Conference Paper

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Publication Date:
2016

Permanent Link:
https://doi.org/10.3929/ethz-a-010645615 →

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Mismatched Decoding: DMC and General Channels

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Abstract—The setup of mismatched decoding is considered. By analyzing multi-letter expressions and bounds on the mismatch capacity of a general channel, several results pertaining to the mismatched discrete memoryless channel W with an additive metric q are deduced: it is shown that Csiszár and Narayan's "product-space" improvement of the random coding lower bound on the mismatched capacity, $C_q^{(\infty)}(W)$, is equal to the mismatched threshold capacity with a constant threshold level. It is also proved that $C_q^{(\infty)}(W)$ is the highest rate achievable when the average probability of error converges to zero at a certain specified rate, which is $o(1/n)$ in the case of q which is a bounded rational metric. Finally a lower bound on the average probability of error at rates above the erasures-only capacity of the DMC is derived.

I. INTRODUCTION

In [1], the mismatch capacity of the DMC with decoding metric q, denoted $C_q(W)$, is considered. It is shown that the lower bound derived previously in [2] and [3] is not tight in general. This is established by proving that the random coding bound for the product channel W^K, denoted $C_q^{(k)}(W)$, referred to as the "product-space" lower bound, may result in strictly higher achievable rates. The rate $C_q^{(k)}(W)$ is given by $C_q^{(k)}(W) = \max_{P_{X^k}} \min_{P_{\tilde{Y}^k|X^k}} \frac{1}{k} I(X^k; \tilde{Y}^k)$, where the minimization is over $P_{\tilde{Y}^k} = P_{Y^k}, \mathbb{E}(q(X^k, \tilde{Y}^k)) \geq \mathbb{E}(q(X^k, Y^k))$ and $(X^k, Y^k) \sim P_{X^k} \times W^k$. Consequently, $C_q^{(\infty)}(W) = \limsup_{k \to \infty} C_q^{(k)}(W)$ is an achievable rate as well. In the special case of erasures-only capacity, $C_q^{(\infty)}(W)$ is shown to be a tight bound.

II. MULTILETTER EXPRESSIONS FOR THE MISMATCH CAPACITY AND CONSEQUENT RESULTS FOR THE DMC

Consider a DMC with a finite input and output alphabets $\mathcal{X}$ and $\mathcal{Y}$, respectively, which is governed by the conditional p.m.f. W. A rate-R block-code of length n consists of 2^{nR} n-vectors $x^n(m)$, $m = 1, 2, ..., 2^{nR}$, which correspond to 2^{nR} equiprobable messages. An additive mismatched decoder for the channel is defined by function $q_n(x^n, y^n) = \frac{1}{n} \sum_{i=1}^n q(x_i, y_i)$ where q is a mapping, referred to as metric, from $\mathcal{X} \times \mathcal{Y}$ to $\mathbb{R}$. The decoder declares that message i was transmitted iff $q_n(x^n(i), y^n) > q_n(x^n(j), y^n), \forall j \neq i$, and if no such i exists, an error is declared. The mismatch capacity of the DMC is defined as the supremum of achievable rates using the mismatched decoder.

In [4], general multi-letter expressions and bounds for the mismatch capacity of general channel with a general metric were derived. In this work we describe briefly results that were deduced for the mismatched DMC, whose derivations rely on these multi-letter expressions.

The first result refers to a threshold decoder. A (q_n, τ_n)-threshold decoder decides that i is the transmitted message iff $q_n(x^n(i), y^n) \geq \tau_n$ and $q_n(x^n(j), y^n) < \tau_n, \forall j \neq i$.

In [5] we prove that for a bounded metric q, $C_q^{(\infty)}(W)$ is equal to the constant-threshold capacity, that is, the supremum of achievable rates with a threshold decoder of a constant value $\tau_n = \tau$. An implication of this result is that Csiszár and Narayan's conjecture [1] that $C_q(W) = C_q^{(\infty)}(W)$ is equivalent to the statement that $C_q(W)$ is equal to the constant threshold capacity. In [4, Theorem 6] a multi-letter expression for the constant threshold capacity was derived for a general channel and a general metric sequence. Specifying this expression for the DMC case, we obtain an alternative expression for $C_q^{(\infty)}(W)$ in [5]. In [6] we prove that for a bounded metric q, every code-sequence of rate R, whose average probability of error with a decoding metric q employed on the output of a DMC vanishes faster than η_n, where $\eta_n = \min_{x^n, \tilde{x}^n, y^n: q_n(\tilde{x}^n, y^n) \neq q_n(x^n, y^n)} |q_n(\tilde{x}^n, y^n) - q_n(x^n, y^n)|$, must satisfy $R \leq C_q^{(\infty)}(W)$. In particular, for rational metrics, it identifies $C_q^{(\infty)}(W)$ as the highest rate achievable with average probability of error which is $o(1/n)$. Since at rates below $C_q^{(\infty)}(W)$ exponential decay of the average probability of error is feasible [1], one can deduce that for a bounded rational metric q, $C_q(W) = C_q^{(\infty)}(W)$ iff for all $R < C_q(W)$ the error exponent is positive.

Finally it is shown in [7] that the erasures only capacity of the DMC, $C_{q_{eo}}(W)$, satisfies $\mathcal{E}_{eo}(R, \Theta) \geq 1 - \frac{C_{q_{eo}}(W)}{R}$, where $\mathcal{E}_{eo}(R)$ is the lowest achievable average probability of error at rate R in the erasures-only setup.

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