Localization of Acoustic Emissions in a numerically simulated T-shaped Concrete Beam using Multi-Segment Path Analysis

Stephan Gollob, Georg K. Kocur, and Thomas Vogel

Abstract Common source localization algorithms assume a straight wave propagation path. If the straight connection of source and sensor passes air, these straight connection surely does not represent the wave propagation path. A crack can be such an air barrier between source and sensor. However, in case of complex specimens (e.g. T-shaped or C-shaped cross-section) the straight connection between source and sensor can also pass the air surrounding the specimen. FastWay is a novel source localization method, based on multi-segmented path analysis and a heterogeneous velocity model of the investigated specimen. In order to determine the source location, the fastest wave travel path between the sensor and an estimated source location is determined. This wave travel path is multilinear, bypasses air and bases on a heterogeneous velocity model of the specimen. In this paper, the estimated source location of acoustic emissions within a numerically simulated T-shaped concrete beam, determined with FastWay is compared with the results provided by a homogeneous and a heterogeneous Geiger method.

Index Terms wave propagation simulation, numerical concrete model, multi-segmented wave propagation path analysis, FastWay.

1 Introduction

The assessment of health conditions of existing structures is becoming more and more important. Acoustic emission (AE) analysis, a non-destructive testing and monitoring method, can be used for that specific purpose. Common source localization algorithms assume a straight wave propagation path. This prerequisite limits the area of application of the AE analysis. If the arrival time of the detected AE is used for the estimation of the source location, the fastest wave propagation path is of interest. The straight connection between the source and a sensor is only the fastest wave travel path if the wave propagates through a homogeneous specimen. Whenever the specimen is made of a heterogeneous material (e.g. concrete) or a combination of different materials (e.g. reinforced concrete), the fastest wave propagation path is usually not a straight path. In addition, the wave will have to bypass obstacles on its way between source and sensors. These obstacles can occur over time (e.g. cracks) or can be geometrical (e.g. T-shaped girder). FastWay is able to consider such obstacles. Moreover, it is possible that the monitored specimen/area is not enclosed by sensors. Sometimes, not all of the specimen’s surfaces are accessible. In case of a T-shaped girder, the bearing element is often loaded at the top, and/or an additional structure is fabricated on top of it. The flanges and/or the web can still be covered by lost framework. Therefore, sensors can only be applied to a part of the monitored specimen, sometimes.

A numerical model of the T-shaped beam combined with a numerical simulated wave propagation is used to investigate the influence of a complex specimen’s shape and different sensor layouts on the wave propagation in general and on the accuracy of estimated source location in particular. The source location is estimated using three arrival-time-based source localization methods: homogeneous Geiger method, heterogeneous Geiger method, and FastWay.

2 Simulations

2.1 Numerical Specimen

For the simulations of elastic wave propagation a numerical model of a T-shaped beam is used. The beam has a length of l_s = 500 mm, a flange of 280 mm by 80 mm and a 80 mm by 180 mm web, as displayed in Fig. 1. The numerical specimen consist of a numerical concrete model (NCM) [6]. The NCM is composed of three phases: cement matrix, aggregates, and air voids. The material properties are listed in Table 1. The aggregate particles are simplified by ellipsoids and the air voids by spheres. The aggregate particles and the air voids are randomly distributed in space as suggested in [9]. The aggregate have a maximum grain size of 16 mm. The material properties of aggregates usually vary within a defined range. This variation is also implemented in the NCM.

Reinforcements and cracks have a significant influence on the wave propagation behavior [4]. The specimen is modeled uncracked and without reinforcement, in order to study the influence of a complex layout of the specimen (T-shaped cross section) on the wave propagation behavior only.

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The specimen is numerically discretized with a grid size of 1 mm on a three-dimensional layout. The T-shaped beam consists of a total number of \(1.84 \times 10^7\) voxels. The edge of a voxel is referred to as 1 grid point (gp). The information of the numerical model is stored in a three-dimensional cell array. Each cell represents one voxel. A certain material is assigned to each voxel. The material properties of the voxel are stored in the corresponding cells. If the information stored in the cell is reduced to only one entry, the p-wave velocity \(c_p\), the cell is simplified to a three-dimensional matrix referred to as \(C_{\text{mo}}\). The matrix \(C_{\text{mo}}\) is the basis for the heterogeneous Geiger method as well as for FastWay.

### 2.2 Sources

The numerical source is defined as an explosion source and applied as a displacement in all directions of space. Two sources (S1 & S4) are located inside the web. Three sources (S2, S3 & S6) are located inside the flange and the sixth source (S5) is located at the intersection of web and flange. The coordinates of the sources were selected randomly and are listed in Table 2. The source locations are displayed in Fig. 1.

#### Table 2 Source coordinates

<table>
<thead>
<tr>
<th></th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
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<td>(y)</td>
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<td>138</td>
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<td>228</td>
<td>14</td>
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<td>204</td>
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</table>

### 2.3 Wave Propagation Simulation

An algorithm developed by Saenger [7] based on the finite-difference method was used to numerically simulate the elastic wave propagation for a duration of 200 \(\mu\)s. Fig. 2 visualizes the wave field 50 \(\mu\)s after the excitation in three cross-sections. The cross-sections are positioned at the location of the source, which is marked with \(
\star\). The wave field propagating in the specimen is clearly visible in all three cross-sections, just like the reflection of the wave front at the specimen’s surface. In the \(y-z\) cross-section, displayed in Fig. 2, it is clearly visible that the wave cannot reach the lower part of the web propagating along a straight line. It has to bypass the air surrounding the specimen. If the specimen is homogeneous, the fastest wave propagation path from the marked source to the lower part of the web will be bilinear. The wave front within the web is circular. However, the epicenter is not the origin of the source, it is the inner edge between web and flange. Moreover, it becomes visible that the AE induced displacements within the web are smaller than within the flange (geometrical dispersion).

### 2.4 Sensors

A total number of twenty-six virtual sensors are installed in the numerical specimen. Displacements perpendicular to the surface of the voxel at the sensor’s positions are calculated analogues to physical measurements of piezo-electrical sensors. Twelve sensors (S1-S12) are located on the lateral surfaces of the web. Six sensors on each side. Three sensors (S13-S15 & S16-S18) are located
on the bottom side of the flange, three on each side of the web. The remaining eight sensors (S19-S26) are located on the top surface of the flange. There are no sensors on the end faces of the specimen or the bottom surface of the web. The coordinates of the sensor positions are listed in Table 3.

Fig. 2 Snapshot of the elastic wavefield propagation due to a Ricker wavelet (source marked with ☆) displayed in three cross sections of a three dimensional numerical specimen at time step T= 50 μs.

3 Source Localization

The arrival time of the wave is determined threshold-based. There is no white noise since it is a numerical simulation. Therefore, a very small threshold can be used. However, a sensor is excluded from the source localization estimation process if the absolute amplitude of the signal calculated at the sensor positions is smaller than ten times the threshold. In addition, AIC-based picking [8] is used to verify the accuracy of the threshold based picking. If the threshold-determined and energy-based determined (AIC picker) onset time diverse significantly (+10/-30 μs) the sensor is also excluded. The onset time determined using the threshold picking is the input for all source localization methods. In addition, the heterogeneous Geiger method as well as FastWay relay on a velocity model of the specimen Cmo.

3.1 Homogeneous Geiger Method

The Geiger method is a stable, efficient and fast way to calculate the source location of an AE in three-dimensional homogeneous space [3]. However, the estimated source location can differ clearly from the actual source location in the case of a heterogeneous or cracked specimen. The governing equation of the homogeneous Geiger method is formulated as follows, where the observed onset time \( t_a(s) \) at sensor \( s \) correlates with the source time \( t_c \) by assuming a straight wave propagation path between source and sensors:

\[
 t_a(s) = \frac{\sqrt{(x_s - x_c)^2 + (y_s - y_c)^2 + (z_s - z_c)^2}}{c_p} + t_c. \tag{1}
\]

The source coordinates \( x_c, y_c, z_c \) and the source time \( t_c \) are unknowns. The known input values are the sensor coordinates, \( x_s, y_s, z_s \), the picked arrival time of the p-wave at each sensor \( t_a(s) \) and the global p-wave velocity \( c_p \).

The data of at least four sensors is needed to determine the four unknown values. The system of equations is overdetermined if the date of more than four sensors is available. In that case, minimizing the residuals between calculated and the observed arrival times at each sensor is the goal. A linear least-squares algorithm can be used to determine the source location iteratively [2]. A covariance matrix \( C \) based on the partial deviations of (1) can also be calculated. The eigenvalues of \( C \) (excluding the values considering the source time) represent the axis ratio, and the eigenvectors represent the orientation of an error ellipsoid. Schechinger recommended to use 68% error ellipsoids [8]. Therefore, with a probability of 68% the source is located within the error ellipsoid. A small error ellipsoid indicates an anticipated accurate estimation of the source location.
Table 3 Sensor coordinates and sensor layouts

<table>
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<tr>
<th>Sensor</th>
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<td>SE26</td>
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3.2 Heterogeneous Geiger Method

If the Geiger method is applied to a heterogeneous specimen, a small modification in (1) can improve the accuracy of the determined source location significantly [5]. Substituting $c_p$ by the average p-wave velocity $c_{p,het}(s,c)$ for the straight connection of the estimated source $c$ and sensor $s$ can be determined.

Fig. 3 Two-dimensional scheme for determination of the average wave travel velocity between source and sensor, based on a heterogeneous velocity model and straight wave travel path (from [4])

The requirements for the determination of the average p-wave velocity $c_{p,het}(s,c)$ are that the coordinates of the sensor, the coordinates of the estimated source location and a numerically velocity model $C_{mo}$, known a priori. The p-wave velocity $c_p(k)$ is assigned to each of the $k$ voxel. The length of the wave travel path within a voxel $l_{pk}(k)$ needs to be determined for each voxel [10]. Subsequently,

$$c_{p,het}(s,c) = \sum_k \frac{l_{pk}(k) \cdot c_p(k)}{l_p}$$

(2)

can be determined.

In order to incorporate a heterogeneous velocity model the average wave velocity between the estimated source location and each sensor has to be determined for each iteration step. Hence, the computational effort increases considerably. Moreover, the method relies on straight wave propagation paths and is not able to consider the influences of cracks or air inclusions [4]. Moreover, in case of complex specimens (e.g. T-shaped beams) the straight connection of source and sensor pass through the surrounding air.


3.3 FastWay

FastWay is a novel approach to estimate the location of AE sources based on a heterogeneous velocity model and multi-segment wave propagation paths. The major difference between FastWay and common arrival time based source localization methods is that the fastest rather than the direct (shortest) wave travel path between the source and a sensor is used to estimate the source location. This new localization method can adapt to the geometry and material properties of the investigated specimen.

In order to identify the fastest wave travel path, a modified Dijkstra algorithm [1] is applied. A heterogeneous velocity model and the sensor coordinates are the input for the Dijkstra algorithm. The output is one matrix \( A_{\text{tot}}(s) \) for each sensor \( s \). The matrix entries are the earliest arrival time of a wave, starting from sensor \( s \) (source time \( t_\text{s}=0 \) \( \mu \text{s} \)), at the center of a voxel. The position of the voxel corresponds to the position of the value within \( A_{\text{tot}}(s) \) [4]. It is assumed that the fastest wave travel path and wave travel duration between two points \((i \& j)\) is regardless of, which of the points is the emitting point (source) and which of the points is the receiving point (sensor). Hence, a potential source time \( t_{\text{pot},x,y,z} \), based on the value \( a_{\text{tot},x,y,z}(s) \) of the matrix \( A_{\text{tot}}(s) \) and the arrival time \( t_\text{s}(s) \) picked for sensor \( s \), can be determined for each voxel with the coordinates \([x \ y \ z]\). The resulting potential source-time matrix \( S_{\text{pot}}(s) \) can be calculated for each sensor. One entry of all potential source-time matrixes should be (approximately) identical. The corresponding voxel, fulfilling

\[
\begin{align*}
& t_\text{a}(s_1) - a_{\text{tot},x,y,z}(s_1) \approx t_\text{a}(s_2) - a_{\text{tot},x,y,z}(s_2) \approx \ldots \approx t_\text{a}(s_n) - a_{\text{tot},x,y,z}(s_n) \\
& \text{(3)}
\end{align*}
\]

is hosting the source location [4].

It has to be mentioned that the size of the voxels used for FastWay is larger than the one used for the wave propagation simulation as well as the heterogeneous velocity model used in the heterogeneous Geiger method. The grid size of the numerical specimen implemented in FastWay is 5 mm. One FastWay voxel is equivalent to 125 wave propagation simulation voxels. FastWay determines the voxel which most probably hosts the source. However, it is not possible to determine the exact location of the source within this voxel. The source localization output represents the coordinates of the voxel’s center. This results in a systematical inaccuracy of (half an edge length) 2.5 mm in each spatial direction or 2.5 mm \( \times \sqrt{3} = 4.33 \text{ mm in total} \) [4].

4 Outcome of the Simulations and Source Localizations

4.1 Localization Setups

Six simulations with different source locations were performed. A total of twenty-six virtual sensors recorded the calculated, time-dependent displacement induced by the artificial sources (see section 2). The recorded discrete functions were used for determining the onset time of the wave at the sensor. The onset times are an input for estimating the source location applying a homogeneous Geiger method, a heterogeneous Geiger method and FastWay. Five different sensor layouts were used to estimate the source location. The layouts are composed of:

- Sensor layout A: all sensors applied to the upper surface of the flange; SE19 – SE26
- Sensor layout B: all sensors applied to the lower surface of the flange; SE13 – SE18
- Sensor layout C: all sensors applied to the web; SE1 – SE12
- Sensor layout D: all sensors applied to the web and the lower surface of the flange; SE1 – SE18

The Layouts are schematically visualized in Fig. 6. The sets consist out of six to twenty-six sensors. For estimating the source location with all three methods data of the six sensors, with the largest absolute amplitudes within the first 20 \( \mu \text{s} \) after the picked arrival time of the wave at the sensor, are used.

4.2 Localization Results

The result of the two mentioned Geiger source localization algorithms are the coordinates of an estimated source location and an error ellipsoid to visualize the estimated accuracy of the determined solution (see section 3). The result of FastWay is a normalized error matrix. The values of this matrix represent the normalized deviations from the condition formulated in (3) and range from 0 to 1. The lowest value corresponds to the voxel which is most probably hosting the source [4]. Hence, the results are not straightforward the coordinates of the estimated source location. The output coordinates are the coordinates of the center of the determined voxel. Instead of error ellipsoids, the visualization of the normalized error matrix is used to represent the reliability and expected accuracy of the determined solution.

The results for source S6 determined with the data provided by sensor layout E are displayed in Fig. 4. A three-dimensional representation of the outcome of the three source localization algorithms has turned out to be impractical. Instead, three cross-sections are used to visualize the projection of the three-dimensional error ellipsoids superimposed with the result of the normalized error matrix. The cross-sections are positioned at the source location estimated with FastWay. The error matrix is visualized for the voxels located in the displayed cross section, only. The Geiger error ellipsoids, the estimated source location determined with the two mentioned Geiger methods, and the sensor locations are displayed on the cross-sections.
Fig. 4 Cross-sections displaying normalized error matrix, error ellipsoids for the Geiger methods, locations of the used sensors, estimated sources and predefined source (S6) marked, using sensor layout E.

Fig. 4 represents one of the most satisfying out of all thirty results. All three methods estimated the source location within a max. deviation of less than 7 mm (see Fig. 6). The determined error ellipsoids are very small, and the predefined source location is located within both error ellipsoids. The voxels with a normalized error matrix value ($e_{norm}$) of 0.11 or smaller, form a spatial area in which the source is located with a high probability. The voxels are emphasized in colors from violet ($0 \leq e_{norm} \leq 0.016$) to a greenish yellow ($0.095 \leq e_{norm} \leq 0.111$). The voxels with $e_{norm} \geq 0.111$ are colored from green to gray (see Fig. 4, Fig. 5).

Fig. 5 Cross-sections displaying normalized error matrix, error ellipsoids for the Geiger methods, locations of the used sensors, estimated sources and predefined source (S3) marked, using sensor layout C.
Fig. 6 Visualization of the deviation between the estimated and the predefined source locations. In the last segment the mean (colored bar) and the median of the deviations for all six sources is visualized.
The area with $e_{\text{norm}} \leq 0.111$ should be as small as possible, if the reliability of the accuracy of the determined source location is high. In case of Fig. 4 the area is very small and two voxels are colored in violet. One of the two voxels is hosting the predefined source. Fig. 5 represents an unsatisfying result. It is clearly visible that the determined error ellipsoids are larger than the specimen itself. Obviously, the predefined source is located within both error ellipsoids. However, the large error ellipsoids indicate that the accuracy of the determined source location is very low. The deviation between the predefined and the estimated source location is 263.3 mm and 200.6 mm for the homogeneous and the heterogeneous Geiger method respectively. The estimated source location determined with FastWay also differ from the predefined source location considerably. The deviation is 41.8 mm, which can still be considered as satisfyingly accurate. The visualization of the normalized error matrix in Fig. 5 indicates that the source is located upper-left corner of the flange ($y$-$x$-cross section). However, a preference of the z-location within the web cannot be identified. The predefined source is located within the spatial area formed by the voxels with $e_{\text{norm}} \leq 0.111$. However in case of Fig. 5, this area is significantly larger than in Fig. 4. Summarizing, the error ellipsoids and the visualization of the normalized error are a reliable way to demonstrate the expected accuracy of the determined source location.

4.3 Localization Accuracy

The deviation between the estimated source location and the controlled, predefined source location is significantly lower, if the estimated source location was determined with FastWay. The mean deviation between the predefined source location and the source location estimated with FastWay (of all thirty source localization setups) is 20.7 mm. The homogeneous Geiger method and the heterogeneous Geiger method provide estimated source locations with a mean deviation from the predefined source location of 119.7 mm and 89.6 mm, respectively. The localization error of FastWay is about one fifth of the localization error of the Geiger methods. FastWay provides twenty-five of thirty of the estimated source location with the smallest deviation from the predefined source location. The homogeneous Geiger method provides four of the most accurate source location prediction and the heterogeneous Geiger method only one. In the last five cases the localization error of the source location predicted with FastWay was between 0.3 mm and 5.0 mm larger than the most accurate prediction. Fig. 6 visualizes the deviations between the estimated and the predefined source locations for all six sources, five sensor layouts, and three localization methods. The bar diagram which displays the deviation is limited to a maximum of 100 mm. A localization error of more than 100 mm is considered as unsatisfying. Determining a satisfying source prediction of source S2, S3, and S4 was not possible using the homogeneous Geiger method, regardless of the sensor layout. These sources are located in both the web (S4) and the flange (S2 & S3). The localization error of S3 and S4 is about 250 mm, which is half of the largest specimen’s dimension ($l_e = 500$ mm). The heterogeneous Geiger method also was not able to predict the source location of S3 and S4 satisfyingly, regardless of the sensor layout. Nevertheless, the localization error was smaller compared to the homogeneous Geiger method. In addition, the localization error of source S1 in combination with sensor layout A and B, as well as source S2 in combination with sensor layout E is more than 100 mm, if determined with the heterogeneous Geiger method.

5 Conclusion

The last row of Fig. 6 emphasizes that FastWay provides significantly more accurate source predictions than the two investigated Geiger methods, with regard to the investigated experimental setups. Moreover, Fastway was the only method that provided a satisfying accurate estimation of the source location for every source and sensor layout. Nevertheless, the deviations of more than 10 mm point out that FastWay bases on a simplified wave propagation model, and does not reflect the real physical wave propagation entirely. However, the accuracy of the estimated source locations can be improved significantly by means of FastWay.

References