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# One-Bit Quantizers for Fading Channels

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**Abstract**—We study channel capacity when a one-bit quantizer is employed at the output of the discrete-time average-power-limited Rayleigh-fading channel. We focus on the low signal-to-noise ratio regime, where communication at very low spectral efficiencies takes place, as in Spread Spectrum and Ultra-Wideband communications. We demonstrate that, in this regime, the best one-bit quantizer does not reduce the asymptotic capacity of the coherent channel, but it does reduce that of the noncoherent channel.

## I. INTRODUCTION

We study the effect on channel capacity of quantizing the output of the discrete-time average-power-limited Rayleigh-fading channel using a one-bit quantizer. This problem arises in communication systems where the receiver uses digital signal processing techniques, which require the analog received signal to be quantized using an analog-to-digital converter (ADC). The effects of quantization are particularly pronounced when high-resolution ADCs are not practical and low-resolution ADCs must be used [1].

We focus on the low signal-to-noise ratio (SNR) regime, where communication at very low spectral efficiencies takes place (as in Spread-Spectrum and Ultra-Wideband communications). For the average-power-limited *real-valued Gaussian channel*, it is well-known that, in this regime, a *symmetric one-bit quantizer* (which produces 1 if the channel output is nonnegative and 0 otherwise) reduces the capacity by a factor of  $2/\pi$ , corresponding to a 2dB power loss [2]. It was recently shown that, by allowing for *asymmetric one-bit quantizers* with corresponding *asymmetric signal constellations*, these two decibels can be recovered in full [3]. A similar result was shown for the average-power-limited *complex-valued Gaussian channel* [4]: using binary on-off keying and a *radial quantizer* (which produces 1 if the magnitude of the channel output is above some threshold and 0 otherwise), one can achieve the low-SNR asymptotic capacity of the unquantized channel by judiciously choosing the threshold and the on-level as functions of the SNR. Here we extend [3], [4] to *Rayleigh-fading channels*. Specifically, we study the capacity per unit-energy [5] of such channels when the channel output is quantized using a one-bit quantizer.

For *coherent* fading channels, where the receiver has perfect channel knowledge, we show that quantizing the channel output with a one-bit quantizer causes no loss in the capacity

per unit-energy. As in [4], the capacity per unit-energy can be achieved using binary on-off keying and a radial quantizer by choosing the threshold as a function of the SNR and the fading, with the threshold and the on-level both tending to infinity as the SNR tends to zero. This result might mislead one to think that quantizing the channel output with a one-bit quantizer causes no loss in the capacity per unit-energy also for *noncoherent* fading channels, where the receiver does not have perfect channel knowledge. Indeed, in the absence of a quantizer the capacity per unit-energy does not depend on whether the receiver has perfect channel knowledge or not [6], [7]. Since this capacity per unit-energy can be achieved using binary on-off keying with diverging on-level, it might therefore seem plausible that also in the presence of a quantizer the capacity per unit-energy would not depend on whether the receiver has perfect channel knowledge or not. But this is not the case: in contrast to the coherent case, quantizing the output of the *noncoherent* Rayleigh-fading channel with a one-bit quantizer reduces the capacity per unit-energy.

The rest of the paper is organized as follows. Section II describes the channel model and introduces the capacity per unit-energy. Section III presents the main results. Section IV discusses the capacity per unit-energy when the real and the imaginary part of the channel output are quantized separately with one-bit quantizers. And Section V presents the proofs of the main results.

## II. CHANNEL MODEL AND CAPACITY PER UNIT-ENERGY

We consider a discrete-time Rayleigh-fading channel whose complex-valued output  $\tilde{Y}_k$  at time  $k \in \mathbb{Z}$  corresponding to the channel input  $x_k \in \mathbb{C}$  (where  $\mathbb{C}$  and  $\mathbb{Z}$  denote the set of complex numbers and the set of integers) is given by

$$\tilde{Y}_k = H_k x_k + Z_k, \quad k \in \mathbb{Z}. \quad (1)$$

Here  $\{Z_k, k \in \mathbb{Z}\}$  and  $\{H_k, k \in \mathbb{Z}\}$  are independent sequences of independent and identically distributed (i.i.d.), zero-mean, circularly-symmetric, complex Gaussian random variables, the former with unit variance and the latter with variance  $\sigma^2$ . We say that the channel is *coherent* if the receiver is cognizant of the realization of  $\{H_k, k \in \mathbb{Z}\}$  and that it is *noncoherent* if the receiver is cognizant only of the statistics of  $\{H_k, k \in \mathbb{Z}\}$ .

The receiver does not have access to the channel outputs  $\{\tilde{Y}_k, k \in \mathbb{Z}\}$  but only to a quantized version thereof. Specifically, the complex channel output  $\tilde{Y}_k$  is fed to a one-bit

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quantizer which produces  $Y_k = 1$  if  $\tilde{Y}_k$  is in the quantization region  $\mathcal{D}$  and  $Y_k = 0$  otherwise, for some Borel set  $\mathcal{D} \subset \mathbb{C}$ . In the coherent case,  $\mathcal{D}$  may depend on the fading  $\{H_k, k \in \mathbb{Z}\}$ .

We assume that the average power of the channel inputs is limited by  $P$ . The capacity of the above channel is [8], [9]

$$C(P) = \sup I(X; Y|H), \quad \text{coherent case} \quad (2)$$

$$C(P) = \sup I(X; Y), \quad \text{noncoherent case} \quad (3)$$

where the suprema on the right-hand side (RHS) of (2) and (3) are over all distributions on  $X$  satisfying  $\mathbb{E}[|X|^2] \leq P$  and over all quantization regions  $\mathcal{D}$ . (Since the above channel is memoryless, we omit the time indices.)

The capacity per unit-energy is given by [5, Th. 2]

$$\dot{C}(0) = \sup_{P>0} \frac{C(P)}{P}. \quad (4)$$

It can be shown that

$$\dot{C}(0) = \lim_{P \downarrow 0} \frac{C(P)}{P}. \quad (5)$$

Thus, the capacity per unit-energy is equal to the slope at zero of the capacity-vs-power curve. It can be further shown that [5, Th. 3] (see also [6])

$$\dot{C}(0) = \sup_{\xi \neq 0, \mathcal{D}} \frac{D(P_{Y|H, X=\xi} \| P_{Y|H, X=0} | P_H)}{|\xi|^2} \quad (6)$$

in the coherent case and

$$\dot{C}(0) = \sup_{\xi \neq 0, \mathcal{D}} \frac{D(P_{Y|X=\xi} \| P_{Y|X=0})}{|\xi|^2} \quad (7)$$

in the noncoherent case. Here  $D(\cdot \| \cdot)$  denotes relative entropy

$$D(P \| Q) = \begin{cases} \int \log\left(\frac{dP}{dQ}\right) dP, & \text{if } P \ll Q \\ \infty, & \text{otherwise} \end{cases}$$

(where  $P \ll Q$  indicates that  $P$  is absolutely continuous with respect to  $Q$ );  $D(\cdot \| \cdot | \cdot)$  denotes conditional relative entropy

$$\begin{aligned} D(P_{Y|H, X=\xi} \| P_{Y|H, X=0} | P_H) \\ = \int D(P_{Y|H=h, X=\xi} \| P_{Y|H=h, X=0}) dP_H(h); \end{aligned}$$

$P_H$  denotes the distribution of the fading  $H$ ;  $P_{Y|X=x}$  denotes the output distribution given that the input is  $x$ ; and  $P_{Y|H=h, X=x}$  denotes the output distribution conditioned on  $(H, X) = (h, x)$ .

By the Data Processing Inequality [10, Th. 2.8.1], the capacity per unit-energy of the quantized channel is upper-bounded by that of the unquantized channel [7], [6]

$$\dot{C}(0) \leq \frac{1}{\sigma^2}. \quad (8)$$

We show that in the coherent case this upper bound holds with equality, while in the noncoherent case it is strict.

### III. MAIN RESULT

We restrict ourselves to *radial* quantizers, for which

$$\mathcal{D} = \{\tilde{y} \in \mathbb{C} : |\tilde{y}| \geq T\}, \quad \text{for some } T > 0. \quad (9)$$

In the noncoherent case—as we show in Section V-B—such quantizers are optimal in the sense that they maximize the relative entropy on the RHS of (7) for every  $\xi \neq 0$ . In the coherent case such quantizers need not be optimal in the above sense. However, they suffice to achieve the capacity per unit-energy. And such quantizers have the practical advantage of not requiring knowledge of the phase of  $\tilde{y}$ .

*Theorem 1:* Consider the above channel model, and assume that the channel output is quantized using a one-bit quantizer.

1) In the *coherent* case,

$$\dot{C}(0) = \frac{1}{\sigma^2} \quad (10)$$

which can be achieved by some radial quantizer (9) with  $T$  depending on  $H$  and  $\xi$ .<sup>2</sup>

2) In the *noncoherent* case,

$$\dot{C}(0) < \frac{1}{\sigma^2} \quad (11)$$

with the inequality being strict.

*Proof:* See Section V. ■

### IV. QUANTIZING THE REAL AND IMAGINARY PART

Instead of quantizing  $\tilde{Y}$  using a one-bit quantizer, often the real and imaginary parts of  $\tilde{Y}$  are quantized separately using a one-bit quantizer for each; see, e.g., [11]–[14], [6]. Thus, the first quantizer produces  $Y_{R,k} = 1$  if  $\text{Re}(\tilde{Y}_k) \in \mathcal{D}_R$  and  $Y_{R,k} = 0$  otherwise, and the second quantizer produces  $Y_{I,k} = 1$  if  $\text{Im}(\tilde{Y}_k) \in \mathcal{D}_I$  and  $Y_{I,k} = 0$  otherwise, for some Borel sets  $\mathcal{D}_R, \mathcal{D}_I \subset \mathbb{R}$ . (Here  $\mathbb{R}$  denotes the set of real numbers,  $\text{Re}(\cdot)$  denotes the real part, and  $\text{Im}(\cdot)$  denotes the imaginary part.) In the coherent case,  $\mathcal{D}_R$  and  $\mathcal{D}_I$  may depend on the fading  $\{H_k, k \in \mathbb{Z}\}$ .

The capacity per unit-energy of this channel is given by (6) or (7), but with  $Y$  replaced by  $(Y_R, Y_I)$ , and with  $\mathcal{D} \subset \mathbb{C}$  replaced by  $(\mathcal{D}_R, \mathcal{D}_I) \subset \mathbb{R} \times \mathbb{R}$ .

For symmetric quantizers, i.e., for

$$\mathcal{D}_R = \mathcal{D}_I = \{u \in \mathbb{R} : u \geq 0\} \quad (12)$$

it follows from [11] and [15, Th. 2] that, in the coherent case,

$$\dot{C}_{\text{sym}}(0) = \frac{2}{\pi\sigma^2}. \quad (13)$$

In the noncoherent case, symmetric quantizers result in zero capacity and hence, by (4), in zero capacity per unit-energy. Indeed, for (12)

$$\Pr(Y_R = 1 | X = x) = \Pr(Y_I = 1 | X = x) = \frac{1}{2}, \quad x \in \mathbb{C}.$$

Since, conditioned on  $X$ , the random variables  $Y_R$  and  $Y_I$  are independent, this implies that the capacity is zero. Thus, quantizing the real and imaginary parts of the Rayleigh-fading

<sup>2</sup>Here and throughout this paper,  $\xi$  refers to the parameter in (6) or (7).

channel using symmetric one-bit quantizers reduces the capacity per unit-energy by a factor of  $2/\pi$  in the coherent case, and it reduces it to zero in the noncoherent case. In the following, we show that if we allow for *asymmetric* quantizers, then we can fully recover the loss of  $2/\pi$  incurred on the coherent Rayleigh-fading channel. For the noncoherent channel, we show that asymmetric quantizers achieve a positive capacity per unit-energy, albeit strictly smaller than  $1/\sigma^2$ .

*Theorem 2:* Consider the above channel model, and assume that the real and imaginary parts of  $\tilde{Y}$  are quantized separately using a one-bit quantizer for each.

1) In the *coherent case*,

$$\dot{C}(0) = \frac{1}{\sigma^2} \quad (14)$$

which can be achieved by some quantization regions

$$\mathcal{D}_R^* = \{u \in \mathbb{R} : u \geq T_R\} \quad (15)$$

$$\mathcal{D}_I^* = \{u \in \mathbb{R} : u \geq T_I\} \quad (16)$$

where  $T_R$  and  $T_I$  depend on  $\text{Re}(H\xi)$  and  $\text{Im}(H\xi)$ , respectively.

2) In the *noncoherent case*,

$$\frac{2Q(1)}{\sigma^2} \leq \dot{C}(0) < \frac{1}{\sigma^2} \quad (17)$$

with the upper bound being strict. Here  $Q(\cdot)$  denotes the Gaussian  $Q$ -function [16, Eq. (1.3)]. The lower bound can be achieved by the quantization regions (15) and (16) with  $T_R = T_I = (|\xi|^2 + \sigma^2)/2$ .

*Proof:* Omitted. ■

## V. PROOF OF THEOREM 1

### A. Part 1)

We show that a radial quantizer (9) achieves the rate per unit-energy  $1/\sigma^2$ . Together with (8), this proves Theorem 1.

To this end, we first note that, conditioned on  $(H, X) = (h, x)$ , the squared magnitude of  $\frac{2}{\sigma^2}\tilde{Y}$  is a noncentral chi-square distribution with degree 2 and noncentrality parameter  $\frac{2}{\sigma^2}|h|^2|x|^2$  [16, p. 8]. Consequently, a radial quantizer yields [16, Sec. 2-E]

$$\Pr(Y = 1 \mid H = h, X = x) = Q_1\left(\sqrt{\frac{2}{\sigma^2}}|h||x|, \sqrt{\frac{2}{\sigma^2}}T\right)$$

where  $Q_1(\cdot, \cdot)$  denotes the first-order Marcum  $Q$ -function [16, Eq. (2.20)]. Furthermore, for  $x = 0$  this becomes

$$\Pr(Y = 1 \mid H = h, X = 0) = e^{-\frac{T^2}{\sigma^2}}.$$

We thus obtain

$$\begin{aligned} D(P_{Y|H,X=\xi} \parallel P_{Y|H,X=0} \mid P_H) &= \mathbb{E} \left[ Q_1\left(\sqrt{\frac{2}{\sigma^2}}|H||\xi|, \sqrt{\frac{2}{\sigma^2}}T\right) \log \frac{1}{e^{-\frac{T^2}{\sigma^2}}} \right] \\ &+ \mathbb{E} \left[ \left\{ 1 - Q_1\left(\sqrt{\frac{2}{\sigma^2}}|H||\xi|, \sqrt{\frac{2}{\sigma^2}}T\right) \right\} \log \frac{1}{1 - e^{-\frac{T^2}{\sigma^2}}} \right] \\ &- \mathbb{E} \left[ H_b\left(Q_1\left(\sqrt{\frac{2}{\sigma^2}}|H||\xi|, \sqrt{\frac{2}{\sigma^2}}T\right)\right) \right] \\ &\geq \mathbb{E} \left[ Q_1\left(\sqrt{\frac{2}{\sigma^2}}|H||\xi|, \sqrt{\frac{2}{\sigma^2}}T\right) \frac{T^2}{\sigma^2} \right] - \log 2 \end{aligned} \quad (18)$$

where  $H_b(\cdot)$  denotes the binary entropy function, i.e.,

$$H_b(p) \triangleq \begin{cases} p \log \frac{1}{p} + (1-p) \log \frac{1}{1-p}, & \text{for } 0 < p < 1 \\ 0, & \text{for } p = 0 \text{ or } p = 1. \end{cases}$$

Here the inequality follows because the second term in the first step is nonnegative, and because the binary entropy function is upper-bounded by  $\log 2$ .

We choose  $T = \mu|h||\xi|$  for some fixed  $\mu \in (0, 1)$  and lower-bound the RHS of (18) using the general lower bound on the first-order Marcum  $Q$ -function [16, Sec. C-2, Eq. (C.24)]

$$Q_1(\alpha, \beta) \geq 1 - \frac{1}{2} \left[ e^{-\frac{(\alpha-\beta)^2}{2}} - e^{-\frac{(\alpha+\beta)^2}{2}} \right], \quad \alpha > \beta \geq 0.$$

We thus obtain for the first term on the RHS of (18)

$$\begin{aligned} &\mathbb{E} \left[ Q_1\left(\sqrt{\frac{2}{\sigma^2}}|H||\xi|, \sqrt{\frac{2}{\sigma^2}}\mu|H||\xi|\right) \frac{\mu^2|H|^2|\xi|^2}{\sigma^2} \right] \\ &\geq \frac{\mu^2 \mathbb{E}[|H|^2] |\xi|^2}{\sigma^2} \\ &- \frac{1}{2} \mathbb{E} \left[ \exp\left(-\frac{|H|^2|\xi|^2}{\sigma^2}(1-\mu)^2\right) \frac{\mu^2|H|^2|\xi|^2}{\sigma^2} \right] \\ &+ \frac{1}{2} \mathbb{E} \left[ \exp\left(-\frac{|H|^2|\xi|^2}{\sigma^2}(1+\mu)^2\right) \frac{\mu^2|H|^2|\xi|^2}{\sigma^2} \right] \\ &\geq \frac{\mu^2 \mathbb{E}[|H|^2] |\xi|^2}{\sigma^2} - \frac{\mu^2}{2e(1-\mu)^2} \end{aligned} \quad (19)$$

where the last step follows because  $0 \leq xe^{-\alpha x} \leq 1/(e\alpha)$  for every  $x \geq 0$  and  $\alpha > 0$ .

Combining (19) with (18), and computing its ratio to  $|\xi|^2$  in the limit as  $|\xi|^2$  tends to infinity, yields

$$\dot{C}(0) \geq \frac{\mu^2 \mathbb{E}[|H|^2]}{\sigma^2} = \frac{\mu^2}{\sigma^2}. \quad (20)$$

Theorem 1 follows then by letting  $\mu$  tend to one.

### B. Part 2)

We first note that, by the Data Processing Inequality for relative entropy [10, Sec. 2.9], the relative entropy on the RHS of (7) is upper-bounded by the relative entropy corresponding to the unquantized channel, i.e., [6, Eq. (64)]

$$\frac{D(P_{Y|X=\xi} \parallel P_{Y|X=0})}{|\xi|^2} \leq \frac{1}{\sigma^2} - \frac{\log\left(1 + \frac{|\xi|^2}{\sigma^2}\right)}{|\xi|^2}. \quad (21)$$

Consequently, the capacity per unit-cost (7) is strictly smaller than  $1/\sigma^2$  unless  $|\xi|$  tends to infinity. It thus remains to show that

$$\overline{\lim}_{|\xi| \rightarrow \infty} \sup_{\mathcal{D}} \frac{D(P_{Y|X=\xi} \parallel P_{Y|X=0})}{|\xi|^2} < \frac{1}{\sigma^2}. \quad (22)$$

To this end, we first note that, for every  $\xi \neq 0$ , the supremum in (22) over all quantization regions  $\mathcal{D}$  can be replaced with the supremum over all radial quantizers (9). Indeed, for every quantization region satisfying

$$\Pr(Y = 1 \mid X = \xi) = \beta, \quad 0 < \beta < 1$$

the relative entropy

$$\begin{aligned} D(P_{Y|X=\xi} \parallel P_{Y|X=0}) &= \beta \log \frac{1}{\Pr(Y = 1 \mid X = 0)} \\ &+ (1 - \beta) \log \frac{1}{1 - \Pr(Y = 1 \mid X = 0)} - H_b(\beta) \end{aligned} \quad (23)$$

is a convex function of  $\Pr(Y = 1 \mid X = 0)$ . Thus, for every  $0 < \beta < 1$ , the RHS of (23) is maximized for the quantization region that minimizes (or maximizes)  $\Pr(Y = 1 \mid X = 0)$  while holding  $\Pr(Y = 1 \mid X = \xi) = \beta$  fixed. By the Neyman-Pearson lemma [17], such a quantization region has the form

$$\mathcal{D}_* = \left\{ \tilde{y} \in \mathbb{C} : \frac{f(\tilde{y}|0)}{f(\tilde{y}|\xi)} \leq \Lambda \right\}, \quad \Lambda > 0 \quad (24)$$

(or the complement thereof), where  $f(\tilde{y}|x)$  denotes the conditional density of  $\tilde{Y}$ , conditioned on  $X = x$ , and where  $\Lambda$  is such that  $\Pr(\tilde{Y} \in \mathcal{D}_* \mid X = \xi) = \beta$ . (Note that for every  $0 < \beta < 1$  there exists such a  $\Lambda$  since, for the above channel model (1),  $\Pr(\tilde{Y} \in \mathcal{D}_* \mid X = \xi)$  is a continuous, strictly increasing function of  $\Lambda > 0$ .) The likelihood ratio on the RHS of (24) is readily evaluated as

$$\frac{f(\tilde{y}|0)}{f(\tilde{y}|\xi)} = \left( 1 + \frac{|\xi|^2}{\sigma^2} \right) e^{-\frac{|\tilde{y}|^2}{\sigma^2} \frac{|\xi|^2}{\sigma^2 + |\xi|^2}}, \quad \tilde{y} \in \mathbb{C}.$$

Consequently,  $\mathcal{D}_*$  is the same as (9) with

$$T = \sigma \sqrt{\left( 1 + \frac{\sigma^2}{|\xi|^2} \right) \log \left( \frac{1 + \frac{|\xi|^2}{\sigma^2}}{\Lambda} \right)}.$$

We thus obtain that, for every  $\xi \neq 0$ , the relative entropy  $D(P_{Y|X=\xi} \parallel P_{Y|X=0})$  is maximized by a radial quantizer (9).

For such a quantizer, we have

$$\Pr(Y = 1 \mid X = x) = \exp\left(-\frac{T^2}{|x|^2 + \sigma^2}\right)$$

which yields

$$\begin{aligned} D(P_{Y|X=\xi} \parallel P_{Y|X=0}) &= e^{-\frac{T^2}{|\xi|^2 + \sigma^2}} \log \frac{1}{e^{-\frac{T^2}{\sigma^2}}} \\ &+ \left[ 1 - e^{-\frac{T^2}{|\xi|^2 + \sigma^2}} \right] \log \frac{1}{1 - e^{-\frac{T^2}{\sigma^2}}} - H_b\left(e^{-\frac{T^2}{|\xi|^2 + \sigma^2}}\right) \\ &\leq \frac{T^2}{\sigma^2} e^{-\frac{T^2}{|\xi|^2 + \sigma^2}} - \left[ 1 - e^{-\frac{T^2}{\sigma^2}} \right] \log\left(1 - e^{-\frac{T^2}{\sigma^2}}\right) \\ &\leq \frac{T^2}{\sigma^2} e^{-\frac{T^2}{|\xi|^2 + \sigma^2}} + \frac{1}{e} \end{aligned} \quad (25)$$

where the second step follows because  $H_b(\cdot) \geq 0$  and  $\exp(-T^2/(|\xi|^2 + \sigma^2)) \geq \exp(-T^2/\sigma^2)$ ; and the third step follows because  $-x \log x \leq \frac{1}{e}$ ,  $0 < x < 1$ .

The first term on the RHS of (25) is maximized for  $T^2 = |\xi|^2 + \sigma^2$ . The RHS of (25) is thus upper-bounded by

$$D(P_{Y|X=\xi} \parallel P_{Y|X=0}) \leq \frac{|\xi|^2}{e\sigma^2} + \frac{2}{e}. \quad (26)$$

Dividing the RHS of (26) by  $|\xi|^2$ , and computing the limit as  $|\xi|$  tends to infinity, yields

$$\overline{\lim}_{|\xi| \rightarrow \infty} \sup_{\mathcal{D}} \frac{D(P_{Y|X=\xi} \parallel P_{Y|X=0})}{|\xi|^2} \leq \frac{1}{e\sigma^2} < \frac{1}{\sigma^2}. \quad (27)$$

This proves Theorem 2.

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