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Energetics of passivity based running with high-compliance series elastic actuation

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ABSTRACT

The efficiency of running gaits in nature results from a passive elastic oscillation on springy legs. We applied this principle to robotic systems by endowing them with high compliance series elastic actuators in which the electric motors are decoupled from the joints by elastic elements. Periodic motor inputs were applied to excite the natural dynamic motion of the robot and create a passivity-based running motion. An optimization algorithm minimized energy expenditure and estimated the necessary initial model states and the coefficients of a parameterized excitation function for the simulations of a two-dimensional hopping monopod and a planar bounding quadruped. Gait synthesis within this framework was analyzed with respect to energy consumption, particularly as a function of running speed. Different solution groups were found, each of them corresponding to a structurally different movement which proved to be most efficient for the corresponding speed range. This shines a different light on the meaning of ‘gait’ in the context of robotics, and directly contributes to a better understanding of the creation and exploitation of different modes of locomotion in legged robotics.

1. INTRODUCTION

The study of passive dynamic walkers [1, 2] has significantly influenced the ways in which we think about legged locomotion. In particular, one can consider steady walking or running as a primarily periodic motion in which the states of the robotic system pass through the same configuration over and over again while the main body is moving forward. In a very abstract mathematical sense, we can consider a legged system as a nonlinear oscillator whose state-variables describe a periodic limit-cycle in phase-space. In order to create an energy efficient gait, this periodic motion should be as passive as possible, i.e., corresponding to natural dynamic oscillations of elastic elements or pendula (as it has been brought to perfection in fully passive dynamic walkers). Actuators should only be used to add sufficient energy to maintain the limit cycle, to stabilize it, and to reject disturbances. These considerations set the foundation for a mutually complementary, three-pronged approach to the control of legged systems that takes full advantage of passive dynamics, applies periodic inputs to excite an energy efficient limit-cycle gait, and adjusts these inputs to account for errors, disturbances, and high-level control inputs.

In this study, we focus on the efficient excitation of such limit cycles in hopping and running systems. While the periodic motion in walking is based on the mechanical dynamics of rigid legs that essentially function as pendula, periodicity in running gaits results from an elastic oscillation of the main body on intentionally springy legs. For this reason, we propose the use of high-compliance series elastic actuators in which the actual actuator and the joint are decoupled by an elastic element. Such actuators have been traditionally employed for force control [3], but are used differently in the context of passive dynamic locomotion: the elastic element is utilized to periodically store and return energy over the course of the gait cycle and is hence designed as part of the entire natural dynamic system. The motor, which acts in series, is only used to excite the natural dynamic motion and to feed energy into the system.

This paper focuses on the synthesis of open-loop control inputs for robotic systems based on such actuators. The inputs are generated to minimize energy consumption while creating a periodic motion of the robotic system. This goal is mathematically expressed as a constrained optimization problem (similar to formulations by Mombaur [4] and Stelzer [5]) and solved numerically. The gait synthesis in this framework is analyzed with respect to energy expenditure, especially as a function of forward velocity. This allows for a different interpretation of the meaning of ‘gait’ in the context of robotics, and directly contributes to a better understanding of the creation and exploitation of different modes of locomotion in robotic systems.
2. METHODS

Two different planar models were considered in this study: a one-legged hopper and a two-legged bounding robot (Fig. 1). The legs of these models consisted of a lower leg segment with mass $m_1$ and rotational inertia $j_1$, and an upper leg segment (with $m_2$ and $j_2$). The two segments were connected by a purely prismatic joint, which defined the total length of the leg $l$. The feet were modeled to be circular (radius $r_{\text{foot}}$), with the center point of the circle being positioned $l$ below the center of gravity (COG) of the segment. A rotational joint (with joint angle $\alpha$) connected the leg with the main body (mass $m_1$, inertia $j_1$) at a distance of $l$ above the COG of the upper leg segment. This rotational joint, the foot center, and the two COGs were all located on a straight line, collinear with the line of action of the prismatic joint. In the monopod case, the leg was attached directly at the COG of the main body, in the bounding case, the two legs were attached at a distance of $l$ from the COG. Again, the two joints and the COG were arranged along a straight line with no lateral offset. The position of the main body COG was given by the states $(x, y)$ and the orientation by the pitch angle $\phi$.

![Diagram of running robots](image)

Figure 1: Two different models of running robots were used in this study, a monopod hopper and a two-legged bounding robot. Both are driven by series elastic actuators, in which a highly compliant spring decouples the joint motion from the actual motor. This arrangement enables passive dynamic oscillations, which are exploited to create energy-efficient locomotion.

The equations of motion (EoM) of this system can be stated as:

$$M(q) \ddot{q} - h(q, \dot{q}) = f + J^T(\dot{q}) \cdot \lambda,$$

(1)

with the coordinate set $q = (x \ y \ \phi \ \alpha \ l)^T \in \mathbb{R}^{5 \times 1}$ for the one-legged hopper and $q = (x \ y \ \phi \ \alpha_{\text{front}} \ l_{\text{front}} \ \alpha_{\text{back}} \ l_{\text{back}})^T \in \mathbb{R}^{7 \times 1}$ for the bounding model. The dynamics of the system are given by the mass matrix $M \in \mathbb{R}^{5 \times (5 \times 5)}$, the differentiable force vector $h \in \mathbb{R}^{5 \times (7 \times 1)}$, the generalized forces in the joints $f \in \mathbb{R}^{5 \times (7 \times 1)}$, and the Jacobian $J = \frac{\partial \mathbf{r}}{\partial \mathbf{q}} \in \mathbb{R}^{4 \times 2 \times (5 \times 1)} \times (7 \times 1)$. The number of feet $k$ that are in contact with the ground defines the dimension of the Cartesian contact position vector $r \in \mathbb{R}^{2 \times k}$, and the vector of corresponding contact forces $\lambda \in \mathbb{R}^{2 \times k}$. As long as a foot is on the ground, it is only allowed to perform a pure rolling motion. No slipping or sliding was modelled. For the contact points this means that

$$r'' = 0$$

$$\dot{r} = J \cdot \dot{q} = 0$$

(2)

must hold for the vertical distance $r''$ and the relative velocity with respect to the ground $\dot{r}$. These constraints are transferred into accelerations, and a uni-directional force element that allows no slippage ($\dot{r} = 0$), no
penetration \( (\dot{r}^y \geq 0) \), and is limited in the normal direction to positive forces \( (\lambda^y \geq 0) \) is assumed as the contact law. The complementary condition \( (\lambda^y \cdot \dot{r}^y = 0) \) combines the requirements that a contact is either active \( (r = 0) \) or opening \( (\dot{r}^y > 0, \lambda = 0) \).

While the opening of a contact (and thus the lift-off of the corresponding foot) is already included in this formulation, the collision that occurs if a new contact is closed, requires some additional consideration. Due to arising external impulsive forces \( \Lambda \), the occurrence of such a collision implies instantaneous changes in velocities. To compute these, the equations of motion (1) are integrated over the duration of the collision:

\[
\int_{[s]} \left[ M\ddot{q} - h - J^T \cdot \lambda \right] dt = M(\dot{q}^+ - \dot{q}^-) - J^T \cdot \Lambda = 0
\]

As the integration is performed over an infinitesimally short time span, the bounded differentiable force vectors \( h \) and \( f \) do not contribute and only the impulsive forces and the velocity changes must be taken into account. Assuming a perfect inelastic collision with a Newton-type collision law [6, 7], the contact points of the feet that are considered part of the collision instantaneously come to a rest (or remain motionless) \( (\lambda = 0, \dot{r}^y > 0) \). For contacts that are opening, the corresponding contact points must leave the ground right after the collision \( (\dot{r}^y \geq 0, \lambda = 0) \). These two alternatives are expressed in a complementary description of the collision in normal direction:

\[
\dot{r}^+ - \dot{r}^- = J \left( \dot{q}^+ - \dot{q}^- \right) = JM^{-1} J^T \cdot \Lambda
\]

\[
\dot{r}^+ < 0, \quad \lambda^+ > 0, \quad \dot{r}^+ \cdot \lambda^+ = 0,
\]

which is solved for the post impact velocities \( \dot{q}^+ \).

In the joints, the motion of two adjoining segments is coupled by linear springs with stiffness \( k_j \) (\( k_a \)) and damping coefficient \( b_j \) (\( b_a \)). These springs are attached to the distal segment while the other end is moved relative to the proximal segment by a servo-controlled motor. Without modeling the detailed dynamics of the motor, it is assumed that it creates a displacement \( u_j \) (\( u_a \)) with velocity \( \dot{u}_j \) (\( \dot{u}_a \)) of the spring. In this arrangement, joint, spring, and motor form a series elastic actuator [8] in which the motor does not impede the passive oscillations of the model, but is only used to excite and shape the natural dynamic motion. The actuator generated joint torques and forces are \( f = f(u) = (0 \quad 0 \quad F_i \quad T_u)^T \) for the one-legged hopper and \( f = (0 \quad 0 \quad 0 \quad F_i,\text{front} \quad T_u,\text{front} \quad F_i,\text{back} \quad T_u,\text{back})^T \) for the bounding model, with

\[
F_i = k_i \left( l_b + u_i - l \right) + b_i \left( \dot{u}_i - \dot{l} \right)
\]

\[
T_u = k_a \left( \alpha_u + u_u - \alpha \right) + b_a \left( \dot{u}_u - \ddot{\alpha} \right).
\]

All states and parameters are normalized with respect to the total mass \( m \), uncompressed leg length \( l_0 \), and gravity \( g \) (Table 1). The resting angle of the legs is given as \( \alpha_o \).

Table 1: Parameters used in this study. Values are normalized to total mass \( m \), uncompressed leg length \( l_0 \), and gravity \( g \).

<table>
<thead>
<tr>
<th>parameter</th>
<th>one-legged model</th>
<th>bounding model</th>
</tr>
</thead>
<tbody>
<tr>
<td>gravitational constant</td>
<td>( g )</td>
<td>( 1 \ \left[ g \right] )</td>
</tr>
<tr>
<td>main body mass</td>
<td>( m )</td>
<td>( 0.85 \ \left[ m \right] )</td>
</tr>
<tr>
<td>mass upper leg segment</td>
<td>( m_1 )</td>
<td>( 0.1 \ \left[ m \right] )</td>
</tr>
<tr>
<td>mass lower leg segment</td>
<td>( m_3 )</td>
<td>( 0.05 \ \left[ m \right] )</td>
</tr>
<tr>
<td>inertia main body</td>
<td>( j_1 )</td>
<td>( 0.4 \ \left[ m \cdot l_0^2 \right] )</td>
</tr>
<tr>
<td>inertia upper leg segment</td>
<td>( j_2 )</td>
<td>( 0.002 \ \left[ m \cdot l_0^2 \right] )</td>
</tr>
<tr>
<td>inertia lower leg segment</td>
<td>( j_3 )</td>
<td>( 0.002 \ \left[ m \cdot l_0^2 \right] )</td>
</tr>
</tbody>
</table>
With respect to their structure and the soft actuation, the two presented models have some similarity with Marc Raibert’s early hoppers [9] and with the Scout II robot [10] from Martin Buehler’s group. However, in contrast to those robots, our models are fully actuated and employ high-compliance series elastic actuation in all their degrees of freedom.

The inputs $u$ of this actuation are defined by a Fourier series which created a parameterized periodic excitation function according to

$$u = u(t) = \sum_i a_i \sin(i \cdot 2\pi f_{\text{hop}} \cdot t) + b_i \cos(i \cdot 2\pi f_{\text{hop}} \cdot t).$$  \hfill (6)

The coefficients $a_i$ and $b_i$ as well as the excitation frequency $f_{\text{hop}}$ are the design variables of this parametric representation. In contrast to other parametric functions (such as splines, polynomials, or piecewise linear/constant functions), the use of a Fourier series ensures that the activation function is periodic and (by limiting the number of terms in the series) does not exceed the closed loop position control bandwidth of the servo motors.

Within this framework, gait synthesis is considered as the search for initial model states $\bar{q}(t_0), \bar{\dot{q}}(t_0)$ and excitation parameters $T_{\text{hop}}, a, b$, that generate a periodic motion of the generalized coordinates $\bar{q} = (y, \varphi, \alpha, l)^T$ and generalized speeds $\bar{\dot{q}} = \dot{\bar{q}} = (\dot{x}, \dot{y}, \dot{\varphi}, \dot{\alpha}, \dot{l})^T$. Only $x$ is allowed to be aperiodic, thus reflecting the desired forward motion of the robot from one step to the next. With these two requirements fulfilled, the models are able to perform a continuous forward motion, or in other words, they exhibit a steady gait. With the additional requirement of minimal energy expenditure defined as a simplified measure of the cost of transportation [11] (i.e., the net positive work per distance traveled and total body weight),

$$c = \frac{1}{x(T_{\text{hop}}) \cdot m \cdot g} \int_0^\infty \sum_r \max(0, F_r \cdot \dot{u}_r) \, dt,$$  \hfill (7)

we transform the search for a periodic hopping/running motion into a constrained optimization problem:

$$\min \left\{ c \left( T_{\text{hop}}, a, b, \bar{q}(t_0), \bar{\dot{q}}(t_0) \right) \right\}$$

s.t. $\bar{q} \left( T_{\text{hop}} \right) = \bar{q}(t_0)$

$\bar{\dot{q}} \left( T_{\text{hop}} \right) = \bar{\dot{q}}(t_0)$,  \hfill (8)

where $\bar{q}(t)$ and $c$ are evaluated by numerical simulation of the presented models using equations (5)-(7).

Once an initial periodic solution is found, we exclude $\dot{x}(t_0)$ as free parameter from the optimization problem and specifically search for periodic solutions with a given initial forward velocity, thereby sweeping a large range of velocities and using previously found solutions as initial guesses. A similar technique is employed for one-legged hopping in place, where the forward velocity was set to zero and a predefined hopping height $y(t_0)$ is enforced. This is done to evaluate the influence of the number of Fourier terms and as a
reference for the actual running gaits. Because for hopping in place the COT approaches infinity, the optimization criterion is confined to the integral of net positive work \( c = \int_0^\infty \sum_i \max(0, F_i \cdot u_i) \, dt \). The constrained optimization problem is solved in all cases numerically using the \texttt{fmincon} routine from the Matlab optimization toolbox \cite{12}.

3. RESULTS

Gaits, i.e. periodic solutions, are identified for both models. As one would expect, increasing the number of terms in the Fourier series improves the performance of the models and reduced energy consumption. For hopping in place, for example, energy consumption per step (expressed in \([mg/h]\)) can be reduced by about 20% by increasing the number of Fourier terms from 1 to 10 (Fig. 2). For a higher number of terms, one can observe an asymptotic saturation, and no further improvement is achieved. The reduction in energy consumption can be attributed to a better adaptation of the actuator motion that minimizes the foot velocity at impact and the motion of the springs, thereby reducing the associated energy losses due to the contact collisions and the viscous damping.

![Figure 2](image)

Figure 2: Optimal limit cycles were identified for a varying number of Fourier terms in the activation function. Increasing the number of terms yielded better adaptation of the excitation inputs which reduced the energy consumption of the system. This example shows the results for monopod hopping in place for different hopping heights. Energy consumption per step increases nearly linear with the hopping height.

According to our expectations, the energy consumption of in-place hopping increased approximately linearly with the hopping height, which provides an indication that the optimization did not get stuck in a local minima, but indeed found the globally optimal solution.

By evaluating the constrained optimization problem of Eq. (8) for varying forward velocities \( \dot{x} \), our approach is able to avoid local minima and gradually detect new and structurally different solutions. To this end, we generate a grid of velocities \( \dot{x} \) on which the optimization is performed and whenever a new solution is found on one grid point, use this solution as an initial guess for optimizations of the neighboring grid points, thereby iteratively generating better solutions. Once the algorithm ceases to create an improved solution on all grid points, the grid is refined between the two neighboring points with the largest difference in COT. The results are presented for one-legged hopping in Figure 3, and for two-legged bounding in Figure 4.
Figure 3: Cost of transportation of the one-legged hopper as a function of forward velocity. The natural passive dynamics of the hopper are best exploited for forward hopping with $0.45 \sqrt{g \cdot t_0}$, for which the COT becomes minimal.

When plotting the cost of transportation over the locomotion speed at apex transit $\dot{x}(t_a)$ of the bounding model, one can identify three groups, for which the optimization has found structurally different solutions (Fig. 3). The first group, at very low running speeds of up to $0.27 \sqrt{g \cdot t_0}$ was characterized by a short phase of double support, and a single oscillation of both legs during a full gait cycle (Fig. 4a). The double support vanishes in the second group (between $0.27 \sqrt{g \cdot t_0}$ and $2 \sqrt{g \cdot t_0}$ ) where phases of single support alternate with phases in which the entire model is in the air (Fig. 4b). A second oscillation of the legs becomes apparent, but is limited to positive values, leading to an asymmetric motion of the legs over time. This second oscillation becomes more pronounced in the last group (for speeds larger than $2 \sqrt{g \cdot t_0}$). The legs swing with twice the frequency of the main body, i.e. they undergo a full additional oscillation while in the air (Fig. 4c) in order to compensate for the increased amount of time the legs are spending in the air.

![Figure 3: Cost of transportation of the one-legged hopper as a function of forward velocity. The natural passive dynamics of the hopper are best exploited for forward hopping with $0.45 \sqrt{g \cdot t_0}$, for which the COT becomes minimal.](image1)

Figure 4: Cost of transportation of the two-legged bounding model as a function of forward velocity. Three distinct solution groups were identified and approximated by quadratic functions. Each group corresponds to a different gait and has an optimal locomotion velocity. The labels (a)-(c) correspond to the time traces in Figs 4a-4c.

All groups have a very distinct parabolic shape, and can be closely approximated by quadratic functions (with an $R^2$-value of 0.96). This shape is reminiscent of studies in biology (see for example [13]), where the metabolic COT (expressed by oxygen consumption per distance traveled) was measured as a function of
walking/running speed. Just as the model presented in this study exploited structurally different solutions for optimal locomotion at varying velocities, nature employs different gaits (such as walking, trotting, or galloping) to minimize the metabolic COT over a larger range of speeds. And similar to the solution groups we identified, every gait has a velocity at which the COT is minimized, and would require a higher COT for slower or faster locomotion.

![Figure 5](image)

Figure 5: Time series for the angles of front and back legs (with respect to vertical) for four consecutive strides. One can clearly distinguish three structurally different solutions (with respect to the foot-strike/lift-off sequence and the oscillation of the legs) that exploit different natural dynamics at different locomotion velocities.

4. DISCUSSION

The presented study deals with the synthesis of efficient running gaits for a monopod hopper and a two-legged bounding robot. The analysis considers a large range of forward velocities and thereby provides an effective means of finding and defining different gaits for robots. This is in contrast to classical biological gait definitions which may not be as suited for robots since they cannot take into account the substantially different morphology of mechatronic systems. This was clearly shown in the bounding motion of a two-legged robot by the double oscillation of the swing legs, which had a huge energy saving potential in the presented models, but cannot be found anywhere in nature.

To improve the presented results, it would be desirable to run the algorithm on a finer grid, which could be facilitated by the use of advanced optimal control techniques, such as Multiple Shooting [14] or Direct Collocation [15], in which integration of the equations of motions is split into smaller subintervals or directly transformed into a set of algebraic constraints, thereby reducing the effect of non-linearities and improving the convergence behavior. Additionally, one important issue that was not considered in this study is the question of stability. While the gaits created in this study were all periodic, there is no guarantee that the underlying limit-cycles are stable. This means that an implementation of these gaits would possibly require a stabilizing feedback controller. Such a controller could be integrated into the presented framework, for example, by periodically modulating the free variables of the excitation function. To this end, the Poincaré map of the system must be linearized with respect to the excitation parameters, which can then be included in a discrete LQR-controller.
REFERENCES