Mismatched Decoding: DMC and General Channels

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Abstract—The setup of mismatched decoding is considered. By analyzing multi-letter expressions and bounds on the mismatch capacity of a general channel, several results pertaining to the mismatched discrete memoryless channel $W$ with an additive metric $q$ are deduced: it is shown that Csiszár and Narayan’s “product-space” improvement of the random coding lower bound on the mismatched capacity, $C_q^{(\infty)}(W)$, is equal to the mismatched threshold capacity with a constant threshold level. It is also proved that $C_q^{(\infty)}(W)$ is the highest rate achievable when the average probability of error converges to zero at a certain specified rate, which is $o(1/n)$ in the case of $q$ which is a bounded rational metric. Finally a lower bound on the average probability of error at rates above the erasures-only capacity of the DMC is derived.

I. INTRODUCTION

In [1], the mismatch capacity of the DMC with decoding metric $q$, denoted $C_q(W)$, is considered. It is shown that the lower bound derived previously in [2] and [3] is not tight in general. This is established by proving that the random coding bound for the product channel $W^K$, denoted $C_q^{(K)}(W)$, referred to as the “product-space” lower bound, may result in strictly higher achievable rates. The rate $C_q^{(K)}(W)$ is given by $C_q^{(K)}(W) = \max_{P_{X^K}} \min_{P_{Y^K}} \frac{1}{n} I(X^K;Y^K)$, where the minimization is over $P_{Y^K}$, $E_r(q(X^K, Y^K)) \geq E_r(q(x^n, y^n))$ and $E_r(X^K, Y^K) \sim P_{X^K} \times W^K$. Consequently, $C_q^{(\infty)}(W) = \limsup_{K \to \infty} C_q^{(K)}(W)$ is an achievable rate as well. In the special case of erasures-only capacity, $C_q^{(\infty)}(W)$ is shown to be a tight bound.

II. MULTILETTER EXPRESSIONS FOR THE MISMATCH CAPACITY AND CONSEQUENT RESULTS FOR THE DMC

Consider a DMC with a finite input and output alphabets $\mathcal{X}$ and $\mathcal{Y}$, respectively, which is governed by the conditional p.m.f. $W$. A rate-$R$ block-code of length $n$ consists of $2^{nR}$ $n$-vectors $x^n(m), m = 1, 2, ..., 2^{nR}$, which correspond to $2^{nR}$ equiprobable messages. An additive mismatched decoder for the channel is defined by function $q_n(x^n, y^n) = \frac{1}{n} \sum_{i=1}^{n} q(x_i, y_i)$ where $q$ is a mapping, referred to as metric, from $\mathcal{X}$ to $\mathcal{Y}$. The decoder declares that message $i$ was transmitted iff $q_n(x^n(i), y^n) > q_n(x^n(j), y^n), \forall j \neq i$, and if no such $i$ exists, an error is declared. The mismatch capacity of the DMC is defined as the supremum of achievable rates using the mismatched decoder.

In [4], general multi-letter expressions and bounds for the mismatch capacity of general channel with a general metric were derived. In this work we describe briefly results that were deduced for the mismatched DMC, whose derivations rely on these multi-letter expressions.

The first result refers to a threshold decoder. A $(q_n, \tau_n)$-threshold decoder decides that $i$ is the transmitted message iff $q_n(x^n(i), y^n) \geq \tau_n$ and $q_n(x^n(j), y^n) < \tau_n, \forall j \neq i$.

In [5] we prove that for a bounded metric $q$, $C_q^{(\infty)}(W)$ is equal to the constant-threshold capacity, that is, the supremum of achievable rates with a threshold decoder of a constant value $\tau_n = \tau$. An implication of this result is that Csiszár and Narayan’s conjecture [1] that $C_q(W) = C_q^{(\infty)}(W)$ is equivalent to the statement that $C_q(W)$ is equal to the constant threshold capacity. In [4, Theorem 6] a multi-letter expression for the constant threshold capacity was derived for a general channel and a general metric sequence. Specifying this expression for the DMC case, we obtain an alternative expression for $C_q^{(\infty)}(W)$ in [5]. In [6] we prove that for a bounded metric $q$, every code-sequence of rate $R$, whose average probability of error with a decoding metric $q$ employed on the output of a DMC vanishes faster than $\eta_n$, where $\eta_n = \min_{x^n, y^n} \max_{q_n(x^n, y^n)} [q_n(x^n, y^n) - q_n(x^n, y^n)]$, must satisfy $R \leq C_q^{(\infty)}(W)$. In particular, for rational metrics, it identifies $C_q^{(\infty)}(W)$ as the highest rate achievable with average probability of error which is $o(1/n)$. Since at rates below $C_q^{(\infty)}(W)$ exponential decay of the average probability of error is feasible [1], one can deduce that for a bounded rational metric $q$, $C_q(W) = C_q^{(\infty)}(W)$ iff all $R < C_q(W)$ the error exponent is positive.

Finally it is shown in [7] that the erasures only capacity of the DMC, $C_{q_{eo}}(W)$, satisfies $\mathcal{E}_{\text{er}}(R, \theta) \geq 1 - \frac{2^{\theta R}}{\mathcal{E}_{\text{er}}(R)}$, where $\mathcal{E}_{\text{er}}(R)$ is the lowest achievable average probability of error at rate $R$ in the erasures-only setup.

REFERENCES