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One-Bit Quantizers for Fading Channels

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Abstract—We study channel capacity when a one-bit quantizer is employed at the output of the discrete-time average-power-limited Rayleigh-fading channel. We focus on the low signal-to-noise ratio regime, where communication at very low spectral efficiencies takes place, as in Spread Spectrum and Ultra-Wideband communications. We demonstrate that, in this regime, the best one-bit quantizer does not reduce the asymptotic capacity of the coherent channel, but it does reduce that of the noncoherent channel.

I. INTRODUCTION

We study the effect on channel capacity of quantizing the output of the discrete-time average-power-limited Rayleigh-fading channel using a one-bit quantizer. This problem arises in communication systems where the receiver uses digital signal processing techniques, which require the analog received signal to be quantized using an analog-to-digital converter (ADC). The effects of quantization are particularly pronounced when high-resolution ADCs are not practical and low-resolution ADCs must be used [1].

We focus on the low signal-to-noise ratio (SNR) regime, where communication at very low spectral efficiencies takes place (as in Spread-Spectrum and Ultra-Wideband communications). For the average-power-limited *real-valued Gaussian channel*, it is well-known that, in this regime, a *symmetric one-bit quantizer* (which produces 1 if the channel output is nonnegative and 0 otherwise) reduces the capacity by a factor of $2/\pi$, corresponding to a 2dB power loss [2]. It was recently shown that, by allowing for *asymmetric one-bit quantizers* with corresponding *asymmetric signal constellations*, these two decibels can be recovered in full [3]. A similar result was shown for the average-power-limited *complex-valued Gaussian channel* [4]: using binary on-off keying and a *radial quantizer* (which produces 1 if the magnitude of the channel output is above some threshold and 0 otherwise), one can achieve the low-SNR asymptotic capacity of the unquantized channel by judiciously choosing the threshold and the on-level as functions of the SNR. Here we extend [3], [4] to *Rayleigh-fading channels*. Specifically, we study the capacity per unit-energy [5] of such channels when the channel output is quantized using a one-bit quantizer.

For *coherent* fading channels, where the receiver has perfect channel knowledge, we show that quantizing the channel output with a one-bit quantizer causes no loss in the capacity

per unit-energy. As in [4], the capacity per unit-energy can be achieved using binary on-off keying and a radial quantizer by choosing the threshold as a function of the SNR and the fading, with the threshold and the on-level both tending to infinity as the SNR tends to zero. This result might mislead one to think that quantizing the channel output with a one-bit quantizer causes no loss in the capacity per unit-energy also for *noncoherent* fading channels, where the receiver does not have perfect channel knowledge. Indeed, in the absence of a quantizer the capacity per unit-energy does not depend on whether the receiver has perfect channel knowledge or not [6], [7]. Since this capacity per unit-energy can be achieved using binary on-off keying with diverging on-level, it might therefore seem plausible that also in the presence of a quantizer the capacity per unit-energy would not depend on whether the receiver has perfect channel knowledge or not. But this is not the case: in contrast to the coherent case, quantizing the output of the *noncoherent* Rayleigh-fading channel with a one-bit quantizer reduces the capacity per unit-energy.

The rest of the paper is organized as follows. Section II describes the channel model and introduces the capacity per unit-energy. Section III presents the main results. Section IV discusses the capacity per unit-energy when the real and the imaginary part of the channel output are quantized separately with one-bit quantizers. And Section V presents the proofs of the main results.

II. CHANNEL MODEL AND CAPACITY PER UNIT-ENERGY

We consider a discrete-time Rayleigh-fading channel whose complex-valued output \tilde{Y}_k at time $k \in \mathbb{Z}$ corresponding to the channel input $x_k \in \mathbb{C}$ (where \mathbb{C} and \mathbb{Z} denote the set of complex numbers and the set of integers) is given by

$$\tilde{Y}_k = H_k x_k + Z_k, \quad k \in \mathbb{Z}. \quad (1)$$

Here $\{Z_k, k \in \mathbb{Z}\}$ and $\{H_k, k \in \mathbb{Z}\}$ are independent sequences of independent and identically distributed (i.i.d.), zero-mean, circularly-symmetric, complex Gaussian random variables, the former with unit variance and the latter with variance σ^2 . We say that the channel is *coherent* if the receiver is cognizant of the realization of $\{H_k, k \in \mathbb{Z}\}$ and that it is *noncoherent* if the receiver is cognizant only of the statistics of $\{H_k, k \in \mathbb{Z}\}$.

The receiver does not have access to the channel outputs $\{\tilde{Y}_k, k \in \mathbb{Z}\}$ but only to a quantized version thereof. Specifically, the complex channel output \tilde{Y}_k is fed to a one-bit

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quantizer which produces $Y_k = 1$ if \tilde{Y}_k is in the quantization region \mathcal{D} and $Y_k = 0$ otherwise, for some Borel set $\mathcal{D} \subset \mathbb{C}$. In the coherent case, \mathcal{D} may depend on the fading $\{H_k, k \in \mathbb{Z}\}$.

We assume that the average power of the channel inputs is limited by P . The capacity of the above channel is [8], [9]

$$C(P) = \sup I(X; Y|H), \quad \text{coherent case} \quad (2)$$

$$C(P) = \sup I(X; Y), \quad \text{noncoherent case} \quad (3)$$

where the suprema on the right-hand side (RHS) of (2) and (3) are over all distributions on X satisfying $\mathbb{E}[|X|^2] \leq P$ and over all quantization regions \mathcal{D} . (Since the above channel is memoryless, we omit the time indices.)

The capacity per unit-energy is given by [5, Th. 2]

$$\dot{C}(0) = \sup_{P>0} \frac{C(P)}{P}. \quad (4)$$

It can be shown that

$$\dot{C}(0) = \lim_{P \downarrow 0} \frac{C(P)}{P}. \quad (5)$$

Thus, the capacity per unit-energy is equal to the slope at zero of the capacity-vs-power curve. It can be further shown that [5, Th. 3] (see also [6])

$$\dot{C}(0) = \sup_{\xi \neq 0, \mathcal{D}} \frac{D(P_{Y|H, X=\xi} \| P_{Y|H, X=0} | P_H)}{|\xi|^2} \quad (6)$$

in the coherent case and

$$\dot{C}(0) = \sup_{\xi \neq 0, \mathcal{D}} \frac{D(P_{Y|X=\xi} \| P_{Y|X=0})}{|\xi|^2} \quad (7)$$

in the noncoherent case. Here $D(\cdot \| \cdot)$ denotes relative entropy

$$D(P \| Q) = \begin{cases} \int \log\left(\frac{dP}{dQ}\right) dP, & \text{if } P \ll Q \\ \infty, & \text{otherwise} \end{cases}$$

(where $P \ll Q$ indicates that P is absolutely continuous with respect to Q); $D(\cdot \| \cdot | \cdot)$ denotes conditional relative entropy

$$\begin{aligned} D(P_{Y|H, X=\xi} \| P_{Y|H, X=0} | P_H) \\ = \int D(P_{Y|H=h, X=\xi} \| P_{Y|H=h, X=0}) dP_H(h); \end{aligned}$$

P_H denotes the distribution of the fading H ; $P_{Y|X=x}$ denotes the output distribution given that the input is x ; and $P_{Y|H=h, X=x}$ denotes the output distribution conditioned on $(H, X) = (h, x)$.

By the Data Processing Inequality [10, Th. 2.8.1], the capacity per unit-energy of the quantized channel is upper-bounded by that of the unquantized channel [7], [6]

$$\dot{C}(0) \leq \frac{1}{\sigma^2}. \quad (8)$$

We show that in the coherent case this upper bound holds with equality, while in the noncoherent case it is strict.

III. MAIN RESULT

We restrict ourselves to *radial* quantizers, for which

$$\mathcal{D} = \{\tilde{y} \in \mathbb{C} : |\tilde{y}| \geq T\}, \quad \text{for some } T > 0. \quad (9)$$

In the noncoherent case—as we show in Section V-B—such quantizers are optimal in the sense that they maximize the relative entropy on the RHS of (7) for every $\xi \neq 0$. In the coherent case such quantizers need not be optimal in the above sense. However, they suffice to achieve the capacity per unit-energy. And such quantizers have the practical advantage of not requiring knowledge of the phase of \tilde{y} .

Theorem 1: Consider the above channel model, and assume that the channel output is quantized using a one-bit quantizer.

1) In the *coherent* case,

$$\dot{C}(0) = \frac{1}{\sigma^2} \quad (10)$$

which can be achieved by some radial quantizer (9) with T depending on H and ξ .²

2) In the *noncoherent* case,

$$\dot{C}(0) < \frac{1}{\sigma^2} \quad (11)$$

with the inequality being strict.

Proof: See Section V. ■

IV. QUANTIZING THE REAL AND IMAGINARY PART

Instead of quantizing \tilde{Y} using a one-bit quantizer, often the real and imaginary parts of \tilde{Y} are quantized separately using a one-bit quantizer for each; see, e.g., [11]–[14], [6]. Thus, the first quantizer produces $Y_{R,k} = 1$ if $\text{Re}(\tilde{Y}_k) \in \mathcal{D}_R$ and $Y_{R,k} = 0$ otherwise, and the second quantizer produces $Y_{I,k} = 1$ if $\text{Im}(\tilde{Y}_k) \in \mathcal{D}_I$ and $Y_{I,k} = 0$ otherwise, for some Borel sets $\mathcal{D}_R, \mathcal{D}_I \subset \mathbb{R}$. (Here \mathbb{R} denotes the set of real numbers, $\text{Re}(\cdot)$ denotes the real part, and $\text{Im}(\cdot)$ denotes the imaginary part.) In the coherent case, \mathcal{D}_R and \mathcal{D}_I may depend on the fading $\{H_k, k \in \mathbb{Z}\}$.

The capacity per unit-energy of this channel is given by (6) or (7), but with Y replaced by (Y_R, Y_I) , and with $\mathcal{D} \subset \mathbb{C}$ replaced by $(\mathcal{D}_R, \mathcal{D}_I) \subset \mathbb{R} \times \mathbb{R}$.

For symmetric quantizers, i.e., for

$$\mathcal{D}_R = \mathcal{D}_I = \{u \in \mathbb{R} : u \geq 0\} \quad (12)$$

it follows from [11] and [15, Th. 2] that, in the coherent case,

$$\dot{C}_{\text{sym}}(0) = \frac{2}{\pi\sigma^2}. \quad (13)$$

In the noncoherent case, symmetric quantizers result in zero capacity and hence, by (4), in zero capacity per unit-energy. Indeed, for (12)

$$\Pr(Y_R = 1 | X = x) = \Pr(Y_I = 1 | X = x) = \frac{1}{2}, \quad x \in \mathbb{C}.$$

Since, conditioned on X , the random variables Y_R and Y_I are independent, this implies that the capacity is zero. Thus, quantizing the real and imaginary parts of the Rayleigh-fading

²Here and throughout this paper, ξ refers to the parameter in (6) or (7).

channel using symmetric one-bit quantizers reduces the capacity per unit-energy by a factor of $2/\pi$ in the coherent case, and it reduces it to zero in the noncoherent case. In the following, we show that if we allow for *asymmetric* quantizers, then we can fully recover the loss of $2/\pi$ incurred on the coherent Rayleigh-fading channel. For the noncoherent channel, we show that asymmetric quantizers achieve a positive capacity per unit-energy, albeit strictly smaller than $1/\sigma^2$.

Theorem 2: Consider the above channel model, and assume that the real and imaginary parts of \tilde{Y} are quantized separately using a one-bit quantizer for each.

1) In the *coherent case*,

$$\dot{C}(0) = \frac{1}{\sigma^2} \quad (14)$$

which can be achieved by some quantization regions

$$\mathcal{D}_R^* = \{u \in \mathbb{R} : u \geq T_R\} \quad (15)$$

$$\mathcal{D}_I^* = \{u \in \mathbb{R} : u \geq T_I\} \quad (16)$$

where T_R and T_I depend on $\text{Re}(H\xi)$ and $\text{Im}(H\xi)$, respectively.

2) In the *noncoherent case*,

$$\frac{2Q(1)}{\sigma^2} \leq \dot{C}(0) < \frac{1}{\sigma^2} \quad (17)$$

with the upper bound being strict. Here $Q(\cdot)$ denotes the Gaussian Q -function [16, Eq. (1.3)]. The lower bound can be achieved by the quantization regions (15) and (16) with $T_R = T_I = (|\xi|^2 + \sigma^2)/2$.

Proof: Omitted. ■

V. PROOF OF THEOREM 1

A. Part 1)

We show that a radial quantizer (9) achieves the rate per unit-energy $1/\sigma^2$. Together with (8), this proves Theorem 1.

To this end, we first note that, conditioned on $(H, X) = (h, x)$, the squared magnitude of $\frac{2}{\sigma^2}\tilde{Y}$ is a noncentral chi-square distribution with degree 2 and noncentrality parameter $\frac{2}{\sigma^2}|h|^2|x|^2$ [16, p. 8]. Consequently, a radial quantizer yields [16, Sec. 2-E]

$$\Pr(Y = 1 \mid H = h, X = x) = Q_1\left(\sqrt{\frac{2}{\sigma^2}}|h||x|, \sqrt{\frac{2}{\sigma^2}}T\right)$$

where $Q_1(\cdot, \cdot)$ denotes the first-order Marcum Q -function [16, Eq. (2.20)]. Furthermore, for $x = 0$ this becomes

$$\Pr(Y = 1 \mid H = h, X = 0) = e^{-\frac{T^2}{\sigma^2}}.$$

We thus obtain

$$\begin{aligned} D(P_{Y|H,X=\xi} \parallel P_{Y|H,X=0} \mid P_H) &= \mathbb{E} \left[Q_1\left(\sqrt{\frac{2}{\sigma^2}}|H||\xi|, \sqrt{\frac{2}{\sigma^2}}T\right) \log \frac{1}{e^{-\frac{T^2}{\sigma^2}}} \right] \\ &+ \mathbb{E} \left[\left\{ 1 - Q_1\left(\sqrt{\frac{2}{\sigma^2}}|H||\xi|, \sqrt{\frac{2}{\sigma^2}}T\right) \right\} \log \frac{1}{1 - e^{-\frac{T^2}{\sigma^2}}} \right] \\ &- \mathbb{E} \left[H_b\left(Q_1\left(\sqrt{\frac{2}{\sigma^2}}|H||\xi|, \sqrt{\frac{2}{\sigma^2}}T\right)\right) \right] \\ &\geq \mathbb{E} \left[Q_1\left(\sqrt{\frac{2}{\sigma^2}}|H||\xi|, \sqrt{\frac{2}{\sigma^2}}T\right) \frac{T^2}{\sigma^2} \right] - \log 2 \end{aligned} \quad (18)$$

where $H_b(\cdot)$ denotes the binary entropy function, i.e.,

$$H_b(p) \triangleq \begin{cases} p \log \frac{1}{p} + (1-p) \log \frac{1}{1-p}, & \text{for } 0 < p < 1 \\ 0, & \text{for } p = 0 \text{ or } p = 1. \end{cases}$$

Here the inequality follows because the second term in the first step is nonnegative, and because the binary entropy function is upper-bounded by $\log 2$.

We choose $T = \mu|h||\xi|$ for some fixed $\mu \in (0, 1)$ and lower-bound the RHS of (18) using the general lower bound on the first-order Marcum Q -function [16, Sec. C-2, Eq. (C.24)]

$$Q_1(\alpha, \beta) \geq 1 - \frac{1}{2} \left[e^{-\frac{(\alpha-\beta)^2}{2}} - e^{-\frac{(\alpha+\beta)^2}{2}} \right], \quad \alpha > \beta \geq 0.$$

We thus obtain for the first term on the RHS of (18)

$$\begin{aligned} &\mathbb{E} \left[Q_1\left(\sqrt{\frac{2}{\sigma^2}}|H||\xi|, \sqrt{\frac{2}{\sigma^2}}\mu|H||\xi|\right) \frac{\mu^2|H|^2|\xi|^2}{\sigma^2} \right] \\ &\geq \frac{\mu^2 \mathbb{E}[|H|^2] |\xi|^2}{\sigma^2} \\ &- \frac{1}{2} \mathbb{E} \left[\exp\left(-\frac{|H|^2|\xi|^2}{\sigma^2}(1-\mu)^2\right) \frac{\mu^2|H|^2|\xi|^2}{\sigma^2} \right] \\ &+ \frac{1}{2} \mathbb{E} \left[\exp\left(-\frac{|H|^2|\xi|^2}{\sigma^2}(1+\mu)^2\right) \frac{\mu^2|H|^2|\xi|^2}{\sigma^2} \right] \\ &\geq \frac{\mu^2 \mathbb{E}[|H|^2] |\xi|^2}{\sigma^2} - \frac{\mu^2}{2e(1-\mu)^2} \end{aligned} \quad (19)$$

where the last step follows because $0 \leq xe^{-\alpha x} \leq 1/(e\alpha)$ for every $x \geq 0$ and $\alpha > 0$.

Combining (19) with (18), and computing its ratio to $|\xi|^2$ in the limit as $|\xi|^2$ tends to infinity, yields

$$\dot{C}(0) \geq \frac{\mu^2 \mathbb{E}[|H|^2]}{\sigma^2} = \frac{\mu^2}{\sigma^2}. \quad (20)$$

Theorem 1 follows then by letting μ tend to one.

B. Part 2)

We first note that, by the Data Processing Inequality for relative entropy [10, Sec. 2.9], the relative entropy on the RHS of (7) is upper-bounded by the relative entropy corresponding to the unquantized channel, i.e., [6, Eq. (64)]

$$\frac{D(P_{Y|X=\xi} \parallel P_{Y|X=0})}{|\xi|^2} \leq \frac{1}{\sigma^2} - \frac{\log\left(1 + \frac{|\xi|^2}{\sigma^2}\right)}{|\xi|^2}. \quad (21)$$

Consequently, the capacity per unit-cost (7) is strictly smaller than $1/\sigma^2$ unless $|\xi|$ tends to infinity. It thus remains to show that

$$\overline{\lim}_{|\xi| \rightarrow \infty} \sup_{\mathcal{D}} \frac{D(P_{Y|X=\xi} \| P_{Y|X=0})}{|\xi|^2} < \frac{1}{\sigma^2}. \quad (22)$$

To this end, we first note that, for every $\xi \neq 0$, the supremum in (22) over all quantization regions \mathcal{D} can be replaced with the supremum over all radial quantizers (9). Indeed, for every quantization region satisfying

$$\Pr(Y = 1 | X = \xi) = \beta, \quad 0 < \beta < 1$$

the relative entropy

$$\begin{aligned} D(P_{Y|X=\xi} \| P_{Y|X=0}) &= \beta \log \frac{1}{\Pr(Y = 1 | X = 0)} \\ &+ (1 - \beta) \log \frac{1}{1 - \Pr(Y = 1 | X = 0)} - H_b(\beta) \end{aligned} \quad (23)$$

is a convex function of $\Pr(Y = 1 | X = 0)$. Thus, for every $0 < \beta < 1$, the RHS of (23) is maximized for the quantization region that minimizes (or maximizes) $\Pr(Y = 1 | X = 0)$ while holding $\Pr(Y = 1 | X = \xi) = \beta$ fixed. By the Neyman-Pearson lemma [17], such a quantization region has the form

$$\mathcal{D}_* = \left\{ \tilde{y} \in \mathbb{C} : \frac{f(\tilde{y}|0)}{f(\tilde{y}|\xi)} \leq \Lambda \right\}, \quad \Lambda > 0 \quad (24)$$

(or the complement thereof), where $f(\tilde{y}|x)$ denotes the conditional density of \tilde{Y} , conditioned on $X = x$, and where Λ is such that $\Pr(\tilde{Y} \in \mathcal{D}_* | X = \xi) = \beta$. (Note that for every $0 < \beta < 1$ there exists such a Λ since, for the above channel model (1), $\Pr(\tilde{Y} \in \mathcal{D}_* | X = \xi)$ is a continuous, strictly increasing function of $\Lambda > 0$.) The likelihood ratio on the RHS of (24) is readily evaluated as

$$\frac{f(\tilde{y}|0)}{f(\tilde{y}|\xi)} = \left(1 + \frac{|\xi|^2}{\sigma^2} \right) e^{-\frac{|\tilde{y}|^2}{\sigma^2} \frac{|\xi|^2}{\sigma^2 + |\xi|^2}}, \quad \tilde{y} \in \mathbb{C}.$$

Consequently, \mathcal{D}_* is the same as (9) with

$$T = \sigma \sqrt{\left(1 + \frac{\sigma^2}{|\xi|^2} \right) \log \left(\frac{1 + \frac{|\xi|^2}{\sigma^2}}{\Lambda} \right)}.$$

We thus obtain that, for every $\xi \neq 0$, the relative entropy $D(P_{Y|X=\xi} \| P_{Y|X=0})$ is maximized by a radial quantizer (9).

For such a quantizer, we have

$$\Pr(Y = 1 | X = x) = \exp\left(-\frac{T^2}{|x|^2 + \sigma^2}\right)$$

which yields

$$\begin{aligned} D(P_{Y|X=\xi} \| P_{Y|X=0}) &= e^{-\frac{T^2}{|\xi|^2 + \sigma^2}} \log \frac{1}{e^{-\frac{T^2}{\sigma^2}}} \\ &+ \left[1 - e^{-\frac{T^2}{|\xi|^2 + \sigma^2}} \right] \log \frac{1}{1 - e^{-\frac{T^2}{\sigma^2}}} - H_b\left(e^{-\frac{T^2}{|\xi|^2 + \sigma^2}}\right) \\ &\leq \frac{T^2}{\sigma^2} e^{-\frac{T^2}{|\xi|^2 + \sigma^2}} - \left[1 - e^{-\frac{T^2}{\sigma^2}} \right] \log\left(1 - e^{-\frac{T^2}{\sigma^2}}\right) \\ &\leq \frac{T^2}{\sigma^2} e^{-\frac{T^2}{|\xi|^2 + \sigma^2}} + \frac{1}{e} \end{aligned} \quad (25)$$

where the second step follows because $H_b(\cdot) \geq 0$ and $\exp(-T^2/(|\xi|^2 + \sigma^2)) \geq \exp(-T^2/\sigma^2)$; and the third step follows because $-x \log x \leq \frac{1}{e}$, $0 < x < 1$.

The first term on the RHS of (25) is maximized for $T^2 = |\xi|^2 + \sigma^2$. The RHS of (25) is thus upper-bounded by

$$D(P_{Y|X=\xi} \| P_{Y|X=0}) \leq \frac{|\xi|^2}{e\sigma^2} + \frac{2}{e}. \quad (26)$$

Dividing the RHS of (26) by $|\xi|^2$, and computing the limit as $|\xi|$ tends to infinity, yields

$$\overline{\lim}_{|\xi| \rightarrow \infty} \sup_{\mathcal{D}} \frac{D(P_{Y|X=\xi} \| P_{Y|X=0})}{|\xi|^2} \leq \frac{1}{e\sigma^2} < \frac{1}{\sigma^2}. \quad (27)$$

This proves Theorem 2.

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