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Calcul des courbes de fragilité sismique par approches non-paramétriques

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Résumé :

En génie civil, les courbes de fragilité sont utilisées pour estimer la vulnérabilité des structures soumises au séisme. Ces courbes représentent les probabilités de défaillance liées à un critère de performance (par exemple, le déplacement inter-étage maximal qui dépasse un seuil admissible) en fonction de l'intensité des séismes. L'approche classique suppose que les courbes de fragilité ont la forme d'une fonction de répartition lognormale, dont les paramètres sont calculés par régression linéaire. On cherche ici à construire les courbes de fragilité par des méthodes non-paramétriques *en ne faisant aucune hypothèse sur la forme des courbes. L'utilisation des densités à noyau est une possibilité. L'autre approche est la simulation Monte Carlo conditionnellement au PGA (peak ground acceleration). Les courbes de fragilité associées à une structure de 3 étages sont calculées par les deux approches et comparées. Le comportement non-linéaire du matériau constitutif est également pris en compte. On compare finalement les résultats avec ceux de l'approche lognormale.*

Abstract :

Fragility curves are commonly used in civil engineering to estimate the vulnerability of structures to earthquakes. The probability of failure associated with a failure criterion (e.g. the maximal inter-storey drift being greater than a prescribed threshold) is represented as a function of the intensity of the earthquake (e.g. peak ground acceleration or spectral acceleration). The classical approach consists in assuming a lognormal shape of the fragility curves. In this paper, we introduce two non-parametric approaches *to establish the fragility curves without making any assumption, respectively the kernel density estimate and the conditional Monte Carlo simulation. As an illustration, we compute the fragility curves of a 3-storey structure. The nonlinear behavior is also taken into account. The curves obtained by different approaches are compared with each other and with the classical lognormal assumption.*

Mots clefs : earthquake engineering, fragility curves, non-parametric approach

1 Introduction

Fragility curves are commonly used in seismic probabilistic risk assessment in order to estimate the vulnerability of structures to earthquakes, *e.g.* the probability that a structure fails to fulfil a safety criterion during the ground motions. The probability of failure associated with a failure criterion (*e.g.* the maximal inter-storey drift Δ being greater than a prescribed threshold δ_0) is represented as a function of the intensity of the earthquake (*e.g.* peak ground acceleration (PGA)) [1].

From the mathematical point of view, a fragility curve represents the conditional probability that a prescribed threshold is attained or exceeded *given the intensity* of an earthquake. The classical approach to compute fragility curves consists in assuming that the curves have a lognormal shape [1], and their parameters are determined by linear regression. This assumption can ease the estimation of the curves. However, the validity of this assumption remains an open question.

For the purpose of validating the classical approach, we introduce two non-parametric approaches to establish the fragility curves *without* making any assumption, respectively the kernel density estimate and the Monte Carlo simulation conditionally to the intensity measure (*e.g.* the PGA).

The computation of fragility curves requires a large number of transient dynamic analysis of the structure under seismic excitations, that are either recorded or synthetic. Due to the lack of recorded signals with the properties of interest (*e.g.* magnitude, duration, etc.), it is common practice to generate suitable samples of synthetic earthquakes. This approach is used in the sequel.

The paper is organized as follows: in Section 2, the approach recently proposed by Rezaeian and Der Kiureghian [2] to generate synthetic earthquakes, which are used in the example in Section 4, is briefly recalled. The different approaches for establishing the fragility curves are presented in Section 3. In Section 4 we compute the fragility curves of a steel frame structure subject to seismic excitations.

2 Seismic generation

In this section, we summarize the parameterized approach proposed by Rezaeian and Der Kiureghian [2] in order to simulate synthetic ground motions. The seismic acceleration is represented as a non-stationary process. Der Kiureghian and Rezaeian separate the non-stationarity into two components, namely a spectral and a temporal one, by means of a modulated filtered Gaussian white noise:

$$a(t) = \frac{q(t,\boldsymbol{\alpha})}{\sigma_h(t)} \int_{-\infty}^{t} h\left[t-\tau, \boldsymbol{\lambda}(\tau)\right] \omega(\tau)\, \mathrm{d}\tau \tag{1}$$

in which $q(t,\boldsymbol{\alpha})$ is the deterministic non-negative *modulating function* and the integral is the non-stationary response of a linear filter subject to a Gaussian white noise excitation. The Gaussian white-noise process denoted by $\omega(\tau)$ will pass a filter $h\left[t-\tau, \boldsymbol{\lambda}(\tau)\right]$ which is selected as the pseudo-acceleration response of a single degree of freedom linear oscillator:

$$\begin{aligned} &h\left[t-\tau, \boldsymbol{\lambda}(\tau)\right] = 0 \quad \text{for} \quad t<\tau \\ &h\left[t-\tau, \boldsymbol{\lambda}(\tau)\right] = \frac{\omega_f(\tau)}{\sqrt{1-\zeta_f^2(t)}} \exp\left[-\zeta_f(\tau)\omega_f(\tau)(t-\tau)\right] \sin\left[\omega_f(\tau)\sqrt{1-\zeta_f^2(\tau)}(t-\tau)\right] \quad \text{for} \quad t \geq \tau \end{aligned} \tag{2}$$

where $\boldsymbol{\lambda}(\tau) = (\omega_f(\tau), \zeta_f(\tau))$ is the vector of time-varying parameters of the filter h. $\omega_f(\tau)$ and $\zeta_f(\tau)$ are the filter's natural frequency and damping ratio at instant τ, respectively. They represent the evolving predominant frequency and bandwidth of the ground motion. The statistical analysis of real signals shows that the $\zeta_f(\tau)$ may be taken as a constant ($\zeta_f(\tau) \equiv \zeta$) while the predominant frequency varies linearly in time:

$$\omega_f(\tau) = \omega_{mid} + \omega'(\tau - t_{mid}) \tag{3}$$

In Eq. (3) t_{mid} is the instant at which 45% of the expected Arias intensity I_a is reached, ω_{mid} is the filter's frequency at instant t_{mid} and ω' is the slope of the linear evolution. After being normalized by the standard deviation $\sigma_h(t)$, the integral in Eq. (1) becomes a unit variance process with time-varying frequency and constant bandwidth. The non-stationarity in intensity is then captured by the modulating function $q(t,\boldsymbol{\alpha})$. This time-modulating function determines the shape, intensity and duration T of the signal. A Gamma-like function is usually used: $q(t,\boldsymbol{\alpha}) = \alpha_1 t^{\alpha_2 - 1} \exp(-\alpha_3 t)$ where $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$ is directly related to the energy content of the signal. For computational purpose, the acceleration in Eq. (1) can be discretized as follows:

$$\hat{a}(t) = q(t,\boldsymbol{\alpha}) \sum_{i=1}^{n} s_i\left(t, \boldsymbol{\lambda}(t_i)\right) U_i \tag{4}$$

where the standard normal random variable U_i represents an impulse at instant $t_i = i \times \dfrac{T}{n}$, $i = 1, \ldots, n$, (T is the total duration) and $s_i(t, \boldsymbol{\lambda}(t_i))$ is given by: $s_i(t, \boldsymbol{\lambda}(t_i)) = \dfrac{h\left[t - t_i, \boldsymbol{\lambda}(t_i)\right]}{\sqrt{\sum_{j=1}^{i} h^2\left[t - t_j, \boldsymbol{\lambda}(t_j)\right]}}$

As a summary, the proposed model consists of 3 temporal parameters $(\alpha_1, \alpha_2, \alpha_3)$, 3 spectral parameters $(\omega_{mid}, \omega', \zeta_f)$ and the standard Gaussian random vector $\boldsymbol{U}$ of size n.

3 Computation of fragility curves

Fragility curves represent the probability of failure given the intensity of an earthquake. In this paper, it is the conditional probability that the maximal inter-storey drift Δ attains or exceeds the admissible threshold δ_0 given PGA:

$$\text{Frag}(PGA) = \mathbb{P}[\Delta \geq \delta_0 | PGA] \tag{5}$$

in which Frag(PGA) denotes the fragility at the given PGA. In order to establish the fragility curves, transient finite element analyses are used to provide paired values $\{(PGA_i, \Delta_i), i = 1, \ldots, N\}$.

3.1 Classical approach

The classical approach to establish fragility curves consists in assuming a *lognormal shape* for the curves. More specifically, the maximal inter-storey drift Δ is modelled by the lognormal distribution in which the log-mean value λ is a *linear function* of $\ln PGA$, say $\ln \Delta \sim \mathcal{N}(\lambda, \zeta)$, $\lambda = A \ln(PGA) + B$. Parameters A and B are determined by means of linear regression in a log-log plot. The same approach is widely applied in the literature, see *e.g.* Choi et al. [3], Padgett and DesRoches [4] among others. Let us denote by e_i the residual between the actual value $\ln \Delta$ and the value predicted by the linear model: $e_i = \ln \Delta_i - A \ln(PGA_i) - B$. Parameter ζ is obtained by $\zeta^2 = \sum_{i=1}^{N} e_i^2 / (N-2)$.

The probability of failure in Eq. (5) is then given by:

$$\text{Frag}(PGA) = \mathbb{P}\left[\ln \Delta \geq \ln \delta_0\right] = 1 - \mathbb{P}\left[\ln \Delta \leq \ln \delta_0\right] = \Phi\left(\frac{\ln PGA + (B - \ln \delta_o)/A}{\zeta/A}\right) \tag{6}$$

where $\Phi(t) = \int\limits_{-\infty}^{t} \exp\left[-u^2/2\right]/\sqrt{2\pi}\mathrm{d}u$ is the Gaussian cumulative distribution function. The above so-called classical approach is *parametric*, *i.e.* it imposes the shape of the fragility curves in Eq. (6) which is similar to a lognormal CDF as a function of PGA. In the sequel, we will propose two *non-parametric* approaches to compute fragility curves without making such an assumption.

3.2 Kernel density estimate

If the conditional distribution $f_{\Delta|PGA}$ was known, the fragility curves in Eq. (5) would be Frag$(a) = \int\limits_{\delta_0}^{+\infty} f_{\Delta|PGA}(\delta; a)\mathrm{d}\delta$. The main idea is to compute the conditional density distribution of Δ given PGA following Bayes' theorem using the kernel density estimate:

$$\hat{f}_{\Delta|PGA}(\delta|PGA = a) = \frac{\hat{f}_{\Delta,PGA}(\delta, a)}{\hat{f}_{PGA}(a)} \tag{7}$$

in which $\hat{f}_{\Delta,PGA}(\delta, a)$ is the joint distribution estimate of Δ and PGA and $\hat{f}_{PGA}(a)$ is the estimate of the marginal PDF of PGA.

Wand and Jones [5] estimate the univariate density distribution of a random variable X based on an i.i.d sample $\{x_1, \ldots, x_N\}$ as follows:

$$\hat{f}_X(x) = \frac{1}{Nh} \sum_{i=1}^{N} K\left(\frac{x - x_i}{h}\right) \tag{8}$$

where h is the *bandwidth* parameter, $K(\cdot)$ is the *kernel* function which integrates to one. Classical kernel functions are the Epanechnikov, uniform, normal or triangular functions. The choice of the kernel is known not to affect strongly the efficiency of the estimate [5] provided the sample set is large enough. In case a standard normal PDF is adopted for the kernel, *i.e.* $K(x) \equiv \varphi(x) = \exp\left[-x^2/2\right]/\sqrt{2\pi}$, the kernel density estimate rewrites:

$$\hat{f}_X\left(x\right) = \frac{1}{Nh}\sum_{i=1}^{N}\varphi\left(\frac{x - x_i}{h}\right) \tag{9}$$

The choice of bandwidth h is crucial for the kernel density estimate. An inappropriate value of h can lead to an oversmoothed or undersmoothed estimated PDF. Eq. (9) is used for estimating $\hat{f}_{PGA}(a)$ in Eq. (7)

Kernel density estimation may be extended to a multivariate random variable $\boldsymbol{X} \in \mathbb{R}^d$ knowing an i.i.d sample $\{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N\}$ [5]:

$$\hat{f}_{\boldsymbol{X}}\left(\boldsymbol{x}\right) = \frac{1}{N\left|\boldsymbol{H}\right|^{1/2}}\sum_{i=1}^{N}K\left(\boldsymbol{H}^{-1/2}(\boldsymbol{x} - \boldsymbol{x}_i)\right) \tag{10}$$

in which $\boldsymbol{H}$ is the *bandwidth matrix* whose determinant is denoted by $|\boldsymbol{H}|$. When a multivariate standard normal kernel is adopted, the joint distribution estimate becomes:

$$\hat{f}_{\boldsymbol{X}}\left(\boldsymbol{x}\right) = \frac{1}{N\left|\boldsymbol{H}\right|^{1/2}}\sum_{i=1}^{N}\frac{1}{(2\pi)^{d/2}}\exp\left[-\frac{1}{2}^{\mathsf{T}}(\boldsymbol{x} - \boldsymbol{x}_i)\boldsymbol{H}^{-1}(\boldsymbol{x} - \boldsymbol{x}_i)\right] \tag{11}$$

The bandwidth matrix $\boldsymbol{H}$ can be computed by means of plug-in and cross-validation estimators, see [6]. Using Eq. (11) to estimate the joint PDF $\hat{f}_{\Delta,PGA}(\delta, a)$ and Eq. (9) for the marginal PDF $\hat{f}_{PGA}(a)$, one can finally rewrite the probability of failure as follows:

$$\begin{aligned}\widehat{\mathrm{Frag}}(a) =& \mathbb{P}[\Delta \geq \delta_0 | PGA = a] = \int_{\delta_0}^{+\infty}\hat{f}_{\Delta}(\delta|PGA = a)\mathrm{d}\delta \\ =& \frac{h_{PGA}}{2\pi\left|\boldsymbol{H}\right|^{1/2}}\frac{\int_{\delta_0}^{+\infty}\sum_{i=1}^{N}\exp\left[-\frac{1}{2}^{\mathsf{T}}\begin{pmatrix}\delta - \Delta_i \\ a - PGA_i\end{pmatrix}\boldsymbol{H}^{-1}\begin{pmatrix}\delta - \Delta_i \\ a - PGA_i\end{pmatrix}\right]\mathrm{d}\delta}{\sum_{i=1}^{N}\varphi\left(\frac{a - PGA_i}{h_{PGA}}\right)}\end{aligned} \tag{12}$$

3.3 Empirical approach

Having at hand a large sample set of pairs $\{(PGA_j, \Delta_j), j = 1, \ldots, N\}$, it is possible to use a non parametric, so-called *empirical approach* to compute the fragility curve. Let us consider a given abscissa PGA_o. Within a small bin surrounding PGA_o, say $[PGA_o - h, PGA_o + h]$ one assumes that the maximal drift Δ is linearly related to the PGA. (Note that this assumption is exact in the case of linear structures but would be only an approximation in the non linear case). Therefore, the maximal drift $\Delta_j \in [PGA_o - h, PGA_o + h]$ related to PGA_j is converted to the drift Δ_o which would be related to a similar input signal having a peak ground acceleration of PGA_o as follows:

$$\Delta(PGA_o) = \Delta_j\frac{PGA_o}{PGA_j} \tag{13}$$

The value of the fragility curve at PGA_o is obtained by a crude Monte Carlo estimator:

$$\widehat{\mathrm{Frag}}(PGA_o) = \frac{N_f\left(PGA_o\right)}{N_s\left(PGA_o\right)} \tag{14}$$

where $N_f\left(PGA_o\right)$ is the number of observations in the vicinity of PGA_o such that $\Delta(PGA_o) \geqslant \delta_0$ and $N_s(PGA_o)$ is the total number of observations that fall into the bin $[PGA_o - h, PGA_o + h]$.

The vicinity is defined by the bin width $2h$, which is selected according to the sample set of observations. In this study, $h = 0.25$ m/s^2 is chosen. This empirical approach is close to the incremental dynamic analysis (IDA) in Vamvatsikos and Cornell [7].

4 Illustration: steel frame structure

We evaluate the fragility curve of a 3-storey 3-span steel frame structure with the following dimensions: storey-height $H = 3$ m, span-length $L = 5$ m. The steel material has a non-isotropic non-linear behavior following the Giuffre-Menegotto-Pinto model as implemented in the finite element software OpenSees. The initial elastic tangent of steel is equal to $E_0 = 205,000$ MPa, and the yield strength is $F_y = 235$ MPa, the strain hardening ratio (ratio between post-yield tangent and initial tangent) is $b = 0.01$. The loading consists of dead-load (from the frame elements as well as the supported floors), and variable load in accordance with Eurocode 1.

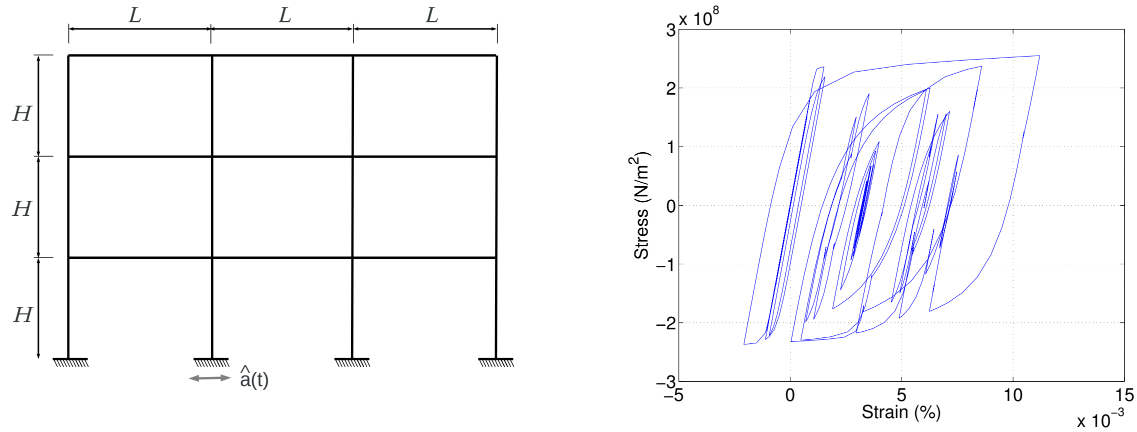

Figure 1: Steel frame structure and nonlinear behavior of steel material

The structure is subject to ground motions modelled by the time-history of acceleration at the ground level. Each ground motion is modelled by 6 randomized parameters $(\alpha_1, \alpha_2, \alpha_3, \omega_{mid}, \omega', \zeta_f)$ and an input vector $\boldsymbol{U}$ made of 500 independent Gaussian random variables U_i. We use the finite element code OpenSees [8] to carry out a large number ($N = 10^4$) of transient dynamic analyses of the frame. The pairs $\{(PGA_i, \Delta_i), i = 1, \ldots, N\}$ are collected and postprocessed following the different approaches to establish the fragility curves.

In the present case, when using the classical lognormal approach, we obtain the following values for the parameters: $A = 0.9676$, $B = -6.8931$ and $\zeta = 0.6027$. Concerning the kernel density approach, we used the smoothed cross-validation estimator provided by Duong (2013) in the package *ks* in R to estimate the bandwidth matrix $\boldsymbol{H} = \begin{bmatrix} 1.082e-7 & 6.125e-5 \\ 6.125e-5 & 8.585e-2 \end{bmatrix}$. The univariate bandwidth $h_{PGA} = 0.25$ m/s^2 is determined by the direct plug-in estimator and "eye-control" taking into account the support $\mathrm{D}_{PGA} = (0, +\infty)$. Figure 2 depicts the resulting fragility curves associated with the different values of δ_0. The curves obtained by the non-parametric approaches (kernel density estimate and empirical) are consistent whatever the damage level is. However, for the high values of PGA (>5 m/s^2), some noise is observed on the empirical curves due to the lack of observations in the considered intervals. This noise may be reduced by adding new observations in this range of PGA.

The lognormal curves differ significantly from the non-parametric curves, especially for high level of damage, *e.g.* $\delta_0 = 1/150, 1/200$. The classical approach tends to underestimate the probability of failure in a range of PGA which is very common, *i.e.* from 2 to 6 m/s^2. The fact that the parameters of the lognormal curve are determined from a sample containing both linear and nonlinear behavior might lead to the error. When a slight damage is of interest, the curve provided by the classical approach is satisfactory.

We also observe a non-monotonic curve (in case $\delta_0 = 1/150$) around high values of PGA (>10 m/s^2). The large confidence interval in this range of PGA shows that the non-monotony might be due to the lack of available observations and can be simply verified by considering a large number of analyses in this range. We should not exclude another aspect that is the correlation between the frequency domain

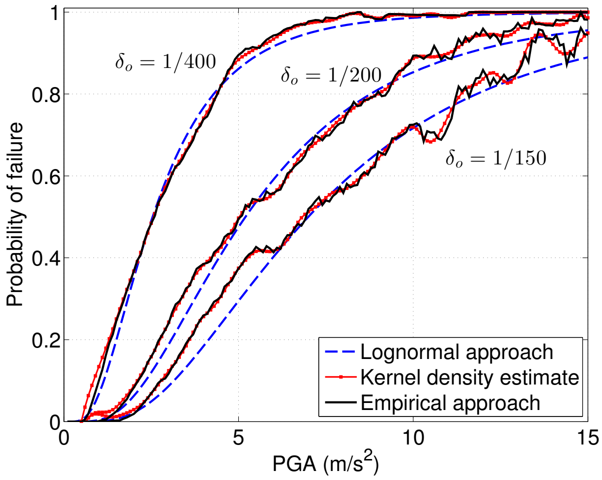

Figure 2: Fragility curves by classical lognormal, kernel density estimate and empirical approaches

and the magnitude of the earthquakes. It may happen that an excitation with higher magnitude, *i.e.* higher PGA, can cause a smaller drift due to the couplings between the structural eigenfrequencies and the frequency content of the excitation. Further research is being pursued in this direction.

5 Conclusions

In this paper, we introduced two non-parametric approaches to establish the fragility curves of a steel frame structure under earthquake excitation as well as to validate the classical lognormal approach.

The classical approach is shown to be satisfactory for estimating the probability that slight damage occurs in a structure of this type. For the high levels of damage, the classical curves underestimate the true failure probability. The decision based on this wrong estimation could lead to unsafe situations. Further investigations need to be carried out to clarify the observed non-monotony of fragility curves in the large range of PGA.

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