

Optimized Noisy Network Coding for Gaussian Relay Networks

Conference Paper

Author(s):

Kolte, Ritesh; Özgür, Ayfer; El Gamal, Abbas

Publication date:

2014

Permanent link:

<https://doi.org/10.3929/ethz-a-010095448>

Rights / license:

[In Copyright - Non-Commercial Use Permitted](#)

Optimized Noisy Network Coding for Gaussian Relay Networks

Ritesh Kolte, Ayfer Özgür, Abbas El Gamal
 Stanford University
 {rkolte, aozgur}@stanford.edu, abbas@ee.stanford.edu

Abstract—In this paper, we provide an improved lower bound on the rate achieved by noisy network coding in arbitrary Gaussian relay networks, whose gap to the cutset upper bound depends on the network not only through the total number of nodes but also through the degrees of freedom of the min cut of the network. We illustrate that for many networks this refined lower bound can lead to a better approximation of the capacity. The improvement is based on a judicious choice of the quantization resolutions at the relays.

I. INTRODUCTION

Characterizing the capacity of Gaussian relay networks has been of interest for long time. Recently, significant progress has been made in [1], [2], [3], [4] which show that compress-forward based strategies (such as noisy network coding) can achieve the capacity of any Gaussian relay network within a gap that is independent of the topology of the network, the SNR and the channel coefficients. However, the gap depends linearly on the number of nodes in the network. This limits the applicability of these results to small networks with few relays.

A natural question is whether the gap to capacity can be made smaller than linear in the number of nodes. In this paper, we provide an improved lower bound on the rate achieved by noisy network coding in arbitrary Gaussian relay networks, which for many networks can lead to an approximation gap which is significantly better than the linear gap in [1], [2], [3], [4]. The improvement is based on the observation that in the compress-and-forward based strategies (such as quantize-map-and-forward in [1] and noisy network coding in [2]) there is a fundamental trade-off involved in the choice of the quantization (or compression) resolutions at the relays. If relays quantize their received signals finely, they introduce less quantization noise to the communication. If they quantize more coarsely however, there is a smaller number of quantization indices that need to be communicated to the destination on top of the desired message. This trade-off is not immediately evident from the development of these strategies in [1] and [2], since the employed decoder does not require the quantization indices of the relays to be uniquely decoded. Therefore it is not clear if the quantization indices are indeed decoded at, and thus communicated to the destination, and therefore whether there is a penalty involved in communicating these indices. Based on the work of [5], we argue that in the optimal distribution for the quantization indices, the quantization indices of all relays can be uniquely decoded at the destination. Moreover, an optimal choice of the quantization indices requires much coarser quantization than the noise level. We then apply the

new lower bound to a class of layered networks with fixed channel coefficients of unit magnitude and arbitrary phases (i.e. each channel coefficient is of the form $e^{j\theta}$ for some arbitrary $\theta \in [0, 2\pi]$) and show that it leads to a capacity gap that is logarithmic in the number of nodes rather than linear.

A similar insight was used earlier in [6], [7] and [8] to obtain improved capacity approximations for other classes of Gaussian relay networks. [6] and [7] provide an approximation for the capacity of the diamond network which is logarithmic in the number of nodes, while [8] considers a layered network with i.i.d. fast-fading links and shows that the gap to capacity increases logarithmically in the depth of the network. However, in both settings there are other strategies which can yield similar performance. For the diamond network, [6] shows that a partial-decode-and-forward strategy also achieves the logarithmic dependence on the number of nodes, and for the fast fading layered network, ergodic computation-alignment over independent realizations of the fading distribution [9] achieves a gap that does not increase with the number of layers. (Note that both these alternative schemes require increased CSI at the relays and the source nodes.) However, for the layered network with fixed channel gains considered in this paper, these schemes are not applicable and we know of no scheme other than compress-forward that can give a constant gap capacity approximation.

II. GAP TO CAPACITY WITH NOISY NETWORK CODING

In this section, we discuss the elements of the gap between the rate achieved by noisy network coding (NNC) and the cutset-upper bound and identify a trade-off between different elements of the gap. Our main result in the next section builds on the understanding of this trade-off.

Consider an arbitrary discrete memoryless network with a set of nodes \mathcal{N} where a source node s wants to communicate to a destination node d with the help of the remaining nodes acting as relays. NNC can achieve a communication rate [2, Theorem 1]:¹

$$\min_{\Omega \subseteq \mathcal{N}} I(X_\Omega; \hat{Y}_{\Omega^c} | X_{\Omega^c}) - I(Y_\Omega; \hat{Y}_\Omega | X_{\mathcal{N}}, \hat{Y}_{\Omega^c}) \quad (1)$$

for any distribution of the form $\prod_{k \in \mathcal{N}} p(x_k) p(\hat{y}_k | y_k, x_k)$; where for brevity of expressions of, \hat{Y}_{Ω^c} is assumed to include Y_d . Comparing this with the information-theoretic

¹In this paper, we need to consider only $s - d$ cuts, which means $s \in \Omega, d \in \Omega^c$. Hence we do not state this explicitly.

cutset upper bound on the capacity of the network given by [10, Theorem 15.10.1]

$$\bar{C} = \sup_{X_{\mathcal{N}}} \min_{\Omega \subseteq \mathcal{N}} I(X_{\Omega}; Y_{\Omega^c} | X_{\Omega^c}) \quad (2)$$

we observe the following differences. First, while the maximization in (2) is over all possible input distributions, only independent input distributions are admissible in (1). The gap corresponds to a potential beamforming gain that is allowed in the upper bound but not exploited by NNC. Second, the first term in (1) is similar to (2) but with Y_{Ω^c} in (2) replaced by \hat{Y}_{Ω^c} in (1). The difference corresponds to a rate loss due to the quantization noise introduced by the relays. Third, there is the extra term $I(Y_{\Omega}; \hat{Y}_{\Omega} | X_{\mathcal{N}}, \hat{Y}_{\Omega^c})$ reducing the rate in (1). One way to potentially interpret this term would be as the rate penalty for communicating the quantized (compressed) observations \hat{Y}_{Ω} to the destination on top of the desired message. Note that this is the rate required to describe the observations Y_{Ω} at the resolution of \hat{Y}_{Ω} to a decoder that already knows (or has decoded) $X_{\mathcal{N}}, \hat{Y}_{\Omega^c}$.

However, it is not completely clear if this interpretation is precise because the non-unique decoder employed by NNC does not require the quantization indices to be explicitly decoded. The non-unique decoder of NNC searches for the unique source codeword that is jointly typical with a (not necessarily unique) set of quantization indices at the relays and the received signal at the destination. The following example in Figure (1) illustrates that in certain cases the decoder can indeed recover the transmitted message even if it can not uniquely recover the quantization index of the relay.²

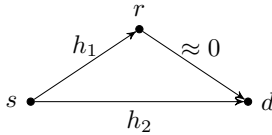


Fig. 1: Example

Consider the classical relay channel with a very weak link from the relay to the destination. Clearly, as long as the source uses a codebook of rate less than the capacity of the direct link, no matter what the operation at the relay is, the destination can always decode the source message by performing a joint typicality test between its received signal and the source codebook (which is subsumed by the non-unique typicality test of NNC). In particular, if the relay quantizes too finely, then there is no way for the destination to recover the relay's quantization index, even though the source message can still be recovered.

On the other hand, this example reveals the following strange property of the expression in (1). While the above discussion reveals that in the setup of Fig. 1, the rate achieved by NNC is equal to the capacity of the direct link independent of the relay's operation (i.e. what \hat{Y}_r is), the rate in (1) is decreasing with increasing resolution for the quantization at the relay (due to the subtractive term $I(Y_{\Omega}; \hat{Y}_{\Omega} | X_{\mathcal{N}}, \hat{Y}_{\Omega^c})$).

²Even though we focus on the extremal case where the $r-d$ link is zero, the discussion extends to the case where this link is sufficiently weak.

This suggests a more careful analysis of the rate achieved by NNC which leads to the following improved rate:

$$\max_{\mathcal{M} \subseteq \mathcal{N}} \min_{\Omega \subseteq \mathcal{M}} I(X_{\Omega}; \hat{Y}_{\Omega^c} | X_{\Omega^c}) - I(Y_{\Omega}; \hat{Y}_{\Omega} | X_{\mathcal{M}}, \hat{Y}_{\Omega^c}). \quad (3)$$

Here only a subset $\mathcal{M} \subseteq \mathcal{N}$ of the relays is considered in the non-unique typicality decoding, while the other relay transmissions are treated as noise.

It has been shown in [5] that if \mathcal{M}^* is the subset that maximizes (3) for a given $\prod_{i \in \mathcal{N}} p(x_i) p(\hat{y}_i | y_i, x_i)$, then the quantization indices of the relays in \mathcal{M}^* can be uniquely decoded at the destination, while the quantization indices of the relays in $\mathcal{N} \setminus \mathcal{M}^*$ cannot be decoded and in fact, it is optimal to treat the transmissions from these relays as noise. Since the transmissions from $\mathcal{N} \setminus \mathcal{M}^*$ are treated as noise in (3), the rate can be further improved if these relays are shut down. Hence, we can conclude that in the optimal distribution $\prod_{i \in \mathcal{N}} p(x_i) p(\hat{y}_i | y_i, x_i)$, some relays can be off (not utilized or equivalently always quantizing their received signals to zero) and some relays can be active, but the quantization indices of all relays can be uniquely decoded at the destination. Thus, $I(Y_{\Omega}; \hat{Y}_{\Omega} | X_{\mathcal{M}}, \hat{Y}_{\Omega^c})$ can indeed be interpreted as the associated rate penalty for communicating these indices.

The above discussion reveals that NNC communicates not only the source message but also the quantization indices to the destination; and while making quantizations finer introduces less quantization noise in the communication, it leads to a larger rate penalty for communicating the quantization indices. This tradeoff is made explicit in the following section.

III. MAIN RESULT

Consider a Gaussian relay network where a source node s communicates to a destination node d with the help of a set of relay nodes. The signal received by node i is given by

$$\mathbf{Y}_i = \sum_{j \neq i} H_{ij} \mathbf{X}_j + \mathbf{Z}_i$$

where H_{ij} is the $N_i \times M_j$ channel matrix from node j equipped with M_j transmit antennas to node i equipped with N_i receive antennas. We assume that each node is subject to an average power constraint P per antenna and $\mathbf{Z}_i \sim \mathcal{CN}(0, \sigma^2 I)$, independent across time and across different receive antennas. Let N be the total number of receive antennas and M be the total number of transmit antennas in the network. Also, define

$$C_Q^{i,i,d}(\Omega) \triangleq \log \det \left(I + \frac{P}{(Q+1)\sigma^2} H_{\Omega \rightarrow \Omega^c} H_{\Omega \rightarrow \Omega^c}^\dagger \right),$$

which is the mutual information across the cut Ω if the channel input distribution at node j is i.i.d. $\mathcal{CN}(0, PI)$ and the noise is i.i.d. $\mathcal{CN}(0, (Q+1)\sigma^2)$. The matrix $H_{\Omega \rightarrow \Omega^c}$ denotes the induced MIMO matrix from Ω to Ω^c and \log denotes the natural logarithm. The main result of this paper is given in the following theorem.

Theorem 1. *The rate achieved by noisy network coding in this network can be lower bounded by*

$$C \geq \bar{C} - d_0^* \log \left(1 + \frac{M}{d_0^*} \right) - \frac{N}{Q} - d_Q^* \log(Q+1),$$

for any non-negative Q where \bar{C} is the cutset upper bound on the capacity of the network given in (2) and d_Q^* is the degrees-of-freedom (DOF) of the MIMO channel corresponding to the cut Ω_Q^* that minimizes $C_Q^{i.i.d.}(\Omega)$, denoted succinctly as

$$d_Q^* = \text{DOF} \left(\arg \min_{\Omega} C_Q^{i.i.d.}(\Omega) \right).$$

The proof of Theorem 1 is presented in Section IV.

Remark 1. The result can be extended to the case of multiple multicast, i.e. when multiple sources are multicasting their information to a common group of destination nodes.

Note that Q in the theorem is a free parameter that can be optimized for a given network to minimize the gap between the achieved rate and the cutset upper bound. $Q\sigma^2$ corresponds to the variance of the quantization noise introduced at the relays; larger Q corresponds to coarser quantization. In previous works [1], [2], Q is chosen to be constant independent of the number of nodes (or antennas) N (i.e. $Q \approx 1$ and the quantization noise $Q\sigma^2$ is of the order of the Gaussian noise variance σ^2). This results in an overall gap that is linear in N . Note that both d_0^* and d_Q^* can be trivially upper bounded by N . However, in many cases, the min cut of the network can have much smaller DOF than M and N and in such cases allowing Q to depend on N can result in a much smaller gap.

For example, in the diamond network with single antenna at each node it is clear a priori that any cut of the network has at most two degrees of freedom, regardless of the number of relays, and therefore $d_Q^* \leq 2$ for any Q . It can be seen immediately from the above theorem that choosing $Q = N$ in this case results in a gap logarithmic in N [6], [7], which compares favorably with a gap that is linear in N . Similarly, for the fast-fading layered network with K single antenna nodes per layer considered in [8], it is the case that $d_Q^* \leq K$ for any Q . If there are D layers in the network so that $N = M = KD$, the above expression tells us that choosing Q to be proportional to D gives a gap that is logarithmic in D instead of linear in D . In Section V, we demonstrate another setting in which applying Theorem 1 with Q increasing with the number of layers in the network allows us to obtain an improved gap. This demonstrates that the rule of thumb in the current literature to quantize received signals at the noise level ($Q \approx 1$) can be highly suboptimal.

IV. PROOF OF THEOREM 1

We know that the rate in (1) is achievable in the Gaussian network for any $\prod_{k \in \mathcal{N}} p(x_k)p(\hat{y}_k|y_k, x_k)$ that satisfies the power constraint. We choose the channel input vector at each node j as $\mathbf{X}_j \sim \mathcal{CN}(0, PI)$ and \hat{Y}_k for each receive antenna in the network is chosen such that

$$\hat{Y}_k = Y_k + \hat{Z}_k \text{ where } \hat{Z}_k \sim \mathcal{CN}(0, Q\sigma^2),$$

independent of everything else.

Consider the achievable rate expression in (1). We first show that $\max_{\Omega \subseteq \mathcal{N}} I(Y_\Omega; \hat{Y}_\Omega | X_{\mathcal{N}}, \hat{Y}_{\Omega^c}) \leq \frac{N}{Q}$. This follows

on similar lines as [8, Lemma 1].

$$\begin{aligned} I(Y_\Omega; \hat{Y}_\Omega | X_{\mathcal{N}}, \hat{Y}_{\Omega^c}) &\leq h(\hat{Y}_\Omega | X_{\mathcal{N}}) - h(\hat{Y}_\Omega | Y_\Omega, X_{\mathcal{N}}) \\ &= \left(\sum_{j \in \Omega} N_j \right) \log \left(1 + \frac{1}{Q} \right) \leq \frac{N}{Q}. \end{aligned} \quad (4)$$

We now lower bound the first term in (1). Let Ω_Q^* denote $\arg \min_{\Omega} C_Q^{i.i.d.}(\Omega)$. Then,

$$\begin{aligned} \min_{\Omega} I(X_\Omega; \hat{Y}_\Omega | X_{\Omega^c}) &= \min_{\Omega} C_Q^{i.i.d.}(\Omega) = C_Q^{i.i.d.}(\Omega_Q^*) \\ &\stackrel{(a)}{\geq} C_0^{i.i.d.}(\Omega_Q^*) - d_Q^* \log(Q+1) \\ &\geq C_0^{i.i.d.}(\Omega_0^*) - d_Q^* \log(Q+1) \\ &\stackrel{(b)}{\geq} \max_{X_{\mathcal{N}}} I(X_{\Omega_0^*}; Y_{(\Omega_0^*)^c} | X_{(\Omega_0^*)^c}) - d_0^* \log \left(1 + \frac{M}{d_0^*} \right) - d_Q^* \log(Q+1) \\ &\geq \max_{\Omega} \min_{X_{\mathcal{N}}} I(X_\Omega; Y_\Omega | X_{\Omega^c}) - d_0^* \log \left(1 + \frac{M}{d_0^*} \right) - d_Q^* \log(Q+1) \\ &= \bar{C} - d_0^* \log \left(1 + \frac{M}{d_0^*} \right) - d_Q^* \log(Q+1), \end{aligned} \quad (5)$$

where

(a) is justified by the following:

$$\begin{aligned} C_Q^{i.i.d.}(\Omega_Q^*) &= \log \det \left(I + \frac{P}{(Q+1)\sigma^2} H_{\Omega_Q^* \rightarrow (\Omega_Q^*)^c} H_{\Omega_Q^* \rightarrow (\Omega_Q^*)^c}^\dagger \right) \\ &\geq \log \det \left(I + \frac{P}{\sigma^2} H_{\Omega_Q^* \rightarrow (\Omega_Q^*)^c} H_{\Omega_Q^* \rightarrow (\Omega_Q^*)^c}^\dagger \right) \\ &\quad - d_Q^* \log(Q+1) \\ &= C_0^{i.i.d.}(\Omega_Q^*) - d_Q^* \log(Q+1), \quad \text{and} \end{aligned}$$

(b) follows from [1, Lemma 6.6] equation (144).

The proof of Theorem 1 follows from (4) and (5). \blacksquare

Remark 2. If there exists a class of cuts \mathcal{A} such that

$$\min_{\Omega} C_Q^{i.i.d.}(\Omega) \geq \min_{\Omega \in \mathcal{A}} C_Q^{i.i.d.}(\Omega) - \kappa$$

for all Q , where κ is a constant, then the gap in Theorem 1 can be possibly improved to

$$\tilde{d}_0^* \log \left(1 + \frac{M}{\tilde{d}_0^*} \right) + \frac{N}{Q} + \tilde{d}_Q^* \log(Q+1) + \kappa, \quad (6)$$

where

$$\tilde{d}_Q^* \triangleq \text{DOF} \left(\arg \min_{\Omega \in \mathcal{A}} C_Q^{i.i.d.}(\Omega) \right). \quad (7)$$

This can be seen by modifying the proof of the lower bound (5) slightly as:

$$\begin{aligned} \min_{\Omega} I(X_\Omega; \hat{Y}_\Omega | X_{\Omega^c}) &= \min_{\Omega} C_Q^{i.i.d.}(\Omega) \\ &\geq \min_{\Omega \in \mathcal{A}} C_Q^{i.i.d.}(\Omega) - \kappa \\ &\geq \min_{\Omega \in \mathcal{A}} C_0^{i.i.d.}(\Omega) - \tilde{d}_Q^* \log(Q+1) - \kappa \\ &\geq \bar{C} - \tilde{d}_0^* \log \left(1 + \frac{M}{\tilde{d}_0^*} \right) - \tilde{d}_Q^* \log(Q+1) - \kappa. \end{aligned}$$

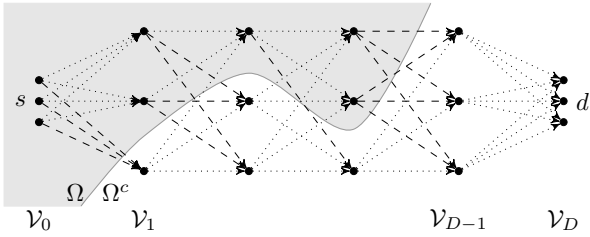


Fig. 2: The cut Ω depicted here $\notin \mathcal{A}$ since the crossing links come from 4 layers, and $4 > K - 1 = 2$.

V. LAYERED NETWORK WITH MULTIPLE RELAYS

In this section, we apply Theorem 1 to obtain an improved approximation for the capacity of a layered network in which a source node with K transmit antennas communicates to a destination node with K receive antennas over $D - 1$ layers of relays each containing K single-antenna nodes (see Figure 2). Since the network is layered, for $0 \leq i \leq D - 1$, the received signal at nodes in \mathcal{V}_{i+1} (or antennas if $i = D - 1$) depends only on the transmit signals of nodes in \mathcal{V}_i and at time t is given by

$$Y_{\mathcal{V}_{i+1}}[t] = H_{\mathcal{V}_i \rightarrow \mathcal{V}_{i+1}} X_{\mathcal{V}_i}[t] + Z_{\mathcal{V}_{i+1}}[t],$$

where $Y_{\mathcal{V}_{i+1}}$ and $X_{\mathcal{V}_i}$ are vectors containing the received and transmitted signals at nodes in \mathcal{V}_{i+1} and \mathcal{V}_i respectively; and $Z_{\mathcal{V}_{i+1}} \sim \mathcal{CN}(0, \sigma^2 I)$. The (k, l) 'th entry of the matrix $H_{\mathcal{V}_i \rightarrow \mathcal{V}_{i+1}}$ denotes the channel coefficient from l 'th relay in \mathcal{V}_i to k 'th relay in \mathcal{V}_{i+1} at time t and we assume that it is an arbitrary fixed complex number with unit magnitude, i.e., of the form $e^{j\theta_{kl}}$ for some $\theta_{kl} \in [0, 2\pi]$. The phases θ_{kl} are arbitrary for different links. All transmitting nodes are subject to an average power constraint P . We can assume that $Y_{\mathcal{V}_0} = 0$ and $X_d = 0$. Note that $N = M = KD$. We have the following lower bound on the capacity C of this network.

Theorem 2. For $K \geq 2$ and $D \geq 2$,

$$C \geq \bar{C} - 2K^2 \log D - K \log K - K. \quad (8)$$

This theorem shows that for a fixed number of nodes K per layer, the gap to capacity grows only logarithmically with the number of layers D . We note that the constants in the gap can be carefully optimized for, however to maintain brevity we do not worry about getting the best constants.

VI. PROOF OF THEOREM 2

We first show that for any Q , $\min_{\Omega} C_Q^{i.i.d.}(\Omega)$ can be approximated upto an additive constant by restricting the minimization to cuts in a particular class. Then, Theorem 2 follows immediately from Remark 2.

For convenience, we call the K^2 entries in $H_{\mathcal{V}_i \rightarrow \mathcal{V}_{i+1}}$ as the links in layer i . With this convention in mind, let \mathcal{A} denote the set of $s - d$ cuts Ω for which the links crossing from Ω to Ω^c come from at most $K - 1$ layers, e.g. see Figure 2.

Lemma 1. We have

$$\min_{\Omega \in \mathcal{A}} C_Q^{i.i.d.}(\Omega) - K \log K \leq \min_{\Omega} C_Q^{i.i.d.}(\Omega) \leq \min_{\Omega \in \mathcal{A}} C_Q^{i.i.d.}(\Omega).$$

Proof: The upper bound is immediate. The lower bound can be proved as follows. For any cut $\Omega \notin \mathcal{A}$,

$$\begin{aligned} C_Q^{i.i.d.}(\Omega) &= \sum_{i=1}^D C_{Q, \text{MIMO}}^{i.i.d.}(H_{(\mathcal{V}_i \cap \Omega) \rightarrow (\mathcal{V}_{i+1} \cap \Omega^c)}) \\ &\stackrel{(a)}{\geq} K \log \left(1 + \frac{P}{(Q+1)\sigma^2} \right) \\ &\stackrel{(b)}{\geq} C_Q^{i.i.d.}(\mathcal{V}_0) - K \log K \\ &\geq \min_{\Omega \in \mathcal{A}} C_Q^{i.i.d.}(\Omega) - K \log K, \end{aligned}$$

where (a) follows since for any cut $\Omega \notin \mathcal{A}$, at least K terms in the summation are non-zero, each lower-bounded by the point-to-point AWGN capacity; and (b) follows by Lemma 2. This concludes the proof of the lemma. ■

Lemma 2. We have

$$C_Q^{i.i.d.}(\mathcal{V}_0) \leq K \log \left(1 + \frac{P}{(Q+1)\sigma^2} \right) + K \log K.$$

$$\begin{aligned} \text{Proof: } C_Q^{i.i.d.}(\mathcal{V}_0) &= \log \det \left(I + \frac{P}{(Q+1)\sigma^2} H_{\mathcal{V}_0 \rightarrow \mathcal{V}_1} H_{\mathcal{V}_0 \rightarrow \mathcal{V}_1}^\dagger \right) \\ &\stackrel{(a)}{\leq} \sum_{i=1}^K \log \left(1 + \frac{P}{(Q+1)\sigma^2} \mathbf{h}_i \mathbf{h}_i^\dagger \right) \\ &\leq K \log \left(1 + \frac{P}{(Q+1)\sigma^2} \right) + K \log K, \end{aligned}$$

where (a) follows by using Hadamard's inequality; \mathbf{h}_i denotes the i 'th row of $H_{\mathcal{V}_0 \rightarrow \mathcal{V}_1}$. ■

The desired result (8) follows from (6) by setting $Q = D - 1$ and noting that the DOF (7) of the MIMO channel created by any cut in \mathcal{A} is trivially upper-bounded by K^2 . ■

REFERENCES

- [1] A. Avestimehr, S. Diggavi, and D. Tse, "Wireless network information flow: A deterministic approach," *IEEE Trans. Inf. Theory*, vol. 57, no. 4, pp. 1872–1905, 2011.
- [2] S. Lim, Y.-H. Kim, A. El Gamal, and S.-Y. Chung, "Noisy network coding," *IEEE Trans. Inf. Theory*, vol. 57, no. 5, pp. 3132–3152, 2011.
- [3] A. Ozgur and S. Diggavi, "Approximately achieving gaussian relay network capacity with lattice codes," in *IEEE Int. Symp. Inf. Theory*, 2010, pp. 669–673.
- [4] G. Kramer and J. Hou, "Short-message quantize-forward network coding," in *Multi-Carrier Systems Solutions (MC-SS), 2011 8th International Workshop on*, 2011, pp. 1–3.
- [5] X. Wu and L.-L. Xie, "On the optimal compressions in the compress-and-forward relay schemes," *IEEE Trans. Inf. Theory*, vol. 59, no. 5, pp. 2613–2628, 2013.
- [6] B. Chern and A. Özgür, "Achieving the capacity of the n-relay gaussian diamond network within $\log n$ bits," in *IEEE Information Theory Workshop (ITW) Lausanne*, 2012, pp. 377–380.
- [7] A. Sengupta, I.-H. Wang, and C. Fragouli, "Optimizing quantize-map-and-forward relaying for gaussian diamond networks," in *IEEE Information Theory Workshop (ITW), Lausanne*, 2012, pp. 381–385.
- [8] R. Kolte and A. Özgür, "Improved capacity approximations for gaussian relay networks," in *IEEE Information Theory Workshop (ITW) Seville*, 2013.
- [9] U. Niesen, B. Nazer, and P. Whiting, "Computation alignment: Capacity approximation without noise accumulation," *IEEE Trans. Inf. Theory*, vol. 59, no. 6, pp. 3811–3832, 2013.
- [10] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. Wiley-Interscience, 2006.