] performed at Havard (group of Prof. Westervelt). They study a 2DEG based quantum point contact that was defined using metallic topgates at 1.7 K shown in Fig. 2.2. They show that electrons flow in branches due to an inhomogeneous background potential instead of spreading smoothly. Quantum mechanical phase
coherence leads to additional ripples in the observed branches. These are due to the interference of electron waves backscattered to the QPC by the tip-induced potential [Fig. 2.2].

Other experiments on QPCs showed similar results, highlighting that the transport through a QPC is strongly influenced to charge traps, impurities and localized states in the vicinity of the point contact [20, 21, 22, 23, 24].

An approach raising the hopes, that it might be possible to alter nano-structures produced via local anodic oxidation in situ came from Rolf Crook (group of Prof. Ritchie, University of Cambridge) [25, 26]. There they showed, how isolating regions could be written and erased in situ.

As the variety of nano-structures and materials studied within the boundaries of this thesis is rather broad, we will give a short review of the literature focusing on measurements connected to the experiments performed within this thesis in the corresponding chapters.

A literature of scanning gate measurements performed on different graphene structures will be given in section 2.3.1. Previous measurements conducted on quantum rings will be presented in section 2.3.2.

Scanning gate experiments focusing on single quantum dots will be reviewed in section 2.3.3.

As no measurements have been performed on intentionally formed double quantum dots, we will shortly review the scanning probe measurements conducted on unintentional double-dot like structures in 2.3.4.
2.3.1 Review: Scanning gate measurements on graphene

Graphene nano-structures have spiked an enormous research frenzy among solid state experimentalists and theorists. Therefore it is not surprising, that even though graphene is still a very "young" material some scanning gate experiments have already been performed on graphene sheets and nano-structures.

The obvious advantage of a graphene nano-structure is its accessibility. As opposed to structures made out of 2DEGs graphene is directly deposited on the surface of the substrate therefore directly accessible via a scanning gate tip. This should among other advantages allow for an significantly increased resolution as the scanning gate tip can be brought closer to the sample.

First scanning gate measurements have been performed in the group of Amir Yacoby (Harvard) by Jens Martin [27]. Those measurements imaged the electron-hole puddles in graphene. They were able to show that the disorder landscape smoothes out at high charge carrier densities, which agrees with the measurement results shown in the following.

Jesse Berezovsky (group of R. M. Westervelt, Harvard) has performed extensive measurements on a graphene Hall bar [28, 29]. They imagine the tip as a single scatterer, that can be moved freely through the complete structure. Using this image they were able to investigate how electron waves interfere leading to universal conductance fluctuations (UCF). They also investigate weak localization recoding a so called spatial fingerprint of the interfering paths.

Romaneh Jalilian (group of Yong Chen, Purdue) has performed scanning gate measurements on a graphene field effect transistor revealing mesoscopic domains of electron-doped and hole doped regions. They investigated the spatial charge carrier density showing that "metal contacts, graphene edges, and resist residues can be important sources of extrinsic doping". [30]

Finally other scanning gate measurements have been performed in the group of Klaus Ensslin as well [31]. There a graphene quantum dot was investigated imaging the localized states in the constrictions.

2.3.2 Review: Scanning gate measurements on a nano-ring

Up to now only one group has carried out and published scanning gate measurements on a quantum ring. All publications concerning those measurements [2, 32, 33, 34] revolve around B. Hackens, F. Martins and simulations from M. G. Pala from Grenoble. We will give a short introduction of their published results and interpretations.
Sample and setup
The investigated rings are electron lithography patterned structures. The material used are InGaAs-InAlAs based two dimensional electron gas (2DEG) wavers. The outer ring diameter is about 600 nm with an arm width of 170 nm, values that are smaller than the electron coherence length at the measurement temperatures. All measurements are carried out at 4.2 K.

Underlying idea
The metallic tip of the AFM is used to locally disturb transport by locally modifying the electric potential in the quantum ring. The transmission and therefore the conductance of the sample should be influenced only by:

- "the modification of the electron wavelength and phase in the perturbed area (coherent effect)"[2].
- "the change in momentum of electrons impinging the local perturbation (semiclassical effect)"[2].

Background features
The most obvious feature visible in the conductance maps is a ring like structure of suppressed conductance. However this ring structure does not coincide perfectly with the geometry of the ring, but rather extends outside the ring. While the ring has an inner diameter of 210 nm the ring structure in the conductance map has a radius of about 400 nm. The maximum radius of the topographical ring is 300 nm while the maximal radius of the ringlike feature in the conductance map is up to 900 nm. The shape of the ringlike feature in the conductance map does therefore not perfectly image the geometry of the ring. It is about a factor of two larger and displays a much stronger conductance change near the left arm of the ring [see Fig. 2.3(a)]. This so called background shape is attributed to a global shift of the local potential as opposed to quantum transport properties.

Fine structure: High pass filtering
Subsequently the conductance maps are high pass filtered and in the following only the high pass filtered conductance maps are investigated. The high pass filtered conductance maps display conductance oscillations with a periodicity around 100 nm as shown in Fig. 2.3(b). The oscillation amplitude changes around the

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1This distorted and not perfectly matching ring shape that extends over the geometric boundaries of the ring is a feature that we will also be observing in the scanning gate measurements carried out on the graphene ring that will be presented.
ring and attributed to an asymmetry in either the ring itself or the tip shape. The fringes are radial in the region of the ring and concentric when the tip is further away. In the following they focus on the origin of these two different kinds of fringes.

The amplitude of the radial fringes changes gradually when the sign of the measurement current is reversed. Reversing the current leads to a rearrangement of the ballistic trajectories inside the quantum ring. This switching behavior therefore reflects the effect of the tip on trajectories inside the ring. The fringes inside the ring can be associated to variations of the local electron density.

The amplitude of the concentric fringes shows a peak at zero current and then gradually decreases for higher absolute current values mirror symmetric to zero $I$. As high measurements currents increase the energy of the electrons, electron-electron scattering increases, suppressing phase coherent effects. Therefore the concentric fringes are attributed to a coherent origin, more specific they mark isophase lines for Aharonov-Bohm interferences.

Simulations

Using quantum mechanical simulations of the electron probability density, including charge defects and the perturbing influence of the tip potential a relation between the conductance maps and the underlying electron probability density maps could be confirmed. Those simulations show that the measurement results could be theoretically reproduced on the basis of a theory that makes the defects responsible for the asymmetric radial fringes. Therefore the conductance maps are directly related to the local density of states (LDOS) as opposed to the current density [Fig. 2.3(c,d)].
As a conclusion "one can view SGM as the analog of STM for imaging the electronic LDOS in open mesoscopic systems" [32].

2.3.3 Review: Scanning gate measurements on quantum dots

In the past a variety of quantum dots has been investigated. The most research in this perspective has been performed on semiconductor quantum dots.

Experiments on quantum dot structures fabricated via local anodic oxidation have been performed by Alessandro Pioda and Slavo Kicin (Group of Prof. Ensslin, ETH Zurich). They have found that this technique allows to move single electrons on and off the quantum dot by the tip. This technique has been brought to its maximum by Parisa Fallahi (Group of Prof. Westervelt, Harvard), who showed current maps of a semiconducting quantum dot, depleted down to the very last electron [35].

Pioda et. al. also showed, that the tip potential is a quantity not easily understood and showed, that it might even consist of voltage dependent and independent parts [36]. This idea was more thoroughly investigated by Arnd Gildemeister (group Prof. Ensslin) who conducted measurements on a similar sample focused on deciphering the tip induced potential [1]. He could show that the tip potential can be described by a superposition of the part that is due to the metallic tip and added parts, which were the effect of charged particles picked up by the tip.

Studies of quantum dots inside nano-wires [37, 38] (group Prof. Westervelt) have focused on the complexity of unintentional multiple-dots formed in nanowires, similar to the charge traps investigated by Gildemeister via charge readout in [39]. These publications have highlighted the disorder and vast amount of charge traps, localized states and so on present in conventional nano-structures.

Meanwhile Burke (group of D.K. Ferry, Arizona State University) has been focused on imaging the inside of an open quantum dot with this technique [40], interpreting the data in view of scarred wave functions inside such a quantum dot.

When Carbon Nanotubes edged closer to the spotlight of the mesoscopic community quantum dots formed inside those tubes were investigated with scanning gate microscopy by the group of Prof. McEuen (University of California, Berkeley)[41, 42]. Here scanning probe microscopy was used to locate the quantum dots inside the carbon nano-tubes and investigate the electrostatic forces associated with the movements of single electrons.

Recently graphene has become the new "in" material and first investigations of a graphene quantum dot have been carried out within a cooperation that was part of this thesis. They showed that localized states can be found inside the constrictions leading up to a graphene quantum dot. More information can be
found in the thesis by Stephan Schnez [43] and the publication mentioned within [31].

2.3.4 Review: Scanning gate measurements on double dot like structures

Until now no measurements have been performed on a well controlled and intentionally formed double quantum dot. However one can find a few scanning gate measurements in the literature that deal with double dot like structures.

Work on unintentionally formed multi-dot arrays in carbon nano-tubes have been performed by M. Woodside [42]. Resistance maps were used to image and locate individual dots inside the nano-tubes. However these unintentional multiple dots are not tunable. In this publication the focus was on electrostatic coupling between dot and tip as well as the electrostatic forces associated with single electrons hopping on and off the quantum dot.

A. Bleszynski has performed scanning probe experiments on InAs nano-wires [37]. In these nano-wires a complex pattern of multiple dots could be observed. Scanning probe microscopy has been used to determine their size and position.

2.4 Conclusion

Scanning gate microscopy has proven to be a powerful tool to investigate transport through a variety of diverse nano-structures. This method enables us to gain a spatial resolution in addition to transport data, helping to understand the processes that take place inside our sample, and ultimately lead to the observed transport properties.
Chapter 3

Transport through nano-structures

3.1 Dimensionality: Quantum effects

All structures investigated in the following chapters are orders of magnitudes larger than single atoms. However, we will still discuss quantum effects such as the interference of waves occurring in those samples. This is possible, because the Fermi wave length $\lambda_F$ and the inelastic mean free path are comparable or larger to the lateral extend of any sample investigated.

For many years solid state physicists have been searching for structures with less than three dimensions. The first step to lower dimensions is a two-dimensional electron gas.

An excellent review on this topic can for example be found in [134].

3.1.1 The two-dimensional electron gas (2DEG)

Electrons in a two-dimensional electron gas are free to move in the xy-plane, however they are strongly confined in z-direction. By now Molecular Beam Epitaxy (MBE) is so advanced that 2DEGs can be grown as part of a heterostructure with mobilities exceeding $10^7$ cm$^2$/Vs at 4.2 K [44]. Since their first realization in MOS-FETs in the 1960s [45], 2DEGs have become a well established system to study quantum effects.

The density of states $\rho$ in an ideal 2DEG, assuming parabolic dispersion is independent of the energy and only depends on the effective mass $m^*$ of an electron [47, 49]:

$$\rho(E) = \frac{m^*}{\pi \hbar^2} = const.$$  \hspace{1cm} (3.1)
Chapter 3. Transport through nano-structures

The Fermi wavelength $\lambda_F$, the Fermi energy $E_F$ and the Fermi vector $k_F$ are determined by the electron sheet density $n_s^{2D}$ (assuming spin degeneracy and a single valley) [48, 49]:

$$\lambda_F = \sqrt{\frac{2\pi}{n_s^{2D}}},$$

(3.2)

$$E_F = \frac{n_s^{2D}}{\rho},$$

(3.3)

$$k_F = \sqrt{2\pi n_s^{2D}}.$$

(3.4)

Typical values for GaAlAs heterostructures are around $n_s^{2D} = 5 \times 10^{15} \text{m}^{-2}$ leading to $E_F = 20 \text{meV}$ and $\lambda_F = 35 \text{nm}$. Finally the mobility is calculated using the momentum relaxation time $\tau_m$ [49]:

$$\mu = \frac{e\tau_m}{m^*},$$

(3.5)

leading to a mean free path [48]

$$l_e = \tau_m v_F,$$

(3.6)

that can be in the order of several microns.

When the lateral dimensions of a structure are of the same order of magnitude as the Fermi wavelength $\lambda_F$, quantum confinement effects start to play a role. The large value of $\lambda_F$ in 2DEGs allows for structures which have a lateral extent of tens of nanometers, as opposed to metals, where one would have to pattern structures which only have the extent of a few atoms in order to see quantum effects.

Nowadays most experiments, including the work shown in chapter 7 and 8 are performed on III-V semiconductors, such as GaAs, AlAs, InP.

Less than ten years ago graphene [chap. 5], which has been studied theoretically for a long time as the prototype of an ideal 2DEG, has been experimentally isolated. However, graphene proved to come with its own set of experimental challenges such as electron-hole-puddles, edge effects, ripples, undesired doping and so on. Therefore research on more conventional 2DEGs is still an interesting topic.

### 3.1.2 Zero dimension (quantum dots)

The ultimate low-dimensional case is a zero-dimensional quantum dot. This is essentially a tiny island, on which electrons are confined in three dimensions. Transport through such quantum dots occurs in multiples of the elementary charge, where only one electron after the other can be placed on the quantum dot.

Transport processes taking place through such zero-dimensional quantum dots will be discussed in chapter 6.
3.2 Diffusive transport (the Drude model)

We can even couple quantum dots in series, forming a double quantum dot. Additionally to the confinement in all three directions these two quantum dots then interact with each other. We can understand this on the basis of single quantum-dot physics. Transport processes through a coupled double quantum dot are explained in chapter 7.

3.2 Diffusive transport (the Drude model)

We describe transport as diffusive, when the lateral extend of the sample is considerably bigger than the length scale on which scattering of charge carriers takes place.

One model that is used to describe diffusive transport is the Drude model.

The Drude model

More in depth accounts of the Drude model can be found in [46, 47].

The Drude model explains current by treating electrons classically, like balls in a pinball machine. Electrons are treated as an ideal electron gas which moves through an array of positively charged ions. Each electron is a point charge with an effective mass $m^*$ and velocity $v_{\text{therm}}$. Electrons do not interact with each other or with the ions. They are thought to collide with ions. These collisions lead to an instantaneous change of electron velocity. These collisions are needed to maintain the thermodynamic equilibrium. On average an electron travels a time $\tau_t$ between two collisions.

When an electric field $E$ is applied, the movement of the electrons changes. The equation of motion reads as follows [47]:

$$ m\dot{v} = -eE - \frac{m}{\tau_t} v_D $$  \hspace{1cm} (3.7)

Without the presence of an electric field, electrons only move with the thermal velocity $v_{\text{therm}}$. With an electric field, they start to move with an isotropic drift velocity $v_D = v - v_{\text{therm}}$ which they gain in addition to their thermal energy. for the stationary case-i.e. $\dot{v} = 0$ we find [47]:

$$ v_D^* = -\frac{e\tau_t}{m^* \mu_e} \vec{E} = -\mu_e \vec{E} $$  \hspace{1cm} (3.8)

where $\mu_e$ is the electron mobility. The current density $j = -env_D$ can be written as:

$$ j = -env_D = n \frac{e^2 \tau_t}{m^*} E $$  \hspace{1cm} (3.9)
Chapter 3. Transport through nano-structures

From that we find the Drude conductivity [47, 46]:

$$\sigma = \frac{j}{E} = n \frac{e^2 \tau_d}{m^*} = \frac{e^2}{h} k_F l_c$$  \hspace{1cm} (3.10)

The Drude model is a purely classical model. Electrons and ions are treated as solid spheres. Furthermore current is carried by all electrons, not only electrons close to the Fermi energy. Its basic assumptions are not fulfilled any more as soon as the electric field fluctuates on the same length scales as $l_c$.

3.3 Ballistic transport (Landauer-Büttiker formalism)

3.3.1 Tunnel junction

An essential building block of many nano-structures that will be discussed in the following is a tunnel junction. A tunnel junction consists of two pieces of conductive material which are separated by a thin insulator. Such a junction is shown in Fig. 3.1. Single electrons can travel from one conductive part to the other by tunneling through the barrier. From this we see immediately, that current flow through nano-structures can be viewed as transmission and reflection processes of charge carriers.

From transport to transmission: The Landauer formula [50, 51, 52]

The Landauer-Buettiker formalism builds upon this idea, that electronic transport can be described in terms of the probability with which a charge carrier is
transmitted through a mesoscopic system. Electrons moving through systems of reduced size can be described with the following wave function:

\[ \Psi_{nk} = \chi(x, z) \frac{1}{L} e^{ik_y y} \]  

We assume that the wave travels along a channel that is infinite in y direction. The confinement leads to \( N \) discrete energy levels (subbands or modes). Electrons travel along these independent modes.

We assume, that the leads are perfectly coupled to a source and a drain reservoir. The source reservoir is held at the electrochemical potential \( \mu_S \) and the drain reservoir on \( \mu_D \). An external bias voltage \( V_{\text{bias}} \) is applied between source and drain: \( \mu_S - \mu_D = eV_{\text{bias}} \). The transport from source to drain can be described according to the idea, that a certain number of electrons are transmitted from source to drain and a certain amount will be reflected back. The current can be calculated by integrating over the difference of incident and reflected states. If the transmission probability is different for all \( N \) modes the sum over all \( N \) modes has to be taken. The expression is normalized so the total transmission probability is 1.

Using the transmission \( T \) we can now calculate the current. The current starting at the source contact is [51]:

\[ I_S = g_s \left( \frac{e}{h} \right) N (\mu_S - \mu_D) \]  

\[ I_D = g_s \left( \frac{e}{h} \right) NT (\mu_S - \mu_D) \]  

From this we can derive the conductance \( G = \frac{eI}{\mu_S - \mu_D} \), giving us the so called Landauer equation (for Temperature= 0 K) [51].

\[ G = g_s \left( \frac{e^2}{h} \right) \sum_{n=1}^{N} T_n \]  

Therefore the conductance is given by the sum of all contributing modes, where each mode has its own transmission probability.

**From two to multi-terminal: Büttiker formula [50, 51, 52]**

A generalization of the Landauer equation for multi-terminal structures is the Büttiker formula. In order to generalize the expression 3.14 we have to define a
Chapter 3. Transport through nano-structures

transmission matrix, where each entry $T_{ij}$ gives the transmission from contact $i$ to contact $j$ [95]:

$$
\begin{pmatrix}
I_1 \\
I_2 \\
\vdots \\
I_N \\
\end{pmatrix} = \frac{e^2}{\hbar} \begin{pmatrix}
N_1 - R_1 & -T_{12} & \cdots & -T_{1N} \\
-T_{21} & N_1 - R_1 & \cdots & -T_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
-T_{N1} & -T_{N2} & \cdots & N_N - R_N \\
\end{pmatrix} \begin{pmatrix}
V_1 \\
V_2 \\
\vdots \\
V_N \\
\end{pmatrix}
$$

(3.15)

$N_i$ is the number of modes in a lead and $R_i$ the backscattering probability. The diagonal entries $T_{ii}$ give the probability of a charge carrier to enter the structure through this contact. [95]

$$T_{ii} = N_i - R_i$$

(3.16)

The transmission matrix is related to the conductance matrix via $G_{ij} = \frac{2e^2}{\hbar} T_{ij}$. For $T = 0$ K the current through one specific contact $i$ is given by [95]:

$$I_j = \sum_i [G_{ij}V_j - G_{ji}V_i]$$

(3.17)

Since the current has to be zero when no voltage is applied on any contact, the matrix has to be symmetric leading to $\sum_j G_{ij} = \sum_j G_{ji}$.

As experimental data is bound to be recorded at finite temperature we have to rephrase this equation for the case of nonzero temperature. To do so we have to take into account the distribution functions $f(E)$ in the contacts. For this case the current through contact $j$ can be found using [95]:

$$I_j = -g_s \frac{e^2}{\hbar} \int \{(N_j(E) - R_j(E))f_i(E) - \sum_{i,i \neq j} T_{ji}(E)f_i(E)\}dE$$

(3.18)

3.3.2 Quantum point contact

The tunnel junction is a specific case of a so-called quantum point contact. A quantum point contact (QPC) in general consists of a constriction connected to a source and a drain, which act as electron reservoirs. The way electrons pass through this narrow constriction can be described by the Landauer formula, where for low temperatures the conductance of the QPC will display quantized values [53]:

$$G = 2 \frac{e^2}{\hbar} \sum_n \tau_n$$

(3.19)

where $\tau_n$ is the transmission probability of the occupied mode. The conductance is found by taking the sum over all occupied modes. For low temperatures the
conductance only depends on the transmission probability at the fermi energy $E_F$. Each mode contributes one or zero conductance quantums $G_0 = 2e^2/h$ to the transport. The conductance is quantized and given by [53]:

$$G = \frac{2e^2}{h} N \quad (3.20)$$

In a less mathematical and more conceptive view, we can imagine transport through a QPC as follows. At low temperatures only electrons at the Fermi energy will contribute to electronic transport. A quantum point contact provides a transverse confinement. Therefore the transverse motion will be quantized. In order for an electron to pass through the point contact it has to be able to interfere constructively, which due to the transverse confinement will only be the case for selected cases, or modes. The number of modes $N$ is roughly given by the width $W$ of the QPC and the Fermi wavelength $\lambda_F$ [54]:

$$N = \frac{2W}{\lambda_F} \quad (3.21)$$

Each mode at the Fermi level has a different group velocity $v_n$, however, as the difference in density of states is given by $\rho_n = 1/(hv_n)$ this cancels out (in 1D). The conductance is given by the mesoscopic equivalent of Ohms law: $G = I/V$. As the current is given by $I = Ve^2\rho_n v_n = Ve^2/h$, the conductance can by found by $G = Ne^2/h$.

Using quantum point contacts as building blocks we can construct many different nano-structures such as quantum dots, double quantum dots and so on which will be discussed later on [chap. 6, 7].
Chapter 4

Experimental setup

In this chapter we present the new atomic force microscope that was built as part of this thesis with the experienced help of Paul Studerus, Cecil Barrengo and Slavo Kicin. The AFM\textsuperscript{1} is operated in a $^3$He system with a base temperature of 280 mK.

4.1 Introduction

The setup consists of two main parts: One is the cryostat including the magnet and the commercially bought part of the insert, and the other is the homebuilt AFM itself, consisting of many different modules.

The cryostat is schematically shown in Fig. 4.1. It consists of a main bath filled with liquid helium (4.2 K) and a nitrogen shield (77 K), used to thermally shield the main bath from the surroundings (280 K). A superconducting magnet [sec. 4.2.2] is included into the setup. Thermometers, cabling and heaters can be used to monitor and control the temperature [sec. 4.3 and 4.4]. As those parts are improved whenever possible, they are subject to changes.

The AFM itself is the most complex part of the setup. It consists of a coarse positioning unit, made up of an x-, y- and z-motor, a scanning sensor mounted onto the scan piezo and a sample holder [Fig. 4.1]. The x-, y- and z-motor are used to coarse position the sensor relative to the sample that is to be investigated. It is important to notice, that the relative position of the different motors shown in Fig. 4.1 is the orientation used in our homebuilt systems. When commercially bought Attocube-motors were used, all three axis were mounted at the place of

\textsuperscript{1}Parts of the predecessor of the described setup and connected physics have been described in [55]. The cryostat described in that reference is the same as the cryostat used for the setup described in this chapter. Unfortunately after an explosion of the setup prior to the start of this thesis the complete insert, including the wiring and the AFM had to be rebuild and characterized.
the xy-motor. This, along with the different concepts of motors, will be discussed in more detail in the following sections 4.6 and 4.5.

As many different versions of individual parts were designed and used during this thesis they will be described below in more detail.

4.2 Cryostat and vibration reduction

The AFM is built into a Janis Research Corporation $^3$He system, with a base temperature of 280 mK.

4.2.1 Vibration reduction

As it is desirable to have as few vibrations in the system as possible in order to increase accuracy in the position of the tip with respect to the sample and resolution while scanning, the whole cryostat is located in a hole in the laboratory floor and placed into a box filled with sand. This box itself rests on a concrete floor, which is detached from the building to further reduce vibrations. As the whole system is additionally located in the cellar of the building no vibrations from the vacillation of the building have to be expected. In order to also reduce microphony effects, the cryostat is wrapped with a rubber mat.

As pumps are known to be an important cause of mechanical vibrations, they are located in the floor beneath the setup. Furthermore all pump lines are flexible, to transmit as few vibrations towards the insert as possible.

As the outer surface of the cryostat is rather large, one has to be careful not to transfer vibrations from the cryostat onto the insert. To do so the insert is suspended from a heavy framework resting on the same floor/sand construction as the cryostat. The only connection between the cryostat and its insert consists of a thick latex cuff.

4.2.2 Magnet

The cryostat is equipped with a superconducting magnet that can be operated at fields up to $\pm 9.2$ T. It is important to know, that the Janis power supply used to control this magnet cannot conveniently be used for sweeping the magnetic field. The sweeping speed of the magnet is connected to the interval size between the initial and the final value, independent of the speed set in the software of the power supply. Figure 4.2 shows the true sweeping speed of the magnet versus the sweep range. Whenever the actual field comes closer than about 1 T to the final field the sweep speed is slowed down.
Figure 4.1: Schematic of the setup (not to scale). (a) shows the cryostat (blue) divided into the main bath (light blue) and the nitrogen shield (dark blue). In the middle one can see the insert (green) on the bottom of which the AFM itself (brown) is fixed. The magnet coil is represented by the red half-cylinder which is located in such a way, that the sample holder is placed in central field. (b) shows a zoom into the AFM itself. It consists of a motor for the z-direction (black) as well as the xy direction (purple). The scan piezo (blue) is located inside the z-Motor and the tuning fork sensor (yellow) is fixed on top of it. The sample holder (green) is mounted upside down on the xy-module. The whole setup is fixed into an AFM-cage (brown).
Figure 4.2: Measured sweep rate of the magnetic field versus the field range that was swept. The sweep rate set in the magnet software is the same for all data points. Each sweep range was started at zero tesla.

Programming interfaces allowing the operation of the magnet using the standard "Step and Log" measurement software (written by Simon Gustavson) have been written.

4.2.3 The $^3$He cryostat

The cryostat used in these experiments is a $^3$He system. It can reach a base temperature of 280mK and be kept at this temperature for up to 5 days when the microscope is included in the setup and no magnetic field is applied. Detailed operation instructions are included in the appendices A, B and C of this work.

Basic principle of a $^3$He cryostat

As can be seen in the schematic drawing in Fig. 4.3 the outermost shield of the cryostat is filled with liquid nitrogen. The main bath itself is filled with liquid helium. By properly adjusting the needle valve of the 1K-Pot, the microscope can be cooled down to about 1.3 K. In order to reach the base temperature of the cryostat a condensation cycle has to be carried out as shown in Fig. 4.4. Such a cycle starts with heating the charcoal sorption pump to a temperature above 40 K to evaporate the $^4$He trapped in there. At the same time the 1K-Pot is kept below 1.7 K by passing liquid $^4$He from the 1K-Pot capillary through the needle valve.
into the 1K-Pot while pumping on it. All available $^4\text{He}$ released from the sorption pump is then condensed when passing through the 1K-Pot and drips down into the $^3\text{He}$-Pot within 3 hours. Upon cooling the sorption pump back to 4 K by passing $^4\text{He}$ through a capillary wound around it, it then acts as a pump thus decreasing the vapor pressure above the liquid $^3\text{He}$. This causes the liquid $^3\text{He}$ to evaporate while cooling itself and its surroundings down to temperatures of 280mK.

As this cooling principle works only as long as there is still liquid $^3\text{He}$ left, the microscope warms up to 2K at the end of each condensation cycle. When we do not utilize the magnet the base temperature can be held for up to 5 days, when both the $^4\text{He}$ bath and the nitrogen shield are filled.
4.3 Cabling

In order to operate a low temperature AFM we do not only need cabling for the sample and the thermometers and heaters, but also for the microscope itself. This leads to a large additional amount of cables needed for the motors including positioning sensors, the scan piezo and the scanning force sensor itself.

4.3.1 Cables

As we want to measure currents of several nA on the one hand, and on the other hand need to drive the motors of the microscope with more than 100 V, we needed to use different cables for different purposes and try to spatially separate the more sensitive (sample) cables from the ones that produce more noise. Furthermore the cables are fixed with Stycast in place to avoid shaking which would produce noise.

The cables that carry the smallest signals and are therefore most sensitive to noise are the sample cables. They consist of 12 twisted pairs of Constantan wires with a diameter of 125 μm each. All 24 wires are placed in a high-grade steel tube with an inner diameter of 2 mm in which they are guided all the way down to the sample holder.

The thermometry and the heaters of the microscope are connected via a cable strap consisting of 16 Constantan wires with a diameter of 125 μm and 6 Cu wires...
4.3. Cabling

(a) (d)(b) (c) (e)

Figure 4.5: (a) Photo of the thermal anchoring of the cables as indicated by the blue arrows. (b) Cables with thermal anchoring. (c,d) Cables fixed into place by teflon tape as indicated by the green arrows. (e) Sorption pump wrapped in teflon.

with a diameter of 100 µm. This cable strap is also enwrapped in a high-grade steel tube.

The 12 high voltage cables needed to drive the motors consist of teflon coated NbTi cable cores with a diameter of 128 µm. They are bundled together in pairs of four and covered with a stainless steel shield.

The six coax cables used for the scanning force sensor as well as for the positioning sensors consist of a stainless steel inner conductor with a diameter of 114 µm followed by a dielectric layer and an outer conductor of stainless steel, leading to a total diameter of 508 µm for each cable.

4.3.2 Thermal anchoring

As all cables start at the vacuum feed through at room temperature it is necessary to thermally anchor them at each temperature stage. To do so the cables are fed through a metal piece that is thermally well coupled to the temperature at that stage or tightly wrapped around a metal pin [Fig. 4.5(a,b)].

In order to be able to operate the microscope at 280 mK it is necessary to make sure that no part of the insert especially none of the cables close to the microscope touches the inner vacuum chamber. Therefore all cables are fixed with teflon tape and isolated with a millimeter of teflon on parts where they are prone to come in contact with the IVC [Fig. 4.5(c,d)]. To thermally isolate the cables from the sorption pump the sorption pump is wrapped in a teflon sheet [Fig. 4.5(e)].

4.3.3 Heat load of the cables

The heat load of each cable can be calculated as follows. Starting from [96]

\[ j = -k \nabla T \] (4.1)
where \( j \) stands for the current, \( k \) the thermal conductivity and for \( T \) the temperature. The thermal conductivity can be written as \([96]\)

\[
k = k_0 \left( \frac{T}{T_0} \right)^\alpha.
\]  

(4.2)

Where \( k_0 \) is the thermal conductivity at the temperature \( T_0 \). Therefore we can write equation 4.1 as follows:

\[
j = -k_0 \left( \frac{T}{T_0} \right)^\alpha \nabla T
\]

(4.3)

\[
dx = -\frac{k_0}{T_0^\alpha} T^\alpha dT
\]

(4.4)

For a cable of length \( L \) that is connected to a temperature bath \( T_1 \) on one side and \( T_2 \) on the other side we can integrate as follows

\[
\int_{x=0}^{L} dx = -\frac{k_0}{T_0^\alpha j} \int_{T=T_1}^{T_2} T^\alpha dT
\]

(4.5)

\[
L = -\frac{k_0}{T_0^\alpha j} \frac{1}{\alpha + 1} \left( T_2^{\alpha+1} - T_1^{\alpha+1} \right)
\]

(4.6)

as the power \( P \) dissipated by a wire with a cross section \( A \) is \([97]\)

\[
P = jA
\]

(4.7)

we can find it with the above consideration to be

\[
P = -\frac{A k_0}{L T_0^\alpha} \frac{1}{\alpha + 1} \left( T_2^{\alpha+1} - T_1^{\alpha+1} \right)
\]

(4.8)

We can now calculate the heat load produced by each of the cables used in our setup. The power is calculated for two different temperature ranges. \( P_1 \) is the power for all cables of one specific kind from the sorption pump to the 1K-Pot and \( P_2 \) from the 1K-Pot to the \(^3\)He-Pot.

From the values shown in table 4.1 we can conclude, that the overwhelming amount of heat load carried by the cables is due to the copper cables which are part of the cable strap bearing the thermometry. To decrease the heat load of those cables the cable strap was cut at the 1K-Pot and replaced with CuNi wires to connect them to the \(^3\)He-Pot. The second biggest heat load arises from the shields of the high voltage (HV)-cables. Therefore these shields were interrupted to stop the heat flow.
4.4 Thermometry and heaters

The cryostat and microscope are equipped with multiple thermometers, to be able to measure the temperature at different temperature ranges as well as at different points of the microscope. The microscope itself is equipped with a PT100 and a RuO or an Allan Bradley thermometer (depending on the xy-motor that is used). The temperature of the sorption pump, the 1K-Pot and the superconducting magnet can be monitored using carbon resistors. Another RuO thermometer is located at the $^3$He-Pot.

Heaters are positioned at the sorption pump and the sample. The sorption pump heater is needed to cool down the system to base temperature. The sample heater is used mostly during cool down to keep water from freezing onto the sample surface, but also for temperature dependent measurements.

4.5 z-motor

The z-motor of the AFM is used to move the tip closer and farther away from the sample surface. After positioning the tip above the nano-structure at ambient conditions, the z-motor is used to retract the tip from the sample. This is necessary because of the mechanical vibrations evoked when pumping the IVC, as well as while lowering the insert into the dewar. These mechanical vibrations can lead to shifts between several micrometers and several hundred micrometers not only in z, but also in x and y direction, where the shift in z-direction generally tends to be smaller than for the other axis. The direction of those shifts is random and cannot be foreseen. Therefore the only possibility to avoid a tip crash because of those mechanical vibrations, is to retract the tip at least 50 μm from the sample.
surface. Another factor leading to unwanted changes in the z-position is the cool down procedure itself. Because the AFM is built of different materials, different parts have dissimilar thermal expansion coefficients. This leads to an inherent drift of the x, y and z-position during the cooldown, which again is in the order of several micrometers. As opposed to the mechanically induced drift, this thermal drift will not necessarily be seen on a positioning sensor. Once the system has reached thermal equilibrium, the z-motor is then used to approach the sample again, to enable us to scan the surface.

4.5.1 Basics of stick-slip motors

All motors used in this setup up till now are different variations of stick-slip motors. The basic principle of a stick-slip motor is illustrated in Fig. 4.6. A slider (here schematically shown in blue) is placed on a piezo (light grey). When the piezo is then slowly elongated via applying a voltage, the slider moves with the piezo to the right. To ensure that the relative position of the slider on the piezo stays the same one has to adjust the system in such a way, that the force due to friction \( F_G = \mu mg \) is bigger than the inertia force \( F_i = m\ddot{x} \).

Subsequently the piezo is very quickly retracted to its initial length. If this is done quickly enough, that the inertia force is now bigger, than the friction force, the slider stays at the same position as in (b), only the piezo retracts beneath it. If those two forces are properly adjusted, one can move the slider to any desired position on the piezo.
4.5. z-motor

4.5.2 Homebuilt stick-slip motor

The z-motor consists of three pairs of piezo stacks that are fixed onto the microscope housing. These piezo stacks move a triangular makor tube. Inside the hollow makor tube the scan piezo is fixed. On top of the scan piezo we mount the sensor. The force due to friction is controlled via the force with which the piezo stacks are pressed against the makor tube. This can be adjusted via the stiffness of a spring. A schematic top view and side view of the z-motor, scan piezo and sensor is shown in Fig. 4.7 (a,b).

Working principle

The working principle is illustrated in Fig. 4.7(c,d). Two piezo stacks are glued onto the microscope cage every 120°. These six piezo stacks in total can move the makor prism (blue) up and down over a range of more than one centimeter using the standard stick-slip principle. The drawback of this motor is the fact, that moving the z-motor mechanically shakes the tuning fork sensor, that is used to detect the surface. For some tuning fork sensors this shaking can lead to instabilities of the frequency shift, making it impossible for them to function properly.

Positioning sensor

The position of the z-positioner can be measured with a capacitive positioning sensor. This positioning sensor consists of two electrically connected plates that are mounted onto the movable prism and a voltage is applied to them, as shown in Fig. 4.9. On the side opposing these two plates another plate is mounted onto
Chapter 4. Experimental setup

Figure 4.8: Working principle of the z-positioning sensor. In the position shown in (a) the signal due to both plates (red and yellow) is the same, and zero will be read of the lock-in as illustrated in (c). In an asymmetric position as shown in (b) however, a finite signal will be observed as illustrated in (d).

the stationary cage of the microscope. The capacitive current is measured using standard lock in techniques.

When the opposing electrode is positioned symmetrically opposing the two counter electrodes no capacitive current is measured, however a shift of the central electrode leads to a finite signal, which scales approximately linear with the z-motor position. With this technique we can achieve an accuracy of about 200 nm.

Challenges

The main challenge of any stick-slip motor is to adjust the right pressure with which the slider is pressed onto the piezo stacks. If the pressure is too high the slider will stick onto the piezo stack even during the fast retraction period, resulting in a "stick-stick" motor. However if this pressure is too low, the slider slides during both the fast and the slow movement of the piezo, which also results in no netto movement.

For this particular stick-slip motor design, the crux lies in firstly finding the spring with the right spring constant, which defines the pressure and then keeping the surfaces off all six piezo stacks very clean and at precisely the same height.

4.5.3 Attocube positioner

The attocube z-motor (model ANP/z51/Res) is a commercially available stick-slip motor.
4.6. xy-motor

All x- and y-motor modules are based on the stick-slip method of movement. However, different designs have been used. We will give a short overview over the tested motor designs, their advantages and disadvantages.

4.6.1 The homebuilt three piezo tube design

Three piezo tubes are glued into a makor ring. The piezo tubes are connected and aligned in parallel. Two plates with sapphire surfaces are pressed onto these tubes from both sides by a spring. Using a conventional stick-slip approach the table can
Figure 4.10: (a) Schematic drawings of the ANP/z51/Res motor. (b) Photo of a motor module. (c) 3D view of the schematic drawings. All images taken from the data sheets from Attocube.

<table>
<thead>
<tr>
<th>Advantage</th>
<th>Challenge</th>
</tr>
</thead>
<tbody>
<tr>
<td>• cheap</td>
<td>• x and y motor direction are strongly coupled</td>
</tr>
<tr>
<td>• homebuilt-i.e. can be altered according to our needs</td>
<td>• the same position cannot be found reproducibly</td>
</tr>
</tbody>
</table>

Table 4.2: Challenges and advantages of the three leg homebuilt motor.

be moved relative to the makor ring, which is fixed onto the microscope frame. A photo and schematic sketch of this motor design is shown in Fig. 4.11.

Table 4.2 lists the advantages and challenges of this motor design.

4.6.2 The homebuilt single module design

The main disadvantage of the three-piezo tube motor design is the fact, that the x and y direction are strongly coupled. i.e. the direction of movement cannot be controlled. It is therefore not possible to reproducibly arrive at the same xy-position. In other words: If we drive 10 steps in plus x direction and then 10 steps in minus x direction, we will arrive at a different real space position.

To eliminate this problem we designed a single-module design. In this design a small piezo stack is wedged between the housing and a spring [Fig. 4.12]. The
piezo is surrounded by a plunger. Around the plunger a slider is clasped. The strength with which the slider is clasped onto the plunger can be adjusted with homemade spiral springs, which press the two plunger parts together.

Unfortunately this design comes with many problems. First of all we use a plunger with an elliptical diameter. Due to this shape it can twist inside the housing, limiting the overall mechanical stability. Especially when mounting two modules on top of each other the inherent twisting and tilting became obvious. Additionally this twisting frequently leads to the plunger getting stuck, preventing the motor from functioning. A quadratic plunger shape would be preferable.

When the piezo is quickly shortened (as it has to do once for every stick-slip step) the plunger is much slower in sliding back to its original position. It is therefore safe to assume, that with each contraction of the piezo the plunger crashes into it, when it returns to its original position. This so called hammering effect lead to a swift destruction of numerous piezos. To avoid this effect the piezo would have to be glued into the housing on one side and to the plunger on the other.

Furthermore it is known that sliding different materials on top of each other is likely to cause problems. However tests with plungers of a multitude of different materials, even though they improved the functionality of the motors, they did not make the motor fully functional.

When the slider reaches the end of its range on the plunger it gets stuck frequently. A spacer to prevent the slider from reaching extremal positions should be incorporated.
Figure 4.12: (a) Schematic of a single module of the motor. (b) Photo of x and y module with the chip carrier.

Finally during the extensive tests of these motor designs the modules showed irreproducible behaviors. I.e. when a functional setup was found (in terms of plunger material, weight on the sample holder, slider version, spring constant, washers under the spring, applied drive voltage and frequency) and it was left with those settings at ambient conditions for several hours it was usually found to not be functional any more but rather needed readjustment for any number of those settings.

Table 4.3 lists the advantages and challenges of this motor design.

### 4.6.3 The attocube design

The method and design of the ANPx51/RES motor correspond to those of the z-module. A detailed characterization is given in appendix H.

Table 4.4 lists the advantages and challenges of this motor design.

### 4.7 Scan piezo

For scanning we use a scan piezo from Staveley Sensors (EBL 3). The scan piezo consists of a cylinder with a central electrode on the inside. By applying a voltage to the inner electrode the scan piezo’s length can be adjusted. The outside is divided into four quadrants, acting as four electrodes. Two opposing electrodes are used for movement in x and y direction by applying voltages with opposite sign to them.
4.7. Scan piezo

<table>
<thead>
<tr>
<th>Advantage</th>
<th>Challenge</th>
</tr>
</thead>
<tbody>
<tr>
<td>• cheap</td>
<td>• does not work reproducibly</td>
</tr>
<tr>
<td>• homebuilt – i.e. can be altered according to our needs</td>
<td>• the same position cannot be found reproducibly</td>
</tr>
<tr>
<td>• x and y direction are decoupled</td>
<td>• adjusting the stiffness of the spring is time consuming</td>
</tr>
<tr>
<td></td>
<td>• piezos are destroyed on a weekly basis (hammering effect)</td>
</tr>
<tr>
<td></td>
<td>• positioning sensor tends to produce shortcuts</td>
</tr>
<tr>
<td></td>
<td>• mechanically unstable (tilt)</td>
</tr>
</tbody>
</table>

Table 4.3: Challenges and advantages of the homebuilt single module motor.

<table>
<thead>
<tr>
<th>Advantage</th>
<th>Challenge</th>
</tr>
</thead>
<tbody>
<tr>
<td>• reliable</td>
<td>• expensive</td>
</tr>
<tr>
<td>• decoupled x, y and z axis</td>
<td>• sensitive to sheer stress or strain</td>
</tr>
<tr>
<td>• reliable positioning sensor</td>
<td>as it might occur when mounting a sample</td>
</tr>
<tr>
<td>• functional closed loop positioning</td>
<td>• tuning fork sensor is fixed in space-stable frequency shift while driving the motors</td>
</tr>
<tr>
<td>• decoupled x, y and z axis</td>
<td>• ability to reproducibly position at the same place on the structure using the positioning sensor</td>
</tr>
</tbody>
</table>

Table 4.4: Challenges and advantages of the attocube motor.

By applying large voltages to either electrode for an extended period of time the piezo can be depolarized. This can be determined by measuring the capacitive coupling of the electrodes [sec. D]. If the capacitive coupling of the four outer electrodes to the inner electrode is noticeably different the piezo can be repolarized [see sec. E]. A depolarization is more likely to happen when large voltages are applied. Scanning at pressures between $10^{-2}$ and $10^{-4}$ mbar is likely to lead to electric discharges which harm the piezo as well and should therefore be avoided.

The dimensions of the scan piezo are the following:

- $t = \text{wall thickness} = 0.508 \text{ mm}$,
- $d_i = \text{inner diameter} = 9.017 \text{ mm}$,
Chapter 4. Experimental setup

\bullet d_o = \text{outer diameter} = 9.525 \text{ mm},

\bullet d_m = \frac{d_i + d_o}{2} = 9.271 \text{ mm},

\bullet d_{31}(293 \text{ K}) = -2.62 \, \text{ÅV},

\bullet d_{31}(4.2 \text{ K}) = -0.33 \, \text{ÅV},

\bullet d_{33}(293 \text{ K}) = 5.83 \, \text{ÅV},

\bullet d_{33}(4.2 \text{ K}) = 0.74 \, \text{ÅV},

\bullet L = \text{length} = 55 \text{ mm}.

We apply the following voltages:

\bullet \text{Voltage applied in } x \text{ or } y \text{ direction } V_{xy} = 200 \text{ V},

\bullet \text{Voltage applied in } z \text{ direction } V_z = 50 \text{ V}.

Using the equations

\[ \Delta xy = 0.9 \cdot d_{33} V L^2 \frac{d_m}{d_m}, \]  

(4.9)

\[ \Delta z = \frac{d_{31} V_z L}{L}, \]  

(4.10)

we can calculate its nominal range in x or y direction \( \Delta x, \Delta y \) and z direction \( \Delta z \) at room temperature to be

\[ \Delta xy(292 \text{ K}) = 60 \, \mu \text{m}, \]  

(4.11)

\[ \Delta z(292 \text{ K}) = 1.4 \, \mu \text{m}, \]  

(4.12)

and at 4 K:

\[ \Delta xy(4.2 \text{ K}) = 7.5 \, \mu \text{m}, \]  

(4.13)

\[ \Delta z(4.2 \text{ K}) = 0.35 \, \mu \text{m}. \]  

(4.14)

4.8 Sensor

4.8.1 Introduction

Most commercially available AFMs employ a laser beam to measure the deflection of a cantilever to detect force. This method cannot be employed because semiconductor nano-structures are sensitive to light. Therefore the measurement of the cantilever deflection using a laser beam would disturb the transport measurement.
Hence we have to use a different force detection mechanism here. The home-built tuning fork sensors measure force on the basis of a purely electrical detection mechanism.

The basis of such a sensor is a quartz tuning fork, as usually used in watches. As they are an article of daily use, they are commercially available at low costs. In order to use them for force detection we excite them at their resonance frequency using the piezoelectric effect. A phase locked loop is used to detect the shift of their resonance frequency due to an external force.

### 4.8.2 Characterization

A typical tuning fork sensor is shown in Fig. 4.13(a,b) The tuning fork is mounted onto a makor block and electrically connected with wires down to the support plate. We glue a PtIr wire to the upper prong of the tuning fork. This PtIr wire is then etched and used as a tip for scanning Fig. 4.13(c). It is electrically connected via the support post next to the top end of the tuning fork. The tuning fork and the tip are later electrically connected using the silver contacts on the support plate.

In order to characterize a tuning-fork sensor we record its oscillation amplitude $A$ and phase $p$ versus drive frequency $f$ as shown in Fig. 4.14(a,b). At the resonance frequency we observe a peak in $A(f)$ and an abrupt phase shift in $p(f)$ at the same resonance frequency $f_{\text{res}}$. The parameters used to characterize the quality of a tuning fork sensor are the oscillation amplitude, the Q-factor (i.e. the steepness of the $p(f)$ phase jump), and the resonance frequency.

All of these parameters show a strong dependence on the pressure of the surrounding medium as well as the temperature. The temperature dependence is shown in Fig. 4.14(c,d,e). Amplitude and Q factor increase greatly when the tuning fork is used in vacuum as opposed to ambient conditions. The quality of the tuning fork increases further when cooling it down where the strongest change can be seen at low temperatures. Similarly we also encounter a dependence of the resonance frequency on temperature. This dependence is especially strong below 100 K.

In summary: All tuning fork characterization parameters depend strongly on the temperature. This dependence is very strong at low temperatures. It is therefore crucial for topography scanning to work at constant temperatures.

### 4.8.3 Optimizing the sensor

For scanning gate microscopy we would like to use a tip that is as clean and sharp as possible [see sec. 7.4.2 for the description of an ideal tip]. However due
Figure 4.13: Complete tuning fork sensor from the side (a) and from the top (b). (c) Tip obtained via wet etching from a PtIr wire. (d) Sharpening of a wet-etched tip via FIB. (e) Nanowire grown on a wet etched tip.

to topography scanning the tip becomes increasingly dull and dirty. In order to avoid this we can follow different approaches:

- minimize topography scanning
- alter the tip in situ
- start with sharper tips

In order to minimize topography scanning we could employ markers on the sample as well as the "scanning feel" method for fine positioning, as well as positioning after all mechanical intrusions but before the cooldown as discussed in section 2.2.4 and section 2.2.3.
Figure 4.14: (a,b) Drive frequency versus oscillation amplitude and phase measured with an excitation voltage of 1 mV. (c,d,e) show the temperature dependence of the characterizing parameters of the tuning fork.

The tip can be altered in situ by mechanical alterations as well as electrical procedures. A more in depth explanation is shown in section 6.4.5 and 7.4.2. However both methods work on the basis of statistics, where the precise outcome is not known in advance.

Finally we can start out with tips that are sharper than the tips usually produced with standard wet etching processes as shown in Fig. 4.13(c). The main goals here are to achieve a high aspect ratio and small tip apex radius.

**FIB tips**

One approach is to sharpen the etched tips using focused ion beam (FIB) micro machining. In this method a focused Ga\(^+\) ion beam can be used to "cut" away nano-parts of the existing tip forming an incredible sharp sensor. Figure 4.13(d) shows the progress on one such tip. (i) displays the wet etched tip that is the starting point for this operation. It has an apex radius of about 1 \(\mu\)m. We gradually cut material away from all sides carving out a much sharper rod (ii). We then apply the same practice on a smaller scale, further narrowing down the last few micro meters of the tip (iii) until finally focusing on the apex (iv). Using this technique we can achieve opening angles of less than 10° and and tip apex radii around 10 nm. An additional advantage of this method is that it allows us to
define a tip apex with a specific shape, i.e., tips that are more symmetric around their rotation axis.

**EBD tips**

The second option tested within this thesis was to use electron beam deposition (EBD) to grow a nano-wire on an existing tip. An example of this method is shown in Fig. 4.13(e). These tips have an aspect ratio that cannot be outbid. Their apex radius is also admirable low, below 10 nm.

However, these new tips still come with some uncertainties. They are grown unto a tip, providing a predetermined breaking point at the tip-nano-wire interface. Furthermore depending on the material used to fabricate them they may not be conducting, which might lead to the need of applying another conductive layer coating onto the wire.

### 4.8.4 Sensor properties

**Magnetic field dependence**

Fig. 4.14 shows how the resonance frequency of the tuning fork depends on temperature. Additionally the tuning fork resonance can also be altered by applying a magnetic field. Figure 4.15 shows a typical shift in resonance frequency versus the magnetic field. This shift is rather strong for low fields and saturates upon reaching a magnetic field of several tesla. As the frequency shift $\Delta f(B)$ is a factor of 20 larger than the normally used frequency shift setpoints around 200 mHz it is out of question to perform any scanning gate or topography scanning while sweeping the magnetic field.

**Surface position**

The z-position at which the surface is found by the sensor depends crucially both on the setpoint and the drive amplitude. In a simple picture we can imagine the tip softly hitting the surface as it oscillates. The force how hard the tip is going to hit the surface is given by the setpoint. Figure 4.16(a) shows the surface position $z_{\text{meas}}$ versus the sensor setpoint at constant drive amplitude. We see that when increasing the setpoint the position at which the surface is "found" by the tip $z_{\text{meas}}$ moves closer to the actual surface $z_{\text{real}}$. In our simple picture this means that we are pushing the tip with more force onto the surface.

When on the other hand increasing the drive amplitude while keeping the setpoint constant we observe that with increasing drive amplitude the surface position $z_{\text{meas}}$ moves further away from $z_{\text{real}}$. At higher drive amplitudes the tip
oscillates with a higher amplitude [Fig. 4.16(c)], therefore hitting the surface at values further away from $z_{real}$ i.e. higher $z_{meas}$ values.

**Noise level**

To perform high resolution topography scan it is essential to optimize the signal to noise ratio. When the tip is positioned on the surface at a constant position the noise in z-position has to be as small as possible. This noise is strongly related to the drive amplitude. Figure 4.16(d) shows the noise in the frequency shift signal versus drive amplitude. Below a certain drive amplitude $A_{drive,C}$ (about 0.5 nm for this particular sensor) the noise of the frequency shift signal is highly dependent on the drive amplitude. For higher drive amplitudes it does not decrease significantly. High drive amplitudes are not desirable, as they in themselves limit the resolution and are more prone to damage the tip. Therefore it is reasonable to perform topography scans with a drive amplitude close to $A_{drive,C}$.

### 4.9 Electronics

The electronics needed to operate the complete setup consists of three parts:

- the cryostat electronics,
- the measurement electronics,
- the AFM electronics.
Figure 4.16: Properties of a tuning fork sensor. (a) Measured surface position versus setpoint. (b) Measured surface position versus drive amplitude. (c) Drive amplitude versus oscillation amplitude $A$ of the sensor. (d) Noise level of the frequency shift versus drive amplitude.

The cryostat electronics consists of a readout utility for the thermometry, heater, helium level and a control unit for the superconducting magnet. The readout electronics for the thermometers measures the voltage drop across the resistors. This voltage drop is then converted to resistance and matched to the corresponding temperature using labview programs. Similarly the helium level is read out by a home written labview routine. Another home written labview program can be used to read and control the magnet power supply.

The measurement electronics consists of standard lock-in set-ups for the AC measurement and Yokogawas and Hewlett Packed multimeters for the DC measurements. The measured signals are feed into the Nanonis setup for the scanning gate measurements and will be explained in more detail in the chapters dealing with the measured data.

The AFM electronics consist of a high voltage supply for the scan piezo, which is triggered by the Nanonis outputs and can therefore be controlled using the Nanonis software. Another part of the electronics is designed to control the piezo-motors and the corresponding positioning sensors. For the homebuilt motors the electronics was homebuild and the positioning sensors were read out using standard lockin measurements. When the homebuilt motors were replaced with the attocube motors, the homebuilt electronics was complemented with the Attocube ”ANC350-3RES” electronics. This electronics supplies the voltage spikes used to
move the piezo motors, when triggered by Nanonis, or by hand. However when triggering the ANC using the Nanonis software, the axis of movement has to be chosen in the Attocube software "Daisy". The ANC350-3RES can also be used to read out the positioning sensors of the motors and determine the capacitance of the piezos. It furthermore offers the option of closed-loop positioning.

Another labview program was written to carry out series of Nanonis scans. Such a series can consist of a set of scan frames for different voltage settings (both controlled by the Nanonis outputs, or an external voltage source), magnetic fields, tip-sample distances or xy-positions. Basic safety features such as a "safe-tip" module and stop and start conditions for a given temperature or magnetic field are included.
Chapter 5

Measurements on a graphene ring

5.1 Introduction

Carbon is a material fascinating for its many different manifestations. Carbon atoms have the unique natural ability to arrange themselves in solid-state structures of different dimensionality. The most commonly known structure is graphite, a three dimensional manifestation, where carbon atoms are arranged in layers that are held together by Van der Waals forces. A single one of these layers can exist as a two-dimensional object. This so-called graphene consists of carbon atoms bound in a honeycomb lattice. Such a carbon sheet can be rolled up forming a carbon nano-tube—the one-dimensional manifestation of a carbon material. When rolling a small part of a carbon sheet into a ball, fullerenes or buckyballs form. These represent the zero-dimensional case of a carbon solid. Images of all theses manifestations are shown in Fig. 5.1.

Figure 5.1: (a) Single block of bulk graphite [68]. A two dimensional graphene sheet is shown in panel (b)[69]. Panel (c) shows a environmental scanning electron micrograph of carbon nano-tubes [70]. A STM scan of a zero dimensional fullerene [71] is shown in (d).
The unique two-dimensional manifestation has spiked intense research interests. While the earlier research interests have focussed on the most basic nanostructures: Graphene constrictions or nano-ribbons [56, 57, 58, 59, 60, 61], soon different groups were able to form increasingly complex structures such as quantum dots [62, 63, 64, 65] and double dots [66]. Despite this increasing control over very complex nano-structures, only one experimental publication on graphene rings is available [67]. There the Aharonov–Bohm effect has been observed for the first time in a two-terminal graphene ring structure [67]. In this experiment the visibility of the Aharonov–Bohm oscillations was found to be less than 1% at low magnetic fields. It was speculated that this small value might be due to inhomogeneities in the two interferometer arms leading to a tunneling constriction that suppressed the oscillations.

Apart from these predominantly theoretical aspects the Aharonov–Bohm effect [72, 73] in graphene has been addressed [74, 75]. One consequence of interference (weak localization which is the coherent backscattering of electron-waves from a disordered potential: Coherent interference between time-reversed paths) has been studied in graphene flakes [76, 77, 78] with the outcome that weak localization can only be observed in the presence of intervally scattering.

On the other hand nano-scale rings have been studied in nano-structures of a variety of different materials including carbon-materials such as carbon nano-tubes [79, 80].

Here we investigate the Aharonov–Bohm effect [sec. 5.3.3] in a graphene nano-ring. We start by giving a short introduction to graphene and the sample under investigation in section 5.2. We then discuss basic transport properties of graphene [sec. 5.3.1, sec. 5.3.2], before discussing the Aharonov–Bohm effect in detail and the effects of gates on it for the rest of section 5.3. We show scanning gate experiments performed on the sample in section 5.4 and interpret them with the help of electrostatic simulations [sec. 5.5].

Parts of the pure transport measurements presented in section 5.3 have been published in [81] and [82].

5.2 Sample and measurement setup

5.2.1 Graphene: A true two dimensional material

In the wake of trying to perform experiments that could be correctly explained with simple analytical models, physicists soon started to hunt for structures with reduced dimensionality. Two-dimensional electron gases as part of heterostructures have been discovered, extensively investigated and still hold a big appeal up to date [chap. 7 and 8] [45]. However big the triumphal procession of 2DEGs both in
5.2. Sample and measurement setup

research and subsequently in industrial applications as well might have been, some points of discussion remained. For one an ideal two-dimensional material should consist of a single layer of atoms, located on the surface of an insulator. Locating such a material on the surface, as opposed to the buried 2DEGs in semiconductors should allow to carve nano-structures with even lower dimensionality out of the material as well as allow for unique investigations due to its accessibility. Problems like those left physicists pondering, if there might be a more perfect true two-dimensional material.

The discovery of graphene

Theoretical physicists predicted, that there should be an answer to this search, that a true two-dimensional material, made from a single atomic layer of carbon atoms with unique electronic features should in fact exist [83, 84, 85].

However, as appealing as this material seemed, it was predicted to be thermodynamically unstable [86, 87]. Therefore it was common sense to expect that this material could not exist in a real experimental surrounding. Putting all skeptics to shame graphene was experimentally realized and identified in a an optical microscope in 2004 by Novoselov et al. [88]. For this groundbreaking discovery Novoselov and Geim have recently been awarded the Nobel price.

With the experimental isolation of graphene physicists now have access to a new true two-dimensional material and research interests on this topic exploded.

Structural properties

Graphene consists of a single atomic layer of carbon molecules. The atoms are regularly arranged in a honeycomb lattice [Fig. 5.2(a)], a unit cell consists of two atoms. The first Brillouin zone in reciprocal space is also hexagonally shaped. The points of high symmetry are shown in Fig. 5.2(b). The points K and K’ will be referred to again when discussing the band structure of graphene.

The bonds are $sp^2$ hybridizations. These $sp^2$ orbitals form strong covalent bonds with their three direct neighbors. These bonds are strong enough to make graphene structurally stable at room temperature [89]. The remaining valence electron per atom is delocalized, accounting for the electrical conductivity of the material.

5.2.2 Sample fabrication

The substrate on which the graphene structure is fabricated consists of a highly doped Si wafer topped with a 295 nm thick silicon oxide. Single-layer graphene
flakes were deposited by mechanical exfoliation employing the Scotch tape technique described in [88]. Flakes which have a high probability of being single layer flakes (i.e. graphene) are identified by the contrast of the optical microscope [90, 91]. These potential candidates are then evaluated using the method of Raman spectroscopy [92, 93].

The Raman scattering process corresponds to the inelastic scattering of a photon. Therefore after scattering the photons have experienced a frequency shift compared to their incident frequency, having usually given energy to the molecules on which they were scattered. This changes the vibrational, rotational or electronic energy of the molecule in question. Raman spectroscopy measures intensity versus the frequency shift of the scattered photons, giving insight into the phonon modes within the system.

These phonon modes are particular for single-layer graphene as opposed to multi-layer graphene. Therefore Raman spectroscopy allows to differentiate between single [Fig. 5.3(d)] and multilayer graphene flakes [93].

In order to fabricate the desired nano-structure onto the graphene we use the method of electron beam lithography (EBL). In a first EBL step the aimed for graphene structure is transferred onto the resist forming a mask of this structure. Subsequent Ar/O\(_2\) reactive ion etching removes the unprotected graphene. Afterwards a fresh layer of resist is applied and with a second EBL step the mask for
the metallic electrodes is defined. Finally the electrodes formed by evaporating a 2 nm sticking layer followed by a 40 nm thick gold layer [63, 64].

5.2.3 The measured sample

The sample is fabricated as described above from a single-layer graphene sheet. It consists of a graphene ring with an inner radius of 200 nm and an outer radius of 350 nm. The arms of the ring have a width of 150 nm to avoid the undesired formation of quantum dots inside the arms occurring on smaller length scales. The sample is shown in Fig. 5.3. Panel (a) and (b) show AFM-scans of the structure at room temperature. The graphene appears in a lighter gray shade, whereas the areas where the graphene has been removed via etching appears almost black.

The ring is connected to four contacts labeled C1 to C4 as can bee seen in Fig. 5.3(a). Each one of those four graphene contacts is then connected to two metallic electrodes which appear as white patches in Fig. 5.3(b). When performing an AFM scan at 1.7 K, these metal contacts are clearly visible. From the positions of the contacts the location of the ring can be deduced [white dashed lines in Fig. 5.3(c)] even when the single layer graphene cannot clearly be resolved.

The side gates are also fabricated from single-layer graphene. They are shaped in an arch around either side of the ring, with a distance of 100 nm from the ring. While performing the transport measurements only the side gate labeled "SGb" in Fig. 5.3(a) was functioning, while the other side gate was kept at $V_{SGa}=\text{ground}$ due to a shortcut in the wiring of the cryostat.

The sample is also equipped with a back gate, which allows a tuning of the charge carriers from electrons to holes as well as their density.

5.2.4 Measurement Setup

All measurements presented in this chapter are performed in a $^3$He cryostat at a base temperature of $T \approx 500$ mK. Standard low-frequency lock-in techniques are used to measure the resistance by applying a constant current. A magnetic field is applied perpendicular to the sample plane.

The scanning gate experiments are performed using the AFM-setup described in chapter 4. When performing the scanning gate measurements the sample resistance was measured in the two-terminal configuration.
Figure 5.3: (a) Detailed AFM scan of the ring. The area where the graphene has been removed and die SiO$_2$ can be seen appear in a darker shade. The contacts are labeled C1 to C4. The side gates are labeled SG$_b$ and SG$_s$. (b) Shows the AFM scan of a larger area of the sample. Each contact C1, C2, C3 and C4 as well as each gate is connected to two metallic electrodes, seen in white. (c) Shows an AFM scan of the sample carried out at 1.5 K. The graphene structures can roughly be recognized by adjusting the color bar accordingly. The metallic contacts are clearly visible. From those the position of the ring can easily be estimated as shown by the white dashed lines. (d) Raman spectrum of the same flake before processing. The spectrum was recorded using a laser excitation wavelength of 532 nm.

5.3 Transport measurements

5.3.1 Basic transport properties of graphene

The tight binding approach

A detailed calculation of the band structure of graphene can be found in a variety of publications, for example [83, 94].

Graphene is constructed from carbon atoms arranged in a hexagonal lattice. Therefore also the Brillouin zone will have a hexagonal shape. At the corners of these hexagons the K and K’ points (Dirac points) are located. Building on this basic geometric framework we can construct a tight-binding Hamiltonian, for the case that electrons can hop to the nearest and next to nearest neighboring atoms.

From this Hamiltonian we can derive the energy bands [94]:

$$E_{\pm}(\vec{k}) = \pm \sqrt{3t f(\vec{k}) - t' f(\vec{k})}$$  \hspace{1cm} (5.1)
where \[ f(\vec{k}) = 2\cos(\sqrt{3}k_ya_0) + 4\cos(\frac{\sqrt{3}}{2}k_ya_0)\cos(\frac{3}{2}k_xa_0) \] (5.2)

t is energy needed for hopping between different sublattices. and \( t' \) is the energy for hopping in the same sublattice. \( a_0 \) is the distance between two neighboring atoms [Fig. 5.2].

When only considering the energy range directly around the Dirac point the dispersion can be obtained by expanding the full band structure equation 5.2 around the K and K'-point. In this case the dispersion becomes approximately linear [83]:

\[ E_\pm(\vec{q}) \approx \pm \hbar v_F|\vec{q}| + O(\frac{|\vec{q}|}{|K|}) \] (5.3)

For a perfect undoped graphene sheet the Fermi energy \( E_F \) lies directly at the Dirac point. In this case the valence band is completely empty while the conduction band is completely filled.

**Band structure of graphene**

The linear dispersion of graphene is a unique feature of this material. Conventional semiconductors show a parabolic bandstructure, with a gap \( E_g \) forming between the conduction band \( E_C \) and the valence band \( E_V \). The conduction band can be described via

\[ E = E_C + \frac{\hbar^2 k^2}{2m^*} \] (5.4)

and analogously for the valence band. As no states can form inside the energy gap the density of states \( D \) will be zero within \( E_g \) and constant within the valence and the conduction band

\[ D_{2D}(E) = \frac{m^*}{\pi \hbar^2} \] (5.5)

This is shown schematically in Fig. 5.4 (a,c). Opposed to this is the linear dispersion of graphene [see Fig. 5.4 (a)]. Here no bandgap forms, making graphene a zero bandgap semiconductor. The valence and the conduction band touch in a single point, called the K-point. Here the dispersion relation of valence and conduction band is linear.

\[ E(\vec{q}) = \pm \hbar v_F|\vec{q}| \] (5.6)

This linear dispersion leads to the density of states that is linear in energy. Due to the fourfold degeneracy (spin and valley) this relation reads as follows:

\[ D(E) = \frac{2|E|}{\pi \hbar^2 v_F^2} \] (5.7)

As can be seen in Fig. 5.4 (b) the density of states vanishes at the Dirac point.
Chapter 5. Measurements on a graphene ring

Figure 5.4: Schematic bandstructure of graphene and a conventional 2D semiconductor in comparison (a). (b) and (c) show the density of state for the respective systems.

The Dirac equation

As a result of this linear energy dispersion at low energies, charge carriers behave like relativistic particles. They can therefore be described by the two-dimensional Dirac equation:

\[-iv_F \vec{\sigma} \nabla \psi(\vec{r}) = E \psi(\vec{r})\]  \hspace{1cm} (5.8)

Here \(\vec{\sigma}\) are the Pauli matrices. When describing graphene the Pauli matrices relate to the pseudospin (i.e. index indicating sublattice A or B). Therefore the charge carriers (i.e. holes and electrons) can be described as massless Dirac fermions in this context.

5.3.2 Sample characterization

Back-gate sweep

Figure 5.5(a) displays the resistance of the ring as a function of applied back gate voltage \(V_{BG}\) measured in a four-terminal configuration. The ring itself is connected via two graphene ribbons of a size of 150 × 350 nm (graphene-leads) to a branching which ends in larger graphene areas, where four gold contacts are used to measure the resistance. The measured resistance is composed of the ring resistance itself and the resistance of the graphene leads.

When evaluating the corresponding two-terminal measurement shown in Fig. 5.5(b) we observe that the resistance is about an order of magnitude higher than for the four-terminal configuration. Additionally features related to the contacts show up as extra peaks. The Dirac point itself can be seen at the same voltage as extracted for both measurement mechanisms.
5.3. Transport measurements

The measured four-terminal resistance \( R_{\text{meas}} \) consists of the following parts:

\[
R_{\text{meas}} = 2R_{\text{gl}} + R_{\text{ring}} \tag{5.9}
\]

where \( R_{\text{gl}} \) is the resistance of one graphene lead and \( R_{\text{ring}} \) is the resistance of the ring itself. In a semiclassical Drude picture these resistances can be calculated from the geometric aspect ratios (i.e. the length \( L \) and the width \( W \)) of the graphene lead (gl) and one arm of the graphene ring (aring) as follows:

\[
R_{\text{meas}} = \left( \frac{2L_{\text{gl}}}{W_{\text{gl}}} + \frac{1}{2} \frac{L_{\text{ring}}}{W_{\text{ring}}} \right) \frac{1}{\sigma} = 7.5 \frac{1}{\sigma} \tag{5.10}
\]

where \( \sigma \) is the conductivity at the given density \( n \) by \( \sigma = n|e|\mu \). Hence the ring contributes about 38% to the measured resistance and the graphene leads about 62%.

As seen in Fig. 5.5(a,b) the charge neutrality point occurs at \( V_{\text{BG}} \approx 10 \text{ V} \) this is due to unintentional doping. The high resistance observed at the charge neutrality point is related to the small width \( W = 150 \text{ nm} \) of the ring arms [60]. However, this width was chosen large enough that strong localization of charge carriers leading to Coulomb-blockade dominated transport in narrow ribbons [60, 61] is not dominant. A rough estimate of the mobility taking into account the geometry of the structure and using the parallel plate capacitor model leads to \( \mu \approx 5000 \text{ cm}^2/\text{Vs} \), comparable to the value quoted for the material used in Ref.[67]. For the typical back gate voltage \( V_{\text{BG}} = -5.8 \text{ V} \) used for most of the presented measurements, we can calculate the sheet carrier density using the parallel plate capacitor model. As the lateral extent of the graphene structure is in the order of one micrometer and therefore significantly larger than the separation between the graphene structure and the back gate \( (d_{\text{GloBG}} = 300 \text{ nm}) \), we can employ this model. The sheet carrier density \( p_s \) is calculated as follows:

\[
p_s = \frac{\epsilon \epsilon_0}{\epsilon d_{\text{GloBG}}} \Delta V_{\text{BG}} = 1.2 \times 10^{12} \text{ cm}^{-2}. \tag{5.11}
\]

Where \( \epsilon = 3.9 \) is the permittivity of SiO\(_2\), \( \epsilon \) is the permittivity of vacuum, \( d_{\text{GloBG}} \) is the graphene-to back gate separation and \( \Delta V_{\text{BG}} = V_{\text{BG,meas}} - V_{\text{BG,Dirac}} \) is the voltage separation between the back gate \( V_{\text{BG,Dirac}} \) voltage of the Dirac point and the evaluated back gate voltage \( V_{\text{BG,meas}} \).

We analyze the different length scales present in the system. By doing this we gain insight about the relevant transport regime. The Fermi wavelength corresponding to the carrier density mentioned above is

\[
\lambda_F = \sqrt{4\pi/p_s} = 33 \text{ nm} \tag{5.12}
\]
Figure 5.5: (a) Four terminal resistance across the ring structure as a function of back gate voltage, with both side gates grounded. The corresponding two-terminal measurement is shown in (b). Both measurements are recorded at $T = 500 \text{ mK}$ with a constant current of $I = 10 \text{ nA}$.

For comparison, at the same density the mean free path is

$$l = \hbar \mu \sqrt{\frac{\pi}{2p_s}}/e \approx 65 \text{ nm} \quad (5.13)$$

This is less than the sample dimensions. For our sample the geometric extent: The width $W = 150 \text{ nm}$ of the arms, the mean ring radius $r_0 = 275 \text{ nm}$ and its corresponding circumference $L = 1.7 \mu \text{ m}$ is much smaller. Therefore, the presented measurements are all close to the diffusive (dirty metal) regime. As a result carrier scattering at the sample boundaries alone cannot fully account for the value of the mean free path.

The relevance of thermal averaging of phase-coherent effects can be judged from the thermal length

$$l_{th} = \hbar v_F l/2k_B T = 700 \text{ nm} \quad (5.14)$$

In our case $l_{th}$ is significantly smaller than $L$. Therefore thermal averaging of interference contributions to the conductance is expected to be relevant.

**Two-point versus four-point measurements**

In a four point measurement a current is applied across the sample via two connections. The current generates a voltage drop across the complete measurements setup i.e. the sample including the contacts and the setup wiring in accordance with Ohms law: $V = IR$. If the second set of connections is placed very close to the actual sample it will only measure the voltage drop across the sample.

Therefore four-terminals allow for a more precise measurement eliminating the contact resistance and effects originating from the contacts and measurement
5.3. Transport measurements

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<td>0.390</td>
<td>good</td>
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<td>0.032</td>
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Figure 5.6: (a) IV-traces of all different contact combinations. (b) shows a table giving the two contacts used for the measurement, the trace this corresponds to in (a) and the average slope of each IV-trace. The yellow marked entries note pairs of contacts located on the same contact pad. The orange marked entries mark pairs of contacts located on one side of the ring. The red marked entries mark pairs located on opposite sides of the ring. Panel (c) notes the different contact numbers schematically.

setup. We can therefore attribute features that only show up in a two-terminal measurement but not in the four-terminal measurement mostly to the contact areas of the sample.

IV traces

A method to evaluate the quality of each contact is current versus voltage measurements, so called IV-traces. We can record IV-traces between each pair of contacts as shown in Fig. 5.6(a). The nano-ring is connected to four contact areas and on each contact area we find a pair of contacts to different leads as schematically shown in Fig. 5.6(c). We observe that the slope of the recorded IV-traces for all these contact combinations varies from almost zero to $0.4 \times 10^4 \text{A/V}$.

Evaluating the slopes of the different IV-curves we can choose the best suited contacts for the measurements to follow.
Figure 5.7: $I(V_{SG}, V_{BG})$ for different voltage ranges. (a) shows single $R(V_{BG})$ traces taken from the measurement shown in (b). The traces are offset in resistance for clarity. (c) shows the same data as in panel (b) but here the trace taken at $V_{SG} = 0$ V is subtracted from all traces, making the shift of the Dirac point more obvious. (d,e) show high resolution measurements of a smaller voltage range.

The contact resistance and therefore the slope and shape of such IV-curves can be altered by passing a large current through a contact, a method called 'current annealing'.

Lever arms

We will now evaluate the relative lever arms of the side gate SGb and the back gate. To do so we record the current versus back gate and side gate voltage as shown in Fig. 5.7. Panel (b) shows an overview over the complete voltage range accessible. We observe a slight shift in the Dirac peak with changing side gate voltage. This becomes more obvious when subtracting the trace recorded at $V_{SG} = 0$ V from all other traces as shown in Fig. 5.6(c). Panel (a) shows $R(V_{BG})$ cuts through this measurement, which are shifted in resistance for clarity.

If we take a closer look at the central voltage-range [Fig. 5.7 (d,e)] we can make out different slopes of features shifting with the gate voltages: $\alpha_{BG}/\alpha_{SG} = 14.9$
and $\alpha_{BG}/\alpha_{SG} = 4.8$. We have to keep in mind that the side gate is located directly next to one arm, and relatively far away from the other arm of the ring. However, we measure the resistance across the complete structure including both ring arms. One possible interpretation is therefore, that one lever arm gives the relative lever arm of both gates on the arm close to the side gate. And the other relative lever arm corresponds to the lever arm of the gates on the ring arm opposite to the side gate.

Reproducibility

Upon stabilizing at certain voltage settings for several days the sample could be measured for several days without a charge rearrangement. Faster changes in the voltages lead to instability making features as the Aharonov-Bohm oscillations less pronounced. We could also observe a tendency that the hole region usually is electrically more stable than the electron regime. Therefore we concentrate on the measurements performed in the hole regime.

5.3.3 Basic Aharonov-Bohm measurements

A detailed review about the Aharonov–Bohm effect can be found in [95].

Introduction to the Aharonov–Bohm effect

One of the concepts that make up the core of quantum mechanics is the principle of superposition. The Aharonov–Bohm effect is one of the most direct manifestations of interference of charge carriers. It therefore offers a direct window into the heart of quantum mechanical principles.

We now consider the setup schematically shown in Fig. 5.8(a). A source emits monochromatic waves. These waves are diffracted by two slits, which have a width narrower than the wavelength of the incident waves. Upon passing through the slits the waves will be displayed on a screen, where an interference pattern will show up. The probability amplitude $t_1$ for transmission through the upper slit taking the path $\gamma_1$ is [95]

$$t_1 = a_1e^{i\theta_1} \quad (5.15)$$

and the probability amplitude $t_2$ for transmission through the lower slit along path $\gamma_2$ is [95]

$$t_2 = a_2e^{i\theta_2} \quad (5.16)$$
\[ T = |t_1 + t_2|^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta \]

where \( \delta = \theta_1 - \theta_2 \)

Figure 5.8: (a) Schematic illustration of an Aharanov–Bohm experiment. (b) Schematic of an Aharanov–Bohm effect ring geometry.

\( a_1 \) and \( a_2 \) are real positive numbers between zero and one. The transmission phase \( \theta_i \) is also a real number. We now can find the intensity on the screen \( T \) as follows [95]:

\[ T = |t_1 + t_2|^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta \]

To observe this interference pattern in semiconductor nano-structures, where particles are detected locally in contacts, as opposed to an interference screen, is not directly possible. Therefore instead of recording \( T \) versus a spatial direction one has to take a different approach. To do so the two paths \( \gamma_1 \) and \( \gamma_2 \) have to enclose a magnetic flux \( \phi \). This magnetic flux can be used to change the relative phase between two paths. We therefore can now record an interference pattern in \( T \) versus \( \phi \) as shown in Fig. 5.8(a). This was first proposed by Aharonov and Bohm [72].

In experiments this theoretical setup is usually realized via a closed ring structure [see Fig. 5.8(b)]. The magnetic flux \( \phi \) described by a vector potential \( \mathbf{A} \) enclosed by both paths leads to a phase difference \( \delta(\phi) \) [95]:

\[ \delta(\phi) = \phi(0) - \frac{|e|}{\hbar} \int_{\gamma_1 - \gamma_2} \mathbf{A} \cdot d\mathbf{s} \]

\[ = \phi(0) - 2\pi \frac{\phi}{\phi_0} \]

(5.18)
where $\phi_0 = h/|e|$ is the magnetic flux quantum. As a consequence the total transmission $T$ through the ring structure depends on the magnetic flux enclosed. If we use this $\delta$ to rewrite equation 5.17 we get the following expression [95]:

$$T(\phi) = a_1^2 + a_2^2 + 2a_1a_2\cos[\phi(0) - 2\pi \frac{\phi}{\phi_0}]$$  \hspace{1cm} (5.19)

This way we can use the spatially fixed detector (i.e. a contact) to detect the interference between two waves by changing the magnetic flux enclosed by their paths.

The magnetic flux enclosed by the paths in this geometry is given by $\phi = BA$, where $A$ is the area enclosed by the paths. Therefore the transmission is periodic in magnetic field with a period [95]

$$\Delta B = \frac{h/e}{A}$$  \hspace{1cm} (5.20)

which we also call $h/e$ periodic oscillations. In case the phase coherence length is long enough that paths can enclose the whole ring more than once also higher order oscillations can be observed. These oscillations are then called $h/je$ periodic oscillations, where $j$ is a real number. Their period is given by [95]

$$\Delta B = \frac{h/(je)}{A}$$  \hspace{1cm} (5.21)

**Review: Measurements on graphene rings**

In 2008 the Aharonov–Bohm effect was for the first time observed in a two-terminal graphene ring by Russo et al. [67]. They showed that $h/e$ and $h/2e$ periodic oscillations could be observed. The amplitude of these oscillations strongly decreased with increasing temperature with $T^{-1/2}$ indicating that below 1 K, the phase coherence length is comparable to or larger than the size of the ring. On the other hand the amplitude of the oscillations increased with higher magnetic fields, which they attributed to the orbital effect of the magnetic field. Additionally they also found a dependence of the oscillation amplitude on the back gate voltage which had no clear microscopic origin.

The visibility of the Aharonov–Bohm oscillations observed in the Russo-experiment was lower, most likely due to the two-terminal setup of their structure.

**Aharonov-Bohm effect in a four terminal graphene ring**

Figure 5.9(a) displays the four-terminal resistance of the ring as a function of magnetic field at $V_{BG} = -5.789$ V. The raw data trace shows a strong modulation
Chapter 5. Measurements on a graphene ring

Figure 5.9: Four-terminal resistance across the ring as a function of magnetic field recorded at $V_{BG} = -5.789$ V with a constant current of 0.5 nA. (a) Displays the raw data. For panel (b) the background has been subtracted as described in the text. (c) Fourier transform of the trace.

of the background resistance on a magnetic field scale of about 100 mT. Clear periodic oscillations can be seen on top of this background. They have a period in magnetic field $\Delta B_{AB} = 17.9$ mT, indicated by the vertical lines. This period corresponds to the $h/e$-periodic Aharonov–Bohm oscillations of a ring structure of 271 nm radius.

This value agrees well with the lithographic dimensions of the ring, which has an inner radius of 200 nm and an outer radius of 350 nm. The average value between the inner and outer radius of the measured ring structure $r_0$ is 275 nm.

Figure 5.9(b) shows the same data with the background resistance subtracted (see next section). The amplitude of the Aharonov-Bohm oscillations is modulated as a function of magnetic field on the same scale as the background resistance, indicating that a finite number of paths enclosing a range of different areas contribute to the oscillations. This observation is compatible with the finite width $W$ of the ring [98].

In Figure 5.9(c) the fast Fourier transform (FFT) of the data in Figure 5.9(a) is plotted. The peak seen at 60 mT corresponds to the $h/e$-periodic Aharonov Bohm effect. The width of this peak is significantly smaller than the range of frequencies expected from the range of possible enclosed areas in our geometry (indicated as a gray shaded region in Figure 5.9(c)).
5.3. Transport measurements

Geometrically the sample has an aspect ratio length $L$ to width $W$ of about $L/W = 7.5$ as discussed above. When evaluating the resistance of the sample at the Dirac point we find it to be eighteen times $h/4e^2$ which yields an aspect ratio of $L/W = 18$, given that most graphene samples have a resistivity of $h/4e^2$ at the charge neutrality point. The geometrical aspect ratio is roughly one third of this aspect ratio estimated from the sample resistance at the charge neutrality point. Similarly the FFT-peak width of one $R(B)$-trace is only one third of the peak width expected from the geometry sample dimensions.

We therefore speculate that the paths contributing to transport in general and to the Aharonov-Bohm effect in particular may not cover the entire geometric area of the ring arms. One possible interpretation is that the sample has rough unordered edges leading to a region along the edges that does not contribute to the electrical transport.

In this four-contact measurement, the oscillations have a relative amplitude $\Delta R_{AB}/R_{total}$ of more than 5%. Considering that the resistance of the ring is only about 40% of the total measured resistance $R_{total}$ this corresponds to a relative change of 12% of the ring resistance.

In general, the observed Aharonov-Bohm oscillations become more pronounced for smaller current levels, as expected. The current level of 5 nA was chosen as a good compromise between the signal-to-noise ratio of the voltage measurement and the visibility of the Aharonov–Bohm oscillations. The lowest current level useful for taking data depends on the signal-to-noise ratio, which itself depends on the back gate voltage and also on the sample history. However, due to limited sample stability, the visibility of the oscillations at a given back gate voltage depends on the back gate voltage history. Therefore measurements presented here were taken only over small ranges of back gate voltage after having allowed the sample to stabilize in this range.

Higher harmonics, especially $h/2e$-periodic oscillations, are neither visible in the magnetoresistance traces, nor do they lead to a clear peak in the Fourier spectrum (less than 1% of the $h/e$-oscillation amplitude). This indicates that the phase coherence length $l_\varphi < 2L$, i.e., it is (significantly) smaller than twice the circumference of the ring. Given the temperature of our experiment, this estimate is well compatible with the phase-coherence lengths reported in Refs. [67, 99, 100, 101].

The measurements were taken in a magnetic field range where the classical cyclotron radius $R_c = h k_F/eB > 640 \text{ nm}$ is bigger than the mean free path $l$, the ring width $W$, and even the ring diameter. At the same time, Landau level quantization effects are negligible, because the sample is studied in the low field regime $\mu B \ll 1$. The only relevant effect of the magnetic field on the charge carrier dynamics is therefore caused by the field-induced Aharonov–Bohm phase.
In diffusive ring-shaped systems, conductance fluctuations can coexist with Aharonov–Bohm oscillations. However, the relevant magnetic field scale of the conductance fluctuations
\[ \Delta B_{\text{CF}} \sim \phi_0/Wl_\varphi \]
with \( \phi_0 = h/e \), can be forced to be well separated from
\[ \Delta B_{\text{AB}} = \phi_0/\pi r_0^2 \]
by choosing a sufficiently large aspect ratio \( r_0/W \). Judging the situation from the measurement traces in Fig. 5.9(a), the only candidates for conductance fluctuations are the magnetic field dependent variations of the background resistance, which occur on a magnetic field scale that is at least a factor of five larger than \( \Delta B_{\text{AB}} \). As far as the amplitude of the modulation of the background can be estimated from figure 5.9(a), it is of the order of the conductance quantum \( e^2/h \) which is reasonable, since the condition \( l_\varphi \sim L \) implies the absence of strong self-averaging over the ring circumference \( L \).

**Aharonov-Bohm measurements: Data evaluation**

Figure 5.10(a) displays two-terminal resistance data as a function of magnetic field with \( V_{BG} = -30 \text{ V} \) and \( V_{SG} = 0 \text{ V} \). The traces show Aharonov-Bohm oscillations with a period of 16.5 mT on top of a varying background resistance. The background can be seen to vary on a larger field scale than the Aharonov-Bohm oscillations.

In order to subtract the background the trace is transformed into Fourier space. The fast Fourier transform is multiplied with a filter function to distinguish the periodic Aharonov Bohm oscillations from the background oscillations. The data is then transformed back into real space. This method is well established and described in more detail in [102].

In order to counter check this evaluation the background is spline-fit and then added to the Aharonov-Bohm oscillations as extracted above. This result is then compared to the raw data trace and checked for any discrepancies. The background was determined by performing a running average over one Aharonov–Bohm period \( \Delta B_{\text{AB}} \). This method was found to lead to no relevant distortion of the oscillations after background subtraction (with some exception around \( B = 0 \text{ T} \), which is of minor importance for the quintessence.)

**Aharonov-Bohm measurements over a large magnetic field range**

Figure 5.10 shows both the two-terminal measurements in panel (a) as well as the four-terminal measurement in (b) for the complete magnetic field range accessible.
Both traces show regularly spaced Aharonov–Bohm oscillations over the complete magnetic field range, which are displayed in Fig. 5.10(c,d). The traces are symmetric in $B$ which is in accordance with the Onsager theorem $R(B) = R(-B)$ \cite{103, 104, 105} valid for a two-terminal measurement. The visibility is up to 10\% for the four-terminal case and 3\% for the two-terminal case due to the increased influence of the contacts.

The Fourier transforms are displayed in 5.10 (e) and (f). One single peak forms around 60 T$^{-1}$ which corresponds to 17 mT. The width of the peak is $60 \pm 15$ T$^{-1}$. From the geometric dimensions of the ring, the Aharonov-Bohm period is expected to be $22 \pm 11$ mT which corresponds to $60 \pm 30$ T$^{-1}$. Therefore, the radius derived from the Aharonov-Bohm period lies well within the geometric dimensions of the structure. The position of the Fourier peak as well as the width are comparable for both the four-terminal and the two-terminal measurement.

The $h/2e$ peak in the Fourier spectrum which is expected around 120 T$^{-1}$ is strongly suppressed. If we assume, that paths which are longer than the phase coherence length $l_{\phi}$ do not contribute to the interference pattern, we estimate the phase coherence length to be below 2 $\mu$m.

Figure 5.11 shows the detailed analysis of the four-terminal data discussed in the section above in terms of the period $\Delta B$ and amplitude $A$ of the Aharonov-Bohm oscillations. The evaluation was carried out as follows. Starting from the Aharonov-Bohm oscillations without background, each extremal position was determined. The separation in magnetic field between two such resistance extrema was taken to be half of the period $\Delta B$ of the Aharonov-Bohm oscillations. This period $\Delta B$ is plotted in Fig. 5.11(a). We observe an average value of about 17.6 mT (as indicated by the gray shaded area).

Subsequently the amplitude is derived by taking the absolute value of the difference in resistance of two adjacent extremal positions. The amplitude $A$ versus magnetic field is plotted in Fig. 5.11b. Taking into account the Aharonov-Bohm oscillations of the complete magnetic field range of 5 T, we see that the average oscillation amplitude rises with increasing magnetic field values by a factor of 3. Such an increase in $A$ at high magnetic fields can also be seen in metallic rings. There it is usually attributed to scattering on magnetic impurities \cite{106, 98}. We observe the same tendencies in the two-terminal data. This tendency is also observed by Russo et al. \cite{67} for the two-terminal case. In their case they have advised, that this increase of amplitude with magnetic field might be of orbital nature. They speculate that it originates from inhomogeneities or defects in the graphene leading to asymmetric ring arms.
Figure 5.10: AB oscillations measured over a large magnetic field range. The two-terminal case is shown in the left column, whereas the four-terminal data can be seen in the right column. (a) and (b) show the resistance traces as measured. Sub figures (c) and (d) show a zoom into the Aharonov Bohm oscillations without the background resistance. It can be seen, that the oscillations are spaced with a period of 16 mT (indicated by the vertical gray lines). (e) and (f) show the fast Fourier transform of the traces shown in (a) and (b), respectively. A single peak at $\hbar/e$ can be observed. Since the $\hbar/2e$ peak is below experimental resolution, we estimate that the phase coherence length is smaller than twice the ring circumference. The gray underlined part represents the full width half maximum of the Fourier transform. Experimental settings: $V_{SG} = 0$ V and $V_{BG} = 30$ V, constant current 5 nA.
5.3. Transport measurements

Figure 5.11: In this figure we show the analysis of the data shown in Fig. 5.10 in terms of frequency $\Delta B$ and amplitude $A$ of the 4-point resistance data. Subfigure (a) shows the period of the Aharonov-Bohm oscillations. The gray shaded area corresponds to the marked area in the Fourier spectrum seen in Fig. 5.10 (f). The period of the Aharonov-Bohm oscillations remains well within the range, that is, expected from the geometric dimensions of the ring. Furthermore, it remains constant over the complete investigated magnetic field range. The amplitude of the oscillation however, shown in (b) is seen to increase by about a factor of 3 over the investigated field range compared to 0 T.

5.3.4 Influence of the side gate

Figure 5.12 displays the four-terminal resistance of the ring as a function of magnetic field and voltage $V_{SG}$ applied to the side gate SG1, for two different back gate voltages without [Fig. 5.12 (a, c)] and with [Fig. 5.12(b, d)] background subtraction. In the raw data [Fig. 5.12(a, c)], a modulation of the background resistance on a magnetic field scale, similar to that in Figure 5.9(a), can be observed. The subtraction of the background (extracted as described before) makes the Aharonov–Bohm oscillations visible [Fig. 5.12(b, d)]. Aharonov–Bohm oscillations at different $V_{SG}$ display either a minimum or a maximum at $B = 0$ T, with abrupt changes between the two cases at certain side gate voltage values. This behavior is compatible with the generalized Onsager symmetry requirement for two-terminal resistance measurements, $R(B) = R(-B)$. Although our measurement has been performed in a four-terminal configuration, the contact arrangement with respect to the ring and the fact, that the contacts are separated by distances $\geq l_\phi$ from the ring lead to a setup where the two-terminal symmetry is still very strong. Closer inspection shows that the part antisymmetric in magnetic field of each trace (not shown) is more than a factor of ten smaller than the symmetric part.

In previous studies on metal rings the effect of electric fields on the Aharonov-Bohm oscillations has been investigated, and two possible scenarios were discussed: [107] on one hand, the electric field may shift electron paths in space and thereby change the interference. On the other hand, the electric field may change the electron density and thereby the Fermi wavelength of the carriers. We discuss the
latter effect in more detail below, since the relative change in the Fermi wavelength is expected to be more pronounced in graphene compared to conventional metals.

In order to estimate which phase change $\Delta \phi$ an electronic wave picks up on the scale of the side gate voltage change $\Delta V_{SG}$ on which Aharonov–Bohm maxima switch to minima, we use the relation

$$\Delta \phi = \Delta k_F L_{\text{eff}}$$  \hspace{1cm} (5.24)

where $L_{\text{eff}}$, being the effective length of a characteristic diffusive path, is assumed to be independent of the side gate voltage, whereas the change in wave number $\Delta k_F$ is assumed to be caused by $\Delta V_{SG}$. The quantity $\Delta k_F$ is found from the density change $\Delta p_s$ using

$$\Delta k_F = \sqrt{\pi/4p_s} \Delta p_s.$$  \hspace{1cm} (5.25)

The density change is related via a parallel plate capacitor model to a change in back gate voltage, i.e.,

$$\Delta p_s = \Delta V_{BG} \epsilon \epsilon_0 / ed$$  \hspace{1cm} (5.26)
(\epsilon: \text{relative dielectric constant of the silicon dioxide substrate}, \ d: \text{thickness of the oxide layer}) leading to

\[ \Delta p_s/\Delta V_{BG} \approx 7.5 \times 10^{10} \text{ cm}^{-2} \text{ V}^{-1}. \]  (5.27)

Finally, \( \Delta V_{BG} \) is related to \( \Delta V_{SG} \) via the lever arm ratio \( \alpha_{SG}/\alpha_{BG} \).

In order to determine this lever arm ratio, we have performed measurements of conductance fluctuations in the plane defined by the back gate voltage \( V_{BG} \) and the side gate voltage \( V_{SG} \) [sec. 5.3.2]. The characteristic slope of fluctuation minima and maxima in this parameter plane allows us to estimate the lever arm ratio \( \alpha_{SG}/\alpha_{BG} \approx 0.2 \). In previous experiments on side-gated graphene Hall bars [108] we found a similar lever arm for regions close to the edge of the Hall bar whose width is roughly comparable to the width of the arms of the ring investigated here.

Using the numbers given above and using the density \( p_s = 1.2 \times 10^{12} \text{ cm}^{-2} \) for Fig. 5.12(b), we find

\[ \Delta k_F \approx 1.2 \times 10^6 \text{ m}^{-1} \text{ V}^{-1} \Delta V_{SG}. \]  (5.28)

In ballistic systems the effective length of a path is given by \( L_{\text{eff}} \sim L \), giving

\[ \Delta \varphi \approx \Delta V_{SG} \pi/1.5 \text{ V}. \]  (5.29)

A phase change of \( \pi \) would imply a change of side gate voltage on the scale of 1.5 V which is large compared with the measurement in Fig. 5.12(b) where this scale is of the order of 100 mV. However, in the diffusive regime, a characteristic path contributing to Aharonov–Bohm oscillations is longer by a factor of \( L/\ell \approx 27 \) due to multiple scattering [109] giving

\[ \Delta \varphi \approx \Delta V_{SG} \pi/55 \text{ mV}. \]  (5.30)

A change of the side gate voltage of typically 55 mV would cause a switch of the Aharonov–Bohm phase by \( \pi \), in better agreement with the observation than the ballistic estimate. The same calculation could be used to estimate the correlation voltage of the conductance fluctuations of the background resistance, in agreement with the observation in Fig. 5.9 and Fig. 5.12. This correlation voltage is on the same scale as the phase jumps of the Aharonov–Bohm oscillations.

### 5.3.5 Influence of the back gate

Figure 5.13 shows magnetoresistance data for varying back gate voltages and \( V_{SG} = 0 \text{ V} \). Similar to the case where the side gate was tuned, we observe variations of the oscillation patterns as well as \( \pi \)-phase shifts. The raw data displayed in Fig. 5.13(a),- shows background fluctuations with \( h/e \)-periodic Aharonov–Bohm
oscillations superimposed. In Figure 5.13(b), the background has been removed. Again, alternating minima and maxima at $B = 0$ T can be observed.

The larger visibility of Aharonov–Bohm oscillations observed in our sample, compared to the work in Ref. [67] is unlikely to be caused by better material or sample quality. Also our measurement temperature is about a factor of four higher than the lowest temperatures reported there. We therefore believe that the smaller ring dimensions in combination with the four-terminal arrangement may be responsible for the larger value of the visibility observed in our experiment. In Ref. [67] the expression

$$\Delta G \propto l_{th}/l_e e^{-\pi r_0/l_e}$$

was invoked to explain the observed $T^{-1/2}$-dependence of the oscillation amplitude. The exponential term on the right hand side contains the radius of the ring $r_0$. A smaller radius will lead to a larger oscillation amplitude which may explain the improved amplitude in our measurements. However, trying to relate the visibilities observed in the two experiments quantitatively (assuming that all experimental parameters except the ring radius are the same) would lead to a phase-coherence length $l_{ph}$ smaller than the ring circumference $L$ and only slightly larger than the ring radius $r_0$. As our experiment demonstrates, a separation of $\hbar/e$-periodic oscillations from background variations due to magnetoconductance fluctuations is still possible in our device despite the aspect ratio $r_0/W$ which is reduced in our device compared to Ref.[67]. A phase-coherence length between $L$ and $r_0$ is also compatible with the observation $\Delta B_{CF}/\Delta B_{AB} \approx 5$.

We also note that the diffusive regime investigated in our device is quite extended in back gate voltage. Assuming diffusive scattering at the edges to become dominant as soon as $l \approx W$, we estimate that this does not occur (for transport in the valence band) until $V_{BG}$ becomes more negative than $-80$ V. Transport may also enter a different regime, when the Fermi wavelength becomes larger than $l$, which is expected to happen (again for transport in the valence band) at back-gate voltages larger than $+2$ V in our sample. An even different regime may be entered at a back gate voltage of $+9.3$ V, where $\lambda_F \approx W$. As a consequence, the ‘dirty metal’ description of the Aharonov–Bohm oscillations should be applicable in the whole range of back-gate voltages shown in Fig. 5.5, except for a region of $\pm 8$ V around the charge neutrality point, where the resistance is maximum.

### 5.3.6 Influence of the temperature

Figure 5.14 shows the evolution of the Aharonov-Bohm oscillations with the background resistance subtracted for different temperatures. As the measurements were carried out in a He3 system, only a limited temperature range down to
5.3. Transport measurements

Figure 5.13: Four-terminal resistance as a function of magnetic field and back gate voltage measured with a constant current of 1 nA. (a) displays the raw data, while for (b) the background has been removed.

Figure 5.14: Aharonov–Bohm-oscillations with background subtracted for different temperatures. An offset is added to the data for clarity. The amplitude $A$ decays with rising temperature.

500 mK was accessible. We see that for this temperature regime the amplitude of the oscillations drops for rising temperature. This behavior is consistent with earlier findings [67] and can also be seen in metallic rings under certain conditions concerning the relative energy scales and the size of the phase coherence length [98]. We have shown above, that the amplitude of the Aharonov-Bohm oscillations rises at high magnetic fields. On the other hand it is known, that sweeping to high magnetic field can induce heating in the measurement system, which would in turn lead to a diminished oscillation amplitude. The rise in the amplitude of the Aharonov-Bohm oscillations seen at high magnetic fields dominates over any such heating effects.
5.3.7 Influence of the measurement current

Figure 5.15 shows the dependence of the Aharonov-Bohm oscillations on the measurement current. Panel (a) shows Aharonov-Bohm traces with the background resistance subtracted for different measurement currents. We see that the oscillation amplitude drops for rising measurement currents \( I \). We see in the evaluation in panel (b) that this drop in amplitude is most significant when measuring at low currents. Therefore the measurement current for all measurements was chosen as low as possible. However with lower current the noise on the signal rises, leading to a minimal current that can be used, to obtain optimal signal to noise ratio.

Both results (i.e. the rise of the Amplitude versus the current and the temperature) are a little surprising. We do expect to observe a saturation of \( A \) for low currents as well as for low temperatures. The amplitude seems to saturate for temperatures below 500 mK, however, because the accessible temperature range is limited this saturation cannot be investigated further. Analogously we observe the maximum amplitude for currents around 1 nA. For lower currents the extracted amplitude saturates. Additionally its extraction becomes less precise due to the increased noise level. From the data available here it is unclear if the rise in amplitude is related to a measurement artifact, or what the actual physical explanation is. Further investigations should be carried out for a broader range of parameters to conclusively answer this question.

5.3.8 Conclusion

Even though graphene rings have been analyzed in detail theoretically, there has been only one experimental study until now [67]. In this work, we have studied the Aharonov-Bohm effect in graphene in a two-terminal ring, but using a four-contact geometry. This increases the relative contribution of the ring resistance to the total measured resistance, and together with the smaller ring radius it allows
to achieve a higher visibility of the oscillations of up to 10%. The data is analyzed by a simple dirty metal model justified by a comparison of the different length scales characterizing the system.

The main advantage of graphene compared to metals for Aharonov-Bohm studies is the reduced screening. We have shown that by changing the voltage applied to one of the side gates, we can induce a phase jump in the oscillations by changing the phase accumulated along this path.

5.4 Scanning gate measurements

5.4.1 Scanning gate measurements on graphene

In the following chapter we will discuss the local imaging and manipulation of transport through the graphene nano-ring described in section 5.2.3.

The transport measurements have shown that the local charge carrier density in graphene can be greatly changed when altering the voltage applied to one gate. We can use the metallic tip of our scanning gate setup to locally manipulate the charge carrier density in the structure.

A short review on the existing scanning gate measurements on both other graphene structures and and nano-rings in semiconducting materials has been given in section 2.3.1 and section 2.3.2. Here we will discuss the most striking features in the resistance maps [sec. 5.4.2] taken on this graphene ring. We will show how the resistance of the sample can be locally increased or decreased using the tip as a movable gate. Depending on the tip voltage and the in-plane gate voltages the tip acts to locally enhance or diminish the charge carrier concentration right beneath it. This leads to a region of lowered or raised resistance in the resistance maps.

We will further investigate the influence of different gate voltages such as the global influence of the back gate in section 5.4.4. We show, how the complexity of the underlying transport mechanisms greatly increases when moving closer to the Dirac point in the sample. This indicates the increased existence of localized states closer to the Dirac point.

We then show how the charge carrier density is changed more locally using the in-plane side gates [sec. 5.4.5]. Furthermore the influence of the magnetic field [sec. 5.4.3] and the tip-sample separation are investigated [sec. 5.4.6].

Finally in analogy to scanning gate measurements performed by other groups we will briefly focus on the fine structure underlying the resistance maps in section 5.4.7.
Chapter 5. Measurements on a graphene ring

Figure 5.16: (a) and (b) show examples of resistance maps recorded at the same back gate voltages $V_{BG}$ but different tip voltages. (a) The tip voltage of $V_{tip} = 0$ V leads to a resistance map, where the ring structure can be seen as a region of diminished conductance. Using a slightly negative tip voltage (b) we can observe the ring as a region of enhanced conductance. The outline of the ring structure in real space is schematically drawn in as white lines. (c) shows a zoom in into the back gate sweep $R(V_{BG})$ where the back gate voltage used to record the resistance maps is marked with a blue arrow. The tip voltage settings are schematically drawn in with green arrows, assuming a relative lever arm $\alpha_{BG}/\alpha_{tip} = 1$.

5.4.2 Local manipulation of the charge carrier density

In graphene structures both local (such as side gates) and global (such as the back gate) gate voltages are used to change the charge carrier density of the structure. We know that changing the voltage applied to a side gate does not change the charge carrier density uniformly in the complete structure, but rather produces a charge carrier density gradient. In a rough estimation this can be described with two conductors in parallel that are influenced differently by a side gate [108].

In scanning gate experiments, the tip acts as a local nano-gate, freely movable in space. The first obvious difference to local gates (side gates) is its movability. Side gates are fixed in space and always act on the same part of the structure. Opposed to that the metallic tip can be moved to any point in space. We can therefore intentionally probe the structure at any desired point. For graphene rings this means that we can locally diminish or enhance the charge carrier density at any point in the sample. The second obvious difference to a side gate is the small size of the tip. While side gates usually extend a few hundred of nanometers along the structure, the tips of sensors such as the sensor used here usually have a diameter around 70 nm. This allows for a much more specific probing than using the more extended side gates.
5.4. Scanning gate measurements

When scanning the tip across the graphene ring, we record the resistance through the ring structure in dependence of the spatial position of the tip, leading to so-called resistance maps. This allows us to see how the resistance of the complete structure changes, when we change the charge carrier density at just one specific point below the tip. No current flows from the tip to the structure.

Figure 5.16(a) shows a typical example of such a resistance map. We see the resistance through the sample versus x and y position of the tip, i.e. \( R(x, y) \). The tip voltage is \( V_{\text{tip}} = 0 \text{ V} \). The outlines of the sample are schematically sketched into the resistance map as white lines. We observe, that when the tip is scanned directly above the ring structure the resistance of the ring increases by about 15 k\( \Omega \). The effective tip voltage diminishes the charge carrier density in the ring structure whenever positioning it directly above the structure, leading to a rise in resistance. Even though the charge carrier density is not globally changed, we can diminish the resistance of the complete structure, by only locally manipulating the charge carrier density on one single arm of the nano-ring.

In the same way we expect to be able to enhance the resistance through the complete structure, when applying a negative voltage to the tip, i.e. diminishing the charge carrier density right below the tip [see Fig. 5.16(c)]. Figure 5.16(b) shows a resistance map recorded at the same back gate voltage, using a negative tip voltage \( V_{\text{tip}} = -0.2 \text{ V} \). The outline of the structure is highlighted with white lines. Along the outline of the ring the resistance in the resistance map is enhanced. Whenever the tip scans across the ring structure it locally decreases the charge carrier density leading to an increase of the resistance of the complete structure. Therefore already a local change in the charge carrier density in only a small part of one arm of the sample leads to a global change in the resistance of the sample.

5.4.3 Effects of an external magnetic field

Figure 5.17 shows resistance maps taken at different magnetic fields. The magnetic field differences are chosen to be small enough to be within one Aharonov–Bohm period of 15 mT. We observe a resistance map that looks almost identical to the current maps recorded by Hackens et. al. in [2] shown in Fig. 2.3(a).

The resistance maps show a single ring-like feature of enhanced resistance with a radius that extends over the geometric radius of the ring.

When increasing the magnetic field we observe a gradual broadening of the ring feature. The radius and amplitude of the ring feature in the resistance map increases with magnetic field.
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5.4.4 Changing the charge carrier density globally: Back gate voltage

The charge carrier density in a graphene sample can be changed by changing the voltage applied to the back gate. When looking at the back gate sweep $R(V_{BG})$ shown in the lower panel of Fig. 5.18 close to the Dirac point, which is around $V_{BG} = -2$ V for this specific cool down cycle we see that the back gate sweep shows a lot of structure. It is not a smooth curve, but rather shows many pronounced and reproducible spikes and peaks.

We now show resistance maps for a region close to the Dirac point highlighted in orange in the back gate sweep in Fig. 5.18. For all different back gate voltages we observe concentric ring features. They range from a single circular region of enhanced resistance such as for $V_{BG} = -3.3$ V to a more complex multi-ring structure such as $V_{BG} = -2.6$ V. Even for the more complex structures all rings are still centered around the same point in space.

When recording a resistance map directly at the Dirac point we observe an even more complex pattern as shown in Fig. 5.19. The resistance map shows various lines, circles and sections of enhanced and suppressed resistance. As a basic feature we still observe a ring-like structure with a radius of about 1 $\mu$m. This oval extends along the symmetry axis of the sample.

Inside this oval we observe a more complex structure. We see different small regions of suppressed resistance. They form single circles similar to the rings observed, when recording the resistance map of quantum dots. One possible explanation for these new regions inside the big circular feature would be the formation of dot-like localized states at different points of the nano-structure. These would lead to single resistance rings. In the center of each of those resistance rings we expect the formation of a localized state.

The smallest features in this resistance map are in the order of magnitude of 50 nm i.e. much smaller than the length scales of the geometric ring.

Figure 5.17: Resistance maps taken at different points within an Aharonov–Bohm cycle. The Aharonov–Bohm oscillation period is about 15 mT.
Figure 5.18: Resistance maps for different back gate voltages. The color bars are adjusted for each resistance map to show optimal contrast. The lower panel shows a $R(V_{BG})$ sweep of the sample. The region in which the resistance maps were taken is highlighted in orange.
Chapter 5. Measurements on a graphene ring

5.4.5 Changing the charge carrier density locally: Side gate voltages

We have seen how the back gate voltage can be used to change the charge carrier density in the sample. By changing the voltage applied to a side gate we will now investigate the change in resistance of the sample, when the charge carrier density is only altered in one arm of the ring structure.

When recording a current map with zero side gate voltage $V_{SGb} = 0$ V we observe that for these specific gate voltage settings the ring is visible as a ring structure of enhanced conductance [Fig. 5.20]. When gradually increasing the side gate voltage on the upper side gate $SGb$ we observe a change in resistance in the region of the ring arm that is close to this side gate. The resistance decreases gradually with gate voltage, until at $V_{SGb} = 0.6$ V we do not observe a ring-
structure in the resistance map any more, but rather a half moon following only
the lower arm of the ring, which is further away from the side gate in question.

At a gate voltage of $V_{SGb} = 0.6\, \text{V}$ the current flows dominantly through
the lower arm of the ring, while the arm close to the ring only contributes minimally
to transport.

If we follow the idea of Hackens et al. that scanning gate maps image the density
of states in the sample, we can interpret the data as follows. When changing the
side gate voltage we expect to change the LDOS in the arm close to the side gate
in question. The arm opposing this side gate is assumed to not change in terms
of its DOS. We can now use the resistance maps to investigate the reach of a side
gate.

At zero side gate voltage the change produced by the tip is large in the ring arm
close to $SG_b$ so also the DOS in this arm should be larger. When increasing the
side gate voltage the LDOS in this arm decreases gradually, until at $V_{SGb} = 0.6\, \text{V}$
next to no charge carriers seem to be present in this arm of the ring. This depletion
starts at the left end of $SG_b$ and then gradually spreads over the complete arm of
the ring.

5.4.6 Z-dependence

Figure 5.21 shows a single line scan across the ring structure (X-direction) versus
the tip-sample separation $d$. The color bar displays the resistance through the
complete ring structure. For large tip-sample separations two broad peaks in the
resistance are visible. These two features stay unchanged by the tip-sample dis-
tance until $d$ becomes smaller than 100 nm. Starting at this tip-sample separation
the features start to shift and change with $d$.

More features of increasingly small feature size appear when moving the tip
closer to the sample.

The closest the tip can possibly be moved to the sample is when the topog-
raphy is scanned, driving the sensor in feedback mode. We can then record the
topography and at the same time the resistance through the structure is recorded.
An example of a single line scan recorded this way is shown in Fig. 5.22. We see
the topography in blue, and the simultaneously recorded resistance trace.

Around $Y = 0\, \mu\text{m}$ the resistance and topography of the sample are strongly
correlated. The resistance trace shows an almost perfect reproduction of the to-
pography. If the same region is scanned with a constant tip-sample separation the
resistance trace becomes flat.

However for higher $Y$ values features in $R(Y)$ and $Z(Y)$ are not correlated.
Features appear in the resistance even though the topography of the structure is
flat. In this region the tip-sample distance is approximately constant. Therefore
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Figure 5.21: Change in the $R(x)$ trace across the sample for different tip-sample separations.

Figure 5.22: Resistance trace (red) recorded while the tip is scanned over the sample in force feedback mode recording the topography (blue) simultaneously.

for this region the features in the resistance are evoked by the tip locally changing the charge carrier density below it.

### 5.4.7 Fine structure

When discussing the measurements on a 2DEG based nano-ring performed by Hackens et. al. we showed the similarity between the raw data resistance maps of those measurements and the measurements performed on a graphene ring shown here. The raw data resistance maps both show a ring-like change in the current, when the tip is scanned across the sample [see Fig. 5.23(a)] This ring feature extends beyond the geometric dimension of the quantum ring.

This most obvious variation of the resistance takes place on a length scale of several hundred nanometers. We will now remove this broad "background". We carried this out using different methods, which essentially all lead to the same result, showing the same fine structure resistance map. Those methods include spline-fits, as well as averaging over the period of the broad background variation.
5.4. Scanning gate measurements

Figure 5.23: (a) Raw data resistance map of the sample. (b) Background resistance obtained from this measurement using the FFT method described in the text. (c) Resistance fringes left, once the background resistance has been removed.

The most efficient method proved to be the same that was used to remove the background variations in the \( R(B) \) traces to obtain the Aharonov Bohm oscillations.

To do so the data is transferred into Fourier space, filtered and subsequently transferred back. This method is described in more detail in the transport section of this chapter.

The result of this method is shown in Fig. 5.23. Panel (a) shows the raw data trace. We observe the ring structure in the resistance extending beyond the geometric ring. Panel (b) displays the broad background in this resistance map. Panel (c) shows the fine structure obtained using this method.

It displays resistance fringes on a length scale of 100 nm. These fringes are very similar in spacing and amplitude to the fringe patterns shown by Hackens in [2]. We can observe these fringes in all resistance maps, recorded after the sample has been kept on the same gate voltages for an extended period of time. They are not distinguishable when the gate voltages have been changed shortly before the measurement was performed and the sample did not have time to settle at those voltage settings.

The origin of those fringes is still an open question at this point and may be subject to further investigation. However, we have to acknowledge the fact that the resistance maps both in their broad background features and their fringe pattern show a striking similarity to the measurements reported previously on the 2DEG ring. We can therefore speculate if a similar interpretation revolving around the LDOS in the sample might also apply for the measurements shown here.
5.4.8 Conclusion

As a conclusion we have shown scanning gate measurements on a graphene nanoring. The most dominant feature in most resistance maps is a ring structure that extends beyond the geometric dimensions of the ring structure. When applying back gate voltages close to the Dirac point the resistance maps becomes increasingly more complex, displaying a multitude of resistance rings on top of the ring features. These smaller features underline the increasing density of localized quantum-dot like states near the Dirac point.

When either side gate voltage is increased we can change from the display of a ring-like structure in the resistance map to a half moon structure, where only one arm of the rings is traced. This confirms how the in-plane side gate changes the charge carrier density only in its very vicinity, while the arm of the ring located across the side gate stays virtually unchanged by the side gate voltage.

We can use the tip to locally change the charge carrier density in the graphene ring. We have shown that we are able to locally enhance and diminish charge carrier density in the ring, thus locally enhancing or diminishing the resistance of the graphene ring locally.

5.5 Simulations

In order to gain insight into the function of the stationary gates as well as gaining an understanding of the electrostatic influence of the tip on the sample, we carried out electrostatic simulations employing the program Comsol Multiphysics.

Comsol Multiphysics uses finite element analysis to solve various, in our case electrostatic problems. It allows for the definition of two- or three-dimensional geometries, along with the desired boundary conditions.

5.5.1 Geometry

Despite the two-dimensional nature of the graphene part of the sample itself, the simulations are carried out in 3d, in order to truthfully incorporate both the back gate as well as the substrate and the AFM tip. The geometry of the sample is known from AFM scans carried out before the measurements. This geometry is then reproduced in Comsol Multiphysics. For simplicity the contacts were omitted, as they are not expected to vitally affect the electric field at the position of the graphene ring itself.

The whole system is considered to be suspended in vacuum, which is a good approximation to the measurement environment which consists of vacuum. The graphene was simulated to be metallic in nature. The surfaces of all metallic parts are on fixed predefined potentials.
5.5. Simulations

The graphene structure is fixed on a 295 nm thick silicon dioxide layer with a dielectric constant of $\varepsilon_r = 4.5$. The lower boundary of this layer is used as a back gate and therefore held at a fixed voltage $V_{BG}$. The simulated area has a diameter of 12 $\mu$m.

The boundary conditions used for this simulations, which are crucial for the results are discussed in the appendix [chap. I.1].

5.5.2 General discussion

In order to understand the influence and interplay of the different gates, which will be discussed in the following sections, we need to thoroughly understand the electric potential for one gate-voltage combination. In order to do so we choose the simulation were zero volt is applied to all gates besides the tip, which is kept at $V_{tip} = 1$ V.

Figure 5.25 (a-d) shows cuts in the xy-plane. Here as well as in the following unless noted otherwise, only a zoom into the graphene-structure is shown although the simulated area extends farther out. Figure 5.25 (d) shows a cut right below the
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Figure 5.25: Cuts in the xy-plane for $V_{\text{tip}} = 1\text{V}$ and zero volt applied to all other gates. Figures (a) through (d) show cuts for different heights $s$ above the sample plane as shown in the schematic side view in (e).

end of the tip as can be seen in (e). We see that the potential has an axial symmetry around the center of the ring, from were on it decays to values below 1 mV within 4 $\mu$m. Even 20 $\mu$m above the sample surface, we find this axial symmetry to be present as seen in Fig. 5.25(c). However when getting close to the surface we can see an influence of the gates evolving. (b) shows the electric potential 2 nm above the sample surface. We see the rotational symmetry being broken by the lateral gates. This effect starts to become faintly visible below 15 nm above the sample surface. In the sample plane [Fig. 5.25(a)] we see the electric potential due to the tip is still about 0.15 V directly below the tip. However 10 nm from the inner ring radius it has already dropped to 4 mV. Outside the ring it only rises to 3 mV in the space between the ring and the lateral gates. Outside the gates the electric potential rises to about 4 mV and then drops to values below 1 mV within 4 $\mu$m.

When looking at a cut in the xz-plane as shown in Fig. 5.26 we observe that around the tip the electrical potential is 1 V and decays with increasing distance to the tip. This decay happens on a smaller scale in the space between sample and tip than far away from the sample in radial direction, as the graphene structures as well as the back gate are fixed on zero volt. The slight bending of the equipotential lines in the vicinity of the end of the simulated area in $+z$ direction is due to the boundary conditions. In order to verify if it has a big impact on the electric potential closet to the graphene structures, another simulation was carried out, where the extent of the simulated area in $+z$ direction was increased by 3 $\mu$m.
5.5. Simulations

Figure 5.26: This figure shows the electric potential along the z-axis of the complete geometry. The voltage applied to the tip is 1 V. The voltage applied to all other gates is zero. The electric potential decays with distance from the tip. It decays faster between the end of the tip and the sample plane, as zero volt is applied to the graphene structure as well as to the back gate.

However, compared to the geometry used, this leads to a change in the electric potential at the position of the ring of less than 1%. Therefore we can conclude that the extent in z-direction shown in Fig. 5.26 is sufficient to produce realistic results. We focus the simulations shown in the following on this area.

5.5.3 Comparison of the different gates

The first important difference between the in-plane gates, and the back gate is that while applying a voltage to the back gate leads to a situation in which the electric potential is axially symmetric with respect to the center of the ring, applying a voltage to either one of the in plane gates brakes this symmetry. When a certain voltage is applied to a side gate, only the potential at the arm close to this gate rises, while the electric potential at the other arm is essentially not influenced. When raising the potential applied to the back gate by the same amount, the electric potential on both arms changes by the same amount.

Figure 5.27 shows the amplitude of the change in electric potential for the different gates. The electric potential at the position of the left ring arm is marked with blue dots, while the electric potential at the position of the right arm is marked with red dots. The average of these two values is shown in green. We show two different cases for the first (lower) case the voltage to all gates but the investigated gate is zero. For the second (upper) case the voltage applied to all gates is changed consecutively.

We observe that the lever arm of a side gate to the adjacent arm is of the same magnitude as the lever arm of the back-gate on both arms. Therefore the effective lever arm of a side gate on the ring is only half as big, as one has to average over both ring arms. It is important to keep in mind, that changing a side gate voltage only influences the part of the structure that is directly adjacent and already decays strongly over the width of the arm of the ring.
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Figure 5.27: Electric potential at the position of the left (blue) and right (red) ring arm, as well as the average electric potential of both of those potions (green) for different gate voltage combinations. For the upper gate one gate after the other is stepped up to three volts, leading to a continually rising electric potential at the discussed points in space. For the lower series each gate is stepped consecutively, while zero volt is applied to all other gates.

When the same voltage is applied to the tip, the potential at the ring arms rises about three times as much as for the other gates discussed above. The lever arm of the tip depends highly on its position relative to the sample and will be evaluated in the next section.

We then evaluate the capacitance of each gate by integrating the surface charge over the according surfaces. The results are shown in Table 5.1. The capacitances of all graphene structures (i.e. the ring $C_{tt}$, and both side gates $C_{SGb,SGb}, C_{SG,SG}$) as well as the capacitance of the tip are in the same order of magnitude, namely around $10^{-16}$ F. The capacitance of the back gate $C_{BG,BG}$ is an order of magnitude larger.

5.5.4 Influence of the tip position

The parameter that is expected to most strongly influence the transport through the structure in our experiment is the position of the tip relative to the sample. In order to understand how the electric potential [Fig. 5.28], the induced charges [Fig. 5.31] and therefore also the capacitances [Table 5.2] change, when the tip is moved with respect to the sample, we carried out a set of simulations, where the
5.5. Simulations

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<th>BG(F)</th>
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</tr>
</tbody>
</table>

Table 5.1: Capacitance matrix of all gates.

The tip was moved away from the center of the ring. The tip is moved in $+y$ direction towards the big side gate first in steps of 100 nm and then in steps of 200 nm. This distance of the tip to the center of the ring in $+y$ direction is called $t$.

Figure 5.28(a) shows the electric potential 1 Å above the sample surface, for the case that $V_{tip} = 1$ V. No voltage is applied to the back gate and the graphene structure is removed. We observe a potential that shows an axial symmetry. When adding the graphene structure as seen in Fig. 5.28(b) where $V_{ring} = V_{SGb} = V_{SGs} = 0$ V, this symmetry is broken by the gates as discussed in more detail before in section 1.2.2. We now increase $t$, breaking the mirror symmetry with respect to the x-axis. The point of highest potential directly below the tip is moved across one arm of the ring, then across the gap between the ring and the side gate and finally onto the side gate.

We investigate the electric potential at the center of the right arm and the left arm as marked by red and blue arrows in Fig. 5.29 (c). The different positions of the tip are marked with green dots. Figure 5.29 (d) shows the result of this simulation series. We show the electric field for each tip position. For the lower arm (red) the tip is moved further and further away. Therefore the electric potential at this position decreases with increasing distance. The electric field on the upper arm (blue) rises, as the tip approaches and reaches it maximum when the tip is directly above it. As the tip is moved further away the electric field decreases. We now compare this development of the curve as a function of the electric field from our simulation with our measurements. Figure 5.29 (a) shows a resistance map of the ring. The resistance of the sample rises when the tip is positioned precisely above the ring and drops, as the tip is moved either to the center of the ring, or away from the ring arms in outward direction. We show different cuts through this measurement in (b). The light blue cut is chosen in a way that it corresponds to the direction in which the tip is stepped in the simulations. These cuts all display a rise in the resistance when the tip is positioned above an arm of the ring. The shape of this resistance is strikingly similar to the development of the electric field taken from our simulations. It also shows a change in the measured quantity on the same length scale as for the simulation.
Figure 5.28: Electric potential in the xy-plane just above the ring structure at $s = 1.1$ nm. (a) shows the electric potential for a geometry, where all graphene parts are removed. Therefore the simulated geometry consists purely of the back gate, covered by 295 nm of SIO$_2$ and the metallic tip, placed at a distance of 100 nm above the SIO$_2$ surface. The tip-voltage is $V_{tip} = 0$ V. The color bar has a range from 0 to 0.8 V. For the other geometries (b) through (h) the scale bar has a range from 0 to 0.4 V as shown next to figure (e). The series (b) to (h) shows a series of geometries where the tip is continuously moved in $+y$ direction. (b) shows the symmetric case where the tip is positioned at the center of the ring, which we call tip position $t = 0$ nm. The electric potential shows the expected cylindrical symmetry. In the following figures the tip position is changed, as noted by the $t$ value noted in the figures, which gives the displacement of the tip from the center of the ring in $+y$ direction.
5.5. Simulations

Figure 5.29: (a) shows a scanning gate image of the sample recorded at 1.5 K. (b) shows a cross section through this scanning gate image along the black line. The cross section is chosen to be perpendicular to the source-drain axis of the sample and therefore stretches from the left gate, over the ring to the right gate. We can see two peaks in resistance which appear, when the tip is positioned just above the ring. (c) The Comsol simulation is carried out in the following way: The tip was moved from the center of the ring in $+y$ direction, increasing the distance $t$ between the tip and the center of the graphene ring. For each position of the tip the electric field directly on the surface of the ring was extracted for both the upper arm at the position of the blue arrow and the lower arm at the position of the red arrow. These electric field values $E$ are shown in (d) in blue for the left arm and red for the right arm versus the corresponding tip position $t$.

An interesting question is now, how we can influence the position of the maximum in electric field and therefore also the electron trajectories in the experiment, by applying a voltage to either side gate. In order to do so we repeated the simulation discussed above for different side gate voltages. Figure 5.30 shows the curves analogously to Fig. 5.29(d) for different side gate voltages. All other gates besides the tip voltage are always set to zero volt.

When a voltage of 1 V is applied to the small side gate, the potential on the upper arm stays the same, as this arm is further away from this gate. Only the potential on the lower arm is lifted by about 0.1 mV. The opposite happens when a voltage is applied to the big side gate. The potential in the center of the ring stays the same, however the potential on the ring arm adjacent to the side gate is lifted. When finding the center of the average potential of both arms, we find, that
Figure 5.30: Electric potential versus tip position $t$ for different side gate voltages. The dots mark the electric potential for the case that $V_{SGb} = V_{SGs} = 0$ V. The squares are used for $V_{SGs} = 1$ V, the diamonds for $V_{SGb} = 1$ V and the asterisks for $V_{SGb} = 2$ V. All blue symbols are used for the lower arm of the ring, the red symbols mark values for the upper ring arm and the green markers the average value of those two. The grey lines are drawn at the maximum of each average electric potential line. They show the peak in the electric potential shows a tendency to be shifted by the side gate voltages.

A voltage of 1 V applied to a side gate leads to a shift of the peak in the electric potential by about 10 nm. Applying a positive voltage to the small side gate shifts the point of maximum potential to higher $t$ values, whereas the application of a positive voltage to the big side gate leads to a shift to lower $t$ values.

The $E_z$, which is proportional to the induced charge is shown in Fig. 5.31. When the tip is still in the center of the ring, the surface charge is spread out symmetrically. As soon as the tip is moved towards the upper arm, the induced charges in the lower arm are considerably reduced. When $t = 200$ nm no charges are induced in the lower arm of the ring any more. The same is true for the upper arm, once the tip reaches the side gate at $t = 600$ nm. As soon as the tip has a distance of more than 300 nm from an arm of the ring, no significant amount of surface charge is induced in this arm any more.

When looking at the capacitances with respect to the tip, we see, that the ring capacitance as well as the capacitance of $SG_s$ decreases, with increasing $t$, while $C_{SGb}$ increases continuously up to a factor of two. The back-gate capacitance shows a slight increase by about 7%, while the tip capacitance drops by about 20% as the tip comes closer to the arm of a ring, and then rises again minimally.
Figure 5.31: $E_Z$ i.e. the surface charge for different tip positions. The position of the tip is marked by the small black circle.
Chapter 5. Measurements on a graphene ring

Table 5.2: Capacitances $C_i$ for different tip positions $t_i$. All capacitances shown here are with respect to the tip, corresponding to the last line in Fig. 5.1.

<table>
<thead>
<tr>
<th>$t$ (nm)</th>
<th>ring(F)</th>
<th>SGs(F)</th>
<th>SGb(F)</th>
<th>BG(F)</th>
<th>tip(F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$-1.42e^{-17}$</td>
<td>$-1.04e^{-17}$</td>
<td>$-1.55e^{-17}$</td>
<td>$-1.52e^{-16}$</td>
<td>$1.97e^{-16}$</td>
</tr>
<tr>
<td>100</td>
<td>$-1.43e^{-17}$</td>
<td>$-1.05e^{-17}$</td>
<td>$-1.86e^{-17}$</td>
<td>$-1.60e^{-16}$</td>
<td>$1.65e^{-16}$</td>
</tr>
<tr>
<td>200</td>
<td>$-1.40e^{-17}$</td>
<td>$-9.69e^{-18}$</td>
<td>$-2.03e^{-17}$</td>
<td>$-1.60e^{-16}$</td>
<td>$1.68e^{-16}$</td>
</tr>
<tr>
<td>300</td>
<td>$-1.31e^{-17}$</td>
<td>$-8.69e^{-18}$</td>
<td>$-2.23e^{-17}$</td>
<td>$-1.60e^{-16}$</td>
<td>$1.67e^{-16}$</td>
</tr>
<tr>
<td>400</td>
<td>$-1.15e^{-17}$</td>
<td>$-8.33e^{-18}$</td>
<td>$-2.46e^{-17}$</td>
<td>$-1.60e^{-16}$</td>
<td>$1.67e^{-16}$</td>
</tr>
<tr>
<td>600</td>
<td>$-8.51e^{-18}$</td>
<td>$-7.33e^{-18}$</td>
<td>$-2.87e^{-17}$</td>
<td>$-1.61e^{-16}$</td>
<td>$1.65e^{-16}$</td>
</tr>
<tr>
<td>800</td>
<td>$-6.72e^{-18}$</td>
<td>$-6.58e^{-18}$</td>
<td>$-2.93e^{-17}$</td>
<td>$-1.63e^{-16}$</td>
<td>$1.65e^{-16}$</td>
</tr>
</tbody>
</table>

5.6 Conclusion and outlook

5.6.1 Summary

In conclusion we have shown that the Aharonov–Bohm effect can be observed in a graphene ring. We have investigated the phase jumps produced in the $R(B)$ resistance traces by the voltages applied to side and back gates. Scanning gate measurements have shown that the charge carrier density can be changed locally by the local tip potential. We can use this method to gain insight into the efficiency of side gates and show that side gates only change the charge carrier density in the ring arm located close to it.

Scanning gate measurements close to the Dirac point show a wealth of additional structure in the resistance maps indicating the existence of localized states close to the Dirac point that smoothen out for high charge carrier densities.

5.6.2 Outlook

Graphene has proven to be a promising material to be used to fabricate nano-structures. It is also a promising candidate to be used in combination with scanning probe methods. Due to its unique position at the substrate surface we are able to locate the tip close to the nano-structure. Additionally in further measurements one might be able to directly access a nano-structure both topologically and electrically, as tunneling into a graphene nano-structure is well within reach.

The first obviously interesting question open to evaluation with scanning probe methods is the formation of dot-like states in graphene constrictions. It would be interesting to evaluate where the localized states form, i.e. if they tend to form inside the constriction or rather on the sides. We should be able to judge, if the amount, extent and position of these localized states can be changed when changing the width, length and shape of a structure is altered. We could investigate if the
amount of localized states changes as indicated by the measurements shown here, with the measurement point with respect to the Dirac point; i.e. the question if localized states become more pronounced and more frequent when close to the Dirac point.
Chapter 6

Measurements on a superconducting SET

6.1 Introduction

In this chapter we will discuss the transport and scanning gate measurements carried out on a superconducting aluminium single electron transistor (SET). The sample has been fabricated with electron beam lithography at the University of Neuchatel by Kasper Suter in the group of Urs Staufer. The sample was produced as part of a project of the ETH in cooperation with the University of Neuchatel where a SET should be fabricated on the tip of an AFM-probe. When such a tip is used for scanning gate measurements one cannot only measure the electronic influence on the sample, but also use the charge state of the SET as a very sensitive charge probe as successfully shown by other groups [110, 111, 27]. As an intermediate step in the process of building such a scanning SET-sensor, SETs first had to be fabricated successfully on a flat surface. This will be discussed in more detail in section 6.2.

Such a SET is investigated here both in terms of transport properties [sec. 6.3] and locally resolved transport using scanning probe methods [sec. 6.4]. Furthermore we use the method of scanning probe microscopy to resolve the spatial variation of parameters such as different capacitances and the charging energy of the SET. The scanning gate measurements are supported with electrostatic simulations using Comsol Multiphysics [sec. 6.5].

Parts of the measurements and simulations that will be discussed in the following have been published in [112].
6.2 Sample and measurement setup

In the following section we shortly introduce the sample on which the measurements were carried out. We give a short overview over the fabrication methods, then describe the sample investigated in this measurement and finally lay out the part of the measurement setup that was particular for these measurements in addition to the AFM-setup, that is described in detail in chapter 4.

6.2.1 Sample fabrication

The sample is fabricated on a silicon substrate, on which a 400 nm thick SiN capping layer is grown via low pressure chemical vapor deposition (LPCVD). Large contact areas are patterned using optical lithography and evaporation. These large contacts consist of a double layer structure. First a 5 nm thick titanium layer is deposited, in order to increase the adhesion of the following layer to the substrate. On top of this layer a 100 nm thick platinum film is deposited.

The SETs themselves are fabricated with electron beam lithography (EBL). Here a two-layer resist system is used to help to develop a good undercut in the resist system which is necessary whenever the method of shadow evaporation is employed to avoid depositing metal onto the resist side walls during the evaporation steps [Fig. 6.1 (a,b)]. For this method material is evaporated onto the sample under an angle allowing for a lateral shift of the structure patterned in the resist mask on the waver. When the same mask is used to evaporate through it under different angles, the same structure will be deposited onto the substrate at different positions. The lateral shift between two evaporation steps is determined by the thickness of the soft resist layer and the angle $\varphi$ under which the evaporation takes place. This is schematically shown for a two-step angle evaporation in Fig. 6.1(c,i-iv). This method allows for several evaporation steps without needing to break the vacuum.

For this structure first a single layer of Aluminum is evaporated, then the sample is exposed to oxygen in order to form the tunnel barriers between the SET and the leads. Finally a second layer of Aluminum is evaporated in order to form the SET-island itself. Each single Aluminum layer has a thickness of 20 nm.

6.2.2 Measured sample

Figure 6.2(a) shows an overview over the complete structure. The dark grey regions are the substrate. The lighter gray lines are the platinum leads. We see that a few thick platinum leads are usually patterned to run in parallel, narrowing down on one end. At the very tip of these leads they are connected to Aluminum leads,
which directly connect to the SET. On their thicker side the leads are eventually connected to bond pads. The Aluminum part is shown in Fig. 6.2(b).

Two layers of Aluminum are distinguishable. The first, wider one results from the first evaporation step. The source $S$ and drain $D$ forming in this evaporation step are marked in the scanning electron microscope picture. The second layer appears as a slightly narrower, brighter layer in the graphic. It forms the SET-island $I$ positioned between the leads. The island has a size of about $60 \times 40 \times 20 \text{ nm}^3$. Oxide layers resulting in tunnel barriers form between source and island as well as between island and drain. These contact areas have a size of about $20 \times 40 \text{ nm}^2$ with an oxide thickness around 1 nm. A gate $G$ consists of a double-
Figure 6.2: Images of a typical sample. (a) Overview over the complete structure. Several SETs are fabricated at once. (b) Zoom in into the red marked area in (a): Topography of the SET, where $S$ is the source, $D$ is the drain, $G$ is the gate, and $I$ is the SET island.

layer from both evaporation steps and is located about 300 nm away from the SET.

The peculiar shape of source, drain, island and gates is chosen first of all in a way that shadow evaporation is possible. Second this was intended to be the first step towards fabricating such an SET on the tip of an AFM cantilever. To do so all contacts, leads and gates had to access the island from one side and spread over only a very small angle. This is described in great detail in the publication by Kaspar Suter et. al. [113].

6.2.3 Measurement setup

The measurements shown here are carried out in a $^3$He cryostat in which the homebuilt AFM setup was constructed, which is described in chapter 4. Standard lock-in techniques are used to record the AC measurements. The AC voltage across the SET is applied via the lock-in Output with a frequency of $f_{\text{meas}} = 33$ Hz. The gate voltage $V_G$ and tip voltage $V_{\text{tip}}$ are applied via Yokogawa voltage sources.

The conductance is recorded via an IV-converter with a feedback resistance of 100 MΩ with a lock-in amplifier for the AC-data and a multimeter for the DC-data [Fig. 6.3].

6.3 Transport measurements

6.3.1 Introduction

The structure described in section 6.2.2 is three to four orders of magnitude larger compared to typical atomic dimensions which are in the of a few Angstroms i.e., 0.1 nm. However, because mean free elastic paths $l_e$ can be up to the order of
microns [for Aluminum films [114] at 4.2 K as well as GaAs (sec. 7.3.1)] we are still able to observe quantum effects.

The essential building block of many nano-structures that will be discussed in the following is a highly resistive barrier. This can for example be a quantum point contact (QPC) operated as a tunnel junction or an oxide layer tunnel junction. Current through a tunnel junction occurs as a result of tunneling processes. Tunneling is a discrete process, i.e. current through a tunnel junction flows in multiples of the charge of electrons. Building upon the idea that current consists of single particles, we can describe current flow through nano-structures as transmission and reflection processes of charge carriers. We can build upon this view of current flow to explain transport through a single quantum dot.

An introduction to electronic transport through quantum dots can be found in various places, for example [115, 116, 118, 119, 120, 121].

6.3.2 Transport through a normal conducting SET

If we connect two QPCs in series, with only a very small volume between them, we form a quantum dot (QD), or in our case a SET. This is essentially a small conducting island, that is electrically connected by tunneling to source and drain

\[ \text{for a more detailed review see [115, 125]} \]
The source has the electrochemical potential $\mu_S$ and the drain $\mu_D$. If no bias voltage $V_{\text{bias}}$ is applied, these two electrochemical potentials will be equal i.e. $\mu_S = \mu_D$. A plunger gate (PG) is capacitively coupled to the quantum dot with a capacitance $C_G$. This gate can be used to shift the quantized energy levels in the quantum dot up and down, essentially loading more or fewer charge carriers onto the dot.

**Conductance resonances**

If we record the current through the SET versus gate voltage we observe that the current through the structure is zero apart from regularly spaced peaks of current, so called conductance resonances as seen in Fig. 6.4(b). Apart from the conductance resonances the current through the structure is zero within the measurement precision. The structure is in a state called Coulomb blockade in the valleys of the suppressed conductance.

For this particular structure we observe that the noise level on resonance is larger than off resonance. During Coulomb blockade the current through the structure is blocked, so the current in this range corresponds to the noise of the setup. The additional noise observed on resonance can most likely attributed to the sample itself.

Charge transport onto the quantum dot occurs by tunneling through the barrier. Therefore the number of charge carriers on the dot has to be a quantized number $N$. The electrostatic energy $E_S$ of an accumulation of charge $eN$ on a capacitance of $C_\Sigma$ is given by $E_S = \frac{e^2N^2}{2C_\Sigma}$. When an additional electron (number $N + 1$) is added onto the quantum dot, its charge $e$ changes the electrostatic potential of the quantum dot by the addition energy $E_A$. The energy needed to add this electron is given by $E_{\text{change}}$ [122]

$$E_{\text{change}} = E_S(N + 1) - E_S(N)$$

$$= \frac{e^2(N + 1)^2}{2C_\Sigma} - \frac{e^2N^2}{2C_\Sigma}$$

$$= \frac{e^2}{C_\Sigma}(N + \frac{1}{2})$$

(6.1)

From this we can determine the addition energy, or charging energy $E_C$. The addition energy is the energy needed to make a transition from an $N$-electron state to an $N + 1$ electron state. It is given by [122]:

$$E_C = E_{\text{change}}(N + 1) - E_{\text{change}}(N)$$

$$= \frac{e^2}{C_\Sigma}$$

(6.2)
6.3. Transport measurements

Figure 6.4: (a) Schematic of a quantum dot. A quantum dot, on which $N$ electrons are assembled is connected to source and drain reservoir. A plunger gate is capacitively coupled to the structure. (b) Current through a quantum dot versus the voltage applied to the plunger gate. The data is recorded on the sample shown in Fig. 6.2 at $T = 300$ mK and $B = 0$ T.

Therefore the next electron $N + 2$ that wants to occupy the quantum dot at the same time as electron $N + 1$ has to have an additional energy of $E_C$ compared to the electron $N + 1$.

In order for Coulomb blockade to manifest itself as described above, $E_C$ has to be larger than the other relevant energy scales. At room temperature the thermal energy $E_{\text{thrm}} = k_B T$ washes out such charging effects. For all measurement data shown and discussed in this chapter $k_B \ll E_C$ holds true.

Furthermore the resistance of the tunnel junctions has to be sufficiently high so we can localize electrons on the island. We can estimate how high their resistance $R_T$ has to be from Heisenberg's uncertainty relation [122]:

$$\Delta E \Delta t \geq h$$

The energy gap associated with a single electron is the addition energy $E_C = e^2 / C$. The characteristic time for charge to load onto a capacitor $C$ is $\Delta t = R_T C$. With those two quantities we can estimate the order of magnitude of the minimum tunnel resistance of a junction needed to confine an electron [122]:

$$R_T \geq \frac{h}{e^2} \approx 25.8 \text{ k}\Omega$$

Capacitance model

In a simple model Coulomb blockade in a metallic island can be considered as an electrostatic problem. The interactions between the electrons and the gates
are defined by their capacitances $C_{ij}$. The charge state of such a system, where a voltage $V_j$ is applied to the gate $j$ [in our case the dot itself, source and drain contacts as well as the plunger gate and in section 6.4 the metallic AFM tip] inducing a charge $Q_i$ on the island can be described using the capacitance matrix $C$ [95]:

$$Q_i = \sum_{j=0}^{n} C_{ij} V_j + Q_i^{(0)}$$  \hspace{1cm} (6.5)

We choose our notation such that index $i, j = 0$ denotes the metallic island. Consequently the self capacitance of the quantum dot is $C_{00} = C_\Sigma = -\sum_{j=0}^{n} C_{0j}$. The indices $i, j = 2$ and 3 correspond to source and drain, and all following indices to the gates.

Charge neutrality demands that [95]

$$\sum_{i=0}^{n} C_{ij} = 0.$$  \hspace{1cm} (6.6)

When zero potential is applied to all gates, the charge on the gates will be $Q_i^{(0)}$. If we solve equation 6.5 for the electrostatic potential applied to the dot we find [95]:

$$V_0 = \frac{1}{C_\Sigma} (Q_0 - Q_0^{(0)} - \sum_{j=1}^{n} C_{0j} V_j)$$  \hspace{1cm} (6.7)

where $Q_0^{(0)}$ is the island when all other voltages are zero.

We are now interested in the electrostatic energy that is needed to add one additional electron to the quantum dot. We find it by integrating from the initial charge on the quantum dot $Q_0^{(0)} - Ne$ to the final charge on the quantum dot $Q_0^{(0)}$. [95]

$$E_s(N) = \int_{Q_0^{(0)} - Ne}^{Q_0^{(0)}} V_0(Q_0) dQ_0$$  \hspace{1cm} (6.8)

$$E_s(N) = \frac{e^2 N^2}{2C_\Sigma} + eN \sum_{j=1}^{n} \frac{C_{0j}}{C_\Sigma} V_j$$  \hspace{1cm} (6.9)

**Constant interaction model**

Apart from the purely classical effect, based on the Coulomb repulsion of same-charged particles, we also have to consider the wave character of electrons. Due to their wave nature, quantized energy states will form when they are confined in a quantum well i.e. for our case: On a SET. They will occupy discrete levels $\epsilon_i$ with a level spacing $\Delta_N = \epsilon(N + 1) - \epsilon(N)$. 
6.3. Transport measurements

The constant interaction model assumes, that the total energy of the quantum with \( N \) electrons \( E(N) \) is given by the electrostatic energy plus the sum of its single-particle energies (confinement energy) [95]

\[
E(N) = \sum_{n=1}^{N} \epsilon_n^{(0)} + E_s(N) \tag{6.10}
\]

The energy the \( N \)th electron needs to possess in order to be added to the quantum dot is the following difference [95]:

\[
\mu_N = E(N+1) - E(N) \tag{6.11}
\]

\[
\mu_N = \sum_{n=1}^{N+1} \epsilon_n^{(0)} + E_s(N + 1) - \sum_{n=1}^{N} \epsilon_n^{(0)} + E_s(N) \tag{6.12}
\]

\[
\mu_N = \epsilon_N^{(0)} + \frac{e^2 N}{C_\Sigma} + e \sum_{j=1}^{n} \frac{C_{0j}}{C_\Sigma} V_j \tag{6.13}
\]

A quantity which is often used to simplify the notation, and is also accessible in the experiment, is the lever arm \( \alpha_i \) of a specific gate \( i \). Conceptually, the lever arm gives the strength of the influence of the gate on the quantum dot. More precisely, the lever arm is the change in energy in the quantum dot when a certain voltage \( V_i \) is applied to gate \( i \). For this model we assume, that the lever arm is constant. [95]

\[
\alpha = -\frac{C_{0i}}{C_\Sigma} \tag{6.14}
\]

Using this simplification we can solve equation 6.13 for the gate voltage \( V_G(N) \) to find the voltage at which a Coulomb peak will occur. There the electrochemical potentials \( \mu_S \approx \mu_D \approx \mu_{QD} \) are approximately equal. The gate voltage distance between two successive Coulomb peaks (peak spacing) is then given by [95]

\[
\Delta V_G(N) = V_G(N) - V_G(N - 1)
= \frac{1}{e\alpha_G} (\epsilon_N - \epsilon_{N-1} + E_C) \tag{6.15}
\]

We now introduce the abbreviation of the single particle spacing: \( \Delta_N = \epsilon_N - \epsilon_{N-1} \)

leading to

\[
\Delta V_G(N) = \frac{1}{e\alpha_G} (\Delta_N + E_C). \tag{6.16}
\]

That means, the spacing between two levels in the quantum dot, the charging energy \( E_C \), only depends on the level spacing and the addition energy.
Chapter 6. Measurements on a superconducting SET

The constant interaction model makes the assumption that lever arms as defined in equation 6.14 are constant for each gate and do not depend on the voltage applied to any gate, i.e. on the state in the dot. It also assumes that the addition energy \( E_C \) is constant for all electron numbers on the dot.

For metallic systems as the Al-SET studied here the Fermi-wavelength is small i.e. in the order of Angstroms. Its confinement energy can be estimated from the energy of an infinite potential well \( \Delta = \frac{\hbar^2 \pi^2}{2m^* d^2} \) (where \( m^* \) is the effective mass and \( d \) the dot diameter) to be below one 1\( \mu \)eV for the sample under investigation. Therefore the spectrum in a metallic SET can be viewed to be continuous. Such a system can be described using only the electrostatic energy.

Energy diagram

From this simple model we see that electrons in the quantum dot will be found on quantized energy levels with the energy \( \mu_{N+k} \), where \( k \) is an integer. The spacing between two levels is given by the charging energy \( E_C \). By changing the gate voltage we can shift the levels of the quantum dot. If we start out with a situation as depicted in Fig. 6.5(a) no current will flow through the quantum dot. The energy level \( \mu_{N-1} \) is filled and no empty levels are located at energies lower than the chemical potential of source or drain. If we now shift the levels using the gate voltage (b) we can reach a situation (c) where the source and drain level are aligned with one \( \mu_N \) level, yielding \( \mu_S \approx \mu_D \approx \mu_N \) as used in the mathematical description. Now an additional electron can be loaded on the level \( \mu_N \), allowing for current to flow through the structure. If the levels in the quantum dot are shifted further up by the in-plane gate, the dot will be again in Coulomb blockade.
6.3. Transport measurements

IV-characteristics

Until now we have focused on the quantized energy levels in SET, which can be shifted by the voltage $V_G$ applied to the in plane gate, bringing energy levels in and out of Coulomb blockade. In addition to shifting the energy levels of the SET relative to $\mu_S, \mu_D$, we can also shift the electrochemical potentials of the source and drain leads $\mu_S, \mu_D$ relative to $\mu_N$. We do so when recording so called IV-traces, where at a fixed gate voltage the bias voltage is varied.

When we choose the gate voltage in such a way, that we are on a resonance peak, the IV-curve displays a linear increase over the measurement range. This can be seen in the pink curve in Fig. 6.6(a). We start at a energy-level configuration [see panel (b)] where $\mu_S = \mu_D = \mu_{N+1}$. When increasing the source drain bias $V_{bias}$ towards point (c), we shift the source and drain levels up and down respectively. This opens the so called bias window marked in yellow. Transport can occur as long as a $\mu_N$ remains within this bias window. As changing $V_{bias}$ does not modify the energy of the levels inside the quantum dot, this will be the case along the complete IV-trace displayed here.

This linear IV-trace, which would occur even at zero temperature is characteristic for an SET due to the continuum of states that can contribute to transport. If we were to look for example at a semiconductor quantum dot we would observe a linear rise, where the slope is given by the temperature and would therefore form into a stepfunction at zero temperature. At a certain voltage the IV-trace becomes constant if no excited states contribute to transport.

Choosing $V_G$ in the Coulomb blockade regime, leads to a nonlinear IV-characteristic as displayed by the orange trace in (a). We now consider the energy schemes of the red trace. No dot level is in resonance with the source or drain level for zero bias (d). When increasing the bias no current flows, until the bias window is big enough that it includes an energy level in the quantum dot. For this case of maximum Coulomb blockade both levels $\mu_N$ and $\mu_{N-1}$ will be included in the bias window at the same bias voltage. Starting at this $V_{bias}$ we will observe a finite current flow [Fig. 6.6 (e)].

The gate voltage range from zero bias to the onset of current for the case of maximum Coulomb blockade corresponds to the charging energy $E_C$, in our case we find $E_C \approx 1 \text{ mV}$.

AC current versus DC current

Most data traces presented in the current chapter are AC-measurements recorded with standard lock-in techniques. The previous measurements have shown the DC current $I_{D,DC}$ through the structure as seen in a sample IV-trace in Fig. 6.7 (a). AC measurements record the differential current through the dot $(dI_{D,AC}/dV)$. (for
Figure 6.6: (a) IV-curves taken at different gate voltages measured at 300 mK. The pink curve is recorded on resonance and the orange curve off resonance. The energy schemes for the points marked (b-e) in (a) are shown on the right. The energy offset between source and drain electrochemical potential is given by the bias voltage: \(-eV_{\text{bias}}\).

simplicity also called \(dI/dV\). As the AC-technique incorporates stronger low-pass filtering than the DC-measurements, the signal to noise level is improved for the data recorded with AC-technique. We show the numerical derivative of the DC-measurement presented in (a) in panel (b). Panel (c) shows the simultaneously recorded AC measurement. The higher quality of the AC data can already be distinguished by naked eye.

Figure 6.7: (a) DC current versus bias voltage. (b) Numerical derivative of this trace. (c) AC measurement recorded simultaneously.
6.3. Transport measurements

Coulomb blockade diamonds

In order to characterize a quantum dot, charge stability measurements (Coulomb blockade diamonds), i.e. \( \frac{dI}{dV}(V_{\text{bias}}, V_G) \) are recorded. We have discussed how resonance peaks occur in \( I(V_G) \) traces shown in Fig. 6.4 while the energy scheme of the sample moves in and out of Coulomb blockade.

The discussed \( I(V_G) \) trace corresponds to the vertical cut marked (1) through the Coulomb blockade diamond shown in Fig. 6.8(a). The cuts marked (2-4) correspond to the IV-traces used to explain the \( V_{\text{bias}} \) dependence of transport through a SET in Fig. 6.6.

The charge on the SET remains unchanged for each region of suppressed current, as marked by the letters \( N, N + 1 \) in Fig. 6.8(b). Along the onset of the finite current, when the system moves out of Coulomb blockade, the electrochemical potential of source or drain is in resonance with the level inside the quantum dot. The corresponding energy schemes along one such line of current onset are shown in Fig. 6.8(c,2-5).

Figure 6.8(c) shows the energy schemes for the different voltage-settings marked with the corresponding numbers in (b). We see that current and its suppression can be understood in terms of the relative positions of source, drain and dot energy levels.

Excited states

Each quantum state in the quantum dot consists not only of the ground state, that has been discussed until now, but rather a series of states. The lowest energy state is the ground state, and a series of excited states appear at higher energies \( \mu_{N+1}^{(m,n)} \).

In general all states that are within the bias window will contribute to transport. One example is the energy level configuration in Fig. 6.8 (c,3), where current flows through the ground state (marked in red) and also through an excited state (orange). Excited states become visible in charge stability diagrams as additional lines of enhanced current, that run in parallel to the borders of the Coulomb diamonds, as schematically marked in Fig. 6.8 by the blue dashed lines. These lines are not visible in this measurement due to thermal smearing.

6.3.3 Transport through a superconducting SET

The sample discussed within this chapter consists of Aluminum, which becomes superconducting below a critical temperature of \( T_C = 1.2 \) K and a critical field \( B_C = 10 \) mT for bulk material. All measurements shown previously in this chapter and the physical explanations [115, 95, 125] given refer to the normal conducting
Figure 6.8: Coulomb blockade diagram for the Aluminum SET at $B = 0.5$ T and $T = 300$ mK. (a) shows the DC raw data, while (b) shows the absolute value of the current, yielding to a more common representation of Coulomb blockade diamonds. (c) Energy diagrams for possible transport processes corresponding to the voltage settings marked in (b).
state. To be able to measure in the normal conducting state a magnetic field of $B = 500 \text{ mT}$ was applied, while measuring at $T = 300 \text{ mK}$. The main focus of the scanning gate measurements [sec. 6.4] will be centered around exploring the superconducting state of the sample via scanning gate microscopy. In order to understand the scanning gate measurements it is important to first thoroughly investigate and understand the pure transport properties of the superconducting SET.

To do so we first give a brief review about superconductivity. We will then show how tunneling occurs through a superconductor-isolator-superconductor (SIS) junction and then discuss the changes we have to make to our picture of transport through a quantum dot, in the case that this quantum dot is not made up from a normal conducting metal, but rather employs superconducting properties.

**Superconductivity**[47, 126]

In a normal conductor that is exposed to an electric field a finite resistance arises from the scattering of free electrons on phonons and lattice defects. As both of these causes cannot be eliminated in a solid body, one has to find a completely different model to explain the phenomenon of zero resistance. For superconducting materials the resistance drops from to a finite value close to zero, when they are cooled down below a critical temperature $T_C$. We can assume, that this sharp drop in resistance coincides with a new phase of the electron gas in the metal, which we call superconductivity.

In a system in the superconducting state electrons form a condensate of Cooper pairs, which are pairs of electrons. Those Cooper pairs exist at the Fermi energy. The density of states of a superconductor as given by the BCS theory consists of bands of states. These bands are separated from the Fermi energy by an energy $\Delta$ as shown in Fig. 6.9(b). This is in contrast to the continues density of state for the normal conducting state [Fig. 6.9(a)] and the discrete energy levels in a SET [Fig. 6.9].

From this we see that the quantity $\Delta$ plays a crucial role. In order to break up a Cooper pair an energy of $2\Delta$ has to be supplied. This process is thought of as creating an excited state of the superconductor. This is in contrast to an excitation in a metal where an electron can be transferred into an excited state by adding any arbitrarily small amount of energy. In a superconductor the Cooper pair has to be broken apart therefore requiring the minimum energy of $2\Delta$.

**SIS junction**[127, 125]

We will now consider how current flows through a tunnel junction, when both sides of this junction are superconducting. We have to differentiate between two
tunneling processes: Either Cooper pairs can tunnel or single electrons ("quasi-particles").

At zero temperature quasiparticle tunneling can only occur, when the bias voltage $V_{bias}$ provides enough energy to create a hole in the source and an electron in the drain [Fig. 6.10(a)]. At $T > 0$ K quasiparticle tunneling can also occur below this threshold, due to thermally excited quasiparticles.

Alternatively Cooper pairs can tunnel through the barrier as a whole. This process is called the Josephson effect. This effect only occurs when the Fermi energies of the superconductor on either side of the barrier are aligned as shown in Fig. 6.10 (b). The Cooper pair sketched into the schematic is slightly misleading as the Cooper pair at the Fermi energy cannot tunnel into any of the other quasiparticle states shown in the energy sketch. The energy coupling of the two superconductors is given by the Josephson energy $E_J$ [123, 124]:

$$E_J = \frac{\hbar I_C}{2e} \approx 0.1 \text{ } \mu\text{eV}$$  \hspace{1cm} (6.17)

This energy is defined by the critical current $I_C$ given by [123, 124]

$$I_C = \frac{\pi \Delta}{2e R_N},$$  \hspace{1cm} (6.18)

where $R_N$ is the normal resistance.

**Coulomb diagram of a superconducting SET**

If we connect two such SIS junctions in series with only a small volume between them we form a SET, just like the one shown in Fig. 6.2. Transport through a
superconducting SET is in principle very similar to transport through a normal conducting SET as described in section 6.3.2 with the additional feature of the superconducting gap $\Delta$.

Figure 6.11 shows the Coulomb diamonds of the normal conducting state (a,b) and the superconducting state (c,d) both in linear and logarithmic scale. We show the differential current through the dot versus gate voltage $V_G$ and source drain bias $V_{bias}$.

The obvious difference is that the Coulomb diamonds for the superconducting state do not close any more at $V_{bias} = 0$ as in the normal conducting state. Rather a rectangular region of suppressed current seems to be inserted at zero source-drain bias. When tracing the current in $V_{bias}$ direction we observe a sharp rise in current from the almost insulating regime to the conducting regime, where transport is dominated by resonant quasi-particle tunneling. This sharp rise in the current will be referred to as current onset (CO) in the following description. Due to finite temperature in our setup, we do not expect to see processes such as Andreev reflections or Josephson quasi-particle processes inside the Coulomb blockaded regions as they were observed in other measurements at lower temperatures [128, 129]. The additional region of suppressed current is marked as "gap" in Fig. 6.11(d). Its extent corresponds to eight times the superconducting gap $\Delta$. The superconducting gap from this measurement is found to be about 0.2 meV. This value is well within the range of other literature values [128, 129, 130].

The relation between $\Delta$ and the region marked "gap" becomes obvious when considering the energy level configuration with schematic sketches as shown in Fig. 6.12. Each sketch consists of the Fermi-level of the source and drain to the left and
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Figure 6.11: (a) Coulomb blockade diamond of the SET in the normal conducting state recorded at a magnetic field $B = 0.5$ T and a temperature $T = 700$ mK. The color bar shows the differential conductance $\log_{10}(dI/dV_{SD})$ with the differential conductance $dI/dV_{SD}$ in units of $e^2/h$ plotted on logarithmic scale, whereas (b) shows the same Coulomb blockade diamond plotted on a linear scale. The color bar shows $dI/dV_{SD}(e^2/h)$. (c) Coulomb blockade diamond of the SET in the superconducting state recorded at $B = 0$ T and $T = 700$ mK, plotted on logarithmic scale. The color bar shows the value of $\log_{10}(dI/dV_{SD})$ with the differential conductance $dI/dV_{SD}$ in units of $e^2/h$. (d) The same Coulomb blockade diamond on linear scale. The color bar shows $dI/dV_{SD}(e^2/h)$.

right as well as the single levels inside the SET shown in the middle and labeled with $\mu_{n+1}, \mu_n, \mu_{n-1}$. Grey shaded intervals represent the superconducting gap $\Delta$. The solid vertical lines represent the tunnel barriers between the island and the leads. At the center of the Coulomb blockade diamond [Fig. 6.12(d)], all levels are detuned. No energy is available to produce quasi-particle tunneling. When moving to higher gate voltages, the levels in the SET move down [Fig. 6.12(c)] until one of them becomes resonant with the source and drain levels [Fig. 6.12(b)]. When moving from this point to finite source drain voltages [Fig. 6.12(g)], the source and drain levels shift antisymmetricaly compared to the SET level. The energy difference between source and drain electrochemical potential for this case is then given by $4\Delta$. This shift becomes large enough to allow quasi-particle tunneling at the current onset. When moving along the current onset to lower $V_G$ [Fig.
6.3. Transport measurements

Figure 6.12: (a) Coulomb diamond of the SET in the superconducting state recorded at \( B = 0 \) T and plotted in logarithmic scale. Columns (b) to (i) show schematics of the energy level structure to illustrate how electric transport takes place.

The source level stays at the same relative position compared to the SET level, while the drain level is shifted further down. At the outermost peak of the current onset [Fig. 6.12(i)] the source and drain levels have been shifted far enough apart, that two levels in the SET can contribute to transport. As shown by [Fig. 6.12(e,f)] processes inside the Coulomb blockaded region are also possible, due to the thermal energy available in the system.

From the period \( p \) of \( dI(V)/dV_{\text{bias}} \) typical values for the gate-island capacitance of

\[
C_G = \frac{e^2}{p} = 0.3 \text{ aF} \quad (6.19)
\]

are determined [Fig. 6.11(a)]. The capacitance of the island \( C_\Sigma \) is derived from the addition energy \( E_A \) using

\[
C_\Sigma = \frac{e^2}{E_A} = 160 \text{ aF} \quad (6.20)
\]

We assume that each of the two junctions has the same capacitance, since the fabrication process and the nominal area are the same. Therefore the junction capacitance can be calculated using

\[
C_{\text{junct}} = \frac{C_\Sigma - C_G}{2} = 80 \text{ aF} \quad (6.21)
\]

This value is consistent with the capacitance one would expect when approximating the junction capacitance using a simple plate capacitor model with areas of 20 × 50 nm, which is calculated to be

\[
C_{\text{calc}} = \frac{\epsilon A}{d} = 77 \text{ aF} \quad (6.22)
\]
Figure 6.13: (a) Decay of the superconducting gap versus magnetic field. (b) Change in the charging energy as a function of magnetic field. The two different symbols mark two data sets, taken in different condensation cycles.

A more in depth evaluation of these capacitances will be shown in section 6.4.7 and simulated in section 6.5.

Figure 6.13 shows the dependence of $E_C$ and $\Delta$ on the magnetic field. The superconducting gap $\Delta$ reduces until the system reaches the normal conducting state between $0.1 - 0.2$ T [Fig. 6.13(a)]. However, the charging energy $E_C$ remains essentially constant for both the superconducting and the normal conducting state as seen in Fig. 6.13(b).

The exact evaluation routine to derive those quantities from the Coulomb blockade measurements will be described later. However, the evaluation routine is slightly changed when reaching a magnetic field of $B = 0.2$ T, as the employed model only works for the superconducting state. Above $B = 0.2$ T the superconducting gap is approximately zero and only the charging energy was deduced from the Coulomb blockade diamonds. As can be seen in Fig. 6.13(b), when changing the evaluation procedure, $E_C$ rises slightly and then decreases indicating, that the superconducting gap might not have reached completely zero below $B = 0.4$ T. However, for all data points below $B = 0.2$ T this analysis shows, that the charging energy is little if not at all influenced by the magnetic field, whereas the superconducting gap clearly is.

### 6.3.4 Energy scales

In the transport measurements described in this chapter so far different energy scales have been discussed. We will give a quick summary of the most important energy scales here.

- The charging energy $E_C \approx 1$ mV [sec. 6.3.2 (IV-characteristics)]
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- The single level particle spacing $\Delta_N < 1 \mu eV$ [sec. 6.3.2 (Constant interaction model)]

- The thermal energy $E_T = k_B T \approx 0.03 \text{ meV}$

- The Josephson energy $E_J = 0.1 \mu eV$ [sec. 6.3.3 (SIS junction)]

- The superconducting gap $\Delta = 0.2 \text{ meV}$ [sec. 6.3.3 (Coulomb diagram of a superconducting SET)]

Due to the metallic nature of the SET we can neglect the single particle spacing as it is small compared to the charging energy $E_C$.

$$E_C \approx E_A = e^2/C_{\Sigma}$$ (6.23)

Therefore the quantities marked in Fig. 6.8(b) can be written as

$$\epsilon p = \frac{1}{\alpha G} \left( \Delta_{N-1} + \frac{e^2}{C_{\Sigma}} \right)_{=E_C}$$

$$\approx \frac{1}{\alpha G} E_C$$

$$\alpha G = \frac{C_G}{C_{\Sigma}}$$

$$= \frac{e^2}{C_G}.$$ (6.24)

From this we can then extract $E_C, C_G$ and $C_{\Sigma}$ as given in the previous section. The lever arm of the gate is

$$\alpha_G = \frac{C_G}{C_{\Sigma}} = 0.002.$$ (6.25)

Excited states contribute to transport through the structure when $eV_{bias} \gg \Delta_N$. In our case $eV_{bias} \approx 1 \text{ meV}$ which is much larger than the single level spacing.

When $E_C \gg E_J$ as it is the case here, Josephson processes are strongly suppressed and transport is dominated by Coulomb blockade. We are therefore able to describe transport through the structure within the energy level schemes shown in Fig. 6.12. As the thermal energy $E_t$ is only one order of magnitude smaller than $\Delta$ thermally activated quasi-particle funneling can take place.
6.4 Scanning gate measurements

In the following section we will discuss the scanning gate measurement performed on the superconducting SET. After a short introduction to the general features visible in scanning gate measurements on a quantum dot [sec. 6.4.1] we will discuss how superconductivity can be observed in scanning gate measurements [sec. 6.4.2]. We will focus on the effects of different voltages [sec. 6.4.3, sec. 6.4.4] and evaluate the tip potential, as well as our possibilities to shape it according to our needs [sec. 6.4.5, sec. 6.4.6].

Finally we will investigate the spatial variation of parameters accessible in an experiment. We will start by evaluating parameters whose spatial dependence can be explained in terms of electrostatic considerations [sec. 6.4.7] as well as more complex quantities [sec. 6.4.8].

6.4.1 Introduction: Scanning gate measurements on a SET

When performing scanning gate microscopy on a quantum dot we use the tip to shift the energy levels inside the quantum dot in and out of Coulomb blockade. We know that we can shift a quantum dot in and out of Coulomb blockade by sweeping the voltage applied to an in plane gate. The gate voltage shifts the quantized levels inside the dot, leading to resonance peaks in the current of $I(V_G)$ traces as shown in Fig.6.14(a).

Similarly we can modify the strength of the influence of the tip either by changing the voltage applied to it, or by changing its distance from the quantum dot. If we look at the current versus the tip position $d$ we observe the same conductance peaks as in an $I(V_G)$ plot [see Fig.6.14(c)]. The path the tip takes for such a measurement is schematically sketched in panel (b).

If the tip is perfectly rotationally symmetric it has the same influence on the dot, whenever the tip-QD-separation is the same. If we keep the tip-surface separation in z-direction constant we can move the tip in a circle around the quantum dot while staying at the same conductance resonance. This is shown in Fig.6.14(b) by the thin red line, which forms a circle with radius $R$ around the SET. Along this circle current will flow through the structure, as the energy levels inside the quantum dot are not shifted with respect to source and drain levels. Therefore we will be on resonance along the whole circle. Such a circle of enhanced conductance is called Coulomb ring.

In the case of an ideal geometry with perfect symmetry a Coulomb ring is in fact a perfect circle. If the tip potential is not perfectly symmetric any more, the Coulomb ring will resemble the symmetry of this potential. If we record a current map $I(x, y)$ we observe several Coulomb circles which essentially show the contour lines of the tip induced potential.
6.4. Scanning gate measurements

Figure 6.14: (a) Current through the SET versus the gate voltage. (b) Schematic sketch of the scanning gate measurement shown in (c) and described in the text. A tip is scanned along the black dashed line across a SET. If we record the current through the quantum dot while scanning the tip along the line we obtain the current versus tip position $d$ as shown in panel (c). In such a perfect geometry the tip potential exerted on the quantum dot is constant along a circle centered around the quantum dot as symbolized by the red circle with radius $R$.

6.4.2 Superconducting vs. normal conducting

Figure 6.15(a) shows the current map $dI/dV(e^2/h)$ of the SET in the normal conducting state. We observe slightly elliptically shaped rings of enhanced currents, the Coulomb rings. All of those Coulomb rings are concentric around the same position, which corresponds to the position of the quantum dot in real space.

The tip potential used to record those measurements is very complex [see sec. 6.4.5]. In particular the tip potential consists of multiple local maxima. It is therefore not possible to derive the electrical position of the quantum dot from those measurements. However, we know the relative orientation of the sample with respect to the scan frame. To envision this relative orientation the outline of source, drain and SET are drawn into Fig. 6.15 as white lines.

We discussed previously that the charge stability diagram of the normal conducting state [Fig. 6.16(a)] and the charge stability diagram of the superconducting state [Fig. 6.16(b)] differ by the fact, that while the first displays closed Coulomb diamonds of suppressed current the second displays an added region of suppressed current due to the superconducting gap. Every current map is taken at a constant bias voltage $V_{bias}$. If we compare cross sections through both charge stability diagrams taken at a finite bias voltage $V_{bias} = 1.5$ mV we observe the fol-
Figure 6.15: Current maps for different conducting states of the SET. Measurement settings: $V_T = 0 \text{ V}$, $V_G = 0 \text{ V}$ and $V_{bias} = 1.5 \text{ mV}$. (a) shows a current map for the normal conducting state taken at $T = 2 \text{ K}$ and $B = 0 \text{ T}$, while (b) is taken at $T = 700 \text{ mK}$ and $B = 1 \text{ T}$. (c) shows a current map of the superconducting state at $T = 700 \text{ mK}$ and $B = 0 \text{ T}$. Insets show cross sections at the position of the gray dotted lines. White lines in (a) mark the approximate position of source, drain, and SET of the structure.

Following: $dI_{\text{Normal Conducting}}/dV(V_G)$ displays a single peak of enhanced current once every period $p$. We always stay outside the reach of complete Coulomb blockade. For the superconducting state, $dI_{\text{Super Conducting}}/dV(V_G)$ we move in and out of Coulomb blockade. In particular we pass the CO twice within each period $p$ for the superconducting state.

If we record a current map, where the sample is continuously cooled down from 2 K to 0.9 K we will witness the phase transition from the normal conducting state to the superconducting state. Such a current map is shown in Fig. 6.16(c).

Figure 6.15(c) shows a scanning gate measurement of the same scan frame as in Fig. 6.15(a) but in the situation where the SET is in the superconducting state. Since the two measurements have been carried out one right after another and no charge rearrangements were observed, we can assume the SET to be in the same now superconducting state until the clearly visible charge rearrangement in the last quarter of the measurement in the superconducting state takes place. The signature of superconductivity at this source drain voltage is the splitting of the resonance rings. Every single ring splits up into two when the SET is scanned in the superconducting state. This finding is consistent with the features observed in the Coulomb diamonds. Looking at the dashed line in Fig. 6.16(b) we can see that for each period $p$ in the superconducting state we expect to cross the line of current onset twice. When looking at the insets in Fig. 6.15(c) we see a difference in the peak height as compared to the normal conducting scanning gate measurement.
6.4. Scanning gate measurements

Figure 6.16: Charge stability diagrams for the normal conducting state (a) and the superconducting state (b). The grey dashed line at $V_{\text{bias}} = 1.5$ mV represents the bias voltage used to record the current map shown in (c). For this current map the temperature is continuously decreased so we can witness the phase transition around the middle of the current map.

This is consistent with the fact that for the normal conducting state we do not reach the regime of total Coulomb blockade for the chosen source drain voltage, whereas for the superconducting state we cross the line of the current onset. The observations made in these scanning gate measurements are consistent with the Coulomb diamonds discussed before.

Since the number of features as well as their spacing is the same for the superconducting and normal conducting state, we know, that we can controllably load single electrons onto the SET by scanning the tip.

The effect of temperature on the scanning gate measurement is investigated in Fig. 6.15(a,c). We observe that the double rings found in the superconducting state merge into single rings when the temperature is sufficiently high to suppress superconductivity. The position of the rings remains unchanged by this transition. This is in good accordance with the measurement at finite magnetic field. However, the resonances in the normal conducting state are broader due to the higher temperature.

6.4.3 Dependence on $V_{\text{bias}}$

The influence of the source drain voltage on the scanning gate measurements is investigated in another series of measurements, parts of which are shown in Fig. 6.17. Row (a) shows the scanning gate measurement taken at a distance $z = 50$ nm of the tip above the surface, $T = 700$ mK, $V_G = 0$ V, $B = 0$ T and the indicated source drain voltage. We observe, that the shape of the rings always stays the same.
However, the shape and number of peaks shown in the cross sections in row (b) is altered by $V_{\text{bias}}$. The measurement taken at $V_{\text{bias}} = 0$ V shows small single peaks with approximately equal spacing. The measurement taken at $V_{\text{bias}} = 0.4$ mV shows double peaks. The other measurements show again different peak structures. In order to make the peak structures more easily visible a cross section through those scanning gate images at the position of the gray dashed line is shown in row (b).

When looking at the Coulomb blockade diamond in Fig. 6.16(b) one can see that for cross-sections at different $V_{\text{bias}}$-values different peak structures have to be expected. In order to make these peak structures more easily tangible the cross sections through the Coulomb diamonds are shown in Fig. 6.17(c). We see that for different $V_{\text{bias}}$ the differential conductances vs. $V_{G}$ traces show different peak structures.

Because our SFM tip acts as a movable gate, changing the voltage on the tip should have a similar effect as changing the voltage applied to the in plane gate. Therefore the cross-section peak structures of the Coulomb diamonds should also be observable in the corresponding scanning gate images [Fig. 6.17 row (a)].

When comparing row (b) to row (c) in Fig. 6.17 we notice a good correlation between those two sets of measurements, with respect to peak shape, height and spacing. Notably the measurements for $V_{\text{bias}} = 0$ mV, 0.6 mV, 1.0 mV show an almost perfect consistency. The correlation seems not quite as good for the measurements with $V_{\text{bias}} = 0.4$ mV, 1.6 mV. For $V_{\text{bias}} = 0.4$ mV the scanning gate measurement shows just barely split double peaks, whereas the cross-section taken from the Coulomb diamond shows double peaks but with minima between them that have all the same depth. For $V_{\text{bias}} = 1.6$ mV the scanning gate image shows the beginning of a peak splitting into double peaks, where we would expect clean single peaks from the Coulomb diamond measurement.

### 6.4.4 Influence of the in-plane sidegate

By applying a voltage to the in-plane gate, we can shift the levels inside the quantum dot. Applying an increasingly positive voltage to the in-plane gate will lead the energy levels inside the dot to shift down, until another electron can be loaded onto the dot.

This shift can be observed in Fig. 6.18. The first scanframe at $V_{G} = 0$ V displays egg-shaped Coulomb rings. The smallest Coulomb ring has an average radius of about 200 nm. We now apply an increasingly positive voltage to the in plane gate. This leads to a shift of the energy levels inside the quantum dot, which becomes apparent by the shift of the Coulomb rings in real space. However, the position of the Coulomb rings in real space changes, while their shape which is determined by the tip potential stays the same. Therefore when a complete cycle
6.4. Scanning gate measurements

Figure 6.17: Current maps for different $V_{bias}$ values, as indicated on the top of each column. The top row (a) shows the scanning gate image. The color bars show the differential conductance in $dI/dV_{bias}$ in $10^{-4}e^2/h$ for the first two columns, in units of $10^{-2}e^2/h$ for the second and third, and in units of $10^{-1}e^2/h$ for the fifth column. The scalebar has a length of 200 nm. Row (b) shows a cross section through this image at the position of the grey dashed line in the scanning gate image. The y-axis shows the differential conductance in $dI/dV_{bias}$ in the same units as the color bar of the scanning gate plot above. Row (c) shows the corresponding cut through the Coulomb diamond. The y-axis shows $dI/dV_{SD}$ in units of $10^{-3}e^2/h$ for the first, $10^{-2}e^2/h$ for the second, $10^{-1}e^2/h$ for the third and $e^2/h$ for the last two columns. Overall a good agreement between the images in row (b) and (c) is reached. However, some of them differ in some finestructure. The tip sample distance is around 50 nm, $T = 700$ mK, $V_G = 0$ V and $B = 0$ T.

of loading another electron onto the quantum dot is completed, the shape of the Coulomb rings is still the same as seen for the case where $V_G = 0.55$ V even though an additional electron is located on the quantum dot.

6.4.5 Altering the tip: Tip flushes

The non-circular shape of the Coulomb rings is presumably due to a tip shape that is not completely rotationally symmetric around the z-axis. This can probably be attributed to the topography scanning carried out before the scanning gate measurement, which led to slight deformations, or the attachment of unwanted particles to the tip. Both effects have been observed before in scanning gate measurements [39].
Figure 6.18: Series of current maps recorded with a tip sample distance of 50 nm, $T = 700$ mK, $V_{\text{bias}} = 1.5$ mV and $B = 0$ T. Between each two current maps the gate voltage $V_G$ is stepped by 50 mV as noted by the superscript $V_G$ on each single scanframe. We observe that the position of the Coulomb rings can be shifted continuously from one scan frame to the next, until the voltage has been shifted enough to make a whole new Coulomb ring, i.e. an additional electron could be loaded onto the quantum dot. The size of the scan frame is $1 \mu m \times 1 \mu m$.

Figure 6.19: Current maps of the sample recorded in the normal conducting state. The current maps are scaled, so $1 \mu m$ corresponds to the same length in the graphic. Between each two maps an in situ field treatment has taken place.
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In the interpretation of the measurement data we have to assume a certain tip shape. Ideally this tip has a cylindrical electric potential. Any derivation from such a potential will make the current maps more difficult to interpret.

The topographical shape of a freshly etched tip is usually very simple and should fulfill this requirement. In order to position the tip over the sample however a rather extensive amount of topography scanning has to be carried out at low temperatures. This scanning makes the tip more blunt and possibly rough leading to multi-tips. Additionally the tip may pick up particles that have accumulated on the sample surface. Such particles can contribute a potential to the tip, which might be a tip-voltage independent contribution to the complete tip potential as discussed in detail by A. Gildemeister [1, 18].

Different attempts can be made to remove such particles or multi-tips and form a single conical tip-potential. One revolves around making a tunnel current flow from the tip to a metallic part of the sample. Such a high current of values up to 100 nA has shown to alter the tip [18]. A second approach employing mechanical vibrations will be shown in section 7.4.2. And lastly we can apply an AC voltage to the tip, which is suspended in vacuum. No current flows from the tip to the sample. We now change the AC-voltage applied to the tip via a lock-in from zero to 10 V. This voltage is applied at different AC-frequencies starting from a few Hz up to a few kHz. When this procedure is carried out at 1.31 kHz, the resonance frequency of the tuning fork is observed to change by 7 mHz indicating that a change of the sensor has taken place. One possible explanation is that a small particle clinging to the tip might have fallen off.

The subsequently recorded current map [Fig. 6.19(b)] and therefore the tip potential had changed considerably from the tip potential observed before this treatment [Fig. 6.19(a)]. The tip can be altered again as seen when comparing (b) and (c).

6.4.6 The tip-induced potential

Lever arm variation in space

In order to infer the influence of the in-plane gate on the dot, we have used charge stability diagrams [sec. 6.3]. From those we could calculate a lever arm $\alpha_G = 0.002$ of the gate, which is a measure for the strength of the influence of the gate.

A central question in any scanning gate measurement is the kind and magnitude of the tip induced potential. In the measurement shown in this chapter we are able to change the tip induced potential by applying a voltage to it, so it could be used as a conventional gate. Figure 6.20(a) shows a charge stability diagram measured while employing the tip as a gate. The tip position was kept constant at the scan frame center throughout the measurement. The charge stability diagram displays
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Figure 6.20: (a) Charge stability diagram recorded while using the tip as a stationary gate with a constant position. (b) Relative lever arm measurements $I(V_G, V_{Tip})$ shows how strongly single Coulomb peaks are affected by the voltage applied to the in plane gate and the tip. From this we can deduce a relative lever arm $\alpha_{Tip}/\alpha_G$. The quantity $d$ notes the lateral distance from the tip to the center of the SET. (c) $\alpha_{Tip}/\alpha_G$ vs. $d$ as determined from the measurements shown in (b). The green trace is one possible fit of a Lorentzian to these data points. However, due to the amount and spacing of the data points this fit is not reliable.

the same transport features as the charge stability diagrams recorded using the in plane gate. The only noticeable difference is the scale of the $V_{Tip}$ axis. From these Coulomb diamonds we can derive the lever arm of the tip for this particular tip-potion in real space to be $\alpha_{Tip} = 0.001$.

Different from the in-plane gate, the tip can change its position relative to the SET. The lever arm of the in-plane gate is given by its geometric position relative to the sample. Therefore it will stay the same throughout all measurements, apparat from screening effects due to the tip position. It can only scarcely be diminished, when shielded from the dot with the tip [sec. 6.4.7]. The lever arm of the tip however can be changed immensely when the tip is moved in real space.
Figure 6.20(b) shows $I(V_G, V_{Tip})$ for different lateral displacements $d$ of the tip from the center of the SET. Those plots show how strongly Coulomb peaks can be shifted in gate voltage by the voltage applied to the tip or to the in-plane gate. The relative lever arm $\alpha_{Tip}/\alpha_G$ is given by the slope of the shift of the Coulomb peaks in these plots. While the $z$-position is kept constant the tip is moved radially away from the quantum dot from each measurement to the next. When the tip is located directly above the quantum dot the tip has an influence that is half as strong as the influence of the tip yielding $\alpha_{Tip}/\alpha_G = 1.8$. When the tip is radially moved away from the quantum dot by 1 $\mu$m its influence drops to one fourth of the gate influence $\alpha_{Tip}/\alpha_G = 4.4$. By 2 $\mu$m the relative lever arm is 7.9 and by $d = 3$ $\mu$m the lever arm is 10.8 times smaller than the gate lever arm. This gives us an almost perfectly linear increase of the relative lever arm over distance $\alpha_{Tip}/\alpha_G(d)$. As the tip potential is expected to be Lorentz-shaped, we show a corresponding fit in Fig. 6.20(c). However, as there are only few data points, spaced over a limited range of lateral positions, this fit is not reliable.

**Imaging the tip-induced potential**

In order to learn more about the tip-induced potential, a measurement of the differential conductance as a function of $V_{Tip}$ is performed while moving the tip along a line across the SET. The approximate position of this trace is shown as a dashed line in Fig. 6.2(b). The height $z$ of the tip above the surface was constant at 200 nm. At each of the 1950 steps along this line a trace of the differential conductance as a function of the tip voltage was recorded. Figure 6.21(a) shows the result of this measurement.

We see how the positions of the Coulomb peaks change, when altering the tip position relative to the SET. We observe a concave and a convex part, which means that our tip potential consists of an attractive and a repulsive component. However, there is no tip voltage value in the investigated regime, where the tip does not induce any charge on the SET at all positions along the line. This is in agreement with the observations made previously on semiconductor quantum dots [21]. One would expect to observe a least invasive tip voltage close to the value one estimates from the work function differences. Contrary to this expectation, we do not find a least invasive voltage here, even though it would be expected at an offset voltage of 1.4 V because of the work function difference between PtIr ($\phi_{Pt} \approx 5.6$ eV) and Al ($\phi_{Al} \approx 4.3$ eV).

The shapes of the single resonance curves are almost identical as seen in the overlay shown in Fig. 6.21(b). This indicates, that for a tip position between zero and 2 $\mu$m the shape of the tip-induced potential is independent of the voltage applied to the tip. A change in $V_{Tip}$ only changes the offset of this potential, not its shape over the length scale of this measurement. This is the manifestation of
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Figure 6.21: (a): Conductance of the SET vs. the voltage applied to the tip along one line over the SET. From this we can see that the tip potential has an attractive and a repulsive part. The tip sample distance is around 50 nm, $T = 700$ mK, $V_{SD} = 0$ V and $B = 0$ T. Panel (b) shows an overlay of the single resonances. Highlighting their similar shape.

Analyzing the tip induced potential

It has been shown in [1] that for scanning gate experiments on a semiconductor quantum dot fine structure could be observed that depended on which quantum state the dot was kept. The exact interpretation, however, remained an open question. When performing scanning gate measurements on a metallic SET, we do not expect to see features connected to individual single particle wave functions since in metals the single-particle energy scale is negligible.

In order to shine further light on this question two scanning gate measurements are performed where all settings were kept exactly the same, except that the gate voltage was changed by one Coulomb diamond period $p$ to reach a different charge state. Figure 6.22 shows two such scanning gate measurements in (a) and (b). Figure 6.22(c) shows the difference of those two measurements. We see that some of the rings do overlap as expected. However, in the center the measurement in (a) shows rings with a smaller radius than the measurement in (b), whereas at the
6.4. Scanning gate measurements

outermost rings the opposite seems to be the case. Therefore a shift in gate voltage by \( p \) does not lead to exactly the same scanning gate image. This is contrary to our expectations, that the Coulomb rings should overlap completely when carrying out two scanning gate measurements for two different charge states in a metallic SET. We therefore have to conclude that the period depends on the position of the tip, i.e., \( p = p(x, y) \). Where \( p \) determines both the charging energy \( E_C \) and the lever arm \( \alpha_G \) both of which are quantities that depend on the tip position as we will discuss in more detail.

Mathematically speaking the condition for two subsequent Coulomb rings is:

\[
\mu_N = -e \cdot \alpha_G(\vec{r}_{\text{tip}}) V_G + \alpha_{\text{tip}}(\vec{r}_{\text{tip}}) V_{\text{tip}} = \mu_{S/D} \quad (6.26)
\]

\[
\mu_{N+1} = -e \cdot \alpha_G(\vec{r}_{\text{tip}}) V_G' + \alpha_{\text{tip}}(\vec{r}_{\text{tip}}) V_{\text{tip}} + \frac{e^2}{C_{\Sigma}(\vec{r}_{\text{tip}})} = \mu_{S/D} \quad (6.27)
\]

where \( \alpha_i(\vec{r}_{\text{tip}}) \) are the lever arms of gate \( i \) which depend on the tip position \( \vec{r}_{\text{tip}} \). The position \( \vec{r}_{\text{tip},0} \) is the tip position that was used to determine the charge stability diagram from which the difference in gate voltage was determined, that was used for the scanning gate measurements. For this position we can write

\[
0 = -e \cdot \alpha_G(\vec{r}_{\text{tip},0}) V_G + \alpha_{\text{tip}}(\vec{r}_{\text{tip},0}) V_{\text{tip}} + e \cdot \alpha_G(\vec{r}_{\text{tip},0}) V_G' - \alpha_{\text{tip}}(\vec{r}_{\text{tip},0}) V_{\text{tip}} - \frac{e^2}{C_{\Sigma}(\vec{r}_{\text{tip},0})} \quad (6.28)
\]

\[
0 = -e \cdot \alpha_G(\vec{r}_{\text{tip},0}) V_G + e \cdot \alpha_G(\vec{r}_{\text{tip},0}) V_G' - \frac{e^2}{C_{\Sigma}(\vec{r}_{\text{tip},0})} \quad (6.29)
\]

\[
V_G' = V_G - \frac{e}{C_{\Sigma}(\vec{r}_{\text{tip},0})} \alpha_G(\vec{r}_{\text{tip},0}) \quad (6.30)
\]

using \( C_G(\vec{r}_{\text{tip},0}) = C_{\Sigma}(\vec{r}_{\text{tip},0}) \alpha_G(\vec{r}_{\text{tip},0}) \) we can write:

\[
V_G' = V_G - \frac{e}{C_G(\vec{r}_{\text{tip},0})} \quad (6.31)
\]

Using this expression for \( V_G' \) we can write:

\[
-e \cdot \alpha_G(\vec{r}_{\text{tip}}) V_G + \alpha_{\text{tip}}(\vec{r}_{\text{tip}}) V_{\text{tip}} = -e \cdot \alpha_G(\vec{r}_{\text{tip}}) [V_G - \frac{e}{C_G(\vec{r}_{\text{tip},0})}] + \alpha_{\text{tip}}(\vec{r}_{\text{tip}}) V_{\text{tip}} + \frac{e^2}{C_{\Sigma}(\vec{r}_{\text{tip}})} \quad (6.32)
\]

\[
0 = \frac{1}{C_G(\vec{r}_{\text{tip},0})} + \frac{e \alpha_G(\vec{r}_{\text{tip}})}{C_G(\vec{r}_{\text{tip}})} \quad (6.33)
\]

This equation is only true for certain cases of \( \vec{r}_{\text{tip}} \). In simple words that means, as \( C_G \) depends on the tip position the Coulomb peaks will show a different period
Figure 6.22: Current maps for two different $V_G$ voltages [plotted blue (a) and red (b) for better comparison. The color bar is the same for both cases, shown here only for (b) for simplicity]. (c) shows the difference of those two measurements. One can see that although only one electron was added onto the SET the scanning gate measurement shows a different spacing of the Coulomb rings. The tip-sample distance is around 50 nm, $T = 700$ mK, $V_{bias} = 1.5$ mV and $B = 0$ T. The scanned area is $2 \mu m \times 2 \mu m$.

$p = \frac{e}{2V_G}$ in $V_G$. As can be seen in the measurement in Fig. 6.22, for most tip positions we observe a shift in the lateral position of the Coulomb rings, when comparing two quantum states.

As a conclusion, we have shown, that the influence the tip exerts on the SET during scanning gate measurements is similar to the influence of a planar gate. Although we have a complex tip potential, we are able to control the occupation of the SET island on the level of single electrons. Furthermore the measurement shown in Fig. 6.22 reveals, that the period $p$ is influenced differently by the position of the scanning tip for different quantum states on the SET.

### 6.4.7 Electrostatics of the system: Dependence of the capacitances on the tip position

We will now investigate the spatial variation of various parameters accessible in this experimental setup. We will start by evaluating the parameters whose spatial variation can be explained within the framework of an electrostatic problem, as will be shown in section 6.5. In particular we will discuss the capacitances present in our setup.
6.4. Scanning gate measurements

The capacitances that can be extracted from our measurements are the capacitance between the tip and the island $C_{\text{tip}}$, the gate and the island $C_G$, the overall capacitance $C_\Sigma$ of the island and the capacitances between the junctions and the island $C_{\text{junct}}$.

**Data acquisition and parameter extraction**

All physical quantities that will be discussed can be extracted from charge stability diagrams. When recording two Coulomb diamond measurements, one where the tip is used as the gate, and one where we use the in plane gate we can extract the capacitances $C_{\text{tip}}, C_G, C_\Sigma, C_{\text{junct}}$ as well as the respective lever arms $\alpha_{\text{tip}}$ and $\alpha_G$. Furthermore we can extract the charging energy $E_C$ and the superconducting gap $\Delta$ for a specific set of external parameters for which the two charge stability diagrams are recorded.

However, recording a charge stability diagram is very time consuming. Because of time constraints the Coulomb blockade diamonds are not recorded with the resolution shown in Fig. 6.16, but rather reconstructed from $V_{\text{bias}}$ sweeps at as few $V_G$ voltages as needed to extract the desired quantities [Fig. 6.23(b)]. We call this kind of measurement a compressed Coulomb diamond measurement (CCD). As the Coulomb diamond is symmetric with bias voltage, only the part for $V_{\text{bias}} \geq 0$ is recorded. Additionally the number of $dI/dV(V_G)$ sweeps is reduced.

The charging energy and the superconducting gap can be read from a Coulomb diamond, when the innermost and outermost position in $V_{SD}$ direction of the CO are known. These positions are marked as inner cross section IC and outer cross section OC in Fig. 6.23(b). To determine those positions the COs are fitted linearly from four $dI/dV_{SD}$ versus $V_{SD}$ sweeps (using two sweeps to determine the rising slope and two to determine the declining slope): We determine the maximum of each $dI/dV(V_G)$ sweep by hand (those values are marked with red dots labeled 1 through 4) and then fit them via linear regression which leads to the dark blue lines. These lines are then shifted in $V_G$ by one period $p$ (light blue lines). We assume this period to be constant over a $V_G$ range of several Coulomb blockade diamonds. From these fits we can directly extract IC and OC.

The superconducting gap is given by:

$$\Delta = 0.25 \cdot IC$$  \hspace{1cm} (6.34)

and is in the order of 0.2 meV. The charging energy is given by

$$E_C = OC - IC$$  \hspace{1cm} (6.35)

and is found to be around 1 meV.

The period $p(x, y)$ of the Coulomb blockade diamonds is determined from a $dI/dV$ versus $V_G$ sweep. To determine it precisely a high resolution $dI/dV(V_G)$
sweep is recorded [Fig. 6.23(d)]. At the given bias voltage two current maxima occur per period \( p \). We determine the period by taking the average value of \( p \) given by the difference \( p_1 = |V_{ga} - V_{gc}| \), \( p_2 = |V_{gb} - V_{gc}| \) and so on.

From this period we can also extract the gate capacitance \( C_G \):

\[
C_G = \frac{|e|}{p} \tag{6.36}
\]

The gate capacitance is in the order of 0.3 aF.

The period \( p_{tip} \) of a charge stability diagram that would have been recorded using the tip as a gate can be extracted from a high resolution \( dI/dV(V_{Tip}) \) sweep shown in Fig. 6.23(c). Again two maxima occur within each period. \( C_{Tip} \) is derived from the average of all possible pairs such as \( p_{Tip,i} = |V_{ta} - V_{tb}| \). The capacitance of the tip \( C_{Tip} \) is then given by:

\[
C_{Tip} = \frac{e}{p_{Tip}} \tag{6.37}
\]

A typical value for \( V_{Tip} \) is in the order of 0.3 aF

The lever arms can be extracted from the capacitances:

\[
\alpha_G = \frac{C_G}{C_{\Sigma}} \tag{6.38}
\]

\[
\alpha_{Tip} = \frac{C_{Tip}}{C_{\Sigma}} \tag{6.39}
\]

Typical values of the lever arms range between 1 and 2.

**Error estimation**

We shortly discuss how precise the extracted parameters are, starting with \( C_{tip} \).

The exact position of the peaks in the \( dI/dV(V_{Tip}) \) peaks i.e. the values \( V_{ta} \) and \( V_{tb} \) are determined by eye, as noise or a limited amount of data point within each trace can lead to faux maxima when taking \( V_{tip} \) at which \( dI/dV \) is at a local maximum. By repeatedly choosing the maxima in a single line trace by hand, and comparing the \( V_{tip} \) values obtained, we found the error to be well below \( \delta V_{ta} = \delta V_{tb} = 0.01 \) V.

Using standard error approximation, we can find the upper limit of the error \( \delta q \) of any function \( q(x, ..., z) \), where each variable is given within an error \( \delta x, ..., \delta z \):

\[
\delta q = \sqrt{\left| \frac{\partial q}{\partial x} \right|^2 \delta x^2 + ... + \left| \frac{\partial q}{\partial z} \right|^2 \delta z^2} \tag{6.40}
\]
6.4. Scanning gate measurements

Figure 6.23: (a) Cross section of two different Coulomb diamonds. Each set of lines of the same color belongs to one Coulomb diamond. Arrows mark the position of the current onset in each trace. (b) through (d) sample set of compressed Coulomb diamond data for one set of external parameters. (b) shows the time-optimized charge stability diagrams with the lines fitted for the parameter extraction. (c) and (d) Show single current vs. gate voltage sweeps used to extract the capacitances of the corresponding gates. (e) shows cross sections of different Coulomb diamonds recorded at different magnetic fields. We see that at higher fields the peak shapes broaden, making the extracted parameters less precise.
In this way we can estimate the upper bound of the error of the tip capacitance \( \delta C_{\text{tip}} \) to be around 4%. As \( C_{\text{tip}} \) is evaluated from no less than two data sets, the upper bound for its error is 2%.

In the same way we can estimate the upper bound for the error of \( C_G \) to be 1%. A similar error evaluation of all other quantities leads to an upper bound for the error of all extracted parameters below 5%. The exact value of these upper bounds will be indicated in the plots by error bars.

**Data sets**

Knowing that the period \( p \) and with it the charging energy depends on the AFM tip position, the next open question is, how other parameters extracted from the Coulomb diamonds depend on the tip position or voltage. For the first measurement (grid measurement) a grid of 36 tip positions is chosen that covers the scan frame shown in Fig. 6.15. The positions are marked with black squares in Fig. 6.24. At each of these 36 positions a compressed Coulomb diamond is recorded and \( \Delta, E_C, C_G \) and \( C_T \) are extracted. In order to reach a good comparability of these data points all 36 points are measured in the same condensation cycle of the fridge.

In order to cover a larger lateral distance the same measurement is carried out for a number of positions distributed along a stretch of 15 \( \mu \)m across the SET (line measurement, see green squares in Fig. 6.24).

Since we also want to investigate the influence of the tip voltage on \( \Delta, E_C, C_G \) and \( C_T \), Coulomb blockade diamond measurements are carried out for a constant tip position but changing tip voltages ranging from 0 V to 7 V. Two sets of measurements were carried out in different condensation cycles, referred to as \( V_{\text{tip}} \)-data 1 and 2.

Finally the influence of the magnetic field \( B \) on all parameters was evaluated. Two sets of measurements were carried out in different condensation cycles, referred to as \( V_B \)-data 1 and 2. For these measurements, evaluating the influence of the tip voltage and the magnetic field, the tip was kept at a constant position marked with the blue spot in Fig. 6.23.

**Inference to be made about the tip potential**

Figure 6.25(b) shows \( C_{\text{tip}} \) and \( C_G \) as derived from single Coulomb blockade diamond measurements performed as the tip is positioned on various points along one line across the SET. We observe a smoothly shaped single peak dependence for both capacitances, with a width of several micro-meters. The change in \( C_G \) is due to the fact, that the gate gets shielded from the SET by the presence of the tip. Therefore the change in \( C_G \) is much smaller than the change in \( C_{\text{tip}} \) and
Figure 6.24: Current map of the complete scan frame. The black crosses and green squares mark the positions at which compressed Coulomb diamond measurements were taken. The tip was positioned at the xy-position marked by the blue circle to evaluate the magnetic field and tip voltage dependence.

Inverse in sign to it. However, the change in $C_{tip}$ by 0.4 aF is small compared to the capacitance of the island $C_\Sigma$, which is around 161 aF.

It is interesting to note, that even though the tip potential looked quite complex at the point of time of this measurement [see current map in Fig. 6.24], the capacitances only show a very smooth Gaussian dependence which extends on a larger length scale than the fine features in the current map. This confirms, that the tip potential is the sum of two independent parts, of which only one depends on $V_{tip}$. 

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The gate capacitances: $C_{\text{tip}}$ and $C_G$

Capacitances in general are highly geometry dependent quantities. It seems therefore reasonable, that those quantities are strongly influenced, when moving the AFM tip in space.

When looking at the tip capacitance $C_{\text{tip}}$ we expect it to increase strongly, when the distance between the tip and the SET is decreased. We can make a crude approximation of the size of the tip, when assuming that the apex of the tip is nearly flat. We then can treat the SET-tip system as a plate capacitor. As we are working in vacuum we can write

$$C_{\text{Tip,PC}} = \frac{\epsilon_0 A}{d}$$

where $A$ is the area of the plates and $d$ the distance between them. When the tip is positioned above the SET and the tip capacitance is measured, we get a value of $C_{\text{Tip}} = 0.8 \text{ aF}$. As the vertical distance $d$ between the tip and the sample is about 80 nm we can now estimate the area of the capacitor plates $A \approx 4.5 \text{ fm}^2$. The size of the SET is about the same (5 fm$^2$). We can therefore conclude, that the minimum size of our tip is the same size of our sample.

From the plate capacitor model we can see that the capacitance of the tip will change dramatically, when the tip-sample distance is changed. A factor of two in tip-sample distance, is at the same time expected to produce a change in the capacitance by a factor of two.

The gate capacitance $C_G$ on the other hand is expected to change little when the position of the tip is altered. Since the gates position is fixed we only observe a change in $C_G$ when the gate is shielded by the presence of the tip. Therefore the change in $C_G$ is expected to be much smaller than the change in $C_{\text{tip}}$ and antiproportional to it.

When looking at the results from the grid measurement in Fig. 6.25(a,b) we see the expected behavior. $C_{\text{tip}}$ rises significantly by a factor of 10 when the tip is moved 2 $\mu$m closer to the SET, while $C_G$ decreases by 10%.

The capacitances do not reflect the complex, multi-maxima tip potential, but rather image a gaussian overall shape of the tip becomes visible in the line measurement in Fig. 6.25(c). For both data sets which evaluate the influence of the tip-position on $C_{\text{tip}}$ and $C_G$ we observe the expected anticorrelation between these two quantities, which is a result of electrical shielding.

No dependence of either capacitance on $V_{\text{tip}}$ or the magnetic field $B$ is observed.
6.4. Scanning gate measurements

Figure 6.25: (a,b) Color plots of the tip and gate capacitances extracted from the grid measurements. (c) $C_{\text{Tip}}$ and $C_G$ for the line measurement. We show both capacitances versus the position of the tip.

**Other capacitances in the geometry: $C_\Sigma$ and $C_{\text{junct}}$**

The capacitance of the island $C_\Sigma$ is derived from the charging energy. If we assume each of the two junctions has the same capacitance, since the fabrication process and the area are the same we find

$$C_{\text{junct}} = \frac{C_\Sigma - C_G}{2} = 80 \text{ aF}$$  \hspace{1cm} (6.42)

We see that $C_\Sigma$ changes by about 30% in our measurements when changing the tip position. When we consider the geometry of our system [Fig. 6.26] we expect to be able to influence the part of the electric field lines that does not lie inside our structure. In a very basic picture that means, that we can only strongly influence the electric field lines, that we can intersect with our tip.

As the tunnel barriers lie within our sample, well-covered by a layer of Aluminum (black regions in Fig. 6.26) we do not expect to change the capacitance of these tunnel barriers by changing the tip position. Therefore the only part of the capacitances that can be strongly influenced by the tip would be $C_G$. We discussed the influence of the tip on the gate capacitance before and found it to be negligible compared to the magnitudes of change observed in $C_\Sigma$. It remains therefore an open question, that, even though is seems highly unlikely from the sample design, this change is nevertheless related to a change in the tunnel barriers, or if it is evoked by another effect.
As $C_\Sigma$ is closely related to the charging energy $E_C$ we will now focus our evaluation on this quantity.

### 6.4.8 Tip dependence of the charging energy and the superconducting gap

Figure 6.27(a) shows the most striking result of the compressed Coulomb diamond measurements. We observe an anticorrelation between $E_C$ and $\Delta$ for different tip positions. For different tip positions and voltages neither $E_C$ nor $\Delta$ stay constant, they rather vary by about 15% and 20% respectively. For situations where the charging energy is large (small) the superconducting gap is small (large). However, because of the complexity of the tip potential it is impossible to make out a certain spatial trend in this variation as seen in Fig. 6.27(b,c).

We compare the cross sections taken at the minimum and the maximum $V_{SD}$ value of the current onset of two Coulomb diamonds, taken at different positions. In Fig. 6.23(a), the maximum value of $V_{bias}$ of the current onset stays almost the same, the minimum value, however, shifts.

In order to verify, that this correlation does not arise from a systematic error, such as the fact that the current onset does not run in a completely straight line as a function of gate voltage, or from noise overlaying the peak structure of the diamonds, we determined the minimum and the maximum possible values for $E_C$
6.5 Simulations

In order to gain further knowledge about the electrostatics of the complete system (sample and AFM-tip), and therefore learn more about the behavior of the capacitances, simulations are carried out with the software tool COMSOL. The

Figure 6.27: \( \Delta \) and \( E_C \) extracted from compressed Coulomb diamond measurements taken at different lateral positions of the tip and different voltages applied to it. The distance from the tip to the surface was kept constant.

and \( \Delta \) that could be extracted when combining the peak positions of the current onset for all cross sections. We found the values of \( E_C \) and \( \Delta \) to vary by less than 4%. However, the change in these values as seen in Fig. 6.27 is more than a factor of 3 larger.

The correlation between \( E_C \) and \( \Delta \) is highly unexpected. Since the SET is metallic, screening should occur on the surface within the first Ångstroms of the sample. Furthermore the junctions are buried and should not be subjected to the influence of the tip. The superconducting properties of the system have their origin in the volume of the system and not on the surface, and should therefore not be influenced by the SFM tip.

In order to verify the validity of our evaluation we also extracted \( E_C(B) \) and \( \Delta(B) \) which have been discussed before in the context of Fig. 6.13. There we observe that the superconducting gap decays with rising magnetic field as expected. Due to the fact, that all features broaden with increasing magnetic field, as shown for the example of the crossections displayed in Fig. 6.23(e), the precision with which both \( \Delta \) and \( E_C \) can be estimated decrease with increasing magnetic field.
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geometry is modeled following the SEM-pictures of the structure [Fig. 6.2]. The tip is approximated as a cone, i.e. having a round cross section closed to the sample with a diameter of 40 nm, which increases when moving further away from the structure. The tip is positioned at different positions with respect to the island and the electric field is calculated. Furthermore the capacitances of the tip, gate source and drain are determined with respect to the island.

Figure 6.28 shows the electric field in the sample plane for different tip positions and voltages. In Fig. 6.28(a) the tip is located directly above the SET as marked by the cross hair. When \( V_{tip} = 1 \) V the electric field at the SET is barely enhanced. When applying higher voltages up to \( V_{tip} = 7 \) V we see a strong increase of the electric field at the SET, which laterally quickly decays. Therefore already the electric field directly at the leads is increased by less than half the amount. When the tip is laterally moved towards the left in -x direction [Fig. 6.28(b,c)] this electric field distribution shifts. When the tip is placed directly above the lead (b) the electric field is still strongest enhanced at the position of the SET, as it is higher than the leads on its edges. When the tip is moved a few micro meter in -x direction the shift in the electric field by \( V_{tip} \) is almost uniform.

When the tip is displaced in -y direction [Fig. 6.28(d)] the position of the maximum induced electric field shifts. Now the electric field is not mainly enhanced at the position of the SET any more, but rather at the position of the gates. This way the tip also acts as an electric shield, from the gate to the SET.

Figure 6.29 shows the magnitude of the electric field in color, whereas the sample outline is marked by the black lines. The apex of the tip is depicted by the black circle and a white cross in the first row. Figure 6.29 (a) shows the electric field when the tip is positioned precisely above the island. The sample geometry is approximately mirror symmetric left to right. This is reflected by the electric field distribution, which is also symmetric. When moving the tip away from this symmetry axis towards the left above one lead, the electric field ceases to be symmetric. Rather a high electric field is now found in the vicinity of the tip around the left lead. The electric field can be influenced by the position [Fig. 6.29(a,b)] and voltage (not shown) applied to the tip. Moving the tip over source or drain leads to an asymmetric electric field with respect to the sample symmetry axis.

Although the shape of the tip in our simulations has been simplified compared to the experimental setup, the capacitances obtained from the simulation show very good agreement with the experimental values. All numbers are within a factor of 2 of the measured values and their relative magnitudes match the experimental data [see table in Fig. 6.29(c)]. The relative change of those values obtained from the simulations also agrees with the experimental data. Experimentally the gate capacitance \( C_G \) decreases by about 50% when placing the tip between the gate and
the island as can be seen in Fig. 6.25, a change that is also seen in the simulations. The tip-island capacitance $C_{\text{Tip}}$ can also be seen to decrease when the tip is moved away from the island in good agreement with experimental data. The capacitances of the junctions ($C_S, C_D$), however, remain unchanged when the tip is placed in different positions.

6.6 Conclusion

6.6.1 Summary

We have presented first scanning gate measurements on a superconducting SET. Excellent correlation between scanning gate measurements and charge stability measurements is observed. Small differences remain. This leads to the conclusion, that the in-plane gate has a slightly different effect on the SET than the movable out-of-plane gate: The period of the Coulomb blockade peaks can be influenced by the lateral position of the AFM-tip. The tip potential consists of two parts, only one of which depends on the voltage applied to it. This supports the findings in [1].

We furthermore observe an anticorrelation between the gate capacitance and the tip capacitance, which can be explained in terms of the screening the tip exerts...
Figure 6.29: (a,b) Magnitude of the electric field in $10^8\text{V/m}$ as obtained from simulations carried out with COMSOL. For (a) the tip is positioned above the SET as indicated by the black circle, whereas for (b) the tip is above the left lead (source). (c) Capacitances with respect to the island obtained from the simulation. The abbreviation S stands for source, D for the right lead (drain), I for island, G for gate and T for the tip. Row A shows the values corresponding to the situations shown in plot (a), whereas row B corresponds to figure (b). The numbers in row C are values obtained from the experimental data.

and is supported by the electrostatic simulations carried out. The electrostatic simulations carried out with Comsol show excellent agreement with the experimental capacitances extracted.

The exact origin of the observed and surprising anticorrelation between the charging energy and the superconducting gap cannot be explained by the electric field configuration of the setup which has been simulated considering the geometry of the tip and the device.

### 6.6.2 Outlook

The most striking open question arising from these measurements is the connection between the superconducting gap and the charging energy. To evaluate this anticorrelation further measurements should be carried out. First of all a detailed analysis of the behavior of the charging energy in the normal conducting state would be beneficial. Ideally a dependence of $E_C$ on the tip position and voltage in the normal conducting state could be explained in terms of classical electrostatics, and supported by further electrostatic simulations.

Second a more in depth investigation of $\Delta(E_C)$ for the superconducting state will be helpful. The interpretation should become easier when dealing with a cleaner single-tip potential, aiming to evaluate the spatial dependence $\Delta(x,y)$. A simpler dependence could be obtained by varying the $z$-position of the tip over the sample. Therefore measurements of $\Delta(z)$ and $E_C(z)$ should help to shine light into the origin of anticorrelation between $E_C$ and $\Delta$. 
Different sample geometries should be evaluated. The tunnel barriers could be made of different sizes and be arranged in a way that they are either well covered maybe with an additional capping layer to minimize the influence of the tip, or expose them as well as possible to the tip. Different island sizes and thicknesses should also provide a change in both the charging energy and $E_C$, helping to decipher their dependencies.

Another extremely interesting experiment would be the local destruction of superconductivity. To do so a magnetic tip would have to be used. When an external magnetic field only shortly below $B_C$ is applied, the additional field produced by the tip should be large enough to locally create normal conducting regions in the sample. It would be fascinating to see the influence of a small normal conducting island, that can be moved through a superconducting nano-structure.
Chapter 7

Measurements on a GaAs double quantum dot

7.1 Introduction

Even though quite a few publications have appeared on scanning probe measurements on single quantum dots [38, 132, 42, 133, 36], no results of scanning gate measurements on an intentionally formed double quantum dot have been published. So far scanning gate microscopy has been used to locate the position of unintentionally formed dots in nano-wires [38]. There Blesynski et al. presented, that the positions of dots can be localized by finding the center of a set of Coulomb rings [sec. 2.3.4]. Using this method on an intentionally formed double quantum dot we expect to be able to localize the position of the quantum dot inside the structure and investigate its shift with different in-plane gate voltages and imaging the coupling behavior.

In the following chapter we will discuss measurements carried out on a double quantum dot defined in a GaAs/AlGaAs heterostructure via local anodic oxidation. The great advantage of scanning gate microscopy over conventional transport measurements lies in its spatial resolution. And the even bigger opportunity when applying this method to a double dot is, that with the dot-dot separation we can measure a relative parameter. This is desirable in this case as such a relative parameter is a much more robust quantity then absolute values. This is opposed to absolute parameters such as the position of a certain Coulomb ring in a scan frame, which can shift when external parameters are changed.

We will start with a short description of the sample and the measurement setup in section 7.2. In order to be able to fully understand the scanning gate measurements that will be the main focus of this chapter we will first show transport measurement data of the sample and use this data to give an introduction
to transport through a double dot [sec. 7.3]. The main focus will then be on the scanning gate measurements of the double dot presented in section 7.4. To further understand the role of the tip which is ideally assumed to be a weak perturbation in the scanning gate measurements, we performed simulations of the electrostatic situation [sec. 7.5].

7.2 Sample and measurement setup

In the following section we will shortly introduce the sample on which the measurements were carried out and describe the measurement setup that was used in addition to the low-temperature AFM setup described in detail in chapter 4.

7.2.1 Sample fabrication

The measurements presented in the following chapter are performed on a nanostructure, that is based on a Ga[Al]As heterostructure. As GaAs and AlAs have different band gaps ($E_g = 1.52$ eV for GaAs vs. $2.23$ eV for AlAs), different potential shapes can be fabricated by tuning the concentration $x$ in the alloy $Al_xGa_{1-x}As$. Figure 7.1 (a) schematically shows the different layers used to fabricate the heterostructure used in this work.

On top of the GaAs-AlGaAs heterojunction a doping layer ($\delta$-doping) is grown and finally a GaAs layer is added as a capping layer on top. The Si Atoms in the $\delta$-doping layer replacing a Ga atom have one loosely bound electron, which the majority of them will donate, leaving ionized donors behind. The conduction band edge of GaAs is lower than that of AlGaAs. Therefore the electrons will diffuse into the GaAs, building up an electric field. This electric field will build up until it is big enough to stop this process, preventing further electrons to diffuse towards the GaAs. However the electrons in the 2DEG thus forming at the heterointerface are free to move in the xy-plane.

The width of this potential is comparable to the Fermi wavelength of electrons (typically around 30 nm), leading to a quantization of the energy in z-direction.

A detailed review about heterostructures can be found in [134].

The heterostructure used for this work was grown by the group of Prof. Werner Wegscheider at the University of Regensburg.

7.2.2 Measured sample

A double quantum dot has been fabricated in an AlGaAs-GaAs heterostructure, which contains a two-dimensional electron gas with density $n_s = 5 \cdot 10^{15} \text{ m}^{-2}$ and mobility $\mu = 40 \text{ m}^2/\text{Vs}$ at 4.2 K. The 2DEG is buried 34 nm below the surface.
7.2. Sample and measurement setup

The double quantum dot structure has been defined by local anodic oxidation [135] at room temperature. With this technique oxide lines are defined by applying a voltage to an AFM tip, that is positioned close to the sample. Below those lines the underlying 2DEG is locally depleted. The width of the oxide lines is typically smaller than the region below, where the 2DEG is depleted. The depletion length tends to be in the order of 50 nm. Thus oxide lines allow to electrically separate adjacent areas in a 2DEG. Fig. 7.2(b,c,d) show SFM scans of the investigated structure carried out at different temperatures. The insulating oxide lines are seen as bright protrusions. The geometric positions at which the quantum dots are expected to form are marked with two red dots and labeled "dot 1" and "dot 2" in Fig. 7.2(d). The double quantum dot is electrically contacted via source (S) and drain (D). The tunnel barriers between source and the first dot as well as between the second dot and drain can be tuned via the gates close to the tunnel barriers marked STG and DTG respectively. The energy levels inside each dot can be shifted by applying a voltage to the areas marked PG1 and PG2. The areas PG1 and PG2 can be used as quantum point contacts, where each can be tuned via an adjacent gate (QPCG1 and QPCG2). A central gate (CG) acts on both double dots at once, tuning the interdot coupling.

Figure 7.1: (a) The layers of the GaAlAs heterostructure. The position where the 2DEG is formed is shown in red. (b) The resulting band structure.
Figure 7.2: Images of the sample with different magnifications. Figure (a) shows a light microscope image taken of the complete sample, including the leads, up to the bond pads. Figure (b) and (c) show images taken at room temperature with a commercial AFM. Figure (d) shows an AFM scan of the sample carried out in situ at 4 K. The labels on the different parts of the structure will be used in the following to specify the gates, dots etc.

The larger structures observable in Fig. 7.2(a) are produced with optical lithography and etching. The 2DEG is contacted via Ohmic contacts realized by diffusing GeAu into the heterostructure.

### 7.2.3 Measurement setup

The measurements shown in the following are all carried out at a cryostat temperature of 300 mK. Standard lockIn techniques are used for the AC measurements. The DC-current is recorded with a multimeter. The voltages used as source-drain voltages, and gate voltages are supplied by Yokogawa power supplies or by the DC-Outputs of a lockIn. The measurement setup is schematically shown in Fig. 7.3.

### 7.3 Transport measurements

In the following section we will discuss conventional transport measurements recorded on the sample. In order to learn from the scanning gate measurements it is essential to first characterize the sample with transport measurements and understand the dominant features.
7.3. Transport measurements

Figure 7.3: Schematic measurement setup used to measure transport through the double quantum dot and one of the quantum point contacts.
7.3.1 Introduction to double quantum dots

A double quantum dot (DQD) as discussed in the following consists of two quantum dots in series, that are weakly coupled to source and drain. Similar to transport through single quantum dots [see chap. 6.3], the transport through a double quantum dot is predominantly governed by Coulomb blockade, meaning that in order to place another electron on a quantum dot, one needs to provide enough energy to overcome the Coulomb repulsion between the new electron and the electrons already present on the quantum dot. The effect is called Coulomb blockade, because Coulomb repulsion dominates the addition energy.

Similar to single quantum dots, that are often compared to artificial atoms, double quantum dots can be compared to artificial molecules. In such a model the type of the molecular bond (i.e. ionic or covalent) can be translated into the interdot coupling.

The electrostatics of a double quantum dot can be modeled with capacitors and resistors as shown in Fig. 7.4. The source (S) is connected via a tunnel barrier to the first dot. This tunnel barrier is represented by a resistor and a capacitor. The first dot is then tunnel coupled to the second dot, which in turn is tunnel coupled to the drain (D). On each dot we find a specific electron number (N,M). The plunger gates PG$_1$ and PG$_2$ are capacitively coupled to the dots. The center gate (CG) and the gates labeled STG and DTG are capacitively coupled and used to tune the tunnel barriers.

Starting from the basic equation $Q = CV$ relating the capacitance matrix $C$ to the charge $Q$ via the voltage $V$ we can regard the investigated double quantum dot system as a combination of a number of $F$ conductors. There exists a capacitance...
between each pair of conductors and additionally from each single conductor. This results in a capacitance matrix [136]:

\[ Q_j = \sum_{k=0}^{F} c_{jk}(V_j - V_k). \] (7.1)

For the charge \( Q_i \) with \( i = 1, 2 \) for the two dots we now write [136]:

\[ Q_1 = C_S(V_1 - V_S) + C_{PG1}(V_1 - V_{PG1}) + C_m(V_1 - V_2) \] (7.2)
\[ Q_2 = C_D(V_2 - V_D) + C_{PG2}(V_2 - V_{PG2}) + C_m(V_2 - V_1). \] (7.3)

Slightly rearranged those equations read

\[ Q_1 + C_S V_S + C_{PG1} V_{PG1} = (C_S + C_{PG1} + C_m) V_1 - C_m V_2, \] (7.4)
\[ Q_2 + C_D V_D + C_{PG2} V_{PG2} = (C_D + C_{PG2} + C_m) V_2 - C_m V_1, \] (7.5)

and can now be written in a matrix:

\[
\begin{pmatrix}
Q_1 + C_S V_S + C_{PG1} V_{PG1} \\
Q_2 + C_D V_D + C_{PG2} V_{PG2}
\end{pmatrix} =
\begin{pmatrix}
C_S + C_{PG1} + C_m & 0 \\
0 & C_D + C_{PG2} + C_m
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2
\end{pmatrix}
\] (7.6)

Solving this equation for \((V_1, V_2)\) we get:

\[
\begin{pmatrix}
V_1 \\
V_2
\end{pmatrix} = \frac{1}{C_1 C_2 - C_m^2}
\begin{pmatrix}
C_2 & C_m \\
C_m & C_1
\end{pmatrix}
\begin{pmatrix}
Q_1 + C_S V_S + C_{PG1} V_{PG1} \\
Q_2 + C_D V_D + C_{PG2} V_{PG2}
\end{pmatrix}
\] (7.7)

In general we can calculate the electrostatic energy \( U \) stored in a number of single capacitors using the capacitance matrix [136]:

\[ U = \frac{1}{2} V \cdot Q = \frac{1}{2} V \cdot C \cdot V = \frac{1}{2} Q \dot{C}^{-1} Q. \] (7.8)

For the case \( V_S = V_D = 0 \) with \( Q_i = -N_i |\epsilon| \) using the capacitance matrix calculated above we find

\[ U(N_1, N_2) = \frac{1}{2} N_1^2 E_{C1} + \frac{1}{2} N_2 E_{C2} + N_1 N_2 E_{Cm} + f(V_{PG1}, V_{PG2}) \] (7.9)

where the charging energy \( E_{C1} \) of each dot is given by [136]

\[ E_{C1} = \frac{e^2}{C_1} \frac{1}{1 - \frac{C_m^2}{C_1 C_2}}. \] (7.10)
The coupling energy \( E_{Cm} \) is the energy change in one dot, when an electron is added to the other dot and can be calculated as follows [136]:

\[
E_{Cm} = \frac{e^2}{C_m} \frac{1}{\frac{C_1 C_2}{C_m} - 1}.
\]

(7.11)

The capacitance of one dot \( C_1 \) or \( C_2 \) is the sum off all capacitances surrounding it [136]:

\[
C_1 = C_L + C_{PG1} + C_m
\]

(7.12)

\[
C_2 = C_R + C_{PG2} + C_m.
\]

(7.13)

### 7.3.2 Influence of the plunger gates: Charge stability diagrams

**Charge stability diagrams: An introduction**

The method of choice to characterize a double quantum dot is, analogously to characterizing a single quantum dot, the charge stability diagram. The features visible in a charge stability diagram of a single quantum dot have been discussed in chapter 6.3. The charge stability diagram of a double quantum dot can be obtained from a colorscale plot of the current through the structure in the plane of both plunger gate voltages \( V_{PG1} \) and \( V_{PG2} \). The features appearing in such a measurement will be discussed in the following.

If we are to construct a charge stability diagram for two uncoupled dots (while neglecting the capacitances of \( PG_1 \) to dot 2 and from \( PG_2 \) to dot 1), the result would resemble the schematic shown in 7.5(a). If the dots are completely decoupled (i.e. \( C_m = 0 \)) the charge on dot one does not influence the electronic state of dot two and vice versa. Additionally plunger gate one only affects dot one, while the plunger gate two only affects dot two. This would lead to a charge stability diagram [Fig. 7.5(a)] where all lines run parallel to gate-voltage axes. Along each line the electrochemical potential of one dot is aligned with the electrochemical potential of source and drain. No current flows in the case of uncoupled dots. Current flows whenever the levels of both quantum dots are aligned with the electrochemical potential of source and drain, i.e. whenever the lines corresponding to a resonance in dot 1 and dot 2 cross. These points are marked with green dots.

However, usually the plunger gates do not only act on the dot positioned directly next to it. In reality there is a cross capacitance where plunger gate 1 also acts on dot 2 and vice versa for dot 1. This leads to finite slopes of the lines where the dot electrochemical potentials are aligned with the electrochemical potential of the leads. This slope is determined by the relative lever arms \( \alpha_i \) of the gates [Fig. 7.5(b)].
7.3. Transport measurements

If the capacitive coupling between the dots is increased a pattern as shown in Fig. 7.5(c) evolves. This is the well known hexagon pattern, that is usually associated with double dots. The intersections of the lines arising from dot one (blue) and dot two (red) now evolve into pairs of triple points which are marked with a green and a black dot. The separation between the two triple points (grey) is determined by the interdot coupling energy $E_{Cm}$ as defined above in equation 7.11. This separation is due to Coulomb repulsion between the dots. If an additional electron is loaded onto dot 1 an increased energy is needed to add an additional electron onto dot 2. At the triple point marked in black the transport process takes place as follows: An electron coming from the source tunnels onto dot one, then moves on to dot two and finally tunnels out through the drain. This corresponds to the following sequence: $(N, M) \rightarrow (N + 1, M) \rightarrow (N, M + 1) \rightarrow (N, M)$. For the second (green) triple point we can imagine the following sequence: $(N + 1, M + 1) \rightarrow (N + 1, M) \rightarrow (N, M + 1) \rightarrow (N + 1, M + 1)$. This is the equivalent of a hole moving from drain to dot 2, then to dot 1 and then to the source.

The dimensions of a honeycomb cell are related to the capacitances as follows [see Fig. 7.6(b) for comparison] [136]:

\[
\Delta V_{PGi} = \frac{|e|}{C_{PGi}}, \quad (7.14)
\]

\[
\Delta V_{PGi}^{m} = \frac{|e|C_{m}}{C_{PGi}C_{j}} = \Delta V_{PGi} \frac{C_{m}}{C_{PGj}}. \quad (7.15)
\]

We will now consider the case of zero source drain bias $V_{bias} = 0$ in more detail. Figure 7.7(k) shows a charge stability diagram around a pair of triple points. The number of electrons on both dots are marked by $(N, M)$ in blue for the electrons on dot 1 and red for the electrons on dot 2. In each region of a single color (yellow, pink, brown or green) the number of electrons on both dots is fixed and no current flows through the structure, i.e. the double dot is in Coulomb blockade. Along the cotunneling line separating the region $(0,0)$ from $(0,1)$ the energy level $\mu(0,1)$ will be in resonance with the source and drain level as visible in panels (e),(f), (g) and (h). When increasing the voltage applied to $V_{PG1}$ along this line, only the levels in dot 1 will shift down. To compensate for the interdot coupling $V_{PG2}$ will simultaneously have to be changed by a much smaller amount to stay ”on” the cotunneling line. Along this cotunneling line current can flow via cotunneling through dot 1.

Analogous considerations can be carried out for the cotunneling line separating $(0,1)$ and $(1,1)$, following the energy schemes sketched out in (a), (b), (g) and (j). We know that at the triple points (b) and (f) current will flow through the structure, as both dot levels are aligned with the electrochemical potential in source and drain. When continuing the cotunneling lines with dashed lines shown
Figure 7.5: Schematic charge stability diagram where (a) shows the case for no interdot coupling, (b) the same case including cross capacitances (c) the case for intermediate interdot coupling and (d) the case for strong interdot coupling.

in (k), we see that at (g) those lines cross resulting in another situation, where both double dot levels are aligned with the Fermi levels of source and drain. However at this point we do not observe transport through the double dot. The levels aligned here [i.e. $\mu_1(1,1)$ and $\mu_2(0,1)$] do not allow for transport through the structure.

Along the line connecting the two triple points [marked grey in (l)] the chemical potentials of both dots are aligned with each other $\mu_1(1,0) = \mu_2(0,1)$. They shift simultaneously with respect to the source and drain levels as shown in the series (f)-(d)-(b).

If a finite source-drain voltage is applied, the triple points expand from being points to becoming triangles. We assume that a finite source-drain bias $V_{bias}$ is applied to the source lead while the drain is connected to ground. This will lead to
7.3. Transport measurements

Figure 7.6: (a) Energy levels of the double quantum dot, when current flows through the structure. (b) Zoom into a single hexagon.

a shift in chemical potential of the source lead to $\mu_S = -|e|V_{\text{bias}}$ while the chemical potential of the drain stays at $\mu_D = 0$ as labeled in Fig. 7.8. Due to the resulting offset between the chemical potential of source and drain, transport can now not only occur at the triple point [see Fig. 7.8 (a)] but inside the whole triangle (a)-(b)-(c), whose extent is determined by the applied source-drain voltage.

A more detailed description of double quantum dots can be found in the excellent review by van der Wiel [136] and the references therein.

Charge stability diagrams: Measurements

The charge stability diagrams measured on this sample do not display the triangular features mentioned above. Instead due to thermal broadening each pair of triple points is visible as an area of enhanced current as can be seen in Fig. 7.9. From the measurement shown we can estimate values within the following order of magnitude:

$$\Delta V_{PG1}^m = 0.5 \text{ mV} \quad (7.16)$$
$$\Delta V_{PG1} = 8 \text{ mV} \quad (7.17)$$
$$\Delta V_{PG2}^m = 1 \text{ mV} \quad (7.18)$$
$$\Delta V_{PG2} = 10 \text{ mV} \quad (7.19)$$

using equation 7.14 this leads to the following capacitances:

$$C_{PG1} = \frac{|e|}{\Delta V_{PG1}} = 20 \text{ aF} \quad (7.20)$$
$$C_{PG2} = \frac{|e|}{\Delta V_{PG2}} = 16 \text{ aF} \quad (7.21)$$
Figure 7.7: Energy levels of source, drain and both dots for different points in the charge stability diagram. (k) and (l) show the region of a charge stability diagram around a pair of triple points. The energy level configurations corresponding to the positions marked (a) through (j) in panel (k) are shown on the left. (l) displays the same part of a charge stability diagram. Here we mark the conditions for the chemical potentials along all lines.

and using equation 7.15 to:

\[
C_m = \frac{\Delta V_{PG1}^m}{C_{PG2} \Delta V_{PG1}} = 1 \text{ fF} \quad (7.22)
\]

\[
C_2 = \frac{|e|C_{PG2}}{C_{PG1} \Delta V_{PG1}} = 16 \text{ aF} \quad (7.23)
\]

\[
C_1 = \frac{|e|C_{PG1}}{C_{PG2} \Delta V_{PG2}} = 20 \text{ aF} \quad (7.24)
\]
7.3. Transport measurements

Figure 7.8: (a) Energy levels of the double quantum dot, leading to triangles at the triple points when measuring at finite bias. (b) Zoom into a single hexagon.

So we can deduce the charging energy $E_{C1}$ of either dot to be in the range of

$$E_{C1} = \frac{e^2}{C_1} \left( \frac{1}{1 - \frac{C_1}{C_1 C_2}} \right) = 8 \text{ meV}$$

(7.25)

$$E_{C2} = \frac{e^2}{C_2} \left( \frac{1}{1 - \frac{C_2}{C_1 C_2}} \right) = 9 \text{ meV}$$

(7.26)

The mutual charging energy is in the order of

$$E_m = 0.5 \text{ meV}.$$  

(7.27)

Measurements carried out on the same sample in a different cooldown showed that at lower temperatures, when the single triple points became distinguishable in the charge stability diagram, the charging energy of both dots was found to be of the same order of magnitude, namely $E_C = 3 \text{ meV}$. The mutual charging energy was found to be $E_m = 0.4 \text{ meV}$. We have to keep in mind that additionally the sample changed a couple of times during the course of the measurement (which will be discussed in more detail later on), which also affected the values given above.

When charge stability diagrams are recorded over the full range of voltage which is accessible, we observe the pattern shown in Fig. 7.10(a). When both plunger gate voltages are very positive (i.e. above about 300 mV) the double dot
is completely opened. No double dot is opened and a current of about 1 nA flows when a source drain bias of 200 \( \mu \text{V} \) is applied, which corresponds to a low resistance of about 200 k\( \Omega \) for the complete structure. When only one gate is set to a very positive voltage, when \( V_{PG2} = 500 \) to 100 mV and \( V_{PG1} = -200 \) to +200 mV or vice versa, single stripes occur in the conductance. In this range the transport is dominated by a single dot, while the other is not clearly confined. In the region of \( V_{PG1} = -100 \) to +100 mV and \( V_{PG2} = -100 \) to +100 mV we observe regularly arranged single points of enhanced currents. When zooming in to this region as shown in panel (b) we can see that this pattern is in fact the hexagon pattern, as discussed before. Due to the measurement temperature of 300 mK the triple points are smeared out and each pair appears as a single, elongated region of enhanced current. We can conclude that in this gate voltage region a double dot is formed. When zooming in even more (panel (c)) we can even observe features inside the pairs of triple points which might be connected to the existence of excited states. They have an energy spacing of about 30 eV. The thermal energy in comparison is in the order of \( E_{\text{therm}} = k_B \cdot T = 30 \mu \text{eV} \). Going to even lower gate voltages the current becomes completely suppressed.

Figure 7.11(b,d) shows the charge stability diagram for both dots recorded separately. In order to do so the voltages of both plunger gates were stepped in parallel along the lines indicated in panel (a). In panel (b) the charge on dot 2 is constant, while electrons are loaded on dot 1; whereas for (d) the charge on dot 1 remains constant. We observe that in the measurement shown in panel (d) the extent of the Coulomb blockaded diamond varies strongly with the electron number. We also find that for the charge stability diagrams of the single dots, the features are broadened as when measuring a charge stability diagram of the complete structure.
7.3. Transport measurements

Figure 7.10: Panel (a) shows the charge stability diagram of the sample over the whole plunger gate range voltage accessible. The green box marks the gate voltage range for which a more detailed measurement was recorded as shown in (b). A zoom in on one pair of triple points is shown in (c), where features that might be connected to be excited states are visible. The bias voltage used to record these datasets is $V_{\text{bias}} = 100 \, \mu\text{V}$

The source-drain bias offset originating from the measurements setup (for example due to the IV-converter, or thermocurrent in the cryostat cabling) was compensated for those and all measurements to follow. Figure 7.11(c) shows the current through the double quantum dot versus the voltage applied to $V_{PG1}$ for different bias voltages. The amplitude of the current decreases with decreasing bias until we observe a virtually flat curve for $V_{\text{bias}} = 90 \, \mu\text{V}$, which is the bias offset that is compensated for in all following measurements. When decreasing the voltage further we observe a negative current.

7.3.3 Influence of STG and DTG

In addition to the plunger gates, the structure is also equipped with gates closet to the tunnel barriers (STG and DTG). These gates can be used to open and close the tunnel barriers, i.e. tune the tunnel coupling from one dot to its adjacent lead. Using this we can tune the double dot from a situation, where the tunnel barriers are so strong, that no current flows through the quantum dot, to a situation, where the complete nano-structure loses quantization and charging effect, with almost no barrier separating one dot from its lead. Therefore we can open either one, or both
Chapter 7. Measurements on a GaAs double quantum dot

Figure 7.11: Panel (a) shows the current through the double dot as a function of the two plunger gates. In order to measure the charge stability diagrams of the single dots as shown in the panels (b) and (d), the individual plunger gates are stepped along the lines indicated in (a). (c) shows a single line of $I(V_{PG1})$ for different source-drain bias voltages, as indicated in the caption.

of the tunnel barriers using the tunnel gates, therefore destroying the confinement of either dot. This will be discussed in more detail in chapter 7.4.

The electronic states of the double quantum dot can also be influenced by tuning the voltages applied to the gates STG and DTG. As a result charge stability diagrams $I(V_{STG}, V_{DTG})$ can be recorded that display very similar features as the charge stability diagrams shown in Fig. 7.10 as can be observed in Fig. 7.12.

7.3.4 Influence of the center gate

The center gate can be used to tune the coupling of the two dots. Due to its very narrow shape we can expect the 2DEG to be depleted up to a distance of about 200 nm from the dot. Therefore the influence of the center gate on the inter-dot-tunnel barrier is rather weak and it is not possible to form a single dot that extends over the complete structure by applying a large positive voltage to this gate. It predominantly acts again on the potential of both dots in parallel.
7.4. Scanning gate measurements

In the following section we will discuss the measurements carried out on the sample using the technique of scanning gate microscopy. After a general introduction to the most dominant features in the scanning gate measurements in section 7.4.1 we will move on to discuss the role of the tip [sec. 7.4.2], as well as the resolution in real space [sec. 7.4.4]. We will further investigate the influence of other parameters such as magnetic field, temperature and gate voltages. We will conclude with an outlook of other measurements that might be a next step, building on the measurements presented here [sec. 7.4.7]

7.4.1 Introduction: Scanning gate measurements on a double quantum dot

The most fundamental form of a scanning gate measurement is recording a current map, i.e. the current through a structure, while sweeping the metallic tip at a constant height $d$ across the sample. The resulting measurement displays $I(x, y)$. When such a current map is recorded for a single quantum dot, single rings of enhanced currents are observed, which are concentric around a certain point. This situation is schematically displayed in Fig. 7.12(a). The green concentric rings are so called Coulomb rings of the quantum dot. Along each Coulomb ring a finite current flows through the sample, which is highlighted by the grey underlay of the complete rings. On a Coulomb ring a quantized level inside the quantum dot is in resonance with both source and drain level. For each adjacent Coulomb ring
Figure 7.13: Schematic: Basic appearance of a scanning gate measurement recorded on a double dot. When performing a scanning gate measurement on a single quantum dot we observe single concentric rings of enhanced current as schematically shown in (a) and in a sample measurement trace in (c). When measuring on a double quantum dot each dot produces its own set of concentric rings (b,d).

another electron is loaded onto or removed from the dot, so a new level comes in resonance with the electrochemical potential of source and drain.

When recording a current map of a double quantum dot, we can understand the basic features of the resulting current map based on our understanding of the single-dot-physics: Each single dot will evoke its own set of Coulomb rings, where one set of Coulomb rings is centered around the electronic center of the left dot and the second set of Coulomb rings is centered around the electronic center of the right dot [as noted by the blue and green sets of rings in Fig. 7.13(b)]. In order to have transport through the double dot structure one level of both dots has to be in resonance with source and drain, as shown in Fig. 7.6(a). However, the levels of the left dot are only aligned with the electrochemical potential of source and drain along the blue Coulomb rings and the levels of the right dot are only aligned along the green Coulomb rings. Therefore we only encounter nonzero current, when both sets of rings intersect. This is the case at all grey highlighted regions in Fig. 7.13(b).

Figure 7.13(c) and (d) show measurements carried out on a single quantum dot and a double quantum dot. The right panel of subfigure (c) displays the raw measurement data. We see approximately concentric rings of enhanced current.
Their shape is almost perfectly circular, owing to a very symmetric tip potential. In the center of the measurement the current is suppressed owing to a repulsive tip. At the center of the Coulomb rings we expect the geometric position of the quantum dot, i.e. the maximum wave function in real space. For better comparability the most dominant Coulomb rings are traced by hand and presented in the left panel. We see that this simplified version of the measurement data agrees very well with the theoretically expected features displayed in (a).

Similarly the right panel of Fig. 7.13(d) displays the raw data of a current map carried out on a double quantum dot. We observe spots and elongated regions of enhanced current. We can now superimpose two sets of rings, as displayed in the left panel to reproduce all features shown in this measurement.

For the current map displayed in Fig. 7.13(d) one can trace out the two sets of Coulomb rings almost by naked eye. This basic underlying structure stays the same for more complex measurements as shown in Fig. 7.14. Panel (c) shows a zoom into the most central region of the current map, and again one can reproduce all the features present in the scanning gate image starting from two sets of Coulomb rings. Panel (a) shows additionally the oxide lines as white lines. We see that the center of each set of Coulomb rings coincides with the position, where a quantum dot would be expected to form from the oxide barriers.

However, from our discussion of transport through double quantum dots in the previous section 7.3 we know, that when measuring two quantum dots in series, we also have to take into account interdot coupling. Therefore the charge stability diagrams does not only display single spots of currents along a square lattice, but rather the well known hexagon structure as displayed in Fig. 7.15(a,b). Even through the single triple points cannot be resolved individually the interdot coupling becomes apparent by the lateral offset of the cotunneling lines from one hexagon to the next. The same interdot coupling becomes obvious in current maps. We now zoom into a current map in a region about 1 μm below the electrical position of the dots [see Fig. 7.15(c)]. We observe that the Coulomb rings from dot 1 (blue) and dot 2 (red) are not continuous Coulomb rings, but rather have a little offset at each crossing. This offset is the same as observed in charge stability diagrams and evoked by the interdot coupling. The current maps therefore displays what we call distorted hexagons.

We know that in charge stability diagrams the gate-voltage region of one hexagon corresponds to a certain number of electrons on each dot marked by \( (I,J) \) in Fig. 7.15(a,b). When crossing a cotunneling line into a neighboring hexagon the number of electrons on one dot changes by one. In addition to the in plane gates, we now have to consider the position of the tip. For the charge stability diagram shown in (b) the tip was located at the position marked with a green cross in (d). Vice versa the green cross in (b) corresponds to the gate voltage
Figure 7.14: (a) Current map of the double dot measured at $V_{PG1} = 0$ mV, $V_{PG2} = 190$ mV, $V_{STG} = V_{DTG} = -40$ mV, $V_{SD} = 210$ µV and $V_{tip} = 0$ V. The tip is scanned about $d = 10$ nm above the surface. The oxide lines that define the structure are schematically shown as white lines. (c) shows a zoom into the central region of the measurement marked with a black rectangle in (a). The underlying structure of two sets of Coulomb rings is additionally displayed in (b).

settings used to record the current map displayed in (d). Therefore the geometric range marked $(N, M)$ in (d) corresponds to the same number of electrons as the region marked $(N, M)$ in (b). To simplify the visualization, hexagons and the corresponding distorted real-space hexagons are marked with different colors in (a) and (c).

7.4.2 Characterization of the tip

In the subsequent sections we will use the wet-etched AFM-tip to probe and manipulate the nano-structure. Ideally one aims to measure the spatial variation of the probability density of the electrons in a dot, or the maximum wavefunction in each quantum dot. This means that we want to use the small perturbation produced by the tip, the tip induced potential or tip potential $\phi_{tip}(x, y)$ to perturb the energy $E_\Psi$ of a particular state $\Psi$. This should lead to a change $\Delta E_\Psi$ in the
Figure 7.15: (a) Schematic of the hexagon pattern formed when measuring $I(V_{PG1}, V_{PG2})$ in the double dot regime. The letters $I$ and $J$ represent the number of electrons in the left and right dot respectively. (b) Charge stability diagram for a source-drain voltage of 210 mV. The green cross marks the gate settings used for the measurement shown in (d). The green cross in (d) marks the position at which the tip was located to record the measurements shown in (b). (c) schematically shows the expected scanning gate pattern. The hexagons observed in (a) are distorted in real space. (d) Scanning gate measurement of the double dot. The tip is scanned about $d = 10$ nm above the surface. Pairs of triple points are visible as regions of enhanced current. The labels $(I, J)$ mark the same electron numbers on the dot as in the charge stability diagram shown in (b). Measurement settings: $T = 300$ mK, $V_{CG} = 120$ mV, $V_{STG} = V_{DTG} = -40$ mV, $V_{tip} = 0$ V.

The quantum state energy given in first order perturbation theory by the convolution of the probability density with the tip potential:

$$\Delta E_{\Psi}(x, y) = \int |\Psi(r)|^2 \phi_{tip}(x, y, r) d^3 r$$

We have to consider the following two length scales: First the extent of the tip induced potential, which is related to the geometric dimension of the tip. Second the length scale on which the probability density changes. For an infinitely narrow tip potential, $\Delta E_{\Psi}(x, y)$ would directly correspond to the probability density $|\Psi(r)|^2$. We can therefore only measure changes in the probability density which are on a larger length scale than the tip potential. Hence the tip potential has to be sharp. Ideally the tip radius should be smaller than the lateral extent of a quantum dot.

This idealized model however considers the tip potential as a small potential perturbation. This is most likely not the case here. The tip potential is rather large compared to the extent of the wave function inside the quantum dot. However, a tip potential does have a relatively sharp potential gradient. From the measure-
ments presented within this chapter we can find an energy gradient which is in the order of magnitude of 1 meV/15 nm. We can therefore investigate (quantum) features, which are sensitive to the energy gradient (as shown in the interference patterns imaged on quantum point contacts for example in [138, 16, 139]).

Finally in order to make the analysis of the measurements as easy as possible the tip should only consist of a single tip. No double tip or charged particles on the tip are desirable.

In conclusion the ideal tip would look as follows:

- tip potential/tip apex radius smaller than the lateral extend of the investigated structure
- simple geometric shape (no double tips/charged dirt particles)
- magnitude of the tip induced potential smaller than the charging energy of the dot

We will now evaluate the tip used in the experiment based on these factors.

**Tip apex radius**

The tip apex radius is accessible only in an indirect way. As each single tip is home-made by wet etching, each tip is different. Inspection of several etched tips by scanning electron microscopy has shown that their radii typically vary between 20 nm and about 100 nm. However, this initial tip radius can be greatly altered
through the topography scanning carried out in order to position the tip directly above the sample. At the time of the measurement the tip radius can be estimated from a topography scan upon reaching the structure employing the method of blind tip estimation [140] from topography scans via the programm SPIP.

Blind tip estimation is a well established algorithm that calculates the shape of a tip from a topography scan. Its ability has been shown in numerous publications, including [141, 142, 143, 144, 145]. As the tip shape is never ideal (i.e. delta shaped) tip-induced distortions are visible in surface scans. The resulting image is a convolution of the actual sample surface and the tip shape. By deconvolving those two parts, the actual tip shape can be obtained. A detailed explanation of this method and the underlying algorithm can for example be found in [144, 140]. A more detailed description of this method can be found in appendix K.

In our case the sample topography was only scanned in detail after all measurements were completed. We can therefore assume that the apex radius derived from these scans is a worst case approximation. During the measurements that will be presented in the next sections, three tip crashes have taken place: One due to a power failure, one due to a crash of the Nanonis Software and one due to mechanical disturbances of the setup. All three tip crashes noticeably changed the tip and the sample. It is therefore reasonable to assume, that the tip apex radius estimated from those scans is considerably worse than the tip radius at the time the measurements were started.

The blind tip approximation was carried out for a series of 14 topography scans carried out one after the other. We determine a tip radius of $39 \pm 11$ nm. As the diameter of a single quantum dot is about 150 nm, our tip radius is at the very least well below the lateral dimension of the sample, therefore fulfilling the first specification pointed out above. The shape of the tip derived from one of those measurements is shown in Fig. 7.16. We observe a very sharp and symmetric single tip. This is consistent with the tip shape estimated from the current maps.

**Altering the tip: Towards a simple geometric shape**

At the beginning of the scanning gate measurements the tip shape was less than simple. We observed clear double tip features in the current maps as seen in Fig. 7.17(a). In section 7.4.1 we discussed how the current map of a double quantum dot can be understood by the interference pattern of two sets of Coulomb rings. The measurement in Fig. 7.17(a) can be understood by 4 sets of Coulomb rings, where we observe two pairs which show the same shape: One pea like shape on the left and one bean like shape on the right. The important point here is to distinguish between features evoked by the double dot and features resulting from a possible double tip. The shape of the Coulomb rings basically mirrors the shape of the tip induced potential. We can therefore think of such a measurement
Figure 7.17: Current maps performed before and after altering the tip. (a) Shows a current map recorded with a double tip. (c) Shows the single Coulomb rings extracted from that measurements. We observe two different sets of Coulomb rings. One pea-shaped set on the left and one bean shaped set on the right. (b) Shows a current map recorded with a single tip. A single set of Coulomb rings that can be used to map this measurement is shown in (d).

as a representation of the equipotential lines of the tip. However, we know that equipotential lines, similar to contour lines on a topographical map, only cross in a saddle point which has to be present for a double tip. Therefore we can distinguish between features of the two quantum dots, which can cross, and features of the double tip, which cannot cross. This can be seen more clearly in Fig. 7.17(c), which displays the two pairs of ring-sets that can be used to model the measurement shown in (a).

Different techniques can be employed to change the shape of the tip. Most often the technique of “flushing”, i.e. positioning the tip on top of a metallic part of the sample and applying a voltage between those two parts has been used successfully [39]. As in this experiment it was not possible to apply a voltage to the tip, we had to take a less invasive approach.

To do so the tip was retracted from the surface by about 2 μm. Then the tuning fork sensor was driven with a drive amplitude that was up to a factor 100
bigger than that used for the scanning topography (about 0.4 mV). Additionally
the frequency with which the tuning fork was driven was slowly changed between
zero and 100 kHz. At some point the frequency shift could be seen to make a
sudden jump. Upon reaching equilibrium the tip was brought back to the surface
and another current map of the sample was recorded. Figure 7.17(b) shows a
typical current map recorded after this treatment of the tip. We now observe only
a single pair of sets of Coulomb rings. We can therefore assume, that the double
tip visible in (a) was due to a particle attached to the tip, that could be shaken
off with the treatment described above.

Therefore we also fulfill the second criterion of measuring with a tip of simple
geometric shape.

Using the tip as a gate: The magnitude of the tip induced potential

In order to quantify the magnitude of the tip induced potential, we measured
charge stability diagrams for different lateral tip positions as indicated in Fig.
7.18(b). Figure 7.18(a,c,d,e) shows charge stability diagrams where the tip was
moved to different positions in the scan frame. We observe, that all charge stability
diagrams shows only lateral shifts of the features of few mV in gate voltages. All
observed features stay the same. This is not unexpected as even though the double
dot structure is not rotationally symmetric, our tip potential is approximately
rotationally symmetric. Therefore the shift of features in the charge stability
diagram is the same, no matter in which direction the tip is moved away from the
double dot.

If we however leave this axial symmetry and put the tip directly in the center
of the scan frame, over the double dot [as schematically sketched out in Fig.
7.18(b)] we observe a strong shift in the charge stability diagram. All pairs of
tripe points shift to more positive plunger gate voltages. Also we see that the
region of suppressed conductance becomes more extended towards positive gate
voltages. Panel (f) shows charge stability diagrams recorded for different tip-
surface distances $d$. We conclude that the tip potential is repulsive.

Figure 7.18(g) shows the lowest gate voltage $V_{PG1}$ for a fixed $V_{PG2}$ voltage,
at which the current through the double dot rises above 10 pA. This value for
$V_{PG1}$ was determined for different tip-sample distances $d$. The influence of the tip
decreases strongly when moving it further away.

Ideally the magnitude of the tip induced potential should be smaller than the
charging energy of the dot which is around $E_C = 1$ meV. Depending on the tip-
sample separation we can observe between one and 25 Coulomb rings. Therefore
the tip induced potential ranges between one and 25 meV. For a typical tip-sample
separation the hight of the tip-induced potential is in the order of 8 meV. Therefore
the tip induced potential is in the same order of magnitude as the charging energy.
Figure 7.18: (a,c-f) Charge stability diagrams for different tip positions as shown in (b). Panel (g) shows the gate voltage $V_{PG1}$ at $V_{PG2} = const$ when the current first rises above 10 pA.
This tip potential is fixed for all measurements, as no voltage can be applied to the tip intentionally.

### 7.4.3 Changing the center gate: Shuffling single electrons through the structure

The center gate is positioned close to the tunnel barrier that connects dot 1 and dot 2. As it is rather narrow the 2DEG might be depleted close to the tip of the gate resulting in a lower lever arm than naively expected from the geometry of the sample. The voltage applied to the center gate can be as much as 0.5 V. Figure 7.19 shows a series of current maps recorded for different $V_{CG}$. In the range of 500 – 300 mV we observe double dot behavior. As visible in the bottom panel the amplitude of the current stays approximately constant in this gate voltage range. When reaching lower voltages between 300 – 150 mV the amplitude of the thirteenth root of the current starts to drop linearly with gate voltage. In addition, the current is completely suppressed in the center of each current map. This region of suppressed current increases as the gate voltage is lowered. When reaching $V_{CG} = 100$ mV current can pass only very scarcely through the structure and is completely suppressed for gate voltages below 50 mV.

We can see from this series, that within the accessible gate voltage range we are not able to form a single dot that expands across the complete double-dot region.

We will now focus on the effect of the center gate by recording current maps for different $V_{CG}$ which are stepped over a much smaller range than in the measurement discussed before. Figure 7.20 shows current maps for center gate voltages between 40 and 120 mV. Panels (a) through (i) show only the right half of the image. As the current maps are virtually mirror symmetric along this axis half a scan frame contains the same information as the complete scan frame for this purpose. We see that when we trace a horizontal line from the outermost triple-point ring in (a) by image (d) or (e) we again encounter a new triple point ring of the same radius and again in (i). To confirm this finding we calculate the cross correlation between the current maps shown in (a) and (b-i) using the following equation:

$$ CC = \frac{\sum_{x,y} I(V_{CG(a)}, x, y) I(V_{CG(i)})}{\sqrt{\sum_{x,y} (I(V_{CG(a)}, x, y))^2} \sqrt{\sum_{x,y} (I(V_{CG(i)}, x, y))^2}} $$

This cross correlation (CC) is shown in Fig. 7.20(j). The self-correlation from (a) with itself starts out at one. Afterward the correlation drops down almost to zero for (c) and then rises to 0.2 again. We see that the cross correlation shows a period of 40 mV. Within this period another electron is loaded onto the double
Figure 7.19: Series of current maps for the complete range of gate voltages that can be applied to the center gate. We see how the complete structure is gradually pinched off, when decreasing the voltage applied to the center gate. The color bar shows $I^{1/13}$. The bottom plot shows the maximum current (yellow squares) and the minimum current (black circles) for each current map versus center gate voltage. Measurement settings: $V_{PG1} = V_{PG2} = 500 \text{ mV}$, $V_{STG} = 300 \text{ mV}$, $V_{DTG} = -100 \text{ mV}$, $V_{bias} = 210 \mu\text{V}$, $d = 50 \text{ nm}$, $T = 300 \text{ mK}$.

dot. However within this gate voltage range all other features of the current map stay essentially the same. Therefore in this gate voltage range the center gate acts homogenously on both dots, loading electrons on both dots.

### 7.4.4 Lateral position and shift of the double quantum dots

The big advantage scanning gate microscopy has over conventional transport measurements is the lateral resolution it provides. Specifically we know that under certain conditions discussed in section 7.4.2 we expect to be able to localize the maximum of the wave function in each quantum dot. This position is expected to be where a Coulomb ring of zero radius (called zeroth Coulomb ring for simplicity) forms. In the following we will discuss the opportunities and limitations
The grid measurements

In order to investigate the influence of the different gates on the double dot, we will now show single line scans, where each line scan is recorded at a different gate voltage or tip-sample distance \( d \). This leads to measurements of the current \( I \) versus a spatial direction such as \( x \) or \( y \) and one gate voltage \( (V_{PG1}, V_{PG2}, V_{CG}, V_{STG}, V_{DTG}) \) or tip-sample distance \( d \). These measurements contain the information of a series of current maps in a very condensed way. The real space axes are chosen as follows: One line labeled (a) in Fig. 7.14 traverses dot 1 and the dot 2 in \( x \)-direction. The other line labeled (b) in Fig. 7.14 is the symmetry axis of the structure in \( y \)-direction.

We will now discuss the construction of a so called grid measurement on the example of \( I(d, x) \). We record a series of current maps for different tip-sample separations \( d \). This series is shown in Fig. 7.21(a). We observe that the first row shown in Panel (a) shows the familiar distorted hexagon pattern. When reaching row four this hexagon pattern has become very hard to recognize and by line five
Figure 7.21: Panel (a) shows a series of current maps recorded for different tip-sample distances $d$. The first current map is recorded at $d = 26$ nm. $d$ is then increased by 10 nm for each scan frame. (b) shows the current $I(x, z)$. It is derived by taking a single line cut along the line marked with a grey dashed line in the first current map for each scan frame. (c) shows the corresponding grid measurement, where the tip is stepped along a line across the double dot and the z-position is swept at each point. Measurement settings: $V_{PG2} = 500$ mV, $V_{PG1} = 500$ mV, $V_{STG} = 300$ mV, $V_{DTG} = -100$ mV $V_{bias} = 210$ μV $T = 300$ mK.
we have reached a current map that is not distinguishable from that of a single quantum dot. We are now interested in the shifts of the center of each Coulomb ring in x or y direction. This information is contained not only in a current map, but in a condensed way already when recording two cuts one in x-direction and one in y-direction across such a current map. If we now extract a cut through each current map along the line shown in the first subfigure of panel a and plot them on top of each other we subsequently form a plot $I(d, x)$ as shown in Fig. 7.21(b). Similar to following the shifts of single Coulomb rings from one current map to the next we can in such a grid measurement follow the Coulomb rings which appear as single bright profiles. We can now increase the resolution of such a measurement by not extracting it from a series of current maps, but rather stepping the tip along one of the two lines mentioned above across the sample and sweeping the desired parameter (gate voltage or $d$) at each point along this line. For the corresponding case this results in a graphic as shown in Fig. 7.21(c). We observe the same features as in the figure shown in panel (b), but with a much higher resolution. In such a measurement we can now easily follow the single profiles of the single Coulomb rings and monitor their shift due to the influence of the external parameter plotted on the y-axis.

Figure 7.22(a) shows $I(V_{PG1}, x)$. We observe two sets of profiles. The profiles represent cuts through the tip-induced potential, where the gate voltage axis can be converted into an energy scale using the appropriate lever arm. Two profiles of the right set are traced with a solid white line. Two profiles of the left set are traced with white dashed lines. The left profiles belong to dot 1, whereas the right profiles can be attributed to dot 2. Shifting and scaling each of these profiles collapses them onto the same curve (see top panel). This confirms that the two dots sense the same tip-induced potential. Increasing the gate voltage, we observe that the amplitude of the profiles increases by a factor of 3 over the measured range. This is due to the gate voltage dependence of the lever arm of PG1. For higher gate voltages the lever arm becomes smaller, therefore the voltage increase needed to cause a particular energy shift of a quantum dot level becomes larger, leading to profiles with a bigger amplitude. When applying a more positive voltage to the gate, the relative potential of the dot compared to the gate becomes increasingly negative. This leads to an increase of the depletion length. Effectively the distance between the dot and the gate increases. The gate moves "further away" leading to an increased lever arm [146].

Furthermore we observe that the amplitude of the profile of dot 1 is a factor of two to three smaller, than the amplitude of the profile related to dot 2 in the same $V_{PG1}$-range. This is due to the fact, that dot 1 is located next to PG1, which has a larger lever arm on dot 1 than on dot 2, leading to a relative lever arm ratio.
Figure 7.22: \( I(V_{\text{Gate}}, \text{position}) \) measurements for x and y direction for both plunger gates (a,b) and the center gate (c) Measurement settings: \( V_{\text{PG2}} = 190 \text{ mV}, V_{\text{PG1}} = 100 \text{ mV}, V_{\text{bias}} = 210 \mu\text{V}, T = 300 \text{ mK} \) and \( d = 10 \text{ nm} \).
Scanning gate measurements

\[ \alpha_{PG1,\text{dot}1}/\alpha_{PG1,\text{dot}2} = 2. \] Analogous observations can be made for PG2 as seen in Fig. 7.22(c).

We now investigate the x-position of the maximum of a profile for different \( V_{PG1} \). We observe that the maxima of the profiles of dot 2 stay at the same x-position. However, the maxima of the profiles of dot 1 shift towards PG1 by about 20 nm/V. This shift is indicated by the grey dashed line which follows the maxima of the profiles of either dot. When applying a positive voltage to PG1, we expect that dot 1 will shift to, or extend towards this gate. This is visible by the lateral shift of the profiles belonging to dot 1.

When sweeping the tip along the symmetry axis of the structure, we obtain \( I(y, V_{PG1}) \) as shown in 7.22(b). Again we observe two sets of profiles with different amplitudes. For better visibility a profile for each dot is traced with a white line. Again we observe that the profile belonging to dot 1 (dashed white line) has a higher amplitude than the one belonging to dot 2 (solid white line). However, their shape is the same as can be seen in the header. Furthermore, we do not observe a shift in space when sweeping the plunger gate voltage. Analogously when evaluating panel (d) we do not observe a shift of either profile when sweeping the voltage applied to PG2. We can conclude that the apparent y-position of the double dot is not influenced by the gate voltage applied to the plunger gates.

When sweeping the voltage applied to the center gate we observe two profiles of the same amplitude when stepping the tip along the x-axis [panel (e)]. The spatial location of their maxima does not shift in x-direction when \( V_{CG} \) is changed. When the tip is stepped along the y-axis [panel (f)] both profiles have the same amplitude, and we observe a slight shift of the maxima towards the center gate in the order of 10 nm/V when increasing the voltage applied to it. This shift is indicated by the grey dashed line.

The bottom panels (g,h) in figure 7.22 show an overlay of the different profiles obtained from the six grid measurements discussed so far. The amplitude and positions of the profiles are shifted and scaled to fit on top of each other. The profiles of all grid measurements show the same shape, confirming that the double dot senses the same tip potential in all cases.

We will now move on to evaluate the influence of the tunnel gates STG and DTG on the double dot structure. Figure 7.23 shows grid measurements for both gates.

The measurement \( I(V_{DTG}, \text{position}) \) shows that for low gate voltages we can again make out two profiles that intersect. For higher voltages these profiles become less defined, owing to the decreasing confinement of dot 1. At low voltages no current flows through the double dot. We observe an analogous behavior for \( I(V_{STG}, \text{position}) \). Furthermore we can see that owing to a tip crash that has taken place between the grid measurement performed using DTG and the grid
measurement using STG, the shape of the tip has been altered as visible by the different profile shape. The measurement shown in panel (c) also shows increased instability of the sample confirming that in addition to the tip potential the tip crash also altered the electronic structure of the sample.

Figure 7.24 shows the grid measurement performed by changing the tip-sample distance $d$. Similar to the measurements where the gate voltage was swept, these measurements show a interference pattern of two different profiles. At high distances the interference pattern is reduced to broad, continuous structures as would be expected to form in case of a single quantum dot. This coincides with the features expected from the current maps, shown in Fig. 7.21. Additionally we observe a shift in the position of the maxima in $-x$ direction, which is connected to the not perfectly symmetric tip potential shape.

We make the following observations: First, we can observe profiles of the same shapes for the two dots independently of which gate is swept. Second, we can
7.4. Scanning gate measurements

Figure 7.24: $I(d, \text{position})$ measurements for x and y direction versus the tip-sample separation $d$. Measurement settings: $V_{PG2} = 190 \text{ mV}$, $V_{PG1} = 500 \text{ mV}$, $V_{bias} = 210 \mu\text{V}$, $V_{CG} = 250 \text{ mV}$, $V_{STG} = 300 \text{ mV}$, $V_{DTG} = -100 \text{ mV} T = 300 \text{ mK}$.

determine the increase in lever arm by the change in amplitude of the resonances for increasing gate voltages as well as the relative lever arms of the gates. And finally, we can quantify the apparent shift of the double dots due to the increased voltage on a gate, which is in the order of ten to twenty nanometer per volt.

An obvious interpretation of this shift is, that a change in the position of the maximum of the profile allows us to measure by how much the distribution of confined electrons can be shifted in real space by an in-plane gate. Although this is qualitatively correct, we have to keep in mind, that such a quantitative deduction of the shift of the electron distribution from the profile maximum might be more complex. The constant interaction model [147] has proven in practice to be an extremely useful approximation in many-electron quantum dots and may therefore serve as a model for a more specific discussion. It assumes the existence of well-defined gate-voltage independent electrostatic lever arms, a gate-voltage independent charging energy, and a set of discrete single-particle quantum levels. In the presence of a scanning gate all these quantities may depend on the position of the tip [133, 36]. It is therefore hard to decide without a thorough self-consistent numerical solution of the problem, which of these quantities, if any, contributes most strongly to the apparent shift of the maxima observed in the experiment. Furthermore it remains to be investigated, how the observed shift relates to the true shift of the electronic density distribution.

In order to make sure that the observed shift is a robust and meaningful experimental quantity, z-dependent measurements were carried out.

To do so, a set of current maps was recorded, where for each current map the tip was kept at a different tip-sample separation, while all voltage settings were
kept the same. From each current map the (x,y) coordinates of positions of both dots were extracted with a new evaluation, the so called ellipse evaluation.

**The ellipse method of evaluation**

To find out if the electrostatic effect of the tip or the shift of the wave function is the main cause for the shift discussed in this section so far, we now evaluate the dependence of of the dot-dot separation \( s(d) \) on the tip-sample separation \( d \). This measurement and evaluation will show that the observed shift is a robust and meaningful quantity. We record a series of current maps, where all gate voltages were kept the same and the tip-sample distance \( d \) was increased after each current map. One series like this is shown in Fig. 7.25. From each current map the (x,y) coordinates of positions of both dots (i.e. the position of the zeroth Coulomb ring) were extracted as follows:

- Guides to the eye where drawn into the current map by hand in order to assign each pair of triple points to the correct Coulomb rings. An example to those rough guides to the eye can be seen in Fig. 7.26(a).

- After assigning all triple points, the maximum positions of all triple points belonging to one Coulomb ring were marked by hand and fitted with an ellipse. Points were added to fit the ellipse until the shift of the center of this ellipse varied by less than 2nm when adding an additional three points.

- Upon completing one ellipse the next bigger one was defined and so on until all ellipses available in this scan frame were fitted via Coulomb rings. The
Figure 7.26: Principle of the ellipse-evaluation. A detailed explanation is given in the text.

- points choosen by hand and the fitted ellipses for the set belonging to dot 2 can be seen in Fig. 7.26(b).

- We observe that not all ellipses have the same center point, but rather follow a linear shift. This can be attributed to the particular shape of our tip. The tip is not perfectly symmetric, as already observed in the grid measurements. Via linear regression we determined the \((x_0, y_0)\) coordinate at which the zeroth Coulomb ring, i.e. an ellipse with zero axis length would form. The x and y coordinates of the ellipses of dot 2 and the linear fit for a typical set are shown in Fig. 7.26(c,d).

- The parameter evaluated is the distance \(s\) between these \((x_0, y_0)\) coordinate for dot 1 and dot 2.

It is important to first draw the guide to the eyes into the current map as assigning a triple point to the wrong Coulomb ring will lead to the whole evaluation to fail. Figure 7.27 shows the correct guide to the eyes in (b) and examples for incorrectly assigned Coulomb rings in (c,d). We see that when drawing in all
Figure 7.27: Different options to assign the ellipses in one current map. (a) shows the raw data. (b) shows the raw data with the correctly assigned ellipses. (c,d) show the raw data with wrongly assigned ellipses. White squares are used to indicate the area, where the ellipses do not coincide with the measurements.

guides to the eye, we will note when a triple point is wrongly assigned as marked by the white square. In all cases we either have no crossing in the guides to the eye, where we observe a current peak in the current map, or miss a current peak at a predefined crossing. Whenever either case occurs, the guide to the eyes have to be redefined, until good agreement is reached. Another hint if the ellipses are defined correctly is the continuity of the centers of a single set of ellipses. If the centers do not approximately fall onto one line as shown in the example [Fig. 7.26(c,d)] but show large derivations, the assignment of the triple points to certain Coulomb rings was evaluated again and if necessary be corrected. Unfortunately in an evaluation that is based on the ability of the human eye/mind to recognize patterns, one can never be one hundred percent sure, that no mistakes are made. The possibility of human error can therefore not be excluded.

Determining the relative position $s$ makes the evaluation robust against shifts of the complete scan frame. The angle of all ellipses is about zero [Fig. 7.28(a)] confirming the approximately circular shape of the tip potential as well as proving the consistency of the evaluation method. If we now plot the x and the y position of the zeroth Coulomb ring versus the tip-sample separation $d$ in Fig. 7.28(b,c) we observe the following: The x-positions of the zeroth Coulomb ring show a continues, almost linear shift towards smaller x-values when $d$ is increased. The red dots correspond to the position of dot 1 and the blue dots to the position of dot 2. At the same time the y-position also shifts to smaller y-values. However this shift occurs on a scale that is about an order of magnitude smaller than the shift observed in x-direction. Additionally due to the general interference pattern of a current map the y-position can be distinguished only with a larger errorbar than the x-position.

In the final step we now want to plot the dot-dot separation $s(d)$ as shown by the red dots in Fig. 7.29. We see that when changing the tip-sample distance,
Figure 7.28: Result of the ellipse evaluation. (a) shows the distribution of the angles of the fitted ellipses versus the length of the major axis. We see that all angles are in the range of ±2°. (b) and (c) show the coordinate of the center ellipse with zero major axis length vs the tip-surface separation \( d \).

The dot-dot separation remains constant as long as the tip-sample separation is below 120 nm. For larger tip-sample separations this evaluation becomes very imprecise. This, together with the decreasing resolution of the measurements leads to a drop in dot-dot separation for tip-sample distances above this value. Additionally the current maps of such large tip-sample separations often only display a single Coulomb ring, leading to a break down of the evaluation method. All current maps and grid measurements used to determine the dot-positions are carried out well below this critical threshold of \( d_c = 120 \) nm. The fact that the dot-dot separation remains constant for different tip-sample separations is a strong indication, that \( s \), the way it is derived here is a robust quantity.
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Figure 7.29: Result of the ellipse evaluation. The figure shows the dot-dot separation versus the tip-sample separation \(d\). The red circles correspond to one set plunger-gate voltage setting, the blue squares to another set with more positive gate voltage settings. We see that the average dot-dot separation increases, when the voltage applied to the plunger gates is increased.

When the same measurement and evaluation are carried out for more positive voltages applied to both plunger gates, we obtain the data set marked with blue squares. Again the dot-dot separation is independent of the tip-sample separation below a certain threshold value. For this measurement we observe a slightly bigger dot-dot separation compared to the first set of current maps. The change in this separation of about 20 nm/V is in the same order of magnitude as the shift extracted from the \(I(V, \text{position})\) discussed above.

We can also use the ellipse evaluation to extract \(\Delta s(V_{\text{gate}})\) from a series of current maps recorded while sweeping a single parameter. Obviously such a series has to be taken with \(d \ll d_c = 120\) nm. When applying this method to the series of current maps recorded for different \(V_{\text{CG}}\) shown in Fig. 7.19 we obtain a shift of 9 nm/V in y direction which is consistent with the result of the grid measurements.

Since the evaluation of the dot-dot separation in dependence of the tip-sample separation \(s\) shows no dependence \(s(d)\) below a threshold \(d_c\) we can conclude that for measurements carried out within this boundary condition, we can rightfully treat \(d\) as a reliable quantity.

Transition from a single to a double dot

In addition to tuning the double dot with the plunger gates PG1 and PG2, we can also tune the tunnel barriers using STG and DTG. All previous measurements are performed in a region where both barriers are tuned to the tunneling regime, leading to a display of Coulomb blockade in both dots. Using one of the tunnel barrier gates, we can tune only one dot to the Coulomb blockade regime, whereas the other dot is open, forming an extension of the respective contact. With this method we can confine either the right dot, or the left dot in this double-dot geometry. Alternatively we can obviously still form a proper double dot. This can
already be deduced from the charge stability diagram in Fig. 7.30(g-j). However, the open question remains where in the structure the single and double dots will form. We will now investigate this question.

Figure 7.30(a) through (c) show current maps carried out in these three regimes. For (a), only dot 2 is confined by the central tunneling barrier and the tunneling barrier near DTG. We observe only a single set of concentric conductance rings. One of these rings is highlighted with a green dotted line and also shown in (d). In Fig. 7.30(b), we observe clear double-dot behavior. We recognize two sets of concentric rings. Again two single resonances are highlighted for clarity. The solid, blue line marks a resonance of dot 1 and the solid, green line marks a resonance of dot 2. When only the dot 1 is confined we observe again single conductance rings as visible in panel (c). The single resonance traced with a blue, dashed line is shown in Fig. 7.30(f). Comparing the 4 traced resonances from Fig. (a-c) in panel (e), we observe that the centers of the two blue resonances coincide. These measurements demonstrate that we can establish the position of each single dot in real space. Furthermore, we learn that the position of such a single dot is not changed when a second dot is established close to it.

In conclusion we can derive the positions of both quantum dots in real space and evaluate the shift of their apparent positions due to in plane gate voltages. We carry out tip-sample-separation dependent measurements and evaluations to find hints, that our measurement is robust against the electrostatic disturbance the tip might cause in the electric field lines. In other words, the goal is to find boundary conditions which allow to interpret the measured values as independent of the tip-sample separation at which they are obtained. We show that we can form a single dot at different places inside the defined structure by applying different gate voltages. The position of a single dot stays the same, even if the second dot is created next to it.

### 7.4.5 Measurements at finite magnetic field

We now record current maps while changing the magnetic field the sample is exposed to. A series of current maps where the gate voltage settings were kept the same while the magnetic field was stepped from 0 T to 9 T in one tesla steps in shown in Fig. 7.31.

We observe that as the magnetic field is increased the current through the double dot becomes suppressed. However, other than that the precise position of all features does not change. If we follow a specific feature (i.e. a position of enhanced current) visible in panel (a) through the series up to panel (j) we see that the feature remains almost the same in shape and position relative to the dot centers (one feature is marked with a black arrow for better comparison). Applying a magnetic field does not lead to a change in the electronic state of the
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Figure 7.30: Current maps for different tunnel gate voltages $V_{STG}$ and $V_{DTG}$. By changing the voltages applied to the gates controlling the tunnel barriers, we can open the tunnel barriers, to lessen confinement of one of the dots. Panel (a-c) show current maps for different tunnel gate settings. Single conductance rings are highlighted with dashed lines for (a) and (c) and solid lines for (b). These conductance rings are again shown in the panels (d-f). Panel (g) shows a charge stability diagram of the double dot. (e-g) show higher resolution measurements of the regions where the Measurement settings: $V_{PG2} = 190$ mV, $V_{PG1} = 100$ mV, $V_{CG} = 120$ mV $V_{bias} = 210$ µV $T = 300$ mK and $d = 50$ nm.
quantum dot or shift of the Coulomb rings. It only suppresses the current through the double dot.

Our measurement does not resolve individual quantum states but rather relies on multi-level transport. Therefore changes of single quantum states with the magnetic field are not resolved i.e. the position of all features stays the same, because the overall electrostatics stays the same. As the external magnetic field changes the tunnel couplings the overall current changes.

7.4.6 Measurements at finite temperature and bias

We will now investigate the influence of temperature and bias voltage on the current maps. Figure 7.32 shows two current maps recorded at different temperatures. While features in the current map recorded at 300 mK display an average width of 30 nm, they broaden visibly when heating the sample to 1.7 K. In panel (b) a normal feature width is of the order of 70 nm.

When on the other hand changing the bias voltage while measuring at \( T = 300 \) mK, we observe that features do not become much sharper when measuring with lower bias voltages. Figure 7.33(a) shows a current map recorded with minimal bias voltage of \( V_{bias} = 10 \) \( \mu \)V. The current level compared to the noise level is worse than for the case shown in panel (b), where a measurement bias of \( V_{bias} = 210 \) \( \mu \)V was applied. Lowering the bias voltage does not change the mea-
Figure 7.32: Current maps recorded at different temperatures. (a) is recorded at base temperature (300 mK) and (b) is recorded at $T = 1.7$ K. Measurement settings: $V_{PG1} = V_{PG2} = 500$ mV, $V_{CG} = 250$ mV, $V_{STG} = 300$ mV, $V_{DTG} = -100$ mV, $V_{bias} = 210$ μV, $d = 40$ nm.

measurement since the electron temperature is above the corresponding energy scale of the bias voltage.

When applying high bias voltages such as 1 mV as shown in 7.33(c) we observe a smearing of the features again similar, but more pronounced to that observed for high temperatures.

### 7.4.7 Beyond the carried out measurements

The measurements presented in this work are the first scanning gate experiments carried out on a intentionally formed double quantum dot. We explain the dominant features present in current maps and show how they connect to transport measurements. We have analyzed the influence of temperature, bias voltage, magnetic field as well as all in plane gates and the tip on the transport properties of the sample. We show investigations of the position of the double dots and how this apparent position is shifted by gate voltages and the confinement of a second dot next to it.

In order to gain more information about the transport through a double quantum dot, we would have to increase the resolution of our data. The tip used for the presented measurements is clean and sharp. Therefore the lateral resolution of the measurements is excellent. The limiting factor in this case is the measurement temperature. Pairs of triple points are resolved in the charge stability diagrams or the current maps. Going to lower temperatures will enable us to resolve single triple points, investigate crossings and anticrossings. It will allow us to gain insight into the behavior of excited states and locally manipulate them.
7.4. Scanning gate measurements

Figure 7.33: Current maps recorded at different bias voltages (a) is recorded at a bias voltage of 10 μV, (b) is recorded at $V_{\text{bias}} = 210$ μV, (c) is recorded at $V_{\text{bias}} = 1000$ μV. Measurement settings: $V_{\text{PG1}} = V_{\text{PG2}} = 500$ mV, $V_{\text{CG}} = 250$ mV, $V_{\text{STG}} = 300$ mV, $V_{\text{DTG}} = -100$ mV, $V_{\text{bias}} = 210$ μV, $d = 40$ nm.

In this measurement we have already seen that the sample can be electrically modified by mechanically altering it using the tip [sec. J.1]. It would be very interesting to expand upon this idea. Using the tip to change the tunnel barriers in situ, maybe even start with a single dot geometry and later add a second dot in situ next to it would be a fascinating measurement. We show here that the sample can be altered mechanically by scratching it with the tip. Another approach would be to alter it by applying a large voltage to the tip, following the concept of in situ electrostatic lithography, which has been successfully carried out by Crook [26],[148],[25].

If a voltage can be applied to the tip, one could determine the least invasive voltage of the tip. This way the disturbance of the quantum system by the probing tip is reduced to a minimum allowing for even less invasive measurements.

The measurements shown in section J.1 hint that it is possible to carry out scanning gate measurements on the double dot and the charge readout simultaneously. Additionally the problem of charge traps will have to be overcome [18]. Building upon that one should be able to measure on a sample, where charge readout and double dot have their working points at the same gate voltages. This would enable us to record the current flow through the double dot in real space using the method of current maps, while simultaneously deciphering the time dependence of the transport processes using charge detecting and counting. This should allow us to gain full, space and time resolved understanding of the transport through a double dot system. Unfortunately in the regime where counting is
experimentally accessible the direct dot current is too small to be measured at the same time. Therefore spatial and time resolved quantities could not be resolved simultaneously.

7.5 Simulations

A question of crucial importance in interpreting the above discussion of the scanning gate data is knowing the exact electrostatics in the system.

On first thought we can assume, that the influence of the tip is considerably weaker than the influence of the in-plane gates due to the following facts. First of all the 2DEG lies buried 34 nm below the surface. It is covered with a layer of GaAs with a dielectric constant of $\epsilon \approx 13$ [149]. This will to a certain degree shield the quantum dots from the influence of the tip. Therefore the electric field lines of the nano-structure should not be significantly disturbed by the tip. Second this experiment is carried out with an extremely clean and sharp single tip. All problems usually created by multiple tips, with different potentials, attractive and repulsive parts are not present in the case presented here. And last in the evaluation presented above, we concern ourselves with relative as opposed to absolute positions. If the position of a quantum dot was to be shifted due to the mere presence of the tip, both identical quantum dots would be subjected to the same shift, as they are both probed by the same tip potential. Therefore the relative position, i.e. the distance between both dots is robust to such an effect.

On the other hand the inter-dot length scales are of the same order of magnitude as the tip-2DEG separation (34 nm vs. 200 nm). This will counteract the favorable effect of the capping layer. In order to quantify which effect is dominant, we carried out simulations of the electrostatics of the complete tip-double dot system.

7.5.1 Geometry and boundary conditions

The simulated geometry is shown in Fig. 7.34. The geometry has been adapted directly from the AFM scan of the structure. It consists of a metallic 2DEG (treated as a metallic layer with a thickness of 5 nm) which is cut in separate parts by oxide barriers with a dielectric constant of $\epsilon_{OB} = 10$ [150]. The shape of these oxide lines is given by the AFM scans carried out before the measurement was conducted. The oxide barriers were given a finite width of about 30 nm. The 2DEG itself is assumed to behave metallic. A layer of GaAs is located below the 2DEG-layer. Another GaAs layer with a thickness of 34 nm and $\epsilon_{GaAs} = 13$ is located above the 2DEG. The tip is assumed to have a round apex and then taper out conically. The tip is surrounded by vacuum with a dielectric constant of $\epsilon_{vac} = 1$. 
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Figure 7.34: Geometry used for the simulations with Comsol. The tip is shown in yellow and surrounded by a box of vacuum (light grey). The 2DEG (green) is separated by Oxide lines (pink). Above the 2DEG is a capping layer (grey). The whole structure is found on a substrate (grey), which as well as the capping layer consists of GaAs.

The different possible boundary conditions have been discussed and evaluated in detail in section I.1. Here we use the boundary condition described there as ”Line-charge approximation”.

7.5.2 Influence of the gates voltages

We will now investigate how strong the influence of the different gates and the tip is on the two quantum dots. To do so a geometry is chosen, where the tip is as close as possible to dot 1. As visible in Fig. 7.34 the tip is placed directly above dot 1 with a tip-dot separation of 70 nm. With this geometry we now carry out a series of simulations, where for each simulation the gate voltage of one gate is set to 1 V while the voltage applied to all other gates is zero volt. The double dot is kept on a fixed potential of 100 μV, which is in the same order of magnitude as the source drain bias voltage $V_{bias}$ used in the measurements discussed earlier in this chapter.
Figure 7.35: Induced charge (a) and electric potential (b) 0.01 nm above the surface of the 2DEG for $V_{\text{tip}} = 1$ V.

Figure 7.35(a) shows the induced charge $\sigma$ when 1 V is applied to the tip, when petitioning it directly above dot 1. The induced charge $\sigma$ is related to the electric potential $\phi$ at a $z$-height $a$ above the 2DEG surface which is positioned at $z = 0$ via

$$\sigma \sim \frac{\phi(a) - \phi(0)}{a}.$$

The induced charge is in the order of magnitude of a few $C/cm^2$.

Figure 7.36 shows the result of this series for all different gates. The black outlines mark the contours of the oxide barriers. The black circle with the crosshair marks the position of the tip projected onto the 2DEG-plane. We always show a cut of the xy-plane. The $z$-position of this plane is located directly at the upper boundary of the 2DEG. The color scale displays the electric potential from zero to 2 mV.

We observe that when one volt is applied to the tip [Fig. 7.36(a)] the electric potential right below the tip in the 2DEG is increased. This potential rapidly decays in radial direction. As the potential of the 2DEG regions is fixed, we observe a much bigger change in the electric potential in the region of the oxide lines, than directly above the 2DEG.

This becomes more obvious when looking at Fig. 7.37. Here we show the results of the simulation shown in 7.36 (a,b) in a different visualization.

We show a 3d-view on the geometry, which is indicated with black lines. Three cuts through the volume are shown simultaneously, one cut for each spatial direction. The color scale shows the electric potential in from zero to one volt. For the case where one volt is applied to the tip [Fig. 7.37(a)] we see that directly at the surface of the tip the measured electric potential is 1 V. Radially in the xy-plane the tip-potential quickly drops. Below the tip, between the end of the tip and the 2DEG-plane the tip induced potential also decreases, which is addition-
Figure 7.36: All figures show the electric potential from zero (black) to 2 mV (white). A voltage of 1 V is applied to each gate while the voltage of all other gates is fixed to zero. We show cuts of the electric potential 0.01 nm above the surface of the 2DEG for these different gate voltage settings.
Figure 7.37: Electric potential from zero (black) to 1 V (white). Panel (a) shows cuts in x, y, and z direction when 1 V is applied to the tip, while zero volt is applied to all other gates. (b) shows the same cuts for the case where $V_{PG1} = 1$ V. (c,d) Cross sections of the electric potential versus x for different z-values (red). Lorentzian fit to the electric potential in green. $z = 0$ nm marks the layer 0.01 nm above the upper boundary of the 2DEG.
Figure 7.38: Electric potential in mV at the position of dot 1 and dot 2 for two different tip-shapes sharp for (a,b) and blunt for (c,d). For each gate a simulation was carried out where a voltage of 1 V was applied to this specific gate while the voltage on all other gates was zero. The numbers in (a) give the electric potential at the position of dot 1, when a voltage of 1 V is applied to the gate on which the number is written. Analogously the numbers in (b) give the electric potential at the position of dot 2 when 1 V is applied to the corresponding gate. The same numbers are evaluated for a blunt tip geometry in (c,d).

ally supported by the existence of the dielectric capping layer above the 2DEG. Consequently the cut through the xy-plane, which is taken at the same tip-height as the cut shown in Fig. 7.36(a) appears to show no influence on the tip voltage, on the color bar chosen for this plot.

Figure 7.37(c,d) display cuts along the x-axis for different z-heights above the 2DEG. As shown in panel (d) we can fit the tip induced potential directly above the sample surface well with a Lorentzian curve shape.

When $V_{PG1} = 1$ V is applied to the plunger gate 1, we observe the electric potential distribution displayed in Fig. 7.37(b). Within the xy-plane the potential quickly decreases, as all other gate voltages are kept on zero volt. In z-direction the electric potential drops on a larger length scale than in the xy-plane.

While Fig. 7.36 shows the electric potential distribution as a whole, allowing us to judge by eye how the decay of the electric potential evoked by the different gate voltages takes place, we will now focus on a more quantitative evaluation of this data set. From each simulation shown above, we extracted the electric potentials at two points. One point below the tip, which marks the geometric center of dot 1 and the second point in the corresponding geometric center of dot 2. Using this evaluation we now obtain two values for the electric potential for each gate-voltage setting. Figure 7.38(a) shows those numbers in meV for dot 1. The electric potential measured at the position of dot 1, when $V_{CG} = 1$ V is marked on the center gate. The electric potential measured at the position of dot 1, when $V_{tip} = 1$ V is marked at the cross hair marking the geometric position of
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the tip. This notation is used for all gates. Panel (b) shows the values measured at dot 2, employing the same notation.

The electric potential measured at the position of dot 1 is a factor of two larger than the electric potential measured at dot 2 when the tip is placed directly above dot 1. For this geometric arrangement the maximal lever arm of the tip is about the same as the lever arm of PG1: $\alpha_{PG1}/\alpha_{tip,max} \approx 1$. This is the maximum value calculated for the position where the tip has the highest possible influence on the sample. As we will discuss below, this value quickly drops when the tip is laterally moved.

We have to note the following: We probe the electric field at a position that has the same xy-coordinates as the apex of the tip. Therefore we are geometrically as close as possible to the tip, at the position where the influence of the tip is at its maximum. The in-plane gate in contrast is about 150 nm separated from this probing position. Therefore the electric field evoked at this position by the in-plane gate will already have significantly dropped. The lever arm ratio $\alpha_{PG1}/\alpha_{tip,max} \approx 1$ is therefore measured at conditions that are all most favorable for the tip. The value given here presents an upper boundary for the lever arm. In a real experiment this value will be smaller depending on the lateral tip position.

The lever arms of the tunnel gates and the QPC gates on their corresponding dot are the same. The difference of the change in electric potential evoked by QPCG1 on dot 1 and dot 2 is by about 10%. The lever arm of the QPC-Gates and the tunnel gates on their corresponding dot, compared to the influence of the plunger gate in question is $\alpha_{PG1}/\alpha_{TG} \approx 6$. Due to shielding of the center gate voltage by the tip, the electric field produced by the center gate at the position of dot 1 is slightly smaller than at the position of dot 2.

7.5.3 Influence of the tip

In order to gain an understanding how big the influence of the precise tip shape on the electric potential produced by the tip is, we repeat the simulation series for a different tip shape. Until now the tip used in the simulation consisted of a cone, that was capped at the bottom with a half sphere with a radius of 50 nm. Now instead of a half sphere the tip was cut, producing a blunter tip shape by removing the half sphere.

Figure 7.38(c,d) shows the electric potential for dot 1 and dot 2 for this blunt tip shape. While the electric potential of the in plane gates is not significantly influenced, the tip induced potential slightly increases for the blunt tip, owing to two facts: First this tip shape incorporates sharp edges leading to field exaggeration at these points. Second the amount of tip-material close to the sample surface increases for this blunt shape.
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Figure 7.39: Electric potential from zero (black) to 2 mV (white) for xy-planes taken at different z-positions. \( z = 0 \) nm marks the layer 0.01 nm above the upper boundary of the 2DEG. The capping layer on top of the 2DEG has a thickness of 34 nm. 1 V is applied to the tip. The voltage applied to all other gates is zero.

We will now look at the z-evolution of the tip potential for the sharp tip shape and the gate-voltage settings of \( V_{\text{tip}} = 1 \) V. We show four representative cuts of the xy-plane for different z-heights above the 2DEG in Fig. 7.39. The electric potential in panel (a) shows a small potential peak right below the tip of less than 1 mV. The potential decreases by a factor of two within several tenth of nanometer in radial direction and is slightly higher in the region of the oxide barriers, due to their dielectric properties. When moving closer to the surface of the sample [panel (b,c)] the shape of this potential stays the same and it slowly increases in amplitude. Upon exiting the capping layer [panel (d)] the tip potential increases strongly.

To investigate the effect of the presence of the tip (i.e. if it the field lines are noticeable distorted by the mere presence of the metallic tip at zero volt) we carry out the following two simulations. First the electric field is determined for the described geometry including the tip at zero volt. For this geometry the electric field at dot 1 and dot 2 was determined. In a second step the same simulation was carried out for a geometry without the tip. Again the electric field at dot 1 and dot 2 was determined. We find, that the electric field at the dots for both cases constant within the calculated accuracy. We can therefore conclude, that the mere presence of the tip does not greatly influence the electric potential at the position of the double dot.

To investigate the influence of moving the tip laterally simulations for the case where \( V_{\text{tip}} = 1 \) V were carried out for four different tip positions shown in Fig. 7.40. The tip was placed at two different tip-dot separations: Once \( d = 70 \) nm and second \( d = 200 \) nm. Additionally the lateral position of the tip was changed. First it was placed directly above dot 1, then it was moved by 450 nm over PG1.
Figure 7.40: Electric potential for a sharp tip shape. The color scale shows the electric potential from 0 (black) to 2 mV (white). The tip is located at two different xy-positions and two different z-positions. The voltage applied to the tip is 1 V. (a) shows the case where the tip is positioned directly above dot 1 with \( d = 70 \) nm. For (b) the tip is retracted to \( d = 200 \) nm. For (c,d) the tip is displaced 450 nm and the geometry is again simulated for \( d = 70 \) nm (c) and \( d = 200 \) nm (d). The table in (e) shows the electric potential at the position of dot 1 and dot 2 for different tip positions. We use the following voltage settings: \( V_{\text{tip}} = 1 \) V, \( V_{\text{STG}} = V_{\text{DTG}} = V_{\text{QPC1}} = V_{\text{QPC2}} = V_{\text{QPC1gate}} = V_{\text{QPC2gate}} = V_{\text{CG}} = 0, V_{\text{DD}} = 1 \cdot 10^{-4} \) V.

<table>
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<tr>
<th>x(nm)</th>
<th>y(nm)</th>
<th>z(nm)</th>
<th>Epot dot1(V)</th>
<th>Epot dot2(V)</th>
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</tr>
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<td>200</td>
<td>2.38E-04</td>
<td>1.89E-04</td>
</tr>
</tbody>
</table>

When the tip is retraced by 130 nm from the surface while placed directly above dot 1 the tip potential felt by both dots changed by a factor of two (from 0.77 mV to 0.33 mV for dot 1 and from 0.36 mV to 0.19 mV for dot 2). When the tip is laterally displaced while kept close to the surface the tip potential at dot 1 changed from 0.77 mV to 0.26 mV. Simultaneously the tip potential at dot 2 changed 0.36 mV to 0.19 mV. This means that the tip potential at dot 2 was altered the same by both tip movements, while a lateral tip-movement of 450 nm proved to have a bigger influence on the tip potential at dot 1, then retracting the tip by 130 nm.

Finally the tip was retracted to 200 nm while placed above PG1. We now find a tip potential of 0.24 mV at dot 1 and of 0.19 mV at dot 2. The tip potential at both dots changes only minimally once the tip is moved further away than a certain threshold of somewhere between 200 – 400 nm.
In short: The mere presence of the tip does not alter the electric potential at the dot significantly. The tip induced potential decays quickly with a length scale of few hundred nanometers. Small changes on the tip geometry do not alter the tip potential.

### 7.5.4 Shape of the tip induced potential

We can now derive the shape of the tip induced potential for $V_{\text{tip}} = 1$ V for the used geometry in the plane $0.01$ nm above the 2DEG.

$$\phi(x,y,d) = \frac{A(d)}{\pi} \frac{0.5\Gamma(d)}{(x-x_0)^2 + (y-y_0)^2 + (0.5\Gamma(d))^2}$$  \hspace{1cm} (7.31)

The dependence of the full width at half maximum $\Gamma$ for the geometry under investigation on the tip-sample separation $d$ is given by:

$$\Gamma(d) = 2.88 \cdot 10^{-9}d + 1.34 \cdot 10^{-7}$$  \hspace{1cm} (7.32)

The dependence of the amplitude $A$ for the geometry under investigation on the tip-sample separation $d$ is given by:

$$A(d) = 74.05d^{-0.982}$$  \hspace{1cm} (7.33)

### 7.5.5 Comparison to the experimental data

The electric potential of the tip at $z = 0$ can be best described by a Lorentzian shaped potential as shown on the example of a single cut in Fig. 7.37(d). From this fit we can derive the full width at half maximum and the amplitude of the tip induced potential at the position of the sample for different tip positions. We carry out this simulation and fit for a series of tip-sample distances $d$. For each tip-sample distance we extract the amplitude of the tip induced potential $E_{\text{fit}}$. The result is shown as the blue data points in Fig. 7.41. We observe a strong decay in $E_{\text{fit}}$ for small tip-sample distances which flattens for tip-sample distances larger than $d = 100$ nm.

The amplitude of the tip induced potential is a quantity that is experimentally accessible. We know that the energy separation between two Coulomb resonances is about 1 meV. From the number of Coulomb rings of each dot in a single current map we can therefore deduce the amplitude of the tip induced potential. We now determine the evolution of this amplitude $E_{\text{tip}}$ for different tip-sample distances from the data set shown in Fig. 7.21. The extracted dependence $E_{\text{tip}}(d)$ is shown in Fig. 7.41 as the red data points.

We observe an almost perfect agreement between the simulated and the measured $E_{\text{tip}}(d)$ behavior. Even in absolute values the simulation is within a factor of two of the measured numbers.
Figure 7.41: Evolution of the amplitude of the tip induced potential in the sample plane derived from the simulation (blue) and the measurement (red).

7.5.6 Conclusion

In conclusion the simulations show, that the upper boundary for the lever arm of the tip $\alpha_{\text{tip, max}}$ is comparable to the lever arm of the in plane plunger gates PG1 and PG2. This maximum lever arm of the tip is reached when the tip is positioned directly above one dot, 70 nm above the dot. The lever arm decays quickly when the tip is moved away from the sample in any direction or when the electric potential is determined at a position that lies not directly below the tip but rather in the vicinity of the plunger gate. When the tip is more than a few hundred nanometers away from the sample, the change in electric potential at the dot position induced by the tip becomes negligible.

The dielectric capping layer shields the sample from the influence of the tip. Therefore the change in lever arm of the other gates in dependence on the position of the tip is very small and the electric field at the position of either dot does changes by less than 3% when a tip with $V_{\text{tip}} = 0$ is positioned above the dot, compared to the geometry without a tip. We can therefore conclude that the electrostatics of the sample is not significatively altered by the presence of the tip in the scanning probe setup.

The shape of the tip showed to have little influence on the lever arm, as long as it was only altered from half spherical to blunt. Therefore small tip alterations
7.6. Conclusion

In conclusion we have presented transport and scanning gate investigations carried out on a GaAs-AlGaAs double quantum dot fabricated by local anodic oxidation [sec. 7.2]. The transport measurements and extracted values are consistent with double-dot measurements carried out by other groups [sec. 7.3].

Additionally, scanning gate microscopy gives local access to the transport properties of the double quantum dot. The most dominant features occurring in a current map can be explained based on the idea of transport levels and interdot coupling leading to the formation of distorted hexagons [sec. 7.4.1].

We characterize the tip and alter it until we can perform measurements with a perfectly clean and sharp single tip, which induces only a small local potential [sec. 7.4.2]. We show that single electrons can be manipulated using the scanning tip.

The center gate can be used to suppress transport through the whole structure [sec. 7.4.3]. Within a small gate voltage range, however, changing the voltage applied to it by $\Delta V_{CG} = 40 \text{ mV}$ allows us to load one single electron onto each of the quantum dots. As Coulomb resonances occur roughly periodically in plunger gate voltage as well as in the scanning gate measurements this is periodic with $\Delta V_{CG}$. The electronic structure observed in a current map is not altered when an additional electron is loaded onto the quantum dot using this method.

We are able to determine the apparent positions of the two quantum dots in real space and shift them using the plunger gates. We show the reliability of this assertion using two different methods, the grid method, where we record $I(V_{gate, \text{tip-position}})$ and follow the maxima of the resonance peaks and second the ellipse method, where current maps for different tip-sample separations and different gate voltages are recorded. Both evaluations show results which are

occurring during a measurement are not expected to significantly change the measurements.

The lever arms of all in-plane gates on their corresponding dot are about a factor of five bigger, than the influence of those gates on the other dot. The lever arm of the plunger gates is also a factor of five bigger, than the influence of the tunnel gates STG and DTG and the QPC gates QPC1Gate and QPC2gate.

We can accurately describe the shape of the tip induced potential as a Lorentz shape. An equation for the shape of the tip induced potential is given in equation 7.31. Its decay with increased tip-sample separation agrees strikingly well with the experimental results. The simulated values are within a factor of two of the measured values. We see that at tip-sample distances above 150 nm the change in amplitude of the tip induced potential becomes very small.
consistent with each other, showing that the lateral position of both dots can be shifted by about 20 nm/V by applying voltages to the in plane gates. At extreme voltage settings we can change from a configuration where only a single dot is formed to a configuration where a double dot is formed. By recording current maps we can distinguish where in the structure the single dot has formed and find that we are able to form a single dot that does not shift laterally when we form a second dot next to it, forming a complete double dot system [sec. 7.4.4].

We investigated the possibilities inherent to such a structure such as being able to combine spatial resolution from the current maps with time-resolution from the charge readout [sec. J.1] as well as altering the electronic structure of the sample in situ by mechanically scratching it with the tip. We elaborate on the possibilities of such measurements, when carried out in future cooldowns and name some follow up measurements in section 7.4.7.

To gain further insight into the electrostatic setting of the nano-structure, specifically the role of the tip, we carry out electrostatic simulations of the precise geometry using Comsol [sec. 7.5]. We find that the lever arm of the tip is maximally as big as the lever arm of the plunger gates and decays quickly when the tip is laterally moved away from the dot position. The voltage of the in-plane gates as measured at the position of both quantum dots is not significantly influenced by the presence of the tip.
Chapter 8

Leakage currents through oxide lines fabricated with local anodic oxidation

8.1 Introduction

The method of local anodic oxidation has proven to be extremely useful to fabricate different nano-structures in a 2DEG [135]. One key point of this technique is, that using the same fabrication method one can define QPCs [151], quantum dots [151], coupled quantum dots [152, 10], quantum rings [153, 12], or any combination of these different nano-structures [154] with high precision.

Despite the obvious opportunities offered by this technique one of its disadvantages is the occurrence of undesired leakage currents between different parts of the structure. This limits the accessible parameter range and therefore tuning options. Sometimes this eliminates the possibility to reach the desired measurement regime without the occurrence of undesired and uncontrolled leakage currents. Ideally no current should flow when applying a voltage to a single isolated part of a structure defined by local anodic oxidation, while keeping the other, electrically isolated parts of the structure grounded. However we observe that above a certain voltage $V_C$ a current (leakage current) can flow from the investigated part of the structure to other parts of the structure, which should be electrically disconnected via potential barriers formed by the oxide lines [sec. 8.3].

These leakage currents are known to occur in all nano-structures fabricated with local anodic oxidation above a sample-dependent threshold voltage. However little is known about the origin of these currents. If a leakage current flows between two adjacent regions it is not known, if this current occurs homogeneously along the complete oxide line or rather on one particular point.
Scanning gate microscopy enables us to investigate where leakage currents flow. When recording a current map while a leakage current flows we will be able to locate this leakage current [sec. 8.4]. Comparison with topography scans enables us to identify at which position the current crosses the isolating oxide barrier. This in turn allows us to make a connection between the topological appearance of the oxide barrier and the most likely position where a leakage current will flow.

By making this connection we are able to pinpoint "weak" points from the topography of AFM-written nano-structures, enabling us to adjust the fabrication processes in a way to minimize leakage and therefore maximize tunability. Ideally this might even give access to measurement ranges not accessible in present samples [sec. 8.5].

8.2 Sample and measurement setup

8.2.1 Measured sample

The sample investigated is a multi-terminal nano-structure fabricated via local anodic oxidation. It consists of a double dot structure with multiple gates and charge readouts as described in chapter 7 in more detail.

Figure 8.1(a) shows an AFM scan of the structures topography recorded with a commercial AFM at room temperature. The image was recorded before any transport or scanning gate measurements. The labels on the different terminals in (a) will be used from now on to refer to specific parts of the structure.

During the measurement the sample has been altered by tip crashes resulting from a power failure as well as mechanical intrusions on the entire measurement cryostat system. Figure 8.1(b) shows an AFM-topography scan recorded after the scanning gate measurements have been performed. The sample has been obviously altered. Probably the tip has been scratched across from PG2 over the center gate \( CG \) to PG1 and QPCG1 as highlighted by the red circle. Material abrasions most likely from the tip have been deposited along this path on the oxide lines.

Additionally a few particles have been deposited on the sample.

8.2.2 Measurement setup

In order to measure the leakage current from one terminal to another terminal a voltage \( V \) was applied to the investigated terminal. All terminals that are not electrically connected to the terminal were connected to ground. Terminals that are not separated from the terminal in question by an oxide barrier were kept floating (such as the source contact \( S \) was kept floating when a voltage was applied to \( D \)).
8.3. Transport measurements

Leakage current $I_L$ is an unintentional current that flows from one terminal to another terminal to which no intentional electrical connection exist. In the case of multi-terminal nano-structures fabricated via local anodic oxidation a leakage current can flow across oxide barriers, when the voltage difference between the two terminals is large enough.

Figure 8.3 shows the leakage current that flows from terminal $QPCG2$ versus the voltage applied to this terminal. We observe a linear rise in current [Fig. 8.3(b)] up to a voltage of about 0.7 V. This current corresponds to a resistance of about 3 $T\Omega$ and is most likely limited by the leakage between cables. The finite current at zero applied voltage results from the input voltage offset or the output voltage offset of the I-V converter.

Above the break-through voltage $V_C$ the current rises exponentially [Fig. 8.3(a,c)] with the applied voltage.

Figure 8.1: Sample on which the leakage currents were investigated. (a) shows a topography scan of the sample before the cooldown. (b) shows a topography scan after the cooldown. The notations in panel (a) will be used in the following to refer to specific terminals of the sample.

The current flowing to ground was recorded using a standard IV converter and a HP multimeter as shown in Fig. 8.2. All measurements were recorded at 300 mK.
Figure 8.2: Measurement setup to record leakage currents in the structure.

Figure 8.3: Leakage current of gate QPC2G versus Voltage applied to this gate $V_{QPC2G}$. (a) complete IV trace. (b) zoom into the linear range. (c) Zoom into the region around $I_L$ where the IV dependence becomes logarithmic. The red trace is plotted on a linear scale, while the blue curve shows the same data plotted on an logarithmic axis.
8.4 Scanning gate measurements

The amplitude of the leakage current is given by the voltage applied to the specific terminal. Leakage current of several hundred nA have been evoked within these measurements. However in order not to alter the sample permanently the leakage currents used for the scanning probe measurements shown here were limited to the order of pA.

As discussed in the last section the leakage current rises exponentially with the voltage once a certain break-through voltage is reached. Our goal is now to find out where in the structure the leakage current flows. In our case a tip inducing a repulsive potential is used to record a current map. Therefore we expect the leakage current to drop, when the tip is positioned right above the location where the current flows through the oxide barrier.

If the leakage current was to flow homogenously along the complete oxide barrier, we would expect a long region of suppressed current along the complete oxide barrier. As the combined length of one complete oxide barrier is several tenths of micrometers, we would additionally expect this to be an extremely small effect. If the current was to flow only at a distinct position we expect to record current maps, where the current is constant except at a single position which marks the position where the oxide barrier is the weakest allowing for current to pass through it. At this position the repulsive tip is expected to lower the leakage current, leading to a single spot of suppressed current in an otherwise featureless current map.

Figure 8.4(a) shows the current map for the leakage current from terminal PG2. The position of the oxide lines of the sample are determined by a topography scan directly after the current map was recorded and sketched in accordingly as black lines. The terminal labeled with the letter PG2 is the terminal the voltage was applied to. All other terminals were connected to ground. The current map displays a single spot of suppressed current in an otherwise featureless current map. The center of this area of suppressed current coincides with the geometric position of an oxide line.

Analogous measurements are performed for the terminal PG1 (d), QPCG1(b), QPCG2(e), STG(f), DTG (c), CG(g) and the double dot itself in (h). All current maps (a, b, c, d, f, g, h) display a single spot of suppressed current. For most current maps this is a rather distinct spot of a diameter of about 1 μm. Only the barrier between STG and PG1 shown in (d) and (f) shows a more extended area of suppressed current.

The position where the leakage current is observed is independent of the voltage applied to the terminal under investigation. Below the break through voltage $V_C$ no leakage current could be observed in the current maps. For voltages above $V_C$ a leakage current can be observed as shown in Fig. 8.4, where the best visibility is
Figure 8.4: Current maps for the leakage current of the various terminals of the structure. The terminal on which the voltage was applied is marked with the corresponding abbreviation. The maps show the current flowing from the investigated gate. The color bar for the measurement (a) is shown next to the corresponding scan frame. The size of the color scale for the measurements (b-h) is adjusted to display maximum contrast for each image usually covering a change of a factor of two to four in current. All measurement voltages are in the order of one volt. (i) Room temperature topography scan of the structure. The positions where leakage currents appear between two adjacent structures within the scan frame that could be evaluated are marked with red circles.
found for voltages that are within a few hundred mV of $V_C$. This voltage proves to be around 1 V for all terminals. When the applied voltage is set to much bigger values leakage currents of several tenths of nA can be observed. As the tip potential then becomes comparably small to the energy scale of the applied voltage, the current maps become less pronounced for higher voltages. However the position where the leakage current is found does not show any voltage dependence.

The position of the leakage current flowing from terminal QPCG2 [see Fig. 8.4(e)] cannot be determined. We can measure a leakage current in the transport measurement, however its position is not in the scan frame available within this measurement.

When we measure the leakage current from DTG we observe that it leaks from DTG to terminal $D$, reversible when applying a voltage to $D$ its leakage current flows to DTG. Therefore this point is the weakest connection all along the connection lines between the Double Dot and all other gates.

On the other hand the leakage current from CG flows to PG2, while the leakage current from PG2 flows to DTG. The barrier between DTG and PG2 is therefore weaker than the barrier between CG and PG2.

We now mark all the positions where we observe leakage current onto a single topography scan shown in Fig. 8.4(i). The red circles mark the positions where we are able to localize leakage currents within the given scan frame. We observe a striking similarity: For five out of the six positions where leakage was observed the leakage current flows at a position where two oxide lines intersect. As a naive guess these crossings would be expected to be more robust to leakage currents than the oxide lines, as the sample should be depleted in a larger region below it. However this seems not to be the case.

Interestingly no leakage current could be observed near the smallest features of the structure i.e. close to the point where the double dot connects to all surrounding gates. Although this part of the structure is usually regarded as the most sensitive, leakage current appear rather on the leads, than in the nano-structure itself.

The favored occurrence of leakage currents at oxide line crossings can be explained as follows: When a first oxide line is written, it forms a pronounced oxide region. When forming the second oxide line, the AFM-tip is moved in feedback over the surface. When crossing the first oxide line, the effective write distance from the tip to the 2DEG is larger, than when writing a line on a 'fresh' part of the substrate, as the tip has to follow the 'hill' formed by the first oxide line. Due to this increased effective write distance the energy barrier of the second oxide line is weaker around the crossing. Thus it is likely that a leakage current will flow in the vicinity of a oxide line crossing.
The sample is produced in such a way that first the nano-structure is formed. Then the first set of oxide lines from the nano-structure to the first crossing is written for all terminals. Afterward the lines between first and second crossing are formed for all terminals and so on. At the position of the first crossing the writing tip is still very sharp, allowing for a more pronounced oxide barrier. When writing the second crossing the tip has already degraded, making those crossings prone to be the weak points within an oxide barrier.

8.5 Conclusion

8.5.1 Summary

In conclusion leakage currents could be measured between different terminals of the nano-structure. With the method of scanning gate microscopy we were able to locate the position at which the leakage current flows. The small repulsive force of the tip is hereby big enough to decrease the leakage current by roughly a factor of three when the tip is placed directly above the position of the leakage. We could observe that a leakage current does not flow uniformly across the oxide lines of a nano-structure along their whole length, but rather crosses the oxide barriers at distinct points.

Surprisingly leakage currents do not arise at the very center of the nano-structure but rather at its leads. Regions especially prone to produce leakage currents are crossings of two oxide lines.

8.5.2 Consequences

Contrary to intuitive expectations most leakage currents do not flow at the most complex part of the nano-structure-i.e. the center of the structure where the double dot connects to all gates. Leakage currents appear with higher probability on the leads, where oxide lines cross. A sample that minimizes leakage currents should therefore be a sample, where the oxide lines defining the leads are ideally continuous lines, with as few crossings as possible.

Another option are the so called hybrid structures. Such samples are patterned by the combination of AFM-produced oxide lines and metallic top gates. The most complex parts of the structure could be patterned using AFM lithography, while the leads farther out could be defined via top gates.
Chapter 9

Outlook

9.1 Summary

In this thesis we have shown the successful construction and operation of a home-built scanning force microscope operated in a $^3$He refrigerator with a base temperature of 300 mK. We have shown its capability to investigate diverse nanostructures ranging from rings to dots and double dots made from materials such as superconducting aluminum, graphene and GaAs/AlGaAs 2DEGs.

We have discussed the setup and the different parts, with special focus on avoiding heat input via the cabling as well as different versions of the stick-slip coarse positioning motors. Operating instructions for the cryostat and the microscope are given in chapter 4 and the Appendix.

The measurements on the graphene ring have shown a high visibility of the Aharonov-Bohm oscillations. Local gates and a back gate were used to locally and globally tune the charge carrier density and allow to observe the electrostatic Aharonov-Bohm effect without the use of tunnel barriers in the arms of the ring. We have shown that by changing the voltage applied to one of the side gates, we can induce a phase jump in the oscillations by changing the phase accumulated along this path. Scanning gate measurements on the same structure reveal an increasingly complex potential landscape when spatially resolving transport close to the Dirac point, as well as underlying fine structure even in the current maps of a more straight forward appearance.

The scanning gate measurements on the SET show excellent correlation between scanning gate measurements and charge stability measurements. Small differences remain. This leads to the conclusion that the in-plane gate has a slightly different effect on the SET than the movable out-of-plane gate: The period of the Coulomb blockade peaks can be influenced by the lateral position of the SFM tip. The tip potential consists of two parts; only one of which depends on the voltage
applied to it. We furthermore observe an anticorrelation between the gate capacitance and the tip capacitance, which can be explained in terms of the screening the tip exerts. The exact origin of the observed and surprising anticorrelation between the charging energy and the superconducting gap cannot be explained by the electric field configuration of the setup which has been simulated considering the geometry of the tip and the device.

Scanning gate measurements on a GaAs double quantum dot give local access to the transport properties of the double quantum dot. We demonstrate that single electrons can be manipulated using the scanning tip. We could derive the positions of both quantum dots in real space and evaluate the shift of their positions due to in-plane gate voltages. We showed that we can form a single dot at different places inside the structure defined by lithography by applying different gate voltages. The position of a single dot stays the same, even if the second dot is created next to it.

Scanning probe experiments on a 2DEG-based structure fabricated via local anodic oxidation aim to investigate leakage currents. We show that leakage currents occur at specific points across the oxide barriers as opposed to occurring along the whole length of a barrier. We can show that the probability for the appearance of leakage currents it highest at positions where two oxide lines intersect.

9.2 Outlook and Visions

9.2.1 Setup Improvements

The goal in scanning gate microscopy is to use the tip—a small potential with a simple shape—to locally probe transport properties of nano-structures. In order to be able to interpret the data on this basis the tip potential has to be small, with little spatial extent and of a simple shape. To achieve this the tip shape has to be sharp and clean. This can be accomplished by two different approaches:

- Start with a sharper tip.
- Minimize wear on the tip prior to the experiment.

Starting with a sharper tip can be archived by optimizing it on the nano-scale using EBD wires grown onto the electrically etched tip, further sharpen it via FIB, or try applying commercially available tips as described in section 4.8. This allows us to start with tips that have an initial tip apex radius of about 14 nm. Additionally all those options provide us with geometrically well defined and symmetric initial tip shapes. Some commercial available tips even employ the possibility of electrical shielding, further refining the tip induced potential.

On the other hand one has to keep the initial tip as sharp and clean as possible during the steps leading up to the scanning probe experiments. One step into
the right direction was the implementation of the ATTOCUBE SYSTEM motors with closed loop positioning option into our system. However to be able to fully explore the possibilities that come with the closed loop positioning system, the investigated samples should always be equipped with markers. In that case the positioning upon reaching 4.2 K could look as follows:

- Perform a topography scan of a single scan frame.
- With a unique marker every 7 μm the position of the tip is then known after the first topography scan.
- Position above the sample using the closed loop system.
- Employ scanning ”feel” methods for fine positioning.

With enough markers only a single topography scan would have to be carried out, which would immensely decrease tip wear compared to the almost 100 scan frames recorded before positioning above the SET and carrying out the measurements presented in chapter 6.

Additionally, due to implementation of the ATTOCUBE SYSTEM motors we have gained about another 1.5 cm of space in z-direction. We could use this new space to build in a new, longer scan piezo. A longer scan piezo comes with a larger scan frame, making positioning easier. As an alternative we could keep the scan frame the same as it is currently and decrease the voltage applied to all electrodes. This should counteract the too frequent depolarization of the scan piezo. Albeit a longer scan piezo would be beneficial for positioning, we have to keep in mind that an increased scan frame comes with the expense of decreased stability and increased noise. The noise could be decreased by using voltage deviders close to the setup, i.e. applying smaller voltages to the scan piezo or using a smaller scan piezo. On the other hand sample positioning would benefit from a scan frame, that was as large as possible. One has to weight both sides and decide on which effect is more important, depending on the experiment at hand. One possibility is certainly to apply large voltages while positioning the tip above the sample and include voltage dividers for the actual measurements.

Currently the setup is in a comparably robust, fully functional state. With a few alterations (markers on the sample, better tips, maybe a larger scan piezo) performing scanning probe experiments is expected to be not nearly as cumbersome as we have known it to be in the past. This should lead the way to accomplish more measurements on different samples, even enabling us to explore the potential of this methods away from the classical ”non contact” scanning probe experiments carried out currently.
9.2.2 Visions

To extract information about the quantum mechanical processes taking place in our nano-structures using scanning probe methods, we have to know exactly the kind of perturbation our probe causes in the sample. Currently the effects of the tip potential and the effects of the transport properties are still entangled in our measurements. If we could differentiate between them, all measurements to come would be open to a completely new interpretation, as we could ”subtract” the features arising from the tip potential and be left only with the features connected to transport processes in the sample. To gain a full understanding of the tip-induced potential it would be useful to take simple structures such as a quantum dot and study the same sample with different sensors (i.e. different shapes, materials, contamination levels...). The difference between each data set can then be attributed to the changed sensor, allowing us to see how different sensors influence the probing mechanisms. In all measurements to come we could thereafter subtract the influence of the tip from the data, leaving us with data showing the pure, unentangled physics [see Fig. 9.1(a)].

We could also perform follow up work on nano-rings. Using the tip as a local gate we could start with a large quantum dot and by ranging the tip voltage $V_{\text{tip}}$ create a potential barrier in the center of the dot changing from a large dot geometry to a ring geometry, allowing us to study the transport properties of the continuous evolution of a ring structure. This allows us to study when the...
Aharanov–Bohm effect becomes observable. We could also investigate how the manifestation of the Aharanov–Bohm effect changes, when we shift the center of the ring to a different position, making one arm wider and the other one smaller or form a deliberate asymmetry in the ring structure along different axis [see Fig. 9.1(b)].

We could also use magnetic tips to study superconducting samples. Magnetic fields destroy superconductivity. If we apply an external magnetic field right below the breakdown field $B_C$, we could use our tip to locally disturb superconductivity. If the sample was a simple superconducting wire we could therefore think of moving a normalconducting island through the wire [Fig. 9.1(d)]. Superconducting rings might be even more fascinating, exploring how transport changes, when for example only one arm of the ring was to be normalconducting. Obviously for such experiments more detailed thoughts taking into considerations the superconducting phase coherence length and the proximity effect would have to be carried out to ensure the feasibility of such a project.

Starting with so called nano-gaps we could use the tip to form a connection between the two sides of the nano-gap [Fig. 9.1(c)] thus forming an in situ modifiable junction that could for example be implemented in a split ring.

When starting with an antidot lattice [Fig. 9.1(f)] we could use the tip as a single additional antidot that we can move through the structure.

The concept of ”erasable electrostatic lithography” has been already shown [26, 25]. Furthermore we have already seen within this work that mechanical ”scratching” of a sample fabricated via local anodic oxidation can greatly alter it. Building upon this one could try to modify nano-structures in situ [Fig. 9.1(g)]. We could start with characterizing the locally resolved transport properties of a single dot, then modify one tunnel barrier intentionally by scratching over it and investigate how this alters the sample.

Graphene also opens a whole zoo of new possibilities to scanning probe methods, as it is directly mechanically accessible. Besides the more obvious questions, such as imaging edge states or even tunneling into the edge states of Hall bars [Fig. 9.1(e)] the most pressing question is the position of the localized states/ puddles in graphene. We could use scanning gate microscopy to identify the position of localized states, find if they form on the edge or in the center of nano-structures such as ribbons, constrictions or any other nano-structure that displays dot-like behavior. We could use different samples to evaluate how different processing mechanisms lead to more or less localized states. Ultimately the goal would be to understand the mechanisms behind the formation of localized states and to be able to control them. As graphene is directly accessible with the tip, we could also tunnel into them, trying to controllably charge or deplete them. However, we could also try more exotic approaches.
When working on suspended graphene flakes [Fig. 9.1(h)] we could try to alter them by rubbing the tip along their edges. Depending on the tip material we might even be able by doing so to change the edge structure via depositing molecules from the tip onto the graphene edges. As graphene is expected to be more stable than the tip material, we could decorate the graphene edges with tip material, or change adsorbed molecules on the edges. We could also push down on the graphene [Fig. 9.1(i)] to investigate how transport through graphene changes with applied tension. And as a far fetched thought: It is a big dream of people working on graphene nano-structures to have ultimate control over the edges of a structure. As control over single atoms has been proven to be possible by people who intentionally move single atoms to designated positions using scanning probe methods, it should in theory be possible to use scanning probe methods to modify and engineer the edges of a graphene nano-structure in situ [Fig. 9.1(j)].

As we have seen scanning probe microscopy holds a sheer infinite amount of opportunities, that will help us to understand how transport through our nano-structures works. The setup built within this thesis is in excellent condition right now and is expected to function well when operated with the appropriate care. By employing the necessary caution and the steps to preserve the tip described above, in combination with samples sporting appropriate markers, we expect that the production of fascinating results is possible. On the one hand “conventional” scanning probe experiments carried out with much sharper tips (and therefore higher energy resolution, as the observed 1 meV/15 nm i.e. charging energy/minimal Coulomb ring spacing) should really enable us to look into the quantum properties of transport, on the other hand more inventive scanning probe methods will open the door to a whole new field of transport understanding through in situ sample modifications.
Appendix A

Instruction: Cooldown from 300 K to 77 K

_starting position:_ Dewar and insert are at RT, insert is pumped to $10^{-5}$ mbar

1. Connect the 1K-Pot pumpline and the sorbtion pump pumpline.
2. Make sure the needelvalve is closed.
3. Make sure the green valve on the cryostat that goes to the pump is closed.
4. Pump the 1K-Pot and the pumpline to the sorbtion pump → pressure in the 1K-Pot goes to zero.
5. Ideally don’t disconnect the pumplines, so the 1K-Pot and the pumpline to the sorbtion pump are pumped at all times!
6. Put the LN$_2$ transfer tube into the He-transfer opening (should go all the way in—there is a red mark about 40 cm above the tube end that indicates that position).
7. Switch the output of the He main bath exit so it goes into air (red and yellow valves on the panel on the wall). The main bath exit has to have an open exit.
8. Open the valves on the right wall, so the main bath and the LN$_2$ shield will fill at the same time.
9. Switch on the sample heater.
10. Make sure the 1K-Pot is pumped.
11. Check if the sniffer pieces of the sorbtion pump and the 1kpot are securely fixed to the insert.

12. Put the insert into the dewar.

13. Fill in LN$_2$ (takes around 1.5h to fill in a complete LN$_2$ dewar).
Appendix B

Instruction: Cooldown from 77 K to 4 K

starting position: Dewar and insert are at LN$_2$

1. Start sample heater.

2. Push out the LN$_2$.
   - Close the main bath exit that goes into air.
   - Put the LN$_2$ transfer tube end that is attached to the wall into something that can hold LN3.
   - Connect the „Gas“exit from the LN$_2$ bottle over the tubes that run on the right wall over the pannell at the wall to the main bath.
   - Check that the LN$_2$-transfer tube is all the way in the main bath (i.e. only about 30cm of the metallic tube are sill outside).
   - Open the LN$_2$-bottle → push LN$_2$ out off the main bath. The pressure of the main bath should not rise above ~ 0.3. When the main bath is empty the pressure drops to zero and no liquid comes out of the tube any more.

3. Flush
   - Connect the He-gas bottle to the main bath.
   - Disconnect and close the path to the LN$_2$ bottle.
   - Start flushing the main bath with He.
   - Remove the LN$_2$ transfer tube.
• Put the He-transfer tube (main bath part) into the main bath (all the way in). Make sure the transfer tube is closed on top!!
• close all the valves to the pumps of the 1kpot and the sorbtion pump on the metal plate.
• Check that the needle valve is closed.
• Open „to pump“valve on the panel → pressure in the main bath goes to -1.
• Close the „to pump“valve.
• Carefully open the „warm bath“valve → main bath pressure goes to +0.2.
• Close „warm He“valve → pressure in the main bath goes to -1.
• Repeat those last 6 steps three times.

4. Fill in He
• Close the „to pump“valve.
• Carefully open the „warm bath“valve → main bath pressure goes to +0.1.
• Put the transfer tube into the He bottle and connect to the other side of the transfer tube already in the dewar.
• Open „He-return“red valve.
• Open sorbtion pump and 1kpot pump on the panel.
• Start to transfer He.
Appendix C

Instruction: Condensation from 4 K to 300 mK

starting position: Dewar and insert are at LHe-Temperature

1. Make sure the green valve to the sorbtion pump on the insert is closed i.e. sorbtion pump is not being pumped.

2. Open the needlevalve a bit → pressure of the 1K-Pot (black clock on the wall) goes up to $10^{-20}$ mbar and the 1K-Pot-Temperature drops $< 1.7$ K (best value currently is about 17mbar, but generally any value below 50mbar can work).

3. Check that all heaters are off.

4. Start the sorbtion pump heater (AxOut3 on the Lockin with GPIB 5). Check on the Temperature-Measurement box, if the switch for the Sorb heater is on. Then start the programm 'keep T-Sorb constant with timer' good settings there: Runtime condensation=3h, T-Sorb=41K, amount heater Voltage will change=0.05V.

5. Once the sorbtion pump has warmed up, check if the 1kPot is still below 1.7K. It has to be below 1.7 K during the whole time period of the condensation, otherwise it will not work. If needed readjust it via the needlevalve.

6. Once done with the condensation cool down the sorbtion pump (check if the programm has put the heater Voltage to zero, when desired open the green valve on the insert to the sorbtion pump about one rotation to pump it. If this is done, the 1K-Pot usually becomes unstable and needs to be readjusted, because it uses the same pump. )
7. Keep the 1K-Pot temperature below 1.7K at all times using the needlevalve.

8. The temperature at the microscope and the RuOHe3 will drop below 1 K to 280 mK (takes about 45 min if the sorbtion pump is not pumped).

9. If the sorbtion pump was pumped close the green to-pump valve of the sorbtion pump on the insert.

10. Keep 1K-Pot temperature below 1.7K during the complete condensation for the optimal base time.
Appendix D

Instruction: Measuring the capacitive coupling of the scan piezo

In order to measure the capacitive coupling of the scanpiezo the setup shown in Fig. D.1 can be used.

To measure the following values have been used:

- phase=0
- frequency=114.38Hz
- amplitude=0.05V

Values for the piezo capacitances after repolarization:

- apply on ZHV⇒ Value on +XHV:
  - R=+0.9020µA
  - Y=+0.9013µA

- apply on ZHV⇒ Value on -XHV:
  - R=+0.8912µA
  - Y=+0.8906µA

- apply on ZHV⇒ Value on -YHV:
  - R=+0.8829µA
  - Y=+0.8823µA
Chapter D. Measuring the scan piezo capacitances

Figure D.1: Setup to measure the capacitive coupling of the scanpiezo

- apply on ZHV ⇒ Value on +YHV:
  
  R = +0.9126 µA
  Y = +0.9121 µA

  If the measured capacitances of any quadrant of the scanpiezo differ from those of the other quadrants by more than 10% (or from previously established values) the piezo should be repolarized [see sec. E].
Appendix E

Instruction: How to repolarize the scan piezo

In order to repolarize the scan piezo a large positive voltage has to be applied to the outer four electrodes, while the inner electrode is kept on ground. This can be archived with the setup shown in Fig. E.1. The voltages should be applied for an extended period of time more than 12h (ideally over the weekend) at RT.

The connectors are assigned as follows:

- A=scan piezo +Y
- B=scan piezo -Y
- C=scan piezo -X
- D=scan piezo +X
- O=scan piezo center Z

Make sure not to accidentally apply 500V to the frame of the microscope as well-put something isolating below the connections (here: banana cables)

Figure E.1: Setup to repolarize the scan piezo
Appendix F

Cabling of the setup

Figure F.1 on the next page shows the cabling of the complete setup outside the cryostat.
Chapter F. Cabling of the setup

Figure F.1: Cabling of the complete setup.
Appendix G

Instruction: Measure the tuning fork resonance in a box

Building a tuning fork sensor into the setup is always a tedious exercise. In order to make this process as efficient as possible, it has proven to be useful to measure the tuning fork resonance of the finished sensor by directly connecting it to the Nanonis PLL connectors. This way we can evaluate if the quality of the sensor is high enough to built it into the setup. The cabling used to test the resonance frequency of a sensor is shown in Fig. G.1.

Figure G.1: Setup to measure the resonance frequency of a tuning fork directly over the Nanonis PLL connectors.
Appendix H

Attocube: Motor properties

H.1 Electronics

H.1.1 Trigger connections from Attocube to Nanonis

<table>
<thead>
<tr>
<th>direction</th>
<th>Nanonis DIO</th>
<th>Nanonis pin</th>
<th>Attoc. pin</th>
<th>Attoc. trigger</th>
<th>cable color</th>
</tr>
</thead>
<tbody>
<tr>
<td>+X</td>
<td>2</td>
<td>7</td>
<td>14</td>
<td>6</td>
<td>green</td>
</tr>
<tr>
<td>-X</td>
<td>3</td>
<td>3</td>
<td>13</td>
<td>5</td>
<td>blue</td>
</tr>
<tr>
<td>+Y</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>4</td>
<td>brown</td>
</tr>
<tr>
<td>-Y</td>
<td>5</td>
<td>4</td>
<td>11</td>
<td>3</td>
<td>red</td>
</tr>
<tr>
<td>+Z</td>
<td>6</td>
<td>9</td>
<td>10</td>
<td>2</td>
<td>grey</td>
</tr>
<tr>
<td>-Z</td>
<td>7</td>
<td>5</td>
<td>9</td>
<td>1</td>
<td>black</td>
</tr>
<tr>
<td>ground</td>
<td>-</td>
<td>1</td>
<td>15</td>
<td>-</td>
<td>white</td>
</tr>
</tbody>
</table>

H.1.2 Peak shape of the motor signal

Measurement of the peak shape of the drive signal with the oscilloscope one time near the attocube ANC350 (inside Pauls HV-box, at the back of the 25D-Sub) and one time near the motor (last plug before the motor). Measurement setup is the same for both measurements. The setup is completely put together and the cryostat cabling is used to connect the motors. The curve shapes are shown in Fig. H.1.

The table below gives the rise and decay time for those two positions as they can be read of from the oscilloscope.

<table>
<thead>
<tr>
<th>position</th>
<th>rise time (μs)</th>
<th>decay time (μs)</th>
<th>Peak height (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Near the ANC350</td>
<td>750</td>
<td>20</td>
<td>28.4</td>
</tr>
<tr>
<td>Near the Motor</td>
<td>700</td>
<td>70</td>
<td>25.6</td>
</tr>
</tbody>
</table>
on the electronics

![Graph](image1.png)

200 μs

20 μs

at the motor

![Graph](image2.png)

200 μs

20 μs

Figure H.1: Peak shape of the drive-voltage signal for the attocube motors. Once measured near the ANC350 and once near the motor. The red arrows indicate the decay time.

### H.1.3 Cabling of the motors

All motors are connected with superconducting wires (two in parallel for each motor direction). The return conductor is the insert itself with a resistance of approximately 0.1 Ω. The superconducting wires have a resistance of 9.2 Ω/m when normal conducting. At room temperature the resistance of a single cable is about 21 Ω. Therefore the total length of the cables is 2.28 m. They run in a straight line for the top part of the cryostat and are wound up inside the beaker [Fig. H.2].

As depending on the amount of He in the cryostat we assume only the wires inside the beaker to become superconducting, we can assume, that all but the top 115 cm are superconducting. We therefore expect the cable resistance of a single cable to drop to about 11Ω. As two cables are connected in parallel this should lead to a total resistance of 6 Ω at low temperatures.
Figure H.2: Only the top 115 cm of the cabling should remain normal conducting, when the insert is cooled down.

H.2 Motor behavior: Single, on the table, air, 300 K

H.2.1 Setup

Each motor-module is operated separately on the table. Each motor is mounted onto a big metal plate and operated.

H.2.2 Minimum drive voltage of the motors

tested at 1 kHz.

The minimum drive Voltage is the voltage at which the motor still covers its full scan range.

<table>
<thead>
<tr>
<th>Minimum drive Voltage</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 V</td>
<td>8 V</td>
<td>20 V</td>
<td></td>
</tr>
</tbody>
</table>

H.3 Motor behavior: Built into the setup, air, 300 K

H.3.1 Setup

The complete setup is put together (including the z-module) and the motors are operated at room temperature and ambient conditions.

H.3.2 Capacitances of the motors

As measured by the Attocube ANC350 electronics.
### H.3.3 Minimum drive voltage of the motors

Tested at 1 kHz.

The minimum drive Voltage is the voltage at which the motor still covers its full scan range.

<table>
<thead>
<tr>
<th>date measured</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacitance data sheet</td>
<td>1.05(\mu)F</td>
<td>1.05(\mu)F</td>
<td>1.05(\mu)F</td>
</tr>
<tr>
<td>Capacitance 2009-06-03</td>
<td>1.144(\mu)F</td>
<td>1.308(\mu)F</td>
<td>1.181(\mu)F</td>
</tr>
<tr>
<td>Capacitance 2009-06-05</td>
<td>1.121(\mu)F</td>
<td>1.249(\mu)F</td>
<td>1.174(\mu)F</td>
</tr>
<tr>
<td>Capacitance 2009-06-08</td>
<td>1.147(\mu)F</td>
<td>1.288(\mu)F</td>
<td>1.208(\mu)F</td>
</tr>
<tr>
<td>Capacitance 2009-09-02</td>
<td>1.142(\mu)F</td>
<td>1.260(\mu)F</td>
<td>1.156(\mu)F</td>
</tr>
</tbody>
</table>

### H.3.4 Minimum drive voltage in dependence of the drive frequency

Test the dependence of the minimum drive Voltage on the drive frequency.

Values for the x-Motor [plot see Fig. H.3]:

<table>
<thead>
<tr>
<th>date measured</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum drive Voltage 2009-06-03</td>
<td>21 V</td>
<td>27 V</td>
<td>42 V</td>
</tr>
</tbody>
</table>
H.3. Motor behavior: Built into the setup, air, 300 K

Figure H.3: Dependence of the minimum drive voltage of the x-Motor on the drive frequency.

<table>
<thead>
<tr>
<th>drive frequency Hz</th>
<th>min drive Voltage V</th>
<th>motor direction</th>
<th>date measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>29</td>
<td>X</td>
<td>2009-06-04</td>
</tr>
<tr>
<td>1900</td>
<td>28</td>
<td>X</td>
<td>2009-06-04</td>
</tr>
<tr>
<td>1800</td>
<td>29</td>
<td>X</td>
<td>2009-06-04</td>
</tr>
<tr>
<td>1700</td>
<td>29</td>
<td>X</td>
<td>2009-06-04</td>
</tr>
<tr>
<td>1600</td>
<td>28</td>
<td>X</td>
<td>2009-06-04</td>
</tr>
<tr>
<td>1500</td>
<td>28</td>
<td>X</td>
<td>2009-06-04</td>
</tr>
<tr>
<td>1400</td>
<td>26</td>
<td>X</td>
<td>2009-06-04</td>
</tr>
<tr>
<td>1300</td>
<td>25</td>
<td>X</td>
<td>2009-06-04</td>
</tr>
<tr>
<td>1200</td>
<td>24</td>
<td>X</td>
<td>2009-06-04</td>
</tr>
<tr>
<td>1100</td>
<td>24</td>
<td>X</td>
<td>2009-06-04</td>
</tr>
<tr>
<td>1000</td>
<td>24</td>
<td>X</td>
<td>2009-06-04</td>
</tr>
<tr>
<td>900</td>
<td>29</td>
<td>X</td>
<td>2009-06-04</td>
</tr>
<tr>
<td>800</td>
<td>27</td>
<td>X</td>
<td>2009-06-04</td>
</tr>
<tr>
<td>700</td>
<td>24</td>
<td>X</td>
<td>2009-06-04</td>
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<tr>
<td>600</td>
<td>25</td>
<td>X</td>
<td>2009-06-04</td>
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<tr>
<td>500</td>
<td>27</td>
<td>X</td>
<td>2009-06-04</td>
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<td>400</td>
<td>25</td>
<td>X</td>
<td>2009-06-04</td>
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<tr>
<td>300</td>
<td>24</td>
<td>X</td>
<td>2009-06-04</td>
</tr>
<tr>
<td>200</td>
<td>26</td>
<td>X</td>
<td>2009-06-04</td>
</tr>
<tr>
<td>100</td>
<td>25</td>
<td>X</td>
<td>2009-06-04</td>
</tr>
</tbody>
</table>
H.3.5  Rough estimation of the positioning sensor calibration at RT

Positioning sensor values for the extremal positions of the motors, as they can be moved to using the electronics. For the calibration the nominal scan range as given in the data sheet is used.

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min Position on the sensor</td>
<td>247.216</td>
<td>50.83</td>
<td>73.08</td>
</tr>
<tr>
<td>Max Position on the sensor</td>
<td>2906.21</td>
<td>2861.45</td>
<td>2403.99</td>
</tr>
<tr>
<td>nominal scan range (from data sheet)</td>
<td>3mm</td>
<td>3mm</td>
<td>2.5mm</td>
</tr>
<tr>
<td>Calibration positioning sensor</td>
<td>886/mm</td>
<td>937/mm</td>
<td>932/mm</td>
</tr>
</tbody>
</table>

H.4  Motor behavior: Built into the setup, vacuum, 77 K

None of the modules moves at 70V at any frequency.

H.5  Motor behavior: Built into the setup, vacuum, 4 K

H.5.1  X-Motor movement

To check the functionality of the X-Module of the Attocube Motors, a series is carried out in which alternatingly a scan of the topography is recorded and the X-Motor is stepped in positive direction. The analyzable part of this series is shown in Fig. H.4. It shows a topography scan for eight positions. On top of each scan the position as can be read of from the positioning sensor is noted in X,Y and Z coordinates. Below the scans is denoted how many steps were taken between two adjacent scans, in which directions those steps were taken, at which driving Voltage of the motor and how much of a change in the positioning sensor reading those steps evoked. We have to note, that even though only the X-Module of the motor was moved deliberately, the Y-Module of the motor moved also up to 10% of the distance.

For better comparison arrows are used to mark a certain easily identifiable feature over a series of scans. The pink arrows for example trace the upper left corner of one grid dip over the scans (a) through (d).

Figure H.5(a) shows the XY-Coordinates of the positions as read of the positioning sensor. The positions are ladled with indices of the different scans, where index (6) corresponds to scan frame (a) shown in Fig. H.4. The range of both axis
is chosen to be the same for better comparison, however the axis are not plotted equal length. We see, that the X-position increases gradually from one position to the next, whereas the Y-position fluctuates on a much smaller scale.

In Fig. H.5(b) we see the change in the X-Positioning sensor relative to the first position at index 6 plotted in green, plotted over the position number (index). As already seen in (a) the X-Value on the positioning sensor increases monotonous from index 6 to index 13. In the same plot we can see the change in Y-component of the vectors used to trace topographical features in Fig. H.4 relative to the vector at index 6. RV stands for the red vector plotted in red, PV for the pink vector and OV corresponds to the values of the orange vector.

Analogous (c) shows the same plot for the Y coordinate of the positioning sensor along with the x-components of the vectors. The values vary statistically on a much smaller scale, than the values shown in (b). Therefore we can conclude, that a step in +X direction with the Motor, moves a point into +Y direction on the scan frame.

We can now derive a step size for the x-motor. To do so we have two options. We can use the room-temperature calibration of the motor [sec. H.3.5] to convert the change in the x-coordinate of the positioning sensor to a distance in μm and divide this by the number of steps. The result of this is shown in Fig. H.5(d) in cyan. Or we can use the change in the y-component of the vectors, using an average value of all vectors whenever possible and divide this value by the number of steps. This result is shown in dark blue. The average value of the latter is shown as a blue dashed line. The average step size of the x-motor at 4 K is 0.1049 μm.

From the latter we can derive a low-temperature calibration of the x-positioning sensor, by relating the change in the y-component of the vector to the change in x-positioning sensor reading, as shown in Fig. H.5(e). We get a low temperature calibration of the x-positioning sensor of 0.9687 PS-units/μm

**H.5.2 Y-Motor movement**

The same evaluation as for the X-Motor was carried out for the Y-Motor. The results are shown in Fig. H.6 and Fig. H.7.

**H.6 Advice Attocube**

- Do not drive the motors with more than 50V at any condition
- Do not move the motors by hand
- Motors are very sensitive to pressure, especially twisting and sheer strength. ⇒ careful when mounting/demounting samples!
Figure H.4: Series of scans. In between the scans, the Motor is moved in +X direction.
Figure H.5: Evaluation of the series shown in H.4. (a) shows the xy-position as displayed by the positioning sensor. (b) Shows the change in position, relative to position 6. The red, orange and pink data points are the Y component of the positions of the arrows, which mark distinguishable topographic features which could be tracked over a series of scans. The green data points are the change in value of the X-positioning sensor with respect to position 6. (c) Shows analogous to (b) the Y component of the positioning sensor and the X component of the vectors. (d) Shows the step size in μm/step for each scan number out of the series (index numbers 6 to 13). The Cyan curve is derived using the Nominal range of Motor and Positioning sensor at room temperature to achieve a sensor units/μm calibration. The Dark blue curve is derived by deriving a shift in μm as calculated from the vectors and relating this to the number of steps taken between two adjacent scans. The dashed line is the average for all indices. (e) Shows the new positioning sensor calibration derived from the scans and change in the y-component of the vectors. The dashed line is the average for all indices. Abbreviations: PS=Positioning sensor; PV=Pink vector; OV=Orange vector; RV=Red vector
<table>
<thead>
<tr>
<th>position 1</th>
<th>position 2</th>
<th>position 3</th>
<th>position 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>X=940.99</td>
<td>X=940.77</td>
<td>X=940.77</td>
<td>X=940.82</td>
</tr>
<tr>
<td>Y=809.53</td>
<td>Y=816.94</td>
<td>Y=818.99</td>
<td>Y=819.75</td>
</tr>
<tr>
<td>Z=256.73</td>
<td>Z=256.81</td>
<td>Z=256.94</td>
<td>Z=256.99</td>
</tr>
</tbody>
</table>

+Y, 3st, 60V
ΔX=-0.22
ΔY=7.41
ΔZ=0.08

+Y, 5st, 50V
ΔX=0
ΔY=2.05
ΔZ=0.13

+Y, 3st, 50V
ΔX=-0.05
ΔY=0.76
ΔZ=0.05

position 5  position 6  position 7
X=940.86  X=940.76  X=940.67
Y=820.37  Y=820.36  Y=822.52
Z=256.95  Z=256.98  Z=256.99

+Y, 6st, 50V
ΔX=0.04
ΔY=0.62
ΔZ=-0.04

+Y, 6st, 50V
ΔX=-0.10
ΔY=-0.01
ΔZ=0.03

+Y, 8st, 70V
ΔX=-0.09
ΔY=2.16
ΔZ=0.01

position 8  position 9  position 10
X=940.64  X=940.58  X=940.72
Y=823.86  Y=824.96  Y=826.25
Z=256.97  Z=256.77  Z=256.80

+Y, 8st, 65V
ΔX=-0.03
ΔY=1.34
ΔZ=-0.02

+Y, 9st, 65V
ΔX=-0.06
ΔY=1.1
ΔZ=-0.02

+Y, 9st, 65V
ΔX=0.14
ΔY=1.29
ΔZ=0.03

Figure H.6: Series of scans. In between the scans, the Motor is moved in +Y direction.
Figure H.7: Evaluation of the series shown in H.4. (a) shows the xy-position as displayed by the positioning sensor. (b) Shows the change in position, relative to position 1. The red, orange and pink data points are the Y component of the positions of the arrows, which mark distinguishable topographic features which could be tracked over a series of scans. The green data points are the change in value of the X-positioning sensor with respect to position 1. (c) shows analogous to (b) the Y component of the positioning sensor and the X component of the vectors. (d) Shows the step size in μm/step for each scan number out of the series (index numbers 6 to 13). The Cyan curve is derived using the Nominal range of Motor and Positioning sensor at room temperature to achieve a sensor units/μm calibration. The Dark blue curve is derived by deriving a shift in μm as calculated from the vectors and relating this to the number of steps taken between two adjacent scans. The dashed line is the average for all indices. (e) shows the new positioning sensor calibration derived from the scans and change in the y-component of the vectors. The dashed line is the average for all indices. Abbreviations: PS = Positioning sensor; PV = Pink vector; OV = Orange vector; RV = Red vector
Appendix I

Electrostatic simulations of a graphene ring

Electrostatic simulation of the scanning gate setup involving a graphene ring have been discussed in section 5.5. For brevity only a very limited amount of simulation results has been shown. Also the simulation details were omitted. Here we discuss the boundary conditions, which are of crucial importance for all electrostatic simulations shown within this thesis in section I.1. The influence of the different gate voltages, which has been shown shortly in section 5.5.3 is shown in the section I.2 in more detail.

I.1 Boundary conditions

Comsol Multiphysics allows to define one of the following boundary conditions for each surface when used to simulate the electrostatics of a geometry in three dimensions:

- electric displacement $\vec{D}_0$ with $\vec{n} \cdot \vec{D} = \vec{n} \cdot \vec{D}_0$
- surface charge $\rho_S$
- zero surface charge $\vec{n} \cdot \vec{D} = 0$
- electric potential $V = V_0$
- ground

All metallic surfaces are held at a fixed potential $V_i$. However care has to be exercised when choosing the boundary conditions for the outer boundaries of the simulated area.
Figure I.1: (a) Electric potential in the xy-plane for $V_{tip} = 1$ V. The voltage applied to all other gates is zero. Zero charge was used as a boundary condition for the outer boundaries. (c) Corresponding electric field lines. (b) shows a zoom in into the area marked in (c). It shows the border of the simulated area. The space that is not included into the simulation is shaded in grey. We see that at the border of the simulated area an unnatural bending of the electric field lines occurs due to the chosen boundary conditions. The area close to the sample however seems not be influenced (b). When for example at the field lines extending from the tip in light blue to the sample we see that as expected all field lines exit the metallic surface at a 90° angle.

I.1.1 Zero surface charge

As the simulated area is more than a factor 10 bigger than the area of interest the boundary condition ” zero surface charge ” is the first obvious choice. However it is a very crude approximation, not taking in account any effects of the specific geometry in use, leads to such artifacts as a bending of the electric field lines at the borders of the simulated regime as can be seen in Fig. I.1(b,c).

However, when zooming into the area right around the ring, this falsification does not have an outstanding effect on the field distribution in this area any more, as can be seen in Fig. I.1(b). The field lines exit all metallic surfaces at an angle of 90° [Fig. I.1(d)]. However, the unrealistical shape of the electric field lines at the borders of the simulated area make it clear, that it is advisable to improve the boundary conditions to be more realistic, taking into account the simulated geometry.

I.1.2 Endless cylinder approximation

The tip of the AFM is etched from a wire which extends for about half a millimeter to a millimeter before being attached to the tuning fork sensor. The electric displacement in z-direction will therefore look as if an endless cylinder would continue on in this direction. The electric field for such an infinite cylinder is:

$$\vec{E} = \frac{\lambda}{2\pi \varepsilon_0 \times r} \quad (I.1)$$
where \( \lambda \) is related to the tip voltage \( V_{\text{tip}} \) by integrating from the inner radius i.e. the radius of the tip \( d_i \) to the outer radius of the simulated area with radius \( d_o \) as follows:

\[
V_{\text{tip}} = \int_{r=d_i}^{r=d_o} E(r)dr
\]

(I.2)

\[
V_{\text{tip}} = \frac{\lambda}{2\pi \epsilon_0} \ln \frac{d_o}{d_i}
\]

(I.3)

The expressions I.1 and I.3 can then be used to determine the expression for the electric field:

\[
\frac{2\pi \epsilon_0 \times V_{\text{tip}}}{\ln \frac{d_o}{d_i}} = 2\pi \epsilon_0 \times E \times r
\]

(I.4)

\[
E = \frac{V_{\text{tip}}}{\ln \frac{d_o}{d_i}} \frac{1}{r}
\]

(I.5)

We therefore use the following boundary conditions:

\[
D_x = D_y = \epsilon_0 \ln \frac{d_o}{d_i} \frac{V_{\text{tip}}}{\sqrt{x^2 + y^2}}
\]

(I.6)

which for the simulated case equates to:

\[
D_x = D_y = \frac{\epsilon_0}{\ln \frac{d_o}{d_i}} \frac{V_{\text{tip}}}{\sqrt{x^2 + y^2}}
\]

(I.7)

\[
D_z = 0
\]

(I.8)

### I.1.3 Endless cylinder and point charge approximation

However the "endless cylinder" which makes up the tip does not extend through the whole simulation frame. The end of the tip is first of all not a cylinder but a conus and secondly ends about 100 nm above the surface in the simulation. Therefore the endless cylinder-approximation is a good approximation when looking at the upper boundaries of the simulated area and up from there. However, in the xy-plane it is not an accurate description.

For the xy-boundaries the tip can be described as a point charge, as in first approximation the extension of the tip is small compared to its distance to the outer boundaries of the simulated area in the xy-plane. The electric potential for a point charge reads:

\[
\phi = \frac{Q}{4\pi \epsilon_0 r}
\]

(I.9)
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Leading to the following proportionality:

\[ \phi = C \frac{V_{\text{tip}}}{4\pi \epsilon_0 [x^2 + y^2 + (z - 2.1 \cdot 10^{-6})^2]} \] (I.10)

At the outside of the tip the voltage applied to the tip has to be equal to this expression:

\[ V_{\text{tip}} = C \frac{V_{\text{tip}}}{4\pi \epsilon_0 [(3 \cdot 10^{-6})^2]} \] (I.11)

\[ C = 1.0009 \cdot 10^{-21} \] (I.12)

which leads to the following expression to be used as a boundary condition:

\[ \phi = 9 \cdot 10^{-12} \frac{V_{\text{tip}}}{[x^2 + y^2 + (z - 2.1 \cdot 10^{-6})^2]} \] (I.13)

I.1.4 Line-charge approximation

As the tip is an extended object, we can make the simulation more precise by incorporating boundary conditions that reflect this geometry as precisely as possible. Most accurately this can be done by assuming charge to be accumulated along a line, which is positioned at a certain distance from a plate, creating a mirror charge as sketched schematically in Fig. I.2. We can write:

\[ \phi(r, z) = \lambda \int_{z_0=a}^{z_0=b} \frac{1}{\sqrt{r^2 + (z_0 - z)^2}} dz_0 - \lambda \int_{z_0=a}^{z_0=b} \frac{1}{\sqrt{r^2 + (z + z_0)^2}} dz_0 \] (I.14)

since \( r^2 = x^2 + y^2 \):

\[ \phi(r, z) = \lambda \int_{z_0=a}^{z_0=b} \frac{1}{\sqrt{r^2 + (z - z_0)^2}} dz_0 - \lambda \int_{z_0=a}^{z_0=b} \frac{1}{\sqrt{r^2 + (z + z_0)^2}} dz_0 \] (I.15)

The substitutions \( \xi = z - z_0 \rightarrow -d\xi = dz_0 \) and \( \mu = z_0 + z \rightarrow d\mu = dz_0 \) lead to:

\[ \phi(r, z) = -\lambda \int_{\xi=-b+z}^{\xi=-a+z} \frac{1}{\sqrt{r^2 + \xi^2}} d\xi - \lambda \int_{\mu=a+z}^{\mu=b+z} \frac{1}{\sqrt{r^2 + \mu^2}} d\mu \] (I.16)

which can be solved analytically:

\[ \phi(r, z) = \lambda \left[ -\ln \frac{-b + z + \sqrt{(-b + z)^2 + r^2}}{-a + z + \sqrt{(-a + z)^2 + r^2}} - \ln \frac{b + z + \sqrt{(b + z)^2 + r^2}}{a + z + \sqrt{(a + z)^2 + r^2}} \right] \] (I.17)
Figure I.2: We show the geometry assumed to model the boundary conditions for the simulation. The potential at the boundaries can be estimated by assuming a rod with charge density $\lambda$ placed between $a$ and $b$ opposing a plate. Inside the plate (marked in grey) a mirror charge will form. Sketch not to scale. The plots (a) and (b) show the resulting function used for the boundary conditions. As the problem has a cylindrical symmetry by nature, only plots for $x$ and $z$ are shown. $y$ looks analogously to $x$.

The integration parameters can be set as follows in our model: $a = \text{tip-sample distance} = 100 \text{ nm}$ and $b = \text{distance from tip to end of the tip} = 100 \mu\text{m}$ as the tip continues far along the $z$-direction. At the position $r = r_i = \text{maximum tip radius}$ the expression $\phi(r, z)$ has to equal $V_{\text{tip}}$:

$$
\lambda \left[ - \ln \frac{-b + z + \sqrt{(-b + z)^2 + r^2}}{-a + z + \sqrt{(-a + z)^2 + r^2}} - \ln \frac{b + z + \sqrt{(b + z)^2 + r^2}}{a + z + \sqrt{(a + z)^2 + r^2}} \right] = V_{\text{tip}} \quad (I.18)
$$
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\[\lambda[-\ln \frac{b + (1.2 \, \mu m) + \sqrt{(b + (1.2 \, \mu m))^2 + r_i^2}}{a + (1.2 \, \mu m) + \sqrt{(a + (1.2 \, \mu m))^2 + r_i^2}} - \ln \frac{b + (1.2 \, \mu m) + \sqrt{(b + (1.2 \, \mu m))^2 + r_i^2}}{a + (1.2 \, \mu m) + \sqrt{(a + (1.2 \, \mu m))^2 + r_i^2}} = V_{\text{tip}} \]  

(I.19)

\[V_{\text{tip}} = 2.000863053 \times \lambda \]  

(I.20)

With this \(\lambda\) the equation for the boundary condition reads as follows:

\[\phi(r, z) = \frac{V_{\text{tip}}}{2.000863053}[-\ln \frac{-b + z + \sqrt{(-b + z)^2 + x^2 + y^2}}{-a + z + \sqrt{(-a + z)^2 + x^2 + y^2}} - \ln \frac{b + z + \sqrt{(b + z)^2 + x^2 + y^2}}{a + z + \sqrt{(a + z)^2 + x^2 + y^2}}] \]  

(I.21)

This function is plotted in Fig. I.2. In the in-plane \(xy\)-direction which is shown in Fig. I.2(a) the potential is highest close to the tip and then decays with about \(r^{-2.5}\) which is close to the decay for a infinite cylinder where \(V_{\text{cylinder}} \sim r^{-2}\). Along the axis of the tip in \(z\)-direction the potential rises from zero to a finite value, at which it saturates for high \(z\)-values.

I.1.5 Top-Bottom approach

In order to verify the chosen boundary conditions, we carried out a simulation of an simplified geometry, which covers a more extended area. It consists of a tip opposing the back gate which is extended in the \(xy\)-plane for 240 \(\mu m\). As the tip in the experiment is formed by etching the end of a wire, we simulate the tip here as a wire, extending for 70 \(\mu m\) narrowing down to a conus close to the surface.

Figure I.3 shows the result of this simulation. The electric potential in the \(xy\)-plane can be seen in Fig. I.3(a). It shows a cylindrical symmetry. The potential is 1 V at the place of the tip and decays to zero, when moving radially away from it [Fig. I.3(c)]. The situation is slightly more complicated for a cut through the \(yz\)-plane as can be seen in Fig. I.3(b). Along the cylindrical part of the tip, the potential at a certain distance from the tip is constant. However when the tip becomes smaller in diameter and a gap is formed between the tip and the metallic surface, the potential along a line parallel to the tip axis as indicated in Fig. I.3(b) decays to zero, as can be seen in more detail in Fig. I.3(d).

When comparing those line cuts shown in Fig. I.3(c,d) with the cross sections of the used boundary conditions, as shown in the inserts in Fig. I.2, we see a good agreement in the potential arising when simulating a large area of a simplified geometry as well as the potential from the calculations using a line charge opposing a plane.
I.2 Influence of the gate voltages

I.2.1 Influence of the tip voltage

We carried out simulations of the geometry shown in Fig. 5.24 for different voltages applied to the various gates. We will now subsequently investigate the influence of each gate. We will start in this section by evaluating the effect of the voltage applied to the tip $V_{\text{tip}}$.

Figure I.4 shows the electric potential directly above the sample surface. Zero volt is applied to all gates. The tip voltage is stepped from zero volts in (a) to 3 volts in (d). For better comparison the cuts for all different tip voltages are shown in the same color scale from 0 to 0.5 V. For $V_{\text{tip}} = 0$ V the electric potential in the whole plane shown in Fig. I.4(a) is zero. When the voltage $V_{\text{tip}}$ is raised to 1 V this voltage induces a cylindrical symmetric electric potential in this plane. The radial symmetry is broken by the existence of the lateral gates and the graphene ring. Therefore we observe a high potential inside the graphene ring, which drops to zero when at the position of the graphene ring and rises outside of the graphene ring, before it drops down to zero again in y-direction when getting close to the side gates as discussed in section 5.5.2.

Figure I.5(a) shows a line cut in y-direction $1 \, \text{Å}$ above the sample surface as a line plot. Here we can see in more detail, how the potential behaves between the in plane graphene structures. For a tip voltage of $V_{\text{tip}} = 1$ V the electric potential
Figure I.4: This figure shows the electric potential 1 Å above the sample surface. The voltage applied to the tip is increased from 0 V in (a) to 3 V in (d). The voltage applied to all other gates is zero volt. The color bar shows the electric potential from 0 to 0.5 V.

inside the ring is still around 0.15 V, so approximately one tenth of the voltage applied to the tip. between the gates and the ring the potential rises to 3 mV, so only one third of the value inside the ring.

The first part in Figure 5.27 with the grey underlay shows the electric potential at the position of the upper and the lower arm of the ring in red and blue respectively. The green curve is the average of these two potentials, so the average electric potential the ring is exposed to. As this is a symmetric case, the electric potential on the right arm is the same as on the left arm. Therefore all curves overlap precisely. When raising $V_{\text{tip}}$ the electric potential at the position of the ring rises linearly.

I.2.2 Influence of the side gates

We now evaluate the influence of the in plane gates for comparison. In order to do so, we simulated the electrostatics for different voltages applied to the small side gate $V_{SGs}$ and the large side gate $V_{SGb}$ once for all other gate voltages set to zero volt [Fig. I.6(a,b,d,f)] and once for a different combination of nonzero gate voltages as shown in Fig. I.6(c,e,g).

For better comparison line cuts along the y direction for different gate combinations are shown in Fig. I.5. When a voltage of 1 V is applied to the large side gate the electric potential right around this gate rises. However, as can be seen in Fig. 5.27 highlighted in orange the electric potential at the position of the ring arm that is further away from this side gate is not noticeably influenced. The electric potential at the ring close to the swept gate rises about 0.1 mV per volt applied to the side gate. This holds for both side gates despite their different shapes and sides as can be seen in Fig. 5.27 (purple and light orange highlighted). When nonzero voltages are applied to the other available gates, this change in potential above
Figure I.5: (a) Electric potential for line cuts in y-direction across the ring and the gate 1 Å above the sample surface. We show line plots for different tip voltages between zero and 3 V as noted above. Analogous line cuts are shown in (b) and (c) for different $V_{SGs}$ values, in (b) for 0 V applied to all other gates, and in (c) for nonzero values on another gate. (e) and (f) show a series for the big side gate. (g) and (h) for the back gate.
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Figure I.6: Electric potential 1 Å above the sample surface. The voltage applied to the large side gate is increased from 0 V in (a) to 3 V in (d). The voltage applied to all other gates is zero volt. for the series shown in (a) through (d), whereas for the series shown from (e) to (h) the voltage applied to the tip voltage is 3 V, while the voltage applied to all other gates is zero. The color bar shows the electric potential from 0 to 3 V.

the rings stays the same, even though it is offset in absolute electric potential due to the other gate voltages as can be seen for two gate voltage combinations in the second and third row in Fig. 5.27 (orange and dark grey highlighted).

I.2.3 Influence of the back gate

When a voltage is applied to the back gate, the electric potential in the sample plane is cylindrical symmetric with respect to the center of the ring. As opposed to the case, when applying a voltage to either one of the side gates. Applying a finite voltage to the back-gate leads to an increased electric potential in the whole sample plane which is only suppressed in the area just around the graphene structures as shown in Fig. I.7.

Figure I.5(f,g) shows cuts along the y-direction. When a voltage of 1 V is applied to the back gate while a voltage of zero volt is applied to all other gates, the potential at the ring rises by about 0.1 mV, so only about half the amount compared to when the same voltage is applied to the tip. The voltage in the center of the ring rises up to about 0.3 V.
I.2. Influence of the gate voltages

Figure I.7: Electric potential 1 Å above the sample surface. The voltage applied to the back gate is increased from 0 V in (a) to 3 V in (d). The voltage applied to all other gates is zero volt for the series (a-d) whereas for the series (e-h) 3 V are applied to all other gates. The color bar shows the electric potential in volt from 0 to 3 V.
Appendix J

Additional scanning gate measurements on a double quantum dot

In this chapter we show additional some scanning gate experiments performed on the GaAs double dot structure described in chapter 7.

J.1 Measurements on the complete structure: QPCs, charge traps and more

Even though the sample consists of not only a double quantum dot, but also two charge readouts, until now only the current through the double quantum dot has been discussed. The hope for this sample was to enable the control over electrons in space and time by combining the method of scanning gate microscopy with the added information archived from charge readouts. In the following we will discuss the functionality of the charge readouts, briefly discuss the role of charge traps and show how the sample could be mechanically altered in situ.

Previously QPCs have successfully been employed in scanning probe measurements with a single quantum dot [18, 39, 133, 21, 155]. On the other hand the capability to perform time resolved measurements with a QPC have also been shown [156].

J.1.1 QPCs

In order to use the quantum point contacts as charge readouts, they have to be conducting in a parameter region where current flows through the double dot. In order to determine the correct gate voltage settings, we recorded a current map,
Figure J.1: Charge stability diagram $I(V_{PG1} + V_{QPCG1} + 0.5 \, V, V_{PG2} + V_{QPCG2} + 0.5 \, V)$ for the current through the double dot (a), the current through QPC1 (c) and the current through QPC2 (d). All three measurements were recorded in parallel. Panel (b) shows a schematic of the gate voltage regions in which each of the three nano-structures conducts. The colorbars show the current in $\log[I_{DD}(A)]$ for (a), $\log[I_{QPC1}(A)]$ for (b) and $\log[I_{QPC2}(A)]$ for (d). Measurement settings: $T = 300 \, \text{mK}$, $V_{bias} = 210 \, \mu\text{V}$.

where the current trough both QPCs and the double dot was recorded in parallel. To reach the maximum accessible gate voltage range $V_{PG1}$ was stepped in parallel with $V_{QPCG1}$ and $V_{PG2}$ was stepped in parallel with $V_{QPCG2}$. Figure J.1(a) shows the current through the double dot. The charge stability diagram shows the same features as discussed in the context of Fig. 7.10. Current flows through the double dot in a region of $V_{PG1} > 0 \, \text{mV}$ and $V_{PG2} > 0 \, \text{mV}$ as schematically shown by the blue region in panel (b). A well balanced double dot forms approximately around $0 \, \text{mV} < V_{PG2} < 100 \, \text{mV}$ and $0 \, \text{mV} < V_{PG2} < 150 \, \text{mV}$.

Current flows through QPC1 when $V_{PG1} < -100 \, \text{mV}$ as marked in Fig. J.1(b) with the pink region. The gate voltage settings at which current flows through QPC2 is marked in orange. We observe that we will not be able to use all three nano-structures simultaneously. However we can try to use one charge readout, while measuring the current through the dot. Since the distance from either charge readout is different to dot 1 than to dot 2, we should still be able to distinguish charging events arising from dot 1 and dot 2.

To determine which QPC is more fit to be used as a charge detector the current through both QPCs versus gate voltage was recorded [Fig. J.2]. We observe that the characteristic of QPC1 shows steplike features, while QPC2 displays many
J.1. Measurements on the complete structure: QPCs ...

Figure J.2: Current through QPC1 (a) and QPC2 (b) versus gate voltage. Bias voltage applied on the QPCs: $V_{Qbias}=50 \mu V$

Figure J.3: Map of the current through the double dot (a) and through QPC 1(b) recorded simultaneously. Measurement settings: $V_{PG1} = -50 \text{ mV}$, $V_{PG2} = 450 \text{ mV}$, $V_{CG} = 500 \text{ mV}$, $V_{STG} = -100 \text{ mV}$, $V_{STG} = -100 \text{ mV}$, $d = 40 \text{ nm}$, $V_{Qbias}=50 \mu V$

different maxima, steps and dips. These features might be due to localized states inside or close to the constriction, in the doping plane or inside the bulk. When evaluating the dependence of such characteristics on the double dot bias, only slight dependence of the current through either QPC on the dot bias voltage (i.e. also on the dot current) could be found, hinting that the QPC current is only very scarcely influenced by the double dot state.

We now record current maps of the current through one charge readout and the double dot at once. Figure J.3 shows a typical measurement of this kind. Figure J.3(a) shows the current map through the double dot, while (b) shows the current map of QPC1. Both current maps are recorded simultaneously. We can now trace the Coulomb rings visible in the current map of the double dot (blue) and those visible in the current map of QPC1 (green) and superimpose them [Fig. J.3(c)]. First we see that the current map of the double dot does not show the distorted hexagon pattern that is the hallmark of a double dot in current maps, but rather a
Figure J.4: Panel (a) and (b) show current maps recorded with the same measurement settings. The colorbar shows the current in pA. The only parameter changed was the scan direction which is indicated by the white arrows. Panel (c) shows the amplitude of the sensor oscillation in (nm). Panel (d) shows the frequency shift of the sensor in milli Herz. Measurement settings: $V_{PG1} = 430$ mV, $V_{PG2} = 230$ mV, $V_{CG} = 470$ mV, $V_{STG} = 200$ mV, $V_{DTG} = 150$ mV, $d = 40$ nm

single set of concentric Coulomb rings. This is expected as the gate voltage settings needed to be able to observe transport through QPC1 are not in the region, where an equitable double dot is anticipated to form from the charge stability diagram. We rather expect a double-dottish behavior where dot 2 dominates the transport properties.

The current map through QPC1 also shows single rings, similar to the Coulomb rings observed for a single dot. If we now look at the overlay of both sets of rings [Fig. J.3(c)] we see that their center, or the corresponding zeroth Coulomb ring as estimated from the visible rings, appear at positions that are about 250 nm separated in real space. It is therefore possible that QPC1 actually mirrors the electronic state of dot 1. However an other option is, that those features are evoked by a charge trap accidentally located in the close vicinity of dot 1.

**J.1.2 Charge traps**

The sample showed some phases (both in time and parameter space) of unstable behavior. This became visible by dragging-stripes seen in the current maps [see Fig. J.4(a) for example] or sudden jumps in the current when recording charge stability diagrams.

Such charge rearrangements proved to show up very frequently for specific gate voltage settings. For some gate voltage settings the sample became more stable upon letting it settle down at a given gate voltage configuration for a couple of hours. Many of the dragging stripes in current maps however proved to be reproducible. I.e. when recording the same line back and forth the current always switched when the tip arrived at a defined position. Therefore it seems likely that
most of these features are not evoked by random events, but rather by loading and unloading specific charge traps. Some of these charge traps remained stable after filling, leading to sample stability to improve over time.

Such instabilities did not only occur when sweeping a gate voltage, but also during and after helium transfer and condensation. A typical trace displaying jumps in the currents is shown in Fig. J.5, bottom panel. We see that the frequency shift of the engaged tuning fork sensor is stable. This means that the position of the tip does not change significantly. No mechanical vibration or even tip crashes are taking place. However the current through the double dot still shows random spikes and jumps.

To prove that these instabilities are not connected to a mechanical disturbance of the sample caused by the scanning tip we record a scanning gate image while the tuning fork sensor is oscillating. The phase locked-loop was engaged, however no amplitude feedback was used. The resulting current map is displayed in Fig. J.4(a). We see a number of these so called dragging stripes obscuring the double-dot distorted hexagon pattern. However the frequency shift (c) shows no structure, neither does the amplitude map (d). If the dragging stripes were connected to mechanical disturbances of the sample by the tip, this should be visible both in the frequency shift and the amplitude signal. The same behavior could be observed when the tip was retracted by more than 100 nm from the sample, making mechanical perturbations highly unlikely. We can therefore conclude, that the
Figure J.6: Charge stability diagram of the sample before (a) and after (b) its mechanical altering. (c) and (d) show current maps recorded at approximately the same gate voltage settings.

sample was not mechanically disturbed while recording these measurements, and the dragging stripes are purely electrical in origin. This is also confirmed by a current map recorded with a scan direction that is turned by ninety degree toward the usual scan direction (b). Again dragging stripes occur with a similar frequency as in (a).

J.1.3 Altering the sample

Due to a crash of the tip into the sample provoked by undesired mechanical disturbance of the setup the sample was significatively altered. Figure J.6(a,b) show the charge stability diagram before (a) and after (b) this mechanical altering of the sample.

The gate voltage region highlighted with a blue square in (a) is the gate-voltage region in which the sample displayed double-dot behavior. The green square marks the gate voltage region in which the charge stability diagram shown in (b) was recorded. Before the mechanical intrusion the nano-structure was completely open at these gate voltage settings. The confinement of the double dot was only scarcely
J.1. Measurements on the complete structure: QPCs ...

recognizable. We see that gate voltage settings needed to form a clean double
dot have been changed by about half a volt. This is also clearly visible when
examining the current maps recorded at similar voltage settings shown in (c,d).
The current map shown in (c) shows an unstable double dot, where both dots are
equally pronounced. For (d) which was recorded after the mechanical altering of
the sample, $V_{PG1}$ was increased by 150 mV and $V_{CG}$ by 80 mV compared to (c).
Dot 1 should therefore be more strongly pronounced than before. However the
current map shows, that transport is dominated by dot 2. Dot 1 only leads to very
faint resonances in the current.

We can therefore conclude, that even though the 2DEG is buried beneath
34 nm of GaAs, a slight push of the tip onto the surface can still significatively
alter the sample.
Appendix K

Blind tip estimation

Every topography image recorded via AFM is a distorted representation of the true surface, produced by the shape and size of the particular tip. Blind tip estimation is a method that estimates the shape of the tip without the knowledge of the true surface topography only from the AFM topography. At extreme ”image protrusions are broadened replicas of those on the specimen. However it is only convention which determines which of the two objects being scanned across one another is the tip and which the specimen.” [140].

In blind tip estimation we assume that the modeled tip is sharp enough that is does not act simultaneously on two surface objects which are separated by a sufficiently large distance. If this is the case each surface object in question is imaged independently. Each of such a surface object this acts as an outer bound on the tip shape. Reconciling all of the bounds produces the bluntest tip that can produce such a topography image. Each recorded topography image therefore contains a superposition of the inverted probe. Mathematically this can be expressed via morphological theory. For the precise analytical operation we refer to the specialized publications on this matter.

The underlying idea of the analytical operation can be understood as follows.: When imaging a sharp feature as shown in Fig. K.1(a), the topography recorded will equal to that presented as a blue line in (b), which is an inverted image of the probe. When scanning with the same tip over two sharp features (c, grey) the recorded topography image will be the envelope of inverted tip profiles (c, blue).

We can expand this idea and imagine any surface to be constructed from a number of such sharp features. In analogy the recorded topography image Fig. K.1(d,blue) is the envelope of many inverted tip profiles. The common volume below all local maxima produces an estimate of the tip geometry.

A more detailed description of the method and the precise underlying algorithm can be found in [140, 144, 145] on which this short summary has oriented itself.
Figure K.1: (a) Tip (red) scanning over a sharp surface profile (grey). (b) Resulting surface scan in blue. (c,d) Expansion of this idea on many sharp features. Image and description text in analogy to [142].
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