CONTINENTAL SURFACE EVOLUTION IN RESPONSE TO DEEP LITHOSPHERIC PROCESSES

ABHANDLUNG
zur Erlangung des Titels

DOKTOR DER WISSENSCHAFTEN

der
ETH ZÜRICH

vorgelegt von
THIBAULT DURETZ
Dipl.-Geowissenschaft., Université Louis Pasteur at Strasbourg
geboren am 9. Mai 1985
aus Frankreich

Angenommen auf Antrag von

Prof. Dr. Paul J. Tackley ETH Zürich Referent
Prof. Dr. Taras V. Gerya ETH Zürich Korreferent
Prof. Dr. Wim Spakman University of Utrecht, Utrecht Korreferent
Dr. Jeroen van Hunen University of Durham, Durham Korreferent

2011
In dedication to my parents
## Contents

1 Introduction

1.1 Plate tectonics and convergent margins ........................................ 1
1.2 Modelling of subduction processes ................................................. 6
1.3 The slab detachment model .......................................................... 7

1.3.1 Insight from seismology .......................................................... 7
1.3.2 Conceptual models ................................................................. 9
1.3.3 Implications of the slab detachment model .................................. 11
1.3.4 Previous studies of slab detachment ........................................... 13
1.4 Thesis structure ........................................................................... 16

2 Discretization errors and free surface stabilization in the finite difference
and marker-in-cell method for applied geodynamics: A numerical study 25

2.1 Introduction ................................................................................. 27

2.1.1 Background ............................................................................ 27
2.1.2 Present work ........................................................................... 29

2.2 Physical problem and numerical method ......................................... 30

2.2.1 Governing equations ............................................................... 30
2.2.2 Numerical method ................................................................... 32

2.3 Discretization errors and convergence ............................................ 41

2.3.1 Errors in approximate solutions of PDE’s ................................ 41
4  Dynamics of slab detachment

4.1  Introduction ................................................................. 113

4.2  Conceptual models of slab detachment .............................. 114

4.3  Insight from mechanical and thermo-mechanical modeling ...... 116

  4.3.1  Methodology ........................................................... 116

  4.3.2  The 2D kinematics of slab detachment .......................... 117

  4.3.3  The quasi pure shear detachments: necking .................... 119

  4.3.4  The contribution of pure and simple shear: shearing and necking 120

4.4  Comparing 2D simulations and 1D analytics ....................... 122

4.5  Impact of shear heating .................................................. 126

  4.5.1  1D solution for viscous slab necking with shear heating ... 129

  4.5.2  Application to slab detachment ................................... 131

4.6  Discussion ................................................................. 132

  4.6.1  Differences between 1D analytical and 2D numerical models 132

  4.6.2  Sensitivity of the results .......................................... 133

  4.6.3  Comparison with previous studies ................................ 134

  4.6.4  Timing of slab detachment, duration and consequences ...... 136

  4.6.5  Three dimensional slab detachment dynamics .................. 137

4.7  Conclusions .............................................................. 137

5  Thermomechanical modelling of slab eduction ..................... 145

5.1  Introduction .................................................................. 146

  5.1.1  Background ............................................................. 146

  5.1.2  The term “Eduction” ................................................ 147

  5.1.3  The term “Eduction” ................................................ 147

  5.1.4  Present work .......................................................... 148

5.2  Numerical modelling ...................................................... 148

  5.2.1  Setup .................................................................. 148

  5.2.2  Potential causes for eduction ..................................... 149

5.3  The plate eduction model ................................................. 151

  5.3.1  Reference model ..................................................... 151
5.4 The dynamics of plate eduction ........................................ 158
5.5 A Corner flow/Torque balance approach .......................... 159
5.6 Simplified 2D setup ...................................................... 161
   5.6.1 Parameters controlling eduction ................................. 162
5.7 Discussion ............................................................... 165
   5.7.1 Low peak temperature in the exhumed crust .................. 165
   5.7.2 Limitations of the density model ............................... 170
   5.7.3 Comparison with the models of coherent nappe exhumation . 170
   5.7.4 Dimensionality and plate motions ............................. 171
5.8 Conclusions ............................................................ 172
5.9 Numerical code description .......................................... 173

6 Crust rheology, slab detachment and topography ..................... 185
   6.1 Introduction .......................................................... 187
   6.2 Modelling approach ............................................... 188
      6.2.1 Methodology .................................................. 188
      6.2.2 Setup .......................................................... 190
   6.3 Strength of the lithospheric model ................................ 192
      6.3.1 Rheological model ............................................ 192
      6.3.2 Rheological profiles ......................................... 193
   6.4 The influence of crustal rheology from 2D subduction-collision experiments 195
   6.5 Weak crust end-member: Slab retreat & delamination ............. 195
   6.6 Intermediate crustal rheology: delamination & slab detachment .... 201
   6.7 Strong crust end-member: slab detachment & eduction ............ 203
   6.8 Discussion ............................................................ 206
      6.8.1 Mechanical coupling within the crust and through the Moho .... 206
      6.8.2 Occurrence of slab detachment ................................ 206
      6.8.3 Crustal slice exhumation ..................................... 208
      6.8.4 Exhumation rates, average surface uplift rates, and instantaneous uplift rates ........................................... 208
      6.8.5 The development of orogenic plateaus ....................... 208
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.8.6 Initial rates of convergence</td>
<td>209</td>
</tr>
<tr>
<td>6.8.7 Crustal rheology and continental subduction</td>
<td>209</td>
</tr>
<tr>
<td>6.8.8 Shape of the margin</td>
<td>210</td>
</tr>
<tr>
<td>6.8.9 Coupled and decoupled style of orogeny</td>
<td>210</td>
</tr>
<tr>
<td>6.8.10 Tectonic forces, model limitations and future perspectives</td>
<td>211</td>
</tr>
<tr>
<td>6.9 Conclusions</td>
<td>211</td>
</tr>
<tr>
<td>7 Conclusions and perspectives</td>
<td>221</td>
</tr>
<tr>
<td>7.1 Thesis summary</td>
<td>221</td>
</tr>
<tr>
<td>7.2 3D models and topography</td>
<td>222</td>
</tr>
<tr>
<td>7.2.1 Background</td>
<td>222</td>
</tr>
<tr>
<td>7.2.2 Model setup</td>
<td>224</td>
</tr>
<tr>
<td>7.2.3 Evolution of the model</td>
<td>226</td>
</tr>
<tr>
<td>7.2.4 Slab detachment in 3D</td>
<td>228</td>
</tr>
<tr>
<td>7.2.5 Comparison with 2D models</td>
<td>229</td>
</tr>
<tr>
<td>7.2.6 Further developments</td>
<td>230</td>
</tr>
</tbody>
</table>
Abstract

This thesis is a compilation of numerical modelling studies related to the field of geodynamics and subduction dynamics. The different studies which comprise this thesis are essentially focussed on the topic of slab detachment in collision zones. The multiple themes that were addressed during this work concern the evolution of subduction-collision zones subjected to slab detachment, both in terms of geodynamic and topographic evolution, the quantification of slab detachment depth and duration, the deformation modes and mechanisms that lead to slab detachment, as well as the dynamic consequences of this geodynamic process such as the variations of surface uplift, exhumation of high-pressure rocks and orogenic extension.

The results presented in this thesis were obtained using a two-dimensional (2D) thermo-mechanical code based on the Finite-difference/Marker-In-Cell (FD-MIC) method. In the first chapter, a study of the numerical accuracy of the FD-MIC methodology is presented. The order of accuracy of the FD-MIC scheme was investigated by means of a grid convergence study. In different test cases, large and sharp variations of viscosity (being a typical feature in geodynamic models) were employed. Simulations that combine mantle flow and topographic evolution require a free surface or an approximate free surface. However, this configuration may result in unphysical oscillations. A stabilisation algorithm that suppresses the instability was therefore derived and tested. The subsequent chapters applied the FD-MIC scheme to study the various aspects of slab detachment.

The evolution of subduction-collision zones subjected to spontaneous slab detachment was investigated by means of two-dimensional (2D) thermo-mechanical modelling. The initial age of the oceanic slab was shown to control the depth of slab detachment which can vary between 40 and 400 km. Since slab detachment can occur over a wide depth range, variations in the tectonic styles of collision can be expected. The results showed that the topographic evolution of these collision zones witness the deep dynamics of subduction. Moreover, a strong feedback between the depth of slab detachment and the rates of surface uplift was highlighted.

In order to better understand the modes of deformation that lead to detachment, a compilation of slab detachment simulations was analysed. The results showed that slab deformation is accommodated by both pure and simple viscous shear deformation. Additionally, our results indicate that the duration of slab detachment events is fast with regard to the geological timescale (< 4 Ma). A comparison between 2D thermo-mechanical experi-
ments and a one-dimensional (1D) necking analytical solution highlighted the importance of ductile necking during slab detachment. The influence of shear heating was investigated in 1D which led to the conclusion that shear heating can accelerate (up to 5 times) the detachment process.

The slab detachment concept implies a significant decrease of the slab pull force, which triggers a dynamic lithospheric response. Part of this response is the potential reactivation of the subduction zone into a normal sense shear zone. This extensional regime, termed eduction, was numerically studied and its implications were described into detail. Moreover, scaling laws predicting plate velocities after slab detachment were derived. The eduction mechanism can partially explain the coherent exhumation of deeply buried crust. The potential applicability of this model to the Western Gneiss Region (Norway) is discussed.

In order to tackle the importance of the crustal rheological structure on slab detachment, a numerical study was carried out. The results showed that collisions involving a strong crust provide a strong coupling at the Moho. This mechanical coupling promotes deep continental subduction and an eventual slab detachment. Conversely, weak coupling at the Moho is the consequence of a weak crust. Such configuration led to delamination and slab retreat, inhibiting slab detachment. Introducing a layered crust (upper/lower crust) resulted in a collision zones that were sequentially affected by delamination and slab detachment. Each of these modes of continental collision were characterised by their detailed topographic evolution and uplift history.

Lastly, a description of some recent three-dimensional (3D) numerical simulations is provided. Although 3D dynamics involves additional complexities in the trench parallel direction, the overall geodynamic and topographic evolution of these preliminary models is consistent with the 2D simulations of slab detachment presented in this thesis.
Résumé

Ce mémoire synthétise une succession d’études numériques en géodynamique aillant pour but commun une étude détaillée des processus de subduction. Les différents chapitres qui constituent cette thèse portent essentiellement sur le sujet des détachements de panneaux plongeants (DPP) dans les zones de collision. Les multiples thématiques abordées dans cette étude concernent l’évolution géodynamique et topographique des zones de collision sujettes aux DPP, la quantification de la durée et la profondeurs auxquelles surviennent les DDP, ainsi que les conséquences dynamiques des DPP telles que les variations des taux d’élévation de la surface, l’exhumation des roches de haute pression, et l’extension orogénique.

La plupart des résultats présentés dans cette thèse ont été obtenus à l’aide de la méthode des Différences Finies/Marqueurs-En-Cellulle (DF-MEC). Le premier chapitre de cette thèse présente les résultats de l’étude numérique du schéma DF-MEC. L’erreur de troncation de la méthode DF-MEC a été calculée à différentes résolutions de maille permettant ainsi d’évaluer la précision de ce schéma numérique. Les différents cas considérés lors de cette étude, impliquant des variations de viscosité de plusieurs ordres de grandeur, sont typiques des cas rencontrés en modélisation géodynamique. Les simulations numériques couplant les écoulements du manteau avec l’évolution de la topographie nécessitent une surface libre (ou approximation). Néanmoins, ce type de calcul peut générer des oscillations non physiques. Pour parer à ce phénomène, un algorithme de stabilisation a été développé et testé. Les chapitres suivants présentent des applications du schéma DF-MEC et sont axés sur l’étude des différents aspects liés aux DPP.

L’évolution des zones de subduction-collision a été étudiée à l’aide de simulations thermomécaniques à deux dimensions (2D). Les profondeurs auxquelles les DPP peuvent survenir varient entre 40 et 400 km. Cependant, ces profondeurs sont fortement influencées par l’âge thermique initial du panneau. Ces différentes profondeurs peuvent donc résulter en une variété de styles tectoniques collisionels mis en évidence par la modélisation. L’analyse de l’évolution topographique de ces simulations a permis d’identifier la contribution des processus profonds liés à la subduction. Un fort couplage entre la profondeur des DPP et les taux d’élévation de la surface a été mis en évidence.

Une compilation de résultats numériques de DPP a été réalisée afin d’étudier le comportement mécanique des DPP. L’analyse des déformations du panneau montre que les DPP sont la conséquence d’une combinaison de cisaillement pur et simple dans le champ rhéologique
visqueux. Subsidiairement, les résultats obtenus indiquent que la durée des DPP est relativement rapide à l'échelle des processus géodynamiques (< 4 Ma). La comparaison entre les résultats numériques 2D et une solution analytique d'amincissement de plaque a permis de mettre en évidence l'importance du boudinage durant les DPP. En complément, une analyse 1D révèle que l'influence du chauffage par cisaillement permet d'accélérer les DPP, au maximum, par un facteur 5.

La conséquence directe des DPP est la diminution, au moins partielle, de la force d’entraînement du panneau plongeant. Le changement du bilan des forces dans la zone de collision peut alors générer une réponse dynamique de la lithosphère. La réactivation potentielle de la faille de subduction dans un régime extensif fait partie de ces conséquences. Les implications de ce contexte tectonique transitoire, appelé éduction, ont été étudiées au moyen de simulations numérique. Subsidiairement, une loi d'échelle permettant de prédire la vitesse de la plaque subdue après un DPP a été établie. L’application de ces simulations numériques à l’évolution tectonique de la Régions des Gneiss de l’Ouest (Norvège) y est discutée.

La structure rhéologique de la croûte influence fortement les styles de collision tectonique, une étude numérique a été menée afin d’évaluer son influence sur les DPP. Les croûtes résistantes permettent un couplage mécanique important au Moho, ce couplage promeut l’enfouissement de la croûte continentale dans les zones de subduction ainsi que les DPP. À l’inverse, les croûtes molles favorisent la délamination au Moho ainsi que le recul du panneau plongeant, empêchant le développement des DPP. L’introduction d’une croûte lithée (croûte supérieure/inferieure) permet des développements orogéniques alliant délamination et DPP.

En guise de conclusion, les résultats de simulations tridimensionnelles (3D) de DPP sont discutés. Bien que ces simulations impliquent la contributions de processus tectoniques dans l’axe de la fosse, leur évolution géodynamique et topographique sont en bonne concordance avec les résultats 2D présentés dans cette thèse.
1.1 Plate tectonics and convergent margins

In the framework of Plate Tectonics theory, the Earth’s surface is divided in a number of tectonic plates (up to 52 (Bird, 2003)). The tectonic plates exhibit rigid body like motion (translation and rotation) that can be tracked throughout Earth’s history (Torsvik et al., 2008a, 2010), these motions are the surface expression of the Earth’s internal dynamics (Torsvik et al. 2008b). The lithosphere of these plates can be identified as being either of an oceanic or a continental nature (Fig. 1.2). Continental lithosphere is generally characterised by a thick crust (<10 km) of an average felsic composition (Rudnick and Gao, 2003), and is mainly responsible for positive topography, continental lithosphere represents approximately 0.4% of the Earth lithosphere. The oceanic lithosphere is composed of a thin crust of an average mafic nature. The oceanic lithosphere represents 0.6% of the Earth’s lithosphere and its average topography is negative, correlating with the location and distribution of the oceans.

Fig. 1.1 shows plate reconstructions and velocities through the last 150 Ma and highlights the differential displacement between the tectonic plates. In order to accommodate this displacement, the tectonic plates are separated by plate boundaries. The interface between plates that show large differential motion are known as active margins, these regions are subject to interplate deformation. Some stable plate boundaries are not affected by deformation, they are termed passive margins. These margins are usually located at the ocean-continent transition (OCT). The three idealised types of active margins are defined according to type of deformation they exhibit.

Transcurrent margins (Fig. 1.3a) separates the plates that are subject to coaxial motion
CHAPTER 1. INTRODUCTION

Figure 1.1: Rigid motion of tectonic plates at the surface of the Earth. The two subfigures show paleographic reconstructions at 10 a) and 150 b) Ma before present. The red line symbolises the plate limits that are used in the reconstruction and the arrows represent the direction and magnitude of the plate motions. This figure was modified after Torsvik et al. (2010).

with regard to their limit. At these boundaries, the plates slide past each other and generate strike-slip deformation (Alpine fault of New Zealand, Dead Sea fault, or San Andreas fault). The divergent plate margins (Fig. 1.3b) are commonly known as oceanic spreading ridges. At these boundaries, the displacement of the plates is perpendicular to the axis of spreading and the deformation is extensional. In spreading ridges, the extension is accommodated by mantle upwelling and partial melting that gives rise to the generation of oceanic lithosphere. A major tectonic force results from the bathymetry produced by
the generation of oceanic floor, this force is coined ridge push. Extensional deformation can also take place within the continents, yielding to rifting (Rhine graben, East African Rift), further extension of the lithosphere can lead to the generation of plate boundaries and the formation of a new ocean (drifting). The convergent margins are characterised by interplate thrusting (Fig. 1.3c) and are generally referred to as subduction zones or subduction-collision zones. When convergent motion takes place between two plates of oceanic nature (Fig. 1.4a), the plate boundary is termed ocean-ocean subduction (Marianas, Izu-Bonin, or Tonga). In regions such as the Cascadian or the Andean margin (Fig. 1.4b), oceanic lithosphere is underthrust beneath continental lithosphere giving rise to an ocean-continent subduction zone. In both of these subduction settings, the sinking of oceanic crust within the Earth’s mantle causes the hydration and the melting of the mantle. This has major consequences for magmatism and the generation of continental/arc crust within the overriding plate. Another important feature of subduction zones is the build up of tectonic force resulting from the burial of the negatively buoyant oceanic lithosphere. This force is termed slab pull and its magnitude is generally estimated as being an order of magnitude larger that the ridge push force exerted by the oceanic ridges (Turcotte and Schubert, 1982).

A convergent tectonic setting can also lead to the collision between continental plates. It is usually considered that collision takes place following to the subduction (closure) of an ocean, for this reason they are also described as subduction-collision zones. Due its more
felsic nature, the continental crust tends to be more buoyant than the oceanic crust, it is therefore resistant to burial in subduction zone and results in widespread deformation at the vicinity of the plate interface. At present times, the Himalaya (Fig. 1.4c) result from the ongoing convergence between the Indian plate and the Eurasian plate and represent the largest active continental collision zone. Older collision zones such as the Variscan and Caledonian belts can be found at the surface of the Earth, these fossil orogens reflect the past dynamics of continental motions and accretions through geological times. Both ocean-continent and continental-collision are associated to the thickening of the crust triggered either by underplating or by internal deformation. This particularity leads these types of margins to be associated to topographic highs and plateaus. In order to investigates what controls the dynamics of convergent margins, geologists and geophysicists have resorted to developing models. Models can be conceptual or physical. Conceptual models (such as in Fig. 1.4) aim at compiling the different types of geological or geophysical observation concerning a specific region into a plausible geodynamic framework. Physical models are employed to quantify and evaluate what are the first order mechanisms that control the evolution of convergent margins. Conceptual models are a great source of inspiration for any kind of physical modelling study, they usually provide the primary hypotheses to be
Figure 1.4: Examples of the different types of convergent margins. a) Ocean-ocean subduction in the Mariana region after Maruyama et al. (2009), BMW, MMF, MF respectively stand for Big Mantle Wedge, Metamorphic-Metasomatic Factory, and Metasomatic Factory. b) Ocean-Continent subduction in the Andes, interpretative cross section through Bolivia after Jaillard et al. (2002). c) Simplified cross-section of the Himalayan Continent-Continent collision after Avouac (2007).

investigated by physical models. Reciprocally, the information obtained by physical mod-
elling influences the design of conceptual models by improving the knowledge about how convergent margins evolve.

1.2 Modelling of subduction processes

Physical models of subduction processes can be subdivided into two main categories, laboratory models (analogue models) or computer simulations (numerical models). Analogue models allow for the investigation of first order physical mechanisms controlling geological processes (Schellart, 2002). Analogue models are carried out at laboratory length-time scales and they necessitate a careful choice of material. Each material involved in the experiments should exhibit rheological behaviour comparable to those of geomaterials (rocks) on a geological timescale (dynamic similarity). An advantage of laboratory models is their natural three-dimensionality. In contrast to analogue experiments, numerical simulations enable one to explore the dynamics of subduction zones in detail. The flexibility of numerical modeling allows simulations that take into account complex rheologies (viscoelasticity, non-Newtonian flow, plasticity), temperature-pressure dependent densities (equation of state, phase transitions), thermomechanical feedback (shear heating), or fluid related processes (melting, fluid transport).

During the last forty years (Gerya, 2011), numerical modelling of subduction processes have focussed on several major topics. The mechanisms that lead to the development of subduction initiation has led to the development of a number of hypotheses (Stern, 2004) and the systematic testing of these hypotheses is therefore an active research field (Matsumoto and Tomoda, 1983; Regenauer-Lieb et al., 2001; Nikolaeva et al., 2010). The generation and exhumation of high pressure (HP) and ultra-high pressure (UHP) rocks (Fig. 1.5a) and their relation to subduction dynamics remain unclear and are subject to a continued modelling investigation (Yamato et al., 2007; Li and Gerya, 2009; Warren et al., 2008). Recent studies have focussed on understanding coupling between slab dehydration (Fig. 1.5c,d) and mantle hydration (Arcay et al., 2005; Zhu et al., 2009; Richard and Bercovici, 2009) and consequently, as mantle hydration might triggers partial melting, much effort is currently underway in developing models that incorporate melt production (Fig. 1.5c,e) and crustal growth (Kimura et al., 2009; Gerya and Meilick, 2010). Several studies have been dedicated to the study of the interaction between the subduction dynamics and their overriding plates (Fig. 1.5f), such as understanding the conditions leading to back-arc deformation or topographic plateau development (Sobolev and Babeyko, 2005; Clark et al.,...
At larger length scales, the dynamics of slab penetration into the Earth’s mantle and the deformation that slab undergo at the phase boundaries was investigated in order to better interpret large scale seismic tomographic images (Christensen 1996; Cížková et al. 2007).

An important aspect of slab deformation is the possibility for a subducting lithosphere to localise deformation and yield under its own negative buoyancy (slab pull force). This mechanism is termed slab detachment and this geodynamic process can potentially affect all the subduction processes listed above. Since the study of slab detachment mechanisms and the associated lithospheric response is an important part of this Ph.D. thesis, a more detailed description of the slab detachment model and its implications will be elaborated upon the following section.

1.3 The slab detachment model

1.3.1 Insight from seismology

During the past decades, the mechanism of slab detachment (or slab breakoff) has gained popularity in both in both geological and geophysical communities. The idea that a slab could detach was born from geophysical observations and was first hypothesised in the late sixties (Isacks and Molnar 1969) to explain seismicity patterns observed within subducting slabs. The location of deep earthquake hypocenters is a powerful technique to track slabs through the mantle and the study of deep seismicity patterns has indicated the existence of non-continuous seismogenic zones (Fig. 1.6) that might be associated to gaps within the slabs (Chatelain et al. 1993; Chen and Brudzinski 2001; Sperner et al. 2001; Kundu and Gahalaut 2011). The slab detachment model has further gained popularity together with the development of seismic tomography. By imaging the perturbation of seismic velocity within the Earth’s mantle, seismic tomography can lead to the detection of portions of slab deep in the mantle (Wortel and Spakman 1992; Widiyantoro and van der Hilst 1996; van der Meer et al. 2010; Rogers et al. 2002; Levin et al. 2002; Schmandt and Humphreys 2011; Zor 2008). A further input from seismic tomography is the discovery, at a regional scale, of positive seismic velocity anomalies beneath collision zones. Such structures were subsequently interpreted as a detached, or detaching slab (Wortel and Spakman 2000; Lippitsch et al. 2003; Martin and Wenzel 2006; Replumaz et al. 2010).
1.3.2 Conceptual models

The model of slab detachment involves the detachment of a portion of slab in a convergent margin setting. Such a yielding of the lithosphere would cause its detached part to sink down into the Earth’s mantle and produce a major thermo-mechanical re-equilibration of the convergent margin. Apart from the models involving the subduction of pre-existing lithospheric structures (fracture zones, transform fault), the slowdown of subduction rate is often proposed to be the driving mechanism of slab detachment. Subduction slowdown is usually related to the subduction of continental crust, for this reason, slab detachment is strongly linked to the evolution of continental collision zones. The partial subduction of buoyant material combined with the negative buoyancy of a previously subducted oceanic lithosphere can result in large extensive stresses within the hanging slab (Fig. 1.7). These
circumstances might lead to progressive deformation of the slab and its subsequent detachment.

![Figure 1.7](image)

**Figure 1.7:** Sketch of a subduction-collision system undergoing the combination of both the positive buoyancy of the crust ($F_b$) and the negative buoyancy of the slab ($F_p$). C, ML, and A respectively stands for continental crust, mantle lithosphere and asthenosphere).

The detachment of a slab is often simplified in conceptual sketches as being the result of either a sharp fracture (e.g. (Nolet 2009)), a shear zone (e.g. Sacks and Secor (1990)), or thinning (e.g. Sacks and Secor (1990)). These schematic detachment models inherently involve contrasting physical modes of slab deformation: tensile brittle failure (Fig. 1.8c) extension along a simple shear zone (Fig. 1.8c), and pure shear necking (Fig. 1.8b). The tensile failure model conveys the idea that the slab behaves as a homogeneous fragile plate for the pressure and temperature conditions of subduction zones. This model implies geologically instantaneous suction of the asthenosphere within the gap opened following the rupture of the slab and is likely to explain the fast heat advection and melting events. The simple shear model (Sacks and Secor 1990) requires localized shear deformation without explicitly suggesting any rheological behaviour of the slab (either viscous, plastic or brittle). The slab necking (Sacks and Secor 1990) implies that the slab deformation is accommodated by pure shear viscous or plastic deformation. In this model, the astheno-
1.3.3 Implications of the slab detachment model

The detachment of a portion of a slab, resulting from the large extensional stresses within the downgoing plate, is likely to have a major influence on the evolution of convergent margins. Two main consequences of slab detachment can be distinguished: (1) a partial or
Figure 1.9: The multiple implications of the slab detachment model.  
a) Response leading to exhumation of high pressure rocks Andersen et al. (1991).  
c) Transition from flysch to molasse in the Alps Sinclair (1997).  
d) Entrainment of crustal slices by detached slab within the mantle Hildebrand and Bowring (1999).

complete loss of the slab pull force and (2) the inflow of hot asthenospheric mantle at the location of the detachment. The first consequence results in the rebalancing of the forces acting on the plate margin that may triggers a wide range of dynamical effects. This slab detachment model was consequently used in the explanation of tectonic processes such as high pressure and ultra-high pressure rock exhumation (Fig. 1.9a) Andersen et al. (1991; Babist et al. 2006; Xu et al. 2010), variations in surface uplift rates Rogers et al. (2002); Morley and Back (2008); Wilmsen et al. (2009) and in the sedimentary record (Fig. 1.9c) Sinclair (1997; Mugnier and Huyghe 2006), orogenic extension Zeck (1996), rapid changes
in plate motions (Austermann et al., 2011), or the reversal of the subduction dip (Regard et al., 2008). The second consequence is usually considered as an efficient mechanism to advect heat at lithospheric, to sub crustal levels (van de Zedde and Wortel, 2001), subsequently triggering partial melting in the mantle (Davies and von Blanckenburg, 1995; Ferrari, 2004; Altunkaynak and Can Genç, 2004) associated to plutonism and volcanism (Fig. 1.9b) (Keskin, 2003; Qin et al., 2008; Ferrari, 2004). It has also recently been suggested that the entrainment of continental material attached to detached slabs (Fig. 1.9d) may play a role in the long term processes of crustal recycling (Hildebrand and Bowring, 1999).

### 1.3.4 Previous studies of slab detachment

There has been many quantitative modelling studies focussed on slab detachment. These studies have been carried out by means of analytical, semi-analytical, analogue and numerical models (Davies and von Blanckenburg, 1995; Ton and Wortel, 1997; van de Zedde and Wortel, 2001; Buiter et al., 2002; Gerya et al., 2004; Cloetingh et al., 2004; Toussaint et al., 2004; Li and Liao, 2002; Andrews and Billen, 2009; Regard et al., 2008; Macera et al., 2008; Schmalholz, 2011). Most of these studies were focussed on describing the evolution of pre-subducted slab in the mantle with or without prescribing the detachment of the slab (Fig. 1.10).

Since slab detachment promotes asthenospheric mantle inflow into the detachment zone, it is important to evaluate at which depth slab detachment can take place. The results available in the literature propose that slabs can detach within a wide range range of depths. The models of van de Zedde and Wortel (2001) suggest that detachments can occur at shallow depths (≈35 km), whereas the results of Gerya et al. (2004) suggest that detachment depths can be greater than 100 km. On the other hand, Andrews and Billen (2009) predict different breakoff depth ranges in relation to activation of different deformation mechanisms. They introduce two categories of slab breakoff: a shallow breakoff mode (≈150 km) involving a weak slab and fast plastic yielding, and a deep (≈300 km) breakoff mode involving stronger slabs characterised by slow thermal yielding. In contrast with preceding studies, Baumann et al. (2009) used a different setup allowing spontaneous slab detachment after a period of oceanic subduction including the effects of phase transitions. In this study breakoff depths ranged between 400 and 600 km.
Figure 1.10: Numerical simulations and analogue experiments of slab detachment. a) Numerical study of slab detachment applied to the Carpathians [Cloetingh et al. (2004)]. b) Laboratory experiment of slab dip reversal resulting from slab breakoff [Regard et al. (2008)]. c) Evolution of thermal field after detachment [van de Zedde and Wortel (2001)]. d) Evolution of the viscosity field during detachment [Andrews and Billen (2009)].

Another important feature of the slab detachment is its duration. Since the velocity of asthenospheric mantle is a function of the detachment duration, it is important to quantify the possible duration of the detachment events. In general, the results indicated that slab detachment last less than 10 Ma [Andrews and Billen (2009), Gerya et al. (2004), even if some experiments have suggested time intervals that are larger than 15 Ma [Baumann et al. (2009)].
After the detachment of the slab and the loss of the slab pull force, a topographic rebound can be expected. Few studies have led to a quantitative estimation of the topographic response to slab detachment. Buiter et al. (2002) predicted topography uplift in the range of 2 to 6 km using an elastic model whereas Gerya et al. (2004) predicted lower uplift values (< 2 km) using a viscoplastic model.

Three dimensional aspects have been investigated via laboratory experiment (Regard et al., 2008) and the result suggested that the mantle flow associated to detachment can lead to reversal of the slab dip angle. Recent three-dimensional numerical simulations of van Hunen and Allen (2011) have investigated what is the amount of time necessary to initiate slab detachment after the onset of continental collision (10-25 My). Burkett and Billen (2011) studied slab detachment triggered by the subduction of a ridge. Both studies have highlighted the tendency for slabs to start detaching from the inside rather than from their edges.

The analytical study of Schmalholz (2011) has brought a new insights in the understanding of slab detachment mechanism. This study provides a one-dimensional analytical solution that reproduce the necking dynamic of a slab under its own weight. It further indicates that detachment duration are a function of the power-law parameters of the slab.

Although many studies have bee dedicated to the modelling of slab detachment and given the potential geological consequences of slab detachment, several questions are yet to be answered:

1. How can we reconcile the different estimations of detachment depths available in the literature?

2. Can we refine the timescale of the slab detachment duration?

3. What are the main deformation mechanism that are active in the deformation of the slab detachment?

4. Do slabs yield because of a slab rupture, distributed pure shear deformation or localised simple shear?

5. What is the magnitude of the topographic signal associated to slab detachment?

6. How does slab detachment depth affects the surface uplift rates?
7. Can the force re-equilibration related to slab detachment partially explain the exhumation of high-pressure rocks?

8. Can slab detachment produce major changes in the direction of plate motions?

9. How does slab detachment interact with other subduction processes, such as slab retreat?

10. Do slab always detach during continental collision?

1.4 Thesis structure

The aim of this thesis is to provide at least partial answers to the different questions listed above. The body of the thesis is composed of five chapters, for which we provide an outline below.

- Chapter 2 introduces the reader to the numerical modelling tool which is employed throughout the thesis. This chapter investigates the numerical accuracy of the Finite Difference - Marker-In-Cell (FD-MIC) and introduces a stabilisation algorithm which is important for simulations involving the evolution of topography with ongoing lithospheric deformation and mantle flow.

- Chapter 3 presents two dimensional results of slab detachment subsequent to the closure of an ocean. It focusses on identifying the rheological mechanisms that are important for the slab detachment and quantifying the surface response of slab detachment both in terms of surface uplift and topography.

- Chapter 4 describes the deformation kinematics of slab detachment and evaluates the amount of time necessary for slabs to detach. It provides a comparison between 2D thermo-mechanical models with a 1D analytical solution of necking under gravity and investigates the impact of shear heating on the detachment durations.

- Chapter 5 introduces the concept of eduction (normal sense reactivation of the subduction plane) as a result of slab detachment. This chapter discusses the conditions under which eduction occurs and the implications for the exhumation of high pressure rocks are discussed.
- Chapter [6] investigates the influence of the crustal rheology on the modes of continental collision and the occurrence, or not, of slab detachment. A detailed description of the topographic evolution of continental collisions, based on their geodynamic evolutions, is provided.

A final perspective chapter (Chapter [7]) provides a summary of the work achieved during the thesis and discusses a number of three-dimensional models of slab detachment and their topographic evolution.

Bibliography


CHAPTER 1. INTRODUCTION


URL http://www.physicalgeography.net/fundamentals/images/lithosphere.gif


Abstract

The Finite Difference/Marker-in-Cell (FD-MIC) method is a popular method in thermo-mechanical modeling in geodynamics. Although no systematic study has investigated the numerical properties of the method, numerous applications have shown its robustness and flexibility for the study of large viscous deformations. The model setups used in geodynamics often involve large smooth variations of viscosity (e.g. temperature dependent viscosity) as well large discontinuous variations in material properties (e.g. material interfaces). Establishing the numerical properties of the FD-MIC and showing that the scheme is convergent adds relevance to the applications studies that employ this method. In this study, we numerically investigate the discretization errors and order of accuracy of the velocity and pressure solution obtained from the FD-MIC scheme using two-dimensional analytic solutions. We show that, depending on which type of boundary condition is used, the FD-MIC scheme is a second order accurate in space as long as the viscosity field is constant or smooth (i.e. continuous).
With the introduction of a discontinuous viscosity field characterized by a viscosity jump \( (\eta^*) \) within the control volume, the scheme becomes first order accurate. We observed that the transition from second order to first order accuracy will occur with only a small increase in the viscosity contrast \( (\eta^* \approx 5) \). We have employed two methods for projecting the material properties from the Lagrangian markers onto the Eulerian nodes. The methods are based on the size of the interpolation volume (4-Cell, 1-Cell). The use of a more local interpolation scheme (1-Cell) decreases the absolute velocity and pressure discretization errors. We also introduce a stabilization algorithm that damps the potential oscillations that may arise from quasi free surface calculations in numerical codes that employ the strong form of the Stokes equations. This correction term is of particular interest for topographic modeling, since the surface of the Earth is generally represented by a free surface. Including the stabilization enables physically meaningful solutions to be obtained from our simulations, even in cases where the time step value exceeds the isostatic relaxation time. We show that including the stabilization algorithm in our FD stencil does not affect the convergence properties of our scheme. In order to verify our approach, we performed time-dependent simulations of free surface Rayleigh-Taylor instability.

\[ 1 \]

---

1 This chapter was co-authored by T. Duretz, D.A. May, T.V. Gerya and P.J. Tackley (Geochemistry Geophysics Geosystems, Vol. 12, Q07004 (2011)).
CHAPTER 2.  FD-MIC SCHEME DISCRETIZATION ERRORS

2.1   Introduction

2.1.1   Background

Unravelling the deformation history from the present day geological record is a challenging problem. To complement field data and traditional interpretation based geodynamic modeling approaches, mathematical models are often employed to develop our understanding of geological processes. The use of thermo-mechanical models which solve the equations of conservation of momentum, mass and energy has a long history in geodynamics. In these models one prescribes (i) a set of mathematically permissible boundary conditions (ii) the geometry of the model domain and (iii) the geometry and rheology (or lithology) of the rocks to be modeled. Using such an approach to simulate geologically realistic scenarios is complicated by several attributes which we discuss in more detail.

At a given length scale, rocks can be extremely heterogenous. Here, the heterogeneities may consist of differing lithology, or in simply the contrast of effective material properties (viscosity, shear modulus, etc). Furthermore, the contrast of effective material properties may be extremely large, and occur over a very small length scale. The geometry of the heterogeneities is also a complex issue to treat in thermo-mechanical models as coherent structures (e.g. layering) may need to be represented. Over geological time scales, rocks are subjected to enormous strains, i.e. large deformation. Given that rocks possess characteristics of both ductile and brittle materials (over a certain time scale), during their deformation they will yield. In contrast to many engineering applications, geologists are interested in the deformation modes both pre and post failure.

This set of physical attributes associated with geological processes has motivated numerous different modeling approaches. Rather than describe all the methods in detail, we instead provide a brief historical overview of the approaches and highlight the merits and shortcomings with regards to the physical attributes identified above. Two broad categories of methods can be defined; (a) those which explicitly models interfaces from which a material domain (volume) is inferred, or (b) those which explicitly model volumes.

We refer to (van Keken et al., 1997; Popov and Sobolev, 2008; Zlotnik et al., 2007; Braun et al., 2008; Samuel and Evonuk, 2010; Schmalzl and Loddoch, 2003; Lin and van Keken, 2006) as examples of methods from category (a). While many methods to represent interfaces exist, developing robust schemes with low numerical diffusion, and which are capable to representing complex structures required by geodynamic models is non-trivial. With the
advent of affordable, distributed memory computer clusters, it is also an important consideration whether a given method can be implemented in 3D and whether the algorithm is suitable to be implemented in a distributed memory environment.

For geological applications, the geometric complexity of the structure needing to be represented, combined with algorithmic difficulties associated with implementing interface based models, has motivated the use of “particle based” methods (category (b)). The term “particles” is deliberately used vaguely as different methods may regard “particles” in different ways. In general, particles are used to represent a given lithology (i.e. material properties) and as such represent volumetric quantities. The huge advantage afforded by particle methods is that they are completely unstructured, and do not possess any connectivity associated with neighboring particles. The use of particles to track complex flow features (e.g. free surface evolution) dates back to the pioneering marker-and-cell (MAC) method (Harlow and Welch, 1965; Pracht, 1971). Here, the particles (or markers) were Lagrangian quantities and were used to represent (discretize) the volume of the fluid. The fluid equations for conservation of mass and momentum were solved via a staggered grid, finite difference method. The markers were used to indicate which cells in the grid were completely filled with fluid, and which contained the free surface, and hence where the free surface boundary condition should be applied. The MAC methodology has been extensively developed in the geodynamics community (Weinberg and Schmeling, 1992; Poliakov and Podladchikov, 1992; Zaleski and Julien, 1992; Fullsack, 1995; Tackley, 1998; Babeyko et al., 2002; Gerya and Yuen, 2003, 2007; Moresi et al., 2003, 2007). These authors follow the underlying concept introduced in the MAC scheme. Namely, the conservation equations are solved on a grid, while complex geometric features are represented with markers. In the geodynamic applications, the markers are not used simply to identify regions of free surface/fluid/air, rather they typically represent different lithologies to which material parameters and a constitutive law is attributed. Other Lagrangian particle based methods used in geodynamics include (Poliakov et al., 1993; Braun and Sambridge, 1994, 1995; Hansen, 2003; Schwaiger, 2007). While different in their methodology, they embody Harlow’s original concept.

The idea of representing complex geometric structures via Lagrangian markers is very appealing, and consequently gained widespread usage in computational geodynamics for a number of reasons: it is very simple to associate lithology and material properties to markers; the numerical implementation of marker fields is algorithmically straightforward; three dimensional implementations of marker methods are not significantly more complex than its 2D counterpart; the approach is amenable to distributed memory environments.
2.1.2 Present work

To obtain reliable results from numerical models, one has to ensure that a sufficiently high “numerical resolution” is used to guarantee that the physics is captured. Depending on the method, numerical resolution might be related to the size of grid cell used in a mesh, or the number of markers used to represent a volume of fluid. Furthermore, one has to establish that the numerical error associated with the method used actually decreases if the resolution is increased. That is, the convergence of the method must be established. Understanding the convergence properties of a method provides some insight into the resolution required to resolve a certain flow feature (for example) and furthermore, it also indicates how rapidly the error is reduced as a function of increasing numerical resolution.

Despite the widespread usage and acceptance of thermo-mechanical modeling as a viable tool to study geology, few studies focus on the accuracy of the numerical methods being employed. In general, it is often regarded that it is difficult to perform a formal error analysis on marker based methods due to their inherent unstructured nature. Furthermore, geological applications often utilize discontinuous material properties (e.g. viscosity). The use of spatially variable coefficients also make formal error analysis more complicated. While the marker-grid methods lack a formal error analysis, numerous numerical studies have been performed. Numerical studies consist of either output comparison between different codes or laboratory experiments (a.k.a. benchmark studies) (Blankenbach et al., 1989; Travis et al., 1990; van Keken et al., 1997; Tackley and King, 2003; Buiter et al., 2006; Schmeling et al., 2008; OzBench et al., 2008; van Keken et al., 2008), or solution comparison between an analytic solution and the model output (Moresi et al., 1996; Deubelbeiss and Kaus, 2008; Popov and Sobolev, 2008; Zhong et al., 2008).

Given the number of practitioners now employing marker-grid style thermo-mechanical numerical models, it is important to thoroughly address the order of convergence of these methods, either through carefully designed numerical experiments or analytical approaches (Nicolaides, 1992; Nicolaides and Wu, 1996). In this work, we consider one such representative marker-grid based approach (Gerya and Yuen, 2003, 2007) and numerically examine the convergence properties of the method. This is achieved by using three different analytic solutions for a Stokes flow problem with continuous and discontinuous viscosity structures. Although idealized, these solutions have sufficient complexity in terms of their lithology and geometry to be regarded as representative of a typically geodynamic application. Using the analytic solutions, the true discretization error can be measured. While using analytic
solutions to measure errors and determine the order of convergence is by no means exhaustive (in a mathematical sense), in the absence of formal convergence proof, the approach is justified if the analytic solutions possess sufficient complexity compared to the intended application of interest. Given the interest in representing free surfaces for modeling topography and the difficulties that are related to the introduction of this surface (Kaus et al., 2010), we adopt a strong form variant of the stabilization technique described in Kaus et al. (2010) for a finite difference scheme. To verify that the use of this algorithm does not affect the convergence properties of our finite-difference/Marker-in-cell scheme (FD-MIC), we again utilize analytic solutions.

The outline of this paper is as follows. In the first section we introduce the physical problem of interest along with the numerical method that we employ. In Section 2.3 we describe the sources of errors and the methodology we employ to analyze the discretization error of our numerical method. The models used in the convergence study and the results obtained using the convergence properties of the FD-MIC method are described in the Section 2.4. Section 2.5 provides an application of our method to solve a time-dependent free surface problem. In Section 2.6 we discuss some future directions and perspectives beyond examining discretization errors within numerical schemes. Lastly, concluding remarks are provided in Section 2.7.

2.2 Physical problem and numerical method

2.2.1 Governing equations

Traditionally, the flow of geomaterials over geological timescales is calculated by solving the momentum equations neglecting inertial terms (Stokes equations)

\[
\frac{\partial \sigma_{ij}}{\partial x_j} = -\rho g_i.
\]  

(2.1)

This equation describes the balance between the viscous force and the body forces for an infinitesimal volume of fluid. The viscous force is formulated as the gradient of the stress tensor \(\sigma_{ij}\) and the body force is written as the product of the fluid density \(\rho\) and the gravitational acceleration vector \(g_i\). Moreover, in the absence of melting and
phase transitions, geodynamic flows are considered as incompressible. Incompressibility is enforced by coupling the aforementioned equations with the continuity equation

\[ \frac{\partial v_i}{\partial x_i} = 0, \]  
(2.2)

where \( v_i \) is the velocity and \( x_i \) is the spatial coordinate. Eqs. 2.1 and 2.2 are valid over the model domain which we denote by \( \Omega \). To close the system the equations for the conservation of momentum and mass are supplemented with two boundary conditions. Decomposing the boundary of \( \Omega \) into two, non-overlapping regions denoted by \( \partial \Omega_N \) and \( \partial \Omega_D \), the boundary conditions are written as:

\[ \sigma_{ij}n_j = a_i, \quad x \in \partial \Omega_N \]  
(2.3)

and

\[ v_i = b_i, \quad x \in \partial \Omega_D. \]  
(2.4)

Here \( n_j \) is the outward point normal to the boundary of \( \Omega \), \( a_i \) is an applied traction and \( b_i \) is a prescribed velocity.

The mechanical behavior of the material is defined by a constitutive relationship. We relate the stress tensor \( \sigma_{ij} \) to the strain rate tensor \( \dot{\epsilon}_{ij} \), using a linear, isotropic viscous rheology given by:

\[ \sigma_{ij} = -p\delta_{ij} + 2\eta\dot{\epsilon}_{ij}, \]  
(2.5)

where \( p \) is the pressure, \( \eta \) is the viscosity and the strain rate is given by

\[ \dot{\epsilon}_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right). \]  
(2.6)

The model domain contains several different material compositions, or lithologies which we denote by \( C(x) \). The compositional field \( C \) does not possess any physical diffusion, and evolves according to

\[ \frac{\partial C}{\partial t} + v_i \frac{\partial C}{\partial x_i} = 0. \]  
(2.7)

Prior to any discretization, the equations above are non-dimensionalized by means of dynamic scaling. This scaling is achieved by firstly defining a set of characteristic units such as a characteristic length (e.g. domain size), a characteristic time (e.g. diffusion time, inverse background strain rate), a characteristic viscosity (e.g. minimum viscosity in the domain) and secondly deriving all the related characteristic units (mass, stress, force...).
In Sec. 2.4, we employ characteristic units that are equal to 1. The results are not scaled back to dimensional units and therefore the velocity errors and pressure are dimensionless. The experiment described in Sec. 2.5 involves processes occurring at Earth-like dimensions and thus the characteristic units differ from 1, the corresponding results are scaled back to dimensional units.

2.2.2 Numerical method

Spatial discretisation

We solve Eqs. 2.1 and 2.2 for the primitive variables $v_i$ and $p$. The discrete solution of the two field formulation is defined by a grid based conservative finite difference scheme. The constraint imposed by incompressibility condition is effectively treated using the classical staggered grid arrangement of the primitive variables (Harlow and Welch, 1965). In the staggered formulation used here, we solve for pressure which is defined at cell centers and for the component of velocity normal to the cell face (Fig. 2.1). The velocity component is located at the centroid of each cell face. For flow problems possessing a spatially variable viscosity, the viscosity is required to be defined at both the vertices, and the centre of each control volume, in order for the discrete equations to conserve stress between neighboring control volumes (Patankar, 1980).

This particular staggered grid difference scheme has been demonstrated to be robust in solving variable viscosity problems in mantle convection by numerous authors (Ogawa et al., 1991; Tackley, 1993; Ratcliff et al., 1995; Harder and Hansen, 2005; Trompert and Hansen, 1996; Stemmer et al., 2006; Tackley, 2008) and in lithospheric dynamics (Zaleski and Julien, 1992; Gerya and Yuen, 2003, 2007; Katz et al., 2007).

The material properties viscosity and density are discretized in space via a set of markers, or particles. For a given property $\phi$ represented via the marker field, we adopt the following interpolant defined on the markers

$$\phi(x) \approx \hat{\phi}^m(x) = \sum_{p=1}^{N_m} \delta(x - x_p)\phi_p^m, \quad (2.8)$$
Figure 2.1: Spatial distribution of the primitive variables \((u, v, p)\) and material properties \((\eta, \rho)\) for a two dimensional staggered grid and example of boundary conditions discretizations (case of the \(u\)-momentum equation). The black symbols represent the nodes that are part of the stencil for the boundary equation discretization. The gray symbols represent the neighboring nodes which are not taken into account in the stencil. A) The extrapolated boundary condition is formulated as a linear combination of two internal \(u\)-nodes. B) The fictitious node boundary condition implementation is achieved by discretizing the equations along the domain boundary. The dashed symbol represent the fictive point used for the formulation of this boundary equation.

where \(\phi^m_p\) is the value of property \(\phi\) (viscosity or density) defined at marker \(p\), \(Nm\) is the number of markers, \(\delta(x - x_p)\) denotes the standard Kronecker delta function and the marker coordinate is \(x_p\). For simplicity we express Eq. (2.8) via

\[
\phi^m(x) = Q^T(x)\Phi^m, \tag{2.9}
\]
where \( \Phi^m \) is the vector of all marker values used to represent the field \( \phi \) and \( Q^T \) is a row vector with each entry defining the Kronecker delta function for each marker \( p \).

To evaluate the finite difference stencil, we are required to interpolate the marker values for viscosity and density onto the relevant locations (cell vertex or centroid) within the FD grid. Here we derive the “marker-to-node” interpolation scheme defined in Gerya and Yuen (2003) by regarding the operation as an \( L_2 \) projection (least squares minimization) of the marker properties onto the vertices of a structured grid. To derive the interpolant in Gerya and Yuen (2003), we first define grid based representation of the field \( \phi \). The grid is constructed from vertices of the cells defining each pressure control volume. Over each cell, the grid representation of field, which we denote by \( \phi^g(x) \), is assumed to vary according to a bilinear function. Denoting the bilinear interpolant at node \( i \) by \( N_i(x) \), we have the following approximation for \( \phi \) over the grid

\[
\phi(x) \approx \phi^g(x) = \sum_{i=1}^{N_n} N_i(x) \phi^g_i = N^T(x) \Phi^g, \tag{2.10}
\]

where \( N_n \) is the number of vertices in the mesh, \( \Phi^g \) is the vector of all nodal values used to represent the field \( \phi \) and \( N^T \) is a row vector with each entry defining the interpolation function \( N_i(x) \) for node \( i \). The interpolating functions \( N_i \) have a compact support \( \hat{\Omega} \), implying the functions are only non-zero over a subset of the entire domain \( \Omega \). In this work, the compact support of each interpolant \( N_i \), is \( \hat{\Omega} \equiv 2\Delta x \times 2\Delta y \), where \( \Delta x, \Delta y \) are the cell dimensions in the \( x \) and \( y \) direction respectively. Furthermore, the grid interpolants \( N_i \) are a partition of unity, that is \( \sum_{i=1}^{N_n} N_i(x) = 1 \). In Fig. 2.2A) we depict the compact support for a node \( i \) (denoted by the gray region) and the shape of the bilinear interpolation function \( N_i \). Outside of the gray region, \( N_i(x) = 0 \).

The projection operator \( \mathcal{P} \) can be defined as

\[
\mathcal{P}: \phi^m(x) \rightarrow \phi^g(x).
\]
Figure 2.2: The “4-Cell” and “1-Cell” schemes for projecting properties defined on the markers (denoted by stars) onto a node (denoted by the solid circle). A) The 4-Cell scheme. The support of the interpolating function \( N_i \) associated with node \( i \) is indicated by the shaded region. Only markers within the support of node \( i \) contribute to the projection operation used to define the nodal value at \( i \). The shape of the bilinear interpolation function for node \( i \) is indicated in the lower frame. B) The 1-Cell scheme. The thick lines in the lower frame indicate the grid used to discretize the Stokes equations, while the thin lines indicate the grid onto which marker properties are projected. The 1-Cell scheme utilizes a compact support of size \( \Delta x \times \Delta y \). The support for nodes \( r, s, t \) are indicated by the shaded regions. Only markers within the nodal support contribute to the projection operation for that node. The vertex and cell centered values on the grid used to discrete Stokes equations (points \( r \) and \( s \) respectively) are directly obtained from the local projection scheme.

Given that the number of markers and vertices in the mesh is not equal, a natural choice to define \( P \) is to use the \( L_2 \) projection of \( \phi^m \) onto \( \phi^g \). The least squares minimization leads us to seek a solution of the following:

\[
\min \left[ \frac{1}{2} \int_{\Omega} (\phi^g(x) - \phi^m(x))^2 \, dV \right] = \min [J],
\]  

(2.11)
where $\Omega$ is the model domain. Substituting Eq. 2.9 and Eq. 2.10 into Eq. 2.11 and computing $\partial J/\partial \Phi_g = 0$ we obtain

$$\left( \int_{\Omega} N(x)N^T(x) \, dv \right) \Phi_g = \int_{\Omega} N(x)Q^T(x)\Phi_m \, dv, \tag{2.12}$$

and inserting Eq. (2.8) yields

$$\left( \int_{\Omega} N(x)N^T(x) \, dv \right) \Phi_g = \sum_{p=1}^{N_m} N(x_p)\phi^m_p. \tag{2.13}$$

The “marker-to-node” interpolation of Gerya and Yuen (2003) is obtained from Eq. 2.13 by making the following approximations; (1) replacing the matrix on the left hand side by a diagonal matrix defined by summing the entries in each row, i.e.

$$N_i(x)N_j(x) \approx N_i(x) \left( \sum_{k=1}^{N_n} N_k(x) \right) \delta_{ij} \tag{2.14}$$

$$= N_i(x)\delta_{ij},$$

where the summation in brackets was eliminated since $N_i$ form a partition of unity and (2) evaluating the integrals using a numerical quadrature defined by the markers within the compact support of each $N_i$. Approximating $NN^T$ by a diagonal matrix makes the projection operation completely local to each node $i$ in the mesh. This choice is predominately made to reduce the computational cost by eliminating the need to solve a matrix problem. Nevertheless, it is usually more desirable to utilize a local $L_2$ approximation, as global $L_2$ tend to produce overly smooth fields. Given that the interpolant used for the marker fields are Kronecker delta functions (see Eq. 2.9), the most natural quadrature scheme to use is a Monte-Carlo method in which the markers coordinates define the abscissa of the quadrature scheme and every quadrature point is assigned the same quadrature weight, with the only constraint that the sum of the weights should equal the volume of the integration domain. Incorporating the above two assumptions, and invoking Eq. 2.9 reduces Eq. 2.12 to the following:

$$\phi^g_i = \frac{\sum_{p=1}^{N_m} N_i(x_p)\phi^m_p}{\sum_{p=1}^{N_m} N_i(x_p)}. \tag{2.15}$$
We note that the $L_2$ interpolant defined in Eq. 2.12 was $O(h^2)$ accurate, where $h$ represents the grid spacing, as the function space onto which we projected the marker field was bilinear. However, the approximated interpolant in Eq. 2.15 possesses a reduced rate due to the choice of quadrature scheme utilized. In the worst case, classical Monte-Carlo quadrature with random abscissa converges like $O(n^{-1/2})$, where $n$ is the number of points, while quasi-Monte Carlo methods using pseudo random abscissa can converge as fast as $O(\sqrt{\log(n)}/n)$. The abscissa used in our quadrature scheme are determined via the coordinates of the markers, which are inherently non-deterministic and are the result of the solution to Stokes flow. Consequently, we anticipate that the quadrature rule employed here will tend toward the classical Monte-Carlo limit and thus the expected order is $O(\bar{H}^{1/2})$, where $\bar{H}$ is the average spacing of the markers. In this work, the projection depicted in Fig. 2.2A will be referred to as the “4-Cell” scheme.

In Gerya and Yuen (2007), a “local” variant of the projection operator in Eq. 2.12 was proposed. This local marker-to-node projection is depicted in Fig. 2.2B. The principal difference is that the marker properties are projected onto a grid of finer resolution (thin lines, bottom frame) than the grid used to discretize Stokes equations (thick lines, bottom frame), thus the compact support of each interpolant $N_i$ is now $\hat{\Omega} = \Delta x \times \Delta y$. While developed heuristically, numerical experiments revealed that this local projection method yielded more accurate results (Gerya and Yuen, 2007). Given that we have shown that the interpolant used by Gerya and co-workers is an approximate $L_2$ projection, it is apparent why the local variant yields more accurate results. Specifically, we note that while the order of the local $L_2$ projection is the same variant depicted in Fig. 2.2A, the discretization parameter $h$ has been reduced by a factor of two, which thus also reduces the error. In the local projection scheme, vertex values and cell center values (denoted respectively by $r$ and $s$ in Fig. 2.2B) are obtained from application of Eq. 2.15 on the finer grid. Note that the mid-side values (denoted by $t$) are not required by the finite difference stencil. The projection depicted in Fig. 2.2B will be referred to as the “1-Cell” scheme.

**Temporal discretisation**

The Lagrangian markers are used to discretize each composition field $C_i$. In the Lagrangian frame of reference, the solution of Eq. 2.7 can be obtained by solving the following two equations:

\[
\frac{DC_i}{Dt} = 0, \quad \frac{dx_i}{dt} = v_i, \quad (2.16)
\]
where $C_p, x_p, v_p$ are the composition, coordinate and velocity vector of marker $p$ respectively. The solution of $DC_p/Dt = 0$ is trivially obtained with the marker representation. The kinematic equation is solved using a 4th order Runge-Kutta (RK4) time stepping scheme applied to each marker $p$. We do not re-evaluate the flow field during the application of RK4, thus the method is fourth order accurate in space only. The velocity field at the marker $v_p$, is obtained by interpolating the flow field computed on the finite difference grid to the marker coordinate $x_p$. After the markers have been advected, the material properties are interpolated from the new marker positions to the Eulerian grid using either the 4-Cell or 1-Cell projection defined by Eq. (2.15). Alternatively, higher-order time integrators such as the predictor-corrector method are employed in the community [Weinberg and Schmeling, 1992; Schmeling et al. 2008], this method achieves second order (in time) accuracy but requires to solve the discrete Stokes equations twice per time integration.

**Boundary condition implementation**

We have used two different techniques to impose boundary conditions (BC) on the non boundary matching nodes of the staggered grid. The first method (extrapolated boundary condition) enables to describe the stress or velocity value at the boundary by setting a condition on two nodes inside the domain (extrapolation). A linear combination of the velocities at those two nodes defines a velocity gradient, the value of the velocity at the boundary is therefore set by the value of the gradient. For instance the $u$-component of a free slip boundary condition at the top of the domain (assuming zero normal velocity) is expressed as

$$\frac{\partial u}{\partial y} \bigg|_{\text{top}} = 0 \implies u_C - u_S = 0,$$

where the node labeling is depicted in Fig. 2.1. For the no slip case, we extrapolate to the boundary, thus

$$u |_{\text{top}} = 0 \implies u_C - \frac{1}{3} u_S = 0.$$

The stencil corresponding to this boundary equation only contains two points and additional constraints are needed while solving for pressure. This condition is usually fulfilled by setting additional pressure constrain such as zero horizontal pressure flux in the corners of the domain (Gerya, 2010). Given the fact that the boundary condition is defined via extrapolation, we expect this implementation to be first order in space. A second method (fictitious node method) is derived by discretizing the momentum equations along the do-
main boundaries. The usual stencil is modified to account for “virtual nodes” outside of
the domain. These virtual nodes are used to define the velocity flux at the boundary of
the domain, they are not explicitly included into the system of equations. The isoviscous
$u$-momentum equation stencil at the top of the domain is written as:

$$
\frac{\partial \sigma_{xx}}{\partial x} + \eta \left( \frac{u_F + u_S - 2u_C}{\Delta y^2} \right) + \eta \frac{\partial^2 v}{\partial y \partial x} - \frac{\partial p}{\partial x} = -\rho g_x,
$$

with

$$
u_F = u_C,
$$

for a free slip case and

$$
u_F = -u_C,
$$

for a no slip case. This method do not require any additional pressure constrains, thus no
boundary conditions are required while solving for pressure. With this discretization of
the boundary condition, each partial derivative is approximated by means of central finite
differences and we therefore expect this method to be second order accurate in space.

**Free surface treatment**

As shown by [Kaus et al. (2010)](#), the introduction of a free surface boundary condition in
geodynamic simulations can induce artificial numerical oscillations in the free surface. This
phenomena was coined the “drunken seaman” instability. The free surface in these models
represents an interface between air and rock, and this is characterized by a sharp jump in
density ($\sim 10^3$ kg.m$^{-3}$). A large displacement of this interface within a single timestep
can therefore give rise to severe “out-of-balanceness” ([Kaus et al. (2010)](#)). Physically, this
imbalance may occur when the employed timestep exceeds the isostatic relaxation time
([Fuchs et al. (2011)](#)). Such a situation is likely to occur when an explicit advection scheme is
used, since the non-linear residual associated with the advected coordinates (at time $t+\Delta t$),
and the evaluation of the force term and stresses (at time $t$) is not guaranteed to be small.
The “drunken seaman” instability can occur when the free surface is explicitly tracked, such
as in a body fitted Lagrangian finite element method, or when the free surface boundary
condition is approximated via the “sticky air” approach ([Schmeling et al. (2008)](#)), which is
commonly used in Eulerian-Lagrangian finite difference methods in which explicit meshing
of the interface is not possible. Several methods can be used to stabilize the evolution
of the free surface, which either involve solving the non-linearity related to advection or
introducing higher-order terms arising from a Taylor series expansion of the momentum balance equation. Here we focus on the second approach applied to our staggered grid finite difference discretization.

Kaus et al. (2010) showed that a higher-order Taylor series expansion of the weak formulation of the momentum balance equation leads to an expression of a correction term which has the form of a boundary traction. This correction term is then discretized and added to the original Stokes stiffness matrix. Following this methodology, we derive a similar correction term for the strong form of the Stokes equations (such as used with the finite difference method) by performing a Taylor series expansion of Eq (2.1) about the point \( x + \theta \Delta t \mathbf{v} \), where \( \Delta t \) is the time step and \( 0 < \theta < 1 \) is an arbitrary parameter used to limit the size of the displacement increment. The expansion of the stress tensor \( \sigma_{ij} \), is given by

\[
\sigma_{ij}(x + \theta \Delta t \mathbf{v}) \approx \sigma_{ij}(x) + \theta \Delta t \, v_k [\sigma_{ij}(x)]_{,k} + O(\Delta t^2),
\]

(2.22)

where the subscript \( ,k \) denotes a partial derivative with respect to \( x_k \). The gradient of the stress tensor \( \sigma_{ij,j} \), is then

\[
\sigma_{ij,j}(x + \theta \Delta t \mathbf{v}) \approx \sigma_{ij,j}(x) + \theta \Delta t \, v_k [\sigma_{ij}(x)]_{,jk} + \theta \Delta t \, v_{k,j} [\sigma_{ij}(x)]_{,k} + O(\Delta t^2).
\]

(2.23)

Assuming that \( \mathbf{g} \) is constant, we expand the density as

\[
\rho(x + \theta \Delta t \mathbf{v}) \approx \rho(x) + \theta \Delta t \, v_k [\rho(x)]_{,k} + O(\Delta t^2).
\]

(2.24)

Inserting Eq. (2.23) and (2.24) into Eq. (2.1) and keeping only the terms which are linear in \( v_k \) and those of \( O(\Delta t) \), we obtain the perturbed momentum balance equation

\[
\sigma_{ij,j}(x) + \theta \Delta t \, \rho(x)_k v_k g_i = -\rho(x) g_i.
\]

(2.25)

Here, the term \( \theta \Delta t \, \rho(x)_k v_k \) represents a higher-order correction which is a function of the velocity field, \( v_k \). We denote the discrete representation of the momentum equation in Eq. (2.25) via

\[
\mathbf{Ku} + \mathbf{Lu} + \mathbf{Gp} = \mathbf{f},
\]

(2.26)

where \( \mathbf{Ku} \) is discrete gradient of the stress tensor, \( \mathbf{Gp} \) is the discrete pressure gradient, \( \mathbf{f} \) is the discrete body force and \( \mathbf{Lu} \) is the stabilization term. The construction of \( \mathbf{L} \) requires the current timestep \( \Delta t \) and the evaluation of the density gradient at each of the \( u, v \) nodes.
In our FD-MIC formulation, this is achieved by central finite differences. We evaluate \( \frac{\partial \rho}{\partial x} \approx \frac{\Delta \rho}{\Delta x} \) and \( \frac{\partial \rho}{\partial y} \approx \frac{\Delta \rho}{\Delta y} \) at the \( u \) and \( v \) nodes respectively. The first order derivatives are computed using values of density defined at the cell centre (i.e. at the pressure node). The density at the cell centre is interpolated from the marker density field, or averaged from a density field defined on the vertices of the grid.

### 2.3 Discretization errors and convergence

#### 2.3.1 Errors in approximate solutions of PDE’s

Errors in the approximate solution of partial differential equations (PDE’s) arise from four main sources.

1. The discretization error (or truncation error), which is defined as the difference between a given mathematical model (analytic solution) and its discretized expression. In the case of the finite difference method, the derivative of a function is numerically estimated by first assuming that the function is continuous, and then by replacing the continuous derivative with a truncated Taylor series about a point. The discretization error is therefore generated by the truncation of the Taylor expansion and is proportional to the remainder \( O(h^n) \). This type of discretization error is termed locally-generated error by Roy (2010).

2. An additional discretization error may occur in the definition of the coefficients within the PDE. Coefficients may consist of terms on the right side of the equation (e.g., density), or terms within the differential operator itself (e.g., viscosity). Coefficients will possess a discretization error if they are obtained by interpolation (e.g. projection of markers properties to nodes), or if they are a function of variables which were discretized. Typical examples of the latter include temperature or strain-rate dependent viscosity where temperature (or velocity) are discretized over a grid, and the viscosity is required at a location which doesn’t coincide with a discretization point in the grid.

3. Round-off errors arising from the numerical representation of real, continuous numbers, with a finite precision representation. This error therefore depends on the
precision (number of digits) which is employed to represent floating point numbers in a numerical scheme.

4. The error resulting from the solution of systems of linear equations. The use of direct solvers (such as LU, Cholesky decomposition) can minimize this type error, at least to the extent possible with finite precision arithmetic (see item 3 above). However, if iterative methods are used, the error induced is a result of stopping condition used to terminate the iterative cycle. Robust stopping conditions should monitor the residual associated with the system and the current estimate of the solution. However, one still needs to specify a measure of when the residual is “small enough” to conclude that the iterative method has converged.

We here focus on the evaluation of the discretization error of the FD-MIC scheme. Our specific interest is to determine the convergence of the method in the case where viscosity jumps (discontinuous coefficients) occur at a sub-grid level. The discretization error of the FD-MIC scheme can be seen as the combination of the truncation error of the staggered grid (including boundary condition discretization) and the error related to the projection of material properties onto the nodes. In this study, the discrete system of equations describing Stokes flow was solved using a direct factorization technique, thus eliminating any error associated with using an iterative method. All calculations were performed using double precision arithmetic.

Measuring convergence of the discretization: Methodology

Several methods are available in order to study the discretization error of numerical schemes (Roache 1997; Roy 2010). We here focus on measuring the order of convergence of the primitive variables of our discrete Stokes problem. To do so, we employed two-dimensional analytic solutions which can be utilized to determine the exact velocity and pressure values at each node of our grids. In order to compute the discretization error, we utilize the $L_1$ norm, which for a scalar quantity $\phi$, is defined as follows

$$\|\phi\|_1 = \int_\Omega |\phi| \, dV,$$

(2.27)

where $\Omega$ is the model domain. For a vector quantity $w = (s, t)$ we have

$$\|w\|_1 = \|s\|_1 + \|t\|_1 = \int_\Omega (|s| + |t|) \, dV.$$

(2.28)
We define the pressure error as

$$\| e_p \|_1 := \| p - p_{\text{exact}} \|_1,$$

(2.29)

where $p_{\text{exact}}$ is the exact value of the pressure. The $L_1$ error for the velocity $u = (u, v)$, is defined via

$$\| e_u \|_1 := \| e_u \|_1 + \| e_v \|_1 = \| u - u_{\text{exact}} \|_1 + \| v - v_{\text{exact}} \|_1,$$

(2.30)

where $u_{\text{exact}}$ and $v_{\text{exact}}$ are the exact values of the $u, v$ velocity components. For numerical computations we approximate the above integrals via a one point quadrature rule, i.e.

$$\| \phi \|_1 \approx \| \phi \|_{1h} := \sum_e |\phi(x_e)| V_e,$$

(2.31)

where $V_e$ is the representative volume for the point $x_e$. For FD schemes, the appropriate volume to use in Eq. (2.31) is the control volume associated with each node. Within staggered grid FD schemes, the control volume associated with the $p$ and $u, v$ degrees of freedom are different. In our results, we utilise two dimensional meshes containing $M \times M$ elements. As a result, we have $(M + 1) \times M$ nodes for $u$, $M \times (M + 1)$ nodes for $v$, and $M \times M$ nodes for pressure. From this, we define the following discrete $L_1$ norm for pressure as

$$\| p \|_1 \approx \sum_{I=1}^{M} \sum_{J=1}^{M} |p_{IJ}| \Delta x \Delta y,$$

(2.32)

where $\Delta x \Delta y$ is the cell volume. For the velocity components we have

$$\| u \|_1 \approx \frac{1}{2} \sum_{J=1}^{M} |u_{IJ}| \Delta x \Delta y + \sum_{I=2}^{M} \sum_{J=1}^{M} |u_{IJ}| \Delta x \Delta y + \frac{1}{2} \sum_{J=1}^{M} |u_{(M+1)J}| \Delta x \Delta y,$$

(2.33)

$$\| v \|_1 \approx \frac{1}{2} \sum_{I=1}^{M} |v_{IJ}| \Delta x \Delta y + \sum_{I=1}^{M} \sum_{J=2}^{M} |v_{IJ}| \Delta x \Delta y + \frac{1}{2} \sum_{I=1}^{M} |v_{I(M+1)}| \Delta x \Delta y.$$

(2.34)

The order of convergence of the discretization is determined by computing the numerical solution defined on a sequence of uniformly refined grids, and computing the $L_1$ norm of $e_u$ and $e_p$ on each grid. Following this, a least squares regression is performed on the log10
of the error norm and the cell size $h$. We relate the convergence rate of the $L_1$ norms for velocity and pressure errors to the order of convergence via

$$
\| e_u \|_1 \leq C h^{r_u}, \quad \| e_p \|_1 \leq C h^{r_p},
$$

(2.35)

where $h$ is the mesh size, $C$ is a constant independent of the grid resolution $h$, and $r_u, r_p$ are the order of convergence for velocity and pressure fields respectively. In all our experiments, the grid possesses the same number of vertices $N$ in each direction. We define our grid sequence using $N = \{41, 81, 101, 201, 301, 401, 501, 601, 701, 801, 901, 1001\}$

### 2.4 Numerical experiments

#### 2.4.1 Idealized models used in the convergence study

To study the error distributions and convergence properties of our numerical scheme, we carried out two sets of experiments. Each of these tests is aimed at studying the effect of a spatially variable coefficient (viscosity) on the discretization. We define a global measure of the viscosity contrast over the entire domain $\Omega$ via $\eta^* = \max(\eta(x))/\min(\eta(x))$. The first test focuses on the buoyancy-driven flow in a box containing a one-dimensional viscosity structure. The second test addresses the influence of a large but smooth variation of viscosity within a box where the flow is driven by buoyancy. The third test investigates a pure shear deformation field, which is perturbed by the presence of a two-dimensional, circular, highly viscous inclusion.

**One dimensional viscosity structure: SolCx**

The first series of convergence test were performed using a two-dimensional analytical solution of a variable viscous Stokes flow problem which we identify as SolCx. The model domain is defined as $\Omega \equiv [0, 1] \times [0, 1]$. The boundary conditions on all sides of the domain are prescribed to be free slip, implying that the normal velocity to each wall is zero, and the tangential stress along the wall vanishes. Internal to the domain, fluid flow is driven by a sinusoidal force given by

$$
f = (0, -\sin(\pi y) \cos(\pi x))^T.
$$

(2.36)
In practice this force is imposed by setting a constant gravity acceleration ($x$-component equal to 0 and $y$-component equal to $-1$) and the allowing the density field to vary in space according to

$$\rho(x, y) = \sin(\pi y) \cos(\pi x).$$

Experiments were carried out for both isoviscous and variable viscosity case, in the latter case the viscosity field is discontinuous and is given by

$$\eta(x, y) = \begin{cases} 
1, & \text{if } 0 \leq x \leq 0.5 \\
10^6, & \text{if } 0.5 < x \leq 1.
\end{cases}$$

This setup allows for sharp viscosity jumps of large magnitude which is a typical requirement for geodynamic applications (Fig. 2.3).

**Figure 2.3:** Material properties and the analytic solution for SolCx. $\rho$ and $\eta$ are the density and viscosity distributions, $u, v$ are the analytic $x, y$ components of velocity respectively, and $p$ is the analytic pressure field. The vertical component of gravity acceleration is 1.

Furthermore, the discontinuous viscosity structure provides a more challenging test of both the discretization and the solver in comparison to solutions with continuous viscosity structures. A complete description of the analytic solution is provided in [Zhong (1996)](zhong1996). The source code used in our study to evaluate the analytic solution is available from the
open source package Underworld (Moresi et al., 2007). Since the flow is driven by the density gradients, we will again use this test to analyze the influence of the free surface stabilization scheme on the convergence of the FD-MAC method.

**Smooth viscosity variation in one dimension: SolKz**

In order to investigate the effect of a large and smooth viscosity variation on our discretization, we utilized the analytic solution from Revenaugh and Parsons (1987), which here we have termed SolKz. This solution allows for an exponential variation of viscosity from the bottom to the top of the model domain which is defined as $\Omega \equiv [0, 1] \times [0, 1]$. All the boundaries are free slip and the flow is driven by a smooth density distribution, the forcing term is expressed as

$$f = (0, -\sin(2y) \cos(3\pi x))^T. \quad (2.39)$$

and the viscosity increases from the bottom to the top of the box according to

$$\eta(y) = \exp(2By) \quad (2.40)$$

with the parameter $B$ controlling the magnitude of the overall viscosity variation. Here we choose $B$ such that over the vertical extend of the domain we have viscosity contrast of $\Delta \eta^* = 10^6$. The density, viscosity distribution and the resulting flow field are depicted in Fig. 2.4. Similarly to the SolCx test, the source code used to evaluate the analytical solution is part of the open source package Underworld.

**Two dimensional viscosity structure: pure shear inclusion test**

The third series of convergence test were carried out using the analytical solution for an inclusion in a weak matrix undergoing pure shear. The derivation as well as the scripts that can be used to compute the analytic are available in Schmid (2002); Schmid and Podladchikov (2003). For this problem, the model domain is defined as $\Omega \equiv [-1, 1] \times [-1, 1]$ and contains a circular shaped inclusion at the origin. The inclusion has a radius, $R_{inc}$ of 10% the length of the domain. The viscosity field is given by

$$\eta(x, y) = \begin{cases} 1, & \text{if } x^2 + y^2 > R_{inc}^2 \\ 10^3, & \text{if } x^2 + y^2 \leq R_{inc}^2 \end{cases}, \quad (2.41)$$
as used in [Deubelbeiss and Kaus (2008)]. The flow is driven by a pure shear strain rate boundary condition and the force vector, $\rho g_i$ is zero (Fig. 2.5). In this test, the circular shape of the inclusion ensures that the viscosity jump is never aligned with cartesian coordinate system, and thus is never aligned via the finite difference stencil. Consequently, this test is particularly relevant since bodies of arbitrary shape (e.g. plumes, slabs) typically develop during the evolution of geodynamic simulations.

### 2.4.2 Staggered grid discretization

In the first set of experiments, we used the analytical solution SolCx to examine the convergence properties of the staggered grid finite-difference discretization. These experiments do not employ markers to represent the material properties $\eta, \rho$. The finite difference stencil was thus defined by directly evaluating Eqs. (2.36) and (2.38). Experiments were performed using the two different boundary condition implementations described in Sec. 2.2.2.
CHAPTER 2. FD-MIC SCHEME DISCRETIZATION ERRORS

Figure 2.5: Viscosity structure ($\eta$) and analytic solution for velocity ($u, v$) and pressure ($p$), for the pure shear inclusion test (Schmid and Podladchikov, 2003). For this setup, the flow is driven by a strain rate boundary condition ($\dot{\varepsilon} = 1$) and the buoyancy forcing term is 0 (e.g. $\rho = 0$ or $g_y = 0$).

SolCx: Isoviscous

Here we consider a spatially constant viscosity field, i.e. $\eta(x, y) = 1$. These tests were performed in order to measure convergence of the method for an optimal case, and to assess the error associated with the different boundary condition implementations. In Fig. 2.6, the error distribution for a $101 \times 101$ grid resolution is shown.

A clear influence of the type of boundary condition can be observed. With both implementations, the maximum error for all fields is confined along the boundary of the domain. However, the error obtained using the extrapolated BC approach is significantly larger. At this resolution, we observe that the velocity and pressure errors are approximately 2 orders of magnitude smaller when the fictitious node BC implementation is used. The $L_1$ errors for a sequence of grid resolutions is shown in Fig. 2.7A), and the computed rates
Figure 2.6: Spatial distribution of the absolute value of the discretization error \((e_u, e_v, e_p)\) for the variables \(u, v,\) and \(p\). Comparison between extrapolated and fictitious node boundary condition implementations. Maximum errors are located at the domain boundaries. The error pattern and magnitudes between the two methods are notably different. The test was carried out using the isoviscous SolCx setup with a grid resolution of 101\(\times\)101 nodes.

As we expected, the velocity and pressure obtained using extrapolated BC discretization converge to the analytical solution with a rate of \(~1\). The fictitious node boundary condition implementation yields \(u, p\) fields which both converge at a rate of \(~2\).

<table>
<thead>
<tr>
<th>(\eta^*)</th>
<th>1(^{st}) order</th>
<th>2(^{nd}) order</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r_u)</td>
<td>1.0648</td>
<td>1.0052</td>
</tr>
<tr>
<td></td>
<td>1.0276</td>
<td>1.0064</td>
</tr>
<tr>
<td>(r_p)</td>
<td>1.0416</td>
<td>1.0026</td>
</tr>
<tr>
<td></td>
<td>2.0297</td>
<td>1.0114</td>
</tr>
</tbody>
</table>

Table 2.1: Order of convergence of the \(L_1\) error norm between analytics and numerics without using interpolation. Results are produced using the analytical solution SolCx. Comparison between two boundary condition implementations. Results are calculated for two different fluid viscosity contrasts.

From Fig. 2.7A, it is apparent that the absolute value of the velocity and pressure errors in \(L_1\) are much lower (for all grid resolutions) when the fictitious node BC is used. Given these results, we will refer to extrapolated BC method and the fictitious node BC implementation as the “first order” and “second order” BC methods in the following sections.
Figure 2.7: Velocity and pressure $L_1$ error norms with increasing resolution for the SolCx test. Viscosity and density are directly sampled and therefore no interpolation is used. A) First order convergence is achieved while using extrapolated boundary conditions. Second order convergence is obtained with fictitious node boundary conditions. B) As soon as a jump in viscosity is introduced, both extrapolated and fictitious node boundary conditions converge at a first order rate.

SolCx: $10^6$ viscosity jump

In this second test, a large discontinuous jump in viscosity ($10^6$ Pa.s) was introduced. The discretization error for a $101 \times 101$ grid resolution is shown Fig. 2.8.

In comparison with the isoviscous results (Fig. 2.6), the location of the maximum error is no longer only confined along the boundary of the domain. Rather, we now observe that maximum errors in $u, v$ occur within the low viscosity region, and share a similar spatial correlation with the isoviscous case if we consider only the low viscosity domain ($x < \frac{1}{2}$). For the $v$ velocity component, the maximum absolute error is located at the jump, with a magnitude of $|e_v| \approx 10^{-4}$ for both boundary condition implementations. The effect of the BC implementation remains visible for both $u$ and $v$, at the boundaries second order BC produces errors which are approximately 1 order of magnitude smaller than for the first order case. Pressure errors are distributed at the jump and close to the boundaries. In contrast with the isoviscous case, the pressure errors are much larger than the velocity errors ($\approx 10^{4}$ to $10^{3}$ times) and significantly larger ($\approx 100$ to $1000$ times) than in the isoviscous case. The discontinuous variation of the viscosity not only has a strong influence on the
Figure 2.8: Absolute value of the discretization error for the primitive variables $u, v,$ and $p$. Results are produced with the analytical solution SolCx with a viscosity jump of $10^6$ and a grid resolution of $101 \times 101$ nodes. Extrapolated and fictitious node boundary condition implementations produce a similar velocity error pattern. The pressure error is dominant at the location of the viscosity jump.

Spatial distribution of errors, but also on the order of convergence of the computed velocity and pressure fields. The $L_1$ errors for the variable viscosity case are shown in Fig. 2.7B), and the computed rates are provided in Table 2.1. The most noticeable feature is that the choice of boundary condition implementation is much less critical. This is apparent as we observe that the convergence of the velocity and pressure errors in $L_1$, for both boundary condition implementations, are approximately first order. The magnitudes of the $L_1$ error for velocity and pressure error are lower when using second order boundary conditions, however the difference is less than $0.25$ of an order of magnitude. For this particular test, we observed different results if an even grid sequence was employed. The even grid sequence corresponds to a situation where the central pressures nodes are located directly on the viscosity jump. Using an even grid sequence with second order boundary conditions lead to an improvement of the velocity and pressure errors in $L_1$, with orders of convergence of $r_u = 2.0$ and $r_p = 1.0$ respectively. Due to the super convergent nature of these rates, we regard meshes with viscosity jumps aligned with pressure nodes as a special case. While it is of interest to understand the reason for the apparent super convergent behavior, in practice we rarely ever encounter material property contrast which can be aligned with the grid, thus we do not study this special case any further here.
2.4.3 Staggered grid / marker-in-cell discretization

In the second set of experiments, we defined the material properties \((\eta, \rho)\) via markers, therefore both viscosity and density fields were projected from the markers onto the grid. The values of material properties at the cell centre and cell vertices are calculated using the interpolation described by Eq. (2.10). We tested both the 1-Cell and 4-Cell interpolation schemes presented in Sec. 2.2.2. In the following tests, we used an average of 64 markers per cell and we only considered using arithmetic viscosity averaging. In order to mimic the geometrically disturbed nature of an advected marker field, the markers were initially laid regularly throughout the grid and then perturbed such that each marker position \(x_p\) was randomly displaced (from the uniform distribution) by 1\% of the grid spacing. The order of convergence in \(L_1\) for velocity and pressure are reported in Table 2.2. Further tests (not shown here) indicated that these results are not sensitive to the perturbation applied to the markers’ coordinates.

<table>
<thead>
<tr>
<th>(I)</th>
<th>1-Cell</th>
<th>4-Cell</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1^\text{st}) order</td>
<td>(2^\text{nd}) order</td>
<td>(1^\text{st}) order</td>
</tr>
<tr>
<td>(\eta^*)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(r_u)</td>
<td>1.0309</td>
<td>1.0016</td>
</tr>
<tr>
<td>(r_p)</td>
<td>1.0323</td>
<td>1.0175</td>
</tr>
</tbody>
</table>

Table 2.2: Order of convergence for the coupled staggered grid/marker-in-cell scheme using an arithmetic viscosity averaging and 64 markers per cell. The results are produced using the analytical solution SolCx. They represent the rate at which the \(L_1\) error norm decreases with increasing resolution. Comparison between two interpolation scheme \((I)\), two boundary condition implementations, and two different fluid viscosity contrasts \((\eta^*)\).

SolCx: Isoviscous

The order of convergence for \(u, p\) in \(L_1\) using different interpolation methods and the two boundary condition schemes are reported in Table 2.2. We note that both the 1-Cell and 4-Cell interpolation methods maintain the order of convergence in \(L_1\) of the staggered grid discretization measured in Sec. 2.4.2. In these isoviscous models, the style of boundary condition implementation influences the order of convergence of both the \(u\) and \(p\) fields. As in Sec. 2.4.2, second order boundary conditions show superiority by providing second order convergence. While both marker interpolation methods provide similar rates of convergence, 1-Cell interpolation is more accurate than 4-Cell for a given number of marker


per interpolation volume (Fig. 2.9A). In this test, the viscosity field is spatially constant, thus the difference in error is therefore related to the interpolation of density (a smooth field in this model) required to assemble the force vector. The offsets between the velocity and pressure solutions with and without introducing marker-to-node interpolation is $\sim 0.5$ an order of magnitude.

![Figure 2.9](image)

**A) Isoviscous**  
**B) $10^6$ viscosity jump**

**Figure 2.9**: Velocity and pressure $L_1$ norms for SolCx test. Comparison of different material properties interpolations schemes (4-Cell, 1-Cell). Results are obtained using 64 markers per interpolation area. For this specific test, we used second order BC. A) With an isoviscous problem, velocity and pressure errors converge at second order. The offsets between the different lines are a result of the density interpolation. For similar marker density per interpolation volume, local interpolation provides more accurate results. B) When a viscosity jump is introduced, all solutions converge at a first order rate. The influence of the interpolation scheme is only noticeable on the pressure error.

**SolCx: $10^6$ viscosity jump**

In Fig. 2.9B) the errors for a sequence of grid resolutions used to solve the variable viscosity case are provided. Similarly to the test in Sec 2.4.2 in which no marker-to-node interpolation was employed, all the simulations converged with a rate of first order for velocity and pressure, regardless of the type of interpolation used. The choice of using either 1-Cell or 4-Cell interpolation method for interpolating the viscosity jump and the smoothly varying density field only appeared to affect the accuracy of the pressure field.
SolKz: $10^6$ smooth viscosity variation

We have run the test SolKz to test the effect of a large continuous variation of viscosity through the domain. We have used second order boundary conditions and two interpolation methods. Second order $L_1$ velocity and pressure convergence were obtained while using 1-Cell interpolation. 4-Cell interpolation provided second order velocity convergence and a pressure convergence order $r_p$, slightly less than 2. For this particular test, the absolute velocity and pressure error are approximately half an order of magnitude more accurate when the 1-Cell method is utilized (Fig. 2.10).
Inclusion test: $10^3$ viscosity jump

This analytical solution requires a strain rate boundary condition (e.g. pure shear) to be applied far away from the centre of the domain where the inclusion is located. In order to avoid any problems related to the treatment of this particular boundary condition, each boundary was treated as a Dirichlet boundary. The analytical solution was evaluated and imposed on the boundaries of our model domain. This approach also has the effect of removing the truncation error introduced while discretizing the strain rate boundary condition.

The tests were run for an inclusion/bulk viscosity contrast of $10^3$, using both the first order and second order boundary condition implementations. Nodal material properties were evaluated from the marker field using 1-Cell and 4-Cell interpolations and both interpolations were carried out using an average of 64 markers per interpolation volume. Given the sharp, two-dimensional viscosity structure (circular), this setup provides a tough test for both the discretization and the interpolation scheme. For all our cases, we obtained first order convergence in $L_1$ for the velocity and pressure fields (Table 2.3).
Table 2.3: Order of convergence for velocity and pressure for the inclusion test ($\eta^* = 10^3$), using the staggered grid/marker-in-cell scheme employing an arithmetic viscosity averaging (64 markers/cell). The order of convergence are observed to be independent of the type of marker-to-node interpolation, and the style of boundary condition implementation.

<table>
<thead>
<tr>
<th>$\mathcal{I}$</th>
<th>1-Cell</th>
<th></th>
<th>4-Cell</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>BC</td>
<td>1st order</td>
<td>2nd order</td>
<td>1st order</td>
<td>2nd order</td>
</tr>
<tr>
<td>$r_u$</td>
<td>1.025</td>
<td>0.94897</td>
<td>1.0634</td>
<td>1.0043</td>
</tr>
<tr>
<td>$r_p$</td>
<td>0.98051</td>
<td>0.94107</td>
<td>0.99427</td>
<td>0.94204</td>
</tr>
</tbody>
</table>

Free surface stabilization

In this section, we investigate the influence that the free surface stabilization scheme described in Sec. 2.2.2 has on the order of convergence of the staggered grid discretization. For this purpose, we used the test SolCx described in section 2.4.1. This test does not include a free surface, but since the flow is driven by density gradients, the contribution of the stabilizing terms in the discretization is therefore non-zero. In order to choose the value of the timestep, here we evaluate the timestep using a Courant criteria (with $\theta = 0.5$) computed from the grid spacing the maximum velocities given by the analytical solution. In practice, for time dependent problems, the value of the timestep used would be that computed from the flow field and mesh configuration from the previous timestep.

We ran a convergence test using the second order boundary condition implementation and 1-Cell interpolation (64 markers per interpolation volume) with a viscosity jump of $10^6$. The stabilization affects the spatial distribution of error (compare Fig. 2.11A with Fig. 2.8). This effect is related to the proportionality between the magnitude of the correction term and the density gradients. With decreasing grid spacing, the numerical scheme with stabilization conserves its first order convergence in $L_1$ for both velocity and pressure, although a slight offset in pressure convergence was observed between simulations including or not the stabilization (dashed lines Fig. 2.11B). Therefore the stabilization term does not noticeably modify the flow discretization, nor the overall convergence of the method.
2.5 Application to a Rayleigh Taylor instability with a free surface

In Sec. 2.4.3, we applied the free surface stabilization algorithm to the model setup of SolCx and observed that the order of convergence was unaltered by the introduction of the stabilization. We considered this test as being suitable for a convergence test, even though the problem does not include a free surface, as the flow is driven by buoyancy, thus possesses spatial variations in density. In order to test the robustness of the free surface
stabilization scheme presented in Sec. 2.2.2 we ran several simulations of a Rayleigh-Taylor
instability with a free surface.

This setup was used in Kaus et al. (2010) and was shown to produce the “drunken seaman”
instability with a finite element code which explicitly tracked the free surface. With our
Eulerian-Lagrangian approach, we approximate the free surface by using “sticky air”, i.e.,
we introduce a layer of zero density, low viscosity incompressible material (Schmeling et al.,
2008). The pseudo free surface is then defined as the interface between the crust and the
“sticky air”, which is free to deform. The setup of the experiment consists of a domain Ω ≡
[−250, 250] × [−600, 0] km, with a resolution of 50² cells (Δx = 10 km, Δy = 12 km ). The
box is filled with a 400 km thick asthenosphere, a 100 km thick layer of lithosphere, and an
additional 100 km thick layer of “sticky air” (Fig. 2.12A). The lithosphere/asthenosphere
interface is offset by a 5 km sinusoidal perturbation which is therefore a subgrid feature.
The boundary conditions used are free slip at the top, left and right side of the domain,
the bottom of the box is a no slip boundary. Omitting the stabilization scheme leads
to the development of a surface instability from the very beginning of the experiment.
The magnitude of the topography produced during the instability is unrealistically large
and its polarity oscillates through time (Fig. 2.12B). Including the stabilization enables
the development of a stable topographic signal, characterized by a sinusoidal shape (Fig.
2.12C).

Following Kaus et al. (2010), we monitored the depth of the lithosphere/asthenosphere
interface with time. In order to do so, we explicitly tracked the left most marker located
on this interface, the results are reported in Fig. 2.13.

We tested the two different boundary condition implementations described in Sec. 2.2.2.
The results show that using the second order boundary condition implementation en-
ables our model to better reproduce the results of Kaus et al. (2010). We suspect that
the deviation observed using first order boundary conditions witnesses of an accumulated
(transport) error resulting from the nature of the BC implementation, affecting therefore
the solution in the vicinity of the domain boundaries. In summary, the stabilization al-
gorithm appears to effectively damp the oscillations that frequently arise when solving
time-dependent problems which contain a strong density contrast. Despite the apparent
stability of the algorithm when time steps defined by the classical Courant criterion are
used, we note that, according to the timescale of interest, accurate topographic modeling
may require the use of time steps which are much smaller than those predicted by the
Courant criterion.
Figure 2.12: A) Setup of the Rayleigh-Taylor instability with a free surface text (not to scale). B) Example of early development of topography without stabilization with a Courant number equal to 0.5. Labels 1, 2, 3 respectively indicate topographic profiles at times 100, 167, 171 ky. The “drunken seaman” instability produces topography of very large amplitude which is not stable during timestepping. C) Early development of topography with stabilization and Courant criteria equal to 0.5. The topographic signal is composed of a single positive topographic bulge in the centre of the domain. Labels 1, 2, 3 respectively indicate topographic profiles at time 100, 200, 300 ky.

2.6 Perspectives

2.6.1 Improving convergence properties

Throughout Secs. 2.4.2, 2.4.3 we have observed the degradation of the order of convergence of the staggered grid solution when discontinuous viscosity structures were introduced. Fig. 2.14 exhibits the decay of the $L_1$ velocity and pressure convergence order with increasing viscosity contrast. Here we observe a reduction of the convergence, from second order to first order, when the viscosity contrast is increased from one (isoviscous) to ten. These results were obtained using the analytic solution SolCx. While these results are representative of a one-dimensional viscosity structure, this type of behavior is commonly observed when solving problems involving large discontinuities in coefficients (Das et al., 1994), or waves such as those occurring in shock dynamics (Banks et al., 2008), where
Figure 2.13: Free surface evolution during the Rayleigh Taylor instability test as presented in Kaus et al. (2010). We monitor the vertical coordinate of a marker initially positioned on the lithosphere/asthenosphere boundary and next to the left side of the box. Results are all computed for a Courant number equal to 0.5. Differences can be seen between the first order and second order boundary condition implementations, the latter follows the results Kaus et al. (2010) more closely.

shocks behaves as discontinuities and locally reduce the order of convergence of the numerical scheme. Restoring the optimal convergence properties of numerical method, prior to the introduction of the discontinuities is appealing. Here we briefly describe some efforts towards this.

In Sec. 2.4.2 we noted that under a specific alignment of our finite difference grid with the one-dimensional discontinuous viscosity structure, the optimal order of convergence for our method was observed. While this was regarded as a “special” case, it does raise the question, under what conditions is the viscosity jump correctly captured by our FD stencil? It has been proposed that different averaging methods (harmonic, geometric) of material properties can lead to improved numerical results (MacKinnon and Carey 1988; Das et al., 1994). Examples of using such methods for viscous flow problems are available in the literature (Das et al. 1994; Deubelbeiss and Kaus 2008; Schmeling et al., 2008), however,
determining which averaging scheme is appropriate remains unsolved. The conclusions to this questions differ with the various model setup (boundary driven flow, buoyancy driven flow, viscosity contrast) and with the structure of the discontinuity (1D versus 2D versus 3D geometries). While such averaging is completely justified in 1D problems, it does not naturally generalize to higher dimensions and consequently is not robust enough for general use. Additionally, the aforementioned averaging methods only consider scalar quantities, or isotropic constitutive tensors. The extension of the approach to tensorial quantities, which may be required for obtaining an effective stress, or an effective orthotropic constitutive tensor is non-trivial. We refer to Cowin and Yang (1997) for further details on this topic.

In contrast to rudimentary averaging of multiscale behavior (or coefficients), homogenization provides an alternative view by decomposing phenomena into two scales: a “macro”
scale (coarse scale) and a “micro” scale (fine scale). The essence of this class of methods is to extract coarse scale equations (or coefficients) which incorporate a multitude of different scales. The idea of scale separation can be naturally adopted to the FD-MIC scheme discussed here, in which we have a model domain discretized via cells (coarse scale) and within each cell we have numerous markers which define variations of the viscosity and density, thereby describing fine scale information. In classical homogenization theory (Bensoussan et al., 1978; Murat, 1978), it is frequently required to assume that the fine scale heterogeneity is periodic in one direction, thus this approach may lack the generality required. Nevertheless, further extensions of classical homogenization are being developed for non-periodic heterogeneities (Capdeville et al., 2010). For a thorough review of homogenization we refer to Hassani and Hinton (1998b, a, c). Numerous alternatives to classical homogenization exist, see for example: Brewster and Beylkin (1995); Abdulle and Weinan (2003); Arbogast (2002); Jenny et al. (2003); Weinan and Engquist (2003); Kouznetsova et al. (2002). These methods also employ coarse and fine scale representation of the problem, however they do not assume that the fine structure is periodic. Many of these methods require the solution of independent cell problems, where local, fine scale solutions are subsequently coupled to a coarse grid solution. Such approaches could used in geodynamic models to resolve fine scale structure defined via the markers, and may also improve the convergence (Masud and Khurram, 2004; Liu and Li, 2006) of our scheme in the presence of discontinuous viscosity fields.

2.6.2 Input data uncertainty estimation and model parameter sensitivity

In the light of establishing that the numerical scheme employed to solve a given set of equations, which describes the geodynamic process of interest, has been deemed to be sufficiently accurate and robust, other challenges await. Geodynamic simulations require prescription of the rheological behavior and its associated parameters for each lithology present in the system. In practice, these parameters and flow laws are frequently derived experimentally. The evaluation of flow parameters at high pressure/temperature is a challenging task and the behavior of crystals such as olivine at depth still remains widely debated (Raterron et al., 2009; Rozel et al., 2011). Moreover, the extrapolation of flow laws from laboratory time scales to geological time scales give rise to additional uncertainties (Paterson, 1987). One interesting direction for geodynamic modeling is the integration
of the material property uncertainties into the simulations, as done in other communities (Laz et al., 2007; Houtekamer et al., 1996; Rabier et al., 1996).

Another avenue to explore could be to consider the geodynamic model as an “optimal design” problem (Hicks and Henne, 1978; Jameson, 1988, 1995; Giles and Pierce, 2000). In such approaches, one seeks to minimize (or maximize) a given objective function \( F \), subject to set of design variables \( U \) which define the model setup. For instance, in the context of a viscous folding model, the design parameters might be: size of the model domain, rheological parameters (viscosity, density), rate of compression applied as a boundary condition, and the objective function could be to minimize the difference between the dominant wavelength from the model and one observed from field data. From such optimal design approaches, one obtains the relativity sensitivity of \( F \) with respect to each design parameter \( U_i \) - or phrased another way, we can answer the question, what is the perturbation in dominant wavelength due the domain size, rheological parameters and boundary condition. Both optimal design and the integration of parameter uncertainties provide a means to quantitatively characterize model parameter sensitivities, thereby further developing our understanding of the system of equations we choose to represent our geological problem.

### 2.7 Conclusions

Computational modeling in geology requires numerical methods that are robust, reliable and accurate when applied to study the deformation of materials which possess discontinuities in their properties. Such variations in material parameters are likely to strongly influence the quality of the solution. While theoretical analysis of such methods is difficult due to the discontinuous nature of the material properties, very few numerical studies focussing specifically on the quality of discrete solutions obtained from geodynamic models have been performed. Quantifying the numerical accuracy for complex models that are relevant to geodynamics is of vital importance if the solution are to be used in any quantitative manner. In this study, we address these issues by examining the discretization errors and convergence characteristics of the discrete solution obtained from the FD-MIC scheme, which is a widely utilized method in the geodynamic community.

The convergence study was carried out using two-dimensional analytical solutions which possess large continuous and discontinuous variations in the viscosity field. Two differ-
ent boundary conditions implementations, namely extrapolated BC and fictitious nodes methods were tested. If the fluid was isoviscous, we found that the fictitious node method provided second order accurate velocity and pressure fields and therefore showed superiority over the extrapolated BC method, which produced first order accurate fields. Smooth but large variations of viscosity throughout the model domain did not affect the second order behavior of the FD-MIC method. However, the introduction of a viscosity jump in the model domain affected the convergence properties of the fictitious node method, resulting in first order velocity and pressure fields. This drop in the order of convergence occurred if the viscosity jump was larger than 5. An essential component of our Eulerian-Lagrangian discretizations is the projection of marker properties onto the finite difference grid. We tested two different marker-to-node projections which differ in their domain of influence. Introducing these projections was not observed to modify the order of convergence of the FD-MIC method. The more local 1-Cell interpolation was shown to be more accurate than the 4-Cell interpolation. For the range of problems considered, our results clearly establish that the FD-MIC scheme converges. That is, increases in the numerical resolution lead to a reduction of the discretization error. Demonstrating that the method is convergent adds robustness to both previous and future geodynamic applications which employ this particular numerical method.

Additionally, we introduced a strong form variant of the free surface stabilization algorithm presented in Kaus et al. (2010). This stabilization method is suitable for finite difference discretizations. By the means of a convergence test, we showed that the stabilization algorithm also does not notably affect the convergence properties of our numerical scheme. Further testing was carried out by performing time-dependent simulations and using a setup which is prone to instability. This test demonstrated that the stabilization suppresses the instability that may occur while running free surface calculations. The stabilization permits larger time steps to be used. Whilst this certainly does not improve the temporal accuracy of the solution, its inclusion is necessary to obtain physically meaningful results from simulations which use a Courant time step. The stabilization method is important for any simulation that includes large density contrasts. In our modeling, the surface of the Earth is a classical example of this type of interface. Thus, this stabilization routine approach is particularly relevant for modeling topography in regional and global scale simulations over geological time periods.

Lastly, we wish to remark that continued research in understanding discretization errors and convergence properties of the numerical tools used to study geological processes is vi-
tally important if we wish to develop the level of reliability and robustness in our modeling technology which exists in the engineering community. However, equally as important as the quality of our numerical solutions, is our ability to understand the dynamics of the system we use to describe geological processes, and this entails understanding the sensitivity of the model output to the underlying flow laws, material parameters and boundary conditions and other input used to define our model.

Bibliography


Schmeling, H., Babeyko, A. Y., Enns, A., Faccenna, C., Funiciello, F., Gerya, T. V.,
Golabek, G. J., Grigull, S., Kaus, B. J. P., Morra, G., Schmalholz, S. M., van Hunen,

Ph.D. thesis, ETH Zurich, Zurich, Switzerland.


Schwaiger, H. F., 2007. An implementation of smoothed particle hydrodynamics for large
deformation, history dependent geomaterials with applications to tectonic deformation.

Stemmer, K., Harder, H., Hansen, U., 2006. A new method to simulate convection with
strongly temperature- and pressure-dependent viscosity in a spherical shell: Applications

Tackley, P. J., 1993. Effects of strongly temperature-dependent viscosity on time-
dependent, three-dimensional models of mantle convection. Geophysical Research Letters
20 (20), 2187–2190.

Tackley, P. J., 1998. Three-dimensional simulations of mantle convection with a thermo-
chemical basal boundary layer: D”? In: Gurnis, M. (Ed.), The Core-Mantle Boundary

Tackley, P. J., 2008. Modelling compressible mantle convection with large viscosity con-
trasts in a three-dimensional spherical shell using the yin-yang grid. Physics of the Earth
and Planetary Interiors 171 (1-4), 7 – 18.

Tackley, P. J., King, S. D., 2003. Testing the tracer ratio method for modeling active com-
positional fields in mantle convection simulations. Geochimistry Geophysics Geosystems
4 (4).

Travis, B. J., Anderson, C., Baumgardner, J., Gable, C. W., Hager, B. H., O’Connell,
methods for infinite Prandtl number thermal convection in two-dimensional Cartesian
geometry. Geophysical and Astrophysical Fluid Dynamics 55, 137–160.


Numerical modelling of spontaneous slab breakoff and subsequent topographic response

Abstract

We conducted a set of numerical experiments to study the evolution of a subduction-collision system subject to spontaneous slab breakoff. The study takes into account complex rheological behaviour including plasticity, viscous creep and Peierls creep. By varying the oceanic slab age and initial plate convergence rate, four different end-members were observed. In this parameter space, breakoff depth can range from 40 to 400 km. Each of those breakoff modes display complex rheological behaviour during breakoff. Peierls creep in olivine turns out to be a key mechanism for slab breakoff, generally causing slabs to break earlier and at shallower depths. Models involving different depths of breakoff are subject to different topographic evolution, but always display a sharp breakoff signal. Post-breakoff uplift rates in foreland and hinterland basins range between 0.1 km/My for deep detachment and 0.8 km/My for shallow detachment. Our systematic study indicates an approximately linear relationship between the depth of breakoff and the rate of uplift. Continental crust subduction was observed in breakoff experiments involving oceanic lithosphere older than 30 My. Different exhumation processes such as slab retreat and eduction occur according to the depth of breakoff. These models are likely to undergo large rebound following breakoff and plate decoupling if the subducted oceanic slab is old enough.

\[1\] This chapter co-authored by T. Duretz, T.V. Gerya, and D.A. May (Tectonophysics Vol. 502 (1–2) (2010), 244–256)
3.1 Introduction

Slab breakoff, or slab detachment, is often referred to as an early collisional process during which a part of subducted slab detaches from subducting plate. Such a rupture would cause the detached slab to sink down into the Earth’s mantle and cause a major thermomechanical re-equilibration at the orogen scale. Slowdown of subduction rate is often thought to be the driving mechanism of slab breakoff. Slowdown can be achieved with different scenarios. For example, ridge subduction or subduction of buoyant continental material may strongly decrease subduction rate. This results in large extensive stresses which can then develop in the hanging slab and subsequent slab breakoff can occur. The concept of slab breakoff comes from the analysis of seismic tomography data where positive seismic velocity anomalies can be observed under collision zones are often interpreted as detached slab segments (Wortel and Spakman, 2000). Many other observations suggest the possibility of slab detachment in subduction-collision setting such as seismicity patterns in subduction zones (Chen and Brudzinski, 2001; Sperner et al., 2001), magmatism geochemistry and surface expression of volcanism (Keskin, 2003; Qin et al., 2008; Ferrari, 2004), uplift data analysis (Wilmsen et al., 2009) and interpretation of structural and petrological data for exhumation of UHP rocks (Andersen et al., 1991). Many quantitative slab breakoff studies have already been carried out by means of analytical, semi-analytical, analogue and numerical models, most of those studies involved the evolution of an already subducted slab (Davies and von Blanckenburg, 1995; Ton and Wortel, 1997; van de Zedde and Wortel, 2001; Buiter et al., 2002; Gerya et al., 2004; Cloetingh et al., 2004; Toussaint et al., 2004; Li and Liao, 2002; Andrews and Billen, 2009; Regard et al., 2008; Macera et al., 2008). Those studies propose different depth ranges where slab breakoff could occur. In van de Zedde and Wortel (2001) detachment occurs at shallow depths (∼35 km), in Gerya et al. (2004) detachment occurs at depths greater than 100 km. On the other hand, Andrews and Billen (2009) predict different breakoff depth ranges related to different mechanisms. They introduce two categories of slab breakoff: a shallow breakoff mode (∼150 km) involving a weak slab and fast plastic yielding, and a deep (∼300 km) breakoff mode involving stronger slabs characterised by slow thermal yielding. In contrast with preceding studies, Baumann et al. (2009) used a different setup allowing spontaneous slab detachment after a period of oceanic subduction including the effects of phase transitions. In this study breakoff depths ranged between 400 and 600 km. Few studies quantitatively examined the topographic response to slab detachment. Buiter et al. (2002) predicted topography uplift
in the range of 2 to 6 km using an elastic model whereas Gerya et al. (2004) predicted lower uplift values (< 2 km) using a viscoplastic model. In this paper, we examine a set of spontaneous slab breakoff numerical experiments using a similar approach to Baumann et al. (2009). We focused the study on the effect of oceanic plate age and convergence rate on slab breakoff, whilst taking into account complex rheological behaviour including Peierls mechanism (Kameyama et al., 1999). Our intent is to (i) identify different slab breakoff modes and characterise the rheological mechanisms responsible for each end-member, and (ii) describe the topographic evolution and orogenic development above a collisional zone subject to slab breakoff.

3.2 Model setup

3.2.1 Numerical code description

The numerical experiments were conducted in two dimensions using the thermo-mechanical code I2VIS (Gerya and Yuen, 2003a). This numerical code is based on conservatives finite differences and non-diffusive marker-in-cell technique taking into account visco-plastic rheologies. Both 2D Stokes flow and continuity equations are solved on a staggered finite difference grid. The system of equation takes the form:

\[
\begin{align*}
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} &= \frac{\partial P}{\partial x} \\
\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \sigma_{zx}}{\partial x} &= \frac{\partial P}{\partial z} - g\rho(T, P, C, M) \\
\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} &= 0
\end{align*}
\]

In equation 1 and 2, \(\sigma_{xx}, \sigma_{zz}, \sigma_{xz}\) are the deviatoric stress tensor components. The density \(\rho\) is an explicit function of the temperature \((T)\), the pressure \((P)\), the composition \((C)\) and the mineralogy \((M)\). The heat equation is solved in a Lagrangian manner and is expressed in the following way:
\[ \rho C_p \frac{DT}{Dt} = -\frac{\partial q_x}{\partial x} - \frac{\partial q_z}{\partial z} + H_r + H_T + H_s \]  
(3.4)

\[ q_x = -k(T, C) \frac{\partial T}{\partial x} \]  
(3.5)

\[ q_z = -k(T, C) \frac{\partial T}{\partial z} \]  
(3.6)

\[ H_T = \left(1 - \rho \left(\frac{\partial H}{\partial P}\right)_T\right) \frac{DP}{Dt} \]  
(3.7)

\[ H_s = \sigma_{xx} \dot{\epsilon}_{xx} + \sigma_{zz} \dot{\epsilon}_{zz} + \sigma_{xz} \dot{\epsilon}_{xz} \]  
(3.8)

\[ C_p = \left(\frac{\partial H}{\partial T}\right)_P \]  
(3.9)

Such a formulation takes into account the effect of radioactive heating \((H_r)\), isothermal compression/decompression effects \((H_T)\), shear heating \((H_s)\). Thermal conductivity \(k(T, C)\) is a function of both temperature and composition (cf table 3.1). \(q_x\) and \(q_z\) are the components of the heat flux vector, \(\dot{\epsilon}_{xx}, \dot{\epsilon}_{zz}, \dot{\epsilon}_{xz}\) are the components of the strain rate tensor, \(H\) is the rock enthalpy at the current timestep. The isobaric heat capacity \((C_p)\) and isothermal compression term are computed at each timestep and takes into account the effect of latent heat due to phase transformations. Evaluation of \(H_T\) and \(H_s\) terms requires information from the pressure and stress tensor components. These terms are thus computed after solving Stokes equations. The heat equation can then be solved taking into account the effect of isothermal compression, phases changes and shear heating at the current timestep.
Material & $k (W/m/K)$ & $H_r (W/m^3)$ & $C_p (J/kg)$ \\
--- & --- & --- & --- \\
Sediments & $0.64 + \frac{807}{T+77}$ & $1.50 \times 10^{-6}$ & 1000 \\
Upper continental crust & $0.64 + \frac{807}{T+77}$ & $1.00 \times 10^{-6}$ & 1000 \\
Lower continental crust & $1.18 + \frac{474}{T+77}$ & $0.25 \times 10^{-6}$ & 1000 \\
Upper oceanic crust & $0.64 + \frac{1293}{T+77}$ & $0.25 \times 10^{-6}$ & 1000 \\
Lower oceanic crust & $1.18 + \frac{1293}{T+77}$ & $0.25 \times 10^{-6}$ & 1000 \\
Mantle & $0.73 + \frac{1293}{T+77}$ & $2.20 \times 10^{-8}$ & 1000 \\
Weak zone & $0.73 + \frac{1293}{T+77}$ & $2.20 \times 10^{-8}$ & 1000 \\

Table 3.1: Thermal parameters of materials used for the experiments. Temperature dependent thermal conductivity $k$ is documented in [Clauser and Huenges (1995)]. $H_r$ is the radioactive heat production and $C_p$ is the isobaric heat capacity.

All the local rock properties (density, heat capacity, thermal expansion) are updated at each timestep according to Gibbs energy minimisation. [Gerya et al. 2004; Baumann et al. 2009].

The rheologies used in the experiments are visco-plastic. Viscous creep is computed in terms of deformation invariants and depends on strain rate, temperature, and pressure [Ranalli 1995]. The viscous component of the deformation is calculated as a combination of diffusion ($\eta_{\text{diff}}$) and dislocation creep ($\eta_{\text{disc}}$).

$$\frac{1}{\eta_{\text{creep}}} = \frac{1}{\eta_{\text{diff}}} + \frac{1}{\eta_{\text{disc}}}$$

(3.10)

The calculation of the viscosity associated with dislocation creep regime is formulated as follows:

$$\eta_{\text{creep}} = (\dot{\epsilon}_{II})^{(1-n)/2n} (A)^{-1/n} \times e^{(\frac{E_a + P V_a}{nRT})}$$

(3.11)

where $\dot{\epsilon}_{II}$ is the second invariant of strain rate, $A$, $E_a$, $V_a$, $n$ are respectively, a material constant, the activation energy, the activation volume, and the stress exponent. Those material properties are determined from laboratory flow experiments and are given in table 3.2. Variations of flow parameters (activation volume) with phase transitions are not included.
A smooth transition between diffusion creep and dislocation creep is assumed to occur at approximately 30 kPa \cite{Turcotte1982}. By equating strain rate invariants for dislocation creep and diffusion creep at the stress transition, an expression for diffusion creep viscosity is derived as a function of the dislocation creep flow law.

Plasticity is implemented using the Druker-Prager yield criterion \cite{Ranalli1995}. The calculated creep viscosity is therefore limited in such manner:

\[
\eta_{\text{creep}} \leq \frac{C + P \sin(\phi)}{(4\dot{\epsilon}_{II})^{1/2}}
\]

(3.12)

where \(C\) is the cohesion and \(\phi\) is the internal angle of friction.

The Peierls mechanisms \cite{Kameyama1999}, or exponential creep, limits the strength of material at high pressure. This mechanism takes over from dislocation creep in areas of large stresses. The viscosity of a material flowing according to Peierls creep can be expressed in such manner:

\[
\eta_{\text{Peierls}} = \frac{1}{A_{\text{Peierls}}} \frac{1}{\sigma_{II}} \exp \left[ \left( \frac{E_a - PV_a}{RT} \right) \left( 1 - \left( \frac{\sigma_{II}}{\sigma_{\text{Peierls}}} \right)^k \right)^q \right]
\]

(3.13)

where \(\sigma_{II}\) is the second stress invariant, \(\sigma_{\text{Peierls}}\) is a stress value that limits the strength of the material. \(A_{\text{Peierls}}, k,\) and \(q\) are material constants derived from laboratory experiments which indicate that \(k\) should range between 0 and 1, and \(q\) should range between 1 and 2 for rocks \cite{Kocks1975}. In subduction modelling, slabs of olivine rheology are subject to large stresses and low temperature in contrast to the surrounding asthenosphere \cite{Katayama2008}. In this context Peierls creep is often activated at stresses which may be much lower than the material strength value, \(\sigma_{\text{Peierls}}\).

Following the approach of coupled petrological-thermo-mechanical modelling \cite{Gerya2006, Baumann2009}, phase transitions are calculated at every timestep based on Gibbs free energy minimisation \cite{Vasilyev2004}. All local material properties such as density, heat capacity, thermal expansion, adiabatic and latent heating are calculated from thermodynamics in order to better constrain the effect of phase transitions on subduction processes.

Surface processes are implemented using a gross-scale erosion sedimentation law. In this approach, the interface crust/air or crust/water is tracked on a eulerian grid. This surface
\[ \eta = \eta_0 \cdot a \cdot \exp \left( \frac{-E_a}{R T} \right) \]

\[ V_a = \frac{E_a}{R T} \]

\[ C = C_0 \cdot \exp \left( \frac{-E_c}{R T} \right) \]

Table 3.2: Rheological parameters of materials used for the experiments. \( \eta \) is the reference viscosity, \( n \) is the stress exponent, \( E_a \) is the activation energy, \( V_a \) is the activation volume, \( \phi \) is the friction angle, and \( C \) is the cohesion. Parameters used for Peierls mechanism are \( A_{\text{Peierls}} = 10^{7.8} \times 10^{-12} \), \( \sigma_{\text{Peierls}} = 2.9 \) GPa for wet olivine rheology (Katayama and Karato 2008), and \( \sigma_{\text{Peierls}} = 9.1 \) GPa for dry olivine rheology (Evans and Goetze 1979).
evolves as an erosion/sedimentation level according to the transport equation (Gerya and Yuen [2003b])

\[
\frac{\partial z_{es}}{\partial t} = v_z - v_x \frac{\partial z_{es}}{\partial x} - v_s + v_e
\] (3.14)

where \(z_{es}\) is the vertical position of the tracked interface, \(v_x\) and \(v_z\) are the material velocities interpolated on this interface, \(v_e\) and \(v_s\) are the gross-scale erosion/sedimentation rates defined with regard to a reference water level. Values used for this study are 0.1 mm/yr for both \(v_s\) and \(v_e\).

### 3.2.2 Setup of the experiment

The experiments were performed in a 4000 \(\times\) 1400 kilometre box. The models used a grid resolution of 1361 \(\times\) 351 nodes with variable grid spacing. This allowed a minimum grid resolution of 1 kilometre in the area subject to largest deformation. 13 million Lagrangian markers carrying material properties were used in each experiment. Two 1700 km long continental plates were separated by a 700 km long oceanic plate. Both continental plates and oceanic are composed of an upper crust, lower crust and lithospheric mantle (Fig. 3.1). The rheological parameters used in the experiments are presented in table 3.2.
Figure 3.1: Description of the model box used for the experiments. Full box initial geometry and crop on the oceanic segment with initial thermal structure (plate age = 20 My). Initial plate rate are applied at $x = 1000$ km for the left plate and $x = 3000$ km for the right plate.

The boundary conditions used in this setup are free slip on all boundaries, except at the lower boundary which is treated as a permeable boundary satisfying an external free slip boundary condition (Burg and Gerya, 2005; Ueda et al., 2008). In order to let the
Figure 3.2: Description of the results of the parametric study. Discrimination of four different end-members, depths of breakoff and time of breakoff for each experiment. Time of breakoff is incremented after forced convergence is stopped.
topography build up, a free surface is simulated at the air/lithosphere interface by setting the viscosity of the air (\(10^{19}\) Pa.s) to be at least two orders of magnitude lower than the crust \cite{Schmeling2008}. The thickness of the air layer used in the simulations was 10 km. The thermal structure of the oceanic lithosphere was calculated from the half space cooling model for a given plate age \cite{Turcotte1982}. The initial continental geotherm was set using values of 0 °C at the top and 1344 °C at the bottom of the lithosphere (140 km thick). The thermal gradient used within the asthenosphere was quasi-adiabatic (0.5 °C/km). In order to initiate subduction a weak zone was imposed on the right ocean-continent transition. The weak zone cuts through the whole lithospheric mantle with an angle of 30 degrees and is characterised by weak plastic strength (1 MPa) and wet olivine rheological parameters \cite{Ranalli1995}. Despite the fact that subduction initiation can be modelled in a geologically more relevant manner \cite{Gerya2008,Ueda2008,Nikolaeva2010}, the use of such a weak zone remains useful for our specific setup. A fixed convergence rate is imposed on both continental plates until 500 km of oceanic crust has been subducted, afterward, subduction and subsequent collision are only driven by slab pull.

### 3.3 Model results

In our study we systematically explored a two dimensional parameter space. The parameters which were varied are the original convergence rate of the plates which is applied until 500 km of convergence is reached and the age of the oceanic lithosphere (Fig. 3.2). The convergence rate is symmetric, each plate is pushed with the same constant velocity until 500 km of total convergence is reached. This enables us to reach a point where the weight of the hanging slab is heavy enough to drive the subduction-collision system. The age of the oceanic segment is constant along the plate, varying this parameter enables us to indirectly vary the original plate strength by modifying its initial temperature structure. By independently varying those parameters, we observed four different end-members of slab breakoff and model evolution. Those different end-members are characterised by different slab breakoff depths linked to activation of different rheological mechanisms. Each model exhibits a different post breakoff evolution which can be monitored in the evolution of the topographic signal. Results of the parametric study can be seen in Fig. 3.2 where different modes of slab breakoff are displayed as a function of the varied parameters.

In all experiments, slab breakoff events can be identified by monitoring stresses in the slab.
A slab detachment episode is responsible for a large stress drop within the slab (Fig. 3.3b) and a very sharp increase of the strain rate within the slab (Fig. 3.3c).

Figure 3.3: a) Second stress invariant map before and after slab detachment. Contours highlight interfaces between atmosphere, crust, mantle lithosphere and asthenosphere. b) Evolution of the average stress (second invariant) in the slab necking zone through time. The dashed line indicates the breakoff event. c) Evolution of the average strain rate (second invariant) in the slab necking zone through time. The dashed line indicates the breakoff event. Time is incremented from the start of the simulation.

3.3.1 Shallow slab breakoff

Our modelling results suggest that shallow slab detachment is likely to occur in young oceanic lithosphere (Fig. 3.4) with fast plate convergence rates. In this particular setting, fast breakoff takes place soon after the continental plates reach the collision stage (~1 My) and continental crust subduction does not develop. Detachment is localised at the ocean-continent transition and the depth of the rupture is very shallow (~40 km). Once the rupture is initiated, detachment of the oceanic lithosphere occurs along the subduction plane, triggering the opening of an asthenospheric window under the collision
zone. Asthenospheric inflow at a subcrustal level may have some major consequences for magmatism and volcanism. Rheological mechanisms responsible for this mode of slab breakoff are viscous creep in the mantle lithosphere, whereas the crust behaves in a plastic (brittle) manner (Fig. 3.5).

The topographic evolution map (Fig. 3.6) displays a convergence stage with the formation of a deep trench and accretionary wedge. A collision signal can be observed as the topography reaches its peak value on the order of 2 km. This is the consequence of squeezing the accretionary wedge between the colliding continental plates. After approximately 2 My, slab breakoff occurs and produces a sharp topographic signal. Extension related to the opening of the asthenospheric window produced a collapse of the topography. An uplift of few kilometres (∼ 4 km) related to breakoff is recorded above the former subduction zone. The topographic response is fast (∼ 5 My) which produces an average uplift rate of 0.8 mm/yr. Finally, the system stabilises and surface processes tend to flatten out the topography over a long timescale (∼ 15 My).
Figure 3.4: Four stages evolution of the shallow slab breakoff end-member. a) Oceanic subduction. b) Continental collision, necking in the mantle lithosphere. c) Breakoff d) Post breakoff rebound and asthenospheric window. Time is incremented from the start of the experiment. Origin of the z distance axis is the top of the box (including the 10 km thick air layer).
Figure 3.5: Rheological mechanisms activated during shallow slab breakoff. Brittle, viscous, and Peierls mechanism strain rates. At breakoff location, the crust deforms in a plastic manner (brittle) and the mantle lithosphere deforms viscously. Dark blue means that mechanism is locally not activated during the process. Contours highlight interfaces between atmosphere, crust, mantle lithosphere and asthenosphere.
3.3.2 Failed shallow slab breakoff

Using a slow initial total convergence rate (2.5 cm/yr), a second end-member of young oceanic slab breakoff was observed (Fig. 3.7). In this particular case, breakoff does not localise at the ocean-continent transition, but instead occurs at a much deeper level (≈ 400 km). As in the shallow breakoff mode, convergence stops after both continental plates meet, the weight of the hanging slab is not sufficient to overcome the buoyancy of continental crust. As the convergence velocity was originally slow, the subducted slab is hotter (i.e. both less strong and less dense) and extensional stresses are not high enough to cause a shallow slab rupture. Similar bifurcation with sharp changes in the slab detachment depth

**Figure 3.6:** Topography evolution through time above the subduction-collision zone for each end-member. For each case, a sharp breakoff signal and subsequent uplift is observed. The overall topographic signal can be divided in time windows corresponding to different geodynamic stages.
were also observed in previous breakoff models with relatively warm slabs (Gerya et al., 2004). Detachment occurs approximately 5 My after the termination of subduction (Fig. 3.8) the main deformation mechanism activated during the breakoff was viscous creep.

The topographic evolution shows similarities with the fast shallow breakoff mode (Fig. 3.6). Topography reaches its peak value during the collision stage (~ 4 km). The slab breakoff topographic signal is more diffuse than in the fast shallow breakoff mode but post breakoff uplift last longer (~ 20 My) and uplift rates are on the order of 0.2 mm/yr. In each of those breakoff end-members involving young oceanic slabs, the topography after breakoff is relatively low and the width of the orogen is narrow (~ 100 km).
Figure 3.7: Evolution of the failed shallow slab breakoff end-member. a) Oceanic subduction. b) Continental collision. c) Necking of the slab d) Breakoff and rebound. Time is incremented from the start of the experiment. Origin of the $z$ distance axis is the top of the box (including the 10 km thick air layer).
Chapter 3. Slab Breakoff and Topographic Response

Figure 3.8: Rheological mechanisms activated during failed shallow slab breakoff. Brittle, viscous, and Peierls mechanism strain rates. At breakoff location, the crust and mantle lithosphere deform viscously. Dark blue means that a mechanism is locally not activated during the process. Contours highlight interfaces between atmosphere, crust, mantle lithosphere and asthenosphere.
3.3.3 Intermediate depth slab breakoff

Models involving an oceanic lithosphere older than 30 My and with total convergence rates faster than 2.5 cm/yr display significantly different behaviour (Fig. 3.9). In this case, the oceanic lithosphere is dense enough to initiate continental crust subduction and strong enough not to yield before the buoyant crust is subducted deep enough. Breakoff occurs at the ocean-continent transition a few million years after the collision stage initiated (∼4 My). The rupture occurs at the depth of approximately 200 km during the stage of slab steepening. The rupture is due to the combined action of viscous creep in the hotter crust and mantle of the two outer layers of the slab and Peierls mechanism in the cold mantle lithosphere of the slab core (Fig. 3.10). As soon as breakoff ends, the orogen undergoes a short period of extension, coeval with eduction of the buoyant continental crust (Andersen et al., 1991).

The topographic signal shows distinct signals for convergence, collision, slab breakoff and subsequent rebound (Fig. 3.6). Trench depth notably increases and reaches a rather unrealistic peak value greater than 10 km. During the collision stage, the overriding plate displayed positive topography of ∼4 km in height. Post breakoff rebound linked to the eduction triggered the formation of positive topography on the subducted plate and further disappearance of the trench. The uplift rate is 0.5 mm/yr with a rebound of 5 km in approximately 10 My. Topography on both side of the suture reached a value of ∼4 km in height. Finally, the width of the orogen reached ∼300 km.
Figure 3.9: Snapshots through the evolution of the intermediate depth slab breakoff end-member. a) Oceanic subduction. b) Continental collision and continental subduction. c) Breakoff d) Eduction and extension. Time is incremented from the start of the experiment. Origin of the $z$ distance axis is the top of the box (including the 10 km thick air layer).
Figure 3.10: Rheological mechanisms activated during intermediate depth slab breakoff. Brittle, viscous, and Peierls mechanism strain rates. At breakoff location, the crust deforms in a visco-plastic (brittle) manner and the mantle deforms both viscously and by Peierls mechanism. Dark blue means that mechanism is locally not activated during the process. Contours highlight interfaces between atmosphere, crust, mantle lithosphere and asthenosphere. Origin of the $z$ distance axis is the top of the box (including the 10 km thick air layer).
3.3.4 Deep slab breakoff

By using an oceanic lithosphere of age greater than 60 My for any convergence rate, or using a younger oceanic lithosphere with slow convergence rate, deep slab breakoff was observed (Fig. 3.11). For this end-member, the depth of the detachment is above 250 km deep and breakoff can occur around 10 My after the start of collision. The strong and dense slab is able to drag down the continental crust to a depth above 200 km before detachment occurs at the ocean-continent transition. Prolonged slab steepening initiates plate decoupling and slab retreat. Breakoff occur during slab retreat in a pure shear manner at the ocean-continent transition. The rupture is triggered by a complex activation of viscous creep (hot outer slab layers) and Peierls mechanism (colder slab core) in the mantle lithosphere and viscous creep in the crust (Fig. 3.12). A short episode of eduction was observed after breakoff but eduction is not the dominant exhumation mechanism is this model. Plate decoupling triggers the exhumation of the buoyant crustal material in the subduction channel and the opening of an asthenospheric window under the orogen.

The evolution of topography above the orogen is mostly similar to the intermediate depth breakoff end-member (Fig. 3.6). The main differences are the prolonged collision stage before breakoff and the post breakoff evolution. As the plates are decoupled, the orogen is free to undergo large scale extension after slab detachment. The uplift rate is 0.2 mm/yr with an uplift of 2 km in 10 My. Most of the topography is accommodated on the subducted plate during extension, with a maximum topography greater than 5 km in height. Total width of the orogen reaches 400 km at this stage.
Figure 3.11: Deep slab breakoff end-member evolution in four stages. a) Oceanic subduction. b) Continental collision and continental subduction. c) Slab retreat, breakoff and eduction d) Plate decoupling, rebound and extension. Time is incremented from the start of the experiment. Origin of the $z$ distance axis is the top of the box (including the 10 km thick air layer).
**Figure 3.12:** Rheological mechanisms activated during deep slab breakoff. Brittle, viscous, and Peierls mechanism strain rates. In the necking zone, the crust deforms viscously and the mantle lithosphere deforms both viscously and by Peierls mechanism. Dark blue means that mechanism is locally not activated during the process. Contours highlight interfaces between atmosphere, crust, mantle lithosphere and asthenosphere.
3.4 Discussion

3.4.1 Modelled and observed surface uplift rates

A strong correlation between average uplift rates and depth of breakoff was observed in the experiments. Fast uplift rates can be expected in the case of shallow slab detachment whereas deeper detachment provides slower uplift rates. The magnitude of the modelled uplift rates ranged from 0.8 to 0.1 mm/yr (Fig. 3.13a).

Geological and geophysical studies suggest natural prototypes where uplift is related to slab breakoff. Such uplift rates are estimated around 0.3 mm/yr in Borneo (Morley and Back, 2008) and 0.25 - 0.5 mm/yr in Central America (Rogers et al., 2002). Those measurements are consistent with uplift rates calculated in our study. According to the surface processes parameters used in the simulations, comparing modelled uplift rates and measured uplift rates may give additional information on the depth and timing of slab detachment in the studied areas. A relation between the time of breakoff and the start of the post breakoff uplift has also been observed in our simulations (Fig. 3.13b). This time offset is strongly dependent on the depth of slab detachment and it decreases as the depth of breakoff decreases.

3.4.2 Rheology of the lower crust

The rheology of the lower crust plays a crucial role on the ability of the crust to support topography (Clark et al., 2005). In our parametric study, we considered a rather strong lower continental crust of plagioclase rheology ($\text{An}_{75}$ from Ranalli, 1995). By using of a weaker material (quartzite flow law) to simulate lower crust, comparable simulations would produce smoother topography. Fig. 3.14 shows a comparison between two intermediate depth breakoff models, one including a plagioclase lower crust and the other including a quartzite lower crust.

The model including a quartzite lower crust display a broader orogen before and after slab detachment. Using a weak lower crust enables the building of topography on the lower plate during subduction, less crustal material is entrained in the subduction channel. This has the consequence to weaken the slab and therefore detachment occurs earlier ($\sim$ 2 My) and shallower ($\sim$ 20 km) than in the models including a plagioclase lower crust. The weak lower crust induces a slower but broader topographic response on the lower plate. Slower
Figure 3.13: a) Correlation between average uplift rate in the foreland basin and breakoff depth. Uplift rates are averaged over the 5 My following slab breakoff. b) Time delay between slab detachment and breakoff depth.
3.4.3 Importance of Peierls mechanism

In contrast to previous slab breakoff studies using the I2VIS numerical code (Gerya et al., 2004; Baumann et al., 2009), this study includes the effects of Peierls mechanism which brings us closer to natural processes. Activation of Peierls mechanism by large extensional stresses in the cold core of the hanging slab is of first order importance for slab breakoff
modelling. Drastic discrepancies were observed between similar model setups in which the only difference was the inclusion of Peierls mechanism. Timing and depth of breakoff, along with the topography were also different. In Fig. 3.15, a model including Peierls mechanism show faster (∼10 My) and shallower breakoff. Topography evolution starts differing ∼10 My after the start of the experiment. Late orogenic development and the final geometry are also strongly affected.

![Figure 3.15: Comparison between models with and without Peierls mechanism. Parameters used in this setup are an oceanic lithosphere age of 40 My and initial total plate rate of 10 cm/y. Origin of the z distance axis is the top of the box (including the 10 km thick air layer).](image)

3.4.4 Metastability of olivine

As our petrological approach is based on Gibbs energy minimisation, the experiments do not take into account the potential effect of metastable reactions. Schmeling et al. (1999) discussed the importance of metastable olivine in subducting slab. In their experiments, old slabs can potentially contain a significant volume of metastable olivine on the prograde
path. A buoyant olivine wedge has a tendency to slow down subduction, hence influencing the dynamics. Subduction slowdown is regarded as one of the mechanism triggering slab detachment (Li and Liao, 2002). Although it is not clear that slabs contain large amount of metastable olivine, its buoyant effect may influence both depth and timing of slab breakoff.

### 3.4.5 Orogenic evolution and exhumation mechanisms

In our study, subduction-collision models involving an oceanic crust older than 30 My are likely to trigger continental crust subduction. Intermediate depth and deep slab breakoff end-members are likely to occur for such slab ages. Both of these end-members display different exhumation mechanisms, namely a coherent crustal nappe exhumation due to eduction and buoyant flow of subducted crust triggered by slab retreat (Fig. 3.16). In our experiments, both eduction and rollback require subduction of a significantly long oceanic slab (∼ 400 km) before the onset of collision.

- For deep slab breakoff, slab steepening initiates plate decoupling and slab retreat, triggering exhumation of the crust in a buoyant flow manner. In this case, exhumation is likely to start before breakoff. Models involving old oceanic lithosphere develop complete plate decoupling and inflow of asthenospheric material at a shallow level (Fig. 3.11) and undergo a major topography building phase during post breakoff extension (Fig. 3.6).

- For intermediate depth breakoff, rupture occurs during slab steepening and thus prevents further steepening to trigger slab retreat and plate decoupling. Once breakoff ends, exhumation of the subducted crust occurs along the subduction plane with a normal sense motion. This mechanism is known as eduction and has already been suggested to explain the exhumation of UHP rocks (Andersen et al., 1991).

### 3.4.6 Evolution of stresses in the lithosphere

A slab detachment event is responsible for a stress redistribution within the lithosphere. This can be observed in Fig. 3.3a which shows the stress distribution within the model lithosphere before and after breakoff. Release of the gravitational stresses during breakoff is followed by an increase of stress in the lower plate. This increase of stress is related to the positive buoyancy of the subducted continental crust which is no longer driven down by
the slab. The increase of stress is controlled by the amount of eduction as this mechanism allows, to some extent, the exhumation of the buoyant crust. An overall increase of stress
(lower and overriding plate) is also noticeable in the crustal part of the model which is subject to topography building and loading.

### 3.4.7 Comparison with preceding studies

In comparison with preceding studies (Davies and von Blanckenburg, 1995; Li and Liao, 2002), we do not observe a systematic correlation between the initial plate rate, or the subduction velocity, and depth of slab detachment. The influence of the initial plate rate is clearly noticeable for young slabs, where a sharp transition between shallow and failed shallow breakoff occurs for initial plate rate lower than 5 cm/yr. For older slabs, depth of detachment does not show such a clear dependance on the initial convergence rate since different breakoff modes can affect slabs of the same age but with a different initial plate rate.

In our study, shallow slab breakoff displays comparable results with van de Zedde and Wortel (2001) in which asthenospheric material can rise up to subcrustal depths. However, in our case, it does not necessarily require continental crust subduction or a slow convergence rate as the failed shallow slab breakoff mode can take over if the initial convergence rate is too slow (less than 7.5 cm/yr).

Intermediate depth slab breakoff displays the same breakoff depths and occurs on the same timescale (5-7 My after stopping the convergence) as the shallow plastic breakoff in Andrews and Billen (2009), despite the fact that they do not utilise the same rheological mechanisms and that initial setups differs from the one used here.

Our deep breakoff depth is comparable to the deep viscous breakoff in Andrews and Billen (2009). However, in our models the timing of rupture is shorter (10-20 My) than in their experiments (15 to 25) for slabs of the same age. This is likely due to the activation of Peierls mechanisms in the mantle lithosphere which enables breakoff to occur earlier (Fig. 3.15).

Rheological mechanisms responsible for slab breakoff are different for each type of end member as suggested by Andrews and Billen (2009), with the exception of the failed shallow slab breakoff end-member in which many deformation mechanisms are likely to be activated during the rupture. Plastic behaviour (Druker-Prager) is only important for shallow slab breakoff however this only affects the crust. Peierls appears to be a key
mechanism for slab breakoff dynamics (Fig. 3.15). This mechanism plays an important role in the mantle lithosphere for intermediate depth and slab breakoff.

Fluid/melt related weakening effects are not taken into account in this study, however they can potentially have a strong influence on the mechanism of breakoff and the overall orogenic evolution. Faccenda et al. (2008) discuss the influence of fluid percolation on the orogenic style (coupled or decoupled plates). In our models, despite the lack of fluids, we observe both coupled and decoupled orogens. Plate decoupling (slab retreat) occurs in the case of old slabs (80 My) which are dense, but strong enough to initiate a rollback event before breakoff. Including the effect of fluids may drastically enhance the amount of rollback.

In our study we frequently observed a deep topography at the subduction trench. Trench depths larger than 10 km are not representative of present-day Earth subduction zones. This effect might be related to the fact that modelling approach employed a visco-plastic rheology, which does not include the effects of elasticity. Including a visco-elasto-plastic rheology enables the production of more realistic topographic profiles throughout subduction simulations (Sobolev and Babeyko 2005).

3.4.8 Possible application to natural prototypes

Different modes of slab breakoff can be identified. They are characterised by the depth of breakoff, timing of the detachment, activation of different rheological mechanisms, style of topographic evolution and orogenic development. These end-members can potentially be applied to many natural prototypes where surface observables such as uplift rates or heat-flow can be measured and where the geophysical data such as tomography or hypocenter location supports the possibility of slab breakoff.

Areas such as Eastern Anatolia (Turkey) where magmatism witnesses from deep heating and where mantle lithosphere is shallow to absent (Keskin 2003) would favour the possibility of shallow slab breakoff. On the other hand, large orogens such as Caledonides where breakoff and eduction were suggested as a possible mechanism of UHP rocks exhumation (Andersen et al. 1991) would rather favour an intermediate or deep slab breakoff scenario.
3.5 Conclusions

Our modelling study provides a wide range of topographic behaviour linked to different slab breakoff end-members. The study takes into account a more realistic complex rheology which includes Peierls mechanisms. This mechanism turns out to play an important role for the depth and timing of slab detachment and is an important deformation mechanism for several breakoff end-member.

The four different slab breakoff modes display significantly different topographic development. The evolution of the models involves different geodynamic processes such as slab steepening or retreat and slab breakoff is responsible for the sharpest signal in the topography evolution.

A correlation between the depth of detachment and the surface uplift rate recorded in the foreland and hinterland basins has been observed; the shallower the breakoff is, the faster the uplift rate will be. In our context, exhumation of subducted buoyant crust can occur by either slab retreat and buoyant flow of crust in the extending subduction channel or by coherent eduction of deeply subducted continental plate.

Bibliography


Dynamics of slab detachment

Abstract

We investigate the dynamics of slab detachment around the detachment zone and evaluate the amount of time necessary for slabs to detach. The study combines results of two-dimensional (2D) state-of-the-art thermo-mechanical numerical simulations and a 1D analytical solution for viscous necking under gravity.

We show that the dominant deformation mechanism during slab detachment is viscous necking, independent of the depth of slab detachment. When the slab dip is moderate (35°–70°), slab detachment is partly affected by localized simple shearing in the colder parts of the slab. Brittle fracturing (breaking) plays a minor role during slab detachment.

Our 2D thermo-mechanical models indicate that the duration of slab detachment, quantified from the onset of slab thinning until the actual detachment (i.e. vanishing of slab-pull force), is relatively short (< 5 Ma) and can occur in less than 0.5 Ma. No clear correlation between the depth and the duration of slab detachment was observed. The simulations suggest that even deep slab detachment (> 250 km) can occur within a short time interval (< 1 Ma) which has implications for geodynamic interpretations using slab detachment as explanation for processes such as melting, exhumation or surface uplift.

The thinning of the slab during detachment, observed in 2D simulations, agrees well with predictions from a 1D analytical solution indicating that the 1D solution captures the first-order features of the detachment process. We also evaluate the impact of shear heating on the duration of slab detachment. The predictions of a simple semi-analytical solution,
based on dimensionless parameters, agree well with our and previously published results.

\[1\]

\[1\]This chapter was co-authored by T. Duretz, S.M. Schmalholz, and T.V. Gerya (submitted to G-cubed)
4.1 Introduction

In the last decades, slab detachment has become a popular geodynamic research topic in both the geological and geophysical communities. This convergent margin process involves the detachment of a portion of the slab during ongoing subduction. The idea of slab detachment was born from the interpretation of geophysical observations and was employed to explain seismicity patterns within the subducting slabs (Isacks and Molnar, 1969; Chatelain et al., 1993; Chen and Brudzinski, 2001; Kundu and Gahalaut, 2011). The slab detachment model further gained popularity with the development of seismic tomography and the detection of slab remnants within the Earth’s mantle (Wortel and Spakman, 1992; Widiyantoro and van der Hilst, 1996; van der Meer et al., 2010; Rogers et al., 2002; Levin et al., 2002; Schmandt and Humphreys, 2011; Zor, 2008) and especially underneath orogens (Lippitsch et al., 2003; Martin and Wenzel, 2006; Replumaz et al., 2010).

It is commonly accepted that slab detachment results from the development of extensional stresses within the downgoing plate. Subduction slowdown is considered to drive this stress build up (Li and Liao, 2002) and is associated to the subduction of ridges or buoyant continental material. Two major consequences of slab detachment can be distinguished: (1) a partial or complete loss of the slab pull force and (2) the inflow of hot asthenosphere at the location of the detachment. The first consequence is commonly used in the explanation of tectonic processes such as exhumation of high pressure rocks (Andersen et al., 1991; Babist et al., 2006; Xu et al., 2010), variations in surface uplift rates (Rogers et al., 2002; Morley and Back, 2008) and in the sedimentary record (Mugnier and Huyghe, 2006), orogenic extension (Zeck, 1996), or rapid changes in plate motions (Austermann et al., 2011). The second consequence is usually considered as an efficient mechanism to advect heat at lithospheric to sub crustal level (van de Zedde and Wortel, 2001), subsequently triggering partial melting and plutonism (Davies and von Blanckenburg, 1995; Ferrari, 2004; Altunkaynak and Can Genç, 2004). It has also been proposed that the entrainment of continental material by detached slabs in the mantle may play a role in the long term processes of crustal recycling (Hildebrand and Bowring, 1999).

Given the potential implications of slab detachment, it is important to better understand the dynamics of slab detachment. In particular, we investigate the first-order thermomechanical process eventually leading to slab detachment and also the duration of slab detachment. Moreover, since slab detachment involves lithospheric-scale strain localization, we investigate the effect of viscous heating on the dynamics of slab detachment.
Whereas thermo-mechanical models enable to test the influence of a variety of complicated model geometries, boundary/initial conditions, or material rheology (Duretz et al., 2011a), simple analytical models give fundamental insights into the dynamics of slab detachment (Schmalholz, 2011). In this study, we analyze and compare results of one-dimensional (1D) analytical and 2D numerical models in order to better understand the mechanism of slab detachment. The results of 2D simulations are used to quantify the duration of slab detachment, i.e. the time interval between the onset of thinning of the slab and the actual detachment of the slab. We refer to this time interval as slab detachment duration. Moreover, we evaluate the influence of shear heating on the slab detachment duration using predictions of a 1D semi-analytical solution.

4.2 Conceptual models of slab detachment

![Diagram of slab detachment](image)

**Figure 4.1:** Typical conceptual illustrations of slab detachment. a) Slab fracture as the result of tensile failure (corresponding to a mode I fracture). b) Simple shear model including the contributions of either plastic or viscous shear zones. c) Necking model resulting from the extension of a (power-law) viscous layer.

The detachment of a slab is often simplified in graphics as being the result of either a sharp fracture (e.g. (Nolet, 2009)), a shear zone (e.g. Sacks and Secor, 1990), or thinning
(necking) (e.g. Sacks and Secor (1990)). Neglecting the role of inherited structures, these schematic detachment models inherently involve contrasting physical modes of slab deformation: tensile brittle failure (Figure 4.1a), extension along a simple shear zone (Figure 4.1b), and pure shear necking (Figure 4.1c). The tensile failure model conveys the idea that the slab behaves as an homogeneous fragile plate for the pressure and temperature conditions of subduction zones. This model implies geologically instantaneous suction of the asthenosphere within the slab’s crack and is therefore very inclined to explain fast heat advection and melting events. The simple shear model (Sacks and Secor, 1990) requires localized shear deformation without explicitly suggesting any rheological behavior of the slab (either viscous, plastic or brittle). The slab necking (Sacks and Secor, 1990) implies that the slab deformation is accommodated by pure shear viscous or plastic deformation. In this model, the asthenosphere advection velocity is proportional to the thinning rate of the slab. The model of Lister et al. (2008) combines necking (boudinage) and shear zones to explain the observed slab’s morphology and the intra-slab seismicity.

On geological timescales, experimental and analytical studies suggest that olivine deforms by viscous deformation mechanisms such as grain-size sensitive and dislocation creep (Hirth, 2002; Karato, 2010; Faul et al., 2011; Rozel et al., 2011). At low temperature (slab-like conditions), the deformation of olivine is expected to occur in the low-temperature plasticity regime (Peierls mechanism) (Evans and Goetze, 1979; Kameyama et al., 1999; Raterron et al., 2004; Katayama and Karato, 2008). Consequently, a lithospheric-scale brittle (or breaking) behavior is not expected through entire slabs (Replumaz et al., 2010). However, the brittle slab break-off model is frequently visualized in sketches and graphics of slab detachment. We consider this break-off model as potentially misleading because it is often applied in geodynamic interpretations concerning both rheology and heat transfer. We consider the viscous shearing and necking models as more realistic thermo-mechanical models for the first-order deformation processes acting during slab detachment.
CHAPTER 4. DYNAMICS OF SLAB DETACHMENT

4.3 Insight from mechanical and thermo-mechanical modeling

4.3.1 Methodology

To study the dynamics of slab detachment, we use state-of-the-art 2D thermo-mechanical models of slab detachment (Baumann et al., 2009; Duretz et al., 2011a). The simulations are performed with the thermo-mechanical code I2VIS (methodology and numerical analysis described in Gerya and Yuen (2003a); Duretz et al. (2011b)). This numerical code solves the incompressible steady state momentum and heat conservation equations. Thermo-mechanical coupling is achieved by employing temperature-stress dependent viscosities and including viscous dissipation in the heat equation. The rheological model combines a diffusion-dislocation creep model with a Mohr-Coulomb stress limiter for each
material. Exponential creep (Peierls mechanism of Katayama and Karato, 2008) can only be activated in the mantle. The effects of mineral phase transitions are taken into account by modifying the material properties according to their current pressure and temperature. The density and heat capacity of the mafic and ultramafic rocks are computed according to Gibbs energy minimization (Connolly, 2005). The model assumes a pyrolitic mantle composition and a basaltic-gabbroic oceanic crust. The thermodynamic database is calculated for the chemical model CaO-FeO-MgO-Al₂O₃-SiO₂ (Gerya et al., 2004; Baumann et al., 2009). The density of the continental crust and sediments evolve according to an equation of state (Gerya and Yuen, 2003b). Our simulations are carried out in a 4000 × 1400 km size domain (Figure 4.2) and a maximum resolution of 1 km is achieved in the central part of the domain. Two 1300 km wide continents are separated by a 500 km wide oceanic basin and a weak zone is used to initiate the subduction. In order to generate sufficient slab-pull force, the model is initially kinematically driven using convergence rates ranging between 1 and 10 cm/a. During this stage, the continents decouple from the lateral sides of the model domain, leaving the space for asthenosphere upwelling and the development of ridges. Once the oceanic lithosphere is subducted, the kinematic constrain is deactivated and the model becomes dynamically driven by slab-pull. We refer to our previous studies (Baumann et al., 2009; Duretz et al., 2011a) for details concerning the setup and physical properties of the materials involved.

An evolved stage of continental collision is presented in Figure 4.2b. The large-scale features of our simulations are the sinking of the slab in the mantle and its interaction with the 660 km phase transition boundary that deviates the slab from its trajectory. The velocity vectors highlight the pattern of mantle flow triggered by the sinking lithospheric slab. As a consequence of the displacement of the continental plates, mantle upwelling and oceanic lithosphere generation takes place on lateral sides of the domain. The enlarged picture focuses on the location of the ongoing slab detachment and velocity arrows indicate the asthenospheric flow into the necking zone. In this simulation, the slab detachment occurs at the buried passive margin at a depth of 190 km.

4.3.2 The 2D kinematics of slab detachment

In a previous study (Duretz et al., 2011a), we showed that, depending on the initial thermal age of the oceanic lithosphere and the initial convergence rate, the depth of slab detachment can range between 35 and 400 km. We also showed that, according to their depth (shallow,
intermediate, deep), these detachments are affected by the activation of different deformation mechanisms within the slab. Figure 4.3 and Figure 4.4 show the detailed temperature and viscosity field during detachment for each of the three representative cases. The white contours emphasize the location of the crust/mantle and lithosphere/asthenosphere interfaces and thus enable distinguishing the geometry of the slabs. A common characteristic of these models is that slab detachment results in distributed weakening within the thinning zone. Another common feature is the progressive weakening of the asthenosphere
throughout the detachment. This shear thinning effect is the consequence of the power-law deformation behavior of the asthenospheric mantle at high stress. On the other hand, according to their depth, these slab detachment models are characterized by deformation kinematics that vary from mostly pure shear (necking) to the combination of simple shear and pure shear (shear necking). These two distinct mechanisms, that eventually lead to slab detachment, are described in the following sections (Sec. 4.3.3 and 4.3.4).

### 4.3.3 The quasi pure shear detachments: necking

![Figure 4.4: Evolution of the viscosity during shallow, intermediate, and deep slab detachment models. These cases are the results of large-scale thermo-mechanical simulations of plate collision. The right column displays the viscosity evolution in a purely mechanical finite element simulation of power-law necking under gravity. The white lines define the interface between the crust and the mantle lithosphere.](image)

The shallow and deep slab detachments both exhibit a lithospheric deformation behavior which is dominated by pure shear thinning. At shallow levels (35-100 km), slab detachment can occur as soon as two converging continental lithospheres are in contact. The thinning
of the slab takes place around the passive margin and is initiated at the lower boundary
of the slab. Necking occurs in a sub-horizontal direction and the slab eventually separates
from the crust by gliding along the subduction shear zone. The deep detachment occurs
at a depth range between 300 and 400 km and can either occur at the location of the
subducted margin or within the oceanic plate. The detachment process is characterized
by symmetric necking that initiates at both the upper and lower boundary of the slab.
In this model, extension occurs along a sub-vertical axis, minimizing the angle to the
vertical (20 to 15°). The rightmost column of Figure 4.4 depicts the viscosity evolution
of power-law necking under gravity obtained from a simple 2D finite-element calculation
(e.g. Schmalholz (2011)). This purely mechanical simulation shares the first-order features
with the thermo-mechanical slab detachment simulations: (1) distributed weakening of
the thinning slab and (2) viscosity reduction of the surrounding mantle together with
ongoing necking. These morphological and rheological observations indicate the similarity
between the complicated thermo-mechanical slab detachment process and simple necking
of a power-law viscous layer.

4.3.4 The contribution of pure and simple shear: shearing and
necking
Figure 4.5: a) Finite strain pattern during intermediate depth slab detachment. The upper plot shows the initial simple shear dominated regime, the lower displays the necking dominated deformation that takes over from simple shear after about 30% of across slab thinning. The white lines correspond to lithological contours (crust, mantle lithosphere, asthenosphere) b) Strain ellipses corresponding the two stage intermediate depth slab detachment, the time frames correspond to those of a). The blue lines correspond to the lithological contours. c) Conceptual sketch showing the interplay of localized shear deformation in the colder parts of the slab and distributed necking in the hotter parts.

Intermediate-depth slab detachment occurs at depths ranging between 150 and 200 km. At these depths, the continental margin that separates the buoyant continental crust from the negatively buoyant slab, enters the bending zone and start deforming. Due to its dip angle (from 35° in the continental segment to 70° in the oceanic part), the slab resides at an angle with regard to vertical gravitational acceleration (55 to 20°). The continental margin is therefore subject to a substantial amount of simple shear deformation during the onset thinning. The first dominant feature is the development of a shear zone through the slab (Figure 4.5a). In the upper (colder) part of the slab, the shear zone is localized and accommodates normal-sense displacement whereas the deformation in the lower (hotter) part of the slab remains diffuse and symmetric. The transition from simple shear to pure shear thinning occurs after about 30% of across slab thinning. At this stage the shear zone starts to bend significantly and the deformation is dominated by necking. This pure shear deformation, particularly noticeable in the flattening of the strain ellipses in the core of the slab (Figure 4.5b), stays dominant and will eventually lead to slab detachment.
4.4 Comparing 2D simulations and 1D analytics

As suggested by Schmalholz (2011), necking of power-law viscous layers is a suitable mechanical process to explain the thinning of non-Newtonian slabs, eventually leading to their detachment. The study showed that a 1D analytical solution for viscous necking can explain the first-order dynamics of 2D necking under gravity (see Figure 4.4). Although this simplified pure shear necking model is valid for a homogeneous, non-rheologically layered slab that vertically dips in the mantle, our 2D thermo-mechanical models also suggest that viscous necking can be the dominant, slab-scale deformation process during slab detachment. We, therefore, compare the thinning observed in the thermo-mechanical 2D simulations with those predicted by the 1D analytical solution of slab detachment.
Figure 4.6: a) Compilation of results of 30 2D thermo-mechanical simulations showing the corresponding slab thickness versus time curves. The vertical axis represents the non-dimensional slab thickness. b) Similar plot as a) with a non-dimensional time axis. In each simulation the necking duration \( t_c = t_{det} \) defines the characteristic necking time used for the non-dimensionalization. c) Comparison between the 1D analytical solution for necking with the thinning measured in the 2D thermo-mechanical simulations. The three solid lines represent the analytical solution plotted for the lowest, largest, and mean values of characteristic \( n \) calculated for our dataset. The shaded area contains all the measured necking curves of the 2D simulations.
The temporal evolution of the slab thickness was recorded in 30 2D thermo-mechanical numerical simulations of slab detachment (see Tab. 4.2). The simulations are chosen such that they cover a wide range of slab detachment depths. They also include detachments occurring at subducted margins and within the oceanic plate (intra-oceanic). A general feature of these simulations is occurrence of slab detachment within 15 Ma after the start of continental collision. The onset of slab detachment is here empirically defined as the moment at which the across slab strain rate yields noticeable thinning \((1 \times 10^{-15} < \dot{\epsilon}_0 < 5 \times 10^{-15} \text{ s}^{-1})\). These initially slow deformation rates are difficult to detect and the effects of variations of \(\dot{\epsilon}_0\) are discussed in Sec. 4.6.2. The end of slab detachment is defined by monitoring the magnitude of total slab-pull force which is defined as:

\[
F_{\text{pull}} = \int \int_A \left( \bar{\rho}_{\text{Ast}}(y) - \bar{\rho}_{\text{Slab}}(y) \right) g \, dx \, dy
\]

where \(\bar{\rho}_{\text{Ast}}, \bar{\rho}_{\text{Slab}}, g\) and \(A\) respectively stand for the mean asthenosphere and slab density at a given depth, the acceleration of gravity and the surface area of the slab, respectively. The mean asthenosphere and slab densities are averaged horizontally (for each horizontal gridline) and the slab area is defined lithologically (upper bound is the top of the basaltic crust) and thermally (lower bound is the 1100 °C isotherm). After detachment, the average density of the slab remains stable due to fast sinking velocity (high Péclet number). However, the slab/asthenosphere density contrast rapidly decreases with the increasing density of the adjacent asthenosphere. Consequently, the end of the slab detachment is related to a drop of the total slab-pull magnitude. We therefore define the detachment duration \(t_{\text{det}}\) as the period lasting between the beginning and the end of the detachment. Since the thickness can vary according to the initial thermal age of the lithosphere, all the thickness measurements are normalized by their initial magnitudes. Figure 4.6a shows the compilation of lithosphere thinning evolution for the 30 simulations. Independent of their depth, the detachment processes are characterized by a thinning acceleration with time. A second peculiarity is the fact that all detachment durations span the narrow range between 0.30 and 3.07 Ma yielding a mean detachment duration of 1.49 Ma and a median duration of 1.37 Ma with a standard deviation of 0.76 Ma.
<table>
<thead>
<tr>
<th>Material</th>
<th>k (W/m/K)</th>
<th>$H_r$ (W/m³)</th>
<th>$C_p$ (J/kg)</th>
<th>Flow law</th>
<th>$\eta_0$ (Paˢ⁻¹)</th>
<th>n</th>
<th>$E_a$ (J)</th>
<th>$V_a$ (J/bar)</th>
<th>$\sin(\phi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sediments</td>
<td>0.64 + $\frac{807}{T+47}$</td>
<td>$1.50 \times 10^{-6}$</td>
<td>1000</td>
<td>wet Qz.</td>
<td>$1.97 \times 10^{17}$</td>
<td>2.3</td>
<td>$1.54 \times 10^{5}$</td>
<td>0.8</td>
<td>0.1</td>
</tr>
<tr>
<td>Upper cont. crust</td>
<td>0.64 + $\frac{807}{T+47}$</td>
<td>$1.00 \times 10^{-6}$</td>
<td>1000</td>
<td>wet Qz.</td>
<td>$1.97 \times 10^{17}$</td>
<td>2.3</td>
<td>$1.54 \times 10^{5}$</td>
<td>0.8</td>
<td>0.1</td>
</tr>
<tr>
<td>Lower cont. crust</td>
<td>1.18 + $\frac{474}{T+47}$</td>
<td>$0.25 \times 10^{-6}$</td>
<td>1000</td>
<td>Pl. (An75)</td>
<td>$4.80 \times 10^{22}$</td>
<td>3.2</td>
<td>$2.38 \times 10^{5}$</td>
<td>1.2</td>
<td>0.1</td>
</tr>
<tr>
<td>Upper oceanic crust</td>
<td>0.64 + $\frac{807}{T+47}$</td>
<td>$0.25 \times 10^{-6}$</td>
<td>1000</td>
<td>wet Qz.</td>
<td>$1.97 \times 10^{17}$</td>
<td>2.3</td>
<td>$1.54 \times 10^{5}$</td>
<td>0.8</td>
<td>0.1</td>
</tr>
<tr>
<td>Lower oceanic crust</td>
<td>1.18 + $\frac{474}{T+47}$</td>
<td>$0.25 \times 10^{-6}$</td>
<td>1000</td>
<td>Pl. (An75)</td>
<td>$4.80 \times 10^{22}$</td>
<td>3.2</td>
<td>$2.38 \times 10^{5}$</td>
<td>0.8</td>
<td>0.1</td>
</tr>
<tr>
<td>Mantle</td>
<td>0.73 + $\frac{1207}{T+47}$</td>
<td>$2.20 \times 10^{-8}$</td>
<td>1000</td>
<td>dry Ol.</td>
<td>$3.98 \times 10^{16}$</td>
<td>3.5</td>
<td>$5.32 \times 10^{5}$</td>
<td>0.8</td>
<td>0.1</td>
</tr>
<tr>
<td>Weak zone</td>
<td>0.73 + $\frac{1207}{T+47}$</td>
<td>$2.20 \times 10^{-8}$</td>
<td>1000</td>
<td>wet Ol.</td>
<td>$5.01 \times 10^{20}$</td>
<td>4.0</td>
<td>$4.70 \times 10^{5}$</td>
<td>0.8</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 4.1: Physical properties for the different lithologies used in the 2D numerical simulations. Qz., Pl., and Ol. correspond to the abbreviations of Quartzite, Plagioclase, and Olivine. k denotes the thermal conductivity, $H_r$ is the radiogenic heat production, $C_p$ is the specific heat capacity, $\eta_0$ is the reference viscosity, n is the stress exponent, $E_a$ is the activation energy, $V_a$ is the activation volume, $\phi$ is the internal friction angle. The cohesion (C) is 1 MPa for each lithology.
The second plot (Figure 4.6b) displays the same dataset of slab detachment normalized by their respective detachment duration and compared to the 1D analytical solution (Equation 4.2). This analytical solution was derived in Schmalholz (2011) and has the form:

\[
\frac{D}{D_0} = \left(1 - \frac{t}{t_{\text{det}}}\right)^{\frac{1}{n}}
\]

where \(D/D_0\) represents the slab thickness normalized by its initial thickness, \(t/t_{\text{det}}\) corresponds to the time normalized by the detachment duration. The parameter \(n\) is the characteristic stress exponent of the slab. This number describes the non-linearity that links strain rate and stress in a material with power-law viscous rheology and controls the shape of the analytical necking curve. For the space defined by the dimensionless coordinates \(D/D_0\) and \(t/t_{\text{det}}\), the numerical simulations of slab detachment form a narrow cluster of thinning versus time curves. These numerical results exhibit a comparable thinning evolution to that of the 1D analytical solution and the numerical models might therefore indicate that necking is the dominant process for most cases of detachment. In order to best explain our thinning data, we performed a grid search for the value of \(n\) used in the analytical solution. The results indicate that the sharpest necking curve can be described by a characteristic \(n = 8.48\) whereas the smoothest one corresponds to a value of 2.27. A mean characteristic \(n = 4.13\) is obtained by fitting all the necking curve and averaging the corresponding \(n\) values. This comparison shows that the 1D analytical solution with \(n\) around 4 is well suited to describe the first-order dynamics of slab detachment, i.e. the evolution of slab thickness with time around the detachment zone under buoyancy stress.

### 4.5 Impact of shear heating

As described in section 4.3.2, slab detachment results from lithospheric-scale deformation involving a combination of pure and simple shearing. Since the rheology of the lithosphere is temperature-dependent, it is important to quantify the amount of mechanical energy dissipated (shear heating) during slab detachment and to understand the thermo-mechanical feedback on the dynamics of slab detachment. Previous 2D numerical studies (Yoshioka et al., 1994; Gerya et al., 2004) have investigated the role of shear heating on slab detachment by comparing simulations in which this feedback is taken into account or not. Both studies concluded that shear heating has an influence on the deformation of the slab and can lead to a 10% acceleration of the detachment duration. We provide here an additional
Run ID | Differences with the reference run
---|---
1 (Reference) | None
2 | $t_{oc} = 80 \text{ Ma}$, $v_{conv} = 2.5 \text{ cm/a}$
3 | $t_{oc} = 20 \text{ Ma}$, $v_{conv} = 10 \text{ cm/a}$
4 | $t_{oc} = 20 \text{ Ma}$, $v_{conv} = 2.5 \text{ cm/a}$
5 | $t_{oc} = 20 \text{ Ma}$, $v_{conv} = 2.5 \text{ cm/a}$, No Peierls
6 | $t_{oc} = 20 \text{ Ma}$, $v_{conv} = 2.5 \text{ cm/a}$, No Peierls
7 | $t_{oc} = 20 \text{ Ma}$, $v_{conv} = 2.5 \text{ cm/a}$, No Peierls
8 | $v_{conv} = 2.5 \text{ cm/a}$
9 | $v_{conv} = 10 \text{ cm/a}$
10 | $t_{oc} = 30 \text{ Ma}$, $v_{conv} = 2.5 \text{ cm/a}$
11 | $t_{oc} = 80 \text{ Ma}$, $v_{conv} = 10 \text{ cm/a}$
12 | $t_{oc} = 80 \text{ Ma}$, $v_{conv} = 7.5 \text{ cm/a}$
13 | $t_{oc} = 80 \text{ Ma}$, $v_{conv} = 2.5 \text{ cm/a}$
14 | $t_{oc} = 80 \text{ Ma}$
15 | $t_{oc} = 60 \text{ Ma}$, $v_{conv} = 7.5 \text{ cm/a}$
16 | $v_{conv} = 7.5 \text{ cm/a}$
17 | $t_{oc} = 60 \text{ Ma}$, $v_{conv} = 2.5 \text{ cm/a}$
18 | $t_{oc} = 60 \text{ Ma}$
19 | $t_{oc} = 60 \text{ Ma}$, $v_{conv} = 10 \text{ cm/a}$
20 | $t_{oc} = 30 \text{ Ma}$, $v_{conv} = 7.5 \text{ cm/a}$
21 | $v_{conv} = 2.5 \text{ cm/a}$, $z_{ADIABAT} = 130 \text{ km}$
22 | $t_{oc} = 20 \text{ Ma}$, $v_{conv} = 10 \text{ cm/a}$, No shear heating
23 | No shear heating
24 | $v_{conv} = 1.25 \text{ cm/a}$, $z_{ADIABAT} = 120 \text{ km}$, dry olivine flow law (cont. crust)
25 | $v_{conv} = 1.25 \text{ cm/a}$, $z_{ADIABAT} = 120 \text{ km}$, dry granulite flow law (cont. crust)
26 | $v_{conv} = 1.25 \text{ cm/a}$, $z_{ADIABAT} = 120 \text{ km}$
27 | $v_{conv} = 1.25 \text{ cm/a}$, $z_{ADIABAT} = 120 \text{ km}$, plagioclase An.75 flow law (cont. crust)
28 | Oceanic plate length: 700 km
29 | $v_{sed} = 10 \text{ cm/a}$
30 | 

**Table 4.2:** List of the 30 two-dimensional numerical simulations of slab detachment and their differences to the reference run (described in Fig. 4.2). The flow laws are taken from [Ranalli 1995](#) and are described in Tab. 4.1.
Figure 4.7: a) Frequency histogram showing the range of slab detachment durations obtained in the 30 2D thermo-mechanical simulations. b) Diagram showing the correlation between the depth of slab detachment and the duration of detachment. The different symbols contrast the simulations in which detachment occurred at different locations: either at the subducted margin or within the subducting oceanic lithosphere. The different filling colours are used to distinguish between the simulations in which both the Peierls mechanism and shear heating are activated and those in which either shear heating or Peierls mechanism is deactivated.
first-order analytical result that aims to better understand and quantify the influence of shear heating of slab detachment.

4.5.1 1D solution for viscous slab necking with shear heating

![Figure 4.8: a) Thinning versus time for $A = 1.5$. b) Contours for the ratio of duration of detachment without shear heating to the duration of detachment with shear heating in the space $n - A$. The two crosses correspond to the two results for $n = 1$ and 5 displayed in a). The gray shaded area indicates the realistic range of values for $A$ expected for lithospheric slabs.](image)

The 1D analytical solution for necking of a viscous slab due to buoyancy stress provides the evolution of the thinning factor at the zone of necking, $D/D_0$, with time, $t$. (Schmalholz
D is the thickness of the slab and $D_0$ is the initial thickness of the slab. The rate of deformation parallel to the slab, $\dot{\epsilon}$, is defined by the rate of thinning of the slab and is related to the stress, $\sigma$, by a standard power-law flow law:

$$
\dot{\epsilon} = -\frac{1}{D} \frac{\partial D}{\partial t} = B \sigma^n
$$

$B$ is a material parameter and $n$ is the stress exponent. Thinning is driven by the buoyancy stress and for this stress-driven necking the stress at the zone of necking is directly proportional to the thickness of the slab because the force balance along the slab requires a constant force. The product of stress times strain rate quantifies the viscous dissipation and is the shear heating source term in the equation for the evolution of the temperature, $T$. For simplicity, we consider here the adiabatic case with no heat conduction (Brun and Cobbold, 1980), which provides a maximum effect of shear heating. The material parameter $B$ depends exponentially on temperature and this exponential dependence is described here by the Frank-Kamenetzky approximation (i.e. $B = B_0 \exp(\gamma T)$). The two ordinary differential equations for the evolution of the thickness $D$ (Schmalholz, 2011) and for the temperature $T$ form a system of two coupled equations and can be written in dimensionless form:

$$
\frac{\partial D}{\partial t} = -D^{1-n} \exp(T) \quad (4.4)
$$

$$
\frac{\partial T}{\partial t} = A \left( \frac{1}{D} \right)^{1+n} \exp(T) \quad (4.5)
$$

The dimensionless parameter $A$ is:

$$
A = \frac{\Delta \rho g D_0 \gamma}{2 \rho c} \quad (4.6)
$$

$\Delta \rho$ is the density difference between the slab and the asthenosphere, $g$ is the gravity acceleration, $\rho$ is the density of the slab and $c$ is the specific heat. The parameter $\gamma = Q/R/T_0^2$ where $Q$ is the activation energy of the applied flow law, $R$ is the universal gas constant and $T_0$ is a reference temperature (e.g. Brun and Cobbold, 1980; Braeck et al., 2009). The applied characteristic scales for the non-dimensionalization are:
\[ D_c = D_0 \quad \text{(length)} \]  
\[ \sigma_c = \frac{1}{2} \Delta \rho g H \quad \text{(stress)} \]  
\[ t_c = \frac{1}{B \sigma_c^n} \quad \text{(time)} \]  
\[ T_c = \frac{1}{\gamma} \quad \text{(temperature)} \]

### 4.5.2 Application to slab detachment

Equations (4.4) and (4.5) have been integrated numerically with explicit finite differences using a sufficiently small time step for stability and accuracy. Typical values for \( \gamma \) for lithospheric conditions are between 0.01 and 0.1 and typical values for \( A \) for lithospheric slabs are between 0.3 and 8. Figure 4.8a shows the evolution of \( D/D_0 \) with \( t/t_c \) for \( A = 1.5 \) and for \( n = 1 \) and 5. For comparison, the thinning-versus-time curves are also shown for the case of no shear heating. Shear heating decreases the duration of detachment and, as expected, accelerates necking. For \( n = 1 \) the duration of detachment is relatively more decreased due to shear heating than for \( n = 5 \). In the applied simple semi-analytical solution, when the thickness goes to zero the stress goes to infinity. This is impossible in nature and the stress is limited by either a plastic yield stress or a brittle failure stress. These stress limiters are ignored in the applied solution because it is used to quantify the duration of detachment. The duration of detachment does not vary considerably once the thinning versus time curve is close to vertical which happens for most parameters when values of \( D/D_0 \) are smaller than about 0.4 and stresses are therefore only moderately increased for \( 0.4 < D/D_0 < 1 \). Figure 4.8b shows a contour map of the ratio of the duration of detachment without shear heating to the duration of detachment with shear heating. In other words, the numbers of the contours specify the factor that quantifies how much shorter the duration of detachment is if shear heating is considered. Shear heating shortens the time of detachment by an order of magnitude for \( A > 10 \). Realistic values of \( A \) for lithospheric slabs are in the approximate range 0.3 < \( A < 8 \) and Figure 4.8b indicates that for this range it is expected that shear heating shortens the duration of detachment by maximal a factor of about 5 for small \( n \). Because heat conduction is ignored the factor of 5 is a maximum estimate and more realistic factors are expected to be smaller. For example, Gerya et al. (2004) predicted that slab detachment will happen 8% earlier after the start of
their experiments (22.7 Ma vs. 24.6 Ma) with shear heating. Using the parameters employed in the simulations of Gerya et al. (2004) ($n = 3.5$, $Q = 532$ kJ/mol, $R = 8.314$ J/mol/K, $c = 1000$ J/kg, $\rho = 3300$ kg/m$^3$, $D = 80$ km), using $T_0 = 773 \div 973$ K and assuming that $\Delta \rho$ is on the order of 50 kg/m$^3$, the non-dimensional quantity $A$ spans between 0.4 and 0.6. For such values of $A$ the semi-analytical solution predicts detachment duration accelerations that are slightly faster than 10% (see Figure 4.8b). This acceleration only applies to the detachment duration. Although this is not directly comparable to the acceleration predicted in Gerya et al. (2004), it is still in good agreement. Therefore, the predictions of the semi-analytical solution regarding the impact of shear heating agree well with results of 2D thermo-mechanical numerical simulations, if the corresponding parameters are used.

4.6 Discussion

4.6.1 Differences between 1D analytical and 2D numerical models

In the simple 2D slab detachment models of Schmalholz (2011), it was shown that the presence of a lid (continent) can be the source of discrepancies between the 2D numerical and 1D analytical results. Although our 2D thermo-mechanical models reproduce the first-order dynamics of necking, it is evident that many processes are likely to affect the necking dynamics, causing the results to deviate from the 1D analytical solution. These processes induce rheological and geometrical complexities which are the major reasons that can explain the difference between the simulations and the analytical results (Figure 4.6c). The characteristic $n$ values obtained from fitting the 1D solution to the 2D results are therefore affected by these processes. Such deviations can result from the way slab-pull is considered in the 1D and 2D simulations. The 1D model neglects the viscous resistance of the asthenosphere and thus assumes that the entire slab-pull is involved in the slab deformation. On the other hand, numerical simulations take into account the sinking of a slab in a finite viscosity asthenosphere. This viscous resistance reduces the net effect of slab-pull (Schellart 2004), therefore just a fraction of the total pull is involved in the detachment. In other words, a simulation that accounts for a partially transmitted pull can lead to underestimated values of $n$ ($< 3.5$) in the sense of the analytical solution. Conversely, the largest value of $n = 8.48$ may result from the activation of Peierls creep within the slab core (Duretz et al. 2011a). It was shown that the activation of Peierls mechanism can
lead to effective stress exponents that are larger than the stress exponents for the standard power-law flow law for dislocation creep in olivine (Schmalholz and Fletcher 2011). The average value of characteristic $n = 4.13$ is relatively close to the power-law dislocation creep stress exponent of olivine.

4.6.2 Sensitivity of the results
The onset of slab detachment is characterized by a very slow thinning rate which complicates the exact determination of its timing in the 2D simulations. Consequently, the final results (slab detachment duration, characteristic n evaluation) depend on the criteria used to infer when detachment starts. As described in Sec. 4.4, the onset of each detachment is determined by the corresponding slab thinning rate. Our 30 data curves are characterized by their initial thinning rates ($\dot{\epsilon}_0$) ranging between 1 and $5 \times 10^{-15}$ s$^{-1}$. In order to evaluate the influence of this detachment onset criterion, we have introduced a variability of the initial thinning rate for each individual experiment. The initial strain rate is defined as: $\dot{\epsilon}_0 \approx \Delta D/D_0/\Delta t_0$. Keeping $\Delta D/D_0$ constant, a change in the initial strain rate is inversely proportional to $\Delta t_0$. Thus, variations of the initial detachment time are taken into account by perturbing the initial strain rates. These perturbations are calculated assuming a normal distribution of standard deviation ($\sigma_{\dot{\epsilon}_0}$) varying between $1 \times 10^{-16}$ and $5 \times 10^{-16}$ s$^{-1}$. A total of 100 dataset perturbations were performed (20 tests for fives values of $\sigma_{\dot{\epsilon}_0}$). A maximum value of $\sigma_{\dot{\epsilon}_0} = 5 \times 10^{-16}$ s$^{-1}$ corresponds to a variation of thinning velocity of about 1 mm/a for a 70 km thick slab. The sensitivity of the minimum, maximum, and mean values of detachment duration and characteristic n are presented in Tab. 4.3. For a the maximum value of $\sigma_{\dot{\epsilon}_0}$, the mean detachment time ($t_{\text{det}}^{\text{mean}}$) of 1.508 ± 0.016 Ma was obtained over the 20 tests. The mean characteristic stress exponent ($n_{\text{det}}^{\text{mean}}$) showed more sensitivity and is equal to 4.169 ± 0.052 for the same value of $\sigma_{\dot{\epsilon}_0}$. Similarly, the other tested parameters ($t_{\text{det}}^{\text{min}}, t_{\text{det}}^{\text{max}}, n_{\text{det}}^{\text{min}}, n_{\text{det}}^{\text{max}}$) are also close to the values presented in the above sections.

4.6.3 Comparison with previous studies

Most of the 2D thermo-mechanical numerical modeling studies indicated that slab detachment is the results of a progressive viscous thinning (Yoshioka et al. 1994, Schott and Schmeling 1998, Gerya et al. 2004, Andrews and Billen 2009, Burov and Yamato 2008, Baumann et al. 2009). Our study yields similar results and detailed inspection of the models highlights the role of simple shearing (Figure 4.5c), especially at the onset of detachment when slabs detach in a moderate depth range (150-200 km). These results are comparable to the model presented by Lister et al. (2008) which proposes the combination of both necking and shearing processes. Concerning the duration of the detachments, our results differs from those of Baumann et al. (2009), leading to detachment durations that are significantly smaller (Figure 4.7a). This can be explained by the different criteria that
### Table 4.3: Sensitivity of the main results for variable initial thinning rate.

The mean value and standard deviation of each parameters are calculated from 20 tests over the whole dataset (30 detachments). Each test consists in a pseudo-random variation (normal distribution) of the initial thinning rates $\dot{\epsilon}_0$ with a corresponding standard deviation $\sigma_{\dot{\epsilon}_0}$.

<table>
<thead>
<tr>
<th>$\sigma_{\dot{\epsilon}_0}$ [s$^{-1}$]</th>
<th>$n^{\text{min}}$</th>
<th>$n^{\text{max}}$</th>
<th>$n^{\text{mean}}$</th>
<th>$t_{\text{det}}^{\text{min}}$ [Ma]</th>
<th>$t_{\text{det}}^{\text{max}}$ [Ma]</th>
<th>$t_{\text{det}}^{\text{mean}}$ [Ma]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-16}$</td>
<td>2.274 ± 0.010</td>
<td>8.501 ± 0.199</td>
<td>4.133 ± 0.008</td>
<td>0.304 ± 0.001</td>
<td>3.068 ± 0.025</td>
<td>1.490 ± 0.002</td>
</tr>
<tr>
<td>$2.10^{-16}$</td>
<td>2.257 ± 0.019</td>
<td>8.499 ± 0.421</td>
<td>4.140 ± 0.018</td>
<td>0.302 ± 0.002</td>
<td>3.092 ± 0.055</td>
<td>1.494 ± 0.006</td>
</tr>
<tr>
<td>$3.10^{-16}$</td>
<td>2.271 ± 0.028</td>
<td>8.783 ± 0.799</td>
<td>4.151 ± 0.034</td>
<td>0.303 ± 0.003</td>
<td>3.074 ± 0.077</td>
<td>1.496 ± 0.009</td>
</tr>
<tr>
<td>$4.10^{-16}$</td>
<td>2.265 ± 0.033</td>
<td>8.436 ± 0.824</td>
<td>4.149 ± 0.035</td>
<td>0.303 ± 0.004</td>
<td>3.212 ± 0.229</td>
<td>1.502 ± 0.019</td>
</tr>
<tr>
<td>$5.10^{-16}$</td>
<td>2.257 ± 0.051</td>
<td>8.920 ± 1.658</td>
<td>4.169 ± 0.052</td>
<td>0.302 ± 0.006</td>
<td>3.297 ± 0.281</td>
<td>1.508 ± 0.016</td>
</tr>
</tbody>
</table>
were used to determine the end of detachments. In this study, we have considered the dramatic decrease of slab-pull as the criterion for determining the end of the detachment periods. However, in the simulations (composition field), the slab might still be visually connected by a thin, unresolved, viscous filament. These weak filaments can remain visually attached for a long period of time but do not transfer any slab-pull to the overriding plates. Correlating the disappearance of such filaments with the end of slab detachment \cite{Baumann2009} yields to considerably longer detachment durations. Our compilation of 2D slab detachment data indicates that the necking duration is independent of depth. The relation between slab detachment duration and depth is depicted in Figure 4.7b. Similarly to the conclusion of \cite{Baumann2009}, no clear correlation was observed between duration and depth of detachment since fast detachments ($t_{\text{det}} < 2$ Ma) happen in the entire considered depth range. On the other hand, slow slab detachments ($t_{\text{det}} > 2$ Ma) are only likely to occur at depths greater than 250 km. Intra-oceanic (away from the margin) slab detachment (squared boxes) only occurs at large depths.

4.6.4 Timing of slab detachment, duration and consequences

Fast detachment durations have major implications for the overlying collision zone. The inflow of asthenosphere in the detachment zone is proportional to the thinning rate of the slab. This heat advection process is expected to occur in a narrow time range ($0.25 - 3.5$ Ma) and has major consequences for the thermal structure of the orogen. The rapid loss of the slab-pull force causes a dramatic change in the force balance within orogens. Once the slab-pull force is no more transmitted to the orogen, the buoyancy of the buried continental margin may become a dominant force. According to the magnitude of the lateral forces acting on the system (e.g. ridge push), the buoyancy of the orogenic root can trigger an internal reorganization of the collision zone (eduction of \cite{Andersen1991}) which can cause widespread extension and might be involved in the exhumation of high-pressure rocks. Another important consequence of slab detachment is the resulting topographic response \cite{Buiter2002, Gerya2003a}. In \cite{Duretz2011a}, a relation between slab detachment depth and surface uplift rates was highlighted. This response affects both forearc and backarc basins and can be related to the dimensions of the lithosphere deflection (induced by oceanic subduction) at the moment of slab detachment. In the case of shallow slab detachment ($z > 35$ km), no continental subduction is involved, the whole lithosphere deflection is relaxed in response to slab detachment. In the deeper
slab detachment models \((z > 100 \text{ km})\), continental subduction precedes slab detachment and reduces the magnitude of the deflection due to oceanic subduction. Since topographic relaxation time scales are inversely proportional to the deflection breadth \({\text{(Melosh 2011)}}\), shallow detachments may result in faster relaxation than deeper ones. The present study shows no relation between depth and duration of slab detachment and thus no relation between detachment duration and the topographic response. Together with the technological advances and increasing precision of geochronological methods, high resolution (time) data is employed in tectonic reconstructions of collisions zones. For this purpose, it is important to take into consideration that slab detachment, and its consequences, can occur in a short timespan.

### 4.6.5 Three dimensional slab detachment dynamics

Although slab detachment may involve three dimensional (3D) effects, the first 3D simulations of slab detachment \({\text{(van Hunen and Allen 2011) Burkett and Billen 2011)}}\) provide evidence that viscous necking remains the dominant process. In some case \({\text{(van Hunen and Allen 2011)}}\), the detachment may occur progressively in the along-trench direction and the detachment has the morphology of laterally variable viscous thinning. These results are in good visual agreement with the viscous necking and shear-necking models observed in the 2D simulations. The criteria used to determine the termination of detachment might be different in 3D models based on whether the slabs are actually detaching progressively in the third dimension. However, the analytical solution might, to some extent, be applicable to study slab thinning in the trench direction. The application of a 1D necking solution may thus be of interest for predicting the first-order dynamics of 3D slab detachment.

### 4.7 Conclusions

The major mechanism leading to slab detachment is viscous creep, independent of the depth of slab detachment. In agreement with previous studies, we show that viscous necking is the dominant mechanical process involved in slab detachment. Slab thinning may also benefit from the contribution of localized simple shearing in the colder parts of the subducted lithosphere.

The duration of slab detachment is defined here as the time interval between the onset of slab thinning and the vanishing of the slab-pull force. The duration of slab detachment
is geologically short (< 4 Ma with $t_{\text{det}}^{\text{mean}} = 1.5 \pm 0.02$) which is of major importance for tectonic reconstructions and geodynamic interpretations. Since deep detachments (> 250 km) can occur in a short timespan ($t_{\text{det}} < 1$ Ma), there is no simple correlation between the depth of slab detachment and its duration. Slab detachment in depths between 35 and 200 km occurs within periods shorter than 2 Ma.

We use a 1D semi-analytical solution to evaluate the impact of shear heating on the duration of slab detachment. The analytical predictions regarding the impact of shear heating on slab detachment agree well with previously published results of 2D thermo-mechanical numerical simulations. The simple semi-analytical results and the corresponding dimensionless parameter are therefore useful to make a fast and reliable assessment regarding the impact of shear heating on the slab detachment duration.

The applied 2D numerical simulations employ a rheological model that reflects the current knowledge of mantle rheology (i.e. viscoplastic considering Peierls mechanism) and the corresponding thermo-mechanical feedbacks. The 1D analytical solution considers only a power-law flow law but describes well the first-order dynamics of slab detachment modeled with the 2D models. The combination of simple analytical models and elaborated numerical models assesses the validity of each model and leads to a better insight and understanding of the dynamics of slab detachment and its potential impact on plate tectonics.

**Bibliography**


Thermomechanical modelling of slab eduction

Abstract

Plate eduction is a geodynamic process characterized by normal-sense coherent motion of previously subducted continental plate. This mechanism may occur after slab detachment has separated the negatively buoyant oceanic plate from the positively buoyant orogenic root. Eduction may therefore be partly responsible for exhumation of high pressure rocks and late orogenic extension. We used two-dimensional thermomechanical modeling to investigate the main features of the plate eduction model. The results show that eduction can lead to the quasi adiabatic decompression of the subducted crust (≈ 2 GPa) in a timespan of 5 My, large localized extensional strain in the former subduction channel, flattening of the slab, and a topographic uplift associated with extension of the orogen. In order to further investigate the forces involved in the eduction process, we ran parametric simulation and compared them to analytic plate velocity estimations. These experiments showed that eduction is a plausible mechanism as long as the viscosity of the asthenospheric mantle is lower than $10^{22}$ Pa.s while subduction channel viscosity does not exceed $10^{21}$ Pa.s. We suggest that eduction can be a viable geodynamic mechanism and discuss its potential role during the orogenic evolution of the Norwegian Caledonides.¹

¹This chapter was co-authored by T. Duretz, T.V. Gerya, B.J.P. Kaus and T.B. Andersen (Submitted to Journal of Geophysical Research - Solid Earth)
5.1 Introduction

5.1.1 Background

The exhumation of high pressure and ultra high pressure (HP-UHP) metamorphic rocks at convergent margins is a very active field of research in tectonics and geodynamics. Understanding the long term dynamics of subduction-collision zones requires conceptual models, which can account for both real Earth geological and geophysical observations and comply with quantitative geomechanical models.

Numerous models specifically focussed on the processes driving the exhumation of HP-UHP rocks are available in the literature. Syn-to-late collisional exhumation models involve mechanisms such as corner flow or buoyant flow within the subduction channel (Cloos and Shreve, 1988; Gerya et al., 2002; Yamato et al., 2008; Li and Gerya, 2009), buoyancy driven crustal stacking and development of normal sense shear zones (Chemenda et al., 1995, 1996), crustal flow associated with the gravitational spreading of orogens (Andersen and Jamtveit, 1990; Vanderhaeghe and Teyssier, 2001) and focussed erosion (Beaumont et al., 2001, 2004, 2006), slab rollback driven by the retreat of the subducting slab (Lister et al., 2001; Brun and Faccenna, 2008; Husson et al., 2009; Bialas et al., 2011) and delamination of the crust (Bird, 1978), or slab extraction involving slab detachment and the decompression of buried material by plate unbending (Froitzheim et al., 2003; Janák et al., 2006).

The slab detachment model necessitates the build up tensional stresses that overcomes the strength of the subducting plate. Such scenario is likely to take place during an attempted ridge subduction or a continental crust subduction/collision. Both of these contexts have been subject to extensive two-dimensional studies (Davies and von Blanckenburg, 1995; Buiter et al., 2002; Andrews and Billen, 2009; Baumann et al., 2009; Duretz et al., 2011) as well as three-dimensional modelling (Burkett and Billen, 2011; van Hunen and Allen, 2011) and analytical studies focussed on necking dynamics (Schmalholz, 2011). All models agree on the fact that slab detachment causes a dramatic change in the orogenic force balance. Yet, the dynamic consequences of this force balance perturbation have not yet been studied in detail.
CHAPTER 5. THERMOMECHANICAL MODELLING OF SLAB EDUCTION

5.1.2 The term “Eduction”

The concept of plate eduction has been introduced by Dixon and Farrar (1980) to describe a mechanism that could lead to the exhumation of subducted/accreted rocks at an ocean-continent margin. Their model, conceived for the exhumation of the Californian Franciscan blueschists, involved the subduction of an actively spreading ridge beneath North America. The subduction of a ridge triggers shallowing of the slab and the ongoing spreading promotes extension in the subducting slab which would eventually lead to the exhumation of the subducted/accreted material along the margin.

Andersen et al. (1991) introduced the subduction-eduction model to explain burial and exhumation of HP-UHP rocks in continent-continent collision. Based on geological data from one the largest and best preserved HP-UHP provinces in the world this work highlighted the potential link between slab detachment, orogenic extension and exhumation of coherent slab of HP-UHP rocks through the evolution of the Norwegian Caledonides. In this model, the continental lithosphere was subducted to the point at which slab detachment occurred causing the removal of the slab pull force. Subsequently, the subducted and vertically stretched continental plate was coherently educted leading to the exhumation of HP-UHP rocks along a large normal-sense shear zone near the former subduction plane.

The concept of eduction has remained a popular concept in the literature focussed on post-orogenic extension, specifically for the case of the Caledonides (Fossen, 2000; Brueckner and van Roermund, 2004; Brueckner, 2009; Rey et al., 1997; Schlindwein and Jokat, 2000) and the Variscides (Schneider et al., 2006).

5.1.3 The term “Eduction”

The concept of plate eduction has been introduced by Dixon and Farrar (1980) to describe a mechanism that could lead to the exhumation of subducted/accreted rocks at an ocean-continent margin. Their model, conceived for the exhumation of the Californian Franciscan blueschists, involved the subduction of an actively spreading ridge beneath North America. The subduction of a ridge triggers shallowing of the slab and the ongoing spreading promotes extension in the subducting slab which would eventually lead to the exhumation of the subducted/accreted material along the margin.

Andersen et al. (1991) introduced the subduction-eduction model to explain burial and exhumation of HP-UHP rocks in continent-continent collision. Based on geological data
from one the largest and best preserved HP-UHP provinces in the world this work highlighted the potential link between slab detachment, orogenic extension and exhumation of coherent slab of HP-UHP rocks through the evolution of the Norwegian Caledonides. In this model, the continental lithosphere was subducted to the point at which slab detachment occurred, causing the removal of the slab pull force. Subsequently, the subducted and vertically stretched continental plate was coherently educted leading to the exhumation of HP-UHP rocks along a large normal-sense shear zone near the former subduction plane. This eduction concept has remained a popular model in the literature focussed on postorogenic extension, specifically for the case of the Caledonides (Fossen 2000; Brueckner and van Roermund 2004; Brueckner 2009; Rey et al. 1997; Schlindwein and Jokat 2000) and the Variscides (Schneider et al. 2006). This definition contrast with that of Dixon and Farrar (1980) since it explicitly implies a period of reversed subduction following slab detachment during continental collision.

5.1.4 Present work

In this paper we concentrate on the large scale geodynamic process of lower plate eduction in a continental collisional context (i.e. definition of Andersen et al. (1991)). Post slab detachment eduction was observed in several previous numerical modeling studies (Mishin et al. 2008; Duretz et al. 2011) but has not yet been subjected to a systematic parametric study. We therefore focus on the description of the major features of the eduction model and study its dynamics by means of two-dimensional numerical modelling. Finally, we discuss the potential role of eduction for the exhumation of HP-UHP rocks and focus on the case of the Norwegian Caledonides.

5.2 Numerical modelling

5.2.1 Setup

We have used a setup consisting of two continents and of, initially, one ocean (Fig. 6.1). The simulations were run with the thermo-mechanical code I2VIS (Gerya and Yuen 2003a), a description which is provided in Appendix A. Each lithology was characterised by a temperature-stress dependent visco-plastic rheology, with rheological and flow parameters as listed in table 5.1. Following the approach of Gerya et al. (2004), all simulations included
the effect of phases changes on material densities. The size of the model domain was $4000 \times 1400$ km and variable grid spacing ($1361 \times 351$ nodes) was employed to attain a 1 km grid spacing in the collision area. We initially imposed a plate convergence rate of 5 cm/a by prescribing velocities inside the domain. As soon as the model reached 500 km of convergence, the kinematic condition was deactivated and the models were subsequently driven by internal forces (e.g. slab pull). All the boundaries of the box were free slip. An additional, 20 km thick layer of sticky air ($\rho_{\text{air}} = 0$ kg/m$^3$, $\eta_{\text{air}} = 10^{18}$ Pa.s) was utilised in order to mimic the effect of a free surface and the development of topography (Schmeling et al., 2008; Crameri et al., 2012). The setup was designed such that the continental plates detach from the sides of the box during convergence, leaving space for the development of oceanic ridges in the vicinity of the domain boundaries (see Fig. 5.3). In the simulations presented here, the initial continental crustal thickness was 35 km. We do not vary this parameter in this study, nevertheless we expect that variations in crustal thickness (and therefore of plate buoyancy) will affect both the burial depth of the continental crust and the amount of subsequent eduction that can be accommodated.

5.2.2 Potential causes for eduction

We assume that the closure of an oceanic basin occurs prior to continental collision and that this closure stage is accommodated by oceanic plate subduction. Depending on the dimensions, convergence rate and density structure of the oceanic basin, subduction gen-
<table>
<thead>
<tr>
<th>Material</th>
<th>$k$ (W/m/K)</th>
<th>$H_r$ (W/m$^3$ K)</th>
<th>$C_p$ (J/kg K)</th>
<th>$\eta_0$ (Pa.s)</th>
<th>$n$</th>
<th>$E_a$ (J)</th>
<th>$V_a$ (J/bar)</th>
<th>$\sin(\phi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sediments</td>
<td>0.64 + 807T</td>
<td>+77 × 10$^{-6}$</td>
<td>1.50 × 10$^{-6}$</td>
<td>0.80</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper cont. crust</td>
<td>0.64 + 807T</td>
<td>+77 × 10$^{-6}$</td>
<td>1.50 × 10$^{-6}$</td>
<td>0.80</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lower cont. crust</td>
<td>1.18 + 474T</td>
<td>+77 × 10$^{-6}$</td>
<td>1.50 × 10$^{-6}$</td>
<td>0.80</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper oceanic crust</td>
<td>0.64 + 807T</td>
<td>+77 × 10$^{-6}$</td>
<td>1.50 × 10$^{-6}$</td>
<td>0.80</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lower oceanic crust</td>
<td>1.18 + 474T</td>
<td>+77 × 10$^{-6}$</td>
<td>1.50 × 10$^{-6}$</td>
<td>0.80</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mantle</td>
<td>0.73 + 1293T</td>
<td>+77 × 10$^{-8}$</td>
<td>1.50 × 10$^{-6}$</td>
<td>0.80</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weak zone</td>
<td>0.73 + 1293T</td>
<td>+77 × 10$^{-8}$</td>
<td>1.50 × 10$^{-6}$</td>
<td>0.80</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1: Thermal and rheological parameters for the lithologies employed in the simulations. Where $k$ is the thermal conductivity, $H_r$ is the radiogenic heat production, $C_p$ is the specific heat capacity, $\eta_0$ is the reference viscosity, $n$ is the stress exponent, $E_a$ is the activation energy, $V_a$ is the activation volume, $\phi$ is the internal friction angle, and $C$ is the cohesion equal to MPa for all lithologies. Qz., Pl., and Ol. respectively means Quartzite, Plagioclase, and Olivine.
erates the slab pull force which is generally considered as a major tectonic force (Turcotte and Schubert, 1982). While continental margins enter into contact, a period of continental subduction can occur. Due to the buoyant nature of the crust, continental subduction is not steady and might terminate by slab detachment. Once slab detachment has occurred, and assuming a competent crustal rheology, the subducted continental lithosphere may exhume coherently due its buoyancy triggering plate eduction.

As presented in Duretz et al. (2011), plate eduction is likely to occur for continental collision involving relatively fast plate velocities ($5 - 10$ cm/a) and initial slab thermal ages ranging between 40 and 60 Ma. Younger slabs are too weak and detach before any continental crust subduction happens. Older slabs lead to a collisional regime dominated by retreat and delamination in the sense of Bird (1978).

It is important to notice that, in these models, eduction develops because no convergent kinematic constrains are imposed during the collision. Although the plates are laterally confined by newly formed oceanic lithosphere, the buoyancy of the root may be sufficiently large to overcome the ridge push force leading to divergent plate motion and inversion of the subduction plane.

## 5.3 The plate eduction model

### 5.3.1 Reference model

**Time evolution**

The model evolution can be decomposed in several successive stages (Fig. 5.3 and 5.2). The earliest stage is the closure of the oceanic basin. During this period the slab pull ($F_p$) builds up progressively as oceanic plate subduction proceeds (Fig. 5.4b). After 11 Ma, continental subduction initiates and continues until the ocean-continent transition (OCT) reaches a depth at which the positive buoyancy of the crust ($F_B$) compensates the negative buoyancy of the slab ($F_p \approx F_B \approx 3 \times 10^{13}$ N/m). This period lasts approximately 6.5 Ma and the OCT is buried to a maximum depth of 170 km. Slab detachment eventually occurs and necking takes place at the OCT separating the negative (oceanic) from the positively buoyant (continental) portions of the slab.

Fig. 5.4c show the average density difference between the subducted continental lithosphere and the surrounding asthenosphere. This density contrast is responsible for the build up of
buoyant stress within the subducted crust and the large magnitude of the root buoyancy force. After the oceanic slab has detached, the slab pull force decreases dramatically (Fig. 5.4b) and the system is driven by the continental root buoyancy and the ridge push ($\approx 1 - 5 \times 10^{12}$ N/m in our models). As a result of the imbalance between these two forces, the subduction plane reverses as a normal-sense shear zone along which the positive buoyancy of the subducted crust is accommodated. Throughout the eduction stage that lasts 5 Ma, the main portion of the slab returns towards the surface in a coherent motion. After a progressive decrease of the root buoyancy force (Fig. 5.4b), the orogen is eventually affected by a period of buoyant flow which causes the continental crust to rise diapirically towards the surface of the model (see Fig. 5.2).
Figure 5.3: Large scale time evolution of the reference subduction/collision model. The two plates converge towards each other leaving space for the development of ridges in the vicinity of the model’s left/right boundaries. The interaction of the detached slab with the 660 km phase boundary is also depicted.

Exhumation of the subducted crust

In this model the exhumation occurs by two different mechanisms: plate eduction and buoyant flow. Eduction allows for a vertical displacement of the tip of the slab of about 60 km. Buoyant flow of the crust overtakes eduction at its latest stage and is responsible for doming at shallow structural level. These two exhumation stages are clearly identifiable on the pressure-temperature-time (PTt) paths of material tracers (Fig. 5.5). These clockwise paths display a pressure peak corresponding to timing of slab detachment followed by a nearly adiabatic decompression corresponding to the eduction stage. The later cooling event occurs when buoyant flow takes over eduction, this process occurs at a crustal level (depth < 35 km). The signal of slab detachment, and subsequent eduction, can also be noticed on the vertical velocity ($v_y$) evolution of the material tracers (Fig. 5.5). Slab detachment is followed by an “instantaneous” peak exhumation rate of 8 cm/a that exponentially decreases to a rate of about 0.8 cm/a through the period of eduction.
Figure 5.4: a) Post slab detachment density structure. The white contours denotes lithological groups (sediments, upper crust, lower crust, mantle lithosphere and asthenosphere). b) Time evolution of slab pull and root buoyancy forces (buried continental crust) throughout the subduction/collision history. The time is incremented from the onset of the experiment. c) Average vertical profile of density difference between the subducted continental lithosphere and the asthenosphere after slab detachment. This average density contrast ($\Delta \rho$) is computed by first evaluating mean mantle and crust densities at each depth level (i.e. horizontal gridlines), and by subtracted one from the other.
Figure 5.5: Pressure, temperature, density, vertical velocity evolution for 4 Lagrangian material tracers located in the subducted crust. The left column represents the location of the markers throughout the exhumation period. Dashed lines indicate the signal corresponding to slab detachment/start of eduction and the end of the eduction/start of buoyant flow. The time is incremented from the onset of the collision.

Topographic evolution

Plate eduction is responsible for a major uplift event or rebound following slab detachment. The topographic evolution of the subduction-collision zone can be divided in four geodynamic phases that are characterized by a specific surface imprint (Fig. 5.6 a)). During the first 11 Ma period, closure of the oceanic basin takes place, collision takes over oceanic subduction and lasts until 18 Ma when slab detachment happens. While continental subduction proceeds, the topographic peak is located on the upper plate whereas a deep subduction trench is located on the lower plate. During the next 6 Ma following slab detachment, the topography builds up on both sides of the suture marking the stage of plate eduction. The uplift also takes place in the neighboring foreland and hinterland.
basins. The smooth transition between eduction and buoyant flow that occurs at around 25 Ma of model time does not result in a sharp topographic feature.

**Finite strain**

Eduction is characterized by a rigid-body (or coherent) motion of the lower plate. The transition from a subduction-relate plate motion to eduction is sharp and is depicted in Fig. 5.6b). This divergent motion causes localized extension within the subduction channel. The finite deformation pattern (Fig. 5.7), produced using the methodology of Huet et al. (2010), reveals the intensity of the strain that accumulated along the subduction channel.
Figure 5.7: Finite strain intensity pattern related to eduction. Most of the deformation is accommodated by the subduction channel and the asthenosphere. The strain is incremented after the slab has detached. The time after collision has started is indicated. The white contours represent the interfaces between lithological groups (sediments, upper crust, lower crust, mantle lithosphere and asthenosphere).

and the asthenospheric mantle underlying the plate. This illustrates the coherent plate motion resulting from the slab pull loss and the absence of kinematic constrains. The educted plate itself does not undergo any significant internal deformation apart from the
pervasive but not intense strain resulting from the unbending. The surface plate velocities also highlight the rigid motion of lower plate throughout the collision. After slab detachment, the lower plate reaches a velocity on the order of $-5 \text{ cm/a}$ during few Ma which coincides with eduction (Fig. 5.6b)).

**Slab unbending/flattening**

Another feature of the eduction model is tendency for the lower plate to unbend throughout the eduction stage. Fig. 5.6c) depicts the evolution of the slab dip angle after slab detachment occurred, the slab dip is here defined as the angle between the interface separating the underthrustsed continental crust and the subduction channel. The normal shear motion along the former subduction interface is accompanied by the unbending of the lower plate and the flattening of the slab. In our reference model, the dip of the slab decreases from 35° to about 22° through the extensional period (5 Ma timespan).

### 5.4 The dynamics of plate eduction

As described above, plate eduction is a dynamic consequence of the loss of slab pull subsequent to slab detachment. In order to investigate which parameters control the post slab detachment tectonic regime, we have combined both an analytic approach (Fig. 5.8a) and a simplified 2D numerical model (Fig. 5.8b). The analytic method enables to test the influence of the force difference acting on a plate whereas the numerical model allows for the independent variation of the orogenic root buoyancy and the ridge push force. These calculations are utilized to discriminate under which force regime subduction or eduction occurs.
**Figure 5.8:** a) Sketch of the corner flow/torque balance model used to calculated plate velocities. The plate is considered as rigid and the velocities are calculated from the balance between the gravitation torque and the hydrodynamic torque obtained from the corner flow model. b) Simplified setup of a post slab detachment configuration. This setup is used to systematically investigate the parameters that control horizontal plate velocity (subduction or eduction) by varying the ridge push and the orogenic root buoyancy as well as the asthenosphere and subduction channel viscosity, the lower plate length ($L$) and the subduction angle ($\theta$).

### 5.5 A Corner flow/Torque balance approach

In order to obtain a first order prediction of subduction velocity we have used the classical torque balance approach ([Stevenson and Turner](1977) [Manea et al.](2006). This approach is isoviscous and allows us to investigate the dependence of subduction velocity on the mantle viscosity and the slab angle. The torque balance formulates the equilibrium between the moments produced by gravity $M_G$ (i.e. slab’s buoyancy) and mantle flow $M_F$ (i.e. “slab’s lifting torque”). This simplistic approach neglects the torque related to slab bending ([Dvorkin et al.](1993) [Lallemand et al.](2008)).
The hydrodynamic torque resulting from the arc and back arc pressures difference \((P_A - P_B)\) is obtained from the isoviscous stream function approach (McKenzie, 1969) expressed in polar co-ordinates \((r, \theta)\) and can be formulated as follows:

\[
M_F = \int_0^L (P_A - P_B) r \delta r, \tag{5.1}
\]

where the arc/back arc pressure difference is expressed as in Stevenson and Turner (1977):

\[
P_A - P_B = \frac{2 \eta_{\text{last}} V}{r} \left[ \frac{\sin \theta_s}{(\pi - \theta_s) + \sin \theta_s} + \frac{\sin^2 \theta_s}{\theta_s^2 - \sin^2 \theta_s} \right]. \tag{5.2}
\]

The gravitational torque results from the density difference existing between the slab and mantle \((\Delta \rho)\):

\[
M_G = \int_0^L \Delta \rho gh \cos \theta_s r \delta r, \tag{5.3}
\]

where \(g\) and \(h\) respectively stands for the gravitational acceleration and the slab thickness. Integrating and solving both expressions for \(V\) leads to the velocity expression:

\[
V = \frac{F K}{4 \eta_{\text{last}}}, \tag{5.4}
\]

where \(F = \Delta \rho ghL\) is the slab’s buoyancy force and \(K\) is a factor that depends on the slab dip:

\[
K = \frac{\cos \theta_s}{\sin \theta_s (\pi - \theta_s) + \sin \theta_s} \frac{\sin \theta_s^2}{\theta_s^2 - \sin^2 \theta_s}. \tag{5.5}
\]

Assuming that the slab is rigid and transmits the far-field ridge push force, we may consider that the slab is effectively driven by the force difference between slab’s buoyancy and ridge push acting in the direction of the slab \((F \propto dF = F_P \cos \theta - F_B \sin \theta)\). Subsequently, the velocity expression can be rewritten as

\[
V \propto \frac{dF K}{4 \eta_{\text{last}}}, \tag{5.6}
\]

where the velocity of plate motion is directly proportional to \(dF\) and which is either positive when subduction occurs or negative when eduction takes place.
5.6 Simplified 2D setup

In order to test the applicability of the torque balance approach, we performed additional numerical simulations for a simplified setup that consists of a rigid newtonian (high-viscosity) slab and a linear viscous mantle. The steady state equation of momentum and incompressibility are discretized using T2P1 elements and solved for the given distribution of density and viscosity, after which the resulting instantaneous flow pattern is used to determine whether the orogen is undergoing subduction \( V_{\text{plate}} > 0 \) or eduction \( V_{\text{plate}} < 0 \). The boundary conditions are set to free slip on the left, right and bottom boundaries, and a free surface condition is applied at the top of the box. All the simulations were run using the code MILAMIN-VEP \([\text{Kaus et al.} 2010]\).

The horizontal (i.e. ridge push) force is prescribed by varying the density gradient between the asthenosphere and the plate, whereas the root buoyancy force is controlled by the density difference between the buried crust and the surrounding mantle (Fig. 5.8b). We define the magnitude of the ridge push force as

\[
F_P = (\rho_{\text{lit}} - \rho_{\text{ridge}}) \ A_{\text{ridge}} \ g, 
\]  

(5.7)

where \( \rho_{\text{lit}} \), \( \rho_{\text{ast2}} \), and \( A_{\text{ridge}} \) respectively stand for the lithosphere density, the ridge and the ridge area. Similarly, we define the magnitude of the orogenic root buoyancy as:

\[
F_B = (\rho_{\text{lit}} - \rho_{\text{root}}) \ A_{\text{root}} \ g, 
\]  

(5.8)

where \( \rho_{\text{root}} \) and \( A_{\text{root}} \) correspond to the density and the area of buried continental crust and are calculated according to the magnitude of each force such as:

\[
\rho_{\text{ridge}} = \rho_{\text{lit}} - \frac{\ F_P }{ A_{\text{ridge}} \ g } 
\]  

(5.9)

\[
\rho_{\text{root}} = \rho_{\text{lit}} - \frac{\ F_B }{ A_{\text{root}} \ g } 
\]  

(5.10)

The tectonic regime is therefore controlled by the forces that are applied to the plates and imposed via the density distribution. In contrast to the isoviscous torque balance described in 5.5, such setup enables to explore the effect of the subduction channel rheology on the plate kinematics.
5.6.1 Parameters controlling eduction

Over the many parameters that affects the velocity of the subducting or educting plates, we have focused on the dip angle of slab and the viscosities of both the asthenosphere and the subduction channel. Both torque balance and 2D finite elements were employed to calculate plates velocity after slab detachment (the results are depicted in Fig. 5.9a). Despite the different assumptions made with each approach, the results are in reasonable agreement, especially when the subduction channel’s viscosity is equal to that of the asthenosphere. Both approaches predict that eduction occurs if the buoyancy of the slab exceeds the ridge push force ($dF < 0$).
Slab dip

The effect of the slab dip was investigated using the torque balance approach. We have used a constant asthenosphere viscosity of $10^{20}$ Pa.s and varied the slab dip from 10 to 70 degrees (Fig. 5.9b). The slab dip angle shows a significant influence of the plate velocities. Plate velocities increase with increasing dip angle up to an optimal angle of $\approx 60$ degrees (close to the results of Stevenson and Turner (1977)). Over this critical dip angle value, the plates velocities decrease with increasing slab dip.

\textbf{Figure 5.9}: a) Plate velocity for variable force difference ($dF$) (using $\eta_{\text{ast}} = 10^{20}$ Pa.s and $\theta = 30^\circ$). The solid lines represent the solution obtained from the 2D simulations for different subduction channel viscosities ($\eta_{ch}$). The dotted line is the prediction of the torque balance approach for an isoviscous mantle. b) Sensitivity of plate velocity to the slab dip angle for variable force difference values. This was computed using the torque balance model. c) Magnitude of the basal drag (red) and channel drag forces (blue) estimated for our 2D setup as a function of plate velocity. These estimations assume a Couette flow in the asthenosphere and the subduction channel. d) Efficiency of the subduction channel drag over the basal drag for various channel aspect ratios and channel/asthenosphere viscosity contrast.
Asthenosphere viscosity

The asthenosphere provides viscous resistance to slab penetration. This resistance can be decomposed in force acting parallel to the motion direction and a basal drag tangential to the slab face ($F_{BD}$). From the torque balance model, the slab velocity is inversely proportional to the asthenosphere viscosity. A viscosity increase of one order of magnitude will therefore produce a velocity decrease of the same magnitude. However, a more realistic model would also take into account effects of the slab bending (Buffet, 2006), the drag torque due to slab curvature, or the slab anchoring due to the upper plate motion (Scholz and Campos, 1995; Lallemand et al., 2008) which would also affect subduction/eduction rates. The magnitude of the basal drag force in 2D can be estimated by considering a Couette flow in the mantle driven by the slab motion in such manner:

$$F_{BD} \propto \eta_{\text{ast}} V \frac{L_{\text{Plate}}}{H_{\text{ast}}}.$$  \hfill (5.11)

Using a plate length ($L_{\text{Plate}}$) of 1500 km and an asthenospheric channel thickness ($H_{\text{ast}}$) of 500 km, Fig. 5.9c shows that the estimated magnitude of basal drag does not exceed $8 \times 10^{11}$ N/m even for plate velocities larger than 8 cm/a. This justifies that the basal drag force plays a minor role in our calculations.

Subduction channel viscosity

Similarly to the basal drag force, a slab tangential force is also generated in the subduction channel ($F_{BD}$). Its influence on plate velocities can not be investigated by the isoviscous torque balance model, however the 2D simulations showed large sensitivity to this parameter (Fig. 5.9a). At first order, the magnitude of this force may also be approximated from a simple Couette flow model driven by the subducting plate and leading to the expression:

$$F_{BD} \propto \eta_{\text{ch}} V \frac{L_{\text{Channel}}}{H_{\text{Channel}}}. $$  \hfill (5.12)

Estimated channel drag forces computed using a channel length ($L_{\text{Channel}} = 300$ km) and thickness ($H_{\text{Channel}} = 15$ km) corresponding to our 2D setup are depicted in Fig. 5.9c. In the ideal case, the channel is considered to provide strong mechanical decoupling between the plates, and hence should not produce a strong drag force (e.g. Fig. 5.9c with $\eta_{\text{ch}} = 10^{18}$ Pa.s). However, its magnitude is highly dependent on the channel viscosity and
may overcome the basal drag force once the channel viscosity tends to the asthenosphere viscosity (e.g. Fig. 5.9e with $\eta_{ch} = 10^{20}$ Pa.s). Another major parameter is the aspect ratio of the channel, Fig. 5.9d shows the ratio of channel/basal drag force magnitudes for variable channel/asthenosphere viscosity ratio and channel aspect ratio. These results indicate that, in the case the channel undergoes strengthening processes and/or geometrical modifications (thickening/thinning), channel drag may largely overcome basal drag and might become a major force ($F_{CH} > 10^{12}$ N/m).

5.7 Discussion

5.7.1 Low peak temperature in the exhumed crust

In our simulations, the reference model presented here fails at producing sufficiently large enough temperatures in the exhumed HP rocks compared to geological examples of HP and UHP rocks formed at corresponding extreme burial depths ($T < 600$ to $750$ °C). These low temperatures can be explained by the short residence time of subducted material at mantle depths which is limited by the onset of extensional processes within the orogen. An unrealistic low geotherm of less than $3$ °C/km during continental subduction is the result of this model. Hence, a fast (cold) subduction/collision is subject to a rapid burial and aborted by slab detachment. In this case, the main exhumation stage occurs after the detachment and the buried continental margin is educted without reaching high temperatures. Since a fast convergence plate eduction models can only explain the exhumation of cold UHP rocks ($T < 500$ °C), we expect that slower convergence models are likely to exhum UHP rocks with higher temperatures. Although our simulations take into account the thermomechanical feedback induced by viscous dissipation, the peak temperatures reached by the exhumed material are approximately $100$ °C too cold to match natural data. These results, comparable to those obtained in previous studies (Yamato et al., 2008; Stöckhert and Gerya, 2005), tend to show that viscous dissipation may not explain common peak temperatures ($T > 600$ °C) of exhumed HP and UHP rocks. In contrast to the results of Li and Gerya (2009); Li et al. (2011), our simulations do not account for partial melting, this may potentially explain our low peak temperatures. We expect the heat advected by the partially molten upwellings and the heat released during crystallization of potential plutons to provide the missing $\approx 100$ °C. As material for discussion, we present additional
subduction-collision models that were obtained following the same methodology as in the simulations presented above (i.e. without including any effect related to melting).

**A Delamination-Eduction model**

![Diagram](image)

**Figure 5.10:** A) Temporal evolution of the delamination/eduction model, the coloured stars correspond to Lagrangian tracers used to record the pressure-temperature history of the subducted continental crust. B) Pressure-temperature evolution of the Lagrangian tracers throughout the collision event.

The models presented in Sec. 5.3 were designed in order to isolate the process of slab eduction. However, more realistic models of orogenic evolution may include delamination (Bird, 1978) or buoyant uplift (Warren et al., 2008b,a) linked to slab rollback. Fig. 5.10A depicts the evolution of an initially slower continental collision (1.25 cm/y). The simulation runs for 40 Ma before continental subduction takes place. The burial stage lasts for about 10 Ma and the subducted crust is affected by delamination as soon as the continental crust reaches its maximum depth of burial (≈ 200 km). Most of the decompression occurs during the delamination period that lasts approximately 7 My before slab detachment eventually occurs triggering a late stage eduction event. As indicated by Fig. 5.10B, the exhumed crust peak temperatures higher than in the case of pure eduction (≈ 660 °C). Another feature of this delamination-eduction model is the flow of the extruded crust towards the
foreland that leads to a stage of orogenic broadening, and the occurrence of a deep-seated thrust that likely to be exposed in the hinterland of the broadened mountain belt. The combination of delamination and eduction gives rise to the exhumation of HP-UHP rocks with peak temperatures of 500 to 650 °C, more akin to natural examples, and we therefore regard this type of exhumation model as more realistic.

Subduction of a thinned continental margin

![Figure 5.11: A) Evolution of the thinned margin subduction model, this feature inhibit slab detachment at the timescale of the collision. The Lagrangian tracers used to record the pressure-temperature history of the subducted continental crust are denoted as coloured stars. B) PT paths corresponding to different initial locations within the passive margin.](image)

Similarly to boudinage instabilities (Schmalholz et al., 2008), slab detachment has been demonstrated to be the result of a viscous necking instability in a power law fluid (Schmalholz, 2011). We therefore expect our slab detachment to be sensitive to the geometry and the thermal state of the subducted margin (amplitude and wavelength of the initial perturbation). Fig. 5.11A displays the time evolution of a model employing a thinned ocean-continent transition. Although, the initial convergence rate is prescribed similarly as for our reference model (5 cm/a), the presence of a stretched ocean-continent transition
delays slab detachment and ultimately favors delamination. The distal part of the margin reaches a maximum depth larger than 200 km from where the delamination of the upper crust is initiated. As the delamination occurs (≈ 10 Ma), several nappes detach from the subducted margin. These buoyant nappes are emplaced adjacent to adjacent stacked within the orogen and record significantly different PT histories (Fig. 5.11B). This model is in agreement with previous studies (Warren et al., 2008a; Li et al., 2011; Yamato et al., 2008) that showed that slab detachment is not a necessary factor for the exhumation of high pressure rocks and that the stage of continental collision does not exceed more than 10 Ma (Yamato et al., 2008).

The Western Gneiss Region, Scandinavian Caledonides.

The idea of continental subduction and eduction following slab breakoff at the terminal stages of a Wilson cycle was put forward in order to explain the geological observations from the Scandian continent-continent collision in the Caledonides (Andersen et al., 1991). The collision produced one of the worlds largest HP-UHP terrains and the main characteristics of the vast Western Gneiss Region (WGR) are:

1. Protoliths of the HP-UHP rocks are Middle Proterozoic (≈1700 to 950 Ma) mostly orthogneisses (e.g. Austrheim et al. (2003); Tucker et al. (2004)), and major parts of the WGR experienced granulite metamorphic conditions at ≈ 1 Ga (e.g. Røhr et al. (2004); Krabbendam et al. (2000)).

2. The uppermost structural level of the WGR is a nearly intact Proterozoic crust, only affected by the Caledonian metamorphism and deformation near the basal Caledonian thrust (Labrousse et al., 2010).

3. The WGR has a metamorphic field-gradient from 600 °C at 1.8 GPa in the east to 750 °C at 2.8 GPa in the west (e.g. Young et al. (2007); Hacker et al. (2010)).

4. The HP-UHP metamorphism in the WGR lasted nearly 20 My (415 to 397 Ma, e.g., Krogh et al. (2011)), whereas Scandian HP in the nappes is older up to 430 Ma.
5. There are no exposed major syn- to post-UHP-metamorphic thrusts (Hacker et al., 2010), and a very large-scale extensional detachment zone separates the HP-UHP rocks in the footwall from lower grade nappes in the hanging wall.

6. The total duration of the Scandian continental collision was approximately 30 Ma (430 to 400 Ma) (Andersen et al., 1990; Corfu et al., 2006).

The absence of large-scale structures perturbing the regional metamorphic zonation within the WGR suggests that it was buried and exhumed as a mostly intact slab (40000 km$^2$) of continental crust. Some very local extreme UHP occurrences recording 3 to 6 GPa (diamond, majoritic garnet and opx-eclogites) within the coesite-grade domains (see review by van Roermund (2009)) cannot be fitted to the regional metamorphic field gradient and probably needs alternative explanations (Vrijmoed et al., 2009), which will not be discussed here.

The eduction model presented in this study (Fig. 5.5) results in a 2D structural geometry of the exhumed HP-UHP rocks, which is quite similar to observations in the WGR (see figure 2 in Andersen et al. (1991)). The duration since on-set of collision is 27 My (Fig 4B), also similar to the Caledonian analog. The HP-UHP rocks also constitute the lowermost observable structural level similar to what is observed in western Norway. There is no large-scale thrust below the UHP rocks, and there is a very large extensional detachment above the HP-UHP rocks. The model presented in Fig. 5.5, however, fails to reproduce the temperatures similar to those seen in the eclogites in the WGR by approximately 150 to 250 °C (Hacker et al., 2010; Labrousse et al., 2004). It is obvious that the reference eduction model used here is too cold to explain the WGR example in full. The alternative delamination-eduction model (Fig. 5.10) also reproduces a number of the field characteristics, and the temperatures obtained are still lower (up to 100°C), but more comparable to those from the WGR. The main structural difference between the eduction and the delamination-eduction models is that the delamination stage produces a major thrust at mid-crustal levels, below the UHP rocks during exhumation (Fig. 5.10). The presence of such thrust(s) has been postulated in several papers on the WGR (e.g. Andersen et al. (1991); Hacker et al. (2010)), but the existence has not been verified by observation in
the field. If present it must be covered by nappes in south-central Norway. Observations that argue against the presence of the deep thrust is that sedimentary cover to the autochthonous Proterozoic basement can be traced almost continuously along the basal thrust from the foreland below the nappes to the WGR in central south Norway. Considering all available geological and geophysical observations (Andersen, 1990), a geomechanical model combining delamination with slab breakoff and eduction provides the best quantifiable model for exhumation of the HP-UHP rocks in the WGR.

### 5.7.2 Limitations of the density model

Although we did not explicitly consider the effect of phase transitions occurring within the continental crust, the crustal densities were calculated according to the transient pressure-temperature conditions. The equation of state (see Appendix 5.9) allows the upper crustal density to smoothly vary between 2700 to 2850 kg/m$^3$ whereas lower crust density ranges between 2800 to 2950 kg/m$^3$ (Fig. 5.4). Both density changes related to either phase changes or the equation of state were considered to be instantaneous and reversible. Natural observation indicates that reaction kinetics can be slow and can lead to partial prograde phase transformations (Krabbendam et al., 2000) and neglecting this effect might lead to an overestimation of the crust density on the prograde path. This would have the effect of overvaluing the buoyancy of the crust and could potentially reduce the peak burial depth of continental crust prior to slab detachment. On the other hand, retrograde density changes may lead to an underestimation of the crust density and therefore overrating the root buoyancy force in the later stages of eduction.

### 5.7.3 Comparison with the models of coherent nappe exhumation

The models proposed by Chemenda et al. (1995, 1996, 1997) involved decoupling of the subducting crust from the mantle lithosphere and exhumation of the coherent nappe by faulting of the crust. Although overall kinematics of exhumation resembles the eduction model, the dynamics of these models are different. In Chemenda et al. (1995), the buried crust decouples from the mantle lithosphere, implying weak mechanical coupling at the Moho. The exhumation of the nappe is coherent and is accommodated by a normal fault close to the suture and a thrust on the lower plate. In our simulations, such normal shear
zone is produced consequently to slab detachment and eduction. Despite slab breakoff occurred in the experiments of Boutelier et al. (2004), no eduction was observed because of two major reasons: (1) the crust had already delaminated from the mantle at breakoff time, preventing coherent lithospheric-scale motion, (2) the system was subject to a continued kinematic push.

5.7.4 Dimensionality and plate motions

Our two-dimensional models do not take into account effects related to three dimensional flow in the mantle. We can expect that the toroidal flow component can have a strong influence on subduction-collision systems. A consequence is the tendency for subduction to roll back, leading to a more decoupled, or retreating, style of collision (Husson et al., 2009; Stegman et al., 2006). The plate eduction end-member is triggered by slab detachment, recent three-dimensional studies of slab detachment (Burkett and Billen, 2011; van Hunen and Allen, 2011) show that the tearing of the slab can happen in an inhomogeneous manner along the trench direction. We thus expect that plate eduction will take place progressively in the along trench direction following the detachment of the slab at depth. This direction of slab tearing should also control the timing and location of emplacement of the exhumed high pressure nappes as well as surface uplift and filling of the adjacent basins. Another 3D aspect of the force balance change related to slab detachment is the possibility to trigger abrupt changes in plate motion (Austermann et al., 2011). In our 2D study, the main consequence of the slab pull is the partial eduction of the subducted slab from the subduction zone. Our results shows that a lower plate horizontal displacement on the order of 100 km can result from eduction. In three dimensions, finite-width slabs associated with progressive (along trench) slab detachment may lead to changes in plate motion directions and potentially to rotations. On the other hand, we could also expect that plate rotations, which can be triggered by oblique continental collision (Bellhasen et al., 2003) can strongly affect slab detachment (van Hunen and Allen, 2011) and potentially help and/or trigger plate eduction in some specific conditions. Another simplification of our 2D results relies in the fact that detachment results in a total loss of the slab pull force. In three-dimensions, we may expect that slab portions might remain attached in the along-trench direction (along strike coupling) and provide additional pull force, this effect may effectively reduced the possibility of eduction. Variations of slab width may also occur during subduction (Guillaume et al., 2010) and might strongly perturb the subduction force lead-
ing to variations of plate velocities, such effect may promote slab rollback and detachment as well as the development of slab tear faults \cite{Wortel2009}.

5.8 Conclusions

The plate eduction model is characterized by the inversion of the subduction plane within continental collision zones. This mechanism is likely to take place after slab detachment has occurred and eliminated the effect of slab pull. Our two-dimensional study demonstrates that this model can partly explain the exhumation of buried continental crust and orogenic extension. Models that isolate the plate eduction mechanism lead to quasi adiabatic decompression of the buried crust ($\approx 2$ GPa) in a timespan of 5 Ma. The coherent extraction of the slab from the subduction zone results in localized extensional strain within the subduction channel. This response to slab detachment is accompanied by the flattening of the slab and the build up of topography on the lower plate. We have derived scaling laws that enable the prediction of the lower plate velocity that is subjected to eduction. The plate velocity is function of the buoyancy of the previously subducted continental crust and of the lateral force that is exerted on the collision zone (e.g. ridge push). Asthenosphere and subduction channel viscosity mantle have a first order influence on the rate of eduction and allow eduction if their respective magnitudes are lower than $10^{22}$ and $10^{21}$ Pa.s. Eduction subsequent to slab detachment can occur in combination to other geodynamic processes (e.g. slab retreat) and may play a significant role in the geodynamic evolution of collision zones.
Appendix

5.9 Numerical code description

The thermo-mechanical code I2VIS solves the two-dimensional steady state Stokes equations and heat conservation equation using the finite-difference(marker-in-cell) method (Gerya and Yuen, 2003a; Gerya, 2010):

\[
\frac{\partial \sigma_{ij}}{\partial x_j} = -\rho g_i \tag{5.13}
\]

\[
\frac{\partial v_i}{\partial x_i} = 0 \tag{5.14}
\]

\[
\frac{\partial}{\partial x_i} \left( k \frac{\partial T}{\partial x_i} \right) = -\rho C_p \frac{DT}{Dt} - H \tag{5.15}
\]

where \( x_j \) represents the coordinates, \( \rho \), the material density (kg/m\(^3\)), \( k \), the thermal conductivity (W/m/K), \( C_p \), the specific heat capacity (J/kg) and \( H \) (J/m\(^3\)/s), the contribution of the different heat sources (radiogenic, shear, and adiabatic heating). The density and heat capacity of each lithology are functions of both pressure \( P \) (Pa) and temperature \( T \) (K). The oceanic crust, lithospheric and asthenospheric densities are pre-computed via Gibbs free energy minimization and updated at each timestep. The densities of continental crust rocks are calculated via the equation of state:

\[
\rho = \rho_0 (1 - \beta (T - 298.15)(1 + \alpha (P \times 10^{-8} - 10^{-3})) \tag{5.16}
\]

where \( \rho_0 \) corresponds to the reference density (2700 kg/m\(^3\) for the upper crust and 2800 kg/m\(^3\) for the lower crust), \( \beta \) to the isothermal compressibility (0.5 1/K for the crust) and \( \alpha \) to the thermal expansivity (1.5 1/kbar for the crust). The thermal conductivity \( k \) depends on the temperature (Clauser and Huenges, 1995), the functions used to evaluate \( k \) are listed in table 5.1

The mechanical solver uses a viscous formulation and the deviatoric stress tensor \( \sigma_{ij} \) relates to the material viscosity \( \eta \) and the rate of deformation tensor \( \dot{\epsilon}_{ij} \) via:

\[
\sigma_{ij} = 2\eta \dot{\epsilon}_{ij} = \eta \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \tag{5.17}
\]
The different lithologies deform according to a visco-plastic rheology. At stresses larger than 30 kPa, most of the flow occurs in the dislocation creep regime \( (\text{Turcotte and Schubert, 1982}) \) and depends on the second invariant of the strain rate tensor \( (\dot{\epsilon}_{II}) \), the temperature and the pressure \( (\text{Ranalli, 1995}) \). The effective viscosity corresponding to a dislocation creep regime is calculated as following:

\[
\eta_{\text{creep}} = \eta_0 \dot{\epsilon}_{II}^{\frac{1}{n-2}} \exp \left( \frac{E_a + PV_a}{nRT} \right) \tag{5.18}
\]

where \( n \) is the stress exponent, \( \eta_0 \), the reference viscosity \( (\text{Pa}^n.s) \), \( E_a \), the activation energy \( (\text{J}) \), \( V_a \), the activation volume \( (\text{J/bar}) \) and \( R \), the gas constant \( (8.314472 \text{ J/mol/K}) \).

Mohr-Coulomb (or Drucker-Prager) plasticity act as a stress limiter in the regions where the second stress invariant \( (\sigma_{II}) \) exceeds the material yield stress. The yield stress depends on the pressure, the cohesion \( C \) \( (\text{MPa}) \), and the internal friction angle \( \phi \). The stress is limited via local viscosity reductions such as

\[
\eta_{\text{creep}} \leq \frac{C + P \sin(\phi)}{2\sqrt{\dot{\epsilon}_{II}}} \tag{5.19}
\]

In the mantle, at sufficiently high stress and low temperature, Peierls plasticity may be the dominant deformation mechanism \( (\text{Evans and Goetze, 1979}; \text{Kameyama et al., 1999}; \text{Raterron et al., 2004}; \text{Katayama and Karato, 2008}) \). This regime has exponential dependance on the second stress invariant and can therefore act as a strong weakening mechanism in the lithospheric mantle. The effective viscosity corresponding to the Peierls creep regime is formulated as:

\[
\eta_{\text{Peierls}} = \frac{1}{A_{\text{Peierls}}} \frac{1}{\sigma_{II}} \exp \left[ \left( \frac{E_a - PV_a}{RT} \right) \left( 1 - \left( \frac{\sigma_{II}}{\sigma_{\text{Peierls}}} \right)^k \right)^q \right] \tag{5.20}
\]

In our simulations, this mechanism becomes active when the Peierls viscosity \( \eta_{\text{Peierls}} \) is inferior to the creep viscosity \( \eta_{\text{creep}} \). We use the dry olivine parameters \( A_{\text{Peierls}} = 10^{7.8} \times 10^{-12} \) and \( \sigma_{\text{Peierls}} = 9.1 \text{ GPa} \) \( (\text{Evans and Goetze, 1979}) \). For practical aspects, the viscosity is limited such as \( 10^{18} < \eta < 10^{25} \text{ Pa.s} \).
The model’s surface $h$ (air/crust interface) evolves following a gross-scale erosion-sedimentation law (Gerya and Yuen, 2003b; Gerya, 2010):

$$\frac{\partial h}{\partial t} = v_y - v_x \frac{\partial h}{\partial x} - \dot{e} + \dot{s}$$

(5.21)

where $v_y$ and $v_x$ are the uplift and advection velocity (m/s) predicted by the tectonic model. $\dot{e}$ and $\dot{s}$ represent prescribed erosion and sedimentation rates (m/s), they are set to 0.1 mm/y in our reference run. Erosion is active above a reference altitude of 1km whereas sedimentation takes place at depth inferior to -1 km.

The results of the calculation of the fluid velocity obtained on the Eulerian grid is interpolated to the Lagrangian markers, the advection equation is the solved explicitly by a coordinate update of the Lagrangian markers:

$$x^{t+1}_j = x^t_j + \Delta t v^t$$

(5.22)

where $v^t$ corresponds to the marker velocity computed via 4th order (in space) Runge-Kutta scheme, $\Delta t$ represent the value of the timestep at the current time integration.

**Bibliography**


Crust rheology, slab detachment and topography

Abstract

The collision between continents following the closure of an ocean can lead to the subduction of continental crust. The introduction of buoyant crust within subduction zones triggers the development of extensional stresses in slabs which eventually result in their detachment. The dynamic consequences of slab detachment affects the development of topography, the exhumation of high-pressure rocks and the geodynamic evolution of collision zones. We employ two-dimensional thermo-mechanical modelling in order to study the importance of crustal rheology on the evolution of spontaneous subduction-collision systems and the occurrence of slab detachment. The modelling results indicate that varying the rheological structure of the crust can results in a broad range of collisional evolutions involving slab detachment, delamination (associated to slab rollback), or the combination of both mechanisms. By enhancing mechanical coupling at the Moho, a strong crust leads to the deep subduction of the crust (180 km). These collisions are subjected to slab detachment and subsequent coherent exhumation of the crust accommodated by eduction (inversion of subduction sense) and thrusting. In these conditions, slab detachment promotes the development of a high (> 4.5 km) and narrow (< 200 km) topographic plateau located in the vicinity of the suture. A contrasting style of collision is obtained by employing a weak crustal rheology. The weak mechanical coupling at the Moho promotes the widespread delamination of the lithosphere, preventing slab detachment to occur. Further shortening leads to buckling and thickening of the crust resulting in the development of topographic bulging on the lower plate. Collisions involving rheologically layered crust are characterised by a decoupling level at mid-crustal depths. These initial condition favours
the delamination of the upper crust as well as the deep subduction of the lower crust. These collisions are thus successively affected by delamination and slab detachment and both processes contribute to the exhumation of the subducted crust. A wide (> 200 km) topographic plateau develops as the result of the buoyant extrusion of the upper crust onto the foreland, this mechanism is further amplified by slab detachment. Our results suggest that the occurrence of both delamination (Apennines) and slab detachment (Himalaya) in orogens may highlight significant differences in their initial rheological structure.  

---

1 This chapter was co-authored by T. Duretz, T.V. Gerya (to be submitted)
CHAPTER 6. CRUST RHEOLOGY, SLAB DETACHMENT AND TOPOGRAPHY

6.1 Introduction

The slab detachment (or breakoff) model has recently been widely employed for the interpretation of both geological and geophysical observation. This model involves the detachment of a portion or the integrity of a subducting slab beneath a convergent margin. The concept of slab detachment was born from seismological studies and was first hypothesised in the late sixties (Isacks and Molnar, 1969) in order to explain seismicity patterns in subduction zones. The study of deep seismicity patterns has indicated the existence of inhomogeneous seismogenic zones that were associated to gaps within slabs (Chatelain et al., 1993; Chen and Brudzinski, 2001; Sperner et al., 2001; Kundu and Gahalaut, 2011). The slab detachment model has further gained popularity with the development of seismic tomography. Tomographic images provide maps of seismic velocity variations within the Earth’s mantle and have lead to the detection of slab remnants in the deep mantle (Wortel and Spakman, 1992; Widiantoro and van der Hilst, 1996; van der Meer et al., 2010; Rogers et al., 2002; Levin et al., 2002; Schmandt and Humphreys, 2011; Zor, 2008). Moreover, regional scale seismic tomography has enabled the detection of positive seismic velocity anomalies beneath collision zones, these structures were consequently attributed to detached or detaching slab (Wortel and Spakman, 2000; Lippitsch et al., 2003; Martin and Wenzel, 2006; Replumaz et al., 2010).

The occurrence of slab detachment is the result of the development of large extensional stresses within the downgoing slab. Slowdown of the subduction rate is a plausible mechanism that can account for such stress build up (Li and Liao, 2002) and is likely to take place following the subduction of a ridge (Andrews and Billen, 2009; Burkett and Billen, 2011) or the subduction of continental material (Baumann et al., 2009; van Hunen and Allen, 2011; Duretz et al., 2011). The slab detachment model has two major consequences: (1) a partial or complete loss of the slab pull force and (2) the inflow of hot asthenosphere at the location of the detachment. The loss of slab pull results in the rebalancing of the forces acting on the plate margin which can potentially triggers a wide range of dynamical effects. The slab detachment model was consequently used in the explanation of tectonic processes such as high pressure and ultra-high pressure rock exhumation (Andersen et al., 1991; Babist et al., 2006; Xu et al., 2010), variations in surface uplift rates (Rogers et al., 2002; Morley and Back, 2008; Wilmsen et al., 2009), and in the sedimentary record (Sinclair, 1997; Mugnier and Huyghe, 2006), orogenic extension (Zeck, 1996), rapid changes in plate motions (Austermann et al., 2011), or the reversal of the subduction dip (Regard et al., 2008).
The inflow of asthenosphere within a detaching slab is usually considered as an efficient mechanism to advect heat at lithospheric to subcrustal level (van de Zedde and Wortel, 2001), subsequently triggering partial melting in the mantle (Davies and von Blanckenburg, 1995; Ferrari, 2004; Altunkaynak and Can Genc, 2004) associated to plutonism and volcanism (Keskin, 2003; Qin et al., 2008; Ferrari, 2004). Many analytical, semi-analytical, analogue and numerical modelling studies have focussed on slab detachment (Davies and von Blanckenburg, 1995; Ton and Wortel, 1997; van de Zedde and Wortel, 2001; Buiter et al., 2002; Gerya et al., 2004; Cloetingh et al., 2004; Toussaint et al., 2004; Li and Liao, 2002; Andrews and Billen, 2009; Regard et al., 2008; Macera et al., 2008; Baumann et al., 2009; Schmalholz, 2011; van Hunen and Allen, 2011; Burkett and Billen, 2011; Duretz et al., 2011). These studies have been designed to evaluate the slab detachment depth (van de Zedde and Wortel, 2001; Gerya et al., 2004; Baumann et al., 2009; Duretz et al., 2011), the duration of slab detachment (Gerya et al., 2004; Andrews and Billen, 2009; Baumann et al., 2009) as well as the topographic expression of slab detachment (Buiter et al., 2002; Gerya et al., 2004; Duretz et al., 2011). Recently, three dimensional aspects of slab detachment have been addressed in laboratory experiment (Regard et al., 2008) and numerical simulations (van Hunen and Allen, 2011; Burkett and Billen, 2011). The analytical study of Schmalholz (2011) further indicates that the mechanism of detachment is a result of the non-Newtonian rheology of the slabs.

However, the importance of rheology of the crust on slab detachment as well as the interaction between slab detachment and slab rollback (Schellart, 2004; Brun and Faccenna, 2008; Husson et al., 2009a) has not yet been studied into detail. Since both slab detachment and slab rollback can trigger strong topographic effects (Duretz et al., 2011; Husson et al., 2011), we use 2D state-of-the-art numerical simulations to investigate the influence of the crustal rheology on the occurrence of slab detachment, the evolution of collision zones and their topographic evolution.

### 6.2 Modelling approach

#### 6.2.1 Methodology

This study was carried out using the thermo-mechanical code I2VIS (Gerya and Yuen, 2003a; Gerya, 2010). This numerical code solves the two-dimensional steady state momen-
CHAPTER 6. CRUST RHEOLOGY, SLAB DETACHMENT AND TOPOGRAPHY 189

<table>
<thead>
<tr>
<th>Material</th>
<th>$k$ (W/m/K)</th>
<th>$H_r$ (W/m$^3$/s)</th>
<th>$C_p$ (J/kg)</th>
<th>$\sin(\phi)$</th>
<th>$C$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sediments</td>
<td>0.64 + $\frac{807}{T^3}$</td>
<td>$1.50 \times 10^{-6}$</td>
<td>1000</td>
<td>0.15</td>
<td>1</td>
</tr>
<tr>
<td>Upper cont. crust</td>
<td>0.64 + $\frac{807}{T^3}$</td>
<td>$1.00 \times 10^{-6}$</td>
<td>1000</td>
<td>0.15</td>
<td>1</td>
</tr>
<tr>
<td>Lower cont. crust</td>
<td>1.18 + $\frac{474}{T^3}$</td>
<td>$0.25 \times 10^{-6}$</td>
<td>1000</td>
<td>0.15</td>
<td>1</td>
</tr>
<tr>
<td>Upper oceanic crust</td>
<td>0.64 + $\frac{807}{T^3}$</td>
<td>$0.25 \times 10^{-6}$</td>
<td>1000</td>
<td>0.00</td>
<td>1</td>
</tr>
<tr>
<td>Lower oceanic crust</td>
<td>1.18 + $\frac{474}{T^3}$</td>
<td>$0.25 \times 10^{-6}$</td>
<td>1000</td>
<td>0.60</td>
<td>1</td>
</tr>
<tr>
<td>Mantle</td>
<td>0.73 + $\frac{1293}{T^3}$</td>
<td>$2.20 \times 10^{-8}$</td>
<td>1000</td>
<td>0.60</td>
<td>1</td>
</tr>
<tr>
<td>Weak zone</td>
<td>0.73 + $\frac{1293}{T^3}$</td>
<td>$2.20 \times 10^{-8}$</td>
<td>1000</td>
<td>0.00</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 6.1: List of thermal and plastic (Mohr-Coulomb) parameters used in the simulations, $H_r$ is the radiogenic heat production, $C_p$ is the specific heat capacity, $\phi$ is the internal friction angle, and $C$ is the cohesion.

The mechanical solver uses a viscous formulation and the deviatoric stress tensor $\sigma_{ij}$ relates to the effective material viscosity $\eta_{eff}$ and the rate of deformation $\dot{\epsilon}_{ij}$ via:

$$\sigma_{ij} = 2\eta_{eff}\dot{\epsilon}_{ij} = \eta \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)$$  \hspace{1cm} (6.4)
All the lithologies can deform according to a visco-plastic rheological model. This rheological model combines several flow rules that contribute to the effective viscosity of the material. A more detailed description of this approach is given in Sec. 6.3.1.

The model’s surface $h$ (air/crust interface) evolves following a gross-scale erosion-sedimentation law (Gerya and Yuen 2003b; Gerya, 2010):

$$\frac{\partial h}{\partial t} = v_y - v_x \frac{\partial h}{\partial x} - \dot{\epsilon} + \dot{s}$$

where $v_y$ and $v_x$ are the uplift and advection velocity (m/s) predicted by the tectonic model. $\dot{\epsilon}$ and $\dot{s}$ represent prescribed erosion and sedimentation rates, in our simulation $\dot{\epsilon} = 0.1$ mm/a if $h > 1$ km and $\dot{s} = 0.1$ mm/a if $h < -1$ km.

The advection equation is solved explicitly by a coordinate update of the Lagrangian markers:

$$x_{j}^{t+1} = x_{j}^{t} + \Delta t v_{j}^{t},$$

where $\Delta t$ represents the value of the timestep at the current time integration and $v_{j}^{t}$ corresponds to the markers velocity computed via 4th order (in space) Runge-Kutta scheme. It is worth noticing that, for the sake of simplicity, our simulations do not take into account the effects of partial melting and hydration/dehydration processes.

### 6.2.2 Setup

Our setup consists of two continental plates separated by an oceanic basin (Fig. 6.1). The different lithologies are represented by temperature-stress dependent visco-plastic rheologies (see Sec. 6.3.1). In order to take into account the effect of phases changes, we employ a precomputed thermodynamic database to update material properties (density, heat capacity) according to their current pressure and temperature similarly to Gerya et al. (2004).

The model domain is contained in a $4000 \times 1400$ km box ($1361 \times 351$ nodes) where all boundaries are free slip. We employ variable grid spacing which enables to reach a 1 km grid resolution in the collision area. In order to study the interplay between slab dynamics and orogenic evolution, we follow the methodology developed in Baumann et al. (2009); Duretz et al. (2011). The subduction-collision system is therefore kinematically prescribed during the stage of oceanic subduction only. After the oceanic basin closure, the system is driven by the slab pull force leading to a dynamic collision between the continental plates and eventually to slab detachment. The development of the oceanic subduction is achieved
Figure 6.1: Initial phase distribution in the model domain (cropped area) and evolved simulation (after 38 Ma of convergence). The slab temperature is computed according to an half-space cooling model given an age of 40 Ma. The continental geotherms are both initially linear. A symmetric total convergence rate of 1.25 cm/y is imposed at the location of the arrows until 500 km of convergence has been reached. The shaded areas and dashed lines represent breaks in the model’s scale.

by imposing a symmetric plate convergence rate of 1.25 cm/a until 500 km of convergence is accommodated. In practice, the velocities are prescribed within the model domain at the location indicated on Fig. [6.1] and during a period of 40 Ma. Once this internal kinematic constrain is deactivated, the system becomes dynamic and is only driven by body forces (e.g. slab pull, ridge push). With ongoing convergence, the two converging plates decouple from the lateral sides of the box, these zones accommodate hot mantle upwellings and are therefore the location where oceanic ridges spontaneously develop (Fig. 6.1). We employ an additional 20 km thick layer of sticky air ($\eta_{\text{air}} = 10^{18}$ Pa.s, $\rho_{\text{air}} = 0$ kg/m$^3$) in order to mimic the effect of a free surface and enable the development of topography (Schmeling et al., 2008). The initial slab temperature field is defined using the half-space cooling model (Turcotte and Schubert, 1982) using a slab age of 40 Ma and a diffusivity of $10^{-6}$ m$^2$.s$^{-1}$. The continental crust geotherms are initially defined as linearly varying from surface ($T = 293$ K) to the bottom of the domain ($T = 1617.6$ K).
6.3 Strength of the lithospheric model

6.3.1 Rheological model

We consider that the strength of the lithosphere is controlled, on the timescale of orogeny, by the combination of both brittle and ductile deformation mechanisms. In our model the brittle part of the lithosphere is controlled by Mohr-Coulomb (or Drucker-Prager) plasticity which display a linear dependance with the pressure or depth in the case of a lithostatic pressure gradient. Mohr-Coulomb plasticity act as a stress limiter in the regions where the second stress invariant ($\sigma_{II}$) exceeds the material yield stress. The yield stress depends on the pressure, the cohesion $C$ (MPa), and the internal friction angle $\phi$. The stress is limited via local viscosity reductions such as

$$\eta_{\text{creep}} \leq \frac{C + P \sin(\phi)}{2\sqrt{\dot{\epsilon}_{II}}} \quad (6.7)$$

Our rheological model takes into account a second semi-brittle deformation mechanism characterised by the exponential flow of olivine (or Peierls mechanism) (Evans and Goetze, 1979; Kameyama et al., 1999; Raterron et al., 2004; Katayama and Karato, 2008). This regime has exponential dependance on the second stress invariant and can therefore act as a strong weakening mechanism in the lithospheric mantle. The effective viscosity corresponding to the Peierls creep regime is formulated as:

$$\eta_{\text{Peierls}} = \frac{1}{A_{\text{Peierls}} \sigma_{II}} \exp \left[ \left( \frac{E_a - PV_a}{RT} \right) \left( 1 - \left( \frac{\sigma_{II}}{\sigma_{\text{Peierls}}} \right)^k \right)^q \right] \quad (6.8)$$

We use the dry olivine parameters $A_{\text{Peierls}} = 10^{7.8} \times 10^{-12}$ and $\sigma_{\text{Peierls}} = 9.1$ GPa (Evans and Goetze, 1979), we allow the activation of Peierls creep for temperature lower that 1373 K.

Ductile (diffusion-dislocation) creep is represented by the temperature and stress-dependent dislocation creep (Ranalli, 1995). This mechanism can be active in any of the lithospheric level as long as the current state of stress ($\sigma_{II}$) does not exceed the Mohr-Coulomb or Peierls yield stresses. At stresses larger than 30 kPa, most of the flow occurs in the dislocation creep regime (Turcotte and Schubert, 1982) and depends on the second invariant of the strain rate tensor ($\dot{\epsilon}_{II}$), the temperature and the pressure (Ranalli, 1995).
The effective viscosity corresponding to a dislocation creep regime is calculated as following:

\[
\eta_{\text{creep}} = \eta_0 \frac{1}{n} \dot{\varepsilon}^{\frac{1}{n}} \exp \left( \frac{E_a + PV_a}{nRT} \right)
\] (6.9)

where \( n \) is the stress exponent, \( \eta_0 \), the reference viscosity (Pa\(^n\).s), \( E_a \), the activation energy (J), \( V_a \), the activation volume (J/bar) and \( R \), the gas constant (8.314472 J/mol/K).

In practice, the effective viscosity corresponding to each flow rule is calculated for every lithology. The mechanism that produces the lowest viscosity is defined as the active deformation mechanism. For practical reasons, the viscosity is limited such as \( 10^{18} < \eta < 10^{25} \) Pa.s.

### 6.3.2 Rheological profiles

The first order contribution of rheological model to the lithospheric strength can be probed by the use of rheological profiles or “Christmas trees” (Jackson, 2002; Afonso and Ranalli, 2004; Burov and Watts, 2006). Although this representation usually assume a constant average strain rate (\( \dot{\varepsilon}_{\text{av}} \)) through the entire lithospheric column (upper/ lower crust (UC/LC), mantle lithosphere (ML)), they can be used as first order indicators of the lithospheric strength (\( F \)) for modelling purposes (Thompson et al., 2001; Toussaint et al., 2004d; Gerbault and Willingshofer, 2004; Schmalholz et al., 2009). We define three main end-members that are characterised by the rheology of the continental crust (Fig. 6.2). The stress profiles a generated for a crustal thickness of 35 km, a Moho temperature of 450 °C and a lithostatic pressure gradient. This set of parameters reflects the initial continental state of stress in our simulations, each lithospheric end-member is characterised by a strong upper mantle rheology that can account for the coherent development of subduction and the long-term support of topography (Burov, 2011; Burov and Watts, 2006). The rheological parameters that are utilised to generate the different rheological profiles are listed in table 6.1 and 6.2. The single layered quartzite crust (Wet quartzite of Ranalli (1995)) represent our weakest crust end member (Fig. 6.2A). It is characterised by a single brittle-ductile transition that occurs between 17 and 20 km depth for an average strain rate varying between \( 10^{-13} \) and \( 10^{-17} \) s\(^{-1}\). Our second lithospheric model is composed of a two-layer crust (Fig. 6.2B) in which the upper 20 km is composed of quartzite rheology and the lower part (15 km thick) of plagioclase (An\(^{75}\) of Ranalli (1995)). Two brittle-ductile transitions occurs within the crust and are located at around 20 and 25 km, respectively. It is worth noticing that this crustal model correspond is similar to the one used in our previous study.
Figure 6.2: Example of rheological profiles for the different strength end-members for 3 different average strain rates. A) Weak crust: quartzite rheology. B) Layered quartzite/feldspar crust. C) Felsic granulite crust. D) Corresponding pressure and temperature profiles. E) Integrated strength versus average strain rate diagram for the three different lithosphere models.

Our strongest crust model (Fig. 6.2C) consists of a single layer of granulitic rheology (felsic granulite of Ranalli (1995)). Within this crust, the brittle-ductile transition takes places at depth ranging from 20 to 30 km for corresponding strain rates of $10^{-13}$ and $10^{-17}$ s$^{-1}$. The Peierls mechanism is active in each presented lithospheric model. It takes over Mohr-Coulomb plasticity at a depth of about 45 km to a maximum depth of 85 km when the average strain rate reaches $10^{-13}$ s$^{-1}$. The integrated strength of each lithospheric model for variable average strain rate is depicted in Fig. 6.2E. The quartzite crust exhibits a strength ranging from $10^{13.3}$ to $10^{13.7}$ N/m whereas the granulite crust is generally almost 1.25 times stronger. Although we do not notice a significant difference of integrated strength between the quartzite/plagioclase and the granulite crust,
we may expect that rheological layering will play an important role on the evolution of our subduction-collision system.

### 6.4 The influence of crustal rheology from 2D subduction-collision experiments

In this section, we model the development of subduction-collision chains in two dimensions using the three lithospheric models described in the section above. We show that the rheology of the crust has a first order importance on the occurrence of slab detachment, the geometry of the collision zone and their pattern of surface topography.

### 6.5 Weak crust end-member: Slab retreat & delamination

**Time evolution**

Our weak crust end-member model is characterised by a crust composed of single layer of quartzite rheology (see Sec. 6.3.2). Fig. 6.3 depicts the evolution of the composition field through time. Although the simulation undergoes compression during the stage of oceanic subduction, extension occurs within the crust of the overriding plate. This thinning event results from the viscous drag exerted by the downgoing plate. Subsequently to oceanic basin close, continental crust subduction initiates and the subducted continental margin reaches a maximum depth of 150 km within 3 Ma. While the crust reaches this depth, it

![Table 6.2: Ductile creep parameters corresponding to the different rheologies employed in the simulations, \( \eta_0 \) is the reference viscosity, \( n \) is the stress exponent, \( E_a \) is the activation energy, and \( V_a \) is the activation volume.](attachment:image.png)
builds up a substantial amount of positive buoyancy ($\sim 10^{13}$ N/m). Since the mechanical coupling across the Moho is weak, delamination of the mantle lithosphere initiates at this interface. The decoupling of the crust from the mantle and its return to the surface triggers pure and simple shear deformation of the crust. Periodical crustal buckling takes place in the towards foreland and propagates with ongoing delamination. The wavelength of these crustal scale fold reaches about 50 km. This deformation leads to a significant thickening of the continental crust (doubling) and the development of 70 km thick orogenic root overlying the asthenosphere. Since the lithosphere totally delaminates from the crust, there is no persistent passive margin subduction and therefore no slab detachment occurs. As no perturbation in the magnitude the slab pull force occurs, slab retreat and delamination are free to develop and positively feedbacks each other.

**Topographic evolution**

The topographic evolution of the simulation is shown on Fig. 6.4A and 6.5A. This topographic shows that each of the geodynamic processes described in the preceding section has its own surface expression. This first feature, which is also shared with the other models (Sec. 6.6 and Sec. 6.5), is the subsidence of the overriding margin during the stage of oceanic basin closure. The subsidence is related to the downward suction exerted by the slab during its descent, it is geographically consistent with extended margin visible on the compositional map (Fig. 6.3). When the continents are sufficiently close to each other the accretionary prism is exhumed and becomes a topographic high. Continental crust subduction initiates soon after ($\sim$ 1 Ma) driving the bulging of the overriding margin and causing the maximum topography to be located close to the plate interface. After 3 Ma of continental burial, the delamination starts driving the thickening and buckling of the lower plate crust. With ongoing slab retreat, the topography develops towards the foreland and reaches its peak altitude 4 Ma after the onset of delamination (7 Ma after collision started). This highest area correspond to region where the mantle lithosphere has been delaminated and where the crust has reached its thickness maximum. High topography is also produced on the upper plate and is located just above the suture where the crust is underthrust by allochtonous crust.
Figure 6.3: Temporal evolution of the weak crust end-member. The crust is composed of a single layer of wet quartzite rheology (Ranalli, 1995). The snapshots represent the compositional field at three different stages: oceanic subduction, continental subduction and onset of delamination, plate decoupling and slab retreat.

Exhumation

In this type of model there is no major HP-UHP exhumation event. Despite the fact that the subducted continental crust has the tendency to flow towards the surface, the material becomes accumulated under the weak thickening orogenic root (see Fig. 6.6A). On a longer
Figure 6.4: Topographic evolution of the three collisional end-members. The different labels correspond to: 1) exhumation of the accretionary prism, 2) onset of continental subduction, 3) start of delamination, 4) slab detachment, s) subsidence of the upper plate, f) foreland basin, h) hinterland basin b) surface trace of crustal buckling. The white dashed line indicates the position of the trench.
Figure 6.5: Surface uplift variation during the evolution of the three collisional modes. The label D corresponds to the surface trace of slab detachment. The symbols f), h) b) respectively indicates the location of the foreland basin, the hinterland basin and the surface trace of crustal buckling. The black solid line symbolises the reference topography (0), the dashed contours indicate altitude levels.
timescale (∼ 100 Ma) and in the absence of any additional tectonic event, we would expect lower crust to be exhumed within the fold syntaxes driven by erosion.

**Figure 6.6:** The three collisional modes and their contrasting styles of exhumation. The composition map is show at the left side, the coloured stars correspond to Lagrangian. The pressure-temperature paths, and vertical velocity of the corresponding Lagrangian material tracers is displayed on the graphs.
6.6 Intermediate crustal rheology: delamination & slab detachment

Time evolution

The intermediate crustal strength model is represented by a two-layer crust of quartzite and plagioclase rheology (Fig. 6.2A), the temporal evolution of this model is described in figure 6.7. In contrast to the weak crust end member (Fig. 6.3), no extension takes place within the overriding margin during the initial stage of convergence. The stage of continental crust subduction last for $\sim 7$ Ma. When the crust reaches the burial depth of $\sim 50$ km, the rheologically weaker upper crust looses it coherency and starts delaminating from the upper crust/lower crust transition. This buoyant flow and thickening of the upper crust leads to the widening of the subduction channel. Consequently, inflow of hot mantle initiates in the deeper parts of the subduction channel and triggers plate decoupling. Slab detachment eventually occurs at a depth of 250 km at the location of the ocean-continent transition. Following the loss of slab pull, eduction occurs leading the subduction to act as normal sense shear zone. Entrained by both eduction and delamination, the formerly subducted crust is extruded towards the surface and spreads over foreland in the form of a channel flow. The lower crust finally delaminates from the slab and diapirically ascends towards the lower part of the orogenic root. At the later stage, the crustal thickness gradually increases from 35 km in the foreland to 75 km in the vicinity of the suture zone.

Topographic evolution

Each of the deep collisional processes described in the previous section have a strong imprint on the topographic development (Fig. 6.4B, 6.5B). Although no extension occurs in the overriding plate during convergence, its margin is affected by subsidence. At the end of the stage of oceanic basin closure, the accretionary is pinched between the converging plates and is responsible for the location of the first topographic high. During the continental subduction period, underthrusting of continental crust yields to a long wavelength topographic bulge leading the topography to develop on either side of the subduction thrust. After $\sim 7$ Ma of continental material burial, the crust start delaminating and the subduction channel widens. The topographic influence of this slab retreat event is noticeable on the lower plate where the orogenic front progressively migrates towards the foreland at a
Figure 6.7: Evolution of the compositional field for the layered crust model. The upper crust (20 km thick) is composed of wet quartzite and the lower crust (15 km thick) of plagioclase (An$_{75}$) (Ranalli [1995]). The collision system successively undergoes delamination, slab detachment and plate eduction.

rate of $\sim$ 2 cm/y. Slab detachment occur 15 Ma after the start of collision and is responsible for the sharpest topographic and surface uplift signals. Since the orogen is already partially decoupled, the topographic response of slab detachment mostly affects the lower
plate topography. At this stage topography grows towards the foreland is associated to the flow of crustal material extruded from the subduction channel.

Exhumation

In this simulation, several mechanisms are responsible for the exhumation of the subducted crust. The first mechanism is the buoyant return flow of the crust within the subduction channel. Decoupling occurs between the upper and lower crust, as a result the upper and lower crustal material follows different exhumation paths (Fig. 6.6B). This mechanism is intrinsically linked to slab retreat and orogen decoupling and can yield to fast return exhumation velocities (here up to 6 cm/a). The second geodynamic event that contributes to exhumation is plate eduction consequently to slab detachment. The loss of slab pull yields the remaining part of slab, which is still attached to continental crust, to be positively buoyant. This rebound leads to partial extraction of the slab from the mantle and therefore to a large scale exhumation of the lower plate material. The contribution of slab detachment is depicted on Fig. 6.6B, the vertical maximum vertical velocity reaches 10 cm/a during a short period (< 3 My) in the lower crust of the educting lithosphere. In this model, the combination of eduction together with the buoyant return flow is responsible for the exhumation of the subducted crust and the development of channel flow of the exhumed crust towards the foreland.

6.7 Strong crust end-member: slab detachment & eduction

Time evolution

The compositional cross sections depicted on Fig. 6.8 represent the time evolution of our strong crust subduction-collision model (Fig. 6.2C). Since the crustal rheology that we employ is strong, the stress exerted by the downing slab is not sufficient to yield extension in the overriding margin. The stage of continental subduction last over a period of ∼ 15 Ma and although the crust has a similar buoyancy has in the previous experiments, its stiffer rheology allows it to remain coherent at depth. Once the subducted margin has reached its maximum burial depth (∼ 200 km), slab detachment takes place and separates the positively and negatively buoyant parts of the slab. Subsequently to slab detachment,
the buoyant part of the slab is educted and the former subduction channel is inverted into a normal sense shear zone. After 3 Ma of coherent plate eduction, the crust finally decouples from the mantle lithosphere to form a slice. This slice is exhumed along shear which is rooted in the Moho of the slab and expressed as foreland dipping thrust in the crust. During exhumation the crust is subject to a substantial amount of thickening leading to the development of a thick orogenic root ($\sim 70$ km) but narrow ($\sim 100$ km) orogenic root.

**Topographic evolution**

Similarly to the other models presented above, it is possible to monitor the influence of the different lithospheric processes on the evolution of topography (Fig. 6.4C, 6.5C). The two first signals that we observe are common feature of these subduction-collision models. These patterns are namely the subsidence of the overriding plate margin during convergence and the exhumation of the accretionary prism prior to collision. During the overall period of continental subduction, the topography develops on the overriding plate as a response to the underthrusting of continental crust. Slab detachment occurs after $\sim 15$ Ma and yields to the geologically instantaneous broadening of the orogen characterised by a strong (15 mm) surface uplift of short duration ($< 0.5$ Ma). Since this type of orogen remains coupled, the topographic rebound associated with slab detachment also affects the upper plate. A noticeable feature of this model is the appearance of the surface trace of the exhumation thrust within the foreland. This trace remains a sharp feature for a period of 5 Ma and witnesses the activation of this deeply rooted shear zone.

**Exhumation**

Two main mechanisms are involved in the exhumation of the buried crust. Each of these mechanisms are intrinsically related to the rheology of the continental crust. The first mechanism, eduction, is the direct consequence of slab detachment. This mechanisms is driven by the buoyancy of the crust and is efficient as long the crust remains coherently coupled to the mantle lithosphere. Since the crust remains coherent during exhumation, the material tracer display a similar retrograde (see Fig. 6.6C), a short ($< 2$ Ma) peak of exhumation rate ($\approx 7$ cm/a) is attained after slab detachment and is the signal of eduction. The second mechanism which finally takes over eduction is the formation and detachment of a crustal slice. This mechanisms requires the crust to be strong enough to enable the
Figure 6.8: Development of the collisional system for a single layered crust of felsic granulite rheology (Ranalli [1995]). The crust remains mostly undeformed during continental subduction, slab detachment occur triggering plate eduction and exhumation of a coherent crustal slice. Another interesting feature of this model is that although the suture dips as a thrust fault it acts as a normal sense shear during the whole period of exhumation (eduction and slice extrusion).
6.8 Discussion

6.8.1 Mechanical coupling within the crust and through the Moho

Our simulation employed different crustal rheologies as well as lithological layering. Such configurations are characterised by different levels and intensity of mechanical coupling within our model lithospheres. We termed mechanical coupling the stress ratio that exist across the different layers of the lithosphere (UC, LC, ML). These levels of decoupling favour the localisation of deformation and thus control the overall evolution of subduction-collision systems. We have used stress profiles in order to monitor the degree of coupling through the subducting lithosphere. Fig. 6.9 shows the state of differential stress across slabs of different crustal rheologies during continental subduction. The wet quartzite crust provides weak coupling at the Moho ($\sigma_{ML}/\sigma_{LC} > 50$) which facilitates the delamination of the lithosphere. The introduction of lithological layered crust (UC/LC) induces an additional decoupling level at mid-crustal depth, this level localises the delamination that occurs at mid-crustal level and promotes the buoyant channelised extrusion of the upper crust. The strong felsic granulite crust exhibits to strong coupling through the Moho ($\sigma_{ML}/\sigma_{LC} < 10$) during continental subduction and favours the deep dragging of continental crust within the mantle. Our results are in agreement with Burov and Yamato (2007) that showed that a weak coupling at the Moho does not promote the formation and exhumation of high pressure rocks (Fig. 6.6a). Moreover, Shemenda and Grocholsky (1992) already suggested that the coupling across the Moho should be sufficiently strong to transfer stress through the crust and produce a pattern of deformation that resembles to what is observed in orogens. In general, mechanical decoupling zones are not only the result of layering but can also be promoted by structural inheritance at crustal scale (Huet et al., 2011) or widespread inherited mechanical anisotropy at lithospheric scale (Vauchez et al., 1998).

6.8.2 Occurrence of slab detachment

The purely retreating collision mode (Fig. 6.3) is characterised by a very weak coupling at the Moho of the lower plate. This promotes the delamination of the lithosphere and results in an quasi freely sinking slab mode. This lack of resistance does not promote the development of strong extensional stresses within the descending slab and therefore delays
and potentially inhibit the occurrence of slab detachment. The geophysical observation of slab detachment in certain collision zones such the Alps, the Carpathians, or the Himlaya (Lippitsch et al., 2003; Cloethingh et al., 2004; Lister et al., 2008) might, to some extent, indicate the degree of coupling at the Moho of the orogen.
6.8.3 Crustal slice exhumation

The exhumation of a buoyant and stiff (Felsic granulite rheology) crust enables the development of foreland dipping thrusts that can partly accommodate its exhumation. Although the crust may suffer significant internal deformation (thickening) during its return to the surface (see Fig. 6.8), its exhumation is bounded by a thrust rooted at the Moho of the descending plate and a normal fault located in the vicinity of the former subduction plane. This coherent style of exhumation is well documented by the similar retrogression pattern in both upper and lower crust (Fig 6.6C). This result is in geometrical agreement with the previous studies (Chemenda et al., 1995; Toussaint et al., 2004a) which describe a similar structural pattern of fault bounded exhumation. In contrast to the results of Chemenda et al. (1995); Toussaint et al. (2004a), our results indicate that the development of such fault bounded exhumation of crustal slice can develop subsequently to slab detachment.

6.8.4 Exhumation rates, average surface uplift rates, and instantaneous uplift rates

In our simulations, the exhumation rates (vertical velocity) of deeply buried rocks are on the order of 5 to 8 cm/a. The typical exhumation signal of slab detachment is associated to the mechanism of eduction and occurs in a short timespan. The fastest exhumation events (10 cm/a) are the results of the combination of both slab detachment and delamination. On the other hand, the instantaneous surface uplift following slab detachment can reach mm/a. Similarly to the exhumation rates, the peak of instantaneous surface uplift is geologically short (< 1 Ma) and decreases exponentially with time. This transient effect yields to time averaged (over a period of 5 Ma) surface uplift rates on the order of 0.2 to 1 mm/a (Duretz et al., 2011).

6.8.5 The development of orogenic plateaus

Delamination of the mantle lithosphere from the overlying crust can result in the development of topography (Bird, 1979). In our simulations, the purely delaminating collisional model (Fig. 6.3), characterised by a weak crust, is associated with pure shear thickening and buckling of the crust resulting in a periodic topographic imprint. The conditions under which our models develop a wide undeformed plateau are (1) the combination of a strong
lower crust (avoiding delamination at the Moho) and (2) a weak upper crust (mid-crustal decoupling). In this model the growth of the plateau coincides with the extrusion of hot crust from the subduction channel onto the foreland, this feature compares to previous studies of plateau growth by channel flow [Gerbault and Willingshofer, 2004; Medvedev and Beaumont, 2006]. Our results are also in good agreement with the study of [Sokoutis and Willingshofer, 2011] that stressed the importance of mid-crustal decoupling on the development of topographic plateaus.

### 6.8.6 Initial rates of convergence

As pointed in [Duretz et al., 2011], the initial rate of convergence plays a role in the evolution of the collisional system. This kinematic constraint controls the velocity at which the oceanic subduction develops, it will therefore influence the thermal structure, strength and buoyancy of the slab. We expect that, for a given initial thermal slab age, a subduction with fast converging plates will generate a larger slab pull force than slower ones. In [Duretz et al., 2011], we presented models of subduction-collision systems driven by initial plate rates larger or equal to 2.5 cm/a. Using a similar slab thermal age (40 Ma) and crustal structure (quartzite/plagioclase crust) but a slower initial plate rate (1.25 cm/a) we notice that delamination may occur prior to slab detachment (see Sec. 6.6). This is the consequence of the reduced slab pull force that is generated by slowly converging plate systems.

### 6.8.7 Crustal rheology and continental subduction

In our 2D simulations, the maximum depth of continental crust burial is dynamically limited by two competing mechanisms: (1) the occurrence of slab detachment or (2) the onset of delamination. Our results indicate that a weak lower crust favours fast delamination from the Moho and moderate burial depth ($\approx 140$ km). On the other hand, a strong lower crust provides sufficient mechanical coupling at the Moho to drag continental crust deep into the mantle ($>180$ km) until detachment eventually occurs. Under the assumption that HP-UHP domains are produced under near-lithostatic conditions [Burov and Yamato, 2007; Yamato et al., 2008], the existence of such domains at the surface of the Earth [Chopin, 2003; Hacker et al., 2006] may witness from a strong coupling between the mantle lithosphere and the crust during periods of continental subduction.
6.8.8 Shape of the margin

Franco et al. (2008) have studied the influence of the passive margin geometry on the early dynamics of continental collision. The compilation provided by the authors shows that passive margins can be characterised by their slopes that can range between 3 and 15 degrees. When entering into subduction, passive margins provide both geometrical (slope) and a thermal variations (transition from continental to oceanic geotherms) that can lead to stress focussing. In continental subduction setting, slab detachment is likely to occur at the passive margin (Duretz et al., 2011) and we can expect passive margin geometries to exert a strong control on slab detachment (Cloethingh et al., 2004). Since slab detachment is the results of a viscous tensional mechanical instability Schmalholz (2011), the slab deformation is likely to take place at passive margins. We therefore expect steep passive margins to provide large and localised stress concentration yielding to faster slab detachment, whereas low angle margins may results in a diffuse stress distribution pattern and delay the timing of slab detachment.

6.8.9 Coupled and decoupled style of orogeny

Faccenda et al. (2008, 2009) have studied the parameters that controls the development of coupled and decoupled orogenic styles. Decoupled orogenic (to post-orogenic) style is usually associated with retreating slabs and widespread extension, examples of such types of orogens are the Apennines (Reutter et al., 1980) or the Aegean domain (Jolivet et al., 2009). In contrast, coupled collisions such as the Western Alps or the Himalaya are characterised by their compressional state of stress and the thickness of their crustal wedge (Faccenda et al., 2009). The authors suggest that slowly converging collisions (2 cm/a) promotes coupled collision and fast convergence (10 cm/a) leads to decoupled collisional style. Their results also showed that plate decoupling is further enhanced by the percolation aqueous fluids in the mantle wedge, sourcing from the dehydration of previously subducted oceanic crust, promote the weakening of the plate boundary and Moho leading to decoupling of the collision zone. Although our models differ significantly from those of Faccenda et al. (2008, 2009) in terms of boundary conditions, our results are in good agreement since both studies predict decoupled orogenic style when the mechanical coupling of the Moho is weak.
6.8.10 Tectonic forces, model limitations and future perspectives

The 2D simulations presented above do not take into account any three dimensional effects. The toroidal component of the asthenospheric flow, which is absent in two-dimensional simulations, can significantly influence subduction dynamics by enhancing slab retreat (decoupled) \((\text{Husson et al., 2009b})\) and even leading to slab dip-reversal following slab detachment \((\text{Regard et al., 2008})\). We also expect that collisions undergoing slab detachment might also exhibit inhomogeneous dynamics in the along trench direction \((\text{Burkett and Billen, 2011; van Hunen and Allen, 2011})\). As pointed out by \((\text{Toussaint et al., 2004b})\), the process of continued convergence following slab detachment remains poorly understood. However, if slab detachment occurs sufficiently deep \((> 300 \text{ km})\), the negative buoyancy of the hanging slab combined with far-field push forces might be sufficient to restart convergence. On the other hand, it is important to take into account three-dimensional effects such slab pull transmission (in the along-trench direction) in areas where slabs are partially detached. This mechanism, which remains to be explored, may lead to the continued convergence in regions where the slab is detached and which are adjacent to regions where the slab can still be attached.

6.9 Conclusions

Our results show that the rheology of the crust, as well as its structure, can by itself determine the evolution of subduction-collision systems. These collisional evolutions can vary from narrow orogens subjected to slab detachment to broad orogens that are dominated by widespread delamination.

A strong crustal end-member enables mechanical coupling at the Moho, this configuration favours the subduction of crust. Deep continental subduction \((180 \text{ km})\) eventually leads to the detachment of the slab at the passive margin and coherent exhumation of the crust by means of eduction and thrusting. These orogens become bivergent subsequently to slab detachment and are characterised by a high \((> 4.5 \text{ km})\) and narrow \((< 200 \text{ km})\) topographic plateau located in the vicinity of the suture.

On the other hand, a weak crustal rheology provides decoupling at the Moho. This type of collision promotes the widespread delamination of the lithosphere with ongoing subduction which can inhibit the occurrence of slab detachment. The buckling of the crust, decoupled
from the mantle lithosphere, triggers the development of periodic (50 km) topographic bulging located on the lower plate.

Layered crustal models introduces a level of decoupling at mid-crustal depths. Such rheological configuration promotes the delamination of the upper crust and the deep subduction of the lower crust. This type of collision can therefore be successively affected by delamination (associated to rollback) and slab detachment. The extrusion of the crust and its channelised propagation onto the foreland (lower plate) results in the development of a wide (> 200 km) topographic plateau. The lateral growth of this plateau is further enhanced by the occurrence of slab detachment.

These results may indicate that the occurrence of both delamination (Apennines) and slab detachment (Himalaya) in orogens is strongly related to initial differences in terms of crustal rheology and structure.

**Bibliography**


Conclusions and perspectives

7.1 Thesis summary

The work has that was achieved during this thesis has been divided into six main topics, a brief summary of each of these thesis chapter follows.

- Chapter 2 provides a numerical study of the Finite-difference/Marker-In-Cell method. A grid convergence study was carried out and allowed to measure the order of accuracy of this numerical method. This study further add to the validity of this method for the modelling of large deformation geophysical flows with strongly variable viscosity. Moreover, a (quasi) free surface stabilisation algorithm was developed. This technique allows for the damping of spurious oscillations that may arise simulations which combines mantle flow with topographic evolution.

- Chapter 3 presents the results of subduction-collision simulations subjected to spontaneous slab detachment. The results indicated that depending on the initial age of the oceanic slab, the depth of slab detachment can vary between 40 and 400 km. The different styles of collision associated to slab detachment developed characteristic topographic evolution reflecting the dynamics of subduction. Furthermore, surface uplift rate, following slab detachment, was show to be a function of the depth of slab detachment.

- Chapter 4 investigates the modes of deformation that lead to slab detachment. The results demonstrated that slab detachment is fast (< 4 Ma) and results from both pure and simple shear deformation. Pure shear deformation takes the form of ductile
necking and becomes the dominant deformation mode at large slab dips. The comparison between 2D thermo-mechanical experiments and 1D analytics showed good agreement and further stressed the role of ductile necking during slab detachment. Additionally, the influence of shear heating was addressed and the result showed that its contribution may speed up the detachment process (up to 5 times).

- Chapter 5 evaluates the dynamic lithospheric response that follows slab detachment. The study showed that slab detachment can lead to the normal sense reactivation of the subduction channel after continental subduction. This extensional response, termed eduction, is the dynamic consequence of the force rebalancing within collisions undergoing slab detachment. This mechanism can partially explain the coherent exhumation of high pressure rocks, a discussion concerning the applicability of such model to the Western Gneiss Region (Norway) was provided.

- Chapter 6 focusses on the impact of the crustal rheological structure on the evolution subduction-collision models. Model involving a strong crust are characterised by a strong coupling at the Moho that promotes deep continental subduction and slab detachment. The introduction of a weak crust favours delamination and slab retreat, resulting in the inhibition of slab detachment. The combination of delamination and slab detachment was achieved by employing a layered crust (upper/lower crust). A detailed description of the topographic evolution and uplift history of these collisional models was presented.

In order to conclude the thesis, and as an introduction to some future steps of subduction modelling, a brief presentation of recent modelling results will be described in the next sections.

### 7.2 3D models and topography

#### 7.2.1 Background

In the previous thesis chapters, we have addressed multiple features of the slab detachment process by means of two-dimensional models. However, the interpretation of seismological observations indicates that slab deformation, ultimately leading to slab detachment, is likely to be inhomogeneous in the along-trench direction. The slab detachment model is
often idealised as a tear propagation that progressively separates the slab from the over-riding plate (Wortel and Spakman, 2000; Nolet, 2009). Moreover, the propagation of slab detachment in the trench parallel direction can potentially have a significant influence on the development of topography and volcanism (Wortel and Spakman, 2000; Ferrari, 2004). For example, Fig. 7.1 show the conceptual geodynamic model for Miocene volcanic belt in Baja California (Mexico) (Pallares et al., 2007). Ridge-trench collision and subsequent slab tear detachment may have led to slab melting and progressive along-trench volcanic pattern.

Figure 7.1: Ridge-trench collision, development of slab tearing and subsequent slab melting and volcanism. (Modified from Pallares et al. (2007))

These geological and geophysical observations have motivated three dimensional numerical studies of slab detachment (van Hunen and Allen, 2011; Burkett and Billen, 2011). Slab detachment subsequently to a ridge trench collision has been investigated by Burkett and Billen (2011), their results indicated that ridge subduction triggers simultaneous thinning of the slab in the along trench direction. The subduction-collision of a ridge perpendicular to the trench direct can lead to the development of slab tears that originates from the slab interior and propagates towards the slab edges (Fig. 7.2a). Models of continental collision leading to slab detachment have been addressed in van Hunen and Allen (2011), their results suggested that detachment can occur between 10 to > 20 Ma after the onset of continental collision. Moreover, their experiments showed that a symmetric collision leads to homogeneous necking of the slab in the trench direction, whereas lateral tearing initiating from the edge of the slab is promoted by an oblique collision (Fig. 7.2b).
However, the pattern of topographic development in response to slab detachment has not yet been addressed in any of the previous three-dimensional studies. In the next section, we introduce recent three-dimensional results of continental-collision with a pseudo free surface, a methodology allows us to track the topographic evolution of the subduction-collision system.

### 7.2.2 Model setup

The following simulations were computed with the 3D numerical code I3VIS (Zhu et al., 2009) which is based on the finite difference / Marker-In-cell technique (Gerya and Yuen, 2009).
Figure 7.3: a) Outline of the lithospheric units initially present in the 3D computational domain. $C_{A,B}$, $O_{A,B}$, $A$, $WZ$, $TF$ respectively stand for continental lithosphere, oceanic lithosphere, asthenosphere, weak zone, transform fault. The label $I$ (inflow), $O$ (outflow), $FS$ (free slip) denotes the boundary conditions that are applied to the faces of the domain. 

b) Distribution of the lithologies (refer to figures from the previous Chapters for the colour legend) and topography at the beginning of the simulation.

The size of the computational domain is 1000 km long, 328 km deep and 200 km wide with a corresponding resolution of $501 \times 164 \times 101$ nodes in the $x, y, z$ directions.
(effective resolution of \(\approx 2 \text{ km}\)). The domain contains two continents which are initially separated by a 500 km long oceanic basin (Fig. 7.3). The left continent \((C_A)\) has a limited width (150 km) and is adjacent to a transform fault. This transform fault (low viscosity zone) separates the ocean into two independent plates \((O_A\) and \(O_B)\) that can decouple and slide past each other. A weak zone is located along the margin of the right continent \((C_B)\), this low viscosity zone dips at an angle of 45 degrees and facilitates the initiation of subduction. The thermal age of the oceanic plate, which is employed to compute the initial thermal field, is 30 Ma. The initial continental geotherm is linear and varies from 0 to 1327 °C at 90 km depth. The mechanical boundary conditions are free slip on the top, front and back boundaries, the left and right sides have are subjected to inflow and outflow (no slip in the \(y\) direction). The right boundary has an inflow rate of 1 cm/a, the left boundary has the particularity of combining inflow (2 cm/a) at the location of the continent \((z > 50 \text{ km})\) and outflow (1 cm/a if \(z < 50 \text{ km}\)) at the other side of the transform fault. The bottom boundary is permeable and assumes free slip at infinity in the \(x\) and \(z\) direction, an outflow rate of 0.8 mm/a is applied in order to satisfy the incompressibility requirement. The thermal boundary conditions are zero flux at left, right, back and front boundaries, the top boundary is fixed temperature (273 K) and the bottom boundary employs a prescribed zero flux at infinity. In order to let the topography develop, a 15 km thick sticky air layer of constant viscosity \((10^{19} \text{ Pa.s})\) \cite{Schmeling et al., 2008} is added on top of the crust.

### 7.2.3 Evolution of the model

The initial phase of convergence is characterised by oceanic subduction (Fig. 7.4a,b). During this period, the left continent slides along the transform fault and approaches the right continent. This period of oceanic subduction last around 20 Ma and is characterised by the development of a deep subduction trench (Fig. 7.5a). As soon as continental collision occurs (Fig. 7.4c), deformation propagates within the overriding continent. The subduction of continental material leads to a large uplift of the overriding margin. After 100 km of burial the upper continental crusts starts delaminate, this delamination process is accompanied by a stage of slab retreat and the broadening of the topography on the overriding plate. It is interesting to notice that with ongoing collision, lateral extrusion develops at the edge of the mountain belt. After approximately 25 Ma of continental collision, slab detachment occurs at 200 km depth (Fig. 7.4d). The detachment of the
slab takes place at the location of the passive margin and trigger an uplift. As a result, topography builds up on the lower plate and a 200 km wide plateau (≈ 4 km of altitude) is established linking both side of the suture (Fig. 7.5f).
7.2.4 Slab detachment in 3D

Similarly to the results obtained by Burkett and Billen (2011); van Hunen and Allen (2011), we notice that slab detachment has the tendency to initiate at the centre of the slab and further propagates towards its edge. Fig. 7.6 shows the contour of the slab (here viscosity contour) during detachment, these results indicate that the viscosity reduction resulting from the necking of the slab is more efficient in the middle of the slab. This viscosity distribution correlates well with the shape of the 1600 K isotherm (Fig 7.4e) which also suggests that the slab is colder, and therefore stronger than its surroundings.


Figure 7.6: Viscosity contour ($8 \times 10^{22}$ Pa.s) of the slab during detachment. The left picture shows the front side of the slab colored by the magnitude of the velocity vector. The right picture shows the back side of the slab coloured by the second strain rate invariant.

7.2.5 Comparison with 2D models

The three dimensional models enables us to study processes in the along trench direction, such as tear propagation and lateral extrusion resulting from collision. Nevertheless, our 3D results are also in good agreement with the two dimensional simulations presented in the preceding chapter. Although the timing can be different, the simulation presented above shows the subsequent occurrence of delamination (with slab retreat) and slab detachment, resulting in the formation of a $\approx 200$ km wide topographic plateau. These results are in sequential agreement with the 2D results obtained in Chapter 6 using a similar crustal layering (Wet quartzite / An$_{75}$ from Ranalli (1995)) that predicted a similar collisional development. In terms of detachment dynamics, these results (as well as those of (Burkett and Billen, 2011; van Hunen and Allen, 2011)) indicate that ductile necking is an important mechanism that contribute to slab detachment and further add to the validity of the results presented in Chapter 4. However, we notice that the duration of the continental collision stage is longer than what was obtained in the 2D simulations (up to twice), the duration of the necking events is also longer than the mean 2D prediction. Both discrepancies might
be related to the underestimation of the slab pull force in this simulation, this effect comes into account as soon as the slab penetrates the permeable lower boundary.

### 7.2.6 Further developments

With the further affordability of 3D geodynamic models, the future studies of slab detachment shall be carried out in a larger model domain. The use of a deeper box will lead to better evaluation of the slab pull force and the study of deep interactions of the slab with the phase transition boundaries. Increasing the size of the model domain in both lateral direction will allow for a better quantification of processes such syn-collisional lateral extrusion and orogen parallel extension. As suggested in Chapter 3 as well as in several previous studies \[\text{[Braun, 2010]} \quad \text{[Matthews et al., 2010]}\], the feedback between mantle processes and the surface of the Earth is strong, and thus deep processes (several hundreds of km's) within the mantle can significantly affect the surface of Earth and the development of topography. For a better understanding of the interplay between surface evolution and mantle dynamics, future subduction modelling studies would benefit from the coupling with landscape evolution models such as Cascade \[\text{[Braun and Sambridge, 1997]}\] or CHILD \[\text{[Tucker et al., 2001]}\]. Such coupled approaches will allow for realistic topographic developments in response to surface motions computed via the lithosphere/mantle-scale modelling code, reciprocally lithospheric deformation will be influenced by the redistribution of topographic loads produced by the landscape evolution model \[\text{[Braun and Yamato, 2010]}\]. Naturally, the future studies should also include model validation consisting of a detailed comparison of the model results with both geological and geophysical observations in collision zones such as the Alps or the Himalaya.

### Bibliography


Education

- Ph.D., ETH Zürich, 2011 (Defended on December 14th, 2011).
  - Committee: Prof. Dr. Wim Spakman, Dr. Jeroen van Hunen, Prof. Dr. Taras Gerya, and Prof. Dr. Paul Tackley.
- Master in Geosciences and Environnement, Université Louis Pasteur, Strasbourg, 2008.
- Bachelor in Earth Sciences, Université Louis Pasteur, Strasbourg, 2006.

Fields of Research Interest

Tectonics, Geodynamics, Numerical and Analogue Modelling, Structural Geology.

Scientific Publications

• Duretz T., S. M. Schmalholz and T. V. Gerya. Dynamics of slab detachment submitted to Geochemistry, Geophysics, Geosystems.

• Duretz T. and T. V. Gerya. Crust rheology, slab detachment and topographic evolution submitted to Tectonophysics.


Teaching Experience

• Teaching assistant: numerical modelling for geodynamics (TOPO-EUROPE Spring course), ETH Zurich 2011.
• Lecturer: Alpine Geology Seminars (extraordinary class), University of Lausanne 2011.

• Field assistant: gravimetry, ETH Zurich 2010.

• Lecturer: Alpine Geology Seminars (extraordinary class), University of Lausanne 2010.

• Teaching assistant: Finite Element method for geodynamics, ETH Zurich 2010.

• Teaching assistant: Finite Difference method for geodynamics (PRODOC Adamello course), ETH Zurich 2009.

Field Experiences

• Magmatic petrology and volcanism, Sierra Nevada/Cascades, US (2010).

• Metamorphic petrology and structural geology, Western Gneiss Region, Norway (2009).

• Magmatic petrology, Adamello massif, Italy (2009).


• Sedimentary processes, Mont Saint-Michel, France (2008).

• Fold-thrust belt geology, Svalbard, Norway (2008).

• General geology, Digne/Barles, France (2007).

• Alpine transect and general geology, Alps/Italy, France (2007).

• General geology, Ardennes, France/Belgique (2005-2006).
Acknowledgements

Deep acknowledgments to my official thesis supervisors Taras and Paul as well as to my (unofficial) supervisors DD and Bo.
I’d like to thank Prof. Wim Spakman and Dr. Jeroen van Hunen for accepting to examine my work.
This work benefited from collaborations with Torgeir and Stefan who are warmly acknowledged.
Massive shout out to my people: Pips, la Tartouze, Laeti, Alban(ounet), Marcel, Gregor, JV, MikieMike, DaddyF, Jed, Nobu, Taka, Prof’s WetWetWet, Prof. Castagnette, Kosuke, Ylona, Kees.
Thanks a lot to my (numerous) office mates: Kata, Jess, Andrey, Sanja, Gang, Toby, and the Lin(s).
Big Danke to the former “Taras kids” who helped me a lot back in the Hönnggerberg days: Manuele, Zhonghai, Irene.
Merci Jean-Pierre pour toutes ces discussions au sujet du “necking”.
Cheers to my “mandates”: Dave, Teo, Darling, and the Matts (yes, both)
Bisous to the (RIP) Andorra crew: Patricia and Manuela
Also, many thanks to the present and former members of GFD, my past and present officemates, Pinar and Mark, the FB crew, the “Adamello kidz”.
Chapter 2 benefited from the reviews of Harro Schmeling and Cedric Thieulot which improved the quality and clarity of the paper.
Chapter 3 has been improved by comments from Sierd Cloetingh and two anonymous reviewers.
Chapter 4 has benefited from the fast and detailed corrections of Vlad Manea, Jeroen van Hunen and the editor Thorsten Becker.
Sincere acknowledgments to DD, Taras, Jean-Pierre, my parents, Marcel, Alban, Dr. Morasse and Greg for their help with proofreading and TeX typesetting.
This work was supported by the TOPO-EUROPE project.