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Stochastic reserve scheduling
and smart charging of Plug-In
Electric Vehicles in power
networks with wind power
generation

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Abstract

In this project we investigate the potential of exploiting Plug-in Electric Vehicles (PEVs) for reserve provision, through a direct control scheme for power networks with wind generation uncertainty. We aggregate the vehicle fleet and represent it as a set of virtual storage units, which is incorporated in a stochastic reserve scheduling and smart PEV charging algorithm. The overall problem is formulated as a chance constrained optimization program and it is solved using a variant of the so called scenario approach, which guarantees constraint satisfaction with a given probability. The performance of our method in terms of operational cost and reliability is evaluated via Monte Carlo simulations. The obtained solution is finally distributed to the individual vehicles and the error due to the aggregation step is quantified.
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1 Introduction

In recent years, the effort for a more sustainable power mix has led to significant growth of power generation from renewable energy sources (RES), such as wind and solar, which are relatively cheap and can play a major role in meeting emission reduction targets. However, as RES are of fluctuating and uncertain nature, their large-scale integration in the power network poses great challenges to power system operation and planning. In principle, power generation has to match power demand at all times. When the RES penetration is high, the total supply is weather-dependent and thus fluctuating and has to match the already fluctuating load. This is managed through ancillary services, including reserves or flexible storages. In particular, reserves are services provided by contracted generators that adjust their power output or are turned on and off on demand, while storage in the system is basically provided by pumped hydro, which is rather limited. In the liberalized power market, the Transmission System Operator (TSO) is responsible for operational tasks including the unit commitment, generation dispatch, reserve scheduling and security of the power system. In order to effectively accommodate RES, it is essential that these concepts are revisited, since the electric power network was traditionally designed for generators with constant in-feed.

Furthermore, the incorporation of plug-in electric vehicles (PEV) has received increasing attention over the last decade, showing high potential in decarbonizing road transport. Nonetheless, a large and uncontrolled adoption of PEVs could have detrimental effects on the system’s operation, since it could induce or increase system peaks and lead to overloading of system components. Most research findings have concluded that these adverse impacts could be minimized, as long as a “smart” charging technique or charging control is implemented. A concept that is of particular interest, showing significant potential, is the synergy of RES and PEVs. As far as ancillary services are concerned, a potential solution for large storage would be the aggregation of PEVs into a virtual storage, which could be used to provide quick-response, high-value electric services, compensating the in-feed error of RES and helping the network to survive unexpected asset failure. Indeed, this is strongly related to the concept of Vehicle-to-Grid (V2G) services, which allow the PEVs to act not only as loads but also as generators.

Taking into consideration the aforementioned possibilities for increased security, cost-efficiency and sustainability in terms of power system operation and individual transportation, the goal of this thesis is to develop a unified framework...
for optimal day-ahead scheduling and operation of power networks with large penetration of wind power, as a specific form of RES, and shares of PEVs.
2 Background and related work

2.1 Basics of frequency control reserves

The proper operation of the power system requires that power generation and power demand are in balance at all times, so that the system frequency is maintained at the target value. In fact, a certain amount of active power, the so-called frequency control reserve, is kept for this task. There are three frequency control schemes: primary, secondary and tertiary [1], [14], [30]. The reserves, being either positive (increase of generation or decrease of load) or negative (decrease of generation or increase of load), are determined by the TSO depending on the size and generation portfolio of each control area.

The automatic control schemes in a power system are the primary and the secondary control. However, the latter one is performed manually in some power networks, including the Nordel system. The primary control action is provided by participating spinning generators, which adjust to disturbances as their speed governors respond to deviations from the nominal frequency through their speed droop characteristics. In fact, the primary control action has to be activated within seconds in order to stabilize the system, e.g. 30 sec in Continental Europe, bringing the frequency back to an acceptable region.

However, as the primary controller is purely proportional (P), a steady state frequency error remains. Therefore, the objective of the secondary controller or the so-called Automatic Generation Control (AGC), is to clear the frequency error, bringing the frequency back to its nominal value after the primary controller has acted. In addition, in the interconnected power system, the secondary controller is responsible for restoring the tie-line power flows between different areas to predetermined values, that have been affected by the active power imbalances and the primary control action. Secondary control reserves are indeed activated by a proportional-integral (PI) controller operated by the TSO, while the participating generators are spinning generators in the control zone of the disturbance solely, which alter their generation set-points proportionally to some participation factors.

The third frequency control scheme, the so-called tertiary control, is activated manually such that the used primary and secondary control reserves are released after a disturbance. This control scheme occurs 10-15 min after a contingency and its goal is to impose new, post-contingency generator setpoints, while respecting generation, demand and transmission power flow limits and ensuring that con-
2.2 Optimal operation in power systems with wind power generation

In the deregulated market, the TSO is responsible for the power network management, determining the optimal combination of generating stations (unit commitment), the power output of each unit (generation dispatch), as well as the reserve providers (reserve scheduling), such that the power balance is maintained on a continuous basis and the system is able to survive unexpected disturbances. This is achieved through the solution of an optimal power flow problem (OPF), which is essentially an optimization problem with the objective of maximizing social welfare, e.g. minimization of generation and reserve costs, minimization of emissions etc. The determination of the optimal reserve power involves a trade-off between the cost of reserves and the cost of allowing power imbalances, which lead to energy spillage or load shedding.

As already mentioned, in case of penetration of wind power in the system, the aforementioned tasks become more challenging. Taking into account the uncertainty of wind power generation, which cannot be perfectly forecasted, a stochastic variant of the standard day-ahead planning problem is adopted in most cases. The underlying optimization problem is a multi-period OPF, which might consist of a market-clearing procedure, incorporating the unit commitment, generation dispatch and reserve scheduling, but may be also formulated in a security-constraint framework. In the latter case, the system operation is typically designed such that it is N-1 secure, which means any possible single component outage will not lead to a set of cascading system failures, avoiding undesired disruption of service.

In the aforementioned stochastic context, significant research has been conducted focusing on the maximization of an expected social welfare. In a security-constrained market-clearing formulation, the stochastic unit commitment and reserve scheduling problem has been addressed in [14], [6], [7], [5] where it has been suggested that the stochastic formulation improves performance by focusing on the most likely forecasted conditions of power generation, rather than being biased by the...
worst-case scenario, as in the deterministic context. Furthermore, a multi-stage stochastic unit commitment problem is formulated in [26], in which the uncertain generation was modeled by using scenarios and the tractability of the problem was achieved through reduction techniques. A similar reasoning was followed in [28], neglecting network constraints. Nonetheless, the above articles do not provide any a-priori guarantee regarding the satisfaction of system constraints and reliability of the solution.

On the contrary, probabilistic certificates for the solution of the underlying stochastic optimization problem were provided in [35], [23], in which the optimal day-ahead generation dispatch and reserve procurement were investigated in a security-constrained context, towards the objective of minimizing the cost of reserves and generation. The resulting OPF problem was a chance constrained, bilinear problem. In [35], the arising tractability problems were tackled by the deployment of the so-called scenario approach for stochastic chance-constraint problems [11] and by reformulating the initial problem in a convex form. In [23], an improved technique was presented, as the authors proposed a heuristic algorithm and a convex reformulation of the initial problem, along with two alternative methods to address the challenge of the chance constraints. In a similar framework, providing probabilistic certificates, [21] proposed a formulation unifying stochastic generation dispatch, reserve scheduling and unit commitment, based on a combination of randomized and robust optimization techniques.

2.3 The role of Plug-In Electric Vehicles

Concerns on the availability of oil, the increasing global impulse towards sustainability and the target for pollution reduction, have all led to a growing interest for electrification of transportation. Indeed, the large-scale introduction of electric vehicles shows great potentials. However, engineers have to meet numerous challenges in order for plug-in electric vehicles (PEV) to become a viable solution, including the significant challenge of power system operation and planning. The mutual dependencies of the power network and transport infrastructure should be investigated in depth. The integration of PEVs can be viewed from different perspectives, as they can be operated in different modes, follow alternative operational objectives and system architectures [16]. These are discussed in the following subsections.

2.3.1 Uncontrolled charging of PEV

A simple method to deploy PEVs is to consider them as inflexible load with varying temporal and spatial distribution, related to the driving patterns, which consumes power whenever connected to the power system. The impacts of such uncontrolled operation of PEVs on both the supply side and the electricity grid
have been widely examined in literature. Initial studies focused on how large fleets of uncontrolled PEV would influence the portfolio of utilities and the load curves, by investigating the importance of timing of PEV recharging \cite{13}. Other studies showed that PEVs, if regarded as inflexible loads, could have a detrimental effect on the distribution network infrastructure, such as inducing unacceptable temperature increase in transformers. More recent investigations highlighted that under uncontrolled conditions, increase in peak demand and in electricity prices is anticipated \cite{17}. This would require expansion of the generation and transmission capacity. According to further studies, electricity spot prices will show an upward trend as PEVs lead to increased electricity demand \cite{36}. The aforementioned effects are expected to pose significant risks for the distribution grid and system security especially, including the acceleration of transformer aging due to high temperatures, low voltages and line overloading \cite{16}.

### 2.3.2 Smart charging of PEV and V2G potentials

In the previous subsection, the potential adverse impacts of considering PEVs as inflexible load have been presented, justifying why a significant part of researchers have focused on the development of smart charging strategies. In fact, this reasoning is driven by the fact that statistically, the fraction of parked vehicles throughout the day is very high \cite{16}, as Fig. 2.1 illustrates for the case of Switzerland on a weekday, according to the 2000 Swiss survey mobility. Therefore, vehicles could be regarded as flexible load and part of the charging could be done during the so-called valley hours, when electricity demand is relatively lower, such as night hours.

![Figure 2.1: Fraction of vehicles being parked on a weekday in Switzerland](image)

Several smart charging techniques have been investigated in literature, distinguished by the objectives they pursue, as well as by the type of their control architecture. In particular, PEVs can be viewed solely as mobile load for the system, with associated end-user constraints, or as both load and distributed
storage. The latter functionality is termed Vehicle-to-Grid (V2G) strategy and shows great potentials in terms of provision of ancillary services and large-scale integration of RES. In addition, the possibility of using PEVs for profit maximization has been examined, in which PEVs discharge at peak hours to exploit differences in electricity prices at different times. This strategy is referred to as trade arbitrage and is typically used with storage resources [32]. However, arbitrage with PEVs has been considered economically ineffective due to high battery degradation costs, leading to drasticical reduce of profit [16]. On the contrary, frequency control and balancing RES services are anticipated to be more profitable and beneficial for the power system operation. These concepts are discussed in the following.

V2G for provision of ancillary services

The studies related to ancillary services provided by PEVs can be categorized in two different types [16]. The first one refers to local services including voltage and reactive power control, as well as frequency control during islanding operating conditions [29]. The second type refers to studies about ancillary services offered system-wise, such as primary, secondary and tertiary control, which have been explained in Section 2.1. In the latter category, studies are commonly based on the aggregation of PEV fleets into one entity, able to provide or draw power to or from the system. To this end, [12] proposed an advanced algorithm for stabilizing the system frequency, in case of a disturbance in a power system with penetration of RES. In specific, systems with RES face stability problems as RES lack stabilizing characteristics, e.g. inertia, which are inherent to conventional generation with synchronous machines. PEV fleets are viewed as aggregated storage or power source, controlled appropriately in order to mimic the inertial behaviour and achieve better frequency control and system stability. Nonetheless, in order for such techniques to become realistic, a method for measuring the frequency locally and accurately should be available.

V2G for balancing RES

Another operational mode envisioned for PEVs is V2G for balancing RES, which can be distinguished from the provision of ancillary services as the two are performed on different time scales [16], [20]. This concept basically focuses on the integration of large shares of RES, which would increase the need for storage capacity and reserves provision. PEVs could act as an aggregated virtual storage in the network, balancing the RES infeed forecast error. In other words, PEVs could draw or offer power back to the network, based on whether the RES generate less or more energy that the forecasted value. At the same time, this implies that PEVs would be charged from emission-free RES, which would constitute the individual transportation sector more environmentally sustainable.
The quantitative study of [20] showed that V2G is a promising technique economically and stability-wise, even in case RES become half of total electrical generation, without large storage costs and increase in the requirement for reserves from conventional generators. The work of [15] presented a coupling between smart charging and V2G for PEVs, in which vehicle fleets are aggregated in a single, large battery with the potential to counter the forecast error of RES and their intrahour generation fluctuations. The authors considered also two conventional flexible generators contributing to the reserve provision, so as to increase the degrees of freedom in the problem. A Model Predictive Control (MPC) scheme was developed, in which the control inputs are the power setpoints of three generators, i.e., the two conventional generators and the aggregated battery of PEVs in V2G mode, with the latter viewed as a virtual generator. In addition, the limitations of the distribution grid and the individual PEV needs for available state of charge (SOC) were included in the problem set-up. The distribution grid constraints should be taken into account because if neglected, the distribution system could be endangered and the drivers would possibly not be able to satisfy their driving demands.

2.3.3 PEV flexible charging architectures

There are two major approaches in how the PEVs could be integrated in the power system framework and market. Indeed, the current environment would need to be adapted or reorganized appropriately. These approaches, referred to as centralized or direct control and decentralized or indirect control, are discussed in the following subsections.

Centralized control approach to PEV flexible charging

The architecture of a centralized control approach for a future power system with PEVs is shown in Fig. 2.2, as envisioned in [16]. The basic actors appearing in the framework are the following: the Aggregator, the Energy Service Providers (ESP), the Transmission System Operator (TSO), the Distribution System Operators (DSO), the Traders, the Balance Group Managers and the individual PEVs. In specific, ESPs act on the wholesale marker by contracting energy in order to supply a portfolio of loads. DSOs are responsible for the planning, operation and maintenance of the medium and low voltage distribution network, including the installation and operation of the charging stations for the electric vehicles. Similarly to the tasks of DSOs, the TSO operates and supervises the high voltage transmission system. Traders’ objective is to make profit through arbitrage, i.e., by buying and selling electricity on the wholemarket, taking advantage of the price variability across time. BGMs submit information on how much a balance group, a cluster of loads and generators, consumes during a time interval to the TSO, so as for the latter to make security and operation
assessments. The aggregator clusters large numbers of PEVs at each load node, which are viewed as single, virtual storages or virtual power plants, similarly to a common technique clustering RES. It is indeed the central component of the centralized approach, as it is in charge of every market operation related to the PEVs, such as the day-ahead forecast of energy demand, the communication with DSOs, BGMs and the TSO. Since the aggregator will exert direct control to individual PEVs, while satisfying the energy requirements for their trips, it needs to have access to spatial and temporal information on their availability and their demand forecast. The aggregator can manage a portfolio of controllable and uncontrollable loads, including inflexible PEVs and other non-PEV loads. Although neglected in some studies, it is highly important that the aggregator collaborates with DSOs, by submitting to them the schedule of spatial and temporal consumption of the vehicles so that a network assessment is performed in advance. In case of congestion and risk of the distribution network, the aggregator would adjust its charging plan and finally submit it to the TSO. For example, [19] investigated the impact of the charging profiles on the distribution grid as a step performed after the optimal charging profiles have been determined. In specific, the scheduled PEV loads, with varying temporal and spatial distribution, are added to the reference load of each distribution grid node and a power flow is performed, seeking to identify asset overloading and voltage drops. Moreover, the centralized approach can incorporate the V2G services for reserve provision, in which the aggregator charges or discharges the virtual storage according to demands of the TSO.

[18], [19] presented a centralized approach for charging PEV fleets. The problem was formulated as a multi-period OPF seeking to minimize generation costs while taking into account transmission network constraints as well as energy constraints.
related to the vehicles’ driving pattern. However, the resulting charging power in such centralized approaches refers to the aggregated level. Therefore, a heuristic algorithm was presented to obtain the individual charging profile for each PEV belonging to a specific network node. This algorithm is performed time step by time step and is based on the current SOC of the vehicle, its energy requirement and its temporal driving schedule. In some cases, the individual charging profiles deviate from the optimal nodal profiles of the aggregated level because of simplification of constraints in the latter level.

Decentralized control approach to PEV flexible charging

An alternative architecture for PEV charging is the decentralized approach \[10\], which could be viewed as in Fig. 2.3. According to this approach, the PEV charging is not directly controlled from an external entity. The objective is to induce a charging behavior to vehicle owners, shifting the charging towards non-peak hours. This is why the central actor in this scheme is the vehicle itself, which makes it more easily acceptable to owners, who retain the authority of control and are in charge of communicating with all the other entities in the power system. The way the charging profiles are determined is typically through the broadcasted temporal and spatial profiles of prices that act as incentives for users. The user performs an optimization algorithm that minimizes the charging costs, given the exogenous prices, his/her driving schedule and the battery size of the vehicle. In

![Figure 2.3: Decentralized control architecture for PEVs](image)

\[19\] such a decentralized, indirect control of PEV fleets was examined through an OPF problem, also highlighting the impact on the distribution network, in the same way as discussed for the centralized approach in the previous subsection.
By using the same OPF approach, investigated the effect of pricing in the network. In fact, the study showed that when the prices were the same system-wide, varying only over time, PEV charging does not induce or increase peaks during busy hours because of the high charging cost. Nonetheless, the high level of charging simultaneity leads to a new shape of load distribution in which peaks are induced overnight. On the other hand, according to the article’s simulation results, imposing different nodal prices leads to charging at different places and different times, finally flattening the load profiles.

2.4 Objective of the thesis

The fact that the electric power system is going through significant changes, such as the large in-feed of RES, dictates the importance of re-examining its optimal operation in terms of reserve scheduling as well as the flexibility of the demand side. Towards that direction, the current thesis examines a potential synergy between spinning generators and PEVs for providing secondary reserves that could compensate any wind power forecast error. The ultimate goal of this work is to develop a unified framework for day-ahead optimal generation dispatch, smart charging of PEVs and reserve scheduling while providing a-priori probabilistic guarantees for the reliability of the solution. Its objective is to build upon the the combination of studies and , by investigating a stochastic DC optimal power flow problem for systems with wind power penetration and operation of PEVs.
3 Problem formulation

In this chapter, the reader is introduced to the problem formulation, while background information is provided regarding DC optimal power flow (OPF) modeling, smart charging of Plug-In Electric Vehicles (PEVs), reserve modeling and real-time deployment as well as the wind power model which is utilized in the current thesis.

3.1 Definitions and preliminaries

For the analysis made in the current project, we consider a power network consisting of $N_G$ generators, $N_L$ loads, $N_L$ lines and $N_b$ buses. The following assumptions are made:

- A linearized version of the network is considered, hence standard DC power flow approach is adopted.
- Wind power generation is located at single node of the system.
- We consider uncertainty only due to wind power generation.
- Wind power generation is a regulated activity and wind producers are not considered as competitive agents in the power market.
- Driving behaviour for PEVs is assumed to be a-priori, fully known.
- No load uncertainty is considered.
- No $N - 1$ security constraints are considered.
- Three modes of operation have been considered in the current framework, in which:
  a) PEVs are optimally charged, while reserves are provided solely by spinning generators.
  b) PEVs are optimally charged, while reserves are provided by both spinning generators and PEVs which increase or decrease their charging.
  c) PEVs are optimally charged, while reserves are provided by both spinning generators and PEVs in V2G mode. The latter mode considers the potential of PEVs providing energy back to the network by de-charging.
The first assumption is rather standard for this type of problems while the second to sixth assumptions are made for the sake of simplification. The seventh one excludes the possibility of component outages and the last one deals with the operational modes considered, especially regarding the PEVs. The problem will be formulated such that it will allow us to determine the day-ahead optimal dispatch of generators, reserve scheduling and charging schedules of PEVs so as to

- minimize generation and reserve costs,
- satisfy network constraints,
- satisfy constraints of PEV end-users,
- leave charging flexibility, in terms of available power and energy, so as to use PEVs for reserves, compensating the wind power prediction errors,
- determine the amount of up and down secondary reserves that shall be purchased in advance by the TSO and
- provide us with a real-time optimal reserve deployment strategy.

### 3.2 DC power flow modeling

In this section, we provide some background on the DC power flow modeling, which is an approximation of the non-linear and non-convex AC power flow model, used for operational and planning purposes. Indeed, the DC approximation of the power flow equations offers the advantage of dealing with a linear and convex system, which is quicker and easier to solve [?]. The standard approximations made in the DC power flow model can be found explicitly in many textbooks such as [?]. Here we provide the main assumptions made:

- The voltage at every network bus remains constant at 1 p.u.
- Active power losses are neglected.
- By considering light load conditions and small values for $\theta_{km}$, which is the difference in voltage angle between buses $k$ and $m$, it can be assumed that $\sin\theta_{km} \approx \theta_{km}$ and $\cos\theta_{km} \approx 1$.

Under these assumptions, the following equations can be written

$$P_f = B_f \theta,$$  \hspace{1cm} (3.1)

$$P_{inj} = B_{bus} \theta,$$  \hspace{1cm} (3.2)
where $P_f \in \mathbb{R}^{N_l}$, $P_{inj} \in \mathbb{R}^{N_b}$ and $\theta \in \mathbb{R}^{N_b}$ denote the power flows across each line, the active power injections and voltage angles in each bus respectively. $B_f \in \mathbb{R}^{N_l \times N_b}$ denotes the matrix containing the imaginary part of the admittance of each network branch and $B_{bus} \in \mathbb{R}^{N_b \times N_b}$ is the nodal admittance matrix of the system. In order to express the power flows $P_f$ as a function of the power injections $P_{inj}$, the angles $\theta$ are eliminated from Eq. 3.1, 3.2. Since $B_{bus}$ is singular with rank $N_b - 1$, we choose one angle as a reference equal to zero. Without loss of generality, we let $\theta_b = 0$. Then, $\tilde{B}_{bus} \in \mathbb{R}^{N_b - 1 \times N_b - 1}$, $\tilde{P}_{inj} \in \mathbb{R}^{N_b - 1}$ and $\tilde{\theta} \in \mathbb{R}^{N_b - 1}$ denote the remaining parts of $B_{bus}$, $P$, $\theta$ after the corresponding entities have been removed. Finally, the power flow across the branches can be written as

$$P_f = B_f[(\tilde{B}_{bus}^{-1}\tilde{P}_{inj}, 0)^T]$$

(3.3)

where

$$\tilde{P}_{inj} = [C_G P_G + C_w P_w - C_L P_L]_{N_b - 1},$$

(3.4)

with operator $[\cdot]_{N_b - 1}$ denoting the first $N_b - 1$ rows of the quantity inside the brackets, which is essentially the power injection in every node. The matrices $C_G, C_w, C_L$ are of appropriate dimension and describe the system configuration by having their element $(i, j)$ equal to "1" if the $j$-th generator/wind power generator/load is connected to the $i$-th bus or equal to "0" if not. $P_G$, $P_w$, $P_L$ represent the power generation dispatch, the actual wind power in-feed and the total power consumed by the loads, consisting of the inelastic reference load which is assumed to be known and the elastic PEV charging load.

### 3.3 Plug-in electric vehicles modeling

This section provides a background on the different modelling levels of Plug-in electric vehicles (PEVs), i.e., the individual level representing the dynamics of each vehicle and the aggregated level, which models the PEVs as clustered virtual batteries.

#### 3.3.1 Individual PEV level

The modeling of the PEVs begins at the individual level, where the available data are the mobility patterns of each vehicle. A mobility pattern is understood as a set of trips performed, their timing, duration and energy consumption, which are all considered fully known for the sake of this thesis. Numerical examples of mobility patterns are generated through the existing transport simulation tool called MatSIM [2]. The mobility patterns extracted by the aforementioned tool are associated with the distribution power network utilized in the system. In specific, each activity is related to a node in the distribution system, which allows us to translate the mobility of PEVs as temporally and spatially varying load
from the perspective of the power system. In addition, the available individual transportation data can be translated into upper and lower bounds for the charging power and energy content of the battery of the vehicles. In particular, the limiting values are

- the minimum state-of-charge and the battery capacity, in terms of energy content of the vehicle.

- the minimum and maximum connection power, in terms of the PEV charging power.

The individual level, describing the mobility pattern of each PEV separately, would provide us with a dynamic model of high precision. However, it would be computationally intractable to use for operational purposes in larger fleets, where the dynamics of a large number of vehicles would be modeled. This dictates the importance of moving to a reduced-order aggregated model, as it will be explained in the next subsection.

3.3.2 Aggregated PEV level

According to the centralized architecture for the control of PEVs, as this is described in Section 2.3.3, the individual PEVs are clustered into a set of virtual batteries by an entity referred to as the Aggregator. Unfortunately, the aggregated model has the drawback of reduced modelling precision as individual PEV dynamics are no longer modeled. However, its reduced size offers the advantage of tractability in practical, everyday planning operations.

Similarly to the individual bounds for charging power and energy content of each vehicle, aggregated bounds for the same quantities need to be computed in order to integrate PEVs in a unified framework for optimal charging and reserve scheduling. Indeed, their computation is based on the individual bounds and on the travelling requirements of the fleet, as presented in the work of [34], [33]. Hence, the aggregated fleet operates as a distributed storage with certain bounds for its charging power and its energy content, which could well be exploited for reserve provision in the secondary reserves power market.

Although operational tasks will be performed at the aggregated level of PEVs, as soon as the results are available, it is essential that they are translated back to the level of individual PEVs. Apparently, this is the only meaningful information from the vehicles’ perspective and their everyday operation. The transition from the aggregated to the individual level will be referred to as the disaggregation procedure and will be presented later in this thesis.
3.4 Reserves modeling

This section presents how reserves are modeled in our problem framework, being either spinning reserves or reserves provided by participating PEVs. In addition, the basic principles and the strategy followed for real-time deployment of reserves are explained.

When a power imbalance arises in the network, which can be caused by various reasons such as a wind power forecast error, this acts as a disturbance to the system leading to frequency deviations from the desired value. In case the power production exceeds power demand, the system frequency increases and hence, down-regulation reserves are required. This can be provided by decreasing power generation or increasing power consumption, as long as the demand side participates in the regulation scheme. On the other hand, when less power is produced than consumed by the loads, this leads to frequency decrease, requiring up-regulation, i.e., increase in power generation or decrease in consumption. Indeed, a disturbance in the system triggers a set of control schemes with different functionalities. Firstly, primary control action is activated locally on the power plant level to bring the frequency back to acceptable values. However, since the primary controller is a purely proportional control action, a steady state error remains, which is essential to be cleared. Therefore, the secondary control scheme is activated, which is a proportional-integral controller, to release the primary control, bring the frequency back to the exact desired value and restore the power flow on tie-lines between different control areas back to scheduled values. Finally, the so-called tertiary controller releases the secondary controller, at specific time intervals such as every 15 minutes.

More specifically, the secondary control scheme, also referred to as the Automatic Generation Control (AGC), adjusts the generation of certain participating generators by distributing its output to the generators in a weighted way, which in the current energy market is the product of contracting agreements between the generators and the TSO. In every new steady state, the setpoint of these generators is changed by a percentage of the active power mismatch.

The above reasoning is taken into account in our problem formulation, in which we consider that the secondary control action has reached a new steady state. This is a reasonable assumption, given that our tasks will be carried out in 24 hourly steps and within the interval of an hour, the frequency deviation will settle to zero due to the secondary reserves provision.

The aforementioned percentage-values belong to a vector which we will call the distribution vector. This is denoted as either $d_{up}$ or $d_{down}$, depending on whether the reserves are for up or down regulation, i.e., the sign of the wind power generation forecast error. In fact, distinguishing the reserves in two cases, i.e., up and down, provides us with more degrees of freedom in the problem, helping to avoid feasibility issues. As already discussed in section 3.1, according to the operation mode, reserves can be provided by spinning generators only or combined
with reserves by PEVs. Hence, in each case the distribution vector is of different dimension. In particular, in the two modes where PEV reserves are available, the distribution vectors contain certain terms corresponding to spinning generators and PEV virtual batteries. In specific, they are defined in the following way:

\[ d_{up} = [d_{up,gen}^T, d_{up,pev}^T]^T, \]
\[ d_{down} = [d_{down,gen}^T, d_{down,pev}^T]^T. \]

Apparently, when only spinning secondary reserves are available, the following holds:

\[ d_{up} = d_{up,gen}, \]
\[ d_{down} = d_{down,gen}. \]

The product of these distribution vectors by the power mismatch in the network is the amount of provided reserves \( R \). According to [23], we define \( R \) to be a linear function of the total generation-load mismatch, which in our case is the wind power forecast error, i.e., the difference between the forecast value and actual wind power in the network. Modeling the steady state behavior of this action, at each time step the reserves are modeled as:

\[ R_t = d_{up,t}\max(0, -P_{m,t}) - d_{down,t}\max(0, P_{m,t}) \]

where

\[ P_{m,t} = P_w - P_{w,t}^f \]

is the wind power forecast error and \( R_t \in \mathbb{R}^{N_G+N_L} \) or \( R_t \in \mathbb{R}^{N_G} \) when only spinning reserves are available.

Similarly to Eq. 3.5 and 3.6, the reserves vector can be distinguished between spinning and PEV reserves as follows:

\[ R = [R_{gen}^T, R_{pev}^T]^T. \]

Furthermore, as the participating generators and PEV batteries compensate for one percentage of the total active power imbalance, the absolution values of the distribution vector entities must sum up to one:

\[ 1^T d_{up,t} = 1, 1^T d_{down,t} = -1, \forall t. \]

In case a generator or PEV virtual battery does not participate in the up/down regulation AGC scheme, the corresponding entity in the distribution vector will be zero. It should also be noted that the entities are not constrained to be non-negative but are instead left free. For example, in case an entity of \( d_{up} \) is negative when \( P_{w,t} - P_{w,t}^f \leq 0 \), in which up regulation reserves are expected, this implies that in contrast with the majority of generators and virtual batteries, the
particular one will provide down-spinning reserves as this is the optimal behaviour of the network to relieve congestion.

The reserves provision will modify the power injection in the buses, as the generation and consumption setpoints will be adjusted in order to provide the required amount of reserves. Hence, the power injection vector, appearing in (3.4) is modified by the correction terms $R_{\text{gen}}$ and $R_{\text{pev}}$ as follows:

\[
P_{\text{inj,new}} = [C_G(P_G + R_{\text{gen}}) + C_wP_w - C_L(P_L - R_{\text{pev}})]
\] (3.13)

Here it should be highlighted that the different sign between spinning and PEV reserves is due to the fact that generation-side and demand-side reserves behave in the opposite way in the presence of a certain mismatch. In specific, when $P_{m,t} < 0$ generators need to increase their power generation, whilst PEVs should either decrease their charging or provide energy back to the network by decharging as long as V2G mode is available. On the contrary, in case of $P_{m,t} > 0$ down regulation reserves are required, so generators decrease power provision, whereas PEVs increase their charging power.

**Assumption:** An underlying assumption in the current framework is that PEVs can provide secondary reserves for a certain fraction of the hour, e.g. first quarter. After this certain time has elapsed, a tertiary control layer is activated to release the PEV secondary reserves, bringing their energy back to their scheduled value. This is actually a reasonable assumption, as it is compatible with current practice of TSOs, which allow for the tertiary control to release reserve provision every 15, 30 or 60 minutes.

By product of the modeling formulation for reserves and in specific, the use of the so-called distribution vectors, an optimal strategy for the real-time deployment of reserves is available. In specific, when the actual wind power generation is available, the wind power forecast error is computed and distributed to participating reserve entities according to the weighting factors given in $d_{\text{up}}$ and $d_{\text{down}}$ vectors. What is more, the product of these weights by the worst-case power imbalance in the system results in the worst-case amount of provided secondary reserves, i.e., the reserves that the TSO needs to purchase day-ahead. In fact, the worst-case scenarios for the power imbalance in the system can be computed in a probabilistic way, as will be presented in the final problem formulation.

To sum up, our reserves modeling offers us the following:

a) An optimal policy for real-time reserve provision.

b) An optimal decision about the amount of day-ahead contracted reserves, i.e., reserves purchased by the TSO in advance to be able to compensate the power imbalances appearing the following day.
3.5 Wind power Marcov-chain model

This section provides an introduction to wind power model utilized in this thesis, that enables us to generate scenarios of the wind power error, while taking its temporal correlation into account. Based on Eq. [3.10], the actual wind power could be viewed as the sum of a deterministic component, which is the forecasted value and a stochastic one modelling the forecast error. Indeed, we assume that the uncertainty is introduced by the stochastic quantity of the forecast error $P_m$. By following the work of [27], [25], in order to generate potential realizations of the stochastic parameter $P_m$, we use discretized historical data of it to obtain an estimation of the transition probability matrix. In other words, recorded data are used to train a 1st-order Marcov chain model, described by a stochastic matrix with entities expressing the probability of transition from one specific state to another. Indeed, by assuming an initial value for $P_m$ it is possible to generate a number of scenarios of its 24-hour evolution.

In our framework, we used normalized five-years hourly measured wind power data, both forecasts and actual values, for the total wind power infeed of Germany over the period 2006 - 2011. According to [27], obtaining a stochastic model directly in the wind power domain leads to a reduced number of states and to a lower order of the Markov Chain at equal power data resolution. In addition, the quality of the stochastic model estimation is also favoured, due to the fact that a lower number of independent parameters is estimated given a certain amount of historical data.

![Figure 3.1: Probability transition matrix of 1st-order Marcov Chain model](image)

In Fig. 3.1 the probability transition matrix of the 1st order Marcov chain model is depicted, in which 41 discrete states have been considered for the stochastic parameter, i.e., the wind power forecast error. A triangular structure can be observed in the matrix, which implies a strong temporal auto-correlation of the...
3.6 Chance-constrained DC OPF formulation

After having introduced the reader to the basic considerations of the current framework, i.e., the DC power flow model, the different levels of PEV modeling, the way reserves are modeled and finally, the wind power Markov-chain model used to represent the stochasticity of the wind power forecast error, this section provides the final formulation of the problem. As already explained, the objective of the thesis is to develop a technique for stochastic reserve scheduling and smart charging of PEVs as well as optimal generation dispatch for day-ahead operation.

The problem is formulated as a DC optimal power flow problem with chance-constraints and an optimization horizon of $T_t = 24$ h, with hourly steps. The vector of decision variables is

$$x = \{P_{G,t}^{T}, P_{L,t}^{T}, R_{up,t}^{T}, R_{down,t}^{T}, d_{up,t}^{T}, d_{down,t}^{T}\}_{t=1}^{T_t}, E_{V,B,0}^{T}, \Delta E^{T}\}^{T},$$

(3.14)

where $P_{G,t} \in \mathbb{R}^{N_G}$ is the generation dispatch, $P_{L,t} \in \mathbb{R}^{N_L}$ represents the load dispatch including the optimal charging schedule of PEVs, $R_{up,t}, R_{down,t} \in \mathbb{R}^{N_G+N_L}$ denote the probabilistically worst-case amount of up and down-regulation reserves respectively that the TSO has to purchase in advance from participating generators and PEV virtual batteries, $d_{up,t}, d_{down,t} \in \mathbb{R}^{N_G+N_L}$ are the distribution vectors and $E_{V,B,0} \in \mathbb{R}^{N_L}$ represents the energy content of each load-virtual battery at $t = 0$. It should be noted that the initial energy value is considered as
an optimization variable in this problem formulation as what we aim to examine in this problem is the potential of the vehicles to provide reserves and how much initial energy should the vehicles have so that they can be optimally charged and provide reserves. Nonetheless, the same procedure can be followed with \( E_{V/B,0} \) fixed, with the difference lying mainly in the aggregated bounds for the energy content of the vehicles, which should take into account the fixed initial value. \( \Delta E \in \mathbb{R}^{N_L} \) is a constant vector which can be translated as an additional flexibility in the evolution of PEV energy content when V2G service is available and therefore, is applicable to the corresponding mode only. Its functionality will be explained in detail after the introduction of the probabilistic PEV constraints for the V2G mode in this section. Also, let \( C_1, C_2 \in \mathbb{R}^{N_G} \) and \( C_{up}, C_{down} \in \mathbb{R}^{N_G + N_L} \) denote generation and up/down reserve cost vectors respectively.

The resulting optimization problem is

\[
\min_x \sum_{t=1}^{T} \left( C_1^T P_{G,t} + C_2^T P_{G,t} C_2 + C_{up}^T R_{up,t} + C_{down}^T R_{down,t} \right) \tag{3.15}
\]

subject to the following:

1) Power balance constraints:

\[
1^T \left( C_G P_{G,t} + C_w P_{w,t} - C_L P_{L,t} \right) = 0, \forall t, \tag{3.16}
\]

which have to be satisfied for the forecasted value of wind power generation \( P_{w,t} = P_{w,t}^f \). In other words, the sum of all generation dispatches of the conventional units and the forecasted power output of the wind generator should balance the total load of the system at all times.

2) Generation limits for each generator:

\[
P_{G,min} \leq P_{G,t} \leq P_{G,max}, \forall t. \tag{3.17}
\]

3) Constraints on the total power consumed by loads:

\[
P_{L,t} = P_{L,ref,t} + P_{L,c,t}, \forall t. \tag{3.18}
\]

where \( P_{L,ref,t} \) is the reference load, without the PEVs, which is considered inelastic and \( P_{L,c,t} \) corresponds to the PEVs elastic load.

4) Charging power limits for the loads corresponding to PEVs:

\[
P_{L,c,min,t} \leq P_{L,c,t} \leq P_{L,c,max,t}, \forall t. \tag{3.19}
\]

where \( P_{L,c,min,t}, P_{L,c,max,t} \) are the aforementioned aggregated bounds for PEVs.

5) Distribution vectors constraints:

\[
1^T d_{up,t} = 1, 1^T d_{down,t} = 1, \forall t. \tag{3.20}
\]
6) Constraint on the evolution of the virtual battery energy content during PEV charging:

\[ E_{VB,t} = E_{VB,t-1} + P_{L,c,t} \Delta t \eta_{VB} - E_{VB,d,t} + E_{VB,a,t}, \forall t, \]  

(3.21)

where parameter \( \eta_{VB} \in \mathbb{R}^{N_L \times N_L} \) is a diagonal matrix containing the average charging efficiency coefficients of the loads, while \( E_{VB,d,t}, E_{VB,a,t} \) denote the energy of the vehicles departing and arriving from and at each node, affecting the total energy of the virtual battery. The two latter parameters are considered fixed based on the assumption that vehicles depart from a node with full battery whilst they arrive to a node with empty battery. Indeed, this is a quite important source of the aggregation error, as the dynamics of individual departing or arriving vehicles are not taken into account with sufficient precision.

7) Energy content constraints imposed by initial conditions:

\[ E_{VB,0} = E_{VB,T}, \]  

(3.22)

such that the virtual batteries of the PEVs are not depleted.

8) Bounds on the energy content of the virtual batteries:

\[ E_{VB,\text{min},t} \leq E_{VB,t} \leq E_{VB,\text{max},t}, \forall t, \]  

(3.23)

where \( E_{VB,\text{min},t}, E_{VB,\text{max},t} \) are the corresponding aggregated bounds for energy content expressing the end-user constraints, as explained earlier.

9) A set of probabilistic constraints, with respect to the probability distribution of the wind power forecast error denoted as \( P_{m,t} \):

\[ \mathbb{P}(P_{m,t} \in \mathbb{R} | f_{P_{m,t}}) > 1 - \epsilon, \forall t, \]  

(3.24)

Since these constraints depend on a stochastic variable, they are called chance constraints. Here, the goal is to provide a solution satisfying them with a certain probability \( 1 - \epsilon \), where \( \epsilon \) is a design parameter, ideally as small as possible.

The first part of the above probabilistic constraints includes a probabilistic version of standard DC OPF constraints related to the network constraints and generation dispatch under reserve provision, as well as certain constraints introduced by the reserve deployment scheme and computation of contracted reserves. These are the following:

\[ P_{m,t} = P_{w,t} - P_{f,t} \]  

(3.25)

\[ R_t = d_{\text{up},t} \max(0, -P_{m,t}) - d_{\text{down},t} \max(0, P_{m,t}) \]  

(3.26)

\[ \tilde{P}_{\text{inj},t} = \left[ C_G(P_{G,t} + R_{\text{gen}},t) + C_w P_{w,t} - C_L(P_{L,t} - R_{\text{pev}},t) \right] N_t \]  

(3.27)

\[ -P_{\text{line},\text{min}} \leq B_f \begin{bmatrix} (\tilde{B}_{\text{bus}})^{-1} \tilde{P}_{\text{inj},t} \end{bmatrix} \leq P_{\text{line},\text{max}} \]  

(3.28)

\[ P_{G,\text{min}} \leq P_{G,t} + R_{\text{gen},t} \leq P_{G,\text{max}} \]  

(3.29)

\[ R_{\text{down},t} \leq R_t \leq R_{\text{up},t} \]  

(3.30)

\[ R_{\text{up},t} \geq 0 \]  

(3.31)

\[ R_{\text{down},t} \geq 0 \]  

(3.32)
Constraints 3.25, 3.26, 3.27 should be clear from the discussion in Sections 3.2 and 3.4. Furthermore, constraint (3.28) encodes the standard transmission capacity constraints for the lines, that should not be endagered while reserves are provided and constraint (3.29) is used to provide guarantees that the dispatched generation plus the contribution of the spinning reserve \( R_{gen,t} \) will not lead to any operating point exceeding the capacity limits of the generators. The computation of contracted reserves that will be purchased by the TSO day-ahead is achieved through the inequality constraint (3.30) which determines the probabilistically worst-case values of the correction term \( R_t \).

The second part of the probabilistic constraints refers to those related to the virtual batteries of PEVs, which appear only when PEVs participate in the reserve provision. When so, these can be distinguished by the operation mode selected. In specific, the following cases appear:

a) When PEVs provide reserves without V2G functionality:

\[
P_{L,c,min,t} \leq P_{L,c,t} - R_{pev,t} \leq P_{L,c,max,t} \quad \text{(3.33)}
\]

\[
E_{V_B,min,t} \leq E_{V_B,t} - R_{PEV,t} \frac{\Delta t}{4} - P_{L,c,t} \eta \frac{3\Delta t}{4} \leq E_{V_B,max,t} \quad \text{(3.34)}
\]

where constraint (3.33) encodes the charging power bounds. Indeed, here the lower bound is zero, whilst the upper bound is the maximum connection power of the PEVs to the grid. Constraint (3.34) contains the virtual energy bounds during the interval while PEVs provide reserves, e.g., fifteen minutes, encoding the end-user constraints and the flexibility of PEV charging and reserve provision in terms of energy content. In fact, the latter inequality is based on the following reasoning: The evolution of the energy content is computed according to the deterministic Eq. 3.21 which does not take into account the provision of reserves by PEVs. However, PEVs would in practice provide \( R_{pev} \). Hence, their energy content would be different than the one computed upon the aforementioned deterministic equation. In addition, Eq. 3.21 refers to the evolution at every hourly time step, whereas the PEV reserve provision will be completed within a fraction of the hour, as assumed in the current framework. So, constraint (3.33) is used to account for this mismatch between the deterministic and the stochastic evolution of the energy content.

b) When PEVs provide reserves with V2G functionality:

\[
P_{L,c,min,v2g,t} \leq P_{L,c,t} - R_{pev,t} \leq P_{L,c,max,t} \quad \text{(3.35)}
\]

\[
E_{V_B,min,v2g,t} \leq E_{V_B,t} - \Delta E - R_{PEV,t} \eta \frac{\Delta t}{4} - P_{L,c,t} \eta \frac{3\Delta t}{4} \leq E_{V_B,max,t} \quad \text{(3.36)}
\]

\[
E_{V_B,min,v2g,t} \leq E_{V_B,t} - \Delta E - R_{pev,t} \eta \frac{\Delta t}{4} + P_{L,c,t} \Delta t \left( \frac{1}{4\eta} - \eta \right) \leq E_{V_B,max,t} \quad \text{(3.37)}
\]

Comparing inequalities 3.35, 3.36, 3.37 to 3.33, 3.34 the reader can observe some significant differences between the two operation modes:
1) The first difference lies at the considered lower bounds of charging power of the PEVs, which need to be modified in the V2G case. In specific, when vehicles operate in V2G mode, they are allowed to provide energy back to the network by de-charging their battery, providing down regulation reserves not only by reducing their charging power but also through their de-charging. This means that their lower charging power is no longer zero but equals the maximum de-sarging power, depending on the connection power capacity.

2) The second difference lies at the considered lower bounds of energy content of PEV virtual batteries, which is due to that fact the V2G mode provides more flexibility to the vehicles in terms of energy. The additional flexibility gained because of V2G operation can be better understood by comparing Fig. 3.3 and 3.4. When V2G is not available, the energy content flexibility of PEVs for providing reserves while charging is given by the upper and lower bounds in Fig. 3.3. These bounds are calculated assuming each PEV reaches a full SOC at some point in time and PEVs with VB daily driving distances smaller than the range of their batteries would not deplete their batteries to the minimum SOC. In other words, their energy content can only drop as much as the total daily energy consumption. On the other hand, when PEVs operate in V2G mode, additional de-charging can occur, allowing for vehicles to drop their energy further than their daily energy consumption requirements. This is the reason for the different lower bound, denoted as $E_{VB,min,v2g,t}$, which is lower than $E_{VB,min,t}$ as can be seen in Fig. 3.4 and which takes into account the energy end-user requirements and the minimum SOC of each vehicle.

What is more, in V2G mode a shift variable is introduced, denoted as $\Delta E$, making sure that there is large enough a margin for additional charging when wind power output exceeds the forecast. Compared to the situation illustrated in Fig. 3.3, this shift implies that batteries should not be scheduled to reach a full SOC. The V2G operation can be better understood by observing Fig. 3.4, in which it is shown that the scheduled energy content of PEVs can only move within the grey area, representing the theoretical case of a perfect forecast. When a wind power forecast error arises in the network, PEVs provide up/down reserves with their energy content slipping out of the grey area, under the condition that it does not exceed the upper and lower bounds $E_{VB,max,t}$ and $E_{VB,min,v2g,t}$ respectively. Essentially, the initial energy content $E_{VB,0}$ would be shifted by $\Delta E$, which is an important adjustment, as the aggregated energy bounds have been determined without taking into account the potential of the PEVs for de-charging. It should be highlighted though that when the problem is a strict control problem with fixed initial energy content, the introduction of $\Delta E$ is no longer applicable, as the extra flexibility will be taken care of by the computation of the aggregated bounds.

3) The third significant difference between the two sets of constraints lies in the appearance of 3.37 in V2G mode. The reasoning behind this additional constraint is similar to the explanation of probabilistic constraint 3.33. In specific, when
vehicles de-charge their battery to provide up-regulation reserves, their energy content evolution is no longer expressed by the deterministic equation 3.21 for the same reasons as the ones mentioned for constraint 3.33 but also because de-charging efficiency is inversely proportional to charging efficiency. Indeed, the energy evolution a virtual battery which is decharging would be given by the following equation:

\[
E_{V_B,t} = E_{V_B,t-1} + \frac{P_{L,c,t} \Delta t}{\eta_{V_B}} - E_{V_B,d,t} + E_{V_B,a,t}, \forall t, \tag{3.38}
\]

Considering the above, constraint 3.37 is included to ensure that the actual energy content will not exceed the bounds while up-regulation reserve provision through de-charging.

Figure 3.3: Energy content bounds

Figure 3.4: Energy content bounds in V2G operation mode
4 Aggregated-level solution

After an essential background on the problem formulation has been provided in Chapter 3, the current chapter is focused on the methodology used to solve the problem, in which PEVs are modeled at an aggregated level. The formulated optimization problem is essentially a chance-constrained, multi-stage DC optimal power flow problem and is solved through a two-step scenario-based technique suited for stochastic optimization problems. As a further step, a sampling and discarding method is implemented in order to deal with the conservatism of the resulting solution.

4.1 Dealing with the chance constraint

The chance-constrained problem introduced in detail in section 3.6 can be rewritten in a compact form as follows:

$$\begin{align*}
P_1 : \min_{x} J(x) \\
\text{subject to:} \\
A_dx + c_d \geq 0 \\
\mathbb{P}(\delta \in \Delta : A_s(\delta)x + c_s(\delta) \geq 0) \geq 1 - \epsilon,
\end{align*}$$

where $x$ denotes the vector of decision variables presented in Eq. 3.14, inequality 4.2 represents the compact form of deterministic constraints of the OPF problem as a function of the decision variables and inequality 4.2 stands for the compact form of chance constraints with respect to the uncertain parameter $\delta = P_{m,t} = P_w - P_{w}^f \in \Delta$, i.e., the wind power forecast error. The objective function of our problem is quadratic in $x$ and $A_d, c_d, A_s, c_s$ are of appropriate dimension.

The objective is to provide a solution satisfying the probabilistic constraints for all realizations of the uncertainty with probability at least $1 - \epsilon$. In order to achieve that without assuming any specific structure for $\mathbb{P}$, i.e., the underlying distribution of the uncertainty $\delta$, we need to transform the chance-constrained problem $P_1$ into a tractable formulation which will be equivalent in some sense. Therefore, we follow a two-step randomized approach, which was inspired by the so-called scenario approach.
4.1.1 Scenario-approach method

Before presenting the 2-stage randomized technique used to solve the chance-constrained problem $P_1$, a small introduction is given on the scenario approach, on which our approach to the problem solution was based. The scenario approach was introduced in [9], [8]. Its main idea was to consider only a finite of scenarios of the uncertain parameters in a stochastic optimization problem and then to solve the problem where only hard constraints are considered, one for each extracted instance (scenario) of the uncertainty. Indeed, there is a lower bound for the number of scenarios extracted in order to obtain some pre-defined probabilistic guarantees for our problem, which was introduced in [11]. In this paper, this lower bound for $N$ generated scenarios is given by the following inequality:

$$N \geq \frac{1}{\epsilon} \frac{e}{\epsilon e - 1} \ln \left(\frac{1}{\beta} + N_w - 1\right), \quad (4.2)$$

where $N_w$ is the number of decision variables and $\epsilon \in (0,1)$, $\beta \in (0,1)$ are design variables related to the desired probabilistic guarantees of our solution, representing the maximum violation of the chance constraints and confidence level of the solution respectively. The scenario approach works under the assumption of convexity with respect to the decision variables, which is satisfied in our case. However, the number of extracted scenarios grows linearly with the number of optimization variables, which hampers the applicability of this approach to large-scale systems, such as the one in our case. To overcome this difficulty, we take advantage of the results presented in [24] and exploited in [21] for a relevant unit-commitment and reserve scheduling OPF problem. The latter results refer to a two-step scenario-based procedure, which is explained in the following section.

4.1.2 Two-stage scenario-based method

1st step: Let $B(p) = \times_{j=1}^{N_t} [p_j^{\text{min}}, p_j^{\text{max}}]$ be a hyper-rectangle of dimension $N_t$, where $N_t = 24$ in our case, as the problem is a day-ahead multi-stage problem with 24 hourly steps. This hyper-rectangle can be parameterized by $p = (p^{\text{min}}, p^{\text{max}}) \in \mathbb{R}^{2N_t}$, where $p^{\text{min}} = (p_1^{\text{min}}, \ldots, p_{N_t}^{\text{min}}) \in \mathbb{R}^{N_t}$ and $p^{\text{max}} = (p_1^{\text{max}}, \ldots, p_{N_t}^{\text{max}}) \in \mathbb{R}^{N_t}$. Instead of directly attempting to solve problem $P_1$, given in Eq. 4.1, we follow an intermediate step in which we solve the following chance-constrained, convex by construction problem:

$$P_2 : \min_{p \in \mathbb{R}^{2N_t}} (p_i^{\text{max}} - p_i^{\text{min}}) \quad (4.3)$$

subject to:

$$\mathbb{P}(\delta \in \Delta : \delta_i \in [p_i^{\text{min}}, p_i^{\text{max}}], \forall i = 1 \ldots N_t) \geq 1 - \epsilon.$$
28 4.1. Dealing with the chance constraint

In this problem, by minimizing the sum of the interval lengths containing every element of the uncertainty $\delta_i$ we parameterize the hyper-rectangle such that the hyper-rectangle $B(p)$ encloses the uncertainty $\delta$ with probability at least $1 - \epsilon$. Indeed, the reason for choosing to minimize the sum of the interval lengths instead of directly the volume of $B(p)$ is that the latter would lead to a non-convex optimization problem. In order to solve problem $P_2$, we exploit the aforementioned standard scenario approach, in which the number of required extracted scenarios is given by formula 4.2 with $N_w = 2N_t = 48$. The scenario program corresponding to problem 4.3 is given by:

$$P_3 : \min_{p \in \mathbb{R}^{2N_t}} (p_i^{\max} - p_i^{\min})$$

subject to:

$$\delta^{(k)}_i \in [p_i^{\min}, p_i^{\max}], \forall i = 1...N_t, \forall k = 1...N_w.$$ 

Following [8], with confidence at least $1 - \beta$, the optimal solution $p_3$ of $P_3$ is feasible for the chance-constrained problem $P_2$, where $\epsilon$ denotes the violation parameter, which along with $\beta$ are the required design parameters for solving $P_2$. As an illustrative example, Fig. 4.1 shows a number of scenarios which have been extracted to solve the simple problem of bounding an uncertainty vector $\delta$ in 2 dimensions, where $B(p)$ is simply a box.

![Figure 4.1: Illustrative example of bounding $\delta$ in 2 dimensions](image)

2nd step: After having determined the hyper-rectangle from the 1st step, the next step is to solve the following robust counterpart of our initial chance-constrained program $P_1$ in Eq. 4.1:

$$P_4 : \min_x J(x)$$

subject to:

$$A_d x + c_d \geq 0$$

$$A_s(\delta)x + c_s(\delta) \geq 0, \forall \delta \in B(p_3) \cap \Delta$$
Problem $P_4$ implies that the constraints have to be satisfied for all possible realizations of the uncertainty $\delta$ within the hyper-rectangle $B(p_3)$ and is not a randomized problem but rather a robust quadratic problem. Indeed, as shown in Proposition 1 of [8], the optimal solution of $P_4$ is feasible for the initial chance-constrained problem $P_1$. Fig. 4.2 illustrates the reasoning behind problem $P_4$ in the simpler case of 2 dimensions, where the constraints need to be satisfied for all scenarios within the green box.

![Illustrative example of problem’s $P_4$ constraints in 2 dimensions](image)

**Figure 4.2: Illustrative example of problem’s $P_4$ constraints in 2 dimensions**

### 4.1.3 Robust reformulation

The methodology outlined in the previous section, i.e., the two-step scenario-based procedure, requires solving problem $P_2$ in Eq. 4.3 by extracting $N_w$ wind power forecast errors and subsequently, solving the robust problem $P_4$ in 4.5. This procedure is tractable as long as $P_4$ is tractable. In order to address this issue, we exploit the approach of [3], [4], which was presented in [21]. In specific, according to the aforementioned work, the condition under which $P_4$ is tractable is that the constraint functions are concave and homogeneous with respect to the uncertainty vector. Indeed, by introducing some additional decision variables and constraints, program $P_4$ can be solved without requiring to enforce the constraints for all possible $2^{N_t}$ vertices on the hyper-rectangle $B(p_3)$ computed in the 1st step. Note that requirement for homogeneity is satisfied in our case but the elements of $A_s$ might not be concave with respect to the uncertainty $\delta$ due to Eq. 3.26 in our formulation. Nonetheless, the structure of the optimization program allows us to tackle this problem through the following procedure: We split the robust set of constraints $A_s(\delta)x + c_s(\delta) \geq 0$ in two sets, which is theoretically sound since the constraints of each stage depend only on the uncertainty elements corresponding to that stage, while the constraints coupling different stages $1, ..., N_t$ are deterministic. Instead of solving the problem for the hyper-rectangle $B(p_3)$ which was previously computed, when splitting the constraints, the first of the two resulting sets refers to those which should be satisfied for all the values of the uncertainty...
vectors $\delta$ with positive elements only, while the second set to those which should be satisfied for $\delta$ vectors with negative elements only. In this way, we have two separate sets of robust constraints, which are now concave and homogeneous.

Similar to the parametrization vector $p$, let $p^+$, $p^-$ be two parametrization vectors corresponding to the hyper-rectangular regions $B^+$, $B^-$ respectively. In order to deal with a set of robust linear constraints, such as the ones appearing in problem $P_4$, we follow the work in [2], whose authors proposed a methodology to replace the robust constraints by a list of linear constraints. To avoid complicating the notation of the followed methodology, in the following we show a single set of robust constraints. However, this set of constraints in problem $P_4$ should be replaced by two sets of robust constraints which correspond to the new hyper-rectangles $B^+$, $B^-$, based on the previous discussion. In other words, the procedure presented in the sequel should be applied for the two aforementioned parametrization vectors $p^+$, $p^-$ rather than for $p$ solely.

Consider $\Delta \in \mathbb{R}^{N_a}$ and let $e_j \in \mathbb{R}^{N_a}$ be a unit basis vector whose j-th element is "1" for all $j = 1,...,N_t$ and $p^0 = 0.5(p^{\text{min}} + p^{\text{max}}) \in \mathbb{R}^{N_n}$ be a vector whose elements are the middle points of each interval $[p^{\text{min}}, p^{\text{max}}]$, for all $j = 1,...,N_t$. In addition, let $n_r$ denote the number of rows of $A_4$ in (4.5) with $A_{s,i}$ corresponding to each row and $Y \in \mathbb{R}^{N_a \times N_s}$, $Q \in \mathbb{R}^{N_s \times N_r}$ represent matrices of auxiliary optimization variables. At this point, we can proceed to the definition of an optimization problem which is essentially, a tractable reformulation of problem $P_4$ in Eq. 4.5:

$$P_5 : \min_{x,Y,Q} J(x)$$

subject to:

$$A_d x + c_d \geq 0 \quad (4.7)$$

$$A^*_{s,i}(e_j e_j^T(p^{\text{max}} - p^0))x + c^*_{s,i}(e_j e_j^T(p^{\text{max}} - p^0)) \geq q_{ij}, \forall i = 1...N_r, j = 1...N_t \quad (4.8)$$

$$A^*_{s,i}(e_j e_j^T(p^{\text{min}} - p^0))x + c^*_{s,i}(e_j e_j^T(p^{\text{min}} - p^0)) \geq q_{ij}, \forall i = 1...N_r, j = 1...N_t \quad (4.9)$$

$$\sum_{t=1}^{T} q_{ij} \geq y_{ij} \quad (4.10)$$

$$A_s(p^0) + c_s(p^0) + y \geq 0 \quad (4.11)$$

where $A^*_{s,i}$ and $c^*_{s,i}$ contain only the elements of $A_{s,i}$ and $c_{s,i}$ which depend directly from the uncertainty vector $\delta$. After having stated the tractable reformulation of $P_4$, we provide an interpretation for it. Aiming at transforming problem $P_4$, we write its constraints as $A_s(p^0 + \Delta P) + c_s(p^0 + \Delta P) \geq 0$ for all $\Delta P$ with $\Delta P_j \in [p_j^0 - p_j^{\text{min}}, p_j^{\text{max}} - p_j^0]$ for $j = 1...N_t$. Under the assumption of convexity and homogeneity of $A_s$ and $c_s$, which only holds when we distinguish the hyper-rectangle $B$ in $B^+$ and $B^-$, the following can be written:

$$A_s(p^0 + \Delta P) \geq A_s(p^0) + c_s \Delta P$$

$$c_s(p^0 + \Delta P) \geq c_s(p^0) + c_s \Delta P$$
Therefore, it is sufficient to show that
\[
A_s p^0 + A_s \Delta P + c_s p^0 + c_s \Delta P \geq 0 \tag{4.12}
\]
for all admissible \(\Delta P\). In order to show that, we need to bound the terms \(A_s \Delta P + c_s \Delta P\). Indeed, the objective of Eq. 4.8 and 4.9 is to bound these terms for the worst-case values of \(\Delta P\), i.e., the worst-case perturbations vectors \(e_j e_j^T (p^{\text{max}} - p^0), e_j e_j^T (p^{\text{min}} - p^0) \in \mathbb{R}^{N_t}\). It should be noted that the aforementioned perturbation vectors have all their elements zero, except for their \(j\)-th position which contains the maximum (and minimum respectively) deviation of the corresponding element from the middle point \(p^0_j\). For each of the constraints \(i = 1...N_r\), Eq. 4.8 and 4.9 impose a bound \(q_{ij}\) on the maximum and minimum worst-case values, while by letting \(y_i\) for \(i = 1...N_r\) it holds that \(A_{s,i} \Delta P + c_{s,i} \Delta P \geq \sum_{j=1}^{N_t} q_{ij} \geq y_i\) for the worst-case superposition of perturbation vectors. The last inequality implies that \(A_s \Delta P + c_s \Delta P \geq Y\) and along with Eq. 4.12 they justify the appearance of constraint 4.11. In this way, the robust problem \(P_4\) is re-formulated into \(P_5\), which is a tractable multi-stage quadratic optimization problem, with \(2(N_t + 1)N_r\) additional optimization variables and \(2(2N_t + 1)N_r\) additional constraints. It should be noted that the factor 2 accounts for the fact that the robust constraints of \(P_4\) are split into sets for \(B^+\) and \(B^-\).

4.2 Simulation results

In the current section, we provide simulation results from the application of the technique discussed in Section 4.1 to the problem of generation dispatch, reserve scheduling and real-time reserve deployment for the test-case 30-bus IEEE power network, as explained in Section 3. Indeed, the simulation results illustrate the efficiency of the algorithm in terms of the probabilistic guarantees provided, which will be discussed furtherly.

As it has been explained in Chapter 3, two different operation modes have been considered regarding reserve provision by PEVs. In particular, the simulation results are distinguished in two operation modes: a) Reserve provision from spinning generators and PEVs without V2G functionality and b) Reserve provision from spinning generators and PEVs with V2G.

The objective of the problem in both operation modes is to minimize the total costs, which include the generation and reserve costs that the TSO is required to pay in advance so as to contract the power generators and reserve providers, i.e., spinning generators and PEVs, for the following day. The numerical values for generation as well as the reserve costs referring to each generator which appear in Eq. 3.15 and which have been considered in the performed simulations are provided in Table 4.2. In addition, the cost of reserve provision by PEVs has been assumed to be lower than the corresponding cost for generators, implying...
that PEVs would provide cheaper reserves than generators, so that their involvement in the reserve market is encouraged. The problem could be also seen from a power market perspective, in which the PEV cost values would not be exogenous as in this case but would rather depend on the operation of the market as well as on the economic incentives given to PEV owners for participating in the service. Nonetheless, this is out of the scope of the present thesis. All optimization problems were solved using the solver CPLEX via the MATLAB interface YALMIP [22].

<table>
<thead>
<tr>
<th>Unit</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_{up}$</th>
<th>$c_{down}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generator 1</td>
<td>2</td>
<td>0.38</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Generator 2</td>
<td>4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Generator 3</td>
<td>4</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Generator 4</td>
<td>4</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Generator 5</td>
<td>4</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Generator 6</td>
<td>4</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>PEV node</td>
<td>-</td>
<td>-</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 4.1: Generation ($c_1, c_2$), up ($c_{up}$) and down ($c_{down}$) reserve costs for each generation unit and PEV nodes (×1000$/MW)

4.2.1 PEV reserve provision without V2G

To begin with, we consider the case where reserves can be provided by spinning generators and $N_{tech} = 1000$ PEVs without V2G functionality. The problem is initially a stochastic optimization problem in the form of Eq. 4.1. As already discussed, in order to solve the problem we generate a number of $N_s$ scenarios, based on Eq. 4.2, in which the chosen design parameters are $\epsilon = 10\%$ representing the maximum percentage violation of constraints and $\beta = 0.1\%$ denoting the confidence of our solution.

Fig. 4.3 presents the results in terms of PEV charging power and energy content of virtual battery for node 17 where a time-variant number of PEVs are connected throughout a day. The reason why the specific node was chosen to be presented is that it shows interesting behaviour. However, similar results can be observed in other nodes in the system to which PEV are connected. In specific, Fig. 4.3(a) shows the PEV charging power of the specific node in magenta colour, which is scheduled for the case where the wind power forecast error is zero, i.e., no reserve power is provided by the PEVs. Under the same assumption, the evolution of energy content of the virtual battery of the aggregated PEVs at node 17 is presented in Fig. 4.3(b) with magenta colour as well. In both figures, the scheduled daily patterns are within the upper and lower bounds of the corresponding quantities, i.e., $P_{min}, P_{max}$ for the charging power and $E_{vb,min}, E_{vb,max}$ for the energy content.
respectively. It should be noted that the explanation of the latter bounds, i.e.,
virtual energy content bounds, was provided in Chapter 3.6 based on Fig. 3.3.
The probabilistic performance of the algorithm is illustrated by carrying out
Monte Carlo simulations for $N = 1000$ different realizations of wind power forecast error, which were generated through the Markov chain model described in Section 3.5. In particular, for each of the 1000 realizations, the daily amount of spinning and PEV reserves was computed based on the real-time reserve deployment strategy which was provided by the OPF result, as explained in Section 3.4. Indeed, Fig. 4.3(c) and Fig. 4.3(d) present the resulting trajectories of charging power and virtual battery energy content respectively, for the aforementioned 1000 scenarios of wind power forecast error. The result of the Monte Carlo test was $\epsilon = 0.4\%$ and was computed as the empirical probability of constraint violation. In other words, the evaluation of the OPF outcome resulted in violation of constraints for only 4 scenarios out of 1000. An example of constraint violation can be seen in Fig. 4.3(d) where the charging power of PEVs in load 17, for two specific realizations of wind power forecast error exceeds the upper bound.

Furthermore, Fig. 4.4 illustrates the performance of the network entities in presence of a specific wind power forecast error. In detail, in Fig. 4.4(a), the wind power forecast is denoted with the red dashed line, whilst the red plain line represents the actual realization of wind power generation for a certain case extracted from the Markov Chain model, which is obviously higher than the forecasted pattern. In the same figure, the scheduled evolution for the total charging power in the system, i.e., the sum of the reference load and PEV charging load, and for power production from spinning generation are represented by the dashed green and blue lines respectively, whereas the plain lines of same colour correspond to the actual response of PEVs and spinning generators under the particular wind power forecast error. As it has been explained, in presence of a forecast error, both PEV virtual batteries and spinning generators adjust their scheduled consumption and generation respectively in order to provide the required reserves for balancing the network. Fig. 4.4(b) specifically shows the behaviour of PEV charging power under reserve provision, since PEV charging power cannot be easily distinguished in Fig. 4.4(a). Here, it can be observed that because of the positive wind power forecast error, shown in Fig. 4.4(a), PEVs increase their charging power compared to the scheduled evolution, providing in this way down spinning reserves to the network. On the contrary, spinning generators provide down spinning reserves by decreasing their power production. Interesting results are illustrated in Fig. 4.4(c) in terms of the distribution of reserves in the event of a wind power forecast error throughout a daily horizon. In specific, the blue line indicates the error which needs to be compensated for by the reserve providers, i.e., spinning generators and PEVs. The total reserves provided are illustrated with the black line, which are the sum of $R_{\text{pev}}$ and $R_{\text{gen}}$ denoted with red and green colour respectively. It should be observed that the total reserves (black) and the wind power forecast error (blue) are indeed opposite, resulting in zero
power mismatch in the network. Also, it should be noted that PEV reserves (red) account for a part of the total reserves (black), not very significant though in the specific simulation, as most of the reserves are spinning (green).

Figure 4.3: Results for 17th node, 1000 PEVs without V2G

The impact of the number of PEVs in the network, when these participate in the reserve provision scheme, is illustrated in the following. Indeed, the results presented in Fig. 4.3, 4.6 show the case of $N_{veh} = 10000$ PEVs in the network. Similarly to Fig. 4.3(c) and Fig. 4.3(d), Fig. 4.5(c) and Fig. 4.5(d) illustrate the scheduled patterns for charging power and virtual battery energy content, combined with the Monte Carlo simulation results. In the specific case, the empirical mean of constraint violation was 0.7%, some of which violations can be observed in the charging power results of Fig. 4.5(c). The interesting observation is this different case is the system behaviour under a certain wind power forecast error, shown in Fig. 4.6. In fact, the positive error presented in Fig. 4.6(c) (blue), is fully compensated by the total amount reserves (black) as previously, but the contribution of PEV reserves (red) is apparently higher than in Fig. 4.4(c) reducing the need for more expensive spinning reserves.
Chapter 4. Aggregated-level solution

(a) System-wise behaviour

(b) Scheduled PEV charging power and reserve provision

(c) Distribution of total reserves for compensation of the wind power forecast error $P_m$

Figure 4.4: System behaviour at a potential wind power forecast error scenario for 1000 PEVs without V2G
4.2. Simulation results

Figure 4.5: Results for 17th node, 10000 PEVs without V2G
Chapter 4. Aggregated-level solution

(a) System-wise behaviour

(b) Scheduled PEV charging power and reserve provision

(c) Distribution of total reserves for compensation of the wind power forecast error $P_m$

Figure 4.6: System behaviour at a potential wind power forecast error scenario for 10000 PEVs without V2G
4.2.2 PEV reserve provision with V2G

After having presented the results of the OPF problem without V2G functionality, this section is focused on the V2G case, in which PEVs can provide up spinning reserves, not only by decreasing their charging power but by de-charging their battery as well. The results that follow refer to the the same node in the network as previously presented, i.e., node 17. However, similar results can be seen at the other nodes to which PEVs are connected. Regarding the design variables required for the solution of the probabilistic problem, these are selected as previously $\epsilon = 10\%$ and $\beta = 0.1\%$.

First, we present the results for the case of $N_{veh} = 1000$ PEVs participating in the reserve provision scheme. Fig. 4.7(a) illustrates the scheduled PEV charging power whereas the nodal response for 1000 wind power forecast error realizations, extracted from the earlier discussed Markov-chain model, is provided in Fig. 4.7(c). Fig. 4.7(b) illustrates the scheduled profile of virtual battery energy content while Fig. 4.7(d) shows the corresponding 1000 Monte Carlo results. Comparing Fig. 4.7 to Fig. 4.10, the main difference lies in the bounds of the two quantities, i.e., charging power and energy content of virtual battery. In specific, as it was explained in section 3.6, in V2G mode, the lower bound for PEV charging power is nonzero and in specific, it is the opposite of the maximum connection power. In addition, the bounds appearing in Fig. 4.7(b) can be understood by the explanation provided in the aforementioned section combined with Fig. 3.4. The plain blue and red lines represent the upper and lower bounds of energy content which should not be exceeded in case of a perfect wind power forecast. On the other hand, the dashed lines of the same colours denote the energy content bounds under V2G reserve provision, which must be satisfied for all times in presence of a forecast error, with a maximum probability of violation equal to the design variable $\epsilon$. Indeed, for the specific Monte Carlo simulation, the empirical mean of constraint violation was computed as 0.7%, some of which violations can be observed in the charging power results of Fig. 4.7(c). Fig. 4.8 follows to describe the behaviour of the system components, spinning generators and PEVs, in the presence of a specific wind power forecast error. In particular, Fig. 4.8(a) shows that a negative forecast error (red lines) leads to up-spinning reserves (blue) combined with up-regulation reserves provided by PEVs. This is achieved through either decreasing PEV charging power or de-charging, as the red line of Fig. 4.8(b) shows. Finally, the V2G contribution to reserve provision is highlighted in Fig. 4.8(c) in which we can see the sum of reserves (black) compensating fully the wind power forecast error (blue) as well as the distribution of reserves in spinning (green) and PEV (red) ones. Apparently, PEV reserves account for a significant part of the total reserves due to the additional operational flexibility allowed for by the V2G functionality.
Chapter 4. Aggregated-level solution

(a) Scheduled charging power

(b) Scheduled VB energy content

(c) Charging power under reserve provision

(d) VB energy content under reserve provision

Figure 4.7: Results for 17th node, 1000 PEVs with V2G
4.2. Simulation results

(a) System-wise behaviour

(b) Scheduled PEV charging power and reserve provision

(c) Distribution of total reserves for compensation of the wind power forecast error $P^w_m$

Figure 4.8: System behaviour at a potential wind power forecast error scenario for 1000 PEVs with V2G
Next, the results presented in Fig. 4.9 show the case of $N_{veh} = 10000$ PEVs in the network in V2G operation, while Fig. 4.10 illustrates a potential system response in presence of a certain wind power forecast error realization. In detail, it is interesting to observe that the negative forecast error (red) shown in Fig. 4.10(a) is compensated for by the total reserves (black) presented in Fig. 4.10(c), which are mainly provided by PEVs, as spinning reserves (green) are significantly lower than PEV ones (red). In fact, the response of PEVs connected to node 17 is to decrease charging or discharge their battery, as shown in Fig. 4.9(c).

Finally, the comparative OPF and Monte Carlo results for three operation modes and $N_{veh} = 1000$ PEVs are presented in Table 4.2.2 in terms of both system costs, represented by the resulting objective value of the problem and Monte Carlo simulation result, i.e., empirical mean of violations of the probabilistic constraints. One can observe that total costs decrease with the introduction of PEVs in the reserve provision scheme, with a more significant decrease appearing in case PEVs provide reserves in V2G mode. The generation costs are almost
4.2. Simulation results

(a) System-wise behaviour

(b) Scheduled PEV charging power and reserve provision

(c) Distribution of total reserves for compensation of the wind power forecast error $P_{m}$

Figure 4.10: System behaviour at a potential wind power forecast error scenario for 10000 PEVs with V2G
not affected, which was expected as power is produced by generators only. As far as the Monte Carlo results are concerned, the violation percentages of all three simulations is lower than the design variable chosen for the probabilistic guarantees of the solution, which was $\epsilon = 10\%$.

<table>
<thead>
<tr>
<th>Operation mode</th>
<th>Total costs</th>
<th>Generation costs</th>
<th>Reserve costs</th>
<th>Violation</th>
</tr>
</thead>
<tbody>
<tr>
<td>No PEV res.</td>
<td>27654</td>
<td>26867</td>
<td>786</td>
<td>0.5</td>
</tr>
<tr>
<td>PEV res. without V2G</td>
<td>27634</td>
<td>26868</td>
<td>766</td>
<td>0.4</td>
</tr>
<tr>
<td>PEV res. with V2G</td>
<td>27512</td>
<td>26866</td>
<td>645</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 4.2: Comparison of results for three operation modes, in terms of cost (×1000$/MW) and percentage of probabilistic constraints violation

### 4.3 Sampling and discarding method

In this section, we present the so-called sampling and discarding method, as a subsequent development of the scenario approach presented in Section 4.1, which achieves better results in terms of conservatism of the OPF solution. After the principle of the technique has been provided, some simulation results follow, which illustrate the efficiency of the method.

#### 4.3.1 Motivation and method

The motivation behind using the sampling and discarding method is the fact that Monte Carlo simulations performed in the previous section resulted in actual percentages of constraint violations which were significantly lower than our design variable $\epsilon = 10\%$. This can be verified from Table 4.2. Nonetheless, this observation is not surprising since the scenario approach introduced in Section 4.1 provides feasibility-type probabilistic guarantees but not optimality ones. In other words, the scenario approach, based in formula (1.2) gives a conservative number of required scenarios, given the desired maximum percentage of constraint violation $\epsilon$. To achieve better optimality-type results, we incorporate the results of [10], [9] when using the scenario approach as explained in subsection 4.1.2, i.e., to determine the minimum volume hyper-rectangle $B$ enclosing the uncertainty $\delta$ with probability at least $1 - \epsilon$.

In specific, given an $N'$ number of scenarios, we seek to discard $r$ number of them, which could for instance correspond to outliers, according to some rule. The hyper-rectangle is then constructed by using $N' - r$ scenarios. Therefore, the constructed hyper-rectangle $B$ will have a lower volume, leading to a less conservative solution than the one obtained through the methodology described in Section 4.1. However, in order to be able to discard $r$ scenarios, the initial
Sampling and discarding method

The number of extracted scenarios is higher than that of Eq. 4.2 and based on [9] it is given by the following formula:

\[
N_r = \frac{2}{\epsilon} \ln\left(\frac{1}{\beta} + 2(N_x + r - 1)\right).
\]

According to [10], the actual way in which constraints are selected is not relevant. The rule that was selected for our simulation study is a greedy approach, with the following steps:

1) Generate \(N_r\) scenarios according to Eq. 4.13.

2) Find the minimum volume 24-dimensional hyper-rectangle including the \(N_r\) scenarios.

3) Identify samples lying on the facets of the hyper-rectangle.

4) Sequentially remove the samples lying on facet \(i\) \((i=1\ldots24)\) and solve the robust problem \(P'\) with new bounds.

5) Remove the scenarios whose removal corresponded to the highest reduction in the objective value of problem \(P'\).

6) Go back to step 2.

In other words, the above methodology aims at reducing the conservatism of the previous solution by sequentially constructing a smaller volume hyper-rectangle to bound the uncertainty \(\Delta\).

4.3.2 Simulation results

After having introduced the reader to the principles of the utilized methodology for sampling and discarding, we provide simulation results from the application of the technique as a way to reduce conservatism of the OPF solution obtained in Section 4.2. As explained previously, the goal of the specific technique in the current simulation study is to generate \(N_r\) scenarios to initially build a hyper-rectangle \(B\) and iteratively discard scenarios from its facets leading to hyper-rectangles smaller and smaller in volume. This procedure is illustrated in Fig. 4.11, which shows the evolution of the upper and lower bounds of the 24-dimensional rectangle, i.e., the bounding box of the uncertainty \(\Delta\), at each iteration of the method. For our study, we chose to discard 80 samples-scenarios of the uncertainty, requiring 14 iterations to be completed, which are presented in Fig. 4.11(a)-4.11(n). It can be observed that the box volume is reduced as the number of completed iterations increases, since the upper bound is lowered throughout the procedure. In fact, the size of the box decreases only from the upper bound side, indicating that the scenarios whose removal leads to the maximum reduction of the objective value of problem \(P'\) are all located at the corresponding facets of the hyper-rectangle. The evolution of the volume of the hyper-rectangle is also presented in Fig. 4.12(a). The first point, \(i = 1\), indicates the value of the quantity without applying the sampling and discarding technique while the second, \(i = 2\), indicates the beginning of the technique at which the volume is
higher than without its application. The latter observation is reasonable, as the sampling and discarding technique initially requires the extraction of a higher number of scenarios $N_r$, based on Eq. 4.13. However, one can observe that after 5 iterations, the box volume is already lower than before applying the technique, indicating a less conservative solution. This is also verified by the evolution of the objective value of the problem, presented in Fig. 4.12(b), which gradually decreases, finally reaching a lower level than before the method’s application after 5 iterations. Furthermore, the reduction of conservatism can be noticed also in terms of actual percentage of violations constraints $\epsilon$, which gradually increases, moving towards the design value of the parameter $\epsilon = 10\%$, as illustrated in Fig. 4.12(c).
4.3. Sampling and discarding method

Figure 4.11: Evolution of bounds of 24-dimensional hyper-rectangle during sampling and discarding method
(a) Volume of hyper-rectangle before and during sampling and discarding iterations

(b) Objective value $J$ before and during sampling and discarding iterations

(c) Empirical probability of constraint isolation $\epsilon$ before and during sampling and discarding iterations

Figure 4.12: Results of sampling and discarding method
5 Individual-level problem solution

After having presented the approach which was adopted for the solution of the OPF at the level of aggregated PEVs, the final step is to allocate the results, to each individual vehicle providing reserves when connected to the power distribution network. Towards this direction, this chapter presents two ways for the disaggregation of charging power, which can be performed day-ahead, as well as a technique for allocating the aggregated reserves of each node to the individual nodes connected, according to the wind power forecast error appearing in real-time. Moving from the aggregated to the individual level, it is essential to assign individual results to vehicles such that both the sum of the scheduled charging power and reserves as well as the energy content of each PEV are within individual bounds. The latter refer to the following bounds: a) minimum state of charge, b) battery capacity, c) minimum charging power and d) maximum charging-decharging power of each PEV.

5.1 Day-ahead disaggregation of charging power
5.1.1 Single optimization method

To begin with, a single optimization technique can be used for the day-ahead disaggregation of charging power results. The goal here is to minimize the disaggregation error in terms of the charging power, i.e., the deviation between the aggregated charging power at the network nodes, provided by the solution of our OPF problem, and the aggregated charging power resulting from the summation of the individual charging power of PEVs connected to each node. The optimization problem is the following:

\[
\min_{P_{Vk,t}, E_{Vk,0}} \sum_{t=1}^{N_t} \sum_{j=1}^{n_{PL}} |e_{Pj,t}|, \tag{5.1}
\]

where the decision variables \(P_{Vk,t}\) and \(E_{Vk,0}\) represent the charging power of each PEV at all times \(t = 1...N_t\) and the initial energy content of each PEV battery respectively. Also, its hold that \(e_{Pj,t} = P_{Le,j,t} - \sum_{k \in \Phi_{Lj,t}} P_{Vk,t}\) and \(P_{Le,j,t}\) is the aggregated-level charging power result which is already known and \(\Phi_{Lj,t}\) is the set of vehicles connected to node \(j\) at time \(t\). The constraints of the OPF problem are the following:
Chapter 5. Individual-level problem solution

1) Individual charging power constraints:

\[ P_{V,k,min,t} \leq P_{V,k,t} \leq P_{V,k,max,t} \tag{5.2} \]

where \( P_{V,k,min,t} \) is the minimum charging power of the vehicle, which in this case is equal to zero, as the PEV cannot be scheduled to decharge and \( P_{V,k,max,t} \) is the connection power to the grid.

2) Individual energy content constraints:

\[ E_{V,k,min,t} \leq E_{V,k,t} \leq E_{V,k,max,t} \tag{5.3} \]

where \( E_{V,k,max,t} \) is essentially the battery capacity of the PEV and \( E_{V,k,min,t} \) is the minimum state-of-charge allowed for the vehicle, e.g. 20%.

3) Evolution of the energy content of each PEV:

\[ E_{V,k,t} = E_{V,k,t-1} + P_{V,k,t} \Delta t \eta_{V,k} - E_{cons,k,t} \tag{5.4} \]

4) Constraint on the initial energy content:

\[ E_{V,k,0} = E_{V,k,24} \tag{5.5} \]

5.1.2 Heuristic-sequential method

In addition, an alternative method for disaggregation was followed, based on the work of [34]. For large fleets, the single optimization technique presented in the previous subsection could prove to be very expensive computationally. For this reason, the following heuristic approach is followed in a sequential way. The objective function of the method is given by:

\[ \min_{P_{V,k,t},E_{V,k,0} \in \Omega_{V,k,t}} c_{L_j \in \Omega_{V,k,t}} P_{V,k,t} \tag{5.6} \]

where \( L_j \in \Omega_{V,k,t} \) describes the load node where the vehicle \( k \) is connected to at time \( t \) and \( c_{L_j \in \Omega_{V,k,t}} \) is a weighting factor which penalizes the charging power result, at time \( t \), for vehicle \( k \) and node \( L_j \), depending on the remaining non-dispatched power and the number of non-dispatched vehicles at the particular node. Indeed, the weighting factor is given by the following formula:

\[ c_{L_j,t,k} = -\frac{P_{L_j,t} - \sum_{\Phi_{L_j,t,k}} P_{V,k,t}}{\sum_{\not\in \Phi_{L_j,t,k}} 1} \tag{5.7} \]

where \( \Phi_{L_j,t,k} \) denotes the set of vehicles connected to node \( L_j,t,k \).

The constraints of the method are the following:

1) Individual charging power constraints

\[ P_{V,k,min,t} \leq P_{V,k,t} \leq P_{V,k,max,t} \tag{5.8} \]
where $P_{V,k,min,t}$ is the minimum charging power of the vehicle, which in this case is equal to zero, as the PEV cannot be scheduled to decharge and $P_{V,k,max,t}$ is the connection power to the grid.

2) Individual energy content constraints

$$E_{V,k,min,t} \leq E_{V,k,t} \leq E_{V,k,max,t},$$

(5.9)

with $E_{V,k,max,t}$ denoting the battery capacity of the PEV and $E_{V,k,min,t}$ the minimum state-of-charge allowed for the vehicle, e.g. 20%.

3) Evolution of the energy content of each PEV

$$E_{V,k,t} = E_{V,k,t-1} + P_{V,k,t} \Delta t \eta_{V,k} - E_{cons,k,t},$$

(5.10)

4) Constraint on the initial energy content

$$E_{V,k,0} = E_{V,k,24}.$$  

(5.11)

5.2 Real-time disaggregation of PEV reserves

After having determined the scheduled values for individual PEV charging power $P_{V,k}$, the next step is the allocation of the aggregated amount of reserves to the individual PEVs connected to each network node. This procedure is performed in real-time, since the actual reserves for each node depend on the actual value of wind power, i.e., the realization of the wind power forecast error. As soon as the wind power value is available, the real-time reserve deployment scheme, i.e. the distribution vectors $d_{up}, d_{down}$ given by the solution of the OPF problem, can be used to compute the actual reserves required to ensure the system’s power balance, as well as the distribution of the total reserves into spinning and PEV reserves. When the aggregated amount of reserves is known at each node of the system, this is translated into the amount of reserves that each PEV should provide, which is added to the scheduled charging power computed in the previous step $P_{V,k}$, as presented in Section 5.1.

A way to solve the real-time problem of disaggregating PEV reserves $R_{pev}$ is through a single optimization program, which is similar to the method described in subsection 5.1.1, aiming at minimizing the disaggregation error for the total charging power, composed by the scheduled value $P_{L,c,j,t}$ and the reserves $R_{pev,j,t}$ at each network node for each time step. The objective function of the problem is given by:

$$\min_{R_{V,k,t}} \sum_{t=1}^{N_t} \sum_{j=1}^{n_{PL}} |e_{R_{pev,j,t}}|,$$

(5.12)

where $e_{P,t} = (P_{L,c,j,t} - R_{pev,j,t}) - \sum_{k \in \Phi_{L,j,t}} (P_{V,k,t} - R_{V,k,t})$, $P_{L,c,j,t}$ is the aforementioned aggregated-level result which is known and $\Phi_{L,j,t}$ is the set of vehicles connected to node $j$ at time $t$. 

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The constraints for the problem are the following:
1) Individual charging power constraints

\[ P_{V_k,\text{min},t} \leq P_{V_k,t} - R_{V_k,t} \leq P_{V_k,\text{max},t}, \]  
\( (5.13) \)

where \( P_{V_k,t} \) is given by the previous step explained in section 5.1 and \( P_{V_k,\text{min},t} \) is the minimum charging power of the vehicle, which can be other zero in non-V2G mode or \(-P_{\text{connect}}\) in V2G mode, implying that the vehicle can decharge with maximum power equal to the connection power to the grid.

2) Individual energy content constraints

\[ E_{V_k,\text{min},t} \leq E_{V_k,t} \leq E_{V_k,\text{max},t}, \]  
\( (5.14) \)

with \( E_{V_k,\text{max},t} \) denoting the battery capacity of the PEV and \( E_{V_k,\text{min},t} \) the minimum state-of-charge allowed for the vehicle, e.g. 20%.

3a) Evolution of the energy content of each PEV during the reserve provision interval \( \Delta t \) without V2G power functionality

\[ E_{V_k,t} = E_{V_k,t-1} + (P_{V_k,t} - R_{V_k,t}) \frac{\Delta t}{4} \eta_{V_k} - E_{\text{cons},k,t}, \]  
\( (5.15) \)

in which PEVs can only charge their battery to provide the required reserves, i.e, \( P_{V_k,t} - R_{V_k,t} \geq 0 \).

3b) Evolution of the energy content of each PEV during the reserve provision interval \( \Delta t \) with V2G functionality

\[ E_{V_k,t} = E_{V_k,t-1} + \max(- (P_{V_k,t} - R_{V_k,t}), 0) \frac{\Delta t}{4} \eta_{V_k} + \max(P_{V_k,t} - R_{V_k,t}, 0) \frac{\Delta t}{4} \eta_{V_k} - E_{\text{cons},k,t}, \]  
\( (5.16) \)

in which PEVs either charge or decharge their battery to provide reserves, with each case following a different formula of energy content evolution.

5.3 Simulation results and comments

Firstly, regarding the results of the disaggregation techniques presented in sections 5.1.1 and 5.1.2 these are relatively satisfying, as the single optimization based method results in an error at the order of 2% in terms of charging power, while the corresponding error for the heuristic method is at the order of 5%. Despite the superiority of the technique based on single optimization, in terms of disaggregation error, for large fleets it could well be computationally expensive and even infeasible, pointing to the fact that the heuristic method might be the preferred method to employ in practical applications. Indeed, the disaggregation error was expected, considering that aggregating individual vehicles into virtual batteries leads to reduced modelling precision of PEV individual dynamics.
However, it should be highlighted that due to the previously mentioned non-zero disaggregation error of $P_{L,c}$ for both aforementioned techniques, the actual energy content of the aggregated virtual batteries at the end of each time step could be different than the scheduled value. In fact, the tertiary controller would in reality provide different amount of energy back to the vehicles to compensate for the energy they have provided due to reserves, as explained in the assumption of section 3.4, which would lead to a mismatch between the scheduled energy content level of virtual batteries and the actual ones. This has an immediate impact on the provided probabilistic guarantees, which no longer hold for the individual level, as a zero disaggregation error is essential for the aggregated OPF solution to guarantee the optimal results.

In order to test the efficiency of the disaggregation algorithm, we perform a Monte Carlo simulation for 100 different scenarios of wind power forecast error to compute the potential aggregated-level PEV results, then disaggregate the solution allocating it to individual vehicles and finally, move back to the aggregated level by computing the nodal charging power out of the individual patterns. In this way, it is possible to check the feasibility of the 100 individual solutions and verify that the aggregated charging power remains within the bounds. The results for a specific node are presented in Fig. 5.1. In specific, Fig. 5.1(a) and 5.1(c) refer to the non-V2G mode presenting the disaggregation results for the scheduled charging power and the Monte Carlo results of charging power under reserve provision for the aforementioned 100 wind power realizations respectively. It can be observed that regardless of the scheduled charging power disaggregation error shown in Fig. 5.1(a), Fig. 5.1(c) illustrates that all potential solutions for the total charging power in presence of a forecast error, which are denoted with dashed black line, are feasible solutions of the initial OPF problem, since they remain within the upper and lower power bounds. The corresponding Monte Carlo simulation for the V2G mode yields similar results, as presented by Fig. 5.1(b) and Fig. 5.1(d), in which all 100 trajectories of charging or de-charging power remain within the quantity’s bounds.
(a) Day-ahead disaggregation of charging power without V2G
(b) Day-ahead disaggregation of charging power with V2G
(c) Real-time disaggregation of PEV reserves without V2G
(d) Real-time disaggregation of PEV reserves with V2G

Figure 5.1: Disaggregation results
6 Conclusion and future work

In the present work, a unified stochastic framework was developed for generation dispatch, PEV smart charging and reserve scheduling for power networks with wind power generation. Special focus was given to the formulation of the problem, in which PEVs were aggregated into virtual batteries providing reserves, as well as to the presentation of a tractable method to approach its solution. In specific, the technique that was employed is a two-step scenario approach based method for the solution of chance-constrained optimization problems, combined with a tractable reformulation required to tackle certain theoretical requirements. Furthermore, in order to reduce conservatism of the solution, the so-called sampling and discarding method was employed in combination with the scenario-based technique and its advantages were discussed. As the OPF results were at available at the aggregated level, finally it was essential to discuss potential disaggregation approaches which provide the information at the level of individual vehicles. Simulation results of a 30-bus IEEE power network illustrated the efficiency of the presented methodology, while disaggregation results highlighted the inherent drawbacks of the aggregation step.

Potential future directions of the current work include firstly, a more thorough study of the disaggregation procedure, aiming at minimizing the disaggregation error or incorporating the information of this error into the aggregated-level OPF solution towards the objective of providing probabilistic guarantees at the individual PEV level. Secondly, the impact of the results on the distribution network could be investigated in order to provide the TSO and DNOs with a realistic and tractable methodology.
Bibliography


