Transport experiments in two-dimensional systems with strong spin-orbit interaction

Author(s):
Nichele, Fabrizio

Publication Date:
2014

Permanent Link:
https://doi.org/10.3929/ethz-a-010268586

Rights / License:
In Copyright - Non-Commercial Use Permitted
Transport experiments in two-dimensional systems with strong spin-orbit interaction

A thesis submitted to attain the degree of
Doctor of Sciences of ETH Zurich
(Dr. sc. ETH Zurich)

presented by

Fabrizio Nichele
Laurea Specialistica in Ingegneria Fisica
Politecnico di Milano

born June 4, 1985 in Como, Italy
citizen of Italy

accepted on the recommendation of:

Prof. Dr. Klaus Ensslin, examiner
Prof. Dr. Charles Marcus, co-examiner
Prof. Dr. Thomas Ihn, co-examiner

2014
Abstract

We experimentally investigate low temperature electronic transport phenomena in two-dimensional semiconductors characterized by strong spin-orbit interaction. The subjects of our studies are two-dimensional hole gases in $p$-type GaAs/AlGaAs heterostructures and tunable electron/hole systems in InAs/GaSb double quantum wells. A brief introduction about the physics of these two material systems is given in Chapter 2. Chapter 3 describes the processing techniques used to fabricate the samples studied in the thesis.

Low temperature experiments in GaAs two-dimensional hole gasses reveal an extremely rich physics, where spin-orbit, coherence and hole-hole interaction effects compete with each other. This thesis work describes how spin-orbit interaction affects hole transport in a variety of situations, from diffusive quantum transport to transport in ballistic, coherent or chaotic nanostructures.

In Chapter 4 we focus on diffusive transport in gated Hall bars, where spin-orbit interaction results in peculiar magnetotransport phenomena. We observe and analyze Shubnikov-de Haas oscillations, classical positive magnetoresistance, weak anti-localization and hole-hole interaction corrections. In Chapter 5, from the temperature dependence of the Shubnikov-de Haas oscillations we separately extract the effective masses of the two spin-orbit split subbands as a function of hole density. We measure a very pronounced spin-orbit induced non-parabolicity of the valence band and we show that the hole effective mass varies up to a factor of three for the two angular momentum eigenstates.

In Chapter 6 we characterize high quality quantum point contacts embedded in $p$-type GaAs. Spin-orbit interaction results in a strong anisotropy of the effective Landé $g$-factor: the application of an external magnetic field splits the energy levels according to the relative orientation between electrical current and magnetic field. On top of this effect, we observe crossings or anti-crossings among energy levels according to their spin quantum number and external magnetic field orientation. We reconcile this observation with a spin-orbit Hamiltonian for heavy holes that contains the well know cubic-in-$k$ Rashba term and a recently proposed quadratic-in-$k$ term. The existence of the quadratic term allows to explain the increasing tendency of the in-plane $g$-factor with quantum point contact subband index.

Using three quantum point contacts we form a mesoscopic cavity in $p$-type GaAs.
The cavity size is smaller than the hole elastic mean free path and coherence length, but two orders of magnitude longer than the spin-orbit length. In such a cavity spin rotational symmetry is completely broken, and spin currents can be generated at zero magnetic field via spin-orbit interaction. In Chapter 7, using a recently proposed measurement scheme, we give evidence for the existence of a pure spin current at zero magnetic field. Nor the generation or the detection technique involve the use of ferromagnetic contacts, large magnetic fields or optical spin injection techniques.

Spin-orbit interaction is also expected to leave signatures in the coherent transport of holes in a ring shaped nanostructures. In Chapter 8 we present measurements of highly visible $\hbar/e$ and $\hbar/2e$ oscillations in Aharonov-Bohm rings. Similarly to previous works we observe the presence of beatings in the $\hbar/e$ and $\hbar/2e$ oscillations that have occasionally been interpreted as signature of spin-orbit interaction. We propose an additional interpretation for the beatings that does not involve spin-orbit interaction but the combined effect of Aharonov-Bohm oscillations and conductance fluctuations. The phase of $\hbar/2e$ oscillations as a function of a gate voltage does not show phase jumps, contrary from what is expected from the Aharonov-Casher effect. We discuss the temperature dependence of the beatings in terms of phase breaking and ensemble averaging.

InAs/GaSb double quantum wells are ambipolar systems where a two-dimensional electron gas can coexist in close proximity with a two-dimensional hole gas. When the electron density is approximately equal to the hole density, the two systems can hybridize opening a small energy gap. When the Fermi energy lies in the hybridization gap, the system turns insulating in the bulk and, interestingly, one-dimensional helical states form at the same edges.

In Chapter 9 we characterize InAs/GaSb quantum wells as a function of charge density and magnetic field, analyzing the low-field magnetoconductance and the Shubnikov-de Haas oscillations. We give evidences for the presence of two subbands in InAs for low electron density. For high electron density the effective mass increases, revealing the expected band non-parabolicity of InAs. Close to the charge neutrality point, we measure distinct signatures of the simultaneous presence of electron and hole bands.

In Chapter 10 we treat the behavior of InAs/GaSb QWs in a high perpendicular magnetic field. At the electron-hole crossover, a strong increase in the sample longitudinal resistivity is found in a large perpendicular magnetic field. Concomitantly with a local resistance exceeding the resistance quantum by an order of magnitude, we find a pronounced non-local resistance of similar magnitude. The coexistence of the two effects is reconciled in a model where counter-propagating and highly dissipative quantum Hall edge channels coexist with a weakly conductive bulk.
Riassunto

In questa tesi investighiamo sperimentalmente il trasporto elettronico a bassa temperatura in semiconduttori bidimensionali caratterizzati da un forte effetto spin-orbita. I soggetti delle nostre ricerche sono gas bidimensionali di lacune in eterostrutture drogate \( p \) di GaAs/AlGaAs e doppie quantum well in InAs/GaSb. Una rapida introduzione sulla fisica di questi due materiali è presentata nel Capitolo 2. Il Capitolo 3 descrive le tecniche sperimentali usate per fabbricare i campioni studiati nella tesi. A bassa temperatura, i gas bidimensionali di lacune rivelano una fisica estremamente ricca dove effetti di spin-orbita, coerenza e interazione lacuna-lacuna competono tra loro. Questa tesi descrive come l’effetto spin-orbita influenza il trasporto di lacune in una varietà di situazioni differenti, dal trasporto diffusivo quantistico al trasporto in strutture balistiche, coerenti o caotiche.


Nel capitolo 6 caratterizziamo contatti di punto quantici di elevata qualità in gas bidimensionali di lacune. L’interazione spin-orbita risulta in una forte anisotropia del fattore \( g \) di Landé: l’applicazione di un campo magnetico esterno separa i livelli energetici a seconda dell’orientazione relativa tra la corrente elettrica e il campo magnetico. In aggiunta a questo effetto, osserviamo attraversamenti o attraversamenti evitati tra stati energetici a seconda del loro numero quantico di spin e della direzione del campo magnetico. Giustifichiamo queste osservazioni con una hamiltoniana di spin-orbita per lacune pesanti contenente il noto fattore Rashba cubico e un fattore quadratico aggiuntivo recentemente proposto. L’esistenza del termine quadratico permette di spiegare la tendenza a crescere del fattore \( g \) planare con il numero di sottobande occupate nel contatto quantico.

Usando tre contatti di punto quantico formiamo una cavità mesoscopica in GaAs drogato \( p \). La dimensione della cavità è più piccola del cammino elastico medio di

È stato teorizzato come l’interazione spin-orbita lasci tracce nel trasporto coerente di lacune in una nanostruttura ad anello. Nel Capitolo 8 presentiamo misure di oscillazioni $h/e$ e $h/2e$ altamente visibili in anelli di Aharonov-Bohm. Compatibilmente con studi precedenti, osserviamo la presenza di battimenti nelle oscillazioni $h/e$ e $h/2e$ che, in certe occasioni, “è stata interpretata come effetto dell’interazione spin-orbita. Proponiamo un’interpretazione alternativa per i battimenti che non richiede la presenza di interazione spin-orbita, ma l’effetto combinato delle oscillazioni di Aharonov-Bohm e delle fluttuazioni mesoscopiche della conduttività. La fase delle oscillazioni $h/2e$ in funzione del potenziale di gate è costante, contrariamente da quanto previsto dall’effetto Aharonov-Casher. Discutiamo la dipendenza dei battimenti dalla temperature in termini di perdita di fase e media di insieme.

Le doppie quantum well di InAs/GaSb sono sistemi ambipolari dove gas bidimensionali di elettroni possono coesistere in stretto contatto con gas bidimensionali di lacune. Quando la densità di elettroni è simile alla densità di lacune, i due sistemi possono ibridizzare e portare all’apertura di una piccola gap energetica. Con l’energia di Fermi nella gap di ibridizzazione, il sistema diventa isolante nel volume e, allo stesso tempo, stati elicali monodimensionali si formano ai suoi bordi.

Nel Capitolo 9 caratterizziamo doppie quantum well di InAs/GaSb in funzione della densità di carica e del campo magnetico attraverso l’analisi della magnetoresistenza per campo debole e delle oscillazioni di Shubnikov-de Haas. Per basse densità elettroniche, evidenziamo la presenza di due sottobande nell’InAs. Per alte densità elettroniche, riveliamo un aumento della massa efficace dell’elettrone, come atteso dalla non-parabolicità della banda di conduzione dell’InAs. Nei pressi del punto di carica neutrale, misuriamo chiare indicazioni riguardo la presenza simultanea di bande di elettroni e di lacune.

Nel Capitolo 10 trattiamo il comportamento delle quantum well di InAs/GaSb in un forte campo magnetico perpendicolare. Vicino al punto di neutralità di carica misuriamo un rilevante aumento della resistività. In concomitanza con una resistenza locale superiore per un ordine di grandezza al quanto di resitenza, troviamo una resistenza non-locale di simile entità. L’osservazione simultanea di questi due fenomeni è spiegata in un modello dove stati di bordo propaganti in direzioni opposte e altamente dissipativi coesistono con un volume debolmente conduttivo.
# Contents

## 1 Introduction

1.1 *p*-type GaAs ................................................. 2
1.2 InAs/GaSb double quantum wells ............................... 3

## 2 Basic concepts

2.1 *p*-type GaAs ................................................. 5
    2.1.1 Band structure of GaAs ................................. 5
    2.1.2 2D confinement ......................................... 6
    2.1.3 Spin-orbit interaction .................................. 8
    2.1.4 Spin splitting .......................................... 10
    2.1.5 Many-body interaction ................................... 11
2.2 InAs/GaSb double quantum wells ............................... 12
    2.2.1 Band alignment in InAs/GaSb quantum wells ........... 12
    2.2.2 Topological phase in InAs/GaSb ........................ 14
2.3 Magnetotransport .............................................. 16
    2.3.1 Classical Drude conductivity ............................ 16
    2.3.2 Shubnikov - de Haas effect ............................. 19
    2.3.3 Integer quantum Hall effect ............................. 20
    2.3.4 Weak localization and weak antilocalization ........... 21
    2.3.5 Interaction effects ..................................... 23
2.4 Electronic transport in semiconductor nanostructures ....... 24
    2.4.1 Conductance quantization in a quantum point contact ... 24
    2.4.2 Aharonov-Bohm effect .................................. 25
    2.4.3 Conductance fluctuations in mesoscopic samples ........ 27

## 3 Sample fabrication

3.1 *p*-type GaAs ................................................. 28
    3.1.1 Ohmic contacts for *p*-type GaAs ................. 30
3.1.2 Electron beam lithography and shallow etching          30
3.1.3 Gating of p-type GaAs heterostructures and nanostructures  31
3.2 InAs/GaSb double quantum wells                           32
  3.2.1 Ohmic contacts to InAs/GaSb double quantum wells     34
  3.2.2 Etching of Hall bar structures                       34

4 Magnetotransport in $p$-type GaAs/AlGaAs heterostructures  36
  4.1 Introduction                                            36
  4.2 Magnetoresistance of a 2DHG                             37
  4.3 Beating of the Shubnikov-de Haas oscillations          38
  4.4 Classical positive magnetoresistance in two-subband system  39
  4.5 $p$-type GaAs 2DHG with Si$_3$N$_4$ gate dielectric    42
  4.6 Low density regime                                      45
    4.6.1 Interactions correction                             45
    4.6.2 Weak anti-localization correction                  47
  4.7 Conclusion                                             51

5 Effective masses in $p$-type GaAs two-dimensional hole gases 53
  5.1 Introduction                                            53
  5.2 Measurements and numerical methods                     55
    5.2.1 Method A                                            56
    5.2.2 Method B                                            58
  5.3 Results                                                61
  5.4 Conclusions                                            62

6 Spin-orbit interaction in $p$-type quantum point contacts  63
  6.1 Introduction                                            63
  6.2 Zeeman energy for anisotropic $g$-factor               65
  6.3 Sample fabrication and measurement technique           65
  6.4 QPC 1                                                  67
    6.4.1 Lever arm                                          67
    6.4.2 Zeeman splitting                                   71
    6.4.3 Level crossing and anti-crossing                  73
  6.5 QPC2                                                  77
    6.5.1 Lever arm                                          77
    6.5.2 Zeeman splitting                                   77
    6.5.3 Level crossing and anti-crossing                  77
7 Spin-to-charge conversion in mesoscopic cavities with strong SOI

7.1 Introduction

7.1.1 Mesoscopic spin Hall effect

7.1.2 Spin-to-charge conversion

7.1.3 Theoretical background

7.2 Experimental realization

7.3 Results

7.3.1 Conductance fluctuations

7.3.2 Spin-to-charge conversion measurements

7.3.3 Sample B

7.4 Discussion

7.5 Conclusion

8 Aharonov-Bohm effect in $\rho$-type GaAs rings

8.1 Geometric phases in ring shaped nanostructures

8.1.1 Beating in the Aharonov-Bohm oscillations

8.1.2 Aharonov-Casher effect

8.2 Introduction to the experiment

8.3 Samples

8.4 Experimental results

8.4.1 Magnetoresistance

8.4.2 Gate dependence

8.4.3 Temperature dependence

8.5 Discussion

8.5.1 Phase jumps and beatings

8.5.2 Decoherence and ensemble averaging

8.6 Conclusion

9 Electrical transport in InAs/GaSb double quantum wells

9.1 Introduction

9.2 Basic characterization

9.2.1 Temperature and in-plane field dependence

9.2.2 Low field magnetoresistance and Shubnikov-de Haas oscillations

9.3 Effective mass and quantum scattering times

87

94

99

108

114

116

119

122

123

124

125

127

128

130

133

135

137

139

140

141

143

148
9.4 Conclusion ................................................................. 150

10 InAs/GaSb quantum wells in the quantum Hall regime 151
   10.1 Introduction ......................................................... 151
   10.2 Electrical transport in the quantum Hall regime .............. 152
   10.3 Non local transport measurements ............................... 155
   10.4 Effect of an in-plane magnetic field .............................. 157
   10.5 Discussion ........................................................... 157

Appendices 161
   A Fourier analysis of the Shubnikov-de Haas oscillations .......... 161
      A.1 Numerical treatment ............................................. 161
      A.2 Analytical form ................................................ 167
   B Quantum spin Hall effect ........................................... 168
   C Analytical solution of the resistor network model ............... 170

Publications 174

Bibliography 176

Acknowledgements 186

Curriculum Vitae 188
# Lists of symbols

<table>
<thead>
<tr>
<th>physical constants</th>
<th>explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_B$</td>
<td>Bohr radius</td>
</tr>
<tr>
<td>$e &gt; 0$</td>
<td>elementary charge</td>
</tr>
<tr>
<td>$m_0$</td>
<td>electron mass</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>dielectric permittivity</td>
</tr>
<tr>
<td>$\epsilon_0$</td>
<td>vacuum dielectric constant</td>
</tr>
<tr>
<td>$h = 2\pi\hbar$</td>
<td>Planck's constant</td>
</tr>
<tr>
<td>$k_B$</td>
<td>Boltzmann constant</td>
</tr>
<tr>
<td>$\mu_B$</td>
<td>Bohr magneton</td>
</tr>
<tr>
<td>$c$</td>
<td>speed of light</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Explanation</td>
</tr>
<tr>
<td>--------------</td>
<td>-------------</td>
</tr>
<tr>
<td>2DEG</td>
<td>two-dimensional electron gas</td>
</tr>
<tr>
<td>2DHG</td>
<td>two-dimensional hole gas</td>
</tr>
<tr>
<td>AAS</td>
<td>Altshuler-Aronov-Spivak</td>
</tr>
<tr>
<td>AB</td>
<td>Aharonov-Bohm</td>
</tr>
<tr>
<td>AC</td>
<td>alternating current</td>
</tr>
<tr>
<td>AFM</td>
<td>atomic force microscope</td>
</tr>
<tr>
<td>BIA</td>
<td>Bulk inversion asymmetry</td>
</tr>
<tr>
<td>CNP</td>
<td>charge neutrality point</td>
</tr>
<tr>
<td>DC</td>
<td>direct current</td>
</tr>
<tr>
<td>DOS</td>
<td>density of states</td>
</tr>
<tr>
<td>FFT</td>
<td>fast Fourier transform</td>
</tr>
<tr>
<td>FWHM</td>
<td>full width at half maximum</td>
</tr>
<tr>
<td>HH</td>
<td>heavy-hole</td>
</tr>
<tr>
<td>HHh</td>
<td>heavy-heavy-hole</td>
</tr>
<tr>
<td>HHI</td>
<td>light-heavy-hole</td>
</tr>
<tr>
<td>HLN</td>
<td>Hikami-Larkin-Nagaoka</td>
</tr>
<tr>
<td>ILP</td>
<td>Iordanskii-Lyanda Geller-Pikus</td>
</tr>
<tr>
<td>LH</td>
<td>light-hole</td>
</tr>
<tr>
<td>LL</td>
<td>Landau level</td>
</tr>
<tr>
<td>MBE</td>
<td>molecular beam epitaxy</td>
</tr>
<tr>
<td>QHE</td>
<td>quantum Hall effect</td>
</tr>
<tr>
<td>QPC</td>
<td>quantum point contact</td>
</tr>
<tr>
<td>QSHE</td>
<td>quantum spin Hall effect</td>
</tr>
<tr>
<td>QW</td>
<td>quantum well</td>
</tr>
<tr>
<td>SdH</td>
<td>Shubnikov de Haas</td>
</tr>
<tr>
<td>SIA</td>
<td>structure inversion asymmetry</td>
</tr>
<tr>
<td>SO</td>
<td>split-off</td>
</tr>
<tr>
<td>SOI</td>
<td>spin-orbit interaction</td>
</tr>
<tr>
<td>WAL</td>
<td>weak anti-localization</td>
</tr>
<tr>
<td>Symbol</td>
<td>Explanation</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$L,W$</td>
<td>system size (length, width)</td>
</tr>
<tr>
<td>$x,y,z$</td>
<td>space directions</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature</td>
</tr>
<tr>
<td>$E$</td>
<td>energy</td>
</tr>
<tr>
<td>$k$</td>
<td>wavevector</td>
</tr>
<tr>
<td>$k_\parallel$</td>
<td>in-plane wavevector</td>
</tr>
<tr>
<td>$p$</td>
<td>momentum</td>
</tr>
<tr>
<td>$l$</td>
<td>orbital quantum number</td>
</tr>
<tr>
<td>$J$</td>
<td>total angular momentum quantum number</td>
</tr>
<tr>
<td>$B$</td>
<td>magnetic field</td>
</tr>
<tr>
<td>$E$</td>
<td>electric field</td>
</tr>
<tr>
<td>$A$</td>
<td>vector potential</td>
</tr>
<tr>
<td>$\alpha_g$</td>
<td>gate lever arm</td>
</tr>
<tr>
<td>$E_F$</td>
<td>Fermi energy</td>
</tr>
<tr>
<td>$I$</td>
<td>current</td>
</tr>
<tr>
<td>$j$</td>
<td>current density</td>
</tr>
<tr>
<td>$V$</td>
<td>voltage</td>
</tr>
<tr>
<td>$G$</td>
<td>conductance</td>
</tr>
<tr>
<td>$R$</td>
<td>resistance</td>
</tr>
<tr>
<td>$\rho$</td>
<td>resistivity</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>resistivity at zero magnetic field</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>conductivity</td>
</tr>
<tr>
<td>$v_F$</td>
<td>Fermi velocity</td>
</tr>
<tr>
<td>$k_F$</td>
<td>Fermi wavenumber</td>
</tr>
<tr>
<td>$\lambda_F$</td>
<td>Fermi wavelength</td>
</tr>
<tr>
<td>$l_e$</td>
<td>elastic mean free path</td>
</tr>
<tr>
<td>$l_\phi$</td>
<td>phase coherence length</td>
</tr>
<tr>
<td>$l_{SO}$</td>
<td>spin-orbit length</td>
</tr>
<tr>
<td>$\tau_e$</td>
<td>Drude scattering time</td>
</tr>
<tr>
<td>$\tau_q$</td>
<td>quantum life time</td>
</tr>
<tr>
<td>$m^*$</td>
<td>effective electron mass</td>
</tr>
<tr>
<td>$m_{hh}$</td>
<td>heavy-hole band mass</td>
</tr>
<tr>
<td>$\mu$</td>
<td>electron mobility</td>
</tr>
<tr>
<td>$n$</td>
<td>carrier sheet density or QPC subband index</td>
</tr>
<tr>
<td>$p &lt; 0$</td>
<td>hole sheet density density</td>
</tr>
<tr>
<td>$n_i$</td>
<td>carrier sheet density of subband $i$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>filling factor</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>cyclotron frequency</td>
</tr>
<tr>
<td>Symbol</td>
<td>Explanation</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\omega_{x,y}$</td>
<td>confinement strength of a harmonic potential</td>
</tr>
<tr>
<td>$r_s$</td>
<td>interaction parameter</td>
</tr>
<tr>
<td>$g_{\parallel}$</td>
<td>in-plane g-factor</td>
</tr>
<tr>
<td>$g_{\perp}$</td>
<td>out-of-plane g-factor</td>
</tr>
<tr>
<td>$S$</td>
<td>spin operator</td>
</tr>
<tr>
<td>$\sigma_{x,y,z}$</td>
<td>component of the spin</td>
</tr>
<tr>
<td>$\alpha_R$</td>
<td>Rashba SOI parameter</td>
</tr>
<tr>
<td>$\beta_R$</td>
<td>Rashba SOI parameter</td>
</tr>
<tr>
<td>$\beta_D$</td>
<td>Dresselhaus SOI parameter</td>
</tr>
<tr>
<td>$F^*$</td>
<td>interaction parameter</td>
</tr>
<tr>
<td>$R_H$</td>
<td>Hall constant</td>
</tr>
<tr>
<td>$\delta\sigma$</td>
<td>conductivity correction</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>magnetic flux</td>
</tr>
<tr>
<td>$K$</td>
<td>scattering matrix</td>
</tr>
<tr>
<td>$I$</td>
<td>identity matrix</td>
</tr>
<tr>
<td>$N$</td>
<td>subband matrix</td>
</tr>
<tr>
<td>$V_{TG}$</td>
<td>Gate voltage</td>
</tr>
<tr>
<td>$\Delta n$</td>
<td>density difference between subband</td>
</tr>
<tr>
<td>$\Delta_{SO}$</td>
<td>spin-orbit energy splitting</td>
</tr>
<tr>
<td>$B_{SO}$</td>
<td>spin-orbit magnetic field</td>
</tr>
<tr>
<td>$B_\phi$</td>
<td>coherence magnetic field</td>
</tr>
<tr>
<td>$B_{tr}$</td>
<td>transport magnetic field</td>
</tr>
<tr>
<td>$\tau_i$</td>
<td>Drude scattering time of subband $i$</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>Luttinger parameter</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Luttinger parameter</td>
</tr>
<tr>
<td>$B_0$</td>
<td>spin-orbit coupling field</td>
</tr>
<tr>
<td>$I^{(\alpha)}$</td>
<td>spin current</td>
</tr>
<tr>
<td>$T$</td>
<td>transmission matrix</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>energy dependent transmission</td>
</tr>
<tr>
<td>$\tau^{(\alpha)}$</td>
<td>spin dependent transmission coefficient</td>
</tr>
<tr>
<td>$N$</td>
<td>number of occupied modes</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

“It has stood the test of time, and emerged unscathed from every experimental challenge. But I cannot believe this is the end of the story [...]. And is entirely possible that future generations will look back, from the vantage point of a more sophisticated theory, and wonder how we could have been so gullible.”

David Griffiths - Introduction to quantum mechanics

Spin-orbit interaction (SOI) is a relativistic effect that couples the motion of an electron to its spin. Similarly to the atomic case, where SOI perturbs the single atom energy spectrum, the electronic band structure of a solid can be strongly influenced by SOI. How SOI affects electron transport in a semiconductor, in particular in systems with low dimensionality (2D, 1D, 0D), is the question that motivated our experimental study. In addition to their importance concerning fundamental research, semiconductor systems with strong SOI are of particular interest regarding possible spintronic applications.

Spintronics, or spin-based electronics, is a new and rapidly growing branch of solid state physics that studies the storage, transport and processing of information using the spin of the electron rather than its charge. With respect to the nowadays charge based electronics, spintronics promises the development of novel devices characterized by faster data processing speed, lower power consumption and increased on-chip integration. The control of single electron spins for quantum computing applications is already a reality in many research laboratories [1, 2]. Spin injection, manipulation and detection in semiconductor materials is traditionally performed with techniques not compatible with large scale integration. They include the use of ferromagnetic contacts, polarized laser beams and large magnetic coils. A novel approach consists in using spin-orbit interaction to generate, address and detect spin currents in semiconductor by purely electrical means. This second approach would offer the advantage of fast and integrable spin manipulation, but a lot of theoretical and experimental work still needs to be done at the fundamental level.
One of the absolutely most important semiconductors concerning fundamental research and everyday life applications is GaAs. The exquisite control of layer-by-layer growth techniques like molecular beam epitaxy (MBE) allows to fabricate samples with exceptionally high electron mobility. The possibility to easily tailor GaAs and its heterostructures with conventional techniques, made GaAs and its two-dimensional electron gasses (2DEGs) the first choice for studying electron transport in semiconductor nanostructures. As a result, most of the recent milestones in semiconductor physics were achieved using 2DEGs in GaAs. Electrons in the conduction band of GaAs are characterized by a very small SOI. The recent interest for SOI effects in nanostructures triggered the research for new host materials where SOI effects could be more easily observed. Among the others, we cite InAs, InGaAs and InSb 2DEGs or nanowires, HgTe inverted quantum wells and $p$-type GaAs two-dimensional hole gasses (2DHGs). The availability of semiconductor materials with strong SOI allowed the development of new branches in fundamental research. Prominent examples concern the observation and manipulation of pure spin-currents generated via the spin-Hall effect [3–5], the discovery of topological insulators [6–8] and the hunt for Majorana fermions [9, 10].

1.1 $p$-type GaAs

Nanodevices implemented in $p$-type GaAs/AlGaAs heterostructures are promising candidates for the realization of quantum information processing and the study of physical phenomena related to carrier-carrier Coulomb interaction and SOI. Holes in the valence band of GaAs are reminiscent of the $p$ symmetry of the atomic wave-functions. As a result, they possess an intrinsic angular momentum $l = 1$, for which SOI corrections are expected to be much stronger than for their electronic counterparts (for which $l = 0$). Furthermore, holes in GaAs have an effective mass several times larger than the conduction band electrons. The lower kinetic energy makes carrier-carrier Coulomb interaction more important, allowing the study of many-body physics. In this thesis we will perform experiments on carbon-doped, (001) grown 2DHGs characterized by a very strong SOI. The competition between confinement potential and SOI splits the valence band degeneracy, so that carriers are effectively described as heavy holes with total angular momentum projection on the growth axis $j = \pm 3/2$. With respect to previously adopted (311) grown wafers, crystal anisotropies in our samples should not be significant. This thesis describes standard magnetotransport experiments performed in diffusive Hall bar geometries where the hole density could be largely tuned with the use of a top gate. We present, analyze and discuss signatures of the strong SOI and hole-hole interaction present in our samples. In particular we measure the effective masses of the two spin-orbit split subbands in the limit of low magnetic field and as a function of density. This study shows the importance of SOI and highlights the complexity of the valence band of GaAs.
The possibility to pattern a 2DHG with similar techniques as the ones used for conventional n-type GaAs nanostructure fabrication allowed the study of complex devices such as quantum point contacts (QPCs) [11–13], quantum dots [14] and Aharonov-Bohm (AB) rings [15]. The combination of ballistic and coherent transport with the strong SOI is expected to give rise to new physical effects. In this thesis we study hole transport in QPCs, AB rings and chaotic cavities. Due to the high quality of our QPCs, and the very strong SOI of the 2DHG they are embedded in, we observe new SOI signatures in their conductance. Our study allows to give experimental evidence for the existence of two predicted terms in the SOI Hamiltonian of heavy holes. The study on the AB in materials with strong SOI aims at the manipulation of the electron’s geometric phase [16, 17]. Our work follows the track of pioneering experiments performed in AB rings in p-type GaAs [15, 18]. Lastly, we investigate hole transport in a three terminal chaotic cavity where the broken spin-rotational symmetry allows the generation of pure spin currents.

1.2 InAs/GaSb double quantum wells

A new paradigm in modern solid state physics are topological insulators: materials with insulating bulk and protected conductive states at their surface (edges). The existence of this new phase of matter is due to the combination of SOI and time reversal symmetry [19]. The first topological insulator to be experimentally observed was the 2D HgTe/CdTe inverted QW [6], where the combination of SOI and confinement potential leads to the quantum spin Hall effect (QSHE). The QSHE manifests itself at zero magnetic field with an insulating bulk and the formation of conductive states at the sample edges. On each physical edge of the sample two counter-propagating edge channels form; in these edge channels spin and momentum are locked so that states with opposite direction carry opposite spin. Such edge states are protected against elastic back-scattering and their conductance is quantized. HgTe based materials suffer from severe technological drawbacks, such as the difficult growth techniques and the difficult nanostructure fabrication procedures. Furthermore, the possibility to observe the topological phase in HgTe/CdTe is solely determined by the HgTe well thickness. InAs/GaSb double quantum wells attract considerable attention as a possible topological insulator[20]. With respect to HgTe/CdTe QWs, it would allow a direct tunability of the topological phase within the same sample by electrical means. On top of this advantage, the possibility to use well-established technologies for III-V materials, would make InAs/GaSb a good 2D topological insulator candidate for processing nanostructures.

In this thesis we characterize by magnetotransport measurement InAs/GaSb broken gap double QWs. For this purpose we fabricated large Hall bars where the carrier type could be tuned from electrons to holes with a top gate. We do not measure any effect based on the expected topological insulating nature of InAs/GaSb. The reasons probably include the large residual bulk conductivity at the charge neutrality.
point and the too extended size of the structures in use. At high perpendicular magnetic field we study a very particular situation where counter-propagating electron and hole quantum Hall edge channels are present at the same time at the sample edges. Our characterization aims to prepare the ground for future more advanced experiments using this material system.
Chapter 2

Basic concepts

In this chapter we give a theoretical introduction to the experiments discussed in the thesis. We review the most important aspect of $p$-type GaAs and InAs/GaSb heterostructures physics. We discuss the most relevant effects governing the magnetoresistance of a two-dimensional electron gas (2DEG), from zero to high perpendicular magnetic field. Finally we introduce two transport effects observable in semiconductor nanostructures: the Aharonov-Bohm effect and conductance quantization. The concepts discussed here are very general and only serve as an introduction to the rest of the work. In the following chapters we will expand the theoretical concepts introduced here and we will relate them to the experiments.

2.1 $p$-type GaAs

2.1.1 Band structure of GaAs

GaAs is a zinc-blende crystal. It is a direct gap semiconductor and its energy gap is 1.42 eV at room temperature [21]. The electronic bands $E_n(\vec{k})$ of bulk GaAs, calculated with the pseudopotential method, are shown in Fig. 2.1 (a). Fig. 2.1 (b) shows a schematic representation of the conduction and valence band close to the energy gap. The tight-binding method approximates the electronic wave functions in a solid as a superposition of the wavefunctions of the isolated atoms the solid is composed of [22]. In such a picture, the conduction band of GaAs is mainly given by the superposition of atomic $s$ orbitals, thus it is reminiscent of their spatial symmetry and their $l = 0$ orbital quantum number. Differently, the valence band is mainly given by the superposition of atomic $p$ orbitals, with $l = 1$. The atomic $p$ orbitals are three times degenerate and their total angular momentum $J$, given by the sum of orbital quantum number $l$ and spin quantum number $s$, can assume the values $3/2$ and $1/2$. The sixfold degeneracy (a factor of two for the spin) of the valence band is lifted in bulk crystals by SOI, resulting in three distinct bands referred to as heavy-holes (HH), light-holes (LH) and split-off (SO) band. Considering the total angular momentum $J$ and its projection along a $z$ axis $j$, one can find that HH
are characterized by \( J = 3/2 \) and \( j = \pm 3/2 \), LH by \( J = 3/2 \) and \( j = \pm 1/2 \) and the SO band by \( J = 1/2 \) and \( j = \pm 1/2 \). As depicted in Fig. 2.1 (b), HH and LH are degenerate in \( \Gamma \) and are characterized by different effective masses. The SO band is lowered in energy by the quantity \( \Delta_{SO} \), called spin-orbit gap. In GaAs \( \Delta_{SO} = 0.34 \) eV. Differently from the conduction band, the dispersion of the valence band in bulk GaAs is highly anisotropic and non parabolic. Accordingly one can not describe a hole system with a simple effective mass approximation, as it is often valid for the case of electrons.

**Figure 2.1:** (a) GaAs bandstructure calculated using the pseudopotential method including spin-orbit interaction [21]. (b) Schematic representation of the conduction and valence band close the fundamental gap, adapted from [23, 24]

### 2.1.2 2D confinement

Size quantization, obtained by confining the carriers in a quasi-2D system, has a profound effect on the structure of the valence band. Results of a numerical calculation of the in-plane dispersion relations \( E(k_{||}) \) of a 15 nm wide GaAs quantum well are shown in Fig. 2.2 (a). Both the HH and the LH bands split into a series of subbands, and the subbands’ degeneracy at \( \Gamma \) is lifted. In this calculation, the HH-LH energy separation at \( \Gamma \) is of the order of 10 meV. The SO band resides considerably lower in energy, and it is usually not relevant in a transport experiment. Fig. 2.2 (b) schematically shows the first HH and LH bands close to \( \Gamma \). In addition to the lifting of the degeneracy in \( \Gamma \), a mass inversion has occurred: the LH mass is now higher than the HH mass and an anti-crossing appears for finite in-plane \( k \)-vectors. For the bound motion along the growth direction (\( z \) axis), the HH effective mass is instead higher than the LH effective mass.
In most of the cases, the hole density is such that the Fermi energy lies, as depicted in Fig. 2.2 (b), considerably higher than the anti-crossing between HH and LH. For this reason, carriers in a two dimensional hole gas (2DHG), have essentially HH character and total angular momentum $J = 3/2$ with possible projection along the $z$ axis $j = \pm 3/2$. Also in the confined situation, as for the bulk, the effective mass is highly anisotropic and sensitively depends on system parameters such as crystallographic growth direction. The calculated density of states (DOS) effective
mass for the topmost HH band in Fig. 2.2 (a) is shown in Fig. 2.2 (c). $m^*$ is a complicated function of energy, where HH-LH mixing gives rise to a pronounced van Hove singularity. Effective mass anisotropies are present both in 2DHGs as well as in the bulk crystal. For a quantum well grown along the (001) direction, they are expected to be less severe than for other low-symmetry cases.

Electrons in the conduction band are not affected by size quantization. The spin dynamics in the conduction band are mainly determined by SOI and Zeeman effect. In the valence band, on the other hand, spin dynamics is given by the HH-LH splitting competing with SOI and Zeeman terms. For small in-plane $k$-vectors the quantization axis of total angular momentum enforced by the size quantization points in the growth direction of the QW. The fundamental limitation in having a second quantization axis strongly suppresses effects that tend to align the spin in the QW plane. For this reason both in-plane Zeeman splitting and spin-orbit splitting are higher-order effects.

### 2.1.3 Spin-orbit interaction

Spin degeneracy in solids is a combined effect of spatial inversion symmetry and time reversal symmetry [25], resulting in a twofold degeneracy of the dispersion relation: $E_+ (\vec{k}) = E_- (\vec{k})$. If the carriers move in a non symmetric spatial potential, then spin degeneracy is removed already at zero magnetic field, obtaining a finite spin splitting. Such an asymmetry can be due to either bulk inversion asymmetry (BIA) of the crystal structure [26] or a structure inversion asymmetry (SIA) of the host heterostructure [27]. Both situations result in a zero field spin splitting, where the spin split energy bands follow Kramer degeneracy only: $E_+ (\vec{k}) = E_- (\vec{k})$. SOI is a purely relativistic effect: an electron moving in a static electric field $E$ will perceive, in its reference frame, a static magnetic field $\mathbf{B}'$ oriented perpendicularly to its velocity $\mathbf{v}$ and to $\mathbf{E}$:

$$\mathbf{B}' = -\frac{1}{c^2} \mathbf{v} \times \mathbf{E}.$$  \hspace{1cm} (2.1)

$\mathbf{B}'$ couples to the electron spin via Zeeman interaction, giving rise to the spin orbit hamiltonian:

$$H_{SO} = -\frac{g\mu_B}{2c^2} (\mathbf{v} \times \mathbf{E}) \mathbf{S} = \frac{g\hbar}{4c^2 m_0^2} (\nabla V (\mathbf{r}) \times \mathbf{p}) \mathbf{S}$$  \hspace{1cm} (2.2)

where $\mathbf{S}$ is the spin operator. The term $1/2$ appearing in 2.2 is called Thomas factor and is derived from the non-relativistic limit of the Dirac equation [28].

The BIA term, also called Dresselhaus term, is present in crystals whose unit cell lacks a center of inversion. A prominent example of such a category are zinc-blende III-V semiconductors. The Dresselhaus term entering the hamiltonian is cubic in wave vector for a bulk crystal, but linear in $\vec{k}_0$ for a two dimensional electron system:
2.1. $p$-type GaAs

\[ H_D = \beta_D (\sigma_x k_x - \sigma_y k_y) \quad (2.3) \]

where $\sigma_x$ and $\sigma_y$ are Pauli spinors and $\beta_D$ is the material dependent Dresselhaus term. The SIA term, also called Rashba term, can be tailored by proper adjustment of the confinement potential of the QW in which the carriers reside. In two dimensional electron systems, the term entering the Hamiltonian is also linear in $\vec{k}$:

\[ H_R = \alpha_R (\sigma_x k_y - \sigma_y k_x) \quad (2.4) \]

$\alpha_R$ is directly proportional to the perpendicular electric field applied to the system, the proportionality constant is material specific. The latter property makes the Rashba term particularly interesting, in fact it allows to tune SOI by electrical means [29–31]. Using these considerations, it follows that the single-particle Hamiltonian for a 2DEG in the conduction band of a zinc-blend material can be written as:

\[ H = H_0 + \alpha_R (\sigma_x k_y - \sigma_y k_x) + \beta_D (\sigma_x k_x - \sigma_y k_y) \quad (2.5) \]

where $H_0$ is the electron kinetic energy.

In the case of holes in the valence band the situation is very different. For typical values of densities, the leading term in the SOI corrections entering the Hamiltonian is of Rashba type and cubic in $\vec{k}$:

\[ H = H_0 + \beta_R (p_3 \sigma_+ - p_3 \sigma_-) \quad (2.6) \]

where $p_\pm = p_x \pm ip_y$, with $p$ the hole momentum, and $\sigma_\pm = \sigma_x \pm i\sigma_y$. As already discussed, the cubic splitting of holes, in contrast to the linear splitting for spin-1/2 electrons, reflects the existence of a perpendicular quantization axis in the system created by subband quantization and HH-LH splitting. Consequently, the heavy-hole (HH) in-plane dispersion relation for the case under study is [23]

\[ E_\pm (k_\parallel) = \frac{\hbar^2}{2m_{hh}} k_\parallel^2 \pm \beta_R k_\parallel^3, \quad (2.7) \]

where $m_{hh}$ is the heavy hole band effective mass, i.e. the band curvature where SOI effects are negligible.

The Rashba parameter $\beta_R$ is determined by the subband densities $n_1$ and $n_2$ and according to

\[ \beta_R = \frac{\hbar^2}{4\sqrt{\pi} m_{hh}} \frac{n_2 - n_1}{n_2^{3/2} + n_1^{3/2}} \quad (2.8) \]

The energy difference between the spin-split subband is then $\Delta_{SO} = 2\beta_R k_\parallel^3$, where $k_\parallel$ is taken to be the in-plane Fermi wave vector of the low density subband, defined as $k_1 = \sqrt{4\pi n_1}$. It is useful to define the spin-orbit length $l_{SO}$ as a length
scale over which the hole spin rotates. Generally one has \( l_{SO} = v_F \tau_{SO} = \hbar v_F / \Delta_{SO} \).

For linear Rashba SOI it results \( l_{SO} = \hbar^2 / (m^* \alpha) \), for cubic Rashba SOI it results \( l_{SO} = \hbar^2 / (2 \beta_R m^* k_F^2) \). It is common to find different definitions, where additional factors of order unity are added. In Chapter 6 we discuss about the existence of additional terms in the SOI Hamiltonian for HH. The relevance of these other terms is restricted to specific situations.

### 2.1.4 Spin splitting

![Figure 2.3: Calculated effective g-factor for a 20 nm wide QW as a function of the growth direction \( \theta \). Adapted from [23].](image)

The anisotropic nature of the hole wavefunctions (reminiscent of atomic \( p \) orbitals) reflects itself in a strong anisotropic Zeeman splitting that characterized 2DHGs. The HH-LH splitting described in Sec. 2.1.2 defines a strong quantization axis pointing in the growth direction of the crystal. An external magnetic field can not create a second quantization axis on to of the already existing one. This results in a competition between Zeeman effects and confinement effects, as we will see in the following. As a result, the effect of an in-plane field on HH states is suppressed for a high symmetry growth direction (in a first order approximation). In the following the effective \( g \)-factor (times \( \mu_B \)) is defined as the prefactor of the linear term in a Taylor expansion of the Zeeman splitting \( \Delta E(B) \). Fig. 2.3 shows the calculated anisotropic effective \( g \)-factor of HH states as a function of the growth direction. For high symmetry growth directions [001] and [111], the in-plane \( g \)-factor vanishes. For low-symmetry growth direction, such as the [113], a relevant \( g \)-factor is expected. Interestingly, another term present in the Hamiltonian is expected to give rise to linear spin splitting for the [001] growth direction if \( k_{||} \neq 0 \). This term couples spin...
2.1. $p$-type GaAs

states with $k_{\parallel}$ states and is, properly speaking, a SOI. It takes the form:

$$H_{SO}^{(2)} = \gamma \left( B_+ p_+^2 \sigma_+ + B_- p_-^2 \sigma_- \right)$$

(2.9)

with $B_\pm = B_x \pm iB_y$. The proportionality constant is $\gamma = 3\gamma_0 \kappa \mu_B / (m_0 \Delta)$, where $\gamma_0$ and $\kappa$ are Luttinger parameters and $\Delta$ is the energy splitting between heavy-holes and light-holes. The relevance of the $H_{SO}^{(2)}$ term, quadratic in $k_{\parallel}$ and proportional to the in-plane components of the applied magnetic field $B$, was recently proposed [32–35]. Such a term is unique for heavy holes and very useful for exploiting SOI for quantum computing applications [32]. In Chapter 6 we will show the importance of this quadratic SOI for describing spin splitting in a $p$-type quantum point contact. Furthermore we will measure its strength and confirm its spin structure. The situation is different in for an out-of-plane field, where the most relevant contribution to Zeeman splitting is given by the bulk $g$-factor. For an HH subbands in a perpendicular field it results $g = 6\kappa = 7.2$, with $\kappa = 1.2$ for GaAs.

2.1.5 Many-body interaction

Holes in the valence band of GaAs have a very large effective mass compared to electrons in the conduction band. The high mass reduces the kinetic energy, making the Coulomb interaction between different holes more relevant. The Hamiltonian of a many-body system in the effective mass approximation reads [22]:

$$H = \sum_{i=1}^{N} \left( \frac{p_i^2}{2m^*} + \sum_{j \neq i}^{N} \frac{e^2/4\pi\epsilon}{|r_i - r_j|} + eV(r_i) \right)$$

(2.10)

Where $\epsilon = \epsilon_0 \epsilon_r$ is the dielectric constant of GaAs. The energy of the system is not solely given by the sum of kinetic and potential energy of its $N$ particles, but also by the Coulomb interactions present between them. The importance of electron-electron interactions is usually quantified by the interaction parameter $r_s$, denoting the ratio between Coulomb interaction and Fermi energy [36]:

$$r_s = \left( \frac{4\pi a_B^* n}{3} \right)^{-1/3}$$

(2.11)

$a_B^* = \frac{4\pi \hbar^2}{m^* e^2}$ is the effective Bohr radius. With respect to the isolated atom, the material dependent $a_B^*$ takes into account the material dielectric constant and the electron effective mass.

Electron-electron interactions are more relevant at low density, and influence transport affecting the screening efficiency of the 2DEG. When an external electrostatic perturbation acts on the 2DEG, the electron density will be locally modified and electrons will move in a new potential that depends on the external perturbation shape and on how the other electrons rearrange in response to it. A prominent example is the effect of a point-charge located close to the 2DEG, and it is usually
treated in the Thomas-Fermi approximation. In such a situation it can be shown [36] that the self-consistent electrostatic potential induced in the 2DEG plane oscillates as a function of the distance from the point-charge. The oscillations have a typical periodicity of \( \lambda/2 \) and are referred to as Friedel oscillations. The implications of many-bodies interactions in the magnetoresistance of a 2DEG will be discussed in Section 2.3.5.

## 2.2 InAs/GaSb double quantum wells

### 2.2.1 Band alignment in InAs/GaSb quantum wells

InAs, GaSb and AlSb belong to the so called 6.1 Å family. As the name suggests, they are characterized by a similar unit cell size that allows to easily combine them to form various heterostructures. On the contrary, their energy gap drastically varies from the 0.36 eV of InAs to the 1.61 eV of AlSb. Combining two or more of these materials allows to obtain very particular band lineups. Fig. 2.4 shows the band lineups of InAs, GaSb and AlSb.

![Band lineup in the 6.1 Å semiconductor family, the energies are expressed in eV. Adapted from Ref. [37].](image)

For example an InAs quantum well between two AlSb barriers offers a huge confinement for electrons and no confinement for holes. InAs 2DEGs between two AlSb barriers were extensively studied in the 90’s, mainly because of their high
2.2. InAs/GaSb double quantum wells

mobility, high g-factor and the expected strong SOI. Such studies included transport experiments in bulk samples [38–40] as well as nanostructures [41–43]. Placing an InAs QW in close proximity to a GaSb QW allows to form an exotic band lineup, where the conduction band of InAs resides lower in energy than the valence band of GaSb. Depending on the thicknesses of the two layers, it is possible to create a situation where a 2DEG and a 2DHG coexist close to each other in equilibrium. For large enough InAs and GaSb layers, the system behaves as a semimetal, where electrons and holes are mainly located in InAs and GaSb respectively. For very thin layers, the first quantized state in the InAs QW can rise higher in energy than the first quantized state in the GaSb QW, obtaining normal ordering and semiconducting behavior. Particularly interesting is the situation that forms in the inverted regime, when in-plane momentum are similar. In this situation the electron and hole wavefunctions extend in both QWs and hybridize [44]. The hybridization leads to the opening of a gap, whose expected size is of the order of a few meV. The description of the coupling mechanism can be found in Refs. [20, 45].

![Diagram](image)

Figure 2.5: (a) Band lineup for an InAs QW and a GaSb QW between two AlSb barriers. Solid lines refer to the bulk bands profile, dashed lines refer to the first quantized energy levels. (b) Schematic representation of the InAs conduction band and GaSb valence band with and without hybridization (solid and dashed lines respectively). Adapted from Ref. [20].

An additional interesting aspect of InAs/GaSb quantum wells is the possibility of tuning the band structure of the system via a perpendicular electric field [46, 47]. A perpendicular electric field results in a shift of the electron and hole bands in opposite directions, that changes both the wavefunctions’ overlap and the in-plane $k$-vector where the hybridization gap opens. For large enough electric field, the system can undergo a semiconductor to semimetal transition. Using a top gate and a side gate it is possible to independently control the energy of the two subbands and the position of the Fermi energy. Fig. 2.6 shows the bandstructure of an InAs/GaSb double QW for four different values of the applied perpendicular electric field. For zero field (Fig. 2.6 (c)) the bands are hybridized, with the opening of a small gap. For a large negative field (Fig. 2.6 (a)) the band overlap disappears and the system is semiconducting. The electric field necessary to change the bands’ ordering in a typical sample are experimentally achievable within nowadays technology.
2.2.2 Topological phase in InAs/GaSb

A 2D topological insulator is a material that, at zero magnetic field, possesses an insulating bulk and robust conductive states at the edges. The edge states are helical, meaning that on a particular edge we have a couple of counter-propagating 1D states where their momentum is locked to their spin. Since back-scattering requires either to flip the spin or to traverse the entire sample width, the edge states are protected against elastic back-scattering. Because of its analogy to the quantum Hall effect, the edge states configuration present in a 2D topological insulator is called quantum spin Hall effect (QSHE). A detailed description of the QSHE is given in Appendix B.

The first experimental realization of a 2D topological insulator was achieved in HgTe/CdTe QWs [6, 48, 49]. Despite many previous investigations of InAs/GaSb quantum wells, this material system recently attracted a renewed interest due to the theoretical prediction that, under certain circumstances, it should exhibit a 2D topological insulator behavior [20]. When the system is hybridized, the band ordering in the bulk is such that the conduction band lies lower than the valence band. Since in vacuum the opposite band ordering is expected, a band crossing is

Figure 2.6: Calculated band structure of an InAs/GaSb double QW for different values of applied perpendicular electric field. Adapted from Ref. [46].
expected to appear close to the sample edges.

2.2. InAs/GaSb double quantum wells

In Fig. 2.7 (a) and Fig. 2.7 (b) we show the comparison between the energy dispersion for a normal insulator and a topological insulator respectively. As it can be seen in Fig. 2.7 (b), the edge states cut through the hybridization gap. The existence of a topological insulator region in HgTe/CdTe QWs mainly depends on the HgTe thicknesses [48]. On the contrary, InAs/GaSb double quantum wells offer the possibility to tune the electronic phase with an applied perpendicular electric field. The application of a combined topgate and backgate voltage allows to reach, within the same sample, semiconducting, semimetallic and topological insulator regions. Fig. 2.7 (c) shows the phase diagram of a 10 nm InAs QW and a 10 nm GaSb QW. The red regions correspond to the inverted regime, the blue regions to the normal regime. Region I and III are the inverted \( p \)-type and \( n \)-type regimes respectively, region IV and VI the normal \( p \)-type and \( n \)-type regimes respectively. Region II is the topological insulator region and region V the normal insulator region. This feature, peculiar for InAs/GaSb, allows great feasibility as it allows to turn the topological behavior on or off within the same sample by electrical means.

As calculated in Appendix B, the QSHE manifests itself with a quantized conductance at zero magnetic field. One of the main problems preventing a clear and direct observation of the QSHE was, so far, the finite bulk conductivity at the CNP that coexists with the edge channel conductance. In Ref. [50], the conductance of Hall bar structures was studied as a function of the Hall bar width and length. It was shown that, in the limit of a short sample (shorter than the coherence length), the conductance tends to the expected quantized value (the precise value depends on the sample geometry). In a more recent work [51], an impurity layer was deliberately added at the InAs/GaSb interface in order to reduce the sample mobility at low temperature. The bulk conductivity, measured in a Corbino geometry, goes to zero at low temperature while the conductance of small Hall bars structure assumes quantized values as expected from their geometry. The length scaling of the...
edge channel conductance indicates a coherence length of 4.4 \mu m. Because of these recent discoveries, a lot of theoretical and experimental effort recently focuses on InAs/GaSb.

2.3 Magnetotransport

Ohm’s law states that the relation between current density \( j \) and electric field \( E \) is \( j(r) = \sigma E(r) \). \( \sigma \) is the electrical conductivity. The inverse of the conductivity is the resistivity \( \rho \). A typical magnetoresistance measurement is performed in a so-called Hall bar geometry, as the one depicted in Fig. 2.8. A current \( I \) is passed along the Hall bar axis, and the longitudinal and transverse voltages (\( V_{xx} \) and \( V_{xy} \) respectively) are measured as in Fig. 2.8. The longitudinal resistivity \( \rho_{xx} \) of the 2DEG is then \( \rho_{xx} = (WV_{xx})/(LI) \), where \( W \) and \( L \) are the width and length of the Hall bar respectively. The transverse resistivity \( \rho_{xy} \) is \( \rho_{xx} = V_{xy}/I \). These relations hold in a diffusive conductor, whose dimensions are much larger than the electron mean free path.

![Hall bar geometry with the typical measurement scheme used for a magnetotransport measurement.](image)

2.3.1 Classical Drude conductivity

One band transport

The application of a current along the Hall bar and of a magnetic field orthogonal to the 2DEG plane results in the well known Hall effect. In such a situation a finite angle \( \theta \) forms between the current density \( j \) and the equipotential lines of the electric field \( E \). Such angle, called Hall angle, depends on magnetic field according to the rule \( \tan \theta = \mu B \), where \( \mu = e\tau_e/m^* \) is the electron mobility and \( e > 0 \). A Hall voltage builds up in the transverse direction, and the conductivity can be expressed
as a symmetric $2 \times 2$ tensor. Introducing the cyclotron frequency $\omega_c = eB/m^*$ we have:

$$\sigma_{xx}(B) = \frac{ne^2 \tau_e}{m^*} \frac{1}{1 + \omega_c^2 \tau_e^2}$$

$$\sigma_{xy}(B) = \frac{ne^2 \omega_c \tau_e}{m^*} \frac{\omega_c \tau_e}{1 + \omega_c^2 \tau_e^2}$$

(2.12)

(2.13)

The relation between current density and electric field takes now the form:

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & -\sigma_{xy} \\ \sigma_{xy} & \sigma_{xx} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

(2.14)

The resistivity is found by tensor inversion:

$$\rho_{xx}(B) = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} = \frac{m^*}{ne^2 \tau_e} = \frac{1}{ne \mu}$$

(2.15)

$$\rho_{xy}(B) = \frac{\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2} = \frac{B}{en}$$

(2.16)

The dependencies of the conductivity and resistivity components as a function of magnetic field are shown in Fig. 2.9 (a) and Fig. 2.9 (b) respectively.

Figure 2.9: (a) Drude conductivities as a function of magnetic field. (b) Drude resistivities as a function of magnetic field.

**Two bands transport**

In case transport occurs in parallel via two independent subbands, each of them characterized by a density $n_i$ and a mobility $\mu_i = e_i \tau_i / m_i$, the low field resistivities are modified. We will consider the Drude scattering times $\tau_i$, the effective masses $m_i$ and the densities $n_i$ to be positive quantities. The charge can either be $e$ or $-e$ for electrons and holes respectively. With $e < 0$ we get a negative mobility, which has the physical meaning of a negative drift velocity.
Chapter 2. Basic concepts

We write the conductivities of the two channels as:

\[
\sigma_{xx}(B) = \frac{e_i n_i \mu_i}{1 + (\mu_i B)^2} \quad (2.17)
\]

\[
\sigma_{xy}(B) = \frac{e_i n_i \mu_i^2 B}{1 + (\mu_i B)^2} \quad (2.18)
\]

The resistivity tensor is given by:

\[
\rho = \begin{pmatrix}
\sigma_{xx1} + \sigma_{xx2} & -\sigma_{xy1} - \sigma_{xy1}
\sigma_{xy1} + \sigma_{xy2} & \sigma_{xx1} + \sigma_{xx2}
\end{pmatrix}^{-1} \quad (2.19)
\]

The longitudinal and transverse resistivities are respectively the first and second element of the inverted matrix. In formulas:

\[
\rho_{xx}(B) = \frac{n_2 \mu_2 + \mu_1 (n_1 + B^2 n_2 \mu_1 \mu_2 + B^2 n_1 \mu_2^2)}{e (n_1^2 \mu_1^2 + 2 n_1 n_2 \mu_1 \mu_2 + (n_2^2 + B^2 (n_1 + n_2)^2 \mu_1^2) \mu_2^2)} \quad (2.20)
\]

\[
\rho_{xy}(B) = \frac{B n_1 \mu_1^2 + B (n_2 + B^2 (n_1 + n_2) \mu_1^2) \mu_2^2}{e (n_1^2 \mu_1^2 + 2 n_1 n_2 \mu_1 \mu_2 + (n_2^2 + B^2 (n_1 + n_2)^2 \mu_1^2) \mu_2^2)} \quad (2.21)
\]

for \(e_1 = e_2 = e\),

\[
\rho_{xx}(B) = \frac{-n_2 (1 + B^2 \mu_1^2) \mu_2 + n_1 (\mu_1 + B^2 \mu_1 \mu_2)}{e (n_1^2 \mu_1^2 - 2 n_1 n_2 \mu_1 \mu_2 + (n_2^2 + B^2 (n_1 - n_2)^2 \mu_1^2) \mu_2^2)} \quad (2.22)
\]

\[
\rho_{xy}(B) = \frac{B n_1 \mu_1^2 - B (n_2 + B^2 (n_2 - n_1) \mu_1^2) \mu_2^2}{e (n_1^2 \mu_1^2 - 2 n_1 n_2 \mu_1 \mu_2 + (n_2^2 + B^2 (n_1 - n_2)^2 \mu_1^2) \mu_2^2)} \quad (2.23)
\]

for \(e_1 = e = -e_2\).

For \(n_1 \neq n_2\), \(\mu_1 \neq \mu_2\) or \(e_1 = -e_2\) we introduce corrections with respect to the single band case. For small magnetic field, the correction in \(\rho_{xx}\) is quadratic in \(B\). For this reason this effect is often referred to as classical positive magnetoresistance. For zero field we recover the single band formula: \(\rho_{xx} = (|n_1 \mu_1 e_1| + |n_2 \mu_2 e_2|)^{-1}\). \(\rho_{xy}\), in the limit of small field, takes a cubic correction in \(B\). For high magnetic field, \(\rho_{xy} \rightarrow B/(e_1 n_1 + e_2 n_2)\).

Fig. 2.10 shows two examples of \(\rho_{xx}\) (blue) and \(\rho_{xy}\) (red) calculated using the two-bands Drude model. The used parameters are indicated in the picture. The only difference from Fig. 2.10(a) to Fig. 2.10(b) is the sign of \(e_2\). When carriers of opposite charge are present in the system, a peculiar non monotonic behavior appears in \(\rho_{xy}\) close to zero field. In both cases the zero field value of \(\rho_{xx}\) is the same. When dealing with ambipolar systems like InAs/GaSb double QWs, it is common practice to use \(n > 0\) for electrons and \(n < 0\) for holes. Caution should be paid since a negative density in Eqs. 2.17 and 2.18 leads to unphysical results.

Inter-subband scattering, that is not considered in this simple treatment, can lead to substantial modifications of the low field magnetoresistance. We will address this problem in Chapter 5 and Chapter 9.
2.3 Magnetotransport

2.3.2 Shubnikov - de Haas effect

Electrons in a high perpendicular magnetic field form closed cyclotron orbits, where interference of the electron with itself quantizes its motion. These quantized states are called Landau levels and have the energy:

\[ E_n^{\pm} = \frac{\hbar}{2} \left( n + \frac{1}{2} \right) \pm \frac{1}{2} g^* \mu_B B \]  

(2.24)

The first term describes the orbital motion of the electrons in a perpendicular magnetic field, the second term takes into account the Zeeman energy of the two spin eigenstates. The degeneracy of each Landau level is given by \( n_L = eB/n \). It is convenient to introduce the filling factor \( \nu \), defined as the number of occupied Landau levels at a specific value of magnetic field. Scattering limits the lifetime of the electronic states, resulting in a broadening of the Landau levels of the order of \( \hbar/\tau_q \), where \( \tau_q \) is the quantum scattering time. The density of states (DOS) for a 2DEG in a perpendicular field will then look as depicted in Fig. 2.11 (a): it will be modulated by broadened Landau levels separated by the cyclotron energy \( \hbar \omega_c \). The Zeeman energy can additionally split spin degenerate states, at the magnetic field \( B \), the separation between spin pairs will be \( g^* \mu_B B \).

For the case of a single band, and in the limit of small magnetic field, it is possible to show [52] that the longitudinal conductivity takes the form:

\[
\rho_{xx}(B,T) = \rho_0 \left[ 1 - 2e^{-(-\pi/\omega_c\tau_q)} \frac{2\pi^2 k_B T}{\hbar \omega_c} \sinh \left( \frac{2\pi^2 k_B T}{\hbar \omega_c} \right) \cos \left( \frac{2\pi}{2eB} \frac{hn}{\hbar} \right) \right].
\]

(2.25)

The oscillating density of states modulates the classical Drude resistivity \( \rho_0 \). The oscillations are the so-called Shubnikov-de Haas (SdH) oscillations. The oscillating component is periodic in \( 1/B \), with periodicity given by \( \Delta = 2e/hn \). The Fourier power spectrum of \( \rho_{xx}(1/B) \) is typically used to determine the electron density. Numerical examples as well as an analytical form of the power spectrum of the SdH oscillations are given in Appendix A.

The oscillations’ amplitude is given by:
Figure 2.11: (a) Schematic representation of the 2DEG density of states in a perpendicular magnetic field. The separation between Landau levels is given by $\hbar \omega_c$, the Zeeman energy further splits spin degenerate Landau levels by $g^* \mu_B B$. (b) and (c) Transverse and longitudinal resistivity respectively as a function of magnetic field measured in a Hall bar geometry. Adapted from [36].

\[
\Delta \rho_{xx}/\rho_0 = 2e^{(-\pi/\omega_c \tau_q)} \frac{2\pi^2 k_B T/\hbar \omega_c}{\sinh (2\pi^2 k_B T/\hbar \omega_c)}
\]  

(2.26)

The exponential term, that reflects the broadening of the Landau level, determines the magnetic field dependent envelop of the oscillations. The fraction describes the temperature dependence of the oscillations. Given a minimum of the SdH oscillations at a specific magnetic field $B$, the oscillation amplitude is solely determined by temperature, effective mass and quantum scattering time.

### 2.3.3 Integer quantum Hall effect

In a very strong perpendicular magnetic field, the longitudinal resistivity $\rho_{xx}$ measured in a Hall bar geometry goes to zero every time $\nu$ Landau level are completely filled. Concomitantly the transverse resistivity $\rho_{xy}$ forms plateaus whose values are given by $\rho_{xy} = \hbar/(\nu e^2)$.

The simpler explanation of the quantum Hall effect considers localization of bulk states by disorder potential and formation of edge channels at the sample edges. When the Fermi energy lies in between two Landau levels, the density of states goes to zero and the bulk conductance is suppressed. Disorder potential and sample edges lift the energy of Landau level allowing for a local crossing with the Fermi energy. A schematic representation is shown in Fig. 2.13, where one dimensional edge channels encircle potential maxima and sample edges. If the bulk states are not connected to each other, transport can only occur along the edge states. The number of edge states is equal to $\nu$ and their motion is chiral: they move in opposite direction on
2.3. Magnetotransport

Figure 2.12: Transverse and longitudinal resistivity as a function of magnetic field measured in a Hall bar geometry. The red numbers indicate the filling factors corresponding to different plateaus in $\rho_{xy}$. Adapted from [36].

opposite edges of the sample. Transport in these edge states is ballistic, and the phenomenology of the quantum Hall effect can be derived from Landauer-Büttiker formalism [53, 54].

2.3.4 Weak localization and weak antilocalization

Weak localization is a coherent effect that manifests itself in large diffusive samples as a resistance peak at small magnetic field. A necessary condition for the observation of this effect is that the coherence length $l_\phi$ is much longer than the elastic mean free path $l_e$. Weak localization can be understood in a Drude picture of transport, where electron motion is ballistic between two successive scattering events. Scattering at impurities is considered to be elastic, thus it does not randomize the phase of the incoming electron. Multiple scattering events at different impurities that lead to closed paths, as depicted in Fig. 2.14 (a). It can be shown that the interference between time reversed paths is always constructive, hence backscattering is enhanced. Breaking time reversal symmetry with the application of a magnetic field suppresses the effect. The shape of the peak depends on the ration $l_\phi/l_e$, in particular larger values of $l_\phi/l_e$ lead to a sharper peak. Three examples are shown in Fig. 2.14 (b). Temperature affects $l_\phi$ more than $l_e$, and this eventually leads to a suppression of
weak localization. In a material with strong spin-orbit interaction the spin is randomized over the spin-orbit length $l_{SO}$. If $l_{SO}$ is smaller than $l_\phi$, the electron spin is rotated after a closed loop is complete leading to the so-called weak antilocalization correction. This regime counts two important changes with respect to the situation where no spin-orbit interaction is present: the amplitude of the correction is half and its sign is reversed [56]. Weak localization and weak antilocalization are additive corrections, and the overall resistivity shape at small magnetic field is given by the relative magnitude of $l_\phi$, $l_e$ and $l_{SO}$. Examples of weak antilocalization correction in the limit of strong spin-orbit interaction are shown in Fig. 2.14 (c). Both weak localization and weak antilocalization affect the longitudinal resistivity only: the transverse resistivity is unaltered.
2.3.5 Interaction effects

Among other effects caused by electron-electron interaction in a 2DEG, we are interested in the corrections to the low-field magnetoresistance. As shown in section 2.3.1, the longitudinal conductivity of a 2DEG is expressed as \( \sigma_{xx} = ne^2 \tau_e m^* \). Interactions affect \( \sigma_{xx} \) via \( \tau_e \) through the combination of different phenomena with different temperature dependencies. In our case, the most important factors are given by energy averaging of the scattering time and temperature dependent screening. Energy averaging of the scattering time makes the conductivity increase with temperature, resulting in an insulating behavior (\( d\sigma_{xx}/dT > 0 \)). Temperature dependent screening makes the conductivity decrease with increasing temperature, resulting in a metallic behavior (\( d\sigma_{xx}/dT < 0 \)). Heterostructures with charged impurities in the 2DEG will have a conductivity dominated by temperature dependent screening and will show metallic behavior. On the contrary, in samples where the resistivity is governed by remote impurities (small angle scattering), the temperature dependence of the resistivity will be dominated by energy averaging of the scattering time, resulting in insulating behavior.

The metallic behavior due to temperature dependent screening can be pictured microscopically considering scattering at Friedel oscillations originating from a single point charge, as depicted in Fig. 2.15 (a). In this situation, since the periodicity of the Friedel oscillations is always given by \( \lambda_F/2 \), the interference between path A and path B is always destructive, resulting in an increase of backscattering. In such a situation it can be shown that the conductivity depends linearly on temperature [57]:

\[
\sigma_{xx}(T) = \frac{ne^2 \tau_e}{m^*} \left( 1 - C \frac{k_B T}{E_f} \right)
\] (2.27)

Extending the analysis to the case of multiple scattering at different impurity sites (Fig. 2.15 (a)) result in another correction of the Drude conductivity, this time with insulating behavior, that takes the form at low temperature:

\[
\delta \sigma_I = -\frac{e^2}{\pi \hbar} \left( 1 - \frac{3}{4} F^* \right) \ln \left( \frac{\hbar / \tau_e}{k_B T} \right)
\] (2.28)

\( F^* \) is an interaction parameter. The transverse resistivity is affected by interaction corrections to the conductivity. The change in slope of the Hall resistance \( R_H \) as a function of temperature can be used to extract \( F^* \) using the relation:

\[
\delta \sigma_I = -\sigma_0 \frac{\delta R_H(T)}{2R_H^0}
\] (2.29)

\( \sigma_0 \) and \( R_H^0 \) are high temperature values, where the interaction effects are suppressed.
2.4 Electronic transport in semiconductor nanostructures

2.4.1 Conductance quantization in a quantum point contact

A quantum point contact (QPC) is a narrow constriction obtained in a 2DEG, its width is of the order of the Fermi wavelength and its length is smaller than the elastic mean free path. Tuning the width of the constriction with electrostatic gates results in a step-like change of its conductance. The conductance values on the conductance steps are multiple of the conductance quantum $2e^2/h$. Since its initial discovery [59] in a GaAs 2DEG, the quantized conductance in a narrow constriction proved to be a universal effect independent of fabrication technology and host material.

Let’s consider a long wire of width $W$ oriented along the $x$ direction connected at its end to two large two dimensional reservoirs (contacts). The additional confinement in the $y$ direction creates a set of quantized transverse modes. In a first approximation, we can assume that each of this mode has a parabolic dispersion as a function of $k_x$:

$$E_n(k_x) = E_n + \frac{\hbar^2 k_x^2}{2m^*}, \quad (2.30)$$

where $E_n$ are the subbands bottom due to energy quantization.

Applying a voltage difference $e(\mu_L - \mu_R)$ between the two leads of the QPC results in a current flow. The conductance of a QPC with $n$ occupied modes can be calculated using Landauer-Büttiker formalism [36]:

$$G = g_s \frac{e^2}{h} \sum_{i=1}^{n} \int_{-\infty}^{\infty} dE T_n(E) [f(E - \mu_L) - f(E - \mu_R)]. \quad (2.31)$$
2.4. Electronic transport in semiconductor nanostructures

Every mode contributes to the total conductance with a factor equal to the integral on the rhs. The integral contains an energy dependent transmission probability $T_n(E)$ and the difference between the Fermi distributions in the two leads due to the application of the voltage difference. The factor $g_s$ takes into account spin degeneracy, $g_s=2$ for a GaAs 2DEG at zero magnetic field. Assuming ideally transmitting modes and very small temperature and applied bias makes the integral go to one for every occupied modes and to zero for every unoccupied mode. In such a situation the conductance of a QPC with $N$ occupied transverse mode is simply given by:

$$G = 2N \frac{e^2}{h}.$$  \hfill (2.32)

A magnetic field breaks spin degeneracy, bringing to the formation of plateaus with values equal to integer multiples of $e^2/h$.

2.4.2 Aharonov-Bohm effect

The Aharonov-Bohm (AB) effect is an interference effect arising when a charged particle encircles a magnetic flux [60]. In this situation the particle acquires a phase proportional to the line integral of the vector potential $\vec{A}$ over the particle’s path. The AB effect is not only a striking manifestation of the wave-particle duality, but also a demonstration of the importance of the vector potential in quantum mechanics. Let’s consider the schematic representation of Fig. 2.16 (a), where an electron moving from left to right can take two different paths, named $\gamma_1$ and $\gamma_2$ and the two paths enclose a magnetic flux $\Phi$. In this example the magnetic field is restricted to the shaded area of the flux tube, hence the electron always travels in regions where there is no magnetic field.

Figure 2.16: (a) Schematic representation of the Aharonov-Bohm effect. (b) (c) and (d) Three possible paths considered for the AB effect in a ring geometry.
Chapter 2. Basic concepts

The AB effect predicts that the acquired phase difference along the two paths is:

\[ \Delta_{AB} = -\frac{e}{\hbar} \oint \vec{A} d\vec{s} = -\frac{e}{h} \phi = -2\pi \frac{\phi}{\phi_0}. \quad (2.33) \]

where \( \phi_0 = h/e \) is the magnetic flux quantum. The closed path, along which the integral is calculated, is given by \( \gamma_1 - \gamma_2 \). As it can be seen from the equation, the bare presence of the magnetic flux affects the quantum mechanical phase of the electrons, even though the electron does not experience any magnetic field. An experimental realization of the AB experiment as depicted in Fig. 2.16 (a) is very challenging. For practical reasons it is more convenient to perform this experiment in a semiconductor nanostructure using a ring geometry. In such a case the electron motion is guided by a circular confining potential, as shown in Fig. 2.16 (b) and a perpendicular magnetic field is homogeneously applied. If the ring transmission is high, we can restrict ourselves to the first three terms in the reflection probability \( (R(E) = 1 - T(E)) \), schematically depicted in Fig. 2.16 (b,c,d). In Fig. 2.16 (b) the electron is reflect at the ring entrance with probability \( r_0 \), in Fig. 2.16 (c) and Fig. 2.16 (d) the electron is reflected after a full loop inside the ring in anti-clockwise and clockwise direction respectively with probability \( r_1 \). The total reflection probability is:

\[ R = |r_0 + r_1 e^{2\pi i \phi/\phi_0} + r_1 e^{-2\pi i \phi/\phi_0} + \text{c.c.}|^2 \quad (2.34) \]

an expansion of Eq. 2.34 leads to:

\[ R = |r_0|^4 + 2|r_1|^4 + 4|r_0||r_1| \cos \delta \cos 2\pi \frac{\phi}{\phi_0} + 2|r_1|^2 \cos 4\pi \frac{\phi}{\phi_0} + \ldots \quad (2.35) \]

As one can see from Eq. 2.35 two different oscillating components are present. The first one, with periodicity \( \phi/\phi_0 \) is the AB effect; its amplitude is reduced by a factor \( \cos \delta \) depending on the phase difference acquired along the two paths at zero magnetic field. The second term, with periodicity \( 2\phi/\phi_0 \) is the so-called Altshuler-Aronov-Spivak (AAS) effect. The cancellation of the \( \cos \delta \) prefactor in the AAS oscillations makes them more robust than the AB oscillations against ensemble averaging or modification of the microscopic electrostatic configuration of the ring. Because of their more robust nature, AAS oscillations were observed before AB oscillations in long metallic cylinders \[61\]. AB oscillations were subsequently observed in metal \[62\] and semiconductor \[63\] rings.

The AB effect has an elecrromagnetic dual: the Aharonov-Casher effect \[64\]. The Aharonov-Casher effect arises when a neutral particle with a spin encircles a line charge. The radial electric field generated by the line charge is seen, in the reference frame of the moving particle as a magnetic field acting on its spin. Experimental realizations of the Aharonov-Casher effect rely on the presence of SOI in the material in which the ring geometry is embedded. We will discuss in more detail about the AB and Aharonov-Casher effect in Chapter 8.
2.4.3 Conductance fluctuations in mesoscopic samples

In mesoscopic samples, whose dimension $L$ is smaller than the coherence length $l_\phi$, interference between different paths can lead to the formation of magnetoconductance fluctuations. For observing this effect it is not necessarily required to have two clearly distinct paths as for a ring structure. Interference effects can also arise for ballistic paths enclosing a finite size area in a linearly connected sample. Considering every couple of paths $m$ and $n$, the total conductance will be given by the sum of their contributions:

$$G_{mn} = \sum_{mn} |t_m||t_n| \cos(\theta_m - \theta_n) \cos \left( \frac{2\pi e B A_{mn}}{h} \right)$$

where $t_m$ and $t_n$ are the transmission probabilities, $\theta_m - \theta_n$ is the zero field phase difference between the two paths and $A_{mn}$ is the area enclosed by the paths. Mesoscopic conductance fluctuations are more visible in small samples. For large samples, where many different paths exist that connect the two ends of the device, the term $\cos(\theta_m - \theta_n)$ leads to a vanishing amplitude of the fluctuations. Conductance fluctuations are stable and reproducible in time and can be observed as a function of magnetic field or gate voltage. A gate voltage results in a change of the Fermi velocity and in a distortion the shape of the sample. Conductance fluctuations can be observed as a function of in-plane field too and result from time-reversal symmetry breaking in finite width 2DEGs with asymmetric confinement potential. The statistics of conductance fluctuations amplitude in mesoscopic cavities proved to be an extremely sensitive tool for studying material properties [65–68].
Chapter 3

Sample fabrication

This chapter describes the fabrication techniques used in this thesis work for sample processing of both $p$-type GaAs heterostructures and InAs/GaSb double quantum wells.

3.1 $p$-type GaAs

We used $p$-type GaAs wafers grown molecular beam epitaxy (MBE) by Andreas Wieck and co-workers in Bochum University and by Werner Wegscheider and co-workers in Regensburg University. The structures are grown on GaAs substrates along the high-symmetry (001) crystallographic direction and remotely doped with carbon. The carbon dopants are always placed above the 2DHG. All the used wafers had similar layer structure, but showed significant differences in density, mobility and spin-orbit interaction strength. The characteristics of each wafer are reported in Table 3.1. The carbon-doped layers are indicated by the letter C.

In all the three cases, the 2DHG is located about 45 nm below the surface. A higher distance from the surface would allow reaching higher mobility values, but tailoring the 2DHG with lithographic techniques, such as wet etching, would be more difficult, in particular for the fabrication of nanostructures.

The wafers were cut in 5.6 $\times$ 5.6 mm$^2$ pieces, with side walls along the (100) and (010) direction. From each piece we could process four different samples. Each cut piece was thoroughly cleaned with warm acetone and isopropanol in an ultrasonic bath before starting the fabrication. This passage allows to remove an eventual protective layer and other impurities present on the surface.

Etching of large Hall bars and mesa structures

Each 5.6 $\times$ 5.6 mm$^2$ piece was further processed with optical lithography to obtain 100 $\times$ 50 $\mu$m$^2$ Hall bars (Fig. 3.1 (a)) or 20 $\times$ 20 $\mu$m$^2$ mesas (Fig. 3.1 (b)). For this purpose we spin-coated a thick positive optical resist over the sample, baked it and irradiated it with UV light in a mask aligner. After exposure, the sample
3.1. p-type GaAs

<table>
<thead>
<tr>
<th>sample ID</th>
<th>growth sequence</th>
<th>density</th>
<th>mobility</th>
<th>Δn/n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bochum20122</td>
<td>5 nm GaAs 15 nm Al$<em>{0.35}$Ga$</em>{0.65}$As:C 15 nm Al$<em>{0.35}$Ga$</em>{0.65}$As 650 nm GaAs</td>
<td>$3.5 \times 10^{15}$ m$^{-2}$</td>
<td>5.0 m$^2$V$^{-1}$s$^{-1}$</td>
<td>0.22</td>
</tr>
<tr>
<td>Bochum University</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D05124A</td>
<td>5 nm GaAs 15 nm Al$<em>{0.35}$Ga$</em>{0.65}$As:C 15 nm Al$<em>{0.35}$Ga$</em>{0.65}$As 650 nm GaAs</td>
<td>$4.5 \times 10^{15}$ m$^{-2}$</td>
<td>30 m$^2$V$^{-1}$s$^{-1}$</td>
<td>0.22</td>
</tr>
<tr>
<td>Regensburg University</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D040817C</td>
<td>5 nm GaAs 15 nm Al$<em>{0.31}$Ga$</em>{0.69}$As:C 15 nm Al$<em>{0.31}$Ga$</em>{0.69}$As 15 nm GaAs Al$<em>{0.31}$Ga$</em>{0.69}$As</td>
<td>$3.0 \times 10^{15}$ m$^{-2}$</td>
<td>65 m$^2$V$^{-1}$s$^{-1}$</td>
<td>0.29</td>
</tr>
<tr>
<td>Regensburg University</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1: The three p-type GaAs wafers used for this thesis work.

is developed with the MF319 developer and rinsed in water. The surface areas that are now exposed to air can be removed via wet etching using a solution of H$_2$O : H$_2$SO$_4$ : H$_2$O$_2$ with concentration 100 : 3 : 1. During this process we etch about 200 nm deep in the wafer. To improve the sharpness of the edges, before exposing the final structure, we exposed and developed the edges of the sample. In this way it is easier to push the chip in close contact to the mask when exposing, allowing to reach a higher resolution.

![Figure 3.1](image1.png)

**Figure 3.1:** (a) Two 100 × 50 μm$^2$ Hall bars oriented perpendicularly to each other. The golden pads circularly disposed around the Hall bars are ohmic contacts. (b) Inner part of a 20 × 20 μm$^2$ mesa structure with 12 arms. The height of the central mesa with respect to the rest of the wafer is about 200 nm.
Chapter 3. Sample fabrication

3.1.1 Ohmic contacts for p-type GaAs

Electrical contact to the 2DHG is ensured by twelve metallic ohmic contacts, as the ones visible in Fig. 3.1 (a). The region of the ohmic contacts is defined by optical lithography (this time we use a negative resist) and developing. To obtain a good contact, the GaAs surface is treated with a low energy oxygen plasma just before evaporation. The evaporated metallic layers are (in the evaporation sequence) Au, Zn, In and Au with thicknesses of 2 nm, 40 nm, 40 nm and 200 nm. The first 2 nm of Au act as sticking layer, to improve the adhesion of the metal structure on the surface. Zn is the metal that allows to contact the 2DHG while In promotes the diffusion of Zn during annealing. The last Au layer is used for bonding. After the evaporation, the remaining photoresist is removed by an acetone lift-off at 50 °C.

To make the metal diffuse into the semiconductor, a thermal annealing is necessary. For this we used an home-made oven where the samples are annealed for 2 minutes at 150 °C to remove water and for 2 minutes at 500 °C. Annealing is performed in a forming gas (95% N₂ and 5% H₂) atmosphere at 0.3 bar. The ohmic contacts produced in this way show a resistance of the order of 200 kΩ at room temperature, and of the order of a few kΩ at 4.2 K. After a sample was finished, we bonded it using an ultrasonic wedge bonder and Au cables. Ohmic contacts produced with this recipe are typically very fragile, and in many cases it was necessary to bond by hand or to manually reinforce the already existing bondings with a silver epoxy glue. The epoxy glue needs an annealing at 150 °C for a few minutes in order to solidify.

3.1.2 Electron beam lithography and shallow etching

Nanostructures are processed starting from 20 x 20 µm² mesas by electron beam lithography and shallow etching. The samples are covered with a 100 nm PMMA layer by spin-coating and baking. Via electron beam lithography and developing we can open narrow trenches in PMMA and use these as a mask for a subsequent wet etching process. For aligning the electron beam writing field to the mesa we used the ohmic contacts as a reference, obtaining an aligning accuracy of the order of 1 µm. Developing is performed by stirring the sample in a solution of isopropanol and MIBK with concentration 3 : 1 and rinsing in pure isopropanol. After developing, the samples are baked at 120 °C for 2 minutes. This step improved the PMMA adhesion to the surface preventing underetching and slightly reduces the width of the trenches. After baking, the samples are carefully checked with an atomic force microscope (AFM). An example of a PMMA pattern of a quantum ring before wet etching is shown in Fig. 3.2 (a). If a defect in the exposed pattern is found, it is possible to remove the PMMA with acetone and start the processing again without any drawback. The samples that showed no defect in the PMMA pattern are briefly exposed to an oxygen plasma to remove eventual impurities on the GaAs surface and are immediately etched. Etching is performed with a solution of H₂O : H₂SO₄ : H₂O₂
3.1. \( p \)-type GaAs

with concentration 600 : 3 : 1. The etching rate of the first few tens of nanometer was found to vary from sample to sample up to a factor of three. For improving the etching success, an etching test is performed on one of the mesa arms. The etching time is tuned in order to reach a depth between 15 and 40nm. Fig. 3.2 (b) shows a successfully etched ring structure. As one can see from the profile depicted in Fig. 3.2 (c), the depth of the etched lines is about 35 nm and their width about 100 nm. The main issue of \( p \)-type processing is the wet etching unreliability to control the etching depth in the few nanometer range. While all the other fabrication steps have a success rate close to one, about two thirds of the chips are lost while etching because of a too low or too deep etching depth. Except the problem related to the variable etching rate, we never experienced underetching.

![Figure 3.2](image)

Figure 3.2: (a) AFM micrograph of a ring structure exposed in PMMA. The dark lines are 100 nm deep trenches. (b) AFM micrograph of the same ring as in (a) after wet etching and resist removal. The dark lines are 35 nm deep trenches, the small white dots are PMMA residues. (c) Height profile along the blue line of (b).

3.1.3 Gating of \( p \)-type GaAs heterostructures and nanostructures

Metallic gates directly deposited on the GaAs surface are widely employed in \( n \)-type systems as they allow great density tunability and limited induced disorder. For \( p \)-type GaAs such a solution is not possible. Previous experiments with metallic gates showed large leakage currents and hysteresis. A solution to avoid leakage currents consists in depositing an insulating layer over the sample surface before the metallic gate is deposited. For this thesis work, we used Ti/Au metallic global gates deposited on two different insulating materials: HfO\(_2\) grown by atomic layer deposition and Si\(_3\)N\(_4\) grown by plasma enhanced chemical vapor deposition. Both methods result in the deposition of highly insulating materials, with smooth surfaces. A systematic study of the tunability offered by an HfO\(_2\) gate dielectric on both large Hall bars and small nanostructures is found in Ref. [69]. Si\(_3\)N\(_4\) offered a great tunability over a bulk Hall bar geometry but resulted in a strong density decrease; no study of \( p \)-type nanostructures covered by a Si\(_3\)N\(_4\) gate dielectric was performed in this thesis work. The metallic topgates were obtained by shadow mask evaporation of Ti and Au.
Chapter 3. Sample fabrication

The shadow mask was obtained by manually punching a hole in a thin aluminum foil using a sharp needle. An example of such a gate is shown in Fig. 3.3. Despite the unregular shape obtained, this method allows not to expose the insulator to any resist, developer and lift-off solvent.

Figure 3.3: Hall bar structure with a global Ti/Au topgate obtained by shadow mask evaporation.

3.2 InAs/GaSb double quantum wells

For the work presented in this thesis, we used a single wafer grown by Christophe Charpentier and Werner Wegscheider at ETH Zurich. The sample was grown by MBE on a heavily doped GaAs substrate. Its active region consisted, from the bottom to the top, of a 50 nm AlSb barrier, a 15 nm InAs QW in proximity to a 8 nm GaSb QW, a second 50 nm AlSb barrier and a 3 nm GaSb capping layer. The layer scheme and a calculation of the valence band (red) and conduction band (blue) profiles along the growth direction is depicted in Fig. 3.4.

We used an AFM to characterize the sample surface before any processing. Fig. 3.5 (a) shows a 10 × 10 µm² AFM micrograph of the sample surface, close to the center of the wafer. Pronounced discontinuities and islands formations decorate the wafer surface, with steps that arrive up to 10 nm. The origin of this inhomogeneities probably comes from the strong lattice mismatch between the GaAs substrate and the subsequently grown buffer layer [37]. As we will show in Chapter 10, despite the disappointing surface roughness, the carriers mobility in this wafer can be surprisingly high. The possibility to pattern nanostructures on such a surface remains, up to date, unknown.
3.2. InAs/GaSb double quantum wells

Figure 3.4: Numerical calculation of the bands lineup in the sample in use. Valence band (red) and conduction band (blue) profile along the growth direction. Dashed lines indicate the first QW eigenstate. Calculation performed with NextNano³.

Figure 3.5: (a) 10 × 10 µm² AFM micrograph of the sample surface before any fabrication process. (b) Height profile along the blue line of (a).

The wafers were cut in 5.6 × 5.6 mm² pieces, from each piece we could process four different samples. Each piece is thoroughly cleaned with warm acetone and isopropanol in an ultrasonic bath before starting the fabrication. This passage allows to remove impurities present on the surface. The back of the sample is covered by a thick layer of gallium, used to hold the sample in place in the MBE chamber. This layer can be removed by covering the top surface with a thick photoresist and manually polishing the back surface with a brush and warm isopropanol. The gallium leftovers can be further removed by dipping the sample in pure HCl for several minutes. We will discuss in the following the recipes used to make ohmic contacts and to etch the sample. Gating was done with identical techniques as
Chapter 3. Sample fabrication

3.2.1 Ohmic contacts to InAs/GaSb double quantum wells

AlSb is known to rapidly oxidize when exposed to air. For this reason, we leave etching to a successive step and we first deposit the ohmic contacts. We deposit a thick negative photoresist and we expose it to open twelve circular holes. Just before depositing the metal, we expose the sample to a low energy oxygen plasma to clean the surface and remove any resist residues. The ohmic contacts are made with the standard recipe for n-type GaAs and are deposited by electron beam evaporation. The evaporated metallic layers are (in the evaporation sequence) Ge, Au, Ni and Au with thicknesses of 18 nm, 50 nm, 40 nm and 100 nm. A thermal treatment is necessary to melt the metals and make them diffuse in the semiconductor. This can be done by annealing the sample at 500 °C for two minutes in formiergas atmosphere. The last step is not strictly necessary to obtain good ohmic contacts, as the successive top gate deposition at 300 °C is enough to ensure a good annealing. Typically the resistance of an ohmic contact at room temperature is of the order of 1 kΩ and stays roughly constant when the temperature is lowered to 4.2 K.

3.2.2 Etching of Hall bar structures

Etching of semiconductor nanostructures in Sb-based material is usually performed with selective chemical wet etching [70]. The etching of AlSb is a particularly crucial step. AlSb was found to easily oxidize when exposed to many chemicals and form a particularly resistant oxide, that furthermore is conductive at low temperature. To avoid the problems related to AlSb oxidation, it is possible to reduce its reactivity by introducing a low concentration of Ga in the barrier [37]. To avoid the problems related to wet etching of InAs/GaSb QWs, we etched our samples using a microwave Ar plasma in an inductively coupled reactive ion etching chamber. The etching mask was obtained in a positive photoresist with standard optical lithography techniques, as described in Section 3.1. The etching time was tuned in order to etch more than 150 nm, so to leave the buffer layer and not the AlSb barrier exposed to air. A prolonged exposure to the plasma (of the orders of five minutes) can make the photoresist cross link, and make it hard to be removed. For allowing long etching times, we cooled the reactive ion etcher chamber to −150 °C with liquid nitrogen. After etching, the photoresist was removed by warm acetone and heavy sonication for several minutes. With this technique we could fabricate structures with rather smooth edges and depths up to 300 nm, where the smaller feature had a lateral dimension of the order of a few µm. To prevent oxidation, it is particularly important to passivate the surface of the sample as soon as possible after the capping layer is removed. Even if the sample is etched down to the buffer layer, the side walls contain AlSb that is laterally exposed to air. To test the effect of air exposure, we etched a sample down to the second AlSb barrier and, after resist stripping, we left
3.2. InAs/GaSb double quantum wells

...it in air. Fig. 3.6 (a) shows an AFM micrograph of a mesa right after dry etching and resist stripping together with a height profile taken along the blue line. The step height is 80 nm, the walls are well defined and the etched surface is smooth. It is possible to see residues close to the edges, probably due to redeposition of sputtered material. After one day we repeated the measurement and observed a decrease in the step height to 40 nm (Fig. 3.6 (b)). Three days after etching, the etched surface grew higher than the original one and we then measured a negative step of 30 nm. The effect of the oxidation was visible by eye as well, and resulted in an overall change of the color of the etched surface from silver to black. This result proves the importance of immediately deposit the insulator material after etching. Alternatively, the samples can be indefinitely passivated by spin-coating PMMA on their surface and baking at 180 °C for at least 15 minutes. At this point, it is unknown if the physical etching process damages the side walls of the sample. Decreasing the sample size to channel widths of the order of 1 µm would make an alternative wet etching technique more favorable than dry etching.

Figure 3.6: AFM micrographs of a InAs/GaSb sample after dry etching and resist stripping together with a height profile taken along the blue line. The pictures have been taken at different moments. (a) Right after etching and resist stripping. (b) One day after (a). (c) three days after (a).
Chapter 4

Magnetotransport in \( p \)-type GaAs/AlGaAs heterostructures

4.1 Introduction

The resistivity tensor of a two-dimensional hole gas (2DHG) as a function of magnetic field contains a rich variety of physical phenomena. In this thesis we measure weak antilocalization (WAL), classical positive magnetoresistance, interaction effects, Shubnikov-de Haas (SdH) oscillations and the quantum Hall effect (QHE). Tuning the hole density with a global top gate allows to explore, in the same device, different regimes where the relative importance of classical, coherence, interaction and spin-orbit effects is drastically different. In the high density regime the sample is characterized by a high mobility and a strong SOI. As described in Section 2.1.3, carriers are effectively described by two non-degenerate bands with projection of total angular momentum along the \( z \)-axis \( \pm 3/2 \). As a consequence, at low magnetic field we observe a peculiar positive magnetoresistance, understandable with classical diffusive transport. At intermediate field we have SdH oscillations, whose spectrum reveals the presence of the two subbands. The high field regime is characterized by QHE physics, where SOI does not play a major role. In the low density regime the hole mobility decreases and the elastic mean free path becomes comparable to the coherence length and the spin-orbit length. In this regime weak antilocalization appears at small magnetic field. At the same time, since the carriers kinetic energy decreases, interaction effects become more and more important.

In this chapter we present the most important experimental methods and numerical techniques used in the rest of the thesis to characterize our 2DHGs. Furthermore we will present and discuss the gate tunability of our 2DHGs concerning the most important quantities, including SOI strength. All the measurement shown here are performed on two samples obtained from the same wafer structure (D040817C). Each of the two samples consists of two perpendicularly placed 50 \( \mu \text{m} \times 100 \mu \text{m} \) Hall bars. One of the two samples remained ungated, while the other was covered by a 200 nm thick \( \text{Si}_3\text{N}_4 \) dielectric grown by plasma enhanced chemical vapor de-
position and a global Ti/Au topgate. The ungated and gated samples are shown in Fig. 3.1 (a) and Fig. 3.3 respectively.

4.2 Magnetoresistance of a 2DHG

Fig. 4.1 shows the magnetoresistance of our ungated 2DHGs measured in a Hall bar geometry at a temperature of 80 mK. We show the longitudinal resistivity $\rho_{xx}$ (blue) and the transverse resistivity $\rho_{xy}$ (red) for a magnetic field up to 12 T and 1.2 T (Fig. 4.1 (a) and Fig. 4.1 (b) respectively). Despite the fact that the distance of the 2DHG from the surface is 45 nm only, its quality is very high as demonstrated by well developed fractional states between $\nu = 2$ and $\nu = 1$.

Figure 4.1: (a) $\rho_{xx}$ (red) and $\rho_{xy}$ (blue) measured in a Hall bar geometry at a temperature of 80 mK. The first three integer filling factors are indicated on the picture. (b) The same as (a) but for small values of magnetic field.

On Fig. 4.1 (a), for a high magnetic field, we observe a tendency of $\rho_{xx}$ to increase
Chapter 4. Magnetotransport in p-type GaAs/AlGaAs heterostructures

with $B$. The origin of such effect is not understood. A possibility could be the presence of a small parallel conductance that gives rise to a classical parabolic background resistance. The effect of this parallel conductance is anyway not measurable for magnetic field lower than 8 T. $\rho_{xx}$ goes to zero the first time for $B = 700$ mT, and stays zero within noise level up to $\nu \leq 2$ every time we have an integer filling factor. In the same wafer we measured quantum point contacts, where the residual conductance at pinch-off was zero within the noise level up to a magnetic field of 12 T. Reducing the total density with a top gate voltage resulted in the suppression of this effect, such that we obtained $\rho_{xx} = 0$ up to the highest magnetic field available. An alternative explanation might be linked to a degradation of the coupling between ohmic contacts and edge channels in a strong magnetic field. Since this effect appeared just in this particular sample, and had no influence on the data discussed in this thesis, no further studies were performed.

4.3 Beating of the Shubnikov-de Haas oscillations

For small magnetic fields, the SdH oscillations do not follow a clear $1/B$ periodicity, as expected from the Ando formula (Eq. 2.25), but rather form a complex beating pattern. The beating is due to the coexistence of two subbands, as explained in Section 2.1.2. The density of the two subbands is revealed by the Fourier power spectrum of $\rho_{xx}(1/B)$. In order to increase the resolution of the Fourier transform, we used standard numerical procedures explained in Appendix A. The result of such an analysis is shown in Fig. 4.2. The horizontal axis has been multiplied by $e/h$ to convert the peak position to spin-split subband densities. The peaks labeled as $n_1$ and $n_2$ represent the two subbands. Their density are $n_1 = 1.05 \times 10^{15}$ m$^{-2}$ and $n_2 = 1.95 \times 10^{15}$ m$^{-2}$. Since the two subbands contribute to transport in parallel, and since charge redistribution between subbands is present at high field, various combinations of $n_1$ and $n_2$ are expected. The method used for the identification of the various peaks in the spectrum is also reported in Appendix A. It should be mentioned that the relative intensity of the peaks depends on the specific magnetic field range used for the analysis. In fact at small enough magnetic field only the subband $n_1$ gives SdH oscillations. The SdH oscillations from $n_2$ start at intermediate field and their combinations $n_2 + n_1$ and $n_2 - n_1$ appear at even higher field. In the quantum Hall regime, the magnetic field at which the filling factors appear is determined by the total density only. A detailed analysis of the power spectrum and the relative importance of its component as a function of magnetic field range is found in Chapter 5. The strength of SOI can be directly quantified without invoking any specific theoretical model as the subband population imbalance $\Delta n = (n_2 - n_1)/(n_2 + n_1) = 0.3$. 

38
4.4 Classical positive magnetoresistance in two-subband system

The classical Drude longitudinal resistivity $\rho_{xx}$ of a system with two types of charge carriers shows a parabolic magnetoresistance around zero magnetic field. Concomitantly the transverse resistivity $\rho_{xy}$ exhibits a small cubic correction. Both corrections are visible in Fig. 4.1 (b) for small magnetic field. This magnetoresistance is a result of a classical effect, whose form can be derived considering two conductive channels contributing to transport in parallel, as described in Section 2.3.1. In case inter-subband scattering is present, a more elaborate theory based on Boltzmann transport equations was developed [71]. This low field feature should not be confused with the weak antilocalization correction expected to be limited, in our high quality samples, to magnetic fields below 1 mT. In a two subbands system with inter-subband scattering, the resistivity tensor components are given by:

$$\rho_{xx} = \frac{m^*}{e^2} \text{Re} \left( \frac{1}{\text{Tr} \left( N \left( K - i\omega_c I \right)^{-1} \right)} \right)$$  (4.1)

$$\rho_{xy} = \frac{m^*}{e^2} \text{Im} \left( \frac{1}{\text{Tr} \left( N \left( K - i\omega_c I \right)^{-1} \right)} \right)$$  (4.2)

where $\text{Tr}$ is the trace operation, $I$ is the $2 \times 2$ unitary matrix, $\omega_c = eB/m^*$, the elements of $N$ are $N_{ij} = \sqrt{n_i n_j}$, $K$ is the scattering matrix, defined as:
Chapter 4. Magnetotransport in p-type GaAs/AlGaAs heterostructures

\[ K = \begin{pmatrix} K_1 & -K_{12} \\ -K_{12} & K_2 \end{pmatrix} \] (4.3)

The elements of the scattering matrix \( K \) are combinations of the first and second coefficients of the Fourier expansion of the (angle dependent) scattering rate. In the limit of vanishing inter-subband scattering rate, \( K_1 \) and \( K_2 \) are the inverse of the Drude scattering time of the first and second subband respectively while \( K_{12} \) is zero. A finite inter-subband scattering rate affects not only \( K_{12} \), but also \( K_1 \) and \( K_2 \). With the use of Eq. 4.1, the positive magnetoresistance in p-type GaAs was fitted and the scattering rates extracted [31].

In a previous work [72] a strong positive magnetoresistance was measured in p-type (311) GaAs 2DHG, but it could not be satisfactorily fitted with a two band model without inter-subband scattering. In Ref. [31] only \( \rho_{xx} \) was fitted, imposing as a constrain that the fitted curve should match the data for \( B = 0 \), as suggested in Ref. [71]. In the fitting procedure \( K_1 \), \( K_2 \) and \( K_{12} \) were the fitting parameters while \( n_1 \) and \( n_2 \) were separately calculated.

\( \rho_{xy} \) was calculated with the results of the fit demonstrating good matching between data and calculated curve.

Eqs. 4.1 and 4.2 consider just one effective mass for the two subbands. This assumption is justified in situations such as a \( n \)-type GaAs QW with two occupied energy eigenstates. As we will see in Chapter 5, the two subbands of our 2DHG are characterized by very different effective masses and a model like the one described in Ref. [71] is not expected to provide quantitative accuracy. For this reason we modify the model of Ref. [71] to take into account two different effective masses \( m_1 \) and \( m_2 \). Eq. 4.1 and 4.2 can be rewritten as:

\[ \rho_{xx} = \frac{1}{e^2} \text{Re} \left( \frac{1}{\text{Tr} \left( N (K - F)^{-1} \right)} \right) \] (4.4)

\[ \rho_{xx} = \frac{1}{e^2} \text{Im} \left( \frac{1}{\text{Tr} \left( N (K - F)^{-1} \right)} \right) \] (4.5)

where

\[ N = \begin{pmatrix} \frac{n_1}{m_1} & \sqrt{\frac{n_1 n_2}{m_1 m_2}} \\ \sqrt{\frac{n_1 n_2}{m_1 m_2}} & \frac{n_2}{m_2} \end{pmatrix} \] (4.6)

\[ F = \begin{pmatrix} \frac{eB}{m_1} & 0 \\ 0 & \frac{eB}{m_2} \end{pmatrix} \] (4.7)

As in Ref. [31], we fit \( \rho_{xx} \) only. \( K_1 \), \( K_2 \) and \( K_{12} \) are fitting parameters while \( n_1 \) and \( n_2 \) are obtained from the Fourier analysis of the SdH oscillations and \( m_1 \) and \( m_2 \) from their temperature dependences (see Chapter 5). Fig. 4.3 (a) shows \( \rho_{xx} \)
4.4. Classical positive magnetoresistance in two-subband system

Figure 4.3: (a) Low field magnetoresistance as a function of temperature. The curve at the lowest temperature (blue) is fitted to Eq. 4.4 (red). (b) Comparison between measured (blue) and calculated (red) $\rho_{xy}$. (c) Scattering rates as a function of temperature for the ungated sample. (d) Scattering rate as a function of temperature for the gated sample with $n = 2.6 \times 10^{-15}$ m$^{-2}$

as a function of temperature between 80 mK and 800 mK together with a fit of Eq. 4.4 (red) to the measured $\rho_{xx}$ (blue) at the lowest temperature. The result of the fit is $K_1 = 3.37 \times 10^9$ s$^{-1}$, $K_2 = 6.87 \times 10^9$ s$^{-1}$ and $K_{12} = -0.570 \times 10^9$ s$^{-1}$. Fig. 4.3 (b) shows the comparison between the measured $\rho_{xy}$ and the one calculated using the densities obtained from the spectrum of the SdH oscillations and from the scattering rates obtained from the fit. The behavior of the scattering rates as a function of temperature is shown in Fig. 4.3 (c). A similar analysis performed on the gated sample, with a top gate voltage such that $n = 2.6 \times 10^{15}$ m$^{-2}$, is shown in Fig. 4.3 (d). Interestingly the temperature dependence in $\rho_{xx}$ is very pronounced: the magnetoresistance drops from 61% at 80 mK to 12% at 80 mK. The behavior is metallic ($d\rho/dT > 0$), as expected from high quality samples dominated by long range impurities scattering.

From Fig. 4.3 (c) and Fig. 4.3 (d) we see that, as already noticed in Ref. [31], $K_{12}$ is small at low temperature, but increases much faster than $K_1$ and $K_2$ as the temperature rises. We also notice how $K_1$ has a stronger temperature dependence than $K_2$. This anomalous behavior clearly emerges when two distinct effective masses are considered, and was not reported in the past. Most of the data analyzed dur-
ing this thesis work show a behavior qualitatively similar to Fig. 4.3 (d), where the scattering rates are positive quantities. In Fig. 4.3 (c) instead $K_{12}$ is negative for low temperature. A negative $K_{12}$ was already reported in Refs. [72, 73] for a similar sample, characterized by strong SOI and high mobility. We checked the result with different values of effective masses and always found a negative $K_{12}$ at low temperature. The appearance of a negative $K_{12}$ should not alarm, since this term contains the inter-subband scattering rate weighted over the momentum transfer. The model used to fit the data contains three free parameters and always converges to a solution that accurately matches the data, independently of the densities chosen as inputs. While in most of the circumstances we obtain reasonable scattering times, there might be additional corrections to the resistivity not taken into account so far. If instead of fitting only $\rho_{xx}$ we fit $\rho_{xx}$ and $\rho_{xy}$ together, the obtained scattering times change by about 30% and the overall matching is poorer than the one shown in Fig. 4.3 (a). Further theoretical work in this direction might be useful to better understand the low field magnetoresistance of our 2DHGs, in particular concerning the angle dependence of the scattering rate and interaction corrections.

4.5 p-type GaAs 2DHG with Si$_3$N$_4$ gate dielectric

Electrical gating of a GaAs 2DHG proved to be challenging in the past. Metallic gates directly deposited on the surface are often leaky and give rise to strong hysteresis. This might be due to a limited Schottky barrier height as compared to $n$-type GaAs, and to the fact that the diode must be forward biased to have depletion. Attempts to use an insulating layer are very limited. In Ref. [31] an evaporated SiO$_2$ layer was used as a gate dielectric providing good tunability. Further attempts to use HfO$_2$ resulted in a good tunability of nanostructures but a limited tunability of a large 2DHG [69]. A similar behavior using an HfO$_2$ gate dielectric is described in Chapter 8. We report here a much improved bulk density tunability using a 200 nm Si$_3$N$_4$ gate dielectric gown by plasma enhanced chemical vapor deposition. The presence of the gate insulator decreases the hole density from $3.0 \times 10^{15}$ m$^{-2}$ to $2.1 \times 10^{15}$ m$^{-2}$. The application of a positive top gate voltage allowed tuning the density to complete pinch-off. We could satisfactorily measure the 2DHG conductance up to a voltage of $V_{TG} = 1.25$ V, where $n = 1.1 \times 10^{15}$ m$^{-2}$ and $\rho_{xx} = 7.5$ kΩ. Measuring at even lower hole densities was not possible as the application of a more positive gate voltage resulted in a pinch off of the Hall bar lateral arms. For a positive top gate voltage, we could sweep to pinch off and back multiple times without any hysteresis or change in the 2DHG properties. An example of two sweeps performed one after the other with positive gate voltage is shown in Fig. 4.4 (a). The application of a negative top gate voltage allowed to accumulate charge, up to a density of $2.8 \times 10^{15}$ m$^{-2}$ at $V_{TG} = -4.5$ V. On the other hand, this resulted in a permanent modification of the sample. Sweeping back to $V_{TG} = 0$ V, a density $1.3 \times 10^{15}$ m$^{-2}$ was found as shown in Fig. 4.4 (b). Once the sample is mod-
ified, we could still sweep the top gate voltage multiple times between \( V_{TG} = 0 \) V and \( V_{TG} = -4 \) V with limited hysteresis and good reproducibility. Interestingly the 2DHG properties are not affected by the hysteresis. Plotting the mobility as a function of density for all the data point shown in Fig. 4.4 (c) results in a straight line where the hysteresis loop is not visible (red dots in Fig. 4.4 (d)). For comparison, we show the mobility of the ungated sample with a blue square.

![Graphs showing hole density and mobility](image)

Figure 4.4: (a) Hole density as a function of top gate voltage for two successive gate voltage sweeps from 0 V to 1.25 V (blue dots) and back (red squares). The sweeps are performed immediately after cooling down the sample. (b) Hole density as a function of top gate voltage for two gate voltage sweeps from 1.25 V to \(-4.5 \) V (blue dots) and from \(-4.5 \) V to 0 V (red squares). (c) Comparison between the first hysteresis loop (empty dots) and the successive ones performed between 0 V and \(-4 \) V (red dots). (d) Mobility as a function of density for all the data points shown in (a), (b) and (c).

We further investigated the SOI tuning with a top gate voltage. In a new
cooldown with respect to the data shown in Fig. 4.4, we measured the tempera-
ture dependence of SdH oscillations at different densities. To avoid any hysteresis,
all the traces were taken in a single gate voltage sweep. For negative values of gate
voltage, it was necessary to wait at least 1 hour after a gate sweep for the sam-
ple to stabilize. The Fourier analysis of the oscillations allowed us to extract the
densities of the spin-orbit split subbands as a function of gate voltage. We further
calculated the Rashba $\beta_R$ parameter and the SOI energy gap $\Delta_{SO}$ as described in
Section 2.1.3. For $m_{hh}$, we used the low density value of the gate dependent effective
masses measured in Chapter 5. In Fig. 4.5 we show the summary of our results, we
will always compare the quantities measured in the gated sample (red dots) with
the ones measured in the ungated sample (blue square). We show only data points
at high densities, since no clear signature of SOI in the SdH oscillations was found.
4.6 Low density regime

4.6.1 Interactions correction

For very low hole density ($n < 1.5 \times 10^{15} \text{m}^{-2}$), the 2DHG magnetoresistance is completely different from what we have presented so far. The Hall bar resistivity for a density of $1.1 \times 10^{15} \text{m}^{-2}$ is shown in Fig. 4.6 (a). The longitudinal resistivity at zero field is two orders of magnitude higher than at high density and only the first two integer filling factors are visible. Fig. 4.6 (b) shows $\rho_{xy}$ measured in the same configuration while Fig. 4.6 (c) and Fig. 4.6 (d) show two zoom-ins of the data in Fig. 4.6 (a) and Fig. 4.6 (b) respectively. Interestingly the temperature dependence at low density is not metallic ($d\rho/dT > 0$) but insulating ($d\rho/dT < 0$) and a WAL peak appears close to zero magnetic field. $\rho_{xy}$ shows a temperature dependence as well, interpreted as a manifestation of hole-hole interaction effects (see Section 2.3.5). For higher densities we always observed a metallic behavior, even though a temperature dependence of $R_H$ was still present. $\rho_{xx}$ as a function of temperature for four different densities is shown in Fig. 4.7. We see that the WAL correction gets less pronounced by increasing the density and becomes invisible within our experimental resolution for a density of $1.94 \times 10^{15} \text{m}^{-2}$.

The conductivity correction due to hole-hole interaction can be calculated from the temperature dependent Hall constant $R_H$ as:

$$\frac{\delta \sigma_I}{\sigma_0} = -\frac{\delta R_H}{2R_H^0}$$

where $R_H^0$ and $\sigma_0$ are the Hall constant and the longitudinal conductivity mea-
Chapter 4. Magnetotransport in p-type GaAs/AlGaAs heterostructures

Figure 4.6: Temperature dependence of \( \rho_{xx} \) and \( \rho_{xy} \) for a density \( n = 1.1 \times 10^{15} m^{-2} \). All the subfigures were simultaneously measured between 75 mK and 870 mK. (a) \( \rho_{xx} \) as a function of magnetic field for different values of temperatures. (b) \( \rho_{xy} \) as a function of magnetic field for different values of temperatures. (c) Zoom-in of (a) for small magnetic field. (d) Zoom-in of (b) in a region where \( \rho_{xy} \) shows a pronounced temperature dependence.

The conductivity correction is always negative, meaning that interactions give a insulating contribution to the total conductivity. The overall metallic behavior observed for the four densities shown in Fig. 4.7 comes from different phenomena discussed later. A linear fit of \( \delta \sigma_I \) as a function of \( \ln T \) allows to extract the interaction parameter \( F^* \). For high density (\( n \geq 1.3 \times 10^{15} m^{-2} \)) \( R_H \) was found to saturate at high temperature, in fact the logarithmic correction to the conductivity is expected to appear only for \( k_B T / \ll \hbar / \tau_e \). For this reason the fit was performed just on the low temperature points, where the temperature decay is approximately logarithmic. The result of the analysis are shown in Fig. 4.8 (a) for the lowest four densities investigated. The small variation of \( R_H \) with temperature, together with the high value of \( \sigma_0 \), did not allow to extract \( F^* \) for higher densities. The obtained values of \( F^* \)
4.6. Low density regime

Figure 4.7: Temperature dependence of $\rho_{xx}$ at very small magnetic field for different hole densities. (a) $n = 1.18 \times 10^{15} \text{m}^{-2}$ (b) $n = 1.31 \times 10^{15} \text{m}^{-2}$ (c) $n = 1.54 \times 10^{15} \text{m}^{-2}$ (d) $n = 1.94 \times 10^{15} \text{m}^{-2}$

are comparable to what was found for $p$-type GaAs [74] and $p$-type SiGe [75, 76].

4.6.2 Weak anti-localization correction

WAL correction manifests itself as a small resistivity dip around zero magnetic field. Studying its shape and temperature dependence allows to measure the coherence length $l_\phi$ and the spin-orbit length $l_{SO}$ [2]. Previous reports of the observation of WAL in semiconductors include $n$-type GaAs [77], InGaAs/InAlAs quantum wells [78], InAs [79] and $p$-type GaAs [80, 81]. Typically the low field conductivity is fitted to either the Hikami-Larkin-Nagaoka (HLN) theory [56] or the Iordanskii-Lyanda Geller-Pikus (ILP) theory [82]. The first is more appropriate when the main spin-relaxation mechanism is the Elliot spin-orbit skew scattering mechanism, the second when the it is Dyakonov-Perel mechanism. Both theories are applicable in diffusive systems, for magnetic field smaller than the transport field $B_{tr} = h/(2el_0^2)$ and they both consider SOI perturbatively. Their applicability to systems with very high
SOI is still not properly addressed. A detailed analysis of WAL in $p$-type GaAs quantum wells in the high density regime can be found in Ref. [31]. There the WAL conductivity peak was fitted with the HLN theory. For a material with spin splitting proportional to $k^3$, as $p$-type GaAs, the HLN theory and the ILP theory are expected to be equivalent.

In this section we study WAL correction for $p$-type GaAs quantum wells. Differently from previous work we investigate the low density regime where WAL coexists with interaction corrections to conductivity. In order to analyze the WAL correction, it has to be isolated from the conductivity background. In order to do that, we first calculate the conductivity $\sigma_{xx}$ by tensor inversion (see Eq. 2.16). Assuming that two-bands transport can be neglected in this low SOI regime, and assuming no magnetic field dependence of the interaction correction, we fit $\sigma_{xx}$ with the equation:

$$\sigma_{xx}(B) = \frac{\sigma_0}{1 + \left(\frac{\tau_e B}{m^*}\right)^2} + \Delta \sigma$$

where $\sigma_0 = n e \tau_e$. $\tau_e$ and $\Delta \sigma$ are the fitting parameters.

Eq. 4.9 contains a classical Drude term and a constant offset taking into account additional contribution (for example an interaction correction contribution). The fit is performed using data point distributed at intermediate field, so that they are not influence by the WAL correction neither the SdH oscillations. An example of such a fit for the lowest temperature is shown in Fig. 4.9 (a), where the deviation from the classically expected behavior at low field is evident. $\delta \sigma_{WAL}$ is obtained subtracting the data and the fit at low magnetic field. The result of the subtraction is shown in Fig. 4.9 (b) (blue line). $\delta \sigma_{WAL}$ was vertically offset to have the apex at zero field at $\delta \sigma_{WAL} = 0$ and fitted using the theoretical model described in Ref. [82]:

$$\delta \sigma_{xx}(B) = \frac{e^2}{\pi h} \left( f_2 \left( \frac{B\phi + B_{SO}}{B} \right) + \frac{1}{2} f_2 \left( \frac{B\phi + 2B_{SO}}{B} \right) - \frac{1}{2} f_2 \left( \frac{B\phi}{B} \right) \right)$$

(4.10)
Figure 4.9: (a) Longitudinal conductivity $\sigma_{xx}$ together with a fit to Eq. 4.9 for data points at intermediate magnetic field. (b) WAL correction obtained subtracting the two curves of (a) (blue) together with two fit of Eq. 4.10 performed on two different magnetic field ranges. (c) Coherence length $l_\phi$ and spin-orbit length $l_{SO}$ as a function of temperature. The fitting range used for the analysis was 100 mT. Spin-orbit length as a function of temperature obtained from the data of (b) fitting in different magnetic field ranges.

Where $f_2(x) = \Psi(\frac{1}{2} + x) - \ln(x)$, $B_\phi = \frac{\hbar}{4e^2}$, $B_{SO} = \frac{\hbar}{4e^2_{SO}}$. The fitting parameters are $l_\phi$ and $l_{SO}$. The result of the fit of Eq. 4.10 is shown in Fig. 4.9 (b) for magnetic field ranges of 100 mT and 300 mT (red line and orange line respectively). For small magnetic field the fit accurately match the data independently of the fitting range used. For higher field the agreement is worse, and the results strongly vary with the chosen fitting interval. Fig. 4.9 (c) shows the obtained $l_\phi$ and $l_{SO}$ as a function of temperature for a fitting interval of 100 mT. In the same temperature range the elastic mean free path $l_e$ was found to be approximately constant and equal to 100 nm. Modifying the fitting interval results in a negligible variation of $l_\phi$, but in a strong change of $l_{SO}$. Fig. 4.9 (d) shows the calculated $l_{SO}$ as a
function of temperature for four different magnetic field ranges used for the fit. The variation of the results are as high as 30% for the fitting intervals choose and indicate the inaccuracy of the procedure in use for extracting $l_{SO}$. These results are consistent with the fact that the WAL shape is determined, for small values of $B$, mainly by $l_\phi$, while $l_{SO}$ determines the behavior at higher values of $B$. The procedure used to subtract the classical background is very crude and does not take into account the $B$ dependence of the interaction correction. For this reason it might result in large inaccuracies and systematic deviations for large values of $B$, where the shape is mainly determined by $l_{SO}$. For low temperature, where the central conductivity peak is very sharp, the results concerning $l_\phi$ are very robust against modifications of the fitting interval. For higher temperatures, when the peak becomes less pronounced, we found a weak dependence of the calculated $l_\phi$ on the magnetic field range where Eq. 4.9 was fitted to subtract the slowly varying background. A similar conclusion was reached in Refs. [31, 73] for similar 2DHGs, but in a completely different transport regime ($k_F l_e \gg 1$). Differently from the fit shown in Fig. 4.9 (b), where two conductivity minima appear at intermediate field, the experimentally measured $\delta \sigma_{WAL}$ does not show any signature of weak-localization on top of the WAL correction. This fact indicates that $l_{SO} \ll l_\phi$, and the $l_{SO}$ estimated from the fit is probably overestimated.

In Fig. 4.10 we show the calculated $l_\phi$ as a function of temperature for the lowest four values of density investigated (markers). Following the argument above, the data at higher temperature suffer from higher systematic errors than the ones at low temperature. $l_\phi$ increases with density (the main limitation in $l_\phi$ in our 2DHGs comes from the strong hole-hole interaction) and fast decays with temperature. We find a coherence length of 2 $\mu$m for a density of $1.54 \times 10^{15}$ m$^{-2}$. This value is very high when compared to similar 2DHGs (for example in Refs. [15, 18, 31] and Chapter 8, a similar value is measured for densities higher than $3 \times 10^{15}$ m$^{-2}$. The temperature decay was fitted with the power law $l_\phi \approx \gamma T^{\alpha}$ (black lines), obtaining $\alpha = -0.52 \pm 0.07$ for the four densities shown in Fig. 4.10 as expected for 2D diffusive systems where coherence length is limited by electron-electron interactions [36].

The lack of an accurate extraction methods for $l_{SO}$ limits further quantitative analysis of the temperature dependent conductivity. It would be interesting to analyze in more detail the temperature dependence of the Drude conductivity, obtained subtracting the interaction correction and the WAL corrections from the raw data. While the interaction correction to $\sigma_{xx}$ can be obtained from the temperature dependence of the Hall slope, identifying the absolute value of the WAL correction requires a more elaborate analysis. In particular a more advance method to subtract the classical conductivity background and the (magnetic field dependenent) interaction correction would be required to properly isolate the WAL correction. In order to do this, a more complete theory of WAL and hole-hole interaction and their magnetoresistance in hole systems with strong SOI is needed.

Despite the calculated interaction corrections $\delta \sigma_I$ give an insulating contribution, the overall temperature dependence for densities $n \geq 1.18 \times 10^{15}$ m$^{-2}$ is metallic.
The metallic contribution is not completely ascribable to the WAL correction. The temperature change in $\rho_{xx}$ is in fact larger than the WAL correction at low temperature and persists also when the WAL correction is negligible (see Fig. 4.7). What remains to be considered is temperature dependence of the Drude conductivity. In the case under consideration, the temperature dependence of the dielectric function leads to a decrease of the Drude scattering time and to a linear decrease of the conductivity with temperature. To a first order approximation, the temperature dependence of the conductivity is linear:

$$\sigma_{xx} = \frac{n e^2 \tau_e}{m^*} \left( 1 - C \frac{k_B T}{E_F} \right)$$

(4.11)

where the constant $C$ depends on the particular scattering mechanism.

![Figure 4.10: Temperature dependence of $l_\phi$ for four different densities where the WAL correction was visible (markers) together with a power law fit (black lines).](image)

4.7 Conclusion

We performed standard magnetotransport measurement on $p$-type GaAs 2DHGs Hall bars. In the high density regime, we could successfully observe the integer and fraction quantum Hall effect, SdH oscillations and a classical positive magnetoresistance. Our measurements, qualitatively similar to what was reported in previous
work, demonstrate the high quality of our 2DHGs and their high SOI strength. With a Si$_3$N$_4$ dielectric and a global metallic topgate we could tune the hole density in a much wider range compared to previous work. We proved the stability of the top gate in depletion, and discuss its limitations concerning accumulation. We proved a very large tuning of the SOI strength via top gate voltage. In the limit of low density, we observed interaction effects and WAL. We estimated the effects of the two corrections and we extracted useful quantities like the interaction parameter $F^*$ and the coherence length $l_\phi$. Limitations in applying the know models to our data are discussed.
Chapter 5

Effective masses in \( p \)-type GaAs two-dimensional hole gases

5.1 Introduction

The understanding of any semiconductor material starts with the knowledge of the carriers’ effective mass and its energy dependence. For the most important semiconductors, such as Si and GaAs, the electron effective mass has been widely investigated using temperature dependent transport and cyclotron resonance experiments [83–89]. For two-dimensional hole gases (2DHG) in GaAs the situation is significantly more complicated. Despite the importance of GaAs for fundamental research and technological applications, a detailed study of the effective mass of holes in GaAs 2DHGs grown along the high symmetry (001) direction remains to be done. The interpretation of the rapidly increasing number of experiments performed in 2DHGs requires a solid understanding of the physics underlying the effective mass value and its dependence on quantities such as hole density and spin-orbit interaction (SOI) strength.

The low-field longitudinal resistivity as a function of the magnetic field \( B \) is described by the Ando formula in the single subband case. The longitudinal resistivity \( \rho_{xx} \) oscillates around the classical Drude resistivity \( \rho_0 \) according to [52]:

\[
\rho_{xx}(B,T) = \rho_0 \left[ 1 - 2 \exp \left( -\frac{\pi}{\omega_c \tau_q} \right) \frac{2\pi^2 k_B T/\hbar \omega_c}{\sinh \left( 2\pi^2 k_B T/\hbar \omega_c \right)} \cos \left( 2\pi \frac{\hbar n}{2eB} \right) \right], \tag{5.1}
\]

where \( \tau_q \) is the quantum scattering time, \( \omega_c \) the cyclotron frequency, \( T \) the temperature and \( n \) the carrier density. The oscillatory term is given by \( \cos \left( 2\pi \frac{\hbar n}{2eB} \right) \), and is periodic in units of \( 1/B \). The carrier density is derived from the periodicity of \( \rho_{xx}(1/B) \). The carriers’ effective mass \( m^* \) in a two-dimensional system can be estimated from the temperature dependence of the low-field Shubnikov-de Haas (SdH) oscillations. Based on Eq. 5.1 the relative amplitude decay \( \Delta \rho_{xx}/\rho_{xx} \) of the oscillations of the longitudinal resistivity \( \rho_{xx} \) at a magnetic field \( B \) can be fitted
with the equation [36]:

$$\frac{\Delta \rho_{xx}}{\rho_{xx}} = 2 \exp \left( -\frac{\pi}{\omega_c \tau_q} \right) \frac{2\pi^2 k_B T / \hbar \omega_c}{\sinh \left( 2\pi^2 k_B T / \hbar \omega_c \right)} \, ,$$

(5.2)

where $T$ is the temperature and $\omega_c = eB/m^*$ the cyclotron frequency. The fitting parameters are $\tau_q$ and $m^*$.

As already discussed in Chapters 2 and 4, the strong SO-splitting in our 2DHGs can be observed from the presence of a beating in the low-field Shubnikov-de Haas (SdH) oscillations [31, 90–94]. In an approximate picture, the beating is due to the presence of different sets of SdH oscillations for the two angular momentum eigenstates (referred to as 1 and 2), that contribute to transport in parallel. Each set $i$ is characterized by a density $n_i$, an effective mass $m_i$, a Drude scattering time $\tau_i$ and a quantum scattering time $\tau_{qi}$. In this situation, and only in the limit of small magnetic field, Eq. 5.1 would then contain an additional oscillating term in the right hand side square bracket. In such a situation, it is more convenient to plot the power spectrum of $\rho_{xx}/(1/B)$ and calculate the subbands densities from the position of the maxima. Upon performing a Fourier transform of the longitudinal resistivity as a function of $1/B$, two peaks corresponding to the two subband densities are observed. The frequency axis $f$ can be directly mapped into densities $n$ by $n = fe/\hbar$. Since they are coupled by SOI, and since scattering and charge redistribution between subbands can be present, various non-linear terms are expected [95].

The presence of two sets of SdH oscillations due to the two subbands makes it difficult to extract the two effective masses separately. If the magnetic field onsets of the oscillations differ, one of the two effective masses can be deduced from the $\rho_{xx}$ oscillations where the contribution of only one subband is relevant. The other effective mass can then be inferred assuming parabolic bands, hence $m_1/m_2 = n_2/n_1$ as in Ref. [91], or assuming $m_1/m_2 = (\tau_2 / \tau_1)$ as in Ref. [92]. In Ref. [93, 94] a filtering technique was used to separate the different contributions in Fourier space, yielding the individual masses without further assumptions. Despite substantial differences in the effective mass values reported by previous works, the low density subband was always assigned a lower effective mass than the high-density subband. Therefore, the low density SO-split subband is referred to as light-heavy-hole (HHI) subband and the high density one as heavy-heavy-hole (HHH) subband. In Ref. [91] and [93, 94] a linear dependence of the effective masses with respect to magnetic field was observed. The origin of the magnetic field dependence remained unclear and the limited density tunability did not allow an investigation of the density dependence. We report here accurate measurements of the effective masses $m_1$ and $m_2$ of the two SO-split $\pm 3/2$ sub-bands in the limit of small magnetic fields. A pronounced difference between $m_1$ and $m_2$ (up to a factor of three) and the absence of any field dependence is observed. While the HHI effective mass is found to be independent of density, the HHHh effective mass shows a strong density dependence.
5.2 Measurements and numerical methods

![Graph](image)

Figure 5.1: (a) Longitudinal resistivity of the ungated sample for various temperatures, from 80 mK (blue line) to 800 mK (red line). (b) Power spectrum of the low temperature magnetoresistance (as a function of $1/B$) shown in (a). (c) Zero-field SO splitting as a function of carrier density for the ungated (square) and gated (dots) sample.

The effective mass measurements are performed on two samples obtained from the same wafer structure (D040817C). Each of the two samples consists of two perpendicularly placed $50 \times 100 \ \mu \text{m}$ Hall bars. One of the two samples remained ungated, while the other was covered by a 200 nm thick Si$_3$N$_4$ dielectric grown by plasma enhanced chemical vapor deposition and a global Ti/Au topgate. The ungated and gated samples are shown in Fig. 3.1 (a) and Fig. 3.3 respectively.

The ungated sample showed a density of $3.0 \times 10^{15} \ \text{m}^{-2}$ and a mobility of $65 \ \text{m}^2\text{V}^{-1}\text{s}^{-1}$. The presence of the gate insulator decreases the hole density to $2.1 \times 10^{15} \ \text{m}^{-2}$, the application of a top gate voltage allowed tuning the density from $2.8 \times 10^{15} \ \text{m}^{-2}$ to less than $1.0 \times 10^{15} \ \text{m}^{-2}$. An extensive discussion of quantities
like density, mobility and SOI in the two samples can be found in Chapter 4.

The experiment was performed in a $^3$He/$^4$He dilution refrigerator with a base temperature of 80 mK using standard low frequency lock-in techniques. Currents below 10 nA were used to avoid heating effects. The gated sample showed hysteresis in the top gate characteristic when the top gate voltage was set to high negative values. For this reason all the measurements were performed in a single gate voltage sweep. We first set the gate voltage to $1.15 \text{ V}$, where a density lower than $1.0 \times 10^{15} \text{ m}^{-2}$ was measured. We successively decreased the gate voltage up to $-4.5 \text{ V}$, where a density of $2.8 \times 10^{15} \text{ m}^{-2}$ was measured. For every value of gate voltage we first let the system stabilize for at least one hour at base temperature and then performed the temperature dependence starting at low temperature. Each temperature dependence consisted of more than twelve curves, with temperatures typically ranging from 80 mK to 800 mK. The entire gate voltage sweep took 31 days.

Fig. 5.1 (a) shows the longitudinal resistivity measured in the ungated sample as a function of magnetic field and temperature. At base temperature (blue line), $\rho_{xx}$ shows a beating pattern in the SdH oscillations while at 800 mK (red line) many SdH minima are completely suppressed and the remaining oscillations have a regular structure with clear $1/B$ periodicity. Fig. 5.1 (b) shows the power spectrum of $\rho_{xx}$ at 80 mK transformed as a function of $1/B$. The peaks corresponding to the HH1 and HH2 subbands are marked as $n_1$ and $n_2$ respectively. The $n_1$ peak is directly assigned since its frequency corresponds to the low-field periodicity of the SdH oscillations. The peak labeled $n_1 + n_2$ accurately matches the total density derived from the Hall effect. The difference in frequency between the $n_1$ peak and the $n_1 + n_2$ peak allows to identify the second subband peak, labeled $n_2$. The peak labeled $n_2 - n_1$ matches the difference between the subband densities while the $2n_1$ peak is a second harmonic of the $n_1$ peak. A detailed description of the numerical procedure used to transform the data can be found in Appendix A. The SO-splitting, quantified here as $\Delta n/n = (n_2 - n_1)/(n_2 + n_1)$ varies with gate voltage [31, 96]. The density dependence of the SO-splitting is shown in Fig. 5.1 (c) for the ungated (blue square) and the gated device (red dots).

5.2.1 Method A

We used two distinct methods to extract the effective masses from the temperature dependence of the SdH oscillations, referred to as Methods A and B. Method A is adapted from Ref. [93, 94] and consists of separating the different spectral components by finite-width spectral filters. Once a peak is isolated, its inverse Fourier transform reveals the corresponding SdH oscillations. The isolated oscillations are added to the slowly varying background, obtained by fitting $\rho_{xx}$ to a low-order polynomial, and the standard procedure to extract the effective mass is applied to the newly obtained data. Windowing the raw data set should be avoided here, since it can substantially modify the amplitude of different frequency components. The pre-
5.2. Measurements and numerical methods

Figure 5.2: Analysis using Method A in the ungated device. (a) Power spectrum of the low temperature magnetoresistance at density \( n = 3.0 \times 10^{15} \text{ m}^{-2} \) together with the filters used to extract the different components. The power spectrum has been normalized in order to compare it with the filters. No windowing is performed. (b) SdH oscillations obtained after inverse Fourier transforming the filtered spectrum, the oscillations have been vertically offset for clarity. (c) Effective masses deduced from the filtered oscillations as a function of magnetic field. \( m_1 \) and \( m_2 \) are the effective masses of the two SO-split subbands, \( m_3 \) and \( m_4 \) are fictitious effective masses describing the temperature dependences of the \( n_2 + n_1 \) and \( 2n_1 \) peak respectively. (d) Quantum scattering times of the two SO-split subbands.

The presented data are obtained using Gaussian windows as filters. The width of each filter is chosen to be as large as possible, to avoid both perturbing the shape of the peak and including spurious frequency components in the filtered data. We checked that the final results are independent of the particular filter shape and robust against moderate modification of the filter width. For very small magnetic field, due to limited oscillation amplitude, we could not satisfactorily fit the model to the data,
hence those points were excluded from the analysis. Fig. 5.2 (a) shows the filters used for analyzing the data of Fig. 5.1 and Fig. 5.2b gives the corresponding SdH oscillations. Fig. 5.2 (c) shows the effective masses obtained by fitting Eq. 5.2 to the minima of the filtered oscillations and Fig. 5.2 (d) the quantum scattering times obtained for \( n_1 \) and \( n_2 \). In contrast to previous works we clearly see that, in the limit of small magnetic field, the effective masses \( m_1 \) and \( m_2 \) do not depend on \( B \). As the magnetic field increases beyond about 350 mT we leave the validity range of Eq. 5.2 since the oscillations’ amplitude becomes comparable to the background level (about 50 Ω). Here, any analysis based on Eq. 5.2 should be avoided. Alternatively, the magnetic field in which the amplitude of the \( n_2 + n_1 \) oscillations becomes relevant (about 350 mT), can be used as the limit for the validity range of the analysis.

From the data points at low magnetic field we estimate \( m_1 = (0.374 \pm 0.003) m_e \) and \( m_2 = (0.88 \pm 0.01) m_e \), \( m_e \) being the free electron mass and \( \tau_{q1} = 23.4 \pm 0.8 \) ps and \( \tau_{q2} = 39 \pm 1.5 \) ps. The quantum scattering times are an order of magnitude lower than the Drude scattering times obtained from the classical positive magnetoresistance. The oscillations in \( m_1 \) and \( \tau_{q1} \) visible at small magnetic field are due to side peaks in the power spectrum in Fig. 5.2 (a). They originate from boundary effects in the Fourier transform and are totally suppressed by windowing the data, as shown in Fig. 5.1 (b). We further investigated the temperature dependence of the \( n_2 + n_1 \) and \( 2n_1 \) peaks, assigning them fictitious effective masses \( m_3 \) and \( m_4 \) respectively. The \( 2n_1 \) peak is the second harmonic of \( n_1 \). As expected, an analysis based on Eq. 5.2 gives an effective mass of \( 2m_1 \) [36]. The \( n_2 + n_1 \) peak has the strongest temperature dependence found, compatible with an effective mass of \( m_1 + m_2 \). The analysis could not be performed on other peaks due to their strong temperature dependence and small amplitude. In particular the \( n_2 - n_1 \) peak cannot be easily filtered from the low frequency background relevant at high temperature. Qualitatively similar results were obtained with the gated sample for densities larger than 2.5 \( \times 10^{15} \) m\(^{-2}\). Fig. 5.3 shows the same analysis performed in the gated sample when the top gate voltage was such that the total density was 2.7 \( \times 10^{15} \) m\(^{-2}\) and the SO splitting 21%. In this case \( m_1 \) takes similar values as above, but \( m_2 \) decreased to \((0.78 \pm 0.02) m_e \). The quantum scattering time are also considerably smaller than in the ungated sample. The analysis was not possible for smaller densities since the decrease in \( \tau_q \) and SO splitting makes the separation between peaks too small to apply sufficiently broad filters and avoid overlaps.

### 5.2.2 Method B

The second method, called Method B, relies on the temperature decay of the peaks in the power spectrum. Given a magnetic field interval, one can numerically construct \( \rho_{xx}(B) \) from the Ando formula [52] and Fourier transform it in order to compare the peak height with the measured data. The zero-field resistivity and the hole density are read from the experimental data while \( m^* \) and \( \tau_q \) are fitting parameters. To provide robustness to the procedure, the fit is performed on the amplitude of a peak
Figure 5.3: Analysis using Method A. (a) Power spectrum of the low temperature magnetoresistance at density \( n = 2.7 \times 10^{15} \text{ m}^{-2} \) together with the filters used to extract the different components. The power spectrum has been normalized in order to compare it with the filters. No windowing is performed. (b) SdH oscillations obtained after inverse Fourier transforming the filtered spectrum, the oscillations have been vertically offset for clarity. (c) Effective masses deduced from the filtered oscillations as a function of magnetic field. \( m_1 \) and \( m_2 \) are the effective masses of the two SO-split subbands, \( m_3 \) and \( m_4 \) are fictitious effective masses describing the temperature dependencies of the \( n_2 + n_1 \) and \( 2n_1 \) peak respectively. (d) Quantum scattering times of the two SO-split subbands.
experimental and the calculated resistivities. Fig. 5.4 shows the procedure for the two extreme cases where the method was applied. On the left side we see how the $n_1$ and $n_2$ peaks decay with temperature, on the right the peak amplitudes (markers) are fitted to the numerical model (solid lines). The results are indicated in the figure (errorbars are within ±5%), and are compatible with the quantitative findings of Method A. In the limit of small magnetic field, the obtained results do not show any dependence on the specific magnetic field windows chosen for the analysis.

Figure 5.4: Analysis using Method B. (a) Temperature dependence of the resistivity power spectrum in the ungated device. Dots and squares indicate the height of the $n_1$ and $n_2$ peak respectively. (b) Peaks heights as a function of temperature together with a fit (line). (c) and (d) The same as in (a) and (b) for the gated sample. The density was $2.1 \times 10^{-15}$ m$^{-2}$ and the SO splitting 13%. The insets are zoom-ins of the $n_2$ peak.
5.3 Results

Fig. 5.5 summarizes the result of our analysis. Both methods proposed here can be applied to obtain the two different effective masses when a clear SO-splitting is present, so for sufficiently high hole densities. Method A requires a larger SO-splitting than Method B, so data points are provided only for higher densities. When both methods are applicable, the obtained results nicely match providing consistency for the analysis performed. At low density only one peak is visible in the spectrum, hence only one effective mass $m_{1,2}$ is resolved. The HH1 effective mass is constant within the density range under study and equal to $0.38 \ m_e$. The HHh effective mass is instead strongly dependent on the carrier density, indicating a SOI induced non-parabolicity of the valence band, with a less than parabolic dependence on $k$. Both methods precisely determine the fitting parameters. The error bars reported here only refer to statistical errors, and are comparable to the estimated systematic errors in the measurements, e.g. a possible calibration error of the temperature read-out.

![Figure 5.5: Effective masses as a function of density. Comparison of the results for $m_1$, $m_2$ and $m_{1,2}$ obtained using Method A, Method B and self-consistent calculations.](image)

The experimental findings are in good agreement with theoretical predictions on the density dependence of the SO-split density-of-states effective masses at the Fermi energy in the limit $B \to 0$ of a GaAs 2DHG grown on the [001] surface. In these self-consistent calculations, performed by Roland Winkler in collaboration with us, the slope of the Hartree potential at the back interface of the quantum well was used as a fitting parameter to reproduce the SO-splitting measured for the density of $3.0 \times 10^{15} \ m^{-2}$. This slope was then kept fixed when modeling the
different densities tuned via a topgate. The final results are shown in Fig. 5.5 (solid lines). The calculated effective masses obtained in the limit $B \to 0$ show good agreement with the low-field experimental results both in terms of magnitude and trends. Caution should be paid when quantitatively comparing experimental results with self-consistent calculations. Different effective mass definitions can give rise to pronounced differences in the calculated results for a material system with strong band non-parabolocities and high anisotropies such as p-type GaAs. We remark that different experimental techniques or theoretical approaches give access to different properties of the system and could therefore result in slightly different effective mass values.

5.4 Conclusions

We extracted the effective masses of SO-split subbands in p-type 2DHGs grown along the [001] direction. Two different methods allow us to obtain the two effective masses separately. The high quality of our samples allows us to measure at very low magnetic field, where Eq. 5.2 is valid, and rule out the linear dependence of the effective mass on magnetic field observed in previous work. In the accessible density range the HHl effective mass is constant and the HHh effective mass shows a strong density dependence due to SOI induced non-parabolocities in the valence band. The experimental results are qualitatively confirmed by self-consistent calculations. The effective masses in hole systems are markedly different for the two SO-split subbands and strongly dependent on sample specific properties such as density and SOI strength. These results highlight the complexity of the valence band of GaAs.
Chapter 6

Spin-orbit interaction in \( p \)-type quantum point contacts

6.1 Introduction

Conductance quantization in a quantum point contact (QPC) is a universal phenomenon (see Section 2.4.1), observed in a large variety of samples and in a wide range of experimental conditions. First discovered for \( n \)-type GaAs [59], its observation was soon extended to other material systems including \( p \)-type GaAs [11–13]. Despite the technical difficulties related to their fabrication, \( p \)-type QPCs have always attracted strong attention. The reasons are multiple and include the strong Coulomb interaction and spin-orbit interaction (SOI) present in 2DHGs. The large effective mass of holes in \( p \)-type GaAs makes hole-hole interaction much stronger compared to their electronic counterparts, allowing the study of many-body related effects. An example in this direction is the so-called 0.7 anomaly, a plateau-like feature observable in many QPCs for conductance values around \( 0.7 \times 2e^2/h \). Despite the agreement on the intrinsic nature of this effect, a unified consensus on the origin of the 0.7 anomaly is still lacking. For an extended review of the topic, the interested reader can consult Ref. [97]. The first clear experimental observation of the 0.7 anomaly in \( p \)-type QPCs was presented in Ref. [98] for a (311)-grown double quantum well. The QPC was induced in the upper quantum well using metallic split gates, and the back well served as a back gate. Further investigations of the 0.7 anomaly in \( p \)-type QPCs included experiments in a high perpendicular magnetic field [99] and magnetic focusing experiments [100].

Another interesting feature of \( p \)-type QPCs, and probably linked to SOI, is their pronounced \( g \)-factor anisotropy: the effective \( g \)-factor varies according to the relative orientation between QPC axis, magnetic field, and crystallographic direction. The anisotropy was first discovered on (311)-oriented quantum wells [101]. A finite Zeeman splitting was observed when the in-plane magnetic field was applied parallel to the wire, but no Zeeman splitting occurred when the field was perpendicular to it. The in-plane \( g \)-factor \( g_\parallel \) for a field oriented perpendicularly to the wire was found to
be always smaller than 1 and to decrease with the plateau number $n$. The authors interpreted their results invoking an interplay of quantum confinement and spin-3/2 physics in hole systems. More recent experiments were performed on (001)-grown substrate, where eventual crystallographic anisotropies should not play a significant role. In Refs. [34, 35, 102] a similar result was obtained: $g_{\parallel}$ is finite if the in-plane field is applied parallel to the wire and vanishing if it is applied perpendicular to it. In the first case, $g_{\parallel}$ is typically smaller than 1 and increases with $n$. The $g_{\perp}$ is found to be much larger than the in-plane one and typically increases with $n$ too. One should remember that a large $g$-factor anisotropy is observed in bulk (001)-grown 2DHGs as well. As already described in Section 2.1.2, Zeeman splitting competes with confinement energy and results in a vanishing $g$-factor at the heavy hole band bottom. For finite $k$-vectors a mixing between heavy holes and light holes takes place and a finite $g_{\parallel}$ is expected. Since the magnetic field is applied along the quantization axis of the system, $g_{\perp}$ is expected to be much larger and a value of $g_{\perp} = 7.2$ is theoretically predicted [23]. The bulk $g$-factor anisotropy makes it challenging to directly measure the out-of-plane $g$-factor in the bulk with conventional techniques. It was argued [102] that, in the limit of high mode number, the out-of-plane $g$-factor of a $p$-type QPC should approach the 2DHG value. So far, despite the tendency of $g_{\perp}$ to increase with $n$ and the higher values of $g_{\perp}$ with respect to $g_{\parallel}$, no values of $g_{\perp}$ larger than 5 were reported. The physics of Zeeman splitting in $p$-type QPCs is still puzzling. In fact neither an explanation of the anisotropy neither the tendency of $g_{\parallel}$ and $g_{\perp}$ to increase with $n$ is given so far. While exchange interaction [12, 103, 104] is discarded as an origin for the $g$-factor enhancement, since it would result in the opposite behavior and since it was never observed in hole systems, the confinement enhancement of the $g$-factor attracts more attention [34, 35, 105].

Recently, new phenomena concerning the effect of SOI are predicted and observed [106–108]. They rely on the modification of the 1D band structure when both a strong SOI and a magnetic field are present. The interplay between the SOI field and the external magnetic field is expected to induce new gaps in the spectrum, hence new features in the conductance. The appearance of these gaps should depend on the relative orientation between the SOI field and the external magnetic field.

In this chapter we present measurements of very high quality QPCs embedded in carbon doped $p$-type GaAs 2DHGs grown along the high symmetry (001) crystallography direction. The high quality of our samples is confirmed by the observation of many conductance plateaus. We systematically measure the QPCs conductance as a function of gate voltage and magnetic field, using various orientations between magnetic field and QPC axis. Similarly to previous work performed on similar QPCs [34, 35, 102], we can measure a very pronounced $g$-factor anisotropy, and a strong dependence of the $g$-factor on the plateau number $n$. As observed in [34, 35, 105], the subbands cross in a high in-plane magnetic field oriented along the QPC axis. For the first time we observe an anti-crossing of states with opposite spin and crossing of states with the same spin when the magnetic field is oriented perpendicularly to the wire. The anti-crossing can be tuned and suppressed as a function of the tilt
angle. The in-plane and out-of-plane $g$-factors are measured for two QPCs. For a large QPC, and in the limit of high mode number we obtain $g$-factors very similar to the theoretically predicted value of 7.2.

The appearance of level crossings in an in-plane field and anti-crossing in an out-of-plane field is reconciled in a model based on the effective coupling terms in the two-dimensional Hamiltonian for heavy holes. We can quantitatively explain the crossing in an in-plane field invoking the existence of a recently proposed term proportional to $Bk^2$ [32, 33] in the effective Hamiltonian. From the QPC conductance in an in-plane magnetic field we characterize the quadratic term in both magnitude and direction. This new SOI term, whose existence was never proved before, explains the increase of $g_\parallel$ with subband index. Furthermore exploiting this new SOI could be useful for possible quantum computation applications using heavy holes [32].

The anti-crossings in an out-of-plane field are due to the cubic Rashba SOI and the resulting coupling between spin-split levels of different parity. A self consistent calculations can reproduce all the relevant features. This chapter describes how a QPC can be used to characterize the different terms in the SOI Hamiltonian. In the light of our results, we justify the approach of using a large tilt angle for measuring the out-of-plane $g$-factor of the QPC.

### 6.2 Zeeman energy for anisotropic $g$-factor

We consider the Zeeman Hamiltonian $H_Z$ for anisotropic $g$-factor in case $B = (B_\parallel, 0, B_\perp)$:

$$H_Z = \frac{1}{2} \mu_B B_\parallel g_\parallel S_x + \frac{1}{2} \mu_B B_\perp g_\perp S_z$$

(6.1)

Where $S_x$ and $S_z$ are Pauli matrices. The Hamiltonian can be expressed in matrix form:

$$H_Z = \frac{1}{2} B \mu_B \begin{pmatrix} g_x \cos \theta & g_x \sin \theta \\ g_x \sin \theta & -g_x \cos \theta \end{pmatrix}$$

(6.2)

The matrix eigenvalues are:

$$E_\pm = \pm \frac{1}{2} \mu_B B \sqrt{(g_\parallel \sin \theta)^2 + (g_\perp \cos \theta)^2}$$

(6.3)

The Zeeman energy is therefore:

$$\Delta E_Z = \mu_B B \sqrt{(g_\parallel \sin \theta)^2 + (g_\perp \cos \theta)^2}$$

(6.4)

### 6.3 Sample fabrication and measurement technique

The QPCs measured in this experiment are obtained from wafer D040817C, and are arranged in the cavity configuration shown in the AFM micrograph of Fig. 6.1. The sample was fabricated by electron beam lithography and shallow wet etching.
Chapter 6. Spin-orbit interaction in $p$-type quantum point contacts

The cavity, that was designed for a different experiment (see Chapter 8), includes three QPCs (named QPC1, QPC2 and QPC3) with two side gates each. QPC1 has a lithographic width of 240 nm, QPC2 and QPC3 have a width of 350 nm. Due to a side depletion of the etched trenches, the real width of the QPCs might be considerably smaller than their lithographic width. QPC1 and QPC2 are aligned along the (010) crystallographic direction, and in the following their orientation with respect to the externally applied in-plane magnetic field will either be parallel or perpendicular. QPC3 is misaligned by 30 degrees with respect to the other two. Each lead of the cavity is attached to at least two wide contacts, so that standard four-terminal measurements can be performed. The typical configuration for measuring the QPC conductance (in this example QPC1) is schematically depicted in Fig. 6.1. It includes an home made IV-converter that can apply a symmetric bias $\pm V_{SD}/2$ at its two terminal and measure at the same time the current $I$ flowing in the circuit. The diagonal voltage $V_D$ building up across the QPC is measured by a commercial low noise differential voltage amplifier. The QPC conductance is defined as $G = I/V_D$. It can be proven [55] that, if a perpendicular magnetic field is applied to the structure, the conductance measured in this way is insensitive to the Landau quantization in the bulk (for a particular magnetic field direction only). The sample also includes a large cross shaped region used to calibrate the angle between the magnetic field and the sample plane via Hall voltage measurements. We checked that the particular shape of the device does not qualitatively influence the effects described here. The voltages applied to the gates far away from the QPC under investigation can be largely varied without modifying the QPC conductance. The

Figure 6.1: AFM micrograph of the sample under study. The dark lines are etched trenches and the white spots are resist residues. The electrical scheme used to measure QPC1 is depicted on top.
6.4 QPC 1

6.4.1 Lever arm

Fig. 6.2 (a) shows the zero field linear conductance of QPC1 as a function of the voltage $V_G$ applied to its two sides gates. In the leakage free regime of the side gates, QPC1 shows five well developed conductance plateaus. Due to the presence of a finite contact resistance in the small 2DHG regions around the QPC, the plateaus values are slightly lower than expected. Fig. 6.2 (b) shows the non-linear differential conductance $G_{AC}(V_{DC})$ where characteristic diamonds and the 0.7 anomaly are visible.

In order to accurately extract the lever arm from the measurement of Fig. 6.2 (b), the total contact resistance must be calculated [102]. In the AC measurement presented in Fig. 6.2 (a), the contact resistance is given by the 2DHG regions between the QPC and the voltage probes. Knowing that the conductance value on a plateau is quantized in units of $2e^2/h$, the four-terminal contact resistance is readily extracted as a function of gate voltage (see Fig. 6.3 (a)). Subtracting the contact resistance from the measured QPC resistance allows one to obtain a QPC characteristic where the plateaus are precisely quantized, as shown in Fig. 6.4 (a).

The two-terminal contact resistance includes the measurement apparatus, the
cabling of the dilution refrigerator, the ohmic contacts and large portions of 2DHG; generally it will be gate voltage and bias voltage dependent. We first calculated the two-terminal contact resistance for zero DC bias, by comparing the two-terminal conductance $I^{AC}/V_{SD}^{AC}$ with the corrected four-terminal conductance obtaining a contact resistance more than an order of magnitude higher than the four-terminal contact resistance (see Fig. 6.3 (b)). The calculation of the bias dependence of the two-terminal contact resistance is not straightforward since both QPC and contacts have a non linear bias dependence. We assumed that, when the QPC is fully open, all the bias dependence is due to the contacts and that this bias dependence is the same for every gate voltage value. In this way we obtain the bias dependent correction to the two-terminal contact resistance shown in Fig. 6.3 (c). Combining Fig. 6.3 (b) and Fig. 6.3 (c) we obtain the color map of the two-terminal contact resistance shown in Fig. 6.3 (d). In this experiment the DC current was directly measured. If this is not possible, one can still calculate it by integrating the measured $\partial I^{AC}/\partial V_{SD}^{AC}$ from

Figure 6.2: (a) Measured linear conductance of QPC1 as a function of the gate voltage applied to the two side gates. (b) Transconductance as a function of gate voltage and source drain DC bias.
Knowing the two-terminal contact resistance as a function of gate voltage and source drain bias, allows one to calculate the effective source drain bias dropping at the QPC as:

\[ V_{\text{eff}}^{\text{DC}} = V_{SD}^{\text{DC}} - I_{\text{DC}} R_{C}^{2T} \]  \hspace{1cm} (6.5)

The measured non-linear conductance presented in Fig. 6.2 (b) is then reshaped and Fig. 6.4 (b) is obtained.

The level spacing can be measured directly from Fig. 6.4 (b) as the vertical extent of the diamond shaped regions. For the first level it is of the order of 1 meV and is reduced for the higher levels, comparably to previous measurements in p-type QPCs [35, 102]. The source drain bias directly modifies the Fermi energy in the QPC, it can then be used as a reference for measuring the energy tunability of the side gates. The lever arm, defined as \( \alpha_g = \frac{\partial V_{SD}}{2 \partial V_g} \) gives the change in Fermi energy in the QPC for a change in energy of 1 eV in the gates. In our case the lever arm.

Figure 6.3: (a) Four-terminal contact resistance of QPC1 as a function of gate voltage. (b) Two-terminal contact resistance of QPC1 as a function of gate voltage. (c) Bias dependent correction of the two-terminal contact resistance of QPC1. (d) Two-terminal contact resistance of QPC1 as a function of gate voltage and DC bias, calculated from (b) and (c).
can be extracted from Fig. 6.4 (b) as half of the slope of the subband levels (dark lines) as indicated in Fig. 6.5 (a) by the colored lines. The factor 1/2 comes from the fact that, in the electrical setup in use, a source drain voltage $V_{SD}$ corresponds to the application of voltages $\pm V_{SD}/2$ at the sample leads. The slope of the lines is reported in Fig. 6.5 (b) as a function of gate voltage together with a polynomial fit used to interpolate $\alpha_g$ on the entire gate voltage axis. For this analysis the right side of the first level (whose slope produces the first point on the right in Fig. 6.5 (b)) was excluded from the analysis due to the presence of a pronounced 0.7 anomaly.

Once the lever arm is extracted, it is possible to convert the gate voltage axis into an energy axis. Assuming an energy $E = 0$ for the gate voltage $V_0$, it results:

$$E(V_g) = \int_{V_0}^{V_g} \alpha dV_g$$  \hspace{1cm} (6.6)

The QPC linear conductance and transconductance as in Fig. 6.4 (b), but with
Figure 6.5: (a) Corrected transconductance of QPC1 as a function of gate voltage and source drain DC bias together with the lines used to extract the lever arm. (b) Side gates lever arm as a function of gate voltage (markers) together with a polynomial fit used to interpolate $\alpha$ on the entire gate voltage axis.

For both QPC1 and QPC2 we found that the lever arm is independent of magnetic field, and just depends on the gate voltage, similarly to what was found in Refs. [102, 110].

6.4.2 Zeeman splitting

The strongly anisotropic Zeeman splitting is one of the peculiarities of $p$-type QPCs. For a QPC in a wafer grown along the (001) direction, a striking difference exists if
the in-plane magnetic field is aligned parallel or perpendicularly to its axis. Fig. 6.7 clearly shows the presence of this anisotropy. In Fig. 6.7 (a) the magnetic field is applied perpendicularly to the 2DHG plane, and the levels show a clear Zeeman splitting. In addition to the Zeeman splitting, a bending of the levels towards higher energy (more negative gate voltage) is observed. The latter is due to the combined effect of Landau quantization and confinement potential in the transverse direction [55]. Fig. 6.7 (b) and Fig. 6.7 (c) show the QPC transconductance for an in-plane magnetic field oriented parallel or perpendicular to the QPC axis respectively. While Fig. 6.7 (b) shows Zeeman splitting, Fig. 6.7 (c) does not show any signature of splitting up to a field of 12 T.

\( g_\parallel \) is obtained by tracing the energy difference between Zeeman split levels \( \Delta E_\parallel \) as a function of magnetic field when the magnetic field is applied in the plane of the 2DHG. We manually identified the position of the levels as a function of gate voltage and magnetic field as shown in Fig. 6.8 (a) and subsequently converted the gate voltage axis into an energy axis. From the relation \( \Delta E_\parallel = g_\parallel \mu_B B \), \( g_\parallel \) can be
6.4. QPC 1

Figure 6.7: QPC1 transconductance as a function of gate voltage and magnetic field for three different magnetic field directions. (a) Magnetic field applied perpendicular to the 2DHG. (b) Magnetic field applied in the plane of the 2DHG and parallel to the QPC axis. (c) Magnetic field applied in the plane of the 2DHG and perpendicular to the QPC axis.

extracted by a linear fit of $\Delta E_{\parallel}(B_{\parallel})$. In Fig. 6.8 (b) we show the energy splitting obtained from Fig. 6.8 (a), and in Fig. 6.8 (c) the relative $g$-factors defined as the slope of a linear fit to the lines of Fig. 6.8 (b) for two different fitting ranges. As it can be seen in Fig. 6.8 (b), the Zeeman energy increases non-linearly for the second and third subbands (blue and green respectively). In such a situation the effective $g$-factor depends on magnetic field, and fitting $\Delta E_{\parallel}$ in two different magnetic field ranges (in this case $B < 2$ T and $B < 0.2$ T) gives profoundly different results.

6.4.3 Level crossing and anti-crossing

The extraction of the out-of-plane $g$-factor $g_\perp$ is more complex. As it can be seen from Fig. 6.7 (a), the levels do not just bend with increasing perpendicular field but have a tendency to repel each other too. This is evident in Fig. 6.7 (a), where states with opposite spins anti-cross. The anti-crossing is suppressed when the magnetic
field is applied in the plane, as in Fig. 6.7 (b) for the highest subbands. For the first three subbands, the available magnetic field does not allow to probe the presence of a crossing point.

We further investigated the effect of an in-plane field on the level anti-crossings. For this purpose, the sample was tilted with respect to the magnet axis and transconductance maps were acquired for different angles $\theta$. Fig. 6.9 shows four of these measurements, where the tilt angle $\theta$ is indicated in red. In this notation $\theta = 0^\circ$ indicates that the field is completely out-of-plane, while $\theta = 90^\circ$ indicates that the field is completely in-plane. For large tilt angles (Fig. 6.9 (a)), the anti-crossings between the 6th and 7th and 8th and 9th spin split levels are suppressed. The anti-crossing between the 10th and 11th levels appears to be still present for $\theta = 80^\circ$. Increasing the tilt angle will make the anti-crossing occur at higher in-plane field,
and could eventually make it disappear as for the lower levels. Unfortunately this possibility was not tested during the measurements.

The out-of-plane $g$-factor $g_\perp$ can not be extracted from the transconductance plot at $\theta = 0^\circ$ since the position of the energy levels in Fig. 6.7 (a) is strongly distorted by the presence of the anti-crossing. An approach as the one described above to extract $g_\parallel$ applied to the transconductance measurement at $\theta = 0^\circ$ would result in an underestimation of $g_\perp$. For this reason we used the data taken at a tilt angle of $\theta = 80^\circ$, based on the observation that at such an angle most of the anti-crossings are suppressed.

We manually tracked the position of the levels as shown in Fig. 6.10 (a) and converted the gate voltage difference between two spin split levels into an energy difference $\Delta E_Z$ using the gate dependent lever arm. The Zeeman energy due to the in-plane magnetic field is calculated using the in-plane $g$-factors found before. The Zeeman energy solely due to the out-of-plane field is given by:

$$\Delta E_\perp = \mu_B B \sqrt{\Delta E_Z^2 - (g_\| \sin \theta)^2}. \quad (6.7)$$
Figure 6.10: (a) QPC1 transconductance as a function of gate voltage and magnetic field as in Fig. 6.9 (a) superimposed to the lines used to calculate the $g$-factor. (b) Zeeman energy as a function of out-of-plane magnetic field for different subband index. The colors are the same as in (a), the curves have been horizontally shifted for clarity. (c) In-plane $g$-factor as a function of subband index when the Zeeman energy is fitted for $B < 0.6$ T (circles) and $B < 0.4$ T (squares), the colors are the same as in (a).

The Zeeman energy due to out-of-plane field is plotted in Fig. 6.10 (b) as a function of $B_\perp$. A linear fit of $\Delta E_\perp(B_\perp)$ allows us to find the mode dependent out-of-plane $g$-factors. One on hand, the fitting should be performed for sufficiently low out-of-plane magnetic field in order to avoid entering the quantum Hall regime, furthermore for high $B_\perp$ the Zeeman splitting becomes larger than the subbands splitting and the lever arm does not provide significant energy differences. On the other hand, the data at very low field are characterized by large uncertainties. In Fig. 6.10 (c) we show the fitted $g_\perp$ for two cases where the maximum field in which the fit was performed was 0.8 T and 0.3 T (circles and squares respectively), in both cases the minimum magnetic field used for the fit was 0.2 T. The results of the fit are shown in Fig. 6.10 (c). Despite the gate voltage dependent lever arm and the non-linear levels bending, the values of $g_\perp$ show a weak dependence of magnetic
field. With respect to previous measurements [34, 35, 102], where an eventual anti-crossing was not taken into consideration, we observe larger values of $g_\perp$. Caution should be paid when extracting the $g$-factors of the higher modes since the anti-crossings, not suppressed for this tilt angle, could result in an underestimation of $g_\perp$.

6.5 QPC2

6.5.1 Lever arm

We now turn our attention to QPC2, shown on the right in the AFM micrograph of Fig. 6.1. The following measurements are performed by simultaneously varying the two side gates of QPC2 in their leakage free range. The gate voltage axis shown here always refers to the rightmost gate of Fig. 6.1. QPC2 has a much larger lithographic width with respect to QPC1, hence a much smaller level spacing is expected. The linear conductance and non linear transconductance, where the effect of the contact resistance is already taken into account, are shown in Fig. 6.11 (a) and (b) respectively. The gate voltage lever arm, calculated as described above, is shown in Fig. 6.11 (c). The lever arm is considerably smaller than the one extracted for QPC1 and, in the available gate voltage range, it changes by one order of magnitude. We repeated the final bias measurements and lever arm extraction for different values of magnetic field and for different magnetic field orientation. As for QPC1, the lever arm in QPC2 is independent of magnetic field.

6.5.2 Zeeman splitting

Similarly to QPC1, also QPC2 shows a pronounced Zeeman splitting anisotropy, as it can be seen in Fig. 6.12. Tracing the level position as a function of magnetic field allows one to find the in-plane $g$-factors (Fig. 6.13 (a)). Differently from QPC1, the levels’ dispersion is linear in magnetic field for all the observed levels (Fig. 6.13 (a)). This more regular behavior with respect to QPC1 might be due, as discussed in the following, to the weaker confinement potential in QPC2. The first level does not split even for an in-plane field of 12 T, we then assign to the first plateau an effective $g$-factor of zero. The obtained $g$-factors have first a tendency to increase with the subband index, as observed in previous works [34, 35, 102]. For the first time we can see that the $g$-factors drop for a level index larger than 6. The tendency reversal naturally follows from the fact that $g_\parallel$ in absence of lateral confinement is, in a first order approximation, vanishing.

6.5.3 Level crossing and anti-crossing

QPC2 has a much larger lithographic width compared to QPC1, this makes the confinement energy smaller than the Zeeman energy already for small magnetic
Chapter 6. Spin-orbit interaction in p-type quantum point contacts

Figure 6.11: (a) Linear conductance of QPC2 as a function of gate voltage. (b) Transconductance of QPC2 as a function of gate voltage and source drain bias. (c) Side gates lever arm as a function of gate voltage (markers) together with a polynomial fit used to interpolate $\alpha$ on the entire gate voltage axis.

Field values. This is an optimum situation for better studying the phenomenology of the newly observed band anti-crossing in an out-of-plane magnetic field. Fig. 6.12 (a) shows the QPC2 transconductance as a function of gate voltage and out-of-plane field, Fig. 6.12 (b) is a zoom-in of Fig. 6.12 (a) for $B_\perp < 2.5 \ T$. It is possible to observe the presence of many level anti-crossings between states with different spin and band crossings of states with the same spin. Differently from all the others, the first level evolves unperturbed by the magnetic field up to $B_\perp = 2.5 \ T$ and subsequently splits into two branches. This anomalous behavior might be based on the same effect that produces the anti-crossings: the high energy spin branch of the
first mode is repelled by the low energy spin branch of the second mode and can not split until the latter moves to sufficiently high energy. A similar transconductance plot, where a delayed Zeeman splitting was observed in $n$-type GaAs QPCs, was interpreted as a manifestation of a spin incoherent regime [111]. The effect that we discuss here is substantially different, in fact in our case the first step at zero field has a conductance of $2e^2/h$ and not of $e/h$ as in Ref. [111].

The values of $g_\perp$ could not be measured from Fig. 6.12(a), due to strong distortions in the linear dependence of the Zeeman splitting introduced by the anticrossings. Therefore, we measure the level-dependent $g_\perp$ using a tilt angle of $86.7^\circ$ (data of Fig. 6.14(a)). Similarly to before, we tracked the levels’ position as a function of energy and total field $B$, obtaining the total Zeeman splitting $E_Z = \mu_B \sqrt{g_\perp^2 B_\perp^2 + g_\parallel^2 B_\parallel^2}$.

Using the values of $g_\parallel$ of Fig. 6.13(c), we extract $g_\perp$ for $n \leq 8$.

The linear fit was performed in two intervals of $B_\perp$ ranging from 0.1 T to 0.6 T and from 0.1 T to 0.4 T (dots and squares in Fig. 6.15(c) respectively). The first few
subbands have a limited Zeeman splitting (in gate voltage), hence we can fit their splitting up to a high value of magnetic field obtaining meaningful results. On the contrary, the high subbands have a Zeeman splitting that quickly becomes larger than the subbands splitting itself. For this reason the fit in the small magnetic field range are more accurate to extract $g_\perp$ for the high subbands, while both intervals should give comparable results for the lower subbands. Differently from the other levels, the first plateau stays unperturbed up to $B_\perp = 2$ T and subsequently splits into two branches. Because of this anomalous behavior, the value of $g_\perp$ was calculated from the splitting observed in Fig. 6.12 (a) for $\theta = 0^\circ$ and $B_\perp > 2$ T. Its value of 3.0 (diamond in Fig. 6.15 (b)) is compatible to our findings for $n > 1$. $g_\perp$ has the tendency to increase with $n$ up to $n = 5$ and subsequently saturates. The saturation, concomitant to the decreasing tendency of $g_\parallel$, is comparable with the theoretical expectation of $g_\perp = 7.2$ expected for heavy-holes in a bulk 2DHG
Figure 6.14: QPC2 transconductance as a function of gate voltage and magnetic field for different tilt angles $\theta$. (a) $\theta = 86^\circ$. (b) $\theta = 78^\circ$. (c) $\theta = 70^\circ$. (d) $\theta = 60^\circ$.

[23]. The residual presence of anticrossings for $n \geq 8$ leads to an underestimation of Zeeman splitting, and to a consequent decrease of the extracted $g_{\perp}$ (as for $N = 8$). As for QPC1, the main source of error in this analysis lies in the determination of $\alpha(V_g)$.

### 6.6 Discussion

Similarly to previous experiments performed on $p$-type QPCs, we observe a strong anisotropic Zeeman splitting and a level-dependent in-plane $g$-factor. As observed in Refs. [34, 35, 105], the subbands cross in an in-plane magnetic field oriented along the QPC axis (Fig. 6.12(c)) independently of their quantum numbers. Interestingly, when the field is out-of-plane, spin-split subbands form a complex pattern where both crossings and anticrossings appear. The high energy spin-split subband anti-cross with the low energy spin-split subbands of the neighboring energy level. After the anti-crossing takes place, spin-split levels approach each other and cross. The existence of these anticrossings for $B$ along $z$ (i.e., out-of-plane) suggests a strong influence of an in-plane SOI coupling field, which has a finite matrix
elements between the Zeeman eigenstates. On the other hand, the absence of anti-crossings for $B$ along $x$ (i.e., along the QPC) is consistent with such SOI coupling being proportional to $\sigma_x$. As we will show below, this interpretation is substantiated through a simple model describing the interplay of a SOI term quadratic in $k_\parallel$ and the cubic Rashba SOI coupling term for holes, which has previously allowed to explain the anomalous spin-polarization observed in hole QPCs through magnetic focusing \[108\]. The theoretical treatment that follows was developed by Stefano Chesi and was triggered by our experimental data.

The SOI Hamiltonian for heavy holes in GaAs given in Ref. \[32, 33\], and derived within the spherical approximation, reads as follow:

$$H_{SO} = \beta p_- p_+ \sigma_+ + i \alpha p_-^3 \sigma_+ + \gamma B_- p_-^2 \sigma_+ + \text{H.c.}$$  \hspace{1cm} (6.8)
where $\alpha$, $\beta$ and $\gamma$ are proportionality constants. $p_\pm = p_x \pm ip_y$, $B_\pm = B_x \pm iB_y$, $\sigma_\pm = \sigma_x \pm i\sigma_y$. The first two terms represent the Dresselhaus SOI and the Rashba SOI respectively. The same expression, only considering Rashba SOI, is given in Section 2.1.3. The third term is due to SOI between light-holes and heavy-holes and combines two effects: orbital coupling via nondiagonal elements in the Luttinger-Kohn Hamiltonian and magnetic coupling via nondiagonal elements in the Zeeman term [23]. The proportionality constant is $\gamma = 3\gamma_0\kappa\mu_B/(m_0\Delta)$, where $\gamma_0$ and $\kappa$ are Luttinger parameters and $\Delta$ is the energy splitting between heavy-holes and light-holes.

We first give evidence for a quadratic spin-orbit coupling of the type $H^{(2)}_{SO} = \gamma B_x (p^2 \sigma_x + \text{h.c.})/2$. To describe transport in the QPC, we assume harmonic confinement potential and a parametric dependence on $x$ of the lateral wavefunction (in the $y$ direction) [112]. The onset of a conductance plateau occurs when $k_x \simeq 0$ at the narrowest point of the QPC constriction and leads us to consider the following unperturbed Hamiltonian:

$$H_0 = \frac{p_y^2}{2m} + \frac{1}{2}m\Omega^2y^2 - \gamma B_x p_y^2 \sigma_x - \frac{g\mu_B}{2} B_z \sigma_z,$$  \hspace{1cm} (6.9)

with $k_\pm = p_x \pm ip_y$, $\sigma_\pm = \sigma_x \pm i\sigma_y$, $\Omega^2 = \omega^2 + \omega_c^2$, and $\omega_c = eB_z/m$. A form of $H^{(2)}_{SO}$, which takes into account crystal anisotropy, was discussed in Ref. [35] and gives the same result of Eq. (6.9). Notice that $H_0$ in Eq. (6.9) is restricted to two specific orientations of $B$, either along $x$ or $z$, while the more general case of $B$ in the $xz$-plane will be discussed later.

Figure 6.16: (a) Crossing fields $B_n^{(i)}$ extracted from Fig. 6.12(c) (red dots) and a fit to Eq. (6.11) assuming harmonic confinement (solid lines) or hard walls confinement (dashed lines). (b) Calculated $k_x = 0$ eigenvalues of $H_0^1 + H^{(3)}_{SO}$. The red arrows indicate anticrossing points between $n, n+1$ states, the black arrows indicate crossing points between $n, n+2$ states. We used $\hbar\omega = 0.4$ meV, $g\mu_B \simeq 7.2$, $m = 0.3m_0$, and $\hbar^2\alpha_{2,3} = 0.08 \times \gamma_{2,3} \text{ eVnm}^3$ (with $\gamma_{2,3}$ Luttinger parameters), close to Refs. [108] and to our findings of Chapters 4.
Chapter 6. Spin-orbit interaction in p-type quantum point contacts

For $B_\perp = 0$, the subbands energies from Eq. (6.9) are

$$E_{n,\pm} = (n - 1/2)\hbar\omega\sqrt{1 \pm B_\parallel / B_0},$$

(6.10)

with $B_0 = (2\gamma m)^{-1}$. The values of $B_\parallel$ at which the 1D levels cross are obtained from the condition $E_{n,+} = E_{n+i,-}$:

$$B_n^{(i)} = B_0 \frac{(n+i-1/2)^2 - (n-1/2)^2}{(n+i-1/2)^2 + (n-1/2)^2},$$

(6.11)

and are compared directly to the experimental results. In Fig. 6.16(a) we perform a fit to the crossing fields obtained from Fig. 6.12(c) up to order four (dots) and show that Eq. (6.11) (solid line) is able to reproduce the experimental crossings over a wide range of values of $n$ and $B_\parallel$. The single fitting parameter $B_0 = 31.2$ T is in reasonable agreement with the spin-orbit coupling strengths obtained from perturbative estimates. Using the formulas of Ref. [35], for a triangular well with the QPC along (010) and 2D density $n_s = 3 \times 10^{15}$ m$^{-2}$, we obtain $B_0 \simeq 80$ T. The obtained value strongly depends on the detailed form assumed for the confinement [35].

A valuable feature of Eq. (6.11) is the weak dependence on the specific form of the lateral potential assumed in Eq. (6.9), which allows one to single out the effect of the quadratic SOI coupling term. In particular, Eq. (6.11) is independent of the QPC confinement potential $\hbar\omega$. Considering an infinite well instead of the harmonic confinement is done substituting $n \rightarrow n + 1/2$ in Eq. (6.11) independently of the width $\Delta y$, which introduces small differences at moderate $n$ and gives the same large-$n$ asymptotic behavior $B_n^{(i)} \simeq B_0 i/n$. As a result, the experimental values of $B_n^{(i)}$ can be reproduced with similar accuracy (dashed line in Fig. 6.16(a)) and a similar value of $B_0 = 33$ T is obtained. In contrast, other quantities are generally rather sensitive to the form of the (unknown) lateral potential. For example, $g_\parallel$ for subband $n$ is obtained as $g_n = (2n - 1)\hbar\omega/(\mu B_0)$ and $g_n = (n\pi\hbar/\Delta y)^2/(m\mu B_0)$ for harmonic and rectangular confinement respectively. Furthermore the values have a strong dependence on the confinement parameters $\omega, \Delta y$ as well as on $n$. Independently of the specific confinement chosen, the increasing value of $g_\parallel$ with $n$ finds agreement with the experiment.

We now consider $B_\parallel = 0$ and the effect of the Rashba spin-orbit coupling. From a third-order perturbative calculation in the two-dimensional subbands, the following form of anisotropic spin-orbit coupling is obtained:

$$H_{SO}^{(3)} = -\left(\alpha_2 \{p_y, (p_x^2 - p_y^2)\} + 2\alpha_3 \{p_x, \{p_x, p_y\}\}\right)\sigma_x$$

$$+ \left(\alpha_2 \{p_x, (p_x^2 - p_y^2)\} - 2\alpha_3 \{p_y, \{p_x, p_y\}\}\right)\sigma_y,$$

(6.12)

where $\{a, b\} = (ab + ba)/2$ and $p_x = -i\hbar k_x - e/B_\perp y$. Taking $\alpha_{2,3} = \alpha$ and $B_\perp = 0$, Eq. (6.12) recovers the more familiar isotropic expression $i\alpha(p_+^2 \sigma_- - \text{h.c.})/2$ [113].
6.6. Discussion

Equation (6.12) allows one to explain the anti-crossings observed when $B_\parallel = 0$ (Fig. 6.12(b)) as due to the non vanishing off-diagonal matrix element between Zeeman eigenstates (for $B_\perp > 0$):

$$\langle n+1, \uparrow | H_{SO}^{(3)} | n, \downarrow \rangle = i(\Omega + \omega_c) (3\alpha_2(\Omega^2 + \omega_c^2) - 2(2\alpha_2 + \alpha_3)\Omega\omega_c) (nh\hbar/2\Omega)^{3/2}$$

(6.13)

Interestingly, $\langle n+2, \sigma' | H_{SO}^{(3)} | n, \sigma \rangle = 0$ since $H_{SO}^{(3)}$ is odd with respect to $y \rightarrow -y$ within our assumption that $k_\perp \simeq 0$. This feature is in agreement with the higher-order crossings or weak anti-crossings observed in the experiment. As shown in Fig. 6.16(b), the behavior of the eigenstates of $H_0 + H_{SO}^{(3)}(k_\perp = 0)$ as a function of $B_\perp$ is remarkably close to the experiment, considering our very simplified modeling of the QPC. Among the other features, our model allows for a non-monotonic behavior of the anti-crossing gaps with $n$, as observed both in Fig. 6.12(b) and Fig. 6.16(b), due to the fact that $\langle n+1, \uparrow | H_{SO}^{(3)} | n, \downarrow \rangle = 0$ at a specific value of $B_\perp$. The features discussed for the $\theta = 86.7^\circ$ data are well reproduced in the numerical results of Fig. 6.17(a). For smaller tilt angles, the general trend observed in Fig. 6.14 is also found in the simulations as visible in Fig. 6.17(b). The unphysical distortions appearing for high $n$ and large $B$ originate from the fact that, in this regime, the typical wavevector $k_y \sim \sqrt{nm\omega_c/\hbar}$ exceeds the small-$k$ validity region of the coupling terms considered here. Finally, the spin dependence of Eq. (6.12) is also in agreement with the absence of anti-crossings in Fig. 6.12(c) as, for $B_\perp = 0$ and $k_x \simeq 0$, $H_{SO}^{(3)}$ simplifies to $\alpha_2p_y^3\sigma_x$ and the spin-orbit perturbation is parallel to $B$. The same is not expected for a general SOI form: the Dresselhaus term derived in Ref. [32] yields $\beta(p_-p_+p_-\sigma_+ + \text{h.c.})/2 \simeq \beta p_y^3\sigma_y$, which would induce anticrossings also when $B$ is parallel to the QPC.

![Figure 6.17](image_url)

**Figure 6.17:** Calculated $k_x = 0$ eigenvalues of $H_0^{\parallel} + H_{SO}^{(2)} + H_{SO}^{(3)}$ for different values of tilt angle $\theta$. (a) $\theta = 86.7^\circ$. (b) $\theta = 78^\circ$.

Both $g_\parallel$ and $g_\perp$ have the tendency to increase with level index while for high level index they both have the tendency to decrease. The initial increasing tendency in $g_\parallel$ and $g_\perp$ was observed in numerous experiments and remained so far unexplained.
Chapter 6. Spin-orbit interaction in p-type quantum point contacts

[34, 35, 101, 102, 105]. Taking into account the quadratic SOI term allows one to naturally explain the $n$ dependence of $g_{\parallel}$. The following tendency reversal in $g_{\parallel}$ follows from the fact that $g_{\parallel}$ in absence of lateral confinement is, in a first order approximation, vanishing. In QPC2, $g_{\perp}$ has the tendency to increase with $n$ up to $n = 5$ and subsequently saturates. The saturation, concomitant to the decreasing tendency of $g_{\parallel}$, is comparable with the theoretical expectation of $g_{\perp} = 7.2$ expected for heavy-holes in a bulk 2DHG [23]. The residual presence of anticrossings for $n \geq 8$ leads to an apparent decrease of Zeeman splitting and a consequent decrease of the extracted $g_{\perp}$. The values $g_{\perp} < 7.2$ are consistent with theoretical predictions for narrow 1D wires with a strong heavy-hole light-hole mixing [105]. Similar reduced values of the bulk $g_{\perp}$ was measured from the optical spectrum of excitons (e.g., $g_{\perp} \simeq 2.5$ in Ref. [114]) and was related as well to light-hole heavy-hole mixing [115, 116].

In this framework, the level dependence of $g_{\perp}$ is compatible with the lower subbands having a stronger lateral confinement, as confirmed by finite bias measurements. As mentioned above, a narrow QPC (i.e., with width comparable to the well thickness) cannot be treated through $k$-dependent SOI terms. This limitation of our model might explain the discrepancy between the anomalous behavior of the first plateau and the large Zeeman splitting obtained for $n = 1$ in Figs. 6.16(b) and 6.17. In the regime where the anticrossings are suppressed and the lateral confinement potential is weak, the experimental $g_{\perp}$ is comparable to the expected value of 7.2.

6.7 Conclusion

We performed transport experiments on high quality $p$-type QPCs embedded in a 2DHG with strong SOI. We characterize the QPCs in terms of lever arm and level-dependent $g$-factor. Differently from previous work, we observe a complex pattern of crossings and anti-crossings in an out-of-plane magnetic field. In an in-plane field the anti-crossings are suppressed. These features are compatible with an effective SOI Hamiltonian where quadratic and cubic terms appear. We show that the in-plane magnetic field at which the levels cross can be used to measure the strength of the quadratic term neglecting other contributions. The cubic term shows its importance in an out-of-plane field, where our calculations are able to reproduce the observed crossings and anti-crossings pattern. The lack of anti-crossings in an in-plane field proves that the cubic and quadratic terms are aligned with respect to each other. To the best of our knowledge, this is the first experimental proof of the existence of the quadratic term in the SOI Hamiltonian for holes. Such a term is useful for exploiting holes system for quantum computing applications. Using a high tilt angle, we can measure $g_{\perp}$ as a function of subband index. In the limit of a very open QPC we recover the predicted $g_{\perp} = 7.2$ for heavy holes.
Chapter 7

Spin-to-charge conversion in mesoscopic cavities with strong SOI

7.1 Introduction

7.1.1 Mesoscopic spin Hall effect

The generation and detection of pure spin currents in a semiconductor nanostructure is of great interest for both fundamental research and possible spintronic applications. This task remained so far very challenging due to various reasons, for example because of the technical difficulties in injecting a spin-polarized current in a semiconductor nanostructure using ferromagnetic contacts or optical means. Alternatively to the use of ferromagnetic contacts or polarized light, the application of a large in-plane field can lead to spin polarization, and pure spin currents could be generated or converted into a voltage [117–120]. A different approach was theoretically considered in Ref. [121]. It was predicted that a longitudinal charge current in a large quantum dot generates a transverse spin current if the spin-orbit time in the dot is shorter than the electron dwell time. The generation mechanism solely relies on SOI in semiconductors and not on magnetic field or ferromagnetic materials. Such spin current has zero average but fluctuates from sample to sample with appreciable variations. Because of its similarity with the conventional spin Hall effect, this mechanism was called mesoscopic spin Hall effect. Ref. [121] gave rise to a lot of interest on the topic. In fact the generation of a pure spin current by all electrical means is highly desirable for spintronic applications and more detailed analysis soon followed [122–124]. Ref. [124] is particularly interesting since it predicts that, in the limit of very strong SOI, the average spin current amplitude is different from zero. The spin current average value is expected to increase with the SOI strength and to depend on the dot geometry.

The main experimental limitation for studying the mesoscopic spin Hall effect
was, so far, the lack of a direct experimental technique to probe the spin current in a standard semiconductor device at zero magnetic field. The realization of a new measurement protocol that relies on small magnetic field, purely electrical setup and semiconducting materials is promising for the development of spintronic applications in future semiconductor devices. In this chapter we discuss on the experimental realization of a recent theoretical proposal, reported in Ref. [125]. A pure spin current is generated in a chaotic cavity whose dimension is smaller than the elastic mean free path and coherence length, but much larger than the spin-orbit length. In addition to the source and drain leads of the cavity, the spin current measurement requires an additional contact. The conditions for the application of the measurement protocol are matched in the limit where the source and drain contacts (through which the electrical current is passed) carry many more modes than the third contact, and when the third contact shows both magnetic field and energy sensitivity. In this situation the symmetry of measured electrical current or voltage via the third contact as a function of an in-plane field reveals the presence or the absence of the spin current.

### 7.1.2 Spin-to-charge conversion

In a multi-terminal geometry, and in the linear regime, Onsager relations apply and any four-terminal conductance measurement is symmetric upon magnetic field and contact inversion. If a current \( I_{k,l} \) is passed between two generic contacts \( k \) and \( l \), and the voltage \( V_{m,n} \) building up between two generic contacts \( m \) and \( n \) is measured, the conductance \( G_{k,l-m,n} = I_{k,l}/V_{m,n} \) follows the rule:

\[
G_{k,l-m,n}(B) = G_{m,n-k,l}(-B) \quad (7.1)
\]

While no specific magnetic field symmetry holds for a generic four terminal conductance, in a two terminal geometry Eq. 7.1 simplifies and the result \( G(B) = B(-B) \) is obtained. The work described here applies in the situation where source and drain carry many modes, while the measurement contact carries one mode or less. In the described situation, Landauer-Büttiker formalism predicts (see the next section for a formal demonstration) that current or voltage measured through the third contact are symmetric in \( B \), unless three conditions simultaneously apply: a spin current is present in the third contact already at zero magnetic field and the measurement contact has both magnetic field and energy sensitivity.

We consider a cavity as the one depicted in Fig. 7.1. The cavity has three contacts (labeled 1, 2 and 3), and each of them is at a voltage \( V_i \) and carries \( N_i \) spin degenerate modes. We consider the situation where \( N_3 \leq 1 \) and \( N_2, N_3 \gg 1 \). By applying a voltage bias \( V_2 - V_1 \) to the large cavity leads, a charge current \( I \) will flow in the cavity. If contact 3 is grounded, a charge current \( I_{3}^{(0)} \) will flow in it. In the following we are interested in the situation in which no charge current flows in contact 3 at zero magnetic field. This situation can be achieved either by applying a voltage \( V_3 \) that sets \( I_{3}^{(0)} \) to zero at zero magnetic field, or by leaving it floating and connected
7.1. Introduction

Figure 7.1: Schematic representation of the considered cavity. The electrical current is driven from the top to the bottom thanks to the voltages $V_1$ and $V_2$. A spin current, indicated by the curved lines, flows in third contact via a QPC. Adapted from [125].

to a volt meter that measures $V_3$. In the first case the application of a magnetic field will make the current vary from zero, in the second case the current will always remain zero and the measured $V_3$ will vary. As we will show in the following, the zero field spin current $I_3^{(\alpha)}$ in contact 3 is directly proportional to the zero-field derivative of $I_3$ or $V_3$ with respect to the in-plane magnetic field:

$$I_3^{(\alpha)} = \frac{\hbar \omega}{\pi \mu} \partial_B I_3^{(0)} |_{B=0}$$  \hspace{1cm} (7.2)

$$I_3^{(\alpha)} = \frac{\hbar \omega e^2}{\pi \mu \hbar} \left(2 - T_{33}^{(0)} \right) \partial_B V_3 |_{B=0}$$  \hspace{1cm} (7.3)

where $\omega$, $\mu$, $T_{33}^{(0)}$ are QPC parameters that can be easily measured. Their meaning will be made clear later on.

Based on the theory described in [125], we realize a three-terminal chaotic cavity in our 2DHGs with strong SOI using three QPCs as leads. The QPCs conductance can be widely tuned, and the necessary conditions for the realization of the experiment are matched. Electrical measurements show the existence of an asymmetry of the voltage measured through the third contact as a function of an in-plane magnetic field. The asymmetry is found to depend on the third contact conductance and spin sensitivity, compatibly with theoretical expectations. Measurements as a function of the current amplitude, temperature and tilt angle support the measurement of a spin current exquisitely generated by SOI. Differently from the theoretical proposal of Ref. [125], the sign of the spin polarization is found not to vary with the cavity shape. The origin of this important difference is discussed in the light of a recent theoretical result for cavities with extremely strong SOI.
7.1.3 Theoretical background

We consider the three-terminal cavity depicted in Fig. 7.1. In the following theoretical treatment we do not include the presence of any orbital effect. The latter assumption can be partially achieved if no out-of-plane magnetic field is applied. An in-plane field can give rise to other orbital effects [66], that will not be accounted for in this section. We will always consider that the voltages applied to the system are within the linear response regime (i.e. very small compared to other energy scales).

The generic current \( I^{(\alpha)}_i \) at a contact \( i \) can be calculated using Landauer-Büttiker formalism, resulting in:

\[
I^{(\alpha)}_i = \sum_j \left( 2N_j \delta_{ij} \delta_{\alpha 0} - T^{(\alpha)}_{ij} \right) V_j
\]

(7.4)

where \( i = 1, 2, 3 \) denotes the leads and \( \alpha = 0, x, y, z \) denotes the spin polarization of the current. The generic transmission coefficient is:

\[
T^{(\alpha)}_{ij} = \sum_{m \in i, n \in j} T \left( t_{mn}^{(\alpha)} \sigma \sigma' t_{mn} \right)
\]

(7.5)

where \( \sigma^\alpha \) are spin matrices, with \( \sigma^{(0)} \) the identity matrix. The \( 2 \times 2 \) matrices \( t_{m,n} \) indicate the probability of an electron entering the cavity from the \( n \)-th mode of QPC \( j \) to exit the cavity from the \( m \)-th mode of QPC \( i \), their elements \( t_{m,n}^{\sigma \sigma'} \) take into account spin flipping. It can be shown that the transmission coefficients for charge and spin are:

\[
\tau^{(0)}_{ij} = \sum_{\sigma \sigma'} \tau^{\sigma \sigma'}_{ij}
\]

(7.6)

\[
\tau^{(\alpha)}_{ij} = \sum_{\sigma \sigma'} \sigma \tau^{\sigma \sigma'}_{ij}
\]

(7.7)

The transmission probabilities from contact 1 or 2 to contact 3 are assumed to take the form:

\[
T_{3i}^{\sigma \sigma'}(B) = \tau_{3i}^{\sigma \sigma'}(B) \Gamma (E_F - \sigma \mu B)
\]

(7.8)

hence they can be separated into a spin dependent part and an energy dependent part. The spin affects the second term only via Zeeman energy. In Eq. 7.8 it was assumed that the QPC has high energy sensitivity, hence \( \Gamma (E_F - \sigma \mu B) \) varies faster than \( \tau_{3i}^{\sigma \sigma'}(B) \) with \( B \). \( \tau_{3i}^{\sigma \sigma'}(B) \) are phenomenological parameters, describing the spin transmission of the QPC when it is fully open. Eq. 7.8 is valid only in the limit when an electron reflected back in the cavity from contact 3 has a negligible probability to come back to contact 3 again. This limit is achieved when \( N_1, N_2 \gg N_3 \).
Using Landauer-Büttiker expression for charge and spin current through contact 3 one has:

\[ I_3^{(0)} = \frac{e^2}{\hbar} \left( T_{31}^{(0)} (V_3 - V_1) + T_{32}^{(0)} (V_3 - V_2) \right) \]  \hfill (7.9)

\[ I_3^{(\alpha)} = \frac{e^2}{\hbar} \left( T_{31}^{(\alpha)} (V_3 - V_1) + T_{32}^{(\alpha)} (V_3 - V_2) \right) \]  \hfill (7.10)

Imposing \( I_3(0) = 0 \) allows to find an analytical form for the spin current:

\[ V_3 = \left( T_{31}^{(0)} V_1 T_{32}^{(0)} V_2 \right) / \left( T_{31}^{(0)} + T_{32}^{(0)} \right) \]  \hfill (7.11)

\[ I_3^{(\alpha)} = \frac{e^2}{\hbar} \left( T_{31}^{(\alpha)} (V_3 - V_1) + T_{32}^{(\alpha)} (V_3 - V_2) \right) \]  \hfill (7.12)

Equations 7.2 and 7.3 are obtained by evaluating \( \partial_B I_3^{(0)}|_{B=0} \) with constant \( V_3 \) and \( \partial_B V_3|_{B=0} \) for \( I_3^{(0)} = 0 \) respectively. For obtaining a simpler analytical form of these expressions we will make an assumption of the QPC transmission \( \Gamma \). The general results of this theory does not depend on the specific choice of \( \Gamma \). The energy dependent transmission probability in the QPC is given by the saddle point model \[126]\):

\[ \Gamma(E_F, V_g, B) = \frac{1}{1 + \exp \left( -2\pi \left( E_F - \alpha V_g - \sigma \mu B \right) / \hbar \omega \right)} \]  \hfill (7.13)

The partial derivative of the QPC transmission with respect to magnetic field is:

\[ \partial_B \Gamma(E_F, V_g, B) = \sigma \mu \partial_{V_g} \Gamma(E_F, V_g, B) \]  \hfill (7.14)

This allows us to write the magnetic field derivative of the charge transmission coefficients \( T_{3i}^{(0)} \) in terms of spin transmission coefficients \( T_{3i}^{(\alpha)} \):

\[ \partial_B T_{3i}^{(0)} = \partial_B \sum_{\sigma\sigma'} T_{3ij}^{\sigma\sigma'}|_{B=0} \]  \hfill (7.15)

\[ = \sum_{\sigma\sigma'} \tau_{ij}^{\sigma\sigma'} \partial_B \Gamma(E_F, V_g, 0)|_{B=0} \]  \hfill (7.16)

\[ = \sum_{\sigma\sigma'} \tau_{ij}^{\sigma\sigma'} \sigma \mu \partial_{V_g} \Gamma(E_F, V_g, 0)|_{B=0} \]  \hfill (7.17)

\[ = \sum_{\sigma\sigma'} \tau_{ij}^{\sigma\sigma'} \sigma \mu \Gamma(E_F, V_g, 0) \frac{\partial_{V_g} \Gamma(E_F, V_g, 0)|_{B=0}}{\Gamma(E_F, V_g, 0)} \]  \hfill (7.18)

\[ = \mu T_{3i}^{(\alpha)} \frac{\partial_{V_g} \Gamma(E_F, V_g, 0)|_{B=0}}{\Gamma(E_F, V_g, 0)} \]  \hfill (7.19)

In the middle of the first slope in the QPC conductance, the energy sensitivity is maximal and it results \( (V_g = E_F) \):
Chapter 7. Spin-to-charge conversion in mesoscopic cavities with strong SOI

\[ \frac{\partial V \Gamma(E_F, V_g, 0)|_{B=0}}{\Gamma(E_F, V_g, 0)} = -\frac{\pi}{\hbar \omega} \]

(7.20)

The zero-field derivative of the charge current is (\(V_1, V_2, V_3\) are constant):

\[ \partial_B I_3^{(0)}|_{B=0} = -\frac{e^2}{\hbar} \left( \partial_B T_{31}^{(0)}(V_3 - V_1) + \partial_B T_{32}^{(0)}(V_3 - V_2) \right) \]

(7.21)

\[ = -\frac{e^2}{\hbar} \left( \mu \frac{\partial V \Gamma(E_F, V_g, 0)|_{B=0}}{\Gamma(E_F, V_g, 0)} \right) \left( T_{31}^{(a)}(V_3 - V_1) + T_{32}^{(a)}(V_3 - V_2) \right) \]

(7.22)

\[ = \mu \frac{\partial V \Gamma(E_F, V_g, 0)|_{B=0}}{\Gamma(E_F, V_g, 0)} I_3^{(a)} \]

(7.23)

\[ = \frac{\pi \mu}{\hbar \omega} I_3^{(a)} \]

(7.24)

Eq. 7.2 is obtained solving Eq. 7.24 for \(I_3^{(a)}\). Similarly, the zero field derivative of \(V_3\) is:

\[ \partial_B V_3|_{B=0} = -\mu \frac{\partial V \Gamma(E_F, V_g, 0)|_{B=0}}{\Gamma(E_F, V_g, 0)} \frac{1}{2 - T_{33}^{(0)}} \frac{\hbar}{e^2} I_3^{(a)} \]

(7.25)

In the point of highest energy sensitivity we have:

\[ \partial_B V_3|_{B=0} = \frac{\hbar}{e^2} \frac{\pi \mu}{\hbar \omega} \frac{1}{2 - T_{33}^{(0)}} I_3^{(a)} \]

(7.26)

Solving 7.26 for \(I_3^{(a)}\) allows to find Eq. 7.3.

It is interesting, in the light of our experimental results, to investigate the behavior of \(\partial_B V_3|_{B=0}\) around the point of highest energy sensitivity. \(2 - T_{33}^{(0)}\) is the exact expression for the charge current transmission coefficient of contact 3. It can be approximated as (see Eq. 7.8):

\[ 2 - T_{33}^{(0)} = T_{31}^{(0)} + T_{32}^{(0)} = \sum_{\sigma \sigma'} \left( \tau_{31}^{\sigma \sigma'} + \tau_{32}^{\sigma \sigma'} \right) \Gamma(E_F, V_g, B) \]

(7.27)

We can apply the approximation from Eq. 7.8 to Eq. 7.10, finding a relation between \(I_3^{(a)}\) and \(\Gamma\):

\[ I_3^{(a)} = -\frac{e^2}{\hbar} \left( \sum_{\sigma \sigma'} \tau_{31}^{\sigma \sigma'}(V_1 - V_3) + \sum_{\sigma \sigma'} \tau_{32}^{\sigma \sigma'}(V_2 - V_3) \right) \Gamma(E_F, V_g, B) \]

(7.28)

Combining the last three equations we get the expression:

\[ \partial_B V_3|_{B=0} = -\frac{\mu \hbar}{e^2 C} \frac{\partial V \Gamma(E_F, V_g, 0)|_{B=0}}{\Gamma(E_F, V_g, 0)} \]

(7.29)
where $C$ is a prefactor containing the voltages $V_i$ and the coefficients $\tau_{3i}^{\sigma\sigma'}$. It is assumed to be constant with respect to gate voltage. The results obtained here can be summarized with the following proportionality relation:

$$\partial_B V_3|_{B=0} \propto \frac{\partial V_g \Gamma(E_F, V_g, 0)|_{B=0}}{\Gamma(E_F, V_g, 0)}.$$  (7.30)

It is worth reminding that Eq. 7.30 is valid in the limit $N_1, N_2 \gg N_3$ and $N_3 \leq 1$ and the coefficients $\tau_{3i}^{\sigma\sigma'}$ are supposed to weakly depend on magnetic field.

In the absence of SOI, reversing the magnetic field direction reverses the sign of the spin polarization $\alpha$. In this case $\tau_{3i}^{(0)}(B) = \tau_{3i}^{(0)}(-B)$ and $\tau_{3i}^{(\alpha)}(B) = -\tau_{3i}^{(\alpha)}(-B)$ (see Eq. 7.6 and 7.7). Since $\Gamma(E_F, V_g, B)$ is an even function of $B$, it results that $T_{3i}^{\sigma\sigma'}(B) = -T_{3i}^{\sigma\sigma'}(-B)$, and $I_3^{(0)}(0) = 0$ (see equation 7.12): in the absence of SOI, both $\partial_B I_3^{(0)}$ and $\partial_B V_3$ vanish.

![Figure 7.2](image)

Figure 7.2: (a). Comparison between the simulated spin current $I_3^{(\alpha)}$ and the zero-field derivative of the charge current $I_3^{(0)}$. (b) Spin and charge current in terminal 3 as a function of magnetic field. Points a to c refer to (a), d is the case with no SOI. Adapted from [125].

In Ref. [125], the results presented above have been numerically verified for a chaotic cavity embedded in $n$-type GaAs 2DEG. From Fig. 7.2 (a) we can see that spin current and magnetic field derivative of charge current in the third contact are always well correlated. Fig. 7.2 (b) takes into account three different situations depicted in Fig. 7.2 (a) concerning the amplitude and sign of the spin current plus a situation where no SOI is present in the cavity. Due to its chaotic origin, the spin current is expected to average to zero in a large ensemble of cavity shapes. Fig. 7.2 (a) clearly shows this behavior: varying the Fermi energy results in spin current fluctuations with zero average and with amplitudes depending on the microscopic shape of the cavity.
Chapter 7. Spin-to-charge conversion in mesoscopic cavities with strong SOI

7.2 Experimental realization

We realized two nominally identical devices from wafer D040817C, named Sample A and Sample B, each consisting of a three terminal chaotic cavity. If not stated otherwise, we will refer to results obtained from Sample A as its QPCs showed more plateaus than Sample B and proved to be electrically more stable than in Sample B.

The lateral dimension of the cavity is about 2 µm, the hole mean free path \( l_e = 4.8 \mu\text{m} \) and the spin-orbit length \( l_{SO} = 36 \text{ nm} \). \( l_{SO} \) is defined as in Ref. [124] as

\[
l_{SO} = \frac{v_F \tau_{SO}}{(\hbar k_F/m^*)(2\hbar/\Delta_{SO})} \quad \text{with} \quad \Delta_{SO} = 2\beta_R k_F^3
\]

\( v_F \) the Fermi wavevector, \( m^* \) the effective mass and \( k_F \) the Rashba parameter. Because of the large difference between \( m^* \) and \( k_F \) of the two spin-orbit split subbands, we use their average values \( m = 0.71 m_e \) and \( k_F = 1.43 \times 10^8 \text{ m}^{-1} \). The cavity is then in the spin chaos regime [124]. The shape of the cavity is intended to lack any spatial symmetry. The three leads consist of QPCs, referred to as QPC1, QPC2 and QPC3 and widely characterized in Chapter 6. Due to its smaller width, QPC3 is intended to serve as measurement QPC. The lithographic width of QPC3 in Fig. 7.3 is 240 nm, while the other two QPCs have a lithographic width of 350 nm. Two in-plane gates (named G4 and G5) are used to tune the conductance of QPC3. They are designed to have little influence on the cavity conductance. On the contrary, the other three gates (G1, G2, G3) not only tune the conductance of QPC1 and QPC2, but also the

![AFM micrograph of the cavity under study. The black lines are etched trenches, defining insulating lines. The depth of the trenches is about 30 nm. The gates and the QPCs are labeled as they will be referred to in the main text.](image)
cavity shape. QPC3 and QPC2 are both oriented along the (010) direction, QPC1 is misaligned by 30° with respect to the other two. The orientation of the in-plane magnetic field will either be parallel or perpendicular to QPC3, leading to a finite or vanishing $g$-factor respectively. The samples also included a large cross shaped region placed close to the cavity, where the tilt angle between the sample and the magnetic field was calibrated via the Hall effect. The samples were measured in a dilution refrigerator with a base temperature of 110 mK. The sample holder allowed in situ tilting the 2DHGs with respect to the magnetic field around a single axis. In-plane rotation of the sample with respect to the magnetic field required to fully warm up the sample and to manually change the bonding configuration.

7.3 Results

For the realization of the theoretical proposal described above, it is important to independently control the number of modes $N_i$ in each of the $i$ cavity leads. The numerical simulation reported in Fig. 7.2 is performed with $N_2$ and $N_3$ going from 2 to 3. The large gates G1, G2 and G3 allow to widely tune the cavity leads and independently address the number of modes in QPC1 and QPC2 with very little influence of QPC3. Fig. 7.4 (a) shows the cavity transconductance (numerical derivative with respect to the horizontal axis in arbitrary units) as a function of the three cavity gates. The horizontal axis shows the voltage applied to both G1 and G2, the vertical axis the voltage applied to G3. The measurement is performed by applying a constant AC bias of 15 µV between QPC1 and QPC2 and measuring at the same time the injected AC current and the four terminal voltage building up across the cavity. We can see a series of plateaus, in every of them the number of modes in both QPC1 and QPC2 is constant. In Fig. 7.4 (a) two different sets of dark lines appear in the transconductance plot, indicating the transition region between plateau in the two QPCs. The vertical lines are affected only by G1 and G2 and not by G3, so they are related to QPC1. The diagonal lines are similarly affected by G3 and G2, hence they belong to QPC2. To assign the mode number to each plateau, we measured the voltage drop across QPC1 and QPC2 using QPC3 as an additional contact. The measurement, performed at the same time as Fig. 7.4 (a), are shown in Fig. 7.4 (b) and Fig. 7.4 (c) respectively. In this two transconductance plots just the lines belonging to the QPC under study appear, and by measuring the QPC conductance one every step, we can directly assign the mode number $N_i$ as indicated in the figures. As we can see, QPC1 can not be tuned to pass less modes than 3 modes in the leakage free regime of the in-plane gates. Using Fig. 7.4 (b) and Fig. 7.4 (c) we can then assign to every plateau visible in Fig. 7.4 (a) a label $(N_1, N_2)$ indicating the number of modes in QPC1 and QPC2. During the measurement we used an additional current source set to a different frequency to constantly check the conductance of QPC3. As expected from the cavity shape, the conductance of QPC3 is only weakly affected by the three cavity gates within the voltage range.
Figure 7.4: (a) Cavity transconductance as a function of the voltages applied to G1, G2 and G3. G1 is always kept at the same voltage as G2. QPC3 is open for all the gate voltage range shown here. The number of spin degenerate modes \((N_1, N_2)\) of QPC1 and QPC2 is indicated on the picture. (b) QPC1 transconductance in the same voltage range as (a). The voltage drop is measured between QPC1 and QPC3. The number of modes is indicated on the picture. (c) QPC2 transconductance in the same voltage range as (a). The voltage drop is measured between QPC2 and QPC3. The number of modes is indicated on the picture.

shown here. In the rest of the chapter, we will always keep G1 and G2 at the same voltage and we will use Fig. 7.4 (a) to determine the number of modes present in the cavity leads.

To quantitatively measure the spin-current, we need to characterize the quantities present in Eq. 7.2 and 7.3 in QPC3. A detailed analysis of the in-plane \(g\)-factors, that can be found in 6.4.2, gave us \(g_\parallel = 0.27\) and \(g_\perp = 0.6\) for the first and second mode respectively. We determine \(\hbar \omega_x\), i.e. the curvature of the harmonic potential, by fitting a saddle point model [126] to the QPC conductance. The equation that we fit is:
7.3. Results

Figure 7.5: (a) Conductance of QPC3 at the first plateau (dots) and a fit to a saddle point model (line). (b) QPC3 conductance for different values of in-plane magnetic field.

\[
G(E) = \frac{2e^2N}{\hbar} \left( 1 + \exp \left( \frac{2\pi(E - E_0)}{(\hbar \omega_x)} \right) \right)^{-1} \tag{7.31}
\]

and the fitting parameters are a constant energy shift \(E_0\) and the potential curvature \(\hbar \omega_x\). \(N\) is the mode number of the plateau under consideration. We find \(\hbar \omega_x = 0.62\) meV and \(\hbar \omega_x = 0.25\) meV for the first and second mode respectively. A comparison between the data and the fitted curves is shown in Fig. 7.5 (a). The expected proportionality factor between spin current and charge current, given by \(\hbar \omega_x / \pi \mu\), is 25.7 T between pinch-off and first plateau and 4.5 T between first and second plateau. In Ref. [125] it was assumed that the point in the QPC characteristic with the highest conductance, is also the point with the highest magnetic field sensitivity. We can see from Fig. 7.5 (b) that this is not the case for QPC3, in fact the half-plateaus observable for high magnetic field do not form in the middle of the zero-field slopes. Furthermore, the first mode shows a pronounced 0.7 anomaly.
and does not form a fully quantized $e^2/h$ plateau even at the highest field available. Because of these non-idealities, an estimation of the magnitude of the generated pure spin current goes beyond Eq. 7.3.

In Ref. [125] two different ways to measure the spin current are proposed. The one mainly discussed in Ref. [125] consists in grounding QPC3 via a voltage generator that applies a constant voltage $V_3$ so that there is no current flowing in QPC3 at zero magnetic field. Sweeping the magnetic field would result in a change of the current $I_3$ flowing in QPC3. In this first configuration, the ohmic contacts of the sample would play a major role in determining the fraction of total current entering in QPC3 (considering the cavity as a simple current divider). Since in $p$-type GaAs the Ohmic contact resistance strongly depends on magnetic field, temperature and current [127], it would have been difficult to assign an eventual asymmetry in $I_3$ to a spin current or to other spurious effects. The second proposed configuration consists in leaving QPC3 attached only to a high impedance voltage amplifier and measuring $V_3$ as a function of magnetic field. The latter configuration is the one chosen for the experiment. In fact, compared to the first one, it does not require any manual tuning of the zero field bias that can result quite tedious if extended to a large number of cavity and QPC configurations. To completely suppress the effects of the ohmic contacts, we apply a constant AC current in the cavity and measure the voltage difference in a four-terminal configuration between QPC3 and the side of the cavity where the ground is. For completeness, the cavity four-terminal resistance is measured at the same time in a four-terminal configuration. Even though a four-terminal resistance is not expected to be symmetric in magnetic field, the large portions of 2DHG around the cavity have a negligible contribution to the total resistance. In this particular situation the cavity resistance is effectively measured in a two-terminal configuration and is expected to be even in magnetic field. The contacts used to pass a current and to measure a voltage are arranged in longitudinal configuration, so that an eventual angle misalignment would not result in the appearance of a Hall slope in the voltage signal.

A typical measurement is performed by setting the three cavity gates to have an integer number of modes $N_1$ and $N_2$ in QPC1 and QPC2 respectively as shown in Fig. 7.4 (a) and recording two dimensional plots of $V_C$ and $V_3$ as a function of in-plane magnetic field and voltage $V_g$ applied to gates of QPC3. Fig. 7.7 shows the result of such a measurement for $N_1 = 5$ and $N_2 = 4$, where Fig. 7.7 (a) is the cavity resistance ($R_C = V_C/I$) and Fig. 7.7 (b) the voltage $V_3$ divided by the current $I$. In Fig. 7.7 (c) and Fig. 7.7 (d) we show selected curves from Fig. 7.7 (a) and Fig. 7.7 (b) respectively. The colored dots in Fig. 7.7 (b) show the QPC conductance where the measurement above were taken. As expected the cavity resistance is symmetric in magnetic field while $V_3$ is not. From all the measurement presented in Fig. 7.7 we see how, on top of a slowly varying background, various quasi-periodic conductance fluctuations are present. The qualitative aspects of our data motivate an analysis in two parts. We first present a systematic study of fluctuations in the QPC and cavity voltage. We analyze their dependence on temperature, cavity charge current,
angle between sample and magnetic field and QPC effective g-factor. The results suggest that the origin of the high frequency fluctuations are phase coherence and orbital effects with respect to in-plane magnetic field and that they are not linked to Zeeman interaction or variability of spin currents in the cavity. In a second step, we focus on the gate voltage dependence of the zero-field derivative of the slowly varying background in the detector QPC voltage.

Figure 7.6: Measurement configuration for the spin-to-charge conversion experiment. A constant current $I$ enters the cavity from QPC1 and exits from QPC2. The voltages $V_C$ and $V_3$ are measured at the same time.

### 7.3.1 Conductance fluctuations

**Angle dependence**

In Ref. [66] it has been shown that in a chaotic system, similar to the one under consideration here, conductance fluctuations arise not only in the presence of an out-of-plane field, but also for an in-plane only magnetic field. The latter case is due to extrinsic effects such as the non-planarity of the 2DEG, as well as intrinsic ones linked to a modification of the energy bands (anisotropic mass enhancement) and breaking of time reversal symmetry. In this framework, the gate dependence is due to a modification of the cavity shape and of the carriers’ Fermi velocity. To exclude the possibility that the conductance fluctuations visible in our data originate from an out-of-plane component of the magnetic field, we compare data acquired with a small sample misalignment with respect to the external in-plane field. For this measurement we used the same settings as in Fig. 7.7 ($N_1 = 5$ and $N_2 = 4$) with the detector QPC set at $G_3 = e^2/h$. As can be seen in Fig. 7.8 (a), where the
Figure 7.7: (a) Cavity resistance as a function of gate voltage and in-plane magnetic field. (b) Voltage $V_3$ measured at the detector QPC divided by the current $I$ as a function of gate voltage and in-plane magnetic field. (c) Selected curves of (a) for different values of QPC3 conductance as shown in (e). The curves have been vertically offset for clarity. (d) Selected curves of (b) for different values of QPC3 conductance as shown in (e). The curves have been vertically offset for clarity.

curves have been vertically offset, characteristic maxima and minima appear at the same value of magnetic field for different sample orientations. In Fig. 7.8 (b) we compare three curves from Fig. 7.8 (a) without any vertical shift. We can see from both Fig. 7.8 (a) that all the curves look very similar, even though in certain cases the out-of-plane components have large and opposite magnitude. This excludes an origin of the conductance fluctuations linked to the out-of-plane component of the magnetic field. Interestingly, also the slowly varying background is not affected by a small out-of-plane field and always shows a positive slope. Fig. 7.8 (c) shows two
curves taken with nominally the same orientation, but with a time interval of nine days in between. During the nine days the sample was tilted several times in both directions. The variability seen in the curves is very limited and proves the stability and accuracy of our measurement setup.

Figure 7.8: (a) $V_3/I$ for different tilt angles. The data was acquired with the same parameters as Fig. 7.7, with the QPC gates set to have $G_3 = e^2/h$. (b) Three selected curves from (a) without vertical offset. (c) Two curves taken with nominally the same tilt angle. The sample was tilted between the two measurements.

**Gate voltage and magnetic field dependence**

In the previous section we showed how the conductance fluctuations visible in our cavity depend only on in-plane magnetic field and gate voltage. In this section we study in more detail their dependence on gate voltage and magnetic field for different cavity shapes. For the sake of comparison, we express the amplitude of the fluctuations in units of conductance $I/V_C$ and $I/V_3$, by dividing the applied current by the measured voltages. We evaluate the typical fluctuations amplitude
by removing the slowly varying background from the data with a digital filtering technique and calculating the root mean square (RMS) of the remaining data. We use a smoothing spline to filter the slowly varying background $y_i(x_i)$ defined as the function $s$ which minimizes the quantity:

$$p \sum_i (y_i - s(x_i))^2 + (1 - p) \int \left( \frac{d^2 s}{dx^2} \right)^2 dx$$  \hspace{1cm} (7.32)

where $p \in [0, 1]$ determines how fast the spline follows the measured curve. For $p = 0$ we have a linear regression, for $p = 1$ a cubic interpolation. For our analysis we choose $p = 0.5$ since this value allowed the fitted spline to follow the slowly varying background but not the oscillations. We checked that the results presented here do not sensitively vary with the value assigned to $p$. We also checked that similar results are obtained when using a completely different numerical technique, where the data are Fourier transformed, high-pass filtered and transformed back. The data presented here are obtained in various configurations of the cavity gates, and the RMS amplitude of the fluctuations is presented either as a function of gate voltage of the detector QPC or magnetic field. We used an AC current of 4 nA. In all the regimes investigated the fluctuations measured through the QPC detector are higher than the fluctuations in the cavity conductance. The ratio of their amplitude varies between 2 and 10. The area of the cavity probed by the QPC detector is about half of the entire cavity area. Since phase coherent phenomena are exponentially suppressed by phase breaking as a function of distance, the different oscillations amplitude does not imply a different physical origin of the conductance fluctuations in QPC voltage and cavity voltage. Furthermore, both fluctuations are characterized by a similar gate voltage and magnetic field quasi period, as it is visible from Fig. 7.7.

The fluctuations’ amplitude in the detector QPC and in the cavity voltage depends on the cavity transmission. The amplitude gets higher the more modes are transmitted in the cavity but do not depend on the detector QPC conductance or magnetic field sensitivity. Fig. 7.9 shows the fluctuations amplitude for four different sample configurations. For every QPC gate voltage the RMS amplitude with respect to magnetic field was calculated. Fig. 7.9(a), (b) and (c) are obtained with $N_1 = 5$ and $N_2 = 4$. Fig. 7.9(a) and (b) are measured in two distinct cool-downs of the sample where the detector QPC is oriented with respect to the in-plane magnetic field in order to have a non zero $g$-factor. Fig. 7.9 (c) is measured in a third cool-down where the sample is rotated with respect to the in-plane magnetic field in order to have zero $g$-factor (for more details see Chapter 6). Fig. 7.9 (d) is measured in the same cool-down as (b) but with the cavity gates set to the highest negative voltage possible without having leakage currents. We see that Fig. 7.9(a), (b) and (c) are characterized by similar fluctuations amplitude and that the amplitude is uncorrelated with the detector QPC conductance. We also see that, in three different cool-downs, the conductance fluctuations are uncorrelated with each other. Fig. 7.9 (d) shows the RMS of the fluctuations for a very open cavity geometry. Also
7.3. Results

Figure 7.9: Oscillations RMS amplitude (expressed as percentage of $e^2/h$) as a function of gate voltage for $B \in [-3 \text{ T}, 3 \text{ T}]$ in the cavity conductance $I/V_c$ (red) and QPC four terminal conductance $I/V_3$ (blue) compared to detector QPC conductance $G_3$ (black). (a) $N_1 = 5, N_2 = 4$, first cool-down. (b) $N_1 = 5, N_2 = 4$, second cool-down. (c) $N_1 = 5, N_2 = 4$, third cool-down with zero $g$-factor in the detector QPC. (d) The cavity gates are set to have the highest transmission in the cavity. Same cool-down as (b).

in this case we do not observe any correlation with the detector QPC conductance. Fig. 7.10 shows the analysis performed on the same data sets as Fig. 7.9, but the RMS is calculated, for every value of magnetic field, over the corresponding gate voltage range of Fig. 7.9. We can see a qualitatively similar behavior as in Fig. 7.9 concerning the RMS amplitude of fluctuations in the detector QPC. In this case the amplitude fluctuations have an RMS amplitude symmetric with respect to magnetic field, as it is expected from Onsager’s reciprocity. The limited amount of different cavity shapes obtainable in this gate voltage range does not allow to perform a
Figure 7.10: Oscillations RMS amplitude (expressed as percentage of $e^2/h$) as a function of magnetic field in the same gate voltage range as the corresponding subfigures in Fig. 7.9. We show the RMS amplitude of the fluctuations in the cavity conductance $I/V_c$ (red) and QPC four terminal conductance $I/V_3$ (blue) compared to detector QPC conductance $G_3$ (black). (a) $N_1 = 5, N_2 = 4$, first cool-down. (b) $N_1 = 5, N_2 = 4$, second cool-down. (c) $N_1 = 5, N_2 = 4$, third cool-down with zero $g$-factor in the detector QPC. (d) The cavity gates are set to have the highest transmission in the cavity. Same cool-down as (b).

meaningful statistical analysis of the fluctuations, as it was done in Ref. [66].

Temperature and current dependence

To prove the phase coherent origin of the conductance fluctuations, we study the temperature and current dependence of their amplitude. We study both the conductance fluctuations in the detector voltage and cavity conductance. This analysis will be useful also in the next part of the chapter, when we will discuss about the slowly varying background and its different temperature dependence. The measurement was performed with $N_1 = 5, N_2 = 4, G_3 = e^2/h$. The temperature dependence was measured with a current of 4 nA, while the current dependence was performed
at the base temperature of the fridge. The summary of our results are shown in Fig. 7.11. The reported values are always expressed in conductance, in percentage of $e^2/h$. Over the measured temperature range, the oscillations in $V_3/I$ and $V_C/I$ decay by one order of magnitude. An exponential fit of the amplitude decay (solid line in Fig. 7.11 (a) and Fig. 7.11 (b)) allows us to find a decay rate of $-6.4 \text{ K}^{-1}$ for both curves. The deviation from exponential behavior visible in our data in the limit of high temperature is partly attributed to the electronic noise present in our measurement apparatus. Repeating the analysis with noiseless low pass filtered data, we find a weaker deviation from exponential decay for high temperatures. These results suggest that these oscillations have a phase coherent origin. Estimating a phase coherence length from these measurement is not straightforward due to the difficulty in finding a typical length scale for the system. The current dependence was performed by varying the amplitude of the AC current injected in the cavity. Fig. 7.11 (c) and Fig. 7.11 (d) show the oscillation amplitude decay in the QPC voltage and in the cavity respectively. Similarly as for the temperature dependence a decaying behavior is found here too, this time with a less than exponential dependence on $I$. In the limit of small AC current we do not observe any saturation of the oscillations amplitude. This result is mainly interesting when it will be compared to the current dependence of the magnetic field slope of the slowly varying background.
Figure 7.11: (a) Oscillations RMS amplitude in $V_3/I$ as a function of temperature with an exponential fit. (b) Oscillations RMS amplitude in $V_C/I$ as a function of temperature with an exponential fit. (c) Oscillations RMS amplitude in $V_3/I$ as a function of current, the inset is a zoom-in for low currents. (d) Oscillations RMS amplitude in $V_C/I$ as a function of current.
Figure 7.12: Measurement and analysis for $N_1 = 5$, $N_2 = 4$. (a) $\partial B V_3 / I$ (red) compared to $\partial V_3 G_3 / G_3$ (blue). (b) $V_3 / I$ for the measurement in (a). (c) The same as in (a) but for a different cool-down. The horizontal axis has been shifted to make the QPC conductance $G_3$ measured in this cool-down match with the one measured in the same cool-down as in (a). (d) $V_3 / I$ for the measurement in (c). (e) Detector QPC conductance $G_3$ for different values of magnetic field. The measurement was taken during the same cool-down as in (a), the same measurement taken during the same cool-down as in (b) looks the same except for an overall gate voltage shift of 0.13 V.
Chapter 7. Spin-to-charge conversion in mesoscopic cavities with strong SOI

7.3.2 Spin-to-charge conversion measurements

We showed that the conductance fluctuations presented in the previous section, whose quasi-period is of the order of few hundred mT, have all the characteristics of orbital effects caused by change of cavity shape and in-plane magnetic field. They do not have any systematic dependence on the detector QPC conductance or magnetic field sensitivity. We will now focus our attention on the slowly varying background visible in Fig. 7.7 (b). Numerical simulations presented in Fig. 7.2 show that, an eventual spin-to-charge conversion in the detector QPC would result in a slope \( \partial_B V_3 \) weakly different from its zero field value for a magnetic field of the order of 1 T. Due to the presence of pronounced conductance fluctuations, we approximate \( \partial_B V_3 \mid_{B=0} \) with a linear regression fit to \( V_3 \) in a symmetric magnetic field range. The specific range limit of the fit is chosen according to the data set under consideration. If not stated otherwise, a fit in a symmetric 2 T range (between −1 T and 1 T) is adopted. The ideal situation is obtained when the range comprises many conductance fluctuations, so that on average they cancel out the effect they have on the slope, but at the same time the highest magnetic field reached does not perturb the zero-field spin current. In order to compare different measurements, and to study the current dependence of the effect, we will normalize the measured slope with the cavity current: the quantity presented here will be \( \partial_B V_3 / I \). Errorbars represent the 95% confidence interval of the estimated slope. The presence of eventual systematic errors is checked by varying the magnetic field range over which the fit is performed. If the magnetic field range is larger than 0.75 T, it typically includes more than five conductance fluctuations and the results are weakly affected by its modification.

Fig. 7.12 shows the summary of our analysis for \( N_1 = 5 \) and \( N_2 = 4 \). Following the discussion of Section 7.1.3, in particular concerning Eq. 7.30, in Fig. 7.12 (a) and Fig. 7.12 (c) we compare the quantities \( \partial_B V_3 / I \) (red) and \( \partial_V G_3 / G_3 \) (blue) for two different cool-downs. In Fig. 7.12 (e) we show the detector QPC conductance measured in the same cool-down as Fig. 7.12 (a). This curve is identical to the one measured during the same cool-down as Fig. 7.12 (b) except for an overall gate voltage shift of 0.13 V. For comparison we shift the horizontal axis in Fig. 7.12 (c) by 0.13 V. In both cool-downs the measured slope is very similar both in terms of magnitude and trends. Two pronounced maxima at \( V_g = 0 \) V and \( V_g = -380 \) mV and a minimum at \( V_g = -280 \) mV are completely reproducible despite the conductance fluctuations of the two cool-downs are uncorrelated, as visible in Fig. 7.12 (b) and Fig. 7.12 (c). The maxima and minima in \( \partial_B V_3 / I \) are slightly shifted to more negative gate voltage with respect to \( \partial_V G_3 / G_3 \). This can be due to the fact that, in our QPC, the point of highest energy sensitivity is not the point of highest magnetic field sensitivity, as implied in Ref. [125]. In fact, the application of an in-plane field does not result in a symmetric splitting of the energy levels. In Fig. 7.12 (e) we see that, for the first two levels, the first half of the steps is unaltered even at an in-plane field of 12 T and just the second half of the step evolves into a spin-split plateau. This non-ideality, linked to the competition between Zeeman splitting and electro-
static confinement, moves the highest magnetic field sensitivity away from the point of highest energy sensitivity. A direct comparison of Fig. 7.12 (a) and Fig. 7.12 (e) indicates how the first two maxima precisely fall on the points where both energy sensitivity and magnetic field sensitivity are high. For $G_3 > 6e^2/h$ it is still possible to notice similarities between $\partial B V_3/I$ and $\partial V_g G_3/G_3$, at least for Fig. 7.12 (a), we anyway remark that caution must be paid when many modes enter the detector QPC since the theoretical analysis is valid only in the limit $N_3 \leq 1$. For this data set it was found that the specific magnetic field range used for the linear regression does not affect the overall result, as long as it is larger than 0.75 T. The same data as Fig. 7.12 (a) was measured several times during the first cool-down, with time intervals up to two weeks. The obtained results are identical within the noise level. We also repeated the same measurement applying a voltage asymmetries of 40, 100 and 400 mV to the two QPC gates. In all those measurement we obtained a completely different conductance fluctuations pattern but the same $\partial B V_3/I$ as reported here (within errorbars).

We repeated the measurement and analysis of Fig. 7.12 for the regime with $N_1 = N_2 = 4$ and the summary for two different cool-downs is shown in Fig. 7.13. In this regime the amplitude of the fluctuations is smaller than with $N_1 = 5$, hence we could fit the data in the magnetic field range between $-0.5$ T and 0.5 T. The signal $\partial B V_3/I$ is shown in Fig. 7.13 (a), while the conductance fluctuations measured through the detector QPC are shown in Fig. 7.13 (a). In agreement with the theoretical predictions, and similarly to Fig. 7.12 (a), three peaks appear in $\partial B V_3/I$. Differently from the data of Fig. 7.12 (a), the data reported here have a constant positive slope as a function of $V_g$. Repeating the measurement in a second cool-down (Fig. 7.13 (c) and Fig. 7.13 (d)), we find a different result. In the latter situation we have a constant background similar to the first cool-down, but only a relevant peak close to the pinch-off of the detector QPC.

Several other cavity regimes were investigated. Fig. 7.14 (a) shows $\partial B V_3/I$ for $N_1 = 4$ and $N_2 = 5$. Similar to before, $\partial B V_3/I$ follows $\partial V_g G_3/G_3$ for the gate voltage range in proximity of the first conductance plateau of the detector QPC. For more negative gate voltage we do not find any correlation. When $N_1$ or $N_2$ get larger than 6, $\partial B V_3/I$ is always small compared to previously investigated regime, and with no correlations with $\partial V_g G_3/G_3$. As an example we show in Fig. 7.14 (b) the situation where the cavity gates were set to the most possible negative voltage, just before the onset of leakage currents.

**Angle dependence**

The regime with $N_1 = 5$ and $N_2 = 4$ shows the highest agreement with the theoretical expectations. We study it further by tilting the sample with respect to the external magnetic field. In this way we can probe if the slope we measure is an orbital effect linked to a residual out-of-plane magnetic field component. Fig. 7.15 summarizes the results of the angle dependence study. In Fig. 7.15 (a) we show
Chapter 7. Spin-to-charge conversion in mesoscopic cavities with strong SOI

Figure 7.13: Measurement and analysis for $N_1 = 4$, $N_2 = 4$. (a) $\partial B V_3/I$ (red) compared to $\partial V_g G_3/G_3$ (blue). (b) $V_3/I$ for the measurement in (a). (c) The same as in (a) but for a different cool-down. (d) $V_3/I$ for the measurement in (c).

$\partial B V_3/I$ acquired for different angles, while in Fig. 7.15 (b) we show its value for the gate voltage value where the highest peak of Fig. 7.12 (a) is visible ($V_g = -0.1$ V, see the vertical line in Fig. 7.12 (a)).

In the angle range studied here $\partial B V_3/I$ is always positive and, in the limit of small tilt angle, shows the same trend independently of the sign of the out-of-plane magnetic field. These results clearly show that the measured effect is not linked to the out-of-plane component of the magnetic field. The deviation from the zero angle value shown in Fig. 7.15 (b) varies according to the point selected for the analysis. The effect of a small out-of-plane magnetic field appears to be random.

Temperature and current dependence

In the regime with $N_1 = 5$ and $N_2 = 4$ we perform current and temperature dependence studies. These studies are an useful tool to probe the origin of the effect under
7.3. Results

Figure 7.14: (a) Spin-to-charge conversion experiment for \( N_1 = 4, N_2 = 5 \). \( \partial B V_3/I \) (red) compared to \( \partial V_g G_3/G_3 \) (blue). (b) Spin-to-charge conversion experiment for a completely open cavity. \( \partial B V_3/I \) (red) compared to \( \partial V_g G_3/G_3 \) (blue).

Figure 7.15: Angle dependence of \( \partial B V_3/I \) (a) \( \partial B V_3/I \) for various tilt angles. The angle of the 2DHG with respect to external magnetic field is displayed close to every curve. The curves were vertically shifted in steps of 100 \( \Omega \text{T}^{-1} \) for clarity. (b) \( \partial B V_3/I \) for \( V_g = -0.1 \text{ V} \) as a function of tilt angle.

consideration, in particular if this is of quantum nature. Fig. 7.16 (a) shows \( \partial B V_3/I \) measured at three temperatures. As we can see, the series of maxima and minima in \( \partial B V_3/I \) is suppressed in amplitude by temperature, converging to a flat background. We also fixed \( V_g \) to the first peak in \( \partial B V_3/I \) and measured \( \partial B V_3/I \) as a function of magnetic field for different temperatures. The result is shown in Fig. 7.16 (b). We see how temperature suppresses the background slope. The linear fit of the curves shown in Fig. 7.16 (b) is shown in Fig. 7.16 (c) in a logarithmic plot. Increasing the temperature from 109 mK to 535 mK suppresses the maximum of \( \partial B V_3/I \) by a factor of three. The decay with temperature appears to be exponential (the black line in Fig. 7.16 (c) is an exponential fit to the data), anyway the limited data range does
not allow us to extract a decay rate. These data suggest that the slowly varying background of \( V_3(B) \) is of quantum origin. Interestingly, albeit conductance fluctuations are totally suppressed at high temperature, we clearly observe the same gate dependence of \( \partial_B V_3/I \) for the three temperatures shown in Fig. 7.16 (a).

![Figure 7.16: (a) Temperature dependence of \( \partial_B V_3/I \) in the same configuration as Fig. 7.12. (b) \( V_3(B) \) for \( V_g = -0.1 \) V and different temperatures. (c) \( \partial_B V_3/I \) as a function of temperature, calculated from (b). (d) Current dependence of \( \partial_B V_3/I \) for \( V_g = -0.1 \) V.](image)

The current dependence was performed by varying the amplitude of the AC current \( I \) injected in the cavity from 500 pA to 50 nA. We can identify to different regimes: for low current the quantity \( \partial_B V_3/I \) is constant as a function of current, for high current \( \partial_B V_3/I \) decays with a less than exponential behavior. The slope decay starts for \( I \approx 5 \) nA. Considering the resistance of the cavity and the base temperature of 110 mK, we estimate that the linear regime is reached for currents lower than about 2 nA. For currents much larger than 2 nA the device is not in the linear response regime, and outside the validity range of the theoretical treatment discussed in Section 7.1.3. Interestingly the current dependence of \( \partial_B V_3/I \) for low
currents is different from the one of the conductance fluctuations, the latter are in fact found to continuously increase in amplitude as the current decreases (at least up to 500 pA). The current dependence highlights that the slowly varying background of $V_3(B)$ has a different origin than the conductance fluctuations. Since the slope is higher for very low currents we also conclude that what we are observing is a linear effect.

**$g$-factor dependence**

The effect we are interested in relies on the high energy and magnetic field sensitivity of the QPC used as detector. The data presented so far show that, in a restricted parameters setting, a finite $\partial_B V_3/I$ emerges when both conditions are satisfied. Tuning the energy sensitivity via gate voltage showed agreement with the theoretical predictions. The question arises whether we can tune the effect via tuning of the magnetic field sensitivity of the detector QPC. We perform this control experiment in two ways. First we check, in the same bonding configuration described above, if a similar signal can be measured using another cavity QPC. For this purpose we use the right QPC of Fig. 7.3 since, it can be completely closed and since, as we show in Chapter 6, the first mode has $g = 0$. Successively we turn the sample by $90^\circ$ with respect to the in-plane magnetic field to take advantage of the strong $g$-factor anisotropy of $p$-type GaAs. In this configuration we can perform the same measurement described above with QPC3 as detector, but with a vanishing magnetic field sensitivity. In both cases we expect $\partial_B V_i/I$ to be uncorrelated with $\partial V G_i / G_i$ and to be vanishing for $N_2 \leq 1$.

Fig. 7.17 shows a spin-to-charge conversion experiment performed using QPC1 and QPC3 as cavity leads and QPC2 as a detector. In this configuration we used the right cavity gate to tune the conductance of QPC2. Similarly to what we showed above, also in this case $V_C/I$ is symmetric with respect to magnetic field while $V_2/I$ is not (see Fig. 7.17 (a) and (b) respectively). Both signal show random conductance fluctuations. As it is evident from the raw data of Fig. 7.17 (b), $V_2/I$ tends to be symmetric with respect to magnetic field in the limit of low QPC2 conductance (right hand side of Fig. 7.17 (b)). Fig. 7.17 (c) shows $\partial_B V_2/I$ for two different cavity gates configuration (red and blue). In both cases there is no relation between $\partial V G_2 / G_2$ (black line) and $\partial_B V_2/I = 0$.

In order to use QPC3 as detector, but with no magnetic field sensitivity, we perform a third cool-down where the entire sample is rotated by $90^\circ$ with respect to the external in-plane magnetic field. In such a configuration QPC3 has zero $g$-factor for all the modes. Fig. 7.18 shows three examples of measurement performed with $g = 0$ in three different cavity configurations. Fig. 7.18 (a) and Fig. 7.18 (b) show $V_3/I$ and $\partial_B V_3/I$ respectively for $N_1 = 5$ and $N_2 = 4$. This is the same cavity configuration as shown in Fig. 7.12. In this case $V_3/I$ is more symmetric with respect to $B$ than in the first configuration, in particular when QPC3 is close to pinch-off (as it can be seen comparing Fig. 7.18 (a) and Fig. 7.12 (d)). $\partial_B V_3/I$ is uncorrelated with
Chapter 7. Spin-to-charge conversion in mesoscopic cavities with strong SOI

7.3.3 Sample B

We fabricated a second sample whose lithographic shape is nominally identical to the one shown in Fig. 7.3. This sample was measured in two different cool-downs, using the same orientation in both of them. The orientation was such that the $g$-factor of the detector QPC was finite. Unfortunately this sample proved to be less tunable than the one characterized above. QPC3 showed just the first conductance plateau and QPC1 and QPC2 could never be completely pinched-off and always stayed in the many modes regime. Furthermore QPC3 suffered from gate voltage dependent charge rearrangements that limited the experiment to only two different $g$-factors.

$\partial V_g G_3/G_3$ for all the gate voltage range and shows a flat background of the order of $20 \, \Omega T^{-1}$. Similar observation are made for $N_1 = 4$ and $N_2 = 5$ (Fig. 7.18 (c)) and $N_1 = N_2 = 4$ (Fig. 7.18 (d)). We remark that $20 \, \Omega T^{-1}$ is the noise level measured in the configuration with finite $g$-factor.

Figure 7.17: Spin-to-charge conversion using QPC2 as detector. (a) $V_C/I$ as a function of magnetic field and gate voltage. (b) $V_2/I$ as a function of magnetic field and gate voltage. $\partial B V_2/I$ as a function of gate voltage for two cavity configurations (red and blue) compared to $\partial V_g G_2/G_2$ (black).
cavity shapes, where the appearance of charge rearrangements was less severe. In this sample we performed similar measurement as for Sample A. A summary of the most important results is shown in Fig. 7.19. Fig. 7.19 (a) and Fig. 7.19 (c) show $V_3/I$ and $V_C/I$ respectively as a function of in-plane magnetic field, Fig. 7.19 (d) the detector QPC conductance as a function of magnetic field. Again we see that $V_C/I$ is always symmetric with respect to magnetic field while $V_3/I$ is not. Furthermore the asymmetry in $V_3/I$ is more pronounced when the detector QPC is close to pinch off. $\partial_B V_3/I$ for the data set of Fig. 7.19 (a) and (b) is shown in Fig. 7.19 (c) and indicated as Shape 1 (red). $\partial_B V_3/I$ for a different cavity configuration is indicated as Shape 2 (blue). These measurement show good agreement with respect to the previous data. For the first cavity shape we get a large slope only in coincidence with the last QPC step, for the second cavity shape the slope is always limited and uncorrelated with the QPC transconductance. With the gate configuration used to obtain the first cavity shape, we performed current, temperature and angle dependence of $\partial_B V_3/I$ obtaining qualitatively the same results as discussed for Sample A.

Figure 7.18: Spin-to-charge conversion using QPC3 as detector in a bonding configuration with $g = 0$. (a) $V_3/I$ as a function of magnetic field and gate voltage. (b), (c) and (d) $\partial_B V_3/I$ as a function of gate voltage for three cavity configurations compared to $\partial_{v_3} G_3/G_3$ (black).
Chapter 7. Spin-to-charge conversion in mesoscopic cavities with strong SOI

Figure 7.19: Spin-to-charge conversion in Sample B. (a) $V_3/I$ as a function of magnetic field for different values of gate voltage. The gate voltage at which the curves were taken goes from $-1.3 \text{ V}$ to $-0.45 \text{ V}$ in equidistant steps. (b) $\partial_{B}V_3/I$ as a function of gate voltage for two cavity configurations (red and blue). (c) $V_C/I$ as a function of magnetic field for different values of gate voltage. The gate voltage at which the curves were taken goes from $-1.3 \text{ V}$ to $-0.45 \text{ V}$ in equidistant steps. (d) Detector QPC conductance as a function of gate voltage for different values of in-plane magnetic field.

7.4 Discussion

The theory discussed in Ref. [125] applies in a very limited range of parameters, the most stringent one is $N_3 \ll N_1, N_2$. In this limit, calculations based on the Landauer-Büttiker formalism predict that $\partial_{B}V_3 = 0$. An exception to this rule arises when a pure spin current flows in the detector QPC already at zero magnetic field and the QPC shows energy and magnetic field sensitivity. Unfortunately we have no means to independently control or measure the magnitude of the spin current. What can be speculated from our observations is simply if the experiments, in
7.4. Discussion

the many but still limited number of different situations investigated, are consistent
with the theoretical expectations in case a pure spin current is (or is not) present.
We measure $\partial_B V_3$ by performing a linear regression of $V_3(B)$ in a carefully chosen
range of magnetic field. Due to the pronounced conductance fluctuations, this can
introduce systematic errors in the result. Since the presence of conductance flu-
ctuations is unavoidable in a ballistic cavity nanostructure, we checked the presence
of eventual systematic errors by varying the range of magnetic field over which the
fit is performed. Our analysis shows that the quantity $\partial_B V_3$ is not strongly affected
by the fit range used, when the fit range is large enough to include at least four
conductance fluctuations. We also tried to extract the low frequency component of
$V_3(B)$ by Fourier techniques or filtering procedures described in Section 7.3.1 before
fitting. The obtained results match the one found with a simple linear fit of the raw
data.

The magnetic field sensitivity of the detector QPC is a fundamental ingredient
for having a finite $\partial_B V_3$. The possibility to chose the $g$-factor by rotating the sample
or changing the detector QPC allows us to verify the first part of the theoretical pre-
diction of [125], hence that $\partial_B V_3 = 0$ when $g = 0$: even if there were a spin-current,
this could not be measured. The data reported in Fig. 7.17 and Fig. 7.18 agree well
with this expectation. In fact $\partial_B V_i = 0$ shows no correlation with the detector QPC
and is very limited ($\partial_B V_i/I \leq 20 \Omega T^{-1}$) when the detector is close to pinch-off.
The measurements with finite $g$-factor in QPC3 are quantitatively and qualitatively
different. They show that $\partial_B V_i/I$ can be much larger than $20 \Omega T^{-1}$ and vary up to
80 $\Omega T^{-1}$ as a function of gate voltage. Fig. 7.12, Fig. 7.13 and Fig. 7.14 (a) show vari-
ous cavity configurations where $\partial_B V_3/I$ systematically increases by about 60 $\Omega T^{-1}$
between the first conductance plateau of QPC3 and its pinch-off. In Fig. 7.14 (c) the
signal appears to oscillate randomly, but is always very limited in amplitude. The
measurements where $\partial_B V_3/I$ increases at the pinch-off show an evident discrepancy
with respect to Ref. [125], in particular there is an apparent horizontal shift between
$\partial_B V_3/I$ and $\partial_V G_3/G_3$. We speculated that this systematic shift is due to the fact
that in our QPCs the first spin-split plateau does not appear in the point of highest
zero-field transconductance, as implied in Ref. [125], but at slightly more negative
gate voltage. In this framework the position of maxima and minima in $\partial_B V_3/I$ is
determined by the interplay of gate dependent energy and magnetic field sensitivity.
Comparing Fig. 7.12 (a) and Fig. 7.12 (c) with Fig. 7.12 (e) shows that the position
of maxima and minima is quantitatively comparable with the gate voltage values
where our non-ideal QPCs show Zeeman splitting. What remains unclear is the
vertical offset of $\partial_B V_3/I$ observed in Fig. 7.13 (a) for the entire gate voltage range.
The same measurement performed in a second cool-down (shown in Fig. 7.13 (b))
shows a less severe background level of less than 20 $\Omega T^{-1}$ at the first plateau of
QPC3. We could speculate that the vertical shift of Fig. 7.13 (a) was due to an
accidental configuration of conductance fluctuations that mimicked a tilted $V_3/I$.
The fact that the shift disappears after thermal cycling favors this hypothesis. A
temperature dependence investigation of this regime could have solved the issue, but
unfortunately it was not performed. It is very suggestive that similar behavior of $\partial_B V_3/I$ for $N_3 \leq 1$ are observed in different cavity configurations, where the conductance fluctuation fingerprints are completely different, and also in different samples. In all our measurements we never observed a high $\partial_B V_3/I$ in correspondence to the first QPC detector plateau that decreased at pinch-off.

The angle dependence of $\partial_B V_3/I$ showed that such a signal is due to the in-plane component of the magnetic field. In fact the sign and the magnitude of $\partial_B V_3/I$ is conserved when a small out-of-plane magnetic field is applied, independently of the sign of the out of plane field. This excludes that the origin of the signal is linked to orbital effects in an out-of-plane field such as Hall effect (this is excluded a priori thanks to the longitudinal voltage configuration) or out-of-plane field conductance fluctuations. $\partial_B V_3/I$ could arise because of conductance fluctuations in a in-plane field. We checked that the temperature and current dependence of $\partial_B V_3/I$ is about a factor of three weaker than for the conductance fluctuations in the same regime, furthermore an asymmetry in $V_3(B)$ is clearly visible in Fig. 7.16 at high temperature when all the conductance fluctuations are suppressed, and the gate dependence of $\partial_B V_3/I$ persists unaltered when all conductance fluctuations are suppressed. The hypothesis that $\partial_B V_3/I$ originates from conductance fluctuations should be excluded also because it is not compatible with the peculiar gate voltage and $g$-factor dependence observed so far. In a semiconductor nanostructure in the ballistic regime there are classical magneto-electric effects arising when the probability of certain classical trajectories is enhanced [55]. The strong temperature dependence suggests that the effect observed is not of classical origin. Finally the current dependence shows that $\partial_B V_3/I$ is constant as a function of current for low current biases. When increasing the current above 5 nA, $\partial_B V_3/I$ gradually decreases. The analysis of the current dependence is not straightforward since two different effects take place. On one hand increasing the current increases the electron temperature in the cavity, leading to a decrease of $\partial_B V_3/I$ as already discussed. On the other hand a high current drives the cavity outside the linear regime, where the theory of Ref. [125] does not make predictions. Focusing on the limit of low currents we can conclude that the onset of $\partial_B V_3/I$ is within the linear regime. Furthermore the observation of an amplitude saturation at low current provides a further evidence that $\partial_B V_3/I$ is not linked to conductance fluctuations, since they are found not to saturate at least up to a current of 500 pA.

Our measurements show two distinct behaviors. Measurements like Fig. 7.12 (a) and (c), Fig. 7.13 (a) and (c), Fig. 7.14 (a) and Fig. 7.19 (b, Shape 1) are compatible with the presence of a measurable spin current. Other measurement like Fig. 7.19 (b, Shape 2) and Fig. 7.14 (b) are compatible with the absence of spin current. Interestingly all the first group of measurement suggests the presence of a spin current of the same sign (since $\partial_B V_3/I$ is always positive). The numerical simulations presented in Ref. [125] instead show a spin-current that randomly varies in amplitude and sign as a function of Fermi energy in the cavity (see Fig. 7.2). This could point to a different mechanism of spin current generation than what is proposed in Ref. [125].
The mechanism of spin current generation of Ref. [125] is based on the mesoscopic spin Hall effect theory presented in Ref. [121], where the transverse spin conductance of a mesoscopic cavity with strong SOI is investigated with the random matrix theory semiclassical approach [65]. It is found that the sign and amplitude of the transverse spin conductance randomly oscillate in amplitude and sign as a function of sample shape and Fermi energy. The average of the fluctuations is vanishing.

A more complete analysis performed in [124] found the existence of spin currents with non-zero average in a cavity with strong SOI. The average value is given by the form and strength of SOI and the geometrical shape of the cavity. A cavity like the ones used here is in the extreme spin chaotic limit, where spin currents with very high non-zero average are expected appear. Our results, where just one sign of spin current was detected, seem to corroborate the validity of the theoretical results of Ref. [124].

\[
I_3^{(S)} = \frac{e^2}{\hbar} \frac{2\hbar \omega_x}{\pi g_B} \left( \frac{\partial B V_3}{I} \right) I = 174 \text{ pA}
\]  

(7.33)

where the harmonic potential and the \( g \)-factor of the first QPC\(_3\) mode were measured to be \( \hbar \omega_x = 0.46 \text{ meV} \) and \( g = 0.27 \) respectively. The spin transmission of QPC\(_3\) calculated for a three terminal cavity in the spin chaos regime is:

\[
\langle T_{33} \rangle = \mathcal{O}(1) (1 + 2\xi) \frac{I_{SO}^{-1}}{2k_F N_1 N_3} \frac{N_1 N_3}{N_1 + N_2 + N_3} \approx 0.137 \mathcal{O}(1)
\]  

(7.34)

To evaluate we used \( \xi = 1 \), appropriate for a ballistic dot, \( N_1 = N_2 = 4, N_3 = 0.5 \). \( \mathcal{O}(1) \) is a system specific prefactor, of the order unity. Neglecting spin flip caused by QPC\(_3\) itself, the expected spin current is \( I_3^{(S)} \approx (e^2/\hbar) T_{33}^{(S)} (V_1 - V_2) = 134 \text{ pA} \) for a charge current of 4 nA, in reasonable agreement with our measurement.

The spin current generation mechanism proposed in Refs. [121, 128] is based on mesoscopic fluctuations of the spin conductance. Similarly to the conventional charge conductance fluctuations, such spin current has universal variation with sample parameters. We estimate the magnetic field correlation length of the spin or charge fluctuations as \( b_\parallel \approx h/|g_\parallel| \mu_B \tau_\phi \), where \( g_\parallel \) is the in-plane \( g \)-factor and \( \tau_\phi \) the coherence time. Based on measurement of similar samples (see Chapters 4, 7, 8), we obtain \( b_\parallel \approx 350 \text{ mT} \), in agreement with our observations. Even though the zero-average spin currents proposed in Refs. [121, 128] could be present in our samples, spin polarization in the high frequency fluctuations of \( V_3 \) could not be confirmed. In particular it was not possible to test the QPC\(_3\) transmission dependence of \( \partial B V_3/I \) for single fluctuations due to the influence of \( g_4 \) and \( g_5 \) on cavity shape.

### 7.5 Conclusion

In conclusion, we investigated mesoscopic three-terminals cavities with strong SOI. Using the spin-to-charge conversion measurement scheme, we detect and quantify
pure spin currents generated by all-electrical means. The nature of the measured spin current, constant in a large range of parameters, is compatible with the expected geometric correlation induced spin current in spin chaotic cavities. Our results pave the way for future electronic devices based exclusively on SOI.
Chapter 8

Aharonov-Bohm effect in $p$-type GaAs rings

8.1 Geometric phases in ring shaped nanostructures

The Aharonov-Bohm (AB) effect is not the only interference phenomenon occurring in a ring nanostructure. If the electron spin is taken into account, various interesting effects arise. Such effects typically take place via a Zeeman interaction between the electron spin and a magnetic field, where the magnetic field is given by the superposition of the externally applied field (typically perpendicular to the plane) and an additional magnetic field present in the ring (radial, tangential or more complex) [16, 17]. An example of such a magnetic field texture is depicted in Fig. 8.1 (a).

Figure 8.1: (a) One dimensional ring in a magnetic field texture. Adapted from [16].
8.1.1 Beating in the Aharonov-Bohm oscillations

The magnetic field presented in Fig. 8.1 (a) is described by the superposition of a perpendicular field and a radial field. An electron encircling the ring will experience a magnetic field constant in magnitude but with position dependent orientation. The angle between the total magnetic field and the electron’s eigenspinor depends on the ring (host material and radius) and on the magnitude of the total magnetic field. An interesting situation is reached in the adiabatic limit, when the electron spin has the time to encircle many times the total magnetic field before leaving the ring. The adiabaticity criterion reads:

$$\omega_B t_0 \gg 2\pi$$  \hspace{1cm} (8.1)

where $\omega_B = g\mu_B B/2\hbar$ is the Bohr frequency and $t_0$ a characteristic time of the electron motion in the ring. In this case the electron’s eigenspinors are aligned in the same direction as the magnetic field once the electron leaves the ring. An electron can thus enter the ring with two possible spin orientations along the $z$ direction and exit with two possible orientations along the direction of the textured field. This spin-dependent transmission probability can be proven to introduce an additional term in the AB phase presented in Section 2.4.2, and called Berry phase. The total transmission will now read:

$$T = 1 + \cos[\pi (1 - \cos \alpha)] \cos \left(2\pi \frac{\phi}{\phi_0}\right),$$  \hspace{1cm} (8.2)

where $\alpha$ is the tilt angle shown in Fig. 8.1 (a). In a ring where a radial magnetic field is present, Berry phase could be probed by measuring the ring transmission as a function of an external perpendicular field. In such a situation the AB oscillations would be modulated in amplitude, as shown in Fig. 8.1 (a). The position of the node (just one node is expected in this simple case) is given by $\alpha = \pi/3$.

The effect considered so far only applies to ballistic structures, but similar physics applies to diffusive systems as well. In the diffusive case the electron will, on average, spend more time inside the ring and, as a consequence, the external magnetic field needed to reach adiabaticity will decrease [129]. In the adiabatic regime the AB oscillations will be modulated similarly to before, and a beating structure is expected to appear in the ring transmission. The experimental realization of the textured magnetic field can be very challenging. A different approach consists in using the effective magnetic field created by spin-orbit interaction. The SOI field in fact points perpendicularly to the electron’s velocity and to the confining potential, hence radially to the ring structure. An important difference compared to the static radial field is that, in this case, the sign of the angle $\alpha$ depends on the sign of the electron velocity.

Experiments in ring-shaped nanostructures embedded in materials with strong SOI include InAs and p-type GaAs [15, 18, 41, 130]. In these experiments a beating pattern in the AB oscillations, or a splitting of the $\hbar/e$ peak in the transmission
8.1. Geometric phases in ring shaped nanostructures

A power spectrum was observed and interpreted as a signature of Berry phase. Examples of such data are shown in Fig. 8.2.

Figure 8.2: (a) Resistance of a p-type GaAs ring as a function of magnetic field. (b) Filtered $h/e$ oscillations obtained from (a). (c) Power spectrum of the ring magnetoresistance. Adapted from [15].

8.1.2 Aharonov-Casher effect

The Aharonov-Casher effect is the electromagnetic dual of the AB effect [64]. While the AB effect arises when a charge encircles a magnetic flux, the Aharonov-Casher effect arises when a magnetic moment encircles a linear distribution of charge. In such a situation the radial electric field produced by the line charge will be seen, in the moving reference frame of the magnetic moment, as magnetic field oriented parallel to the line. The realization of this experiment in a semiconductor nanostructure is again very challenging. A similar implementation consists in using a ring structure as the one described above, where the effective magnetic field is given by SOI, and study its transmission without the application of any external magnetic field. With respect to the original framework of the Aharonov-Casher effect, in this case the effective magnetic field does not point out of the plane, by radially in the plane. The absence of the external magnetic field results in non-adiabatic motion: having the electron’s eigenspinor aligned with the SOI field requires a ring with infinite radius or infinitely large SOI. It can be shown [131] that the transmission probability is modulated by the strength of SOI:

$$T = 1 - \cos \left( \pi \sqrt{1 + Q^2} \right),$$

(8.3)

with $Q = 2m^*\alpha_R r_0/h^2$. In this case $\alpha_R$ is the Rashba parameter (see Section 2.1.3) and $r_0$ the ring radius. Changing the value of $\alpha_R$, for example with an electrostatic gate, would result in a modulation of the ring transmission at zero magnetic
field. A first attempt to measure the Aharonov-Casher effect in a semiconductor nanostructure was realized in a HgTe ring [132]. In this case the superposition of AB and AAS oscillations created a complicated pattern, whose interpretation is not straightforward. Other experiments rely on large arrays of AB rings [133–135] embedded in InGaAs. Because of the dynamic phase term entering the AB phase (see Eq. 2.35), the $h/e$ contribution from the large ensemble of ring will tend to average to zero and the only magnetic field depending component will be the $h/2e$ oscillations. The amplitude of the $h/2e$ oscillations at zero magnetic field can then be used to probe the presence of SOI induced effects. An example in this direction is given in Fig. 8.3. Fig. 8.3 (a) shows the magnetoresistance of a large ring array as a function of perpendicular magnetic field. The oscillations are associated with the AAS effect. Fig. 8.3 (b) shows the array resistance as a function of both perpendicular field and top gate voltage. A clear modulation of the AAS oscillations at zero magnetic field is present (see Fig. 8.3 (c)). As expected from Eq. 8.3, the period of the modulation changes with the ring radius. The AAS oscillations amplitude as a function of SOI strenght is shown in Fig. 8.3 (d) for different ring radii.

![Figure 8.3](image_url)

**Figure 8.3:** (a) Magnetoresistance of a large array of rings. Only the AAS oscillations are visible. (b) 2D map of the resistance of a large array of rings as a function of magnetic field and top gate voltage. (c) Section of (b) at zero magnetic field. (d) Amplitude modulation of the AAS oscillations for different samples with different ring radii. Adapted from [134].

### 8.2 Introduction to the experiment

Motivated by recent experiments reported in [15], we study AB rings embedded in p-type GaAs heterostructures. A fabrication technology slightly different from the work reported in Ref. [15], allows a much larger electrical tunability of the nanostructure. The overall quality of the fabricated rings is demonstrated by the measurements of highly visible $h/e$ and $h/2e$ oscillation at different gate voltage settings.
Like in Ref. [15], a clear beating pattern of the $h/e$ and $h/2e$ oscillations is present in the magnetoresistance, producing split peaks in the Fourier spectrum. The magnetoresistance evolution is presented and discussed as a function of temperature and gate voltage. It is found that sample specific properties have a pronounced influence on the observed behavior. For example, the interference of different transverse modes or the interplay between $h/e$ oscillations and conductance fluctuations can produce the features mentioned above. In previous work they have occasionally been interpreted as signatures of spin-orbit interaction (SOI)-induced effects. In the light of these results, the unambiguous identification of SOI-induced phase effects in AB rings remains still an open and challenging experimental task. As we will show in detail, the identification and, possibly, the suppression of sample specific features in the future is crucial.

8.3 Samples

We have used two carbon-doped heterostructures grown on [001] oriented substrate in two different MBE systems. In both cases the layer sequence is identical and the 2DHG lies 45 nm below the surface. Sample A was obtained from D05124A while sample B from Bochum20122. At 75 mK sample A and sample B showed densities of $4.5 \times 10^{15}$ m$^{-2}$ and $3.5 \times 10^{15}$ m$^{-2}$ respectively and mobilities of $30$ m$^2$V$^{-1}$s$^{-1}$ and $5.0$ m$^2$V$^{-1}$s$^{-1}$ respectively. For both wafers the SOI strength was extracted from the Fourier transform of Shubnikov-de Haas oscillations measured in a Hall bar geometry and found to be comparable, with a Rashba parameter $\beta$ of $10 \times 10^{-29}$ eVm$^{-3}$ for sample A and $8.9 \times 10^{-29}$ eVm$^{-3}$ for sample B. The SOI energy splitting is calculated from $\Delta_{SO} = 2\beta k_1^3$, where $k_1$ is the smaller Fermi wave vector of the two spin subbands. The splitting $\Delta_{SO}$ results in $1.13$ meV and $0.8$ meV for sample A and B respectively. The internal magnetic field can be estimated comparing the energy splitting given by SOI to that produced by a Zeeman field. The value of the g-factor in p-type 2DHGs is still under debate [102, 136, 137]. Assuming a g-factor of 1, we find a corresponding Zeeman splitting of $19.4$ T for sample A and $13.8$ T for sample B.

Two nominally identical rings were processed with standard electron beam lithography and wet chemical etching for defining insulating trenches. An atomic force microscope scan of sample A is shown in Fig. 8.4(d). The dark areas indicate the etched parts, where the 2DHG underneath the surface is depleted. The area of the AFM scan is $5 \times 5$ $\mu$m$^2$ and the mean radius of the ring is 360 nm. Six in-plane gates allow tuning the transmission in the two leads and in the two arms of the ring. Further contacts are present on the same mesa on which the ring is defined (not shown in Fig. 8.4) to allow a four-terminal measurement of the ring resistance and the characterization of the 2DHG properties in a Hall bar geometry. The depth of the trenches is $33$ nm for sample A and $8.5$ nm for sample B. In both cases the trenches are deep enough to provide confinement for holes, but the low etching depth
Chapter 8. Aharonov-Bohm effect in $p$-type GaAs rings

Figure 8.4: (a) Measured four terminal magnetoresistance of the ring (black curve) together with the low frequency background which will be subtracted from the data (red curve). (b) Extracted $h/e$ oscillations. (c) Period of the $h/e$ oscillations calculated from the separation between two successive extrema. The position of the phase jumps in the $h/e$ oscillations is indicated by the blue dashed lines. (d) AFM micrograph of sample A. The black lines are 33 $rmnm$ deep etched trenches, the scan frame has a lateral size of $5 \times 5 \mu m^2$. (e) Fourier spectrum of the raw data. (f) Extracted $h/2e$ oscillations.

of sample B resulted in limited electrical tunability within the leakage-free range. In order to extend its tunability, sample B was entirely covered with an insulator and a metallic top gate. The details regarding the top gate fabrication are described elsewhere [69]. The samples were measured in a $^3$He/$^4$He dilution refrigerator at a base temperature of 75 $mK$ (unless separately specified). The resistance was measured with conventional low-frequency lock-in techniques by applying a constant current
8.4. Experimental results

of 400 pA through the ring and measuring the four-terminal voltage. The low value of the current was necessary to prevent sample heating. In order to have a low resistance background, all the data discussed here have been taken with the four gates defining the transmission of the leads at the most negative possible value (just before the onset of leakage currents).

For the rest of the discussion, unless explicitly stated, we will focus our attention on sample A, because it showed the highest amplitude AB oscillations. The two samples proved to be very similar, both in terms of oscillation amplitude and structure of the beatings. The electrical tuning of the ring was found to produce similar results using a combination of top gate and side gates (sample B) or just side gates (sample A). The top gate on sample B was found to have a much larger influence on the ring density than on the bulk density. This is probably due to the reduced screening in the ring as compared to the bulk and to edge effects in the ring enhancing the electric fields. In addition, the presence of the etched trenches around the ring allows an easier lateral penetration of the field lines coming from the top gate and thus leads to an improved tunability of the ring density compared to the bulk. For this reason we believe that tuning the ring with a top gate is similar to tuning it with side gates: in both cases the gates mainly change the Fermi energy and any change in the SOI has to be attributed predominantly to the change in the holes density rather than to the change of the external electric field [23, 31].

8.4 Experimental results

8.4.1 Magnetoresistance

Fig. 8.4(a) shows a typical magnetoresistance measurement of the ring, Fig. 8.4(e) shows its Fourier transform. While $h/e$ and $h/2e$ oscillations are visible in the raw data, $h/3e$ oscillations appear as a small peak in the Fourier spectrum. By filtering the Fourier spectrum of the data it is possible to separate the $h/e$ oscillations from low and high frequency components (background resistance and noise respectively). The slowly varying background that is subtracted from our data is depicted in Fig. 8.4(a) with a red line. The extracted $h/e$ oscillations are displayed in Fig. 8.4(b) while the period of the $h/e$ oscillation (calculated from the separation of successive extrema) is displayed in Fig. 8.4(c). We notice the presence of clear and strong beatings in the $h/e$ oscillations similar to the results reported in Ref. [15]. In our data the beating nodes are located at 12 mT, 100 mT and 230 mT. We see in Fig. 8.4(c) that the $h/e$ period fluctuates around a mean value of about 10 mT with deviations of up to 50% when a beating occurs. A closer look at Fig. 8.4(c) shows that the period remains constant between two beating nodes and slightly changes every time a new beating occurs. Very similar behavior is observed for $h/2e$ oscillations as one can see in Fig. 8.4(f). A clear amplitude modulation is present and produces a node at 90 mT. Unfortunately a more quantitative analysis of the $h/2e$ oscillations is not possible, because the signal to noise ratio of the
Chapter 8. Aharonov-Bohm effect in p-type GaAs rings

experiment does not allow us to distinguish features smaller than a few Ohms. The beating of the magnetoconductance leads to the splitting of the $h/e$ and $h/2e$ peaks into many different sub-peaks in the Fourier spectrum shown in Fig. 8.4(e). In order to calculate this spectrum, we collected data in a magnetic field interval from $-650$ mT to $650$ mT, thus resolving structure in the Fourier spectrum narrower than $1$ T$^{-1}$. In performing the Fourier analysis we used conventional techniques to increase the Fourier transform resolution and to suppress the borders’ contribution such as removing the low-frequency background, adding zeros to both ends of the data set and windowing the data. We used different windows and different ranges of magnetic field to check that all the main features of Fig. 8.4(e) are genuine and not due to any finite-size effect.

The slowly varying background subtracted from the data (the red line in Fig. 8.4(a)) is due to the superposition of a classical magnetoresistance and aperiodic quantum conductance fluctuations widely discussed in the literature [55]. The origin of the conductance fluctuations are spurious interference effects in the finite-size areas present along the device and their importance in this context will be addressed later on.

8.4.2 Gate dependence

We first tuned the side gates asymmetrically to find the configuration in which the amplitude of the $h/e$ oscillations is maximized (i.e. the transmissions in the two arms are equal). Once the voltage difference that allowed having equal transmissions was found, we performed a gate dependence study by sweeping the two gates symmetrically. This is justified by the experimental observation that the two gates have very similar capacitance per unit area on the ring transmission. In Fig. 8.5(a) we show the filtered $h/e$ oscillations as a function of the side gates voltages (symmetrically tuned): one can see that the $h/e$ oscillations are strongly affected by a change in electrostatic configuration which results in an amplitude modulation and aperiodic phase jumps of $\pi$ located at zero or finite magnetic field. In Fig. 8.5(b) we plot the $h/e$ period calculated from the same data set (but displayed for a larger interval of magnetic field values). The extracted $h/e$ period homogeneously adopts a value of $10$ mT except for particular lines where a beating occurs and the period deviates on the order of $50\%$. Comparing Fig. 8.5(a) and Fig. 8.5(b) we observe that all the phase jumps visible in Fig. 8.5(a) occur at the position of a beating of the $h/e$ oscillation.

In Fig. 8.5(c) we plot the amplitude of the filtered $h/e$ oscillations (defined as the difference of the resistance in neighboring extrema) alone (left) and we compare it to the lines where a beating occurs (red dots superimposed to the picture on the right). We find that the two data sets are strongly anti-correlated, i.e. whenever a beating occurs, the $h/e$ oscillations experience a minimum in their amplitude. The aperiodic conductance fluctuations mentioned before are sensitive to the gate voltage too, and result in a complex evolution of the background. This effect is visible in Fig. 8.5(d)
Figure 8.5: (a) Gate dependence of the $h/e$ oscillations. (b) Calculated period of the $h/e$ oscillations in an extended field range compared to (a). The red dashed lines indicate the magnetic field range shows in (a). (c) Extracted amplitude of the $h/e$ oscillations alone (left) and compared (right) with the position of the beatings (red dots). (d) Second derivative of the slowly varying background alone (left) and compared (right) with the position of the beatings (red dots).

where we plot the second derivative of the resistance background alone (left) and superimposed to the position of the beatings (right). Comparing the evolution of the background fluctuations with the beatings we observe many similarities. Although the correspondence is not perfect, in many regions the beatings are aligned with the local extrema of the background and both evolve parallel to each other along the gate voltage axis.
We also studied the behavior of our rings in a much larger range of gate voltage. The summary is shown in Fig. 8.6 for both sample A (the three plots on the left, as a function of the symmetric combination of the side gate voltage) and sample B (the three plots on the right, as a function of the top gate voltage). The superposition of $h/e$ and $h/2e$ oscillations produces a complex pattern in the magnetoresistance where certain features appear to be quasi-periodic along the voltage axis. Decomposing the different spectral contribution we can see that, even though the $h/e$ oscillations experience many phase jumps, we were never able to observe a phase jump for the $h/2e$ oscillations [see Figs. 8.6(e) and (f)].

8.4.3 Temperature dependence

Further analysis was performed by measuring the oscillations’ evolution with respect to temperature. It is known that temperature affects an AB experiment in terms of phase breaking [138] and energy averaging. Both effects eventually result in a suppression of the phase-coherent effects [139]. Increasing the temperature of the dilution refrigerator mixing chamber from 75 mK to 300 mK resulted in a suppression of the $h/e$ and $h/2e$ oscillation amplitude. We fitted the amplitude of the oscillations with the exponential law:

$$A(T) = A(T_0) \exp \left( -\frac{nL}{l_\phi(T)} \right)$$

(8.4)

where $n$ is the winding number of the oscillations, $T$ is the temperature, $l_\phi$ is the phase-coherence length and $L$ is the ring circumference. The effect of phase-breaking is superimposed onto the effect of energy averaging. The latter is known to introduce significant corrections in the exponential dependence of all the $h/ne$ oscillations with $n$ odd, but to leave the oscillations resulting from the interference of time-reversed paths (i.e. the $h/ne$ oscillations with $n$ even) less affected [139]. Fitting the amplitude decrease of the $h/2e$ oscillations with Eq. 8.4 allows us to calculate the phase-coherence length of holes in our ring. We find 2 $\mu$m at 75 mK, consistent with Ref. [15]. It should be mentioned that the coherence length calculated with this method oscillates between 1.5 $\mu$m and 4.5 $\mu$m, depending on the magnetic field window in which the amplitude is computed. In particular, we found that close to a node in the $h/2e$ oscillations the coherence length is maximized, while close to a maximum of the envelope it is minimized (similar results are obtained for the $h/e$ oscillations). This behavior can be interpreted considering the temperature evolution of the beatings, as we will discuss later.

The conductance fluctuations present in the background show a strong temperature dependence of their peak amplitude, that can be fitted with an exponential law. The extracted exponents are very different for different fluctuations and gate configurations. In certain regimes the conductance fluctuations decay faster than the $h/e$ oscillations (as in Fig. 8.7), in other cases they were still present when the $h/e$ oscillations were completely suppressed.
8.4. Experimental results

Figure 8.6: (a) \( h/e \) and \( h/2e \) oscillations for sample A. (b) \( h/e \) and \( h/2e \) oscillations for sample B. (c) Extracted \( h/e \) oscillations for sample A. (d) Extracted \( h/e \) oscillations for sample B. (e) Extracted \( h/2e \) oscillations for sample A. (f) Extracted \( h/2e \) oscillations for sample B.

Fig. 8.7 shows a comparison of the temperature evolution of the \( h/e \) oscillations (Fig. 8.7(a)) and of the background (Fig. 8.7(b)). All curves have been vertically shifted for clarity and the amplitude of the \( h/e \) oscillations has been normalized to one. The bottom curves of Fig. 8.7 are taken at the lowest temperature and show similar features as those in Fig. 8.4. With increasing temperature we notice a gradual evolution of the beating positions along the magnetic field axis and two phase jumps at \( B = 0 \). At the lowest temperature the oscillations have a maximum
Chapter 8. Aharonov-Bohm effect in p-type GaAs rings

at $B = 0$ that develops into a minimum at $T = 92$ mK and again into a maximum after $T = 146$ mK. At the highest temperature the background fluctuations are completely suppressed and the $h/e$ oscillations have a regular behavior with no phase jumps (see how the periodic grating corresponds to the oscillation maxima for every period). This behavior is not found in all the regimes investigated: in other gate configurations the conductance fluctuations decayed slower than the $h/e$ oscillations and it was not possible to observe fully suppressed beatings as in this case. We did not find any regime where beatings in the $h/e$ oscillations where still present when the background conductance fluctuations where completely suppressed. We interpret the gradual shift of the beating position, as well as the temperature dependent phase jumps at zero field, as an effect of energy averaging, as we will discuss in the next section.

Figure 8.7: (a) Temperature evolution of the $h/e$ oscillations superimposed to a grid with the same periodicity as the $h/e$ oscillations. Each curve has been vertically rescaled and vertically offset for clarity. The temperatures are reported on the right. The thicker vertical black line marks zero magnetic field. (b) Extracted slowly varying background for the same data set. The data are vertically offset for clarity, the vertical black line marks zero magnetic field.
8.5 Discussion

8.5.1 Phase jumps and beatings

In the framework of geometric phase effects, the beatings in the AB oscillations, as well as the splitting of the Fourier peaks, are due to the superposition of two different oscillations in the magnetoconductance of the ring. These oscillations originate from the two spin species precessing in the magnetic field texture of the ring. Berry’s phase emerges in the adiabatic limit, i.e. when the electron’s spin precesses many times around the effective magnetic field vector before the electron leaves the ring. In this regime, and for the case of electron rings, it was predicted that Berry’s phase will make the oscillations vanish at special values of the external magnetic field, where the total magnetic field (given by the vectorial sum of the SOI induced magnetic field and the external perpendicular magnetic field) will assume “magic” tilt angles [129, 140]. In Ref. [129] it is shown that an experimental observation of a splitting in the Fourier spectrum due to Berry phase would require the application of an external perpendicular field comparable to the SOI induced field. This is hard to reach in most of the experiments within the low field AB regime. In order to reach adiabaticity, the quantity $\kappa$, often referred to as adiabaticity parameter and defined as:

$$\kappa = \frac{\omega_B L^2}{(2\pi)^2 D}$$  \hspace{1cm} (8.5)

needs to be much larger than unity. In Eq. 8.5, $\omega_B$ is the cyclotron frequency of the electron in a magnetic field $B$ and $D$ is the diffusion constant. To reach adiabaticity, rings with a large radius and materials with strong SOI are needed. The diffusive motion of the holes can slow down their propagation around the ring compared to ballistic motion, thus making adiabaticity easier to reach. Using the values for our two samples we calculate that adiabaticity would be reached for a magnetic field larger than ten Tesla, where the magnetoconductance of the ring is dominated by the quantum Hall effect. The diffusion constant used in this estimate is calculated for a bulk 2DHG. In the confined geometry we use, disorder induced scattering might be significantly enhanced making the rings adiabatic for smaller magnetic fields. Another complication might arise due to the presence of different transverse modes in the arms of the ring and their interplay with each other in determining the total conductance. From the lithographic width of the ring’s arm we calculate the presence of 3-4 transverse modes, with an energy separation of the order of 30 $\mu$eV. However the disorder potential, that we estimate as $\hbar/\tau_q$ where $\tau_q$ is the quantum scattering time, is of the order of 60 $\mu$eV, comparable to the modes’ energy separation. This indicates that the modes are partially mixed, and the ring can not be considered as an ideal 1D device. It should be mentioned, however, that in the same wafer we could process, with the same fabrication technology, quantum point contacts where we could observe a well developed first quantized mode.

The presence of beatings in $h/e$ oscillations and splittings in the Fourier spectrum
Chapter 8. Aharonov-Bohm effect in p-type GaAs rings

were often interpreted as a signature of Berry’s phase as described above. However, there are other phenomena, whose origin is not related to SOI, that can lead to similar effects and that were rarely taken into account. In Ref. [141] it is shown that the interplay between different modes leads to the formation of strong beatings even in materials with low SOI. In that work it is pointed out that the beatings’ evolution, if connected to mode mixing, could be affected by the electrostatic potential present in the ring and thus be modified via a gate voltage. Numerical simulations show how a phase jump of $\hbar/e$ oscillations at $B = 0$ leads to the onset of a beating that gradually evolves at finite magnetic field. The period change by 50% is also presented and explained as a result of phase rigidity imposed by the Onsager relations. This effect is very similar to the measurements in our rings where, as discussed above, we estimate the presence of a few transverse modes. The strong relation between phase jumps at zero field and beatings at finite field is evident from Fig. 8.5. The small period change that occurs after a phase jump can be related to the presence of different modes with different effective radii: when the main contribution to the conductance changes from one mode to the other, also the effective area of the ring changes and the total phase might experience a phase jump of $\pi$.

In Fig. 8.6 we show the gate voltage dependence in a much larger range for the two samples under study. The superposition of $\hbar/e$ and $\hbar/2e$ oscillations forms a complex pattern, but a line-by-line Fourier analysis shows us that only $\hbar/e$ oscillations experience phase jumps along the gate voltage axis. A similar pattern was observed in the work of others and interpreted as a signature of the Aharonov-Casher effect [132, 134]. In both Ref. [132] and Ref. [134] the contribution of $\hbar/e$ and $\hbar/2e$ oscillations was not separated. The interpretation is then not straightforward because it is not clear which features are produced by genuine phase jumps in the $\hbar/2e$ oscillations alone and which are produced by phase jumps in the $\hbar/2e$ oscillations or by the interplay of $\hbar/e$ and $\hbar/2e$ oscillations. Assuming a ballistic single-mode 1D ring, when the density in the ring is symmetrically tuned in the two arms, $\hbar/e$ oscillations will experience phase jumps of $\pi$ with a periodicity in $k_F$ given by $\Delta k_F L = 2\pi$. In order to convert the density change into a gate voltage change we measured the variation of the transmission of the ring with respect to the in-plane gates at high perpendicular magnetic field. The lever arm we estimate by tuning the transmission through many Landau levels is in agreement with a simple capacitor model. Assuming ideal conditions we estimate an average gate voltage spacing between successive phase jumps of 29 mV, while the mean spacing we observed is 20 mV. Any deviation from ideality will, in general, introduce additional phase jumps that are difficult to predict.

The situation is different for $\hbar/2e$ oscillations: since they arise from the interference of exactly the same time-reversed path, they should not acquire any dynamical phase when traversing the ring so they are the ideal candidates to prove the relevance of a geometric phase. The existence of the Aharonov-Casher term was predicted for the case of electrons [131] and elegantly observed in a large array of rings in a recent experiment [142]. Interestingly, in the case of holes this effect should show a distinct
8.5. Discussion

signature, namely the frequency of the Aharonov Casher oscillations increases as a function of the spin orbit splitting [143]. In our case we always observe a resistance minimum of $h/2e$ oscillations at $B = 0$, which proves the existence of a Berry phase of $\pi$ in our rings. Apart from that, even though their amplitude shows a dependence on the gate voltage, which may have various different reasons, as one can see from Fig. 8.6, the $h/2e$ oscillations do not go through any phase jump in the voltage range accessible in our devices. The reason for the lack of phase jumps in the $h/2e$ oscillations is not clear yet. On one hand the theory of Aharonov-Casher effect in heavy holes system was always limited to ballistic single-mode rings, on the other hand also in the latter ideal situation the frequency of phase jumps in the $h/2e$ oscillations is expected to increase with increasing SOI. Having larger SOI and larger density tunability might allow the observation of beatings in $h/2e$ oscillations also in a heavy hole ring.

8.5.2 Decoherence and ensemble averaging

When the temperature of the experiment is increased two main phenomena can be observed: the suppression of phase coherent effects ($h/e$ oscillations, $h/2e$ oscillations and conductance fluctuations) and a significant modification of the beating pattern, including the phase jumps of $\pi$ at zero field. The first effect is expected from decoherence and allows to extract a coherence length for holes in our rings. The temperature dependence of the background conductance fluctuations also confirms their phase-coherent origin and thus a possible interplay with the $h/e$ oscillation phase. We may expect that every time a conductance fluctuation completes half a period (i.e. it changes in sign), the superimposed $h/e$ oscillations will be modulated as well with a resulting phase change of $\pi$. This hypothesis is in agreement with the observation that beatings in the $h/e$ oscillations often appear to be correlated to the background conductance fluctuation, as shown in Fig. 8.4 and Fig. 8.5. Furthermore in Fig. 8.7 we show that it is possible to suppress a conductance fluctuation by increasing the temperature and, once the conductance fluctuation is suppressed, the superimposed $h/e$ oscillation recovers a regular behavior. Sweeping the magnetic field we will have a superposition of different conductance fluctuations with various widths and height that will produce an aperiodic modulation of the magnetoresistance of the rings.

Energy averaging produces a suppression of the oscillations as well, but with a weaker temperature dependence than decoherence. The thermal length is defined by the relation $l_T = \sqrt{\hbar D/k_BT}$, and indicates the average distance after which two states separated in energy by $k_BT$ dephase by $2\pi$. The relative importance of the latter for our results can be understood considering the short thermal length of holes ($l_T = 2.6 \mu m$) and comparing it with the phase-coherence length of $2 \mu m$ estimated before. In order to simulate the effect of thermal averaging we performed numerical averaging along the gate voltage axis on the same data set of Fig. 8.7. We first considered a single line taken at a gate voltage of $-1.072 V$ and then we performed
numerical averaging in a gate voltage range of gradually increasing size. The final size of the averaging window we used was 73.5 mV, corresponding to a temperature interval of 1.5 K. The results are summarized in Fig. 8.8. In Fig. 8.8(a) we can see the evolution of the extracted $h/e$ oscillations as a function of the magnetic field and averaging window size. One can see that the oscillations undergo various phase jumps of $\pi$ (both at zero or finite magnetic field) as the averaging window size is increased. Fig. 8.8(b) shows the calculated $h/e$ period for the oscillations shown in Fig. 8.8(a), where we can again observe that the beatings’ position is strongly affected and shifts along the magnetic field axis as more curves are averaged. Finally Fig. 8.8(c) shows three selected sections taken from Fig. 8.8(a), where we compare the first not-averaged curve with the curve obtained after averaging in a gate voltage range of 39 mV and the resulting curve obtained at the end of the averaging. One can immediately see that the amplitude of the oscillations changes by a factor of two in an interval of 1.5 K (calculated using the gate capacitance per unit area), a very small suppression if compared to the one attributed to phase breaking. The main result is, however, the significant modification of the position of the beating nodes, visible already for a small averaging window, comparable to the temperature range used in the experiment. We can also see a phase jump in Fig. 8.8(a) at zero magnetic field (located between 10 and 30 mV).

We believe these results are important regarding future analysis of SOI induced effects in AB rings since, as we show here, temperature can effectively modify the beating patterns. To the best of our knowledge the temperature dependence of the beating was never taken into account in the works regarding SOI induced effects in AB rings. An alternative way to perform energy averaging experimentally would consist in increasing the bias voltage applied to the ring. This option is not feasible in our case since Joule heating strongly suppresses the oscillations for currents larger than 1 nA.

It was recently argued [144] that a way to suppress sample specific features in AB experiments and to observe the expected signatures of SOI induced effects is to study the average of Fourier spectra performed in an ensemble of statistical independent measurements (i.e. taken with a significant gate voltage difference). In fact, if the average is performed over the absolute value of the Fourier spectrum, it should completely suppress the phase differences of the $h/e$ oscillations present in different configurations and still not result in a cancellation of the oscillations. We checked this analysis on a data set consisting of 62 magnetoresistance traces measured by stepping the side gates voltage by 15 mV and sweeping the magnetic field from $-650$ mT to $650$ mT. The result of the analysis is shown in Fig. 8.8(d). We made sure that the averaged curves were statistically independent observing the decay of the $h/e$ amplitude upon successive averaging (the average of N statistically independent curves produces a $1/\sqrt{N}$ decay, as we show in Fig. 8.8(e)). The data obtained in this way showed clearly defined $h/e$ and $h/2e$ peaks (black line in Fig. 8.8(d)), all the small features present on the peaks are irrelevant since their size is comparable to the uncertainty given by the standard deviation (the red and blue line in Fig. 8.8(d))
represent the negative and positive limits of the error bar respectively), calculated as described in Ref.[144]. The lack of a splitting in the average of the Fourier spectra shows that the various beatings we observed as a function of gate voltage and magnetic field are features not robust against energy averaging. However caution must be paid since, as shown above, beatings of the $h/e$ oscillations can survive after averaging in a gate voltage window of moderate size.

8.6 Conclusion

We measure large-amplitude AB oscillations in highly tunable p-type GaAs rings. In our experiments we can qualitatively reproduce various features observed in previous work (splitting of Fourier peaks, beatings, gate-dependent phase jumps) that were interpreted as signatures of SOI induced effects. Based on the discussion of the gate voltage and the temperature dependence of the features cited above, we propose an alternative origin that does not involve SOI. In particular we focus our attention on transverse mode mixing, energy averaging and interplay of the AB phase with the phase coherent conductance fluctuations. We point out that the temperature is a parameter to be taken into account, since a small energy averaging can lead to a substantial modification of the beatings. Finally we have tried to extract traces of SOI induced effects from the average of Fourier spectra taken in large ensembles of data. The results indicate that in our case most features can at least qualitatively be explained by sample-specific features.
Figure 8.8: (a) \( h/e \) oscillations calculated by averaging the line at \(-1.072\, V \) with the nearby curves in a voltage window of increasing size. (b) Calculated period of the \( h/e \) oscillations in (a). (c) Three lines extracted from (a). (d) Averaged modulus of the power spectrum for a large data set (black lines) together with the higher and lower boundary of the corresponding error bar (blue and red line respectively). (e) Amplitude of the \( h/e \) oscillations as a function of the inverse square root of the number \( N \) of curves in which the average is performed (stars) together with a linear fit (dotted line).
Chapter 9

Electrical transport in InAs/GaSb double quantum wells

9.1 Introduction

In this chapter we discuss standard magnetotransport experiments in InAs/GaSb double quantum wells (QWs). The experiments reported here are performed on two devices, named Device A and Device B, obtained from the same wafer. The wafer structure was grown on a heavily doped GaAs wafer. Its active region consisted, from the bottom to the top, of 50 nm AlSb barrier, a 15 nm InAs QW in proximity to a 8 nm GaSb QW, a second 50 nm AlSb barrier and a 3 nm GaSb capping layer. The layer scheme and a calculation of the valence band (red) and conduction band (blue) profiles along the growth direction is depicted in Fig. 9.1.

Device A consisted of a single Hall bar with a width of 25 µm and a separation between lateral arms of 50 µm. Device B consisted of two Hall bars in series, oriented perpendicularly to each other. Their width is 25 µm and the lateral voltage probes have various separations, the shortest being 50 µm. Device A was covered by a 200 nm thick Si₃N₄ insulating layer, device B by a 40 nm thick HfO₂ layer. On both samples a Ti/Au topgate was deposited in order to tune the charge density. Except for the different capacitance per unit area due to the different dielectrics, the two devices showed comparable densities and mobilities. The two samples were measured in different systems, hence certain measurements like the in-plane field dependence or the high temperature (T > 1 K) measurement, were performed on Sample B only. In both samples the back gate electrode was found to be shorted to the transport layer, with Ohmic resistances of the order of 100 MΩ. For this reason the back gate could not be used, and was left floating during the entire experiment.
Chapter 9. Electrical transport in InAs/GaSb double quantum wells

9.2 Basic characterization

The samples showed very pronounced ambipolar field effect as a function of top gate voltage. Fig. 9.2 (a) shows the top gate voltage characteristic of Sample A at a temperature of 80 mK. The top gate voltage allowed tuning the characteristic from electrons (very positive top gate voltage) to holes (very negative top gate voltage) passing from a resistance peak associated to the charge neutrality point (CNP). The transverse resistivity $\rho_{xy}$ for different values of gate voltage is shown in Fig. 9.2 (b). The sign change in its slope is a direct indication of the electron-hole crossover observed in our samples. Except for a different capacitance per unit area, due to a different dielectric, Sample B showed similar characteristics.

For Sample A, the top gate voltage could be swept from 8 V to $-7$ V and back without appreciable hysteresis. Applying a gate voltage outside of the above mentioned range resulted in a permanent introduction of time dependencies and fluctuations in the sample resistance. The initial situation could be recovered by warming up the sample at room temperature. Similar considerations apply for Sample B, that had an available top gate voltage range between 5 V and $-3$ V.

From the Hall slope far away from the charge neutrality point we can estimate the total carrier density. For this we took a linear fit of $\rho_{xy}(B)$ for sufficiently high magnetic field to avoid the two-bands transport feature observed at small magnetic field. The results are shown in Fig. 9.3 (a). This estimation is consistent with the one based on the periodicity of the SdH oscillations as reported later one in this chapter. Both densities have a linear dependence on gate voltage with equal ca-
9.2. Basic characterization

Figure 9.2: (a) Longitudinal resistivity as a function of top gate voltage at zero magnetic field and 1.4 K. (b) Transverse resistivity as a function of magnetic field for the top gate voltage values indicated in (a) with the dots.

pacitances per unit area. Under the assumption of a constant density of states, a linear extrapolation of the data points indicates a partial band overlap and residual carrier densities of \((1.2 \pm 0.09) \times 10^{14} \text{ m}^{-2}\) at the CNP [145] (see the solid lines in Fig. 9.3 (a). The energy overlap \(\Delta\) between bands is equal to the Fermi energy when \(p = 0\), in our case we obtain \(\Delta = 6.4\) meV. In Fig. 9.3 (b) we show the carriers’ mobility as a function of gate voltage. Close to the CNP, the mobility is about 2 \(\text{m}^2/\text{Vs}\), and reaches 30 \(\text{m}^2/\text{Vs}\) for high electron density. These value are comparable to those reported in Ref. [146]. For this analysis we assumed that electrons and holes do not coexist away from the CNP. We will see later on that this is not the case for negative top gate voltage, where our analysis gives a small and constant underestimation of the hole density.

9.2.1 Temperature and in-plane field dependence

In order to probe the existence of an energy gap, it is useful to measure the temperature and in-plane magnetic field dependence of the longitudinal resistivity. Fig. 9.4 (a) shows the temperature dependence of \(\rho_{xx}\) measured on Sample A in a temperature range going from 1.3 K to 95 K. A lower temperature was not necessary, since \(\rho_{xx}\) saturates to its maximum value of 1.8 k\(\Omega\) already at 5 K. \(\rho_{xx}\) decreases by a factor of five against a two orders of magnitude temperature change. Such a limited temperature dependence is not compatible with an activated behavior, as expected from an insulator. The lack of activated behavior might originate from the absence of a gap, or the presence of a small gap masked by impurity states. Although self consistent calculations of the band structure for a nominally identical sample suggest the presence of a gap, a similar temperature dependence was observed in Ref. [47, 146] and understood with similar arguments. Fig. 9.4 (b) shows
Figure 9.3: (a) Electron and hole densities (red dots and blue squares respectively) as a function of top gate voltage. The solid lines are fits to the data points, the vertical dashed line indicates the CNP. (b) Electron and hole mobilities (red dots and blue squares respectively) as a function of top gate voltage.

Figure 9.4: (a) Temperature dependence of $\rho_{xx}$ for sample A. (b) In-plane magnetic field dependence of $\rho_{xx}$ for sample B. The temperature was 100 mK and the perpendicular magnetic field zero.

$\rho_{xx}$ for Sample B as a function of in-plane magnetic field. From 0 T to 12 T we observe a decrease in peak resistivity by a factor of three. On the tails of the peak the resistivity is weakly affected. Similar observations in Ref. [147] are interpreted as a manifestation of an electron-hole coupling modification. In particular a pronounced change in peak resistivity (similar to what is observed in our measurements), is interpreted as a direct proof of the presence of an hybridization gap. In this framework the in-plane field makes the system undergo a semiconductor-to-semimental transition. On the contrary, the same experiment performed on a semiconducting sample did not show any appreciable in-plane field dependence up to 12 T. In Ref. [146] a
9.2. Basic characterization

Sample similar to the one discussed here did not show any change in peak resistivity with an in-plane magnetic field up to 12 T. The only relevant modification induced by the in-plane field was the appearance of a resistivity dip on the side of the CNP. Such feature was interpreted as a van Hove singularity in the density of states and was used for estimating an energy gap. This feature is not present in our data.

9.2.2 Low field magnetoresistance and Shubnikov-de Haas oscillations

The magnetoresistance, \( \rho_{xx} \), shows SdH oscillations as a function of perpendicular magnetic field. The study of the SdH oscillations allows to measure important quantities like the electron density, SOI strength, effective mass and quantum scattering times.

Fig. 9.5 (a) shows \( \rho_{xx} \) as a function of magnetic field for different values of top gate voltage at a temperature of 80 mK (the curves have been offset and normalized). Fig. 9.5 (b) shows the power spectrum of \( \rho_{xx}(1/B) \) for the same gate voltage values shown in Fig. 9.5 (a) (the curves have been offset and normalized). Appendix A explains in detail the numerical procedure adopted to calculate the power spectrum of the SdH oscillations. The frequency axis of Fig. 9.5 (b) was multiplied by \( e/h \) to directly show the spin-split density. For a top gate voltage of 8 V, the sample is deep in the electron regime. At this density we resolve many SdH oscillations and their onset lies at 800 mT. The power spectrum of the SdH oscillations for \( V_{TG} = 8 \) V indicates the presence of a single peak, whose position matches the density \( n = 2 \times 8 \times 10^{15} \text{ m}^{-2} = 16 \times 10^{15} \text{ m}^{-2} \) measured from the Hall slope. Decreasing the electron density results in a gradual suppression of the relative amplitude of the SdH oscillations. For a top gate voltage between \(-0.5 \) V and \(-3 \) V \( \rho_{xx} \) strongly increases with magnetic field. This behavior is described in more detail in the next chapter. In the hole regime there are only few weak SdH oscillations that do not allow an analysis based on their power spectrum.

Interestingly, at a top gate voltage of 2 V the main peak splits into two peaks. The sum of the densities obtained from the peaks position is compatible with the total density measured from the Hall slope. For the same gate voltage range, \( \rho_{xx} \) and \( \rho_{xy} \) show signatures of two bands transport as discussed in Section 4.4. Using the two densities \( n_1 \) and \( n_2 \) derived from the SdH oscillations, together with an effective mass \( m^* = 0.03m_e \), we can use the two-bands model with inter-subband scattering [71] to reproduce the low field magnetoresistance in both \( \rho_{xx} \) and \( \rho_{xy} \). The comparison between the low field magnetoresistance and the fit is shown in Fig. 9.6 (a). For example, with a top gate voltage of 1.5 V, the positions of the split peaks indicate two densities of \( n_1 = 2.50 \times 10^{15} \text{ m}^{-2} \) and \( n_2 = 2.95 \times 10^{15} \text{ m}^{-2} \). The modest density imbalance, together with the low number of oscillations periods visible for this density, makes it difficult to see a beating as a function of magnetic field. The scattering rates obtained from the fit are \( K_1 = 1 \times 10^{12} \text{ s}^{-1} \), \( K_2 = 0.43 \times 10^{12} \text{ s}^{-1} \), \( K_{12} = 0.36 \times 10^{12} \text{ s}^{-1} \), where \( K_1 \) refers to the low density subband and \( K_2 \) to the high
Chapter 9. Electrical transport in InAs/GaSb double quantum wells

Figure 9.5: (a) Normalized longitudinal resistivity as a function of magnetic field for different values of top gate voltage. The top gate voltage is indicated close to each curve. (b) Normalized power spectrum as a function of charge density for the curves in (a).

density subbands. We show the results of the two-bands model fit in Fig. 9.6 (c) as a function of top gate voltage. The black line is the Drude scattering rate $1/\tau_e$ obtained from an analysis based on a one-band Drude model (i.e. the mobilities shown in Fig. 9.3 (b)). For high electron density, the magnetoresistance is well described by a one-band Drude model and we get $K_1 \simeq K_2$ with $K_{12} \simeq 0$. For low electron density $K_{12}$ tends to be of the same order as $K_1$ and $K_2$, so that the two transport channels can not be considered independent. For top gate voltage values lower than 1 V the fit did not lead to satisfactory results.

The low field magnetoresistance shape of $\rho_{xy}$ for the curves shown in Fig. 9.6 (b) indicates that the two densities under consideration refer to electron bands. The
9.2. Basic characterization

Figure 9.6: (a) $\rho_{xx}$ as a function of magnetic field for different values of top gate voltage (blue), together with a fit using the two-bands model with inter-subband scattering. (b) Comparison between the measured $\rho_{xy}$ and the one calculated using the fit of (a). Drude scattering rates as a function of top gate voltage. We compare the scattering rates obtained using the two-bands model to the average scattering rate obtained from the sample mobility.

The simultaneous presence of an hole band and an electron band would result in a slope change close to zero magnetic field, as we will discuss later on. The physical origin of the two bands might be linked to sample inhomogeneities or SOI. In an InAs 2DEG, the SOI energy splitting is expected to increase as $\Delta_{SO} = 2\alpha_R k_\parallel$. Because of the linear dependence on $k_\parallel$, the ratio $\Delta_{SO}/E_F$ should increase with decreasing density. On the other hand $\alpha_R$ increases with the applied electric field and also depends on the self-consistent potential determined by the total density. A calculation of $\alpha_R$ as $\alpha_R = \frac{\hbar^2}{(2m^*)[\sqrt{4\pi n_1} - \sqrt{4\pi n_2}]}$ results in $\alpha_R = 1.33 \times 10^{11}$ eVm$^{-1}$ for
$V_{TG} = 2$ V that increases to $\alpha_R = 2.78 \times 10^{11}$ eVm$^{-1}$ for $V_{TG} = 1$ V. The data reported here shows similarities with Refs. [38, 148]. In Ref. [38], SdH oscillations in InAs/AlSb QWs were measured as a function of top gate voltage and a beating pattern was identified. An analysis based on SOI splitting lead to the conclusion that $\alpha_R$ stays constant or slightly increases as a function of density. Ref. [148] studied SdH oscillations at a fixed density in GaSb/InAs/GaSb QWs. Also in that case a beating in the SdH oscillations was identified as originating from SOI. The values of density splitting and $\alpha_R$ parameter that we measured is compatible with the one reported in Refs. [38, 148]. With respect to Ref. [38], we obtain the opposite trend: $\alpha_R$ increases for low electron density. The opposite trend with respect to what is expected from an InAs/AlSb QWs indicates that, if SOI is the origin of the effects described here, the presence of the GaSb QW creates a strong asymmetry in the confinement potential of the InAs QW close to the CNP. The presence of a considerable spin-splitting close to the CNP in InAs/GaSb double quantum wells was recently calculated using $k \cdot p$ theory [149]. An alternative explanation is that, in the limit of low electron density, the sample becomes inhomogeneous and two zones with different densities contribute to transport in parallel. A detailed study of the Sdh oscillations at mK temperature was performed on Sample A only, that consisted of a single Hall bar. Even though we tested that $\rho_{xx}$ is identical measuring on the two sides of the Hall bar, we can not exclude that the two-bands behavior and the split-peak in the SdH oscillations is due to a particular inhomogeneities configuration. If the effect we are seeing arises due to SOI, a clear proof would be the modification of the density splitting with an in-plane field. We leave the verification of this hypothesis to a future experiment.

In the hole regime both $\rho_{xx}$ and $\rho_{xy}$ show signature of two-bands transport in their low field magnetoresistance. Four examples of $\rho_{xx}$ and $\rho_{xy}$ are shown as blue lines in Fig. 9.7(a) and (b) respectively. Differently from the low density electron regime, we observe a slope change in $\rho_{xy}$ close to zero magnetic field. This feature persists up to the most negative top gate voltage value used for the experiment. Based on the two-bands Drude model, with or without inter-subband scattering, such feature can only be interpreted with the coexistence of an electron and a hole band. Qualitatively, at high magnetic field the slope in $\rho_{xy}$ is given by the total density $n + p$ ($p < 0$) and, at low field, depends on both the densities and the mobilities. Since the extraction of the hole density from the SdH oscillations is not accurate in this low mobility regime, we fit both $\rho_{xx}$ and $\rho_{xy}$ with a two-bands Drude model without inter-subband scattering time (see Section 2.3.1). The fitting parameters are the electron and hole densities and the electron and hole mobilities. There might be other contributions important to understand the situation that are not taken into account. Two examples are the inter-subband scattering between electrons and holes and an eventual spin-orbit splitting of the hole subband. Despite the complexity of the problem, the fit gives good results. It is interesting to note that, even when the sample is deep in the hole regime, we always detect an electron density of $2 \times 10^{15}$ m$^{-2}$, constant with gate voltage. Such a residual density might
9.2. Basic characterization

Figure 9.7: (a) $\rho_{xx}$ as a function of magnetic field for different values of top gate voltage (blue), together with a fit using the two-bands model without inter-subband scattering time (blue). The curves are vertically offset for clarity. (b) $\rho_{xy}$ as a function of magnetic field for different values of top gate voltage (blue), together with a fit using the two-bands model without inter-subband scattering time (red). The curves are vertically offset for clarity. (c) Electron (red) and hole (blue) mobility as a function of top gate voltage. (d) Electron (red) and hole (blue) density as a function of top gate voltage. The total density calculated from the Hall slope at high magnetic field is shown in black.

originate from the fact that the InAs QW sits below the GaSb QW. Approaching the CNP from positive gate voltage, we increase the hole density up to a point where the holes completely screen the top gate electric field, leaving the residual electrons unaffected. The sum $n+p$, matches the total density measured from the Hall slope, as reported in Fig. 9.7 (d).
9.3 Effective mass and quantum scattering times

The temperature dependence of the SdH oscillations allows us to extract the carrier effective mass $m^*$ and quantum scattering time $\tau_q$. The knowledge of $m^*$ and its energy dependence allows to study the band structure of the material under consideration. $\tau_q$ is an important parameter for estimating the wafer quality and disorder potential. Chapter 5 describes the determination of $m^*$ and $\tau_q$ in a different material system, but with similar numerical methods. These measurements were performed in Sample B in a variable temperature insert, with a base temperature of 1.3 K.

![Figure 9.8:](image)

Figure 9.8: (a) $\rho_{xx}$ as a function of magnetic field for different values of temperature. The curves have not been offset. (b) Power spectrum of the curves shown in (a). (c) Power spectrum amplitude as a function of temperature (blue dots) together with a fit to the model described in the text (red). (d) Effective mass (blue) and quantum scattering time (red) as a function of density.

For different values of gate voltage, we measured SdH oscillations as a function of
9.3. Effective mass and quantum scattering times

magnetic field and temperature up to 25 K. In order to calculate the effective mass shown here we used the procedure that in Chapter 5 is referred to as Method B. It consists of using the power spectrum amplitude of $\rho_{xx}$ as a function of temperature for calculating $m^*$ and $\tau_q$ basing the analysis on the Ando formula [36]. Even though the standard approach gave similar results, this method was used since it is less susceptible to small shifts from a curve to the other due to hysteresis in the magnet. Caution was paid to only choose the points at low enough magnetic field, an extensive discussion on the validity of the procedure adopted can be found in Chapter 5. Fig. 9.8 (a) shows $\rho_{xx}$ as a function of magnetic field for different values of temperature (only few selected curves are shown here). Increasing the temperature resulted in a suppression of the oscillations and in an overall increase in the classical background. The power spectra of the curves in Fig. 9.8 (a) are shown in Fig. 9.8 (b). As expected the peak related to the SdH oscillations decreases as the temperature increases. The amplitude of the peaks in Fig. 9.8 (b) as a function of temperature is shown in Fig. 9.8 (c) (blue) together with a fit resulting from our theoretical treatment (red). The quality of the fit is excellent and allows to extract the effective mass and the quantum scattering times. The results are shown in Fig. 9.8 (d) as a function of electron density. The obtained values of $m^*$ and $\tau_q$ are in line with the ones reported in literature, for example in Ref. [39] for InAs/AlSb QWs or in Ref. [146] for electrons in InAs/GaSb QWs. In the explored density range, the effective mass increases up to a factor of two. A similar increase of $m^*$ with density was observed for InAs/AlSb by optical means [150] and interpreted as an effect of band non-parabolicity in InAs. For very high density $\tau_q$ drops by a factor of two because of the population of a second subband in the InAs QW. This is also confirmed by the onset of a positive magnetoresistance at low field.

Figure 9.9: $\rho_{xx}$ as a function of magnetic field for different values of temperature. The curves are taken for a top gate voltage of $-6$ V.

For lower densities we could not satisfactorily extract an effective mass, mainly because the amplitude of the SdH oscillations gets very small. A sample with higher quality would allow to perform this analysis closer to the CNP and understand
more about how the presence of the GaSb QW affects electrons in InAs. The same argument applies for the hole regime. Even at the higher hole density, we could resolve very few SdH oscillations and their temperature change was very modest which made it difficult to fit an exponential curve. Fig. 9.9 shows the temperature dependence of $\rho_{xx}$ for a top gate voltage of $-6$ V and a total density of $8.8 \times 10^{15}$ m$^{-2}$. In the available temperature range the two minima at high magnetic field decrease only by a factor of three. This effect originates from the higher effective mass of holes in GaSb with respect to electrons in InAs and, probably, from a low crystal quality in the GaSb QW.

### 9.4 Conclusion

We characterized by magnetotransport measurement an InAs/GaSb double QW. Our fabrication technology allows for the fabrication of top gated Hall bar structures, where the density can be reliably tuned without hysteresis. As a function of top gate voltage the devices show a pronounced ambipolar behavior, where the main carrier type can be tuned from electrons to holes passing from the CNP. The study of the SdH oscillations and low field magnetoresistance allowed to determine the number and type of the occupied subbands in the system. In one of the two samples, our measurement shows the presence of two electronic subbands close to the CNP, and their possible origins are discussed. In the electron regime it was possible to determine the effective mass and the quantum scattering time as a function of density.
Chapter 10

InAs/GaSb quantum wells in the quantum Hall regime

10.1 Introduction

In this chapter we discuss transport experiments performed in InAs/GaSb double quantum wells in a strong perpendicular magnetic field. Beyond the topological insulator properties, that manifest themselves at zero magnetic field, the fate of topological edge states at finite magnetic field has not been investigated so far. It was recently argued that InAs/GaSb topological insulators could be characterized via spin Chern invariants and be protected also under broken time-reversal symmetry conditions [151–153]. A detailed study of the high-field behavior of tunable InAs/GaSb double quantum wells, and the onset of edge channel transport beyond the quantum spin Hall picture is then of importance to understand the physics of 2D topological insulators. Similarly to other semi-metals like graphene [154, 155] or CdHgTe/HgTe quantum wells [156, 157], electron and hole Landau levels (LLs) can coexist close to the CNP [158, 159]. A detailed understanding of the expected hybridization of LLs [160] and its manifestation in a transport experiment is still missing.

At the electron-hole crossover tuned by a gate voltage, a strong increase in the longitudinal resistivity is observed with increasing perpendicular magnetic field. Concomitantly with a local resistance exceeding the resistance quantum by an order of magnitude, we find a pronounced non-local resistance signal of almost similar magnitude. The co-existence of these two effects is reconciled in a model of counter-propagating and dissipative quantum Hall edge channels providing back-scattering, shorted by a residual bulk conductivity. We investigate the transport properties in this regime using different measurement configurations, and as a function of magnetic field and temperature.
10.2 Electrical transport in the quantum Hall regime

The experiments were performed on two devices (named device A and device B) obtained from the same wafer as described in Chapter 9. In Ref. [146] a nominally identical structure was used, and a hybridization gap of 3.6 meV was reported. Hall bar structures were fabricated by photolithography and argon plasma etching. Device A consisted of a single Hall bar with a width of 25 \( \mu \text{m} \) and a separation between lateral arms of 50 \( \mu \text{m} \). Device B consisted of two Hall bars in series, oriented perpendicularly to each other. Their width is 25 \( \mu \text{m} \) and the lateral voltage probes have various separations, the shortest being 50 \( \mu \text{m} \). Device A was covered by a 200 nm thick \( \text{Si}_3\text{N}_4 \) insulating layer, device B by a 40 nm thick \( \text{HfO}_2 \) layer. On both samples a Ti/Au topgate was deposited in order to tune the charge density. Except for the different capacitance per unit area due to the different dielectrics, the two devices showed comparable densities and mobilities, and a similar high magnetic field behavior.

The experiments above a temperature of 1 K were performed in a \( ^4\text{He} \) system with a maximum magnetic field of 7 T. The experiments at lower temperature were conducted in a \( ^3\text{He}/^4\text{He} \) dilution refrigerator with a base temperature of 70 mK and a maximum magnetic field of 12 T. Electric measurements were performed by low-frequency lock-in techniques using constant biases smaller than 20 \( \mu \text{V} \) to avoid sample heating.

Fig. 10.1 (a) shows the longitudinal resistivity of device A as a function of top gate voltage and perpendicular magnetic field measured at 70 mK. The device shows a pronounced ambipolar behavior where the occupation can be tuned from electrons in InAs to holes in GaSb (right and left side of Fig. 10.1 (a) respectively). In the electron regime well developed SdH minima and spin-splitting are visible above 2 T. Oscillations in the hole regime can only be resolved at larger magnetic fields due to the higher effective mass. From the Hall slope it is possible to extract the electron and hole densities \( (n \) and \( p \) respectively) far from the CNP. A more extensive analysis is performed in Chapter 9, here we report the final result only in Fig. 10.1 (b). Both densities have a linear dependence on gate voltage with equal capacities per unit area. Under the assumption of a constant density of states, a linear extrapolation of the data points indicates a partial band overlap and residual carrier densities of \( (1.2 \pm 0.09) \times 10^{14} \text{ m}^{-2} \) at the CNP [145] (see the lines in Fig. 10.1 (b)). The energy overlap \( \Delta \) between bands is equal to the Fermi energy when \( p = 0 \), in our case we obtain \( \Delta = 6.4 \text{ meV} \). In Fig. 10.1 (d) we show the carriers’ mobility as a function of gate voltage. Close to the CNP, the mobility is about 2 m\( ^2/\text{V}s \), and reaches 30 m\( ^2/\text{V}s \) for high electron density. These values are comparable to those reported in Ref. [146].

To estimate the disorder potential, we extracted the quantum scattering time \( \tau_q \) from the temperature dependence of the low-field SdH oscillations in the electron regime. From \( \tau_q = 0.15 \text{ ps} \), we estimate the amplitude of the disorder potential to be \( \hbar/\tau_q = 6 \text{ meV} \), comparable to the band overlap. The large dimensions of the Hall
bars and the large disorder potential do not allow the observation of the potential presence of helical edge states at zero magnetic field.

Fig. 10.1c shows two horizontal cuts of Fig. 10.1 (a) for magnetic fields of 11 T and 0 T (solid and dashed blue line respectively) together with the transverse conductivity $\sigma_{xy}$ at 11 T obtained by tensor inversion (red line). Similarly to Refs. [50, 161], the zero field resistivity shows a peak of the order of a few kΩ at the CNP. In a high magnetic field a prominent peak develops in $\rho_{xx}$ at the CNP and a $\nu = 0$ plateau appears in $\sigma_{xy}$. The well-developed minima in $\rho_{xx}$ and plateaus in $\sigma_{xy}$ at 11 T away from the CNP indicate that the regime under study is governed by quantum Hall effect physics. The high resistivity peak (346 kΩ at 11 T) is peculiar

Figure 10.1: (a) Longitudinal resistivity of sample A as a function of top gate voltage and magnetic field. The minima are labeled with the corresponding filling factor. The horizontal lines are cuts of the data visible in (c). (b) $n$ and $p$ as a function of top gate voltage, the lines are linear extrapolations of the data. (c) Resistivity at 11 T and 0 T (solid and dashed blue line respectively) together with the transverse conductivity at 11 T (red line). (d) Carriers’ mobility as a function of top gate voltage.
Chapter 10. InAs/GaSb quantum wells in the quantum Hall regime

since it exceeds the resistance quantum by an order of magnitude.

Fig. 10.2 (a) shows the longitudinal resistivity of device A at the CNP as a function of magnetic field. It increases from less than 2 kΩ at B = 0 T to 25 kΩ at B = 4 T. For magnetic fields between 4 T and 6 T, a plateau-like feature, labeled plateau B appears in $\rho_{xx}$ whose value at low temperature is close to 25 kΩ. Above 6 T the resistivity abruptly increases to a few hundred kΩ (insulating state) and above 7.5 T shows another plateau-like feature, labeled plateau A, with reproducible fluctuations as a function of both magnetic field and gate voltage. The appearance of plateaus in $\rho_{xx}$ is another strong indication of the presence of quantum Hall edge channels in the regime under investigation.

![Figure 10.2](image)

Figure 10.2: (a) Temperature dependence of the resistivity at the CNP. (b) Arrhenius plot of the resistivity for different values of magnetic field.

To check if the onset of the insulating state at the CNP is linked to the opening of an energy gap, we measured the temperature dependence of $\rho_{xx}$ at the CNP as a function of magnetic field. For small magnetic fields the temperature dependence is weak, and gets stronger as the magnetic field is increased. In Fig. 10.2 (b) an Arrhenius plot of the CNP resistivity is shown for two different magnetic fields. The data at 7 T has been measured up to 100 K, while the data at 12 T only up to 900 mK. In the limit of high temperatures $\rho_{xx}$ strongly varies with temperature, indicating activated behavior while at low temperature the dependence is weak, which might indicate the onset of hopping transport. From the high-temperature
slope (dashed line) an activation energy of 7.5 meV was estimated. This value is of the same order of magnitude as the energy gaps between different LLs expected at 7 T in this system. The logarithm of the low temperature resistivity can be fitted with a power law of the kind $T^{-1/\alpha}$, with $\alpha < 4$. The limited data range did not allow to determine $\alpha$ with good precision. Hence the underlying hopping mechanism could not be identified.

### 10.3 Non local transport measurements

To confirm the presence of edge channels, we performed four-terminal measurements in device B using various contact configurations. A scheme (not to scale) of device B is shown in the inset of Fig. 10.3 (a). Such measurements are performed by passing a current $I_{i-j}$ between the pair of contacts $i$ and $j$, and measuring the voltage difference $V_{k-l}$ between the pair of contacts $k$ and $l$. The four-terminal resistance is then defined as $R_{i-j,k-l} = V_{i-j}/I_{k-l}$. Fig. 10.3 (a) shows a contour plot of $R_{2-11,3-10}$ as a function of magnetic field and gate voltage. In this case the voltage contacts are placed 50 $\mu$m away from the direct path between the current contacts. At the CNP and at low magnetic field, the non-local resistance is smaller than 100 $\Omega$. In correspondence to plateau A, a giant non-local response develops above 6 T, whose value of about 2 M$\Omega$ is much larger than the resistance quantum and comparable to the two-terminal resistance measured in the same configuration.

To study the dependence of the non-local response on the separation between current and voltage contacts, measurements have been performed with separations ranging from 50 $\mu$m to 600 $\mu$m. Since device B has two Hall bars oriented perpendicularly to each other, we measure the distance between lateral arms along the central axis of the Hall bar. The results are shown in Fig. 10.3 (b) for a magnetic field of 8 T. The signal decays as a function of length with a behavior that could be fitted with the exponential law (blue dashed line):

$$ R_{i-j,k-l}(x) = R_0 e^{-x/l_0}, \quad (10.1) $$

where $R_0$ is a constant prefactor and $l_0$ is the decay length. We could successfully fit the decay with Eq. 10.1 for every magnetic field value above 7.75 T, obtaining values for $l_0$ close to 180 $\mu$m (see Fig. 10.3 (c)). This value of $l_0$ is surprisingly high. From standard diffusive transport a decay length of $W/\pi = 8 \mu$m is expected [162]. This provides strong evidence for the presence of edge channel transport. Similar behavior was observed for any other measurement configuration in the L-shaped Hall bar, ruling out any transport anisotropy linked to the crystallographic orientation of the device. On both sides of the CNP, we observe a fan of side peaks originating from the well-known non-local transport in the quantum Hall regime [163, 164]. While the giant non-local peak at the CNP is always well visible, the side peaks can be distinguished from the noise just for the smallest distance between current and voltage probes.
Fig. 10.3 (d) shows the temperature dependence of $R_{2-11,3-10}$ at 12 T. Similarly to $\rho_{xx}$, the non-local response is suppressed by temperature. The length $l_0$ decreases as well as the temperature is increased and, above a temperature of 1 K, a complete suppression of the non-local four-terminal resistance within the first 50 $\mu$m is observed (see Fig. 10.3 (e)).

Figure 10.3: (a) Contour plot of the four-terminal resistance $R_{2-11,3-10}$ as a function of magnetic field and top gate voltage for a 50 $\mu$m contact separation. The inset shows a sketch of the device B. (b) Non local resistance at the CNP as a function of contact separation, the data is taken using various measurement configurations. The blue line is a fit to an exponential curve. (c) Decay length as a function of magnetic field. (d) Four-terminal resistance $R_{2-11,3-10}$ as a function of temperature at a constant magnetic field of 12 T. (e) Decay length as a function of temperature.
10.4 Effect of an in-plane magnetic field

The non-local response was further characterized as a function of in-plane magnetic field for a fixed perpendicular field of 9 T, the results are summarized in Fig. 10.4. A moderate in-plane field leaves $\rho_{xx}$ and the four-terminal non-local resistances mostly unaffected while, for fields exceeding 2 T, a strong resistance suppression sets in. For an in-plane field of 7.5 T, $\rho_{xx}$ decays to about 20 kΩ and $R_{2-11,3-10}$ to 1 kΩ. We could successfully fit the decay in non-local resistance in length up to a field of 4.5 T. Our results indicate that the decay length is suppressed by about a factor of two in that in-plane field range. After 4.5 T the signals are too small to obtain an accurate value of $l_0$.

![Diagram](a) Four-terminal resistance peak for a perpendicular field of 9 T and various values of in-plane field in a non-local configuration for two different separation between current and voltage terminals. (b) Decaying length as a function of in-plane field for a constant perpendicular field of 9 T.

10.5 Discussion

The effects described here can be understood in terms of hybridization of electron and hole LLs. If a band overlap is present, at high magnetic field LLs might hy-
bridize [160, 165]. The expected LL spectrum (without hybridization) is sketched in Fig. 10.5 (a). In Fig. 10.5 (b) (left) the first two electron and hole LLs are represented along a cross section of the Hall bar. The confinement potential at the sample edges pushes the LLs up or down depending on the charge sign and edge channels form. On the right of Fig. 10.5 (b) the same situation is depicted including the hybridization of LLs. The hybridization creates a gap over the whole sample width. Shifting the Fermi energy through this gap we expect a transition from a situation where two counter-propagating edge channels are simultaneously present (as sketched in Fig. 10.5 (c)) to one where transport is blocked. The latter case is never observed, since $\rho_{xx}$ always shows a measurable conductance and edge-channel transport is present. To account for the observed behavior the disorder potential has to be taken into account. A disorder potential comparable to the gap size can locally suppress the insulating state and give rise to carrier hopping between adjacent conducting puddles, leading to a finite resistance [158, 159] (see Fig. 10.5 (d)).

To study the different contributions in the resistance arising from the edges and bulk, we use an electrical model based on a resistor network as that depicted in Fig. 10.5 (c). The Hall bar is divided into finite elements, each having two local edge channel resistances $R_1$ and a local bulk resistance $R_2$. Applying a voltage $V_i$ to a contact with respect to a ground placed on the opposite side of the sample makes a current flow in the network. The four-terminal resistance, measured in the configuration described above, depends on $R_1$ and $R_2$. Considering the network to be of infinite length in both directions, it is found that $V_i$ scales exponentially with the element number $i$ such that a decay length $l_0$ and a prefactor $R_0$ can be defined (see Eq. 10.1). The calculation of the resistor model gives the following relations:

$$R_1/R_2 = \cosh (W/l_0) - 1$$

$$R_0 = R_1 \left( 1 + \sqrt{1 + 2R_1/R_2} \right) / 2.$$  \hspace{1cm} (10.2)

(10.3)

The derivation of these results is presented in Appendix C. Using $W = 25 \mu m$ and $l_0 = 180 \mu m$, we obtain $R_1/R_2 = 1/100$ at base temperature, confirming that current preferentially flows along the edges. These results confirm the applicability of the infinite chain model: since the Hall bar is much longer than $l_0$, all the current passes from one side to the other via the bulk, and not via edge channels. If no bulk conductance is allowed, the model should be modified into a finite chain of $R_1$ resistors and the voltages $V_i$ would then linearly decay over distance. Within our model, $l_0$ indicates the characteristic distance over which the edge channels on the two opposite edges of the sample equilibrate through bulk scattering.

Since the insulating state in $\rho_{xx}$ occurs between $\nu_e = 1$ and $\nu_h = 1$, we associate plateau A with the situation when $\nu_e = \nu_h = 1$, as indicated in Fig. 10.5 (a) by region 1. Plateau B can originate from the situation where $\nu_e = \nu_h = 2$, as indicated by region 2. A very high magnetic field would bring the first electron LL above the first hole LL, as indicated by region 0, turning the sample into a normal band insulator. Our experimentally accessible magnetic field range does not allow probing this
scenario. Temperature facilitates hopping transport along the edges and suppresses the localization in the bulk (both \( R_1 \) and \( R_2 \) decrease). \( \rho_{xx} \) is mainly determined by the smaller, in our case \( R_1 \), but the non-local resistance depends on both \( R_1 \) and \( R_2 \) and is rapidly shunted by the onset of small bulk conduction. This argument is consistent with the fact that the energy gaps of the lowest electron and hole LLs are completely overcome for a temperature of 850 K.

We conclude that transport predominantly occurs at the sample edges and, since the two involved edge channels have different chirality, helical edge channels are expected. Checking the individual potential of every contact at high magnetic field, we found that the potential distribution along the edges follows opposite chirality for the electron and hole regime respectively. At the CNP no preferential direction is observed, confirming the helical nature of the edge channels under investigation. This situation is particularly interesting since it combines non-local transport, typically an effect arising from transport through ballistic edge channels, with a two-terminal resistance exceeding by far the resistance quantum, which usually governs diffusive transport. The coexistence of electron and hole LLs is a necessary requirement for the realization of this peculiar state, hence it can only be observed in a restricted class of materials. Similarly to other experiments performed in GaAs [163, 164, 166], the amplitude of the non-local response is determined by a competition between edge channels transmission and bulk conduction. The phenomena under consideration strongly differ from the ones known from standard electron transport. This is due to the different chirality and temperature dependent transmission of both bulk and edge channels.

The effect of in-plane magnetic field is different, since it suppresses the CNP resistivity and the non-local resistance, but only weakly affects \( l_0 \). The in-plane field mainly acts on spins, changing the energy of the spin-split LLs, and on the orbital motion of carriers by modifying the Fermi contours [167, 168]. On one hand, a decrease in \( \rho_{xx} \) by one order of magnitude would be naturally identified with a similar decrease in the edge channel dissipation (\( R_1 \) decreases by one order of magnitude). On the other hand, a decrease in \( R_1 \) would result in an increase of \( l_0 \), the contrary of what is observed in the present case. An alternative explanation is that the in-plane field results in a strong reduction of the bulk resistivity (\( R_2 \) decreases by at least two orders of magnitude) and the edge channel resistivity stays constant. A strong decrease in \( R_2 \), so that \( R_2 \simeq R_1 \) would weakly affect \( l_0 \) and \( R_0 \) with respect to the case \( R_2 = 100R_1 \) (see Eq. 10.2 and 10.3). The amplitude of the non-local response is anyway exponentially dependent on \( l_0 \), and this explains the suppression by four orders of magnitude of \( R_{2-11.3-10} \) and \( R_{2-11.5-7} \) shown in Fig. 10.4(a). The microscopic origin of this effect is, up to date, non completely understood. New insights in the in-plane field response of this system in the quantum Hall regime could be obtained from a Corbino structure, where the absence of edge channels would allow to directly identify the bulk contribution.

In conclusion, we investigated the behavior of InAs/GaSb QWs at the CNP and at high perpendicular magnetic field. A strong resistivity increase accompanied by
Figure 10.5: (a) LLs for electrons (red) and holes (blue) without taking hybridization into account. (b) (Left) LLs’ energy for electrons (red) and holes (blue) as a function of position along the sample. (Right) The same as on the left, but taking into account a hybridization between LLs. (c) ideal situation where two perfect counter-propagating edge channels are present. (d) Our case, where transport occurs along non-ideal edge channels. (e) The resistor model described in the text.

the onset of a giant non-local response was observed and studied. The results are interpreted in terms of helical quantum Hall edge states forming at high magnetic field.
Appendices

A  Fourier analysis of the Shubnikov-de Haas oscillations

This supplementary material section provides additional data analysis and technical information. We describe the mathematical procedure adopted to perform the Fourier analysis of the Shubnikov-de Haas oscillations and identify the various peaks present in the power spectrum. We discuss numerical methods commonly used to suppress mathematical artifacts due to the discreteness of the sampled data points. For completeness, we derive an analytical form of the Fourier transform of the Ando formula.

A.1  Numerical treatment

In order to transform our data, we used a discrete Fourier transform based on a fast Fourier transform algorithm included in the software Matlab. To suppress eventual numerical artifact originating from the discreteness and finite dimension of the data range, it is common practice in this kind of analysis, to subtract a slowly varying background, pad the data with zeros and multiply them with a smooth windowing function \[169\]. The subtraction of the slowly varying background suppresses the low frequency contribution in the power spectrum. The resolution of the power spectrum depends on the real space extension of the data points to be transformed. To increase the number of points in the discrete frequency spectrum, we extended the data range with vectors of zeros (ten times longer than the original data). Padding the data with zeros results in a smooth interpolation of the transformed spectrum that does not increase the true resolution. Zero-padding the data does not preclude further analysis. The multiplication of the data with a magnetic field dependent function (also called windowing) corresponds, in the frequency domain, to the convolution between the spectrum of the data and the Fourier transform of the window. The bare selection of a particular range in magnetic field corresponds to window the data with a rectangular function. This operation creates hard boundaries on the data set to be transformed and, as a consequence, spurious side peaks might appear in the final result. In order to minimize the contribution of the boundaries it is common practice to window the data with a smooth function that minimizes the contribution
of the edges. A detailed discussion about the existing windowing functions and their properties goes beyond the scope of this text.

We perform this exemplary analysis in the magnetic field range between 0.2 T and 1.2 T. The procedure described here produces Fig. 5.1 (a), but the quantitative analysis of the effective mass presented in Sec. 5.2.1 is done in larger intervals of magnetic field. Fig. A.2 shows $\rho_{xx}$ (blue) plotted as a function of $1/B$ in the magnetic field range of interest. The data are interpolated with a cubic spline in order to obtain equally spaced points in $1/B$. In our example, we pass from 3101 data points before interpolation, to 41001 points after interpolation. The data interpolation in a newly defined axis where the points are equally spaced is mandatory in order to perform the fast Fourier transform algorithm. We use a least mean square fit of a sixth order polynomial (red curve in Fig. A.2) as a slowly varying background to be subtracted from the data. The result of the subtraction is shown in Fig. A.3 (blue). The windowing function used for the analysis is the symmetric Hamming window (red) [169, 170]. The discrete windowing function consists of the same number $N$ of data points as the data set, and is given by:

$$H(n) = 0.54 - 0.46 \cos \left(2\pi \frac{n}{N}\right), \quad (A.1)$$

with $0 \leq n \leq N$. Fig. A.4 shows the result after windowing the data and padding the result with zeros. The amplitude of the function to be transformed is reduced at the boundaries. We tried different windowing functions, obtaining
qualitatively similar results.

Fig. A.4 summarizes the analysis results. Subfigures (a) and (b) show the raw data and the corresponding power spectrum respectively in case no particular operation is performed. Subfigures (b) and (c) show the raw data after zero-padding and the corresponding power spectrum respectively. Subfigures (e) and (f) show the raw data after zero-padding and subtraction of a sixth order polynomial and the corresponding power spectrum respectively. Subfigures (g) and (h) show the raw data after zero-padding, subtraction of a sixth order polynomial and multiplication with an Hamming window and the corresponding power spectrum respectively. The spectrum in Fig. A.4 (f) shows many side peaks that are absent in Fig. A.4 (h), as indicated by the arrows.

Windowing is very useful in situations where a specific frequency component has to be identified from the background. It is important to notice that the relative amplitude of the peaks changes if windowing is performed. For this reason no windowing function is used for Method A in the paper, where an inverse fast Fourier transform is performed on the filtered spectra. For method B the Hamming window is used for both the measured and the calculated data. The operations previously performed leave the amplitude of the peaks of interest unchanged, hence they do not affect the quantitative analysis presented in the main text.

The frequency axis can be directly converted into a (spin-resolved) density axis using the relation $n = f e / h$, as shown in Fig. A.5. In the following we will show how
all the peaks in the power spectral densities can be consistently interpreted. The highest peak in the power spectrum of Fig. A.4 (d) matches the periodicity of the low-field SdH oscillations, visible in Fig. A.4 (a) above 4 T$^{-1}$ and can be identified with the first subband oscillations. The total density $n = n_1 + n_2$ is obtained from the Hall slope. The transverse resistivity $\rho_{xy}$ is not linear at small magnetic field due to the presence of the two subbands [71]. We therefore determine the density from the slope of $\rho_{xy}$ between 80 mT and 200 mT, where it corresponds to the total density, and obtain $2.95 \times 10^{15}$ m$^{-2}$. At this density, indicated by the green dot in Fig. A.5, the power spectrum shows a peak. The density of the second subband is obtained by $n_2 = n - n_1$. The result (blue dot) accurately matches the position of another peak in the spectrum of Fig. A.5. The peaks located at frequencies $2n_1$ and $n_2 - n_1$ derive from combinations of $n_1$ and $n_2$. The peak located at a frequency corresponding to the average density $(n_1 + n_2)/2$ cannot be obtained by multiplication of sinusoidal functions. This anomalous peak, already observed in Ref. [94], is interpreted with non-adiabatic effects [171]. The peak visible for a density smaller than $0.5 \times 10^{15}m^{-2}$ is due to a residual of the slowly varying background present in our data after the subtraction of the low order polynomial described above.
Figure A.4: (a) Raw data to be transformed. (b) Power spectrum of the data in (a). (c) Data in (a) after zero-padding. (d) Power spectrum of the data in (c). (e) Data in (c) after subtraction of a sixth order polynomial. (f) Power spectrum of the data in (e), the arrows indicate side peaks that are absent in (h). (g) Data in (e) after multiplication with an Hamming window. (h) Power spectrum of the data in (g). Note that the horizontal extension of the data in (b), (d) and (g) is much larger than what shown in the graphs.
Figure A.5: Power spectrum as a function of density. The dots indicate the subband densities calculated from the analysis of $\rho_{xx}$ and $\rho_{xy}$. The excellent agreement between the calculated densities and the power spectrum peaks proves consistency for the analysis.
A.2 Analytical form

After considering the numerical treatment, it is interesting to give the analytical solution of the same problem: the Fourier transform of $\rho_{xx}(1/B)$. For simplicity, we rewrite the Andro formula as:

$$\rho_{xx}(x) = -\left(2e^{-cx} \frac{ax}{\sinh(ax)} \cos(dx)\right)H(x)$$ (A.2)

$$x = \frac{1}{B} \quad a = \frac{2\pi^2 k_B T m^*}{\hbar e}$$ (A.3)

$$c = \frac{\pi m^*}{e \tau_q} \quad d = \frac{\pi \hbar n}{e}$$ (A.4)

where $H(x)$ is the Heaviside function. The Fourier transform is:

$$\mathcal{F}\left\{\rho_{xx}\right\}(f) = \frac{1}{2a} \left(\Psi\left(1, \frac{a + c + i(2\pi f - d)}{2a}\right) + \Psi\left(1, \frac{a + c + i(2\pi f + d)}{2a}\right)\right)$$ (A.5)

where $\Psi(1, x)$ is the first derivative of the logarithm of the gamma function. Considering the monolateral power spectrum only, we obtain the result:

$$\left|\mathcal{F}\left\{\frac{\rho_{xx}}{\rho_0}\right\}(f)\right|^2 = \left|\frac{1}{2a} \left(\Psi\left(1, \frac{a + c + i(2\pi f - d)}{2a}\right)\right)\right|^2$$ (A.6)

In Fig. A.6 we plot the power spectrum just calculated. The physical parameters are reported in the figure.

![Figure A.6: Analytical calculation of the power spectrum of $\rho_{xx}(1/B)$ for the parameters indicated in figure.](image)

The peak height as a function of temperature takes the form:
\[ F \left( \frac{\rho_{xx}}{\rho_0} \right) \left( \frac{d}{2\pi} \right) \right|^2 = \left| \frac{1}{2a} \Psi \left( 1, \frac{a+c}{2a} \right) \right|^2 \]  \hspace{1cm} (A.7)

In many practical situations, the SdH oscillations are transformed in a limited \(1/B\) range. In this case the effect of windowing should be taken into account. Considering a rectangular window of unitary height, width \(\Delta\) and central coordinate \(x = \overline{x}\), we obtain:

\[ F \left( \frac{\rho_{xx}}{\rho_0} \text{rect} \left( \frac{x - \overline{x}}{\Delta} \right) \right) = F \left( \frac{\rho_{xx}}{\rho_0} \right) \ast \Delta \text{sinc}(\Delta f) e^{-2\pi i \overline{x} f} \]  \hspace{1cm} (A.8)

where \(\ast\) is the convolution operation. For the typical situations discussed in this thesis, the selection of a specific range in magnetic field makes the peak height decrease by about one order of magnitude. The value of the last equation in the peak maximum can be easily calculated numerically as a function of \(\tau_q\), in the cases where the effective mass is precisely known. Comparing the experimentally found value of the power spectrum maximum with the calculated one allows an estimation of \(\tau_q\) that does not involve any temperature dependence study.

**B Quantum spin Hall effect**

In this appendix we derive the current and voltage distribution of a six terminal Hall bar with two counter-propagating edge channels (see Fig. B.1) using Landauer-Büttiker formalism. Such a situation applies to the quantum spin-Hall effect (QSHE) regime, expected from a 2D topological insulator at zero magnetic field or to the situation described in Chapter 10 at high magnetic field. We will not consider the case in which the counter-propagating edge channels are couple or the case in which the edge channels are shorted by a conductive bulk.

Such a problem can be solved using standard multi-terminal Landauer-Büttiker formalism. The current flowing out from contact \(i\) is:

\[ I_i = \frac{e^2}{\hbar} \left( (N_i - R_i)V_i - \sum_{j \neq i} T_{ij} V_j \right) \]  \hspace{1cm} (B.1)

where \(N_i\) are the modes generated at terminal \(i\) and \(R_i\) the modes back-scattered into it. \(T_{ij}\) is the transmission probability from mode \(i\) to mode \(j\). In matrix form:

\[
\begin{pmatrix}
I_1 \\
I_2 \\
I_3 \\
I_4 \\
I_5 \\
I_6 \\
\end{pmatrix} = \frac{e^2}{\hbar} \begin{pmatrix}
2 & -1 & 0 & 0 & 0 & -1 \\
-1 & 2 & -1 & 0 & 0 & 0 \\
0 & -1 & 2 & -1 & 0 & 0 \\
0 & 0 & -1 & 2 & -1 & 0 \\
-1 & 0 & 0 & 0 & -1 & 2 \\
\end{pmatrix} \begin{pmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
V_5 \\
V_6 \\
\end{pmatrix}
\]  \hspace{1cm} (B.2)
Figure B.1: Schematic representation of a six-terminal Hall bar with counter-propagating edge channels.

Typically the longitudinal resistance is measured by applying a constant bias $V_1$ from contact 1 and leaving contact 4 grounded ($V_4 = 0$) while the other contacts are floating ($I_2 = I_3 = I_5 = I_6 = 0$). A current $I$ is injected by contact 1 and absorbed by contact 4.

$$
\begin{pmatrix}
I \\
0 \\
0 \\
-I \\
0 \\
0
\end{pmatrix} = \frac{e^2}{h} \begin{pmatrix}
2 & -1 & 0 & 0 & 0 & -1 \\
-1 & 2 & -1 & 0 & 0 & 0 \\
0 & -1 & 2 & -1 & 0 & 0 \\
0 & 0 & -1 & 2 & -1 & 0 \\
-1 & 0 & 0 & 0 & -1 & 2
\end{pmatrix} \begin{pmatrix}
V_1 \\
V_2 \\
V_3 \\
V_5 \\
V_6
\end{pmatrix}
$$

(B.3)

It follows that:

$$I = \frac{2e^2}{3h} V_1 = \frac{2e^2}{h} (V_2 - V_3) \quad \text{(B.4)}$$

$$R = \frac{V_1}{I} = 3h \quad \text{(B.5)}$$

$$\rho_{xx} = \frac{V_2 - V_3}{I} = \frac{V_6 - V_5}{I} = \frac{h}{2e^2} \quad \text{(B.6)}$$

$$\rho_{xy} = \frac{V_2 - V_6}{I} = \frac{V_3 - V_5}{I} = 0 \quad \text{(B.7)}$$

It is instructive to consider the case in which a current is passed from contact 2 to contact 6, in this case we have:
\[
\begin{pmatrix}
0 \\
I \\
0 \\
0 \\
0
\end{pmatrix}
\begin{pmatrix}
0 & -1 & 0 & 0 & 0 & -1 \\
-1 & 2 & -1 & 0 & 0 & 0 \\
0 & -1 & 2 & -1 & 0 & 0 \\
0 & 0 & -1 & 2 & -1 & 0 \\
0 & 0 & 0 & 0 & -1 & 2
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
V_5 \\
0
\end{pmatrix}
= \frac{e^2}{h}
\begin{pmatrix}
2 \\
0 \\
0 \\
-1 \\
0
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
V_5 \\
0
\end{pmatrix}
\]

That leads us to:

\[I = \frac{3e^2}{4h} V_2\]  \hspace{1cm} (B.9)

\[R_{2,6-2,6} = \frac{V_2}{I} = \frac{4h}{3e^2}\]  \hspace{1cm} (B.10)

\[R_{2,6-3,5} = \frac{V_3 - V_5}{I} = \frac{2h}{3e^2}\]  \hspace{1cm} (B.11)

A general result obtained in QSHE regime is that, fixing a voltage difference \(V = V_1 - V_2\) between two arbitrary contacts with all the other contacts floating, the potential will drop stepwise at each contact in between. In the QHE regime the chirality of the edge states will maintain half of the contacts at a voltage \(V_1\) and the other half at the voltage \(V_2\). In Fig. B.2 we show the potential distribution for the two examples considered in this appendix.

![Figure B.2: Potential distribution for a six-terminal Hall bar for two different contact configuration, as described in the text.](image)

\section{Analytical solution of the resistor network model}

In this chapter we provide an analytical solution of the resistor network model used in Chapter 10 to describe transport in an InAs/GaSb double quantum well in a high perpendicular magnetic field.

We consider an infinite resistor network as the one depicted in Fig. C.1 (a). It includes two different resistors named \(R_1\) and \(R_2\). A voltage \(V_{in}\) is applied in one node and a ground is placed on the symmetric node on the other side of the network.
A current $I$ will flow from the top to the bottom and will generate a potential drop $V_{n,u}$ on the upper $n$-th node of the network and a potential drop $V_{n,d}$ on the lower $n$-th node of the network. We are interested in finding an analytical solution that links the quantity $(V_{n,u} - V_{n,d})/I$ to $n$.

For simplicity, we consider a semi-infinite network. The voltage $V_{in}$ and the ground are applied on the first upper and lower node respectively. We will see that the extension of the results to the infinite network will be straightforward. The semi-infinite network can be divided into blocks composed of two $R_1$ resistors and one $R_2$ resistor, as indicated in red in Fig. C.1 (a). For such a block we can consider the ingoing and outgoing currents entering its nodes and the voltage drops at them (with respect to ground). These quantities are shown in Fig. C.1 (b). The current $I_n$ flowing in the upper line at node $n$ must be equal to the current flowing in the lower node $n$, but opposite in sign. This is a result of the current conservation condition: the semi-infinite network is closed, and no current can be lost or generated in it.

Applying the well-known Kirchhoff rules, we can find a matrix $M$ that links the electrical quantities before a block to the ones after it:

\[
\begin{pmatrix}
  V_{n+1,u} \\
  V_{n+1,d} \\
  I_{n+1}
\end{pmatrix}
= 
\begin{pmatrix}
  1 & 0 & -R_1 \\
  0 & 1 & R_1 \\
  -1/R_2 & 1/R_2 & 1 + 2R_1/R_2
\end{pmatrix}
\begin{pmatrix}
  V_{n,u} \\
  V_{n,d} \\
  I_{n}
\end{pmatrix}
\]

(C.1)

The matrix $M$ can be written as $M = VDV^{-1}$, where $D$ is a diagonal matrix and $V$ is a basis transformation matrix. This is convenient when we consider the series of $k$ blocks.

\[
\begin{pmatrix}
  V_{k,u} \\
  V_{k,d} \\
  I_{k}
\end{pmatrix}
= M^k \begin{pmatrix}
  V_{in} \\
  0 \\
  I
\end{pmatrix}
\]

(C.2)

The explicit calculation of Eq. C.2 yields:
\[
\begin{align*}
\begin{pmatrix}
V_{k,u} \\
V_{k,d} \\
I_k
\end{pmatrix}
&= 
\begin{pmatrix}
V(1 + \lambda^k)/2 \\
V(1 - \lambda^k)/2 \\
I\lambda^k
\end{pmatrix}
\tag{C.3}
\end{align*}
\]

\[
\lambda = 1 + \frac{R_1}{R_2} - \sqrt{\frac{R_1}{R_2} \left( 2 + \frac{R_1}{R_2} \right)}
\tag{C.4}
\]

The current \( I \) entering the semi-infinite network is given by the applied voltage \( V_{in} \) divided by the total resistance of the network \( R_T \). The latter is obtained imposing that adding one block at the beginning of the network does not change the total resistance:

\[
R_T = 2R_1 + R_2||R_T
\tag{C.5}
\]

\[
I = \frac{V}{R_1 \left( 1 + \sqrt{1 + 2\frac{R_2}{R_1}} \right)}
\tag{C.6}
\]

\( R_2||R_T \) indicated the harmonic sum operation.

We now extend the solution to a continuum model, where the typical length scale is longer than the physical dimension of each element. Assuming that each element has height and width both equal to \( W \), we can calculate the non-local resistance at a distance \( x \). From Eq. C.3 we have:

\[
\begin{align*}
V_u(x) &= \frac{V}{2} \left( 1 + e^{-\frac{x}{l_0}} \right) 
\tag{C.7}
\end{align*}
\]

\[
\begin{align*}
V_d(x) &= \frac{V}{2} \left( 1 - e^{-\frac{x}{l_0}} \right) 
\tag{C.8}
\end{align*}
\]

\[
\begin{align*}
I(x) &= \frac{V}{R_1 \left( 1 + \sqrt{1 + 2\frac{R_2}{R_1}} \right)} e^{-\frac{x}{l_0}} 
\tag{C.9}
\end{align*}
\]

\[
\begin{align*}
l_0 &= \frac{W}{\ln \left( 1 + \frac{R_1}{R_2} - \sqrt{\frac{R_1}{R_2} \left( 2 + \frac{R_1}{R_2} \right)} \right)} 
\tag{C.10}
\end{align*}
\]

Where \( l_0 \) is the characteristic decay length of the system. The non-local resistance is given by:

\[
\begin{align*}
R_{NL} &= \frac{V_u(x) - V_d(x)}{I} = R_0 e^{-\frac{x}{l_0}} 
\tag{C.11}
\end{align*}
\]

\[
\begin{align*}
R_0 &= R_1 \left( 1 + \sqrt{1 + 2\frac{R_2}{R_1}} \right) e^{-\frac{x}{l_0}} 
\tag{C.12}
\end{align*}
\]
With some algebra on Eq. C.10 we obtain the solution of the semi-infinite network:

\[
\cosh \left( \frac{W}{l_0} \right) = 1 + \frac{R_1}{R_2} \tag{C.13}
\]

\[
R_0 = R_1 \left( 1 + \sqrt{1 + 2 \frac{R_2}{R_1}} \right) e^{-\frac{\phi}{l_0}} \tag{C.14}
\]

The infinite network depicted in Fig. C.1 (a), consists of the parallel of two semi-infinite networks and a \( R_2 \) resistor. In the situation described in the main text \( R_2 \gg R_1 \), and the resistance of the infinite network is simply half of the resistance of the total network. Keeping the same \( V_{in} \), the current flowing in the circuit will be twice as high and the only modification arises in Eq. C.12, that has to be multiplied by 1/2.
Publications

A graphene nanoribbon memory cell
Stützel E, Burghard M, Kern K, Traversi F, Nichele F, Sordan R
Small 24 2822 (2010)

Aharonov-Bohm rings with strong spin-orbit interaction: the role of sample-specific properties
New Journal of Physics 15 033029 (2013)

Counting statistics of hole transfer in a p-type GaAs quantum dot with dense excitation spectrum
Physical Review B 88 035417 (2013)

Suppression of bulk conductivity in InAs/GaSb broken gap composite quantum wells

Insulating state and giant non-local response in an InAs/GaSb quantum well in the quantum Hall regime

Spin-orbit splitting and effective masses in p-type GaAs two-dimensional hole gases
Physical Review B 89 081306(R) (2014)
Characterization of spin-orbit interactions of GaAs heavy holes using a quantum point contact

Accepted in Physical Review Letters (2014)

Pure spin currents generation and detection in mesoscopic cavities with strong spin-orbit interaction

In preparation
Bibliography


Acknowledgements

The results presented in this thesis work are not only the outcome of motivation and hard work but, in particular, the result of strong professional collaborations in a friendly environment. I am deeply grateful to Klaus Ensslin for giving me the unique chance to work in his research group. Accepting me as a PhD student at ETH Zurich changed my life and opened me the way for new opportunities. His support was constant and energetic, from my first steps in the laboratory to the writing of my final results. He encouraged me to follow my ideas and intuitions, giving me all the tools needed to arrive to the results presented here. Thomas Ihn’s deep understanding of physics, rational approach to problems and kindness will always be an example to me. The patience and time he dedicated to my experiments is unquantifiable, as the notions I have learned from our discussions or from his book. A special thanks goes to Charles Marcus for accepting to be a member of the exam committee and for critical reading of my thesis. It was a great honor to have him present during my defense.

I would like to thank Werner Wegscheider, Christophe Charpentier, Christian Reichl, Christian Gerl, Andreas Wieck and Dirk Reuter. Without their high quality wafers, the experiments presented here would not have been possible. Roland Winkler, Stefano Chesi, Daniel Loss, Philippe Jacquod and Peter Stano contributed to the interpretation of our results. Their theoretical understanding of spin-orbit interaction strongly enriched the message of this thesis and the quality of my publications. Yashar Komijani and Miklós Csontos introduced me to the world of holes, cryogenics and sample processing. Team-working with them was a pleasure, and I hope I have been a good successor to them. I wish good luck to the Top Team, composed now by Susanne Müller and Atindra Nath Pal. Together we took the first steps in the land of topological insulators, and I am sure the best is yet to come.

I wish to thank Szymon Hennel, Patrick Pietsch and all the other students who completed their thesis under my supervision. Their strong commitment and interest for the subject allowed to sensitively improve the outcome of the experiments, make the fridge run smoothly and systematically collect and analyze plenty of data. I wish them all the best for their future.

The equipment in our group was always the best I could desire. This is due to the enviable skills of the technical team: Paul Studerus, Cecil Barengo and Peter Märki. A special mention also goes to the FIRST team. Thanks to their great
professionalism, the amount of time I spent fabricating samples in the clean room remained limited and pleasant.

Theodore Choi, for whom electron beam lithography has no secrets, made the E2 office feel like home. The final thank goes to all the members of the Nanophysics Group, that contributed to make my time in Zurich unforgettable.
Curriculum Vitae

Name: Fabrizio Nichele
born: June 04, 1985 in Como (Italy)
citizen of Italy

07/2010-07/2014 Ph. D. at the Laboratory of Solid State Physics (ETHZ) in the group of Prof. Klaus Ensslin

03/2008-05/2010 M.Sc. Physical Engineering
Politecnico di Milano, Milano, Italy

03/2009-06/2010 Master Diploma Thesis
LNESS, Politecnico di Milano, Como, Italy
Fabrication and characterization of graphene nanoelectronic devices in the group of Prof. Romand Sordan

10/2004-03/2008 B.Sc. Physical Engineering
Politecnico di Milano, Milano, Italy

09/1999-07/2004 Scientific High School
Collegio Gallio, Como, Italy