Master Thesis

Numerical Modelling of Flow-Mechanics Coupling in Fractured Reservoirs

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Numerical Modelling of Flow-Mechanics Coupling in Fractured Reservoirs

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Professor: Dr. Patrick Jenny
Nomenclature

Roman Symbols

\begin{itemize}
\item \(N_f\) Number of discrete fracture domains \\
\item \(B\) Skempton’s pore pressure coefficient \\
\item \(M_{IV}\) Moment magnitude \\
\item \(t\) Time \\
\item \(b\) Aperture \\
\item \(S\) Total slip \\
\item \(L_x\) Fracture segment length along the fracture direction \\
\item \(L_z\) Fracture segment length perpendicular to \(L_x\) \\
\item \(q\) Source term \\
\item \(S_0\) Cohesive force on the fracture plane \\
\item \(c_f\) Fluid compressibility \\
\item \(p\) Pressure \\
\item \(G\) Shear modulus \\
\item \(E\) Young’s modulus \\
\item \(K_s\) Bulk modulus of the solid grain \\
\item \(K_f\) Fluid bulk modulus \\
\item \(K_{dr}\) Drained bulk modulus \\
\item \(M_o\) Seismic moment \\
\item \(\tilde{I}\) Unit tensor \\
\item \(\tilde{k}\) Permeability tensor \\
\item \(\tilde{z}\) Depth \\
\item \(\vec{u}_r\) Rock displacement vector \\
\item \(\vec{F}\) Force vector \\
\item \(\vec{u}\) Fluid Velocity \\
\item \(\vec{g}\) Gravitational acceleration vector \\
\item \(\vec{f}_r\) Rock force density vector
\end{itemize}

Greek Symbols

\begin{itemize}
\item \(\phi\) Porosity \\
\item \(\alpha\) Biot’s effective stress coefficient (isotropic)
\end{itemize}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>SI Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ</td>
<td>Slip friction coefficient</td>
<td>–</td>
</tr>
<tr>
<td>Ω</td>
<td>Domain of the fracture/damaged matrix</td>
<td>–</td>
</tr>
<tr>
<td>σ₀</td>
<td>Static friction coefficient</td>
<td>–</td>
</tr>
<tr>
<td>σ₃</td>
<td>Dynamic friction coefficient</td>
<td>–</td>
</tr>
<tr>
<td>ν</td>
<td>Poisson’s Ratio</td>
<td>–</td>
</tr>
<tr>
<td>νᵤ</td>
<td>Undrained Poisson’s Ratio</td>
<td>–</td>
</tr>
<tr>
<td>ρᶠ</td>
<td>Fluid Density</td>
<td>kg m⁻³</td>
</tr>
<tr>
<td>ρᵣ</td>
<td>Rock Density</td>
<td>kg m⁻³</td>
</tr>
<tr>
<td>μ</td>
<td>Fluid dynamic viscosity</td>
<td>kg m⁻¹ s⁻¹</td>
</tr>
<tr>
<td>λ</td>
<td>Lamé constant</td>
<td>N m⁻²</td>
</tr>
<tr>
<td>Ψᵢ→d</td>
<td>Fluid exchange from the fracture to the damaged matrix</td>
<td>m s⁻¹</td>
</tr>
<tr>
<td>Ψ₃→i</td>
<td>Fluid exchange from the damaged matrix to the fracture</td>
<td>m s⁻¹</td>
</tr>
<tr>
<td>σₑ</td>
<td>Compressive Stress</td>
<td>N m⁻²</td>
</tr>
<tr>
<td>̇ε</td>
<td>Strain Tensor</td>
<td>–</td>
</tr>
<tr>
<td>̇σ</td>
<td>Stress Tensor</td>
<td>N m⁻²</td>
</tr>
<tr>
<td>̇τ</td>
<td>Shear Stress Tensor</td>
<td>N m⁻²</td>
</tr>
</tbody>
</table>

**Other Notations**

- Xᵣ: Damaged matrix properties
- Xᵢ: Discrete fracture properties

**Abbreviations**

- ETH: Eidgenössische Technische Hochschule, Zürich
- a-HFR: adaptive hierarchical fracture
- PDE: Partial differential equation
- DFM: Discrete fracture model
- AGM: Algebraic multigrid
- EGS: Enhanced geothermal system
- MMS: Moment magnitude scale
Abstract

The modelling of flow in fractured reservoirs has become the subject of active research in recent years. This is mainly due to the fact that there exist a plethora of industries and engineering applications which depend heavily on the use of such simulators. Numerical models for fractured reservoirs find application in oil industries, natural gas applications, subsurface CO$_2$ sequestration sites etc. In recent years, the need for including geomechanical effects on the flow in fractured reservoirs has been demonstrated. The main objective of this thesis is to develop a coupling module which is capable of simulating the physics of such subsurface systems accurately.

Initially, the two in-house geomechanics and fluid flow solvers were studied. Next, a literature survey was done in order to determine the best coupling methodology which could be applied to the two solvers. It was found that the explicit and iteratively coupled methods were most suited for coupling these solvers. Coupling algorithms were applied to both the porous media (damaged matrix) and the discrete fractures.

The damaged matrix coupling was done using both, the explicit and the iterative coupling algorithms. The accuracy and computational cost of these coupling methods were tested using simple test cases. The discrete fractures were coupled using a variation of the explicitly coupled method. A bisection algorithms was introduced into the explicit coupling methodology which is required to accurately predict the actual time at which the fracture slip event occurs. This coupled system was then used on a test case involving the shear stimulation of a single fracture.

This thesis shows the need for geomechanics and fluid flow coupling in order to accurately simulate the flow physics in fractured reservoirs. This coupled algorithm can be effectively used in predicting the flow in a compressed porous media and also in determining the fracturing events of the discrete fractures which are present in the system. Further improvements which can be made to this work are also explored.
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I would like to thank my tutor, Prof. Dr. Leonhard Kleiser for guiding me thought my masters studies at ETH. I would like to thank him for guiding me in the selection of my courses and his constant support ensured that my masters studies at ETH was pleasurable and intellectually rewarding.

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I would like to thank my supervisor, Rajdeep Deb for guiding me through the entire duration of my thesis. I would like to thank him for his patience in the explanation of various concepts related to the numerical coupling of fractured reservoirs. The numerous discussions which we have had together has certainly helped me understand the various processes involved in the coupled system. His help made sure that my thesis was both enjoyable and intellectually engaging. Also his guidance was very helpful in ensuring that my thesis was always on track and this helped me to complete my thesis on time, without any difficulties.

I also thank Dr. Dimitrios C. Karvounis for helping me to understand the fluid flow simulator. The various meetings and discussions which we had helped me to understand the layout of the fluid simulator quickly and he was also invaluable in helping me to understand the physics of the flow the fractured reservoirs.

I would also like to thank Bianca Maspero for helping me with all the administrative issues during my stay at ETH. Her help made sure that my experience working at the institute of fluid dynamics was pleasurable and had no problems.

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Chapter 1. Introduction

1. Introduction

This chapter introduces the major task of this thesis. The need for geomechanics and fluid dynamic coupling is explored.

1.1. Motivation for damaged matrix coupling

![Skempton effect in porous media](image)

Figure 1.1.: Skempton effect in porous media

The damaged matrix refers to the porous media and these two terms are used interchangeably in this thesis. The need for flow simulators in porous media is seen in the case of reservoir simulators. Most oil and gas companies use sophisticated reservoir simulators to predict the flow properties in underground reservoirs. These simulators are used to predict the performance and effective operational conditions of these reservoirs. These simulators also play a critical role in other engineering applications such as in subsurface CO\textsubscript{2} sequestration.

Thus, it is not surprising to find that research in reservoir simulators have become widespread in recent years. Most of the research conducted upto this point have involved exploring the flow physics in the porous media. These have involved development of sophisticated models to handle the effects of multiphase flows and thermal transport in the porous media. The development of numerical methods for porous media flows have also been a subject of active research.
1.2. Motivation for discrete fracture coupling

Only in recent years has the effect of geomechanics on the porous flow taken the attention of researchers. It has been demonstrated that the effect of mechanical stresses on the porous domain plays a significant role in the flow physics of porous media. This is because most reservoirs are located deep inside the earth, in which the deformation of the porous rock is significant. These effects become critical when considering the extension of production in reservoirs, which has significant geomechanical considerations to be taken into account.

The Skempton effect and the Mandel-Cryer effect are two such effects which are seen in the porous media because of the geomechanical stresses applied on it. The Skempton effect occurs in because of the porosity reduction in compressed porous rock. Figure 1.1 indicates, schematically this effect. This causes an increase in fluid pressure inside the porous domain. The Mandel-Cryer effect is also observed when drainage conditions are applied on the porous media. The effect of geomechanics can cause non-monotonic pressure variation inside the porous media. A test case recreating this effect can be used to validate the coupled fluid-geomechanics solver.

1.2. Motivation for discrete fracture coupling

![Figure 1.2.: Fracture aperture increase on shearing](image)

In recent years the need to predict the failure of discrete fractures has become very important for a large number of applications. Large discrete fractures are present in EGS reservoirs and in natural gas extraction systems. These discrete fractures are prone to failure either by necessity (creation phase in the EGS reservoir), or accidentally during the normal operation of the EGS system. Most, recently, it was seen that such a failure in the fracture lead to a magnitude 3.4 earthquake at Basel, Switzerland in 2006.
Thus, it is critically important to develop simulators which can be used to predict accurately, the failure of such fractures in these systems. This is crucial in order to prevent the triggering of fractures which might lead to truly devastating seismic events. Such simulators could also help gain the public acceptance for the operation of the EGS reservoirs which could potentially be of significant importance for the energy demands of the future.

Fracturing in EGS reservoirs is usually initiated when a high pressure fluid is injected into the fractured domain. This pressure tends to destabilize the fractures and it induces seismicity when the pressure inside the fractures reaches a certain critical value. This causes the fracture to slip and the static friction acting on the fractures drops down to the dynamic limit and the system readjusts itself to a new equilibrium position. This slipping process tends to increase the aperture of the fracture which is schematically shown in figure 1.2. This aperture change introduces an increase in volume and permeability of this fractured node and this in turn significantly affects the pressure in the system. It is noted that the fracturing process, unlike the effect of stresses on the porous media, is a discrete process. Thus, the traditional algorithms applied for the porous media cannot be applied for this case. Thus, better algorithms have to be developed which are capable of predicting, accurately the exact times of these fracture events.

1.3. Objectives

The objectives of this thesis was to develop and implement a flow-mechanics coupling module into an already existing flow and geomechanics simulators. This task involved the following steps.

- Get acquainted with the two in-house flow and geomechanics simulators.
- Perform a literature survey and decide on the best coupling methods which could be implemented to these solvers.
- Combine the two solvers and introduce a coupling module which is capable of handling independent calls to both these solvers. The coupling module should also be capable of performing the various interpolations between the fluid and geomechanics grids.
- Apply the coupling algorithms to both the damaged matrix and the discrete fractures.
- Study the coupling algorithms using simple test cases.
- Demonstrate the need for coupling flow and geomechanics in physical systems.
1.4. Conclusion

This chapter has introduced the topic which has to be dealt with in the rest of the thesis. The need for flow-geomechanics coupling was briefly introduced. Also, the objectives of the thesis and the steps involved in completing the task were defined.
2. Introduction to the Fluid and Geomechanics solvers

2.1. Introduction

In this chapter, the solvers used for solving the fluid flow [Karvounis] and the geomechanics [Deb and Jenny] are introduced. This chapter only deals briefly with these solvers, for more details, the reader is referred to the cited documentation.

2.2. Fluid Flow Solver

The solver used to calculate the flow in a fractured reservoir is an in-house solver developed at ETH. This solver uses the a-HFR modelling approach. This solver treats large dominant fractures differently than smaller fractures. In this solver, the large fractures are treated discretely and the smaller fractures are averaged into the continuum description of the underlying porous medium. This porous medium is referred to as the damaged matrix.

The figure 2.1 shows how a typical fractured domain is upscaled into a DFM. Upscaling is the process in which the smaller fractures are averaged into the damaged matrix and the large fractures are treated discretely. The energy equation is not presented in this work because the effect of temperature on the coupled model is neglected.

2.2.1. Mathematical Formulation

The velocity of the flow in the porous medium is approximated using Darcy’s Law, represented in equation 2.1.

\[ \tilde{u} = -\frac{k}{\mu} \cdot \left( \nabla p - \rho_f g \right) \] (2.1)

In this code, the flow is assumed to be laminar and having a single phase. Also, the Oberbeck-Boussinesq assumption is applied which states that the density variations can be neglected in any term of the PDE, if that term is not multiplied by the gravitational vector. This approximation is considered to be accurate for subsurface flows.
Figure 2.1.: Fractured domain with the corresponding upscaled DFM [Karvounis]
2.2. Fluid Flow Solver

Using these approximations, the mass conservation in a Cartesian grid is given in equation 2.2.

\[
\frac{\partial \rho_f \phi}{\partial t} - \nabla \cdot \left( \frac{k_f}{\mu} \left( \nabla p - \rho_f g \right) \right) \rho_f = \rho_f q_f \quad (2.2)
\]

For incompressible flow, this equation reduces to 2.3.

\[
\frac{\partial \phi}{\partial t} - \nabla \cdot \left( \frac{k}{\mu} \left( \nabla p - \rho_f g \right) \right) = q_f \quad (2.3)
\]

The a-HFR approach uses a set of coupled equations which resolve the flow in the fractured reservoir. This approach uses kernel functions to account for the discontinuities between the discrete fractures and the damaged matrix.

2.2.2. Flow in the Damaged matrix

Using both Darcy’s law and the Oberbeck-Boussinesq assumption, the mass conservation in the damaged matrix is given in equation 2.4

\[
\frac{\partial \phi^d}{\partial t} - \nabla \cdot \left( \frac{k^d}{\mu} \left( \nabla p^d - \rho^{d,f} g \right) \right) = q^d + \sum_{i=1}^{N_f} \int_{\Omega_i} \Psi^{i\rightarrow d}(x',x) \hat{G}(x,x') dA(x') \quad (2.4)
\]

The term in the integrand accounts for the flow exchange from the discrete fracture to the damage matrix. The loss of fluid into the pore space as well as injection and production through wells are captured by the volumetric source term \( q^d \). For more details please refer [Karvounis].

2.2.3. Flow in fractures

As mentioned before, the large scale fractures are discretized independently of the damaged matrix. The mass conservation in the fracture domain is given in equation 2.5

\[
\frac{\partial \phi^i}{\partial t} - \nabla \cdot \left( \frac{k^i}{\mu} \left( \nabla p^i - \rho^{i,f} g \right) \right) + \Psi^{i\rightarrow d}(x',x) = q^d + \sum_{i=1}^{N_f} \int_{\Omega_i \cap \Omega_j} \Psi^{j\rightarrow i}(x',x') \hat{G}(x,x') dS(x') \quad (2.5)
\]

Here, the term in the integrand represents the fluid flow between fracture elements.
2.2.4. Transmissibility

A model is needed to describe the terms dealing with fluid exchange between the fractures and the damaged matrix. In this code, it is assumed that a linear relationship exists between the fluid exchange terms and the pressure difference. Accordingly, these relationships are expressed in equation 2.6

\[ \Psi^{i\rightarrow d} = C^{id}(p^i - p^d) \]  
\[ \Psi^{j\rightarrow i} = C^{ji}(p^j - p^i) \]  

Where, \( C^{id} \) and \( C^{ji} \) are connectivity coefficients which depend on the matrix permeability, fluid viscosity and on the shape of the kernel function.

2.2.5. Remarks

This solver is capable of solving two and three dimensional problems. It has the capability of using a Gauss Seidal or AGM methods to solve for the fluid pressure. Because of the use of the a-HFR, this code allows for easy addition and manipulation of the discrete fractures without having to implement time consuming meshing routines. This modular structure is thus very suitable for coupling with an external geomechanical solver.

2.3. Geomechanics solver

The geomechanics solver is also an in-house solver developed at ETH. This solver assumes the fractured reservoir to be a continuum domain with embedded fracture manifolds. The solver allows for stress or displacement boundary conditions to be applied on the domain. Fracture locations, where slip can occur are predefined during the pre-processing stage of the solver. This solver solves only for steady state problems and all time dependencies are neglected.

2.3.1. Mathematical Formulation

\[ \nabla \cdot \tilde{\sigma} + \tilde{f}_r = \rho_r \frac{\partial^2 \tilde{u}_r}{\partial t^2} \]  

The equation 2.7 represents the force balance in the continuum domain. Further, this solver solves for the equilibrium problem in the EGS, i.e. it assumes that all perturbations
2.3. Geomechanics solver

Figure 2.2.: Representation of a fractured domain in an elastic medium in the geomechanics solver [Deb and Jenny]

are almost immediately propagated in the whole domain. Thus, the time dependent term is neglected in the domain, and the final stress equation takes the form of equation 2.8. It is noted the the discrete fractures are initially assumed to be closed and do not affect the stress solution in any way (until the point of failure).

\[ \nabla \cdot \tilde{\sigma} + \tilde{f}_r = 0 \]  \hspace{1cm} (2.8)

2.3.2. Closure using linear elasticity

A closure model relating stress and strain is required for equation 2.7. For most geomechanical processes, a linear relation between stress and strain is assumed to be sufficient. The linear elasticity model used in the solver is shown in equation 2.9.

\[ \tilde{\sigma} = \lambda \left( \nabla \cdot \tilde{u}_r \right) \mathbf{I} + G \left( \nabla \tilde{u}_r + \nabla \tilde{u}_r^T \right) \]  \hspace{1cm} (2.9)

The elastic strain is defined as given in equation 2.10.

\[ \tilde{\epsilon} = \frac{1}{2} \left( \nabla \tilde{u}_r + \nabla \tilde{u}_r^T \right) \]  \hspace{1cm} (2.10)
2.3. Geomechanics solver

Sometimes it is convenient to express these tensors in terms of their volumetric and deviatoric components. The volumetric components are shown in equation 2.11 and the deviatoric components are shown in equation 2.12.

\[
\begin{align*}
\epsilon_v &= \frac{\epsilon_{ii}}{3} \quad (2.11a) \\
\sigma_v &= \frac{\sigma_{ii}}{3} \quad (2.11b)
\end{align*}
\]

\[
\begin{align*}
\tilde{\epsilon}_d &= \tilde{\epsilon}_{ij} - \frac{\epsilon_{ii}}{3} \tilde{I} \quad (2.12a) \\
\tilde{\sigma}_d &= \tilde{\sigma}_{ij} - \frac{\sigma_{ii}}{3} \tilde{I} \quad (2.12b)
\end{align*}
\]

2.3.3. Failure criterion and slip solution

The geomechanics code allows for shear failure to occur in the discrete fractures. Shear failure occurs when the fracture is unable to sustain traction forces applied on it due to local shear stresses. This causes high shear stresses at the tips of the fracture which leads to further failure. Tensile failure on the other hand occurs when the normal stress on the fracture surface becomes positive.

A model based on static to dynamic friction transition is applied to study these modes of failure.

\[
\begin{align*}
\sigma_c &= -(\hat{n} \cdot \tilde{\sigma} \cdot \hat{n}) \quad (2.13) \\
\tilde{\tau} &= (\tilde{I} - \hat{n}\hat{n}) \cdot \tilde{\sigma} \cdot \hat{n} \quad (2.14) \\
|\tilde{\tau}| &\leq \left( S_0 + \varsigma_s \sigma_c \right) \quad (2.15)
\end{align*}
\]

Equation 2.13 defines the compressive force on the fracture plane. Equation 2.14 defines the traction forces acting on the fracture. These forces are compared to the static friction limit for a given compressive force. If the traction force exceeds the maximum static friction limit, then failure occurs and the friction coefficient is reset to its dynamic value.
2.4. Assumptions

The slip criterion is applied as shown in equation 2.15. The slip solution is then obtained by solving an additional equation which balances the total shear traction to the dynamic friction limit of normal compressive stress. It is also noted that the time taken for the slipping motion of the fracture is also neglected.

2.3.4. Remarks

It is seen that this code solves only for a steady state problem and it uses the Gauss-Seidal method for this purpose. At the moment, the solver supports only two dimensional problems. Also, the code is limited by the fact that it is unable to predict the failure of intersecting fractures. The code only predicts the stress distribution due to mechanical forces and it does not take thermal effects into account. Thus, the two way coupling between these two solvers will be constrained by the functionality offered by the geomechanical module, although the coupling can be easily updated when new features are made available in the geomechanical module.

2.4. Assumptions

Some of the assumptions made in solving the coupled problem using these solvers are listed in this section.

- The flow in the entire domain is assumed to be incompressible and single phase.
- The effect of temperature on the fluid flow is neglected.
- The time scale for the geomechanics is assumed to be much smaller than the time scale for the fluid flow.

2.5. Conclusion

This chapter briefly introduces the solvers which are used to study the coupled flow problem. The basic equations solved and the features offered by both these solvers were introduced. For the sake of brevity, the exact discretization and linearisation procedures used in these solvers was not provided and the interested reader can refer to the cited documentation for further elaboration.
Chapter 3. Coupling Methodology

3. Coupling Methodology

3.1. Introduction

In this chapter, the basic coupling methodology used is explained. First, a survey of various coupling techniques is done and the most suitable technique is selected for the current solvers. Next, the interpolations and modifications made to these solvers in order to make them compatible for the two way coupling is explained.

3.2. Coupling Strategies

There are four basic coupling strategies which are used in the literature to couple the geomechanics and fluid flow solvers. These are called the fully coupled (monolithic schemes), iteratively coupled (sequential schemes), explicit (single-pass sequential schemes) and the loosely coupled methods. A good literature survey of the authors using each these methods is given by [Kim] and [Kim et al.].

3.2.1. Fully coupled methods

![Figure 3.1: Schematic of fully coupled methods](image)

Fully coupled methods solve the coupled equations for the flow and geomechanics simultaneously at each time step. Usually, an implicit scheme is applied to the coupled system and both sets of equations are solved at the same time. This approach ensures that the
coupled system is stable and convergent. A drawback of this method is the need for a unified flow and geomechanics simulator, which can be difficult to construct and tends to be computationally expensive. Also, further constraints such as the use of the same grid and time-stepping for both problems makes this method highly rigid and inefficient.

3.2.2. Iteratively coupled methods

In this coupling method, either the fluid or the geomechanical problem is solved first and then the other problem is solved using intermediate information from the problem which was solved first. This procedure is iterated over the same time step until convergence. This iterative process ensures that the solution obtained is virtually identical to that obtained from the fully coupled system. The advantages of this kind of coupling methodology is plentiful. This type of coupling allows both of the problems to be solved, virtually independent of each other. Each problem can have its own grid, time steps and implicit solution algorithms with interpolation sub-routines which enable coupling within each time step. Thus, iteratively coupled systems are highly flexible and gives the user a great deal of freedom in constructing the problem in the most efficient manner possible. The main drawback of this coupling methodology is its lack of stability and convergence in cases where the problem is not clearly defined.

3.2.3. Explicitly Coupled methods

In this coupling method, either the fluid or the geomechanical problem is solved first and then the other problem is solved using intermediate information from the problem which was solved first. This procedure is iterated over the same time step until convergence. This iterative process ensures that the solution obtained is virtually identical to that obtained from the fully coupled system. The advantages of this kind of coupling methodology is plentiful. This type of coupling allows both of the problems to be solved, virtually independent of each other. Each problem can have its own grid, time steps and implicit solution algorithms with interpolation sub-routines which enable coupling within each time step. Thus, iteratively coupled systems are highly flexible and gives the user a great deal of freedom in constructing the problem in the most efficient manner possible. The main drawback of this coupling methodology is its lack of stability and convergence in cases where the problem is not clearly defined.
3.3. Coupling methods used in this work

Explicit, also known as the staggered schemes are a subclass of the Iteratively coupled methods. This scheme does not iterate the solution within each time step. Thus, these schemes are called as the single-pass sequential schemes where the coupling only takes place once in every time step. These schemes are used in cases where iterative schemes are deemed expensive and inefficient. This is the case when the effect of fluid flow on the geomechanics is small.

3.2.4. Loosely coupled methods

In this method, the coupling between the two solvers is established only after a predefined number of time steps. This method is computationally less expensive than the other solution methods but is far less accurate. It also requires the programmer to estimate the time-steps at which this coupling is to be performed, which is not a simple task for complex problems.

3.3. Coupling methods used in this work

In the present work, iteratively coupled and explicit methods are used.

Some of the advantages of using these coupling methods are:

- Cost for software development is low.
- Both solvers can have their own independent, specialized numerical algorithms.
- Greater flexibility is provided by allowing both the problems to have their own independent parameters such as grid and time-step sizes.
- Provides greater accuracy than the loosely coupled method.
3.4. Conclusion

Some of the disadvantages of using these coupling methods are:

- The solution is not as accurate as the fully coupled system.
- The solver is not unconditionally stable as the fully coupled system.

These coupling methods allow us to easily use the geomechanics and fluid flow solvers which were introduced in the previous chapter. Also, since the iteratively coupled scheme is employed, the results obtained are very close to those which could be theoretically obtained using the fully coupled algorithm. Thus, an external coupling module is introduced into these solvers which communicates between these solvers whenever required. This kind of coupling further allows the user to upgrade the functionality of each of the solvers independently and only minor changes will be needed in the coupling functions in order to simulate the coupled problem.

3.4. Conclusion

The survey of the various coupling methods used is done in order to determine the best form of coupling which could be employed for the current work. It was concluded that the explicit and the iteratively coupled scheme offered a number of advantages which far outweighed the disadvantages, and was thus chosen. The use of these coupling methods to study the effect of flow-mechanics coupling on the damaged matrix and on individual fracture segments are to be investigated. The next two chapters deal with the application of these coupling methods to the current solvers.
4. Damaged Matrix Coupling

4.1. Introduction

This chapter deals with the flow-mechanics coupling in the porous media, also known as the damaged matrix. First, the mathematics involved in coupling of the porous media is introduced. Next, the coupling algorithm used in this scenario is explored in detail. Finally, the coupling implementation is checked by simulating the drainage in a compressed, porous media, which exhibits the well documented Mandel-Cryer Effect.

4.2. Modifications to the Geomechanics Solver

The mathematical description of the flow in porous media was formulated using Biot’s theory of consolidation in a fluid saturated, deformable media. Details of this formulation are found in [Yang] and [Kim].

The changes in the geomechanics solver are introduced by the effect of pressure on the effective stress terms in the force balance equation in the continuum domain. According to [Biot], the effective stresses are calculated as given in equation 4.1. It is noted that the compressive stresses are taken to be negative by convention.

\[
\tilde{\sigma}' = \tilde{\sigma} + \tilde{I} \alpha p
\]  

(4.1)

The effective stresses described follow the constitutive law given in equation 2.9. Some of the steps required to implement this equation in the geomechanics code are given below.

\[
\tilde{\sigma} + \tilde{I} \alpha p = \lambda \left( \nabla \cdot \tilde{u}_r \right) \tilde{I} + G \left( \nabla \tilde{u}_r + \nabla \tilde{u}_r^T \right)
\]  

(4.2)
4.2. Modifications to the Geomechanics Solver

From equation 4.2, the stress components are obtained as given in equation 4.3.

\[ \sigma_{xx} = \lambda \left( \frac{\partial u_{rx}}{\partial x} + \frac{\partial u_{ry}}{\partial y} \right) + 2G \frac{\partial u_{rx}}{\partial x} - \alpha p \] (4.3a)

\[ \sigma_{xy} = \sigma_{yx} = G \left( \frac{\partial u_{rx}}{\partial y} + \frac{\partial u_{ry}}{\partial x} \right) \] (4.3b)

\[ \sigma_{yy} = \lambda \left( \frac{\partial u_{rx}}{\partial x} + \frac{\partial u_{ry}}{\partial y} \right) + 2G \frac{\partial u_{ry}}{\partial y} - \alpha p \] (4.3c)

Figure 4.1.: Geomechanics grid nomenclature [ Deb and Jenny ]

The discretization applied to this system of equation is consistent with that used in the Geomechanics code. The force balance equation in the X-direction is given in equation 4.4. This equation is discretized in as given by the terms in equation 4.5. The notations used are illustrated in figure 4.1.

\( (\sigma_{xx}|_E - \sigma_{xx}|_W) \Delta y + (\sigma_{xy}|_N - \sigma_{xy}|_S) \Delta x = 0 \) (4.4)

\( (\sigma_{xx}|_E - \sigma_{xx}|_W) = (\lambda + 2G) \left( \frac{\partial u_{rx}}{\partial x}|_E - \frac{\partial u_{rx}}{\partial x}|_W \right) + \lambda \left( \frac{\partial u_{ry}}{\partial y}|_E - \frac{\partial u_{ry}}{\partial y}|_W \right) - \alpha \left( p|_E - p|_W \right) \) (4.5a)

\( (\sigma_{xy}|_N - \sigma_{xy}|_S) = G \left( \frac{\partial u_{rx}}{\partial y}|_N - \frac{\partial u_{rx}}{\partial y}|_S \right) + G \left( \frac{\partial u_{ry}}{\partial x}|_N - \frac{\partial u_{ry}}{\partial x}|_S \right) \) (4.5b)
A similar set of equations can be derived analogously for the force balance in the Y-direction as well. Further, the quantities and derivatives at each face have to be calculated and these are obtained using a simple two point cell-centred scheme around each face. For further details regarding the discretization, please refer [Deb and Jenny].

4.3. Modifications to the Fluid Solver

The mass conservation equation as given in equation 2.2 has to be modified to account for the coupling. The major derivation steps are given below. For more details please refer [Kim].

Equation 2.2 is written in its simplified form in equation 4.6. This serves as the starting point of the derivation. Please note that the additional strain term is needed to account for the motion of the solid skeleton of the damaged matrix. A similar formulation is also used by [Rutqvist et al.].

\[
\begin{align*}
\frac{\partial \rho_f \phi}{\partial t} + \nabla \cdot \rho_f \vec{u} + \rho_f \phi \frac{\partial \epsilon_v}{\partial t} &= \rho_f q \\
\phi \frac{\partial \rho_f}{\partial t} + \rho_f \frac{\partial \phi}{\partial t} + \nabla \cdot \rho_f \vec{u} + \rho_f \phi \frac{\partial \epsilon_v}{\partial t} &= \rho_f q \\
\phi \frac{\partial \rho_f}{\partial p} \frac{\partial p}{\partial t} + \rho_f \frac{\partial \phi}{\partial t} + \nabla \cdot \rho_f \vec{u} + \rho_f \phi \frac{\partial \epsilon_v}{\partial t} &= \rho_f q \\
c_f &= \frac{1}{\rho_f} \frac{\partial \rho_f}{\partial p}
\end{align*}
\]  

From the definition of fluid compressibility which is defined in equation 4.9, equation 4.8 is further modified as given in equation 4.10.

\[
\phi \rho_f c_f \frac{\partial p}{\partial t} + \rho_f \frac{\partial \phi}{\partial t} + \nabla \cdot \rho_f \vec{u} + \rho_f \phi \frac{\partial \epsilon_v}{\partial t} = \rho_f q
\]

The variation of porosity with time, for a single phase flow is given in equation 4.11.

\[
\frac{\partial \phi}{\partial t} = \frac{\alpha - \phi}{K_s} \frac{\partial p}{\partial t} + (\alpha - \phi) \frac{\partial \epsilon_v}{\partial t}
\]
4.3. Modifications to the Fluid Solver

Where, the Biot coefficient is defined as given in equation 4.12 and the drained bulk modulus is defined in equation 4.13. The relationship between the poisson’s ration and the shear modulus and Lamé constant is expressed in equation 4.14.

\[
\alpha = 1 - \frac{K_{dr}}{K_s} \tag{4.12}
\]

\[
K_{dr} = \frac{E}{2(1 + \nu)(1 - 2\nu)} \tag{4.13}
\]

\[
\nu = \frac{\lambda}{2(\lambda + G)} \tag{4.14}
\]

Using these definitions, equation 4.10, is further simplified as given in equation 4.15.

\[
\rho_f \left\{ \frac{\partial p}{\partial t} \left( \phi c_f + \alpha - \phi \right) + \alpha \frac{\partial \varepsilon_v}{\partial t} \right\} + \nabla \cdot \vec{u} = \rho_f q \tag{4.15}
\]

The equation 4.15 can be reformulated as a function of the mean volumetric stress using the constitutive relation between effective stress and strain, this is shown in equation 4.16.

\[
\left( \frac{1}{M} + \frac{\alpha^2}{K_{dr}} \right) \frac{\partial p}{\partial t} + \alpha \frac{\partial \sigma_v}{\partial t} + \nabla \cdot \vec{u} = q \tag{4.16}
\]

Where,

\[
M = \phi c_f + \frac{\alpha - \phi}{K_s} \tag{4.17}
\]

\[
\sigma_v = \frac{1}{3} \left( \sigma_{xx} + \sigma_{yy} + \sigma_{zz} \right) \tag{4.18}
\]

Usually, the permeability changes in the porous media are assumed to be negligible, but the current coupling module also provides an option to take this variation into account. The permeability is assumed to be a function of porosity as given in equation 4.18 ([Rutqvist et al.]).

\[
\tilde{k} = \tilde{k}_0 \exp \left( C_1 \left( \frac{\phi}{\phi_0} - 1 \right) \right) \tag{4.19}
\]

Where, \( \tilde{k}_0 \) and \( \phi_0 \) are the permeability and porosity at zero stress respectively and the constant \( C_1 \) has to be determined experimentally.
4.4. Coupling Algorithm

Both the explicit and the iteratively coupled methods are implemented and tested for the damaged matrix. Figures 4.2 and 4.3 show a simple flow chart for the explicit and iteratively coupled methods respectively.

4.5. Validation using the Mandel problem

The coupling algorithm is validated using the Mandel problem. The Mandel setup consists of a porous medium which is compressed between two flat plates. This constant compression produces a uniform pressure rise inside the porous medium by the Skempton effect \(p_0\). Please note that the final pressure due to the Skempton effect is taken as the initial conditions for this test case. Next, this pressure rise is allowed to drain out of the system by applying drainage boundaries \(p_{\text{drain}}\) on the left and right edges of the domain. Thus, the pressure near the edges slowly dissipate due to the drainage conditions and at a very large time, the pressure inside the domain must become equal to the drainage pressure conditions. The effect of gravity is neglected in all cases.

It was observed by Mandel that due to the sudden drop of pressure near the boundaries of the domain, the pore pressure increases beyond the initial value predicted by the Skempton effect. This effect is dominant during the initial stages of the draining process but it is soon exceeded by the drainage process itself, and thus at later times, the pressure decreases everywhere in the domain. This effect has been well documented, both numerically and experimentally. This effect is usually called the Mandel-Cryer effect, after the authors who first noticed this phenomena in porous media.

This test cases serves as an effective benchmark in the validation of the coupled algorithm because of the existence of an analytical solution for this problem. The analytical solution of this case is shown in Appendix A.

The Mandel problem was set up and a number of simulations were done in order to check the accuracy of the coupled system. Most of the results were obtained by using a probe at the center of the computational domain. The results obtained were compared to the analytical solution of the same setup.

The figures 4.5 and 4.6 show sample pressure fields inside the entire domain at two different times. The Mandel-Cryer effect can be easily observed in this case with the pressure at the center of the domain increasing beyond the initial pressure produced by the Skempton effect. This kind of pressure variation can only be observed using a coupled system. This behaviour is exactly what is expected of these schemes.
4.5. Validation using the Mandel problem

Figure 4.2.: Explicit coupling algorithm
4.5. Validation using the Mandel problem

**Figure 4.3:** Iterative coupling algorithm
4.5. Validation using the Mandel problem

4.5.1. Geometry and boundary conditions

The general setup of the Mandel problem, including some of the notations used, are shown in figure 4.4. The current test case uses a square domain of side 1000 m. The initial and boundary conditions used for the coupled system are summarised in table 4.1.

4.5.2. Time Convergence

Both the explicit and iteratively coupled methods were checked for time convergence using variable time steps. In this case, a 100 × 100 grid was chosen and this was kept constant for all of the simulations.

Figure 4.7 shows the pressure variation at the center of the domain, for an explicitly coupled system with varying time-step sizes. It is clearly seen that the solutions converge to a unique solution with decreasing time step sizes.
4.5. Validation using the Mandel problem

**Figure 4.5.:** Pressure solution of the Mandel problem at time = 1 s

**Figure 4.6.:** Pressure solution of the Mandel problem at time = 76 s
4.5. Validation using the Mandel problem

Figure 4.7.: Time convergence of the explicitly coupled system

Figure 4.8.: Comparison of explicit and iterative schemes with $\Delta t = 100$ s
4.5. Validation using the Mandel problem

**Figure 4.9.** Comparison of explicit and iterative schemes with $\Delta t = 50$ s

**Figure 4.10.** Comparison of explicit and iterative schemes with $\Delta t = 1$ s
4.5. Validation using the Mandel problem

Table 4.1: Initial and boundary conditions for the Mandel problem

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_0)</td>
<td>0.3</td>
<td>–</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>(k_0)</td>
<td>(5 \times 10^{-11})</td>
<td>m(^2)</td>
</tr>
<tr>
<td>(\mu)</td>
<td>(1 \times 10^{-3})</td>
<td>kg m(^{-1}) s(^{-1})</td>
</tr>
<tr>
<td>(\rho_f)</td>
<td>(1 \times 10^3)</td>
<td>kg m(^{-3})</td>
</tr>
<tr>
<td>(\rho_r)</td>
<td>(2.7 \times 10^3)</td>
<td>kg m(^{-3})</td>
</tr>
<tr>
<td>(G)</td>
<td>(1.45 \times 10^9)</td>
<td>N m(^{-2})</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>0</td>
<td>N m(^{-2})</td>
</tr>
<tr>
<td>(c_f)</td>
<td>(0.5 \times 10^{-9})</td>
<td>N(^{-1}) m(^2)</td>
</tr>
<tr>
<td>(K_{dr})</td>
<td>(1.45 \times 10^9)</td>
<td>N m(^{-2})</td>
</tr>
<tr>
<td>(p_0)</td>
<td>(2 \times 10^6)</td>
<td>N m(^{-2})</td>
</tr>
<tr>
<td>(p_{\text{drain}})</td>
<td>0</td>
<td>N m(^{-2})</td>
</tr>
<tr>
<td>(\frac{F}{\pi})</td>
<td>(5 \times 10^6)</td>
<td>N m(^{-2})</td>
</tr>
</tbody>
</table>

The figures 4.8 to 4.10 compare the capability of the explicit and the iteratively coupled methods. It is observed that at higher time step sizes, the iteratively coupled method performs better than the explicitly coupled method. But, at smaller time step sizes, both the iterative and the explicitly coupled methods tend to converge.

4.5.3. Grid Convergence

The grid convergence study was carried out using the explicitly coupled system with a time-step size of 1 s. Figure 4.12 pressure profile at the center of the domain using a \(100 \times 100\) and a \(200 \times 200\) grid. It is seen that these solutions are also convergent.

4.5.4. Properties at an offset position

In this subsection, the results for porosity and pressure variations obtained at the center of the domain and a point 105 m away from the left edge of the domain are shown.

Figure 4.12 shows the pressure variation at the offset position. This is also compared to the analytical solution and it also shows a relatively good match. Figures 4.13 and 4.14 show the variation of porosity and pressure at these locations.

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4.5. Validation using the Mandel problem

Figure 4.11.: Grid convergence of the explicitly coupled system

Figure 4.12.: Pressure variation at an offset position
4.5. Validation using the Mandel problem

**Figure 4.13.**: Porosity variation in the Mandel problem

**Figure 4.14.**: Pressure variation in the Mandel problem
4.5. Validation using the Mandel problem

4.5.5. Effect of permeability variation

Another point to be noted is the fact that the analytical solution assumes that the permeability variation in the porous domain is negligible. But the code developed has the capability of taking this variation into account as well. To show this, the same test case was run again, but this time, the variation of permeability with porosity (equation 4.19) is taken into account. In this case $C_1 = 22.2$, which is consistent with the experimental results obtained for sandstone [Rutqvist et al.].

![Permeability variation in the Mandel problem](image)

**Figure 4.15.** Permeability variation in the Mandel problem

Figure 4.15 shows the permeability variation at the center of the domain.

Figure 4.16 shows the difference in pressure at the center of the domain when the permeability effects are taken into account. It is seen that because of the variation in permeability, the pressure field inside the domain takes a slightly longer time to drain. This is attributed to the fact that the average permeability inside the domain reduces, thus making the drainage flow slower.
4.5. Validation using the Mandel problem

Figure 4.16.: Pressure difference due to variable permeability

Figure 4.17.: Simulation time for the explicit and iteratively coupled methods
4.5.6. Efficiency of the explicit and iteratively coupled methods

Figure 4.17 shows the time needed for one simulation step for the explicit and iteratively coupled methods. It is seen that the iteratively coupled method is far more computationally expensive than the explicitly coupled methods. Thus, iteratively coupled methods should only be used when high accuracy is required, or when the time step or grid sizes used are not sufficient for convergence.

4.5.7. Remarks on the Mandel problem

There are several factors which have to be noted while assessing the results of the Mandel test case. The first point to be noted is the fact that although the results obtained show a very good fit with the analytical data, nevertheless, they never converge exactly with the analytical solutions even with further time and grid refinement. This is attributed to the fact that the analytical solution assumes that the variation of flow properties along the Y-direction is negligible. But this assumption is never fully valid in the case of the numerical code. Such a differences between the analytical and numerical solutions are also pointed out by [Winterfeld et al.]. It is also noted that the permeability changes are neglected in the analytical solution of the Mandel problem.

4.6. Application of the coupled code

In this section, the coupled code is applied to a two dimensional stressed porous media with a constant rate of production. These results are compared to the uncoupled case and significant differences in the solution are seen. This indicates the importance of the fluid-geomechanics coupling in the simulation of these types of problems.

4.6.1. Geometry and boundary conditions

Figure 4.18 shows the geometry and stress conditions applied to this problem. The initial and boundary conditions used for this setup are summarised in table 4.2.

4.6.2. Results

The results obtained for this case is presented in this section. The results are both grid and time convergent and the simulations were run using a $50 \times 50$ and a $100 \times 100$ grid and the time step sizes were 20s and 50s.
Figure 4.18.: Two dimensional production setup
Table 4.2.: Initial and boundary conditions for the production problem

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_0$</td>
<td>0.3</td>
<td>–</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>$k_0$</td>
<td>$1 \times 10^{-13}$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$5 \times 10^{-3}$</td>
<td>kg m$^{-1}$ s$^{-1}$</td>
</tr>
<tr>
<td>$\rho_f$</td>
<td>$1 \times 10^3$</td>
<td>kg m$^{-3}$</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>$2.7 \times 10^3$</td>
<td>kg m$^{-3}$</td>
</tr>
<tr>
<td>$G$</td>
<td>$0.833 \times 10^9$</td>
<td>N m$^{-2}$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$0.556 \times 10^9$</td>
<td>N m$^{-2}$</td>
</tr>
<tr>
<td>$c_f$</td>
<td>$1 \times 10^{-9}$</td>
<td>N$^{-1}$ m$^2$</td>
</tr>
<tr>
<td>$K_{dr}$</td>
<td>$1.38 \times 10^9$</td>
<td>N m$^{-2}$</td>
</tr>
<tr>
<td>$p_0$</td>
<td>$3.468 \times 10^6$</td>
<td>N m$^{-2}$</td>
</tr>
<tr>
<td>$E$</td>
<td>$5 \times 10^6$</td>
<td>N m$^{-2}$</td>
</tr>
<tr>
<td>flow rate</td>
<td>$-8 \times 10^{-6}$</td>
<td>m$^3$ s$^{-1}$</td>
</tr>
</tbody>
</table>

Figure 4.19 shows the contours of pressure in this case. It is seen that because of the imposed flow rate out of the domain at the well, the fluid flows out of the domain and this leads to a decrease in pressure in the domain which varies radially from the well location. Figure 4.20 shows the porosity variation in the domain. Because the porosity decreases near the well, this causes an effect similar to the Mandel-Cryer effect and this causes the pressure drop to occur much slower than when compared to the uncoupled problem. This effect is clearly shown in figure 4.21. This effect is attributed to the porosity variation differences between the coupled and the uncoupled codes and this is shown in figure 4.22. These results follow, qualitatively, the expected trend for such simulations as discussed by [Yang]. (Note that the exact simulation as discussed by [Yang] could not be reproduced as the flow rate mentioned in this paper was ambiguous)

Thus, this case clearly shows the need for fluid-geomechanical coupling while simulating flows within a stressed porous media.

4.7. Conclusion

This chapter has dealt with the coupling applied to the porous media. The mathematical formulation involved and the coupling algorithms have been introduced. Both the explicit and the iteratively coupled methods were implemented and tested. It was seen that although the iterative scheme performs better than the explicitly coupled scheme, its prohibitive computational cost makes it inefficient to use, unless high accuracy is required.
Figure 4.19.: Pressure contours of the production problem

Figure 4.20.: Porosity contours of the production problem
4.7. Conclusion

Figure 4.21.: Pressure variation at the center of the domain

Figure 4.22.: Porosity variation at the production point
4.7. Conclusion

The damaged matrix coupling was verified using the Mandel problem and it was seen that the numerical solution agreed closely with analytical solution. Next, this coupled system was used to study a production case in a stressed porous media. It was seen that the coupled code is necessary to accurately simulate the physics of such problems.
5. Discrete Fracture Coupling

5.1. Introduction

This chapter deals with the flow-mechanics coupling when dealing with slipping in single discrete fractures. First, the mathematics involved are introduced and next, the coupling algorithm used in this scenario is explored in detail. This algorithm is then applied to the shear stimulation of a single discrete fracture.

5.2. Modifications to the Geomechanics Solver

When dealing with the two way fracture coupling, modifications in the code are required for both the solvers. In the geomechanical solver, the main modification to be made is the change of the slip criteria. The injection of fluid into the domain has the effect of destabilizing the fractures. When the fluid is injected into the system, the pressure inside the fracture increases, this in turn reduces the effective compressive stresses acting on the fracture. Thus, smaller traction forces are able to induce failure in the fractures. This effect of fluid pressure has to included in the slip criteria. This change is shown in equation 5.1.

\[
|\bar{t}| \leq \left[ S_0 + \varsigma_s (\sigma_c - p) \right]
\]  

(5.1)

5.3. Modifications to the Fluid Solver

Failure of the fracture results in a change in its aperture. The aperture change is modelled as a linear function of slip in each fracture segment and is shown in equation 5.2. \( \theta \) is a constant coefficient whose values usually lie in the range of \([3^\circ - 10^\circ]\). This is a simplified model of the aperture change. More exact models like the one implemented by [McClure and Horne] can be introduced easily into the solver if required.

\[
\Delta b = \Delta S \tan(\theta)
\]  

(5.2)
5.4. Quantification of earthquake magnitude

\[ k^i = b^2/12 \]  (5.3)

This change in the fracture aperture affects the fracture permeability and fracture volume, which in turn affect the fracture transmissibility. In the fluid code, the permeability is modelled as given in equation 5.3. For more details on transmissibility changes which have to be done to the fluid code, please refer [Karvounis].

### 5.4. Quantification of earthquake magnitude

Seismic moment and the moment magnitude are two functions which are used by modern seismologists to quantify the magnitude of a failure event.

\[ M_o = G\Delta SL_x L_z \]  (5.4)

The seismic moment is a measure of the size and energy release of an earthquake. The mathematical formulation of seismic moment is given in equation 5.4. Here, \( L_x L_z \) is the area of the fracture segment which slips during the fracture process. In our case, since we are dealing with a two dimensional domain, we need to provide a reliable estimate for the value of \( L_z \).

\[ M_W = \log_{10}M_o - 6.07 \]  (5.5)

MMS (\( M_W \)) was developed in the 1970’s to indicate the magnitude of an earthquake. This scale was developed in order to improve on some of the drawbacks which were faced by the Richter scale. The mathematical formulation of the moment magnitude is given in equation 5.5.

### 5.5. Two point interpolation

These code modifications have to be done and their values interpolated between both grids. A coupling module is inserted between the two solvers which handles the data transfer and interpolation between the time steps. At the moment, a two point linear interpolation is found to be sufficient for the fracture segments, but a more accurate interpolation scheme can be employed later if necessary. The accuracy of the two point scheme is shown in figure 5.1.
5.6. Coupling Algorithm

The Discrete fracture coupling algorithm is shown schematically in figure 5.2. This algorithm is essentially an explicit algorithm except for the fact that it has a bisection step introduced when it detects a fracture.

5.6.1. Bisection Algorithm

The bisection algorithm is initiated when the code detects fracture failure at a particular time step. In this case the code resets the time step to just before the fracture took place and it uses a bisection algorithm to detect the exact point of fracture (with a certain tolerance) in this time step. The bisection algorithm is necessary because the fracturing is a discrete event and it is required to find, accurately the exact time of failure. This algorithm enforces time convergence of the coupled code even when relatively large time step sizes are used. Figure 5.3 shows, schematically, the working of the bisection algorithm. The black circle indicates the point at which the bisection algorithm checks for failure. The green colour indicates that the fracture has not occurred yet and the red colour
5.6. Coupling Algorithm

**Figure 5.2:** Discrete fracture coupling algorithm
5.6. Coupling Algorithm

Figure 5.3.: Working of the bisection algorithm
indicates a fracture slip. It is clearly seen that the bisection algorithm iterates over this
time step till the fracture point is determined within a given tolerance limit.

5.6.2. Fracture time scale division

After detecting the point of fracture with the bisection algorithm, the geomechanics code is
run to determine the magnitude of slip in the fracture elements. This slip is interpolated
into the fluid solver and this slip corresponds to a change in volume of the fracture
segment. This change in volume is directly interpolated from the geomechanics code and
the influence of pressure on the volume is neglected during the fracturing process. It is
also assumed that this fracturing process takes place over a small time interval which
is called the fracture time scale and this is taken to be much smaller than the global
time scale. It must be noted that a very small time scale cannot be chosen in this case
because the fracturing process causes a large jump in volume of the fracture nodes. This
jump in volume causes a large sink term to be introduced into fluid PDEs and thus,
stability is not ensured in this process. Thus, a good estimate of the fracture time scale
which does not cause instability is required to estimate this process. In order to enforce
stability in the fluid code when large changes in volume take place, this volume change is
further divided into a number of steps which divide the volume change of the fracture into
smaller segments. Figure 5.4 shows this process schematically. In this figure, the fracture
is divided into three segments but the number of segments can be varied depending on
the magnitude of volume change expected during the fracturing process.
5.7. Shear stimulation of a single fracture

The explicit fracture simulation algorithm is applied to the case of a single horizontal fracture. This case consists of a horizontal fracture embedded in the damaged matrix. In this case, the effect of geomechanics on damaged matrix is neglected. Thus, the coupling interpolations are needed only within the nodes of the fracture segment.

5.7.1. Geometry and boundary conditions

Figure 5.5.: Shear stimulation setup

Figure 5.5 shows the configuration used for the shear stimulation problem. A constant compressive and shear stresses are applied on the rock domain. A horizontal fracture is placed at the center of the domain with an injection well placed at its center. Zero pressure boundary conditions are applied along the boundary edges of the domain. A constant flow rate is injected through the well which allows the pressure to build inside the domain. The pressure rises in the domain until it reaches the critical value at which
5.7. Shear stimulation of a single fracture

Shear stimulation of a single fracture slips. This causes the volume of the fracture to increase, which in turn affects the pressure in the domain. A $161 \times 161$ grid is used for all of the simulations. The fractures are divided into 81 fracture segments which gives a fracture segment length of around $6.17m$ which is approximately that which is used by [McClure and Horne]. The effect of gravity is neglected in all cases. It is noted that the well radius influences the production index of the well. The well model used in the fluid flow solver is discussed in detail by [Karvounis].

Table 5.1.: Initial and boundary conditions for the shear stimulation problem

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_0$</td>
<td>0.001</td>
<td>–</td>
</tr>
<tr>
<td>Fracture storativity</td>
<td>$5 \times 10^{-14}$</td>
<td>–</td>
</tr>
<tr>
<td>Porous media storativity</td>
<td>$5 \times 10^{-9}$</td>
<td>–</td>
</tr>
<tr>
<td>$\theta$</td>
<td>3°</td>
<td>–</td>
</tr>
<tr>
<td>$b$</td>
<td>0.0001</td>
<td>m</td>
</tr>
<tr>
<td>$L_z$</td>
<td>200</td>
<td>m</td>
</tr>
<tr>
<td>$k_0$</td>
<td>$2 \times 10^{-15}$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$1 \times 10^{-3}$</td>
<td>kg m$^{-1}$ s$^{-1}$</td>
</tr>
<tr>
<td>$\rho_f$</td>
<td>$1 \times 10^{3}$</td>
<td>kg m$^{-3}$</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>$2.7 \times 10^{3}$</td>
<td>kg m$^{-3}$</td>
</tr>
<tr>
<td>$G$</td>
<td>$15 \times 10^{9}$</td>
<td>N m$^{-2}$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$15 \times 10^{9}$</td>
<td>N m$^{-2}$</td>
</tr>
<tr>
<td>$p_0$</td>
<td>0</td>
<td>N</td>
</tr>
<tr>
<td>$S_0$</td>
<td>0</td>
<td>N</td>
</tr>
<tr>
<td>$\varsigma_d$</td>
<td>0.55</td>
<td>N</td>
</tr>
<tr>
<td>$\varsigma_s$</td>
<td>0.60</td>
<td>N</td>
</tr>
<tr>
<td>$\xi_a$</td>
<td>$50 \times 10^{6}$</td>
<td>N m$^{-2}$</td>
</tr>
<tr>
<td>Shear</td>
<td>$10 \times 10^{6}$</td>
<td>N m$^{-2}$</td>
</tr>
<tr>
<td>Well radius</td>
<td>0.18</td>
<td>m</td>
</tr>
<tr>
<td>flow rate</td>
<td>$2 \times 10^{-4}$</td>
<td>m$^3$ s$^{-1}$</td>
</tr>
</tbody>
</table>

Table 5.1 lists the initial and boundary conditions used for the shear stimulation case.

5.7.2. Time convergence

The figure 5.6 shows the increase of fracture area with time. The sharp jumps denote the points at which fracture slipping occurs and a corresponding increase in fracture area is seen. The global time step sizes chosen were 40s, 80s, 160s, and 320s respectively. It is seen that the algorithm predicts an almost identical fracture propagation irrespective of
5.7. Shear stimulation of a single fracture

Figure 5.6.: Fracture area propagation

Figure 5.7.: Fracture area propagation close-up
5.7. Shear stimulation of a single fracture

the time step sizes chosen. This is attributed to the accurate working of the bisection algorithm.

Figure 5.7 shows a close up view of one of the fracture points. It is seen that the algorithm predicts that the fracturing occurs a few minutes apart. This is attributed to the fact the fluid solver gives a slightly non-converged pressure when using different time step sizes. Although this effect is present, it was concluded that error introduced by this is negligible when compared to the overall time scale of the problem.

![Fracture aperture time convergence](image)

**Figure 5.8.:** Fracture aperture time convergence

Figures 5.8 and 5.9 shows the fracture aperture and pressure profile inside the fracture respectively after a time of 11 days (represented by the final magenta bar in figure 5.6). Good convergence properties are seen with the aperture and pressure profiles as well.

Figures 5.10 and 5.11 show the aperture and pressure profiles at various stimulation times (indicated by the magenta bars in figure 5.6). It is seen that the fracture and pressure propagate along the horizontal fracture as time progresses. From the aperture plot it is seen that it is possible that one segment keeps undergoing failure (in this case the central point) which might lead to unphysical aperture values. This can be corrected by using a more accurate function (equation 5.2) to estimate the fracture aperture change from the slip values.

Figures 5.12 and 5.13 show the plots of the seismic moment and the moment magnitude respectively. It is noted that the moment magnitude has the same order of magnitude
5.7. Shear stimulation of a single fracture

Figure 5.9.: Fracture pressure time convergence

Figure 5.10.: Aperture propagation with time
5.7. Shear stimulation of a single fracture

Figure 5.11.: Pressure profile propagation with time

Figure 5.12.: Seismic moment of the fracture
5.7. Shear stimulation of a single fracture

Figure 5.13.: Moment magnitude of the fracture

to what is expected for this kind of fracture stimulation. [McClure and Horne] show a moment magnitude in the range of 1 to 3, although in this case, a constant pressure injection was used.

5.7.3. Effect of fracture time scale

In this section the fracture time scale was varied in order to check the consistency of the code. Three different time scales of 0.1s, 0.3s and 1s were tested.

Figure 5.14 and 5.15 shows the effect of fracture time scale on the fracture area propagation and the total slip respectively. It is seen that the fracture time scale effect on the solution is negligible, this is exactly how the code is expected to perform. But, it is noted that taking a very small time scale could lead to instabilities in the Fluid solver.

5.7.4. Effect of fracture division segments

In this section the number of fracture division segments were varied in order to check the consistency of the code. The code was tested by using four different division steps of 1, 3, 5 and 10 respectively.
5.7. Shear stimulation of a single fracture

Figure 5.14.: Effect of time scale on fracture area propagation

Figure 5.15.: Effect of time scale on total slip
5.7. Shear stimulation of a single fracture

**Figure 5.16.** Effect of number of division segments on fracture area propagation

**Figure 5.17.** Effect of number of division segments on traction

Time = 949760 s / 11 days
5.7. Shear stimulation of a single fracture

Figure 5.18 and 5.17 shows the effect of the number of fracture division segments on the fracture area propagation and the traction respectively. It is seen that effect of dividing the fracture segments into a number of discrete portions on the solution is negligible. Thus, it is possible to stabilize the fluid code to manage very large changes in aperture by dividing the volume change into a number of discrete segments.

It is seen in figure 5.17, the traction magnitude increases at the edge of the fractured segments. This increase in traction destabilizes the fracture and these segments are capable of failing at smaller pressure magnitudes, depending on the magnitude of traction. The decrease in traction (depending on $\varsigma_d$) of the slipped segments causes them to stabilize and a larger pressure is required to fracture these segments again.

5.7.5. Effect of flow rate variation

In this section the injection flow rate through the well was varied in order to check its effect on the fracture propagation. The code was tested by using three different flow rates of 0.0002 m$^3$ s$^{-1}$, 0.00025 m$^3$ s$^{-1}$ and 0.0003 m$^3$ s$^{-1}$ respectively.

![Figure 5.18: Effect of flow rate on fracture area propagation](image)

It is seen that as the flow rate increases, the fracture process occurs at a faster rate and the fracture area increases at a faster rate as the the well injection flow rate increases.
5.8. Conclusion

In this chapter, an algorithm used to simulate the shear injection of a fracture was developed. The algorithm developed is ensured to have time convergence and is able to handle the large changes of aperture which is introduced by the fracturing process. Further research is required to determine the effect of fracture segment sizes on the accurate prediction of fracture slips. Also, accurate fracture time scales and aperture change functions can be implemented into code.
This work has dealt with the implementation of a coupling module between a geomechanical and a fluid flow solver. Both of these solvers were independently developed at ETH using the c++ platform. A coupling module was introduced as a mediator between these solvers. The coupling module is capable of handling independent execution of both these solvers and also able to interpolate the required quantities between the two solvers.

A literature study of the various coupling methods which could be applied to these solvers was performed. The survey showed that there were four basic types of coupling methods which could be used in the present work. These were the fully, explicitly, iteratively and loosely coupled methods respectively. It was further concluded that the explicit and iterative method were the ones best suited for the current work. Coupling algorithms was applied to couple both the porous media (damaged matrix) and the discrete fractures. Changes in the mathematical implementation was required in each of these solvers in order to simulate these scenarios.

The damaged matrix was coupled using both the explicit and iterative coupling algorithms. The accuracy of the coupling module was tested by comparing the numerical solution obtained with the analytical solution of the Mandel problem. The results obtained agreed very well with the analytical solution. Thus, the coupling algorithm was concluded to work accurately. Both the explicit and iteratively coupled algorithms seem to give comparable results with the iterative method performing better in certain cases. It was also seen that the computational resources needed for the iterative algorithm was significantly larger than for the explicit algorithm. Thus, it was concluded that the explicit algorithm is sufficient for most practical applications. This coupled code was then applied to the case of a production problem which demonstrated the need for the fluid-geomechanical coupling in order to simulate the physics of such problems accurately.

The discrete fractures were coupled using a slight modification of the explicitly coupled algorithm. The explicit algorithm was improved by adding a bisection algorithm and a fracture division algorithm. The bisection algorithm ensured time convergence of the fracture stimulation problem even while using relatively large global time steps. The fracture division algorithm was introduced in order to make the fluid code more stable when handling large changes in fracture volume during the fracturing process. This algorithms was then applied to the case of the shear stimulation of a single horizontal fracture. The results showed that the fracture propagation was relatively time convergent even while using significantly large differences in global time step sizes. The fracture
division algorithm was also found to work well giving little variation while changing the relevant parameters.

Further improvements in the coupling module are possible. At the moment, the damaged matrix coupling is implemented in such a way that both the geomechanical and fluid flow solver have overlapping grids. An interpolation function could be implemented such that both the solvers use a non-overlapping grids with a different number of grid points. This could make the coupling module more efficient. Presently a very simple function is used to determine the effect of slip on the fracture aperture, this model can be improved to ensure a more physically accurate fracture aperture change. A study to find the effective fracture segment size which closely approximates the actual fracturing process can be carried out. Also test cases can be implemented which coupled both the damaged matrix and the discrete fractures. This can be used to study the effect of the damaged matrix on seismic stability of the discrete fractures.
BIBLIOGRAPHY

Bibliography


A. Analytical solution of the Mandel problem

The general analytical solution for the Mandel problem was derived by [Abousleiman et al.]. In this appendix, only the final results for the special case of an isotropic and incompressible solid constituents \((K_s \to \infty)\) of the damaged matrix is reproduced. This is the kind of problem which is numerically analysed in this thesis. For further details, and a more general formulation please refer to the mentioned paper.

The analytical solution for pressure is given in equation A.1

\[
p = \frac{2F}{aA_1} \sum_{i=1}^{\infty} \frac{\sin \beta_i}{\beta_i - \sin \beta_i \cos \beta_i} \times \left( \cos \frac{\beta_i x}{a} - \cos \beta_i \right) \times \exp\left( -\frac{\beta_i^2 c_1 t}{a^2} \right) \quad (A.1)
\]

Where,

\[
A_1 = \frac{3}{B(1 + \nu^2)} \quad (A.2)
\]

\[
A_2 = \frac{\alpha(1 - 2\nu)}{1 - \nu} \quad (A.3)
\]

\[
c_1 = \frac{k_1 M_{11}}{\mu M_{11}^a} \quad (A.4)
\]

\[
M = \frac{K_f}{\phi} \quad (A.5)
\]

\[
M_{11} = \frac{E(1 - \nu)}{1 - \nu - 2\nu^2} \quad (A.6)
\]

\[
M_{11}^a = M_{11} + \alpha^2 M \quad (A.7)
\]
Also, $\beta_i$ has to satisfy the characteristic equation given in equation A.8. But in order to avoid messy singularities in the numerical calculation of the roots of this equation, equation A.9 is used instead.

$$\frac{\tan \beta_i}{\beta_i} = \frac{A_1}{A_2}$$  \hspace{1cm} (A.8)

$$\sin \beta_i - \left(\frac{A_1}{A_2}\right) \cos \beta_i = 0$$  \hspace{1cm} (A.9)

**Figure A.1.:** Characteristic roots of the Mandel problem

The solution of the equation A.9 is graphically represented as the intersection of the two curves shown in figure A.1. A numerical code in Matlab was written in order to find the solution of equation A.1 upto a desired accuracy.