Refunding in climate treaties

two variants

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Refunding in Climate Treaties: Two Variants

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In this thesis we design climate treaties that address the fundamental weaknesses of the Kyoto Protocol, the only existing global climate treaty. First, the Kyoto Protocol is plagued by free-riding, as the mitigation of climate change constitutes a global public good. Therefore, an efficient climate treaty has to provide incentives for countries to participate in the treaty, and to reduce emissions in a socially optimal way. Second, while current climate change is mainly caused by industrial countries, developing countries are extremely burdened by the adverse effects of global warming. Equity concerns regarding each country’s share of mitigating climate change play a central role in the negotiation of climate treaties. They have to be taken into account when designing such treaties.

The essential idea of the climate treaties we propose is a refunding scheme. It works as follows. The countries participating in the treaty pay an initial fee into a global fund. The fund may be additionally financed by emission tax revenues or interest on assets. In each period, part of the fund is redistributed to the member countries, and the share that a country receives depends on how much emissions it has reduced, compared to overall emission reductions. When the treaty ends, the remainder of the fund is paid back to the participating countries.

In the first part of this thesis, we use a static model with two heterogeneous countries, a developing country and an industrial country, to analyze the following refunding scheme: Only the industrial country has to pay an initial fee. Both countries choose emission tax rates. The tax revenues are paid into the fund. Both countries receive refunds according to their relative emission reductions. The remainder is given back to the industrial country. We show that this refunding scheme can achieve efficiency and equity objectives at the same time, since countries implement socially optimal emission reductions, and the developing country participates voluntarily, as it is better off under refunding than under a no-treaty solution. We also examine a version of this refunding scheme where member countries keep their tax revenues. It achieves socially optimal emission reductions if the
relationship between marginal damages and marginal abatement costs is the same across countries.

In the second part of this thesis, we use a dynamic model with an arbitrary number of homogeneous countries to explore the potential of a refunding scheme that is financed by initial fees from all participating countries and interest yields on assets. We show that if initial fees and refunds per period are chosen appropriately, countries implement socially optimal emission reductions in each period. We also show that emission reductions converge to the socially optimal level under a refunding scheme with constant refunds if initial fees are chosen in an appropriate way.
Kurzzusammenfassung


Im zweiten Teil der Dissertation benutzen wir ein dynamisches Modell mit einer beliebigen Anzahl homogener Länder, um das Potenzial eines Rückerstattungssystems zu untersuchen, das durch Anfangsbeiträge aller Mitgliedsländer und Zinseinnahmen finanziert wird. Wir zeigen, dass die Länder in jeder Periode sozial optimale Emissionsreduktionen umsetzen, falls die Anfangsbeiträge und die Rückzahlungen pro Periode passend gewählt werden. Wir zeigen außerdem, dass Emissionsreduktionen in einem Rückerstattungssystem mit konstanten Rückzahlungen zum sozial optimalen Niveau konvergieren, wenn die Anfangsbeiträge richtig bestimmt werden.
1. Introduction

1.1. Motivation

Many human activities require the combustion of fossil fuels, which results in emissions of carbon dioxide, the most important greenhouse gas. It is widely recognized and documented by the Intergovernmental Panel on Climate Change (IPCC) that the man-made increase in greenhouse gases in the atmosphere leads to an increase in global temperature (IPCC, 2007a). This anthropogenic climate change will have far-reaching consequences if global emissions stay on the business-as-usual path. Most of them are detrimental. According to the IPCC (2007b), the major damages are:

- Decrease in crop productivity due to the extension of drought-affected areas and due to reduced water-availability in regions that depend on glacial water supply
- Damages from floods in low-lying coastal areas and small islands due to the sea-level rise
- Damages from floods due to heavy precipitation and cyclones
- Negative health effects caused by malnutrition, heat waves and higher concentration of ground-level ozone related to climate change
- Negative consequences for biodiversity due to major changes in ecosystem structure and function

As greenhouse gases travel freely around the world, the harm they cause is independent from their source. Conversely, this means that emission reductions to mitigate climate change benefit everybody wherever they are undertaken. They constitute a global public good. As there is no central authority that is able to force countries to reduce emissions, the provision of the global public good “emission reductions” is prone to free-riding: Countries may try to free-ride on other countries’ mitigation efforts. As a consequence,
all countries may reduce much less emissions than if a social planner could dictate and enforce abatement decisions.

To combat anthropogenic climate change, emissions can be reduced in the following areas, see IPCC (2007c):

- Energy supply, e.g. improved supply and distribution efficiency, fuel switching from coal to gas, nuclear power, renewable energy
- Transport, e.g. more fuel efficient vehicles, shift from road transport to rail and public transport
- Buildings, e.g. improved insulation, more efficient heating and cooling devices, efficient lighting
- Industry, e.g. heat and power recovery, material recycling
- Agriculture, e.g. improved rice cultivation techniques and nitrogen fertilizer application techniques
- Forestry, e.g. afforestation, reduced deforestation

These emission abatement efforts involve costs which are typically convex (Nordhaus, 1991). The convexity of abatement costs calls for the coordinated efforts of all countries to reduce emissions. Hence climate policy has to overcome free-riding by solving the difficult task of providing incentives for countries to participate in a treaty and to comply with emission-reduction objectives. Moreover, while it is mainly developed countries that are responsible for the current climate change, this climate change impacts most on the developing world. On average, it suffers more from the increasing temperature (World Bank, 2010), mainly due to its dependence on agriculture and its lack of means to adapt to climate change. Therefore, climate policy-makers are confronted with equity concerns and development objectives. To cope with those diverse problems makes climate change one of the most complex policy challenges.

Up to now, there exists only one global treaty with legally-binding emission targets, the Kyoto Protocol under the United Nations Framework Convention on Climate Change (UNFCCC), initially adopted in 1997 and entered into force in 2005. In a nutshell\(^1\), according to the principle of “common but differentiated responsibilities”, developed countries, so-called Annex B countries, voluntarily commit to reducing their emissions.

\(^1\)The complete text and discussions of the Kyoto Protocol can be found at http://www.unfccc.de.
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by 5.2% on average, compared to 1990 levels, for the commitment period 2008–2012. The Kyoto Protocol specifies three mechanisms how countries can reach their emission reduction targets if they fail to reduce emissions sufficiently in their own country. First, it allows emission trading, i.e. countries and firms can buy emission rights from countries that emit less than their assigned portion. Second, countries can earn emission rights by investing in emission reduction projects in other Annex B countries, a mechanism which is called Joint Implementation. Third, emission reductions also count if carried out in developing countries, which is then called Clean Development Mechanism (CDM).

The Kyoto Protocol suffers from a number of weaknesses and is therefore often considered to be ineffective (Nordhaus and Boyer, 1999; Schelling, 2001; Böhringer and Vogt, 2003). First of all, one of the main contributors of greenhouse gas emissions, the U.S., have not ratified the Kyoto Protocol. Second, although developing countries, including China and India, account for more than one half of CO$_2$ emissions by now (Energy Information Administration, 2010), they have no obligation to reduce emissions.\(^2\) Third, even countries that ratified the Kyoto Protocol will probably not comply with the targets they agreed on.\(^3\) Forth, the mechanisms provided by the Kyoto Protocol to achieve the emission targets, like the CDM, can create perverse incentives (Keohane and Raustiala, 2010). For instance, developing countries may refuse to reduce emissions by their own reduction measures to increase their chances to obtain CDM projects. Moreover, CDM projects often do not yield additional emission reductions.

The Kyoto experience shows that the international community has not been able to set up and agree on a climate treaty that substantially mitigates climate change. There exists a growing literature on alternative proposals for climate treaties that could serve as a successor of the Kyoto Protocol (see for example Aldy and Stavins (2010) for an overview). Those alternative treaties range from treaties that develop dynamic national emission targets for all countries to treaties using national carbon taxes or treaties establishing a global CDM.

The aim of this thesis is to design climate treaties that address the core problems that hamper the implementation of international climate treaties: The lack of incentives to participate, the lack of incentives to comply with emission reduction objectives, and equity concerns. The central idea of the climate treaties developed in this thesis is a

\(^2\)Moreover, it is expected that CO$_2$ emissions in China will rise significantly in the next decades, see Blanford et al. (2010).

\(^3\)Canada, for example is about to miss its emission reduction target for the period 2008–2012, see Böhringer and Rutherford (2010).
1. Introduction

refunding scheme which works as follows. Countries participating in the treaty set up a global fund. The fund is financed by initial fees paid by the joining countries. It may also comprise emission tax revenues of the participating countries and interest on assets. Emission reductions of member countries are rewarded by payments from the fund. Specifically, each participating country receives a refund that depends on the emission reductions the country achieved compared to overall emission reductions. At the end, the fund is closed, and the remainder of the fund is paid back to the member countries. Signing the treaty involves agreement of the participating countries on the initial fees and the refunding formula.

The idea to design a climate treaty with refunding was first proposed by Gersbach (2005). Gersbach and Winkler (2007) analyse a global refunding scheme in a static setting with symmetric countries using emission taxes as a policy instrument to reduce emissions. Gersbach and Winkler (2011) study refunding in an international permit market without relying on initial fees.

There already exist global funds in the context of international climate treaties. The Global Environmental Facility (GEF), for example, established in 1991, serves as the financial mechanism of the Framework Convention on Climate Change since the United Nations (UN) climate change conference held in Rio in 1992. It aims at financing environmental projects with global implications in developing countries (Streck, 2001). Recently, at the UN climate change conference 2010 in Cancún, countries formally agreed on the establishment of a “Green Climate Fund” that should guarantee long-term financing of climate change mitigation and adaptation in developing countries, with developed countries acting as donor countries.4

Those existing funds exhibit a variety of disadvantages:

- They do not create incentives for countries to strongly reduce emissions associated with economic activities in their own country.5
- Countries are not free to choose how emission reductions are implemented.6
- The existing funds are subject to repeated renegotiations with respect to the financial contributions of the donor countries.

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4See Report of the Conference of the Parties on its sixteenth session, held in Cancún from 29 November to 10 December 2010, p. 16–18, on http://www.unfccc.de.
5Evaluation of the GEF has shown its modest impact on climate change mitigation (Global Environmental Facility Evaluation Office, 2010).
6Donor countries might influence the decisions how the money is spent (Mace, 2005).
Our treaties with refunding compensate these deficiencies. First, through the competition for refunds, they provide incentives for countries to reduce their emissions efficiently. Second, countries can freely decide by which means they want to reduce emissions. Third, we design treaties that can, in principle, last forever without requiring renegotiations.

The remainder of this chapter is organised as follows. In the next section, we formulate our research goals. Section 1.3 provides summaries of the different chapters of the thesis.

1.2. Research Questions

The research questions we want to address in this thesis are: How should climate treaties be designed that

- **Q1** provide incentives for countries to participate?
- **Q2** satisfy equity objectives?
- **Q3** induce compliance?
- **Q4** yield efficient emission reductions?
- **Q5** can last for many periods without requiring renegotiations?

We characterize treaties that achieve simultaneously the goals formulated either in Q1–Q4 (Chapter 2–4) or in Q3–Q5 (Chapter 5).

1.3. Structure of the Thesis

**Climate Policy and Development (Chapter 2)**

Developing countries, especially China and India, contribute a large and increasing share of greenhouse gas emissions today. As outlined in Section 1.1, they already account for more than one half of CO₂ emissions. In addition, developing countries provide “low hanging fruits” for emission abatement, for example by reducing emissions from deforestation and forest degradation (REDD) as one fifth of global greenhouse gas emissions
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stems from deforestation. Those emissions can be cut down without any elaborate technological development (Engel et al., 2011). Hence participation of developing countries is crucial for any efficient climate treaty.

At the same time, climate treaties have to satisfy equity objectives. The reason is that current climate change can be mainly attributed to the economic activity of industrial countries. They have caused the accumulation of most of the greenhouse gases in the atmosphere, while developing countries suffer more, on average, from the adverse effects of climate change due to their geographical position and their incapability to adapt than the developed world (IPCC, 2007b).

We propose a treaty with refunding, called Development-Compatible Refunding System (DCR) to simultaneously address efficiency and equity objectives of climate policy. The DCR works as follows. Industrial countries that want to participate in the DCR pay an initial fee into a global fund. Each country chooses the national carbon emission tax it wants to levy. Part of the global fund is refunded to the participating countries, in proportion to the relative emission reductions they achieve. The countries receive refunds net of tax revenues. Then the remainder of the fund is paid back to the industrial country.

We analyze the DCR with a model set-up that consists of the following ingredients:

- two heterogeneous countries, a developing country and an industrial country
- the countries are characterized by
  - emissions that add to the stock of greenhouse gases in the atmosphere
  - emission abatement costs
  - damages from the stock of greenhouse gases

We make two crucial assumptions to reflect the relationship of abatement costs and damages in the developing and the industrial country mentioned above. First, the developing country has lower marginal emission abatement costs than the industrial country. Second, marginal damages are higher for the developing country compared to the industrial country.

The model replicates the standard underprovision of the public good “emission reductions” if there is no international coordination.
We show that the DCR simultaneously satisfies efficiency and equity objectives. Both countries implement socially optimal emission taxes. The developing country will participate voluntarily, as it can be made better off under the DCR compared to the no-treaty solution. Moreover, the system induces compliance, as the countries will always reduce emissions to compete for refunds. Hence the goals formulated in Q1–Q4 in Section 1.2 are fulfilled.

We also examine a simplified version of the DCR where countries keep their tax revenues and the fund is solely financed by initial fees. We show that this simplified system achieves efficiency and equity if the relationship between marginal damages and marginal abatement costs is the same across countries.

**Climate Policy and Development – Generalizations and Relaxing Critical Assumptions (Chapter 3 and 4)**

In these two chapters, we further examine the Development-Compatible Refunding System (DCR) proposed in Chapter 2. To this end, we generalize several aspects of the model that is used to analyse the treaty. In addition, we analyze the consequences of relaxing Assumption 1 imposed in Chapter 2. This assumption has considerably simplified the subsequent analysis. However, it restricted the model parameters to a certain range of values.

Our main insights are as follows. First, the model framework can be extended to more than two countries. We show (a) that for a large number of countries, socially optimal emission taxes are implemented if marginal damages are not too low compared to marginal abatement costs and if the parameters of the treaty are chosen appropriately. We show (b) that under the simplified DCR that does not claim tax revenues all countries set socially optimal emission reductions if the parameters of the treaty are chosen appropriately.

Second, the model does not rely on quadratic cost functions. We obtain results for the simplified version of the DCR that are similar to those derived in Chapter 2 if we choose arbitrary convex increasing cost functions.

Third, whether we impose Assumption 1 or not, we show that the DCR is always able to induce the countries to choose maximal emission reductions. This can be achieved by setting the initial fees of the industrial country at a sufficiently high level.
Fourth, we show that positive refunds are a necessary requirement for the implementation of socially optimal taxes under the DCR, while a negative initial fee or a negative remainder of the fund does not exclude socially optimal emission reductions. This holds without any restrictions on the model parameters.

**Sustainable Climate Treaties (Chapter 5)**

Greenhouse gases, once emitted, stay in the atmosphere for a long time.\(^7\) This implies that every decision on emission reductions made today has long-term effects.

In this chapter, we design a treaty that can last for many periods and that focuses on intertemporal decisions on emission reductions. The treaty works as follows. Before the first period, the participating countries pay an initial fee into a global fund that is invested in long-run assets. In a typical period, part of the fund is distributed among the member countries according to the amount of emission reductions they have achieved in this period, compared to overall emissions reduced. After \(T\) periods, the remainder of the fund is resolved and paid back in equal parts to the member countries.

The model we use to analyze the treaty comprises the following elements:

- \(n\) identical countries
- countries are characterized by
  - emissions
  - emission abatement costs
  - damages from the stock of greenhouse gases
- a time horizon of \(T\) periods

In each period, emissions of the respective period add to the stock of greenhouse gases, and the stock is subject to a natural decay. Moreover, future outcomes are discounted with an exogenously given discount factor.

Using the dynamic programming method for discrete times, we derive socially optimal abatement decisions for all countries and all periods. Moreover, we calculate the subgame

\(^7\)While the IPCC states that the climate impact of the greenhouse gas emissions lasts from decades to centuries (IPCC, 2007a), carbon-cycle models reviewed by Archer and Brovkin (2008) suggest that a significant fraction of 20–60\% of \(\text{CO}_2\), once released, remains in the atmosphere for thousand years or longer.
1. Introduction

perfect Nash equilibria where each country minimizes the net present value of its own costs, given the abatement decisions of all other countries. Comparing these benchmark cases, we observe the standard underprovision of the public good “emission abatement” if no cooperation or coordination takes place: Under the decentralized system, the stock of greenhouse gases in the long run is higher than under the social planner system.

Signing the treaty involves the countries’ agreement on its parameters, namely initial fees and reimbursements per period. We show that those parameters can be determined in such a way that countries choose efficient emission reductions in each period. This kind of treaty is called first-best sustainable. We also identify a second-best sustainable treaty that relies on constant reimbursements (equal to the interest earned per period) and involves only initial fees as its parameter. For an appropriate choice of the initial fees, emission reductions converge to the socially optimal level.

Both types of treaties, the first- and the second-best sustainable treaty, satisfy the goals formulated in Q3–Q5 in Section 1.2, as they require only one-time negotiation, and because countries will implement efficient mitigation efforts either in all periods or in the long run, thus the treaties fulfil compliance.

Finally, as initial fees tend to be high, we suggest ways to raise money for the payment of initial fees which are neutral for the tax payers and for international capital markets.
2. Climate Policy and Development*

2.1. Introduction

Developing countries in Latin America, Asia and Africa are exceptionally burdened by climate change. First, they will be affected by the most severe damages when temperatures in the atmosphere rise. Second, developing countries currently provide the greatest opportunities for low-cost emission reductions, and they are likely to account for more than one-half of greenhouse gas emissions in the next decade.

As industrial countries have caused the bulk of current man-made greenhouse gas concentrations and as emission reductions in developing countries would further aggravate poverty, many argue that industrial countries alone should bear the cost of mitigating climate change. While this fairness argument is at the heart of the negotiations for an international agreement following the Kyoto Protocol, it is unclear how efficiency and fairness considerations in the mitigation of climate change can be combined in such a way that they do not conflict with each other. The policy debate and the major issues on current global climate change policy are discussed e.g. in Bosetti et al. (2009), Karp and Zhao (2009), Whalley and Walsh (2009), Chatterji and Ghosal (2009) and in Sinn (2008). A fundamental question is the participation of developing countries.

Moreover, greenhouse gases travel around the world, so mitigation of climate change is a global public good. Accordingly, achieving both efficiency and fairness objectives in climate policy is difficult. In this chapter we propose a simple scheme that can incorporate efficiency and fairness objectives into the mitigation of climate change. The scheme works as follows:

- Industrial countries pay an initial fee into a global fund.

*This chapter is based on Gersbach and Hummel (2009).
2. Climate Policy and Development

- Countries decide on their emission taxes. Their emission tax revenues are payable to the global fund.

- A fraction of the fund is redistributed to the participating countries according to a sharing rule. The sharing rule specifies that the refund for each country is proportional to the relative emission reductions it achieves. Countries receive refunds minus tax revenues. If that amount is negative, countries have to clear their debts.

- The remaining fraction of the global fund is paid back to industrial countries.

Such a refunding scheme is called “development-compatible refunding” (henceforth called DCR), as developing countries do not have to pay an initial fee and abatement by developing countries is voluntary.

We explore whether a DCR can simultaneously achieve the equity and efficiency objectives of climate policy, thus serving as a base for designing an international treaty. We consider a model with a representative industrial and a representative developing country. The developing country has equal or higher marginal damage from greenhouse gas emissions compared to the industrial country, but has equal or lower marginal abatement costs. Both countries join a development-compatible refunding scheme. Each country receives refunds in proportion to the relative emission reductions it has achieved. The relative shares may be varied by a weighting factor allowing for the increase or decrease of the relative refunding claims of the developing and the industrial country. The fraction of the fund that is not distributed among the countries is paid back to the industrial country.

Our main insight is that a suitably designed DCR can achieve efficiency and equity objectives under a variety of circumstances. Regarding efficiency objectives, the DCR induces both the industrial and the developing country to set abatement levels that are socially optimal. The reason is as follows: A sufficiently high initial fee from the industrial country motivates both countries to set taxes at the socially optimal level as one additional tax dollar and associated emission reduction generates more dollars in refunds. This property stems from the refunding formula as a marginal increase of the tax rate by one country and the associated emission reduction will increase the refunding share for this country at the expense of the other country. When both countries set taxes at the socially optimal level, no country will want to deviate as it would suffer a high
marginal decline in refunds. These refunding losses (and the higher marginal damages) outweigh high marginal abatement costs.

Regarding equity objectives, the developing country does not have to pay an initial fee and abates voluntarily. In addition, the refunds to the developing country can be increased by varying the weighting factors in the refunding formula. In particular, choosing an appropriate weighting factor will make the developing country a net receiver of funds. Moreover, as a rule, the developing country is better off than it would be when an international treaty fails and each country decides independently how much to abate.\(^1\) Moreover, we outline the circumstances for which both the industrial and the developing country are better off under a DCR.

We also explore the potential of a simple refunding scheme that renounces claiming tax revenues from countries. In this simple scheme refunds are solely financed by initial fees. Such a simple refunding scheme yields socially optimal abatement levels if the relationship between marginal damages and marginal abatement costs are similar across countries.

The remaining chapter is organized as follows: In the next section we relate our analysis to the literature. In Section 2.3 we present the model. In Section 2.4 we calculate the social optimum and the decentralized solution as benchmark cases. In Section 2.5 we introduce the development-compatible refunding scheme and characterize its properties. Special cases are discussed in Section 2.6. In Section 2.7 we consider a simple refunding scheme that does not claim tax revenues. In Section 2.8 we address extensions to many countries and different policy instruments, and we discuss critical features of our scheme. Section 2.9 concludes.

### 2.2. Relation to the Literature

Our analysis starts from the observation that most economists working on climate policy argue that current generations need to reduce emissions which, in turn, requires a reduction in current consumption. The assessment of whether and how much climate policies requires sacrifice has attracted a lot of attention. Leading contributions such as

\(^1\)As argued by Spence (2009), it is essential that developing countries get compensated for their abatement efforts in a global agreement as otherwise their growth prospects would be seriously delayed which, in turn, would make it impossible to reach such an agreement in the first place.
Stern (2006), Nordhaus (2007), Karp (2005), Fujii and Karp (2008), Rezai et al. (2009), Hoel and Sterner (2007), Tol (2008), and Weitzman (2009) develop different perspectives and identify the pitfalls when such assessments are made. Here we assume that the current generation around the globe collectively would like to reduce emissions to some degree. The problem is that nations would like to free ride on abatement efforts by other countries and developing countries need to be compensated for the abatement sacrifices.

It is well known that achieving significant emission reductions through Kyoto-style agreements with emission targets is a very difficult undertaking. As a consequence, various other approaches to international coordination have been suggested. Aldy et al. (2003) summarize the alternatives, which include an international carbon tax and international technology standards. More recently, Gersbach (2005) and Gersbach and Winkler (2007) have proposed and discussed a global refunding system in which all countries are treated equally. All countries have to pay an initial fee and they all will receive the same refund. The authors show that, if countries are identical, such a scheme will achieve the social global optimum. In this chapter we propose a development-compatible refunding scheme in which only industrial countries have to pay an initial fee. Our refunding formula is such that efficient abatement levels are achieved in industrial and in developing countries even if developing countries do not have to pay initial fees, suffer higher marginal damages from climate change and have lower marginal abatement costs.

Using game-theoretic models, a considerable body of research has examined the formation of international environmental agreements. This literature mainly focuses on the circumstances fostering the building of coalitions through multilateral agreements. Such agreements must be self-enforcing, since there is no supranational authority to ensure compliance (for a recent contribution, see the important article by Asheim et al., 2006). The literature reaches two main conclusions: It is unlikely that a grand coalition will be formed, and if it is, it will achieve very little. Moreover, sub-coalitions may be better for their members than the grand coalition, and regional agreements can Pareto-dominate a regime based on a global treaty. Our approach complements this literature by suggesting a refunding scheme that balances the efficiency and equity objectives of a mitigation of climate change.
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2.3. The Model

We consider a world consisting of two countries, an industrial country \( I \) and a developing country \( D \). They are characterized by an emission function \( E \), an abatement cost function \( C \), and damages. Throughout the chapter countries are indexed by \( i \) and \( j \) \((i, j \in \{I, D\})\).

The emissions of country \( i \) are assumed to equal some “business as usual” emissions \( \bar{e} \) minus emission abatement \( a^i \):

\[
E^i(a^i) = \bar{e} - a^i, \quad \text{with} \quad a^i \in [0, \bar{e}], \quad i \in \{I, D\}. \tag{2.1}
\]

We assume that these emissions are caused by a representative firm in each country which faces convex abatement costs:

\[
C^i(a^i) = \frac{1}{2\phi^i}(a^i)^2, \quad \text{with} \quad \phi^i > 0, \quad i \in \{I, D\}. \tag{2.2}
\]

We assume that countries use emission taxes as a policy instrument. As developing countries provide the best opportunities for low-cost emission reductions, we assume \( \phi^I \leq \phi^D \). Each country \( i \) individually sets per-unit emission taxes \( \tau^i \). Cost minimizing behavior by the representative firm implies that marginal abatement costs will equal the emission tax:

\[
\tau^i = \frac{a^i}{\phi^i}, \quad \text{with} \quad \tau^i \in \left[0, \frac{\bar{e}}{\phi^i}\right], \quad i \in \{I, D\}. \tag{2.3}
\]

Thus both emissions \( E^i \) and abatement costs \( C^i \) of country \( i \) can be expressed in terms of the emission taxes \( \tau^i \):

\[
E^i(\tau^i) = \bar{e} - \phi^i \tau^i, \quad \text{with} \quad i \in \{I, D\}, \tag{2.4}
\]
\[
C^i(\tau^i) = \frac{\phi^i}{2}(\tau^i)^2, \quad \text{with} \quad i \in \{I, D\}. \tag{2.5}
\]

The sum of the emissions of both countries instantaneously accumulate the stock of

\footnote{For simplicity, we assume that \( \bar{e}^I = \bar{e}^D = \bar{e} \). This assumption reflects the fact that both industrial and developing countries contribute a significant share to global greenhouse-gas emission.}

\footnote{This is a standard short cut to capture aggregate abatement costs in country \( i \) (see, e.g., Falk and Mendelsohn, 1993).}

\footnote{See Morris et al. (2008) and Criqui et al. (1999).}
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greenhouse gases $s$:

$$s = \sum_{j \in \{I,D\}} E^j(\tau^j). \quad (2.6)$$

Note that for ease of presentation we assume that the initial stock is zero.$^5$

The damage caused by the stock of greenhouse gases is given by

$$\frac{\beta^i}{2} s^2, \quad \text{with } \beta^i > 0, \quad i \in \{I, D\}, \quad (2.7)$$

where $\beta^I \leq \beta^D$ as developing countries are more affected by damages caused by climate change than industrial countries (IPCC, 2007b). The parameters $\beta^i, i \in \{I, D\}$, represent marginal damages.$^6$

Besides the differences in marginal abatement costs and marginal damages, developing countries are associated with low income per capita, and we assume that it is impossible for them to pay an initial fee. This is obvious by the case for the poorest countries in Africa, as such payments would either be impossible to collect or would severely affect the citizens of that country.

2.4. Social Optimum and Decentralization

We first characterize the social optimum and the decentralized solution. The social optimum is the efficiency goal of an international agreement. The decentralized solution is the outcome that prevails if no agreement is achieved.

2.4.1. Social Optimum

Consider a social planner seeking to maximize total welfare, i.e., to minimize the net present value of the total costs of emission abatement and the sum of national damages stemming from greenhouse-gas emissions. Accordingly, the social planner wants to

---

$^5$Adding an initial stock $s_0 \neq 0$ would not affect our results qualitatively.

$^6$Marginal damages and marginal abatement costs can be expressed in utility or common currency units. Both interpretations can be applied to our model. It might be easiest to think in monetary units as taxes and transfers are usually expressed in units of money.
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minimize

\[ F^{SO}(\tau^I, \tau^D) := \sum_{j \in \{I, D\}} \frac{\phi^j}{2}(\tau^j)^2 + \frac{\beta^j}{2} s^2 \]  \hfill (2.8)

with respect to \( \tau^I \) and \( \tau^D \), subject to equation (2.6), and \( \tau^i \in [0, \frac{\tau^i}{\phi^i}] \), \( i \in \{I, D\} \). Momentarily we focus on interior solutions. We will provide a sufficient condition for the existence of interior solutions at the end of this subsection.

If we insert (2.6) into \( F^{SO} \), the first-order conditions for an optimal solution are

\[ \frac{\partial F^{SO}}{\partial \tau^i} = \phi^i \left( \tau^i - (\beta^I + \beta^D)s \right) = 0, \quad i \in \{I, D\}. \]  \hfill (2.9)

Due to the strict convexity of \( F^{SO} \) these necessary conditions are also sufficient for a unique solution. Equation (2.9) reveals that both countries set the same emission taxes \( \tau \) in the social optimum. These are given by the following proposition:

**Proposition 2.1 (Social optimum)**

(i) The optimal emission tax \( \tau^* \) for both countries equals

\[ \tau^* = \frac{2\bar{c}(\beta^I + \beta^D)}{1 + (\beta^I + \beta^D)(\phi^I + \phi^D)} . \]  \hfill (2.10)

(ii) The optimal stock \( s^* \) is given by

\[ s^* = \frac{2\bar{c}}{1 + (\beta^I + \beta^D)(\phi^I + \phi^D)} = \frac{\tau^*}{(\beta^I + \beta^D)} . \]  \hfill (2.11)

(iii) The abatements \( a^i* \) of both countries are given by

\[ a^I* = \frac{2\bar{c}(\beta^I + \beta^D)\phi^I}{1 + (\beta^I + \beta^D)(\phi^I + \phi^D)} = \phi^I \tau^* , \]  \hfill (2.12)

\[ a^D* = \frac{2\bar{c}(\beta^I + \beta^D)\phi^D}{1 + (\beta^I + \beta^D)(\phi^I + \phi^D)} = \phi^D \tau^* . \]  \hfill (2.13)

The proof of Proposition 2.1 is straightforward. Proposition 2.1 reveals the well-known property of a social optimum: Tax \( \tau^* \) is set at a level at which marginal costs of abatement equal aggregate marginal damages from the greenhouse-gas stock. Both countries face the same tax. The developing country abates more and benefits more from aggregate abatement efforts.
We assume for the remainder of the chapter

**Assumption 1**

\[
\bar{e} - \frac{\beta^D \phi^I + 2 \beta^D \phi^D + \beta^I \phi^D}{\beta^I + \beta^D} \tau^* > 0.
\]

The assumption implies that in the social optimum the abatement level \(a^I^*\) is below \(\bar{e}\) and the abatement level \(a^D^*\) is below \(\frac{2}{3} \bar{e}\). Hence we focus on circumstances for which socially desirable emission reductions are below 50\% in industrial countries and below 66.7\% in developing countries. Assumption 1 also implies that the social optimum is always an interior solution.

Note that Assumption 1 is equivalent to the following condition expressed solely in terms of the exogenous parameters of the model:

\[
1 + \beta^I \phi^I > \beta^D \phi^I + 3 \beta^D \phi^D + \beta^I \phi^D.
\]

### 2.4.2. Decentralized Solution

Next we examine a decentralized system where the government in each country seeks to minimize its own costs and damages. We look for Nash equilibria when countries simultaneously choose their emission taxes. Given the choice of the other country, country \(i\) minimizes

\[
F^{DS,i}(\tau^i) := \frac{\phi^i}{2} (\tau^i)^2 + \frac{\beta^i}{2} s^2
\]

with respect to \(\tau^i\) and subject to equation (2.6), and \(\tau^i \in \left[0, \frac{\bar{e}}{\phi^i}\right]\).

If we insert (2.6) into \(F^{DS,i}\), we obtain the first-order condition

\[
\frac{\partial F^{DS,i}}{\partial \tau^i} = \phi^i (\tau^i - \beta^i s) = 0.
\]

Analogously to Section 2.4.1, this necessary condition is also sufficient for a unique solution due to the strict convexity of \(F^{DS,i}\), and Assumption 1 rules out corner solutions.

The set of necessary and sufficient conditions (2.15) for both countries \(i \in \{I, D\}\) determines the Nash equilibrium. Solving for the tax rates yields
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**Proposition 2.2 (Decentralized solution)**

There exists a unique Nash equilibrium characterized by $\hat{\tau}^i$ for each country $i \in \{I, D\}$:

$$\hat{\tau}^i = \frac{2\bar{e}\beta^i}{1 + \beta^I \phi^I + \beta^D \phi^D}. \quad (2.16)$$

This yields the following equilibrium stock $\hat{s}$:

$$\hat{s} = \frac{2\bar{e}}{1 + \beta^I \phi^I + \beta^D \phi^D}. \quad (2.17)$$

The proof of Proposition 2.2 is straightforward. Proposition 2.2 implies

**Corollary 2.1**

We have

$$\hat{s} - s^* = 2\bar{e} \frac{\beta^I \phi^D + \beta^D \phi^I}{(1 + \beta^I \phi^I + \beta^D \phi^D)(1 + (\beta^I + \beta^D)(\phi^I + \phi^D)}) > 0.$$

Corollary 2.1 indicates the well-known finding that decentralized decisions on contributions to the public good “emission reduction” lead to underprovision. Abating emissions in one country creates a positive externality for the other country, as it reduces damages in all countries. In a decentralized solution countries are not compensated for these externalities.

It is also useful to compare tax rates in the social optimum and in the decentralized solution:

**Corollary 2.2**

We have

$$\hat{\tau}^I < \tau^* \text{ for all } 0 < \beta^I \leq \beta^D, \ 0 < \phi^I \leq \phi^D,$$

$$\hat{\tau}^D < \tau^* \iff \beta^I + (\beta^I)^2 \phi^I > (\beta^D)^2 \phi^D.$$
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2.5. Development-Compatible Refunding Scheme

We now design a development-compatible refunding scheme (DCR). The scheme works as follows: If it decides to join the DCR, the industrial country is required to pay an initial fee of $f_I^0 \geq 0$ into a fund. Members are free to choose national emission taxes $\tau_i$. All countries owe the fund their emission tax revenues. A fraction $\alpha \in [0, 1]$ of the fund is reimbursed to the participating countries in proportion to the relative emission reductions they have achieved, whereas the remaining fraction $(1 - \alpha)$ of the fund’s assets is paid back to the industrial country if it is a DCR member. At the end, the country receives or pays the difference between its tax revenues and the amount of money redistributed from the fund.

The parameter $\alpha$ provides the additional flexibility necessary to balance the incentives of the developing and the industrialized country. Specifically, $\alpha < 1$ is needed to induce the industrial country to choose socially optimal taxes if the developing country obtains very high refunds. The latter may be needed to ensure that the developing countries are net beneficiaries.

In the following we analyze the potential of a DCR to mitigate climate change. We explain the rules and the timing of payments and refunds in detail and derive conditions under which member countries of the DCR implement socially optimal taxes.

2.5.1. Rules and Timing of the DCR

The timing of the DCR is illustrated in Figure 2.1. At the outset countries sign the DCR, which is managed by an administering agency (AA). Signing the agreement involves

- Payment of an initial fee $f_I^0$ into the fund by the industrial country.
- Tax revenues are liabilities of countries.
- Agreement to a refunding formula with parameters $\{\alpha, \omega\}$ which are explained below. Country $i$ receives refund $r_i$ minus national tax revenues.\(^8\)

\(^7\)An example of parameter constellations for which an efficient DCR fails to exist when $\alpha = 1$ is given by $\beta_I = 0.01$, $\beta_D = 0.5$, $\phi_I = 0.001$, $\phi_D = 0.5$ (calculations are available upon request).

\(^8\)If the difference is negative, country $i$ has to clear its debt.
countries agree on $\mathcal{P} = \{\alpha, f^i_0, \omega\}$, industrial country pays initial fee $f^i_0$

industrial country pays initial fee $f^i_0$

countries set $\tau^i$, members owe the global fund their tax revenues

AA pays (collects) $r^i$—national tax revenues to (of) country $i$

AA pays $(1 - \alpha)f$ back to industrial country

Figure 2.1.: An illustration of the timing of the development-compatible refunding scheme.

For the refund $r^i$ a member country $i$ receives we assume the following refunding rule:

$$
    r^i = \alpha f \frac{\bar{\omega}^i a^i}{\sum_{j \in DCR} \bar{\omega}^j a^j} = \alpha f \frac{\omega^i \tau^i}{\sum_{j \in DCR} \omega^j \tau^j}, \quad i \in DCR,
$$

where we have set $\omega^i = \bar{\omega}^i \phi^i$, and $DCR$ denotes the set of countries that joined the refunding system. The formula captures the basic idea behind refunding: The refund a country $i$ receives is proportional to the relative emission reductions it achieves. Varying the weights $\omega^i$ ($\omega^i > 0$) makes it possible to heighten or to lower the size of the refund that country $i$ obtains if it chooses a particular tax level $\tau^i$ and corresponding abatement $a^i = \tau^i \phi^i$. Since only the ratio of the weights $\omega^i$ matters, we set $\omega := \omega^I / \omega^D$ and thus

$$
    r^I = \alpha f \frac{\omega}{\omega + \frac{\tau^D}{\tau^I}} = \alpha f \frac{\tau^I \omega}{\tau^I \omega + \tau^D} ;
$$

$$
    r^D = \alpha f \frac{\tau^D}{\omega + \frac{\tau^D}{\tau^I}} = \alpha f \frac{\tau^D}{\tau^I \omega + \tau^D} .
$$

The assets of the fund $f$ before refunds are given by

$$
    f = \sum_{j \in DCR} \left( f^j_0 + \tau^j (\bar{e} - \bar{\phi}^j \tau^j) \right) ,
$$

where $f^D_0 = 0$. 

Global warming coupled with the refunding scheme introduces reciprocal and unidirectional externalities:

**Reciprocal externalities**
- Tax externality (D↔I): If \( \tau^i < \bar{e} / (2\phi^i) \): the higher the tax rate in one country, the higher the taxes this country has to pay into the fund, which represents a positive tax externality.\(^9\) If \( \tau^i > \bar{e} / (2\phi^i) \), the tax externality is negative.
- Negative refunding externality (D↔I): the higher the abatement in one country, the lower is the refund for the other country, given its tax choice.\(^10\)
- Positive environmental damage externality (D↔I): the higher the abatement in one country, the lower is the damage for the other country.

**Unidirectional externalities**
- Positive initial fee externality (I→D): the higher the initial fee paid by the industrial country, the higher is the refund to the developing country.
- Residual refund externality (D→I): If \( \tau^D < \bar{e} / (2\phi^D) \), this externality is positive, i.e. the higher the tax of the developing country, the higher is the ultimate residual fund for the industrial country. If \( \tau^D > \bar{e} / (2\phi^D) \), this externality is negative.

The idea behind the scheme is to choose the parameters so that the externalities balance and both the developing country and the industrial country will choose socially optimal taxes.

Throughout the remaining section we assume that both countries join the DCR. We can summarize the treaty by the policy parameters

\[
\mathcal{P} := \{ f_0^I, \alpha, \omega \}, \quad (2.20)
\]

as \( \mathcal{P} \) fully determines the monetary flows that will occur. Now we define

**Definition 2.1 (Feasible \( \mathcal{P} \))**
*The set of policy parameters \( \mathcal{P} = \{ \alpha, f_0^I, \omega \} \) is called feasible if*

\[
\alpha \in [0, 1], \quad f_0^I \geq 0 \quad \text{and} \quad \omega > 0.
\]

\(^9\)Formally, \( \partial f / \partial \tau^i > 0. \)
\(^10\)Formally, \( \partial \sum_{j \neq i} \omega^j / \omega^i / \partial \tau^k < 0, \ k \neq i \)
While one can solve for the countries’ tax rates, abatement levels and damages for any feasible set of policy parameters $P$, we directly examine whether one can find a feasible $P$ that implements the socially optimal solution. For this purpose we define

**Definition 2.2 (Tax goal of DCR)**

The DCR’s tax goal is given by the socially optimal tax rate (2.10), i.e. by

$$\tau^I = \tau^D = \tau^* = \frac{2\bar{e}(\beta^I + \beta^D)}{1 + (\beta^I + \beta^D)(\phi^I + \phi^D)}.$$ (2.21)

We can then define

**Definition 2.3 (Socially optimal $P$)**

A given set of policy parameters $P$ is called socially optimal if it is feasible and the DCR members implement the tax goal under this $P$.

### 2.5.2. General Characterization

We now examine whether it is possible to find feasible policy parameters $P$ for which the countries implement the tax goal. The industrial country $I$ minimizes its total costs

$$F^I(\tau^I) := \frac{\phi^I}{2}(\tau^I)^2 + \beta^I s^2 + \tau^I(\bar{e} - \phi^I \tau^I) - \alpha f \sum_j \frac{\omega^I j \tau^I}{\omega^I j \tau^I} + f_0^I - (1 - \alpha)f$$ (2.22)

with respect to $\tau^I$, subject to equation (2.6) and $\tau^I \geq 0$, given the set of policy parameters $P$ and the choices of the other country. The developing country $D$ minimizes its total costs

$$F^D(\tau^D) := \frac{\phi^D}{2}(\tau^D)^2 + \beta^D s^2 + \tau^D(\bar{e} - \phi^D \tau^D) - \alpha f \sum_j \frac{\omega^D j \tau^D}{\omega^D j \tau^D}$$ (2.23)

with respect to $\tau^D$, subject to equation (2.6) and $\tau^D \geq 0$, and given $P$ and $\tau^I$.

To construct a socially optimal $P$, the policy parameters $\alpha, f_0^I$ and $\omega$ have to fulfil the feasibility conditions, and first- and second-order conditions have to hold under the assumption that countries implement the tax goal. This existence problem is dealt with in Appendix A.1. There, we give a simple sufficient condition for the existence, and we show that non-existence of socially optimal policy schemes can occur when $\phi^I$ is very small. In such circumstances, the industrial country has very high abatement costs and
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thus high emissions. Therefore, it has limited incentives to choose the socially optimal
tax rate, as it owes the fund high tax revenues while the impact on damages is small. In
that case, it may be impossible to induce the industrial country to choose the socially
optimal tax rate without distorting the incentives for the developing country.\footnote{However, even if socially optimal policy parameters do not exist, we show in Appendix A.1 that there are feasible policy parameters that induce the industrial country to set socially optimal taxes and the developing country to abate maximally.}

2.5.3. Developing Country as Net Receiver

In the next step we examine equity objectives. In particular, we derive conditions under
which the developing country is a net receiver of funds, which means that its refunds are
higher than or equal to its tax payments. Hence the developing country is a net receiver
if

\[
\alpha f \frac{\omega^D \tau^D}{\sum_j \omega^j \tau^j} \geq \tau^D (\bar{e} - \phi^D \tau^D).
\]

We obtain one of our main results:

**Proposition 2.3**

Suppose there exists a socially optimal policy scheme \( P = \{\alpha, f^I_0, \bar{\omega}^{NR}\} \) for some \( \omega^{NR} > 0 \). Then, by agreeing on a sufficiently small \( \omega \leq \bar{\omega}^{NR} \), the countries can always find a socially optimal policy scheme such that the developing country is a net receiver of funds.

The proof can be found in Appendix A.2.

Proposition 2.3 shows that one can align efficiency and equity objectives by choosing a
socially optimal policy scheme with a sufficiently small value of \( \omega \). This increases the
refunding share of the developing country, thus becoming a net receiver of funds. In
Section 2.6.1 we will see that if countries are homogeneous with respect to abatement
costs and damages, the developing country is a net receiver under every socially optimal
policy scheme.

The initial fee \( f^I_0 \) tends to infinity and \( \alpha \) is strictly smaller than one if \( \omega \) approaches zero. Hence a socially optimal policy scheme for which the developing country is a net
receiver of funds requires a high initial fee to generate a sufficient amount of refunds
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for the industrial country to preserve their incentives to abate at the socially desirable level.

In the following corollary, we will see that $\omega$ does not need to be very small to make the developing country a net receiver.

**Corollary 2.3**

Suppose that $\omega^i = \phi^i, i \in \{I, D\}$, i.e. $\omega = \phi^I / \phi^D$. Then, under any socially optimal policy scheme $P$, the developing country is a net receiver of funds.

The proof can be found in Appendix A.2.

2.5.4. Developing and Industrial Country as Net Beneficiary

In the next step we examine socially optimal policy schemes for which the developing country is a net beneficiary of the international treaty with refunding scheme, i.e. it is not worse off with the DCR than with the decentralized solution. This can be expressed formally by

$$F^{DS,D}(\hat{\tau}^D) \geq F^D(\tau^*) \, .$$

(2.25)

Analogously to Proposition 2.3, we obtain:

**Proposition 2.4**

Suppose there exists a socially optimal policy scheme $P = \{\alpha, f^I_0, \bar{\omega}^{NB}\}$ for some $\bar{\omega}^{NB} > 0$. Then, by agreeing on a sufficiently small $\omega \leq \bar{\omega}^{NB}$, we can always find a socially optimal policy scheme such that the developing country is a net beneficiary of the DCR.

The proof can be found in Appendix A.2.

We note that there exist constellations of exogenous model parameters for which a socially optimal policy scheme makes the developing country worse off, compared to the decentralized solution. This occurs when $\phi^I$ is very small in comparison with $\phi^D$, and $\omega$ is not low enough.\(^{12}\) Then the developing country almost bears the entire costs of abatement and receives a small refund. However, this is not a concern, as Proposition 2.4 implies that the countries can always agree on a sufficiently low value of $\omega$, such that

\(^{12}\)For the parameter values $\beta^I = 0.14, \beta^D = 0.15, \phi^I = 1 \cdot 10^{-9}, \phi^D = 1$, the conditions from Lemma A.1 in Appendix A.1 represent an upper bound on $\omega$ equal to $1.817$. Inserting the equilibrium values and the upper bound of $\omega$ yields $F^{DS,D}(\hat{\tau}^D) < F^D(\tau^*)$.  

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the developing country can be compensated for its large share of abatement costs by maximizing its refunding share.

Note also that the DCR can make both the industrial and the developing country net beneficiaries of the DCR:

**Corollary 2.4**

Suppose there exists a socially optimal policy scheme \( \mathcal{P} = \{\alpha, f_I^0, \omega\} \) such that the developing country is not a net beneficiary. Then there exist socially optimal policy schemes that make both the developing and the industrial country better off compared to the decentralized solution.

The proof can be found in Appendix A.2. The assumption of Corollary 2.4 is comparatively weak as the welfare of the developing country can always be lowered by setting \( \omega \) sufficiently high.

In the next section, we look at a variety of special cases.

### 2.6. Special Cases

#### 2.6.1. Homogeneous Countries

In this subsection we assume that countries are symmetric regarding the parameters describing their damage and abatement costs, i.e. we assume \( \beta^I = \beta^D = \beta \) and \( \phi^I = \phi^D = \phi \). We obtain

**Proposition 2.5**

Suppose \( \beta^I = \beta^D = \beta \) and \( \phi^I = \phi^D = \phi \). Then there always exists a socially optimal set of policy parameters \( \mathcal{P} \). Such policy schemes satisfy \( \omega \leq 1 \), i.e. \( \omega^I \leq \omega^D \). For \( \omega^I = \omega^D \) we have \( \alpha = 1 \).

The proof can be found in Appendix A.2.

Proposition 2.5 implies that a DCR exists when countries are identical regarding damages and abatement costs. Such a scheme requires a refunding rule where the weight of the developing country is larger than or equal to that of the industrial country. The intuition for this runs as follows: From an efficiency point of view, both countries should abate to the same degree, i.e. set their taxes to \( \tau^I = \tau^D = \tau^* \). For \( \alpha < 1 \), the industrial country
has higher incentives to tax emissions than the developing country, as it will receive the residual fund at the end which is larger the larger the tax revenues are. In order to induce the developing country to set the same emission tax if \( \alpha < 1 \), the weight in the refunding formula has to be higher for the developing country in order to increase its refunds.

The following corollary shows that the developing country is a net receiver of funds and a net beneficiary of the DCR for any choice of \( \omega \) that belongs to a socially optimal policy scheme.

**Corollary 2.5**
Suppose \( \beta^I = \beta^D = \beta \) and \( \phi^I = \phi^D = \phi \). The developing country is a net receiver of funds and a net beneficiary under any socially optimal policy scheme.

The proof can be found in Appendix A.2.

As the refunding scheme implements the socially optimal emission tax, the developing country benefits in the homogeneous case from the scheme in two ways: It is a net receiver of money and its total costs (abatement costs and damages) are lower than in the decentralized solution.

### 2.6.2. Identical Abatement Costs and Heterogeneous Damages

In this subsection we assume that the countries display identical abatement costs, i.e. \( \phi^I = \phi^D = \phi \), and that damages are extremely unequal, i.e. \( \beta^I = 0 < \beta^D = \beta \). We obtain:

**Proposition 2.6**
Suppose that \( \phi^I = \phi^D = \phi \) and \( \beta^I = 0 < \beta^D = \beta \). Then there always exists a socially optimal set of policy parameters \( \mathcal{P} \). In particular, scheme \( \mathcal{P} = \{\alpha, f_0^I, 1\} \) with

\[
\alpha = \frac{\bar{e} - 3\phi \tau}{\bar{e} - 2\phi \tau},
\]

\[
f_0^I = 2\phi \tau^2 \frac{\bar{e} - \phi \tau}{\bar{e} - 3\phi \tau},
\]

where \( \tau = \tau^* \), is socially optimal.

The proof can be found in Appendix A.2.
Note that in contrast to Subsection 2.6.1, it is possible to construct socially optimal policy schemes with $\omega = 1$ and $\alpha < 1$.

**Corollary 2.6**
Suppose $\phi^I = \phi^D = \phi$ and $\beta^I = 0 < \beta^D = \beta$. Under a socially optimal policy scheme, the developing country is a net receiver of funds if and only if $\omega^I \leq \omega^D$. Moreover, the developing country is a net beneficiary under any socially optimal policy scheme.

The proof can be found in Appendix A.2.

**2.6.3. No Initial Fees and Complete Refunding**

It is important to stress that the presence of initial fees $f^I_0 > 0$ is in general necessary to induce socially optimal abatement levels. We illustrate this fact by considering the case $f^I_0 = 0$ and $\alpha = 1$.\(^\dagger\)

**Proposition 2.7 (No initial fees and no residual fund)**
Suppose $f^I_0 = 0$ and $\alpha = 1$. Then a socially optimal policy scheme $P$ exists only if

\[
\begin{align*}
4\tau(\bar{e} - (\phi^I + \phi^D)\tau)^2(\bar{e} - \frac{\phi^D \beta^I + 2\beta^D}{\beta^I + \beta^D} \tau) & - \tau(2\bar{e} - (\phi^I + \phi^D)\tau) \\
\frac{2\tau(\bar{e} - (\phi^I + \phi^D)\tau)(\bar{e} - 2\phi^D\tau)}{\bar{e} - \frac{\beta^I \phi^D + 2\beta^D}{\beta^I + \beta^D}\tau} & = 0 ,
\end{align*}
\]

where $\tau = \tau^\ast$. Such a policy scheme will always fulfil $\omega \geq 1$.

The proof can be found in Appendix A.2. Proposition 2.7 indicates that it is only possible in knife-edge cases to induce socially optimal abatement levels when no initial fees are paid by the industrial country. An example of such a knife-edge case is that of identical countries $\phi^I = \phi^D$ and $\beta^I = \beta^D$, shown in the following corollary. Hence, initial fees from industrial countries help to achieve socially optimal emission abatements and equity objectives.

\(^\dagger\)We note that the theoretical case $f^I_0 = 0$ and $\alpha < 1$ would imply that the industrial country receives residual funds even if it does not pay an initial fee. As this would be a dramatic violation of a development-compatible refunding scheme, we neglect this case.
Corollary 2.7
For homogeneous countries \( \phi^I = \phi^D = \phi, \beta^I = \beta^D = \beta \), there exists a socially optimal policy scheme \( P \) where no initial fees are paid and \( \alpha \) is equal to one. It is given by \( P = \{1, 0, 1\} \).

The proof can be found in Appendix A.2. For almost all other constellations of exogenous model parameters, a socially optimal policy scheme \( P \) with \( f_0^I = 0 \) and \( \alpha = 1 \) does not exist. Examples are countries with identical abatement costs \( \phi^I = \phi^D \) and heterogeneous damages \( \beta^I < \beta^D \).

The interpretation of the property \( \omega \geq 1 \) in Proposition 2.7 is as follows: As \( \phi^I \leq \phi^D \), the industrial country abates less than the developing country when taxes are equal and therefore contributes more to the fund. Furthermore, as \( \beta^I \leq \beta^D \), the industrial country is less affected by damages caused by higher emissions than the developing country. Hence the industrial country has fewer incentives to select the socially optimal tax rates. To counteract these weaker incentives, the industrial country receives a higher share of the fund than the developing country.

2.7. Refunding Schemes without Tax Revenues

In this section we consider the potential of a refunding scheme that foregoes claiming tax revenues from the member countries.

The simplest refunding scheme is when the industrial country pays an initial fee of \( f_0^I \) that is then refunded to the countries according to the relative emission abatement they achieve.

Proposition 2.8
Under a refunding scheme without drawing on tax revenues, there exists a socially optimal policy scheme \( P = \{\alpha, f_0^I, \omega\} \) if and only if it holds that

\[
\frac{\beta^I}{\phi^I} = \frac{\beta^D}{\phi^D}. \tag{2.27}
\]

The proof can be found in Appendix A.2.

The reason why condition (2.27) has to hold can be identified by investigating the externalities at work. We focus on the case \( \omega^I = \omega^D \). First, there is the positive
environmental damage externality: If one country abates more, the damage for the other country decreases. As $\beta_D \geq \beta_I$, the developing country benefits more from abatement by the industrial country. Second, for equal taxes $\tau_I = \tau_D$, the industrial country abates less than the developing country because its abatement costs are higher ($\phi_I \leq \phi_D$). These two effects balance each other if the relationship between marginal damages and marginal abatement costs is equal for both countries, as given in equation (2.27). By varying the level of $f_0^I$ and by exploiting the negative refunding externality, the abatement levels of both countries can be raised to socially optimal levels.

**Corollary 2.8**

Suppose a socially optimal refunding scheme $P = \{\alpha, f_0^I, \omega\}$ without claiming tax revenues exists.

(i) Then, the developing country is a net receiver under such a scheme if and only if

$$\omega \leq \frac{2\beta_D \phi_I}{1 + \beta_I \phi_I - \beta_D \phi_D - 2\beta_D \phi_D}. \quad (2.28)$$

(ii) There exists a $\bar{\omega}_{NT} > 0$ such that the developing country is a net beneficiary under such a scheme if and only if $\omega \leq \bar{\omega}_{NT}$.

The proof can be found in Appendix A.2.

As in Proposition 2.3 and 2.4, the developing country will be a net receiver of funds and a net beneficiary if its weight $\omega_D$ in the refunding formula is sufficiently large relative to the weight $\omega_I$ of the industrial country.

### 2.8. Discussion

We have suggested an approach to international negotiations that can solve the compliance and participation problems of developing countries with regard to climate treaties. In this section, we address several concerns as to the applicability of the proposed scheme.

**Other policy instruments**

While our basic scheme relies on emission taxes, the variant presented in Section 2.7 that forgoes claiming tax revenues can operate with any policy instrument a country
uses to reduce emissions. The reason is that for the refunding scheme in Section 2.7, only the initial fund and emission abatements matter for refunding. Hence, cap-and-permit trade, command-and-control regulation or emission taxes can be employed by a country, and international coordination of policy instruments is not needed.\footnote{Using emission taxes as policy instrument has the advantage that refunding is less vulnerable to cheating as refunds can be based directly on tax rates (see equation (2.18)). Cheating on tax rates is more difficult than manipulating abatement efforts as tax rates are single numbers in a governmental law.}

**Multiple countries**

The proposed scheme can be generalized to multiple countries of each type. The refunding formula for a set \( \Omega_I \) of industrial countries and a set \( \Omega_D \) of developing countries is given by

\[
  r^i = \alpha f \left( \frac{\omega^i}{\sum_{j \in \Omega_D} a^j} + \frac{\omega^I}{\sum_{j \in \Omega_I} a^j} \right),
\]

where \( r^i \) is the refund to country \( i, \ i \in \{I, D\} \).\footnote{Classification of countries into the groups \( \Omega_I \) and \( \Omega_D \) is given for the majority of countries, but is less clear for the group of emerging markets. A GDP per capita threshold can serve as classification criterion. A tight definition of industrial countries would classify emerging markets as developing countries.} The following observations show how our results from a two-countries setting generalize to the present setting. As \( r^i \) determines the incentive of country \( i \) to abate, the more countries involved, the smaller the share an individual country receives from \( \alpha f \). Incentives are restored if \( \alpha f \) is increased accordingly. This occurs automatically when tax revenues are collected and if one half of the countries is industrialized and the sum of initial fees thus increases accordingly. If only a few industrial countries are present, the initial fees per country would have to increase to sustain socially optimal incentives to abate.

The presence of multiple industrial countries, however, raises concerns about their willingness to pay initial fees, which we will address next.

**Participation of industrial countries**

As reducing greenhouse gases is a global public good, the development-compatible refunding scheme does not eliminate the incentives of an industrial country to free-ride on other industrial countries’ initial fee payments. However, it relocates the problem to one time and place, i.e. in the payment of the initial fees. With more than one country, this requires coordination. The common procedure to achieve such coordination among a smaller group of countries, say twenty larger industrial countries, must make each
2. Climate Policy and Development

country pivotal for success. The problem can be described by the following two-stage game:

**Stage 1**

a.) Signing of the treaty and payment of initial fees.

b.) The treaty becomes effective if all industrial countries sign it. Otherwise it is cancelled.\(^{16}\)

**Stage 2** Abatement decision.

It is straightforward to see that there exists a unique subgame perfect equilibrium in which all countries sign and the industrial countries pay initial fees.

**Participation of developing countries**

As developing countries participate voluntarily, special efforts to induce them to participate are not necessary. However, the opposite problem may occur. There may be instances where the dictator of a developing country achieves emission reductions by means that should not allow to claim refunds. For example, a dictator may ruin the economy of his country, thereby reducing emissions. Granting refunds in such circumstances would perpetuate poverty. Hence, it may be required that developing countries loose their right to claim refunds, or may not be admitted to the refunding system at all if they pursue such a strategy.

**Frictions in implementation**

We have assumed that in developing and industrial countries, emission reductions can be verified without any costs. However, measuring the emission reductions achieved by a country is a non-trivial task in practice. Countries have incentives to report higher emission abatement than what they actually achieved. Therefore the agency administering the refunds must be equipped with a review board that is able to monitor these reductions properly.

**2.9. Conclusion**

The successor to the Kyoto Protocol should promote voluntary abatement by developing countries. Our proposal calls for industrial countries to set up a global fund. Competition of industrial and developing countries for refunds yields the socially optimal solution.

\(^{16}\)If countries have already paid initial fees, those would be returned.
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The development-compatible refunding system still requires coordination among industrial countries to pay the initial fees into the global fund. It would appear, however, that such coordination is a substantially smaller problem than world-scale negotiations in the style of the Kyoto Protocol.
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Generalizations

3.1. Introduction

In this chapter we extend the analysis presented in Chapter 2. We generalize the model that is used to examine the proposed climate treaty with refunding in several ways. First, the basic model of Chapter 2 is restricted to two different countries, one developing and one industrial country. In Section 3.2, we introduce an arbitrary number of countries to the model. We focus on the case of two groups of countries having the same size, and the countries differ between the two groups but not within a group. We gain the following insights. If the number of countries is large, we show that the refunding scheme achieves socially optimal taxes if the marginal damages of the countries are not too low compared to the marginal abatement costs. Then we show that under a refunding scheme with many countries that does not claim tax revenues, countries choose socially optimal tax rates if the refunds the countries receive are weighted appropriately.

Second, in the basic model of Chapter 2 countries are characterized by quadratic abatement and damage cost functions. In Section 3.3, we show that the results derived in Chapter 2 for a refunding scheme without tax revenues are preserved if we replace the cost functions by general increasing convex functions.

For the ease of presentation, we summarize the main idea of the refunding scheme described in detail in Chapter 2 and give a list of the key equations and definitions.

Refunding scheme

The rules of the refunding scheme (RS) with two countries are given as follows:

- If the industrial country decides to join the RS, it has to pay an initial fee $f_0^i$ into a global fund $f$. 

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- Members of the RS individually set their emission taxes $\tau^i$, and they owe the fund their tax revenues.

- A fraction $\alpha \in [0, 1]$ of the fund is reimbursed to the participating countries according to how much emission reductions they achieved compared to overall emission reductions.

- The remaining fraction $(1 - \alpha)$ of the fund’s assets is paid back to the industrial country.

We also analyze a simplified version of the RS where the fund consists of the initial fees of the industrial country.

**Key equations and definitions**

The refund $r^i$ country $i$ receives is determined by the following refunding formula:

$$
r^i = \alpha f \sum_{j \in RS} \frac{\omega^i \tau^i}{\omega^j \tau^j}, \quad i \in RS,
$$

(3.1)

where $a^i$ denotes the emission abatement of country $i$, and $\omega^i = \tilde{\omega}^i \phi^i$. $RS$ denotes the set of countries that joined the refunding system. Let $\omega = \omega^I / \omega^D$ be the relation between the weighting factors of the two different types of countries.

The RS can be summarized by the set of policy parameters $\mathcal{P} = \{\alpha, f^I_0, \omega\}$, as these parameters completely describe the flow of money within the refunding scheme.

**Definition 3.1 (Feasible $\mathcal{P}$)**

The set of policy parameters $\mathcal{P} = \{\alpha, f^I_0, \omega\}$ is called feasible if

$$
\alpha \in [0, 1], \quad f^I_0 \geq 0 \quad \text{and} \quad \omega > 0.
$$

We will examine whether one can find a feasible $\mathcal{P}$ that implements the socially optimal solution. For this purpose we define

**Definition 3.2 (Tax goal of RS)**

The RS’s tax goal is given by the socially optimal tax rate, i.e. the solution of the social planner problem where global abatement costs and damages are minimized.

We can then define
Definition 3.3 (Socially optimal $P$)
A given set of policy parameters $P$ is called socially optimal if it is feasible and the RS members implement the tax goal under this $P$.

3.2. Many Countries

The analysis of the refunding scheme presented in Chapter 2 draws on a world consisting of two countries, an industrial and a developing country. This set-up reveals a variety of qualitative properties of the refunding scheme. In this section we examine whether some of those characteristics can be translated into a world of more than two countries. We consider $k$ replicates of the economy, i.e. we consider $k$ industrial and $k$ developing countries, $k \geq 1$. Industrial countries face the same abatement cost function and the same damage cost function. The same holds true for developing countries. We denote the set of industrial and developing countries by $\Omega_I$ and $\Omega_D$ respectively. The abatement cost functions are characterized by the marginal abatement parameters $\phi^i$ with $\phi^i = \phi^I \ \forall \ i \in \Omega_I$ and $\phi^i = \phi^D \ \forall \ i \in \Omega_D$, while the damage cost functions are characterized by the marginal damage parameters $\beta^i$ with $\beta^i = \beta^I \ \forall \ i \in \Omega_I$ and $\beta^i = \beta^D \ \forall \ i \in \Omega_D$. Finally, $\Omega = \Omega_I \cup \Omega_D$ denotes the set of all countries. The stock equation of the basic set-up of the model changes to

$$s = \sum_{j \in \Omega} E^j(\tau^j) = \sum_{j \in \Omega_I} (\bar{e} - \phi^I \tau^j) + \sum_{j \in \Omega_D} (\bar{e} - \phi^D \tau^j).$$

We again assume that the initial stock is equal to zero as this does not alter our results qualitatively.

The remaining section is organised as follows. In the next subsection we derive the social optimum and the decentralized solution as the two benchmark cases of the fully cooperative and the no-treaty solution. Then we extend the refunding scheme to many countries in Subsection 3.2.2. In Subsection 3.2.4 we explore the potential of the refunding scheme that renounces tax revenues of the member countries.
3.2.1. Social Optimum and Decentralized Solution

The social planner wants to minimize the global costs of abatement and damages. Hence the social planner problem reads

\[
\min_{\tau^i, i \in \Omega} \sum_{j \in \Omega} \frac{\phi^j}{2} (\tau^j)^2 + \frac{\beta^j}{2} s^2
\]

s.t. \ (3.2), \ \tau^i \in \left[0, \frac{\bar{e}}{\phi^i}\right], \ i \in \Omega .

We obtain

Proposition 3.1 (Social optimum for many countries)

Let us define \( k^* = \sqrt{(\beta^I + \beta^D)(\phi^D - \phi^I)^{-1}} \) with \( k^* = \infty \) if \( \phi^I = \phi^D \). Let \( k \in \mathbb{N} \).

(i) If \( k \leq k^* \), the optimal emission tax for both types of countries is given by

\[
\tau^* = \frac{2k^2 \bar{e}(\beta^I + \beta^D)}{1 + k^2(\beta^I + \beta^D)(\phi^I + \phi^D)}.
\]

The optimal stock equals

\[
s^* = \frac{2k\bar{e}}{1 + k^2(\beta^I + \beta^D)(\phi^I + \phi^D)} = \frac{\tau^*}{k(\beta^I + \beta^D)}.
\]

(ii) If \( k > k^* \), the optimal emission tax for the \( k \) industrial countries is given by

\[
\tau^*_{cI} = \frac{k^2 \bar{e}(\beta^I + \beta^D)}{1 + k^2(\beta^I + \beta^D)\phi^I},
\]

and the optimal emission tax for the \( k \) developing countries is given by

\[
\tau^*_{cD} = \frac{\bar{e}}{\phi^D}.
\]

The optimal stock equals

\[
s^*_{c} = \frac{k\bar{e}}{1 + k^2(\beta^I + \beta^D)\phi^I} = \frac{\tau^*_{cI}}{k(\beta^I + \beta^D)}.
\]

The proof is given in Appendix B.
Proposition 3.1 shows that two cases can occur. In case (i), the socially optimal tax rate $\tau^\star$ is the same across countries. Marginal abatement costs of developing and industrial countries differ. Therefore, developing countries contribute more to overall emission reductions than industrial countries in the social optimum. If the number of countries per group, $k$, exceeds the threshold value $k^\star$, the developing countries would have to abate more than baseline emissions $\bar{e}$ if we apply the first-order conditions. This is not feasible. As a consequence, case (ii) occurs and the corner solution applies. This effect becomes more severe if marginal abatement costs of industrial countries increase compared to those of the developing countries. Then case (ii) of Proposition 3.1 already applies for a smaller number of countries as the industrial countries’ contribution to overall emission reductions decreases and developing countries cannot increase their contribution any further due to the feasibility constraint.

The loss in emission reductions due to the feasibility constraint of the developing countries is only partly absorbed by the industrial countries, which can be seen in

**Corollary 3.1**

If $k > k^\star$, we have $s^\star < s^\star_e$ and $\tau^\star_I > \tau^\star > \tau^\star_D$.

The proof of Corollary 3.1 is straightforward and therefore omitted.

Note also that if marginal abatement costs are the same for both types of countries, i.e. if $\phi^I = \phi^D = \phi$, only case (i) applies. The reason for this runs as follows: In case of symmetric marginal abatement costs, not only the optimal tax rate but also the optimal abatement is symmetric across countries. For all $k < \infty$ it is never optimal to have an emission stock equal to zero as decreasing maximal abatement slightly would yield only a small increase in damages, but abatement costs would be reduced considerably. Therefore, countries will never abate maximally, i.e. in the optimum it will never occur that all countries set the corner solution $\tau^i = \bar{e}/\phi$.

In the decentralized solution, each country seeks to minimize its own costs of abatement and damages, given the choice of all other countries. Therefore, the problem of country $i$ reads

$$\min_{\tau^i} \frac{\phi^i}{2} (\tau^i)^2 + \frac{\beta^i}{2} s^2 \quad \text{(3.9)}$$

s.t. \ (3.2), $\tau^i \in \left[0, \frac{\bar{e}}{\phi^i}\right]$, given $\tau^j$, $j \in \Omega \setminus \{i\}$.
We obtain

**Proposition 3.2 (Decentralized solution for many countries)**

Let us define \( \hat{k} = (\beta^D \phi^D - \beta^I \phi^I)^{-1} \) with \( \hat{k} = \infty \) if \( \beta^I = \beta^D \) and \( \phi^I = \phi^D \). Let \( k \in \mathbb{N} \).

(i) If \( k \leq \hat{k} \), there exists a unique Nash equilibrium characterized by \( \hat{\tau}^i \) for each country \( i \in \Omega \):

\[
\hat{\tau}^i = \frac{2k\bar{e} \beta^i}{1 + k(\beta^I \phi^I + \beta^D \phi^D)}.
\]

The equilibrium stock equals

\[
\hat{s} = \frac{2k\bar{e}}{1 + k(\beta^I \phi^I + \beta^D \phi^D)}.
\]

(ii) If \( k > \hat{k} \), there exists a unique Nash equilibrium characterized by the optimal emission tax for the \( k \) developing countries, given by

\[
\hat{\tau}^D_c = \frac{\bar{e}}{\phi^D}.
\]

and the optimal emission tax for the \( k \) industrial countries, given by

\[
\hat{\tau}^I_c = \frac{k\bar{e} \beta^I}{1 + k\beta^I \phi^I}.
\]

The equilibrium stock equals

\[
\hat{s}_c = \frac{k\bar{e}}{1 + k\beta^I \phi^I}.
\]

The proof can be found in Appendix B.

We observe from (3.10) that similar to the social optimum, but even more pronounced, the contribution to overall emission reductions is higher for the developing countries. The reason is that developing countries have higher incentives to reduce emissions than industrial countries as abatement is cheaper and they benefit more from emission reductions due to higher damages. If \( k \) exceeds the threshold \( \hat{k} \), emission reductions by the developing countries according to the first-order conditions would violate the feasibility constraint. Then the corner solution in case (ii) of Proposition 3.2 applies. Note that if case (i) occurs, countries abate more in total than if case (ii) occurs:
Corollary 3.2
Let $k > \hat{k}$. Then we have $s < \hat{s}_c$ and $\hat{\tau}_I^c > \hat{\tau}_I^I$, $\hat{\tau}_D < \hat{\tau}_D^I$.

The proof of Corollary 3.2 is straightforward and therefore omitted.

Following a similar argument as for the social optimum, case (i) of Proposition 3.2 applies for all $k$ if countries are homogeneous, i.e., if $\beta_I = \beta_D$ and $\phi_I = \phi_D$.

Comparison of the social optimum with the decentralized solution yields

Corollary 3.3 (Comparison of social optimum and decentralized solution)
(i) The emission stock of the decentralized solution is always larger than the emission stock of the social optimum.

(ii) $s^*/s$ and $s^*_k/s_k$ is monotonically decreasing in $k$.

(iii) $\hat{\tau}_I < \tau^*$ and $\hat{\tau}_c^I < \tau_c^I$.

(iv) $\hat{\tau}_D < \tau^* \Leftrightarrow k\beta_I + (k-1)\beta_D + k^2\phi_I(\beta_I + \beta_D)(\beta_I - \beta_D) > 0$ and $\hat{\tau}_c^D = \tau_c^D$.

The proof can be found in Appendix B.

Corollary 3.3 (i) and (ii) reveal the standard underprovision of the public good “emission abatement” in the decentralized solution. The underprovision increases in relative terms if the number of countries increases. Note that (i) holds for all $k$. Corollary 3.3 (iii) and (iv) show that industrial countries abate always less in the decentralized solution, compared to the social optimum, whereas for the developing countries it can be ambiguous. If the marginal damages of the industrial countries are very low compared to the marginal damages of the developing countries, $\hat{\tau}_D$ may exceed $\tau^*$. The intuition for this runs as follows: The industrial countries abate less due to relatively small damage costs. Then, developing countries compensate for the resulting higher stock of GHGs and therefore higher damages by abating even more than in the global optimum where the costs are borne by both types of countries. This effect becomes more severe if the number of countries increases.

3.2.2. Refunding Scheme for Many Countries

Now we introduce the refunding scheme for many countries (RSMC). It works as follows:
If an industrial country decides to join the RSMC, it has to pay an initial fee $f_i^0$ into a global fund $f$.

Members of the RSMC individually set their emission taxes $\tau^i$, and they owe the fund their tax revenues.

A fraction $\alpha \in [0, 1]$ of the fund is reimbursed to the participating countries according to how much emission reductions they achieved compared to overall emission reductions.

The remaining fraction $(1 - \alpha)$ of the fund’s assets is paid back in equal parts to the industrial countries that participate in the RSMC.

Two changes compared to the original set-up with two countries occur. First, the refunding formula (3.1) changes to

$$r^i = \alpha f \frac{\tilde{\omega}^i a^i}{\tilde{\omega}^i + \sum_{j \in \Omega^RS_I} \omega^j + \sum_{j \in \Omega^RS_D} \omega^j} = \alpha f \frac{\omega^i \tau^i}{\omega^i + \sum_{j \in \Omega^RS_I} \tau^j + \omega^D \sum_{j \in \Omega^RS_D} \tau^j}, \quad i \in \Omega,$$

where $\Omega^RS_I$ and $\Omega^RS_D$ denote the set of industrial and developing countries, respectively, participating in the refunding scheme.

Second, the global fund now consists of the initial fees of all participating industrial countries and the tax revenues of all participating countries. Hence

$$f = \sum_{i \in \Omega^RS_I} f_i^0 + \sum_{i \in \Omega^RS_I} \tau^i (\bar{e} - \phi^I \tau^i) + \sum_{i \in \Omega^RS_D} \tau^i (\bar{e} - \phi^D \tau^i). \quad (3.15)$$

We assume in the following that all $2k$ countries participate in the refunding system, i.e. $\Omega^RS_I = \Omega_I$ and $\Omega^RS_D = \Omega_D$. Note that, as industrial countries are homogeneous, it is reasonable to assume that all industrial countries pay the same amount of initial fee $f_i^0$, denoted by $f_0^I$. Hence $\sum_{i \in \Omega_I} f_i^0 = kf_0^I$.

As in the basic set-up with two countries, the refunding scheme with many countries can be summarized by the set of policy parameters $\mathcal{P} = \{\alpha, f_0^I, \omega\}$, which is feasible if it fulfils the conditions given in Definition 3.1. The tax goal is determined in Proposition 3.1 and given by (3.4) if $k \leq k^*$ and by (3.7) and (3.6) if $k > k^*$. As mentioned in the introduction to this chapter, the set of policy parameters $\mathcal{P}$ is socially optimal, if it is feasible and if countries implement the tax goal under this $\mathcal{P}$.
3. Climate Policy and Development – Generalizations

Now let us consider the optimization problems of the countries. An industrial country \( i \in \Omega_I \) chooses \( \tau_i \) to minimize its total costs of

\[
\frac{\phi_I}{2}(\tau_i)^2 + \frac{\beta_I}{2}s^2 + \tau_i(\bar{e} - \phi_I \tau_i) + f_0^I - r_i - \left(1 - \alpha\right)\frac{f_I}{k},
\]

subject to (3.2) and \( \tau_i \in \left[0, \frac{\bar{e}}{\phi_I}\right] \), and given the tax rates of the other countries. The developing country \( i \in \Omega_D \) chooses \( \tau_i \) so as to minimize its total costs of

\[
\frac{\phi_D}{2}(\tau_i)^2 + \frac{\beta_D}{2}s^2 + \tau_i(\bar{e} - \phi_D \tau_i) - r_i,
\]

subject to (3.2) and \( \tau_i \in \left[0, \frac{\bar{e}}{\phi_D}\right] \), and given the tax rates of the other countries.

In the following subsections, we determine under which conditions we can find socially optimal policy parameters \( \alpha, f_0^I, \) and \( \omega \).

3.2.3. Large Number of Countries

In this subsection, we examine the question whether countries implement the tax goal under the RSMC if the number of countries, \( k \), is large. We first want to restrict our attention to the tax goal given by Proposition 3.1 (i), therefore we assume that countries are homogeneous with respect to their marginal abatement costs, i.e. \( \phi_I = \phi_D \). As we have seen, case (i) then applies for all \( k \in \mathbb{N} \) since \( k^* = \infty \).

We obtain:

**Proposition 3.3 (RSMC with \( k \) large, \( \phi_I = \phi_D \))**

Assume \( \phi_I = \phi_D = \phi \) and \( k \), the number of countries, large.

(i) If \( \beta_I \phi > 1 \), there exists a socially optimal set of policy parameters.

(ii) If \( \beta_I \phi \leq 1 < \beta_D \phi \), there exists a feasible set of policy parameters such that countries implement \( \tau_j^* = \bar{e}/\phi, j \in \Omega_I \), \( \tau_j^* = \tau^* \), \( j \in \Omega_D \).

(iii) If \( \beta_D \phi \leq 1 \), there exists a feasible set of policy parameters such that countries implement \( \tau_j^* = \bar{e}/\phi, j \in \Omega \).

The proof can be found in Appendix B.
The reason for \( \beta^i \phi > 1 \) being necessary to induce implementation of the tax goal in country \( i \) can be explained as follows. If \( \beta^i \phi > 1 \), marginal damages of country \( i \) are sufficiently high compared to marginal abatement costs, which implies incentives for country \( i \) to increase its tax rate to a socially optimal level as it benefits sufficiently from emission reductions.

In Proposition 3.3 (ii) and (iii), following the reasoning above, either the industrial countries or both types of countries would prefer to change the tax rate as benefits from emission reductions are too low. However, we can find feasible policy parameters such that those countries have incentives to choose maximal abatement under the refunding scheme.

Now we extend our analysis to \( \phi^I < \phi^D \). As we consider large \( k \), we can always assume \( k > k^* \). In this case, the socially optimal tax rates are given by Proposition 3.1 (ii). We obtain

**Proposition 3.4 (RSMC with \( k \) large, \( \phi^I < \phi^D \))**

Assume \( \phi^I < \phi^D \) and \( k \), the number of countries, large. Specifically, assume \( k > k^* \).

(i) If \( \beta^I \phi^I > 1 \), there exists a socially optimal set of policy parameters.

(ii) If \( \beta^I \phi^I \leq 1 \), there exists a feasible set of policy parameters such that countries implement \( \tau^j = \tau^*_c, j \in \Omega^D \), \( \tau^j = \bar{e}/\phi^I, j \in \Omega^I \).

The proof can be found in Appendix B.

Hence also in the case of different marginal abatement costs, we can implement the socially optimal tax rates if marginal damages are not too low. In the case where marginal damages of the industrial countries are not high enough, there still exists a feasible set of policy parameters that induces countries to abate maximally.

In the next section, we show that a refunding system with many countries where countries do not owe their tax revenues to the global fund always achieves socially optimal tax rates.

### 3.2.4. Refunding Scheme without Tax Revenues

We consider a version of the RSMC where countries participating in the scheme do not owe their tax revenues to the fund. Then \( f \) only consists of the initial fees paid by the industrial countries, i.e. \( f = kf^I_0 \). We obtain
Proposition 3.5 (RSMC without tax revenues)
Suppose $k > 1$. Then, under the RSMC without tax revenues, there always exists a socially optimal set of policy parameters $P = \{f_0^I, \alpha, \omega\}$. Specifically, it satisfies

(i) If $k \leq k^*$, the policy parameter $\omega$ has to be chosen according to

\[
\omega \geq 1 \iff \phi^I - \phi^D + \frac{\beta^D \phi^D - \beta^I \phi^I}{k(\beta^I + \beta^D)} \leq 0. \tag{3.18}
\]

(ii) If $k > k^*$, the policy parameter $\omega$ has to be chosen small enough and the policy parameter $f_0^I$ high enough.

The proof can be found in Appendix B.

Investigation of (3.18) reveals that $\omega > 1$ can only hold if $\beta^I$ is not too close to $\beta^D$. Intuitively, if marginal damages differ, increasing emission taxes to the socially optimal level has a smaller positive effect on the industrial countries than on the developing countries. Therefore, industrial countries need to be compensated for the increase of taxes by receiving a higher share of refunds. This is done by setting $\omega > 1$. If, however, $\beta^I$ is close enough to $\beta^D$, the effect of increasing the tax rates is similar for both types of countries with respect to the costs of damages. As marginal abatement costs may still differ, industrial countries will only compete sufficiently for refunds and setting socially optimal taxes if their refunds are weighted downwards. This is done by setting $\omega < 1$.

Note also that in contrast to the case of $k = 1$ (see Proposition 2.8 in Chapter 2), i.e. only one developing and one industrial country, no restriction on the parameters of the model occurs. Policy parameters can be varied to induce countries to choose socially optimal taxes. If $k = 1$ and no taxes are owed to the fund, the effect of refunding is exactly the same for both countries, regardless of the weighting factor $\omega$, and the developing and the industrial country have to be similar with respect to the ratio $\beta^I/\phi^I$ to balance the different externalities at work when the tax goal is implemented. For $k > 1$, not only the abatement decision of the $k$ countries of the other type play a role, but also the decision of the $k - 1$ countries of the own type. Therefore, we can achieve the tax goal by varying the policy parameter $\omega$ appropriately.

Corollary 3.4 (No tax revenues and homogeneous countries)
If all countries are homogeneous, i.e. $\phi^I = \phi^D = \phi$ and $\beta^I = \beta^D = \beta$, then the socially optimal policy parameters of the RSMC without tax revenues satisfy
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(i) $\omega = 1$.

(ii) $f_0^I \geq 2\phi(\tau^*)^2$.

The proof can be found in Appendix B. The intuition for this result runs as follows. Both types of countries are equal with respect to their marginal abatement costs and damages. Therefore the share of refunds the countries obtain has to be the same across countries, i.e. $\omega = 1$. The lower boundary on initial fees, given in part (ii), is ensures sufficient competition for refunds of the countries.

3.3. Generalized Cost Functions

In this section, we generalize the basic set-up of the model given in Chapter 2 by replacing the original quadratic abatement and damage cost functions by arbitrary increasing convex abatement and damage cost functions.

First, we rewrite the model equations using the generalized functions. Second, we derive the social optimum and the decentralized solution as the two benchmark cases of a cooperative and a non-cooperative solution. Third, we show that a refunding scheme where the fund is solely financed by the initial fee of the industrial country achieves socially optimal emission reductions if the relation of abatement decisions of the two countries equal the relation of marginal damages in the social optimum.

As in the basic set-up, we consider a world consisting of two countries, a developing country $D$ and an industrial country $I$ that are characterized by emissions, an abatement cost function and a damage cost function. We index countries by $i$ and $j$, where $i, j \in \{I, D\}$.

Emissions are given by

$$E^i(a^i) = \bar{e} - a^i,$$

with $a^i \in [0, \bar{e}]$, $i \in \{I, D\}$.

That means they equal certain business-as-usual emissions $\bar{e}$ minus abatement $a^i$ of country $i$. We assume that in each country, emissions are produced by a representative firm that faces increasing convex abatement costs $C^i(a^i)$, $i \in \{I, D\}$, i.e. $(C^i)'(a^i) > 0$.
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and \((C^i)'(a^i) > 0\) for all \(a^i > 0\). We furthermore assume that

\[
(C^I)'(a) \geq (C^D)'(a) \quad \forall \, a \geq 0 ,
\]

(3.20)
as abatement costs are higher in industrial than in developing countries. Emissions accumulate the stock of greenhouse gases (GHGs) \(s\), i.e.

\[
s = \sum_{i \in \{I,D\}} E^i(a^i) ,
\]

(3.21)

where we have assumed that the initial stock is equal to zero. The damage caused in country \(i\) by the stock of GHGs is described by a damage cost function \(D^i(s)\), \(i \in \{I,D\}\), with \((D^i)'(s) > 0\) and \((D^i)''(s) > 0\) for all \(s > 0\). As developing countries suffer more from climate change, we assume

\[
(D^D)'(s) \geq (D^I)'(s) \quad \forall \, s \geq 0 .
\]

(3.22)

3.3.1. Social Optimum and Decentralized Solution

In this section, we derive the social optimum and the decentralized solution as benchmark cases of the fully cooperative outcome and the no-treaty solution.

The social planner seeks to minimize total costs of emission abatement and damages, hence the social planner problem can be written as

\[
\min_{a^i, a^D, s} \sum_{j \in \{I,D\}} C^j(a^j) + D^j(s) \quad \text{s.t. (3.21),} \quad a^i \in [0, \bar{e}] , \quad i \in \{I,D\} .
\]

(3.23)

We obtain

**Proposition 3.6 (Social optimum for the general model)**

There exists a unique solution \(((a^*I, a^*D), s^*)\) to the social planner problem (3.23). It satisfies \(a^*I \leq a^*D\).

The proof can be found in Appendix B.

Proposition 3.6 shows that in the social optimum the developing country abates more than the industrial country. This follows from having lower abatement costs and higher
benefits from overall emission reductions due to higher damage costs than the industrial
country.

In the decentralized solution each country minimizes its own costs of emission abatement
and damages, given the choice of the other country. Therefore, the problem of country
\(i, i \in \{I, D\}\), can be written as

\[
\min_{a^i} C^i(a^i) + D^i(s) \tag{3.24}
\]

s.t. \((3.21)\), \(a^i \in [0, \bar{a}]\), given \(a^j, j \neq i\).

We have

**Proposition 3.7 (Decentralized solution for the general model)**

There exists a unique solution \((\hat{a}^I, \hat{a}^D, \hat{s})\) to the decentralized problem \((3.24)\). It satisfies \(\hat{a}^I \leq \hat{a}^D\).

The proof can be found in Appendix B.

Comparison of the decentralized solution with the social optimum reveals the standard
underprovision of the public good “emission abatement”:

**Proposition 3.8 (Comparison)**

We have

\[
s^* < \hat{s}. \tag{3.25}
\]

The proof can be found in Appendix B.

### 3.3.2. Refunding Scheme

In this section, we examine a refunding scheme where the fund only consists of the initial
fee paid by the industrial country, \(f = f^I_0\). We focus on the question whether such a
scheme can create socially optimal emission reductions.

The refunding formula that describes the refund \(r^i\) country \(i\) obtains is given by

\[
r^i = \alpha f^I_0 \frac{\omega^i a^i}{\sum_{j \in \{I, D\}} \omega^j a^j} . \tag{3.26}
\]
As only the relationship $\omega = \omega^I / \omega^D$ of the weighting factors matters, we can rewrite the refunding formulas as

$$
I^I = \alpha f_0^I \frac{\omega a^I}{\omega a^I + a^D},
$$

$$
D^D = \alpha f_0^I \frac{a^D}{\omega a^I + a^D}.
$$

We obtain

**Proposition 3.9 (Refunding system for generalized cost functions)**

*There exists a feasible set of policy parameters $\mathcal{P} = \{\alpha, f_0^I, \omega\}$ such that countries choose socially optimal emission abatement under the refunding system if and only if it holds that

$$
\frac{a^*^D}{a^*^I} = \frac{(D^D)'(s^*)}{(D^I)'(s^*)} \quad (3.27)
$$

The proof can be found in Appendix B.*

To induce the countries to set socially optimal emission reductions under refunding, it is necessary and sufficient that the relation between abatement decisions equals the relation of marginal damages of the two countries in the social optimum. This condition can be interpreted as follows. If marginal damages of the developing country are high compared to those of the industrial country, also the abatement decision of the developing country has to be accordingly larger, so that the developing country will benefit from high refunds in order to be compensated for the losses from damages.

### 3.4. Conclusion

We showed that the results from Chapter 2 can be extended to a more general setting in several ways. First, the refunding scheme can achieve socially optimal emission reductions for more than two countries. We give conditions for the existence of a refunding scheme with many countries under which countries implement the socially optimal solution if the number of countries is very large. Moreover, a refunding scheme for many countries that renounces tax revenues always induces countries to set socially optimal taxes if the policy parameters are selected appropriately. Second, we show that the results of Chapter 2 on a refunding scheme without tax revenues generalizes to arbitrary increasing convex abatement and damage cost functions.
4. Climate Policy and Development –
Relaxing Critical Assumptions

4.1. Introduction

In this chapter, we draw on the model presented in Chapter 2 that is used to analyze a refunding scheme for its ability to improve on the non-cooperative solution. The basic set-up remains unchanged: we assume a world consisting of two countries, a developing and an industrial country that are characterized by emissions, by abatement and damage cost functions. In Chapter 2 Assumption 1 was imposed to simplify the subsequent analysis. This assumption restricted the space of (exogenously given) model parameters considerably. Now we extend the analysis to the case where Assumption 1 does not hold to cover the whole range of possible values the model parameters can take.

Our main results are as follows. First, we show that the refunding scheme can be designed in a way that it always implements the tax rates a social planner would choose if the socially optimal solution yields the corner solution. This is achieved by setting the initial fee of the industrial country sufficiently high. The corner solution was ruled out before by Assumption 1. It occurs if the marginal costs of the industrial country are high compared to those of the developing country, and if in addition marginal damages of the countries are not too low.

Second, the refunding scheme can induce a country to abate maximally, which effectively means that emissions are set equal to zero in that country. Such a refunding scheme entails high initial fees, which are a necessary incentive for a country to increase its tax rate to the maximal level. Maximal emission reductions in a country can be useful to enforce if the socially optimal levels of abatement are never cost-minimizing in that country by any design of the refunding scheme.
Third, we show that the implementation of socially optimal taxes always implies positive refunds. This ensures that the refunds are never liabilities of the countries.

Finally, the result of Chapter 2 on the refunding scheme without tax revenues (i.e. the fund is solely financed by the initial fee of the industrial country) does not rely on Assumption 1 and can be extended to the whole model parameter space.

For presentational convenience, we summarize the rules of the refunding scheme, restate Assumption 1, and we give a list of key equations and definitions. By and large, we use the same notation as introduced in Chapter 2.

**Refunding scheme**

The refunding scheme (RS) is described in detail in Chapter 2. We repeat its rules:

- If the industrial country decides to join the RS, it has to pay an initial fee \( f^I_0 \) into a global fund \( f \).
- Members of the RS individually set their emission taxes \( \tau^i \), and they owe the fund their tax revenues.
- A fraction \( \alpha \in [0, 1] \) of the fund is reimbursed to the participating countries according to how much emission reductions they achieved compared to overall emission reductions.
- The remaining fraction \( (1 - \alpha) \) of the fund’s assets is paid back to the industrial country if it participates in the RS.

**Assumption 1**

Let us assume

\[
\bar{e} - \frac{\beta^I \phi^D + 2 \beta^D \phi^D + \beta^D \phi^I}{\beta^I + \beta^D} \tau^* > 0 ,
\]

where \( \tau^* \) is the tax goal given by \( \tau^* = 2\bar{e} (\beta^I + \beta^D) / (1 + (\beta^I + \beta^D)(\phi^I + \phi^D)) \). Assumption 1 can be expressed in terms of model parameters:

\[
1 + \beta^I \beta^D > \beta^I \phi^D + 3 \beta^D \phi^D + \beta^D \phi^I .
\]
Key equations and definitions

The refund $r^i$ country $i$ receives is determined by the following refunding formula:

$$r^i = \alpha f \frac{\tilde{\omega}^i\phi^i}{\sum_{j \in RS} \tilde{\omega}^j\phi^j} = \alpha f \frac{\omega^i\tau^i}{\sum_{j \in RS} \omega^j\tau^j}, \quad i \in RS,$$

where $\omega^i = \tilde{\omega}^i\phi^i$, and $RS$ denotes the set of countries that joined the refunding scheme. Let $\omega = \omega^I/\omega^D$ denote the relation between the weighting factors of the two different types of countries.

The RS can be summarized by the set of policy parameters $P = \{\alpha, f^I_0, \omega\}$, as these parameters completely describe the flow of money within the refunding scheme.

**Definition 4.1 (Feasible $P$)**

The set of policy parameters $P = \{\alpha, f^I_0, \omega\}$ is called feasible if

$$\alpha \in [0, 1], \quad f^I_0 \geq 0 \quad \text{and} \quad \omega > 0.$$

We will examine whether one can find a feasible $P$ that implements the socially optimal solution. For this purpose we define

**Definition 4.2 (Tax goal of RS)**

The RS’s tax goal is given by the socially optimal tax rate $\tau^*$, i.e. the solution of the social planner problem where global abatement costs and damages are minimized.

We can then define

**Definition 4.3 (Socially optimal $P$)**

A given set of policy parameters $P$ is called socially optimal if it is feasible and the RS members implement the tax goal under this $P$.

Existence of a socially optimal $P$ is guaranteed if implementation of socially optimal tax rates minimizes the costs of the participating countries under refunding. Therefore we consider the cost functions of the industrial and the developing country. The industrial country $I$ wants to minimize abatement costs and damages, the tax payments, and the initial fee, and it wants to maximize the own refund and the remainder of the fund it receives at the end. Hence, the industrial country minimizes

$$F^I(\tau^I) := \frac{\phi^I}{2}(\tau^I)^2 + \frac{\beta^I}{2}s^2 + \tau^I(\bar{e} - \phi^I\tau^I) - \alpha f \frac{\omega^I\tau^I}{\sum_{j} \omega^j\tau^j} + f^I_0 - (1 - \alpha)f$$

(4.2)

with respect to \( \tau^I \), subject to the stock equation

\[ s = \sum_{i \in \{I,D\}} (\bar{e} - \phi^i \tau^i) \]  \hspace{2cm} (4.3)

and \( \tau^I \in \left[0, \frac{\bar{e}}{\bar{\phi}^I} \right] \), given the policy parameters \( P \) and the choices of the other country. Similarly, the developing country wants to minimize abatement costs, damages, and the tax payments, and it wants to maximize the own refund. Hence it minimizes

\[ F^D(\tau^D) := \frac{\phi^D}{2} (\tau^D)^2 + \frac{\beta^D}{2} s^2 + \tau^D (\bar{e} - \phi^D \tau^D) - \alpha \sum_j \frac{\omega^D \tau^D}{\omega^j \tau^j} \]  \hspace{2cm} (4.4)

with respect to \( \tau^D \), subject to equation (4.3) and \( \tau^D \in \left[0, \frac{\bar{e}}{\phi^D} \right] \), given \( P \) and \( \tau^I \).

**4.2. Social Optimum and Decentralized Solution**

We start by characterizing the social optimum and the decentralized solution. The social optimum is the efficiency goal of any international agreement. The decentralized solution is the outcome that prevails if no agreement is achieved.

**4.2.1. Social Optimum**

Consider a social planner seeking to maximize total welfare, i.e., to minimize the total costs of emission abatement and the sum of national damages stemming from greenhouse-gas emissions. Hence the social planner solves

\[
\min_{\tau^I, \tau^D} F^{SO}(\tau^I, \tau^D) = \sum_{j \in \{I,D\}} \frac{\phi^j}{2} (\tau^j)^2 + \frac{\beta^j}{2} s^2 \\
\text{s.t. } (4.3), \quad \tau^i \in \left[0, \frac{\bar{e}}{\phi^i} \right], \quad i \in \{I,D\}.
\]  \hspace{2cm} (4.5)

We obtain:

**Proposition 4.1 (Social optimum)**

(i) Suppose \((\beta^I + \beta^D)(\phi^D - \phi^I) \leq 1\). Then, the optimal emission tax \( \tau^* \) for both
countries equals
\[ \tau^* = \frac{2\bar{e}(\beta^I + \beta^D)}{1 + (\beta^I + \beta^D)(\phi^I + \phi^D)}, \]  

(4.6)

and the optimal stock \( s^* \) is given by
\[ s^* = \frac{2\bar{e}}{1 + (\beta^I + \beta^D)(\phi^I + \phi^D)} = \frac{\tau^*}{(\beta^I + \beta^D)}. \]  

(4.7)

(ii) Suppose \((\beta^I + \beta^D)(\phi^D - \phi^I) > 1\). Then, the optimal emission tax for the industrial country equals
\[ \tau^*_{cI} = \frac{\bar{e}(\beta^I + \beta^D)}{1 + (\beta^I + \beta^D)\phi^I}, \]  

(4.8)

and the optimal emission tax for the developing country equals
\[ \tau^*_{cD} = \frac{\bar{e}}{\phi^D}. \]  

(4.9)

The optimal stock \( s^*_{c} \) is given by
\[ s^*_{c} = \frac{\bar{e}}{1 + (\beta^I + \beta^D)\phi^I} = \frac{\tau^*_{cI}}{(\beta^I + \beta^D)}. \]  

(4.10)

The proof is given in Appendix C. As a consequence, we immediately derive:

**Corollary 4.1**

If \((\beta^I + \beta^D)(\phi^D - \phi^I) > 1\), we have \( s^*_{c} > s^* \) and \( \tau^*_{cI} > \tau^* > \tau^*_{cD} \).

The intuition for the result of Corollary 4.1 runs as follows. In case of \((\beta^I + \beta^D)(\phi^D - \phi^I) > 1\), i.e. if the difference in marginal abatement costs is high enough, together with total marginal damages not being too low, the first-order conditions for the developing country would imply negative emissions since abating emissions is cheap and it suffers from the damages. As negative emissions are not feasible, the developing country chooses the corner solution of maximal abatement \( \bar{e} \) or, equivalently, the maximal tax rate \( \bar{e}/\phi^D \).

Knowing this, the industrial country increases its tax rate to reduce damages of a high stock of GHGs. But since its abatement costs are high, the industrial country will not fully compensate for the loss in emission reductions. Therefore the stock of GHGs is higher in case (ii) of Proposition 4.1 than in case (i).

4.2.2. Decentralized Solution

Next we examine the decentralized system where the government in each country seeks to minimize national costs and damages. We look for Nash equilibria when countries simultaneously choose their emission taxes. Given the choice of the other country, country $i$ minimizes

$$F^{DS,i}(\tau^i) := \frac{\phi^i}{2}(\tau^i)^2 + \frac{\beta^i}{2}s^2$$

(4.11)

with respect to $\tau^i$, subject to equation (4.3), and $\tau^i \in \left[0, \frac{\bar{s}}{\phi^i}\right]$.

We obtain:

**Proposition 4.2 (Decentralized solution)**

(i) Suppose $\beta^D\phi^D - \beta^I\phi^I \leq 1$. Then, there exists a unique Nash equilibrium characterized by the tax rate $\hat{\tau}^i$ for each country $i \in \{I, D\}$,

$$\hat{\tau}^i = \frac{2\bar{e}\beta^i}{1 + \beta^I\phi^I + \beta^D\phi^D},$$

(4.12)

and the following equilibrium stock $\hat{s}$:

$$\hat{s} = \frac{2\bar{e}}{1 + \beta^I\phi^I + \beta^D\phi^D}.$$  

(4.13)

(ii) Suppose $\beta^D\phi^D - \beta^I\phi^I > 1$. Then, there exists a unique Nash equilibrium given by the tax rate for the industrial country,

$$\hat{\tau}^I_c = \frac{\bar{e}\beta^I}{1 + \beta^I\phi^I},$$

(4.14)

and the tax rate for the developing country,

$$\hat{\tau}^D_c = \frac{\bar{e}}{\phi^D}.$$  

(4.15)

The corresponding equilibrium stock $\hat{s}_c$ is:

$$\hat{s}_c = \frac{\bar{e}}{1 + \beta^I\phi^I}.$$  

(4.16)

The proof of Proposition 4.2 is similar to the proof of Proposition 4.1 and is therefore omitted. Proposition 4.2 immediately implies
Corollary 4.2
If $\beta^D \phi^D - \beta^I \phi^I > 1$, we have $\hat{\tau}^I_c > \hat{\tau}^I$, $\hat{\tau}^D_c < \hat{\tau}^D$ and $\hat{s} < \hat{s}_c$.

Comparing the equilibrium stocks of the social optimum and the decentralized solution immediately yields:

**Corollary 4.3**
We have $\hat{s} - s^* > 0$ and $\hat{s}_c - s^*_c > 0$.

Corollary 4.3 indicates the well-known finding that decentralized decisions on contributions to the public good “emission reduction” lead to underprovision. Abating emissions in one country creates a positive externality for the other country, as it reduces damages in all countries. In a decentralized solution, countries are not compensated for these externalities, and therefore the equilibrium stock of GHGs is always higher in the decentralized solution.

It is also useful to compare tax rates in the social optimum and in the decentralized solution:

**Corollary 4.4**

(i) $\hat{\tau}^I < \tau^*$ and $\hat{\tau}^I_c < \tau^*_c$ for all $0 < \beta^I \leq \beta^D$, $0 < \phi^I \leq \phi^D$.

(ii) $\hat{\tau}^D < \tau^* \iff \beta^I + (\beta^I)^2 \phi^I > (\beta^D)^2 \phi^I$ and $\hat{\tau}^D_c = \tau^*_c$.

The proof of Corollary 4.4 is straightforward and therefore omitted here.

The decentralized tax rate of the industrial country is always lower than the socially optimal tax rate, but note that $\hat{\tau}^D$ may exceed $\tau^*$. This occurs if the marginal damage of the industrial country compared to that of the developing country is low. Then, the industrial country chooses little abatement in the decentralized solution which, in turn, induces the developing country to set high emission taxes because it suffers from high damages stemming from a high stock of GHGs. Hence, in the decentralized solution the developing country has an even higher incentive to increase its carbon emission tax compared to the social optimum where the costs of abatement are borne by both countries.

4.3. Refunding Scheme and Existence

In this section, we examine whether the refunding scheme outlined in the introduction to this chapter and described in detail in Chapter 2 can achieve socially optimal taxes. This means we study the existence of a socially optimal set of policy parameters. The set of policy parameters $\mathcal{P}$ is socially optimal if the policy parameters $\alpha, f_0^I$, and $\omega$ fulfil feasibility conditions as given in Definition 4.1. At the same time first- and second-order conditions of minimizing both countries’ costs must hold, assuming that both countries implement the tax goal which is given by (4.6) if $(\beta^I + \beta^D)(\phi^D - \phi^I) \leq 1$ and by (4.8) and (4.9) if $(\beta^I + \beta^D)(\phi^D - \phi^I) > 1$.

If the social optimum happens to be a corner solution as derived in Proposition 4.1 (ii), existence of a socially optimal set of policy parameters is guaranteed. This is stated in the following proposition.

**Proposition 4.3**

Suppose $(\beta^I + \beta^D)(\phi^D - \phi^I) > 1$. Then there always exists a socially optimal set of policy parameters $\mathcal{P}$. It is characterized by a lower boundary on the initial fee $f_0^I$.

The proof is given in Appendix C. The result of Proposition 4.3 can be interpreted as follows. From Corollary 4.1 we know that if the corner solution of the social optimum applies, the industrial country chooses higher abatement than if the interior solution applies. Intuitively, high initial fees are needed to create incentives for the industrial country to increase its tax rate under refunding to the socially optimal level, especially when marginal abatement costs are high for the industrial country compared to the developing country (note that $(\beta^I + \beta^D)(\phi^D - \phi^I) > 1$ can only hold for marginal abatement costs not being too similar).

Hence when analyzing the existence problem, we can concentrate in the following on a socially optimal tax rate as given in Proposition 4.1 (i). For the remainder of Chapter 4 we therefore assume $(\beta^I + \beta^D)(\phi^D - \phi^I) \leq 1$.

We turn to the question whether first-order conditions of the countries’ cost minimization and implementation of the tax goal can be satisfied simultaneously. This is achieved if the policy parameters are chosen appropriately:
Lemma 4.1

Assuming that both countries implement the tax goal $\tau^*$, first-order conditions of the countries’ cost minimization hold if and only if the policy parameters $\alpha$ and $f_I^0$ satisfy

$$\alpha = \frac{(\omega + 1)\left(\bar{e} - \frac{\beta D \phi^I + 2\beta D \phi^D + \beta^I \phi^D}{\beta^I + \beta^D} \tau^*\right)}{2(\bar{e} - (\phi^I + \phi^D)\tau^*)}, \quad (4.17)$$

$$f_I^0 = \frac{2(\omega + 1)\tau^*(\bar{e} - (\phi^I + \phi^D)\tau^*) - (\omega + 1)\tau^*(\bar{e} - 2\phi^D \tau^*)}{\omega (\bar{e} - \frac{\beta D \phi^I + 2\beta D \phi^D + \beta^I \phi^D}{\beta^I + \beta^D} \tau^*)}. \quad (4.18)$$

The proof is given in Appendix C.

In the next subsection, we study the existence problem under the assumption that second-order conditions of the countries’ cost minimization are violated.

4.3.1. Failure of Second-Order Conditions

Let us consider the second-order conditions of the cost minimization of the countries. They guarantee that the solution characterized by the first-order conditions really constitutes a Nash equilibrium of our problem. We start by noting

Lemma 4.2

Assuming tax goal implementation, second-order conditions never fail for both countries at the same time.

The proof is given in Appendix C. Lemma 4.2 states that if the second-order condition is violated for one country, it will hold for the other country. That means implementation of the tax goal minimizes the costs of at least one country if we assume feasible policy parameters $P$. Now we can state

Proposition 4.4

If the second-order condition of country $i$ is violated, we can always find a feasible set of policy parameters $P$ such that country $i$ implements $\tau^i = \bar{e}/\phi^i$ and country $j$, $j \neq i$, implements $\tau^j = \tau^*$, $i, j \in \{I, D\}$. It is characterized by

(i) $\alpha$ close to one and $\omega$ close to zero if $i = I$, and
(ii) $\alpha$ close to zero if $i = D$.

In both cases (i) and (ii), $f_0^I$ has to be set appropriately high.

The proof is given in Appendix C.

Proposition 4.4 shows that even if the second-order conditions are not satisfied, we can induce the countries to set the emission tax at the highest possible level. Countries abate the maximally possible amount of emissions if the policy parameters are chosen appropriately. In the case of the industrial country, this is achieved by setting $\alpha$ close to one and $\omega$ close to zero. The latter condition means that the industrial country’s contribution to emission reductions is weighted downwards, therefore it has to set emission taxes accordingly high to still obtain sufficiently high refunds. $\alpha$ close to one means that the industrial country mainly benefits from competition for refunds, while the remainder of the fund it receives can be neglected. This provides an additional incentive for the industrial country to increase its emission tax to the maximal level. The developing country will abate maximally if $\alpha$ is close to zero. A small $\alpha$ implies small refunds, thus the developing country has to set high emission taxes to benefit sufficiently from refunds.

The next subsection concentrates on the policy parameter $\alpha$ and studies implications of $\alpha$ being feasible or infeasible.

### 4.3.2. Policy Parameter $\alpha$ and Refunds

Throughout this subsection, we assume $\omega$ to be feasible, i.e. $\omega > 0$. Then it depends on the parameters of the model $\beta^i, \phi^i, i \in \{I, D\}$, whether we can choose the policy parameter $\alpha$ in a feasible way. This is expressed by the following lemma:

**Lemma 4.3 (Feasibility of $\alpha$)**

The policy parameter $\alpha$ can be chosen to be feasible, i.e. $\alpha \in [0, 1]$, if and only if it holds that

$$2(\beta^D \phi^D - \beta^I \phi^I) \leq |1 - (\beta^I + \beta^D)(\phi^I + \phi^D)|.$$

(4.19)

The proof is given in Appendix C. Investigation of inequality (4.19) reveals that it is satisfied if marginal abatement costs are high (i.e. $\phi^i, i \in \{I, D\}$, small) together with marginal damages being small, as then also the difference in marginal abatement costs and damages between countries diminishes. If, however, there is a large difference
in marginal abatement costs or damages between both countries, condition (4.19) is violated. Then the tax goal implementation under refunding can only be sustained for a negative $\alpha$ or $\alpha > 1$.

Feasibility of $\alpha$ is always coupled with feasibility of the initial fee:

**Lemma 4.4 (Feasibility of $f^I_0$)**

If we can choose $\alpha$ to be in $[0,1]$, we can also choose $f^I_0$ to be non-negative.

The proof is given in Appendix C. The intuition for this result runs as follows. Whenever $\alpha \in [0,1]$ is satisfied, a non-negative initial fee creates incentives for at least one country to set socially optimal taxes, as countries compete for refunds and/or for the remainder of the fund, both depending positively on the initial fee.

Non-feasibility of the policy parameter $\alpha$ means that either $\alpha < 0$ or $\alpha > 1$. In the remaining section, we show that $\alpha < 0$ and $\alpha > 1$ always lead to positive refunds, i.e. $r^i > 0$ for $i \in \{I,D\}$. So even if the policy parameter $\alpha$ can not be chosen in a feasible way due to the exogenous parameters of the model, refunds are feasible in the sense that they are never liabilities of the countries to the fund. This is not the case for the remainder of the fund: Positive refunds in combination with $\alpha < 0$ or $\alpha > 1$ imply that the remainder of the fund is negative, i.e. liabilities of the industrial country to the fund.

We start by considering the case of $\alpha < 0$:

**Lemma 4.5**

If $\alpha < 0$, negative refunds $r^i < 0$, $i \in \{I,D\}$ lead to a failure of the second-order condition of the developing country.

The proof is given in Appendix C. Thus, whenever $\alpha < 0$ and negative refunds occur, implementation of the tax goal does not minimize the costs of the developing country. That means negative refunds never lead to competition for refunds and setting taxes at the socially optimal level.

Stated differently, Lemma 4.5 implies that if we can only choose $\alpha < 0$, but the second-order condition of the developing country is satisfied, we always have positive refunds $r^i$, $i \in \{I,D\}$. Note that the second-order condition of the industrial country always hold in case of the model parameters being such that $\alpha < 0$ occurs. Note also that positive refunds in combination with $\alpha < 0$ imply that the initial fee $f^I_0$ is negative.

Next we consider the case of $\alpha > 1$:

**Lemma 4.6**

If $\alpha > 1$, we can always choose $f_0^I \geq 0$.

The proof is given in Appendix C. Hence if $\alpha > 1$, we always have positive refunds $r^i$ as the fund $f$ consists of the initial fee $f_0^I$ and tax revenues, and $\omega$ was assumed to be positive.

The considerations of the last two sections are summarized in

**Proposition 4.5**

(i) If second-order conditions hold, there always exists a set of policy parameters such that countries implement socially optimal taxes and refunds are positive.

(ii) If the second-order condition of country $i$ is violated, there always exists a feasible set of policy parameters such that country $i$ chooses maximal abatement and country $j$, $j \neq i$, implements the tax goal.

### 4.4. Refunding Scheme without Tax Revenues

In this section we examine a refunding scheme that renounces tax revenues of the member countries. All rules remain the same except that the participating countries do not owe their tax revenues to the fund. Hence the fund is solely financed by the initial fee of the industrial country.

We obtain the following result:

**Proposition 4.6**

(i) Suppose $(\beta^I + \beta^D)(\phi^D - \phi^I) \leq 1$. There exists a socially optimal set of policy parameters $\mathcal{P}$ for the refunding scheme without tax revenues if and only if

$$\frac{\beta^I}{\phi^I} = \frac{\beta^D}{\phi^D}. \quad (4.20)$$

(ii) Suppose $(\beta^I + \beta^D)(\phi^D - \phi^I) > 1$. There exists a socially optimal set of policy parameters $\mathcal{P}$ for the refunding scheme without tax revenues if and only if

$$\frac{\beta^I}{\phi^I} \leq \left(\frac{\tau^*}{\tau^*_{cD}} \phi^D \phi^D \right)^2 \quad (4.21)$$
The proof is given in Appendix C.

We observe that in the case (i) of Proposition 4.6 where the tax goal is given by the interior solution of the social optimum, the result equals the result of Proposition 2.8 in Chapter 2: The relationship of marginal damages and marginal abatement costs has to be the same across countries to make implementation of socially optimal emissions possible under refunding without tax revenues.

In the case of part (ii) of Proposition 4.6 where the tax goal is given by the corner solution of the social optimum, the relationship of marginal damages and marginal abatement costs of the industrial country has to stay below an upper boundary. Intuitively, the industrial country only has incentives to increase its emission reductions to the socially optimal level if marginal abatement costs are sufficiently small (i.e. $\phi^I$ high enough) compared to marginal damages.

### 4.5. Conclusion

In this chapter we provide a detailed analysis of the refunding scheme presented in Chapter 2 if Assumption 1 is abandoned. We extend the existence analysis of a refunding scheme under which countries implement socially optimal emission taxes to the whole model parameter space. We show that, even if we allow infeasible policy parameters, a refunding scheme that achieves socially optimal emission taxes always entails positive refunds. Furthermore, we show that a refunding scheme that renounces tax payments of the member countries can achieve socially optimal emission taxes if the ratio between marginal abatement costs and damages of the two countries satisfy a certain relationship to each other.
5. Sustainable Climate Treaties*

5.1. Introduction

International treaties on the provision of global public goods are plagued by the fundamental free-riding problem: each country’s contribution will benefit all countries in a non-exclusive and non-rival manner. This prisoner’s dilemma aspect and the absence of a supranational authority makes international coordination both crucial and exceptionally difficult to achieve. Countries may either lack the incentive to sign an agreement and benefit from the signatories’ contributions or they may have incentives not to comply with promises made in an agreement.

In long-run problems extending over decades or even centuries, such as mitigating anthropogenic climate change, a second problem arises. Even if the free-riding problem has been solved, little is achieved if the international community fails to agree on a subsequent agreement when the first has expired. With respect to anthropogenic climate change, this is precisely the problem we face today. Although the end of the Kyoto Protocol is nigh,¹ the international community failed to agree on a subsequent international agreement to reduce greenhouse gas emissions both in December 2009 in Copenhagen and a year later in Cancún.

In this chapter we propose and analyze a treaty design involving refunding, which we call a refunding scheme (henceforth RS). The main idea of the RS is that all countries pay an initial fee into a global fund that is invested in long-run assets. Countries maintain full sovereignty over how much emissions they abate each year and what policy measures they use to do so. At the end of each year, part of the fund is paid out to countries

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*This chapter is based on Gersbach et al. (2011).

¹In the Kyoto Protocol – the first international treaty on reducing greenhouse gas emissions with binding emission targets – the industrialized countries of the world, so called Annex B countries, committed themselves to a reduction of greenhouse gas (GHG) emissions by 5.2% against 1990 levels over the period from 2008 to 2012.
in proportion to the relative GHG emission reductions they have achieved in that year. We show that a suitably selected RS establishes a sustainable solution to the free-rider problem, so the fund will never be exhausted. A sustainable solution can be achieved in two ways. Emissions can be at the socially optimal level in each period (first-best sustainable solution) or emissions converge to their socially optimal level in the long run (second-best sustainable solution). In the latter case, the refund is equal to the interest earned on the fund. Both these sustainable solutions share one property: once the refunding scheme is established, no further coordination is required, except in administering the system, measuring reductions, and investing and distributing money. By construction, both schemes will last forever, as the fund will never be exhausted.

Our main formal results are as follows: First, the globally optimal solution minimizing the discounted values of global abatement costs and global damages prescribes uniquely determined emission abatement efforts for each period and each nation such that the global stock of GHGs converges to a steady state called long-run desired stock. If there is no treaty, countries will choose levels of abatement that fall considerably short of the globally optimal solution, while the stock of GHGs converges to a steady state well above the long-run desired stock.

Second, we show that initial fees and a feasible sequence of refunds can be devised in such a way that the RS implements socially optimal abatement levels in each period. We call this treaty the first-best sustainable RS. We also explore the potential of a particularly simple RS. In each period, the interest yields of the initial fees invested at a constant interest rate are refunded to the countries. We determine the amount of initial fees that induces convergence to the long-run desired stock. We call such a scheme second-best sustainable RS, as although countries do not choose socially optimal abatement levels in all periods, in the long run the abatement levels and also the stock of GHGs will converge to the social optimum. Both treaties provide a sustainable solution for the climate change problem. The main intuition is as follows: Nations are free to choose low or even zero abatement levels in one or more periods, but then they will forfeit refunds for that period. If the initial fees are sufficiently large, countries will choose high abatement levels, thereby benefiting from correspondingly large refunds. In the first-best sustainable RS, refunds can be adjusted so that countries choose socially optimal abatement levels in each period, whereas in the second-best sustainable RS, initial fees are chosen in such a way that the countries’ emission abatement levels will converge to the socially optimal levels in the long run.
5. Sustainable Climate Treaties

Third, as the initial amount of money the countries have to pay may be quite large, especially in the case of the second-best sustainable RS, we show that the same solution can be obtained if countries periodically pay a fixed amount into the fund, which will be smaller, the shorter the duration is between two payments made by a nation. Additionally, we suggest different ways of financing the initial fees that are neutral to taxpayers and international capital markets.

Fourth, as countries may want to renounce paying the initial fee and not sign the treaty, initial participation requires that countries be pivotal for the formation of the RS. That is, if any country defects, the treaty will fail. Then initial participation will be part of a subgame perfect equilibrium.

The starting point for our scheme and its analysis is the large body of game-theoretic literature on the formation of international and self-enforcing environmental agreements as there is no supranational authority to ensure participation and compliance. This literature has provided valuable insights into the potentialities and limitations of international environmental agreements. The literature also suggests that a large coalition will achieve only modest abatement efforts or will fail to enter into force (Asheim et al., 2006). Our approach complements this literature by suggesting a procedure that enables a coalition to achieve its emission reduction objectives after the coalition has been formed. We suggest that even large coalitions can achieve substantial emission reductions if they are able to set up an agency that has the power to administer a refunding scheme on which coalition members have previously agreed.

The remaining chapter is organized as follows: In the next section, we set up our model, for which in Section 5.3 we derive the social optimum and the decentralized solution as benchmark cases. The refunding scheme is introduced in Section 5.4, where the existence of the first- and second-best sustainable RS is also established. In Section

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2In practice, of course, only industrial countries will be called upon to set-up the climate fund. But even the participation of industrial countries remains a thorny issue, as we will discuss further in Section 5.6.


4Taking up a suggestion by Gersbach (2005), Gersbach and Winkler (2007) focus on refunding schemes in a two-period setting in which participating countries pay emission taxes into a global fund. In this chapter, we examine a refunding scheme in which countries only pay initial fees and focus on the long-run properties of such a scheme.
5.5 we illustrate our model numerically. In Sections 5.6 and 5.7 we discuss practical aspects of the RS, such as initial participation and how to raise initial fees. Section 5.8 concludes.

5.2. The Model

We consider a world with \( n \geq 2 \) identical countries characterized by an emission function \( E \), an abatement cost function \( C \), and a damage function \( D \) over a finite time horizon \( T \). As we consider \( T \) to be arbitrarily large, we shall also investigate the limit \( T \to \infty \).

Throughout the chapter countries are indexed by \( i \) and \( j \), and time is indexed by \( t \).

Emissions of country \( i \) in period \( t \) are assumed to equal “business-as-usual” emissions \( \epsilon \) (i.e., emissions arising if no abatement effort is undertaken) minus emission abatement \( a_i^t \):

\[
E(a_i^t) = \epsilon - a_i^t , \quad \text{with} \quad a_i^t \in [0, \epsilon] , \quad i = 1, \ldots, n , \quad t = 1, \ldots, T .
\] (5.1)

We assume that emission abatement \( a_i^t \) is achieved by enacting some national environmental policy, which induces convex abatement costs in country \( i \):

\[
C(a_i^t) = \frac{\alpha}{2} (a_i^t)^2 , \quad \text{with} \quad \alpha > 0 , \quad i = 1, \ldots, n , \quad t = 1, \ldots, T .
\] (5.2)

Global emissions, which are the sum of the emissions by all countries, accumulate the stock of greenhouse gases, \( s_t \), according to the following equation of motion:

\[
s_{t+1} = (1 - \gamma) s_t + \sum_{i=1}^{n} E(a_i^t) , \quad \text{with} \quad 0 < \gamma < 1 , \quad t = 1, \ldots, T ,
\] (5.3)

where \( \gamma \) denotes the constant and positive natural decay rate of greenhouse gases in the atmosphere. The initial stock of greenhouse gases is denoted by \( s_1 \).

The global stock of greenhouse gases in period \( t, s_t \), gives rise to strictly increasing and strictly convex damage for each country \( i \):

\[
D(s_t) = \frac{\beta}{2} s_t^2 , \quad \text{with} \quad \beta > 0 , \quad t = 1, \ldots, T .
\] (5.4)

\[5\]This is a standard short-cut way of capturing aggregate abatement costs in country \( i \) (see, e.g., Falk and Mendelsohn, 1993).
Finally, countries are assumed to discount outcomes in period $t$ with the discount factor $\delta^{t-1}$ with $0 < \delta < 1$.

5.3. Social Optimum and Decentralized Equilibrium

Before we introduce the refunding scheme (RS) in the next section, we characterize the global social optimum and the decentralized solution when no international agreement has been reached. As is well-known, the latter is inefficient because the emissions of each individual country impose negative externalities on all other countries that an individual country does not take into account when choosing the extent of its emission abatement.

Both the global social optimum and the decentralized outcome are important benchmarks in evaluating the performance of potential international agreements. While the decentralized outcome is realized if no agreement takes place, the social optimum is the ultimate goal an international agreement seeks to implement. Obviously, any agreement has to outperform the decentralized outcome in order to be seriously considered, and it is the “better,” the closer its outcome is to the global social optimum.

5.3.1. Global Social Optimum

Consider a global social planner seeking to maximize global welfare, i.e., seeking to minimize the net present value of total global costs consisting of global costs of emission abatement and the sum of national environmental damages stemming from the pollution stock.

To solve the social planner’s problem, we introduce the following recursive value function in period $t$:

$$V_t(s_t) = \max \left\{ a_i^t \right\}_{i=1}^n \left\{ \delta V_{t+1}(s_{t+1}) - \sum_{j=1}^n \frac{a_i^j}{2} (a_i^j)^2 - \frac{n\beta^2 s_t^2}{2} \right\}, \quad (5.5)$$

where $V_t(s_t)$ represents the negative of the total global costs accruing from period $t$ onwards discounted to period $t$. The social global planner’s problem is to maximize $(5.5)$ for $t = 1, \ldots, T$ subject to equation (5.3) and $V_{T+1}(s_{T+1}) \equiv 0$. Assuming an interior solution$^6$, i.e., $a_i^t \in (0, \epsilon)$ for all $i = 1, \ldots, n$ and $t = 1, \ldots, T$, the following

$^6$Interior solutions $a_i^t > 0$ are guaranteed by the quadratic abatement cost and damage functions.
first-order conditions are necessary for a global optimum:

\[ \alpha a_i^t = -\delta V_{t+1}'(s_{t+1}) , \quad t = 1, \ldots, T . \]

Differentiating \( V_t(s_t) \) with respect to \( s_t \) and applying the envelope theorem yields

\[ -V_t'(s_t) = n\beta s_t - \delta (1 - \gamma) V_{t+1}'(s_{t+1}) , \quad t = 1, \ldots, T . \]

Recursive evaluation of equation (5.7) implies that the negative of the first derivative of the value function, \( -V_t'(s_t) \), equals the net present value in period \( t \) of all future damages from one additional marginal unit of the pollution stock \( s_t \) summed up over all countries. Then first-order condition (5.6) says that in the global social optimum the costs of abating an additional marginal unit of emissions have to equal the net present value of all mitigated future damages caused by this additional marginal unit by decreasing the pollution stock \( s_t \). As abatement in period \( t \) only influences the damages in the period \( t + 1 \) and the world ends after period \( T \), abatement in the terminal period does not pay off, implying \( a_i^T = 0 \). According to the following proposition, there exists a unique global optimum:

**Proposition 5.1 (Global Social Optimum)**

For any time horizon \( T \leq \infty \) there exists a unique social global optimum characterized by identical sequences of emission abatements for all countries \( i \) in all periods \( t \), \( a_i^* = a_t^* \), and a sequence for the greenhouse gas stock \( s_i^* \) (\( i = 1, \ldots, n; \ t = 1, \ldots, T \)).

The proof is given in Appendix D. The proof is constructive in the sense that we not only show the existence and the uniqueness of the social global optimum but also determine closed-form solutions for the sequences \( a_i^* \) and \( s_i^* \). As we are particularly interested in the long run, we state the following corollary:

**Corollary 5.1 (Global Social Optimum in the Long Run)**

For \( T \to \infty \) the sequences of emission abatement and the greenhouse gas stock in the global social optimum converge to their steady state levels

\[ a^{SO} = \frac{n^2\beta\delta \epsilon}{\alpha \gamma [1 - \delta (1 - \gamma)] + n^2\beta \delta} \quad \text{and} \quad s^{SO} = \frac{n\alpha \epsilon [1 - \delta (1 - \gamma)]}{\alpha \gamma [1 - \delta (1 - \gamma)] + n^2\beta \delta} . \]

The proof is given in Appendix D.
5.3.2. Decentralized Solution

Next we examine a decentralized system in the absence of an international treaty, where a local planner in each country (e.g., a government) seeks to minimize the total local costs consisting of local abatement costs and local environmental damages. We are looking for subgame perfect Nash equilibria in pure strategies for this game.

We solve the game by backward induction, starting from period $T$. It is useful to consider a typical step in this procedure. To this end, suppose that there exists a unique subgame perfect equilibrium for the subgame starting in period $t + 1$ with a stock of greenhouse gases $s_{t+1}$. For the moment, this is assumed to hold in all periods $t + 1$ and will be verified in the proof of Proposition 5.2. Other details of the history of the game apart from the level of the greenhouse gas stock $s_{t+1}$ do not matter, as only $s_{t+1}$ influences the payoffs of the subgame starting in period $t + 1$ and the equilibrium is assumed to be unique.

Given the unique subgame perfect equilibrium for the subgame starting in period $t + 1$ with the associated equilibrium payoff $W^i_{t+1}(s_{t+1})$, country $i$’s best response in period $t$, $\bar{a}^i_t$, is determined by the solution of the optimization problem

$$V^i_t(s_t) | A^{-i}_t = \max_{a^i_t} \left\{ \delta W^i_{t+1}(s_{t+1}) - \frac{\alpha}{2} (a^i_t)^2 - \frac{\beta}{2} s^2_t \right\}, \quad (5.9)$$

subject to equation (5.3), $W^i_{t+1}(s_{T+1}) = 0$, and given the sum of abatement efforts by all other countries $A^{-i}_t = \sum_{j \neq i} a^j_t$.

As we will show in the proof of Proposition 5.2, country $i$’s optimization problem in period $t$ is strictly concave. Thus, differentiating equation (5.9) with respect to $a^i_t$ and restricting our attention to interior solutions, we obtain an implicit function for country $i$’s best response

$$\alpha \bar{a}^i_t = -\delta W^i_{t+1}'(\bar{s}_{t+1}) , \quad (5.10)$$

where $\bar{s}_{t+1} = (1 - \gamma)s_t + n\epsilon - \bar{a}^i_t - A^{-i}_t$. In addition, applying the envelope theorem to equation (5.9) yields

$$-V^i_t(s_t) | A^{-i}_t = \beta s_t - \delta (1 - \gamma) W^i_{t+1}'(\bar{s}_{t+1}) . \quad (5.11)$$

We observe that $W^i_t(s_t) = V^i_t(s_t) | \hat{A}^{-i}_t$ with $\hat{A}^{-i}_t = \sum j \neq i \hat{a}^j_t$ denoting the sum of emission abatement in period $t$ in the subgame perfect Nash equilibrium for all countries $j \neq i$. 

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Recursive evaluation of equation (5.11) implies that the negative of the first derivative of the payoff function in period \( t \), \(-W_i'(s_t)\), equals the net present value of all future damages in country \( i \) from one additional marginal unit of the pollution stock \( s_t \) in period \( t \). Then, the best-response function (5.10) says that country \( i \)'s costs of abating an additional marginal unit of emissions have to equal the net present value of all mitigated local future damages caused by this additional marginal unit by decreasing the pollution stock \( s_t \). Note that country \( i \)'s best-response function only depends on the sum of the emission abatement of all other countries \( A_{T-i} \) and not on the contribution of individual countries \( a^T_i \).

The following proposition establishes the existence and uniqueness of a subgame perfect Nash equilibrium:

**Proposition 5.2 (Decentralized Solution)**

*For any time horizon \( T < \infty \), there exists a unique subgame perfect Nash equilibrium characterized by identical sequences of emission abatements for all countries \( i \) in all periods \( t \), \( \hat{a}_i^t = \hat{a}_t \), and a sequence for the greenhouse gas stock \( \hat{s}_t \) (\( i = 1, \ldots, n; t = 1, \ldots, T \)).*

The proof is given in Appendix D. In the proof we also determine closed-form solutions for the sequences \( \hat{a}_t \) and \( \hat{s}_t \).

Again, we are interested in the long run and take the limit for \( T \to \infty \). The reason is that this equilibrium approximates the equilibrium for very large, but still finite time horizons \( T \).

**Corollary 5.2 (Decentralized Solution in the Long Run)**

*For the limit \( T \to \infty \), the sequences of emission abatement and the greenhouse gas stock in the unique subgame perfect Nash equilibrium of the decentralized solution for finite time horizons \( T \) converge to the steady state levels

\[
a^{DS} = \frac{n\beta\delta\epsilon}{\alpha\gamma[1 - \delta(1 - \gamma)] + n\beta\delta}, \quad (5.12a)
\]
\[
s^{DS} = \frac{n\alpha\epsilon[1 - \delta(1 - \gamma)]}{\alpha\gamma[1 - \delta(1 - \gamma)] + n\beta\delta}. \quad (5.12b)
\]

\[\text{In infinite horizon models, further equilibria and even a continuum of equilibria can occur (Tsutsui and Mino, 1990; Rowat, 2007). However, it can be shown that the equilibrium we achieve by taking the limit \( T \to \infty \) is the unique Markov perfect equilibrium in affine strategies (Lockwood, 1996).}\]
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The proof is given in Appendix D.

From Corollaries 5.1 and 5.2 we observe that $a^{SO} > a^{DS}$ and $s^{SO} < s^{DS}$, which reflects the well-known underprovision of emission abatement in the decentralized case due to the incentives for each country to free-ride on the emission abatements of all other countries. As these incentives increase with the number of countries $n$, the underprovision of abatement becomes more severe, the higher $n$ is.

5.4. Refunding Scheme

In the following, we introduce a refunding scheme (RS) and analyze its potential for improving on the decentralized solution. The essential idea is that a global fund be established that refunds interest earnings to member countries in each period, proportionally to their relative emission reductions.

5.4.1. Rules of the RS

We consider a three-step procedure. First, participating countries negotiate the parameters of the RS. In particular, this includes duration $T$ of the treaty, the level of an initial fee $f_0^i$ payable into a global fund by each participating country, and reimbursements $R_t$ for all periods $t = 1, \ldots, T - 1$. Second, in each period $t = 1, \ldots, T - 1$ the fraction $R_t$ of the fund is reimbursed to the participating countries in proportion to the emission reductions they have achieved relative to overall emission abatements in this period. The remaining assets of the fund are invested at the constant interest rate $\rho$ per period, and the returns add to the global fund in the next period $t + 1$. Finally, in period $T$ the fund is equally redistributed to the participating countries.

Thus, the fund at the end of period $t$ reads

$$f_t = (1 + \rho)f_{t-1} - R_t, \quad t = 1, \ldots, T - 1,$$

with an initial fund $f_0 = \sum_{i \in S} f_0^i$, where $S$ denotes the set of countries participating in the RS. Note that $f_T = 0$, or equivalently $R_T = (1 + \rho)f_{T-1}$. In addition, the refund $r_t^i$
5. Sustainable Climate Treaties

A member country $i$ receives in period $t$ yields

$$r_t^i = \begin{cases} \frac{R_t}{\sum_{j \in S} a_t^j}, & t = 1, \ldots, T - 1, \\ \frac{R_T}{|S|}, & t = T. \end{cases}$$  \hspace{1cm} (5.14)

A set of initial fees $f_t^i \geq 0$ and refunds $R_t \geq 0$ is feasible if $f_t \geq 0$ for all $t = 1, \ldots, T$ holds.

In order to analyze the potential of an RS to mitigate climate change, we proceed as follows: First, assuming that all countries participate in the RS in step one, we show that for any feasible set of initial fees $f_0^i$ and refunds $R_t$ and any time horizon $T < \infty$ there exists a unique and symmetric subgame perfect Nash equilibrium for steps two and three. Second, we show that there exists a feasible set of initial fees $f_0^i$ and refunds $R_t$ for which the subgame perfect Nash equilibrium resembles the global social optimum. This is called a first-best sustainable RS. Third, we show that a refunding scheme in which all returns of the fund are fully redistributed in each period converges to the social global optimum in the long run for an appropriate choice of initial fees $f_0^i$. This treaty is called second-best sustainable RS.

### 5.4.2. Subgame Perfect Equilibrium

Given that all $n$ countries have joined the RS and given an arbitrary but finite time horizon $T$ and a feasible set of initial fees $f_0^i$ and refunds $R_t$, we now analyze the subgame perfect equilibria of the RS. We assume that all countries set a sequence of local abatement efforts $a_t^i$ so as to minimize local abatement costs and environmental damages and to maximize refunds $r_t^i$.

Proceeding as in Subsection 5.3.2, we assume momentarily that there exists a unique subgame perfect equilibrium for the subgame starting in period $t + 1$ with a greenhouse gas stock $s_{t+1}$. Thus, we can write $W_{t+1}^i(s_{t+1})$ for country $i$’s equilibrium payoff for this subgame. Then country $i$’s best response in period $t$, $\bar{a}_t^i$, is determined by the solution of the optimization problem

$$V_t^i(s_t)|A_t^{-i} = \max_{a_t^i} \left\{ \delta W_{t+1}^i(s_{t+1}) - \frac{\alpha}{2}(a_t^i)^2 - \frac{\beta}{2}s_t^2 + r_t^i \right\},$$  \hspace{1cm} (5.15)
subject to equation (5.3), \( W_{i_{T+1}}(s_{T+1}) = 0 \), and given the sum of the abatement efforts of all other countries \( A_{T-i} = \sum_{j \neq i} a_{j}^{t} \). In the proof of Proposition 5.3 we will establish that the optimization problem of country \( i \) in period \( t \) is strictly concave. Thus, differentiating equation (5.15) with respect to \( a_{i}^{t} \) and restricting our attention to interior solutions, we obtain an implicit function for country \( i \)'s best response

\[
\alpha \bar{a}_{i}^{t} = -\delta W_{i_{t+1}}'(\bar{s}_{t+1}) + \frac{\partial r_{i}^{t}}{\partial a_{i}^{t}} \bigg|_{a_{i}^{t} = \bar{a}_{i}^{t}},
\]

where \( \bar{s}_{t+1} = (1 - \gamma)s_{t} + n \epsilon - \bar{a}_{i}^{t} - A_{T-i}^{t} \) and

\[
\frac{\partial r_{i}^{t}}{\partial a_{i}^{t}} = \begin{cases} 
R_{t}^{i} \frac{A_{T-i}^{t}}{(a_{i}^{t} + A_{T-i}^{t})^{2}}, & t = 1, \ldots, T - 1, \\
0, & t = T .
\end{cases}
\]

Applying the envelope theorem yields equation (5.11), as in the decentralized solution. Thus, the best response function (5.16) implies that country \( i \)'s costs for abating an additional marginal unit of emissions have to equal the net present value of all mitigated local future damages caused by this additional marginal unit by decreasing the pollution stock \( s_{t} \) plus the refunds induced by abating this additional marginal unit. In case of the RS as well, the best-response function of country \( i \) only depends on the sum of the emission abatement of all other countries \( A_{T-i}^{t} \) and not on the contribution of individual countries \( a_{j}^{t}, j \neq i \).

Again, we can show the existence and uniqueness of a subgame perfect Nash equilibrium:

**Proposition 5.3 (Refunding Scheme)**

Given a time horizon \( T < \infty \) and a feasible set of initial fees \( f_{0}^{i} \) and refunds \( R_{t} \), there exists a unique subgame perfect Nash equilibrium characterized by identical sequences of emission abatements for all countries \( i \) in all periods \( t, \bar{a}_{i}^{t} = \bar{a}_{t} \), and a sequence for the greenhouse gas stock \( \bar{s}_{t} \) \((i = 1, \ldots, n; t = 1, \ldots, T)\).

The proof is given in Appendix D.
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5.4.3. First-Best Sustainable RS

We now show that there exists a feasible set of initial fees $f^*_0$ and refunds $R_t$ such that the unique Nash equilibrium of Proposition 5.3 is identical to the global social optimum characterized in Proposition 5.1. We call a treaty that exhibits this property first-best sustainable refunding scheme.

To prove the existence of a first-best sustainable RS, we first look for the sequence of refunds $R_t$ that ensures that the unique Nash equilibrium under the refunding scheme equals the outcome of the social global optimum. In a second step, we calculate the minimal initial fund $f_0$ for which this sequence of refunds is feasible.

From equation (D.31) in the proof of Proposition 5.3, we know that in the unique and symmetric subgame perfect Nash equilibrium the following condition for the refund in period $t$, $R_t$, holds:

$$R_t = \frac{n^2 \bar{a}_t}{n-1} \left[ \alpha \bar{a}_t + \delta W_{t+1}(\bar{s}_{t+1}) \right], \quad (5.18a)$$

where $\bar{a}_t$ and $\bar{s}_t$ denote the levels of abatement and the pollution stock in period $t$ in the subgame perfect Nash equilibrium. Recursively applying condition (D.33), we can rewrite equation (5.18a) to yield

$$R_t = \frac{n^2 \bar{a}_t}{n-1} \left\{ \alpha \bar{a}_t - \delta \beta \sum_{k=t+1}^T [\delta(1-\gamma)]^{k-(t+1)} \bar{s}_k \right\}. \quad (5.18b)$$

Inserting the sequences $a^*_t$ and $s^*_t$ of the global social optimum, as characterized by Proposition 5.1, yields the sequence of refunds $R^*_t$ for which the unique subgame perfect Nash equilibrium of the RS and the social global optimum coincide:

$$R^*_t = \frac{n^2 a^*_t}{n-1} \left\{ \alpha a^*_t - \delta \beta \sum_{k=t+1}^T [\delta(1-\gamma)]^{k-(t+1)} s^*_k \right\}. \quad (5.19)$$

To determine the minimal initial global fund $f^*_0$ for which the sequence of refunds $R^*_t$ is feasible, we re-write the recursive equation (5.13) to yield

$$f_0 = \sum_{t=1}^T \left[ \frac{R_t}{(1+\rho)^t} \right] + \frac{f_T}{(1+\rho)^T}. \quad (5.20)$$
Thus, the minimal fund necessary to support the sequence of refunds $R^*_t$ is given by

$$f^*_0 = \sum_{t=1}^{T-1} \left[ \frac{R^*_t}{(1 + \rho)^t} \right].$$  \hfill (5.21)

Note that it is optimal to empty the fund as early as period $T-1$, as the equal distribution of the remainder of the fund in period $T$ does not influence the countries' abatement decisions in the last period. Hence $R^*_T = 0$.

As a consequence, the following proposition holds:

**Proposition 5.4 (Existence of First-best Sustainable RS)**

For any time horizon $T < \infty$, the unique and symmetric subgame perfect Nash equilibrium of the RS coincides with the social global optimum for $f_0 \geq f^*_0$ and $R_t = R^*_t$ ($t = 1, \ldots, T$), where $f^*_0$ and $R_t = R^*_t$ are given by equations (5.21) and (5.19).

Proposition 5.4 says that for any initial global fund equal to or exceeding $f^*_0$ the social global optimum can be implemented by setting $R_t = R^*_t$. Note that the levels of abatement $a^*_t$ and the greenhouse gas stock $s^*_t$ are analytically solvable, as shown in the proof of Proposition 5.1. As a consequence, the levels of the minimal initial global fund $f^*_0$ and the sequence of refunds $R^*_t$ are also analytically solvable.

### 5.4.4. Second-Best Sustainable RS

As it may be difficult to agree ex ante on both an initial global fund $f_0$ and a sequence of refunds $R_t$, we now analyze a simplified version of the RS in which the total initial fund $f_0$ is invested and the refunds in each period equal the interest payments per period

$$R_t = \rho f_0 \equiv R, \quad t = 1, \ldots, T - 1.$$  \hfill (5.22)

Then, the fund in period $t$ is given by the initial fund, i.e. $f_t = f_0$ for all periods $t = 1, \ldots, T - 1$. Note that in the last period, the remainder of the fund is redistributed equally to the participating countries, so that $f_T = 0$ or $R_T = (1 + \rho) f_0$.

We show that under these conditions we can construct an RS for which the level of the greenhouse gas stock, $s_t$, converges to the global social optimal level $s^*_t$ in the long run. We call such a treaty *second-best sustainable refunding scheme* as the global social optimum is only reached in the limit $t \to \infty$. 

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First, we calculate the long-run levels of abatement and the greenhouse gas stock under the RS that correspond to a particular level of the global fund $f_0$. Again, we analyze the unique Nash equilibrium of the RS for $T < \infty$ in the limit $T \to \infty$, which approximates the equilibrium of arbitrarily large but still finite time horizons $T$.

**Proposition 5.5 (RS in the Long Run)**

For the limit $T \to \infty$, the sequences of emission abatement and the greenhouse gas stock in the unique subgame perfect Nash equilibrium of the RS for finite time horizons $T$ converge to the steady state levels

\[
a^{RS} = \frac{n\beta\delta\epsilon + \sqrt{n^2\beta^2\delta^2\epsilon^2 + 4\gamma R \alpha n [1 - \delta(1 - \gamma)]}}{2\alpha \gamma [1 - \delta(1 - \gamma)] + n \beta \delta},
\]

\[
s^{RS} = \frac{n}{\gamma}(\epsilon - a^{RS}).
\]

The proof is given in Appendix D.

Second, comparing the steady state levels $(a^{RS}, s^{RS})$ with the corresponding levels in the global social optimum $(a^{SO}, s^{SO})$, we obtain

**Corollary 5.3 (Second-best Optimal Level of the Global Fund $f$)**

The steady state of the RS $(a^{RS}, s^{RS})$ coincides with the steady state of the global social optimum $(a^{SO}, s^{SO})$ if and only if

\[
f_0^{SB} = \frac{\alpha n^5 \beta^2 \delta^2 \epsilon^2}{\rho \alpha \gamma [1 - \delta(1 - \gamma)] + n^2 \beta \delta^2}.
\]

**Proof:**

Solving $a^{RS} = a^{SO}$ for the fund $f_0^{SB}$ yields (5.24).

Proposition 5.5 together with Corollary 5.3 immediately imply

**Corollary 5.4 (Existence of Second-best Sustainable RS)**

Given that a treaty with an initial global fund $f_0^{SB}$ has been signed, it holds that

\[
\lim_{t \to \infty} \tilde{s}_t = s^{RS} = s^{SO}.
\]

Corollary 5.4 shows that there exists a RS that induces a convergence to the socially desired stock of greenhouse gases in the long run. Hence, the second-best sustainable RS
provides a sustainable solution for the provision of the global public good of mitigating climate change in the long run. However, the path of abatement levels does not, in general, coincide with the socially optimal level. Although this difference in abatement levels between the second-best sustainable RS and the social global optimum vanishes over time, it may be substantial in the short run. The following proposition examines this difference:

**Proposition 5.6 (Difference Second-best Sustainable RS – Social Optimum)**

In the linear approximation around the steady state of the second-best sustainable RS, the following statements hold:

(i) The difference in the levels of the greenhouse gas stock between the second-best sustainable RS and the social global optimum is given by

\[
\Delta s_t = \tilde{s}_t - s^*_t = (s_1 - s^{SO}) \left[ \nu_2^{t-1} - \lambda_2^{t-1} \right],
\]

with

\[
\nu_2 = \frac{1}{2} \left[ 1 - \gamma + \frac{\alpha(2n - 1) + n^2 \beta \delta}{\alpha(2n - 1) \delta (1 - \gamma)} - \sqrt{\left( \frac{1 - \gamma + \alpha(2n - 1) + n^2 \beta \delta}{\alpha(2n - 1) \delta (1 - \gamma)} \right)^2 - \frac{4}{\delta}} \right],
\]

\[
\lambda_2 = \frac{1}{2} \left[ 1 - \gamma + \frac{\alpha + n^2 \beta \delta}{\alpha \delta (1 - \gamma)} - \sqrt{\left( \frac{1 - \gamma + \frac{\alpha + n^2 \beta \delta}{\alpha \delta (1 - \gamma)}}{\delta} \right)^2 - \frac{4}{\delta}} \right].
\]

(ii) We have

\[
\Delta s_t \begin{cases} < 0, & \text{if } s_1 < s^{SO}, \\ > 0, & \text{if } s_1 > s^{SO}. \end{cases}
\]

The proof is given in Appendix D.

Proposition 5.6 says that (at least in a sufficiently small neighborhood around the steady state) the saddle point path of greenhouse gas emissions in the second-best sustainable RS is below (above) the global social optimum if the initial stock of greenhouse gases is below (above) the long run steady state. Thus, the second-best sustainable RS induces inefficiently high (low) abatement levels for \( s_1 < s^{SO} \) (\( s_1 > s^{SO} \)).
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5.5. Numerical Illustration

To give an idea of the order of magnitude needed for the initial fund $f_0$ to implement the first- and second-best sustainable RS, we run a little numerical exercise. However, due to the highly stylized model, the results are rather a numerical illustration than a quantitative analysis.

Following similar numerical illustrations, such as those provided by Goulder and Mathai (2000), we adopt the slightly more complex but also slightly more realistic stock equation by Nordhaus (1994):

$$s_{t+1} = (1 - \gamma)s_t + \theta \sum_{i=1}^{n} (\epsilon_i - a_i),$$  \hspace{1cm} (5.29)

where $s_t$ denotes the CO$_2$ stock above the pre-industrial level of 278 ppm, $\gamma = 0.008$, and $\theta = 0.64$. Thus, only the fraction $\theta$ accumulates the atmospheric stock. Moreover, we assume time-dependent business-as-usual emissions $\epsilon_i$. Global business-as-usual emissions rise exponentially from approximately 8 GtC (3.75 ppm) in 2010 until they peak at 26 GtC (12.2 ppm) in 2125 and flatten out to 18 GtC by 2200.\(^8\) As countries are identical, all countries exhibit the same business-as-usual emissions, which are $1/n$-th of the global business-as-usual emissions.

Of course, our assumption of identical countries drastically oversimplifies real-world affairs. For the numerical illustration, we assume that the world’s economic activity is symmetrically distributed among ten identical countries. This is driven by the observation that the ten largest economies emitted more than 70% of global greenhouse gas emissions in 2006. This ratio is likely to rise further.\(^9\) Thus, an international agreement among the 10 largest greenhouse gas emitters would essentially solve the problem of anthropogenic climate change. The parameters $\alpha$ and $\beta$ are calibrated such that (a) the CO$_2$ stock in the global social optimum equals 2.5 times the pre-industrial concentration (which is close to the optimal scenario in Nordhaus 1994) and (b) a doubling of the CO$_2$ concentration in the atmosphere against pre-industrial levels amounts to environmental damages of 1.33% of world GDP per year (Nordhaus, 1994; Goulder and Mathai, 2000).

According to the CIA World Factbook this equalled $6.5 \cdot 10^{13}$ US $\$ in 2007. The values of the interest rate $\rho$ and the discount factor $\delta$ are in line with Nordhaus (1994). Table 5.1 summarizes the parameter values used for our numerical example.

\(^8\)This business-as-usual emission scenario is similar to Goulder and Mathai (2000).

\(^9\)The ten economies are China, USA, European Union, Indonesia, India, Russia, Brazil, Japan, Canada, and Mexico.
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Figure 5.1.: Global business-as-usual emissions in ppm are shown in the top left corner. Total abatement in ppm (top right), relative abatement (middle left), marginal abatement costs in US $ per tC (middle right) and the atmospheric stock in ppm above pre-industrial level (bottom left) for the decentralized solution (dot-dashed), the first-best sustainable RS (dotted) and the second-best sustainable RS (dashed). The bottom right corner shows total yearly refunds in % of world GDP for the first-best sustainable RS (dotted) and the second-best sustainable RS (dashed).
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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial stock (above pre-industrial level)</td>
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<td>112</td>
</tr>
<tr>
<td>Abatement cost parameter</td>
<td>$\alpha$</td>
<td>$2.21757 \cdot 10^{12}$ $\frac{$/a}{(ppm/a)^2}$</td>
</tr>
<tr>
<td>Damage cost parameter</td>
<td>$\beta$</td>
<td>$9.94312 \cdot 10^6$ $\frac{$/a}{ppm}$</td>
</tr>
<tr>
<td>Decay rate of GHGs</td>
<td>$\gamma$</td>
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</tr>
<tr>
<td>Discount factor</td>
<td>$\delta$</td>
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</tr>
<tr>
<td>Fraction of GHGs that accumulates $s_t$</td>
<td>$\theta$</td>
<td>0.64</td>
</tr>
<tr>
<td>Interest rate</td>
<td>$\rho$</td>
<td>0.05</td>
</tr>
<tr>
<td>Number of countries</td>
<td>$n$</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 5.1.: Summary of the parameter values used in the numerical example

Figure 5.1 illustrates the business-as-usual path and shows total abatement, relative abatement, marginal abatement costs, the atmospheric stock for the decentralized solution, the first-best, and the second-best sustainable RS. In addition, yearly total refunds in % of 2007 world GDP are shown for the first-best and the second-best sustainable RS. Figure 5.1 gives rise to the following observations: First, we see that with respect to abatement levels and atmospheric CO$_2$ concentrations the decentralized solution falls dramatically short of the social global optimum (which is identical to the first-best sustainable RS).

Second, we observe that while in the long run the second-best sustainable RS converges to the social global optimum, abatement levels and marginal abatement costs differ dramatically for the first 125 years. Compared to the social global optimum, the second-best sustainable RS is in fact overambitious with respect to emission reductions. This is in line with Proposition 5.6, as the initial atmospheric CO$_2$ concentration is below the long run levels. As a consequence, the initial fund of the second-best sustainable RS amounts to 71.6 % of the world GDP in 2007 compared to 10.8 % for the first-best sustainable RS. Thus, from the current perspective the second-best sustainable RS seems rather unattractive.

However, it is possible, or even plausible, that countries may fail to reduce greenhouse gas emissions in the near future. Countries might also become more ambitious with respect to the long-run level of greenhouse gases in the atmosphere. For example, limiting global warming to a maximum of 2 °C roughly translates into a long-run concentration of greenhouse gases in the atmosphere amounting to 450 ppm. In both cases, the greenhouse gas concentration will move above the long-run level of the socially optimal
stock. Then the second-best sustainable RS would ensure achievement of the long-run goal at lower abatement costs than the first-best sustainable RS. Third, we observe that while the total initial fund in the case of the first-best sustainable RS is also substantial (almost 11% of world GDP), the yearly refunds for the first-best sustainable RS start at a moderate level of around 0.3% of world GDP in 2010 and rise to around 3.75% in 2170.

5.6. Initial Participation

So far, we have focused on the capacity of an RS to induce countries to follow (first-best sustainable RS) or to converge to (second-best sustainable RS) a socially optimal abatement path. To achieve this, all countries have to agree on the appropriate parameters (initial fees and, in the case of the first-best sustainable RS, a sequence of refunds in all periods) and on initial monetary commitments. We observe that a sustainable RS, as developed in this chapter, transforms the intertemporal climate-policy problem into a standard, static public-goods problem. Once all countries have made their initial contribution, a first-best allocation (or at least a long-run convergence to it in the case of the second-best sustainable RS) is ensured for all later occasions, as countries would be worse off by forfeiting refunds. In the following, we discuss how solution procedures developed in the literature on the private supply of public goods can be applied to motivate countries to make initial payments.

5.6.1. The Ideal Solution

At the initial level, when countries are pondering whether to sign the treaty and to pay the initial fee, the free-rider problem remains present. Especially if the number of countries \( n \) is large, each country may be better off by not signing the treaty. If all other countries participate, the country would benefit from all other countries’ abatement efforts, without having to pay the initial fee and to compete for refunds.

To solve this free-rider problem, the RS could be incorporated in a two-stage game. In the first stage, countries decide whether to participate in a sustainable RS by paying the initial fee \( f_0 \). The treaty only becomes effective if all countries sign and pay the initial fee. Otherwise, the treaty is cancelled and initial fees already paid are returned.
If all countries have signed, we can proceed to the second stage, in which the treaty is executed as outlined in Section 5.4.

It is straightforward to see that it is a weakly dominant strategy for all countries to sign the treaty and pay the initial fee. All countries are better off with the social global optimum achieved if the treaty becomes effective than they are with the decentralized solution that results if the treaty is cancelled.  

5.6.2. Difficulties in Achieving Initial Participation

In practice, the preceding conceptual solution has to be supplemented by additional considerations. This holds, in particular, when countries are not identical, as this paper assumes.

Making larger countries pivotal

The ideal solution lies in making countries – and in particular large countries – pivotal for the formation of the RS. In order to achieve such a scenario, about ten to twenty of the largest greenhouse gas emitters must coordinate on the agreement that the RS will fail if any of them defects.

Sequential procedures

As full participation by all countries at once is unlikely, it is useful to resort to sequential procedures where a subset of countries makes a start and the others follow later (see Andreoni, 1998; Lange, 2006; Varian, 1994). For the RS we might envision four steps. First, as suggested in the last paragraph, a set of large and mainly wealthy countries could initiate the system by paying initial fees. Second, smaller rich countries could follow, which would increase the initial wealth. In the third and fourth steps, larger and smaller developing countries could be invited to join the RS. Regarding the payment of initial fees, they should be treated differently, as we will discuss next.

10While an improvement on the decentralized solution is obvious for the first-best sustainable RS, it is not clear a priori whether the second-best sustainable RS will improve on the decentralized solution in initial periods. However, there always exists a time period at which the second-best sustainable RS will be a Pareto improvement over the decentralized system for the entire remaining lifetime.

11In practice countries must be mutually stubborn and insist on full participation by this core group before going ahead.
Renouncing initial fees for developing countries

Developing countries lack the necessary wealth to pay the initial fees. To induce participation, payment of initial fees could be forgone. Indeed, the RS works in the same way if a subset of \( k \) countries (\( 1 < k < n \)) pays an initial fee equal to \( f^*_0/k \) and \( n \) countries are eligible to the refunds. Once the RS has been initiated, incentives to abate are independent of initial contributions. In such circumstances, developing countries would voluntarily join the system, as they can always choose the same emission reduction policy under the RS as without, but they can benefit from the refunds if it is in their interest. Indeed, the prospect of earning refunds will motivate them to abate much more. Moreover, the discrimination of initial fees is a powerful tool in implementing transfers to the RS. If countries are heterogeneous, that may be necessary to render the RS a Pareto improvement.

5.7. Raising Initial Fees

The sustainable refunding scheme relies on the payment of initial fees. As such fees tend to be quite large, especially in the case of the second-best sustainable RS, we outline two ways in which such fees might be financed.

5.7.1. Repeated Payments

Paying the initial fees in full at the beginning of the treaty is not necessary. We can also achieve the first- and second-best sustainable RS by repeatedly paying a smaller amount of money.

**Proposition 5.7 (RS with Repeated Payments)**

(i) For every time span \( \Delta > 0 \) there exists a sequence of fees \( f_t(\Delta) \) defined by

\[
f_t(\Delta) = \sum_{\tau=1}^{\Delta} \frac{R^*_t+\tau}{(1 + \rho)\tau}
\]

\(12\) On average, the economic and social consequences of climate change also tend to be more severe in developing countries.
such that this RS implements the same solution as the first-best sustainable RS with initial payments larger or equal to \( f^0_0 \) if \( f_t(\Delta) \) is paid into the fund at times \( t = 0, \Delta, 2\Delta, \ldots \).

(ii) For every time span \( \Delta > 0 \) there exist fees \( f(\Delta) \) defined by

\[
f(\Delta) = \frac{(1 + \rho)^\Delta - 1}{(1 + \rho)^\Delta} f_{SB}^0
\]

such that this RS implements the same solution as the second-best sustainable RS with initial payments \( f_{SB}^0 \) if \( f(\Delta) \) is paid into the fund each \( \Delta \) periods.

The proof is given in Appendix D.

With the repeated payments scheme we face a trade-off between high initial fees and the property of the sustainable RS to transform an intertemporal climate-policy problem into a static public-goods problem. In particular, if the time span \( \Delta \) is short, the solution of the climate-change problem relies on the repeated commitment of all countries, as the initial participation problem would have to be solved whenever new payments have to be made. Therefore \( \Delta \) should not be too small.\(^{13}\)

### 5.7.2. Borrowing and Capital Markets

If the repeated solution to the initial participation problem turns out to be a major obstacle to international cooperation, raising the initial fees by allowing countries to borrow money may be more advisable. Countries could then borrow either from the international capital market or directly from the administering agency of the RS. As the administering agency invests the wealth of the climate fund on the international capital market, both ways by countries to borrow are equivalent when capital markets are perfect, as the following proposition shows:

**Proposition 5.8 (Borrowing Initial Fees)**

Suppose capital markets are perfect and all countries borrow the entire amount of the initial fees \( f^0_0 \) required for the sustainable RS. Use \( \mu \) (\( 0 \leq \mu \leq 1 \)) to denote the fraction of \( f^0_0 \) borrowed from the international capital market, implying that the remainder \( 1 - \mu \frac{13}{13}\)Sustained participation can also be fostered by not completely depleting the fund and hence making exit costly.
5. Sustainable Climate Treaties

\( \mu f_0 \) is borrowed from the administering agency running the RS. Then the following statements hold:

(i) For all values of \( \mu \), borrowing by countries does not affect international capital markets.

(ii) For \( \mu = 0 \), the first- and second-best sustainable RS can be implemented without the flow of money.

**Proof:**

Recall that perfect capital markets imply that countries may borrow or lend freely at the per-period interest rate \( \rho \). To see part (i), observe that borrowing by countries increases demand for capital on the international capital market by a total of \( \mu f_0 \). As the administering agency lends a total of \( (1 - \mu) f_0 \) to the countries, it can invest \( \mu f_0 \) in the capital market and thus supply increases by the same amount. Hence, the equilibrium in the capital market is not affected.\(^{14}\) Part (ii) follows from the observation that in the (first-best and second-best) sustainable RS the initial fee \( f_0 \) is the net present value of all future refunds the country receives in the unique subgame perfect equilibrium. So in each time period \( t \), the net present value of the two claims exactly offset each other. No actual money flow is necessary in the polar case \( \mu = 0 \).

Proposition 5.8 says that raising the money needed for the initial payments is no problem under the assumption of perfect capital markets. In practice, two types of deviations from perfect capital markets have to be taken into account. A country may default against the administering agency or default in general. First, if \( \mu \) is small, countries might be tempted to renounce high abatement efforts and to default on their interest-rate obligations to the administering agency. The country would lose all claims to refunds. However, as such refunds are small when abatement efforts are small, such a strategy may be profitable. For say \( \mu = 0 \), a country could choose to default against the administering agency and could free-ride on the abatement efforts of other countries even if it has signed the treaty and has borrowed from the administering agency. Such considerations suggest that countries should borrow mainly on the international capital market.

Second, if countries borrow a large amount on international capital markets, the default risk may rise if outstanding government debt is already at a high level. If the country

\(^{14}\)As the administering agency needs to invest its wealth in the capital market, borrowing by countries to pay the initial fee does not crowd out private investments.
needs to pay a larger interest rate than the risk-free rate, as investors demand a positive risk premium, further borrowing may increase the default risk as refunds are insufficient to cover interest-rate payments. In such cases, it is useful for part of the initial fund to be raised by taxes so as to foster abatement. Then the additional interest-rate burden can be kept smaller than refunds, thereby keeping or reducing default risk on capital markets.

5.8. Conclusion

The RS provides a simple blueprint for an international treaty on climate change. It is governed by a very small number of parameters. The RS is no panacea, as free-rider problems have no perfect solutions, but it might be wiser to focus attention on systems like the RS than on Kyoto-style treaties in which little can be done to induce countries to fulfil their promises.

The practical implementation of the refunding schemes developed in this chapter requires a variety of additional considerations. In the last two sections, we have discussed how to achieve initial participation, and we have outlined several ways of raising initial fees. Other issues, such as governance of the administering agency, uncertainty regarding damage and abatement costs, or heterogeneous countries will need thorough investigation in future research.
A. Appendix to Chapter 2

Notation

Throughout Appendix A we work with the following abbreviations:

\[ B = \beta^I + \beta^D, \]
\[ P = \phi^I + \phi^D, \]
\[ A = \frac{\beta^D P + \phi^D B}{B}, \]
\[ \tau = \tau^* \text{ (tax goal)}. \]

A.1. Existence

Appendix A.1 deals with the existence of a socially optimal policy scheme \( P = \{\alpha, f_0^I, \omega\} \). It is organized as follows: In the next section we derive the conditions for which \( \alpha, f_0^I \) and \( \omega \) are socially optimal, and we analyze how those conditions can hold simultaneously. Section A.1.2 provides a simple sufficient condition that always ensures existence, while Section A.1.3 presents a counterexample where existence fails. In Section A.1.4 we propose a solution to the case when non-existence occurs. We show that there are feasible policy parameters that induce the industrial country to implement the socially optimal tax rate and the developing country to choose maximal abatement.

A.1.1. General Conditions for Existence

Socially optimal policy parameters \( \alpha, f_0^I \) and \( \omega \) have to satisfy the feasibility conditions of Definition 2.1, first- and second-order conditions. This is summed up in the following lemma:
Lemma A.1 (Conditions for socially optimal $\mathcal{P}$)

The set of policy parameters $\mathcal{P}$ is socially optimal if and only if $\omega$ satisfies

\begin{align*}
(i) & \quad 0 < \omega \leq \frac{\bar{e} - \beta^P \phi^I + 2 \beta^I \phi^I + \beta^I \phi^D}{\bar{e} - A \tau}, \\
(ii) & \quad \frac{2(\omega + 1)(\bar{e} - \phi^D \beta^I + 2 \beta^D)}{\omega(\bar{e} - A \tau)} - \frac{(\omega + 1)(\bar{e} - 2 \phi^D \tau)}{\omega} \geq 0 , \\
(iii) & \quad \phi^D (-1 + \beta^D \phi^D + \frac{\bar{e} - A \tau}{\bar{e} - P \tau}) + 2(\bar{e} - \phi^D \beta^I + 2 \beta^D) \frac{1}{\omega(\omega + 1) \tau} \\
& \quad \frac{(\bar{e} - A \tau)(\bar{e} - 2 \phi^D \tau)}{(\bar{e} - P \tau) \tau} > 0 .
\end{align*}

Proof:

Optimization of the objective functions $F^I$ and $F^D$ given in (2.22) and (2.23) respectively yields the first-order conditions

\[ \frac{\partial F^I}{\partial \tau^I} = \bar{e} - \phi^I \tau^I - \beta^I \phi^I s - \alpha(\bar{e} - 2 \phi^I \tau^I) \sum \omega^I \tau^j - \alpha \sum \omega^I \omega^D \tau^D \frac{\omega^I \omega^D \tau^D}{(\sum \omega^I \tau^j)^2} - (1 - \alpha)(\bar{e} - 2 \phi^I \tau^I) = 0 , \]
\[ \frac{\partial F^D}{\partial \tau^D} = \bar{e} - \phi^D \tau^D - \beta^D \phi^D s - \alpha(\bar{e} - 2 \phi^D \tau^D) \sum \omega^D \tau^j - \alpha \sum \omega^I \omega^D \tau^D \frac{\omega^I \omega^D \tau^D}{(\sum \omega^I \tau^j)^2} = 0 . \]

Assuming that both countries implement the tax goal, i.e. $\tau^I = \tau^D = \tau$, the first-order conditions are equivalent to

\begin{align*}
0 & \quad = \bar{e} - \phi^I \tau - \frac{\beta^I \phi^I}{B} \tau - \alpha(\bar{e} - 2 \phi^I \tau) \frac{\omega}{\omega + 1} - (1 - \alpha)(\bar{e} - 2 \phi^I \tau) \\
& \quad \quad - \alpha(\tau f^I + \tau (2 \bar{e} - P \tau)) \frac{\omega}{(\omega + 1) \tau} , \\
0 & \quad = \bar{e} - \phi^D \tau - \frac{\beta^D \phi^D}{B} \tau - \alpha(\bar{e} - 2 \phi^D \tau) \frac{1}{\omega + 1} \\
& \quad \quad - \alpha(\tau f^D + \tau (2 \bar{e} - P \tau)) \frac{\omega}{(\omega + 1) \tau} ,
\end{align*}
where we used $\omega = \omega^I / \omega^D$. Solving these for $\alpha$ and $f_0^I$ leads to

$$
\alpha = \frac{(\omega + 1)(\bar{e} - A\tau)}{2(\bar{e} - P\tau)},
$$

(A.4)

$$
f_0^I = \frac{2(\omega + 1)\tau(\bar{e} - \phi^D \frac{\beta^I + 2\beta^D}{B}(\bar{e} - P\tau) - (\omega + 1)\tau(\bar{e} - 2\phi^D\tau)}{\omega(\bar{e} - A\tau)} - \frac{\tau(2\bar{e} - P\tau)}{}.
$$

(A.5)

Note that Assumption 1 guarantees that $\alpha$ and $f_0^I$ are well defined, as $\bar{e} - P\tau > \bar{e} - A\tau > 0$ and hence the denominators occurring in the expressions for $\alpha$ and $f_0^I$ are always non-zero.

According to Definition 2.1, the policy parameters $\alpha$ and $f_0^I$ have to satisfy

$$
\alpha \in [0, 1],
$$

$$
f_0^I \geq 0.
$$

Condition $\alpha \geq 0$ is satisfied under Assumption 1.

Condition $\alpha \leq 1$ applied to (A.4) and rearranging terms, together with the feasibility condition $\omega > 0$, leads to (i). Condition $f_0^I \geq 0$ applied to (A.5) and rearranging terms leads to (ii).

Now we derive the second-order conditions ensuring that the solution obtained from the necessary conditions is indeed a minimum. The second derivatives of the objective functions $F^I$ and $F^D$ are

$$
\frac{\partial^2 F^I}{(\partial \tau^I)^2} = -\phi^I + \beta^I(\phi^I)^2 + 2\alpha\phi^I \frac{\omega^I \tau^I}{\sum \omega^j \tau^j} - 2\alpha(\bar{e} - 2\phi^I \tau^I) \frac{\omega^I \omega^D \tau^D}{(\sum \omega^j \tau^j)^2} + 2\alpha f \frac{(\bar{e} - 2\phi^D \tau^D)}{(\sum \omega^j \tau^j)^3} + 2(1 - \alpha) \phi^I,
$$

$$
\frac{\partial^2 F^D}{(\partial \tau^D)^2} = -\phi^D + \beta^D(\phi^D)^2 + 2\alpha\phi^D \frac{\omega^D \tau^D}{\sum \omega^j \tau^j} - 2\alpha(\bar{e} - 2\phi^D \tau^D) \frac{\omega^I \omega^D \tau^I}{(\sum \omega^j \tau^j)^2} + 2\alpha f \frac{(\bar{e} - 2\phi^D \tau^D)}{(\sum \omega^j \tau^j)^3}.
$$
Using $\tau^I = \tau^D = \tau$ and inserting the expressions for $\alpha$ and $f_0^I$, the second-order conditions can be written as

$$0 < \phi^I(1 + \beta^I \phi^I - \frac{\bar{e} - A\tau}{\bar{e} - P\tau}) - \frac{(\bar{e} - A\tau)(\bar{e} - 2\phi^I\tau)}{\bar{e} - P\tau} \frac{\omega}{(\omega + 1)\tau}$$

$$+ 2(\bar{e} - \phi^I\beta^I + 2\beta^D) \frac{\omega}{(\omega + 1)\tau} - \frac{(\bar{e} - A\tau)(\bar{e} - 2\phi^D\tau)}{\bar{e} - P\tau} \frac{\omega}{(\omega + 1)\tau},$$

$$0 < \phi^D(-1 + \beta^D \phi^D + \frac{\bar{e} - A\tau}{\bar{e} - P\tau}) - \frac{(\bar{e} - A\tau)(\bar{e} - 2\phi^D\tau)}{\bar{e} - P\tau} \frac{\omega}{(\omega + 1)\tau}$$

$$+ 2(\bar{e} - \phi^D\beta^I + 2\beta^D) \frac{1}{(\omega + 1)\tau} - \frac{(\bar{e} - A\tau)(\bar{e} - 2\phi^D\tau)}{\bar{e} - P\tau} \frac{1}{(\omega + 1)\tau}.$$
only if the right-hand side of (A.6) is negative. But this holds always true as
\[
\bar{e} - 2\phi^D \tau - 2 \frac{\bar{e} - P\tau}{\bar{e} - A\tau} (\bar{e} - \frac{\phi^D \beta^I + 2\beta^D}{B} \tau) \leq -\bar{e} + \frac{2\beta^D \phi^D}{B} \tau < -\bar{e} + A\tau,
\]
and \(-\bar{e} + A\tau < 0\) because of Assumption 1.

Condition (A.3) is equivalent to
\[
\omega \left\{ \phi^D \tau (-1 + \beta^D \phi^D + \frac{\bar{e} - A\tau}{\bar{e} - P\tau}) - \frac{(\bar{e} - A\tau)(\bar{e} - 2\phi^D \tau)}{\bar{e} - P\tau} \right\}
\]
\[
> \frac{(\bar{e} - A\tau)(\bar{e} - 2\phi^D \tau)}{\bar{e} - P\tau} - 2(\bar{e} - \frac{\phi^D \beta^I + 2\beta^D}{B} \tau) - \phi^D \tau (-1 + \beta^D \phi^D + \frac{\bar{e} - A\tau}{\bar{e} - P\tau}).
\]

We again observe that the term in curly brackets on the left-hand side is negative if the right-hand side of (A.7) is positive. Hence, we can find a \(\omega > 0\) that satisfies condition (A.3) if and only if the right-hand side of (A.7) is negative.

These considerations are summed up in the following lemma:

**Lemma A.2**
A socially optimal policy scheme \(P\) exists if and only if the following conditions hold:
\[
\frac{(\bar{e} - A\tau)(\bar{e} - 2\phi^D \tau)}{\bar{e} - P\tau} - 2(\bar{e} - \frac{\phi^D \beta^I + 2\beta^D}{B} \tau) - \phi^D \tau (-1 + \beta^D \phi^D + \frac{\bar{e} - A\tau}{\bar{e} - P\tau}) < 0.
\]

Closer investigation of the inequalities (A.6) and (A.7) also reveals that if one of them is satisfied for some value \(\bar{\omega} > 0\), the same condition is satisfied for all \(\omega \in (0, \bar{\omega}]\). This yields

**Fact 1**
Conditions (A.1), (A.2), and (A.3) impose an upper bound on \(\omega\). The lower bound is equal to zero.

The intuition why a DCR imposes an upper bound on \(\omega\) runs as follows: If the level of \(\omega\) is very high and thus \(\omega^I\) is large relative to \(\omega^D\), the industrial country would abate insufficiently relative to the industrial country, as it receives high refunds in any case. Therefore it would be impossible to induce the countries to choose the socially optimal abatement levels.
A.1.2. A Sufficient Condition

A simple sufficient condition for the existence is $\phi^I$ being close enough to $\phi^D$:

**Lemma A.3**

Suppose that $\phi^I$ is sufficiently close to $\phi^D$. Then there always exists a socially optimal policy scheme $P = \{\alpha, f_0^I, \omega\}$.

**Proof:**

As we have seen in Lemma A.2, a socially optimal policy scheme exists if and only if the condition on the model parameters, (A.8), is satisfied. Inserting $\phi^I = \phi^D = \phi$ into (A.8) as we consider the case of $\phi^I$ being not too small compared to $\phi^D$ yields

$$-\bar{e} + 2\phi \tau - \phi \tau (\beta^D \phi + \frac{\bar{e} - A \tau}{\bar{e} - 2\phi \tau}) < 0.$$ 

This holds by Assumption 1, hence existence is ensured in the case of $\phi^I = \phi^D = \phi$. By continuity, the existence of a socially optimal policy scheme is also guaranteed when $\phi^I$ is slightly decreased. \hfill \Box

A.1.3. A Counterexample

Non-existence of socially optimal policy schemes can occur when $\phi^I$ is very small. An example is $\phi^D = 1, \phi^I = 10^{-6}, \beta^D = 0.25, \beta^I = 0.2$. For these parameter values, (A.8) from Lemma A.2 is not satisfied, and hence, the second-order condition for the minimization problem of the developing country cannot be fulfilled for any feasible set of policy parameters $P$.

A.1.4. Corner Solution

Even if the second-order condition of the developing country fails, we can find feasible policy parameters $\alpha, f_0^I$ and $\omega$ such that the industrial country implements the socially optimal tax rate $\tau$ and the developing country abates maximally, i.e. sets the corner solution $\bar{e}$.
Lemma A.4
There always exists a feasible set of policy parameters $\mathcal{P} = \{\alpha, f_0^I, \omega\}$ such that countries implement $\tau^I = \tau^*, \tau^D = \frac{\bar{e}}{\phi^D}$ under the DCR. It is given by $\alpha$ and $\omega$ close to zero, and $f_0^I$ correspondingly large.

Proof:
The countries implement $\tau^I = \tau, \tau^D = \frac{\bar{e}}{\phi^D}$ if, evaluated at $\tau^I = \tau, \tau^D = \frac{\bar{e}}{\phi^D}$,

$$\frac{\partial F^I}{\partial \tau^I} = 0, \quad \frac{\partial^2 F^I}{(\partial \tau^I)^2} > 0, \quad \frac{\partial F^D}{\partial \tau^D} \leq 0$$

hold. Solving $\frac{\partial F^I}{\partial \tau^I} = 0$ for $f_0^I$ yields

$$f_0^I = \frac{\omega \tau + e}{\alpha \omega \frac{\bar{e}}{\phi^D}} \left\{ \frac{\beta^D \phi^I}{B} \tau + \alpha (\bar{e} - 2 \phi^I \tau) \frac{\frac{\bar{e}}{\phi^D}}{\omega \tau + \frac{\bar{e}}{\phi^D}} - \alpha \sigma (\bar{e} - \phi^I \tau) \frac{\omega \frac{\bar{e}}{\phi^D}}{(\omega \tau + \frac{\bar{e}}{\phi^D})^2} \right\}$$

which is positive for $\omega$ close to zero and arbitrary $\alpha \in [0, 1]$. Evaluating $\frac{\partial F^D}{\partial \tau^D}$ at $\tau^I = \tau, \tau^D = \frac{\bar{e}}{\phi^D}$, we obtain

$$\frac{\partial F^D}{\partial \tau^D} = -\frac{\beta^D \phi^D}{B} \tau + \alpha \bar{e} \frac{\frac{\bar{e}}{\phi^D}}{\omega \tau + \frac{\bar{e}}{\phi^D}} - \alpha f \frac{\omega \frac{\bar{e}}{\phi^D}}{(\omega \tau + \frac{\bar{e}}{\phi^D})^2}$$

which is non-positive if $\alpha$ is close to zero. Now we consider $\frac{\partial^2 F^I}{(\partial \tau^I)^2}$:

$$\frac{\partial^2 F^I}{(\partial \tau^I)^2} = \phi^I (1 + \beta^I \phi^I) + 2 \alpha \phi^I \frac{\frac{\bar{e}}{\phi^D}}{\omega \tau + \frac{\bar{e}}{\phi^D}} - 2 \alpha (\bar{e} - 2 \phi^I \tau) \frac{\omega \frac{\bar{e}}{\phi^D}}{(\omega \tau + \frac{\bar{e}}{\phi^D})^2} \left( \frac{\omega \frac{\bar{e}}{\phi^D}}{(\omega \tau + \frac{\bar{e}}{\phi^D})^2} \right)^2$$

$$+ 2 \alpha f \frac{\omega^2 \frac{\bar{e}}{\phi^D}}{(\omega \tau + \frac{\bar{e}}{\phi^D})^3}$$

which is positive for $\alpha$ being close to zero. Hence by setting $\alpha$ and $\omega$ close to zero, we can find a feasible albeit large $f_0^I$ such that the countries will implement $\tau^I = \tau, \tau^D = \frac{\bar{e}}{\phi^D}$. Hence by setting $\alpha$ and $\omega$ close to zero, we can find a feasible albeit large $f_0^I$ such that the countries will implement $\tau^I = \tau, \tau^D = \frac{\bar{e}}{\phi^D}$. □

Lemma A.4 implies that even if it is impossible to implement the socially optimal solution $\tau^I = \tau^D = \tau$ under the DCR, the scheme can still induce the countries to abate emissions considerably. This is achieved by setting $\omega$ small, which increases the developing country’s share of refunds to an extent such that the developing country has an incentive to abate maximally, and by setting $\alpha$ small so that the industrial country
A. Appendix to Chapter 2

benefits from a large leftover in the fund.

A.2. Proofs

A.2.1. Proof of Proposition 2.3

If both countries choose the socially optimal tax rate $\tau^I = \tau^D = \tau$, the developing country is a net receiver if and only if

$$\alpha f \frac{1}{\omega + 1} \geq \tau (\bar{e} - \phi^D \tau).$$  \hspace{1cm} (A.9)

Inserting $\alpha$ and $f_0^I$ as given in (A.4) and (A.5), respectively, into (A.9) yields

$$\frac{(\omega + 1)\tau}{\omega} \left\{ (\bar{e} - \phi^D \beta^I + 2 \beta^D) B^{-\tau} \right\} \geq \tau (\bar{e} - \phi^D \tau).$$

By construction of a socially optimal policy scheme, the term in curly brackets on the left-hand side is positive. Furthermore, the term $(\omega + 1)/\omega$ tends to infinity if $\omega$ tends to zero. The right-hand side of inequality (A.9) is independent of $\omega$. Thus inequality (A.9) is satisfied if $\omega$ is sufficiently small.

From Fact 1, we know that there exist socially optimal policy schemes for all $\omega \in (0, \bar{\omega}^{NR})$ if there exists an optimal policy scheme associated with some $\bar{\omega}^{NR}$. Hence, if there exists a socially optimal scheme, we can always find one for which the developing country is a net receiver.

A.2.2. Proof of Corollary 2.3

The developing country is a net receiver if and only if

$$\alpha f \sum_j \omega^j \tau^j \frac{\omega^D \tau^D}{\omega^D \tau^D} \geq \tau^D (\bar{e} - \phi^D \tau^D).$$

Assuming equal weights in the refunding formula, i.e. $\omega^i = \phi^i, i \in \{I, D\}$, inserting the tax goal $\tau^I = \tau^D = \tau$, and $\alpha$ and $f_0^I$ as given in (A.4) and (A.5), respectively, this is
equivalent to
\[
\frac{(1 + \beta^I \phi^I - \beta^I \phi^D - 3 \beta^D \phi^D - \beta^D \phi^I)(\phi^D - \phi^I)}{(1 - BP)(1 + BP)} \geq 0 .
\]
This holds true for all parameters \( \phi^I, \phi^D, \beta^I, \beta^D \) that fulfill Assumption 1. Hence, for a refunding formula with equal weights, the developing country is a net receiver for any socially optimal policy scheme.

A.2.3. Proof of Proposition 2.4

The developing country is a net beneficiary if and only if
\[
\frac{\phi^D}{2}((\hat{\tau}_D)^2 - (\tau^*)^2) + \frac{\beta^D}{2}((\hat{s})^2 - (s^*)^2) - \tau^*(\bar{c} - \phi^D \tau^*) + \alpha f \frac{1}{\omega + 1} \geq 0 . \quad (A.10)
\]
We proceed as in the proof of Proposition 2.3. With the exception of the last term on the left-hand side of (A.10), all terms are independent of \( \omega \). If we insert \( \alpha \) and \( f^I_0 \) as given in (A.4) and (A.5), respectively, the last term tends to infinity if \( \omega \) approaches zero. Hence, for a \( \omega > 0 \) small enough, inequality (A.10) is satisfied. As the existence of a socially optimal policy scheme with \( \bar{\omega}^{NB} > 0 \) implies the existence of such a scheme for all \( \omega \in (0, \bar{\omega}^{NB}) \), we can always find one for which the developing country is a net beneficiary.

A.2.4. Proof of Corollary 2.4

Note first that the socially optimal tax rate minimizes total abatement costs and damages. Second, total costs under socially optimal refunding yield the same minimal total abatement costs and damages as monetary flows will net to zero. Third, from Proposition 2.4 we know that by decreasing \( \omega \) we can always obtain \( F^{DS,D}(\hat{\tau}_D) = F^D(\tau^*) \) and thus \( F^{DS,I}(\hat{\tau}_I) > F^I(\tau^*) \). As country-specific total costs are continuous in \( \omega \), lowering \( \omega \) slightly will make both countries better off, which completes the proof.
A.2.5. Proof of Proposition 2.5

For homogeneous countries $\beta^I = \beta^D = \beta$, $\phi^I = \phi^D = \phi$, the parameters $\alpha$ and $f_0^I$ as given in (A.4) and (A.5), respectively, change to

$$\alpha = \frac{\omega + 1}{2},$$
$$f_0^I = \frac{1 - \omega}{\omega} (\bar{e} - \phi \tau).$$

Since Assumption 1 simplifies to $\bar{e} - 2\phi \tau > 0$ in the case of homogeneous countries, both $\alpha \leq 1$ and $f_0^I \geq 0$ are equivalent to $\omega \leq 1$ (imposing $\omega > 0$). The second-order conditions are

$$0 < \beta \phi^2 + \frac{\omega}{\omega + 1},$$
$$0 < \beta \phi^2 + (\bar{e} - \phi \tau) \frac{1 - \omega}{(\omega + 1) \tau} + \frac{\omega}{\omega + 1} \phi.$$

They are satisfied for any $\omega \leq 1$. Hence, for a refunding rule with $\omega^I \leq \omega^D$, there exist socially optimal parameter sets $\mathcal{P}$.

For $\omega^I = \omega^D$ and hence $\omega = 1$ we obtain $\alpha = 1$.

A.2.6. Proof of Corollary 2.5

Consider homogeneous countries $\beta^I = \beta^D = \beta$, $\phi^I = \phi^D = \phi$. The developing country is a net receiver if and only if

$$\alpha f \sum_j \omega^D j \omega^D j \geq \tau^D (\bar{e} - \phi \tau^D).$$

Inserting the tax goal $\tau^I = \tau^D = \tau$, the policy parameters $\alpha$ and $f_0^I$ derived in the proof of Proposition 2.5 adn rearranging terms yields

$$(\bar{e} - \phi \tau) \frac{1 - \omega}{2 \omega} \geq 0.$$  \hfill (A.11)

As a socially optimal policy scheme satisfies $\omega \leq 1$ (see Proposition 2.5), (A.11) holds.
The developing country is a net beneficiary if and only if
\[
\frac{\phi}{2}((\hat{T}^D)^2 - (\tau^*)^2) + \frac{\beta}{2}((\hat{s})^2 - (s^*)^2) - \tau^*(\bar{e} - \phi\tau^*) + \alpha f \frac{1}{\omega + 1} \geq 0. \tag{A.12}
\]

The sum of the first two terms in (A.12) is non-negative, as countries are homogeneous and thus the social optimum minimizes both the sum and each country’s total costs. Inserting the tax goal \(\tau^I = \tau^D = \tau\) and the policy parameters \(\alpha\) and \(f_0\) derived in the proof of Proposition 2.5, the sum of the last two terms simplifies to
\[
\frac{1 - \omega}{2\omega} \tau(\bar{e} - \phi\tau).
\]
Again, as \(\omega \leq 1\), this expression is non-negative, and hence (A.12) holds.

**A.2.7. Proof of Proposition 2.6**

Suppose identical abatement costs \(\phi^I = \phi^D = \phi\) and heterogeneous damage costs \(\beta^I = 0 < \beta^D = \beta\). Then the parameters \(\alpha\) and \(f_0\) as given in (A.4) and (A.5), respectively, change to
\[
\alpha = \frac{(\omega + 1)(\bar{e} - 3\phi\tau)}{2(\bar{e} - 2\phi\tau)}, \tag{A.13}
\]
\[
\frac{f_0^I}{\omega} = \frac{\tau(\omega + 1)(\bar{e} - \phi\tau)(\bar{e} - 2\phi\tau)}{\omega(\bar{e} - 3\phi\tau)} - 2\tau(\bar{e} - \phi\tau). \tag{A.14}
\]

Assumption 1 simplifies to \(\beta\phi < \frac{1}{4}\).

For a socially optimal parameter set \(\mathcal{P} = \{\alpha, f_0, \omega\}, \omega > 0\) must hold. The condition \(\alpha \geq 0\) is satisfied under Assumption 1, and \(\alpha \leq 1\) is equivalent to
\[
\omega \leq \frac{\bar{e} - \phi\tau}{\bar{e} - 3\phi\tau}. \tag{A.15}
\]
Moreover, \(f_0^I \geq 0\) has to hold. This can be written as
\[
\bar{e} - 2\phi\tau - \omega(\bar{e} - 4\phi\tau) \geq 0. \tag{A.16}
\]
We only have to examine the case $\bar{e} - 4\phi \tau > 0$, as for $\bar{e} - 4\phi \tau \leq 0$, (A.16) holds. Inequality (A.16) is then equivalent to

$$\omega \leq \frac{\bar{e} - 2\phi \tau}{\bar{e} - 4\phi \tau}. \tag{A.17}$$

Comparison of (A.15) and (A.17) reveals that (A.15) is the stronger condition.

We now turn to the second-order conditions. For identical abatement costs $\phi^I = \phi^D = \phi$ and heterogeneous damage costs $\beta^I = 0 < \beta^D = \beta$ they can be written as

$$0 < \frac{\phi^2 \tau}{\bar{e} - 2\phi \tau} + 2\phi \frac{\omega}{(\omega + 1)},$$

$$0 < -\phi^2 \frac{\tau}{\bar{e} - 2\phi \tau} + \beta \phi^2 - \frac{\bar{e} - 3\phi \tau}{\tau} \frac{2(\bar{e} - 2\phi \tau)}{(\omega + 1)\tau} =: g(\omega).$$

The former inequality always holds, since $\bar{e} - 2\phi \tau > 0$ by Assumption 1 and $\omega > 0$. For the latter we calculate the derivative with respect to $\omega$:

$$g'(\omega) = - \frac{2(\bar{e} - 2\phi \tau)}{(1 + \omega)^2 \tau} < 0.$$ 

Hence it is strictly monotonically decreasing in $\omega$. Consider $g(\omega)$ evaluated at $\omega = 0$. Using $\tau = \frac{2\beta^3}{1 + 2\beta^3}$, we find

$$g(0) = \frac{-4\beta^3 \phi^3 - 2\beta^2 \phi^2 - 2\beta \phi + 1}{2\beta(1 - 2\beta \phi)},$$

which is positive for $0 < \beta \phi < 1/4$. On the other hand, if we evaluate the second-order condition at $\omega = \frac{\bar{e} - \phi \tau}{\bar{e} - 3\phi \tau}$, we obtain

$$g \left( \frac{\bar{e} - \phi \tau}{\bar{e} - 3\phi \tau} \right) = -\beta \phi^2 \frac{1 + 2\beta \phi}{1 - 2\beta \phi},$$

which is negative for $0 < \beta \phi < 1/4$. Due to strict monotonicity with respect to $\omega$, there exists a unique $\omega \in (0, \frac{\bar{e} - \phi \tau}{\bar{e} - 3\phi \tau})$ for which the sign of $g(\omega)$ changes. By setting $g(\omega) = 0$ and solving for $\omega$, we see that it is given by

$$\omega = \frac{4\beta^3 \phi^3 + 2\beta^2 \phi^2 + 2\beta \phi - 1}{-4\beta^3 \phi^3 - 10\beta^2 \phi^2 + 6\beta \phi - 1}.$$
Hence the second-order conditions are satisfied for all $\omega$ in the interval
\[
\left(0, \frac{4\beta^3 \phi^3 + 2\beta^2 \phi^2 + 2\beta \phi - 1}{-4\beta^3 \phi^3 - 10\beta^2 \phi^2 + 6\beta \phi - 1}\right).
\]
The function $h(x) = \frac{4x^3 + 2x^2 + 2x - 1}{-4x^3 - 10x^2 + 6x - 1}$ is always larger than 1 for $0 < x < \frac{1}{4}$ (recall that Assumption 1 is equivalent to $\beta \phi < \frac{1}{4}$). Therefore there exists a socially optimal policy scheme $P = \{\alpha, f^I_0, 1\}$, and $\alpha$ and $f^I_0$ simplify in this case to
\[
\alpha = \frac{\bar{e} - 3\phi \tau}{\bar{e} - 2\phi \tau},
\]
\[
f^I_0 = \frac{2\phi \tau^2 \bar{e} - \phi \tau}{\bar{e} - 3\phi \tau}.
\]
This completes the proof.

### A.2.8. Proof of Corollary 2.6

Recall that the developing country is a net receiver if and only if
\[
\alpha f \sum_j \omega^D_j \tau^D_j \geq \tau^D (\bar{e} - \phi \tau^D).
\]

We insert the tax goal $\tau^I = \tau^D = \tau$ and the policy parameters $\alpha$ and $f^I_0$ derived in Proposition 2.6 for homogeneous marginal abatement costs $\phi^I = \phi^D = \phi$ and heterogeneous marginal damages $\beta^I = 0 < \beta^D = \beta$, and obtain
\[
\frac{\omega + 1}{2\omega} \geq 1,
\]
which is equivalent to $\omega \leq 1$ and hence $\omega^I \leq \omega^D$. This proves the first point.

The developing country is a net beneficiary if and only if
\[
\frac{\phi}{2} ((\tau^D)^2 - (\tau^*)^2) + \frac{\beta}{2} ((\bar{s})^2 - (s^*)^2) - \tau^* (\bar{e} - \phi \tau^*) + \alpha f \frac{1}{\omega + 1} \geq 0.
\]

Inserting the tax rates, $\alpha$ and $f^I_0$ for $\phi^I = \phi^D = \phi$ and $\beta^I = 0 < \beta^D = \beta$, this inequality is equivalent to
\[
\omega (6\beta^2 \phi^2 + 3\beta \phi - 1) + (1 + \beta \phi) \geq 0.
\]
We distinguish two cases. Case 1: If $6\beta^2\phi^2 + 3\beta\phi - 1 > 0$, then all terms on the left-hand side of (A.18) are positive and the inequality is fulfilled. Case 2: If $6\beta^2\phi^2 + 3\beta\phi - 1 < 0$, we insert the upper bound of $\omega$ from Proposition 2.6, $\frac{4\beta^3\phi^3 + 2\beta^2\phi^2 + 2\beta\phi - 1}{4\beta^3\phi^3 - 10\beta^2\phi^2 + 6\beta\phi - 1}$, into the left-hand side of (A.18) and obtain

$$\frac{\beta^2\phi^2(24\beta^3\phi^3 + 20\beta^2\phi^2 - 6)}{-4\beta^3\phi^3 - 10\beta^2\phi^2 + 6\beta\phi - 1},$$

which is positive as $\beta\phi < 1/4$ from Assumption 1. Thus the left-hand side is positive for all $\omega \in \left(0, \frac{4\beta^3\phi^3 + 2\beta^2\phi^2 + 2\beta\phi - 1}{4\beta^3\phi^3 - 10\beta^2\phi^2 + 6\beta\phi - 1}\right)$ since it is decreasing in $\omega$. This completes the proof.

A.2.9. Proof of Proposition 2.7

Recall from Lemma A.1 that a necessary condition for a socially optimal policy scheme $P$ is that $\alpha$ and $f_0^I$ fulfill

$$\alpha = \frac{(\omega + 1)(\bar{e} - A\tau)}{2(\bar{e} - P\tau)},$$

$$f_0^I = \frac{2(\omega + 1)\tau(\bar{e} - \phi^D\frac{\beta^I + 2\beta^D}{B}\tau)(\bar{e} - P\tau)}{\omega(\bar{e} - A\tau)} - \frac{(\omega + 1)\tau(\bar{e} - 2\phi^D\tau)}{\omega} - \tau(2\bar{e} - P\tau).$$

Setting $\alpha$ to 1 yields

$$\omega = \frac{\bar{e} - \frac{\beta^I\phi^D + 2\beta^I\phi^D + \beta^D\phi^I}{B} \tau}{\bar{e} - \frac{\beta^I\phi^D + 2\beta^I\phi^D + \beta^D\phi^I}{B} \tau},$$

which is always $\geq 1$ under Assumption 1. Inserting this into the equation for $f_0^I$ and setting it to zero yields condition (2.26).

A.2.10. Proof of Corollary 2.7

For homogeneous countries $\beta^I = \beta^D = \beta$, $\phi^I = \phi^D = \phi$, condition (2.26) simplifies to

$$\frac{4\tau(\bar{e} - 2\phi^D)^2(\bar{e} - \frac{2}{3}\phi^D)}{(\bar{e} - 2\phi^D)^2} - \frac{2\tau(\bar{e} - 2\phi^D)^2}{\bar{e} - 2\phi^D} - 2\tau(\bar{e} - \phi^D) = 0,$$
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which always holds true. Recall from Proposition 2.5 that for homogeneous countries there always exists a socially optimal policy scheme provided that $\omega \leq 1$. Now, if $\alpha = 1$ and $f_0^l = 0$, it follows that $\omega = 1$ since

$$
\alpha = \frac{\omega + 1}{2},
$$

$$
f_0^l = \frac{1 - \omega}{\omega}(\bar{e} - \phi \tau)\tau .
$$

This completes the proof.

A.2.11. Proof of Proposition 2.8

The industrial country wants to set its emission tax $\tau^I$ such that

$$
\phi^I (\tau^I)^2 + \frac{\beta^I}{2} s^2 - \alpha f_0^l \sum_j \omega_j \tau^I + f_0^l - (1 - \alpha)f_0^l
$$

is minimized, whereas the developing country minimizes

$$
\phi^D (\tau^D)^2 + \frac{\beta^D}{2} s^2 - \alpha f_0^l \sum_j \omega_j \tau^D
$$

with respect to $\tau^D$. The first-order conditions then are

$$
0 = \phi^I \tau^I - \beta^I \phi^I s - \alpha f_0^l \omega^I \omega^D \tau^I (\sum_j \omega_j \tau)^2 ,
$$

$$
0 = \phi^D \tau^D - \beta^D \phi^D s - \alpha f_0^l \omega^I \omega^D \tau^I (\sum_j \omega_j \tau)^2 .
$$

Assuming implementation of the tax goal $\tau^I = \tau^D = \tau$ and using $\omega = \omega^I / \omega^D$, they simplify to

$$
0 = \phi^I \tau - \frac{\beta^I \phi^I}{B} \tau - \alpha f_0^l \frac{\omega}{(\omega + 1)^2} ,
$$

$$
0 = \phi^D \tau - \frac{\beta^D \phi^D}{B} \tau - \alpha f_0^l \frac{\omega}{(\omega + 1)^2} .
$$

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These two equations can only hold simultaneously if
\[ \beta^I \phi^D = \beta^D \phi^I. \]
This is equation (2.27).

If condition (2.27) holds, the first-order conditions above reduce to one equation, from which we can express \( \alpha \) in terms of \( \omega \) and \( f_0^I \):

\[ \alpha = \frac{(\omega + 1)^2 r^2}{\omega f_0^I} \beta^D \phi^I. \]  
(A.21)

We have
\[ \alpha > 0 \iff \omega, f_0^I > 0, \]
\[ \alpha \leq 1 \iff f_0^I \geq \frac{(\omega + 1)^2}{\omega} \beta^D \phi^I \frac{\beta^I}{B} r^2 \quad \text{(assuming } \omega > 0). \]

The second-order conditions are given by
\[ \phi^I + \beta^I (\phi^I)^2 + 2 \alpha f_0^I \frac{(\omega^I)^2 \omega^D \tau^D}{(\sum_j \omega^j \tau^j)^3} > 0, \]
\[ \phi^D + \beta^D (\phi^D)^2 + 2 \alpha f_0^I \frac{(\omega^D)^2 \omega^I \tau^I}{(\sum_j \omega^j \tau^j)^3} > 0. \]

Inserting the tax goal and \( \alpha \) from (A.21), we obtain
\[ \phi^I + \beta^I (\phi^I)^2 + \frac{2 \beta^D \phi^I}{B} \frac{\omega}{\omega + 1} > 0, \]
\[ \phi^D + \beta^D (\phi^D)^2 + \frac{2 \beta^D \phi^I}{\beta^I + \beta^D} \frac{1}{\omega + 1} > 0, \]
which holds true for all \( \omega > 0 \). Hence it is always possible to find socially optimal policy parameters, given that condition (2.27) is satisfied.
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A.2.12. Proof of Corollary 2.8

As to part (i), recall that the developing country is a net receiver if and only if

\[ \alpha_f \sum_j \omega_j \tau_j \geq \tau^D (\bar{e} - \phi^D \tau^D) . \]  

(A.22)

For a refunding scheme without tax revenues, according to Proposition 2.8, under tax goal implementation we have

\[ \alpha_f = \frac{(\omega + 1)^2 \tau^2 \phi^I \beta^D \omega B}{\omega B} . \]

Inserting this expression into (A.22) and rearranging terms yields

\[ \omega \left( \bar{e} - \frac{\beta^I \phi^D + \beta^D \phi^D + \beta^D \phi^I}{B} \tau \right) \leq \frac{\beta^D \phi^I}{B} \tau . \]

Inserting \( \tau = 2\bar{e}B/(1 + BP) \) and taking into account Assumption 1 leads to inequality (2.28).

As to part (ii), the developing country is a net beneficiary if and only if

\[ \frac{\phi^D}{2} ((\tau^D)^2 - (\tau^*)^2) + \frac{\beta^D}{2} ((s)^2 - (s^*)^2) - \tau^*(\bar{e} - \phi^D \tau^*) + \alpha_f \frac{1}{\omega + 1} \geq 0 , \]

which can be rewritten as

\[ \omega \left\{ \frac{\phi^D}{2} ((\tau^D)^2 - (\tau^*)^2) + \frac{\beta^D}{2} ((s)^2 - (s^*)^2) - \tau^*(\bar{e} - \phi^D \tau^*) + \frac{\beta^D \phi^I}{B} (\tau^*)^2 \right\} \geq -\frac{\beta^D \phi^I}{B} (\tau^*)^2 \]  

(A.23)

As the right-hand side of inequality (A.23) is negative, we can always find a \( \omega > 0 \) that satisfies (A.23). Moreover, tedious algebraic manipulations reveal that the term in curly brackets on the left-hand side is negative.\(^1\) Therefore, (A.23) provides an upper boundary \( \bar{\omega}^{NT} > 0 \) on \( \omega \), such that the developing country is a net beneficiary for all \( \omega \leq \bar{\omega}^{NT} \).

\(^1\)Details are available upon request.
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Notation

Throughout Appendix B we work with the following abbreviations:

\[ B = \beta^I + \beta^D, \]
\[ P = \phi^I + \phi^D, \]
\[ A = \frac{\beta^D P + \phi^D B}{B}, \]
\[ \tau = \tau^* \text{ (tax goal, given by (3.4))}, \]
\[ \tau^I_c = \tau^{*I}_c \text{ (tax goal, given by (3.6))}, \]
\[ \tau^D_c = \tau^{*D}_c \text{ (tax goal, given by (3.7))}. \]

B.1. Proof of Proposition 3.1

First note that if we sort the vector of choice variables \( \tau^i, i \in \{I, D\} \), appropriately, the Hessian of the social planner problem equals

\[ H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}, \tag{B.1} \]

where the \( k \times k \)-block matrices \( H_{11}, H_{12}, H_{21}, \text{ and } H_{22} \) are given by

\[ (H_{11})_{ij} = \phi^I(\delta_{ij} + k\phi^I B), \quad i, j = 1, \ldots, k, \]
\[ (H_{12})_{ij} = (H_{21})_{ij} = kB\phi^I\phi^D, \quad i, j = 1, \ldots, k, \]
\[ (H_{22})_{ij} = \phi^D(\delta_{ij} + k\phi^D B), \quad i, j = 1, \ldots, k. \]
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Here we used the Kronecker notation

\[
\delta_{ij} = \begin{cases} 
1, & \text{if } i = j , \\
0, & \text{if } i \neq j .
\end{cases}
\]  

(B.2)

It is easy to verify that the Hessian is positive definite as

\[
x^T H x = \phi^I \sum_{i=1}^{k} x_i^2 + \phi^D \sum_{i=k+1}^{2k} x_i^2 + k B \left( \phi^I \sum_{i=1}^{k} x_i + \phi^D \sum_{i=k+1}^{2k} x_i \right)^2 > 0
\]

for all vectors \( x = (x_1, \ldots, x_{2k})^T \neq 0 \). Hence the social planner problem is strictly convex and has therefore a unique minimum.

The first-order conditions

\[
\tau^i = k B s \quad \forall \ i \in \Omega .
\]  

(B.3)

imply symmetric taxes for all countries. Inserting (3.2) and solving for \( \tau^i = \tau^* \) yields (3.4). Then we can derive the optimal stock \( s^* \) which yields (3.5).

We observe that

\[
\tau^* = \frac{2k^2 \bar{e} B}{1 + k^2 B P} > \frac{\bar{e}}{\phi^D} \quad \iff \quad k > \frac{1}{\sqrt{B(\phi^D - \phi^I)}} .
\]  

(B.4)

Hence if the number of countries, \( k \), exceeds the threshold \( k^* := \sqrt{B(\phi^D - \phi^I)}^{-1} \), solving the first-order conditions of the developing countries would violate the constraint \( \tau^i \leq \bar{e}/\phi^D, i \in \Omega_D \). Developing countries will then choose the corner solution \( \tau^*_{Dc} = \bar{e}/\phi^D \) instead of \( \tau^* \). Inserting this into the first-order condition for the industrial countries and solving for \( \tau^i = \tau^*_I, i \in \Omega_I \), yields (3.6). Then we can calculate the optimal stock for the corner solution, \( s^*_c \), yielding (3.8). Note also that \( \tau^* \leq \bar{e}/\phi^I \) holds for all \( i \in \mathbb{N} \). This finishes the proof.

B.2. Proof of Proposition 3.2

Note first that country \( i \)'s problem is strictly convex. Hence there exists a unique Nash equilibrium of the local planner problem.
Solving the first-order conditions $\tau^i = \beta^i s$ for all countries $i \in \Omega$ simultaneously yields (3.10) and (3.11).

Second, note that

$$\hat{\tau}_D = 2\bar{e}k \beta D \frac{1}{1 + k(\beta I \phi I + \beta D \phi D)} > \frac{\bar{e}}{\phi D} \iff k > \frac{1}{\beta D \phi D - \beta I \phi I}.$$  \hspace{1cm} (B.5)

Hence if the number of countries, $k$, exceeds the threshold $\hat{k} := (\beta D \phi D - \beta I \phi I)^{-1}$, the tax rate given by the first-order conditions violates the constraint of the developing countries. Thus, the developing countries will choose the corner solutions $\hat{\tau}_D^c = \bar{e}/\phi D$ instead of $\hat{\tau}_D$. Inserting $\hat{\tau}_D$ into the first-order condition for the industrial countries and solving for $\tau^i = \hat{\tau}_I^i$, $i \in \Omega_I$, yields (3.13). Inserting the tax rates $\hat{\tau}_D^c, \hat{\tau}_I^c$ into the stock equation yields (3.14). Note also that $\hat{\tau}_I^c \leq \bar{e}/\phi I$ holds for all $k \in \mathbb{N}$. This finishes the proof.

**B.3. Proof of Corollary 3.3**

We only show (i) as the remaining parts are straightforward to see. Note that $s^*_c$ is relevant only for $k > k^*$, and $\hat{s}_c$ only for $k > \hat{k}$. Hence we have to show

\begin{align*}
s^* &< \hat{s} \quad \forall k \leq \min\{k^*, \hat{k}\}, \\
s^*_c &< \hat{s}_c \quad \forall k > \max\{k^*, \hat{k}\}, \\
s^* &< \hat{s}_c \quad \forall \hat{k} < k \leq k^*, \\
s^*_c &< \hat{s} \quad \forall k^* < k \leq \hat{k}.
\end{align*}

It is straightforward to see that $s^* < \hat{s}$ and $s^*_c < \hat{s}_c$ for all $k \geq 1$. If $k > \hat{k}$, we know from Corollary 3.2 that $\hat{s} < \hat{s}_c$ and hence we also have $s^* < \hat{s}_c$. In case of $k^* < k \leq \hat{k}$, we can conclude $s^*_c < \hat{s}$, as it is equivalent to

$$1 + 2k^2 B \phi I - k(\beta I \phi I + \beta D \phi D) > 0,$$

which holds true for that range of $k$. This completes the proof.
B.4. Proof of Proposition 3.3

We start by proving (i). We calculate first-order conditions for both types of countries by differentiating the cost functions given in (3.16) and (3.17) and setting them equal to zero. As we assume that the tax goal is implemented, we insert \( \tau_i = \tau \forall i \in \Omega \). Solving for \( \alpha \) and \( f_{I0} \) leads to

\[
\alpha = \frac{k(\omega + 1)(MA^D - NA^I)}{M(\bar{e} - 2\phi^D\tau) + N(\bar{e} - 2\phi^I\tau)}, \tag{B.7}
\]

\[
f_{I0} = \frac{(\omega + 1)\tau(A^D(\bar{e} - 2\phi^D\tau) + A^I(\bar{e} - 2\phi^I\tau))}{MA^D - NA^I} - \tau(2\bar{e} - P\tau), \tag{B.8}
\]

where

\[
A^D = \bar{e} - \phi^D\tau - \frac{\beta^D\phi^D}{kB}\tau,
\]

\[
A^I = \bar{e} - \phi^I\tau - \frac{\beta^I\phi^I}{kB}\tau - \frac{1}{k}(\bar{e} - 2\phi^I\tau),
\]

\[
M = (k - 1)\omega^2 + k\omega,
\]

\[
N = k - 1 + k\omega.
\]

Setting \( \phi^I = \phi^D \) and taking the limit with respect to \( k \) as we assume large \( k \) yields

\[
\alpha = \frac{\beta^D}{\beta^I + \beta^D}(\omega + 1),
\]

\[
f_{I0} = 0.
\]

Note that \( f_{I0} = 0 \) is feasible for all \( \omega \), whereas to obtain \( \alpha \leq 1 \), we need to impose \( \omega \leq \beta^I/\beta^D \).

Calculating the second-order derivatives of the cost functions, inserting the expressions for \( \alpha \) and \( f_{I0} \) given in (B.7) and (B.8), respectively, and taking the limits with respect to \( k \) yield the second-order conditions

\[
\phi(\beta^I\phi - 1) > 0, \tag{B.9}
\]

\[
\phi(\beta^D\phi - 1) > 0,
\]

where we again used \( \phi^I = \phi^D \). As \( \beta^I \leq \beta^D \), both conditions are satisfied if (B.9) holds. Hence if (B.9) is fulfilled, we can find a feasible set of policy parameters such that all
countries implement the socially optimal tax rate. This implies (i).

Let us turn to (ii). Here, the second-order condition for large $k$ is fulfilled for the developing countries, but not for the industrial countries. Therefore, the industrial countries would not implement the tax goal. However, if we can find feasible policy parameters such that

- the first derivative of the cost function of the industrial countries is non-positive,
- the first derivative of the cost function of the developing countries is equal to zero, and
- the second derivative of the cost function of the developing countries is positive

at $\tau^j = \bar{e}/\phi$, $j \in \Omega_I$, $\tau^j = \tau$, $j \in \Omega_D$, this represents a local minimum, and countries would not deviate. Calculating the first derivative of the cost function of the developing countries, inserting $\tau^j = \bar{e}/\phi$, $j \in \Omega_I$, $\tau^j = \tau$, $j \in \Omega_D$ and solving for $f_0^I$ yields

$$f_0^I = \frac{k(\omega \bar{e} + \tau)^2(\bar{e} - \phi \tau)(1 - k\beta^D \phi)}{\alpha(\omega \bar{e} + (k - 1)\tau)} - \frac{(\omega \bar{e} + \tau)\tau(\bar{e} - 2\phi \tau)}{k\omega \bar{e} + (k - 1)\tau} - \tau(\bar{e} - \phi \tau).$$

If we set $\alpha = 1$ and $\omega = 0$, we obtain

$$f_0^I = \frac{\phi \tau}{k - 1} ((1 + k^2 \beta^D \phi)\tau - k^2 \beta^D \bar{e}),$$

which is positive for all $k > 1$ and $\lim_{k \to \infty} f_0^I = 0$. Calculating the first derivative of the cost function of the industrial countries, inserting $\tau^j = \bar{e}/\phi$, $j \in \Omega_I$, $\tau^j = \tau$, $j \in \Omega_D$, the expression for $f_0^I$ derived above and $\alpha = 1, \omega = 0$ yields

$$-k\beta^I \phi(\bar{e} - \phi \tau),$$

which is non-positive for all $k > 1$. Calculating the second derivative of the cost function of the developing countries, inserting $\tau^j = \bar{e}/\phi$, $j \in \Omega_I$, $\tau^j = \tau$, $j \in \Omega_D$ and $\alpha = 1, \omega = 0$ yields

$$\phi(\beta^D \phi - 1 + \frac{2}{k}) + \frac{2(k - 1)\phi}{k^2} + \frac{2(k - 1)}{k^2 \tau^2} f_0^I,$$

which is positive for all $k > 1$ as we assumed $\beta^D \phi > 1$, and we already showed $f_0^I \geq 0$ for $\alpha = 1, \omega = 0$. By continuity all properties hold also true in a neighbourhood of $\omega = 0$. 

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In case (iii), the second-order conditions for both types of countries fail for large $k$, and industrial and developing countries will not implement socially optimal taxes for any feasible $\mathcal{P}$. However, if we find feasible policy parameters such that the first derivative of both cost functions is non-positive at $\tau^j = \bar{e}/\phi$, $j \in \Omega$, then this constitutes a local minimum and countries will not deviate. Calculating those derivatives, inserting $\tau^j = \bar{e}/\phi$, $j \in \Omega$ and solving the inequalities for $f_0^I$ yields

$$f_0^I \geq \frac{\bar{e}^2(\omega + 1)}{\phi} \max \left\{ \frac{\omega + 1 - \alpha}{\alpha((k - 1)\omega^2 + k\omega)}, \frac{1}{k\omega + k - 1} \right\},$$

which is a feasible restriction on $f_0^I$ for all $k > 1$, $\alpha \in (0, 1]$ and $\omega > 0$. In the limit $k \to \infty$ it imposes a lower bound on $f_0^I$ that is equal to zero.

### B.5. Proof of Proposition 3.4

Let us first assume $\beta^I \phi^I > 1$. As we assume $k > k^*$, the tax goal is given by $\tau^j = \tau^I_c$, $j \in \Omega_I$, $\tau^j = \tau^D_c$, $j \in \Omega_D$. Countries implement the tax goal if there exist feasible policy parameters $\alpha, f_0^I$ and $\omega$ such that

- the first derivative of the cost function of the industrial countries is equal to zero,
- the first derivative of the cost function of the developing countries is non-positive, and
- the second derivative of the cost function of the industrial countries is positive evaluated at $\tau^j = \tau^I_c$, $j \in \Omega_I$, $\tau^j = \tau^D_c$, $j \in \Omega_D$. Calculating the first derivative of the cost function of the industrial countries, inserting the tax goal, setting it equal to zero and solving for $f_0^I$ yields

$$f_0^I = \frac{k(\omega \tau^I_c + \tau^D_c)^2}{\alpha((k - 1)\omega^2 \tau^I_c + k\tau^D_c)} \left\{ \bar{e} - \phi^I \tau^I_c (1 + \frac{\beta^I}{kB}) - \frac{1}{k}(\bar{e} - 2\phi^I \tau^I_c) \right. \right.
\left. + \alpha(\bar{e} - 2\phi^I \tau^I_c) \frac{\tau^D_c}{k(\omega \tau^I_c + \tau^D_c)} \right\} - \tau^I_c(\bar{e} - \phi^I \tau^I_c). \right.$$}

The term in curly brackets is positive for $\alpha = 0$ and all $k > 1$. Hence $f_0^I$ is non-negative for $\alpha$ close to zero as then the first term tends to $+\infty$ and the second term is independent
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of α. Calculating the first derivative of the cost function of the developing countries, inserting the tax goal and \( f_0^I \) as derived above yields

\[
- \frac{k \omega \tau_c^I}{(k - 1) \omega^2 \tau_c^I + k \tau_c^D} \left\{ \bar{e} - \phi^I \tau_c^I (1 + \frac{\beta^I}{kB}) - \frac{1}{k} (\bar{e} - 2 \phi^I \tau_c^I) \right\} + \alpha (\bar{e} - 2 \phi^I \tau_c^I) \frac{\tau_c^D}{k (\omega \tau_c^I + \tau_c^D)} - \beta D \frac{\phi^D}{kB} \tau_c^I + \alpha \frac{\bar{e} \tau_c^D}{k (\omega \tau_c^I + \tau_c^D)},
\]

which is negative for \( \alpha = 0 \) and for all \( k > 1 \). If we calculate the second derivative of the cost function of the industrial countries, insert the tax goal, the expression for \( f_0^I \) and \( \alpha = 0 \), we obtain

\[
\phi^I (\beta^I \phi^I - 1 + \frac{2}{k}) + \frac{2 \omega}{k^2 (\omega \tau_c^I + \tau_c^D)} \left\{ \bar{e} - \phi^I \tau_c^I (1 + \frac{\beta^I}{kB}) - \frac{1}{k} (\bar{e} - 2 \phi^I \tau_c^I) \right\}, \tag{B.11}
\]

which is positive for large \( k \) as we assumed \( \beta^I \phi^I > 1 \). Due to continuity, every statement remains true in a neighbourhood of \( \alpha = 0 \). This finishes the proof of (i).

From (B.11) we see that the second-order condition of the industrial countries fail for \( k \to \infty \) if \( \beta^I \phi^I \leq 1 \). But we can show that there exist feasible policy parameters such that the first derivative of the cost functions of both types of countries are negative if countries implement \( \tau^j = \bar{e} / \phi^I, j \in \Omega_I, \tau^j = \bar{e} / \phi^D, j \in \Omega_D \): Calculating both derivatives, inserting the tax rates and solving the inequalities for \( f_0^I \) yields

\[
f_0^I \geq (\frac{\omega}{\phi^I} + \frac{1}{\phi^D}) \bar{e}^2 \max \left\{ \frac{\omega}{\phi^I} + \frac{1 - \alpha}{\phi^D} \frac{1}{\alpha ((k - 1) \frac{\omega^2}{\phi^I} + \frac{k}{\phi^D})}, \frac{\phi^D}{k (\omega \phi^I + (k - 1) \frac{1}{\phi^D})} \right\}, \tag{B.12}
\]

which is a feasible constraint on \( f_0^I \) for all \( \alpha \in (0, 1], \omega > 0 \) and for all \( k > 1 \). This finishes the proof of (ii).

B.6. Proof of Proposition 3.5

Under the RSMC without tax revenues, an industrial country \( i \in \Omega_I \) wants to minimize

\[
\frac{\phi^I}{2} (\tau^i)^2 + \frac{\beta^I}{2} s^2 - \alpha k f_0^I \sum_{j \in \Omega_I} \frac{\omega \tau^i}{\omega \tau^j + \sum_{j \in \Omega_D} \tau^j} + f_0^I - (1 - \alpha) k f_0^I, \tag{B.13}
\]

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whereas a developing country $i \in \Omega_D$ wants to minimize

$$
\frac{\phi^D_i}{2} (\tau^i)^2 + \frac{\beta^D_i}{2} s^2 - \alpha k f^I_0 \sum_{j \in \Omega_D} \omega_j \tau^j + \sum_{j \in \Omega_D} \tau^j,
$$

(B.14)
given the tax rates of the respective other countries and the usual constraints.

Assume first that $k \leq k^\star$. Calculating the first-order conditions and inserting the tax goal leads to the following two equations that have to hold in the optimum:

$$
0 = \phi^I \tau - \frac{\beta^I \phi^I}{kB} \tau - \alpha f^I_0 (k - 1) \omega^2 + k \omega \\
0 = \phi^D \tau - \frac{\beta^D \phi^D}{kB} \tau - \alpha f^I_0 \frac{k - 1 + k \omega}{k(\omega + 1)^2}. 
$$

Solving both equations for $\alpha$ and setting them equal yields

$$
\phi^I \left(1 - \frac{\beta^I}{k(\beta^I + \beta^D)}\right) = \phi^D \left(1 - \frac{\beta^D}{k(\beta^I + \beta^D)}\right). 
$$

(B.15)

Note that (B.15) can hold for positive $\omega$ as the nominators of both sides are positive for all $k > 1$. The denominator of the left-hand side of (B.15) is larger than the denominator of the right-hand side if and only if $\omega > 1$. (B.15) can therefore hold if and only if the nominator of the left-hand side is larger than the nominator of the right-hand side. This yields (3.18).

We now turn to the expression for $\alpha$. From the first-order conditions, we derive

$$
\alpha = \frac{k(\omega + 1)^2 \omega^2 \phi^I \left(1 - \frac{\beta^I}{k(\beta^I + \beta^D)}\right)}{f^I_0 ((k - 1) \omega^2 + k \omega)} = \frac{k(\omega + 1)^2 \tau^2 \phi^D \left(1 - \frac{\beta^D}{k(\beta^I + \beta^D)}\right)}{f^I_0 (k - 1 + k \omega)}
$$

(B.16)

We see that $\alpha$ is always positive and can be made $\leq 1$ by increasing the initial fees $f^I_0$ sufficiently.

We have to make sure that the second-order conditions can also be satisfied. Calculating the second derivatives of (B.13) and (B.14) and inserting the tax goal, we obtain the
following second-order conditions:

\[
0 < \phi^I + \beta^I (\phi^I)^2 + 2\alpha k f_0^I \frac{(k - 1)\omega^3 + k\omega^2}{k^3(\omega + 1)^3\tau^2},
\]

(B.17)

\[
0 < \phi^D + \beta^D (\phi^D)^2 + 2\alpha k f_0^I \frac{k - 1 + k\omega}{k^3(\omega + 1)^3\tau^2},
\]

(B.18)

which hold always true for feasible \(\alpha, f_0^I\) and \(\omega\), and \(k > 1\). This completes the proof of (i).

Now assume \(k > k^*\). The tax goal is then given by the corner solution \(\tau^j = \tau_c^I, j \in \Omega_I\), \(\tau^j = \tau_c^D, j \in \Omega_D\), and it is implemented if and only if

- the first derivative of the cost function of the industrial countries is equal to zero,
- the first derivative of the cost function of the developing countries is non-positive,

and

- the second derivative of the cost function of the industrial countries is positive,

evaluated at the tax goal. Calculating \(\partial F^I / \partial \tau^i\), inserting the tax goal and solving \(\partial F^I / \partial \tau^i = 0\) for \(f_0^I\) yields

\[
f_0^I = \frac{k\phi^I \tau_c^I (1 - \beta_I^I) (\omega \tau_c^I + \tau_c^D)^2}{\alpha ((k - 1)\omega^2 \tau_c^I + k\omega \tau_c^D)} ,
\]

which is non-negative for all feasible \(\alpha\) and \(\omega\), and all \(k > 1\). Calculating \(\partial F^D / \partial \tau^i\), inserting the tax goal and the expression for \(f_0^I\) derived above yields

\[
\phi^D \tau_c^D - \beta^D \phi^D \tau_c^I - \frac{\phi^I \tau_c^I (1 - \beta_I^I) (k\omega \tau_c^I + (k - 1)\tau_c^D)}{(k - 1)\omega^2 \tau_c^I + k\omega \tau_c^D)}
\]

(B.19)

which becomes non-positive for \(\omega\) small enough, as then the last term of expression (B.19) tends to \(-\infty\). Calculating \(\partial^2 F^I / (\partial \tau^i)^2\) and inserting the tax goal yields

\[
\phi^I (1 + \beta^I \phi^I) + 2\alpha f_0^I \frac{(k - 1)\omega^3 \tau_c^I + k\omega^2 \tau_c^D}{k^2(\omega \tau_c^I + \tau_c^D)^3} ,
\]

(B.20)

which is positive for all feasible \(\alpha, \omega\) and \(f_0^I\), and all \(k > 1\). This finishes the proof of (ii).
B.7. Proof of Corollary 3.4

We start by noting that for homogeneous countries, the necessary condition for the existence of socially optimal policy parameters, (B.15), implies $\omega = 1$. Hence we showed (i). Then, we see from (B.16) that for homogeneous countries and $\omega = 1$, 

$$\alpha = \frac{2\phi \sigma^2}{f^I_0}. \quad (B.21)$$

As we need $\alpha \leq 1$, this implies (ii).

B.8. Proof of Proposition 3.6

We show that the objective function of the social planner problem (3.23) is strictly convex. This is the case if its Hessian is positive definite. The entries of the Hessian $H$ are given by

$$H_{ll} = (C^i)''(a^i) + \sum_{j \in \{I, D\}} (D^j)''(s), \quad l = 1, 2, i = I \text{ if } l = 1, i = D \text{ if } l = 2 \quad (B.22)$$

$$H_{lk} = \sum_{j \in \{I, D\}} (D^j)''(s), \quad l, k = 1, 2, l \neq k. \quad (B.23)$$

The Hessian is positive definite if the upper left entry and the determinant are positive. The upper left entry is positive as the abatement and damage cost functions are assumed to be strictly convex. The determinant of $H$ is given by

$$\det(H) = \sum_{j \in \{I, D\}} (C^j)''(a^j) \cdot \sum_{j \in \{I, D\}} (D^j)''(s) + \prod_{j \in \{I, D\}} (C^j)''(a^j), \quad (B.24)$$

which is also positive due to the strict convexity of $C^i$ and $D^i, i \in \{I, D\}$. Hence, there exists a unique solution to the social planner problem (3.23), given by $(a^{*I}, a^{*D})$ and the corresponding equilibrium stock $s^*$. 

Now we show $a^{*I} \leq a^{*D}$. Without loss of generality, we can assume an interior solution, i.e. $a^{*i} \in (0, \bar{e}), i \in \{I, D\}$. Due to the strict convexity of the social planner problem, the first-order conditions then characterize the socially optimal solution. They can be
written as
\[(C^i)'(a^i) = \sum_{j \in \{I, D\}} (D^j)'(s), \quad i \in \{I, D\}. \tag{B.25}\]

They are the same for both countries, hence
\[(C^I)'(a^*I) = (C^D)'(a^*D). \tag{B.26}\]

From the assumption of 
\[(C^I)'(a) \geq (C^D)'(a) \quad \forall \quad a \geq 0 \quad \text{and} \quad (C^i)''' > 0, \quad i \in \{I, D\}\]
we can conclude \(a^*I \leq a^*D\). This finishes the proof.

\[\textbf{B.9. Proof of Proposition 3.7}\]

Existence and uniqueness of the solution to the decentralized problem follows from the
assumption of strict convex abatement and damage cost functions, i.e. from 
\((C^i)''' > 0, \quad (D^i)''' > 0, \quad i \in \{I, D\}\). This implies strict convexity of the objective functions of both
countries. Hence, there exists a unique solution to the decentralized problem (3.24),
given by \((\hat{a}^I, \hat{a}^D)\) and the corresponding equilibrium stock \(\hat{s}\).

Now we show \(\hat{a}^I \leq \hat{a}^D\). Without loss of generality, we can assume an interior solution,
i.e. \(\hat{a}^i \in (0, \bar{e}), \quad i \in \{I, D\}\). Due to the strict convexity of the local planner problem, the
first-order conditions then characterize the decentralized solution. They can be written
as
\[(C^i)'(a^i) = (D^i)'(s), \quad i \in \{I, D\}. \tag{B.27}\]

We show \(\hat{a}^I \leq \hat{a}^D\) by contradiction: Assume \(\hat{a}^D < \hat{a}^I\). It follows that
\[
(D^D)'(\hat{s}) = (C^D)'(\hat{a}^D) \quad \text{(first-order condition for D)}
< (C^D)'(\hat{a}^I) \quad \text{(since} \quad \hat{a}^D < \hat{a}^I, \quad \text{and} \quad (C^D)''' > 0) \\
\leq (C^I)'(\hat{a}^I) \quad \text{(we assume} \quad (C^D)'(a) \leq (C^I)'(a) \quad \forall \quad a \geq 0) \\
= (D^I)'(\hat{s}) \quad \text{(first-order condition for I)}.
\]

This contradicts the assumption \((D^D)'(s) \geq (D^I)'(s) \quad \forall \quad s \geq 0\). Hence \(\hat{a}^I \leq \hat{a}^D\) has to
hold. This finishes the proof.
B.10. Proof of Proposition 3.8

We prove the claim by contradiction. Suppose $s^* \geq \hat{s}$. On the one hand, $s^* = \sum_{j \in \{I,D\}} (\bar{e} - a_j^*)$ and $\hat{s} = \sum_{j \in \{I,D\}} (\bar{e} - \hat{a}_j)$, which then implies

$$\hat{a}_I + \hat{a}_D \geq a_I^* + a_D^*. \quad \text{(B.28)}$$

On the other hand, we have for country $I$

$$(C_I)'(a_I^*) = (D_I)'(s^*) + (D_D)'(s^*) \quad \text{(first-order condition of social optimum)}$$

$$> (D_I)'(\hat{s}) \quad \text{as } (D_I)'' > 0$$

$$\geq (D_I)'(\hat{a}_I) \quad \text{as } s^* \geq \hat{s}$$

$$= (C_I)'(a_I^*) \quad \text{(first-order condition of decentralized solution)}.$$

Hence $a_I^* > \hat{a}_I$ as $(C_I)'' > 0$. The same reasoning applies for country $D$. Together, we obtain $a_I^* + a_D^* > \hat{a}_I + \hat{a}_D$. This contradicts (B.28). Therefore, $s^* < \hat{s}$ has to hold.

B.11. Proof of Proposition 3.9

The industrial country wants to set emission abatement $a^I$ such that

$$C^I(a^I) + D^I(s) + f_0^I - \alpha f_0^I \frac{\omega a_I^I}{\omega a_I^I + a_D} - (1 - \alpha) f_0^I$$

is minimized, whereas the developing country chooses $a^D$ such that

$$C^D(a^D) + D^D(s) - \alpha f_0^D \frac{a_D}{\omega a_I^I + a_D}$$

is minimized. Second-order conditions are given by

$$0 < (C^I)''(a^I) + (D^I)''(s) + 2\alpha f_0^I \frac{\omega^2 a^D}{(\omega a_I^I + a_D)^3}, \quad \text{(B.31)}$$

$$0 < (C^D)''(a^D) + (D^D)''(s) + 2\alpha f_0^I \frac{\omega^2 a^D}{(\omega a_I^I + a_D)^3}, \quad \text{(B.32)}$$
which hold true for all feasible $\alpha$, $f_0^I$ and $\omega$ due to the strict convexity of $C^i$ and $D^i$, $i \in \{I, D\}$. Hence the unique solution to the problem is determined by the first-order conditions. Those are given by

\begin{align}
(C^I)'(a^I) &= (D^I)'(s) + \alpha f_0^I \frac{\omega a^D}{(\omega a^I + a^D)^2}, \quad (B.33) \\
(C^D)'(a^D) &= (D^D)'(s) + \alpha f_0^I \frac{\omega a^I}{(\omega a^I + a^D)^2}. \quad (B.34)
\end{align}

Inserting socially optimal abatement decisions, solving both equations for $\alpha f_0^I$, setting them equal and using

\begin{equation}
(C^I)'(a^*) = \sum_{j \in \{I, D\}} (D^j)'(s^*) = (C^D)'(a^D) \quad (B.35)
\end{equation}

yields (3.27). We furthermore obtain

\begin{equation}
\alpha = \frac{(C^I)'(a^*) (\omega a^I + a^*)^2}{f_0^I \omega (a^* + a^D)} = \frac{(C^D)'(a^* D)(\omega a^I + a^* D)^2}{f_0^I \omega (a^* I + a^* D)}, \quad (B.36)
\end{equation}

which is always positive since $(C^i)' > 0$, $i \in \{I, D\}$. Note that we can achieve $\alpha \leq 1$ by setting $f_0^I$ or $\omega$ high enough.


C. Appendix to Chapter 4

Notation

Throughout Appendix C we work with the following abbreviations:

\[ B = \beta^I + \beta^D, \]
\[ P = \phi^I + \phi^D, \]
\[ A = \frac{\beta^D P + \phi^D B}{B}, \]
\[ \hat{A} = \frac{\beta^I P + \phi^I B}{B}, \]
\[ \tau = \tau^* \text{ (tax goal, given by (4.6))}, \]
\[ \tau^I_c = \tau^*_{c^I} \text{ (tax goal, given by (4.8))}, \]
\[ \tau^D_c = \tau^*_{c^D} \text{ (tax goal, given by (4.9))}. \]

C.1. Proof of Proposition 4.1

Solving the first-order conditions \( \tau^i = Bs, i \in \{I, D\} \), for \( \tau^I \) and \( \tau^D \) yields the optimal tax rates

\[ \tau^I = \tau^D = \tau^* = \frac{2\bar{e}B}{1 + BP}. \]

Inserting those into (4.3) yields the socially optimal stock (4.7). Hence we showed (i).

Regarding (ii), observe that

\[ \tau^* \leq \frac{\bar{e}}{\phi^D} \Leftrightarrow (\beta^I + \beta^D)(\phi^D - \phi^I) \leq 1. \]
Hence if \((β^I + β^D)(φ^D − φ^I) > 1\), the tax rate determined by the first-order conditions exceeds the feasibility constraint of the developing country. Then, the developing country will set the corner solution \(τ_{c^D}^* = \bar{e}/φ^D\). Inserting this into the first-order condition of the industrial country and solving it for \(τ^I\) yields

\[
τ_{c^I}^* = \frac{\bar{e}B}{1 + Bφ^I}.
\]

Inserting \(τ_{c^I}^*\) and \(τ_{c^D}^*\) into (4.3), we obtain the socially optimal stock (4.10). This completes the proof.

**C.2. Proof of Proposition 4.3**

If \((β^I + β^D)(φ^D − φ^I) > 1\), the social optimum is given by the corner solution where the developing country abates maximally, i.e. sets \(\bar{e}/φ^D\) as socially optimal emission tax. If both the industrial and the developing country are supposed to implement the socially optimal corner solution under refunding, the first-order conditions are given by

\[
\begin{align*}
\frac{\partial F^I}{\partial τ^I} &= 0, \quad \frac{\partial F^D}{\partial τ^D} \leq 0,
\end{align*}
\]

evaluated at the tax goal \(τ^I = τ_{c^I}^*, \; τ^D = τ_{c^D}^*\). Calculation of the first derivatives of the objective functions \(F^I\) and \(F^D\) given in (4.2) and (4.4) respectively yields

\[
\begin{align*}
\frac{\partial F^I}{\partial τ^I} &= \bar{e} - φ^I τ^I - β^I φ^I s - α(\bar{e} - 2φ^I τ^I) \frac{ω^I τ^I}{\sum ω^I τ^I} - αf \frac{ω^I φ^D τ^D}{(\sum ω^I τ^I)^2} \\
- (1 - α)(\bar{e} - 2φ^I τ^I), \\
\frac{\partial F^D}{\partial τ^D} &= \bar{e} - φ^D τ^D - β^D φ^D s - α(\bar{e} - 2φ^D τ^D) \frac{ω^D τ^D}{\sum ω^D τ^D} - αf \frac{ω^I φ^D τ^D}{(\sum ω^I τ^I)^2}.
\end{align*}
\]

Inserting the tax goal into \(\partial F^I/\partial τ^I\), setting it equal to zero and solving it for \(α\), we derive

\[
α = \frac{β^D φ^D (ω τ_{c^D}^I + τ_{c^D}^2) τ_{c^I}}{B(τ_{c^I}^D(ω f_0^I + φ^I (τ_{c^I}^D)^2) - τ_{c^D}^D(\bar{e} - 2φ^I τ_{c^D}^D))},
\]

(C.1)
We can achieve $\alpha \leq 1$ by setting $f^I_0$ high enough. Note that $f^I_0 \to \infty$ implies $\alpha \downarrow 0$. Calculating $\partial F^D/\partial \tau^D$, inserting the tax goal and (C.1) yields
\[
\frac{\beta^D \phi^I \tau_c^I [\phi^D(\tau_c^D)^2(\omega \tau_c^I + \tau_c^D) - \omega \tau_c^I (f^I_0 + \phi^I(\bar{\varepsilon} - \phi^I \tau_c^I))]}{B \tau_c^D [\omega (f^I_0 + \phi^I(\tau_c^I)^2) - \tau_c^D (\bar{\varepsilon} - 2\phi^I \tau_c^I)]} - \frac{\beta^I \phi^I}{B} \tau_c^I ,
\]
which can be made non-positive by increasing $f^I_0$ sufficiently.

Now let us examine the second-order conditions of the minimization problem. Due to the corner solution, they are given by the following inequality:
\[
\left(\frac{\partial^2 F^I}{(\partial \tau^I)^2}\right) > 0 ,
\]
evaluated at the tax goal. Calculating the second derivative of $F^I$ and inserting (C.1), we obtain
\[
\phi^I (1 + \beta^I \phi^I) + \frac{2\omega \beta^D \phi^I \tau_c^I}{B (\phi^I \tau_c^I + \tau_c^D) B [\omega (f^I_0 + \phi^I(\tau_c^I)^2) - \tau_c^D (\bar{\varepsilon} - 2\phi^I \tau_c^I)]},
\]
which is positive for $f^I_0$ high enough as the last term tends to zero for $f^I_0 \to \infty$ and the other two terms are positive for all feasible $\omega$. These considerations imply existence of a socially optimal set of policy parameters, provided that $f^I_0$ chosen sufficiently high.

C.3. Proof of Lemma 4.1

We assume that both countries implement the tax goal given by (4.6) (interior solution). Hence we insert $\tau^I = \tau^D = \tau$ into the first derivatives derived in the proof of Proposition 4.3 and set them equal to zero. Rearranging terms yields
\[
0 = \bar{\varepsilon} - \phi^I \tau - \frac{\beta^I \phi^I}{B} \tau - \alpha (\bar{\varepsilon} - 2\phi^I \tau) \frac{\omega}{\omega + 1} - (1 - \alpha)(\bar{\varepsilon} - 2\phi^I \tau)
\]
\[
-\alpha (f^I_0 + \tau (2\bar{\varepsilon} - P \tau)) \frac{\omega}{(\omega + 1)^2 \tau}
\]
\[
0 = \bar{\varepsilon} - \phi^D \tau - \frac{\beta^D \phi^D}{B} \tau - \alpha (\bar{\varepsilon} - 2\phi^D \tau) \frac{1}{\omega + 1}
\]
\[
-\alpha (f^I_0 + \tau (2\bar{\varepsilon} - P \tau)) \frac{\omega}{(\omega + 1)^2 \tau} .
\]
Solving these equations for $\alpha$ and $f_I^I$ leads to

$$\alpha = \frac{(\omega + 1)(\bar{e} - Ar)}{2(\bar{e} - P\tau)},$$

$$f_I^I = \frac{2(\omega + 1)\tau(\bar{e} - \phi^I\beta^I + 2\beta^D\tau)(\bar{e} - P\tau)}{\omega(\bar{e} - A\tau)} - \frac{(\omega + 1)\tau(\bar{e} - 2\phi^D\tau)}{\omega} - \tau(2\bar{e} - P\tau).$$

### C.4. Proof of Lemma 4.2

We first derive the second-order conditions ensuring that the solution obtained from the necessary conditions is indeed a minimum. The second derivatives of the objective functions $F^I$ and $F^D$ stated in (4.2) and (4.4), respectively, are given by

$$\frac{\partial^2 F^I}{(\partial \tau I)^2} = -\phi^I + \beta^I(\phi^I)^2 + 2\alpha\phi^I \frac{\sum \omega^I \tau^I}{(\sum \omega^I \tau^I)^2} - 2\alpha(\bar{e} - 2\phi^I\tau) \frac{\sum \omega^D \tau^D}{(\sum \omega^D \tau^D)^2},$$

$$\frac{\partial^2 F^D}{(\partial \tau D)^2} = -\phi^D + \beta^D(\phi^D)^2 + 2\alpha\phi^D \frac{\sum \omega^D \tau^D}{(\sum \omega^D \tau^D)^2} - 2\alpha(\bar{e} - 2\phi^D\tau) \frac{\sum \omega^I \tau^I}{(\sum \omega^I \tau^I)^2},$$

Inserting $\tau^I = \tau^D = \tau$ and the expressions for $\alpha$ and $f_I^I$ given in Lemma 4.1, the second-order conditions can be written as

$$0 < \phi^I(1 + \beta^I\phi^I - \frac{\bar{e} - Ar}{\bar{e} - P\tau}) \frac{\omega}{(\omega + 1)\tau} - \frac{(\bar{e} - Ar)(\bar{e} - 2\phi^I\tau)}{\bar{e} - P\tau} \frac{\omega}{(\omega + 1)\tau},$$

$$+2(\bar{e} - \phi^D\beta^I + 2\beta^D\tau) \frac{\omega}{(\omega + 1)\tau} - \frac{(\bar{e} - Ar)(\bar{e} - 2\phi^D\tau)}{\bar{e} - P\tau} \frac{\omega}{(\omega + 1)\tau},$$

$$0 < \phi^D(-1 + \beta^D\phi^D + \frac{\bar{e} - Ar}{\bar{e} - P\tau}) \frac{\omega}{(\omega + 1)\tau} - \frac{(\bar{e} - Ar)(\bar{e} - 2\phi^D\tau)}{\bar{e} - P\tau} \frac{\omega}{(\omega + 1)\tau},$$

$$+2(\bar{e} - \phi^D\beta^I + 2\beta^D\tau) \frac{1}{(\omega + 1)\tau} - \frac{(\bar{e} - Ar)(\bar{e} - 2\phi^D\tau)}{\bar{e} - P\tau} \frac{1}{(\omega + 1)\tau}.$$
They further simplify to
\[ 0 < \phi^I(1 + \beta^I\phi^I - \bar{\epsilon} - A\tau) + 2\beta^D\phi^I \frac{\omega}{B \omega + 1}, \]  
(C.2)
\[ 0 < \phi^D(-1 + \beta^D\phi^D + \frac{\bar{\epsilon} - A\tau}{\bar{\epsilon} - P\tau}) - \frac{(\bar{\epsilon} - A\tau)(\bar{\epsilon} - 2\phi^D\tau)}{(\bar{\epsilon} - P\tau)\tau} \]
\[ + 2(\bar{\epsilon} - \phi^D\beta^I + 2\beta^D\bar{\epsilon} - P\tau) 1 \frac{1}{(\omega + 1)\tau}. \]  
(C.3)

Next we distinguish between two cases: \( \alpha \) feasible and \( \alpha \) non-feasible. \( \alpha \) feasible means either \( \bar{\epsilon} - A\tau \geq 0 \) or \( \bar{\epsilon} - \bar{A}\tau \leq 0 \), see proof of Lemma 4.3. The former inequality implies that (C.2) holds for any positive \( \omega \), as \( \bar{\epsilon} - A\tau \geq 0 \) ensures \( \bar{\epsilon} - A\tau \geq \bar{\epsilon} - P\tau \leq 1 \) due to \( A \geq P \). The latter inequality, \( \bar{\epsilon} - \bar{A}\tau \leq 0 \), however, implies that (C.3) can hold for a positive \( \omega \). This can be seen by rearranging terms of (C.3),
\[ \omega \left\{ \phi^D\tau(-1 + \beta^D\phi^D + \frac{\bar{\epsilon} - A\tau}{\bar{\epsilon} - P\tau}) - \frac{(\bar{\epsilon} - A\tau)(\bar{\epsilon} - 2\phi^D\tau)}{\bar{\epsilon} - P\tau} \right\} \]
\[ > \frac{(\bar{\epsilon} - A\tau)(\bar{\epsilon} - 2\phi^D\tau)}{\bar{\epsilon} - P\tau} - 2(\bar{\epsilon} - \phi^D\beta^I + 2\beta^D\bar{\epsilon} - P\tau) - \phi^D\tau(-1 + \beta^D\phi^D + \frac{\bar{\epsilon} - A\tau}{\bar{\epsilon} - P\tau}). \]  
(C.4)

We observe that the term in curly brackets on the left-hand side of the inequality (C.4) is positive for \( \bar{\epsilon} - \bar{A}\tau \leq 0 \). This follows from \( \bar{\epsilon} - A\tau \leq \bar{\epsilon} - P\tau \leq \bar{\epsilon} - \bar{A}\tau \) or equivalently \( (\bar{\epsilon} - A\tau)/(\bar{\epsilon} - P\tau) \geq 1 \) and from \( \bar{\epsilon} - 2\phi^D\tau \leq \bar{\epsilon} - \bar{A}\tau \) and therefore \( -(\bar{\epsilon} - A\tau)(\bar{\epsilon} - 2\phi^D\tau)/(\bar{\epsilon} - P\tau) \geq 0 \). Hence we can always find a \( \omega > 0 \) that satisfies inequality (C.4) and thus inequality (C.3).

If \( \alpha \) is non-feasible, we either have \( \bar{\epsilon} - A\tau < 0 < \bar{\epsilon} - P\tau \) or \( \bar{\epsilon} - P\tau < 0 < \bar{\epsilon} - \bar{A}\tau \), see again proof of Lemma 4.3. In the former case (C.2) holds true for any positive \( \omega \) as then \( -(\bar{\epsilon} - A\tau)/(\bar{\epsilon} - P\tau) > 0 \), and the remaining terms on the right-hand side of the inequality are positive, too. In the latter case, (C.3) can hold for a positive \( \omega \), following the same reasoning as above, where we showed positivity of the term in curly brackets in (C.4). This time, it follows from \( \bar{\epsilon} - A\tau \leq \bar{\epsilon} - P\tau \) or equivalently \( (\bar{\epsilon} - A\tau)/(\bar{\epsilon} - P\tau) \geq 1 \) and from \( \bar{\epsilon} - 2\phi^D\tau \leq \bar{\epsilon} - P\tau \) and therefore \( -(\bar{\epsilon} - A\tau)(\bar{\epsilon} - 2\phi^D\tau)/(\bar{\epsilon} - P\tau) \geq 0 \).

Hence second-order conditions can not fail for both countries at the same time.
C.5. Proof of Proposition 4.4

To show that there exists a refunding scheme such that country $i$ implements $\tau^i = \bar{e}/\phi^i$ and country $j$ the tax goal $\tau^j = \tau$, we have to verify that

$$\frac{\partial F^i}{\partial \tau^i} \leq 0, \quad \frac{\partial F^j}{\partial \tau^j} = 0, \quad \text{and} \quad \frac{\partial^2 F^j}{(\partial \tau^j)^2} > 0$$

(C.5)

evaluated at $(\tau^i, \tau^j) = (\bar{e}/\phi^i, \tau)$ hold for a feasible choice of $\alpha, f^I_0, \text{and} \omega$. Note that implementation of $\tau^i = \bar{e}/\phi^i$ requires decreasing costs for country $i$ instead of zero marginal costs and second-order conditions. This is due to the tax rate being at the right-hand side of the feasible interval. Then country $i$ can not deviate further to higher emission abatement as negative emissions are not possible, and it does not want to decrease its tax rate as this would incur higher costs.

Let us first assume $i = I$ and $j = D$. Solving $\partial F^D/\partial \tau^D = 0$, evaluated at $(\tau^I, \tau^D) = (\bar{e}/\phi^I, \tau)$, for $f^I_0$ yields

$$f^I_0 = \frac{(\omega \frac{\bar{e}}{\phi^I} + \tau)^2}{\alpha \omega \frac{\bar{e}}{\phi^I}} \left\{ \bar{e} - \phi^I \beta^I + \frac{2\beta^D}{B} \tau - \alpha (\bar{e} - 2\phi^D \tau) \frac{\tau}{\omega \frac{\bar{e}}{\phi^I} + \tau} \right\}$$

$$- \alpha \tau (\bar{e} - \phi^D \tau) \frac{\omega \frac{\bar{e}}{\phi^I}}{(\omega \frac{\bar{e}}{\phi^I} + \tau)^2}$$

(C.6)

Setting $\alpha = 1$ and $\omega = 0$, the term in curly brackets yields $\phi^D \beta^I \tau/B$, which is positive. By continuity, positivity remains true for $\alpha$ close to 1 and $\omega$ close to 0, hence we can achieve $f^I_0 \geq 0$ for feasible $\alpha$ and $\omega$.

Evaluating $\partial F^I/\partial \tau^I$ at $(\tau^I, \tau^D) = (\bar{e}/\phi^I, \tau)$ and inserting (C.6), we obtain

$$\bar{e} - \frac{\beta^I \phi^I}{B} \tau - \alpha \bar{e} \frac{\tau}{\omega \frac{\bar{e}}{\phi^I} + \tau} - \frac{\phi^I \tau}{\bar{e}} (\bar{e} - \phi^D \beta^I + \frac{2\beta^D}{B} \tau) + \frac{\phi^I \tau}{\bar{e}} \alpha (\bar{e} - 2\phi^D \tau) \frac{\tau}{\omega \frac{\bar{e}}{\phi^I} + \tau}.$$  

By setting $\alpha = 1$ and $\omega = 0$, this term is positive, and by continuity positivity remains true also for $\alpha$ close to 1 and $\omega$ close to 0.

Finally, we have to consider $\partial^2 F^D/(\partial \tau^D)^2$. Evaluation at $(\tau^I, \tau^D) = (\bar{e}/\phi^I, \tau)$ and
inserting (C.6) yields

\[
\phi^D(\beta^D\phi^D - 1) + 2\alpha\phi^D \frac{\tau}{\omega \bar{e}^D + \tau} - 2\alpha(\bar{e} - 2\phi^D\tau) \frac{\omega \bar{e}^D}{(\omega \phi^D + \tau)^2} + 2\alpha f \frac{\omega \bar{e}^D}{(\omega \phi^D + \tau)^3}.
\]

Again, when setting \(\alpha = 1\) and \(\omega = 0\), the term is positive, and by continuity also for \(\alpha\) close to 1 and \(\omega\) close to 0. Hence by choosing the policy parameters \(\alpha\) close to 1 and \(\omega\) close to 0, the countries implement \((\tau^I, \tau^D) = (\bar{e}/\phi^I, \tau)\).

A similar analysis applies for \(i = D\) and \(j = I\). The policy parameter \(\alpha\) being close to 0 implies

\[
\frac{\partial F^D}{\partial \tau^D} \leq 0, \quad \frac{\partial F^I}{\partial \tau^I} = 0, \quad \text{and} \quad \frac{\partial^2 F^I}{(\partial \tau^I)^2} > 0
\]
evaluated at \((\tau^I, \tau^D) = (\tau, \bar{e}/\phi^D)\) and for any feasible choice of \(f_0^I\) and \(\omega\). This finishes the proof.

### C.6. Proof of Lemma 4.3

From Lemma 4.1 we know that under tax goal implementation, the policy parameter \(\alpha\) has to satisfy

\[
\alpha = \frac{(\omega + 1)(\bar{e} - A\tau)}{2(\bar{e} - P\tau)}.
\] 

(C.7)

In the following, we assume feasibility of \(\omega\), i.e. \(\omega > 0\). Then, non-negativity of \(\alpha\) implies that \(\bar{e} - A\tau\) and \(\bar{e} - P\tau\) must have the same sign, and as \(P \leq A\), this means that either \(\bar{e} - A\tau \geq 0\) or \(\bar{e} - P\tau < 0\) has to hold. Hence non-feasibility arises if \(\bar{e} - A\tau < 0 < \bar{e} - P\tau\), or

\[
0 < \bar{e} - P\tau < \frac{\beta^D\phi^D - \beta^I\phi^I}{B} \tau.
\] 

(C.8)

The condition \(\alpha \leq 1\), already using non-negativity, implies

\[
\omega \leq \frac{\bar{e} - \bar{A}\tau}{\bar{e} - A\tau}.
\]

(C.9)

The right-hand side of (C.9) can be negative for \(\bar{e} - P\tau < 0 < \bar{e} - \bar{A}\tau\), or

\[
0 < P\tau - \bar{e} < \frac{\beta^D\phi^D - \beta^I\phi^I}{B} \tau.
\]

(C.10)
Then we can not achieve $\alpha \leq 1$ with a positive $\omega$. Therefore, non-feasibility is excluded if and only if (C.8) and (C.10) do not hold, or equivalently,

$$\frac{\beta^D \phi^D - \beta^I \phi^I}{B} \tau \leq |\bar{e} - P\tau|.$$ \hspace{1cm} (C.11)

Inserting the expression for $\tau$ and rearranging terms implies (4.19).

C.7. Proof of Lemma 4.4

From Lemma 4.1 we derive that $f^I_0 \geq 0$ is equivalent to

$$\omega \left\{ \frac{2(\bar{e} - \phi^D \beta^I + 2\beta^D)}{B} \tau - \frac{2(\bar{e} - 2\phi^D \tau)}{\bar{e} - P\tau} - \frac{(\bar{e} - A\tau)(2\bar{e} - P\tau)}{\bar{e} - P\tau} \right\} \geq \frac{(\bar{e} - A\tau)(\bar{e} - 2\phi^D \tau)}{\bar{e} - P\tau} - 2(\bar{e} - \phi^D \beta^I + 2\beta^D) \frac{\tau}{B}.$$ \hspace{1cm} (C.12)

We claim that if the right-hand side of (C.12) is positive, then the term in curly brackets on the left-hand side is negative. This can be seen as follows. Assuming that $\alpha$ is feasible, $\bar{e} - A\tau$ and $\bar{e} - P\tau$ must have the same sign. $2\bar{e} - P\tau$ is always positive. Hence the last term in the curly brackets on the left-hand side of the inequality in (C.12) is negative. The remaining two terms appear on the right-hand side of the inequality with the reverse sign. Therefore, positivity of the right-hand side implies negativity of the term in curly brackets. Hence $f^I_0 \geq 0$ can be satisfied for positive values of $\omega$ if and only if the right-hand side of (C.12) is negative.

From Lemma 4.3 we know that $\alpha$ is feasible if either $\bar{e} - A\tau \geq 0$ or $\bar{e} - \bar{A}\tau \leq 0$ holds. Therefore we show feasibility of $f^I_0$ in three steps.

Step 1: $\bar{e} - A\tau > 0$

Under this assumption, the right-hand side of (C.12) is always negative as multiplying it by $(\bar{e} - P\tau)/(\bar{e} - A\tau)$ does not change the sign and yields

$$\bar{e} - 2\phi^D \tau - 2 \frac{\bar{e} - P\tau}{\bar{e} - A\tau} \left( \bar{e} - \phi^D \beta^I + 2\beta^D \right) \leq -\bar{e} + 2\frac{\beta^D \phi^D}{B} \tau < -\bar{e} + A\tau < 0.$$
Step 2: \( \bar{e} - A\tau = 0 \)
Then \( f_0^I \geq 0 \) is equivalent to \( \omega \geq -1 \), which can always be satisfied.

Step 3: \( \bar{e} - \tilde{A}\tau \leq 0 \)
Note first that if \( \bar{e} - \phi D^{\beta I + 2/3 D} B \tau \geq 0 \), the right-hand side of (C.12) is always negative as multiplying it by \( (\bar{e} - P\tau)/(\bar{e} - A\tau) \) does not change the sign and yields

\[
\bar{e} - 2\phi D \tau - \frac{2(\bar{e} - P\tau)}{\bar{e} - A\tau} \left( \frac{\bar{e} - \phi D^{\beta I + 2/3 D} B \tau}{\bar{e} - A\tau} \right) \leq -\frac{\beta I \phi D}{B} \tau < 0 .
\]

Hence we assume in the following that \( \bar{e} - \phi D^{\beta I + 2/3 D} B \tau < 0 \) holds.

Now we define

\[
\phi := \phi D, \quad \Rightarrow \phi^I = x\phi \quad \text{with} \quad x \in (0, 1],
\]

\[
\beta := \beta^D, \quad \Rightarrow \beta^I = y\beta \quad \text{with} \quad y \in (0, 1],
\]

\[
p := \beta \phi .
\]

With those definitions we can rewrite every expression of the model in terms of \( x, y \) and \( p \). Inserting the expression for \( \tau \), using \( x, y \) and \( p \) and rearranging terms, the right-hand side of (C.12) being non-negative is equivalent to

\[
F(x, y, p) := (1 - p(3 - x + 3y - xy))(1 - p(3 + x + y - xy))
-2(1 - p(3 - x + y - xy))(1 - p(1 + x + y + xy)) \leq 0 .
\]

Note that we used \( \bar{e} - A\tau \leq \bar{e} - \tilde{A}\tau \leq 0 \). Sorting for the powers of \( p \) yields

\[
F(x, y, p) = (3 - x + y - 3xy)(1 - x)(1 + y)p^2 + 2(1 + xy)p - 1 \leq 0 .
\]

Note that \( 3 - x + y - 3xy \geq 0 \) and \( 1 - x \geq 0 \) as \( x, y \in (0, 1] \), hence \( F(x, y, p) \) is a quadratic convex function in \( p \) (resp. strictly convex if \( x \neq 1 \)) with roots

\[
p_{1/2} = \frac{-1 + \sqrt{1 + xy} \pm \sqrt{(1 + xy)^2 + (3 - x + y - 3xy)(1 - x)(1 + y)}}{(3 - x + y - 3xy)(1 - x)(1 + y)}, \quad (C.13)
\]

and \( F(x, y, p) \leq 0 \iff p \in [p_2, p_1] \). Note also that if \( x = 1 \), we have \( F(1, y, p) \leq 0 \iff p \leq 1/(2(1 + y)) \). As \( p > 0 \) and \( p_2 < 0 \), \( F(x, y, p) \) is non-positive only if \( p \in (0, p_1] \) (resp. \( p \in (0, 1/(2(1 + y))] \) if \( x = 1 \).
Now we examine whether it is possible to simultaneously satisfy $p \in (0, p_1]$ (resp. $p \in (0, 1/(2(1 + y))]$ if $x = 1$) and $\bar{e} - \phi^D \beta^I \frac{\phi^D \beta^D \tau}{B} < 0$, where the latter is equivalent to

$$p > \frac{1}{3 - x + y - xy} .$$

For this to be able to hold, we would need

$$p_1 > \frac{1}{3 - x + y - xy} \quad \text{(resp.} \quad \frac{1}{2(1 + y)} > \frac{1}{2} \text{ if } x = 1) .$$

Inserting $p_1$ from (C.13) into the inequality and rearranging terms yields

$$4xy < 0$$

which never holds. Also the condition for $x = 1$ can never be satisfied as $y \geq 0$. Therefore, the right-hand side of (C.12) is always negative for feasible $\alpha$, which implies feasibility of $f_0^I$. This completes the proof.

\section*{C.8. Proof of Lemma 4.5}

First note that $\alpha < 0$ occurs if $\bar{e} - A\tau < 0 < \bar{e} - P\tau$. In that case, the second-order condition of the industrial country is always satisfied. The second-order condition of the developing country is equivalent to (C.4), see the proof of Lemma 4.2. We know $3\phi^D > A$, hence $\bar{e} - 3\phi^D \tau < 0$, and from $\bar{e} - P\tau > 0$ we can conclude $-1 + \beta^D \phi^D < 0$. The term in curly brackets in (C.4) equals

$$\phi^D \tau \left( -1 + \beta^D \phi^D \right) + \frac{\bar{e} - A\tau}{\bar{e} - P\tau} \left( -\bar{e} + 3\phi^D \tau \right) < 0 .$$

Hence it is always negative. This implies that the second-order condition of the developing country fails if and only if the right-hand side of (C.4) is positive.

For the right-hand side of (C.4) to be negative, we necessarily need

$$\frac{(\bar{e} - A\tau)(\bar{e} - 2\phi^D \tau)}{\bar{e} - P\tau} - 2(\bar{e} - \phi^D \beta^I \frac{\phi^D \beta^D \tau}{B} \tau) < 0 ,$$

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which is equivalent to (remind $\bar{e} - P\tau > 0$)

$$
\alpha f = \frac{(\omega + 1)^2\tau}{2\omega(\bar{e} - P\tau)} \left( 2(\bar{e} - \phi^D \beta^I + 2\beta^D \tau) - (\bar{e} - 2\phi^D \tau)(\bar{e} - A\tau) \right) > 0
$$

for all feasible $\omega > 0$. Hence we can conclude that the second-order condition of the developing country can hold only if $\alpha f > 0$. This finishes the proof.

C.9. Proof of Lemma 4.6

We know that $\alpha > 1$ occurs if $\bar{e} - P\tau < 0 < \bar{e} - \bar{A}\tau$. Knowing $P \leq A$, we obtain

$$
2(\bar{e} - P\tau) - (\bar{e} - A\tau) = \bar{e} - \bar{A}\tau > 0 \Rightarrow \frac{2(\bar{e} - P\tau)}{\bar{e} - A\tau} < 1.
$$

In the proof of Lemma 4.4, we saw that $f^I_0$ can be achieved if the right-hand side of (C.12) is negative. But under the assumption of $\alpha > 1$, the right-hand side of (C.12) is equivalent to

$$
\bar{e} - 2\phi^D \tau - (\bar{e} - \phi^D \beta^I + 2\beta^D \tau) \left[ \begin{array}{c} > 0 \\ < 1 \end{array} \right] < -\beta^I \phi^D \frac{\tau}{B} < 0,
$$

hence $f^I_0$ can always be chosen in a feasible way.

C.10. Proof of Proposition 4.6

Under a refunding scheme without tax revenues, the industrial country wants to minimize

$$
\frac{\phi^I}{2} (\tau^I)^2 + \frac{\beta^I}{2} s^2 + f^I_0 - \alpha f^I_0 \frac{\omega\tau^I}{\omega\tau^I + \tau^D} - (1 - \alpha) f^I_0
$$

with respect to $\tau^I$, whereas the developing country wants to minimize

$$
\frac{\phi^D}{2} (\tau^D)^2 + \frac{\beta^D}{2} s^2 - \alpha f^D_0 \frac{\tau^D}{\omega\tau^I + \tau^D}
$$

with respect to $\tau^D$, given the tax rate of the respective other country. To show (i), see the proof of Proposition 2.8 in Appendix A.2 which also applies here as it does not rely
on Assumption 1.

Regarding part (ii), countries will implement the tax goal \( \tau^I = \tau^I_c, \tau^D = \tau^D_c \) if and only if

- the first derivative of (C.14) with respect to \( \tau^I \) is equal to zero,
- the first derivative of (C.15) with respect to \( \tau^D \) is non-positive, and
- the second derivative of (C.14) with respect to \( \tau^I \) is positive,

evaluated at the tax goal. Calculating the first derivative of (C.14), inserting the tax goal, setting it equal to zero and solving it for \( f^I_0 \) yields

\[
f^I_0 = \frac{\beta^D \phi^I \tau^I_c (\omega \tau^I_c + \tau^D_c)^2}{B \tau^D_c \alpha \omega}.
\]  

(C.16)

Note that \( f^I_0 \) is non-negative for all feasible \( \alpha \) and \( \omega \). Inserting (C.16) and the tax goal into the first derivative of (C.15), imposing non-positivity and rearranging terms, we obtain

\[
\beta^I \phi^D (\tau^D_c)^2 \leq \beta^D \phi^I (\tau^I_c)^2.
\]  

(C.17)

This is equivalent to (4.21).

Finally we consider the second derivative of (C.14), evaluated at the tax goal. It yields

\[
\phi^I + \phi^I (\phi^I)^2 + 2\alpha f^I_0 \frac{\omega \tau^D_c}{(\omega \tau^I_c + \tau^D_c)^3},
\]

which is positive as \( f^I_0 \) is non-negative for any \( \alpha \in [0, 1] \) and \( \omega > 0 \). This finishes the proof.
D. Appendix to Chapter 5

D.1. Proof of Proposition 5.1

We show existence and uniqueness of the social global optimum by solving the optimization problem (5.5) for any time horizon $T$.

In each period $t$, the right-hand side of equation (5.6) is identical for all countries $i$. This implies that abatement in period $t$ is equal among all countries, i.e., $a^i_t = a_t$ for all $i \in \{1, \ldots, n\}$ and $t = 1, \ldots, T$. We rewrite (5.6) to yield

$$V'_{t+1}(s_{t+1}) = -\frac{\alpha}{\delta}a_t.$$  (D.1)

Inserting into (5.7), we eliminate the value function and obtain

$$a_{t-1} = \delta(1 - \gamma)a_t + \frac{n\beta\delta}{\alpha}s_t.$$  (D.2)

Equation (D.2) together with the equation of motion (5.3) yields the system of linear first-order difference equations

$$
\begin{pmatrix}
a_{t+1}^* \\
s_{t+1}^*
\end{pmatrix} =
\begin{pmatrix}
\frac{\alpha + n^2 \beta \delta}{\alpha \delta (1 - \gamma)} & -\frac{n\beta}{\alpha} \\
-n & 1 - \gamma
\end{pmatrix}
\begin{pmatrix}
a_t^* \\
s_t^*
\end{pmatrix} +
\begin{pmatrix}
-\frac{n^2 \beta \delta}{\alpha (1 - \gamma)} \\
\frac{n \epsilon}{\alpha}
\end{pmatrix}$$  (D.3)

the general solution of which is given by

$$
\begin{pmatrix}
a_{t+1}^* \\
s_{t+1}^*
\end{pmatrix} =
\begin{pmatrix}
a^{SO} \\
s^{SO}
\end{pmatrix} + B_1(T)v_1 \lambda_1^t + B_2(T)v_2 \lambda_2^t,$$  (D.4)

where $(a^{SO}, s^{SO})$ are the stationary states obtained by setting $a_{t+1}^* = a_t^* = a^{SO}$ and
\[ s_{t+1}^* = s_t^* = s^{SO} , \text{ hence} \]

\[ a^{SO} = \frac{n^2 \beta \delta \epsilon}{\alpha \gamma [1 - \delta(1 - \gamma)] + n^2 \beta \delta} , \quad (D.5a) \]

\[ s^{SO} = \frac{n \alpha \epsilon [1 - \delta(1 - \gamma)]}{\alpha \gamma [1 - \delta(1 - \gamma)] + n^2 \beta \delta} , \quad (D.5b) \]

and \( v_i \) and \( \lambda_i, \ i = 1, 2 \) are the Eigen vectors and the Eigen values, respectively, of the \( 2 \times 2 \) matrix in (D.3). \( B_i(T), \ i = 1, 2 \) are constants that have to be determined in such a way that the initial and final conditions are satisfied.

Calculating Eigen values and vectors yields

\[ \lambda_{1/2} = \frac{1 + \delta(1 - \gamma)^2 + \frac{n^2 \beta \delta \alpha}{\alpha} \pm \sqrt{\left[1 + \delta(1 - \gamma)^2 + \frac{n^2 \beta \delta \alpha}{\alpha}\right]^2 - 4 \delta(1 - \gamma)^2}}{2 \delta(1 - \gamma)} \]

\[ (D.6a) \]

\[ v_1 = \left( \frac{1}{n \frac{1}{1 - \gamma - \lambda_1}} \right) , \quad v_2 = \left( \frac{1}{n \frac{1}{1 - \gamma - \lambda_2}} \right) . \]

\[ (D.6b) \]

Setting \( C = 1 - \delta + \delta \gamma^2 + \frac{n^2 \beta \delta}{\alpha} \) and \( D = \left[1 + \delta(1 - \gamma)^2 + \frac{n^2 \beta \delta}{\alpha}\right]^2 - 4 \delta(1 - \gamma)^2 \), the Eigen values read

\[ \lambda_{1/2} = 1 + \frac{C \pm \sqrt{D}}{2 \delta(1 - \gamma)} . \]

\[ (D.7) \]

As \( C, D > 0 \) and \( \sqrt{C} > D \), we obtain \( \lambda_1 > 1 > \lambda_2 > 0 \).

Then the general solution is given by

\[ a_t^* = a^{SO} + B_1(T)\lambda_1^t + B_2(T)\lambda_2^t , \quad (D.8a) \]

\[ s_t^* = s^{SO} + \frac{n B_1(T)}{1 - \gamma - \lambda_1} \lambda_1^t + \frac{n B_2(T)}{1 - \gamma - \lambda_2} \lambda_2^t . \]

\[ (D.8b) \]

We determine \( B_1(T) \) and \( B_2(T) \) from the initial greenhouse gas stock, \( s_1 \), and the ter-
minal condition for emission abatement, \( a_T = 0 \):

\[
B_1(T) = \frac{(1 - \gamma - \lambda_1) \left( -na^{SO} - (1 - \gamma - \lambda_2)(s_1 - s^{SO}) \right)}{n \left( 1 - \gamma - \lambda_1 \right) - \frac{\lambda_2^{T-1}}{\lambda_1^{T-1}} (1 - \gamma - \lambda_2)} , \quad (D.9a)
\]

\[
B_2(T) = \frac{(1 - \gamma - \lambda_2) \left[ (1 - \gamma - \lambda_1)(s_1 - s^{SO}) + na^{SO} \right]}{n \left( 1 - \gamma - \lambda_1 \right) \lambda_2 - \frac{\lambda_2^{T}}{\lambda_1^{T-1}} (1 - \gamma - \lambda_2)} . \quad (D.9b)
\]

Inserting back into equations (D.8) yields the unique global social optimum.

**D.2. Proof of Corollary 5.1**

For large \( T \) we obtain for \( B_1(T) \) and \( B_2(T) \) from the proof of Proposition 5.1

\[
B_1^\infty \equiv \lim_{T \to \infty} B_1(T) = 0 , \quad B_2^\infty \equiv \lim_{T \to \infty} B_2(T) = \frac{(1 - \gamma - \lambda_2)(s_1 - s^{SO})}{n \lambda_2} , \quad (D.10)
\]

implying for the solution (D.8) in the limit case \( T \to \infty \)

\[
a^*_t = a^{SO} + \frac{(1 - \gamma - \lambda_2)(s_1 - s^{SO})}{n} \lambda_2^{-1} , \quad (D.11a)
\]

\[
s^*_t = s^{SO} + (s_1 - s^{SO}) \lambda_2^{-1} , \quad (D.11b)
\]

which we can also write as a policy rule \( a_t(s_t) \):

\[
a^*_t(s^*_t) = a^{SO} + \frac{(1 - \gamma - \lambda_2)(s^*_t - s^{SO})}{n} . \quad (D.12)
\]

From the proof of Proposition 5.1 we know that \( 0 < \lambda_2 < 1 \). Thus we obtain from (D.11)

\[
\lim_{t \to \infty} a^*_t = a^{SO} , \quad \lim_{t \to \infty} s^*_t = s^{SO} . \quad (D.13)
\]

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D. Appendix to Chapter 5

D.3. Proof of Proposition 5.2

Before we show the existence of a unique and symmetric subgame perfect Nash equilibrium by backward induction, note that the optimization problem of country $i$ in period $t$ is strictly concave if and only if

$$\delta W_{t+1}'(s_{t+1}) \cdot \alpha < 0 \ . \quad (D.14)$$

Starting in period $T$, recall that $W_{T+1}(s_{T+1}) \equiv 0$, which implies that $\hat{a}_T = 0$ is the best response of all countries independently of the emission abatement choices of all other countries. As a consequence, $\hat{a}_T = \hat{a}_T$ is the unique and symmetric Nash equilibrium for the subgame starting in period $T$ given the level of the greenhouse gas stock $s_T$. The equilibrium pay-off $W_T(s_T) = V_T(s_T) | \hat{A}_T$ is identical for all countries and is strictly concave:

$$W_T(s_T) \equiv W_{T}'(s_T) = -\frac{\beta}{2} \frac{s_T^2}{\delta_T} \Rightarrow W_{T}'(s_T) = -\beta \ . \quad (D.15)$$

Now assume there exists a unique and symmetric subgame perfect Nash equilibrium for the subgame starting in period $t + 1$ with a greenhouse gas stock of $s_{t+1}$ yielding symmetric equilibrium pay-offs $W_{t+1}(s_{t+1}) = W_{t+1}'(s_{t+1})$ with $W_{t+1}''(s_{t+1}) < 0$. Then the optimization problem in period $t$ is strictly concave for all countries $i$, implying there exists a unique best response $\bar{a}_t^i$ for all countries $i$, given the emission abatements of all other countries $j \neq i$, which is given implicitly by

$$\alpha \bar{a}_t^i = -\delta W_{t+1}'(\bar{s}_{t+1}) \ . \quad (D.16)$$

As the right-hand side is identical by assumption for all countries, so is the left-hand side as well. This implies that emission abatement in equilibrium is symmetric and unique, $\hat{a}_t = \hat{a}_t$. As a consequence, the equilibrium pay-off is also identical for all countries $i$, $W_t(s_t) = W_t'(s_t)$. Differentiating (5.11) with respect to $s_t$, we obtain

$$V_t''(s_t) | A_{t-1} = \delta (1 - \gamma)^2 W_{t+1}''(\bar{s}_{t+1}) - \beta \ . \quad (D.17)$$

As $W_t''(s_t) = V_t''(s_t) | \hat{A}_t$, this implies that the equilibrium pay-off $W_t(s_t) = W_t(s_t)$ is strictly concave. Working backwards to $t = 1$ yields a unique symmetric sequence of emission abatements $\hat{a}_t^i = \hat{a}_t$ and the corresponding sequence of the greenhouse gas stock $\hat{s}_t$ ($i = 1, \ldots, n; t = 1, \ldots, T$) that constitute the unique and symmetric subgame
perfect Nash equilibrium of the decentralized system.

As the subgame perfect Nash equilibrium is symmetric and unique, we obtain the following system of first-order difference equations from equations (5.10), (5.11) and (5.3):

\[
\begin{pmatrix}
\hat{a}_{t+1} \\
\hat{s}_{t+1}
\end{pmatrix}
= \begin{pmatrix}
\frac{\alpha + n\beta\delta}{\alpha (1 - \gamma)} & -\frac{\beta}{\alpha} \\
-n & 1 - \gamma
\end{pmatrix}
\begin{pmatrix}
\hat{a}_t \\
\hat{s}_t
\end{pmatrix}
+ \begin{pmatrix}
-\frac{n\beta\epsilon}{n\epsilon}
\end{pmatrix}
.
\]  

(D.18)

Following the same solution technique as described in the proof of Proposition 5.1, we derive for the sequences of emission abatement and the greenhouse gas stock in the subgame perfect Nash equilibrium

\[
\begin{align*}
\hat{a}_t &= a^{DS} + B_1(T)\mu_1^t + B_2(T)\mu_2^t, \\
\hat{s}_t &= s^{DS} + \frac{nB_1(T)}{1 - \gamma - \mu_1}\mu_1^t + \frac{nB_2(T)}{1 - \gamma - \mu_2}\mu_2^t,
\end{align*}
\]

where \((a^{DS}, s^{DS})\) denote the steady state of (D.18) given by

\[
\begin{align*}
a^{DS} &= \frac{n\beta\delta\epsilon}{\alpha\gamma [1 - \delta (1 - \gamma)] + n\beta\delta}, \\
s^{DS} &= \frac{n\alpha\epsilon [1 - \delta (1 - \gamma)]}{\alpha\gamma [1 - \delta (1 - \gamma)] + n\beta\delta},
\end{align*}
\]

(D.20a)

(D.20b)

\(\mu_1\) and \(\mu_2\) equal

\[
\begin{align*}
\mu_{1/2} &= 1 + \delta (1 - \gamma)^2 + \frac{n\beta\delta}{\alpha} \pm \sqrt{\left[1 + \delta (1 - \gamma)^2 + \frac{n\beta\delta}{\alpha}\right]^2 - 4\delta (1 - \gamma)^2} \\
&= 1 + \frac{1 - \delta + \delta \gamma^2 + \frac{n\beta\delta}{\alpha} \pm \sqrt{\left[1 - \delta (1 - \gamma)^2 + \frac{n\beta\delta}{\alpha}\right]^2 + \frac{4\delta^2 (1 - \gamma)^2 \beta n}{\alpha}}}{2\delta (1 - \gamma)},
\end{align*}
\]

(D.21)

which immediately implies \(\mu_1 > 1 > \mu_2 > 0\), and \(B_1(t)\) and \(B_2(T)\) depend on the time
horizon $T$ and the initial stock of greenhouse gases $s_1$

$$
B_1(T) = \frac{(1 - \gamma - \mu_1) \left[ -\frac{na}{\mu_1^t} - \frac{\mu_2^{t-1}}{\mu_1^t} (1 - \gamma - \mu_2)(s_1 - s^{DS}) \right]}{n \left[ (1 - \gamma - \mu_1) - \frac{\mu_2^{t-1}}{\mu_1^t} (1 - \gamma - \mu_2) \right]} ,
$$

(D.22a)

$$
B_2(T) = \frac{(1 - \gamma - \mu_2) \left[ (1 - \gamma - \mu_1)(s_1 - s^{DS}) + \frac{na}{\mu_1^t} \right]}{n \left[ (1 - \gamma - \mu_1)\mu_2 - \frac{\mu_2^{t-1}}{\mu_1^t} (1 - \gamma - \mu_2) \right]} .
$$

(D.22b)

D.4. Proof of Corollary 5.2

For large $T$ we obtain for $B_1(T)$ and $B_2(T)$ from the proof of Proposition 5.2

$$
B_1^\infty \equiv \lim_{T \to \infty} B_1(T) = 0 , \quad B_2^\infty \equiv \lim_{T \to \infty} B_2(T) = \frac{(1 - \gamma - \mu_2)(s_1 - s^{DS})}{n\mu_2} ,
$$

(D.23)

implying for solution (D.19) in the limit case $T \to \infty$

$$
\hat{a}_t = a^{DS} + \frac{(1 - \gamma - \mu_2)(s_1 - s^{DS})}{n} \mu_2^{t-1} , \quad (D.24a)
$$

$$
\hat{s}_t = s^{DS} + (s_1 - s^{DS})\mu_2^{t-1} , \quad (D.24b)
$$

which we can also write as a policy rule $a_t(s_t)$:

$$
\hat{a}_t(\hat{s}_t) = a^{DS} + \frac{(1 - \gamma - \mu_2)(\hat{s}_t - s^{DS})}{n} . \quad (D.25)
$$

From the proof of Proposition 5.2 we know that $0 < \mu_2 < 1$. Thus, we obtain from (D.24)

$$
\lim_{t \to \infty} \hat{a}_t = a^{DS} , \quad \lim_{t \to \infty} \hat{s}_t = s^{DS} ,
$$

(D.26)
D.5. Proof of Proposition 5.3

Again, we start by noting that the optimization problem of country \( i \) in period \( t \) is strictly concave if and only if

\[
\delta W_i^{t+1,\prime\prime}(s_{t+1}) - \alpha + \frac{\partial^2 r_i^t}{(\partial a_i^t)^2} < 0 ,
\]

with

\[
\frac{\partial^2 r_i^t}{(\partial a_i^t)^2} = \begin{cases} 
-2R_i \frac{A_i^{-i}}{(\bar{a}_i^t + A_i^{-i})^3}, & t = 1, \ldots, T - 1 , \\
0, & t = T .
\end{cases}
\]

Solving for the subgame perfect Nash equilibria by backward induction, we start in period \( T \). By virtue of the first-order condition (5.16), \( W_i^T(s_T + 1) \equiv 0 \) implies that \( \bar{a}_T = 0 \) is the best response for all countries independently of the emission abatement choices of all other countries. As a consequence, \( \bar{a}_T = \bar{a}_T^i \) is the unique and symmetric Nash equilibrium for the subgame starting in period \( T \), given the level of the greenhouse gas stock \( s_T \). The equilibrium pay-off \( W_T^i(s_T) = V_T^i(s_T)|\bar{A}_T^i \) is identical for all countries and is strictly concave:

\[
W_T(s_T) \equiv W_T^i(s_T) = -\frac{\beta}{2}s_T^2 + \frac{R_T}{n} \quad \Rightarrow \quad W_T^{\prime\prime}(s_T) = -\beta .
\]

In addition, \( W_T'(s_T) = -\beta s_T < 0 \).

Now assume there exists a unique and symmetric subgame perfect Nash equilibrium for the subgame starting in period \( t+1 \), with a greenhouse gas stock of \( s_{t+1} \) yielding symmetric equilibrium pay-offs \( W_{t+1}^i(s_{t+1}) \equiv W_{t+1}^i(s_{t+1}) \) with \( W_{t+1}^i(s_{t+1}) < 0 \) and \( W_{t+1}^{\prime\prime}(s_{t+1}) < 0 \). Then the optimization problem in period \( t \) is strictly concave for all countries \( i \), implying there exists a unique best response \( \bar{a}_t^i \) for all countries \( i \) given the emission abatements of all other countries \( j \neq i \), which is given implicitly by

\[
\alpha \bar{a}_t^i - R_t \frac{A_i^{-i}}{(\bar{a}_t^i + A_i^{-i})^2} = -\delta W_{t+1}^i(s_{t+1}) .
\]

As the right-hand side is identical by assumption for all countries, so is the left-hand side as well. This implies that emission abatement in equilibrium is symmetric, \( \bar{a}_t \equiv \bar{a}_t^i \) for all \( i = 1, \ldots, n \). Summing up over all \( n \) countries and multiplying by the total amount of abatement \( A_t \) (which is strictly positive, as corner solutions are ruled out) yields a
necessary condition that has to hold in the subgame perfect Nash equilibrium

\[ \alpha \tilde{A}_t^2 + \delta n W'_{t+1}(\tilde{s}_{t+1})\tilde{A}_t - (n - 1)R_t = 0. \]  
\[ (D.31) \]

This equation yields a unique solution, as total emissions have to be non-negative and, in addition, \( W'_{t+1}(s_{t+1}) < 0 \) holds:

\[ \tilde{A}_t = -\frac{\delta n W'_{t+1}(\tilde{s}_{t+1}) + \sqrt{(\delta n W'_{t+1}(\tilde{s}_{t+1}))^2 + 4\alpha(n - 1)R_t}}{2\alpha}. \]  
\[ (D.32) \]

We already know that emission abatement levels in the subgame perfect Nash equilibrium are symmetric, so this implies unique and symmetric abatement levels \( \tilde{a}_t = \tilde{A}_t/n \) and also identical equilibrium pay-offs for all countries \( i, W_t(s_t) = W_t(s_t) \). The envelope theorem yields equation (5.11), from which we obtain

\[ W'_t(s_t) = V'_t(s_t)|\tilde{A}_t^{-i} = -\beta s_t + \delta(1 - \gamma)W'_{t+1}(\tilde{s}_{t+1}) < 0. \]  
\[ (D.33) \]

In addition, differentiating with respect to \( s_t \), we observe

\[ W''_t(s_t) = V''_t(s_t)|\tilde{A}_t^{-i} = \delta(1 - \gamma)^2W''_{t+1}(\tilde{s}_{t+1}) - \beta < 0, \]  
\[ (D.34) \]

implying that the equilibrium pay-off \( W_t(s_t) \) is strictly concave.

Working backwards to \( t = 1 \) yields a unique symmetric sequence of emission abatements \( \tilde{a}_i^t = \tilde{a}_t \) and the corresponding sequence of the greenhouse gas stock \( \tilde{s}_t \) \((i = 1, \ldots, n; t = 1, \ldots, T)\) that constitute the unique and symmetric subgame perfect Nash equilibrium of the RS.

\section*{D.6. Proof of Proposition 5.5}

In the subgame perfect Nash equilibrium, the first-order condition (5.16) reads

\[ \alpha a_t = -\delta W'_{t+1}(s_{t+1}) + \frac{(n - 1)R_t}{n^2 a_t}, \]  
\[ (D.35) \]

where we drop the tilde notation \( " \) for presentational convenience. Together with equation (D.33), this implies that the sequences \( a_t \) and \( s_t \) in the subgame perfect Nash equilibrium of the RS are given by the solution of the following system of first-order
difference equations:

\[
\begin{align*}
R \frac{n-1}{n^2a_t} - \alpha a_t &= \delta(1 - \gamma)(R \frac{n-1}{n^2a_{t+1}} - \alpha a_{t+1}) - \delta s_{t+1}, \quad (D.36a) \\
sv_{t+1} &= (1 - \gamma)s_t + n(\epsilon - a_t), \quad (D.36b)
\end{align*}
\]

with the boundary conditions \( s_1 \) and \( a_T = 0 \) for finite time horizons \( T \). Solving equation (D.36a) with respect to \( a_{t+1} \) yields

\[
a_{t+1} = \frac{g(a_t) - \beta\delta(1 - \gamma)s_t + \sqrt{[g(a_t) - \beta\delta(1 - \gamma)s_t]^2 + 4\alpha\delta^2(1 - \gamma)^2R\frac{n-1}{n^2}}}{2\alpha\delta(1 - \gamma)}, \quad (D.36c)
\]

with \( g(a) \equiv (\alpha + n\beta\delta)a - R(n - 1)/(n^2a) - n\beta\delta\epsilon \). Solving for the steady state and taking into account the fact that negative abatement levels are infeasible yields equations (5.23).

We now analyze the system dynamics for \( T \to \infty \). We show that the system dynamics split into three different regimes. (i) For any initial stock \( s_1 \) there exists a corresponding initial level of abatement \( a_1^{SP} \) such that the sequences \( a_t \) and \( s_t \) converge to their steady state values \( (a^{RS}, s^{RS}) \). (ii) If the initial level of abatement is above \( a_1^{SP} \), then the system dynamics will converge to the corner solution \((\epsilon,0)\) in which emission abatement is maximal \( a_{max} = \epsilon \), i.e. all emissions are abated, and the greenhouse gas stock converges to 0. (iii) If the initial level of abatement is below \( a_1^{SP} \), then the system dynamics will converge to the corner solution \((0,n\epsilon/\gamma)\) in which no emissions are abated and the greenhouse gas stock converges to its maximum value \( s_{max} = n\epsilon/\gamma \).

To see (i), we show that the steady state \((a^{RS}, s^{RS})\) is a saddle point. Denoting the Jacobian of the system of difference equations evaluated at the steady state by

\[
J(a^{RS}, s^{RS}) = \begin{bmatrix}
\frac{\partial a_{t+1}(a^{RS}, s^{RS})}{\partial a_t} & \frac{\partial a_{t+1}(a^{RS}, s^{RS})}{\partial s_t} \\
\frac{\partial s_{t+1}(a^{RS}, s^{RS})}{\partial a_t} & \frac{\partial s_{t+1}(a^{RS}, s^{RS})}{\partial s_t}
\end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\
J_{21} & J_{22} \end{bmatrix}, \quad (D.37)
\]
we obtain

\[ J_{11} = \frac{\alpha + R \frac{n-1}{n^2(a^{RS} s)^2} + \delta \beta n}{\delta(1 - \gamma) \left( \alpha + R \frac{n-1}{n^2(a^{RS} s)^2} \right)}, \]  

(D.38a)

\[ J_{12} = -\frac{\beta}{\alpha + R \frac{n-1}{n^2(a^{RS} s)^2}}, \]  

(D.38b)

\[ J_{21} = -n, \]  

(D.38c)

\[ J_{22} = 1 - \gamma, \]  

(D.38d)

where we have derived (D.38a) and (D.38b) by differentiating (D.36a) with respect to \( a_t \) and \( s_t \) respectively and inserting the steady state. Then the characteristic equation in the linearization around the steady state reads

\[ \nu^2 - (J_{11} + J_{22})\nu + J_{11}J_{22} - J_{12}J_{21} = 0, \]  

(D.39)

which yields the eigenvalues

\[ \nu_{1/2} = \frac{J_{11} + J_{22} \pm \sqrt{(J_{11} + J_{22})^2 - 4(J_{11}J_{22} - J_{12}J_{21})}}{2}. \]  

(D.40)

It can be shown that

\[ J_{11} + J_{22} > \sqrt{(J_{11} + J_{22})^2 - 4(J_{11}J_{22} - J_{12}J_{21})} > J_{11} - J_{22}, \]  

(D.41)

implying that

\[ \nu_1 > J_{11} > 1, \quad 0 < \nu_2 < J_{22} = 1 - \gamma < 1. \]  

(D.42)

Thus, the steady state \((a^{RS}, s^{RS})\) is a saddle point. As a consequence, for any initial value of the greenhouse gas stock \( s_1 \) there exists a corresponding initial level of abatement \( a_1^{SP} \) such that the sequences \( a_t \) and \( s_t \) converge to the steady-state values \((a^{RS}, s^{RS})\).

Figure D.1 sketches the system dynamics. The isoclines are given by

\[ a_{t+1} - a_t = 0 \quad \Leftrightarrow \quad a_t = \frac{n\beta \delta s_t + \sqrt{(n\beta \delta s_t)^2 + 4\alpha \rho f \frac{n-1}{n^2}[1 - \delta(1 - \gamma)]^2}}{2n\alpha[1 - \delta(1 - \gamma)]}, \]  

(D.43a)

\[ s_{t+1} - s_t = 0 \quad \Leftrightarrow \quad a_t = \epsilon - \frac{\gamma}{n} s_t, \]  

(D.43b)

They determine the combinations of \( s_t \) and \( a_t \) for which \( a_{t+1} - a_t = 0 \) and \( s_{t+1} - s_t = 0, \)
Figure D.1.: Phase diagram of the system dynamics of the system of difference equations (D.36) for $T \to \infty$. The steady state $(a^{RS}, s^{RS})$ is a saddle point. The $a$- and $s$-isoclines divide the feasible space into four areas I–IV.

respectively, and divide the $a_t$-$s_t$-plane into four areas. In area I, which is above the $s$- and below the $a$-isocline, $a_{t+1} > a_t$ and $s_{t+1} > s_t$. In area II, given by the segment above both isoclines, $a_{t+1} > a_t$ and $s_{t+1} < s_t$. Area III is below the $s$- and above the $a$-isocline, in which case $a_{t+1} < a_t$ and $s_{t+1} < s_t$ hold. Finally, area IV is below both isoclines, so $a_{t+1} < a_t$ and $s_{t+1} > s_t$. The saddle point path lies in area I for $s_1 < s^{RS}$ and in area III for $s_1 > s^{RS}$. Thus, the saddle point path starts with abatement levels $a^{SP}_1$ which are below (above) the steady-state abatement level $a^{RS}$ and increase (decrease) over time to converge to $a^{RS}$ for $s_1 < s^{RS}$ ($s_1 > s^{RS}$).

We also observe that all paths, although they may start in area I, II or III, for which $a_1 > a^{SP}_1$ eventually reach the maximum abatement level $a_{t'} = a_{max} = \epsilon$. In this case the system dynamics changes regimes, as no abatement levels above $\epsilon$ are feasible. As a consequence, $a_t = \epsilon$ for all $t \geq t'$. Then the stock of greenhouse gases, as determined by equation (D.36b), converges to 0 for $t \to \infty$. In a similar vain, all paths for which the initial abatement level is smaller than the initial saddle point path abatement level $a^{SP}_1$ eventually hit the lower boundary $a_{t'} = 0$. Again, the system dynamics changes regimes, as abatement levels below 0 are infeasible, so $a_t = 0$ for all $t \geq t'$. As a consequence, the greenhouse gas stock converges to its maximum value $s_{max} = (n\epsilon)/\gamma$ for $t \to \infty$.

By assumption, the corner solutions $a_t = 0$ and $a_t = \epsilon$ cannot be best responses for any country $i$ to all feasible greenhouse gas stocks $s_t$ and any given abatement levels of
all other countries $j \neq i$. As a consequence, all paths that either converge to $(\epsilon,0)$ or $(0,(ne)/\gamma)$ are not subgame perfect Nash equilibria, leaving the saddle point path as the only subgame perfect Nash equilibrium.

D.7. Proof of Proposition 5.6

For $f_0 = f_0^{SB}$ as given by equation (5.24), the linear approximation around the stationary state of the system dynamics of the second-best sustainable RS is given by

\begin{align}
\tilde{a}_t &= a^{SO} + \frac{(1 - \gamma - \nu_2)(s_1 - s^{SO})}{n} \nu_2^{t-1}, \\
\tilde{s}_t &= s^{SO} + (s_1 - s^{SO})\nu_2^{t-1},
\end{align}

where $\nu_2$ denotes the smaller eigenvalue of the Jacobian (D.37)

\begin{equation}
\nu_2 = \frac{J_{11} + J_{22} - \sqrt{(J_{11} + J_{22})^2 - 4(J_{11}J_{22} - J_{12}J_{21})}}{2}.
\end{equation}

Evaluating $\nu_2$ for $f_0^{SB}$ yields equation (5.27a). Comparing equation (D.44b) for the greenhouse gas stock in the second-best sustainable RS with the corresponding equation (D.11b) of the social global optimum yields equation (5.26), where (5.27b) states the corresponding eigenvalue $\lambda_2$ in the social global optimum.

It is easy to see that $\nu_2 > \lambda_2$ for $n > 1$ which implies (5.28).

D.8. Proof of Proposition 5.7

(i) A treaty with repeated payments $f_t(\Delta)$ implements the same solution as the first-best sustainable RS with initial payments $f^*_t$ if the same refund $R^*_t$ is distributed among the countries in period $t$. At the end of each $\Delta$-th period, the fund is reduced to zero and will be refilled by $f_{t+\Delta}(\Delta)$. Hence, it must hold that

\begin{equation}
0 = f_{t+\Delta} = (1 + \rho)f_{t+\Delta-1} - R^*_t.
\end{equation}

Inserting recursively back to period $t$ and solving for $f_t$, we obtain the amount of money that should be in the fund in period $t$, which equals (5.30).
(ii) A treaty with repeated payments $f(\Delta)$ implements the same solution as the second-best sustainable RS with initial payments $f_0^{SB}$ if all countries receive $(\rho f_0^{SB})/n$ from the fund in each period. At the end of each $\Delta$-th period, the fund is reduced to zero and will be refilled by $f(\Delta)$. At the beginning of each period, the remainder of the fund is invested and earns interest $\rho$. Therefore it holds that

$$0 = f(\Delta t)(1 + \rho)^{\Delta t} - (1 + (1 + \rho) + \ldots + (1 + \rho)^{\Delta - 1})\rho f_0^{SB}.$$  \hspace{1cm} (D.47)

Applying the formula for the finite geometric series

$$\sum_{k=0}^{\Delta - 1} (1 + \rho)^k = \frac{1 - (1 + \rho)^\Delta}{1 - (1 + \rho)},$$  \hspace{1cm} (D.48)

and solving for $f(\Delta t)$, we obtain (5.31).
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Bibliography


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Bibliography


Curriculum Vitae

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