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The use of post-earthquake residual displacements as a performance indicator in seismic assessment

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The Use of Post-Earthquake Residual Displacements as a Performance Indicator in Seismic Assessment

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Preface

Modern performance based earthquake engineering addresses design, evaluation, construction, maintenance and decommission of structures and requires that their performance during an earthquake can be predicted with a known degree of accuracy. In this framework the importance of residual displacements is twofold: On one hand they represent a significant source of damage affecting mainly the serviceability and the repairability of the structure. On the other hand, after an earthquake, residual displacements are often the only measurable indicator of the shaking occurred during the earthquake. In the framework of a pre-earthquake design or assessment process, the expected residual displacements need to be predicted; while during a post-earthquake assessment the actual residual displacements can be measured.

Within his present doctoral dissertation, Mr. Yazgan developed and validated against experimental evidence a new and original methodology to improve the estimate of maximum deformations occurred during an earthquake taking into account both observable damage and measurable residual displacements. The methodology is based on the Bayesian probabilistic theory and, as a particular novelty, explicitly considers model error. To estimate model error Mr. Yazgan carried out a very extensive study on the numerical modelling of structures under earthquake, which also allowed the formulation of practical modelling recommendations. The quantification of model error is of course of fundamental importance also for any realistic pre-earthquake prediction of residual displacements.

The thorough and comprehensive work presented by Mr. Yazgan allows a deep understanding of the challenges relevant to the use of residual displacements in the seismic performance assessment of structures and his developments represent a new effective way to achieve this important goal.

Zurich, January 2010

Dr. Alessandro Dazio
Summary

Safety assessment of damaged structures is a pivotal part of the post-earthquake recovery process. As a result of the deformation histories that have occurred during the damaging earthquake, the key structural properties of the columns, beams and walls that contribute to the seismic resistance change. The key structural properties include stiffness, strength and deformation capacity. An accurate estimation of the residual key structural properties is crucial in identifying vulnerability of the damaged structure.

These residual structural properties are known to be strongly dependent on the maximum deformations that have occurred. A new post-earthquake assessment method is developed following this premise. After an earthquake, the maximum deformations experienced by a damaged structure can be estimated using the developed method.

The essential idea behind the method is to probabilistically estimate the experienced maximum deformations based on the post-earthquake residual displacements and the visible structural damage. The major uncertainties related to the estimated maximum deformations are explicitly treated in the method. The sources of uncertainties include: (1) errors in the response prediction due to imperfections of the structural model, (2) lack of knowledge of the experienced ground motion, and (3) incompleteness of knowledge of the parameters of the structural analysis model — e.g. material properties, damping behavior, gravity loads. The assessment method is applied to two example structures tested on shaking tables. Comparison of the estimated maximum displacements with those measured during the test confirms the effectiveness of the method.

In order to assess the probable error in the results of nonlinear time-history analysis, shaking table tests are numerically reproduced. The predicted maximum and residual response parameters are compared with those measured during the test. Considered response parameters include the average drift ratio, the ground story drift ratio and the rotations at the plastic hinging regions. A set of alternative modeling approaches is utilized in the analyses. The results suggest that, for the considered response parameters, accuracies of the predicted residual values are noticeably lower than that of the maximum values.

Sensitivity analyses are carried out to investigate the influence of a range of model parameters on the predicted maximum and residual deformations. Following factors are found to influence the predicted residual displacements: the axial load considered in the model, the assumed damping ratio, the adopted finite-element discretization scheme, and the utilized reinforcement stress-strain model.

The method proposed in this study can be utilized as a sub-component in any post-earthquake decision-making process. The proposed method would serve as a consistent and rational basis for the assessment of the risks associated with the damaged structures.
Zusammenfassung

In der Wiederaufbauphase nach einem Erdbeben spielt die Standsicherheitsbeurteilung beschädigter Bauwerke eine zentrale Rolle. In Folge der Verformungsgeschichte, welcher das Bauwerk während des Erdbebens ausgesetzt war, verändern sich die wesentlichen mechanischen Eigenschaften von Stützen, Riegeln und Tragwänden, welche zum Erdbebenwiderstand beitragen. Zu diesen mechanischen Eigenschaften gehören die Steifigkeit, die Tragfähigkeit, sowie das Verformungsvermögen der Bauteile. Die möglichst genaue Abschätzung der verbleibenden mechanischen Eigenschaften ist entscheidend für die Bestimmung der Verletzlichkeit eines beschädigten Bauwerks.


Es werden Sensitivitätsanalysen durchgeführt, um zu untersuchen, welchen Einfluss eine Reihe von Modellierungsparametern auf die maximalen und bleibenden Verformungen hat. Bei den folgenden Faktoren wurde ein merklicher Einfluss auf die bleibenden Verformungen ausgemacht: Die modellierte Normalkraft, das angenommene Dämpfungsmass, die gewählte finite Elemente Diskretisierung und die verwendete Spannungs-Dehnungs-Beziehung des Bewehrungsstahls.
Die hier vorgeschlagene Methodik kann als Teilkomponente eines jeglichen Entscheidungsprozesses nach einem Erdbeben verwendet werden. Das Verfahren stellt eine konsistente und rationale Basis für die Beurteilung der von beschädigten Bauwerken ausgehenden Risiken dar.
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1 Introduction

1.1 Defining the problem

Post-earthquake assessment of damaged structures is a critical and complex problem. In the immediate aftermath of an earthquake, it is the engineer’s responsibility to judge, if a structure is safe enough to continue to function. In the subsequent phase, the responsibility to identify the most cost effective repair actions for the damaged structure again belongs to the engineer. Evaluating the effects of the damage on the structural properties and expected future seismic performance is pivotal in this respect. The problem considered in this report is the identification of the extent of damage to a structure based on the visible damage and the sustained residual displacements.

Structures and the soil supporting them deform when subjected to strong ground motions. If the deformations experienced by the constituent materials of the structures and soils are larger than their elastic limits, the materials undergo plastic deformations and their stiffnesses degrade. Often, some part of these plastic deformations remains permanent after the shaking is over. The residual displacements are the permanent displacement of the structure and soil system. These displacements can be measured after the earthquake. This report is concerned with the residual displacements that are due to the plastic deformations of structures. Throughout this report, the term residual displacement refers to the permanent relative displacement of a structure with respect to its foundation.

It is known that damaging earthquakes are very often followed by a series of aftershocks and sometimes by other mainshocks. Past earthquakes have shown that when urban areas are hit by damaging earthquakes, a significant percentage of structures attain light to moderate damage. Moreover, it is known that structures that sustained some damage during a prior seismic event may collapse during a succeeding event. Such unfortunate events have claimed many lives. Therefore, these structures impose a potential risk to human life, economic assets and the environment. Thus, making decisions regarding the post-earthquake functionality and repair of the damaged structures is a critical part of the post-earthquake recovery process.
After damaging earthquakes, post-earthquake safety evaluations have to be carried out for a large number of structures with varying severity of damage, different functions, structural and site properties, etc. In the current practice, groups of trained engineers evaluate the safety of damaged structures. Often the safety is evaluated based on the visible damage to the structure like extend of cracking, spalling, etc. The evaluation process relies mainly on the expert opinion. Today, there exists no generally accepted method for predicting the safety of earthquake damaged structures.

Estimation of the future vulnerability of a damaged RC structure is subject to several uncertainties. The stiffness, strength and deformation capacity are the fundamental uncertain properties that have profound influence on the expected performance of a structure under seismic actions. Therefore, estimating these structural properties accurately is crucial when predicting the likely future performance —and hence the safety— of a damaged structure. The stiffness, strength and deformation capacities of RC components under seismic actions have been the subject of numerous investigations. The fundamental finding that is shared by all these studies is that the degradation of stiffness and strength of RC components is strongly related to the experienced maximum deformations. Therefore, an accurate estimation of the maximum deformations experienced by a structure during an earthquake, is crucial for estimating the actual effects of that earthquake on the seismic resistance.

After the maximum deformations are estimated, the vulnerability of the structure can be assessed with a higher accuracy and better informed decisions can be made on the possible improvement of the seismic resistance. For example, the critical components of the structure that are likely to sustain significant damage during future earthquake ground motions may be identified. Accordingly, the required immediate structural interventions may be designed to reduce the deformation demands on these components. Subsequently, the overall behavior of the structure may be improved to achieve a satisfactory overall seismic performance during a future earthquake. The knowledge of the experienced maximum deformations is a critical parameter in this process.

1.2 Objective and scope

The primary objective of this report is to develop a method for estimating, after an earthquake, the maximum deformations experienced by a structure taking into account the residual displacements and the visible damage. The major prevailing uncertainties related to the problem are treated in the estimation process. These include the uncertainties related to the model parameters (e.g. excitation, material properties) and the model error associated with the predicted response parameters (i.e. maximum and residual displacements and rotations). The developed method is suitable for the assessment of structures for which the response can be estimated with reasonable accuracy. The method, in its form presented in this report, can be applied to individual structures.
However, it can be conveniently extended for the assessment of groups of buildings.

Another fundamental objective of this report is to investigate the accuracy of state-of-the-art nonlinear time history analysis (NLTHA) methods in predicting the seismic response of RC structures. For this purpose, model structures that were tested dynamically on shaking tables are considered. The tests are numerically reproduced by adopting a set of alternative modeling approaches. The accuracies of the predicted maximum and residual values of the global and local response parameters (e.g., displacements, rotations) are evaluated. Moreover, the sensitivities of the response parameters to several model parameters and modeling assumptions are investigated. Note that the discussion presented in this report is mostly written from the standpoint of the post-earthquake assessment. However, the findings regarding the accuracy of NLTHA methods are applicable to various engineering tasks (e.g., design, performance assessment, reliability evaluation, system identification through finite-element model updating) where the response of a RC structure to seismic actions needs to be predicted by means of NLTHA.

### 1.3 Limitations

Application of the post-earthquake assessment method proposed in this report is limited to structures whose response can be predicted by an analytical method with reasonable accuracy.

The finite element models considered during the NLTHA carried out in this study are suitable for capturing the flexural response of RC components. If other force-deformation mechanisms (e.g., shear, torsion, reinforcement slippage, dowel action, infill wall interference) are expected to have a significant influence on the response of the structure, the modeling approaches adopted here should be modified to take them into account.

The findings presented in this study are applicable to structures where the deformation of the supporting soil can be neglected. The residual displacements that are caused by the poor performance of the supporting soil (e.g., differential settlements, liquefaction, landslides) are beyond the scope of the findings presented here. However, note that in principle the proposed method can be extended to take into account foundation flexibility and soil structure interaction effects given that reliable models are available for capturing these effects.

The findings of this investigation are primarily applicable to structures that deform predominantly along one horizontal direction. The response of structures that undergo large bi-directional plastic deformations is not discussed in detail.

The accuracy of the NLTHA predictions of the response parameters is evaluated based on the data obtained from the dynamic testing of model RC structures and components on shaking tables. The response of these model structures can only represent
the complex response of a real structure to a limited extent. This should be kept in mind while estimating the probable accuracy of the considered NLTHA approaches in predicting the response of other structures based on the findings presented here.

In order to numerically reproduce the dynamic response exhibited during a specific shaking table test, a reliable recording of the excitation is needed. The acceleration histories recorded on the shaking tables invariably contain some noise. In this study, the major part of this noise has been removed by means of baseline adjustment. However, the remaining part of the noise may still have an effect on the accuracy of the simulation.

1.4 Overview

The report consists of six chapters. In the next chapter, the states-of-the art in the two topics are presented: (1) post-earthquake assessment, and (2) the residual displacements. First, an overview of the existing post-earthquake damage assessment methods is given. Subsequently, the previous studies addressing residual displacements are reviewed.

The investigation of the accuracy of the maximum and residual displacements — predicted by means of NLTHA — is presented in the third chapter. The results of the sensitivity analyses are also discussed in this chapter.

The newly developed post-earthquake assessment method is presented in the fourth chapter. The method consists of four steps. The procedures that should be followed during each step are presented in detail in separate sections. The application of the method to sample model structures is demonstrated in the fifth chapter. The results obtained using the method are evaluated in light of the experimental data available for the considered sample structures.

In the final chapter, the report is summarized and the conclusions are presented. Additionally, the outlook for further research is provided. The details of the considered numerical models and the new assessment method are presented in the appendices.
2 State of the art

2.1 Introduction

A number of studies have contributed to the fields of post-earthquake assessment of damaged structures and of residual displacements. In this chapter, the key developments in these fields are presented.

In the following section, an overview of the existing post-earthquake assessment methods is provided. The principle ideas that form the basis of the major existing methods are presented. The limitations of the different methods are summarized in the third section.

The existing literature on residual displacement of structures is presented in the fourth section. The research efforts on this topic can be roughly grouped under three subjects: (1) prediction of residual displacements, (2) consideration of residual displacements in design and assessment, and (3) approaches to mitigate the residual displacements.

2.2 Overview of existing post-earthquake assessment methods

The existing important post-earthquake assessment methods are briefly reviewed in this section. The methods are grouped into three categories: (1) quick inspection methods, (2) detailed evaluation methods, and (3) non-conventional methods. The first two groups consists of methods that have been put into actual practice. The methods forming the third group are those that introduce novel ideas but have not yet gained popularity. After the existing methods are presented, the contributions of the present study to the existing literature on the subject will be explained.

This study focuses particularly on the prediction of the effects of seismic damage on the structural properties. Therefore, special emphasis is placed on the damage evaluation approach employed in the existing assessment methods. The advantages and limitations
of the approaches are pointed out.

2.2.1 Quick inspection methods

The quick inspection methods are developed to assess the usability of damaged structures after an earthquake. The assessment duration per structure is in the range from 10 to 40 minutes. During the assessment, experts inspect the damage to the structure and its foundation. Usually, only the damage indicators that can be inspected with bare eyes and simple tools are considered. Spalled concrete cover, buckled or ruptured reinforcement bars, as well as crack sizes and orientations are examples of such indicators. The damage severity is identified by comparing the observed damage with damage descriptions provided in the relevant documents. Based on the observed damage, the capacity of the damaged structure to resist future loads and deformations is evaluated. Subsequently, the safety is judged.

The differences between the alternative inspection methods are mostly due to the damage severity classifications and to the set of considered indicators. These two differences are directly related to the problem of estimating the residual seismic resistance. Note that, also the reference shaking intensity considered when judging the safety may differ for each method.

The reference shaking intensity is the ground motion intensity considered when deciding whether the damaged structure has the required capacity to meet the future seismic demand or not. The key characteristics of some well known inspection methods are presented below.

**ATC-20 method** One of the earliest well documented procedures for post-earthquake safety evaluation was published by the Applied Technology Council (ATC, 1989) under the title: *ATC-20: Procedures for Postearthquake Safety Evaluation of Buildings*. The document contains guidelines for evaluation of earthquake damaged buildings regarding the safety of its occupants. The procedure guides the damage inspector in assigning a post-earthquake occupancy status (i.e. safe, restricted and unsafe) to a damaged building. Safety assessment forms are provided in form of checklists. The occupancy status is assigned based on the estimated potential of the damaged building to withstand a repetition of the damaging ground motion without collapse. The central focus is the evaluation of the safety for the immediate use of the damaged building.

**AeDES method** Aiming for a harmonization in the post-earthquake safety evaluation of buildings within Europe, the document *Field Manual for Post-earthquake Damage and Safety Assessment and Short Term Countermeasures (AeDES)* was published by the Joint Research Center (JRC) in Ispra (Baggio et al., 2007). AeDES method was orig-
inally developed in 1997 by a joint group of experts from the National Seismic Survey (SSN) and the National Group for the Defense against Earthquakes (GNDT) in Italy. A series of forms is provided to document important properties of the structure and the site, as well as the extent of damage to the structural elements, to the non structural components and to the foundation. In the manual, damage descriptions are provided for a series of damage severities. Within these descriptions residual displacements are also addressed. For example, for RC frame buildings, the damage is classified as “medium-severe” if the residual drift is perceptible but smaller than 1%. Furthermore, the damage is classified as “very heavy” if the residual drift is in the range between 1 and 2%. The reference shaking intensity that is considered when judging the usability of a building is defined as the maximum shaking that may be experienced at the site due to after-shock hazard. Past experience shows that at some sites the shaking experienced during aftershocks may be more intense than that during the mainshock. The usability of a building is evaluated based on the collected assessment data. The considered sources of risk are those related to the condition of: (1) structural members of the building, (2) non-structural components, (3) adjacent buildings and (4) soil and foundation. Additionally, in the document some short term countermeasures (e.g. temporary supports, propping) are provided to help the inspectors implementing measures to reduce the risks in the post-earthquake phase.

Other methods An overview of the different damage assessment practices adopted in Italy, Turkey, California, and Japan over the years is presented by Goretti and Di Pasquale (2008). The studies they address are mainly aimed at the assessment of post-earthquake usability and of financial losses. The form-based usability assessment practice adopted in Greece is presented by Anagnostopoulos and Moretti (2008).

The common limitation shared by the major quick inspection methods is the fact that the final judgment of the safety relies to a large extent on the inspector’s opinion. As a result, a uniform evaluation of the structural stock cannot be ensured; particularly if different groups of inspectors carry out the evaluations. Moreover, since the result is significantly dependent on the expert’s opinion, it is very difficult to identify and correct the inadequacies of these methods. For example, even if it is observed that for a group of structures the evaluation results turns out to be excessively conservative (or the opposite), it is difficult to identify whether the bias is due to the method or the expert.

Another important limitation of quick inspection methods is related to the estimation of the critical residual structural properties —e.g. stiffness, strength and deformation capacity. Since the time that can be allocated on a structure is very limited, the key properties of the damaged structural components —i.e. their deformation capacity and boundary conditions— are not explicitly accounted for during the assessment. Usually, the structure is judged to be safe or unsafe based only on the observed damage indications. However, indicators such as the maximum crack width cannot be used to estimate the future seismic performance of the critical structural components unless their
deformation capacities are known (Hanson, 1996).

2.2.2 Detailed evaluation methods

The detailed evaluation methods aim at predicting the likely seismic performance of the structure in the future and identifying the cost-effectiveness of alternative repair and rehabilitation strategies. The time required to evaluate a damaged structure is in the range of weeks or months. The future performance of the damaged structure is analyzed by accounting for the effects of damage on the structural properties. An analytical model of the structure is utilized for this purpose. For each damaged structural component, the force-deformation behavior is estimated by modifying the behavior estimated for their intact states. The modification is determined based on the inelastic behavior mode (e.g. flexural, diagonal tension) and on the severity of the experienced damage. The appropriate modifications are established based on component testing data and expert opinion. The modification factors are usually provided as deterministic values.

The detailed evaluation methods differ in the way the seismic performance is defined and measured. For a given damaged structure, some methods aim at identifying the likely deformations corresponding to a set of hazard levels, while the others aim at estimating the residual energy dissipation capacity. The residual energy dissipation capacity is defined as the total energy the structure can dissipate under monotonically increasing lateral deformations. Consequently for a given structure, it is very likely that different optimal repair strategies are identified using the alternative evaluation methods.

As a result of the difference presented above, the way the structural properties are modified according to damage severity, also differs for each method. Some methods guide the engineer to reduce the stiffness and strength by given factors, whereas others provide the ratio of the residual energy dissipation capacity to the original capacity. The key properties of two widely known evaluation methods are presented below.

FEMA306 In order to establish a consistent criteria to analyze the future performance and the repair of buildings damaged during an earthquake, in 1999 ATC prepared the document *FEMA306: Evaluation of Earthquake Damaged Concrete and Masonry Wall Buildings*. The approach is based on selecting a set of performance objectives for a building and analyzing the relative performance of the building in its intact, damaged and repaired states. (Figure 2.1a). The severity of the damage is identified based on observable indications of damage —e.g. crack widths and concrete cover spalling. Figure 2.1a presents the recommendations to estimate the force-deformation behavior of an RC wall with “moderate damage”. After the force-deformation behaviors of all structural components are predicted, the performance of the damaged structure is evaluated. Nonlinear static or dynamic analyses are utilized for this purpose. Finally, the effectiveness of the restoration option is evaluated by appropriately modifying the model and assessing
Performance Index Method  The topic of evaluating damage to a structure and identifying technically and economically sound repair actions is addressed by JBDPA (2001). For this purpose, the manual *Guideline for Post-earthquake Damage Evaluation and Rehabilitation* was prepared. It was originally published in 1991 and revised in 2001. An overview of this procedure is presented by Maeda et al. (2004). The procedure is based on estimating a measure of capacity named “seismic performance index $I_S$”. Three factors are considered when determining $I_S$: (1) the residual total energy $E_r$ a structure in a given damage state can dissipate under monotonic loading (Figure 2.2a), (2) a parameter related to stiffness discontinuity along the height and (3) reduction factor for strength degradation. $I_S$ is estimated both for the intact and the damaged states. In order to calculate $E_r$ of a damaged structure, energy dissipation capacities of the structural components are reduced based on the measured maximum residual crack width. Such a relationship proposed by JBDPA (2001), is plotted in Figure 2.2b. The damage level of the structure is evaluated based on the residual seismic capacity ratio which is equal to $I_S$ estimated considering the post-earthquake state of the structure divided by that estimated considering its original state.

In the conventional detailed evaluation methods, the residual structural properties of the components are assigned based on the presence/absence of visible damage indicators. The uncertainty in the estimated residual structural properties is not explicitly taken into account. In fact, even when very detailed inspection results are available, the residual structural properties of a damaged component can be estimated with limited precision only. The effectiveness of alternative repair strategies essentially depends on the residual structural properties. Therefore, if the potential errors in the estimated properties are not taken into account, the cost-effectiveness of repair options can not be properly evaluated.
The existing detailed evaluation methods do not take into account the residual structural displacements (e.g., roof displacement). However, these displacements may provide critical information related to the structural performance. Furthermore, unlike other damage indicators that are often subjected to the engineers’ judgment, residual displacements can be measured.

2.2.3 Non-conventional methods

Evaluation methods that have introduced noteworthy ideas, but have not gained a wide acceptance so far, are reviewed in this section. All methods in this group employ a probabilistic approach. The essential differences are the adopted strategy and the underlying assumptions.

Heredia-Zavoni et al. (2000)  In some cases, structures fail due to cumulative damage exceeding a critical threshold. Fatigue failure is a typical example of this phenomenon. Heredia-Zavoni et al. (2000) developed a method to estimate the cumulative damage sustained by an instrumented structure. Heredia-Zavoni et al. assume a cumulative damage-based hysteresis in the formulation of their method. In the hysteretic model, the stiffness degrades as a function of increasing cumulative damage. The rate of the degradation is controlled by two parameters. These two parameters and the cumulative sustained damage after an earthquake are considered as random variables. The assessment method by Heredia-Zavoni et al. aims at estimating the probability distributions of these variables using Bayesian analysis. In the method, prior distributions are assumed for the uncertain variables. Subsequently, these distributions are updated based on the
force-deformation behavior measured during the earthquake. Heredia-Zavoni et al. argue that the acceleration histories recorded on an instrumented building can be used to identify the experienced force-deformation behavior. A mathematical model representing a one-story RC frame is considered by Heredia-Zavoni et al. to demonstrate the method. The results of time history analysis are assumed as the measured force-deformation response of an instrumented building. The hysteretic model that is employed in the formulation of the method is adopted in the analysis.

In the study by Heredia-Zavoni et al., the hysteretic model is assumed to accurately predict the inelastic dynamic behavior of a structure given that the correct values of the model parameters are used. This critical assumption is also shared by the large majority of the methods that aims at identifying the optimal parameter values that result in the best agreement between the simulated and the measured response histories. In these methods, the only source of uncertainty in the response is due to incomplete knowledge of the model parameters. The likelihood of any bias in the model itself is not taken into account. However, calibrating a model to predict accurately a given response parameter does not always guarantee that other response parameters of interest are predicted with the same accuracy. A model that can accurately predict the peak relative accelerations, for example, may underestimate the peak interstory displacements. This is also suggested by the results that will be presented in Chapter 3. Therefore, for a given structure unless the established model is validated to predict all important response parameters, the likelihood of probable bias in the model should be regarded. Otherwise, misleading results may be obtained.

The methods that are similar to Heredia-Zavoni et al. rely on the measured response histories and do not account for the direct consideration of visible inspection results—e.g. detection of cracking or spalled cover concrete—to reduce the uncertainty in the evaluated damage. A probabilistic method to take into account the visible damage indicators in damage evaluation will be proposed in Chapter 4.

Bazzurro et al. (2004) Bazzurro et al. (2004, 2006) developed a method to assess the seismic performance of buildings. The method can be used for two different purposes: (1) evaluating the safety of a damaged building and (2) assessing the expected seismic performance of a structure by taking into account the likely post-earthquake occupancy status. Here, only the application of this method to damaged buildings is addressed.

The method by Bazzurro et al. aims at estimating, during an aftershock, the likelihood of the roof deformation exceeding a specific deformation limit. The intensity of the aftershock ground motion is measured in terms of the pseudo-spectral acceleration (PSA) of an elastic single-degree of freedom system having a period of vibration equal to the fundamental period of the structure. The threshold PSA level that results in exceedance of the deformation capacity is predicted using SPO2IDA (Vamvatsikos and Cornell, 2002).
SPO2IDA is a simple spreadsheet tool that is developed to predict the statistical properties of incremental dynamic analysis (IDA) (Vanvatsikos and Cornell, 2002) curves for a SDOF system based on its idealized pushover curve. Using the SPO2IDA tool, the median value and the record-to-record variability of the PSA level corresponding to the exceedance of the considered deformation limit can be estimated. In order to estimate these, the idealized pushover curve of the building is required.

In the method by Bazzurro et al., the PSA level to induce collapse is estimated both for the mainshock damaged as well as its pre-mainshock (undamaged) states of the building. Two different pushover curves are established for these two states. The proposed procedure to establish the pushover curve for the damaged building is summarized in the next paragraph. After the two PSA levels are estimated, the mean annual frequencies of exceedance of these two PSA levels are predicted by means of probabilistic seismic hazard analysis. Lastly, the occupancy status for the damaged building is assigned based on the two mean annual frequencies: (1) the frequency of collapse estimated for the pre-mainshock (undamaged) state and (2) the frequency of collapse estimated for its mainshock damaged (present) state.

Estimating the pushover curve for the damaged building is a critical step of the method by Bazzurro et al. (2004, 2006). The stiffness and the strength degradation that have taken place during the mainshock ground motion should be taken into account while carrying out the pushover analysis. Two different approaches are recommended for this purpose: (1) quasistatic cyclic analysis and (2) analysis with damaged components. Both approaches require an initial pushover analysis of the intact building. Based on the results of this analysis, the roof displacement and the base shear corresponding to the observed global damage state of the building is identified. In the quasistatic cyclic approach, a displacement cycle is applied to “damage” the model. Subsequently, the pushover analysis is carried out using this damaged model. In the analysis with damaged components approach, the stiffnesses and strength of the failed structural elements are multiplied by reduction factors. The details of this approach are presented by Maffei et al. (2005).

In the method by Bazzurro et al. (2004, 2006), the presence of a residual roof drift is accounted for while estimating the median aftershock PSA that results in exceedance of the deformation capacity. A simple approach proposed by Luco et al. (2004), is utilized for this purpose. Using this approach, the estimated aftershock PSA level is adjusted by a factor which is a function of the residual roof displacement. Luco et al. developed and verified this approach by simulating the response of a SDOF system to 900 mainshock-aftershock ground motion pairs.

The uncertainty of the predicted response due to the imperfection of the adopted structural analysis procedure is taken into account in the method by Bazzurro et al. (2004, 2006). It is considered in the evaluation of the dispersion of the estimated aftershock PSA. This dispersion is considered to be dependent on the likely inaccuracy of the adopted analysis procedure and on the record-to-record variability of the response. The
record-to-record variability is obtained from the SPO2IDA results. The likely inaccuracy of the adopted structural analysis procedure is estimated based on the interviews with three experienced practicing engineers. It is suggested that the degree of dispersion depends on the type of structural system (e.g., steel moment resisting frame), irregularity of the structural system and the complexity of the structural analysis model. Bazzurro et al. recommend a series of dispersion values for the various cases.

The probable inaccuracy of the numerically predicted response is also one of the main focus points of this report. However, the aim of the evaluation and the adopted strategy is different than the study by Bazzurro et al. (2004, 2006) presented above. In this report, the accuracies of the simulated response parameters (e.g., displacements, rotations) are investigated. In order to evaluate this accuracy, a set of shaking table tests are numerically reproduced and the simulated response is compared with the measured one. The details of this evaluation are presented in Chapter 3.

Yeo and Cornell (2004) Strong earthquakes are typically followed by a series of aftershock earthquakes that occur on the same fault or on other faults nearby. Within the duration of aftershock activity, the rate of earthquakes with small magnitudes is higher than the usual rate. This increased rate of earthquakes results in an increased seismic hazard that may last for several weeks after the mainshock. Yeo and Cornell (2004, 2005b,a, 2009) present a methodology for evaluating the safety of damaged buildings based on their collapse probabilities due to aftershock hazard. They propose a method to estimate the time-dependent aftershock ground motion hazard at a site based on the magnitude and distance of the mainshock earthquake and the number of days passed since the mainshock. In their methodology, the minimum aftershock PSA that can induce collapse is evaluated based on the method by Luco et al. (2004). Yeo and Cornell propose a building tagging criterion that takes into account an estimate of the aftershock hazard at the site, a measure of collapse probability and the function (lifeline, commercial, residential, etc.) of the building.

Accessing buildings that are designated as unsafe can be very dangerous during the aftershock phase. An elaborate method to limit the risk due to occupying damaged buildings is proposed by Yeo and Cornell (2004, 2005b,a, 2009). The method can be used to organize a working schedule for the emergency workers. Specifically, the suitable starting day and the optimal working duration are estimated. The method takes into account three factors: (1) a specified acceptable risk level, (2) the expected building performance and (3) the aftershock hazard estimated for the site. With respect to the evaluation of the extent of structural damage, the method by Yeo and Cornell (2004, 2005b,a, 2009) is similar to that proposed by Bazzurro et al. (2004).

They propose a method to develop a residual displacement based reconnaissance tool. The tool can be used to estimate the residual axial load carrying capacity. More specifically, the likelihood of the residual capacity being lower than a specific percentage of the total design capacity is estimated. In order to estimate this likelihood, a series of time history simulations are carried out. After each simulation, the vertical loading in the bridge model is gradually increased until the maximum resistance is reached. This resistance is recorded as the residual axial load carrying capacity.

2.3 Remarks on the existing methods

The major existing post-earthquake assessment methods were discussed in the previous sections. In all methods, a different approach is adopted when evaluating the severity of structural damage. Common limitations shared by the existing approaches are briefly summarized in the following:

- The local damage indicators and the measured residual displacements are not jointly taken into account.
- The methods are restricted to considering only one residual displacement parameter per application. Jointly considering several measured residual displacement values is not possible using these methods.
- The probable biases in the response parameters predicted by means of time history analysis are either not taken into account or assumed to be the same both for the maximum and the residual values of the parameters.
- The uncertainties associated with the estimated residual structural properties are not explicitly treated.

A new method that does not share these limitations is suggested in Chapter 4. This new method can be used to estimate probabilistically the maximum deformations experienced by a damaged structure. Based on the knowledge of experienced maximum deformations, the residual structural properties can be identified with better accuracy.
2.4 Review of previous research on residual displacements

Residual displacements are one of the key response parameters considered in this report. For this reason, a comprehensive review of previous studies addressing residual displacements is provided in this section. The scope of the review extends beyond the scope of post-earthquake assessment and includes the seismic design of new structures.

2.4.1 Parameters influencing residual displacements

Definitions

It is conventional to normalize deformation parameters with other important parameters that have also the units of length (e.g. yield displacement, story height). For the case of residual displacements $d_r$, several normalization alternatives have been proposed by different researchers making use of the following basis parameters:

- Maximum possible residual displacement, $d_{mr}$. The $d_{mr}$ is the maximum possible residual displacement based on slow unloading from the peak displacement.
- Maximum inelastic displacement, $d_m$
- Elastic spectral displacement, $S_d$
- Yield displacement, $d_y$

Based on these basis parameters, the following normalizations are introduced:

**Permanent displacement ductility**, $\mu_p$ (Mahin and Bertero, 1981; Farrow and Kurama, 2001, 2003)

$$\mu_p = |d_r/d_y|$$  \hspace{1cm} (2.1)

This ratio is referred to as residual displacement ductility, $\mu_r$ by Farrow and Kurama (2001, 2003).

**Residual displacement ratio**, $d_{rr}$ (MacRae and Kawashima, 1997; Kawashima et al., 1998)

$$d_{rr} = |d_r/d_{mr}|$$  \hspace{1cm} (2.2)

This ratio is referred as $SRDR$ by Kawashima et al. (1998).

**Residual displacement to maximum displacement ratio**, $d_{rm}$ (Borzi et al., 2001; Pampanin et al., 2002b; Christopoulos et al., 2003, 2004; Ruiz-Garcia, 2004)

$$d_{rm} = |d_r/d_m|$$  \hspace{1cm} (2.3)
This ratio is referred as residual ratio, \( \gamma \) by Ruiz-Garcia (2004); Ruiz-Garcia and Miranda (2005).

**Residual displacement ratio, \( C_r \) (Ruiz-Garcia, 2004; Ruiz-Garcia and Miranda, 2005, 2006d,a,e)**

\[
C_r = \frac{d_r}{S_d}
\]  

(2.4)

In the following, the findings of the previous studies are presented in two parts: the first one discussing the median value of the residual displacements and the second one discussing their dispersion.

**Findings related to the median value of residual displacements**

**Post-yield stiffness ratio** The post-yield stiffness ratio \( p \) is the ratio of the post-yield stiffness of a SDOF system to its elastic stiffness. A \( p \) value larger than zero implies hardening behavior and smaller than zero implies softening behavior. Several studies show that systems with higher \( p \) tend to sustain smaller residual displacements than those with lower post-yield stiffness (Riddell and Newmark, 1979; MacRae and Kawashima, 1997; Kawashima et al., 1998; Borzi et al., 2001; Pampanin et al., 2002b; Farrow and Kurama, 2003; Ruiz-Garcia and Miranda, 2005; Fu and Menun, 2006). The concept of “hysteresis center curve” is introduced by MacRae and Kawashima (1997) to explain the tendency of SDOF systems to yielding toward or away from zero displacement depending on their post-yield stiffness. Priestley et al. (1996, p.427) address the P-\( \Delta \) effect and its influence on the post-yield stiffness of bridges. They discuss the tendency of SDOF systems with negative post-yield stiffness to yield in a single direction. Furthermore, they state that for SDOF systems with post-yield stiffness greater than zero, the residual displacement decreases with further cycling, and no preferential direction for cumulative displacement develops.

**Hysteretic behavior** The residual displacements calculated for stiffness-degrading SDOF systems are reported to be on the average smaller than those for non-stiffness degrading systems (Riddell and Newmark, 1979; Mahin and Bertero, 1981; Pampanin et al., 2002b; Ruiz-Garcia and Miranda, 2006d). Moreover compared to maximum displacements, the prediction of residual displacements by means of numerical analysis is found to be more sensitive to the hysteretic rule parameters (Pampanin et al., 2002b; Dazio, 2004). Considering Takeda hysteretic model, Pampanin et al. (2002b) show that simulated residual displacements decrease with the increasing reloading stiffness. The residual displacements for systems with larger unloading stiffness are found to be slightly larger compared to those with smaller unloading stiffness (Pampanin et al., 2002b). On the other hand, Dazio (2004) and Ruiz-Garcia and Miranda (2005) report the increase in the residual displacements with increasing unloading stiffness to be much more significant. Residual interstory drift ratios are reported to be more sensitive to the member
hysteretic behavior than the maximum drifts (Ruiz-Garcia and Miranda, 2005, 2006c). The importance of the definition of the hysteretic rules for small amplitude (non-yielding) cyclic response is addressed by Dazio (2004). Furthermore, Dazio investigated the cyclic behavior of RC walls and identified the parameters that influence the re-centering properties of these walls. The relative contribution of the axial load to the flexural strength is reported to be a critical parameter that controls the re-centering property of a RC wall (Dazio, 2004).

Peak displacement ductility The influence of the peak displacement ductility demand on the $S_{RDR}$ is assessed by MacRae and Kawashima (1997) and Kawashima et al. (1998). Their results do not show a significant dependence between the average $S_{RDR}$ and the peak displacement ductility. Similarly, the average $d_{rm}$ and the ductility demand are reported to be only slightly correlated (Christopoulos et al., 2003; Dazio, 2004). On the other hand, Farrow and Kurama (2001, 2003) report correlation coefficients in the range from 0.48 to 0.97, between the peak displacement ductility demand and the $\mu_p$. Moreover, they propose regression based relationships to model this dependence. The $d_{rm}$ is reported to be increasing with increasing displacement ductility for systems with very low post-yield stiffness ratios ($p < -0.2$) (Borzi et al., 2001).

Strength ratio The strength ratio is defined as the elastic strength demand divided by the yield strength of the system. The dependence of $d_{rm}$ to the strength ratio is investigated by Ruiz-Garcia and Miranda (2005). They report that for elastoplastic SDOF systems with strength ratios smaller than 3, the mean $d_{rm}$ ratios increase as the system becomes weaker —i.e. when the strength ratio increases. However for systems with strength ratios in the range of 3 to 6 (weak systems), residual displacements are on average equal to half of the peak inelastic displacements —i.e. $d_{rm}$ equals 0.5, independent of the period of the system. Ruiz-Garcia and Miranda (2005, 2006d,a,e) investigated the dependence of the median of $C_r$ to the strength ratio of SDOF systems. They report that for elastoplastic systems with periods shorter than 0.5 s, the mean $C_r$ ratios are highly sensitive to the strength ratio. For systems with periods longer than 1s, some sensitivity is observed only if the strength ratio is smaller than 3. In this period range, weaker elastoplastic systems (i.e. systems with strength ratios higher than 3) attain residual displacements that are on average equal to half of the elastic displacement demand —i.e. $C_r$ equals 0.5.

Ground motion characteristics The influence of the site class on the residual displacements is investigated by several researchers. Kawashima et al. (1998) report that the mean $d_{rr}$ is not dependent on the site class. Farrow and Kurama (2001, 2003) compare mean $\mu_r$ values identified for two ground motion ensembles. The first ensemble consists of near-fault ground motions, whereas the second ensemble consists of far-fault ground motions. Farrow and Kurama report that for a given strength ratio, $\mu_r$ values
obtained for the near-fault ensemble are larger in the period range from 0.5 to 1.1 s. Ruiz-Garcia and Miranda (2005, 2006d) investigated the dependence of $C_r$ on the site class. They report that $C_r$ is sensitive to the site class for the periods shorter than 1 s. In addition, Ruiz-Garcia and Miranda investigated the dependence of $C_r$ to the magnitude of the earthquake and to the distance of the site to the causative fault. They report no significant dependence on these parameters. The $d_{rm}$ is also reported not to be very sensitive to the site class (Ruiz-Garcia and Miranda, 2005). Residual displacements of SDOF systems subjected to ground motions with strong velocity pulses were investigated by Fu and Menun (2006). They conclude that strong velocity pulses can lead to large residual displacements. However, no apparent relationship is found between the period of the velocity pulse and the residual displacement to maximum displacement ratio $d_{rm}$. The results obtained from three shaking table tests is presented by Phan et al. (2007). The units are subjected to two ground motions with the same peak-acceleration but different velocity history characteristics. One of the motions displayed a clear and definite high amplitude pulse in the velocity history whereas the other one did not display such a pulse. The results indicate that the ground motion with an asymmetrical velocity pulse yields residual displacements that are significantly larger than those resulting from the other ground motion. Gicev and Trifunac (2007) investigate the permanent deformations of elastoplastic shear-beam models of buildings. The model aims at representing the shear wave propagation along tall buildings. Gicev and Trifunac report that the permanent deformations localize at the bottom of the shear-beam if it is subjected to large displacement pulses. On the other hand, the deformations localize closer to the top of the shear-beam if it is subjected to small displacement pulses.

Higher mode effects The amplification of residual drift due to higher modes compared to the amplification of maximum peak interstory drift was investigated by Pampanin et al. (2002b). They report that the amplification factors obtained for the residual drifts are more scattered and larger than those obtained for the maximum drifts.

Deformation mechanism Pampanin et al. (2002b) report that soft story mechanisms likely to occur in gravity load designed frames result in higher residual to maximum drift ratios $d_{rm}$. They further state that an equivalent SDOF system based approach would lead to poor predictions of the seismic performance of multi-degree-of-freedom (MDOF) frames featuring irregular inelastic deformation mechanisms. Ruiz-Garcia and Miranda (2005, 2006c,b) report that building models developing beam-hinge mechanisms experience smaller residual drifts compared to those developing column-hinge mechanisms.

Number of stories The residual interstory drifts of a set of steel frame models having 3, 9 and 20 stories were investigated by Medina and Krawinkler (2003). Their results suggest that the median residual drift ratios obtained using ground motions scaled to a specific hazard level are the same, independently from the total number
of stories. Similarly, Ruiz-Garcia and Miranda (2005) report that for a given relative shaking intensity similar residual roof rotations (i.e. the residual roof displacement divided by the building height) are sustained by multi-story frames having 3 to 18 stories. Ruiz-Garcia and Miranda define the relative shaking intensity as the peak inelastic displacement of an equivalent elastoplastic SDOF system divided by the yield displacement of this equivalent system.

**In-plan asymmetry** The effect of in-plan asymmetry on the residual deformations was investigated by Pettinga et al. (2007b). In particular, the influence of varying the levels of stiffness, strength, and mass eccentricity on the residual deformations is evaluated. They conclude that residual diaphragm rotations increase with increasing asymmetry.

**Bi-directional loading** Effects of bi-directional loading on the residual displacements were evaluated by Nagata et al. (2004). They experimentally investigate the seismic behavior of columns that are subjected to eccentric axial gravity loads. Columns that were subjected to bidirectional excitation sustained significantly larger residual displacements compared to those subjected to unidirectional excitation. Similarly, Maekawa et al. (2003, p.274) state that residual deformations of RC columns under multi-directional excitation may be significantly larger than those under uni-directional excitation. This is also confirmed by the results of the dynamic tests carried out by Hachem et al. (2003).

**Design approach** Park et al. (2003) investigate the residual displacements of bridge piers that are designed according to the U.S. specifications by AASHTO (1995) and the Japanese specifications by JRA (1990), respectively. The approach presented by AASHTO is based on an ultimate strength approach, while the specifications by JRA are based on a working stress approach. In this study, Park et al. (2003) consider three columns: two JRA-designed columns and one AASHTO-designed column. Compared to the JRA-designed columns, the AASHTO-designed column is relatively more flexible and has a denser transverse reinforcement. The scale models of the columns were subjected to a set of successive earthquake ground motions with varying intensities. The AASHTO-designed column is reported to attain larger residual displacements compared to the JRA-designed columns. However, it is also noted that the AASHTO-designed column was able to retain its vertical load carrying capacity throughout many subsequent tests unlike the JRA-designed columns.

**Repeated excitation** The investigation of the response of RC bridges under recorded mainshock-aftershock ground motion sequences is presented by Ruiz-Garcia et al. (2008). They state that the predicted level of increase in the peak and residual drift demands
for a bridge subjected to mainshock-aftershock ground motion sequences depend on the adopted hysteretic model and on the level of ground motion intensity.

**Findings related to the record-to-record variability of residual displacements**

The residual displacements of SDOF systems that experience specific levels of maximum displacement ductility (constant ductility approach) was investigated by Kawashima et al. (1998). The coefficient of variation (COV) of the $S_{RDR}$ ratios for elastoplastic systems is found to be about 0.5.

Mahin and Bertero (1981) investigated the dispersion of the residual displacements of SDOF systems. In their analyses, Mahin and Bertero assume a peak-ductility for each SDOF system. Subsequently, they adjust the strength using the design spectra and ductility-strength-period relationships developed by Newmark and Hall (1982). The COV of the $\mu_p$ values, is reported to be in the range from 0.8 to 1.8. Note that this range of COVs correspond to approximately twice the COV they calculate for the maximum displacement ductility values identified for the same SDOF systems.

Farrow and Kurama (2001, 2003) consider the ductility demands of SDOF systems having yield strength equal to a constant proportion of the minimum strength required to maintain the system elastic (constant-strength approach). They present COV values for $\mu_r$ that are in the range from 0.6 to 1.2 for systems having a post-yield stiffness ratio equal to 0.1 and periods longer than 0.5 s. On the other hand, for periods shorter than 0.5 s, COV values are found to be larger than 1.5.

Christopoulos et al. (2004) investigated the COV of the $d_{rm}$ ratio by adopting the constant ductility approach. The $d_{rm}$ is found to be smaller for the SDOF systems with negative post-yield stiffness compared to those with positive post-yield stiffness. Their results suggest that COV values are in the range from 0.5 to 1.

Ruiz-Garcia and Miranda (2005, 2006d,a,e) investigated the residual displacements of SDOF systems by adopting the constant-strength approach. Ruiz-Garcia and Miranda (2005) report that the COV of the $d_{rm}$ is in the range from 0.46 to 0.71, independent of the period of vibration and of the strength ratio. The sensitivity of $C_r$ to the periods of the SDOF system is presented by Ruiz-Garcia and Miranda (2005, 2006d,e). For the period range shorter than 0.5 s, they report $C_r$ to be sensitive to the period of the system. For periods longer than 1 s this sensitivity is not observed. They state that the COV of $C_r$ is in the range from 0.8 to 2.0. Moreover, Ruiz-Garcia and Miranda (2006d,e) report that—in the case of near-field ground motions—the variability is larger if the period of the system is lower than the period of the velocity pulse of the considered ground motions.

The variability of the residual-to-maximum diaphragm rotation ratio is addressed
by Pettinga (2006). The dispersion of this ratio for a single story structural system is identified to be influenced by the level of torsional restraint. The COV values are reported to be in the range from 0.36 to 1.06 for systems with a radius of gyration of strength larger than the mass radius of gyration. For other systems, the COV values are found to be varying in the range from 0.39 to 1.76.

### 2.4.2 Definition of performance levels

Limits related to residual displacements have been provided in several documents that propose definitions of seismic performance. These limits are briefly reviewed in the following.

The necessity to consider the residual displacements in seismic performance assessment is addressed in the *Vision 2000: Performance-Based Seismic Engineering of Buildings* document by SEAOC (1995). Five performance levels are defined: (1) Fully operational, (2) Operational, (3) Life Safe, (4) Near collapse, and (5) Collapse. The proposed performance definitions include criteria related to the expected safety of the building after a damaging earthquake. In the document, general damage descriptions, and maximum and residual drift ranges are presented for the five performance levels (Table 2.1).

<table>
<thead>
<tr>
<th>Performance level</th>
<th>Fully operational</th>
<th>Operational</th>
<th>Life Safe</th>
<th>Near Collapse</th>
<th>Collapse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall building damage</td>
<td>Negligible</td>
<td>Light</td>
<td>Moderate</td>
<td>Severe</td>
<td>Complete</td>
</tr>
<tr>
<td>Permissible transient drift</td>
<td>&lt; 0.2%</td>
<td>&lt; 0.5%</td>
<td>&lt; 1.5%</td>
<td>&lt; 2.5%</td>
<td>&gt; 2.5%</td>
</tr>
<tr>
<td>Permissible permanent drift</td>
<td>Negligible</td>
<td>Negligible</td>
<td>&lt; 0.5%</td>
<td>&lt; 2.5%</td>
<td>&gt; 2.5%</td>
</tr>
<tr>
<td>Effect on occupancy</td>
<td>No effect</td>
<td>Continuous occupancy possible</td>
<td>Short term to indefinite loss of use</td>
<td>Potential permanent loss of use</td>
<td>Permanent loss of use</td>
</tr>
</tbody>
</table>

In the guidelines *NEHRP Commentary on the Guidelines for the Rehabilitation of Buildings (FEMA273)* by FEMA (1997), four performance levels are defined. Indicative values for the maximum and residual drift values associated with different performance levels are provided in Table 2.2. In the succeeding *FEMA356* guidelines, the same drift values were provided (FEMA, 2000).

Kawashima et al. (1998) present the requirements of Japanese seismic design specifications for bridges (JRA, 1996) related to residual displacements. The specifications state that the expected residual drift should be limited to 1%. If the expected residual drift is found to be larger than this limit, either the peak ductility demand is reduced or the post-yield stiffness is increased.

Priestley (1993) states that the maximization of energy dissipation at the critical regions to obtain a “better” seismic performance may result in large residual displacements
Table 2.2: Indicative maximum and residual drift values by FEMA (1997, 2000)

<table>
<thead>
<tr>
<th>Performance Level</th>
<th>Operational</th>
<th>Immediate occupancy</th>
<th>Life safety</th>
<th>Collapse prevention</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall damage</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RC Frame</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum drift</td>
<td>-</td>
<td>1%</td>
<td>2%</td>
<td>4%</td>
</tr>
<tr>
<td>Residual drift</td>
<td>None</td>
<td>Negligible</td>
<td>1%</td>
<td>~4%</td>
</tr>
<tr>
<td>RC wall</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum drift</td>
<td>-</td>
<td>0.5%</td>
<td>1%</td>
<td>2%</td>
</tr>
<tr>
<td>Residual drift</td>
<td>None</td>
<td>Negligible</td>
<td>0.5%</td>
<td>~2%</td>
</tr>
</tbody>
</table>

that would lead to repair of the structure being infeasible. Priestley (2000) addresses the importance of including residual displacements in the definition of performance states for performance based seismic design. It is proposed that the definition of the fully operational limit state should be based on an acceptable residual crack width and crushing of concrete. Residual crack widths that are in the range from 0.5 mm to 1 mm are suggested as suitable limits to define the fully operational limit state.

The Displacement-based seismic design of reinforced concrete buildings report by fib (2003) addresses the importance of residual drifts. It is suggested that residual drifts on the order of 1% or more can result in a structure being uninhabitable and the repair may be prohibitively expensive.

Kwan and Billington (2003a) propose that the residual displacement being smaller than 1% should be ensured —together with the criteria related to maximum deformations— while assessing the functional-level displacement capacity of bridge piers.

McCormick et al. (2008) addressed the issue of discomfort of the occupants of a building due to residual deformations. They report that significant discomfort is felt by the occupants of buildings with residual inclinations above 0.8% —mainly based on past experiences from buildings that had undergone differential settlement. McCormick et al. propose a permissible residual deformation limit of 0.5% to be considered in performance-based seismic design. This deformation limit is reported as the minimum perceptible inclination angle of the floor for an average person.

2.4.3 Performance assessment methods

A number of seismic performance assessment methods taking into account residual displacements have been proposed. In the following, some of these methods are briefly reviewed.

A simple design oriented method is proposed by Kawashima et al. (1998) to predict the residual displacements of RC bridge piers. In order to predict the residual displace-
ments of a pier based on its post-yield stiffness ("bilinear factor" \( r \)) and the expected maximum displacement, they developed the residual displacement response spectrum shown in Figure 2.3. Making use of this spectrum, a method to predict the residual displacements of eccentrically loaded RC columns is proposed by MacRae (1998).

![Residual displacement response spectrum proposed by Kawashima et al. (1998)](image)

Figure 2.3: Residual displacement response spectrum proposed by Kawashima et al. (1998)

Pampanin et al. (2002b,a, 2003) and Christopoulos et al. (2003, 2004) discuss the inadequacies of available performance assessment methods which are only based either on the peak response or on the cumulative dissipated energy. They introduce the concept of “Residual Deformation Damage Index” as a complementary indicator of structural performance. This index is defined as a function of both the residual deformations attained by primary structural elements and non-structural elements. In the assessment method proposed by Pampanin et al., the seismic performance is defined by taking into account both the residual and the maximum displacements. As a result, the engineer gains better insight on the post-earthquake usability and reparability of the structure.

A simple empirical relationship to predict the residual drift ratios for concrete-filled steel box columns is proposed by Ge et al. (2003). The relationship is developed based on the results from 15 columns that were tested pseudo dynamically. It is aimed to provide a practical estimate of the residual drift ratio based on the estimated yield drift and the maximum displacement.

Mackie and Stojadinovic (2004, 2005, 2006) present probabilistic models for evaluating the ability of a highway bridge to function after an earthquake. They propose alternative methods to assess the magnitude of traffic loads that can be carried by a damaged bridge. They report that the uncertainty of the predicted loss of vertical load carrying capacity is lower if the residual displacement is utilized as an intermediate response parameter while relating the maximum displacements to the capacity loss.

A probabilistic approach to estimate residual drift demands for the performance based assessment of existing buildings is presented by Ruiz-Garcia and Miranda (2005, 2008). The approach aims to estimate the mean annual frequency of exceeding residual
drift demands for a given building. The procedure is based on estimating the probability of residual interstory drift ratios for a multi-story building exceeding a specific residual drift level conditional on the level of shaking intensity at the site. The maximum displacement of a representative inelastic SDOF system is considered as the measure of shaking intensity. The residual drift demand is assumed to be a lognormally distributed random variable. For a given building, parameters of the distribution are identified as a function of maximum displacement of the representative inelastic SDOF system. The residual drift ratio hazard for the building is estimated based on the relationship between the residual drift demand of the building and the maximum displacement of the inelastic SDOF system and the maximum displacement hazard estimated for the inelastic SDOF system. The uncertainty due to record-to-record variability of the residual drift demand is taken into account in the procedure.

Uma et al. (2006a,b) present a probabilistic formulation for consideration of residual displacements together with maximum displacements in performance-based design and assessment. Bivariate joint lognormal distribution is assumed to describe the joint occurrence of maximum and residual deformations. Based on this joint distribution, fragility curves are established to estimate the probability of exceeding specific maximum and residual deformation levels for a given level of ground motion intensity.

A framework for consideration of residual displacements during design and post-earthquake safety assessment is presented by Phan et al. (2007). In particular, the method aims to predict the allowable live load carrying capacity that is expected to remain after an earthquake. The residual drift response spectra are proposed for this purpose. The spectra are developed using a new rule-based hysteretic model and a set of synthetic ground motion records, generated using a composite source model.

2.4.4 Damage observations

Post-earthquake damage evaluations that specifically put an emphasis on observations related to residual displacements are reported in the literature. Some of these observations are the following.

Anderson and Johnston (1998) report the damage of a soft-story steel building that was affected by the Northridge 1994 earthquake. They state that the residual displacement of the ground story of the building was 7.6 cm and it was judged unsafe. However, laboratory testing of the moment connections revealed a good moment resistance and deformation capacity for the damaged beam column joints.

Okada et al. (2000) present the evaluation of reinforced concrete school buildings damaged by the 1995 Hyogo-Ken Nambu Earthquake. They note that six of the school buildings for which the residual plastic deformation capacity was estimated to be adequate, had to be decommissioned due to their large residual drift which exceeded 2%. 

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Altan et al. (1998, 2001) present application of photogrammetric data acquisition technologies to measure the residual deformations of earthquake damaged buildings. Their primary objective is to provide a consistent basis for the damage documentation. They report the measurements obtained for a set of buildings that were damaged during the 1995 Dinar, Turkey $M_w$ 6.2 earthquake.

### 2.4.5 Accuracy of time-history analysis

One of the major focus points of this study is the evaluation of the accuracy of time-history analysis methods. In particular, their accuracy in predicting residual displacements and rotations is investigated. In the following, previous studies addressing this issue are presented.

Paret et al. (1998) investigate the permanent deformations of steel buildings that were damaged during the 1994 Northridge, California $M_w$ 6.7 earthquake. They establish SDOF system models for a sample set of damaged buildings and numerically reproduce their response. The time-history simulation results do not confirm the damage observed in the field —i.e. residual displacements being mainly oriented toward the direction of fault block movement. Paret et al. attribute this lack of agreement to the limitations of the utilized analytical models in predicting the response.

Kanvinde (2003) investigated the accuracy of nonlinear dynamic analysis for predicting the collapse and capturing the response until collapse for a simple small scale structural model. He considers the unidirectional dynamic response of structural models that exhibit non-degrading stiffness. Kanvinde compares the computed maximum and residual drift values against the actual values obtained from shaking table tests. It is concluded that the analysis predicts the maximum and residual drifts with reasonable accuracy.

In order to assess the accuracy of the residual displacements predicted using fiber elements, Dazio (2004) numerically reproduces the dynamic response of a RC wall. The comparison of the simulation results against measured response indicates that the finite-element model could capture the actual maximum displacement of the wall with a good accuracy, but lead to an underestimation of the actual residual displacement. The differences between the simulated and the measured hysteretic behavior are attributed to the modeling of cracking. The cracks that opened in the test unit were “rough cracks” whereas “smooth cracks” were assumed in the adopted fiber-element model. As a result, the simulated reloading behavior differs from the measured behavior.

The accuracies of alternative analytical models are evaluated by Hachem et al. (2003). They numerically reproduce the dynamic response of four RC bridge piers tested on a shaking table. Hachem et al. (2003) conclude that the accuracy of the analytical models in predicting the peak displacements to be significantly higher than that for the residual
displacements. They report that the residual displacements predicted using fiber element models underestimate the measured values whereas the models with a nonlinear hinge located at the plastic hinge region are overestimating them. The data related to these four columns are also utilized in this investigation. For this reason, further details of these columns are presented in Chapter 3.

Berry and Eberhard (2008) propose modeling strategies to model the behavior of RC columns under seismic loading. The models they propose are calibrated based on the data from 37 quasi-static cyclic column tests. Additionally, Berry and Eberhard numerically reproduce the dynamic response of the four columns tested by Hachem et al. (2003). They state that the proposed modeling strategies provide an accurate means of estimating response maxima, hysteretic energy dissipation, and cover spalling. However, their accuracy in predicting the residual displacement is found to be very low.

Methods to predict the residual displacements of concrete-filled steel box columns are investigated by Ge et al. (2003). They numerically reproduce 15 pseudo-dynamic tests by means of SDOF system models as well as fiber-element models. For the SDOF system models, they consider two alternative rule-based hysteretic models: bilinear and trilinear. Ge et al. state that the residual displacements predicted using the SDOF systems with the trilinear hysteretic model are in better agreement with the test results than those predicted using the other modeling approaches.

Pettinga (2006) (and later Pettinga et al. (2008)) investigated the response of single-story buildings with in-plan asymmetry. A series of shaking table tests of model structures with fuse-type replaceable steel plastic hinges and adjustable eccentricity was carried out. Pettinga (2006) simulated the dynamic response of these systems and remarks noticeable differences between the predicted and the measured residual diaphragm rotations.

Phan et al. (2007) tested two RC columns dynamically and simulated their response using a new rule based hysteretic model. The hysteretic model is referred as O-Hyst (“O” stands for “offset”). In general terms, the model is similar to the Clough model (Clough and Johnston, 1966). The major difference is the proposed new force level corresponding to the change of stiffness during reloading. This reloading behavior results in fatter cycles than those obtained using the Clough model. Phan et al. show that the residual displacements estimated using the O-Hyst model are in better agreement with the test results compared to those estimated using either the Clough hysteretic model or the Q-Hyst model (Saiidi and Sözen, 1979).

2.4.6 Mitigation measures

Numerous approaches have been proposed to mitigate residual displacements. Some of these methods involve the use of new strategies to proportion members, whereas others
involve totally new design and construction methods. The studies presented in the following are only a small fraction of these methods, since a comprehensive investigation of this topic is beyond the scope of this study.

The behavior of precast beam-to-column RC joints with unbonded tendons under static cyclic loading was investigated by Priestley and MacRae (1996). These joints are reported to sustain smaller residual displacements compared to conventional RC joints. The results show that precast beam-to-column RC joints attain static residual displacements that are only 2% of the maximum displacement. The dynamic behavior of an RC precast frame is studied by Priestley et al. (1999). For this purpose, they pseudo-dynamically tested a 5-story RC precast frame. The model with unbonded prestressing tendons attained significantly smaller residual displacements than those typical for conventional RC systems.

As a result of a series of SDOF time-history analyses, MacRae and Kawashima (1997) recommend the increase of the post-yield stiffness and the reduction of the maximum ductility as the two most effective methods to mitigate the residual displacements while designing a structure. Additionally, MacRae (1998) suggests that the flexural strength of eccentrically loaded columns should be designed to be larger in the direction of the eccentricity. This way the likelihood of plastic deformations accumulating in one direction may be reduced.

A strategy aiming at reducing the residual displacements of RC walls is proposed by Dazio (2004). For this purpose, the $\alpha_n$ parameter—which is defined as the bending strength of the element due to axial load divided by the total bending strength—is introduced. Dazio proposes that the residual displacement may be reduced by designing the structure and the walls to have optimally large $\alpha_n$ values. In Figure 2.4, the static cyclic response simulated for two walls that have $\alpha_n$ values equal to 0.2 and 0.5, respectively, are presented. Both walls have the same material properties, length, and thickness. However, their longitudinal reinforcement contents and the axial loads are different. The wall with larger $\alpha_n$ ($\alpha_n=0.5$) has higher axial load but lower longitudinal reinforcement ratio compared to the other wall. Note that, the bending strengths of the walls are equal. The wall with larger $\alpha_n$ sustains smaller residual displacements than the other wall (Figure 2.4).

Zatar and Mutsuyoshi (2000, 2002) investigated the use of prestressed RC bridge piers to minimize the residual displacements and propose a new rule-based force-displacement hysteretic model to predict the residual displacements of such piers. Kwan and Billington (2003a,b) analytically investigated the performance of unbonded post-tensioned bridge piers and remarked that the residual displacements attained by these systems are smaller than those of conventional RC piers that experience similar peak displacements.

A parametric study on the residual displacements of RC bridge columns is presented by Sakai and Mahin (2004). They investigated the effects of the axial load ratio and of the longitudinal reinforcement ratio. Sakai and Mahin state that the increase in the
axial load ratio and the reduction in the longitudinal reinforcement ratio reduce the residual displacements. Prestressing the bridge column using an unbonded prestressing strand located at the center is recommended as the optimal mitigation for residual displacements. The effectiveness of this mitigation strategy is confirmed by the results of shaking table tests (Sakai et al., 2006). Test results indicate that the residual displacements of the columns designed according to the suggested approach are less than 10% of the residual displacements of equivalent conventional RC bridge columns.

Pettinga et al. (2006, 2007a) address methods to increase the post-yielding stiffness of RC structures for mitigating residual displacements. They state that the post-yield stiffness of the monotonic tensile stress-stress curves is higher for steels of higher grades. Based on a series of time-history analyses, they conclude that the residual displacements of RC frames are smaller when the post-yield stiffness of the reinforcement steel is higher. Furthermore, Pettinga et al. investigate the benefits of distributing longitudinal reinforcing bars along the height of beam sections instead of placing them at mostly near the top and the bottom of the section which corresponds to conventional detailing of an RC section. Based on time-history analyses carried out on frame models, they conclude that the distributed arrangement of longitudinal reinforcements results in smaller residual displacements while the maximum displacements are not significantly affected. Moreover, similarly to Kiggins and Uang (2006), Pettinga et al. evaluate the effectiveness of introducing a secondary elastic frame acting together with the primary structural system. They present an approach to proportion the secondary frame based on a target global post-yield stiffness value.

Kiggins and Uang (2006) state that the residual displacements of buckling-restrained braced frames can be reduced by designing them as dual systems —i.e. together with a special moment resisting frame. In essence, this design approach introduces an additional
2.5 Remarks on the existing literature on residual displacements

The importance of accounting for the residual displacements in performance-based design and assessment is addressed in several codes and guidelines. However, a detailed evaluation of the accuracy of the simulated residual displacements and rotations of RC structures is missing. In this study, an attempt is made to evaluate this accuracy. Furthermore, the sensitivity of the simulated residual displacements to important modeling parameters and assumptions are investigated.

The findings presented in the previous section suggest the simulated residual displacements to be sensitive to a number of structural properties (e.g. post-yield stiffness, period, hysteretic behavior). For this reason, these governing structural properties were identified in a systematic way for each considered structure while establishing the numerical models in this study.
3 Numerical simulation of the seismic response of RC structures

3.1 Introduction

The large majority of the structures located in seismically active regions are designed to deform beyond their elastic limits under severe strong ground motions. When predicting the extent of probable damage to a structure, estimating the likely magnitudes and locations of the maximum and the residual deformations is essential.

Estimating the maximum and residual displacements of a structure subjected to an earthquake is a complex process. When only static loads are considered, the response of the structure is a function of the properties of the components of the structure (e.g. columns, beams, walls, joints and foundation) and its boundary conditions (e.g. the supporting soil and the loads acting on the structure). On the other hand, when the response to dynamic loads is of interest, the interrelated variations of the essential properties (e.g. stiffness, strength, deformation capacity) of the structural components in time need to be predicted.

In order to establish analytical models to predict the dynamic response of a structure, it is inevitable to simplify the problem by introducing a series of assumptions/idealizations. The degree of sophistication/simplification of the model depends on the following factors:

- the response parameters to be predicted,
- the required accuracy,
- the available time and computational resources,
- the available knowledge regarding the structure and its dynamic behavior.

The complexity of the problem makes it difficult to predict the changes in the accuracy of the results that result from different modeling assumptions being introduced. Hence,
it is difficult to quantify the magnitude of the likely error in the predicted response based on the adopted idealization. As a result, most of the time the accuracy of the predicted response is estimated based on engineering judgment.

In earthquake engineering, the applications that require the prediction of the seismic response of structures can be broadly classified into two groups in terms of their objective: (1) design applications and (2) assessment applications. In design applications, the primary objective of the structural engineer is to ensure that a specific level of safety against seismic actions is provided. This is achieved by providing the necessary stiffness, strength and ductility to the structure. During design, the engineer has—at least to some extent—control over the distribution of the stiffness, the strength and the ductility capacities to the individual structural components. Consequently, a conservative approach can be adopted by introducing a series of load and resistance factors. Given that the introduced factors yield sufficiently conservative estimates of critical response parameters, the accuracy of the predicted response is not overly critical for ensuring that the required minimum safety is provided.

In seismic assessment applications the objective of the analysis is the identification of the likely seismic performance of the structure subjected to different intensities of shaking. Based on the identified performance, the optimal decisions can be made regarding the structure. Moreover, when the assessment is conducted to verify the safety of a damaged structure, the accuracy of the adopted model has a critical importance. The effects of the structural damage on the properties of the structure (i.e. the reduction in stiffnesses and strengths, and the elongation of the periods) can only be captured if a sufficiently accurate model of the structure is used. Otherwise, the analysis may lead to misperception of the risks and adoption of ineffective retrofitting strategies.

The accuracy and sensitivity of important predicted response quantities that are typically used in earthquake engineering are investigated in this chapter. To this end, 12 shaking table tests are reproduced through nonlinear time-history analysis. Predicted residual deformations are given a central role in this investigation.

The chapter commences with an overview of the important seismic response characteristics that lead to residual deformations. After that, some important state-of-the-art analytical modeling approaches are briefly presented. Subsequently, the major properties of the considered test units are presented together with the response they exhibited. Six alternative finite element models are established for each test unit and time history simulations are carried out. The resulting estimates of the key structural response parameters are compared with the experimental evidence. The reliabilities of alternative finite element models are evaluated based on their accuracy in terms of predicting the maximum and the residual deformations. Subsequently, the sensitivities of the predicted maximum and residual deformations to the modeling idealizations are assessed. Finally, the findings of the study are summarized and a set of modeling recommendations are proposed for the dynamic analysis of RC structures.
It is important to note that this study aims to provide a first step toward a better understanding of the limitations of the common modeling approaches. The accuracies identified here should not be considered as conclusive. The results are limited to the considered shaking table tests and the investigated models. In reality, a very large number of potential sources of error which are not considered in this study may strongly affect the accuracy of the predicted response. Nevertheless, the findings are believed to be valuable in terms of identifying some critical points related to the prediction of the seismic response.

3.2 Cyclic response and the residual displacements

The likelihood of attaining residual deformations for a structure subjected to a ground motion depends on the cyclic behavior characteristics of the structure and on the characteristics of the excitation. In this section, issues that are related to the cyclic behavior of structures are discussed. Given that a particular structure attains a specific peak displacement, the likely magnitude of the residual displacement it will sustain is influenced by two characteristic features of the structure: (1) its self-centering capability and (2) its post-yield stiffness. Structures exhibiting a self-centering hysteresis behavior attain negligible—if any—residual displacements following an earthquake. For structures that do not exhibit such a hysteresis (e.g. conventional reinforced concrete and steel structures), the post-yield stiffness strongly influences the likely magnitude of residual displacements. This applies particularly to structures exhibiting a negative post-yield stiffness since they have a higher tendency to deform predominantly in a single direction compared to those exhibiting a positive post-yield stiffness. In the following, this tendency is explained using the results of a shaking table test as an example.

In Figure 3.1 the average drift ratio of the structural wall WDH4 tested by Lestuzzi et al. (1999), is plotted. The response plotted in this figure was measured during the first test Test 1 for this wall. It is seen that a peak drift of 1.52% is attained after about 7.5 seconds from the beginning of the test. At the end of the response the unit attains a residual drift of 0.21% which corresponds to roughly 14% of the peak drift. Lestuzzi et al. (1999) report that the test unit had deformed predominantly under flexure and had failed after a longitudinal reinforcement bar had fractured. In order to present the critical issues related to the residual displacement, the response of the unit during the following cycles is investigated in higher detail:

**Cycle1** - the first yielding cycle taking place in the time interval from 1.31 to 2.12s.
**Cycle2** - a non-yielding cycle taking place in the interval from 3.30 to 3.99s.
**Cycle3** - a yielding cycle taking place in the interval from 4.81 to 5.81s.
**Cycle4** - a non-yielding cycle taking place in the interval from 11.3 to 12.3s.
Figure 3.1: Average drift ratio measured for the wall WDH4 during the Test 1 conducted by Lestuzzi et al. (1999)

In Figure 3.2, the same four cycles are highlighted in the base moment versus average drift ratio hysteresis. In these plots, the points that correspond to noticeable changes in stiffness are indicated. The same points are also indicated in Figure 3.1.

The first time that the component attains a permanent deformation corresponds to the first unloading from a point beyond the yield displacement. This is exhibited during Cycle 1. In the moment versus average drift plot of Cycle 1, the system deforms in the elastic range with nearly a constant stiffness from point A1 to B1 (Figure 3.2a). Subsequently, the estimated yield drift (0.55%) is exceeded and the hysteresis reaches point C1 which corresponds to the peak moment and the peak drift in this half-cycle. The unloading from point C1 takes place towards point D1 along a slightly parabolic line. Due to the plastic deformations in the reinforcement steel, a permanent deformation is attained. The distance between points A1 and D1 is the permanent deformation of the system due to the yielding that has taken place during the segment B1-C1. From point D1 the system continues to deform toward point E1 where again a drop in stiffness is observed. It can be seen that during the reloading phase (D1-E1), in the first phase the stiffness drops below the unloading stiffness. Subsequently, as the system approaches point E1, the stiffness gradually increases.

As the system deforms further beyond point E1, the stiffness drops again until the reversal of the loading direction occurs at point F1. This point corresponds to a displacement ductility of about 1.5. Subsequently, the unloading takes place toward point G1. Similar to the previous unloading branch, the unloading takes place along a slightly parabolic path. However, this time with a lower stiffness when compared to that exhibited along C1-D1.

The step-like behavior in the flexural hysteresis is due to the axial load (Aoyama, 1964). After the direction of deformation is reversed, the system starts to unload following a slightly parabolic path. The deformation in this region is primarily related to the shortening of the reinforcement bars that had yielded just before the load reversal. At the point where the compressive stresses in these bars reach the level of yielding,
Figure 3.2: Measured average drift ratio versus moment at the base plots for wall WDH4 tested by Lestuzzi et al. (1999) during: Cycle 1 (a), Cycle 2 (b), Cycle 3 (c) and Cycle 4 (d), respectively.
the stiffness of the system reduces significantly. However, the stiffness increases again as the cracks that had opened along the component gradually close and the concrete starts to resist the deformation. This gradual change of stiffness results in pinching of the moment-deformation hysteresis loops. The degree of this pinching has an important influence on the residual displacement attainment (Dazio, 2004). Essentially, the possible range of residual deformations that may be sustained at the plastic hinging region is inversely proportional to the degree of pinching of the moment-deformation hysteresis loops.

In the subsequent phase, a number of non-yielding cycles are experienced by the wall. Cycle 2 is one of such cycles (Figure 3.1). The stiffness is nearly constant through this cycle (Figure 3.2b). Furthermore, the measured base moment versus drift behavior forms a line between the points of peak deformations that were attained before (C1 and F1). The displacements at the beginning and at the end of Cycle 2 (points A2 and D2) are the same, i.e. the cycle does not result in a significant change in the permanent deformation.

In the time interval from 4.81s to 5.81s, the second yielding cycle (Cycle 3) takes place. This cycle results in a larger permanent deformation. The shape of the base moment versus average drift curve for Cycle 3 presented in Figure 3.2c resembles the one of Cycle 1. However, this time, yielding occurs at larger drift values than Cycle 1. The points of displacement direction reversal correspond to displacement ductilities of about 2.5 in the negative direction and 3.7 in the positive direction, respectively. The displacements at these points correspond roughly to the previous peak displacements (Figure 3.1). Beyond these points, the behavior of the wall features the post yield stiffness. In this part of the hysteresis loop, the plastic strain in the reinforcement steel increases. The plastic strains locked in the reinforcement steel are the primary cause for the permanent deformations. The post-yield stiffness exhibited along segment E3-F3 is nearly the same as the post-yield stiffness of the corresponding segment E1-F1 along Cycle 1. The stiffness at these segments is controlled primarily by the strain-hardening behavior of the reinforcement steel, by changes in the axial load resisted by the wall, as well as by the P-Δ effects.

For components exhibiting a positive post-yield stiffness, the forces that are required to introduce more post-elastic strains into the reinforcement increase with every yielding cycle. Hence during the dynamic response, the direction in which the smaller displacement was experienced is always the weaker direction for a component that experiences unequal peak displacements in the two loading directions. For example, during Cycle 3, just after the wall reaches point C3 the moment required to yield the component in the negative direction is 15% larger than that in the positive direction (point E3). Given that during the shaking, the forces acting on the component are in the same range of magnitudes in the positive and negative directions, the likelihood of yielding in the weaker direction is essentially higher than in the stronger direction. As the yielding takes place
in this weaker direction, the shifted baseline —i.e. the drift value around which the unit vibrates— returns towards the original configuration. The tendency of balancing the yielding deformations in the two directions reduces the likely magnitude of the residual deformations. For the components with negative post-yield stiffness, the opposite holds true. The strength required to yield the component in a given direction decreases as the yielding progresses in that direction. Therefore, the likelihood of yielding taking place predominantly in a single direction increases with the increasing inelastic deformation. This behavior of components with negative post-yield stiffness significantly increases the probability of sustaining large residual deformations.

In the unloading segments $C3-D3$ and $F3-G3$, the stiffness of the unit is lower than in previous unloading segments. By comparing the unloading histories in the two large cycles, it is seen that the increasing post-elastic deformations lead to a reduction of the unloading stiffnesses. A gradual increase in the stiffness is again exhibited in the reloading segments $A3-B3$ and $D3-E3$.

Just before the vibration starts to diminish, Cycle 4, —which is a non-yielding cycle— takes place (Figure 3.2d). The stiffness of the wall is lower than Cycle 2. Moreover, the stiffness throughout the cycle is rather constant. Similar to Cycle 2, the vibration takes place along the line that passes approximately through the previous points of peak response. This observation may be considered as an indication of the residual displacement being a valuable indicator for estimating the remaining stiffness in RC components.

### 3.3 Modeling approaches

In the order of increasing level of sophistication, modeling approaches for earthquake engineering applications may be classified into three groups:

- **Global models** where the response of the structure as a whole is idealized by a force deformation relationship,

- **Member-by-member models** where the responses of individual structural components are idealized,

- **Material-level models** where the structure is discretized into an assemblage of small elements for which the response is idealized according to the principles of continuum mechanics.

There is a number of references that present these approaches at various levels of detail (e.g. Bathe, 1996; CEB, 1996a,b). This study focuses on the member-by-member modeling approach which is one of the most common modeling approaches adopted to
predict the seismic response of structures. In this approach, the responses of individual structural components are defined according to a specific set of functions called “element formulation”. These functions relate the assumed local force-deformation or stress-strain behavior to the force-deformation behavior of the whole element. A global system of equations is established for the structure based on the interaction between the individual elements. The response of the structure at the defined degrees of freedom is simulated by solving this system of equations iteratively. The resulting force-deformation behavior predicted for the individual structural components depend primarily on the adopted cyclic hysteretic model and the applied idealization regarding the distribution of inelastic deformations along the components. In this study, only the response of RC structures that deform predominantly under flexure is considered. Other deformation mechanisms are omitted in the following.

3.3.1 Flexural response of a section: Hysteretic models

Simulation of the cyclic force-deformation — i.e. the hysteretic — behavior of structural members is a fundamental aspect in the prediction of the seismic response. The role of a hysteretic model is to simulate the degradation of stiffness, deterioration of strength, and the dissipation of energy due to inelastic action. In this study, three of the most commonly used hysteretic models to describe the sectional response of RC components are considered. These are: (1) the bilinear model, (2) the modified Takeda model and (3) the fiber-section model.

Bilinear hysteretic model

The bilinear model is one of the simplest and most frequently used hysteretic models. It is defined by three parameters: the yield deformation $d_y$, the initial stiffness $K_{ini}$ and the post-yield flexural stiffness ratio $b_{EI}$. An example of the bilinear force-deformation hysteresis is presented in Figure 3.3a. Due to its simplicity, it is computationally highly efficient. The fundamental relationships that relate the expected strength of components to the expected ductility demand were derived by Newmark and Hall (1982) using the bilinear hysteretic model. This model has also been utilized in the nonlinear dynamic multi-degree-of-freedom analyses carried out to evaluate the reliability of simpler response prediction methods for RC buildings (ATC, 1996). Furthermore, the design criteria provided in the Japanese specifications for highway bridges (JRA, 1996) related to residual displacements are based on the bilinear hysteretic model (Kawashima, 1997).

The degradation of stiffness that is exhibited by RC components is not captured by the bilinear model. Therefore, the plausibility of this model for refined nonlinear analyses of RC structures is known to be limited (Hidalgo and Clough, 1974). However, it is still considered in this investigation since by comparing the results obtained with a
bilinear hysteretic rule to results obtained with more advanced models the importance of the hysteretic model in predicting the response can be illustrated.

![Normalized displacement, $d / d_y$ vs. Normalized force, $F / F_y$](a)

![Normalized displacement, $d / d_y$ vs. Normalized force, $F / F_y$](b)

Figure 3.3: Moment curvature relationships obtained using bilinear model (a) and modified Takeda (Q-hyst) model (b)

**Modified Takeda hysteretic model**

The modified Takeda model considered in this study is a simplified version of the original phenomenological hysteretic model developed by Takeda et al. (1970). Unlike the bilinear model, the Takeda hysteretic model aims to capture the changes in the component stiffness based on its deformation history. The original Takeda hysteretic model is defined by 16 rules. In order to reduce this complexity, several simpler versions of the original Takeda model have been proposed (Otani, 1974; Litton, 1975; Powell, 1975; Saiidi and Sözen, 1979). In this study the version proposed by Saiidi and Sözen (1979)—which is also known as the Q-hyst model—is considered. Originally, this hysteretic model was proposed to simulate the force-displacement response of SDOF oscillators. However, Saiidi (1982) has shown that the response simulated using the Q-hyst model has a remarkable correlation with that simulated using the original Takeda model.

The modified Takeda model has been used in several studies investigating the residual displacements of RC structures (Pampanin et al., 2003; Christopoulos et al., 2003; Dazio, 2004; Ruiz-García, 2004). In addition, Yazgan and Dazio (2006) have compared the residual displacement predictions obtained using the modified Takeda hysteretic implementations available in different analysis programs. The predicted residual displacements were found to be sensitive to the adopted small cycle hysteretic rules which differ between the different implementations.
The modified Takeda model considered in this study is based on the *Hysteretic* model implemented in OpenSees (McKenna et al., 2007). The model is defined by four parameters: (1) the yield deformation $d_y$, (2) the initial stiffness $K_{ini}$, (3) the post-yield flexural stiffness ratio, $b_{EI}$, and (4) the unloading stiffness parameter $\gamma$. A sample hysteresis simulated using this model is presented in Figure 3.3b. In this study, the unloading stiffness parameter $\gamma$ is taken as 0.5 following the study by Saiidi and Sözen (1979). The model used in the this study reloads to the point of peak curvature in the loading direction.

**Fiber-section model**

The fiber-section model is generally assumed to be the most promising of the member-type modeling approaches to capture the post-elastic flexural cyclic response of RC sections (CEB, 1996b). In a fiber model, the cross section of the component is discretized into smaller subregions, referred to as fibers. A uniaxial cyclic stress-strain model is assigned to each fiber depending on the material it represents. The cyclic response of the cross section is obtained from the uniaxial stress-strain behavior of the individual fibers. For the details of the different versions of the model the reader is referred to other sources (Kaba and Mahin, 1983; Taucer et al., 1991; Bathe, 1996; Maekawa et al., 2003). Fundamentally, similar to the phenomenological hysteretic models presented above, the main objective of the fiber-section model is the simulation of the flexural hysteretic behavior. However, the fiber model differs from those models in the sense that it does not constitute a set of hysteresis rules to simulate the moment-curvature response, but it introduces a modification to the element-formulation. This modification allows the direct simulation of the section response based on the stress-strain response models adopted for the constituent materials and the section geometry. Moreover, as a result of this modification the axial force-bending moment interaction, unlike the rule-based hysteretic models, is directly taken into account by the model. Therefore, a key component of fiber-section models is the set of cyclic stress-strain relationships utilized to characterize the behavior of the materials.

A large number of uniaxial stress-strain models with different degrees of complexity have been proposed to capture the cyclic response of reinforcement steel (CEB, 1996a). In this study, the Giuffrê-Menegotto-Pinto model (Menegotto and Pinto, 1973) is employed. An example of the stress-strain response simulated using this model is plotted in Figure 3.4a. In this model, the properties of the backbone curve, particularly the yield stress, the modulus of elasticity and the strain hardening behavior are controlled by the parameters: yield strength $\sigma_y$, Young’s modulus $E_s$ and post-yield stiffness ratio $b_s$, respectively. Moreover, the shape of the transition curve that is followed from the point of strain direction reversal to the target stress-strain point in the opposite direction, is controlled by three parameters: $R_0$, $A_1$ and $A_2$. This curved transition allows a representation of the cyclic behavior of the reinforcement steel in the inelastic range, particularly the Bauschinger effect (Restrepo-Posada et al., 1994). Menegotto and Pinto
(1973) present the analytical expressions and the fundamental rules governing the behavior the model.

Unfortunately, the Giuffrè-Menegotto-Pinto model is known to suffer from the fact that—depending on the cyclic response history—a very large number of strain direction reversal points may need to be stored and recalled for each single fiber to achieve a plausible simulation. When this is not accomplished, the simulated response may exhibit some unsuitable “jumps”. This issue is called the “memory-effect” (Filippou et al., 1983) and its probable implications in terms of residual deformations are discussed by Yazgan and Dazio (2006). Moreover, Sakai and Mahin (2004) propose a modification to the original model to prevent these inappropriate jumps. In this study, the material model Steel02 in OpenSees (McKenna et al., 2007)—i.e. an implementation of the Giuffrè-Menegotto-Pinto model—is adopted for the steel fibers (Figure 3.4a).

![Figure 3.4: Sample stress-strain simulations obtained using the uniaxial material models Steel02 (a) and Concrete04 (b)](image)

Over the years, several different hysteretic models have been proposed to capture the cyclic response of confined and unconfined concrete (CEB, 1996a). In this study, the stress-strain model Concrete04 available in OpenSees (McKenna et al., 2007) is employed. The backbone curve of this model is based on the model proposed by Mander et al. (1988). The cyclic behavior of the model in the compressive region is based on the linearization of the model by Karsan and Jirsa (1969). An example stress-strain response simulated using this model is presented in Figure 3.4b. An important issue related to modeling the stress-strain behavior of concrete fibers is the confinement effect. In this study, the increase in the compressive strength and in particular the strain capacity due to confinement is estimated based on the model by Mander et al. (1988) as presented in Priestley et al. (2007).
3.3.2 Simulation of the element flexural response: Member models

Post-elastic flexural deformations typically do not localize to the critical section but spread along the RC components that are subjected to cyclic loads. Several member models have been proposed to capture this spreading (CEB, 1996b). Broadly, these models can be grouped into two classes based on the approach that is adopted in their derivation:

1. Displacement-based element models
2. Force-based element models

In this report the performance of these two element formulations in terms of capturing the flexural deformations of RC elements is investigated.

In the displacement-based element formulation (also known as the stiffness-based approach), the curvature and the axial deformation at any section along the element are a function of the member end-deformations (Bathe, 1996). This relationship is defined by means of displacement shape interpolation functions. Generally, cubic Hermitian polynomials are utilized as the shape interpolation functions for the displacements transverse to the element axis. As a result, the curvature is assumed to vary linearly in a single element. Therefore to be able to capture the nonlinear spread of the post-elastic curvatures along the RC component, the component must be discretized into a number of displacement-based elements. This element formulation has been adopted in several FE codes such as OpenSees (McKenna et al., 2007), SeismoStruct (SeismoSoft, 2007b), Zeus-NL (Elnashai et al., 2001) and Rechenbrett 2D (Dazio, 2000).

The force-based element formulation (also known as the flexibility-based approach) is based on the interpolation of member end-forces to identify the internal section forces (Kaba and Mahin, 1984; Spacone et al., 1996; Neuenhofer and Filippou, 1997). As a result, for RC components without transverse member loads, the linear variation of the moment along the component can be captured by utilizing a single element even beyond the elastic limit. The simulated response however is still sensitive to the adopted discretization scheme due to inelastic deformation localization issues (Coleman and Spacone, 2001). Therefore, in order to establish plausible models for inelastic time-history analysis the components should again be discretized into several force-based elements if conventional element integration schemes (e.g. Gauss-Legendre, Gauss-Lobatto) are utilized.
3.4 Shaking table test data

Shaking table testing of RC components provides valuable information regarding the seismic response of RC structures. In this study, the results of shaking table tests are utilized to assess the reliability of the results obtained using the alternative modeling approaches presented in Section 3.3. More data is needed to simulate the response of shaking table tests than is need for static cyclic tests. A reliable record of the shaking table acceleration is mandatory for a plausible simulation. In order to collect shaking table test data, contacts were established with a number of researchers. With their generous help, the data for 12 RC test units were collected. The general geometries of the tests units are presented in Figure 3.5. The equivalent plastic hinge lengths that are estimated for the units according to Priestley et al. (2007) are also presented in the figure. The section geometries of the units are available in Figure 3.6. The longitudinal reinforcement ratios of the units were in the range 0.47 to 1.17%. Similarly, the volumetric ratios of the transverse steel were in the range 0.61 to 2.75%. For all test units, the shaking resulted in moments around the local-y axis shown in Figure 3.6, except for the two columns A2 and B2 which were subjected to bidirectional excitation. The axial load ratios at the critical sections were in the range 1.6 to 9%.

![Figure 3.5: Geometries of the 12 test units (Dimensions are in mm)](image)

3.4.1 Test Units: A1, B1, A2 and B2

The test units A1, B1, A2 and B2 are four circular RC columns tested on the shaking table by Hachem et al. (2003). They are 1:4.5 scale models of a realistic bridge pier prototype. They were designed according to the CALTRANS Bridge Design Specifica-
Figure 3.6: Cross section geometries of the test units (Dimensions are in mm)
tions (Caltrans, 1990). Both the effects of multi-direction loading and the influence of different ground motion characteristics were investigated.

All four columns had the same geometry. The columns A1 and B1 were tested unidirectionally and the columns A2 and B2 were tested bidirectionally. The column A1 was subjected to the strike normal component of the Olive View record that was recorded during the 1994 Northridge $M_w 6.7$ earthquake, whereas the column B1 was subjected to both horizontal ground motion components simultaneously. A strong velocity pulse is present in the velocity time-history series of this record. The columns B1 and B2 were subjected to the Llolleo record from the 1985 Chile $M_w 7.8$ earthquake. This record has a longer duration and was taken from a location 64 km away from the epicenter. Again, the column B1 was only subjected to the component with the higher peak ground acceleration while B2 was subjected to both horizontal ground motion components. Through the entire test program the test units were subjected to a set of strong shaking tests with varying intensities. For the test units A1 and A2, the first test that had a shaking intensity close to the assumed design earthquake was the test Run 2. Similarly for the test units B1 and B2, the intensity of shaking during the test Run 5 was close to the design ground motion intensity. Hachem et al. (2003) report that mainly horizontal cracks opened up during the tests of all columns. This may be considered as a confirmation of the units' primary deformation mode being flexure.

3.4.2 Test Unit: EBII07

The test unit EBII07 is a rectangular RC column that was tested on the ETH shaking table on December 12, 2007. The unit was designed in order to behave primarily in flexure and the plastic hinge zone was detailed so as to achieve a ductile behavior. The technical properties of the shaking table are available in Bachmann et al. (1999). The stronger component of the Caleta de Campos ground motion record that was recorded during the 1985 Michoacan $M_w 8.1$ earthquake was considered as the table excitation. The response of the column EBII07 during Test 3 is considered in this verification study. During this test, the reference ground motion was applied to the unit at 100% intensity. This test was the first to cause spalling of the cover concrete. Moreover, during the excitation, mainly horizontal cracks opened up in the bottom segment of the column.

3.4.3 Test Units: WDH1, WDH2, WDH3, WDH4, WDH5 and WDH6

The test units WDH1 to WDH6 are the six reinforced concrete walls tested on the ETH shaking table by Lestuzzi et al. (1999). An overview of the tests and the relevant results are also presented by Lestuzzi and Bachmann (2007). The test units are models at scale 1:3 of RC structural walls bracing a 3 story reference building. In the test series, the test parameters (1) ground motion characteristics, (2) ductility properties of the
reinforcement steel, (3) adopted design method and (4) longitudinal reinforcement ratio were considered. The walls WDH1 to WDH4 were designed for a displacement ductility capacity of 3 according to the capacity design principles outlined in Bachmann (1995). The wall WDH5 was conventionally designed according to SIA 162 (1993) and its displacement ductility capacity was estimated to be about 2. The detailing of the wall WDH6 was the same as for WDH5, except that the spacing of the transverse reinforcement had been reduced by 40% which improved the stability of vertical reinforcement against buckling.

Two design-spectrum-compatible acceleration series were generated as input for the shaking table. The two series differed regarding their target pseudo-acceleration spectrum and their duration. The first ground motion record is representative of hazard at medium-stiff sites and was generated based on the elastic design spectrum provided in SIA 160 (1989) for the medium-stiff sites and 5% damping. This acceleration series was used in the tests of unit WDH1. The second ground motion record is representative of the hazard at soft sites in Europe and was generated based on the elastic design spectrum in Eurocode 8 (1998) for soft sites and 5% damping. The units WDH2 to 6 were subjected to this acceleration series.

In order to identify the specific tests to be considered in this study, the intensities of shaking and the measured peak values of the critical response parameters were reviewed. The tests during which either the shaking intensity was similar to the design intensity or the yielding of the unit took place were identified to be suitable. For the wall WDH1, Test 3 is found to be suitable for the planned investigation. During this test the target acceleration series was scaled to 70%.

For the walls WDH2 to WDH6, the first dynamic test (Test 1) is included in the investigation. During Test 1 for the walls WDH2 and WDH3, the reference acceleration series was targeted directly. On the other hand, for the walls WDH4 to WDH6 during Test 1, the target acceleration series was scaled to 80%. In summary, the wall pairs WDH1-WDH2 and WDH3-WDH4 have practically the same detailing but were subjected to different excitations. The wall pair WDH5-WDH6 are subjected to the same acceleration series, but are different in terms of the transverse reinforcement ratio. Furthermore, it can be stated that for all six walls, the shaking intensities were close to their design shaking intensities. Finally, Lestuzzi et al. (1999) report that the cracks that opened during the considered tests were mainly horizontal and located in the bottom segment of the walls, hence confirming that they behaved predominantly in flexure.

3.4.4 Test Unit: CAMUS3

The test unit CAMUS3 is a RC structural wall that was tested by Combescure and Chaudat (2000). The test unit is a 1:3 scale model of a 5 story building. It consists of two RC
walls connected by 6 flat-slab floors (including the one connected to the footing). The walls were designed according to Eurocode 8 (1998). The test \textit{MRr2} is considered in this investigation. During this test, the test unit was subjected to the \textit{Melendy Ranch} ground motion record from the 1972 Stone Canyon earthquake. The record was assumed to represent a near field ground motion. The test \textit{MRr2} was the first test that resulted in residual deformations. During this test, the peak acceleration measured on the table was 1.35g. The tests that were performed prior to \textit{MR2r2} had resulted in only minor damage. Combescure and Chaudat (2000) report that during this test cracks had opened in the first 3 stories. However, Combescure and Chaudat (2000) also note that even before the tests some relatively small cracks were already present in the regions where the floors are connected to the walls.

Combescure and Chaudat (2000) report that the displacement at the top story was not recorded properly during the test \textit{MR2r2}. Therefore, in this study, the displacements that are recorded at the 4\textsuperscript{th} story and the height (4.395 m) of this story are considered as an approximation in the average drift calculations.

3.4.5 Brief review of the test results

This study is primarily concerned with evaluating the accuracies of the common modeling tools in terms of predicting the deformations and accelerations experienced by structures subjected to seismic actions. To this end, the evaluation is carried out primarily based on the displacements, rotations and accelerations measured during the shaking table tests. Moreover, both the maximum and the residual values of the displacement and rotation are considered.

The maximum values of the response parameters are indicated with the subscript “\textsubscript{m}” and refer to the absolute maximum values of these parameters. To indicate the residual values, the subscript “\textsubscript{r}” is used. The residual values refer to the absolute values of the displacement levels to which the vibrations converged at the end of the response. If the vibration had not completely stopped at the end of the measured response data, the residual value was assumed to be the average of the two peaks in the last vibration cycle. Note that this study is concerned with the absolute values of the response parameters. In some cases, the permanent deformations may lie in the opposite direction of the maximum deformation experienced by the test unit. The test units WDH1 and WDH2 exhibit such a response. However, for the sake of clarity only the absolute values of the deformations parameters are considered for all test units in this study.
In this study, to make the different test results comparable, displacements are expressed as drift ratios. The average drift ratio $d_a$ is defined as the relative horizontal displacement of the highest center of mass divided by its height above the foundation (Figure 3.5). The maximum average drift ratios $d_{a,m}$ experienced by the test units are presented in Table 3.1, together with the corresponding residual drift values $d_{a,r}$. The largest residual-to-maximum drift ratio ($d_{a,r}/d_{a,m}$) is exhibited by the CAMUS3 wall where 15% of the maximum drift is attained at the end of the response as the residual. The average drift ratios estimated for the yielding and the ultimate deformation limits, $d_{a,y}$ and $d_{a,u}$ respectively, are also available in Table 3.1. These limits are estimated based on the equivalent plastic hinge length approach presented by Priestley et al. (2007). The details are presented in the next section.

The imposed displacement ductility demands $\mu_{\Delta,D}$ cover a large range of values from high to moderate ductility (Table 3.1). In the neighboring column, the displacement ductility capacities $\mu_{\Delta,C}$ are presented. Only for the case of wall WDH2, the imposed demand exceeded the predicted ductility capacity. This is due to the fact that during Test 1 one of the longitudinal reinforcement bars of wall WDH2 ruptured and excessive deformations were attained (Lestuzzi et al., 1999). For the other tests considered in this study, the damage to the units after the test did not exhibit any signs of exceeding the ultimate flexural deformation capacity (i.e. rupture of reinforcement bars, crushing of confined concrete, etc.). The peak displacement ductilities experienced by the units are plotted against the maximum-to-residual average drift ratios in Figure 3.7a. The general trend —excluding WDH2— shows that larger residual displacements are exhibited by the test units that have attained higher ductilities. However, this positive correlation is not very strong. Consider for example walls WDH3 and WDH4 that have the same detailing and were subjected to the same acceleration series scaled to 100% and 80%, respectively. As expected, wall WDH3 —that experiences the stronger shaking— attains a peak displacement that is approximately 20% larger than that of WDH4. However, the residual displacement for wall WDH3 is 40% smaller than that for WDH4. On the other hand, the circular columns A1 and A2 that attained larger peak deformations also exhibited larger residual deformations compared to the columns B1 and B2.

The ground story drift ratio $d_{gs}$ is obtained by dividing the relevant relative displacement by the height of the 1st story above the foundation level. The ground story drift ratio is an important performance parameter. In many instances, the seismic damage to RC structures manifests itself in the ground stories. As a result, an accurate prediction of the peak ground story drift is useful in terms of capturing the seismic performance of a structure properly. In light of this view the maximum and residual ground story drifts, $d_{gs,m}$ and $d_{gs,r}$ respectively, are taken into account in the evaluation. The relative horizontal displacement of the ground story was measured for the walls WDH2 to 6 and CAMUS3. The measured values are presented in Table 3.1.
Table 3.1: Measured displacements and the predicted deformation capacities

<table>
<thead>
<tr>
<th>Test unit</th>
<th>Experiment</th>
<th>Analysis†</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$d_{a,m}$</td>
<td>$d_{a,r}$</td>
</tr>
<tr>
<td>A1</td>
<td>5.10</td>
<td>0.55</td>
</tr>
<tr>
<td>B1</td>
<td>3.56</td>
<td>0.12</td>
</tr>
<tr>
<td>A2</td>
<td>4.98</td>
<td>0.62</td>
</tr>
<tr>
<td>B2</td>
<td>2.98</td>
<td>0.07</td>
</tr>
<tr>
<td>EBII07</td>
<td>8.12</td>
<td>0.38</td>
</tr>
<tr>
<td>WDH1</td>
<td>0.71</td>
<td>0.01</td>
</tr>
<tr>
<td>WDH2</td>
<td>2.22</td>
<td>0.08</td>
</tr>
<tr>
<td>WDH3</td>
<td>1.84</td>
<td>0.13</td>
</tr>
<tr>
<td>WDH4</td>
<td>1.52</td>
<td>0.21</td>
</tr>
<tr>
<td>WDH5</td>
<td>1.20</td>
<td>0.11</td>
</tr>
<tr>
<td>WDH6</td>
<td>1.22</td>
<td>0.12</td>
</tr>
<tr>
<td>CAMUS3</td>
<td>0.83</td>
<td>0.13</td>
</tr>
</tbody>
</table>

† Displacement limits predicted using the plastic hinge method by Priestley et al. (2007)

Figure 3.7: Peak ductility demands versus residual-to-maximum values of the deformation indices average drift ratio $d_a$ (a) and rotation of the base segment $\Theta$ (b)
Rotations

For ductile structures that are designed according to modern codes, the inelastic flexural deformations are expected to occur in the potential plastic hinging regions. As a result, given that the failure modes other than flexure are inhibited, an accurate prediction of the peak flexural deformations experienced in these hinging regions allows a reliable assessment of the likely damage in these regions — i.e. the reduction of the stiffnesses and the strength.

In this study, the rotation $\Theta$ measured at a height slightly higher than the predicted equivalent plastic hinge length is considered as a flexural deformation index. The heights at which these rotations were measured are indicated in Figure 3.5. Both the maximum ($\theta_m$) and the residual ($\theta_r$) values of the rotation index are taken into account in the evaluation. In order to identify these rotations, the measurement scheme adopted in each test unit was reviewed and the most suitable pair of displacement transducer channels was identified. The rotations were deduced by dividing the relative displacement obtained from the channel pair by the horizontal distance between the points of measurement. Naturally, this deformation parameter does not represent the localization of the strains at the hinging region in detail. However, compared to the average drift ratio this rotation index may be assumed to better represent the local response at the plastic hinging region level.

The maximum and residual rotations are presented in Table 3.2 for 8 test units. For the other 4 units, the data needed to deduce the rotations was not available to the author at the time of the study. Due to the fact that the inelastic action is primarily controlled by the behavior of the plastic region, for each unit the residual-to-maximum rotation ratio is similar to the ratio $d_{a,r}/d_{a,m}$ (Table 3.1), except for wall WDH2. For this wall, the rupture of a reinforcement bar occurred resulting in a residual rotation equal to 20% of the peak. For the same wall the permanent top displacement is only 4% of the peak. This example shows that the damage to the plastic hinge zone manifests itself more noticeably in the local deformation measures than in the global ones.

Accelerations

A major cause of damage to non-structural components in structures are the excessive accelerations (Villaverde, 2004). Typical non-structural components consist of the architectural elements (e.g. partitions, ceilings) and mechanical equipment (e.g. machinery, piping). Due to this fact, the horizontal accelerations exhibited at the different levels within a structure are considered to be important structural response parameters. Non-structural damage depends on experienced story absolute acceleration. This acceleration equals to sum of two accelerations: (1) the acceleration exhibited at the foundation and (2) the relative acceleration of the story with respect to the foundation. The acceler-
Table 3.2: Measured maximum and residual rotations ($\Theta_m$ and $\Theta_r$) predicted yield rotation ($\Theta_y$) for the base segment and measured accelerations ($a_{t,rel}$)

<table>
<thead>
<tr>
<th>Test unit</th>
<th>$\Theta_m$ [permil]</th>
<th>$\Theta_r$ [permil]</th>
<th>$\Theta_r/\Theta_m$</th>
<th>$\Theta_y$ [permil]</th>
<th>$\mu_{\Theta,D}$</th>
<th>$a_{t,rel}$ [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>EBH07</td>
<td>78.72</td>
<td>3.93</td>
<td>0.05</td>
<td>11.42</td>
<td>6.9</td>
<td>-</td>
</tr>
<tr>
<td>WDH1</td>
<td>4.22</td>
<td>0.10</td>
<td>0.02</td>
<td>1.65</td>
<td>2.6</td>
<td>-</td>
</tr>
<tr>
<td>WDH2</td>
<td>13.79</td>
<td>2.63</td>
<td>0.19</td>
<td>1.64</td>
<td>8.4</td>
<td>1.07</td>
</tr>
<tr>
<td>WDH3</td>
<td>17.15</td>
<td>1.42</td>
<td>0.08</td>
<td>2.28</td>
<td>7.5</td>
<td>0.28</td>
</tr>
<tr>
<td>WDH4</td>
<td>13.46</td>
<td>2.16</td>
<td>0.16</td>
<td>2.29</td>
<td>5.9</td>
<td>0.39</td>
</tr>
<tr>
<td>WDH5</td>
<td>9.72</td>
<td>1.13</td>
<td>0.12</td>
<td>3.11</td>
<td>3.1</td>
<td>0.32</td>
</tr>
<tr>
<td>WDH6</td>
<td>9.82</td>
<td>1.03</td>
<td>0.10</td>
<td>3.02</td>
<td>3.2</td>
<td>0.41</td>
</tr>
<tr>
<td>CAMUS3</td>
<td>7.44</td>
<td>1.07</td>
<td>0.14</td>
<td>1.51</td>
<td>4.9</td>
<td>3.14</td>
</tr>
</tbody>
</table>

The measured accelerations exhibited at the foundation are primarily related to the intensity of the shaking experienced at that site. The properties of the structure do not significantly influence this acceleration if soil-structure interaction effects are negligible. On the other hand, the relative accelerations of the stories are directly related to the response of the structure. For this reason, the accuracy of the relative accelerations predicted by means of time-history analysis is evaluated in this study. In Table 3.2, the relative horizontal accelerations measured at the top stories of the test units are presented.

### 3.5 Modeling of the test units

The analytical models of the test units were established following the state-of-the-art modeling approaches presented in Section 3.3. The responses of the test units were simulated using the two different element formulations discussed in Section 3.3 —i.e. displacement-based and force-based formulation—, each combined with three different hysteretic models. The six resulting models considered in this study are named as follows:

- **DB**: Displacement-based element model with Bilinear hysteresis
- **DT**: Displacement-based element model with modified Takeda hysteresis
- **DF**: Displacement-based Fiber-section element model ($Steel02$, $Concrete04$)
- **FB**: Force-based element model with Bilinear hysteresis
- **FT**: Force-based element model with modified Takeda hysteresis
- **FF**: Force-based Fiber-section element model ($Steel02$, $Concrete04$)

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3.5.1 Modeling the section flexural response

The characteristic sectional flexural response properties are predicted by moment-curvature analysis using OpenSees. For this purpose, a fiber representation of each section is established for each test unit (Figure 3.8). The confined concrete regions for the walls were discretized into 50 fiber layers. For the column EBI07, 60 fiber layers are utilized to represent the confined core. The circular core regions of the columns A1-2 and B1-2 are discretized into 20 and 10 segments, along the radial and circumferential directions respectively. For each unit, the moment curvature analysis results are considered to judge the adequacy of the fiber discretization density. The sections were discretized with a sufficiently density so that further increase in the discretization density does not have a considerable effect on the simulated moment curvature response. Note that if only maximum displacements are of concern the use of a coarser discretization will have only a minor effect on the results. However, if also the residual displacements are of interest, the density of the fiber discretization is likely to have a noticeable effect.

Figure 3.8: Fiber cross-sections of the models

The uniaxial stress-strain behavior of the reinforcement fibers is simulated using the “Steel02” model available in OpenSees (Figure 3.4a). The parameters of the steel material model are identified based on actual test results. In Figure 3.9 the stress-strain values that are reported for two sample reinforcement types are presented together with the fitted model. The plots related to other reinforcement bars are available in Appendix A. Since the reinforcements that are used in the test units were manufactured in different countries, differences can be noticed in their strain-hardening properties. The stress-strain behavior of the reinforcement bars under cyclic loading was not available for any of the test units. Therefore, for the parameters that control the shape of the curved transition segments, the values proposed by Menegotto and Pinto (1973) are adopted \((R_0 = 20, A_1 = 18.5\) and \(A_2 = 0.15\)).
Figure 3.9: Sample stress strain plots obtained for the No.4 (Ø12.7) reinforcement bar used in test units A1 (Hachem et al., 2003) (a) and for the Ø6 bar used in WDH1 (Lestuzzi et al., 1999) (b)

Table 3.3: Material model parameters for the steel

<table>
<thead>
<tr>
<th>Test unit</th>
<th>Size</th>
<th>σ_y</th>
<th>E_0</th>
<th>b_s</th>
<th>ε_y</th>
<th>ε_su</th>
<th>ε_sur</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1, B1, A2, B2 No.4 (Ø12.7)</td>
<td>506</td>
<td>226</td>
<td>1.5</td>
<td>0.22</td>
<td>12.2</td>
<td>7.3</td>
<td></td>
</tr>
<tr>
<td>EBI07 Ø8</td>
<td>545</td>
<td>180</td>
<td>1.3</td>
<td>0.30</td>
<td>11.9</td>
<td>7.1</td>
<td></td>
</tr>
<tr>
<td>WDH1 Ø4.2</td>
<td>565</td>
<td>202</td>
<td>0.7</td>
<td>0.28</td>
<td>5.0</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>WDH1 Ø6</td>
<td>509</td>
<td>206</td>
<td>0.5</td>
<td>0.25</td>
<td>6.8</td>
<td>4.1</td>
<td></td>
</tr>
<tr>
<td>WDH2 Ø4.2</td>
<td>560</td>
<td>198</td>
<td>0.9</td>
<td>0.28</td>
<td>3.8</td>
<td>2.3</td>
<td></td>
</tr>
<tr>
<td>WDH2 Ø6</td>
<td>514</td>
<td>203</td>
<td>0.4</td>
<td>0.25</td>
<td>7.2</td>
<td>4.3</td>
<td></td>
</tr>
<tr>
<td>WDH3 Ø5.2</td>
<td>479</td>
<td>215</td>
<td>1.2</td>
<td>0.22</td>
<td>7.1</td>
<td>4.3</td>
<td></td>
</tr>
<tr>
<td>WDH4 Ø5.2</td>
<td>486</td>
<td>219</td>
<td>1.0</td>
<td>0.22</td>
<td>8.7</td>
<td>5.2</td>
<td></td>
</tr>
<tr>
<td>WDH5 Ø5.2</td>
<td>489</td>
<td>220</td>
<td>1.0</td>
<td>0.22</td>
<td>7.4</td>
<td>4.4</td>
<td></td>
</tr>
<tr>
<td>WDH5 Ø8</td>
<td>580</td>
<td>216</td>
<td>1.1</td>
<td>0.27</td>
<td>5.9</td>
<td>3.5</td>
<td></td>
</tr>
<tr>
<td>WDH6 Ø5.2</td>
<td>497</td>
<td>219</td>
<td>0.9</td>
<td>0.23</td>
<td>8.3</td>
<td>5.0</td>
<td></td>
</tr>
<tr>
<td>WDH6 Ø8</td>
<td>581</td>
<td>218</td>
<td>1.1</td>
<td>0.27</td>
<td>6.7</td>
<td>4.0</td>
<td></td>
</tr>
<tr>
<td>CAMUS3 Ø4.5</td>
<td>561</td>
<td>197</td>
<td>1.5</td>
<td>0.29</td>
<td>1.7</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>CAMUS3 Ø6</td>
<td>595</td>
<td>192</td>
<td>0.7</td>
<td>0.31</td>
<td>3.2</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td>CAMUS3 Ø8</td>
<td>483</td>
<td>196</td>
<td>0.5</td>
<td>0.25</td>
<td>15.6</td>
<td>9.4</td>
<td></td>
</tr>
</tbody>
</table>

*ε_sur/ε_su=0.6, tension strain limit reduction factor (Priestley et al., 2007)
The material model parameters for the unconfined concrete fibers are determined from the relevant test reports. Plots of the stress-strain relationship however, were not presented in all reports. For the unconfined concrete, the compressive strain limits corresponding to the maximum compressive stress and the ultimate compressive stress are assumed to be equal to 0.2 and 0.4%, respectively, according to the available testing data and the study by Kent and Park (1971). The tensile strength of concrete is neglected in the analyses. The strength and strain capacities estimated for the confined concrete using Mander’s model (Priestley et al., 2007) are presented in Table 3.4.

### Table 3.4: Material model parameters for the concrete

<table>
<thead>
<tr>
<th>Test unit</th>
<th>$E_c$ [GPa]</th>
<th>$\sigma_{c0}$ [MPa]</th>
<th>$\sigma_{cc}$ [MPa]</th>
<th>$\epsilon_{cc}$ [%]</th>
<th>$\epsilon_{cu}$ [%]</th>
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<tr>
<td>A1, A2, B1, B2</td>
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<td>39.2</td>
<td>57.5</td>
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<td>1.92</td>
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<tr>
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<td>40.0</td>
<td>55.1</td>
<td>0.58</td>
<td>2.30</td>
</tr>
<tr>
<td>WDH1</td>
<td>31.5</td>
<td>39.7</td>
<td>50.4</td>
<td>0.47</td>
<td>1.59</td>
</tr>
<tr>
<td>WDH2</td>
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<td>40.4</td>
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<td>0.46</td>
<td>1.57</td>
</tr>
<tr>
<td>WDH3</td>
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<td>36.5</td>
<td>43.6</td>
<td>0.39</td>
<td>1.29</td>
</tr>
<tr>
<td>WDH4</td>
<td>30.1</td>
<td>36.3</td>
<td>43.8</td>
<td>0.41</td>
<td>1.38</td>
</tr>
<tr>
<td>WDH5</td>
<td>30.2</td>
<td>36.5</td>
<td>42.1</td>
<td>0.35</td>
<td>1.00</td>
</tr>
<tr>
<td>WDH6</td>
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<td>42.6</td>
<td>51.7</td>
<td>0.41</td>
<td>1.21</td>
</tr>
<tr>
<td>CAMUS3</td>
<td>31.5</td>
<td>39.6</td>
<td>69.1</td>
<td>0.95</td>
<td>4.02</td>
</tr>
</tbody>
</table>

1Based on the confinement model by Mander (Priestley et al., 2007)

Axial loads acting on the member have a critical influence on the flexural behavior. The axial load relevant for each test unit is determined based on the test reports and the recorded response. For the test units A1, A2, B1, B2 and CAMUS3, the axial loads acting on the plastic hinge regions were mainly due to gravity forces and partly due to the vertical acceleration of the masses located on the shaking table. As a result, a plausible axial load value could be identified easily. For the test units that had been tested on the ETH shaking table, namely the walls WDH1 to 6 and the column EBI07, the axial load was simulated by post-tensioning cables and the masses were located on rails. This testing scheme has the benefit of allowing the response to be investigated at different axial load levels, independent of the dynamic mass attached to the test unit. However, the forces in the post-tensioning cables that are used to simulate the axial loads change as the test unit deforms. In order to identify a plausible value for the axial load level, the load histories of these cables reported by Lestuzzi et al. (1999) were investigated. The estimated time histories of the axial load levels at the bottom of the test units are presented in Appendix B. The axial load ratios $n_{ax} = N/A\sigma_{c0}$ that are considered in the moment curvature analysis are summarized in Table 3.5.

The bilinear idealization of the flexural response of the test units is established based on the recommendations by Priestley et al. (1996). Two sample bilinear idealizations
are presented in Figure 3.10. The others are presented in Appendix C. The ultimate curvature capacity for all sections is controlled by the ultimate strain capacity of the reinforcement. This strain limit is assumed to be 60% of the strain at the peak tensile stress reported in the material tests in accordance with the recommendations by Priestley et al. (2007). In Figure 3.9 this strain limit is indicated with a dashed line. The list of the identified values for the characteristic parameters: effective-to-uncracked stiffness ratio \( \frac{E_{I_{\text{eff}}}}{E_{I_{\text{unc}}}} \), effective stiffness \( E_{I_{\text{eff}}} \), post-yield stiffness ratio \( b_{E_I} \), yield curvature \( \phi_y \), ultimate curvature \( \phi_u \), yield moment \( M_y \) and ultimate moment \( M_u \) are presented in Table 3.5. The parameters for the bilinear and the modified Takeda hysteretic models were obtained from the bilinear idealization of the moment curvature relationship. Furthermore, for the modified Takeda hysteretic model the \( \gamma \) parameter that controls the unloading stiffness is taken as 0.5 (Saiidi and Sozen, 1979).

### Table 3.5: Moment curvature idealization parameters

<table>
<thead>
<tr>
<th>Test unit</th>
<th>( n_{ax} )</th>
<th>( \frac{E_{I_{\text{eff}}}}{E_{I_{\text{unc}}}} )</th>
<th>( E_{I_{\text{eff}}} )</th>
<th>( b_{E_I} )</th>
<th>( \phi_y )</th>
<th>( \phi_u )</th>
<th>( M_y )</th>
<th>( M_u )</th>
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<tr>
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<td>0.33</td>
<td>13.7</td>
<td>1.3</td>
<td>11.3</td>
<td>254.1</td>
<td>154.4</td>
<td>199.0</td>
</tr>
<tr>
<td>EBIIO7</td>
<td>9.0</td>
<td>0.21</td>
<td>0.398</td>
<td>0.73</td>
<td>42.6</td>
<td>809.7</td>
<td>17.0</td>
<td>19.2</td>
</tr>
<tr>
<td>WDH1</td>
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<td>0.14</td>
<td>36.0</td>
<td>1.3</td>
<td>4.1</td>
<td>45.6</td>
<td>146.4</td>
<td>166.3</td>
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<tr>
<td>WDH2</td>
<td>2.5</td>
<td>0.14</td>
<td>37.7</td>
<td>1.2</td>
<td>4.0</td>
<td>49.0</td>
<td>152.4</td>
<td>172.9</td>
</tr>
<tr>
<td>WDH3</td>
<td>1.8</td>
<td>0.13</td>
<td>24.2</td>
<td>1.7</td>
<td>4.1</td>
<td>54.0</td>
<td>98.6</td>
<td>119.6</td>
</tr>
<tr>
<td>WDH4</td>
<td>1.5</td>
<td>0.13</td>
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<td>1.4</td>
<td>4.1</td>
<td>66.0</td>
<td>96.2</td>
<td>117.3</td>
</tr>
<tr>
<td>WDH5</td>
<td>1.8</td>
<td>0.16</td>
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<td>1.9</td>
<td>4.6</td>
<td>44.6</td>
<td>134.2</td>
<td>156.1</td>
</tr>
<tr>
<td>WDH6</td>
<td>1.6</td>
<td>0.15</td>
<td>29.9</td>
<td>1.8</td>
<td>4.5</td>
<td>50.3</td>
<td>135.7</td>
<td>159.6</td>
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<tr>
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<td>2.3</td>
<td>48.6</td>
<td>393.1</td>
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#### 3.5.2 Modeling the component flexural response

**Displacement-based element models: DB, DT, DF**

The displacement-based element models considered in this study are built using the \texttt{dispBeamColumn} element available in OpenSees. This element is based on the Euler-Bernoulli beam theory for geometrically linear behavior. The discretization of the critical regions where the inelastic deformations are expected to occur is a critical step in modeling RC structures using displacement-based elements. In the following, the influence of the discretization scheme on the simulated response is discussed and the employed discretization strategy is introduced.

The discretization of structures into finite elements influences the simulated response. In particular, discretization of the regions that attain inelastic deformations plays a critical role. When modeling RC structures with beam-column elements, two issues
Figure 3.10: Bilinear idealizations established for test units EBI07 (a) and WDH5 (b)
should be considered: (1) strain localization in FE models and (2) spread of plasticity in RC members. The strain localization issue in FE modeling refers to the sensitivity of the simulated strain distribution to the adopted displacement interpolation functions and the numerical integration schemes. The spread of plasticity refers to the spreading of deformations in plastic hinge regions of RC components that yield in flexure.

In order to demonstrate the strain localization issue, three alternative models of unit EBI07 are considered (Figure 3.11a). In all three models, two displacement-based fiber-section elements (DF) are used. In each element, two Gauss-Legendre integration points are utilized. Cubic Hermitian polynomials are used to represent the displacement interpolation functions for transverse displacements. The curvature variation is obtained by taking the second derivatives of the interpolation functions. This derivation yields a linear function. Hence, the curvature varies linearly along each element. In all three models, the lowest node (i.e. the fixed support) is located at the strain penetration depth $L_{sp}$ (Priestley et al., 2007). The length $L_1$ of the lowest element is different for each model. The equivalent plastic hinge length $L_p$ is employed as an index value for $L_1$ and it is estimated as 162 mm using the relationship by Priestley et al. (2007). For the first, second and third model, $L_1$ is equal to $0.5L_p$, $L_p$ and $2L_p$, respectively.

![Discretization schemes for modeling column EBI07 (a) and the simulated pushover curves (b)](image)

Figure 3.11: Discretization schemes for modeling column EBI07 (a) and the simulated pushover curves (b)

The responses of the models subjected to a monotonically increasing horizontal force (applied at the top node) are simulated using OpenSees (McKenna et al., 2007). The force versus drift ratio behaviors obtained using the three models are plotted in Figure 3.11b. The drift ratio corresponding to a significant drop in shear resistance is marked for each model. Note that the moment curvature analysis of the section indicates that the flexural deformation capacity is controlled by crushing of concrete. The drops in the resistance correspond to exceedance of ultimate curvature $\phi_u$ at the lowest integration section (S1). Specifically, the drop corresponds to the exceedance of ultimate
compressive strain for the confined concrete fibers at S1. The drift ratio exhibited when the resistance drops, increases with the increasing of $L_1$.

The primary reason behind the sensitivity of the simulated drift capacity is the dependence of the simulated curvature distribution to the discretization. Simulated curvature distributions are plotted in Figure 3.12a. Specifically, the curvature distributions simulated when the lowest integration section reaches the ultimate curvature $\phi_u$ are plotted. In all models, the plastic deformations are localized within the lowest element. The large curvatures in this element are distributed over a longer region when $L_1$ is increased. Thus, the drift corresponding to the failure at S1 is the largest for the model with the longest $L_1$.

The discretization scheme influences the simulated yield force (Figure 3.11b). The yielding of the models is controlled by the exceedance of yielding moment at S1. The section S1 moves upward with increasing $L_1$. As a result, the simulated yield force increases (Figure 3.11b).

The simulated post-yield stiffness is sensitive to the discretization scheme (Figure 3.11b). This sensitivity is due to the differences in the curvature profile and in the location of S1 for the three models. The post-yield stiffness reduces with increasing $L_1$. As it was noted previously, the post-yield stiffness strongly influences the likelihood of sustaining large residual displacements.

The divergence of the axial forces at the integration sections is an important issue related to discretization of RC components using displacement-based elements. This divergence is a numerical inconsistency that arises when weakening systems are modeled using displacement-based fiber-section elements. The details of this issue are discussed by Zeris and Mahin (1988). In Figure 3.12b, the simulated section axial forces are plotted against the section curvature for the two integration points along the lowest element. For each model, the axial forces diverge symmetrically from the mean value which is equal to the axial load imposed on the element (i.e. 113 kN). The axial force resisted by the element is calculated by taking the weighted sum of the axial forces simulated at the integration sections. Hence, the global equilibrium is satisfied for the element even when its section axial forces diverge. The divergence of axial forces increases with increasing $L_1$. For all models, the divergence reaches a peak when the curvature is approximately 0.13 m$^{-1}$. These peaks correspond to the onset of softening of the cover concrete fibers due to ultimate compression strain being exceeded. This softening is visible in the moment-curvature behavior simulated at the section S1 (Figure 3.12c). For section S1, the simulated compressive axial forces increase with increasing $L_1$. The increase in the compressive force leads to larger yield moments and slightly smaller $\phi_u$.

The simulated cyclic behavior is influenced by the discretization scheme. In order to demonstrate this influence, the models are subjected to a static cyclic loading history. This history consists of a large displacement cycle followed by a small half-cycle. The simulated horizontal force versus drift ratio values are plotted in Figure 3.13a. The stiff-
nesses simulated in the unloading and in the reloading phases decrease as $L_1$ increases. This stiffness reduction is a result of the localization of strains at S1. In Figure 3.13b, the stress-strain histories simulated for a reinforcement fiber at S1 are plotted. The fibers in the models with shorter $L_1$ attain larger peak tensile strains and larger permanent strains upon unloading. As a result when unloaded from the peak displacement, larger permanent displacements are obtained for the models with shorter $L_1$. Similarly, the reloading stiffness exhibited during the small half cycle is larger when a short $L_1$ is used.

Figure 3.13: The cyclic force deformation responses (a) and the steel stress-strain behaviors (b) simulated by adopting alternative discretization schemes (unit: EBII07)

In RC members deforming under flexure, the post-elastic deformations spread along
members. The ultimate displacement capacity of a member is proportional to the length over which the plastic deformations spread. A common approach for predicting the ultimate displacement capacity of a component is to assume an idealized curvature distribution (Figure 3.14a). The method of estimating the ultimate displacement based on an idealized curvature distribution is referred to as the “plastic hinge method”. In this method, the length over which plastic curvatures are assumed to spread is called the plastic hinge length \( L_p \). In the last 60 years, several equations have been proposed to estimate \( L_p \) (see Hines, 2002). The factors that are reported to influence \( L_p \) are: the moment gradient along the member, the depth of the section, the hardening of reinforcement steel, the inclination of the shear-flexure cracks at the plastic hinge region, the penetration of reinforcement bar strains into neighboring elastic regions, the axial load acting on the member, the section shape and the loading direction (see e.g. Priestley and Park, 1987; Zahn, 1986; Hines, 2002; Fardis and Biskinis, 2003; Beyer, 2007; Dazio et al., 2009a).

The ultimate drift capacity \( d_u \) of a cantilever column is estimated based on the idealized curvature distribution, as follows:

\[
\begin{align*}
    d_u &= \phi_y \left( \frac{L_{\text{eff}} + L_{sp}}{3L_{\text{eff}}} \right)^2 + (\phi_u - \phi_y)L_p \\
    \text{(3.1)}
\end{align*}
\]

where \( L_{\text{eff}} \) is the effective length indicated in Figure 3.14a. Note that, \( d_u \) of the cantilever column EBI107 is equal to the ultimate displacement divided by \( L_{\text{eff}} \). Using Equation 3.1, the \( d_u \) of column EBI107 is estimated as 16.8% (Table 3.5).

The idealized curvature distribution in Figure 3.14a has a shape similar to that of the simulated curvature distribution in Figure 3.12a. This similarity suggests \( L_1 = L_p \) to be a suitable discretization strategy. For the DF model with \( L_1 = L_p \), the curvature
\( \phi_1 \) at S1 reaches \( \phi_u \) when drift is equal to 14.9\%. This value is very close to —i.e. approximately 89\% of— the \( d_u \) predicted using Equation 3.1.

The suitability of this discretization scheme was also investigated by considering the other test units. Pushover analyses were carried out for each test unit. In each analysis, \( L_1 \) was made equal to a series of values in the range 0.5\( L_p \) to 1.5\( L_p \). The average drift ratio obtained in the analyses when \( \phi_1 = \phi_u \) are divided by the \( d_u \) estimated for that unit using Equation 3.1. The \( L_1 \) value corresponding to the point where this ratio is equal to 1, is the optimal \( L_1 \) value for that particular unit and that particular model. The results related to DF model employing 2 Gauss-Legendre integration points are presented in Figure 3.15a. In Figures 3.15b and 3.15c, the results obtained considering 3 and 5 integration points are presented. In all these plots \( L_1 = L_p \) is noted as the optimal value. The same analyses were repeated by utilizing Gauss-Lobatto integration. The results obtained from this latter set of analyses are presented in Figure 3.16. Similar to the previous results, these indicate the discretization scheme \( L_1 = L_p \) to be the optimal scheme.

![Figure 3.15: Ratio of the drift when \( \phi_1 = \phi_u \) to \( d_u \), obtained by utilizing 2-point (a), 3-point (b) and 5-point (c) Gauss-Legendre integration schemes](image)

Apart from the discretization scheme, the simulated curvature distribution is also sensitive to several factors such as: the section modeling approach, the numerical integration rule and the number of integration points. The sensitivity of the simulated curvature distribution to the section modeling approach is presented in Figure 3.17a. The first series in this figure is the curvature distribution simulated using the DF model with \( L_1 = L_p \). The second series is obtained using the DT model. This model has the same discretization scheme but employs modified Takeda section hysteretic behavior instead of the fiber section model. For both the DF and the DT models, the curvature distributions simulated when the lowest integration section reaches \( \phi_u \) are plotted. The curvature at the second lowest integration section (S2) of DT model is about 40\% of that of DF model. Furthermore, the drift ratio for the DT model is 40\% smaller than the \( d_u \) estimated using Equation 3.1.
Figure 3.16: Ratio of the drift when $\phi_1 = \phi_u$ to $d_u$, obtained by utilizing 2-point (a), 3-point (b) and 5-point (c) Gauss-Lobatto integration schemes

Figure 3.17: Simulated curvature distributions obtained when $\phi_1 = \phi_u$ using the DT and the DF models with $L_1 = L_p$ (a), section moment-curvature behaviors simulated at S1 and S2 (b) and curvature distributions obtained by adopting the two discretization strategies for the DT model (c) (unit: EBII07)

The reason behind the differences in the two simulated distributions presented in Figure 3.17a is the way the section moment-curvature response is modeled. The moment-curvature values simulated at S1 and S2 are plotted in Figure 3.17b. For the DF model, different moment curvature relationships are simulated at S1 and S2. This difference is due to the divergence of the sectional axial loads which was addressed previously. For the DT model, the axial load and the flexural response are uncoupled. As a result, the same moment-curvature response is simulated at S1 and S2. The values that are simulated when S1 attains $\phi_u$ are marked on the plots. The marked points correspond to the converged section moment-curvature values which satisfy the global force equilibrium and displacement compatibility. The converged curvature values at S2 in the two cases are
significantly different, since the divergence of section axial forces lead to different section behaviors at the integration sections. Note that the difference between the simulated curvature profiles is less sensitive to the modeling of the section when the section exhibits a post-yield flexural stiffness that is larger than that of column EBII07.

For modeling RC components featuring low post-yield stiffness, two discretization strategies are proposed:

1. Use $L_1 = L_p$ and consider the average curvature $\text{avg}(\phi)$ along the critical element
2. Use an adjusted $L_1$ and consider curvature $\phi_1$ at the most critical integration section

The curvature distributions obtained using these two strategies are presented in Figure 3.17c.

The first strategy corresponds to the first series in Figure 3.17c. The curvature distribution simulated using DT model when the average curvature along the lowest element $\text{avg}(\phi)$ reaches $\phi_u$, is plotted. The $\text{avg}(\phi)$ is calculated by taking the mean value of the curvatures simulated at S1 and S2 or by dividing the element rotation by the element length. The drift ratio when $\text{avg}(\phi)$ reaches $\phi_u$ is 17.4%. This value is very close to $d_u$ estimated using Equation 3.1. Moreover, the drift ratios corresponding to the load step when $\text{avg}(\phi) = \phi_u$ were identified for the cases with the following integration schemes: (1) 2-point Gauss-Legendre, (2) 2-point Gauss-Lobatto, and (3) 5-point Gauss-Legendre. For all considered test units and integration schemes, the identified drifts are larger than the estimated $d_u$s (Figure 3.18).

![Figure 3.18: Ratio of the drift when $\text{avg}(\phi) = \phi_u$ to $d_u$ (DT model, $L_1 = L_p$)](image.png)

The primary drawback of the first strategy is the excessive curvatures exhibited at S1. In the rule based hysteretic models such as the Takeda hysteretic model, the degradation of flexural stiffness is a function of the simulated peak curvature divided by the yield
curvature. Therefore, excessive curvatures simulated at S1 may lead to an improper hysteretic behavior.

The results obtained for the second discretization strategy — i.e. using an adjusted $L_1$ — are plotted in Figure 3.17c (2nd series). The curvature distribution obtained for the DT model when $\phi_1$ reaches $\phi_u$ is presented. The length of the lowest element is iteratively adjusted so that the drift at the top node is equal to $d_u$ when $\phi_1 = \phi_u$. After some iterations, $L_1 = 1.93L_p$ was found to be the optimum for column EBII07.

The most important drawback of the second strategy is the requirement for iterations to identify $L_1$. In order to identify the optimal $L_1$ values for all test units, a series of pushover analyses was carried out. These analyses are very similar to those related to Figure 3.15 and 3.16. However, in this case DT models are used instead of DF models. The $L_1$ values considered in the analyses range between $0.5L_p$ and $2.5L_p$. The results are presented in Figures 3.19 and 3.20. The optimal $L_1$ values differ depending on the number of integration points. When 2 integration points are considered, the discretization scheme $L_1 = L_p$ is the most suitable. On the other hand, $L_1 = 2L_p$ is the optimal scheme when 5 integration points are considered.

![Figure 3.19: Ratio of the drift when $\phi_1 = \phi_u$ to the $d_u$ obtained by utilizing 2-point (a), 3-point (b) and 5-point (c) Gauss-Legendre integration (DT model)](image)

In conclusion, the discretization scheme $L_1 = L_p$ is identified as a suitable scheme for modeling RC components using displacement-based elements. In general, the behavior simulated by adopting $L_1 = L_p$ is in agreement with that predicted using the plastic hinge method. For the models employing the Takeda hysteretic model, a general optimal discretization scheme cannot be identified, however, the scheme $L_1 = L_p$ is suitable when 2 integration points are utilized. The DF, DT and DB models utilized in the time-history analyses presented in this study are established according to the $L_1 = L_p$ scheme (Figure 3.21a).

For the numerical integration, 2-point Gauss-Legendre integration is employed in
the time history analysis of the DF, DT and DB models. From the point of view of computational mechanics, using 2-point integration for evaluating the stiffness and force matrices of an element is not appropriate if the element deforms into the inelastic range. Essentially, a higher number of integration points are needed to accurately evaluate the integrals. On the other hand, the assumption of linear curvature variation —forming the basis of the displacement-based element formulation— along the plastified regions is a gross idealization. Consequently, an accurate integration of the stiffness and force matrices for a model featuring such an idealization do not necessarily lead to accurate predictions of the actual response. Moreover, the use of 2 integration points is a common practice in earthquake engineering. In some inelastic structural analysis programs —i.e. SeismoStruct (SeismoSoft, 2007b), ZeusNL (Elnashai et al., 2001)— 2-point Gauss-Legendre integration is adopted by default in the code and cannot be modified by the user.

**Force-based element models: FB, FT, FF**

The *nonlinearBeamColumn* elements available in OpenSees are used in the force-based element models considered in this study. This element formulation is presented by Neuenhofer and Filippou (1997). It is based on the linear interpolation of forces and moments along the element. The distribution of curvature is not constrained by any specific deformation shape function. At each integration section, the curvatures are determined from the interpolated sectional moments and axial forces. As a result, equilibrium is strictly satisfied at all points along the element. The numerical problem causing the deviation of axial forces affecting the behavior of DF elements is not exhibited by force-based elements.

The discretization of components using force-based elements is a critical step in the
(a) Models with displacement-based elements: DB, DT and DF

(b) Models with force-based elements: FB, FT and FF

Figure 3.21: The finite element models of the test units (all dimensions are in mm)
FE modeling of structures. It influences the locations and amplitudes of the simulated inelastic curvatures. Hence, it affects the simulated response. In the following, important issues related to the discretization are discussed and a discretization strategy is proposed.

In the force-based element formulation, the end displacements are related to the section deformations in an integral sense (Coleman and Spacone, 2001). At any given analysis step, the element end displacements that are free of the rigid body modes can be identified by numerically integrating the section deformations. For the numerical integration, a tributary length which is proportional to the weight of the integration section is considered for each section. Note that this integration is an outcome of the element formulation and it is not carried out as a part of the state determination algorithm which was proposed by Neuenhofer and Filippou (1997).

The sensitivity of the predicted response to the discretization scheme is demonstrated by analyzing the alternative models of column EBII07. Again the same three discretization schemes that were presented in Figure 3.11a are considered. A 2-point Gauss-Lobatto integration scheme is utilized in all elements. Therefore, each element has two integration points located at the two ends. The horizontal shear force versus drift ratio values obtained by means of pushover analysis, are plotted in Figure 3.22a. The models with $L_1 = 0.5L_p$ and with $L_1 = L_p$, fail at drift ratios smaller than the $d_u = 16.8\%$ estimated using Equation 3.1. On the other hand, the model with $L_1 = 2L_p$ fails at a drift ratio of 18.5%. The curvatures simulated at the sections S1 and S2 are plotted against the drift ratio in Figure 3.22b. The models fail when the curvature at the lowest integration section S1 exceeds $\phi_u$. After the curvature $\phi_1$ at S1 exceeds the yield curvature $\phi_y$, the curvature increments at the two sections start to diverge.

In the force-based element formulation, the moments at each section are in equilibrium with the forces applied at the nodes. As a result if the column had zero post-yield stiffness, the moment and the curvature at S2 would remain constant after S1 has yielded. However, column EBII07 features a positive post-yield stiffness and the curvature at S2 continues to increase even after S1 exceeds $\phi_y$ (Figure 3.22b). Even so, the curvatures at S2 for all three models are considerably smaller than $\phi_1$ when $\phi_1$ reaches $\phi_u$. Thus, the large portions of the inelastic curvatures localize at S1.

Simulated curvature distributions similar to the idealized profile shown in Figure 3.14a can be obtained by setting the tributary length $L_{t,S1}$ of the section S1 equal to the plastic hinge length $L_p$. Accordingly, $L_1$ is calculated as follows:

$$L_1 = \frac{L_p}{w_1}$$

where $w_1$ is the weight of the first integration point. Note that this modeling approach was also presented by Coleman and Spacone (2001). The weight $w_1$ depends on the utilized quadrature rule and on the number of integration points. In Table 3.6, $L_1$ values calculated using Equation 3.2 for the cases up to 5 integration points are given. The $L_1$ values identified for the Gauss-Lobatto integration rule are larger than their Gauss-Legendre counterparts if more than 2 integration points are considered.
Figure 3.22: Simulated pushover curves (a) and section curvature versus drift ratio values (b) for different $L_1$

Table 3.6: Values of $L_1$ calculated using Equation 3.2

<table>
<thead>
<tr>
<th>$n_{IP}$</th>
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<th>Gauss-Legendre $L_1$</th>
<th>Gauss-Lobatto $L_1$</th>
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<td>2</td>
<td>$w_1 = 1/2$</td>
<td>$2L_p$</td>
<td>$2L_p$</td>
</tr>
<tr>
<td>3</td>
<td>$w_1 = 5/18$</td>
<td>$3.6L_p$</td>
<td>$6L_p$</td>
</tr>
<tr>
<td>4</td>
<td>$w_1 = \frac{18-\sqrt{30}}{72}$</td>
<td>$5.7L_p$</td>
<td>$12L_p$</td>
</tr>
<tr>
<td>5</td>
<td>$w_1 = \frac{322-13\sqrt{70}}{1800}$</td>
<td>$8.4L_p$</td>
<td>$20L_p$</td>
</tr>
</tbody>
</table>

$n_{IP}$: Number of integration points

The geometry of the structure constrains the length $L_1$. The finite element nodes must be located at specific locations (e.g. center of masses, beam-column joints). Therefore, $L_1$ cannot exceed the maximum length set by these constraints. The shortest plausible $L_1$ is obtained when 2 integration points are considered and it is equal to $2L_p$ (Table 3.6). If more than 2 points are considered, $L_1$ must be longer than or equal to $3.6L_p$. For RC walls bracing multi-story buildings, the story height may prohibit the use of a $L_1$ longer than $3.6L_p$. As an example, for the CAMUS3 wall shown in Figure 3.21, the longest possible $L_1$ is equal to $1.6L_p$ due to the constraining story height.

The drift ratio simulated when the curvature $\phi_1$ at S1 reaches $\phi_u$ is a critical parameter. If this drift ratio is smaller than the estimated ultimate drift capacity $d_u$, during the analysis the failure may be predicted at unrealistically small drift levels. The drift ratios corresponding to exceedance of $\phi_u$ at S1 were identified for the FE models of all
the test units. The units were modeled using force-based elements with 2 integration points. Therefore, $L_1$ was set equal to $2L_p$ in all models. Three cases were investigated: (1) fiber-section model with Gauss-Legendre integration, (2) fiber-section model with Gauss-Lobatto integration, and (3) Takeda hysteretic model with Gauss-Lobatto integration. For each model, the drift ratio obtained when $\phi_1 = \phi_u$ was normalized by the $d_u$ estimated using Equation 3.1. The resulting values are presented in Figure 3.23. The drift ratio when $\phi_1$ reaches $\phi_u$ exceeds $d_u$ for all models.

In the time-history analyses of FB, FT and FF models, 2-point Gauss-Lobatto integration scheme was utilized. The Gauss-Lobatto integration is frequently utilized in studies dealing with force-based elements (e.g. Neuenhofer and Filippou, 1997). Since 2 integration points were considered, the discretization scheme $L_1 = 2L_p$ was adopted in all models except those related to CAMUS3 — where $L_1$ was equal to $1.6L_p$ (Figure 3.21b). The use of 2 integration points may be deemed to yield inaccurate results when judged from a numerical analysis point of view. However, it is important to note that the discretization strategy proposed here aims at modeling RC components taking into account the spreading of plasticity. This spreading is not accounted for in the Euler-Bernoulli beam theory that forms the basis of the adopted element formulation. Hence, analyzing Euler-Bernoulli beam models of RC columns or walls with great formal accuracy do not necessarily lead to accurate predictions of their actual behavior.

### 3.5.3 Other idealizations

Modeling the damping behavior of the components is a challenging issue. Suitable models are identified for each test unit by examination of the test reports and the measured response. The last-committed (tangent) stiffness and mass proportional Rayleigh damping model was adopted in the analyses (Otani, 1981). For a given time-step in the time-
history analyses, last-committed stiffness matrix refers to the final global stiffness matrix that was established at the end of the previous time-step. The periods of vibration are needed to determine the parameters of this damping model (Clough and Penzien, 1993). The modal properties are evaluated by means of eigenvalue analysis of the mass and stiffness matrices. The stiffness and mass matrix coefficients are identified based on the modal properties and the measured response. Details of this procedure are presented in Appendix D for each test unit.

The proper representation of possible P-Δ effects is critical to capture the behavior of the test units if large deformations are expected to occur. For the test units that were tested on the ETH shaking table, the post-tensioning mechanism used to simulate the axial loads is fixed at the anchorage locations at the top and bottom of the wall. Therefore the deformation of the wall does not result in any significant P-Δ moment at the critical section. Due to this fact, in the models of the test units WDH1 to 6 and EBII07 P-Δ effects are not considered in the analysis. For the remaining test units, P-Δ effects are considered while transforming the element forces and deformations between the local and global coordinate systems.

In the time-history analyses, the time integration method for constant average acceleration ($\gamma_N=1/2$ and $\beta_N=1/4$) by Newmark (1959) is adopted. The time integration step size is equal to 5% of the period of the 2nd mode. For the test unit EBII07 which has only a single mode it is assumed to be equal to 2% of the fundamental mode. These time steps were identified by simulating the response repetitively for a series of decreasing time step sizes. This process was repeated until the predicted displacement values started to converge and became independent of the time step size. This issue will be further discussed in the following sections. The Newton-Raphson method with normalized displacement increment control is utilized to solve the nonlinear system of equations in the analysis (Chopra, 2006).

The acceleration series recorded on the shaking-tables exhibited a baseline error in their velocity traces. This error is corrected for each test by applying a linear baseline correction using the program SeismoSignal (SeismoSoft, 2007a). In order to assess the plausibility of the acceleration obtained after this adjustment, the second integration of this series was compared with the measured table displacements. A satisfactory agreement was obtained.

### 3.6 Evaluation of the models

The performance of the models is evaluated based on their capability to predict critical response parameters. The simulated values of these response parameters are compared against the experimental evidence of the tests shown in Section 3.4. The response parameters considered in the evaluation can be grouped into the following three groups:
(1) displacements, (2) rotations and (3) accelerations. The ratio of the value measured during the test to the value predicted from the time-history simulations has been calculated for each simulation. This ratio being larger than unity implies that the model underestimates the measured value. The probabilistic character of the ratio is assessed for each modeling approach. This ratio is assumed to be a log-normally distributed variable. The two parameters of the log-normal distribution model are identified according to the maximum likelihood method. The data from all the test units are considered in the identification of the distribution parameters, except the data related to WDH2. Wall WDH2 failed during testing due to rupture of a longitudinal reinforcement bar. Since none of the models considered in this study are intended to capture the severe changes in the structural properties due to this failure mode, this test unit was not considered when assessing the accuracy of the models. However, the results obtained for WDH2 are still included in the scatter plots to display as an example the probable inaccuracy exhibited in this case.

The detailed data related to the time history results are presented in Appendix E.

3.6.1 Displacements

In order to predict the seismic performance of a structure, it is critical to predict both the global and local deformations. Accordingly, two displacement parameters are considered in this evaluation: the average drift ratio $d_a$ and the ground story drift ratio $d_{gs}$.

Maximum average drift ratio

The simulated maximum average drift ratio values are plotted against the measured values in Figure 3.24. In the figure, the estimated probabilistic character of the test-to-simulation ratio is presented by a dashed line and a shaded region. The dashed lines correspond to the median ratios and the shaded areas highlight the region lying between the 10 and the 90 percentile values. At the top of each figure the median and the coefficients of variation (COVs) are presented.

The dispersions of the identified test-to-simulation ratios obtained using bilinear hysteretic models (DB and FB) are noticeably larger than those obtained using the other models (Figure 3.24). On the other hand, the median ratios predicted for all the models are in the range 0.9 - 1.06. The accuracies of the results obtained using the modified Takeda and the fiber-section models are similar. In the figures related to these two models, it is seen that all models underestimate the maximum displacement of test unit WDH2 by a factor of 0.5. Within the results related to the bilinear hysteretic model, it is seen that the maximum deformation is significantly over-predicted for test units B1 and B2, whereas the prediction for test units A1 and A2 had an acceptable deviation. The major difference between these test unit pairs is the duration of shaking. Units B1 and
B2 experience a longer strong shaking than units A1 and A2. Therefore, the difference between the accuracies obtained for these two test unit pairs may be a further indication of the importance of taking into account the stiffness degradation for the cases where the unit experiences several yielding cycles. Furthermore, the results indicate that for a given hysteretic model the errors associated with the predicted maximum displacements are not significantly influenced by the adopted element formulation.

Figure 3.24: Comparison of the measured and the simulated values of the maximum average drift ratio $d_{a,m}$ [%] for different modeling approaches

Residual average drift ratio

In Figure 3.25 the predicted residual average drift values are compared to the measured values. The accuracy of the models in terms of predicting the residual deformations is significantly lower compared to the accuracy achieved for the maximum deformations; this applies for all models. This is expectable because the accuracy of the simulation at any time step depends on the accuracy of the response predicted from the beginning of the simulation up to that time step. As a result, the accuracy of the predicted residual displacements depends on the errors exhibited during the full-course of the simulation process. On the contrary, the peak deformation is typically attained at a much earlier
time during the response. Moreover, the lower accuracy is also due to the fact that the residual displacements are much smaller compared to the maxima. For the test units considered in this study, the residual average drift ratios are in the range of 2 to 15% of the maxima (Table 3.1). As a result, if the simulation fails to predict both the maximum and the residual deformations by a specific amount, the resulting relative error associated with the predicted residual value is much larger than that associated with the maximum.

The hysteretic model utilized in the analysis has a stronger influence on the predicted residual displacements compared to the adopted element formulation (Figure 3.25). The models that employ bilinear and modified Takeda hysteresis result in an overestimation of the residual displacements. On the other hand, the models that employ the fiber section model underestimate the residual displacements. This observation supports the findings by Dazio (2004). Nevertheless, the models with fiber section elements provide estimates of the residual displacements that are more accurate than those obtained using the other models. Comparing Figures 3.24 and 3.25 it can be concluded that, for the test units considered here, when only the maximum displacements are of interest the modified Takeda model provides estimates that are comparable to those that are obtained using fiber section models in terms of accuracy. However, when the residual average drift is of interest the fiber section models yield the most accurate estimates.

Maximum ground story drift ratio

The simulated peak ground story drift ratios are compared to the measured values for the five multistory RC wall test units in Figure 3.26. The general trends of the values are similar to those observed for the maximum average drift ratio. The test units considered in this study are regular in terms of distribution of mass and stiffness along the height. As a result, the higher modes do not contribute excessively to the story displacements. Due to this fact, the fundamental mode shape is the dominant deformation mechanism for all the walls considered in this study. Hence, the ground story displacement is directly related to the top story displacement. This is also supported by other investigations (Gülkan and Akkar, 2002).

The dispersion of the error and the bias of the models are found to be larger for the peak ground story drift than those identified for the maximum average drift ratio. This is mainly a result of the underestimation of the maximum ground story drift ratio for the CAMUS3 wall by all models. The results obtained using model DF show an underestimation of the maximum ground story drift by 51% whereas the maximum average drift is overestimated by only 14%. The results identified for the model DF indicate that the model may tend to overestimate the ground story drift on the average. However, a larger sample set is needed to derive solid conclusions on this issue.
Residual ground story drift ratio

In Figure 3.27 the predicted residual ground story drift ratios $d_{gs,r}$ are plotted against the measured ones. The $d_{gs,r}$ values computed using different models for the same test are noticeably different from one another. These differences confirm that the sensitivity of the residual displacements to the modeling assumptions is larger than for the maximum displacements. The predictions obtained using the models with fiber-section elements tend to underestimate the residual ground story drift, as it is expected from the findings related to the residual average drift ratio. Furthermore, the degree of this underestimation is larger than that identified for the residual average drift ratio. On the other hand, for the predictions obtained using the modified Takeda hysteretic model the overestimation trend that is identified with regard to the residual average drift ratio is not present in the residual interstory drift ratio predictions. The residual ground story drift predictions obtained using modified Takeda model is more accurate than the average residual interstory drift estimates. This is not only due to the different sample sets being considered in the two cases. It is also seen when the measured-to-predicted values identified for the average and the ground story drift ratios are compared individually for each test unit.
Figure 3.26: Comparison of the measured and the simulated values of the maximum ground story drift ratio $d_{gs,m}$ [%]
Figure 3.27: Comparison of the measured and the simulated values of the residual ground story drift ratio $d_{gs,r}$ [%]

### 3.6.2 Rotations

The peak and residual rotations that are measured during the tests are compared against the rotations that are simulated by means of time-history analysis. For the considered rotation measurements, the locations at which the rotations were measured do not coincide with any of the degrees-of-freedom of the model. In order to predict rotations that are comparable to those measured during the tests, different approaches were followed for the force-based and the displacement-based element models.

The displacement and rotations obtained from the models with displacement-based elements are essentially based on the displacement interpolation functions. These functions may also be used to calculate an approximate prediction of the rotations at any location along the element. For the sake of brevity, in the following only the formulation for an element with two transverse and two rotational degrees of freedom is presented. For the displacement-based elements, the rotation $\theta(x)$ at a section $x$ along the element can be estimated as follows:

$$\theta(x) = \sum_{i=1}^{4} \varphi_i'(x) \bar{q}_i \quad (3.3)$$
where $\varphi_i'(x)$ is the derivative of transverse displacement interpolation function associated with the $i^{th}$ element nodal displacement and $\vec{q}_i$ is a vector collecting nodal transverse displacements and rotations. The displacement-based elements implemented in OpenSees employ the cubic Hermitian polynomials as the displacement interpolation functions. After substituting the derivatives of the polynomials into Equation 3.3 and rearranging the terms, the following relationship is obtained:

$$\theta(x) = \frac{x^2}{L^3} [2(\vec{q}_1 - \vec{q}_3) + L(\vec{q}_2 + \vec{q}_4)] + \frac{2x}{L^2} [3(\vec{q}_3 - \vec{q}_1) - L(2\vec{q}_2 + \vec{q}_4)] + \vec{q}_2$$  (3.4)

where $x$ is the distance of the section from the first node, $L$ is the element length, $\vec{q}_1$ and $\vec{q}_3$ are the transverse displacements, and $\vec{q}_2$ and $\vec{q}_4$ are the rotations at the element ends. The rotation time-histories at the heights of measurement are estimated for each model employing displacement-based elements using this equation. Subsequently, the peak and residual values are identified.

For the force-based models considered in this study, the relative rotation at any point along the element is estimated based on the curvature values simulated at the integration sections. In order to achieve this, the tributary length $L_{t,S_i}$ is calculated for each integration section as follows:

$$L_{t,S_i} = w_i L$$  (3.5)

where $w_i$ is the weight of the integration section $i$ (Si) and $L$ is the total element length. The curvature $\phi_i$ simulated at the integration section $i$ is assumed to be uniformly distributed over the relevant tributary length $L_{t,S_i}$. Based on this assumption, the rotation at any point along the element relative to the first node of the element can be calculated as:

$$\Theta(x) = \int_0^x \phi(x) \, dx$$  (3.6)

where $\phi(x)$ is the assumed segmental curvature distribution. The force-based elements considered in this study utilize the Gauss-Lobatto integration rule. As was noted in Section 3.5.2, two integration points are considered along each element. Based on the assumptions presented above, the curvature distribution along the element $\phi(x)$ is defined as follows:

$$\phi(x) = \begin{cases} 
\phi_{IS1} & \text{if } x < L/2 \\
\phi_{IS2} & \text{if } x \geq L/2
\end{cases}$$  (3.7)

where $\phi_{S1}$ and $\phi_{S2}$ are the curvatures predicted at the first and second integration sections that are located at the two end points of the element, respectively. This approach can also be expanded for any number of integration points. It should be noted that Equation 3.6 provides an approximation of the relative rotation. In order to estimate the absolute rotation, the rotation of the first node should be added to this value. For the models employing force-based elements, Equations 3.6 and 3.7 are used to estimate the rotation time-histories.
Maximum rotation

The predicted maximum rotations are presented together with the corresponding measurement results in Figure 3.28. The rotation measurements were available for 8 test units (Table 3.2). Therefore, the results should be considered as preliminary indications. In general, the results are similar to the findings regarding the maximum deformation indexes presented in the previous section. The models that employ either the modified Takeda hysteresis or the fiber section model result in more accurate estimates of maximum rotation estimates than those employing the bilinear hysteretic model.

![Comparison of the measured and the simulated values of the maximum rotation \( \Theta_m [%] \).](image_url)

**Residual rotation**

In Figure 3.29, the estimates of the residual rotations are plotted against the corresponding measured values. Overall, results are similar to those identified for the residual ground story drift ratio with the difference that the dispersion of the measured-to-predicted values is higher. The tendency of fiber-section models to underestimate the residual quantities that is apparent in the residual deformation index estimates (see Fig-
ures 3.25 and 3.27) is also seen in the results related to residual rotation. For the models that employ the modified Takeda hysteresis, the median model bias is much smaller than that identified for the residual average drift. Note however that this is partly due to the set of considered test units being different in the two cases.

![Graphs showing comparison of measured and simulated residual rotation values](image)

Figure 3.29: Comparison of the measured and the simulated values of the residual rotation $\Theta_r \,[\%]$.

### 3.6.3 Accelerations

All models provide reasonably accurate estimates of the peak relative horizontal accelerations $a_{t, {\text{Rel}}}$ measured at the top stories of the 6 RC wall test units (Figure 3.30). This is mainly due to relative accelerations being bounded by the lateral strength of the walls. For the test units considered in this study, the first mode of vibration is predominant. Hence, the peak relative accelerations can be predicted with acceptable accuracy given that the model properly captures the peak resistance. Moreover, for the wall WDH2 that failed during the test, the presented models underestimate the peak accelerations by about 30–50%.
The models that employ the Takeda hysteretic model are found to be capable of capturing the maximum displacements with a reasonable accuracy for the structures considered in this study (Figures 3.24b and 3.24e). However, the residual displacements predicted using this hysteretic model tends to underestimate the actual values considerably (Figures 3.25b and 3.25e). The predicted residual displacements are overestimated for the columns A1, A2, B1, B2 and EBII07. In order to identify the cause of this overestimation, the predicted response histories were further investigated.

The response of unit A1 is considered as an example time history. In Figure 3.31a, the displacement time histories simulated using the two alternative models DT and DF are plotted over the measured response. The maximum displacement predicted using the DT model is accurate; however, the residual displacement is overestimated. The displacement cycle exhibited in the time period from 7.4s to 8.7s is marked in Figure 3.31a. During this displacement cycle, the model DT exhibits a peak displacement in the positive direction that is higher than the corresponding peak within the other
response histories. After the completion of the cycle, the DT model starts to vibrate around a position that is away from the initial position. The other two response histories do not exhibit such a large shift toward the positive direction. As a result, they sustain smaller residual displacements.

Figure 3.31: Response history of test unit A1: displacement versus time (a) and average drift versus base moment (b)

The moment versus drift ratio plots obtained using the DT and DF models are compared with the actual measured response in Figure 3.31b. The response exhibited during the previously mentioned cycle —i.e. from 7.4s to 8.7s— is plotted with bold lines, whereas thin lines are used to indicate the response prior to this period. For the sake of clarity, the response exhibited after the considered displacement cycle is not plotted. During this cycle, the models and the actual unit exhibit small half-cycle response in the positive direction. The term small cycle refers to the cycles during which the previous peak displacements are not exceeded. The model DT reloads in the positive direction with an average stiffness lower than the other two response histories. This smaller stiffness results in a large residual displacement. For the other test units B1, A2, B2 and EBII07 a similar behavior is observed for the response histories predicted using the DT model (Appendix F). Note that due to dynamic effects, the response histories are not smooth and the stiffnesses exhibited by different models cannot be clearly identified from figures such as Figure 3.31b.

Static cyclic analyses were carried out to further investigate the small cycle behavior. A simple static cyclic loading history that consists of a large displacement cycle followed by a smaller one is considered. In Figure 3.32 the hysteretic behavior obtained using the models DT —employing Takeda hysteresis— and DF —employing fiber-section model— are plotted. The large cycle starts from the initial undeformed position (Point O). Subsequently, both models are deformed to a peak average drift of 4% (Point A) in the positive direction and to a drift of -2% (Point B) in the opposite direction. The models
unload to the same point (Point $C$) at the completion of the large cycle. Afterwards, the models are reloaded to a base shear ratio $V/W$ of 0.14 (Points $D_1$ and $D_2$). The reloading stiffness of the DT model is smaller —approximately 30%— than the average reloading stiffness of the DF. As a result, the peak displacement (Point $D_1$) obtained using the DT model is larger than that obtained using the DF model (Point $D_2$). Similarly, the displacement at the unloaded state is larger for the DT model (Point $E_2$) compared to that for the DF model (Point $E_1$).

![Figure 3.32: Differences between the small cycle responses predicted using the Takeda (DT) and fiber-section (DF) hysteretic models](image)

3.8 Sensitivity analysis of the predicted response

The idealizations introduced to model the behavior of reinforced concrete structures inevitably affect the predicted response. In order to identify these effects on the predicted response parameters, a series of sensitivity analyses are conducted. In the analyses, the peak and the residual values of the average drift ratio are considered to identify the sensitivity of the simulated response. The models presented in Section 3.5 are considered as the “reference models” and the parameter values used in these models are designated with the subscript “Ref”. In order to assess the relative change in the predicted maximum and residual values, the values that are simulated using the modified models are divided by those obtained using the associated reference model.

In the sensitivity analyses, the influence of both the model parameters and the modeling idealizations are assessed. The model parameters accounted for are the vertical loading intensity, the strain-hardening and the cyclic behavior coefficients of the steel
hysteretic model, as well as the elastic viscous damping ratio. Additionally, following modeling idealizations are considered: the discretization scheme and the time-integration step size.

### 3.8.1 Axial load

The axial load acting on the critical section of a member has a profound influence on the post-elastic cyclic behavior of the member. For statically indeterminate structures, the changes in the stiffness of the structural members and their support conditions are expected to lead to a redistribution of axial loads. As a result, the axial loads that are resisted by the critical sections under gravity loads are difficult to estimate precisely. Furthermore, under seismic actions the axial loads acting on the critical sections change dynamically due to frame action, vertical accelerations, and second order effects. Thus, there is a large uncertainty related to the estimation of the level of axial load that will be resisted by the critical sections in structural members. In view of these facts, the sensitivity of the predicted response to the adopted axial loading intensity $n_{ax} = N/A\sigma_{c0}$ is investigated. The response predicted using the fiber-section elements under different axial load levels are compared against each other.

The predicted static-cyclic flexural response of column A1 is presented for different load levels in Figure 3.33a. The response of the section is simulated for a large deformation cycle followed by a smaller one. The peak curvature ductilities reached are approximately 9 and 3 in the large and in the small cycle, respectively. The strength capacity and the effective stiffness of the section increase with increasing axial load whereas the yield curvature stays approximately constant —as expected. Moreover, the moment at which the concrete fibers start contributing to the stiffness of the section decreases with increasing axial load. In the analytical model, the points at which concrete fibers start to contribute to the resistance represent the “closure of the cracks” (Figure 3.33a). A small change in the moment corresponding to “closure of cracks” results in a remarkably different cyclic flexural behavior. A 50% reduction in the axial load results in approximately 10% increase in the permanent curvature attained when the system unloads from curvature ductility of 9. The moment curvature values predicted for the smaller deformation cycles diverge from each other at the points where concrete fibers start contributing to the resistance. These profound effects of axial load on the cyclic behavior of RC sections have also been discussed by other researchers (Aoyama, 1964; Dazio, 2004). In Figure 3.33a it is seen that during the small cycle higher peak moments are attained when higher axial loads are assumed. A 50% increase in the axial load results in a 12% increase in the peak moments predicted for the smaller cycle. The effective stiffness of the system in the smaller cycle may be roughly defined as the slope of the line that connects the two peak points. Based on this definition, it can be concluded that the 50% percent increase in axial load results in a 12% increase in the predicted effective stiffness for the smaller deformation cycle. The stiffness predicted for
the latter is important to accurately estimate the residual displacement. For the sake of completeness, the analysis presented for column A1 was repeated also for other test units. The results of this investigation are presented in Appendix G.

Figure 3.33: The predicted moment versus curvature (a) and base shear versus average drift ratio (b) behavior for column A1 under different axial load intensities \( n_{ax} \).

The simulated base-shear versus average drift values are presented in Figure 3.33b for the same three axial loading intensities. One noticeable difference in these plots is the decrease in the post-yield stiffness with increasing axial loads. This decrease is due to the P-\( \Delta \) effects being more pronounced for the cases where the axial load is larger (MacRae, 1994). As discussed by Kawashima et al. (1998) and briefly presented in Section 3.2, the likelihood of attaining large residual displacements is higher for systems with smaller post-yield stiffness. The results presented in Figure 3.33b indicate that the increase in the axial loads leads to an increase in both the initial and the residual effective stiffnesses. Furthermore, it is seen that the displacements attained after the system unloads to zero base shear from a displacement ductility of 4, is approximately the same for all the models. This difference in the identified unloading behaviors between the response of critical section and the response in the global level (Figures 3.33 (a) and (b)) are due to P-\( \Delta \) effects. At the points where the base-shear equals zero, the moment at the critical section is equal to the moment due to P-\( \Delta \) effects. These points are indicated with markers on the plots. In conclusion, it is seen that the P-\( \Delta \) moments may have an important influence on the predicted displacements at the unloaded states.

In order to assess the sensitivity of the predicted maximum and residual average drift values to the adopted axial loading intensity, the time-history simulations are repeated for each test unit by scaling the intensity of the axial loads corresponding to the reference model (Table 3.5). It should be noted that the post-tensioning mechanism —utilized in
the tests of the walls WDH1 to 6 and the column EBII07—do not lead to P-Δ moments. Therefore, the second order effects are not considered in the analysis of these test units. For the other test units, the lateral deformations result in P-Δ moments and these effects are taken into account in the transformation of the forces between the global and the local coordinate systems. The peak average drift values that are normalized by the predictions obtained from the reference model, are presented in Figure 3.34a. The results in this plot are obtained using the DF models. It is seen that, for most of the units, the predicted peak deformations tend to decrease with increasing axial loading intensity. This may be directly attributed to the increases in the strength capacity which leads to an increase in the effective stiffness. The sensitivity of the predicted residual average drift values are presented in Figure 3.34b. It is seen that the predicted residual displacements are highly sensitive to the axial loading intensity. This sensitivity is both due to the differences in the peak deformations, post-yield stiffnesses and the unloading behaviors that are predicted for the different axial load levels.

![Figure 3.34: Sensitivity of the predicted maximum (a) and the residual (b) average drift values to the axial load n_{ax} (DF model)](image)

The sensitivities identified using the FF models are presented in Figure 3.35. Overall trends that are similar to the previous cases are seen in the results. However, when the sensitivities identified using different alternative models are compared for a specific test unit, differences can be noticed. These differences indicate that the sensitivities of the residual displacements to the axial load depend strongly on the utilized element formulation.

In order to identify the sensitivity of the expected accuracy, the influence of the axial loading on the test-to-simulation ratios was investigated. For this purpose, the median and the COV values of the test-to-simulation ratios were identified. In Figure 3.36, the medians and the COVs obtained for the DF and the FF models are plotted against the
axial loading intensity. The accuracy of the simulated $d_{a,r}$ is much more sensitive to the assumed intensity than that of the simulated $d_{a,m}$. The median values in Figures 3.36a and 3.36b indicate that the simulated $d_{a,m}$s and the $d_{a,r}$s underestimate on average their actual values when the axial loading intensity assumed for the model is larger than the true value. In addition, the COV of the test-to-simulation ratios increase as the assumed intensity diverges from the true value.
3.8.2 Discretization scheme

In order to identify the sensitivity of the maximum and the residual deformation levels predicted using the DF models to the adopted meshing scheme, the time history analyses are presented in Section 3.6 repeated for all test units by adopting a different $L_1$ value in each analysis. The results indicate that the predicted maximum average drift values tend to increase with increasing $L_1$ (Figure 3.37a). The predicted residual average drift values are found to be remarkably more sensitive to the adopted meshing scheme compared to the predicted maximum values (Figure 3.37b). An increase of 50% in $L_1$ results in dispersions of the predicted residual drift values in the range from -20% to +70%.

![Graph](image-url)  
(a) Maximum average drift ratio, $d_{amp}$-DF  
(b) Residual average drift ratio, $d_{arm}$-DF

Figure 3.37: Sensitivity of the predicted maximum (a) and residual (b) average drift values to the adopted discretization scheme. Results obtained using models with displacement-based fiber-section elements (DF) ($L_1$: length of the element at the plastic hinge zone)

The sensitivity of the residual displacements to the meshing scheme can be explained by considering the response of column A1. The curvature histories simulated at the lowest integration section $S1$ are plotted in Figure 3.38. The curvatures presented in this figure correspond to three simulations obtained using three different $L_1$ values: $0.5L_p$, $L_p$ and $1.5L_p$. The maximum curvature simulated at $S1$ increases with decreasing $L_1$. Similarly, the peak curvatures exhibited in the negative direction also increases with decreasing $L_1$. Due to the nonlinearity of the behavior, the residual curvature does not increase proportional to the maximum curvature. After the shaking is over, the three models converge to similar residual curvatures that are in the range from $6 \times 10^{-3}$ to $7.2 \times 10^{-3}$ m$^{-1}$.

The curvature profiles simulated at the point of maximum deformation and the residual deformation are presented in Figure 3.39. At the time step (i.e. $t=7$ s) when the
models reach maximum displacement, the largest curvatures localize at the lowest element (Figure 3.39a). Some part of these large curvatures remain as residual when the vibrations are over. The residual drift ratio of the column is proportional to the residual relative rotation of the lowest element. Since the residual curvatures for all models are similar in terms of magnitude, the model with the longest $L_1$ length results in the largest residual rotation. This localization of the residual curvatures can be suggested as a cause for the sensitivity of the simulated residual drift ratios to the discretization scheme (Figure 3.37b).

Figure 3.38: Curvatures simulated at the lowest integration section $S_1$

Figure 3.39: Maximum (a) and residual (b) curvature profiles simulated using the DF model
The sensitivity of the maximum and residual deformations predicted using the FF models to the adopted meshing scheme is presented in Figure 3.40. The sensitivity of the predicted maximum drift ratio values to the adopted meshing scheme is found to be in the same order of magnitude as for the models with displacement-based elements. On the other hand, the results indicate the predicted residual drift ratios to be highly sensitive to the adopted meshing scheme. Even changes in \( L_1 \) that are as low as 25\%, results in drastic changes in the predicted residual deformation values. Note that in the models employing force-based elements \( L_1 \) equals \( 2L_p \). In other words, the reference \( L_1 \) for the FF models is twice that for the DF models. As a result, when \( L_1 \) values of both models are scaled by a specific factor, the absolute increase of \( L_1 \) for the FF models is twice that for the DF models. In conclusion, the results show that the idealized distribution of the curvatures along the element has a strong influence on the predicted residual displacements. This influence is considerably smaller when the maximum displacements are considered.

Figure 3.40: Sensitivity of the predicted maximum (a) and residual (b) average drift values to the adopted discretization scheme. Results obtained using models with force-based fiber-section elements (FF) (\( L_1 \): length of the element at the plastic hinge zone)

The sensitivity of the median and the COV of the test-to-simulation ratios to \( L_1 \) is presented in Figure 3.41. The expected accuracy of the simulated \( d_{a,m} \) is only slightly sensitive to \( L_1 \). On the other hand, the median and the COV of the test-to-simulation ratios related to \( d_{a,r} \) increase with decreasing \( L_1 \). When \( L_1 \) is reduced to half of the reference value, the median underestimation of \( d_{a,r} \) doubles while the COV of the expected error triples. The median test-to-simulation ratios obtained by using the DF and the FF models decrease as \( L_1 \) increases.
3.8.3 Steel material model

The post-elastic cyclic behavior of the reinforcement steel has a significant influence on the cyclic response of the plastic hinge region. The behavior of the reinforcement depends strongly on the details of the manufacturing process and the chemical composition. The differences in post-yield behavior under monotonic loading are visible, for example, in the plotted in Figures A.1 and A.2.

Testing the behavior of reinforcement bars under large inelastic cyclic deformations that simulate the seismic action is associated with important technical difficulties. As a result, the current knowledge and evidence on the cyclic behavior of reinforcement bars is rather limited. Nevertheless, a number of hysteretic models have been proposed to capture the essential properties of the uniaxial cyclic response based on the available data. However, the prediction of the major cyclic response characteristics for a given type of reinforcement bar is subject to large uncertainties. As a result, estimating suitable values for the steel cyclic hysteresis model is not a straightforward process. In this study, the two fundamental features of the stress strain hysteresis response of the reinforcement steel are considered: (1) the strain-hardening ratio and (2) the Bauschinger effect.

Strain-hardening stiffness ratio

In this study, the post-yield branch of the monotonic stress-strain curve is characterized by a single stiffness parameter $b_s$. However, the response identified from tests on steel
coupons, indicates that the stiffness is essentially not constant in the post-yield region. As a result, particularly when the measured behavior exhibits a clear yield plateau followed by a noticeable strain hardening region, identification of a suitable post-yield stiffness becomes difficult. This applies for example to the following reinforcement bars: A1-No4, CAMUS3-$\varnothing$8 and WDH4 to 6-$\varnothing$5.2 (Appendix A).

The uniaxial cyclic stress-strain response simulated using different values of post-yield stiffness $b_s$ are presented in Figure 3.42a. Next to it, the static cyclic responses of column A1 simulated by adopting the same post-yield stiffness values are plotted (Figure 3.42b). It is seen that the post-yield stiffness of the component—as expected—increases with increasing strain-hardening of the reinforcement. Furthermore, the results show that the response simulated for the small non-yielding cycles is the same for the different cases. This is primarily due to the fact that differences in the steel stress-strain response are not very significant unless the stresses approach the monotonic envelope.

The effect of the strain-hardening stiffness value on the predicted maximum and residual average drift ratios is presented in Figure 3.43. The maximum displacement predictions are practically independent of the strain-hardening stiffness $b_s$. As a result, the median and the COV values of the test-to-simulation ratios are insensitive to $b_s$ (Figure 3.44a). On the other hand, the predicted residual displacement values are sensitive to $b_s$ (Figure 3.44b). It is seen that a 25% change in the post-yield stiffness value, results in up to 20% difference in the predicted residual displacement. The median and the COV of the bias in the predicted $d_{a,r}$ are only slightly sensitive to $b_s$ (Figure 3.44b). The results presented in Figures 3.42, 3.43 and 3.44 are obtained using the DF model. Note that similar results are obtained when the FF models are used.

**Bauschinger effect**

An important feature of the cyclic response of reinforcement steel is the curved unloading/reloading path exhibited when subjected to a stress reversal beyond the yielding point. This feature of polycrystalline metals is referred to as the Bauschinger effect. Which properties of reinforcement steel control the degree of softness of the stress-strain curves exhibited under stress reversals is still an open question. Dodd and Restrepo-Posada (1995) have discussed the influence of the carbon content of the steel on the shape of the Bauschinger effect partially based on the findings by Bate and Wilson (1986). Likewise, Balan et al. (1998) have identified the steel grade to have an influence on this shape. However, both studies concluded that more cyclic test results were needed to identify the most important parameters controlling the behavior.

The uniaxial stress-strain hysteretic (MP) model proposed by Menegotto and Pinto (1973) which was introduced in Section 3.5.1, is a commonly adopted model to simulate the post-elastic response of reinforcement steel. Menegotto and Pinto have proposed the coefficients $R_0 = 20$, $A_1 = 18.5$ and $A_2 = 0.15$ as suitable values to capture the
Figure 3.42: The moment versus curvature (a) and the base shear versus average drift ratio (b) behavior of the column A1 for different strain-hardening stiffnesses $b_s$.

Figure 3.43: Sensitivity of the predicted maximum (a) and residual (b) average drift values to the strain-hardening stiffness $b_s$ of the reinforcement model.
Bauschinger effect. The stress-strain relationship obtained using these values is plotted in Figure 3.45a under the name: Case 1. Additionally, in order to investigate the sensitivity of the simulated response to the softness of the transition curve (Bauschinger effect), two additional cases are considered: (1) the case of a bilinear hysteretic model that fails to capture any softness (Case 3) and (2) the case when the parameters of the model are adjusted to simulate a curved transition that lies in between the Case 1 and Case 3 (Case 2). In order to simulate the response for Case 2, the parameter \( A_2 \) is taken as 0.6 while the other parameters retain the same value set for Case 1. It is seen that the monotonic loading envelope is identical for all cases whereas the path that is followed under stress reversals is noticeably different (Figure 3.45a). It should be noted that due to the technical difficulties associated with testing reinforcing bars under cyclic action, usually only monotonic tensile tests are conducted to identify the essential material parameters. The results of the monotonic tests do, however, not provide much information regarding the softness exhibited under cyclic loading.

The simulated quasi-static cyclic response of column A1 for the three different cases introduced above is plotted in Figure 3.45. The results show that the backbone curves obtained for the different cases are the same and that differences first appear in the unloading branches. As the lateral force drops to zero, the model that employs a bilinear hysteretic rule unloads with a higher stiffness compared to the other two. In the ensuing reloading branch, the predictions obtained using different models follow different paths until the concrete fibers start contributing to the stiffness at the critical section. After this point, the models join together and a very similar backbone curve is obtained. During the subsequent small non-yielding cycle, the force-deformation paths predicted
using different models diverge. The model that employs the bilinear hysteretic model results in higher reloading stiffnesses being predicted compared to the other models. Furthermore, the energy dissipated during this small cycle is larger compared to the other models. This difference between the predicted small cycle hysteretic behaviors is a direct result of the different stiffness reduction due to the different modeling of the Bauschinger effect.

![Cyclic stress-strain and force-displacement behavior simulated using different transition curve idealizations to capture the Bauschinger effect](image)

Figure 3.45: Cyclic stress-strain (a) and force-displacement (b) behavior simulated using different transition curve idealizations to capture the Bauschinger effect

In order to investigate the effects of the softness of the transition curve on the predicted displacements, the response of the considered test units is simulated for the two alternative cases, Case 2 and Case 3. Since in the reference model the Menegotto-Pinto model is adopted with the original set of values the results obtained using the reference model correspond to Case 1. In order to identify the sensitivity, the maximum and the residual displacements that are predicted using the different models are divided by the values predicted using the reference model. In Figure 3.46a, it is seen that the differences in the predicted maximum displacement are not very significant. However, the residual displacements predicted using alternative steel hysteretic models are remarkably different (Figure 3.46b). In general, smaller residual displacements are predicted when a bilinear hysteretic model is adopted for the steel. The results clearly indicate that the parameters that control the Bauschinger effect also control the small cycle hysteresis behavior and have therefore a strong influence on the predicted residual deformation values.

The displacement histories for column A1 simulated using three alternative steel hysteresis models are plotted in Figure 3.47. The three models provide the same displace-
Figure 3.46: Sensitivity of the predicted maximum (a) and residual average drift values (b) to the shape of the transition curve (Bauschinger effect) of the reinforcement hysteretic model.

The accuracy of the $d_{a,r}$ is highly sensitive to the modeling of the reinforcement hysteresis models.
First yielding cycle

Curvature, $\phi \times 10^{-3}$ [1/m]

Moment, $M$ [kNm]

Case 1 - MP, $A_2 = 0.15$

Case 2 - MP, $A_2 = 0.6$

Case 3 - Bilinear

Non-yielding cycle

Curvature, $\phi \times 10^{-3}$ [1/m]

Moment, $M$ [kNm]

Case 1 - MP, $A_2 = 0.15$

Case 2 - MP, $A_2 = 0.6$

Case 3 - Bilinear

Figure 3.48: Moment curvature response simulated at the lowest integration section $S1$ during the first yielding cycle (a) and the subsequent non-yielding cycle (b) (DF model of unit A1)

When a bilinear hysteretic model is employed (i.e. Case 3), the actual $d_{a,r}$ are underestimated. For the three cases considered, the underestimation is the lowest for the case when the Bauschinger effect parameters recommended by Menegotto and Pinto (1973) are used to model the cyclic behavior (i.e. Case 1).

Figure 3.49: Sensitivity of the expected accuracy of the simulated maximum (a) and residual (b) average drift values to the shape of the transition curve (Bauschinger effect) of the reinforcement hysteretic model
3.8.4 Damping

In time-history analysis, the energy dissipation due to sources other than the hysteretic behavior of the structural components is usually taken into account by means of viscous damping. This modeling approach leads to a simple system of equations, since the damping forces are solely dependent on the relative velocities. However, inevitably this simple representation involves gross simplifications. Existing approaches to represent damping forces and their limitations have already been discussed by other researchers (e.g. Otani, 1981; Hall, 2006; Priestley et al., 2007; Petrini et al., 2008). In this study, the sensitivity of the predicted maximum and residual displacements to the assumed viscous damping level $\zeta$ is investigated.

The average drift response histories simulated for three different damping levels are plotted together with the measured displacement history in Figure 3.50a. The results show that during the larger cycles all displacement histories are in agreement. However, during the smaller cycles they diverge. At the end of the response the residual displacements attained by all models are smaller than the measured value. In Figure 3.50b, the average drift versus base moment hysteresis obtained for the time interval 4.9-5.9s is presented. For a linear elastic system exhibiting a steady-state harmonic response with constant viscous damping, the damping force attains its peak values at the points of peak relative velocity. These points correspond to the zero relative displacement points for a linear elastic system in steady-state vibration. Therefore, for such systems, the increase in viscous damping results in larger energy dissipation per cycle and thicker hysteresis loops. Moreover, if the Rayleigh damping model (based on the tangent stiffness) is adopted —like in the reference models— and a nonlinear hysteretic model is utilized, the damping force at any time step becomes proportional to the relative velocity and the current stiffness. However, an increase in viscous damping ratio is still expected to yield thicker hysteresis loops. The peak displacements attained during the maximum displacement cycle decrease with the increasing damping (Figure 3.50b). Furthermore, the results show that at zero displacement, where the relative velocity is expected to be the highest, the simulated hysteresis curves converge again. In this case the displacements at the unloaded states for a given vibration cycle are more sensitive to reduction of the peak displacements predicted with increasing viscous damping compared to the thickening of the hysteretic shape due to the same reason.

In order to identify the sensitivity of the predicted displacement quantities to the assumed viscous damping level, the time-history analyses are repeated for each test unit changing the critical damping ratio in each simulation. The maximum deformation values identified using different damping levels are presented in Figure 3.51a. The results show that the variation of the damping by $\pm 50\%$, results in a change in the predicted maximum deformations by up to $\pm 25\%$. As expected, the maximum deformations decrease linearly with the increasing damping. In Figure 3.51b, the influence of the adopted damping ratio on the predicted residual displacements is presented. The results indi-
citate that the predicted residual displacements are more sensitive to the adopted critical damping ratio than the maximum displacements. Both the maximum and the residual deformations are underestimated when large damping ratios $\zeta$ are assumed (Figure 3.52). Moreover, the COV of the test-to-simulation ratios increase as the assumed $\zeta$ increases.

Figure 3.50: Wall WDH4. Average drift ratio versus time (a) and the base moment versus average drift cycles within the time interval 4.9-5.9s (b) predicted using different damping levels $\zeta$

![Figure 3.50](image)

Figure 3.51: Sensitivity of the predicted maximum (a) and residual (b) average drift values to the elastic viscous damping parameter $\zeta$

![Figure 3.51](image)
Maximum average drift ratio, $d_{a,m} - DF$

Residual average drift ratio, $d_{a,r} - DF$

Figure 3.52: Sensitivity of the expected accuracy of the simulated maximum (a) and residual (b) average drift values to the elastic viscous damping parameter $\zeta$

3.8.5 Time integration step size

A fundamental component in nonlinear time-history analysis is the time-integration of the equation of motion. The average-acceleration method by Newmark (1959) is an implicit and unconditionally stable direct integration method and it is one of the most frequently used methods in practice. In this study, all dynamic simulations are carried out based on this method. The selection of a suitable time-step size $\Delta t$ has a critical influence on the predicted response. For linear elastic systems, the time-step size is typically chosen by considering the fundamental period of the system. On the other hand, for systems with nonlinear stiffness and damping properties the selection of the time-step size becomes more critical. In general, a time-step size smaller than or equal to 10% of the fundamental period of vibration is considered to be suitable (Clough and Penzien, 1993). However, investigations focusing on the time-step size selection have examined primarily the objectivity of the peak deformations and force quantities. In this study, the problem is evaluated by considering the precision in terms of predicting the residual deformations.

In the reference models, the time-step size $(\Delta t_{Ref})$ is set to 2% of the fundamental period of vibration for single-degree of freedom units, and 5% of the period of the second mode for multi-degree-of-freedom units. These time-step sizes were identified by repeating the simulations with smaller and smaller time-steps until the predicted residual displacement values stabilized. In order to demonstrate the sensitivity of the predicted displacements to the time-integration step size, the simulations are now repeated using a different time-step size in each simulation. In Figure 3.53a, the sensitivity of the predicted maximum displacement values are presented. It is seen that the maximum
values are not sensitive to the time-step size in the considered range of values, whereas the predicted residual displacements diverge from the values identified using the reference model as the time-step size increases (Figure 3.53b). For some test units, a 50% increase in the time-integration step size leads to a change in the predicted residual drift up to ±50%. Moreover, the results indicate that for all test units, except WDH2, the predicted residual deformations stabilize for the range of time-step sizes smaller than the one adopted in the reference model. It should be noted that the approximate rule introduced in this study to estimate the time integration step size may not be suitable for structures where higher modes have a remarkable influence on the predicted dynamic response. The best practice to identify a plausible value is to repeat the simulations decreasing the step size until the results become insensitive to time-step size.

![Figure 3.53: Sensitivity of the predicted maximum (a) and residual (b) average drift values to the time-integration step size $\Delta t$](image)

The median and COV of the test-to-simulation ratios related to $d_{a,m}$ and $d_{a,r}$ are insensitive to the time step size $\Delta t$ (Figure 3.54). This is due to the fact that as the utilized $\Delta t$ increases some simulations become more accurate while the others become less accurate. On the whole, the net change in the expected accuracy becomes negligible.

### 3.9 Note on the simulation of residual displacements

The results presented in the previous sections indicate that it is very difficult to estimate the residual displacements accurately. The accuracy of the estimated residual displacements is lower than that of the maximum deformations. Moreover, the influence of the details of FE modeling on the simulated residual displacements is significantly larger.
When dealing with engineering problems that involve several uncertain parameters, a major challenge is to identify the critical parameters which should be further refined so that the overall uncertainty in the prediction of the likely performance of the system can be reduced with the least effort. The likely performance refers to the likelihood that the system will fulfill its objectives. Therefore, the cost-effectiveness of making an effort to improve the certainty on any parameter strongly depends on the definition of the objectives.

In the seismic design of structures, estimation of the residual displacements is crucial if the design objectives include the post-earthquake usability. Such objectives are critical for the lifeline systems that are expected to be in use immediately after a damaging earthquake. For example for the San Francisco-Oakland Bay Bridge, an objective was to ensure that the bridge can carry traffic after a seismic event corresponding to a return period of approximately 1500 years (Seible et al., 2005). In regard to this, the allowable average residual deformation of the bridge was limited to 300 mm.

In the post-earthquake damage evaluation, an objective may be to evaluate the residual stiffness and the strength of the damaged structure. For RC structures, the residual strength and stiffness are strongly correlated with the maximum displacements sustained during the earthquake. The residual displacements are the only measurable indicators related to the maximum displacements that can readily be measured after the earthquake. A post-earthquake damage evaluation method which takes into account the sustained residual displacement is proposed in Chapter 4. Obviously, the relationship between the residual and the maximum displacements is not one-to-one but rather a one-to-many re-
relationship. Still however, the results obtained using the proposed method demonstrate the effectiveness of residual displacements in terms of reducing the uncertainty in the evaluation of the structural damage. These results and the example applications are presented in Chapter 5.

3.10 Summary and Conclusions

The comparison of the calculated response histories against those measured during shaking table tests indicated the following points:

- The accuracy of the considered finite-element models, in terms of estimating the residual displacements is found to be much lower compared to that identified for the peak values.

- The influence of the adopted hysteretic rule on the accuracy of the predicted displacements —both maximum and residual displacements— is found to be much stronger than that of the employed finite element formulation.

- The models that employ either a fiber-section or a modified Takeda hysteretic model are found to yield accurate estimates of the peak average drift ratio. The results obtained using bilinear hysteretic model, are found to have a very low accuracy compared to the other two.

- The residual average drift ratios predicted using the fiber-section elements tend to underestimate the actual values, whereas the opposite is true for the predictions obtained using the modified Takeda hysteretic model. The lowest accuracy is exhibited by the results of the bilinear hysteretic model which tends to highly overestimate the residual average drift.

The assessment of the models capabilities in predicting local deformation quantities —such as ground story drift ratios and rotations of plastic hinge zones— was carried out. Since a total number of 12 test units are considered, the results only serve as preliminary indicators. Based on the available data, the following points are identified:

- The accuracy of the peak ground story drift ratio estimates is found to be lower compared to that of the peak average drift ratio estimates. The residual ground story drift ratios predicted using the modified Takeda hysteretic model are found to be more accurate than those obtained using other hysteretic models. The results indicate that the simulated residual ground story drift values are sensitive both to the adopted element formulation and to the hysteretic model.
• The peak rotation at the base segment was captured with accuracies comparable to those identified for the ground story drift ratios. The accuracy of all models is found to be limited in terms of estimating the residual rotation values. However, the predictions obtained using the modified Takeda model were better compared to the others.

• The relative accelerations exhibited at the top degree-of-freedom of the structural walls could be predicted with a reasonable accuracy using all models. The relative accelerations are captured with a good accuracy even using the bilinear hysteretic model—which had relatively low accuracy in terms of capturing the deformation indexes.

The results of the sensitivity analysis indicate that the following points are critical while predicting the peak and the residual displacements of RC structures:

• The sensitivity of the predicted residual displacements to all modeling parameters and idealizations is found to be higher compared to the sensitivity of the maximum displacements.

• The vertical loads have a very large influence on the predicted residual displacements. The peak displacements are less sensitive to the axial loads than the residual displacements.

• The adopted discretization scheme had a larger effect on the predicted residual displacements compared to the maxima. The sensitivity of the models employing force-based elements to the discretization is found to be stronger compared to the models with displacement-based elements.

• The strain-hardening of the steel was found to have a relatively small influence on the predicted residual displacements compared to the other parameters considered in the sensitivity analysis. Moreover, its effect on the predicted maximum displacements is insignificant.

• The shape of the transition curve assumed to capture the Bauschinger’s effect is found to have a large influence on the predicted residual displacements. The adopted shape is found to influence both the predicted hysteresis for the large yielding cycles and the subsequent smaller non-yielding ones, as well. The effect of this transition curve on the estimated maximum displacement is found to be minor.

• The adopted damping ratio is identified to have an important influence on the predicted residual displacements.

• It is found that a smaller time-integration step size is required to properly predict the residual displacements compared to the one needed to properly predict the peak
displacements. If this requirement is not accounted for in the analyses, estimates of residual displacements that depend on the adopted step-size are obtained.

### 3.11 Modeling recommendations

Modeling recommendations are suggested for analysis of RC structures. The first two recommendations are related to the finite element discretization of RC structures for predicting the inelastic response. They concern the strain localization problem. The fundamental strategy is to establish a numerical model that is in agreement with the equivalent plastic hinge length approach presented by Priestley et al. (2007). In particular, it is aimed that the curvatures simulated using the numerical model are close to those estimated using the plastic hinge length approach.

**Finite element discretization** The columns and walls—that may sustain inelastic deformations—should be discretized into finite elements in accordance with the adopted element formulation. The length $L_1$ and the number of integration sections $n_{IS}$ of the element located at the plastic hinging zone should be chosen according to the following criteria:

**Displacement-based elements**: The integration section with the largest curvature should reach the ultimate curvature when the component reaches its ultimate deformation capacity. The ultimate curvature may be identified by means of moment curvature analysis. Ultimate deformation capacity of the component can be identified using the equivalent plastic hinge length approach. For example, $L_1$ should be equal to $L_p$ if Gauss-Legendre integration rule is utilized and 2 integration points are considered (i.e. $n_{IS} = 2$). The $L_p$ can be estimated using the relationship presented by Priestley et al. (2007).

**Force-based elements**: The tributary length $L_{t,IS1}$ of the integration point that attains the largest curvature should be equal to the estimated equivalent plastic hinge length $L_p$. The length $L_{t,IS1}$ is calculated using Equation 3.5. According to this criterion, $L_1$ should be equal to $2L_p$ if the weight of the integration point is 0.5.

**Identifying rotations** The rotations at the specific points along the elements should be identified based on the curvatures simulated at the integration sections and the employed element formulation. The two approaches that are presented in Section 3.6.2 can be used when the displacement-based or the force-based formulations are utilized.

The recommendations presented below are more specific than the ones presented
above. The following recommendations are primarily related to the prediction of residual displacements by means of time-history analysis.

**Hysteretic behavior of the steel** The hysteretic model should account for the softening of the steel under cyclic loading in the post-elastic range —i.e. Bauschinger effect. The model by Menegotto and Pinto (1973) adequately represents this softening.

**Strain-hardening of the steel** It is suggested that the strain-hardening stiffness should be identified by fitting a line to the segment of the stress-strain curve between the yield strain $\epsilon_y$ and tension limit strain $\epsilon_{sur}$. The $\epsilon_{sur}$ corresponds to 60% of the strain $\epsilon_u$ at the peak tensile stress (Priestley et al., 2007).

**Section hysteretic response** The hysteretic response of the section should account for the axial load-flexure interaction. In particular, the influence of the axial load on the small cycle behavior is critical in estimating the residual displacements. In the fiber-section modeling approach, the axial-flexure interaction is automatically accounted for. On the other hand, the rule based hysteretic models such as modified Takeda model fall short of capturing this interaction.

**Time integration** Based on the results obtained in this study, it is suggested that the average acceleration method by Newmark (1959) should be used in the time-history analysis. It is recommended that the time step size $\Delta t$ should be equal to or smaller than the smaller of: (1) 2% of the fundamental period $T_1$ of vibration and (2) 5% of the second mode period $T_2$. 
4 Assessment of the seismic performance using residual displacements

4.1 Introduction

After damaging earthquakes, the assessment of safety is a complex problem except for the structures with two types of damage grades: severe damage and light damage. Photo of an example reinforced concrete building that sustained severe damage during an earthquake is presented in Figure 4.1a. The building was partially collapsed following the 1999 El Quindío, Colombia $M_w$ 6.2 earthquake (EERI, 1999). It is hardly questionable that this partially collapsed building as well as the neighboring buildings are unsafe, and the damage cannot be repaired. Shortly after the first photo is taken, the anticipated collapse of the structure takes place due to an aftershock. This even is seen in the photos in Figures 4.1b and 4.1c.

Similar to structures with severe damage, often the assessment of those with light damage is not very difficult. In Figure 4.2, the photo of a reinforced concrete frame building with light damage is presented. This building stood just next to a building that collapsed during the 1999 İzmit, Turkey $M_w$ 7.4 earthquake. Inspection of the building with light damage revealed that the building had deformed within the elastic limits (Sezen et al., 2000). For such buildings, often stiffness, strength and deformation capacity are assumed to be changed very little following the earthquake. Therefore, the likely performance under future seismic loads is assumed to be the same as that before the earthquake.

Structures that sustain moderate damage are the most difficult ones to assess after an earthquake. For these structures, the sustained damage is not so excessive that the structures can be immediately judged to be unsafe. On the other hand, the damage is also not so insignificant that its effects on the future seismic performance can be neglected. Photos of two RC buildings with moderate damage are presented in Figure 4.3. In these two example cases, the sustained damage did not render the buildings completely unstable. However, the visible cracks suggest that the deformations sustained were not
Figure 4.1: Photos of a partially collapsed reinforced concrete building after the main-shock (a), during an aftershock (b) and after the same aftershock (c) of the 1999 El Quindío, Colombia $M_w$ 6.2 earthquake (Photos from EERI (1999))

Figure 4.2: Photo of a six story RC building with light damage (Photo by Whittaker, A. S., from NISEE)

negligible.

When evaluating structures with moderate damage, the typical issues of concern are the following:

- Can the structure safely resist probable future ground motion?
Figure 4.3: Photos of RC buildings with moderate damage: following the 1999 İzmit $M_w$ 7.4 earthquake (a) and following the 2009 L’Aquila $M_w$ 6.3 earthquake. (The second photo by F. Braune is taken from Dazio et al. (2009b))

- Is it possible to repair the structure and restore its safety?
- What is the most cost-effective strategy to repair the structure?

In order to evaluate the structure, the residual stiffness, strength and deformation capacity of the structure must be estimated. Often, the damage sustained by the structural components is inspected for this purpose. Photos of typical damage sustained by structural components are presented in Figure 4.4. For a comprehensive list of the typical damage types exhibited by structural components the reader is referred to ATC (1999).

When evaluating the damaged components, the major challenge is to determine the extent to which their stiffness and strength have degraded and to assess the implications of this degradation on the likely performance of the structure under static and/or dynamic loads.

Past research on the cyclic behavior of RC structures has indicated the degradation of stiffness and strength to be strongly related to the maximum deformations sustained. Therefore, the maximum deformations sustained are key parameters in the damage evaluation. After an earthquake, these deformations can only be determined with certainty for the small group of structures which are instrumented. Additionally for the small percentage of structures that are located in the close vicinity ($<100$ m) of free field strong motion stations, the maximum deformations can be estimated by means of response history analysis. For the large majority of structures, neither the maximum deformations sustained nor the excitation can be determined with certainty. However, they can be estimated probabilistically based on the available information. A method is proposed here in order to achieve this goal.

The proposed method aims to estimate, after a damaging earthquake, the maximum
average drift ratio experienced. In particular, the likelihood of the experienced maximum average drift being in a drift interval of interest is estimated (Figure 4.5). Visible damage indicators and the known residual deformations are directly taken into account in the method to improve the estimates. It is important to note that in principle the proposed method can be applied to estimate any deformation parameter of interest relevant to the considered structure, and that for this reason, in this section, a general average drift ratio is addressed.

The method takes into account the uncertainties arising from the following sources:

- Limited knowledge about the properties of the structure and of the site,
- Errors associated with the response prediction and with the displacement capacity prediction models,
- Uncertainty in the excitation experienced at the site,
- Probable inaccuracy of the identified residual deformations,

These uncertainties are directly considered to identify the probabilistic character of the response quantities. Estimated distribution of the maximum drift ratio is updated in two stages: Initially, based on the visible damage to the components and/or the structure. Subsequently, the residual drift ratio attained by the structure is taken into account to further improve the estimated distribution.

Figure 4.4: Examples of earthquake damage to a reinforced concrete column (a) and to a beam (b) (The second photo by F. Braune is taken from Dazio et al. (2009b))
4.2 Performance assessment method

The maximum average drift ratio $MA$ experienced by the damaged structure is considered as a random variable according to the Bayesian estimation framework used herein. The method aims to provide an accurate estimate of the probability of $MA$ being in a specific drift interval from $ma_i$ to $ma_{i+1}$. This event, $M_i$, is defined as follows:

$$M_i = \{ma_i < MA \leq ma_{i+1}\} \quad (4.1)$$

where $ma_i$ and $ma_{i+1}$ are the lower and upper limits of the $i^{th}$ maximum drift interval.

A basic schematic presentation of the method is provided in Figure 4.6. First, the prior probabilities $Pr(M_i)$ for the event $M_i$ are estimated. After that, these probabilities are updated based on the visible damage inspection results. As a result, the probabilities $Pr(M_i|I)$ of $M_i$ conditional on the inspection results $I$ are obtained. Finally, the probabilities $Pr(M_i|I \cap MR)$ of $M_i$ conditional on the inspection results $I$ and the measured residual deformations $MR$ are estimated by updating the estimates based on the residual deformations.

The method comprises the following steps:
Step 1: Modeling of the structure

Step 2: Estimating the prior probability distribution of the maximum drift ratio

Step 3: Updating prior probability distribution of the maximum drift ratio based on the visible damage inspection results

Step 4: Updating the posterior probability distribution of the maximum drift ratio obtained at Step 3 based on the measured residual displacements

The essential parts of each step are presented in detail in the following sections.

4.2.1 Step 1: Modeling of the structure

First, a deterministic analytical model of the structure is established. The model should allow the estimation of the maximum and the residual displacements experienced by the structure for a given excitation and a given set of structural properties. Any suitable modeling approach can be adopted into the proposed method. However, for the sake of clarity the prediction of the response by means of nonlinear dynamic time-history analysis based on the finite element (FE) method is addressed here. The whole set of deterministic models that are employed to simulate the response of the structure with certain properties to a given excitation history are referred to as the FE model in the following sections. The set of assumptions and idealizations introduced while establishing the FE model is referred to as the modeling approach. For example, employing a specific hysteretic model for simulating the behavior of the reinforcing steel and the procedure for identifying suitable values for its parameters is a part of the modeling approach.

The estimated accuracy of the FE model in predicting the maximum and the residual displacements is directly taken into account in the assessment process. The results obtained so far indicate that the effectiveness of the proposed method in terms of updating the damage estimates is directly proportional to the accuracy of the adopted modeling approach.

The probable inaccuracy of the simulated response parameters — e.g. displacements, rotations, accelerations — is taken into account using correction factors $C$. A correction factor is a random variable representing the ratio of the true value of a response parameter to the value that is simulated by adopting a specific modeling approach. For a given response simulation $S_n$, the probability distribution (or density function) of the actual maximum average drift $MA_n$, residual average drift $RA_n$ and residual rotation $RR_n$ are estimated as follows:

\begin{align*}
MA_n &= C_{am} d_{a,m}(n) \quad (4.2a) \\
RA_n &= C_{ar} d_{a,r}(n) \quad (4.2b) \\
RR_n &= C_{rr} \theta_r(n) \quad (4.2c)
\end{align*}

In the equations above $C_{am}$, $C_{ar}$ and $C_{rr}$ are the random variables representing the
correction factors, $d_{a,m}(n)$, $d_{a,r}(n)$ and $\theta_r(n)$ are the maximum average drift, residual average drift and residual rotation values resulting from the $n^{th}$ response history simulation $S_n$ (Figure 4.7). Note that, $RA_n$, $RR_n$, $C_{ar}$ and/or $C_{rr}$ are only required if the measured residual displacements and/or rotations are to be considered.

Figure 4.7: Simulated response history and the probability distributions of the actual maximum and the actual residual drift ratios

The probabilistic characters of the correction factors should be identified based on experimental evidence and expert opinion. Shaking table tests are the most suitable sources of experimental evidence for this purpose. In order to assess the accuracy of a specific modeling approach in predicting the dynamic response, a set of dynamically tested units should be identified. The set of test units should be sufficiently large so as to prevent results being strongly biased to specific construction or testing practices. Moreover, the considered test units should have response characteristics similar to the structure under assessment.

The correction factor $c_{am,i}$ for a single test $i$ is calculated by dividing the measured maximum average drift by the deterministic prediction $d_{a,m}(i)$ resulting from the relevant simulation. Consequently, a set of sample correction factors $\{c_{am,1}, c_{am,2}, c_{am,3}, \ldots\}$ is established based on the results from the considered set of tests. In addition to these analyses based on experimental data, the important differences between the dynamic response of the considered test units and the structure under assessment should be identified. The probable influence of these differences on the predicted response parameters should be estimated based on expert opinion. Subsequently, the probabilistic character of the correction factors ($C_{am}$, $C_{ar}$ and $C_{rr}$) should be identified based on considering both: (1) the accuracy of the simulated response in reproducing the experimental data and (2) the representativeness of the considered experimental data for the structure under assessment.
4.2.2 Step 2: Estimating the prior probability distribution of the maximum drift ratio

In order to estimate the probabilistic character of the structural response parameters, all major sources of uncertainty that influence the predicted response —e.g. the structural properties, the site response and the excitation at the site— should be considered in the assessment process. It is recommended that the uncertainty in the predicted response due to uncertain model parameters is captured by Monte Carlo simulations. For this purpose, the major uncertain model parameters should be identified for the structure under assessment. The major uncertain model parameters are the ones that show considerable variability and have significant influence of the predicted response. The degree of influence on the model parameters can be identified from the results of sensitivity analysis —similar to those presented in Chapter 3. After the major uncertain model parameters are identified, their probability models should be established.

Some key points that should be considered while establishing the probability models for the model parameters are presented in the following. For the sake of clarity the parameters are categorized into two groups: (1) parameters related to the ground motion and loading conditions that caused damage and (2) parameters related to the properties of the structure and the site.

Ground motion and loading conditions

The ground motion experienced at the site —or its essential characteristic properties— must be estimated by taking into account the available relevant information. If a reliable record of the ground motion at the site is available, this record should be used while predicting the response. Most of the time, such reliable records are not available for the damaged structure. Usually, there are either no reliable ground motion records available or there are some that have been recorded at a distance from the structure under assessment. Some approaches that may be followed in these two cases are briefly addressed in the following.

If no reliable record of the damaging earthquake is available, the intensity of the ground shaking experienced at the site can be estimated using ground motion prediction models (GMPM) (also known as attenuation relationships). In this study, the pseudo-spectral acceleration (PSA) of a single-degree-of-freedom system with a period equal to the fundamental period $T_1$ of the structure and a damping ratio of 5% is adopted to measure the intensity of the ground motion experienced at the site. More specifically, the GMRotI50 value as defined by Boore et al. (2006) is used to measure the ground motion intensity at the site.

Conventionally, the PSA is assumed to be a lognormal random variable. The GMPMs provide both an estimate of the median value and the standard deviation of the PSA. The typical essential input parameters are the magnitude of the earthquake $M_w$, the distance
of the source to the site $R$, the type of faulting and the local site conditions. With the current global seismic monitoring systems, the former two parameters are determined within a short time after a major earthquake. The type of faulting can be identified based on the resulting surface deformations. The characteristic properties of the site can be identified based on the construction documents or on microzonation studies, if available. If any of these parameters is not available, a set of likelihoods should be estimated for each parameter and the analysis should be conducted accordingly.

An excitation history is needed if the response of the structure is to be predicted by response history analysis. To this end, a set of representative ground motion records should be selected. Optimally, the set of ground motions should be capable of reflecting all the important characteristics of the shaking that influence the predicted response. Furthermore, the set should be comprehensive enough to adequately represent the uncertainty arising from the absence of the actual ground motion at the site of the structure. A plausible way of establishing such a set would be to identify ground motions recorded during similar previous event featuring similar site properties.

The representative ground motions should be scaled. The scaling is done to assure that the intensity of shaking assumed in the analysis is in agreement with the intensity experienced. Since the intensity experienced can only be estimated probabilistically, the records should be scaled by a random variable. The probability distribution models for the random scaling factors should be established such that the distribution of the resulting PSA values is in agreement with the PSA distribution estimated for the site. This can be done by initially scaling all the ground motions to yield a PSA for $T_1$ equal to the median estimated using GMPM. The dispersion of the estimated PSA should be modeled by defining a random scaling factor. This factor should have a lognormal distribution with a median equal to 1 and a log-standard deviation equal to that obtained using GMPM.

If free-field ground motions that have been recorded at a distance from the site are available, these records may be taken into account to estimate the ground motion experienced at the site. The information that may be deduced from these records depends on several factors ranging from the propagation pattern of the seismic waves to the site response exhibited at the considered location. Evidence from past earthquakes has shown that during an earthquake, the strong ground motions recorded at closely spaced (~100m) locations may exhibit significant variations in both amplitude and phase (Steidl, 1993). In essence, the most suitable approach to estimate the shaking experienced at a site based on records taken at a distance is a strongly case dependent issue. Any method that is found to be suitable can be adopted into the proposed assessment method (e.g. Vanmarcke et al., 1993; Boore et al., 2003; Park et al., 2007). The details of these methods are not repeated here, but the approach by Park et al. (2007) is briefly presented in Appendix H. Following their approach, the PSA experienced at the site can be estimated by taking into account other measured PSAs (at distances closer than approximately 7~10km to the site). An example application of this approach is provided.
in Chapter 5.

For the damaged structures conforming modern seismic codes, often the largest contribution to the overall uncertainty associated with predicted response is due to the damaging ground motion not being exactly known. Therefore, a reduction in the uncertainty relevant to the shaking intensity directly leads to a significant reduction in the overall uncertainty related to the predicted response.

The assumed intensity of the axial load acting on structural elements has been identified to have a significant influence on the predicted response, particularly the residual displacements. Often the axial forces resisted by the vertical members of a hyperstatic structural system cannot be precisely estimated. The temporary changes in the distribution and the intensity of the vertical loading and the local changes between the support conditions of individual members are some of the main factors that inhibit a precise estimation. Therefore, the vertical loading intensity should be considered as random when it cannot be precisely estimated. State-of-the-art methods can be adopted to model this uncertainty (see e.g. Galambos et al., 1982; JCSS, 2001).

## Structural properties

The critical uncertain parameters related to the FE model should be identified. The critical uncertain parameters are the parameters that are expected to have a significant influence on the predicted response and that can not be precisely estimated or measured for the structure under assessment. Essentially, all the parameters that influence stiffness, strength, hysteretic behavior, damping and mass assumed for the structure have some degree of influence on the predicted response.

A number of clues related to the seismic vulnerability of a structure can be collected from the apparent damage after an earthquake. These clues should be taken into account while establishing the response prediction model and the probability distributions of its parameters. For example, the predominant deformation mode of the structure may be identified from the apparent damage. The distribution and the orientations of the cracks reveal localized plastic deformations. From these deformations it may be possible to infer the inelastic deformation modes (e.g. shear, flexure) of the critical components. Moreover, spalling of cover concrete at the regions of plastic hinging may reveal the as-built detailing.

The results presented in Section 3.6, indicate that the predicted response — particularly the residual displacements — are sensitive to the hysteretic model employed. Properly estimating the stress-strain behavior of concrete and steel is of primary importance in modeling the hysteretic behavior. Often the parameters controlling this behavior are highly uncertain. Probability distributions for these materials may be established by adopting any approach suitable for the problem at hand (see e.g. Mirza and MacGregor,
1979; Mirza et al., 1979; JCSS, 2001; Nowak and Szerszen, 2003). Furthermore, in-situ test results and material survey results, if available, should also be taken into account while establishing the probability models. In some cases, the material grades or the quality of workmanship can not be identified easily. Furthermore, the important material properties may change over the years due to time related effects. In these cases, a logic tree approach should be adopted and the likelihoods should be estimated for the material grades or the values of the important properties.

**Predicting the Response**

The probabilistic character of the response parameters should be estimated in accordance with the probabilistic models established for the model parameters and for the correction factors. The Monte Carlo simulation method is a useful tool for investigating numerically the probabilistic character of the response parameters. It is particularly of use in cases where the structure is expected to deform beyond its elastic limit. The Monte Carlo simulation method involves the repetition of the simulation process using in each simulation a particular set of values of the random variables generated in accordance with the corresponding probability distributions (Ang and Tang, 1984).

The Monte Carlo simulation approach is adopted here to estimate the probability distributions of the response parameters. To this end, sets of \( N_r \) random realizations are generated for the uncertain model parameters. Note that, the random ground motion scaling factor is included in the uncertain model parameters. For each ground motion record considered, response should be simulated repetitively by considering in each run a single realization of input parameters. The simulation process should be repeated for the whole set of \( N_g \) representative ground motion records. As a result, a total of \( N \) response simulations are carried out \( (N = N_r \times N_g) \).

The result of each simulation is essentially an individual numerical prediction of the maximum drift ratio (Figure 4.8). Taking into account the random correction factor \( C_{am} \), the probability \( \Pr(M_i|S_n) \) of \( M_i \) given the \( n^{th} \) simulation \( S_n \) can be estimated as follows:

\[
\Pr(M_i|S_n) = \int_{\frac{m_{a+1}}{m_a(n)}}^{\frac{m_i}{m_a(n)}} f_{C_{am}}(x) \, dx
\]  

(4.3)

In the equation above, \( ma_i, ma_{i+1}, d_{a,m}(n) \) and \( C_{am} \) were already defined in Equations (4.1) and (4.2a). This probability reflects the uncertainty in the results for a single simulation due to the limited accuracy of the numerical model. Based on the results of \( N \) simulations, the probability of \( M_i \) can be calculated by utilizing the total probability theorem as follows:

\[
\Pr(M_i) = \sum_{n=1}^{N} \Pr(M_i|S_n) \Pr(S_n)
\]  

(4.4)

In the equation above the probability \( \Pr(S_n) \) is equal to \( 1/N \), since all simulations are
assumed to yield equally likely predictions. In this resulting probability, uncertainties due to: (1) the inherent randomness and the limited statistical knowledge of the model parameters as well as, (2) the inaccuracy of the numerical model are taken into account.

4.2.3 Step 3: Updating the prior probability distribution of the maximum drift ratio based on the visible damage indicators

Visible damage indicators provide valuable information on the degree of inelastic action that has taken place in the structure during strong ground shaking. The term visible damage indicators refers to the indications that are related to peak deformations experienced by the structure and that can be visually inspected by trained structural engineers after an earthquake (Figure 4.9). In accordance with this definition, the presence of cracks in the regions of plastic hinging, spalled cover concrete, crushed core concrete, ruptured or buckled reinforcing bars and signs of pounding to neighboring structures are considered visible damage indicators. Since these indicators are strongly related to the maximum deformations experienced, their detection can be taken into account to improve the prior probability estimates Pr(Mi). To this end, the deformation limits associated with the detection of these indicators should be estimated.

Figure 4.9: Visible damage indicators related to buckling (a), reinforcement fracture (b) and crushing of concrete (c) (Photos by Dazio et al. (1999))
Models that relate deformation limits (DL) of structural components to specific damage indicators can be adopted for this purpose (see e.g. Panagiotakos and Fardis, 2001; Fardis and Biskinis, 2003; Berry and Eberhard, 2003). Often the DL associated with a given damage indicator can not be predicted with certainty. There are two major sources of uncertainty related to DL: (1) the probable inaccuracy of the DL prediction model and (2) the uncertain character of the parameters required to predict the DL. The probabilistic character of DL should be estimated by taking into account these two uncertainties. Accordingly, the deformation limit $L_x$ associated with the detection of a damage indicator $x$ is assumed to be equal to the product of two random variables as follows:

$$L_x = \eta_x L'_x$$  \hspace{1cm} (4.5)

In the equation above, $\eta_x$ is the random correction factor associated with the DL values obtained using a given prediction model, $L'_x$ is a random variable representing the variability in the predicted DL values due to uncertain input parameters. The probabilistic character of the associated random correction factor $\eta_x$ should be estimated based on the error statistics available for the prediction model. The probabilistic character of $L'_x$ should be estimated based on the probability models of the FE model parameters presented in the previous section. For each set of randomly generated FE model parameters, the limit associated with the indicator $x$ should be predicted deterministically. After this, the distribution of $L'_x$ should be estimated based on the resulting set of values.

DL prediction models that have been verified against components featuring deformation characteristics similar to the ones being analyzed would be the optimal ones to adopt. Several studies propose models to predict the DL associated with specific damage states and compare the predicted values against the experimental data for a set of tests (see e.g. Panagiotakos and Fardis, 2001; Berry and Eberhard, 2003).

The maximum displacement estimates are updated by conditioning their distribution on the detected damage indicators. In other words, the probability $Pr(M_i|I_x)$ of $M_i$ conditioned on the detection of the damage indicator $x$ is computed. The event $I_x$ represents the DL associated with the damage indicator $I_x$ being smaller than $MA$. In this respect, the event $I_x$ is defined as follows:

$$I_x = \{L_x < MA\}$$  \hspace{1cm} (4.6)

Here, for the sake of simplicity this event is assumed to always lead to the detection of indicator $x$ during the inspection, i.e. inspection errors are neglected.

Suppose that during the post-earthquake inspection of the structure indication of yielding is detected (Figure 4.10). More specifically, unclosed cracks or spalled cover concrete are observed. Let event $I_y$ represent detection of these indicators. Similarly, let the continuous random variable $L_y$ represent the DL beyond which the indications of yielding become visible. The probability distribution $F_{L_y}$ of $L_y$ can be estimated using a DL model by following the procedure outlined above. In consistence with these
definitions, the probability $\Pr(M_i|I_y)$ is estimated using the Bayes’ Theorem as follows:

$$
\Pr(M_i|I_y) = \frac{\Pr(I_y|M_i) \Pr(M_i)}{\sum_j \Pr(I_y|M_j) \Pr(M_j)}
$$

(4.7a)

where $\Pr(I_y|M_i) = F_{L_y}(mc_i)$

(4.7b)

In which $F_{L_y}(m)$ is the probability of $L_y$ being less than or equal to the central value $mc_i$ of the considered drift interval between $ma_i$ and $ma_{i+1}$, $mc_i = (ma_i + ma_{i+1})/2$.

Similarly, damage indicators not being detected can also be considered to update the maximum drift estimates. For this purpose, the conditional probability $\Pr(M_i|\overline{I_y})$ is estimated. The event $\overline{I_y}$ represents not detecting the damage indicator $x$ during the inspection. This is shown for not detecting the indication of exceedance of ultimate deformation capacity. The term “indications of exceedance of ultimate deformation capacity” refers to the presence of ruptured reinforcing bar or of crushed confined concrete. In accordance with the definitions above, let $\overline{I_y}$ represent the event of not detecting these indicators. Similar to the previous case, $L_u$ represents the DL beyond which the indications are detectable. The probability $\Pr(M_i|I_y \cap \overline{I_y})$ is estimated as follows:

$$
\Pr(M_i|I_y \cap \overline{I_y}) = \frac{\Pr(\overline{I_y}|M_i \cap I_y) \Pr(M_i|I_y)}{\sum_j \Pr(\overline{I_y}|M_j \cap I_y) \Pr(M_j|I_y)}
$$

(4.8a)

where $\Pr(\overline{I_y}|M_i \cap I_y) = 1 - F_{L_u|I_y}(mc_i)$

(4.8b)

In the equations above $F_{L_u|I_y}(mc_i)$ represents the cumulative probability distribution of $L_u$ conditional on $I_y$. $\Pr(M_i|I_y)$ and $mc_i$ were already defined in Equation 4.7a.

Often the correlation of the model errors $\eta_y$ and $\eta_u$ related to the prediction of $L_y$ and $L_u$ are not reported. As a result, the cumulative probability $F_{L_u|I_y}(mc_i)$ can not be determined. One simple solution is to calculate $F_{L_u|I_y}(mc_i)$ approximately using $F_{L_y}(mc_i)$ —which can be directly obtained using a typical DL prediction model for $L_u$. This is achieved as follows:

$$
F_{L_u|I_y}(mc_i) = \begin{cases} 
0 & \text{if } F_{L_y}(mc_i) = 0 \\
\frac{F_{L_y}(mc_i) - F_{L_y}(mc_i)}{F_{L_u}(mc_i)} & \text{if } F_{L_y}(mc_i) > 0
\end{cases}
$$

(4.9)
This approximation leads to acceptably accurate estimates of $\Pr(T_u|M_i \cap I_y)$ when the ratio of the median values for $L_u$ and $L_y$ is larger than 2.5. In essence, this ratio is the expected displacement ductility capacity of the structure. For structures with median ductility capacity lower than 2.5, the probabilities $\Pr(T_u|M_i \cap I_y)$ should be estimated using the general procedure provided in Appendix H. This general procedure allows estimation of the probability $\Pr(T_2|M_i \cap I_1)$ for a given pair of DLs $L_1$ and $L_2$. The procedure results in a noticeable change in the estimated probabilities $\Pr(T_2|M_i \cap I_1)$, only if the estimated DLs are: (1) noticeably correlated ($\rho > 0.5$), (2) conditionally dependent and (3) associated with similar range of deformations (e.g. the ratio of the median values $T_2$ and $T_1$ is in the range $1 \sim 2$).

This process of updating $\Pr(M_i)$ conditional on the detected visible damage can be repeated using Equations 4.7a and 4.8a for a number of damage indicators. The only requirement is that their DLs $L_1, L_2, \ldots, L_n$ must be available. In order to simplify the notation, let $I$ represent the joint event of all inspection results. Therefore, $I$ is defined as follows:

$$I = \{I_1 \cap I_2 \cap I_3 \cap \ldots\}$$

where $I_x$ and $\overline{I_x}$ represent a damage indicator $x$ being detected and not detected, respectively. As a result, the probability $\Pr(M_i|I)$ refers to the probability of $M_i$ conditional on the inspection results. This probability estimate can also be interpreted as a posterior probability of $M_i$.

4.2.4 Step 4: Updating the posterior probability distribution of the maximum drift ratio obtained at Step 3 based on the known residual displacements

Identifying the suitable residual deformation parameters

The residual displacements attained by a structure are usually the only measurable indicator that is directly related to the response history experienced by the structure. Within the scope of this study the term residual displacements (RD) refer to all the measurable residual deformations attained by the structure. The RDs attained by a structure subjected to a damaging earthquake are utilized to update the probability $\Pr(M_i)$. Both global and local RDs that are measurable after an earthquake can be utilized for this purpose. In essence, the method allows taking into account any given set of RD parameters. However, for the sake of clarity here the case of two RD parameters is presented. The residual average drift ratio $d_{a,r}$ of the structure is considered as a global RD parameter. As a local parameter, the residual relative rotation $\theta_{r}$ at the plastic hinging region is considered. In the following, the term residual rotation refers to this rotation. The length of the segment across which rotation is measured should include the region of plastic hinging. However, apart from this constraint, their specific height can be chosen indecently of the utilized discretization scheme. The method presented in Chapter 3 can be used to identify the predicted displacements at any section along the
elements, when a degree-of-freedom of the FE model does not coincide with the location of the RD measurement.

Similar to the case of $MA_n$, the actual residual average drift $RA_n$ and the actual residual rotation $RR_n$ are assumed to be random variables unless they are precisely determined. Likewise, the RD’s $RA_n$ and $RR_n$ are related to the results of the simulation $S_n$—i.e. $d_{a,r}(n)$ and $\theta_r(n)$—through the random correction factors $C_{ra}$ and $C_{rr}$, respectively (Equation 4.2a). The joint event $R_{j,k}$ is defined as follows:

$$R_j = \{ ra_j < RA \leq ra_{j+1} \}$$

(4.11a)

$$T_k = \{ rr_k < RR \leq rr_{k+1} \}$$

(4.11b)

In the equation above $RA$ is the actual average residual drift ratio while $ra_j$ and $ra_{j+1}$ are the lower and upper limits of the $j^{th}$ residual drift interval. $RR$ is the actual residual rotation while $rr_k$ and $rr_{k+1}$ are the lower and upper limits of the $k^{th}$ residual rotation interval.

In order to estimate the probability of $M_i$ conditional on the measured RDs, the joint probability $Pr(M_i \cap R_j \cap T_k)$ must be estimated. In essence, the approach followed is very similar to that in Equation 4.4. However, in this case the joint distribution $f_{C_{am}, C_{ar}, C_{rr}, \ldots}$ of the three random correction factors is taken into account. The correction factors $C_{am}$, $C_{ar}$ and $C_{rr}$ are the factors that were already defined in Section 4.2.1. For each single simulation, the joint probability $Pr(M_i \cap R_j \cap T_k|S_n)$ is estimated conditional on that simulation as follows:

$$Pr(M_i \cap R_j \cap T_k|S_n) = \frac{G_n(i,j,k)}{c_n}$$

(4.12a)

where $c_n = \sum_i \sum_j \sum_k G_n(i,j,k)$

(4.12b)

$$G_n(i,j,k) = \left\{ \begin{array}{ll}
\iiint \cdot f_{C_{am}, C_{ar}, C_{rr}}(x,y,z) \, dx \, dy \, dz & \text{if } (ma_{i+1} > ra_j) \\
0 & \text{elsewhere}
\end{array} \right.$$  

(4.12c)

$$V := \left\{ \left( \frac{ma_i}{d_{a,m}(n)} < x \leq \frac{ma_{i+1}}{d_{a,m}(n)} \right) \cap \left( \frac{ra_j}{d_{a,r}(n)} < y \leq \frac{ra_{j+1}}{d_{a,r}(n)} \right) \cap \left( \frac{rr_k}{\theta_r(n)} < z \leq \frac{rr_{k+1}}{\theta_r(n)} \right) \right\}$$

(4.12d)

In the equation above, $d_{a,r}(n)$ and $\theta_r(n)$ are the residual average drift ratio and residual rotation values resulting from the simulation $S_n$. The parameters $ma_i$, and $ma_{i+1}$ were defined in Equation 4.1. Similarly, $d_{a,m}(n)$ was introduced in Equation 4.2a while the parameters $ra_j$, $ra_{j+1}$, $rr_k$ and $rr_{k+1}$ were introduced in Equations 4.11a and 4.11b.

The joint probability $Pr(M_i \cap R_j \cap T_k)$ is calculated based on the conditional probabilities $Pr(M_i \cap R_j \cap T_k|S_n)$ estimated for each simulation, as follows:

$$Pr(M_i \cap R_j \cap T_k) = \sum_{n=1}^{N} Pr(M_i \cap R_j \cap T_k|S_n) \cdot Pr(S_n)$$

(4.13)
Similar to Equation 4.4, in Equation 4.13 the probability $Pr(S_n)$ is equal to $1/N$, since all simulations are assumed to yield equally likely predictions. This joint probability is expected to reflect the uncertainties considered in the parameters of the Monte Carlo simulations as well as the uncertain error in the simulation results due to the inaccuracy of the model when simulating the maximum and the residual drift ratios.

The joint probability $Pr(M_i \cap R_j \cap T_k)$ is estimated based on the results of time-history analysis and the distributions assumed for the corrections factors. The visible damage inspection results should be reflected on the estimated joint probability $Pr(M_i \cap R_j \cap T_k)$. Similar to the approach pursued in Equations 4.7a and 4.8a, this is achieved by calculating the conditional distribution $Pr(M_i \cap R_j \cap T_k | I)$. For the sake of consistency, the same damage indicators that were considered in the sample applications presented in Chapter 4 are also considered here. The joint probability $Pr(M_i \cap R_j \cap T_k | I_y)$ conditioned on $I_y$ is estimated as follows:

$$Pr(M_i \cap R_j \cap T_k | I_y) = \frac{Pr(I_y | M_i \cap R_j \cap T_k) Pr(M_i \cap R_j \cap T_k)}{\sum_l \sum_m \sum_n Pr(I_y | M_l \cap R_m \cap T_n) Pr(M_l \cap R_m \cap T_n)}$$

(4.14)

Thereafter, the joint conditional probability of $Pr(M_i \cap R_j \cap T_k | I_y)$ should be further conditioned on $\overline{T_u}$ as follows:

$$Pr(M_i \cap R_j \cap T_k | I_y) = \frac{Pr(\overline{T_u} | M_i \cap R_j \cap T_k \cap I_y) Pr(M_i \cap R_j \cap T_k | I_y)}{\sum_l \sum_m \sum_n Pr(\overline{T_u} | M_l \cap R_m \cap T_n \cap I_y) Pr(M_l \cap R_m \cap T_n \cap I_y)}$$

(4.15)

In the equation above, $I$ is the joint set of detection events defined as $\{I_y \cap \overline{T_u}\}$ for the considered case. In Equations 4.14 and 4.15, $Pr(I_y | \cdot)$ and $Pr(\overline{T_u} | \cdot)$ are the probabilities of detecting exceedance of $L_y$ and not detecting the exceedance of $L_u$, respectively. Deformation limit prediction models that provide the probabilistic character of both the maximum and the residual drift limits associated with the identified damage indicators are the optimal ones to be adopted in Equations 4.14 and 4.15. If such models are not available, the conditional probabilities $Pr(I_y | M_l)$ and $Pr(\overline{T_u} | M_l)$ that are computed using Equations 4.7b and 4.8b can also be utilized.

The cost-effectiveness of measuring the RDs of a structure depends basically on two parameters: (1) the resources required to carry out the measurements and, (2) the effectiveness of residual displacements in terms of improving the maximum drift ratio estimates —i.e. the degree of probable improvement in the estimated maximum drift distribution. This probable degree of improvement can be assessed based on the computed joint conditional probabilities $Pr(M_i \cap R_j \cap T_k | I)$ by means of the mutual information $MI$. In simple terms, $MI$ is a measure of the amount of information that two random variables share. In technical terms, $MI$ is a measure of mutual dependence between two random variables (Papoulis, 1991) —i.e. it is a measure of the likely reduction in the uncertainty of one of the random variables by knowing the other variable.

Mutual information between any two random variables has a value between zero and one. $MI$ being equal to zero implies total statistical independence, i.e. knowing the true

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state of one of the variables would have no effect on the knowledge regarding the other variable. The other extreme, $MI = 1$, means that the two random variables share the same information. In this case, knowing one of the variables would eliminate all the uncertainty regarding the other variable.

For the problem at hand, there are two measurable response parameters — residual average drift and residual rotation. If there is more than one measurable response parameter, the mutual information can be considered as a predictive measure of the relative effectiveness of these parameters. In other words, it is used to predict the relative reduction in the uncertainty of the maximum average drift ratio, resulting from measurement of an RD parameter. For the problem at hand, the mutual information $MI(MA; RA)$ of maximum average drift $MA$ and residual average drift $RA$ is calculated as follows:

$$MI(MA; RA) = \sum_i \sum_j Pr(M_i \cap R_j | I) \log \left( \frac{Pr(M_i \cap R_j | I)}{Pr(M_i | I) Pr(R_j | I)} \right)$$

(4.16a)

where

$$Pr(M_i \cap R_j | I) = \sum_k Pr(M_i \cap R_j \cap T_k | I)$$

(4.16b)

$$Pr(M_i | I) = \sum_j \sum_k Pr(M_i \cap R_j \cap T_k | I)$$

(4.16c)

$$Pr(R_j | I) = \sum_i \sum_k Pr(M_i \cap R_j \cap T_k | I)$$

(4.16d)

Likewise, the mutual information of the maximum average drift $MA$ and the residual rotation $RR$ at the plastic hinge location can be calculated as:

$$MI(MA; RR) = \sum_i \sum_k Pr(M_i \cap T_k | I) \log \left( \frac{Pr(M_i \cap T_k | I)}{Pr(M_i | I) Pr(T_k | I)} \right)$$

(4.17a)

where

$$Pr(M_i \cap T_k | I) = \sum_j Pr(M_i \cap R_j \cap T_k | I)$$

(4.17b)

$$Pr(T_k | I) = \sum_i \sum_j Pr(M_i \cap R_j \cap T_k | I)$$

(4.17c)

Regarding the measurement of the actual residual average drift ratio $RA$, higher values of mutual information $MI(MA; RA)$ imply that the new information on the maximum drift ratio gained by the measurement is potentially more valuable. On the other hand, the $MI(MA; RA)$ being zero implies that the measurement of the residual drift would not provide any additional information about the maximum drift. A typical case yielding low mutual information is the assessment of a structure with strong re-centering characteristics. On the contrary, ground motions that tend to deform a structure in one direction only — e.g. a near field motion with a high velocity pulse — are likely to yield high mutual information, i.e. making the proposed method particularly effective.
Updating the estimates

Residual deformations sustained by structures can be identified by means of photogrammetric measurement techniques. Application of such techniques to document the extent of seismic damage to reinforced concrete buildings is reported in the literature (e.g. Altan et al. (2001)). The accuracy of the measurements on the deformed state of the structure is influenced by several factors such as: (1) the adopted photogrammetric measurement technique, (2) the site conditions and, (3) the complexity of the deformation pattern of the structure. Furthermore, in order to deduce the residual relative deformations from measurements carried out on the damaged structure, assumptions regarding its pre-earthquake geometry are often required. In summary, often there is a noticeable measurement uncertainty associated with the actual RDs attained by the structure.

Additional information gained by measuring the RDs is considered to update the estimated probabilities $\Pr(M_i|I)$ (Figure 4.11). The event $MR$ represents the measurement of the two RD parameters, $RA$ and $RR$. The joint probability $\Pr(R_j \cap T_k|MR)$ of $R_j$ and $T_k$ conditional on the measurement $MR$ is obtained after the residual displacements are measured. This conditional probability depends on the accuracy of the measurement and on the data available on the pre-earthquake geometry of the structure. Note that $MR$ represents the actual state of the whole group of parameters that have an influence on the conditional probability $\Pr(R_j \cap T_k|MR)$—e.g. the adopted measurement technology and the assumptions made regarding the pre-earthquake geometry of the structure.

![Residual Displacements](image1)

![Pr($R_j$|MR)](image2)

![Pr($M_i$|I $\cap$ MR)](image3)

Figure 4.11: Updating the maximum displacement estimates based on the known residual displacements

The probability distribution estimated for the experienced maximum average drift is updated based on the measured residual displacements. To this end, the conditional
probability $Pr(M_i | I \cap MR)$ is calculated as follows:

$$Pr(M_i | I \cap MR) = \sum_j \sum_k \frac{Pr(M_i \cap R_j \cap T_k | I)}{Pr(R_j \cap T_k | I)} \cdot Pr(R_j \cap T_k | MR) \quad (4.18)$$

The derivation of Equation 4.18 is presented in Appendix J.

Alternatively, it may be decided to measure only the residual average drift ratio but not the residual rotation. The possible reasons behind such a decision may be a very low $MI(MA; RR)$ value or insufficient resources to measure the residual rotation. Let $MR_{RA}$ represent the event of measurement of $RA$. In this case, only the probability $Pr(R_j | MR_{RA})$ of $R_j$ conditional on $MR_{RA}$ is considered to update the probability $Pr(M_i | I)$. The probability $Pr(M_i | I \cap MR_{RA})$ is calculated as follows:

$$Pr(M_i | I \cap MR_{RA}) = \sum_j \frac{Pr(M_{i} \cap R_j | I)}{Pr(R_j | MR_{RA})} \cdot Pr(R_j | MR_{RA}) \quad (4.19a)$$

where

$$Pr(M_i \cap R_j | I) = \sum_k Pr(M_i \cap R_j \cap T_k | I) \quad (4.19b)$$

and

$$Pr(R_j | I) = \sum_i \sum_k Pr(M_i \cap R_j \cap T_k | I) \quad (4.19c)$$

### 4.3 Comparison of the proposed method with conventional methods

The probable errors associated with the simulated response parameters are directly taken into account in the proposed procedure. Other available post-earthquake performance assessment methods that utilize residual displacements do not take into account this uncertainty explicitly.

Visible damage indicators are taken into account to improve the estimates of the experienced maximum deformations. In this process, the probable inaccuracy of the deformation limits predicted for the indicators is explicitly taken into account. In conventional damage evaluation approaches, the relationship between the visible damage and the changes in structural properties is defined by means of deterministic models. Such approaches fail to capture the uncertainty associated with the predicted properties.

The relationship between the maximum and residual deformations parameters is identified based on probabilistic analysis of the structural response. The method is not based on any assumed relationship between the two parameters. Their joint probability distribution is established by taking into account the major sources of uncertainty. This step of the method requires the greatest computational demand. Most of the assessment methods that utilize residual displacements are based on some relationship between the maximum and residual response parameters. One approach is to assume
a linear relationship between these variables and estimate the probabilistic character of
their ratio. The results obtained from the time history analyses presented in Chapter
5 do not confirm such a linear relationship. Another approach is to assume the maxi-

mum and residual displacements to be jointly distributed lognormal random variables.
This approach results in assuming positive probabilities even for the event of resid-

dual displacement being greater than the maximum —which is impossible by definition.
Another approach is to separately relate the maximum and residual displacements to
a ground motion intensity measure. Typically, the two displacements are modeled as
statistically independent lognormally distributed random variables for a given shaking
intensity (e.g. spectral displacement). Similar to the joint distribution assumption, this
approach results in positive probabilities being assumed for the impossible event of a
residual displacement exceeding the maximum.

4.4 Summary and conclusions

A post-earthquake assessment method is presented in this chapter. The method can
be applied to obtain improved estimates of the damage to a structure through a better
prediction of the maximum displacements attained during the earthquake. This goal
is reached by taking both visible damage and post-earthquake residual displacements
directly into account. The flowchart of the method is plotted in Figure 4.12. The
resulting improved damage estimates are expected to lead to better informed decisions
regarding the post-earthquake usability and/or repairability of the structure.

The method is formulated to allow direct consideration of the uncertainties —related
to the excitation, the material properties and the dynamic response— in the assessment
process. Furthermore, the probable model error associated with the adopted model-
ing approach is also directly taken into account. The method allows information from
different sources being considered to improve the estimates of the maximum drift ratio.
Figure 4.12: Post-earthquake damage assessment method
5 Application of the method

5.1 Introduction

In this chapter the post-earthquake assessment procedure developed in Chapter 4 is applied to two example structures. The structures are RC units tested on shaking tables. The model structures are assumed to be damaged structures that need to be assessed after an earthquake. The maximum drift ratio —where the drift ratio is defined as the relative horizontal displacement of the center of mass of the superstructure divided by its height above the foundation— is assumed to be unknown and its value is to be estimated by taking into account the information which may be available in a typical post-earthquake evaluation situation.

In the sample applications, it is assumed that structural drawings are available and the grades of the construction materials are known. Concerning the excitation that caused the damage different assumptions are introduced for the two sample applications. For the first application it is assumed that a reliable record of the ground motion experienced by the structure is not available but all essential properties of the earthquake and the site are known. The term “essential properties” refers to the properties that are needed to predict the intensity of shaking using a ground motion prediction model (GMPM). For the second application it is assumed that in addition to the essential properties of the earthquake and the site a number of ground motions that have been recorded at neighboring sites are available.

In the following two sections the two example applications are presented. The discussion of the results is presented after these two sections. Lastly, the conclusions are summarized.
5.2 Example: Wall WDH4

5.2.1 Introduction

The method is applied to predict the maximum average drift ratio attained by wall WDH4 during Test 1 (Figure 3.1). Lestuzzi et al. (1999) report the actual maximum average drift ratio to be 1.52% for this test. The geometry and the detailing of the wall WDH4 have already been presented in Chapter 3.

The response parameters considered in this example application are the following:

**Maximum average drift ratio** $MA$ experienced: Relative horizontal displacement of the 3rd story divided by the height of that story above the foundation level (average drift ratio).

**Residual average drift ratio** $RA$: Residual value of the average drift ratio that is sustained after the shaking is over.

**Residual rotation at the base segment** $RR$: Rotation of the section that is located 550 mm away from the top of the foundation surface.

The case of both $RA$ and $RR$ being measured after the shaking is considered in this example.

The set of maximum drift ratio intervals $ma_i$ is established by dividing the range of drift values from 0 to 3% into intervals of 0.2% drift. Additionally, an interval that covers the range from 3% to infinity is defined. Similarly, the set of residual drift intervals $ra_j$ is defined by subdividing the drift range 0–0.3% into intervals of 0.05%. The residual rotation intervals $rr_k$ are obtained by subdividing the rotation range 0–4° into 0.5° intervals. The event $M_i$ represents $MA$ being in the interval from $ma_i$ to $ma_{i+1}$ as it was already defined in Equation 4.1. Likewise, $R_j$ represents $RA$ being in the residual average drift interval $ra_j$ to $ra_{j+1}$ and $T_k$ represents $RR$ being in the residual rotation interval interval $rr_k$ to $rr_{k+1}$.

5.2.2 Step 1: Modeling of wall WDH4

The hysteretic models, element formulation and the meshing scheme adopted to carry out the time history analyses is based on the model DF (Section 3.5.2). The finite element analysis code OpenSees (McKenna et al., 2007) is used for the time history analyses.

The accuracy of the model in predicting: (1) the maximum average drift ratio $d_{a,m}$ (2) the residual average drift $d_{a,r}$ and (3) the residual rotation at the base segment $\theta_r$, is
assessed. The correction factors $C_{am}$, $C_{ar}$ and $C_{rr}$ are related to the simulated maximum drift, residual drift and residual rotation, respectively. The probabilistic characters of these random variables are identified based on the results presented in Chapter 3. For each test unit the correction factors are calculated by dividing the measured values of a given response parameter with the value obtained using the DF model. The correction factors are assumed to be log-normally distributed random variables and the distribution parameters are estimated using the maximum likelihood method (Figure 5.1). The predicted distribution parameters are presented in Table 5.1.

The correction factors identified for the residual average drift $C_{ar}$ and for the residual rotation $C_{rr}$ are found to be highly correlated. This is expectable since, given that a particular modeling approach results in over- or underestimation of the residual average drift by a specific factor, it would be likely for the same model to over- or underestimate the residual rotation by a similar factor. On the other hand, no significant correlation is identified between $C_{am}$ and these factors. It should be noted that a limited number of sample points are considered in the analysis and that the dispersion of the parameters is very large. Therefore, the statistical significance of the estimated distributions for the correction factors is rather low. By further research, statistically more significant distributions may be established by considering a larger set of structural tests or developing modeling approaches that predict the residual deformations with higher accuracy.

| Table 5.1: Distribution parameters of the correction factors $C_{am}$, $C_{ar}$ and $C_{rr}$ |
|-----------------|-----|-----|-----|-----|
| $C_{am}$        | 1.06| 0.063| 0.063| 1    |
| $C_{ar}$        | 1.85| 0.814| 0.713| 1    |
| $C_{rr}$        | 1.90| 0.791| 0.697| 0.99 |

5.2.3 Step 2: Estimating the prior distribution of the maximum drift ratio

Seismic excitation

Usually, the actual ground motion that damaged the structure is not precisely known for the large majority of structures. However, the major properties —e.g. magnitude, epicenter, depth— of the seismic event are often known and the likely intensity of shaking experienced at the site of the structure can be estimated based on this data. The wall WDH4 was subjected to an artificially generated acceleration series. Since the acceleration series was not originated by any particular seismic event, the properties of the event had to be assumed. In order to identify a suitable seismic event the pseudo-acceleration (PSA) spectrum calculated using the record was compared against the PSA spectrum predicted using the ground motion prediction model (GMPM) by Campbell and Bozorgnia (2007). The set of properties presented in Table 5.2 is found to be suitable for the assumed event.
Correction factor for the maximum average drift
Cumulative probability
Observed values
Fitted model
(a)

Correction factor for the residual average drift
Cumulative probability
Observed values
Fitted model
(b)

Correction factor for the residual rotation
Cumulative probability
Observed values
Fitted model
(c)

Figure 5.1: Cumulative distributions of the correction factors for the maximum average drift ratio $C_{am}$ (a), the residual average drift ratio $C_{ar}$ (b), and the residual rotation $C_{rr}$ (c)
Table 5.2: Properties of the seismic event and the site

<table>
<thead>
<tr>
<th>Seismic event</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnitude, $M_w$</td>
<td>7</td>
</tr>
<tr>
<td>Faulting</td>
<td>Strike-slip</td>
</tr>
<tr>
<td>Rake angle</td>
<td>0°</td>
</tr>
<tr>
<td>Dip angle</td>
<td>90°</td>
</tr>
<tr>
<td>Depth-TOR, $Z_{TOR}$</td>
<td>5 [km]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Site</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance-RUP, $R_{rup}$</td>
<td>15 [km]</td>
</tr>
<tr>
<td>Joyner and Boore Distance, $R_{JB}$</td>
<td>10 [km]</td>
</tr>
<tr>
<td>Site Class</td>
<td>Soft soil</td>
</tr>
<tr>
<td>Average shear wave velocity, $V_{s30}$</td>
<td>180 [m/s]</td>
</tr>
<tr>
<td>Sediment depth, $Z_{2.5}$</td>
<td>2</td>
</tr>
</tbody>
</table>

1. *Depth to the top of the coseismic rupture plane*
2. *Closest distance to the coseismic rupture plane*
3. *Depth to the 2.5 km/s shear-wave velocity horizon*

The uncertainty in the intensity of shaking experienced at the site due to the assumed seismic event is estimated using the same GMPM by Campbell and Bozorgnia (2007). The median and the logarithmic standard deviation $\sigma_{\log(S_a)}$ of the PSA for a randomly oriented single-degree-of-freedom (SDF) system having a period of 1.27s and 5% damping are estimated as 0.26g and 0.64, respectively. Note that the fundamental period considered here corresponds to the fundamental period of the prototype $T_{pr}$ and not the period $T$ of the tested model.

In order to take into account the uncertainty in the structural response due to the actual excitation being unknown, a set of representative ground motion records is established. The ground motions that have been recorded at sites similar to the assumed seismic event are included in the set. A total of 16 ground motion components are identified. The ground motions used in the analysis are presented in Table 5.3. Each ground motion is scaled by a scaling factor $s_1$ so that the PSA at the period of 1.27s is equal to median value estimated using the GMPM. In Figure 5.2, the median and the plus and minus one standard deviation estimates of the 5% damped PSA spectrum that is obtained using the GMPM are presented (1st and 2nd series). Additionally, the PSA values identified for the acceleration series that was recorded on the shaking table is also presented in the same figure (3rd series). Note that this spectrum is obtained after the time-scaling that was applied by Lestuzzi et al. (1999) is inverted. Thus, the PSAs that correspond to the prototype are obtained. The PSA for the period of 1.27s is 0.34g. This value is 30% larger than the median estimate obtained using the GMPE. The PSAs calculated for the set of representative ground motion records are also presented in Figure 5.2 (4th series). The presented PSA spectra are obtained after the records were scaled with the factors $s_1$ (Table 5.3).

The unit WDH4 is a model at 1:3 scale of a 3 story building. While designing
the test, Lestuzzi et al. (1999) adopted the *lumped mass system* similitude approach presented by Moncarz and Krawinkler (1981). According to the similitude theory the strains developed in the test unit and the prototype become identical if a time scale equal to \(1 : \sqrt{3} = 0.5773\) is used. For this reason, the time step size of the table input motion was reduced according to this time scale. In this study, the same time scale modification was applied to the ground motion records that were utilized in the time history analysis.

Figure 5.2: The pseudo-spectral acceleration values predicted using the attenuation relationship by Campbell and Bozorgnia (2007), the actual table acceleration as well as the set of considered ground motion records

**Vertical loading and damping**

The axial load \(N_{ax}\) resisted by the wall WDH4 varied during the actual test (see Section 3.5.1). The results of the sensitivity analyses presented in Section 3.8.1 suggest the predicted residual displacements to be highly sensitive to the intensity of the vertical loading. Due to these reasons, \(N_{ax}\) is also considered as a random variable in the analysis (Table 5.4).

The results of the sensitivity analyses presented in Section 3.8.4 indicate that both the predicted peak and the residual displacements are sensitive to the assumed damping ratio \(\zeta\). In accordance with this observation, the expected variability of \(\zeta\) is also taken into account in the analysis (Table 5.4).
Table 5.3: Set of ground motion records used in the analysis

<table>
<thead>
<tr>
<th>ID</th>
<th>(Year)</th>
<th>Earthquake</th>
<th>$M_w$</th>
<th>Station name</th>
<th>$R_{ep}$</th>
<th>$R_{rup}$</th>
<th>Site</th>
<th>Class</th>
<th>f$_{LC}$</th>
<th>f$_{HC}$</th>
<th>$s_1$</th>
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<td>1</td>
<td></td>
<td>Landers</td>
<td>7.3</td>
<td>Morongo Valley</td>
<td>21.3</td>
<td>17.3</td>
<td>D</td>
<td>000</td>
<td>-</td>
<td>-</td>
<td>1.16</td>
</tr>
<tr>
<td>2</td>
<td>(1992)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>3</td>
<td></td>
<td>Coolwater</td>
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<td></td>
<td>19.7</td>
<td>D</td>
<td>LN</td>
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<td>30</td>
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<td>19.1</td>
<td>D</td>
<td>000</td>
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<tr>
<td>6</td>
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<td>12.8</td>
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<td>000</td>
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<td>8</td>
<td>(1989)</td>
<td></td>
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<td>23</td>
<td>0.70</td>
</tr>
</tbody>
</table>

*All ground motions files were downloaded from the PEER-NGA database (http://peer.berkeley.edu/nga/)*

Material properties

The grades of the materials that were used in the construction of the wall are identified based on the information provided by Lestuzzi et al. (1999). Since the longitudinal reinforcement was imported from Italy, the steel is assumed to comply with the FeB44k type steel specifications by the MLP (1996). The concrete is assumed to be of grade B45/35 as specified by SIA 162 (1993). The characteristic values of the material properties are obtained from the relevant codes.

The uncertainty due to actual material properties being unknown is modeled by establishing probability distribution models for the major material parameters. Recommendations by the JCSS (2001) were followed to establish the probabilistic models of the material parameters. Estimated distribution parameters are presented in Table 5.4.

Generated input parameters

Sets of 500 random realizations are generated for each uncertain model parameter in accordance with their probability models. Random numbers are generated using MATLAB® (The Mathworks, 2007). The effect of confinement on the ultimate concrete compressive strain and stress capacities of concrete, as well as the plastic hinge length $L_p$ are estimated according to the recommendations provided by Priestley et al. (2007). For each simulation, the length of the beam-column finite element located at the plastic hinging region is set equal to twice the $L_p$ estimated for that realization. The
bilinear idealization of the flexural response is established according to the recommendations given by Priestley et al. (1996).

The periods of vibration at the first two modes are computed based on the effective initial flexural stiffness identified for that particular realization, i.e. the slope of the elastic branch of the bilinear idealization. The damping coefficients are calculated for each simulation based on the damping value and the periods of the first two modes computed for that realization. The resulting sets of input parameters are presented in Figure 5.3.

**Predicted response parameters**

A total of 8000 (i.e. 16 ground motion components and 500 randomly generated model parameters) nonlinear time-history analyses are carried out. An additional zero-acceleration time frame of 3s is added at the end of each time-history analysis to allow the system to converge to a more or less constant displacement. The average of the peak displacements attained in the last cycle of free vibration is taken to be equal to the residual displacement. Simulated maximum and residual average drift values, $d_{a,m}$ and $d_{a,r}$ are presented in Figure 5.3. Additionally, simulated maximum and residual rotations, $\theta_m$ and $\theta_r$ values are also presented in the same figure.

The prior probability distribution of the maximum drift ratio is obtained from the application of Equation 4.4 to the results of the nonlinear time-history analyses. The resulting probability distribution is presented in Figure 5.4a (1st series). Additionally, the cumulative probability estimated for the experienced maximum drift is presented in Figure 5.4b (1st series).

---

Table 5.4: Characteristic parameters of the distribution models of important parameters relevant to the test of wall WDH4

<table>
<thead>
<tr>
<th></th>
<th>Normal distribution</th>
<th>Lognormal distribution</th>
<th>Uniform distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>COV</td>
<td>Correlation coeff., $\rho$</td>
</tr>
<tr>
<td>Yield strength, $\sigma_y$</td>
<td>513</td>
<td>0.058</td>
<td>1</td>
</tr>
<tr>
<td>Ultimate strength, $\sigma_u$</td>
<td>579</td>
<td>0.069</td>
<td>0.85</td>
</tr>
<tr>
<td>Ultimate tensile strain, $\epsilon_{su}$</td>
<td>8.5</td>
<td>0.090</td>
<td>-0.5</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>COV</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>Concrete strength, $\sigma_0$</td>
<td>50.9</td>
<td>0.135</td>
<td>3.93</td>
</tr>
<tr>
<td>Excitation scaling factor, $s_2$</td>
<td>1</td>
<td>0.621</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>COV</td>
<td>a</td>
</tr>
<tr>
<td>Damping ratio, $\zeta$</td>
<td>3</td>
<td>0.385</td>
<td>1</td>
</tr>
<tr>
<td>Axial load, $N_{ax}$</td>
<td>67</td>
<td>0.215</td>
<td>42</td>
</tr>
</tbody>
</table>
Figure 5.3: Histograms and scatter plots of the model input parameters and of the time-history analysis results for wall WDH4
Figure 5.4: Probability distributions estimated for the maximum drift ratio experienced by the wall WDH4

5.2.4 Step 3: Updating the distribution of the maximum drift ratio based on visible damage indicators

The estimated probability distribution of $MA$ is updated based on the visible damage to the wall. Lestuzzi et al. (1999) report that a number of horizontal cracks were opened during the test and at the end of the test they were mostly closed (Figure 5.5). From the remaining permanent cracks, it is possible to infer that the system had deformed beyond yielding during the test. Likewise, since no reinforcing bar fractured nor the confined concrete core crushed, it is obvious that the maximum deformation experienced by the wall during the test did not exceed its ultimate deformation capacity. For this sample application, $I_y$ represent the event of damage indicators associated with yielding of the wall being detected. Additionally, $I_u$ represent the damage indicators associated with ultimate deformation of the wall not being detected. As a result, the joint event of inspection results $I$ is defined as:

$$I = \{I_y \cap \overline{I_u}\} \quad (5.1)$$

In order to update the maximum displacement estimates based on the inspection results, the maximum average drift values beyond which the damage indicators become visible are estimated. For the wall, the yield and the ultimate deformation limits are predicted using the models developed by Fardis and Biskinis (2003). It is assumed that the damage indicators that are associated with these drift limits are detected if these limits are exceeded.

In Fardis and Biskinis (2003) a number of alternative models are proposed and verified based on a large database of structural component test results. The equations used in this example application to estimate the yield $d_{a,y}$ and the ultimate $d_{a,u}$ deformation
Signs of exceedance of yield deformation
- Visible flexural cracks
- No ruptured reinforcement
- No crushed confined concrete

Visible Damage Indicators

Figure 5.5: Visible damage indicators for the wall WDH4 after Test 1 (Photo by Lestuzzi et al. (1999))

limits are the following:

\[
d_y = \phi_y \frac{L_{\text{eff}}}{3} + 0.0025 + a_{sl} \frac{\epsilon_y}{d - d'} \frac{0.2d_y \sigma_y}{\sqrt{\sigma_{cl}}}\] \hspace{1cm} (5.2a)

\[
d_{a,u} = d_{a,y} + (\phi_u - \phi_y) L_{p,FB03} \left( 1 - \frac{0.5L_{p,FB03}}{L_{\text{eff}}} \right) \] \hspace{1cm} (5.2b)

\[
L_{p,FB03} = 0.06L_{\text{eff}} + 0.035L_w + a_{sl} \frac{\sigma_y}{25} d_b \] \hspace{1cm} (5.2c)

In the equations above, \(a_{sl}\) is the reinforcement pull-out parameter, \(d_b\) is the diameter of the longitudinal reinforcement, \(d\) and \(d'\) are the effective depths to the tension and the compression reinforcements, respectively. The \(L_{p,FB03}\) is the equivalent plastic hinge length. Fardis and Biskinis (2003) note that the confinement model presented in CEB/FIP Model Code 90 should be used to calculate the \(\phi_u\) in Equation 5.2b. Additionally, they recommend an equivalent plastic hinge length equation that is to be used with this confinement model. When estimating the ultimate deformation limit for the wall, this plastic hinge length equation is used.

The following sources of uncertainty are taken into account while predicting the deformation limits of structural components: (1) the actual material properties, (2) the axial load resisted at the plastic hinging regions, and (3) the deformation limit prediction models. The random variables \(L'_y\) and \(L'_u\) represent the variability in the predicted yield and ultimate drifts due to the first two sources of uncertainty. In order to estimate their probabilistic character, two drift limits are computed for each random realization and log-normal distributions are fitted to the resulting sets of predictions. The empirical cumulative distribution calculated based on the whole set of realizations are presented in Figure 5.6 (1st and 4th series). Additionally, the distributions of \(L'_y\) and \(L'_u\) are also presented in the same figure (2nd and 5th series). The random variables \(\eta_y\) and \(\eta_u\) represent the model uncertainty for the deformation limit prediction model. The probability models for these variables are estimated based on the error statistics
reported in Fardis and Biskinis (2003). The definition of the yield deformation limit \( L_y \) is presented in Equation 4.5. Here, \( L_y \) is considered as a lognormally distributed random variable and its distribution parameters are calculated as follows:

\[
\lambda_{L_y} = \lambda_{L_y} + \lambda_{\eta_y} \quad (5.3a)
\]

\[
\zeta_{L_y} = \sqrt{\zeta^2_{L_y} + \zeta^2_{\eta_y}} \quad (5.3b)
\]

In the equations above, the expressions \( \lambda \) and \( \zeta \) are used to denote the logarithmic mean and the logarithmic standard deviation of the random variables. The probabilistic character of the ultimate deformation limit \( L_u \) is also estimated in the same way. The distributions estimated for \( L_y \) and \( L_u \) are plotted in Figure 5.6 (3rd and 6th series). The dispersion of \( L_u \) is significantly larger than that of \( L_y \). The ductility capacity of the wall is estimated to be around 6 based on the median values identified for the yield and ultimate deformation limits.

![Figure 5.6: Probability distributions estimated for the yield and ultimate drift ratios](image)

The probability distribution of the maximum average drift conditional on the inspection results \( I \) are calculated using Equation 4.8a (Figure 5.4a, 2nd series). Since the predicted displacement ductility capacity of the wall is larger than 2.5, Equation 4.9 is used for estimating the probability of \( L_u \) conditional on \( I_y \).

5.2.5 Step 4: Updating the maximum drift ratio distribution based on known residual displacements

In the final step of the application, the estimated maximum average drift distribution is updated based on the known residual displacements. For this purpose, the joint probabilities \( \Pr(M_i \cap R_j \cap T_k) \) are established using Equation (4.13). Thereafter, the resulting joint probability is updated based on the inspection results. The same deformation limit prediction model that was used in Step 3 is used in the updating.
Both the residual average drift ratio and the residual rotation are known for the wall WDH4. Lestuzzi et al. (1999) report that the residual average drift ratio and the residual rotation were 0.21% and 0.22%, respectively (Table 3.2). However, still for the sake of completeness, the mutual information between the maximum average drift and the two RD parameters are evaluated using Equation 4.16. The mutual information $MI(MA; RA)$ between $MA$ and $RA$ is estimated as 0.163 based on the joint probability distribution $Pr(M_i \cap R_j | I)$ (Figure 5.7). Similarly, the mutual information $MI(MA; RR)$ between $MA$ and $RR$ is predicted as 0.147. The higher the mutual information, the more significant is the effect of updating based on the measured residual response parameter. Therefore, if only one deformation parameter would have to be measured to estimate the maximum drift ratio for this example structure, measurement of the residual average drift would be more useful compared to the residual rotation at the base segment.

![Figure 5.7: Joint probability distribution of the maximum and the residual average drift ratio conditional on the damage inspection](image)

The probability distribution of the maximum average drift is estimated using Equation 4.18 based on the observed residual average drift and the rotation values (Figures 5.4a and 5.4b, 3rd series). This distribution is expected to reflect the uncertainties arising from the actual ground motion record being unavailable, exact material and dynamic properties being unknown and the model errors being present in the simulation results. Furthermore, it is expected to reflect the uncertainties that remain after the inspection results and despite the fact that the residual displacements are known. Additionally, the cumulative probability $Pr(MA \leq d_{m,a} | I \cap MR_{RA})$ of $MA$ conditional
only on the observed residual average drift is presented in Figure 5.4b (4th series). The discussion of the results obtained in this sample application is presented in Section 5.4 together with the results obtained from the second example application.

The residual structural properties (e.g. stiffness, strength) can be predicted based on the estimated maximum drift. An approximate method to estimate the residual stiffness $K_r$ and strength is presented in Figure 5.8a. In this method, the residual strength is estimated for the wall using its pushover curve obtained by means of a numerical simulation. The base moment obtained when the drift is equal to the considered estimate of the sustained maximum drift $d_{a,m}^*$ is identified. The residual strength $M_r^*$ of the wall is assumed to be equal to this base moment identified on the pushover curve. The residual stiffness $K_r^*$ for the wall is obtained by connecting the residual displacement point with the residual strength point identified. The variability of the residual stiffness can be evaluated by repeating the procedure considering a range percentiles of $MA$.

The median value of $MA$ is considered to predict the median strength and stiffness in Figure 5.8a. The median $K_r$ is found to be equal to 0.38 times the undamaged (cracked) stiffness $K_0$. The stiffness $K_0$ is calculated using the yield moment $M_y$ and the yield drift $d_{a,y}$ values presented in Tables 3.1 and 3.5.

The damaged wall WDH4 was subjected to a second test Test2 after Test1 (Lestuzzi et al., 1999). This second motion can be considered as an aftershock that excites the damaged structure. The response of WDH4 measured during Test2 in the time interval from 0 to the 5.5 s is plotted in Figure 5.8b. Additionally, the approximate median residual stiffness and strength are indicated in the same figure. The agreement between the estimated residual stiffness and the actual response of the damaged unit seems to confirm the effectiveness of the method for this example case.

Figure 5.8: Estimation of the residual stiffness (a) and comparison of the estimate with the measured response (b)

The damaged wall WDH4 was subjected to a second test Test2 after Test1 (Lestuzzi et al., 1999). This second motion can be considered as an aftershock that excites the damaged structure. The response of WDH4 measured during Test2 in the time interval from 0 to the 5.5 s is plotted in Figure 5.8b. Additionally, the approximate median residual stiffness and strength are indicated in the same figure. The agreement between the estimated residual stiffness and the actual response of the damaged unit seems to confirm the effectiveness of the method for this example case.
The residual stiffness and strength obtained for the damaged unit can be utilized in several ways. The likely performance of the damaged structure may be evaluated based on these estimated residual structural properties. Moreover, the retrofitting schemes can be optimized to work well with the damaged structure. Furthermore, the forces that will be transferred to the foundation can be effectively predicted to estimate the likely performance of the substructure located beneath the retrofitted superstructure.

In some applications, numerical simulation of the hysteretic behavior of the damaged components may be of interest. In this case, the numerical model established for the undamaged component may be “damaged” analytically —i.e. modified to reflect the effects of the damage. This may be achieved by applying a prior quasi-static displacement cycle with a peak that is equal to the maximum deformation estimated using the proposed procedure.

5.3 Example: Column A1

5.3.1 Introduction

The procedure presented in the previous section is now applied to a bridge pier model tested on a shaking table. The reinforced concrete column A1 tested by Hachem et al. (2003) is considered as the example structure. Specifically, the response of column A1 during the test Run 2 is considered. Hachem et al. (2003) report the actual maximum drift ratio to be 5.1% for this test. The geometry and the detailing of the column A1 have already been presented in Chapter 3. Similar to the previous sample application, the maximum drift ratio experienced by the unit is assumed to be unknown and is estimated.

The response parameters considered in this example application are defined as follows:

**Maximum average drift ratio** $MA$ **experienced** : relative horizontal displacement of the center of mass of the mass-block divided by its height above the foundation.

**Residual average drift ratio** $RA$ : residual value of the average drift ratio.

The case of $RA$ being measured as the residual displacement parameter is considered here. The main difference in this application —compared to the previous one— is the treatment of the uncertainty associated with the ground excitation. Here, the ground motions recorded at neighboring sites are considered to improve the estimates of the PSA experienced at the site. However, it is again assumed that a reliable record of the ground motion that was experienced at the site is not available.

The treatment of the uncertainty associated with structural properties is very similar
to the previous example — i.e. the structural drawings and the grades of the construction materials are assumed to be known.

5.3.2 Step 1: Modeling of column A1

The modeling approach adopted to carry out the time-history simulations is based on the model DF (similar to the previous example). The details of this model were already introduced in Section 3.5.2. Since the modeling approach is the same as that adopted in the previous example, the probabilistic characters of the correction factors $C_{am}$ and $C_{ar}$ are also assumed to be the same (Figure 5.1 and Table 5.1).

5.3.3 Step 2: Estimating the prior distribution of the maximum drift ratio

Seismic excitation

The shaking experienced at the site of the structure is estimated based on the ground motions recorded at neighboring free-field strong motion stations. In particular, the PSA experienced by the structure is estimated. Thereafter, a suite of representative ground motions is selected and scaled accordingly.

The PSA experienced at the site is predicted using the approach presented in Appendix H. During the test, column A1 was subjected to the stronger horizontal component of the Olive View, Northridge 1994 record. For this reason, the free-field strong motion stations that are located close to the Olive View station are identified (Figure 5.9). Thereafter, the distances $\Delta$ of all the other stations that recorded the Northridge event to the Olive View station are calculated. The coordinates of the stations are obtained from the PEER NGA database. During the Northridge earthquake, eight strong motion stations were located at distances closer than 10 km to the Olive View station (Table 5.5). Note that the limit 10 km is adopted from Wald et al. (2006).

The prior estimates of the PSAs for all sites are obtained using the GMPM by Campbell and Bozorgnia (2007). The pairwise correlation coefficient $\rho_{\varepsilon,\varepsilon}$ of $\varepsilon$s are estimated using the model by Goda and Hong (2008) for each site pair. Some of these coefficients are presented in Table 5.5. The coefficients decrease with increasing $\Delta$. The $\tilde{\varepsilon}$s listed in Table 5.5 are calculated using Equation H.7.

The distributions estimated for the PSA experienced at the site, are presented in Figure 5.10. In this figure, the 1st series is the prior probability distribution obtained using the GMPM. The median of this distribution is 0.3 g. This median underestimates the actual PSA measured at the site by 50%. Note that, this underestimation is larger than the degree of underestimation identified for all the neighboring sites (Table 5.5).
Subsequently, the prior distribution is updated based on the PSAs measured at the neighboring strong motion stations. Three posterior PSA distributions are presented in Figure 5.10. These distributions are obtained when: (1) only the closest station (i.e. 3.7 km distant from the site) is considered (2\textsuperscript{nd} series), (2) the 7 closest stations are considered (3\textsuperscript{rd} series) and (3) all 8 stations are considered (4\textsuperscript{th} series). A gradual reduction in the dispersion is observed as the number of stations considered in the analysis increases. The largest drop in the distribution and increase in the median is obtained when only the closest station is considered. On the other hand, taking into account station 8, which is located 10 km away from the site, has an insignificant influence on the estimated distribution. The final posterior distribution has a considerable variability even after 8 neighboring stations are taken into account in the updating. Moreover, the median PSA of the final posterior distribution underestimates the actual measured value by 44%.

Table 5.5: Set of ground motion stations considered in the updating

<table>
<thead>
<tr>
<th>Station ID</th>
<th>Station name</th>
<th>Site class</th>
<th>$\Delta$</th>
<th>$S_{a}(T=1.65s,\zeta=5%)$ [g]</th>
<th>$\rho_{el}$</th>
<th>$\rho_{el}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sylmar - Converter Sta East</td>
<td>C</td>
<td>3.7</td>
<td>0.36 0.57 0.37 0.70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Pacoima Dam (downstr)</td>
<td>A</td>
<td>4.5</td>
<td>0.32 0.12 0.09 0.34</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Sylmar - Converter Sta</td>
<td>D</td>
<td>4.5</td>
<td>0.32 0.77 0.65 0.26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Jensen Filter Plant Generator</td>
<td>C</td>
<td>5.2</td>
<td>0.30 0.50 0.37 0.47</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Rinaldi Receiving Sta</td>
<td>D</td>
<td>5.9</td>
<td>0.27 0.60 0.38 0.73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Pacoima Kagel Canyon</td>
<td>C</td>
<td>7.2</td>
<td>0.24 0.15 0.17 -0.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>LA - Sepulveda VA Hospital</td>
<td>C</td>
<td>9.1</td>
<td>0.20 0.29 0.30 -0.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Arleta - Nordhoff Fire Sta</td>
<td>D</td>
<td>10.0</td>
<td>0.18 0.18 0.25 -0.53</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 Distance between the site and the ground motion station
2 Median PSA estimated using the GMPM by Campbell and Bozorgnia (2007)

In order to model the uncertainty in the frequency content, phasing and the duration of the actual excitation, a set of 16 representative ground motions is selected (Table 5.6). The site of the Olive View strong motion station is designated as a C class site in the NGA database, according to the NEHRP classification scheme (FEMA, 2003). Therefore, only ground motions recorded on C class sites are included in the database. Eight ground motion records are the ones that are measured at close sites during the Northridge earthquake. The other eight records have major properties — e.g. magnitude, distance, site class — similar to the considered event and site. In Table 5.6, the scaling factors $s_{1,i}$ are presented for each ground motion record. The scaling factor $s_{1,i}$ for the record $i$ is calculated as follows:

$$s_{1,i} = \frac{s_{a,p} s_{t}}{s_{a_i}}$$

(5.4)

where $s_{a,p}$ is the median PSA value predicted for the site using only the GMPM (Figure 5.10, median of the 1\textsuperscript{st} series), $s_{t}$ is the scaling factor that is applied to amplify the original ground motion for the test and $s_{a_i}$ is the PSA calculated for the record $i$. For column A1, $s_{t}$ is equal to 1.09, since the original record that was input into the table was amplified by this ratio during the test Run 2 (Hachem et al., 2003). In Figure 5.11, the spectral ordinates obtained using GMPM are presented (1\textsuperscript{st} and 2\textsuperscript{nd} series). Note
Figure 5.9: Map of the site, the rupture zone of the Northridge 1994 earthquake and the freefield strong motion stations
that the obtained ordinates are multiplied with the factor $s_t$. Additionally, the spectral ordinates calculated for the actual acceleration series recorded on the table are presented in Figure 5.11 as the 3rd series. In order to construct this spectrum, the time scale of the measured acceleration is converted back to the prototype’s scale. The PSA at the fundamental period is 0.68 g. Finally, the PSA spectra — obtained after the records are scaled with $s_1$ — are presented in the same figure (4th series).

The second scaling factor $s_2$ is applied to take into account the variability of the PSA experienced at the site (Table 5.7). Specifically, it represents the variability estimated by taking into account the PSAs measured at the neighboring sites. Let $SA’$ represent the posterior distribution estimated for the PSA, by considering the PSAs measured at the neighboring strong motion stations (Figure 5.10, 4th series). In accordance with this definition, the logarithmic mean $\lambda_{s_2}$ of $s_2$ is calculated as:

$$\lambda_{s_2} = \lambda_{SA’} - \log(s_{ap})$$ (5.5)

where $\lambda_{SA’}$ is the logarithmic mean of the $SA’$ while $s_{ap}$ was already introduced in Equation 5.4. The logarithmic standard deviation $\zeta_{s_2}$ is equal to the logarithmic standard deviation $\zeta_{SA’}$ of the $SA’$. Note that the median estimate of the $SA’$ times $s_t$ is equal to $(0.34 \text{ g} \times 1.09 =) 0.37 \text{ g}$. The actual PSA measured on the table is approximately 84% larger than this posterior estimate.

Figure 5.10: Probability distributions estimated for the experienced pseudo-spectral acceleration
Table 5.6: Set of ground motion records used in the analysis

<table>
<thead>
<tr>
<th>ID</th>
<th>Earthquake (Year)</th>
<th>$M_w$</th>
<th>Station name</th>
<th>$R_{ep}$ [km]</th>
<th>$R_{rup}$ [km]</th>
<th>Site Class</th>
<th>Comp.</th>
<th>$f_{LC}$ [Hz]</th>
<th>$f_{HC}$ [Hz]</th>
<th>$s_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Northridge (1994)</td>
<td>6.7</td>
<td>Jensen Filter Plant Generator</td>
<td>13.0</td>
<td>5.4</td>
<td>C</td>
<td>022</td>
<td>0.083</td>
<td>50</td>
<td>0.77</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>LA - Sepulveda VA Hospital</td>
<td>8.5</td>
<td>8.4</td>
<td>C</td>
<td>270</td>
<td>0.083</td>
<td>50</td>
<td>0.80</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>Pacoima Kagel Canyon</td>
<td>19.3</td>
<td>7.3</td>
<td>C</td>
<td>090</td>
<td>0.140</td>
<td>23</td>
<td>2.24</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td>Sylmar-Converter Sta East</td>
<td>13.6</td>
<td>5.2</td>
<td>C</td>
<td>018</td>
<td>-</td>
<td>-</td>
<td>0.55</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td>Northridge 6.7</td>
<td>288</td>
<td>288</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.53</td>
</tr>
<tr>
<td>6</td>
<td>Kobe (1995)</td>
<td>6.9</td>
<td>Nishi-Akashi</td>
<td>8.7</td>
<td>7.1</td>
<td>C</td>
<td>000</td>
<td>0.100</td>
<td>23</td>
<td>1.97</td>
</tr>
<tr>
<td>7</td>
<td>Loma Prieta (1989)</td>
<td>6.9</td>
<td>Corralitos</td>
<td>7.2</td>
<td>3.9</td>
<td>C</td>
<td>060</td>
<td>0.200</td>
<td>40</td>
<td>1.68</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td>Saratoga - W Valley Coll.</td>
<td>27.1</td>
<td>9.31</td>
<td>C</td>
<td>000</td>
<td>0.100</td>
<td>38</td>
<td>0.90</td>
</tr>
<tr>
<td>9</td>
<td>Cape Mendocino (1992)</td>
<td>7.0</td>
<td>Petrolia</td>
<td>4.51</td>
<td>8.18</td>
<td>C</td>
<td>000</td>
<td>0.070</td>
<td>23</td>
<td>1.57</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td>Kojima</td>
<td>270</td>
<td>270</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.74</td>
</tr>
</tbody>
</table>

Ground motions files were downloaded from the PEER-NGA database (http://peer.berkeley.edu/nga/)

Figure 5.11: The pseudo-spectral acceleration values predicted using the attenuation relationship by Campbell and Bozorgnia (2007), the actual table acceleration as well as the set of considered ground motion records
Table 5.7: Characteristic parameters of the distribution models of important parameters relevant to the test of column A1

<table>
<thead>
<tr>
<th>Normal distribution</th>
<th>Mean</th>
<th>COV</th>
<th>Correlation coeff., ρ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield strength, ( \sigma_y )</td>
<td>516</td>
<td>0.058</td>
<td>1</td>
</tr>
<tr>
<td>Ultimate strength, ( \sigma_u )</td>
<td>733</td>
<td>0.055</td>
<td>0.85</td>
</tr>
<tr>
<td>Ultimate tensile strain, ( \epsilon_{su} )</td>
<td>12</td>
<td>0.090</td>
<td>0.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lognormal distribution</th>
<th>Median</th>
<th>COV</th>
<th>λ</th>
<th>ξ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete strength, ( \sigma_{c0} )</td>
<td>37.0</td>
<td>0.141</td>
<td>3.61</td>
<td>0.140</td>
</tr>
<tr>
<td>Excitation scaling factor, ( s_2 )</td>
<td>1.19</td>
<td>0.535</td>
<td>0.178</td>
<td>0.502</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Uniform distribution</th>
<th>Mean</th>
<th>COV</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damping ratio, ( \zeta )</td>
<td>3</td>
<td>0.385</td>
<td>1</td>
<td>5 [%]</td>
</tr>
</tbody>
</table>

Material and damping properties

Grades of the materials that were used to build column A1 are identified based on the test report by Hachem et al. (2003). It is reported that the longitudinal and transverse reinforcing steels were grade 60 (ASTM A706) and 80 (ASTM A82), respectively. Likewise, the grade of the concrete is reported to be normal weight concrete with 28-day nominal strength equal to 28 MPa (4000psi).

The probabilistic models of the material parameters are established in accordance with the recommendations by JCSS (2001). The resulting distribution models are presented in Table 5.7. The damping ratio is assumed to be the same as that adopted for the previous example. Note that the axial load is considered as a certain parameter for column A1.

Generated input parameters

Sets of 500 random realizations of parameter values are generated following the same procedure as in previous example. Similarly, the remaining model parameters are calculated for each random realization in a similar way to the previous example.

Predicted response parameters

A zero-acceleration time frame of 5 s is considered at the end of each time-history analysis. Similar to the previous example, for each time-history simulation the residual displacement is identified as the average of the peak displacements attained during the last cycle of free vibration.

Randomly generated model parameters and the simulated response parameters are presented in Figure 5.12.
Figure 5.12: Histograms and scatter plots of the model input parameters and of the time-history analysis results for column A1
The prior probability $\Pr(M_i)$ for each maximum drift interval is estimated using Equation 4.4 (Figure 5.13a).

$$\begin{align*}
0 & \quad 0.5 & \quad 1 & \quad 1.5 & \quad 2 & \quad 2.5 & \quad 3 & \quad 3.5 & \quad 4 & \quad 4.5 & \quad 5 & \quad 5.5 & \quad 6 & \quad 6.5 & \quad 7 & \quad 7.5 & \quad 8 \\
\text{Probability} & \\
0 & \quad 0.1 & \quad 0.2 & \quad 0.3 & \quad 0.4 & \quad 0.5 & \quad 0.6 & \quad 0.7 & \quad 0.8 & \quad 1 \\
\end{align*}$$

(a)  

Figure 5.13: Probability distributions estimated for the maximum drift ratio experienced by the column A1

5.3.4 Step 3: Updating the distribution of the maximum drift ratio based on observable damage indicators

Photos of the damage to column A1 after the test are provided by Hachem et al. (2003). The visible damage to column A1 indicates that the yield displacement was exceeded and the ultimate displacement was not exceeded during the test (Figure 5.14). The probability distribution estimated for $MA$ is updated based on the inspected damage. Similar to the previous example, two damage indicators are considered: (1) the indicators related to exceedance of yielding and (2) the indicators related to exceedance of the ultimate deformation capacity.

In order to take into account the detection of spalling, the probability distribution of the drift limit associated with the onset of spalling needs to be established. The drift ratio at the onset of cover concrete spalling is estimated for the column using the simple expression proposed by Berry and Eberhard (2003). The spalling drift limit is calculated as 2.4% based on the axial load ratio ($NA/\sigma_0 = 4.7\%$) and the aspect ratio ($L/D = 6$) of the column. Note that while calculating the axial load ratio, the median values of the concrete strength probability distribution is used. The probable error related to the predicted drift limit is identified based on the error statistics by Berry and Eberhard (2003). They compare the predicted spalling drift limit values with those measured during the static cyclic tests of circular RC columns with spiral reinforcement. The predicted-to-measured ratios are reported to have a mean of 1.07 and a COV of 35%. The probability distribution of the drift $L_s$ at the onset of spalling is estimated based on the predicted drift value and the probable error associated with the predicted value.
The drift limit $L_u$ related to the detection of the ultimate deformation exceedance indicators are estimated based on the study by Fardis and Biskinis (2003) already used in the framework of the first example. In essence, the approach followed to predict this deformation limit is the same as in previous example. The distribution estimated for $L_u$ is presented in Figure 5.15.

In this sample application, the inspection $I$ represents the joint event defined as follows:

$$I = \{I_s \cap \overline{I_u}\} \quad (5.6)$$

where $I_s$ represents spalled cover concrete being detected and $\overline{I_u}$ represents indicators associated with exceedance of ultimate deformation not being detected. The probability $\Pr(M_i|I)$ of $M_i$ conditional on the inspection results is calculated using Equation 4.8a and is shown in Figure 5.13 (2nd series).

Figure 5.14: Visible damage indicators for column A1

Figure 5.15: Probability distributions estimated for the drift ratio of column A1 at the onset of spalling and the exceedance of ultimate deformation capacity
5.3.5 Step 4: Updating the maximum drift ratio distribution based on known residual displacements

The estimated maximum average drift distribution is updated based on the measured residual drift ratio. For this purpose, the joint probabilities $Pr(M_i \cap R_j | I)$ of $M_i$ and $R_j$ conditional on $I$ are calculated using Equations 4.13 and 4.15. The resulting joint probability distribution is presented in Figure 5.16. Based on this distribution, the mutual information $I(MA; RA)$ between the maximum and the residual average drift is found to be 0.23. Note that, this value is higher than that obtained for the previous example. It serves as an indicator of the worthiness of measuring the residual deformations.

The residual drift ratio of the column A1 is reported to be 0.55 % (Hachem et al., 2003). The probability $Pr(M_i | I \cap MR)$ of $M_i$ conditioned on the detected damage indicators and the known residual displacement are estimated by means of Equation (4.19a). The resulting probability distribution is presented in Figure 5.13 (3rd series) together with the distributions obtained in the previous steps. In the next section, the results obtained for the sample applications are discussed.
5.4 Evaluation of the results

5.4.1 Overview

The probabilities $\Pr(M_i |)$. obtained based on different sets of evidence differ noticeably from each other (Figures 5.4 and 5.13). The median, the mean and the COV of $MA$ were estimated by considering each set of probabilities. The resulting values are presented in Figure 5.17. The actual measured average drifts are also indicated in the Figure. The median estimates of $MA$ approach to the actual measured drift as more evidence is accounted for. Moreover, the COV (i.e. the dispersion) of the estimated $MA$ distribution decreases with the increasing evidence. This is an indication of the effectiveness of the method in updating the maximum displacement estimates.

![Figure 5.17](image)

Figure 5.17: The median, mean and coefficient of variation (COV) of the maximum drift $MA$ obtained using the probability distributions $\Pr(M_i |)$ for the example applications: wall WDH4 (a,b) and column A1 (c,d).

When the prior probabilities $\Pr(M_i)$ are considered, the median values of $MA$ underestimate the actual drift (Figure 5.17). This underestimation is equal to 62% and is the largest for the case of unit A1. The COVs obtained using the prior probabilities are 0.81 and 0.63 for the units WDH4 and A1, respectively. These COVs are larger
than the COVs assumed for the pseudo-spectral acceleration values considered in the Monte Carlo simulations (Tables 5.4 and 5.7). This difference is a result of the dynamic response and the characteristics of the structure, as well as the uncertain accuracy of the response prediction models.

The conditional probabilities $\Pr(M_i|I)$ are obtained by reflecting the visible damage inspection results to the predicted prior probabilities $\Pr(M_i)$. As expected, the probabilities estimated for drift ratios smaller than the predicted yield drift $L_y$ are significantly reduced after this updating (Figure 5.4a, 2nd series). Likewise, some reduction is observed in the probabilities estimated for the drift ratios greater than the predicted ultimate drift $L_u$ capacity. However, this reduction is smaller than the reduction observed for the range of smaller drift values. This is due to the fact that $L_u$ has a larger variability than $L_y$ (Figures 5.6 and 5.15). In general, the lower the variability of the predicted drift limit the higher is the effectiveness of considering the indicator in updating. To this purpose, note that the drift limits associated with indications of pounding or spalling of cover concrete can be predicted with a higher certainty compared to $L_u$.

For the second example dealing with column A1 similar observations can be made (Figure 5.13a). However, in this case the detection of spalled cover concrete is considered instead of indications of yielding. Once again, a significant reduction is obtained in the probabilities estimated for the drift values smaller than the drift $L_s$ at the onset of spalling of the cover concrete (Figures 5.4b and 5.13b, 2nd series).

Accounting for the visible damage inspection data results in significant improvements in the median $MA$ estimates (Figures 5.17, 2nd series). These estimates approach the actual measured maximum drift ratios. Moreover, COVs identified using the updated probabilities $\Pr(M_i|I)$ are lower than those identified using the prior distribution $\Pr(M_i|I)$. The reductions in the COVs of $MA$ for the wall WDH4 and for the column A1 are about 44% and 33%, respectively.

The second set of conditional probabilities $\Pr(M_i|I \cap MR)$ is obtained by taking into account the measured residual deformations (Figures 5.4 and 5.13, 3rd series). Both for wall WDH4 and for column A1, the median $MAs$ approach to the actual drift ratio (Figures 5.17, 3rd series). The resulting median $MA$ for WDH4 is, by practical means, equal to the actual measured drift. For the unit A1, even though the median $MA$ approaches to the actual value still it is 25% lower than the actual test result.

In all cases, the agreement of the predicted $MA$ with the measured drift increases with increasing amount of conditioning information (Figure 5.17). Moreover, the COVs of the estimated $MA$ reduce with the increasing of conditioning information. The COVs of $MA$ obtained using the sets of conditional probabilities $\Pr(M_i|I \cap MR)$ are approximately half of those obtained using the prior probabilities. Hence, both the accuracy and the precision of the estimated $MA$ increase with the increasing conditioning information.

The mutual information values are found to be in the range 0.15–0.23 for the two
structures. In both example applications, updating the estimates based on residual deformations had a noticeable impact on the estimated probabilities. Therefore, it can be concluded that if the mutual information is larger than 0.15, measuring the residual deformations is expected to lead to a noticeable updating of the estimated maximum drift ratio distribution. However, a series of additional cases should be investigated in order to derive solid recommendations on interpreting the mutual information values.

In brief, the results indicate that the method is effective in improving the estimates based on the additional information. Another important issue in this regard is the sensitivity of the estimates to the introduced assumptions. This sensitivity is discussed in the next section.

5.4.2 Sensitivity of the results

Measured residual drift ratio

The sensitivity of the probabilities $Pr(M_i|I \cap MR)$ to the known values of the residual drift $d^*_{r,a}$ is investigated. For this purpose, step 4 of the second example application is repeated by considering different values for $d^*_{r,a}$. The cumulative distributions estimated for the maximum drift ratio $MA$ are presented in Figure 5.18a. As expected, the median $MA$ increases with the increasing $d^*_{r,a}$. This sensitivity shows the importance of properly identifying the residual displacements. When $d^*_{r,a}$ is assumed to be 0.1%, the median $MA$ is estimated as 2.2%. On the other extreme, if $d^*_{r,a}$ is equal to 0.9% the median $MA$ is equal to 4.1%. Note that the dispersion of $MA$ increases with increasing measured residual drift $d^*_{r,a}$.

Often the exact residual drift ratio can not be identified. The reasons for this may be the unknown pre-earthquake configuration of the structure or the inaccuracy of the adopted measurement technique. The proposed method allows the impreciseness of the known residual drift to be taken into account. In order to demonstrate this, a uniform distribution is assumed for the actual residual drift ratio $d^*_{r,a}$. The lower and upper bounds of this distribution are 0.2 and 0.8%, respectively. Subsequently, the distribution of $MA$ is estimated using the probabilities $Pr(M_i|I \cap MR)$ obtained using Equation 4.19a. This distribution is plotted in Figure 5.18b (1st series). In addition, the distribution that is obtained for the case of residual drift being precisely known as 0.5% is also plotted in the same figure (2nd series). Note that this latter residual drift corresponds to the median of the assumed uniform distribution. It is seen that the difference between the two $MA$ distributions is negligible. Therefore, it may be concluded that the estimated maximum drift distribution is sensitive to the median value of the adopted residual drift distribution, but not so much to its dispersion.
Accuracy of the predicted response

It was shown that the method directly takes into account the likely accuracy of the simulated maximum and residual drift values (Equation 4.13). Here, the influence of the accuracy of the adopted modeling approach on the results is investigated. For this purpose, two hypothetical modeling approaches are assumed. The first hypothetical modeling approach is referred to as the Better Modeling (BM) approach. It is assumed that the residual drift ratios predicted using BM have a higher accuracy compared to the modeling approach DF which was adopted for the sample applications presented in the previous section. The performance of BM in predicting the maximum drift is assumed to be the same as DF. The second hypothetical modeling approach is referred as the Worse Modeling (WM) approach. This modeling approach is assumed to have the poorest performance in predicting the maximum and the residual drift values. The assumed distributions of the corrections factors for the modeling approaches BM and WM are presented in Figure 5.19.

The calculations for column A1 are repeated based on the two sets of assumed correction factor distributions. The resulting probability estimates are presented in Figure 5.19a. The dispersion of the distributions obtained for the BM is smaller compared to those obtained for WM and DF (Figure 5.20a). This difference is most significant for the case of the final distribution obtained after the measured residual drift is taken into account. On the other hand, the influence of taking into account the measured residual drift is less significant for modeling approach WM that has the poorest accuracy. Therefore, it is seen that the effectiveness of the method is directly proportional to the accuracy of the modeling approach. This holds in regard to the improvement of the estimates of the maximum drift based on the known residual drift.
Correction factor for the maximum average drift, $C_{a,m}$

Cumulative probability

Figure 5.19: Assumed probability distributions of the correction factors for the simulated maximum average drift $C_{a,m}$ (a) and for the simulated residual average drift $C_{a,r}$ (b) (DF: Displacement-based element fiber-section modeling, BM: better modeling approach and WM: Worse modeling approach)

Maximum average drift ratio, $d_{a,m}$ [%]

Probability

(a) Better modeling (BM)

(b) Worse modeling (WM)

Figure 5.20: Probability distributions of the maximum drift ratio identified based on two hypothetical modeling approaches: better modeling (a, BM) and worse modeling (b, WM)
Number of simulations

In order to represent the variability of the uncertain model parameters in the estimation of the prior probabilities, a large number of simulations may need to be carried out in Step 2 of the proposed method. Here, the sensitivity of the estimated maximum drifts to the total number of time-history simulations is investigated for example column A1. For this purpose, the procedure is repeated each time considering a larger set of simulations.

Initially, a small number of simulations are considered. A set of \( N_r = 10 \) input parameter sets are randomly selected from the whole set of generated 500 input parameter realizations. The maximum and residual drift values simulated for the \( N_g = 16 \) ground motions are taken into account for all the considered parameter sets. Based on these \( N_r \times N_g = 160 \) simulations, the drift ratios that correspond to the cumulative probabilities of exceedance of 16%, 50% and 84% are identified. These drift ratios are identified both for the established prior distribution and for the one conditional on the measured residual drift. As a result, six drift ratios are obtained for a single set of simulation results. This procedure is repeated by establishing a larger set of random parameter realizations —i.e. greater \( N_r \)— until the entire set of generated parameter values is taken into account.

The drift values estimated for the given exceedance probability levels are plotted against the total number \( N_r \) of considered parameter realization sets in Figure 5.21. It can be seen that the estimated median value and the dispersion is significantly sensitive to \( N_r \) for the range \( N_r < 50 \). Beyond that region, the median and 16 percentile estimates do not seem to be significantly affected by the number of realizations. However, the 84 percentile estimates are not very stable in the range \( N_r < 100 \). The sensitivities of the prior and the posterior estimates to \( N_r \) seem to have a similar trend. Note that the total number of Monte-Carlo simulations \( N \) is equal to \( N_r \) times \( N_g \).

Ground motion set

The prior joint distribution of the maximum and the residual drift ratios has a pivotal function in the last step of the procedure. Misleading results may be obtained using the proposed method if the prior distribution is significantly biased. A potential source for this bias is the utilization of an insufficient ground motion set for the analysis. If the ground motion experienced at the site is not known, a sufficiently large set of ground motion records should be considered in the analyses. The set should be large enough to cover the likely frequency content, phasing and duration properties of the experienced shaking. The ground motion characteristics that were found to influence the predicted maximum and residual displacements were presented in Section 2.4.1.

Record-to-record variability of the simulated maximum and residual drift values are presented by considering two ground motion records from the Northridge 1994 earth-
The total number of parameter sets, $N_r$, is 0.84.

Figure 5.21: Sensitivity of the maximum drift estimates to the number of parameter set realizations $N_r$.
using the JF record that was recorded just 1.6 km away. In this framework, an important observation concerns the spatial variability of ground motion properties and its effect on the residual versus maximum drift IDA curves. On the curves plotted in Figures 5.22c and 5.23c, consider the case when the simulated residual drift is 0.1%. Maximum drift ratios that correspond to this residual drift are 3% and 4.5% for the SC and JF records, respectively. Therefore, even ground motion records that are measured only a few kilometers away from the structure may not be representative of the actual shaking experienced at the site. In this regard, the best practice to adopt is to consider a set of ground motion records in the analysis. Thus, the uncertainty due to frequency and phasing of the experienced ground motion can be properly taken into account.

Figure 5.22: Maximum and residual drift ratios simulated using the ground motion *Sylmar - Converter Sta East 018* (SC)

Figure 5.23: Maximum and residual drift ratios simulated using the ground motion *Jensen Filter Plant Generator 022* (JF)

In order to assess the influence of the considered set of ground motions, the analyses related to example column A1 are repeated. In each repetition, a different set of ground motions is considered. The maximum drift estimates corresponding to cumulative prob-
abilities of exceedance of 16%, 50% and 86% are identified from the results. The drift ratios identified are plotted in Figure 5.24 for a series of alternative ground motion sets. For each set, the prior estimates are plotted above the posterior estimates.

The topmost pair of lines in Figure 5.24 (Table acc.) are the estimates obtained using the actual table acceleration measured during the Test Run 2 of column A1. In essence, this analysis represents the case when the actual ground motion that affected the site is known. The variability in this case is due to the uncertainty related to properties other than the excitation. In this case, the prior maximum drift estimates are predicting the experienced maximum drift (5.1%) with a reasonably good precision and accuracy. The posterior estimates are nearly the same with the prior ones. This may be an indication of the limited effectiveness of the method when the experienced ground motion is precisely known. However, this limitation is directly dependent on the accuracy of the adopted modeling approach.

The three pairs of estimates: “Site 1”, “Site 4” and “Site 6” are obtained considering the ground motions components recorded at these sites individually (Figure 5.9). The ID’s of these ground motions records in Table 5.5 are 7, 1 and 5, respectively. The three ground motion stations have the same site class as the considered structure. It is seen that if any of the three records is considered individually, highly biased prior distributions are obtained. In essence, the significant difference between prior estimates obtained using the ground motion records “Site 1” and “Site 4” is in agreement with the noticeable difference between the maximum drift ratio versus PSA plots presented for these two cases in Figures 5.22a and 5.23a.

The ground motion set referred as the “Northridge set” contains 8 components from the Northridge 1994 earthquake (Table 5.5). The remaining 8 ground motions from the same table form the “Non-Northridge set”. Lastly, the “Original set” includes all 16 records considered in the example application. The 14, 50 and 86 percentile estimates obtained using these three sets are not significantly different from each other. This independence indicates that if a sufficiently large set of records is considered in the analysis, the resulting estimates become independent from the individual ground motions that are selected to form the set. The results obtained here suggest that, for this example application using a number of ground motions smaller than 16 could also be sufficient to estimate these three percentile values. However, it is not possible to generalize this statement. The number of ground motions that are needed to represent the variability of the maximum and residual displacement values—which are likely to occur at specific PSA levels—is a complex problem that depends on several parameters. Further research is needed to identify widely applicable solutions to this problem.
Figure 5.24: Sensitivity of the maximum drift estimates to the considered ground motion record set

### 5.5 Conclusions

The application of the proposed method to two example structures is presented in this chapter. The considered example structures are scale-models that had been tested under dynamic loads. For the application, it is assumed that the set of basic information such as geometry, structural material grades as well as basic properties of the site and of the earthquake are available. Additionally, the ground motion intensities measured at neighboring sites are taken into account in the second example.

The results indicate that the method effectively improves the estimates of the maximum drift by taking into account the visible damage and the measured residual displacement. Comparison of the estimated maximum drift ratios with those measured during the test confirms the effectiveness of the method.

For the first example application, the estimated median maximum drift ratio is around 1.51%. This estimate is very close to the actual maximum drift measured during the test which is 1.52%. In this sample application, two damage indications that are related to the exceedance of the yielding and of the ultimate deformation limit, respectively, are considered.

The median maximum drift predicted for the second example structure is around
3.74%. The actual maximum drift measured during the test is 35% greater than this median estimate. Note that these estimates are obtained knowing neither the actual ground motion nor the exact material properties. Unlike the previous example, the PSAs measured at neighboring sites and the spalling of the cover concrete at the plastic hinging region are taken into account as additional information.

The accuracy of the adopted response modeling approach is shown to have a significant influence on the effectiveness of the proposed method. The results show that the updating of the maximum drift estimates is much more effective when both the adopted modeling approach yields accurate predictions of both the maximum and the residual drift.

It is found that the estimated maximum drift is not very sensitive to the considered set of ground motions given that a sufficiently large number of records is considered in the analysis. Results obtained for the example application showed that in this particular case 8 ground motions are sufficient.
6 Summary and Conclusions

6.1 Summary

The primary objective of this investigation was to assess the potential benefits of taking into account residual displacements in the post-earthquake assessment of structures. A post-earthquake seismic performance assessment method that takes into account residual displacements is developed. In the method the imperfection of the analysis methods in predicting the maximum and residual displacements of structures is explicitly accounted for. To this end, the accuracy of the state-of-the-art time-history analysis methods is investigated. Lastly, the performance of the post-earthquake assessment method is investigated for a set of damaged model RC structures.

In order to investigate the accuracy of the time history analysis results, a set of RC model structures tested on shaking tables is considered. Sets of alternative finite element models are established for each test unit by adopting different modeling strategies. Each modeling strategy is defined by a set of modeling guidelines. The maximum and residual response parameters (e.g. displacement, rotation) obtained from the time-history analyses are compared with the actual measured values. Additionally, the sensitivities of the simulated maximum and residual deformations to the modeling parameters are investigated.

The post-earthquake damage assessment method proposed here can be applied to obtain improved estimates of the damage to a structure. In particular, the maximum drift experienced during the earthquake by the damaged structure is estimated. For this purpose, the observable damage as well as post-earthquake residual displacements are taken into account. The maximum drift estimates are updated based on the damage information by means of Bayesian analysis. The resulting improved damage estimates are expected to lead to better informed decisions regarding the post-earthquake usability and/or reparability of the damaged structure.

The proposed method is applied to model RC structures tested on shaking tables. The structures are assumed to be typical damaged structures that are to be assessed
after an earthquake. The maximum drifts experienced during the tests are predicted using the proposed method. In the course of the application, first a priori estimation of the experienced maximum drift is made by means of dynamic time history analysis on a FE model of the structure. Uncertainty in the model parameters are taken into account through Monte Carlo simulation. Subsequently, based on the visible damage reported in the test reports, the estimates are improved. Finally, the measured residual displacements of the test units are considered to obtain further improved estimates of the experienced maximum displacements. The estimates are compared to the values measured during the test.

6.2 Conclusions

The investigation of the accuracy of the maximum and residual response parameters obtained for RC structures using time-history analysis leads to the following findings:

- The accuracy of the state-of-the-art finite-element modeling approaches in terms of estimating the residual displacements is found to be much lower compared to that identified for the peak displacements.

- The influence of the adopted hysteretic rule on the accuracy of the predicted displacements—both the maximum and the residual displacements—is found to be much stronger than that of the employed element formulation (i.e. displacement-based versus force-based formulation).

- The models that employ fiber-section models and modified Takeda hysteretic models are found to yield accurate estimates of the peak average drift ratio. The deformation estimates obtained using the bilinear hysteretic model have significantly lower accuracy compared to models that account for the degradation of stiffness—i.e. the fiber-section model and the modified Takeda hysteretic model.

- The residual average drift ratios predicted using fiber-section models tend to underestimate the actual values, whereas the opposite is true for the predictions obtained using the modified Takeda hysteretic model. The lowest accuracy is exhibited by the results of the bilinear hysteretic model which tends to highly overestimate the actual residual average drift.

- The accuracy of the peak ground story drift ratio estimates is found to be lower compared to that of the peak average drift ratio estimates. The simulated residual ground story drift values are found to be sensitive both to the adopted element formulation and to the hysteretic model.

- The peak rotation at the base segment was captured an accuracy that is comparable to that for the ground story drift ratio. On the other hand, the residual rotations
estimated using all models had a limited accuracy. It was seen that the predictions obtained using the modified Takeda model were better compared to others.

The modeling sensitivity studies related to the predicted maximum and residual displacements resulted in the following conclusions:

- The sensitivity of the predicted residual displacements to the modeling parameters and idealizations is found to be higher compared to the sensitivity of the maximum displacements.
- The intensity of the axial load acting on the structural members has a very large influence on the residual displacements predicted using fiber-section models. The peak displacements are not as sensitive to the vertical loading intensity as the residuals.
- The adopted discretization scheme had a larger affect on the predicted residual displacements compared to the maximum displacements.
- The residual displacements predicted using the force-based elements are found to be sensitive to the adopted meshing scheme.
- While modeling the structure with displacement-based elements, placing a single element at the estimated plastic hinge length is found to provide results that are in agreement with the equivalent plastic hinge length approach.
- A set of modeling recommendations is developed based on the findings presented in Chapter 3. The recommendations address a number of issues such as the discretizing RC structures by means of FEs and the estimation of the local deformations at arbitrary locations along structural components.

Related to the proposed post-earthquake damage assessment method, the following conclusions can be drawn:

- The proposed assessment method is formulated to allow direct consideration of the uncertainties related to: (1) the excitation, (2) the material properties and (3) the dynamic characteristics of the structure. Additionally, the probable errors associated with the adopted models are also directly taken into account.
- The method allows information from different sources (e.g. visible damage inspection, residual displacement measurement and free-field ground motions measured at neighboring sites) to be considered in order to improve the estimates of the maximum deformations experienced by the structure during the earthquake.

Results obtained from the application of the method to models of RC structures lead to the following findings:
The method can be used to obtain estimates of the maximum drift ratio based on the visible damage indicators and on the measured residual displacements.

The ground motions recorded at neighboring sites can be taken into account to improve the estimates of the intensity of the ground motion experienced at the site.

The effectiveness of the proposed method is directly proportional to the accuracy of the adopted structural response prediction approach.

Given that a sufficiently large set of ground motions are considered in the analysis, the maximum deformation estimates are relatively independent of the considered set of ground motions.

6.3 Outlook

The research presented in this report revealed a number of topics that should be further investigated. These topics are briefly summarized in the following:

The developed method can be adopted as a useful tool in the management of post-earthquake risks on a regional scale. In order to achieve this goal, a set of structural classes needs to be identified for the structures and the infrastructure in the region. Structural classes should comprise the structures which are expected to attain similar deformations when subjected to the same ground excitation. For each structural class, an analytical model relating the ground excitation to the structural deformations has to be developed. Once the structural classes are identified, the method can be applied to establish the prior distributions of the maximum and residual drift ratios. Updating of the estimated probability distributions based on the observed damage can be achieved using the relationships proposed by Rossetto and Elnashai (2003), which relate the structural response to the damage state. The estimated probability can be further improved by measuring the residual displacements of individual structures. The prior distributions may be identified for a set of probable earthquake scenarios. In the aftermath of an earthquake, the prior probabilities identified for the relevant seismic event can be quickly updated based on the collected post-earthquake reconnaissance data. As a result, the extent of damage to the region can be assessed in a consistent and rational way. The described tool may be implemented into other, existing, regional post-earthquake assessment tools (e.g. Wald et al., 2008).

A post-earthquake usability evaluation method may be developed based on the proposed method. For this purpose, a method should be adopted to estimate the risk due to continued use of a damaged structure. Three factors should be estimated to identify this risk: hazard, vulnerability and consequences. Note that both the risk due to seismic
hazard and to the gravity load hazard should be taken into account in the usability evaluation.

The accuracy of the structural response prediction models should be more comprehensively assessed. For this purpose, a larger set of shaking table test results that cover a wider range of possible response histories is needed. Results of such an investigation can reveal critical modeling assumptions which significantly influence the accuracy of the predicted response parameters. Likewise, the results would indicate the modeling approaches that are more likely to yield accurate response estimates. Moreover, the statistical reliability of the estimated correction factor distributions can be improved by increasing the number of shaking table tests considered in the analysis.

Response histories of instrumented buildings shaken by real earthquakes can be considered in the identification of the likely range of errors in the predicted maximum and residual response parameters. Particularly, buildings that are instrumented to measure relative deformations should be considered (Çelebi et al., 1999).

The processing of strong ground motions records is known to influence the response predicted for the nonlinear systems. The existing research on this issue focuses primarily on the evaluation of the sensitivity of the predicted maximum deformations to the adopted filtering scheme. On the other hand, the sensitivity of the residual displacements is yet to be investigated. Based on the results of such an investigation, guidelines may be developed for processing and selection of ground motion records.

The foundation flexibility and the soil-structure interaction influence the response. These effects on residual displacements should be evaluated. For this purpose, reliable analytical models that can simulate the response of soil-structure systems are needed. Results of such an investigation would be highly beneficial both for the post-earthquake damage assessment as well as for performance-based design applications.

Structural systems other than reinforced concrete should be considered in order to enhance the applicability of the developed method. Particularly, the investigation of the performance of the method when applied to steel moment resisting frames is a promising future research direction. These frames in general behave in a ductile manner and prone to residual deformations. Moreover, steel moment resisting frames are typically known to be costly structures. Hence, the decisions concerning their post-earthquake usability/repairability are critical financial decisions.
Bibliography


MLP (1996). Norme tecniche per il calcolo, l’esecuzione ed il collaudo delle strutture in cemento armato, normale e precompresso e per le strutture metalliche. 09/01/96. (Technical norms for the design, execution and test of reinforced concrete structures, normal and pre-stressed, and for steel structures. 09/01/96). Ministero dei Lavori Pubblici (Ministry of Public Works), Rome, Italy.


Newmark, N. M. and Hall, W. J. (1982). Earthquake spectra and design. engineering monographs on earthquake criteria, structural design, and strong motion records. Technical report, Earthquake Engineering Research Institute, University of California, Berkeley, California.


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SIA 162 (1993). Betonbauten (Concrete Structures), Norm, Schweizerischer Ingenieur – und Architekten – Verein, SIA (Swiss Society of Engineers and Architects). Zurich.


A Steel hysteresis models

A.1 Idealizations and the test results

The parameters of the steel hysteresis model were identified for each unit considered in the framework of this investigation based on actual material test results. The resulting stress-strain models and the relevant test results are presented in Figures A.1 and A.2. The model parameters are presented in Table 3.3.
Figure A.1: Measured reinforcement stress-strain relationships (gray lines) of test units A1-2, B1-2, EBII07 and WDH1-5 as well as corresponding bilinear idealizations (black lines)
Figure A.2: Measured reinforcement stress-strain relationships (gray lines) of test units WDH5-6 and CAMUS3 as well as corresponding bilinear idealizations (black lines)
B Axial load variation

B.1 Measured axial loads

The axial loads acting on the units were not constant during the shaking table testing of the reinforced concrete column EBII07, the walls WDH1 to WDH6 and CAMUS3. The causes for the variable axial loads are presented in Section 3.5.1. The maximum and the minimum axial load ratios identified based on the recorded data are presented in Table B.1. The axial load histories are plotted in Figures B.1 and B.2.

Table B.1: The maximum and the minimum axial load ratios

<table>
<thead>
<tr>
<th>Unit</th>
<th>$A_{c}f_{co}$ [kN]</th>
<th>max($n_{ax}$) [%]</th>
<th>min($n_{ax}$) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>5075</td>
<td>-5.7</td>
<td>-5.7</td>
</tr>
<tr>
<td>B1</td>
<td>5075</td>
<td>-5.7</td>
<td>-5.7</td>
</tr>
<tr>
<td>A2</td>
<td>5075</td>
<td>-5.7</td>
<td>-5.7</td>
</tr>
<tr>
<td>B2</td>
<td>5075</td>
<td>-5.7</td>
<td>-5.7</td>
</tr>
<tr>
<td>EBII07 - Test3</td>
<td>1260</td>
<td>-7.3</td>
<td>-11.3</td>
</tr>
<tr>
<td>WDH1 - Test3</td>
<td>3970</td>
<td>-2.0</td>
<td>-2.4</td>
</tr>
<tr>
<td>WDH2 - Test1</td>
<td>4040</td>
<td>-1.8</td>
<td>-3.8</td>
</tr>
<tr>
<td>WDH3 - Test1</td>
<td>3285</td>
<td>-1.2</td>
<td>-3.1</td>
</tr>
<tr>
<td>WDH4 - Test1</td>
<td>3267</td>
<td>-1.3</td>
<td>-2.8</td>
</tr>
<tr>
<td>WDH5 - Test1</td>
<td>3285</td>
<td>-1.5</td>
<td>-2.6</td>
</tr>
<tr>
<td>WDH6 - Test1</td>
<td>3834</td>
<td>-1.4</td>
<td>-2.4</td>
</tr>
<tr>
<td>CAMUS3 - MRr2</td>
<td>4039</td>
<td>1.2</td>
<td>-8.8</td>
</tr>
</tbody>
</table>

$n_{ax} = N/(A_{c}f_{co})$, Axial load ratio
Figure B.1: Variation of the average drift ratio (black series) and of the axial load ratio (gray series) throughout the test for test units EBII07 (a) and WDH1 (b)
Figure B.2: Variation of the average drift ratio (black series) and of the axial load ratio (gray series) throughout the test for test units WDH2-6 (a-e) and CAMUS3 (f)
C Moment curvature idealization

C.1 Moment curvature analysis results

In order to establish idealized flexural hysteresis models, moment curvature analyses are carried out. The bilinear idealizations are established based on the recommendations by Priestley et al. (1996). The results of the moment curvature analyses and of the relevant idealizations are plotted in Figures C.1 to C.5. The yield and the ultimate curvatures as well as the corresponding bending moments are summarized in Table 3.5.

Figure C.1: Moment curvature idealization established for the test units A1-2, B1-2 (a) and EBI07 (b)
Figure C.2: Moment curvature idealization established for the test units WDH1 (a) and WDH2 (b)

Figure C.3: Moment curvature idealization established for the test units WDH3 (a) and WDH4 (b)
Figure C.4: Moment curvature idealization established for the test units WDH5 (a) to WDH6 (b)

Figure C.5: Moment curvature idealization established for the test unit CAMUS3
D Dynamic properties and damping model parameters

D.1 Dynamic properties of the test units

The mode shapes and the periods of vibration were identified for the units by means of eigenvalue analysis (Chopra, 2006). The effective flexural stiffness values —based on moment curvature analysis— are utilized to establish the stiffness matrices. In the following, the estimated characteristic dynamic properties are presented for each test unit.

D.1.1 Columns A1, B1, A2 and B2

The columns A1 and B1 were subjected to a unidirectional excitation and have two dynamic degrees of freedom: (1) the translation and (2) the rotation of the mass. As a result, they have two modes of vibration. On the other hand the columns A2 and B2 featuring the same geometry as A1 and B1 but were subjected to a bidirectional excitation. As a result, A2 and B2 exhibit four modes of vibration. The results are presented in Table D.1.

Table D.1: Periods and mode shapes estimated for the columns: A1 and B1

<table>
<thead>
<tr>
<th>Mode 1</th>
<th>Mode 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period of vibration, $T_i$</td>
<td>0.779</td>
</tr>
<tr>
<td>Modal shapes</td>
<td></td>
</tr>
<tr>
<td>Translation along x-axis, $\psi_{1,1x}$</td>
<td>1</td>
</tr>
<tr>
<td>Rotation around y-axis, $\psi_{1,1ry}$</td>
<td>0.562</td>
</tr>
</tbody>
</table>

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D.1.2 Column EBII07

The column EBII07 has a single dynamic degree of freedom. The mass corresponding to this degree of freedom is that of the attached steel cart and is approximately equal to 12 t. The mass is free to move only in the longitudinal direction. The fundamental period of vibration is estimated to be around 0.958 s.

D.1.3 Walls WDH1 to 6

The walls WDH1 to 6 have three dynamic degrees of freedom (i.e. the horizontal translation of the story masses in the longitudinal direction of the wall). The masses corresponding to these degrees of freedom are the masses of the steel carts, the connecting steel elements and the walls’ own mass (Lestuzzi et al., 1999). The modal properties identified for these six walls are presented in Table D.2.

Table D.2: Periods and mode shapes estimated for the walls WDH1 to 6

<table>
<thead>
<tr>
<th>Wall</th>
<th>WDH1</th>
<th>WDH2</th>
<th>WDH3</th>
<th>WDH4</th>
<th>WDH5</th>
<th>WDH6</th>
</tr>
</thead>
<tbody>
<tr>
<td>First mode period, $T_1$</td>
<td>0.607</td>
<td>0.593</td>
<td>0.735</td>
<td>0.733</td>
<td>0.681</td>
<td>0.674 [s]</td>
</tr>
<tr>
<td>Second mode period, $T_2$</td>
<td>0.092</td>
<td>0.090</td>
<td>0.112</td>
<td>0.112</td>
<td>0.104</td>
<td>0.103 [s]</td>
</tr>
<tr>
<td>Third mode period, $T_3$</td>
<td>0.035</td>
<td>0.034</td>
<td>0.042</td>
<td>0.042</td>
<td>0.039</td>
<td>0.038 [s]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>First mode shape</th>
<th>$\psi_{1,1x}$</th>
<th>$\psi_{1,2x}$</th>
<th>$\psi_{1,3x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_{1,1x}$</td>
<td>0.141</td>
<td>0.519</td>
<td>1</td>
</tr>
<tr>
<td>$\psi_{1,2x}$</td>
<td>0.141</td>
<td>0.520</td>
<td>1</td>
</tr>
<tr>
<td>$\psi_{1,3x}$</td>
<td>0.140</td>
<td>0.518</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Second mode shape</th>
<th>$\psi_{1,1x}$</th>
<th>$\psi_{1,2x}$</th>
<th>$\psi_{1,3x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_{1,1x}$</td>
<td>0.762</td>
<td>1</td>
<td>-0.630</td>
</tr>
<tr>
<td>$\psi_{1,2x}$</td>
<td>0.763</td>
<td>1</td>
<td>-0.626</td>
</tr>
<tr>
<td>$\psi_{1,3x}$</td>
<td>0.753</td>
<td>1</td>
<td>-0.626</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Third mode shape</th>
<th>$\psi_{1,1x}$</th>
<th>$\psi_{1,2x}$</th>
<th>$\psi_{1,3x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_{1,1x}$</td>
<td>1</td>
<td>-0.640</td>
<td>0.193</td>
</tr>
<tr>
<td>$\psi_{1,2x}$</td>
<td>1</td>
<td>-0.641</td>
<td>0.193</td>
</tr>
<tr>
<td>$\psi_{1,3x}$</td>
<td>1</td>
<td>-0.634</td>
<td>0.191</td>
</tr>
</tbody>
</table>

D.1.4 Wall: CAMUS3

The CAMUS3 wall has five reduced dynamic degrees of freedom. These correspond to the horizontal translation of the masses located at each floor level. The modal properties identified for the five modes of this wall are presented in Table D.3.
Table D.3: Periods and mode shapes estimated for the CAMUS3 wall

<table>
<thead>
<tr>
<th>Period, $T_i$</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
<th>Mode 4</th>
<th>Mode 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_{i,1x}$</td>
<td>0.059</td>
<td>0.387</td>
<td>0.870</td>
<td>1</td>
<td>0.938</td>
</tr>
<tr>
<td>$\phi_{i,2x}$</td>
<td>0.220</td>
<td>0.963</td>
<td>1</td>
<td>-0.181</td>
<td>-1</td>
</tr>
<tr>
<td>$\phi_{i,3x}$</td>
<td>0.449</td>
<td>1</td>
<td>-0.456</td>
<td>-0.732</td>
<td>0.870</td>
</tr>
<tr>
<td>$\phi_{i,4x}$</td>
<td>0.717</td>
<td>0.274</td>
<td>-0.960</td>
<td>0.857</td>
<td>-0.510</td>
</tr>
<tr>
<td>$\phi_{i,5x}$</td>
<td>1</td>
<td>-0.934</td>
<td>0.659</td>
<td>-0.323</td>
<td>0.148</td>
</tr>
</tbody>
</table>

D.2 Identification of the damping model parameters

D.2.1 Columns A1, B1, A2 and B2

A representative damping ratio could be identified for the test units based on their response during the free vibration phase after the forced seismic excitation. Two exponentially decaying displacement envelope functions are fitted to the displacement trace recorded during the shaking table test. The decaying functions are defined as follows:

$$
\begin{align*}
    u_1(t) &= u_o + \rho_1 e^{-\zeta \omega_n t} \\
    u_2(t) &= u_o + \rho_2 e^{-\zeta \omega_n t}
\end{align*}
$$

where $u_1(t)$ and $u_2(t)$ are the upper and lower displacement envelope boundaries, $u_o$ is the identified residual displacement around which the free vibration takes place, $\rho_1$ and $\rho_2$ are envelope factors for the upper and lower boundaries, $\zeta$ is the damping ratio, $\omega_n$ is the angular frequency of vibration and $t$ is the time from the beginning of the free vibration phase.

It should be noted that $\omega_n$ in Equation D.1 is the angular frequency of the system during the free vibration phase at the end of the response. The frequency $\omega_n$ is estimated based on the identified peaks of the vibration cycles in the free vibration phase. For the systems that have deformed beyond yielding, the stiffness decreases and the period of vibration elongates. Therefore, $\omega_n$ used in Equation D.1 is typically smaller than the value estimated using the effective stiffness of the test unit before the test. The nonlinear minimization function available in MATLAB is used to identify the optimal set of parameter values.

The response envelopes are presented in Figure D.1. The coefficients of mass matrix $a_M$ and the stiffness matrix $a_K$ are estimated based on the damping ratio $\zeta_1$ assumed for the first and second modes of vibration featuring the period $T_1$ and $T_2$. The values of the parameters identified for the Rayleigh’s damping model are listed in Table D.4. It should be noted that in the time-history analysis carried out as a part of this study the
last-committed (i.e. tangent) stiffness matrix is used to establish the damping matrix.

Table D.4: Damping model parameters

<table>
<thead>
<tr>
<th>Test unit</th>
<th>$\zeta_1$</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$a_M$</th>
<th>$a_K\times 10^{-4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1.96</td>
<td>0.779</td>
<td>0.114</td>
<td>0.2759</td>
<td>6.185</td>
</tr>
<tr>
<td>B1</td>
<td>2.40</td>
<td>0.779</td>
<td>0.114</td>
<td>0.3373</td>
<td>7.562</td>
</tr>
<tr>
<td>A2</td>
<td>3.33</td>
<td>0.779</td>
<td>0.114</td>
<td>0.4691</td>
<td>10.516</td>
</tr>
<tr>
<td>B2</td>
<td>2.29</td>
<td>0.779</td>
<td>0.114</td>
<td>0.3223</td>
<td>7.227</td>
</tr>
<tr>
<td>EBII07</td>
<td>2.00</td>
<td>0.958</td>
<td>0.090</td>
<td>0.2398</td>
<td>5.238</td>
</tr>
<tr>
<td>WDH1</td>
<td>4.50</td>
<td>0.607</td>
<td>0.092</td>
<td>0.8090</td>
<td>11.491</td>
</tr>
<tr>
<td>WDH2</td>
<td>4.50</td>
<td>0.593</td>
<td>0.090</td>
<td>0.8269</td>
<td>11.242</td>
</tr>
<tr>
<td>WDH3</td>
<td>4.50</td>
<td>0.735</td>
<td>0.112</td>
<td>0.6681</td>
<td>13.907</td>
</tr>
<tr>
<td>WDH4</td>
<td>4.50</td>
<td>0.733</td>
<td>0.113</td>
<td>0.6694</td>
<td>13.881</td>
</tr>
<tr>
<td>WDH5</td>
<td>4.50</td>
<td>0.681</td>
<td>0.104</td>
<td>0.7205</td>
<td>12.910</td>
</tr>
<tr>
<td>WDH6</td>
<td>4.50</td>
<td>0.674</td>
<td>0.103</td>
<td>0.7276</td>
<td>12.786</td>
</tr>
<tr>
<td>CAMUS3</td>
<td>3.34</td>
<td>0.197</td>
<td>0.031</td>
<td>1.8405</td>
<td>2.855</td>
</tr>
</tbody>
</table>

*: Assumed to establish a suitable model

D.2.2 Column EBII07

The shaking table equipment used in the tests of EBII07 involves a steel cart to simulate the mass. The cart can freely move on rails in the horizontal direction. The friction forces generated during the movement of the cart dissipate energy during shaking. This type of damping is referred to as Coulomb friction damping (Chopra, 2006). In this type of damping, the damping force is only dependent on the magnitude of the Coulomb friction force and the direction of motion. Unlike the viscous damping approach, the magnitude of the damping force does not depend on the relative velocity of the system. However, defining an equivalent viscous damping to represent the energy lost due to friction is a favorable approach. It allows simplifying the model noticeably. The equivalent viscous damping should have an effect on the response similar to that due to Coulomb friction damping.

The friction force can be estimated based on the acceleration measured on the cart and the axial force measured at the link between the test unit and the cart. The difference between the estimated inertial force acting on the cart and the force in the link can be used as an estimate of the friction force. In Figure D.3a the estimated friction force versus the absolute displacement values are plotted for the first cart which was also utilized in the tests for EBII07. It is seen that the friction force associated with the first cart is approximately 0.6 kN.

In order to identify the critical damping ratio for the equivalent viscous damping
Figure D.1: Displacement response envelopes fitted to the free-vibration phases for the columns A1 (a), B1 (b), A2 (c) and B2 (d)
system, the response simulated for the Coulomb friction damped system is compared to that for the viscously damped system. The ratio of critical damping which results in the closest match between the two simulated displacement time-histories is identified to be the optimal damping ratio. For the unit EBII07, an equivalent damping ratio of 0.55% is found to provide the closest match. The responses simulated using the two alternative damping models are presented in Figure D.3b. It should be noted that the part of damping that is estimated from these calculations does not include the contribution of the damping associated with the response of the reinforced concrete test unit.

A base value of 2% damping was recommended as a suitable value for RC structures that respond within elastic limits (Gülkan and Sözen, 1974). Clough and Gidwani (1976) subjected a two story RC frame model to a series of shaking table tests. For the first test a damping of 1.55% was identified. As the structure attained damage the damping at the fundamental mode increased and reached 6.56% in the last test. Cantieni et al. (1986) reports the results of a series damping identification tests performed on 213 RC highway bridges deforming within elastic limits. The damping values they identified were distributed in the range 1.9 to 36% and the median was equal to 8.2%. Based on these observations, the value of viscous damping is assumed to be equal to 2% at the fundamental mode of vibration for the EBII07 specimen. As a result a total viscous damping ratio of 2% is assumed in the simulation of the response of the test unit EBII07.

**D.2.3 Walls WDH1 to 6**

The testing scheme used in the tests by Lestuzzi et al. (1999) incorporated three steel carts to simulate the inertia of story masses. The carts had a mass of 12 t each and
could freely move horizontally on rails. Similar to the case for EBII07, the friction forces and the movement of the masses on the rails dissipate some energy. However unlike EBII07, the system has three dynamic degrees of freedom. In order to identify the amount of damping due to friction an equivalent SDOF system is defined. Properties of the equivalent system are based on the estimated ratio of two energies: (1) the energy that is lost due to friction in a single cycle of vibration within elastic limits, $E_F$ and (2) the energy that is needed to deform the system from its initial position to that level of vibration, $E_S$. As a reference point, the energies corresponding to the yield displacement are estimated. The ratio, $E_{F,y}/E_{S,y}$ is found to be in the range from 0.2 to 0.3 for all test units (Table D.5). Consequently, the level of friction force that results in the same $E_F/E_S$ ratio is identified for the equivalent SDOF system. After the properties of the equivalent SDOF system with Coulomb damping have been identified, the procedure that is presented in the previous section for test unit EBII07 is followed and the damping ratio of the viscously damped SDOF system is estimated. The values of the parameters are presented in Table D.5.

### D.2.4 Wall CAMUS3

The CAMUS3 test unit was subjected to a low level ($\leq 0.05g$) white noise test after each main seismic test (Combescure and Chaudat, 2000). Identified periods of vibration and damping ratios are plotted in Figure D.5a. The periods of vibration and the damping ratios that are measured before and after the test “Melendy r2 (MR2)” are considered in the estimation of damping model parameters. The plot of the model that is found to provide a suitable fit is presented in Figure D.5b.
Table D.5: Parameters related to damping of the walls WDH1 to 6

<table>
<thead>
<tr>
<th></th>
<th>WDH1</th>
<th>WDH2</th>
<th>WDH3</th>
<th>WDH4</th>
<th>WDH5</th>
<th>WDH6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strain energy required to yield, $E_{S,y}$</td>
<td>319</td>
<td>355</td>
<td>239</td>
<td>224</td>
<td>339</td>
<td>347</td>
</tr>
<tr>
<td>Energy lost due to friction, $E_{F,y}$</td>
<td>69</td>
<td>75</td>
<td>71</td>
<td>67</td>
<td>74</td>
<td>75</td>
</tr>
<tr>
<td>Energy ratio, $E_{F,y}/E_{S,y}$</td>
<td>21.5</td>
<td>21.1</td>
<td>29.8</td>
<td>30.0</td>
<td>21.8</td>
<td>21.6</td>
</tr>
<tr>
<td>Norm. friction displacement, $d_f/d_y$</td>
<td>2.69</td>
<td>2.64</td>
<td>3.73</td>
<td>3.75</td>
<td>2.73</td>
<td>2.70</td>
</tr>
<tr>
<td>Viscous damping representing the friction, $\zeta_F$</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>Viscous damping due to other sources, $\zeta_O$</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Total viscous damping, $\zeta_n$</td>
<td>4.5</td>
<td>4.5</td>
<td>4.5</td>
<td>4.5</td>
<td>4.5</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Figure D.4: Friction forces associated with the second (a) and third (b) carts of walls WDH2 to 6

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Figure D.5: Identified dynamic properties (a) and the adopted Rayleigh’s model (b) for test unit CAMUS3
E Simulated response

The simulated response parameters and the errors are presented in this section. The considered response parameters are maximum average drift $d_{a,m}$, residual average drift $d_{a,r}$, maximum rotation $\Theta_m$, residual rotation $\Theta_r$, maximum ground story drift $d_{gs,m}$, residual ground story drift $d_{gs,r}$ and relative horizontal acceleration $a_{t,rel}$. The error for each prediction is calculated as follows:

$$Err(x) = \frac{x_{exp}}{x_{sim}}$$  \hspace{1cm} (E.1)

In the equation above $x_{sim}$ is the simulated value of the response parameter and $x_{exp}$ is the actual value measured during the shaking table experiment.

Table E.1: Displacement-based elements with bilinear hysteresis DB

<table>
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<th>Unit</th>
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<th>B1</th>
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205
Figure E.1: Experiment-to-simulation ratios for the maximum (a) and the residual (b) average drift ratios predicted using displacement-based elements with bilinear hysteresis DB

Table E.2: Displacement-based elements with modified Takeda hysteresis DT

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Table E.3: Displacement-based fiber-section element model DF

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Figure E.2: Experiment-to-simulation ratios for the maximum (a) and the residual (b) average drift ratios predicted using displacement-based elements with modified Takeda hysteresis DT.
Figure E.3: Experiment-to-simulation ratios for the maximum (a) and the residual (b) average drift ratios predicted using displacement-based fiber-section elements DF

Table E.4: Force-based elements with bilinear hysteresis FB

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<td>0.251</td>
<td>-0.351</td>
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<td>1.503</td>
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<td>0.982</td>
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Figure E.4: Experiment-to-simulation ratios for the maximum (a) and the residual (b) average drift ratios predicted using force-based elements with bilinear hysteresis FB

### Table E.5: Force-based elements with modified Takeda hysteresis FT

<table>
<thead>
<tr>
<th>Unit</th>
<th>A1</th>
<th>B1</th>
<th>A2</th>
<th>B2</th>
<th>EBII07</th>
<th>WDH1</th>
<th>WDH2</th>
<th>WDH3</th>
<th>WDH4</th>
<th>WDH5</th>
<th>WDH6</th>
<th>CAMUS3</th>
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<tbody>
<tr>
<td>$d_{a,m}$</td>
<td>3.092</td>
<td>4.304</td>
<td>4.956</td>
<td>3.064</td>
<td>-9.002</td>
<td>0.668</td>
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<td>-1.905</td>
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<td>-1.451</td>
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<td>0.965</td>
<td>0.965</td>
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<td>0.833</td>
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<td>1.858</td>
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<td>0.025</td>
<td>-0.131</td>
<td>-0.149</td>
<td>-0.114</td>
<td>-0.088</td>
<td>-0.084</td>
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<td>0.052</td>
<td>0.175</td>
<td>0.056</td>
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<td>1.876</td>
<td>1.594</td>
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[209]
Figure E.5: Experiment-to-simulation ratios for the maximum (a) and the residual (b) average drift ratios predicted using force-based elements with modified Takeda hysteresis FT

<table>
<thead>
<tr>
<th>Unit</th>
<th>A1</th>
<th>B1</th>
<th>A2</th>
<th>B2</th>
<th>EHI07</th>
<th>WDH1</th>
<th>WDH2</th>
<th>WDH3</th>
<th>WDH4</th>
<th>WDH5</th>
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<tbody>
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<td>5.499</td>
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<td>4.923</td>
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<td>-1.255</td>
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Figure E.6: Experiment-to-simulation ratios for the maximum (a) and the residual (b) average drift ratios predicted using force-based fiber-section elements FF
Simulated displacement histories

Simulated response histories are presented in this section. The response histories for test units EBII07, B1, B2 and A2 are included. The responses simulated using the DT and DF models are compared with the actual measured response. The deformation cycles along during which the predictions diverge are marked in each time history plot. In the drift versus base moment plots the response exhibited along these non-yielding cycles are plotted with bold lines. It is seen that often DT model reloads in the positive direction with an average stiffness lower than the other two response histories (i.e. DF model and experiment). This smaller stiffness results in large residual displacements. A detailed discussion of this issue is provided in Section 3.7.

Figure F.1: Response history of test unit EBII07: drift versus time (a) and drift versus base moment (b)
Figure F.2: Response history of test unit B1: drift versus time (a) and drift versus base moment (b)

Figure F.3: Response history of test unit B2: drift versus time (a) and drift versus base moment (b)
Figure F.4: Response history of test unit A2: drift versus time (a) and drift versus base moment (b)
G Simulated static cyclic response

The influence of the axial load on the simulated monotonic and cyclic response histories are presented in this section. The sensitivity of the simulated maximum and residual displacements to the assumed axial load is discussed in Section 3.8.1. The response simulated under diminishing deformation cycles show that the small cycle stiffness is influenced by the axial load. The stiffness exhibited during the non-yielding deformation cycle increases with increasing axial load.

Figure G.1: Predicted base shear versus drift ratio response under different axial load intensities - Test unit: A1, Model: DF
Figure G.2: Predicted base shear versus drift ratio response under different axial load intensities - Model: DF
Figure G.3: Predicted base shear versus drift ratio response under different axial load intensities - Model: DF
Figure G.4: Predicted base shear versus drift ratio response under different axial load intensities - Model: DF
The approach presented by Park et al. (2007) is adopted here to update the pseudo-spectral acceleration (PSA) estimated for a site based on the PSAs measured at nearby stations. Their approach is based on the assumption that for two closely spaced sites and for a given earthquake, the differences between the actual PSA and the median value estimated using a ground motion prediction model (GMPM) are correlated. In this approach, the difference between the PSA felt at the site and the median PSA predicted using the GMPM is referred to as the error. This error is considered to have two components: (1) inter-event component and (2) intra-event component. The inter-event error represents the error in the prediction that is related to particular characteristics of a specific seismic event that is not captured by the GMPM. The intra-event (site-to-site) error represents the random error associated with the predictions made for different sites that are shaken by the same earthquake. In accordance with these definitions, the following relationship is established:

$$\ln(SA_{i,j}) = \ln(sa_{i,j}) + \tau \eta_i + \sigma_s \varepsilon_{i,j}$$ \hspace{1cm} (H.1)

In the equation above, $SA_{i,j}$ is the PSA that is experienced during the earthquake $i$ at the site $j$, $sa_{i,j}$ is the median PSA predicted using the GMPM, $\tau$ and $\sigma_s$ are the standard deviations of the inter- and intra-event error terms, $\eta_i$ and $\varepsilon_{i,j}$ are the standard normal random variables—with mean equal to 0 and standard deviation equal to 1. Alternatively, if a single error term is considered, the following relationship is obtained:

$$\ln(SA_{i,j}) = \ln(sa_{i,j}) + \sigma_T \tilde{\varepsilon}_{i,j}$$ \hspace{1cm} (H.2)

In the equation above, $\sigma_T$ is the standard deviation of the total error —i.e. it represents both the inter- and the intra-event variability— and $\tilde{\varepsilon}_{i,j}$ is the standard normal random variable associated with the total error. By substituting Equation H.1 into H.2, the following relationship can be established for the total error:

$$\tilde{\varepsilon}_{i,j} = \frac{\tau \eta_i + \sigma_s \varepsilon_{i,j}}{\sigma_T}$$ \hspace{1cm} (H.3)

For a pair of closely spaced sites that are located at a distance to the rupture plane, it is reasonable to expect that the $\tilde{\varepsilon}_{i,j}$'s are correlated. In other words, the differences
between the experienced PSA and the predicted median value are positively correlated. If the GMPM results in an underestimation of the PSA at a given site, it is reasonable to expect that it underestimates the PSA at another site nearby. Typically, it is assumed that the correlation between the \( \hat{\varepsilon}_{i,j} \) values estimated for two sites \( a \) and \( b \) depends on the separation distance \( \Delta_{a,b} \) between the sites and the period \( T \) of interest. Based on this fact, the correlation \( \rho_{\hat{\varepsilon}_{i,a},\hat{\varepsilon}_{i,b}}(\Delta_{a,b},T) \) of the \( \hat{\varepsilon} \)'s for the two sites \( a \) and \( b \) that are affected by earthquake \( i \) is:

\[
\rho_{\hat{\varepsilon}_{i,a},\hat{\varepsilon}_{i,b}}(\Delta_{a,b},T) = \frac{\tau^2 + \sigma^2 \rho_{\varepsilon_{i,a},\varepsilon_{i,b}}(\Delta_{a,b},T)}{\sigma^2_T} \tag{H.4}
\]

Note that, \( \eta_i \) is the same for all the sites that are effected by the same earthquake \( i \) —i.e. it is perfectly correlated for all sites.

Several models have been proposed to estimate the correlation coefficient \( \rho_{\varepsilon_{i,a},\varepsilon_{i,b}} \) (e.g. Boore et al., 2003; Wang and Takada, 2005; Goda and Hong, 2008). The model proposed by Goda and Hong (2008) has the following form:

\[
\rho_{\varepsilon_{i,a},\varepsilon_{i,b}}(\Delta_{a,b},T) = \exp \left[ - \left( c_1 \ln(T) + c_2 \right) \Delta_{a,b}^c \right] \tag{H.5}
\]

For the equation above, the coefficients \( c_1 \), \( c_2 \) and \( c_3 \) are estimated to be equal to -0.16, 0.62 and 0.5, respectively, based on regression analysis on ground motion records from California. Note that this model is proposed for \( T \) smaller than 3 s and that \( \Delta_{a,b} \) in Equation H.5 is in km. Correlation coefficients resulting from this model are presented in Figure H.1. The correlation decreases more sharply for shorter periods. Goda and Hong (2008) note that the identified correlations do not differ significantly depending on the GMPM adopted in the regression analysis.

If the PSAs are known for a number of neighboring sites, the probability distribution of \( \hat{\varepsilon}_{i,a} \) conditional on all the known PSAs can be established based on the correlation \( \rho_{\varepsilon_{i,a},\varepsilon_{i,b}}(\Delta_{a,b},T) \). For this purpose, the following steps should be followed:

1. The correlations coefficients \( \rho_{\hat{\varepsilon}_{i,x},\hat{\varepsilon}_{i,y}}(\Delta_{x,y},T) \) are estimated for all the site pairs using Equation H.4.

2. The covariance matrix \( \mathbf{C} \) for the joint distribution of the \( \hat{\varepsilon} \)s for the considered sites is established. The matrix \( \mathbf{C} \) consists of 4 components defined as follows:

\[
\mathbf{C} = \begin{bmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{C}_{21} & \mathbf{C}_{22} \end{bmatrix}
\]

where \( \mathbf{C}_{11} = \begin{bmatrix} 1 & \hat{\rho}_{1,2} & \ldots & \hat{\rho}_{1,N_s} \\ \hat{\rho}_{1,2} & 1 & \ldots & \hat{\rho}_{2,N_s} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\rho}_{1,N_s} & \hat{\rho}_{2,N_s} & \ldots & 1 \end{bmatrix} \)

\[
\mathbf{C}_{12} = \mathbf{C}_{21}^T = \{ \hat{\rho}_{1,a} \ \hat{\rho}_{2,a} \ \ldots \ \hat{\rho}_{N_s,a} \}^T
\]

\[
\mathbf{C}_{22} = 1
\]
In Equation H.6, the notation $\hat{\rho}_{x,y}$ refers to the correlation coefficient $\rho_{\tilde{\varepsilon}_{i,x},\tilde{\varepsilon}_{i,y}}(\Delta_{x,y}, T)$. Note that, $\tilde{\varepsilon}$s are standard normal variables with means equal to 0 and standard deviations equal to 1.

3. Observed values $\tilde{\varepsilon}_{i,x}^*$ are identified for the sites where the PSA is known. These fixed values are calculated as:

$$\tilde{\varepsilon}_{i,x}^* = \frac{\ln sa_{1,x}^* - \ln sa_{i,x}}{\sigma_T}$$

where $\tilde{\varepsilon}_{i,x}$ is the fixed value of the random variable $\tilde{\varepsilon}_{i,x}$, $sa_{1,x}^*$ and $sa_{i,x}$ are the known value of the PSA and the median value obtained for site $x$ using the GMPM, respectively. Subsequently, the vector $\mathbf{E}$ is established as follows:

$$\mathbf{E} = \{ \tilde{\varepsilon}_{i,1}^*, \tilde{\varepsilon}_{i,2}^*, \ldots, \tilde{\varepsilon}_{i,N_s}^* \}^T$$

4. The distribution of $\tilde{\varepsilon}_{i,a}$ conditional on $\mathbf{E}$ is established. Let $\tilde{\varepsilon}'$ represent a random variable with this conditional distribution. The mean $\mu_{\tilde{\varepsilon}'}$ of this random variable is estimated as:

$$\mu_{\tilde{\varepsilon}'} = \mu_{\tilde{\varepsilon}_{i,a}|\mathbf{E}} = \mathbf{C}_{21} \mathbf{C}_{11}^{-1} \mathbf{E}$$

Similarly, the standard deviation $\sigma_{\tilde{\varepsilon}'}$ of $\tilde{\varepsilon}'$ is calculated as:

$$\sigma_{\tilde{\varepsilon}'} = \sigma_{\tilde{\varepsilon}_{i,a}|\mathbf{E}} = (\mathbf{C}_{22} - \mathbf{C}_{21} \mathbf{C}_{11}^{-1} \mathbf{C}_{12})^{1/2}$$

Figure H.1: Spatial correlation model for $\varepsilon_{i,a}$ and $\varepsilon_{i,b}$ according to Goda and Hong (2008)
5. Finally, the posterior distribution of the PSA estimated for site $a$ is established based on the distribution of $\tilde{\varepsilon}'$. Let $SA'$ represent the PSA estimated for site $a$ conditional on the known PSAs. The variables $SA'$ and $\tilde{\varepsilon}'$ are related as follows:

$$\ln(SA') = \ln(sa_{i,a}) + \sigma_T \tilde{\varepsilon}'$$  \hspace{1cm} (H.11)

In Equation H.11, $sa_{i,a}$ is the median PSA estimated for site $a$ using the GMPM. In accordance with Equation H.11, the median value $sa'$ of $SA'$ is estimated as:

$$\text{Median}(SA') = \exp \left[ \ln(sa_{i,a}) + \sigma_T \mu_{\tilde{\varepsilon}'} \right]$$  \hspace{1cm} (H.12)

Similarly, the logarithmic standard deviation of $SA'$ is calculated as:

$$\sigma_{\ln(SA')} = \sigma_T \sigma_{\tilde{\varepsilon}'}$$  \hspace{1cm} (H.13)

An example application of this method is presented in Section 5.3.3. The application aims to predict the PSA experienced at a particular site during the 1994 Northridge $M_w$ 6.7 earthquake.
 Updating procedure for closely spaced and correlated deformation limits

The procedure presented here can be used to estimate the likelihood of a damage indicator not being detected conditional on the experienced maximum deformation and on the detection of another damage indicator. The method is particularly suited when the two indicators are sequential —i.e. the second one can only be detected after the first has been detected—— and the random deformation limits (DLs) associated with them are statistically correlated.

Suppose that $I_1$ represents the detection of a damage indicator associated with exceedance of an uncertain DL $L_1$. Moreover, let $\overline{I_2}$ represent the damage indicator associated with an uncertain DL $L_2$, not being detected. Finally, let $M_i$ represent the case when the maximum deformation equals the central value $m_{c_i}$ of the drift interval between $ma_i$ and $ma_{i+1}$, $m_{c_i} = (ma_i + ma_{i+1})/2$. Note that events $I_1$ and $I_2$ can be defined as follows:

$$I_1 = \{ L_1 < MA \} \quad \text{(I.1a)}$$

$$\overline{I_2} = \{ L_2 > MA \} \quad \text{(I.1b)}$$

If the conditional distribution $f_{L_2|I_1}(\cdot)$ is available, the probability $Pr(\overline{I_2}|M_i \cap I_1)$ can be directly obtained as follows:

$$Pr(\overline{I_2}|M_i \cap I_1) = 1 - \int_0^{m_{c_i}} f_{L_2|I_1}(x) \, dx \quad \text{(I.2)}$$

However, often only the marginal distributions $f_{L_1}(\cdot)$ and $f_{L_2}(\cdot)$ can be obtained using DL prediction models. In this case, two issues should be taken into account while estimating the probability $Pr(\overline{I_2}|M_i \cap I_1)$: (1) the probable correlation $\rho_{L_1,L_2}$ between the uncertain DLs $L_1$ and $L_2$ and (2) the condition of $L_2 \geq L_1$ for all possible $L_1$-$L_2$ pairs. In order to estimate the correlation $\rho_{L_1,L_2}$, the major uncertain parameters that contribute to the dispersions of $L_1$ and $L_2$ can be identified. If these parameters are
common for the two DLs, the correlation $\rho_{L1,L2}$ is expected to be high. The effect of the assumed correlation value on the estimated probabilities is presented in the following.

The joint distribution $f_{L1,L2}(.,.)$ can be established after the correlation $\rho_{L1,L2}$ is estimated. Note that this approximate joint distribution does not necessarily need to obey the condition $L2 \geq L1$. Based on the joint distribution $f_{L1,L2}(.,.)$, the probability $\Pr(I2 | Mi \cap I1)$ can be calculated as follows:

$$\Pr(I2 | Mi \cap I1) = \frac{\Pr(I2 \cap I1 | Mi)}{\Pr(I1 | Mi)}$$  \hspace{1cm} (I.3a)

$$= \frac{\Pr(L2 > mc_i \cap L1 < mc_i)}{\Pr(L1 < mc_i) - \Pr(L1 > L2 | Mi)}$$  \hspace{1cm} (I.3b)

$$= \frac{\int_{A1} f_{L1,L2}(x,y) \, dx \, dy}{\int_{0}^{mc_i} f_{L1}(x) \, dx - \int_{A2} f_{L1,L2}(x,y) \, dx \, dy}$$  \hspace{1cm} (I.3c)

where $A1 := \{0 < x < mc_i \cap mc_i < y < \infty\}$

$A2 := \{0 < x < mc_i \cap 0 < y < x\}$

In the equations above, $Mi$ represents the experienced maximum deformation being equal to $mc_i$. An important probability that should be noted is the probability $\Pr(L1 < L2 | Mi)$ which corresponds to the probability of the improbable set of $L1$ and $L2$ pairs conditional on the $mc_i$. The volumes corresponding to the conditional probabilities $\Pr(I2 \cap I1 | Mi)$ and $\Pr(I1 | Mi)$ are shown in Figure I.1.

Figure I.1: Volumes corresponding to the conditional probabilities $\Pr(I2 \cap I1 | Mi)$ and $\Pr(I1 | Mi)$
The influence of the correlation between the deformation limits $L_y$ and $L_u$ on the predicted conditional probability $\Pr(\overline{T_u}|M_i \cap I_y)$ is investigated by considering a set of alternative cases. First, the case of median values of the deformation limits $L_y$ and $L_u$ being close is considered. Secondly, the case when the median value of the deformation limit $L_u$ being considerably larger than $L_y$ is considered. This latter case is typical for ductile structures.

For the first case, the median values $L_y$ and $L_u$ are assumed to be 1.5% and 2.25%, respectively. Consequently, the median estimate of the deformation ductility capacity is around 1.5. Moreover, the log-standard deviations of the variables $\zeta_{L_y}$ and $\zeta_{L_u}$ are assumed to be 0.3 and 0.5, respectively. The conditional probabilities $\Pr(\overline{T_u}|M_i \cap I_y)$ are calculated by assuming four different values for the correlation coefficient $\rho_{L_y,L_u}$ between $L_y$ and $L_u$ (Figure I.2a). The conditional probability $\Pr(\overline{T_u}|M_i \cap I_y)$ is sensitive to the assumed $\rho_{L_y,L_u}$ for the small drift range ($d_{a,m} < 2.2\%$) and not very sensitive beyond that range.

The dispersions of $L_y$ and $L_u$, and the median value of $L_y$ are assumed to be the same as the previous case. However, the median $L_u$ is assumed to be 4.5%. This results in a median deformation ductility capacity of 3. Similar to the previous case, the conditional probabilities $\Pr(\overline{T_u}|M_i \cap I_y)$ are calculated by assuming four different values for the correlation coefficient $\rho_{L_y,L_u}$ between $L_y$ and $L_u$ (Figure I.2b). The influence of $\rho_{L_y,L_u}$ on the estimated conditional probabilities $\Pr(\overline{T_u}|M_i \cap I_y)$ is negligible in this case.

Figure I.2: Influence of the correlation $\rho_{L_y,L_u}$ on the estimated conditional probability $\Pr(\overline{T_u}|M_i \cap I_y)$ for closely spaced $L_y$-$L_u$ ($\mu_\Delta \cong 1.5$) (a) and distant $L_y$-$L_u$ ($\mu_\Delta \cong 3$) (b)
J Derivation

Derivation of Equation 4.18

Let \( \Pr(M_i \cap R_j \cap T_k | I) \) be the joint probability of \( M_i, R_j, \) and \( T_k \) conditioned on the event \( I \). The probability of \( M_i \) conditioned on \( R_j, T_k, \) and \( I \) can be calculated as follows:

\[
\Pr(M_i | R_j \cap T_k \cap I) = \frac{\Pr(M_i \cap R_j \cap T_k | I)}{\Pr(R_j \cap T_k | I)} \tag{J.1}
\]

In addition to this, let us define the joint conditional probability \( \Pr(R_j \cap T_k | MR \cap I) \) of \( R_j \) and \( T_k \) conditioned on the events \( MR \) and \( I \). This joint conditional probability is established based on the measurement of the residual deformations \( MR \). The probability of \( M_i \) conditioned on the events \( I \) and \( MR \) can be calculated as follows:

\[
\Pr(M_i | I \cap MR) = \sum_j \sum_k \Pr(M_i | R_j \cap T_k \cap I \cap MR) \Pr(R_j \cap T_k | I \cap MR) \tag{J.2}
\]

For the problem at hand, further simplifications can be introduced. Given that the events \( R_j \) and \( T_k \) are known to be true (or false), the event \( M_i \) is independent of \( MR \) (measurement of the residual drift). In other words, the estimated maximum drift is independent from the details of the residual displacement measurement process if the residual deformations are known. Based on this fact, the following equality can be introduced:

\[
\Pr(M_i | R_j \cap T_k \cap I \cap MR) = \Pr(M_i | R_j \cap T_k \cap I) \tag{J.3}
\]

Moreover, considering the fact that the results of the damage inspection have no effect on the joint probability of \( R_j \) and \( T_k \) obtained from the measurement of residual displacements, the following equality can be established:

\[
\Pr(R_j \cap T_k | I \cap MR) = \Pr(R_j \cap T_k | MR) \tag{J.4}
\]

Therefore, Equations J.4, J.3, and J.1 may be substituted into Equation J.2 yielding Equation J.5 which corresponds to Equation 4.18.
\[ \Pr(M_i | I \cap MR) = \sum_{j} \sum_{k} \frac{\Pr(M_i \cap R_j \cap T_k | I)}{\Pr(R_j \cap T_k | I)} \Pr(R_j \cap T_k | MR) \]  

(J.5)
## Notation

### Roman symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{t,rel}$</td>
<td>Relative horizontal acceleration at the top story of the test unit</td>
</tr>
<tr>
<td>$a_{sl}$</td>
<td>Reinforcement pull-out parameter</td>
</tr>
<tr>
<td>$A$</td>
<td>Gross section area</td>
</tr>
<tr>
<td>$A_1$</td>
<td>Parameter controlling the shape of the transition curve in the steel hysteresis model</td>
</tr>
<tr>
<td>$A_2$</td>
<td>Parameter controlling the shape of the transition curve in the steel hysteresis model</td>
</tr>
<tr>
<td>$C_{am}$</td>
<td>Correction factor related to the maximum average drift</td>
</tr>
<tr>
<td>$C_{ar}$</td>
<td>Correction factor related to the residual average drift</td>
</tr>
<tr>
<td>$C_{rr}$</td>
<td>Correction factor related to the residual rotation</td>
</tr>
<tr>
<td>$E_0$</td>
<td>Modulus of elasticity for the reinforcement steel</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Modulus of elasticity for the concrete</td>
</tr>
<tr>
<td>$EI_{eff}$</td>
<td>Effective flexural stiffness identified from moment curvature analysis</td>
</tr>
<tr>
<td>$EI_{unc}$</td>
<td>Uncracked flexural stiffness</td>
</tr>
<tr>
<td>$E_{S,y}$</td>
<td>Strain energy required to yield</td>
</tr>
<tr>
<td>$E_F$</td>
<td>Energy dissipated by friction</td>
</tr>
<tr>
<td>$\overline{\epsilon}$</td>
<td>Vector of identified $\varepsilon_{i,j}^{x}$ values</td>
</tr>
<tr>
<td>$d$</td>
<td>Effective depths to the tension reinforcement</td>
</tr>
<tr>
<td>$d'$</td>
<td>Effective depth to the compression reinforcement</td>
</tr>
<tr>
<td>$d_a$</td>
<td>Average drift ratio</td>
</tr>
<tr>
<td>$d_{a,m}$</td>
<td>Maximum average drift ratio</td>
</tr>
<tr>
<td>$d_{a,r}$</td>
<td>Residual average drift ratio</td>
</tr>
<tr>
<td>$d_{a,u}$</td>
<td>Average drift ratio at the ultimate deformation limit</td>
</tr>
<tr>
<td>$d_{a,y}$</td>
<td>Average drift ratio at the yield deformation limit</td>
</tr>
<tr>
<td>$d_b$</td>
<td>Diameter of the longitudinal reinforcement</td>
</tr>
<tr>
<td>$d_f$</td>
<td>Displacement due to friction force</td>
</tr>
<tr>
<td>$d_{gs}$</td>
<td>Ground story drift ratio</td>
</tr>
</tbody>
</table>
Maximum ground story drift ratio
Residual ground story drift ratio
Post-yield flexural stiffness ratio
Strain-hardening stiffness ratio
Ultimate drift capacity
Yield deformation
Gravitational acceleration
Friction force
Corner frequency of the high-cut filter
Corner frequency of the low-cut filter
Height from the strain penetration depth
Visible damage inspection being carried out
Initial stiffness
Cracked stiffness of the undamaged structure
Residual stiffness of the damaged structure
Total element length
Length of the element at the plastic hinge zone
Length of the element at the plastic hinge zone in the reference model
Effective length
Equivalent plastic hinge length
Strain penetration depth
Tributary length of the \( i^{th} \) integration point
Length of the reinforced concrete wall section
Deformation limit that is related to the detection of indicator \( x \)
Deformation limit that is related to the detection of indicator \( x \) (considering only the parameter uncertainty)
Dynamic mass
Lower limit of the \( i^{th} \) maximum drift interval
Mean value of the drift interval from \( ma_i \) to \( ma_{i+1} \)
Moment
Event of \( MA \) being in the range between \( ma_i \) and \( ma_{i+1} \)
Yield moment
Residual strength of the damaged component
Ultimate moment
Maximum average drift
Event representing measurement of residual displacements
Event representing measurement of only the residual average drift
Axial load action on the critical section
\( n_{ax} \) Vertical loading intensity
\( n_{ax,i} \) Vertical loading intensity of the differentiated model
\( n_{ax,Ref} \) Vertical loading intensity of the reference model
\( n_{ax,Sim} \) Axial load simulated at the integration section
\( n_{ax,Target} \) Axial load at the integration section that satisfies the equilibrium with in the element

\( N \) Total number of simulations
\( N_r \) Number of randomly generated set of parameter values
\( N_g \) Number of ground motion components
\( \bar{q}_i \) The nodal displacement for the \( i^{th} \) element degree-of-freedom
\( r a_j \) Lower limit of the \( j^{th} \) residual average drift interval
\( r r_k \) Lower limit of the \( k^{th} \) residual rotation interval
\( R_0 \) Parameter controlling the shape of the transition curve in the steel hysteresis model
\( R_j \) Event of \( RA \) being in the interval from \( ra_j \) to \( ra_{j+1} \)
\( RA \) Residual average drift
\( RR \) Residual rotation
\( s a_{i,j} \) Median pseudo-spectral acceleration estimated considering earthquake \( i \) and site \( j \)
\( s a^*_{i,j} \) Measured pseudo-spectral acceleration during earthquake \( i \) at site \( j \)
\( S_n \) \( n^{th} \) response simulation
\( S A_{i,j} \) Pseudo-spectral acceleration estimated considering earthquake \( i \) and site \( j \)
\( t \) Time
\( T \) Period of vibration
\( T_k \) Event of \( \Theta_r \) being in the interval \( rr_k \) to \( rr_{k+1} \)
\( V_B \) Base shear
\( V_{B,y} \) Base shear at yield deformation
\( w_i \) Weight of the \( i^{th} \) integration point
\( w_n \) Angular frequency

**Greek symbols**

\( \beta_N \) Newmark’s method: \( \beta \) parameter
\( \Delta \) Distance between two sites
\( \Delta t_i \) Time-increment step size of the differentiated model
\( \Delta t_{Ref} \) Time-increment step of the reference model
\( \epsilon \) Strain
\( \epsilon_{cc} \) Strain corresponding to peak compressive strength of the confined concrete
\( \epsilon_{cu} \) Ultimate compressive strain capacity of the confined concrete
\( \epsilon_s \) Steel strain
\( \epsilon_{su} \) Ultimate monotonic tensile strain capacity of the steel
\( \varepsilon_{\text{sur}} \) Reduced ultimate monotonic tensile strain capacity of the steel
\( \varepsilon_y \) Yield strain of the reinforcement
\( \varepsilon_{i,j} \) Standard normal random variable associated with the intra-event error of the pseudo-spectral acceleration predicted considering earthquake \( i \) and site \( j \)
\( \tilde{\varepsilon}_{i,j} \) Standard normal random variable associated with the total error of the pseudo-spectral acceleration predicted considering earthquake \( i \) and site \( j \)
\( \tilde{\varepsilon}_{i,j}^* \) Observed values of the standard normal random variable associated with the total error of the pseudo-spectral acceleration predicted considering earthquake \( i \) and site \( j \)
\( \eta_x \) Correction factor related to deformation limit prediction model
\( \eta_i \) Standard normal random variable associated with the inter-event error
\( \gamma \) Parameter controlling the unloading stiffness of the modified Takeda hysteresis model
\( \gamma_N \) Newmark’s method: \( \gamma \) parameter
\( \mu_{\Delta} \) Displacement ductility
\( \mu_{\Delta,C} \) Displacement ductility capacity
\( \mu_{\Delta,D} \) Displacement ductility demand
\( \mu_{\epsilon} \) Strain ductility
\( \mu_{\phi} \) Curvature ductility
\( \mu_{\Theta,D} \) Rotation ductility demand
\( \phi \) Curvature
\( \phi(x) \) Curvature at the point \( x \) along the finite-element
\( \phi_y \) Yield curvature
\( \phi_u \) Ultimate curvature
\( \varphi'_{i}(x) \) Derivative of displacement interpolation function associated with the \( i^{th} \) element nodal displacement
\( \psi_{i,j} \) Modal displacement for the mode \( i \) mode and degree-of-freedom \( j \)
\( \rho_{\varepsilon,\varepsilon} \) Correlation between the \( \varepsilon \) values for two neighboring sites
\( \rho_{\tilde{\varepsilon},\tilde{\varepsilon}} \) Correlation between the \( \tilde{\varepsilon} \) values for two neighboring sites
\( \rho_{\tilde{\varepsilon}} \) Correlation between the \( \tilde{\varepsilon} \) values
\( \tau \) Standard deviation of the inter-event error
\( \Theta \) Rotation
\( \Theta_m \) Maximum rotation
\( \Theta_r \) Residual rotation
\( \Theta(x) \) Rotation at the point \( x \) along the finite-element
\( \sigma_{cc} \) Compressive strength of the confined concrete
\( \sigma_y \) Yield strength of the reinforcement
\( \sigma \) Stress
\( \sigma_s \) Steel stress (Chapter 3), standard deviation of the intra-event error
$\sigma_{c0}$ Compressive strength of the unconfined concrete
$\sigma_{cc}$ Compressive strength of the confined concrete
$\sigma_T$ Standard deviation of the total error
$\zeta_F$ Viscous damping representing the friction
$\zeta_0$ Viscous damping due to sources other than friction
$\zeta_n$ Total viscous damping
$\zeta_i$ Viscous damping level of the differentiated model
$\zeta_{Ref}$ Viscous damping level of the reference model