

Integrated timetabling and rolling stock scheduling

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Eidgenössische Technische Hochschule Zürich
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Integrated timetabling and rolling stock scheduling

Master Thesis

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Abstract

In the last decades the passenger demand in public railway networks has been constantly increasing. As a consequence also the number of trips per track per day has increased, requiring efficient usage of track capacity and rolling stock resources. In railway planning the steps of timetabling and rolling stock scheduling are usually carried out sequentially, which can lead to suboptimal rolling stock utilization. This issue could be eliminated by an integrated planning step for both tasks. In this thesis we study the integration of rolling stock scheduling for so-called periodic timetables based on the *Periodic event scheduling problem* (PESP). As shown in previous work, integrated periodic timetabling and rolling stock scheduling can be modeled as a mixed-integer program with a quadratic objective function, which we call RS-PESP. We introduce a linearization for this model using big- M constraints and McCormick envelopes. Furthermore, we derive an iterative heuristic to obtain feasible solutions of the RS-PESP. As a post-processing step, we propose a model that can be used to make an existing rolling stock circulation more robust without increasing the number of required rolling stock compositions. We investigate the performance of the introduced models and algorithms in computational studies with a small and a medium sized real-world instance.

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Chapter 1

Introduction

Public transport planning has been of increasing importance in the last decades with the growth of public transport networks and passenger numbers. A major portion of the passenger demand is satisfied by railway networks, creating a need for public railway companies to constantly improve their services. For example in the Swiss Federal Railways (SBB) the person-kilometers per day have increased by more than 20% and the average number of trains per track per day increased by more than 10% in the last decade [SBB15]. Since many of the arising problems in railway planning can be formulated as mathematical models, railway optimization has been investigated in operations research for over 20 years. The complete planning task comprises several aspects that can be complex even when considered individually. Therefore, it is usually decomposed into hierarchical planning steps that are performed sequentially, as shown in Figure 1.1. In this thesis the focus lies on the two consecutive steps of Timetabling and Rolling stock scheduling. Designing the physical infrastructure of the railway network has a planning horizon of at least five to ten years [BCH10]. Given the network plan, the success of a public transport network is determined by service quality and operational cost, both of which heavily depend on the timetable, the rolling stock schedule and crew schedule. A timetable serves as input for the subsequent step of rolling stock scheduling and therefore implies restrictions on the rolling stock schedule. Since timetables of similar quality with respect to timetabling criteria may admit rolling stock schedules of different impact on the operational cost, performing both steps sequentially can lead to deficient rolling stock utilization.

The question arises as to whether it is possible to optimize a timetable and a rolling stock schedule simultaneously, in order to avoid the described suboptimality. Alternatively, a satisfactory approach would be to integrate certain aspects of rolling stock scheduling into the timetabling step, such that a subsequent step for rolling stock scheduling can be performed unrestricted.

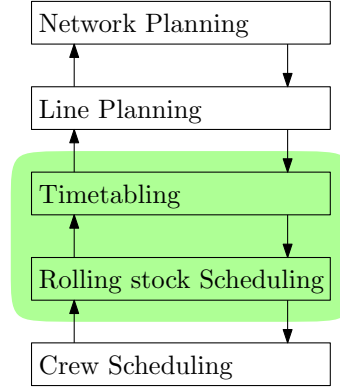


Figure 1.1: Hierarchical planning in public transport [LM07]. The downward arrows indicate the sequential order of the planning steps. The additional upward arrows symbolize dependencies between planning steps that exist, but are not regarded in sequential planning, e.g. a line plan based on passenger demand is an input for a timetable, which may again influence the passenger demand.

In timetabling, periodic and aperiodic timetables are distinguished, where periodic timetables are typically used for “dense” railway networks, i.e. the events happen with high frequency and the distances between stations are short, e.g. as in urban railway networks in Switzerland. A commonly used model for periodic timetables is the *Periodic event scheduling problem* (PESP) by Serafini and Ukovich [SU89], which is a mixed-integer linear program (MILP). The railway network is represented by an *event-activity network* (EAN) with events for arrivals and departures and activities modeling trips and other requirements such as safety and connection constraints. The model features continuous event-times in a period T as variables and constraints for the activity time-differences. These periodic time-differences correspond to the differences of event-times modulo T and can be modeled with so-called shift-integer variables, yielding linear constraints.

For rolling stock scheduling among many aspects mainly the topology of the railway network and the type of rolling stock define the variants of the problem. In this thesis the focus lies on periodic timetables and rolling stock schedules for self-propelled rolling stock compositions in dense railway networks.

Contribution of the thesis

Based on an augmented event-activity network for the PESP and as an application of a general resource assignment for periodic activities [SU89], a circulation of rolling stock compositions is modeled as a matching problem. Combining the PESP and the rolling stock circulation into one optimization problem gives the RS-PESP, an integrated model for a periodic timetable and rolling stock circulation, proposed by [LP02]. This model has a quadratic ob-

jective function and is intractable in practice. To overcome these issues, we investigate several approaches for solving the RS-PESP:

First, we develop a linearization based on big- M constraints and McCormick envelopes. The linearization is an exact formulation and enables us to obtain feasible solutions for the integrated model using a MIP-solver. In order to improve the performance in solving the integrated model we introduce additional constraints to reduce the feasible region by cutting off non-optimal integer solutions. Furthermore, we exploit a special structure in the objective function to prune nodes in a branch-and-cut tree.

Secondly, we formulate a heuristic that iteratively solves instances for the timetable and the rolling stock circulation separately and obtains a local optimum of the integrated model. Computational experiments are used to evaluate both approaches for solving the RS-PESP.

The number of rolling stock cycles that a circulation uses is one possible indicator for the robustness of a combination of timetable and rolling stock circulation. We develop a model to optimize the number of rolling stock cycles for a given timetable, which preserves the quality of an initial rolling stock schedule. This planning step is designed to be performed as post-processing after obtaining a feasible solution of the RS-PESP.

Outline

The thesis consists of eight chapters: Chapter 2 introduces the PESP and its application to model periodic railway timetables. A model for rolling stock schedules for periodic timetables is formulated in Chapter 3, based on a resource assignment for the PESP and resulting in a rolling stock schedule represented by so-called *rolling stock cycles*. The integration of periodic timetabling and rolling stock scheduling is addressed in Chapter 4. Section 4.3 shows the RS-PESP, a combination of the periodic timetabling and rolling stock scheduling models, and discusses different approaches to solve the integrated model RS-PESP in Section 4.5. Furthermore, Section 4.4 reviews the approach of implicitly integrating a rolling stock schedule into the PESP by [Pee03; Vil06; Kro+13]. Periodic events with different frequencies are considered in Chapter 5. Two steps for post-processing are studied in Chapter 6. First, processing a solution given by the variables of the RS-PESP. Secondly, a model to maximize the number of rolling stock cycles is presented in Section 6.2.

Chapter 7 explains the computational experiments, which have been done to benchmark the models and algorithms, and summarizes the results of the experiments. Lastly, Chapter 8 draws conclusions of the thesis, describes remaining questions and shows options for further research.

Timetabling

In timetabling, both periodic and aperiodic timetables are studied, being called cyclic and non-cyclic timetables, too. Aperiodic timetabling often focuses on optimally exploiting some part of the railway infrastructure that could be considered a bottleneck of the network, e.g. a single track section [HKF96]. An example is given by Brännlund et al. [Brä+98] and Caprara et al. [CFT02], who consider ILP formulations for a railway track section connecting two cities.

The periodic timetabling problem, however, is more often used to construct a timetable for a complete railway network. Periodic timetables are common in several European countries, e.g. Switzerland and Germany. Also they are possibly favored by passengers, because they are easily memorable, since the departures and arrivals occur at the same minute every period, e.g. at minutes 13 and 43 with a period of $T = 30$ minutes. Several properties of a railway system, such as connections and travel times, are determined by the timetable. In this thesis we consider the *Periodic event scheduling problem* (PESP) presented by Serafini and Ukovich as the basis for periodic railway timetables.

2.1 Literature review – PESP model formulation and properties

In the following section we present the *periodic event scheduling problem* (PESP) and some of its properties in detail. The PESP was introduced by Serafini and Ukovich in 1989 [SU89] and shown to be NP-complete, by giving a polynomial reduction from the Hamiltonian cycle problem to the PESP. Serafini and Ukovich indicate several applications of the PESP such as traffic light scheduling and transportation scheduling. In addition, they develop a method to assign resources to the periodic activities.

The periodic event scheduling problem can be stated in the following way: A set of periodic events N that recur after a period $T \in \mathbb{N}$ should be scheduled. Any pair of events $(e_i, e_j) \in N \times N$ can be linked by an activity $a = (e_i, e_j)$, which takes at least l_a and at most u_a time units. The set of events N and the set of all linking activities $A \subseteq N \times N$ form a directed graph $D = (N, A)$ that is called *event-activity network* (EAN). In railway timetabling the events represent the arrivals and departures. The activities model e.g. trips. A more detailed discussion about modeling a railway timetable with an EAN is given in Section 2.2. In order to cope with events of different frequencies, i.e. events that happen multiple times during the period, Serafini and Ukovich define the *extended PESP* (EPESP), which incorporates the frequencies of events in the related constraints. Nachtigall [Nac96] also addresses different frequencies of events. We discuss frequencies in Chapter 5.

To model a schedule, let $\pi(e) \in \mathbb{R}$ denote the event-time of an event $e \in N$. Due to periodicity we can assume that $\pi(e) \in [0, T]$. For an activity $a = (e_i, e_j)$ the lower and upper bounds l_a, u_a on the activity duration imply a constraint on the event times $\pi(e_i)$ and $\pi(e_j)$:

$$l_a \leq (\pi(e_j) - \pi(e_i)) \bmod T \leq u_a, \quad (2.1)$$

where

$$x \bmod T = x - \left\lfloor \frac{x}{T} \right\rfloor T \quad \text{for } x \in \mathbb{R}.$$

Then PESP is the problem of determining a feasible schedule given the above information. E.g. let $T = 60$ and let $a = (e_i, e_j)$ be an activity, which is required to take between 10 and 20 minutes, i.e. $l_a = 10$ and $u_a = 20$. Then the event times $\pi(e_i) = 50$ and $\pi(e_j) = 5.5$ are a feasible schedule because $(5.5 - 50) \bmod 60 = 15.5$ and $10 \leq 15.5 \leq 20$.

Definition 2.1 (Periodic event scheduling problem) *Given an instance of the PESP $I = (D, l, u, T)$, where $D = (N, A)$, find a schedule $\pi : N \rightarrow [0, T)$ such that $l_a \leq (\pi(e_j) - \pi(e_i)) \bmod T \leq u_a$ for all $a = (e_i, e_j) \in A$.*

In order to formulate the PESP as mixed integer feasibility program, the modulo operation in the constraints (2.1) can be modeled by using so-called shift-integers z that are multiplied with the period T . The schedule is represented by continuous variables $\pi \in [0, T]^N$.

Definition 2.2 (PESP as mixed integer feasibility program) *Given an instance of the PESP $I = (D, l, u, T)$, where $D = (N, A)$:*

$$\text{FIND } \pi, z$$

$$\text{s.t.}$$

$$l(a) \leq \pi(e_j) - \pi(e_i) + z(a)T \leq u(a) \quad \forall a = (e_i, e_j) \in A \quad (2.2a)$$

$$\pi(e) \in [0, T] \quad \forall e \in N \quad (2.2b)$$

$$z(a) \in \mathbb{Z} \quad \forall a \in A \quad (2.2c)$$

For the timetabling application it might be desirable that events, i.e. arrivals and departures, are always scheduled on the full minute. The existence of an integral feasible schedule π for a feasible PESP instance with integral lower and upper bounds was shown by Odijk [Odi96]. Odijk also proves that the PESP is NP-complete, by a reduction from graph coloring.

Theorem 2.3 (Odijk) *Let $I = (D, l, u, T)$ be an instance of the PESP with l, u and T integral. Then, if I is feasible, there is an integral feasible schedule $\pi \in \{0, \dots, T-1\}^N$ and $z \in \mathbb{Z}^A$.*

As a consequence of this theorem the domain of the event times (2.2b) may be restricted to $\pi(e) \in [0, T-1]$ for all $e \in N$, since $0 \equiv T \pmod T$. We present a proof by Peeters [Pee03].

Proof Let $\pi \in [0, T)^N$ and $z \in \mathbb{Z}^A$ denote a feasible solution for I . Consider the MILP (2.2) with fixed values of z , then the constraints (2.2a) reduce to

$$l'(a) \leq \pi(e_j) - \pi(e_i) \leq u'(a) \quad \forall a = (e_i, e_j) \in A \quad (2.3)$$

with l' and u' being integral. Each constraint corresponds to an arc (e_i, e_j) in the EAN, where the head e_j and the tail e_i have coefficients of 1 and -1 respectively. It follows that the constraints (2.3) can be represented using the arc-vertex incidence-matrix of the EAN, when written in matrix notation. Let $M \in \{-1, 0, 1\}^{A \times N}$ be the arc-vertex incidence-matrix of D . Then the constraints (2.3) for $\pi \in \mathbb{R}^N$ are equivalent to

$$\begin{bmatrix} M \\ -M \end{bmatrix} \pi \leq \begin{bmatrix} u' \\ -l' \end{bmatrix}. \quad (2.4)$$

M is totally unimodular, because it is the arc-vertex incidence-matrix of a directed graph [Sch03a]. It follows that also $\begin{bmatrix} M & -M \end{bmatrix}^T$ is totally unimodular and since the constraints have integral right-hand sides, the feasible region of the PESP for a fixed z has integral vertices [Sch03b]. \square

The unsatisfactory LP-relaxation is an issue of the MILP formulation (2.2) of the PESP [Cap+07]. For a continuous variable z the constraint (2.2a) can be easily fulfilled regardless of the values of π . A stronger formulation can be obtained by representing the PESP in terms of activity-durations, so-called tensions, instead of event-times.

Definition 2.4 (Tension of an activity) *Let $a = (e_i, e_j) \in A$ be an activity, then its duration is defined as the tension x_a of the arc in the EAN:*

$$x_a := \pi(e_j) - \pi(e_i) + z(a)T. \quad (2.5)$$

$z(a)$ is called a shift-integer variable.

The substitution of event-time variables π and shift-integers z with tension variables x , requires the introduction of additional integer variables. The sum of tensions along a cycle in the EAN, weighted according to the orientation of the arcs, has to amount to an integer multiple of T for all cycles in the EAN [Odi96], a property that is called *cycle periodicity*. In further studies, Odijk [Odi96] finds that the property of cycle periodicity is a necessary and sufficient condition for a feasible PESP-solution and that a cutting plane algorithm for the PESP can be formulated based on cuts that can be deduced from the cycle periodicity.

Definition 2.5 (Cycle periodicity) *Let C be a cycle in the EAN $D = (N, A)$ and let the arcs in and opposite to the direction of C be denoted by C^+ and C^- respectively. Then C fulfills the property of cycle periodicity if there exists a $q_C \in \mathbb{Z}$ such that*

$$\sum_{a \in C^+} x_a - \sum_{a \in C^-} x_a = q_C T. \quad (2.6)$$

As a variable, q_C is called the *cycle integer variable* of C . Resubstituting the tensions with the event-times and shift-integers shows that q_C corresponds to the “oriented” sum of shift-integers and that cycle periodicity for every cycle C in the EAN is a necessary condition for a feasible PESP solution:

$$\sum_{a \in C^+} x_a - \sum_{a \in C^-} x_a \quad (2.7a)$$

$$= \sum_{a=(e_i, e_j) \in C^+} [\pi(e_j) - \pi(e_i) + z(a)T] - \sum_{a=(e_i, e_j) \in C^-} [\pi(e_j) - \pi(e_i) + z(a)T] \quad (2.7b)$$

$$= \left(\sum_{a \in C^+} z(a) - \sum_{a \in C^-} z(a) \right) T \quad (2.7c)$$

$$= q_C T. \quad (2.7d)$$

Since cycle periodicity is sufficient for a feasible PESP solution, equation (2.6) can be used as a constraint with variables x and q . Nachtigall [Nac99] transforms the PESP into a formulation in terms of the tensions based on the property of cycle periodicity, called the *cycle periodicity formulation* (CPF). Values of π and z , corresponding to a timetable given by x and q , can be obtained by fixing an event-time $\pi(e)$ for one event e and then computing the other event-times using the tensions along the paths in a spanning tree of the EAN. Given π , the shift-integer variables z can be chosen to satisfy constraints (2.2a) [Pee03]. Since the EAN may have exponentially many cycles in the number of activities, the usage of cycle periodicity constraints for all cycles would make this approach inefficient. However, here it can be exploited that cycle periodicity holds for every cycle, if it holds for every cycle in an integral cycle base of the EAN [LP09].

As shown by Liebchen and Peeters [LP09], the cycle periodicity constraints of an integral cycle base of the event-activity network are sufficient to characterize the feasible region of the CPF. An integral cycle basis $\mathcal{C}(E)$ of a graph $G = (V, E)$ is a minimal set of cycles in the graph such that every cycle can be represented as integer linear combination of the cycles in $\mathcal{C}(E)$. The number of elements in an integral cycle basis is the dimension of the cycle space of the graph G , also called the circuit rank of G , which is given by $|E| - |V| + c$, where c denotes the number of connected components in G .

Using one integer variable per cycle of an integral cycle basis of the EAN $D = (N, A)$ reduces the number of integer variables from $|A|$ for shift-integers to the circuit rank of D , i.e. $|A| - |N| + 1$ assuming D is connected. As already mentioned, a tension based formulation allows for addressing the weak LP-relaxation of the PESP. For a cycle C , the lower and upper bounds of the tensions of the cycle imply lower and upper bounds on the cycle integer variable q_C .

Lemma 2.6 ([Odi94]) *Let C be a cycle in an EAN $D = (N, A)$ for the PESP with lower and upper bounds l and u , then*

$$a_C = \left\lceil \frac{\sum_{a \in C^+} l_a - \sum_{a \in C^-} u_a}{T} \right\rceil \leq q_C \leq \left\lfloor \frac{\sum_{a \in C^+} u_a - \sum_{a \in C^-} l_a}{T} \right\rfloor = b_C. \quad (2.8)$$

Proof

$$\begin{aligned} q_C &= \frac{1}{T} \left(\sum_{a \in C^+} x_a - \sum_{a \in C^-} x_a \right) \geq \frac{1}{T} \left(\sum_{a \in C^+} l_a - \sum_{a \in C^-} u_a \right) \\ &\geq \left\lceil \frac{1}{T} \left(\sum_{a \in C^+} l_a - \sum_{a \in C^-} u_a \right) \right\rceil \end{aligned}$$

The first inequality follows from the bounds on the tensions $l_a \leq x_a \leq u_a$ and the second inequality holds because q_C is integral. The upper bound on q_C can be shown analogously. \square

As a consequence, it is beneficial to choose an integral cycle basis with small ranges for the integer variables, as discussed in [Lie03, LiePee09]. Formulations based on the cycles in the event-activity network, also presented by Peeters and Kroon [PK01], have proven to perform better in practice than the PESP [Cap+07]. Concluding, this gives the *cycle periodicity formulation* (CPF) of the PESP. Liebchen [Lie08] uses the CPF to obtain a periodic timetable, which was operated in the Berlin subway.

Definition 2.7 (CPF) Given an EAN $D = (N, A)$, lower and upper bounds $l, u \in \mathbb{Z}^A$, a period T and an integral cycle base $\mathcal{C}(A)$ of D :

$$\text{FIND } x_a \quad \forall a \in A$$

s.t.

$$l_a \leq x_a \leq u_a \quad \forall a \in A \quad (2.9a)$$

$$\sum_{a \in C^+} x_a - \sum_{a \in C^-} x_a = q_C T \quad \forall C \in \mathcal{C}(A) \quad (2.9b)$$

$$a_C \leq q_C \leq b_C \quad \forall C \in \mathcal{C}(A) \quad (2.9c)$$

$$q_C \in \mathbb{Z} \quad \forall C \in \mathcal{C}(A) \quad (2.9d)$$

$$x \in \mathbb{R} \quad \forall a \in A \quad (2.9e)$$

2.2 Modeling a periodic timetable with the PESP

In this section we discuss how to use the PESP as a model for periodic timetabling in a public railway network. The main tasks are to determine the events and activities of the EAN and corresponding lower and upper bounds on the tensions of the activities. As indicated in Figure 1.1, the physical railway network and the service lines are already known.

The events can be deduced from the itineraries of the service lines, as well as a set of basic activities. Let L denote the set of all service lines and S denote the set of stations. An arrival resp. departure of a service line $l \in L$ at a station $s \in S$ is modeled by an event that is denoted by arr_l^s resp. dep_l^s . These events are connected by an alternating path of so-called trip and dwell activities representing the itinerary of each service line. Then the EAN is augmented with other types of activities that model various requirements for the railway network. An overview of these types of activities is given below. Figure 2.1 shows an example.

Trips A trip activity models the journey of a service line from one station to another. The lower bound on the activity duration is given by the minimum time it takes to complete the journey, which is determined by several factors of the railway network, e.g. the distance of the two stations and speed limits on the railway section. The upper bound amounts to the longest journey time that would still provide a service of reasonable quality to the passengers.

Dwells A stay of a train at a platform of an intermediate stop of a service line is represented by a dwell. The constraints on the duration of the stay should allow for comfortable boarding depending on the respective station and passenger demand. On the other hand the dwell-time should not exceed the maximum reasonable waiting time for passengers already on board.

2.2. Modeling a periodic timetable with the PESP

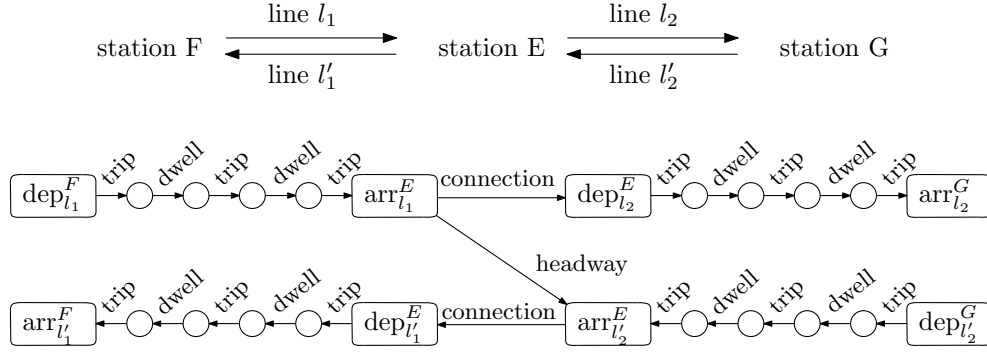


Figure 2.1: The lower graph depicts the EAN for the line-plan given above, showing the paths of trips and dwells for two service lines and respective opposite directions with connection and headway activities at station E . The square events represent the arrivals and departures at the terminal stations of the service, which are of special interest for the integration of rolling stock scheduling (see Section 3.2). The circled events are the arrivals and departures at intermediate stations.

Headways A headway activity implements a safety constraint that requires a time of at least h minutes between two events. Usually this is necessary when two trains use the same part of the infrastructure, e.g. two trains using the same platform at a station should arrive at least 3 minutes apart. A headway of at least h minutes can be ensured by lower and upper bounds of h and $T - h$ respectively on the periodic time difference between the two events, regardless of their sequence.

Connections Important connections that are assumed to be used by a certain amount of passengers, and thus should be guaranteed, can be modeled with an activity in the EAN. Let l_1 and l_2 be two service lines arriving and departing at a station s . Then the arc $(arr_{l_1}^s, dep_{l_2}^s)$ represents a passenger connection from service l_1 to l_2 . The duration of the activity is constrained by the minimum time for the passenger to change the platform in station s and the maximum acceptable waiting time for the connection.

Separations A timetable may require that two events are separated by exactly s minutes, e.g. two departures of a service line that is operated multiple times within one period.¹ It has to be specified which event is considered as the first and which as the second. Let e_i and e_j denote the first and the second event respectively, then the constraint of e_j taking place exactly s minutes after e_i is given by an arc $a = (e_i, e_j)$ in the EAN with associated lower and upper bounds $l_a = u_a = s$.

An objective function for the PESP can be used to favor some feasible timeta-

¹Service lines with different frequencies can be modeled with several approaches, which are discussed in Chapter 5.1, taking into account the needs of an integrated rolling stock schedule.

bles over others. Typically the total travel time of all passengers is minimized, which can be achieved by using the sum of the tensions of trip and dwell activities of all service lines as objective function. Other objective functions also take into account the tensions of passenger connections, and consider different types of activities with different weight coefficients. As discussed in Section 4.2, it is also possible to minimize the operational cost of the timetable, if a rolling stock schedule is already known.

Chapter 3

Rolling stock

In rolling stock scheduling the main goal is to assign the available rolling stock to service lines, satisfying a given timetable, passenger demand and constraints for rolling stock operations, such as coupling, uncoupling, shunting and rotations. Caprara et al. [Cap+07] summarize various modeling approaches and contexts for rolling stock scheduling, where the underlying problem is called *Rolling stock circulation problem* (RSCP). A rolling stock circulation should minimize the operation costs of the railway network, e.g. by minimizing the total amount of carriage kilometers or by using as few rolling stock units as possible.

Depending on the type of rolling stock units and the structure of the railway network, different variants of the RSCP have to be considered. Concerning the rolling stock, the two options are either locomotives, which can be combined with carriages, or self-propelled complete trains. On the network side, the requirements for a rolling stock circulation differ for sparse and dense networks, where dense networks are characterized by high frequencies and relatively short distances. In this thesis we consider the RSCP for self-propelled trains, which we call rolling stock compositions, in dense railway networks. These rolling stock compositions can move in both directions, simplifying and shortening turnarounds at terminal stations of lines. Caprara et al. call the rolling stock circulation in a dense network *anonymous*, i.e. the circulation determines sets of rolling stock compositions that perform the same activities, rather than assigning concrete rolling stock compositions to services of lines. Since the ultimate goal is the integration of rolling stock aspects into the periodic timetabling problem, the assumption of dense networks coincides with the situation for periodic timetables, which are often used for urban networks that also exhibit short distances and high frequencies.

3.1 Overview of the RSCP

In the following section, we give an overview of a few results for the RSCP in dense networks. As already mentioned, an exhaustive overview is given in [Cap+07]. Schrijver [Sch93] introduced a model to schedule two rolling stock types in order to satisfy a given passenger demand for a single service line. The model is based on an integer multicommodity flow and was applied for the dutch railway company NS Reizigers. This paper initiated a line of research, in which models for the RSCP were improved and extended. Alfieri et al. [Alf+06] also consider a single line, where the rolling stock circulation adapts to peak and off-peak passenger demand with coupling and uncoupling operations. Their model respects the order of rolling stock units in a rolling stock composition, using shunting constraints for an integer multicommodity flow. The concept of rolling stock composition changes was modeled with a transition graph in [PK07], where the transitions between compositions are given by the possible coupling and uncoupling operations for a rolling stock composition. (We will discuss a different notion of transition graph later in this chapter.) Fioole et al. [Fio+06] extend the model of Peeters and Kroon by allowing coupling and uncoupling of rolling stock units also during the itinerary of a service line. Furthermore, they consider several objectives for operational cost, service quality and reliability of the rolling stock circulation.

The multi-commodity flow models typically use a time-expanded graph for the railway network, which represents the timetabled services for one day or a week of service, and determine paths for the rolling stock in the time-expanded network that represent a cost-efficient circulation. In contrast to routing the rolling stock, Abbink et al. [Abb+04] allocate different types of rolling stock to service lines in order to meet the passenger demand especially during a rush hour, as a step preceding the circulation problem.

Cadarso and Marín [CM11] increase the robustness of a rolling stock circulation by penalizing complicated shunting operations and empty rolling stock movements during rush hours.

Many of the models for the RSCP are very complex, some even specialized for specific railway networks/companies and are required to handle a lot of details, such as maintenance scheduling and depot planning. The planning horizon, given by a time-expanded network, typically spans a day or even a week. These aspects can hardly be considered within a period T for periodic timetabling, which usually lasts at most a few hours. Thus it is the main goal in this thesis to include certain aspects of the RSCP into timetabling, such that the resulting timetable allows for a good rolling stock circulation, e.g. by reserving a timeframe for generic rolling stock operations in the timetable. In this context, a model for a resource schedule for periodic activities, introduced by Serafini and Ukovich [SU89], is used as a basis.

3.2 Modeling rolling stock cycles

When introducing the PESP, Serafini and Ukovich also give a model that assigns a set of resources to the periodic activities. It generates a so-called resource schedule, consisting of a cyclic sequence of activities for each resource. The underlying model is a matching problem that matches the end of an activity with the beginning of an activity if both activities are performed consecutively by the same resource [SU89].

This approach can be adapted to assign rolling stock compositions to the trips of service lines and model a rolling stock schedule for a periodic timetable that was generated by the PESP. In contrast to a generic resource assignment the rolling stock compositions are not assigned to a single activity in the EAN. Since the rolling stock is usually not changed during the itinerary of a service line, a rolling stock activity corresponds to the whole itinerary of service line, i.e. a path of alternating trips and dwells in the EAN (see Section 2.2).

We use the following notation when modeling rolling stock cycles based on the EAN: Let $S_{\text{terminal}} \subseteq S$ denote the set of terminal stations and let $l \in L$ be a service line going from $s \in S_{\text{terminal}}$ to $t \in S_{\text{terminal}}$. Then, given the corresponding EAN $D = (N, A)$ for the PESP, the beginning and the end of the itinerary of l are denoted by $\text{dep}_l^s \in N$ and $\text{arr}_l^t \in N$ respectively. The set of service arcs $A_T \subseteq A$ contains all trip and dwell activities of all service lines. Let $N_{\text{terminal}} \subseteq N$ denote the set of all departures and arrivals of service lines at terminal stations, which can be partitioned into the arrival events N_{arr} and the departure events N_{dep} , i.e. $N_{\text{terminal}} = N_{\text{arr}} \cup N_{\text{dep}}$.

Definition 3.1 (Rolling stock activity) *Let $l \in L$ be a service line with terminal stations $s, t \in S_{\text{terminal}}$. Then serving l is a rolling stock activity and corresponds to an alternating path of trips and dwells in the EAN going from $\text{dep}_l^s \in N_{\text{terminal}}$ to $\text{arr}_l^t \in N_{\text{terminal}}$.*

Similar to the generic resource schedule in [SU89], a rolling stock schedule consists of the information which rolling stock activities are performed consecutively by the same resource. However, switching from one rolling stock activity to another is also an activity that has to be scheduled, because the rolling stock composition needs to perform a shunting or rotation movement in order to be able to start the next activity. For simplicity, we restrict the rolling stock schedule by not allowing deadheading, i.e. the next activity of a rolling stock composition has to start at the station where the previous activity ended. Instead of a single transition activity, deadheading would require to schedule an empty trip between two stations, using trip activities and safety activities like headways, in order to avoid track capacity and safety issues.

Definition 3.2 (Transition activity) Let $l \in L$ be a service line arriving at terminal station $s \in S_{\text{terminal}}$ and $l' \in L$ be service line departing from s . Then the activity $(\text{arr}_l^s, \text{dep}_{l'}^s)$ is called a transition activity.

A transition activity $(\text{arr}_l^s, \text{dep}_{l'}^s)$ represents the possibility that a train arriving in s while serving line l , may serve l' on its departure. Note that this also covers a turn-around, i.e. l being the opposite direction of l' . In preparation for the rolling stock scheduling, the transition activities can be added to the EAN and scheduled with the PESP. It is necessary to define lower and upper bounds on the duration of a transition activity.

Lower and upper bounds for transition activities Assume that for a transition $a = (\text{arr}_l^s, \text{dep}_{l'}^s)$ we are given a minimum time s_a , which is needed for a rolling stock composition to perform the necessary rotation or shunting. s_a can be used as lower bound for the tension of the transition activity. For a seamless integration into the PESP the transition activities also need upper bounds. It is reasonable to assume that a rolling stock composition does not spend more than a period in addition to the minimum shunting time in a terminal station. Then the tension of a is constrained by

$$s_a \leq \pi_{\text{dep}_{l'}^s} - \pi_{\text{arr}_l^s} + z_a T \leq s_a + T - 1 \quad (3.1)$$

where $z_a \in \mathbb{Z}$ is a shift-integer variable modeling the modulo operation as in the general PESP-constraints (2.2a). The tension is the actual time spent in the station by a rolling stock composition between two rolling stock activities.

As described above, the transition activities can simply be added to the PESP as additional activities. Furthermore, a rolling stock schedule can be modeled based on a graph formed by the transition activities. Let A_R denote the set of all transition activities and let $s \in S_{\text{terminal}}$ be a terminal station, then A_R contains a transition activity from every arrival event to every departure event in station s .

Definition 3.3 (Transition graph) Let N_{terminal} be the set of all arrivals and departures at terminal stations and A_R the set of all transition activities. Then the network $R = (N_{\text{terminal}}, A_R)$ is called transition graph.

Since every transition starts at an arrival and goes to a departure, R is bipartite with N_{arr} and N_{dep} being the two classes of events. See Figure 3.1 for an illustration of a part of such a graph. Note that the components of R are complete bipartite graphs and that every component corresponds to one terminal station, because the number of arrivals and departures at a terminal station are equal.

In a periodic rolling stock schedule every arriving train needs to depart again, and every departure has to be handled by a train that previously

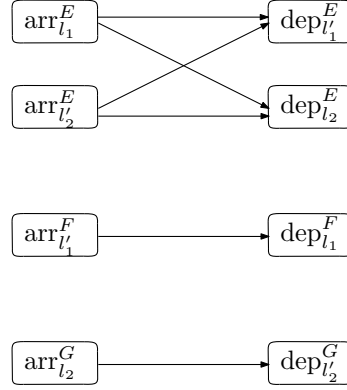


Figure 3.1: The transition graph $R = (N_{\text{terminal}}, A_R)$ for two lines l_1, l_2 with opposite directions l'_1, l'_2 respectively and a line-plan as in Figure 2.1. The subgraph induced by nodes that belong to the same station is a complete bipartite graph.

arrived. This translates to a perfect matching in the transition graph R . For the purpose of this matching it is not necessary to consider the orientation of the arcs. They are just used for integration into the EAN for the PESP. In conclusion, we define the following variant of a rolling stock circulation, which is used in this thesis.

Definition 3.4 (Rolling stock circulation) *Let $D = (N, A \cup A_R)$ be an EAN for the PESP, which is augmented with transition activities. Then a rolling stock circulation is given by a perfect matching in the transition graph $R = (N_{\text{terminal}}, A_R)$. The circulation consists of the rolling stock activities (see Definition 3.1) and the transition activities in the matching.*

Though the rolling stock schedule is given by a matching, it is called a rolling stock circulation for the following reason: As shown by Serafini and Ukovich for generic resource schedules, such a matching induces resource cycles [SU89]. Similarly, for each rolling stock activity the matching edge incident to its arrival indicates the next rolling stock activity in the schedule. Since there is only a finite number of rolling stock activities, each activity has to recur after a finite number of steps. Additionally the sequence of activities is fixed by the matching, so the rolling stock schedule implies cyclic sequences of activities called *rolling stock cycles*. An algorithm to compute the rolling stock cycles, given a matching as input, is described in Section 6.1.

3.3 Rolling stock cycles for a fixed timetable

As described in the previous section, a rolling stock schedule is given by a perfect matching in the transition graph. In this section we discuss how to compute an optimized rolling stock schedule using a weighted perfect matching, i.e. how a cost function for the transition activities may be chosen.

A rolling stock cycle is a cyclic sequence of activities that are performed by the same rolling stock composition. However, it might be necessary that multiple rolling stock compositions follow the same rolling stock cycle as shown e.g. in [PK01].

Observation 3.5 (Number of rolling stock compositions for a RS cycle)

Let $D = (N, A)$ be an EAN, A_R the set of corresponding transition activities and $C \subseteq A \cup A_R$ be a rolling stock cycle. By cycle periodicity (2.6) we have $\sum_{a \in C} x_a = q_C T$ for some $q_C \in \mathbb{Z}$. Now consider a rolling stock composition that serves C , starting at some fixed event e . It needs exactly a time-span of $q_C T$ time to return for the next start at e . Since e happens once every period, q_C rolling stock compositions are required to serve C .

Given a solution to the PESP (in CPF) by tensions x and cycle integer variables q , we want to compute resource cycles that need a minimum number of rolling stock compositions. Let $n_{\mathcal{R}}$ denote the number of rolling stock compositions that are needed to realize a rolling stock schedule. The following lemma shows a relationship between $n_{\mathcal{R}}$ and the rolling stock schedule.

Lemma 3.6 ($n_{\mathcal{R}}$ required by a rolling stock schedule [LP02])

Let $D = (N, A)$ be an EAN and $R = (N_{\text{terminal}}, A_R)$ the corresponding transition graph. Furthermore, let $x \in \mathbb{R}^{A \cup A_R}$ be the tensions of a timetable generated with the CPF for the augmented EAN $D_R = (N, A \cup A_R)$. Then a rolling stock schedule given by a perfect matching $M \subseteq A_R$ in R requires

$$n_{\mathcal{R}} = \frac{1}{T} \left(\sum_{a \in A_T} x_a + \sum_{a \in M} x_a \right) \quad (3.2)$$

rolling stock compositions, where A_T is the set of service activities.

Proof Let $\mathcal{C}_{\mathcal{R}}$ denote the set of all rolling stock cycles induced by M . Then for $n_{\mathcal{R}}$ we have

$$n_{\mathcal{R}} \cdot T = \sum_{C \in \mathcal{C}_{\mathcal{R}}} \sum_{a \in C} x_a \quad (3.3a)$$

$$= \sum_{C \in \mathcal{C}_{\mathcal{R}}} \left(\sum_{a \in A_T \cap C} x_a + \sum_{a \in A_R \cap C} x_a \right) \quad (3.3b)$$

$$= \sum_{a \in A_T} x_a + \sum_{a \in M} x_a. \quad (3.3c)$$

The first equation follows from cycle periodicity and as a consequence of Observation 3.5. In the second equation, every cycle is split into service activities and transition activities. Finally, all service activities from all cycles

are exactly all service activities, because every service activity is contained in exactly one cycle. The transition activities from all cycles are the transition activities that are in the matching, yielding the last equation. \square

It follows that a minimum-weight perfect matching in $R = (N_{\text{terminal}}, A_R)$ with the tensions x_a as the weight of the transition activities $a \in A_R$ minimizes the number of rolling stock compositions, since the tensions of the service activities in (3.2) are already fixed by the timetable. We define the Rolling stock circulation problem in the following way:

Definition 3.7 (Rolling stock circulation problem) *Let $x \in \mathbb{R}^{A_R}$ denote the tensions of the transition activities of a given timetable. Then the Rolling stock circulation problem is to compute a minimum-weight perfect matching by obtaining an optimal vertex solution to the following linear program for the matching variables m_a for $a \in A_R$.*

$$\text{MIN } \sum_{a \in A_R} x_a m_a \quad (3.4a)$$

s.t.

$$\sum_{a \in \delta^+(e)} m_a = 1 \quad \forall e \in N_{\text{arr}} \quad (3.4b)$$

$$\sum_{a \in \delta^-(e)} m_a = 1 \quad \forall e \in N_{\text{dep}} \quad (3.4c)$$

$$m_a \geq 0 \quad \forall a \in A_R \quad (3.4d)$$

$\delta^+(e)$ and $\delta^-(e)$ denote the outgoing and incoming arcs of an event e respectively. Constraints (3.4b)–(3.4d) characterize the perfect matching polytope for the bipartite graph R . The domain of the matching variables m_a in (3.4d) can be relaxed from $\{0, 1\}$, because the above constraints correspond to the vertex-arc incidence matrix of the bipartite transition graph. Such a matrix is totally unimodular, implying a polytope with integral vertices for constraints with integral right-hand sides [Sch03c].

Another method to compute a minimum-cost rolling stock schedule for a fixed timetable is to schedule the rolling stock compositions first-in first-out based on the event times of arrivals and departures in each terminal station [LP02].

An example The circulation of the available rolling stock composition on only one service line and its opposite direction, may be considered as the “default” rolling stock circulation. The example in Figure 3.2 shows that even for an already fixed timetable the computation of rolling stock cycles may reveal ways to operate a timetable with less rolling stock compositions than needed by the default rolling stock cycles. Kroon et al. [Kro+13] show

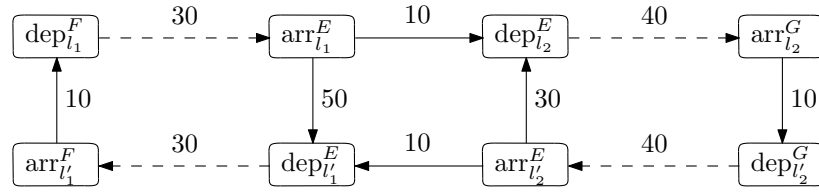


Figure 3.2: The tensions indicated above the arcs are given by a fixed timetable for $T = 60$. Trip arcs are indicated by dashed arrows and solid arrows correspond to the transition graph of Figure 3.1, admitting two matchings. One implies two rolling stock cycles of length $2T$ each, while the other matching implies a single RS cycle of length $3T$.

a similar example to motivate their method of PESP-implicit matching of events, which is discussed in Section 4.4.

3.4 Integrating other specific rolling stock requirements

It is possible to implement case-specific rolling stock requirements in the Rolling stock circulation problem. In one of the case studies (see Chapter 7) it was required that a certain service line l and its opposite direction l' are operated by a separate rolling stock cycle. This was achieved by removing some arcs from the transition graph R defined in Section 3.2. Let $a, b \in S_{\text{terminal}}$ be the two terminal stations of l and l' , so N_{terminal} contains the events $\text{arr}_l^a, \text{dep}_{l'}^a, \text{arr}_{l'}^b$ and dep_l^b . All arcs incident to these nodes, except those representing transitions between the lines l and l' are removed, so for these four nodes the only remaining incident arcs are $(\text{arr}_l^a, \text{dep}_{l'}^a)$ and $(\text{arr}_{l'}^b, \text{dep}_l^b)$. See Figure 3.3 for an illustration of the modified transition graph.

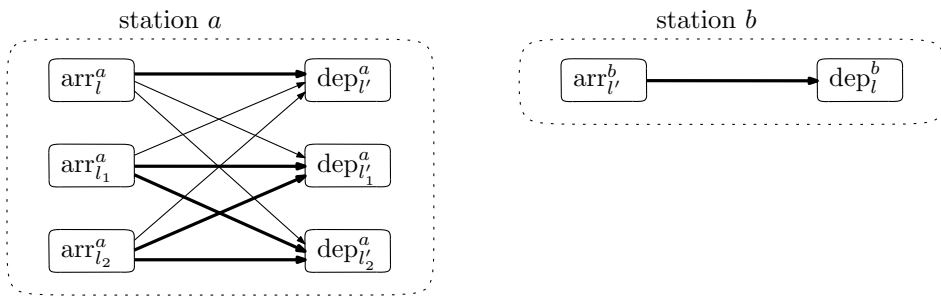


Figure 3.3: Separating a rolling stock cycle for the lines l and l' . The modified transition graph contains only the bold arcs, where the example shows a subgraph for stations a and b , assuming that some lines l_1, l_2 with opposite directions l_1', l_2' also pass through a .

The formulation of the Rolling stock circulation problem as linear program (see Definition 3.7) then contains the perfect matching constraints for the modified transition graph. In the smaller instance of the case studies this

3.4. Integrating other specific rolling stock requirements

concept was applied to create separate rolling stock cycles for two service lines and their respective service lines in the opposite direction. Though the modified transition graph implies a restriction on the general rolling stock circulation, in the case study it did not lead to an increase of the objective value corresponding to the best solution found (see Section 7.2).

Integrated periodic timetabling and RSCP

In the preceding chapters we considered a model for periodic timetabling and a model for rolling stock scheduling for a periodic timetable. As indicated in Figure 1.1, rolling stock scheduling is the subsequent step of timetabling and, thus, uses a timetable as input, which restricts the rolling stock schedule. This may lead to suboptimal rolling stock utilization, since two timetables, which exhibited similar quality in the timetabling step, can require a different number of rolling stock compositions. An example is given in Figure 4.1.

An integrated model for both planning steps could avoid the described suboptimality. One attempt was made by Cadarso and Marín [CM12], who integrate a timetable into their aperiodic rolling stock scheduling model. In this chapter we study integrated periodic timetabling and rolling stock scheduling.

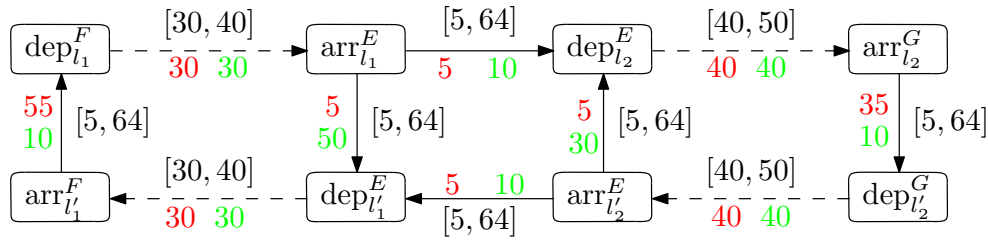


Figure 4.1: A timetabling instance with two feasible timetables (red and green). The green timetable was already used as example in Figure 3.2 to show that computing the matching for a fixed timetable may be beneficial. Both timetables minimize the total travel time. Note that also the sums of total travel time and all transition activities are equal. The network has two possible rolling stock circulations, both of which need 4 rolling stock compositions for the red timetable. For the green timetable, however, there exists a rolling stock circulation, which requires only 3 rolling stock compositions.

4.1 Literature review on integrated periodic timetabling and rolling stock scheduling

Many attempts to integrate aspects of rolling stock scheduling into the PESP exploit the fact that the duration of a rolling stock cycle determines the number of required rolling stock compositions (see Observation 3.5 and Lemma 3.6), and aim at minimizing the number of rolling stock compositions, which are required by the timetable. The main aspect, in which the results have progressed, is to what extent and in which way the rolling stock schedule and trip and dwell times of the service lines are fixed.

The initially considered variant did not allow line changes for the rolling stock, i.e. every rolling stock cycle is given by a service line and its opposite direction. Nachtigall [Nac99] shows how to find timetables that require a minimum number of rolling stock compositions for every service line (and opposite direction) separately by imposing additional upper bounds on the turnaround times at the terminal stations. It is assumed that all lower bounds for trip and dwell activities are met, which corresponds to fixing the trip and dwell times for the service lines a priori.

For fixed trip and dwell times and no line changes the number of rolling stock compositions can differ by at most one for each service line, because every rolling stock cycle contains exactly two turnaround activities, and at a turnaround, a rolling stock composition has to wait at most one period for the next departure [LP02]. When removing the assumption of fixed trip and dwell times, timetables that require an additional rolling stock composition may become feasible. These timetables can be avoided by penalizing the turnarounds [LM07]. Lindner et al. [Lin00] extend the problem by including a model to assign train types and coaches to minimize the operation cost for known circulation on single service lines, i.e. without line changes.

Disallowing line changes for the rolling stock may increase the number of required rolling stock compositions [LP02], which can also be seen in the example in Figure 3.2. A first step towards a more general rolling stock circulation is given by considering arbitrary but fixed rolling stock cycles, which may contain line changes. Peeters and Kroon [PK01] penalize timetables that need more than the minimum number of rolling stock compositions for fixed rolling stock cycles, as discussed in Section 4.2

Softening the restriction of a fixed rolling stock circulation, Peeters [Pee03] models the possibility of joining the rolling stock cycles of two service lines and opposite directions that share a terminal station. Exploiting a special property of PESP-constraints, this method can be interpreted as a matching of four events of the PESP, namely the arrivals and departures of the services in the shared terminal station. This method is generalized to a matching of arbitrarily many events, and can thus be used to model arbitrary rolling

stock cycles [Vil06; Kro+13]. We review the PESP-implicit matching in detail in Section 4.4.

Finally, arbitrary rolling stock cycles can also be represented as an actual matching problem as in Section 3.2 and be integrated with the PESP [LP02]. This approach exhibits similarities to the periodic resource assignment given in [SU89]. It leads to the model presented in Section 4.3 with quadratic objective function that was linearized using a heuristic.

4.2 Penalizing expensive schedules for fixed RS cycles

The operational cost of a timetable is related to the number of rolling stock compositions that are required to operate it. For a given rolling stock schedule, a timetable that minimizes the number of rolling stock compositions can be generated by choosing an appropriate objective function for the PESP as described e.g. in [PK01; LM07; LP02]. Let $\mathcal{C}_{\mathcal{R}}$ be the set of given rolling stock cycles. Then by cycle periodicity (2.6) the length of all activities in a single cycle equals to an integer multiple of T , i.e. there exists $q_C \in \mathbb{Z}$ such that

$$\sum_{a \in C} x_a = q_C T.$$

All tensions are added with the same sign, because a rolling stock cycle is a directed cycle, as the arcs correspond to all activities performed by some rolling stock composition. According to Observation 3.5, q_C is the number of rolling stock compositions required to serve the cycle C . Then as a consequence of Lemma 3.6 the total number of rolling stock compositions can also be expressed in terms of the cycle integers q_c for the rolling stock cycles, i.e.

$$n_{\mathcal{R}} = \sum_{C \in \mathcal{C}_{\mathcal{R}}} q_C. \quad (4.1)$$

Solving the PESP with (4.1) as objective function generates a timetable that requires the minimum number of rolling stock compositions required by the timetable.

4.3 PESP-integrated RS cycles – the RS-PESP

Two timetables that minimize the travel time can require a different number of rolling stock compositions as shown in Figure 4.1. Thus, the goal is to compute rolling stock cycles simultaneously with a PESP-timetable. As in Section 3.2 the EAN is augmented with all transition activities A_R . The lower and upper bounds for a tension x_a with $a \in A_R$ are given by constraint (3.1).

Furthermore the model is extended with matching variables $m_a \in \{0, 1\}$ for $a \in A_R$ to model a perfect matching in the transition graph that induces the rolling stock cycles as in optimization problem (3.4).

In contrary to Section 3.3 the tensions are not fixed, so the sum of the durations of all rolling stock cycles needs to be considered to determine the number of rolling stock compositions required by the schedule. According to Lemma 3.6 this can be achieved by choosing

$$\sum_{a \in A_T} x_a + \sum_{a \in A_R} x_a m_a \quad (4.2)$$

as objective function. In conclusion, this gives an integrated model RS-PESP for the PESP and the RSCP, which was presented in [LP02], shown in Figure 4.2. Since the tension x_a and the matching value m_a are variables for every transition $a \in A_R$, the objective function is quadratic.

$$\text{MIN } n_{\mathcal{R}} T = \sum_{a \in A_T} x_a + \sum_{a \in A_R} x_a m_a \quad (4.3a)$$

s.t.

$$l_a \leq x_a \leq u_a \quad \forall a \in A \cup A_R \quad (4.3b)$$

$$\sum_{a \in C^+} x_a - \sum_{a \in C^-} x_a = q_C T \quad \forall C \in \mathcal{C}(A \cup A_R) \quad (4.3c)$$

$$a_C \leq q_C \leq b_C \quad \forall C \in \mathcal{C}(A \cup A_R) \quad (4.3d)$$

$$\sum_{a \in \delta^+(e)} m_a = 1 \quad \forall e \in N_{\text{arr}} \quad (4.3e)$$

$$\sum_{a \in \delta^-(e)} m_a = 1 \quad \forall e \in N_{\text{dep}} \quad (4.3f)$$

$$x_a \in \mathbb{R} \quad \forall a \in A \cup A_R \quad (4.3g)$$

$$q_C \in \mathbb{Z} \quad \forall C \in \mathcal{C}(A \cup A_R) \quad (4.3h)$$

$$m_a \in \{0, 1\} \quad \forall a \in A_R \quad (4.3i)$$

Figure 4.2: RS-PESP

A heuristic with fixed matching values Liebchen and Peeters [LP02] linearize the RS-PESP by fixing the matching values, thus transforming the problem to the PESP for the augmented transition graph $D_R = (N, A \cup A_R)$. Remember that the transition graph consists of “station-wise” complete bipartite graphs. It follows that a feasible fractional perfect matching is given by

$$m_a = \frac{1}{\deg(e_i)} = \frac{1}{\deg(e_j)} \quad \forall a = (e_i, e_j) \in A_R \quad (4.4)$$

where $\deg(\cdot)$ denotes the degree of an event in the transition graph. An example w.r.t. the transition graph given by Figure 3.1 is $m_a = \frac{1}{2}$ for activities

with both events in station E and $m_a = 1$ for the two activities that have both events in station F and G respectively. Note that the heuristic matching value respects some structure of the transition graph. Furthermore, Liebchen and Peeters introduce a penalty for the transition activities in the objective in order to optimize a trade-off between travel times and transition times.

4.3.1 Relaxing the matching variables of the RS-PESP

Similar to the situation with the event-times in Theorem 2.3, fixing all other variables reduces the constraint matrix for the variables m_a to a totally unimodular matrix, because only constraints (4.3e)–(4.3f) remain. As already mentioned in Section 3.3, these constraints correspond to the vertex-edge-incidence matrix of an undirected bipartite graph, which is totally unimodular.

Proposition 4.1 *The domain (4.3i) of the matching variables m_a in the RS-PESP can be relaxed to $m_a \geq 0$.*

Proof Consider the RS-PESP with constraint (4.3i) replaced with $m_a \geq 0$ and let $x \in \mathbb{R}^{A \cup A_R}$, $q \in \mathbb{Z}^{C(A \cup A_R)}$ and $m \in \mathbb{R}^{A_R}$ be a feasible solution. From total unimodularity it follows that the vertices of the polytope remaining for fixed x and q are integral. Furthermore, for a fixed x the objective function becomes linear, implying the existence of a feasible solution given by x , q and $m' \in \{0, 1\}^{A_R}$ as least as good as the solution given by x , q and m . \square

4.4 Review on PESP-implicit matching of events

In this thesis we consider a rolling stock schedule that is represented by a matching of events of the PESP. Thus far, this matching was modeled with binary matching variables. In this section, we review a method to model a matching with activities in the PESP only, which has been studied in [Pee03; Vil06; Kro+13]. In general, the idea of this approach is that an activity between two events is in the matching if the value of its tension lies in a certain interval. We review the method in order to compare it with the model proposed in Section 4.3 and the approaches for solving it in Section 4.5.

Based on the assumption that rolling stock circulates on one service line and its opposite direction only, Peeters creates the possibility to join the rolling stock cycles of two lines and their respective opposite directions by a matching of four events [Pee03]. Generalizing this concept, Villumsen [Vil06] shows how to model a matching inside a terminal station (also shown in [Kro+13]), as described in Section 3.2, using PESP constraints. In order to simplify the description, we only consider the arrival and depar-

ture events of service lines in one common terminal station. Other terminal stations can be included analogously.

Following the notation of [Vil06], let I and J denote the sets of arrivals and departures respectively, where $|I| = |J| = n$. It is assumed that I and J are cliques, i.e. for all $i_1, i_2 \in I$ there exists an activity between i_1 and i_2 in the EAN and for J analogously. These activities represent separation constraints between all arrivals and between all departures with lower and upper bounds on the tensions denoted by s^- and s^+ , where $s^+ = T - s^-$ and $s^- \leq \frac{T}{2} \leq s^+$. (These are the common bounds for separation constraints, see Section 2.2.) Furthermore for all $i \in I, j \in J$ there is an activity (i, j) , which is in fact a transition activity as in Definition 3.2. For each of these activities (i, j) there is a so-called disjunctive constraint for the tension x_{ij} :

$$x_{ij} \in [d^-, d^+] \cup [d^- + s^-, d^+ + s^+] . \quad (4.5)$$

The interval of $[d^-, d^+]$ corresponds to the lower and upper bounds on the shunting-time in the station. The disjunctive constraints are used to model a perfect matching on the transition activities in the following way: A transition activity is part of the matching if its tension lies in the interval $[d^-, d^+]$. Otherwise the tension lies in $[d^- + s^-, d^+ + s^+]$ and the arc is not in the matching.

Disjunctive constraints can be implemented in the PESP, since the union of two periodic intervals can also be represented as the intersection of two periodic intervals [SU89]. For the right-hand side of constraint (4.5) we have

$$[d^-, d^+] \cup [d^- + s^-, d^+ + s^+] = [d^-, d^+ + s^+] \cap [d^- + s^-, d^+ + T] . \quad (4.6)$$

It follows that disjunctive constraints can be achieved by imposing two regular PESP-constraints. In conclusion, the model features a complete bipartite graph of transition activities between the arrivals and departures in a terminal station with a disjunctive constraint for each transition activity. Furthermore, both set of arrivals I and the set of departures J are cliques of separation activities.

Under the additional assumption that

$$d^+ + s^+ < d^- + n s^- \quad (4.7)$$

the tensions of the transition activities indicate a perfect matching between I and J . Making use of the separation constraints and assumption (4.7), Villumsen shows that for every $i \in I$ exactly one tension of a transition incident to i lies in $[d^-, d^+]$, and that for every $j \in J$ exactly one tension of an incident transition lies in $[d^-, d^+]$ [Vil06].

Exploiting the separation constraints, the sum of all tensions in the matching can be expressed without knowing the matching [Vil06; Kro+13]. It follows

that the sum of tensions of all rolling stock cycles, as in (4.2), can be used as objective function for the PESP, where the matching is modeled implicitly by the disjunctive constraints.

Advantages of this method are that it relies entirely on the properties of the PESP and that it has a linear objective function. On the other hand, assumption (4.7) implies a lower bound on the separation of the events:

$$d^+ + s^+ < d^- + n s^- \quad (4.8a)$$

$$d^+ + T - s^- < d^- + n s^- \quad (4.8b)$$

$$\frac{d^+ - d^- + T}{n + 1} < s^- . \quad (4.8c)$$

Since $d^+ - d^- > 0$, it follows that all arrivals (and departures) have to be distributed almost evenly over the period. From a modeling perspective, this may be an undesirable restriction on the timetable. That is why we consider the RS-PESP and study approaches to solve it in the next section. Both, the RS-PESP and the PESP-implicit matching add a constant number of constraints for each transition activity to the PESP. The main difference in terms of additional constraints and variables is that the RS-PESP needs matching variables. Although they can be relaxed, they still lead to a quadratic objective function. This problem will be addressed in the following section.

4.5 Approaches for solving the RS-PESP

The RS-PESP (Figure 4.2) is stated to be intractable in practice, because of the quadratic objective function by Liebchen and Peeters [LP02]. In the following section, we address this issue with two approaches: first, with a heuristic and secondly with a linearization of the problem.

4.5.1 Iterative approach

The objective function of the RS-PESP in (4.3a) becomes linear for a fixed timetable, i.e. fixed tensions of the transition activities, and it also becomes linear for fixed matching variables. The following iterative heuristic exploits this fact and finds a solution for the RS-PESP by solving two sub-problems, which have linear objective functions.

First, a timetable is computed for fixed matching variables, and secondly the matching problem is solved for the computed timetable. These two problems are denoted by RS-PESP(m) and RS-PESP(x) respectively and shown in Figure 4.5.

Since the values computed in one of the sub-problems serve as input for the other sub-problem, the two steps may be alternatingly iterated. We propose the iterative method, given in pseudocode in Listing 4.1.

$$\begin{array}{ll}
 \text{MIN } \sum_{a \in A_T} x_a + \sum_{a \in A_R} x_a m_a & \text{MIN } \sum_{a \in A_R} x_a m_a \\
 \text{s.t.} & \text{s.t.} \\
 \text{PESP constraints for} & \sum_{a \in \delta^+(e)} m_a = 1 \quad \forall e \in N_{\text{arr}} \\
 D = (N, A \cup A_R) & \sum_{a \in \delta^-(e)} m_a = 1 \quad \forall e \in N_{\text{dep}} \\
 & m_a \geq 0 \quad \forall a \in A_R
 \end{array}$$

(a) RS-PESP(m) (b) RS-PESP(x)

Figure 4.5: The two sub-problems of the iterative method. Note that (b) is formulated as linear program as discussed in Section 3.3.

```

1   $m \leftarrow m_{\text{init}};$ 
2  until(no progress)
3       $x \leftarrow \text{solve RS-PESP}(m);$ 
4       $m \leftarrow \text{solve RS-PESP}(x);$ 
5  end
    
```

Listing 4.1: Pseudocode of the iterative method for finding solutions of the RS-PESP. $x \in \mathbb{R}^{A_R}$ denotes the tensions of the transition activities and $m \in \mathbb{R}^{A_R}$ denotes the matching variables. The method is terminated if there is no progress between two iterations, i.e. solving RS-PESP(m) in line 3 returns the same objective value in two consecutive iterations.

Initialization A possible starting point m_{init} is given by the fractional matching value in (4.4), also used in [LP02]. The influence of different initial matchings was investigated in a computational study, described in Section 7.3.

Convergence Unfortunately the iterative method is not guaranteed to converge to the global optimum of the RS-PESP, but it can be shown, that it converges eventually.

Lemma 4.2 *In the iterative method given in Listing 4.1, the value of $\sum_{a \in A_T} x_a + \sum_{a \in A_R} x_a m_a$ decreases monotonically.*

Proof Let (x_i, m_i) be the solution after iteration i . Then the solution (x_{i+1}, m_i) , where x_{i+1} is obtained by line 3 in Listing 4.1, is at least as good as (x_i, m_i) , since x_i is feasible for RS-PESP(m_i) and gives an upper bound. Analogously (x_{i+1}, m_{i+1}) is at least as good as (x_{i+1}, m_i) , because m_i provides a feasible solution for RS-PESP(x_{i+1}). It follows that the objective value after iteration i bounds the objective value after iteration $i + 1$ from above. \square

The integrality of $\text{RS-PESP}(x)$ implies that the computed matching values are always integral, therefore Lemma 3.6 is applicable. It follows that $\sum_{a \in A_T} x_a + \sum_{a \in A_R} x_a m_a = n_{cur} T$ for some $n_{cur} \in \mathbb{Z}$ after each iteration, where n_{cur} corresponds to the number of rolling stock compositions required by the current timetable. Let n_{opt} denote the minimum number of rolling stock compositions required by any timetable, i.e. $n_{opt} = n_{\mathcal{R}}$ for an optimal solution of the RS-PESP. Since $n_{cur} \geq n_{opt}$ and n_{cur} monotonically decreases according to Lemma 4.2, the value of $\sum_{a \in A_T} x_a + \sum_{a \in A_R} x_a m_a$ converges to $n_{conv} T$ for some $n_{conv} \in \mathbb{Z}$ and $n_{conv} \geq n_{opt}$.

However, it is not clear if the method has converged if the objective value remains constant for several iterations. It can happen that the iterative method cycles through a set of feasible solutions that have the same objective value. Furthermore, it may have reached a local optimum, whose existence is shown later in this section. For these reasons, we terminate the iterative method if two consecutive timetabling steps, i.e. solving $\text{RS-PESP}(m)$, produce the same objective value.

Regarding the time complexity of the iterative method, consider that $\text{RS-PESP}(m)$ is a MILP, which corresponds to the original PESP extended with the transition activities, so one step of the iterative method only adds solving a linear program to the effort of solving the standard PESP. However, it has to be investigated after how many iterations the iterative method converges.

Visited vertices of the matching polytope In order to investigate the behavior of the iterative method, we studied the integral matchings visited during the iterations. To establish a relationship between different perfect matchings, consider the neighborhood-relation: Two perfect matchings are adjacent vertices of the perfect matching polytope if and only if the symmetric difference forms exactly one cycle [Sch03d]. Since the transition graph has one component for every terminal station, the vertices of the perfect matching polytope exhibit a special structure. A vertex is given by choosing a perfect matching in every component of the transition graph. It follows from the neighborhood-relation that two adjacent vertices can only differ in variables that correspond to one component, i.e. one terminal station.

In conclusion, the vertices of the perfect matching polytope are given by the cross-product of the vertices for the component-wise perfect matching polytopes. The neighborhood-relation is as follows: Two vertices are adjacent, if and only if they differ only in one component and the projections onto this component are adjacent in the perfect matching polytope for this component. See Figure 4.6 for a small example.

For two consecutive perfect matchings of the iterative method, the objective function is changed by the intermediate timetabling step, so in general they are not necessarily neighboring matchings. In the computational study for

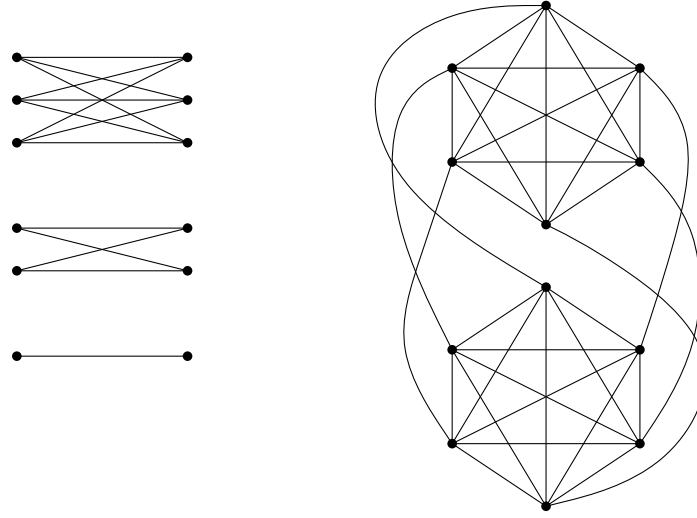


Figure 4.6: An example of a transition graph is shown on the left side, and the corresponding graph of the matching neighborhood is shown on the right side. Each perfect matching is a combination of one of the six perfect matchings in the upper component and one of the two perfect matchings in the middle component of the transition graph. The third component does not have any influence, because it has just one perfect matching.

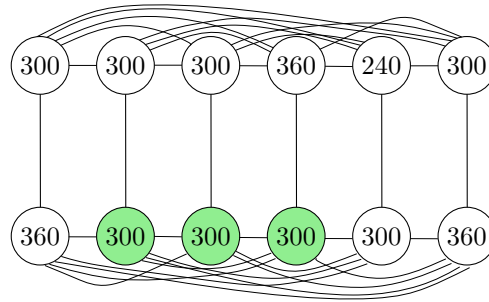


Figure 4.7: The graph shows the same matching neighborhood graph as depicted in Figure 4.6 with the objective value of the best timetable for each matching. The green vertices are local minima w.r.t. the matching neighborhood. The timetabling input consisted of two service lines that share one terminal station, where one line is operated twice per period.

the iterative method (Section 7.3) it was observed that steps to adjacent vertices happen rarely, but the question arises, whether it could be beneficial to explore the neighborhood of a vertex instead of jumping in the matching polytope by changing the objective function.

Existence of local minima w.r.t. the matching neighborhood It would be interesting to formulate an algorithm based on adjacent vertices of the perfect matching polytope. However, this approach has two major problems. First, the neighborhood of a vertex is large. Any subset of at least two matching transitions from the same station can be completed to a cycle using tran-

sitions from the same station. Then flipping this cycle yields a neighboring matching, so the number of neighbors might be exponential in the number of transition activities A_R .

Secondly, a small example in Figure 4.7 shows, that there might exist vertices that are local minima w.r.t. the best timetable existing for a matching, i.e. vertices for which the best timetable for every neighbor needs at least as many rolling stock compositions as the best timetable for the considered vertex.

4.5.2 Linearization of the RS-PESP

In this section we develop a linearization of the RS-PESP (see Figure 4.2). The goal is to obtain a formulation that is tractable for MIP-solvers, but allows for a variable timetable and rolling stock schedule simultaneously.

Note that the quadratic term in the objective of the RS-PESP is a sum of bilinear terms, where each bilinear term is of the form $x_a m_a$ for a transition activity $a \in A_R$. For bilinear terms valid bounds on the value are given by McCormick [McC76] envelopes in form of linear constraints.

Definition 4.3 (McCormick envelopes [McC76]) *Let $f(x_1, x_2) = x_1 x_2$ be a bilinear term, then for $l_1 \leq x_1 \leq u_1$ and $l_2 \leq x_2 \leq u_2$ the term $f(x_1, x_2)$ is bounded by*

$$\begin{aligned} f(x_1, x_2) &\geq l_1 x_2 + x_1 l_2 - l_1 l_2 \\ f(x_1, x_2) &\geq u_1 x_2 + x_1 u_2 - u_1 u_2 \\ f(x_1, x_2) &\leq u_1 x_2 + x_1 l_2 - u_1 l_2 \\ f(x_1, x_2) &\leq l_1 x_2 + x_1 u_2 - l_1 u_2 . \end{aligned}$$

Note that these bounds imply that $f(x_1, x_2) = x_1 x_2$ if the variables meet their lower or upper bounds, i.e. $x_1 \in \{l_1, u_1\}$ and $x_2 \in \{l_2, u_2\}$. The McCormick envelopes can be applied on the bilinear terms $x_a m_a$, since the tension x_a has lower and upper bounds as in (3.1) and $m_a \in \{0, 1\}$ can be constrained by $0 \leq m_a \leq 1$.

In order to linearize the term $\sum_{a \in A_R} x_a m_a$, we introduce a new variable $w_a \in \mathbb{R}$ for each $a \in A_R$, which serves as approximation for $x_a m_a$. Let w_a be constrained by the McCormick envelopes for $x_a m_a$, i.e.

$$w_a \geq l_a m_a \tag{4.9a}$$

$$w_a \geq u_a m_a + x_a - u_a . \tag{4.9b}$$

Bounds from above are not necessary, because w_a will be minimized. Unfortunately these constraints destroy the property of total unimodularity for a fixed timetable, i.e. fixed x and q (see Section 4.3.1), so we cannot consider a

version with relaxed variables m_a . However, with one of the two variables of the bilinear term being binary, constraints 4.9 exhibit the following special property:

Lemma 4.4 *Let $w_a \in \mathbb{R}$ for a transition activity $a \in A_R$ be constrained by inequalities 4.9, where $x_a \in \mathbb{R}$ and $m_a \in \{0, 1\}$. If w_a is minimized, then $w_a = x_a m_a$, i.e. the description of w_a by the McCormick envelopes is exact.*

Proof If $m_a = 0$ inequalities 4.9 reduce to $w_a \geq 0$ and $w_a \geq x_a - u_a$. We have that $x_a - u_a \leq 0$, since $x_a \leq u_a$, so the minimum w_a is zero and $x_a m_a = 0$. Otherwise, if $m_a = 1$ we have that $w_a \geq l_a$ and $w_a \geq x_a$. Since $l_a \leq x_a$, minimizing w_a yields x_a and $x_a = x_a m_a$. \square

Remark 4.5 (big-M constraint) *Constraint (4.9b) can also be derived as so-called big-M constraint. Such a constraint enforces a lower bound of x_a if $m_a = 1$ for a sufficiently large M .*

$$w_a \geq x_a - (1 - m_a)M$$

An additional constraint $w_a \geq 0$ is necessary for the case of $m_a = 0$. In our case M can be chosen as $M = u_a$ and $w_a \geq 0$ is implied by constraint (4.9a). We will use the big-M representation of constraint (4.9b), because it is more semantically representative, i.e. a big-M constraint “enables” and “disables” the cost of a transition w_a according to its matching value m_a .

In conclusion, we obtain a version linearized of the RS-PESP with linearized bilinear terms shown in Figure 4.8.

Theorem 4.6 *The linearized RS-PESP (shown in Figure 4.8) is equivalent to the RS-PESP (shown in Figure 4.2).*

Proof The feasible region for the variables x , q and m remains unchanged. Let $x \in \mathbb{R}^{A \cup A_R}$, $q \in \mathbb{Z}^{C(A \cup A_R)}$ and $m \in \{0, 1\}^{A_R}$ denote a feasible solution of the RS-PESP. There exist many solutions of the linearized RS-PESP that take these values, because the variables w_a for $a \in A_R$ are not bounded from above. However, the solution of the linearized RS-PESP that minimizes the objective function has the same objective value as the corresponding solution of the RS-PESP according to Lemma 4.4. \square

The linearized RS-PESP extends the original PESP in the following way: It introduces $|A_R|$ new activities, which amounts to $|A_R|$ continuous variables and $|A_R|$ constraints in the model. Furthermore, we have $|A_R|$ additional binary variables for the matching and $|N_{\text{terminal}}|$ matching constraints. The linearization needs $|A_R|$ continuous variables and $2|A_R|$ envelopes. In total, the number of added variables and constraints is proportional to the number of transition activities.

According to the number of added variables, especially the additional binary variables, the relaxed RS-PESP appears to be more difficult to solve

$$\text{MIN } n_{\mathcal{R}} = \sum_{a \in A_T} x_a + \sum_{a \in A_R} w_a \quad (4.10)$$

s.t.

$$l_a \leq x_a \leq u_a \quad \forall a \in A \cup A_R \quad (4.11)$$

$$\sum_{a \in C^+} x_a - \sum_{a \in C^-} x_a = q_C T \quad \forall C \in \mathcal{C}(A \cup A_R) \quad (4.12)$$

$$a_C \leq q_C \leq b_C \quad \forall C \in \mathcal{C}(A \cup A_R) \quad (4.13)$$

$$\sum_{a \in \delta^+(e)} m_a = 1 \quad \forall e \in N_{\text{arr}} \quad (4.14)$$

$$\sum_{a \in \delta^-(e)} m_a = 1 \quad \forall e \in N_{\text{dep}} \quad (4.15)$$

$$w_a \geq l_a m_a \quad \forall a \in A_R \quad (4.16)$$

$$w_a \geq x_a - (1 - m_a)u_a \quad \forall a \in A_R \quad (4.17)$$

$$x_a \in \mathbb{R} \quad \forall a \in A \cup A_R \quad (4.18)$$

$$q_C \in \mathbb{Z} \quad \forall C \in \mathcal{C}(A \cup A_R) \quad (4.19)$$

$$m_a \in \{0, 1\} \quad \forall a \in A_R \quad (4.20)$$

$$w_a \in \mathbb{R} \quad \forall a \in A_R \quad (4.21)$$

Figure 4.8: linearized RS-PESP with McCormick envelopes

than the PESP extended with a PESP-implicit matching as described in Section 4.4. Despite its complexity we consider the relaxed RS-PESP in this thesis, because it does not use assumptions that restrict the timetable.

Problem: Weak LP-relaxation

Solving MIPs that use big- M constraints for modeling can lead to practical problems. Apart from numerical issues, a weak LP-relaxation caused by constraints such as (4.17) is a typical problem. The binary variable m_a is used to “activate” or “deactivate” the constraint $w_a \geq x_a$. However, in the relaxation w_a can take values that are smaller than $x_a m_a$ for $0 < m_a < 1$, as illustrated in Figure 4.9. For this reason the objective value of a LP-relaxation of a node in a branch-and-cut tree for solving the MIP can have a big difference to the objective value of the best integral solution. A node can be pruned, if the value of its LP-relaxation is greater or equal than the currently best known value of an integral solution. With a weak LP-relaxation this situation might occur less frequently, slowing down the performance of the MIP-solver.

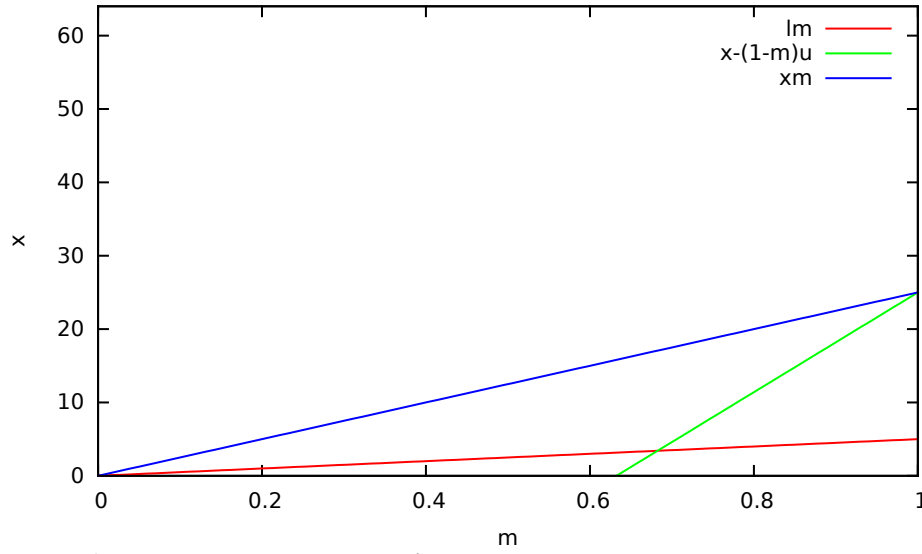


Figure 4.9: Lower bounds for w , where $l = 5, u = 64$ and $x = 25$. When $m = 0.5$, $w = 2.5$ is the minimum value, while the “real cost” is $x \cdot m = 25 \cdot 0.5 = 12.5$

4.6 Improving branch-and-cut for solving the RS-PESP

As observed in (4.1) and (4.2) the objective of the RS-PESP corresponds to an integer multiple of the period length $n_{\mathcal{R}}T$, where $n_{\mathcal{R}}$ is the number of rolling stock compositions required by the schedule. This fact can be exploited when solving the RS-PESP with the branch-and-cut method.

Let x_{rel} be the objective value of the solution for a relaxation P_{rel} of the RS-PESP, that was solved in the branch-and-cut method. Furthermore let $n_{cur}T$ be the value of the currently best known integral solution, which is an integer multiple of T . x_{rel} is a lower bound for the objective value of any integral feasible solution of P_{rel} , so if

$$\left\lceil \frac{x_{rel}}{T} \right\rceil \geq n_{cur} \quad (4.22)$$

it is not necessary to branch at P_{rel} , since the relaxation cannot contain an integral feasible solution with an objective value less than the best already known objective value $n_{cur}T$. Pruning all children of P_{rel} in the branch-and-cut tree may decrease the size of the tree and the running time.

Service lines with different frequencies in the RS-PESP

Often the lineplan, which is used as input for the periodic timetabling, requires that some service lines are offered multiple times within one period, i.e. the service lines exhibit a local periodicity. Then the period T for the whole railway network corresponds to the least common multiple of local periods. In this section, we investigate how to incorporate different frequencies in the RS-PESP by evaluating several approaches for doing so in the PESP. The integration of frequencies into the RS-PESP was necessary for the computational studies in Chapter 7.

5.1 Modeling frequencies

Let f_i denote the frequency per period for a service line L_i . Then every arrival and departure of L_i is supposed to occur f_i times during each period of length T with a local periodicity, i.e. every $\frac{T}{f_i}$ minutes. Goerik and Siebert [SG13] compare two approaches for modeling train lines with different frequencies. First, the extended PESP (EPESP), in which a new period T_{ij} is created for two events e_i and e_j that have different frequencies. It was introduced by Serafini and Ukovich [SU89] and also studied in [Nac96]. We refer to the method of the EPESP as *frequency as attribute* (FA) as in [SG13]. Secondly, in the approach of *frequency as multiplicity* (FM) an event e_i with frequency f_i is copied f_i times [SG13]. We investigate the suitability of FA and FM with regard to the RS-PESP for integrated periodic timetabling and rolling stock cycles (see Figure 4.2).

In FA a PESP-constraint as in (2.1) on an activity between two events e_i, e_j with frequencies f_i, f_j is replaced with the following constraint:

$$l_{ij} \leq \pi_j - \pi_i + z_{ij}T_{ij} \leq u_{ij} , \quad (5.1)$$

where z_{ij} is a shift-integer and $T_{ij} = \gcd\left(\frac{T}{f_i}, \frac{T}{f_j}\right)$ is the local period. Assume that this method is used for a transition activity $a = (e_i, e_j)$ of the RS-PESP, i.e. e_i is an arrival at a terminal station and e_j is a departure at the same terminal station. Then we have a matching variable m_a for the transition activity. Furthermore, assume that the events happen with different frequencies $f_i \neq f_j$. Note that m_a refers to all f_i arrivals per period represented by the event e_i , implying that all f_i rolling stock compositions have to follow the same behavior, e.g. serving departure e_j if $m_a = 1$. However the event e_j occurs f_j times per period, which makes the FA approach inapplicable for $f_i \neq f_j$.

In an attempt to solve this issue, consider a formulation where m_a is not a binary variable, but represents the number of rolling stock compositions that use the transition a from some occurrence of e_i to some occurrence of e_j , i.e. constraints (3.4b)–(3.4d) and (4.3i) are replaced with the following constraints:

$$\sum_{a \in \delta^+(e)} m_a = f_e \quad \forall e \in N_{\text{arr}} \quad (5.2a)$$

$$\sum_{a \in \delta^-(e)} m_a = f_e \quad \forall e \in N_{\text{dep}} \quad (5.2b)$$

$$0 \leq m_a \leq \min(f_{e_i}, f_{e_j}) \quad \forall a = (e_i, e_j) \in A_R \quad (5.2c)$$

$$m_a \in \mathbb{Z} \quad \forall a \in A_R. \quad (5.2d)$$

Then the number of rolling stock compositions performing an event corresponds to the frequency of the event. The RS-PESP uses the tension as cost-function for the transitions, which is given by constraint (5.1) in the FA approach. As shown in [SG13], constraint (5.1) finds the shortest tension between all occurrences of e_i and e_j . However, using it as cost-function does not reflect the real costs of a rolling stock schedule, because not all of possibly many rolling stock compositions using a transition can actually realize this shortest connection time. See Figure 5.1 for an example.

Therefore, we consider the FM modeling approach, which is as follows: an event e_i in the original EAN with frequency f_i gets replaced by the events $e_i^0, \dots, e_i^{f_i-1}$, and also the corresponding activities are copied. The occurrences of e_i are distributed evenly over the period by separation constraints with lower and upper bound $\frac{T}{f_i}$. This modification of the EAN is called *periodic rollout* in [SG13]. The EAN after the periodic rollout can be used in an instance for the RS-PESP. For this approach, the tension of a transition activity amounts to its actual duration, so it can be used as cost-function for the matching. A disadvantage is, however, that the periodic rollout leads to more tension constraints, tension variables and matching variables, and it requires additional separation constraints, which can make the problem

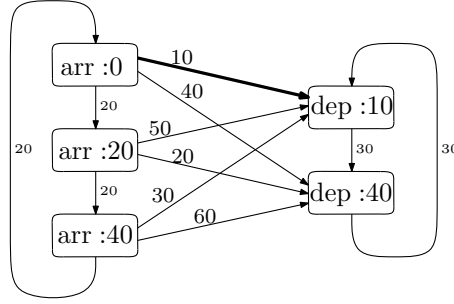


Figure 5.1: Transition activities between an arriving service with frequency 3 and a departing service with frequency 2, where $T = 60$. The arcs between the arrivals and the departures indicate the separation of $\frac{T}{f_i}$ between arrival events and departure events respectively. Only the bold arc has the shortest connection time found by constraint (5.1), so it cannot be used as cost-function for all transitions between the arrivals and the departures. A similar example is shown in [SG13].

computationally more demanding. In order to limit this effect of FM for the RS-PESP, the periodic rollout was only performed for all events at terminal stations N_{terminal} and the transition activities A_R . Other aspects of the PESP such as passenger connections can still be modeled with the FA approach. Since the periodic rollout is not performed for trip-activities of a service line L_i , which also happen with frequency f_i , the frequency of trip activities is integrated in the objective function by introducing f_i as a coefficient for their tensions.

5.2 Additional constraints for the RS-PESP with frequencies

Typically, cutting planes are used to exclude fractional solutions that are not in the convex hull of integer solutions. However, in the case of RS-PESP, the matching constraints (4.3e)–(4.3f) exactly describe the convex hull of all integral solutions, implying that no fractional matching solution can be cut off, without also excluding an integral matching solution.

Thus, it is our goal to show for some of the feasible matchings that there is always another matching, which is at least as good. Then we can safely exclude these matchings from the feasible region. In the following section, we develop additional constraints in order to limit the increase of complexity in the matching after the periodic rollout described in Section 5.1.

First departure of other service

The general idea is as follows: When deciding to match an arrival of a service line with a departure of another service line, it will be assumed that

the arrival is matched with the next occurrence of the departing line and not any later occurrence. More formally we have the following situation: Let $l_1 \in L$ and $l_2 \in L$ be two service lines sharing a terminal station $s \in S_{\text{terminal}}$, where l_1 arrives in s and l_2 departs from s . The frequencies of l_1 and l_2 are $f_1 > 1$ and $f_2 > 1$ respectively in a global period T . It follows that we have the events $A = \{\text{arr}_{l_1}^0, \dots, \text{arr}_{l_1}^{f_1-1}\}$ and $D = \{\text{dep}_{l_2}^0, \dots, \text{dep}_{l_2}^{f_2-1}\}$, where the events A and D are separated by $\frac{T}{f_1}$ and $\frac{T}{f_2}$ respectively. Furthermore, let $x \in \mathbb{R}^{A_R}$ denote the tensions of the transition activities.

Proposition 5.1 *Let $a \in A$ be an arbitrary but fixed arrival and let (a, d^*) for some $d^* \in D$ be the fastest transition from a to D . Furthermore let $m \in \{0, 1\}^{A_R}$ be a matching with $m_{(a,d)} = 1$ for some $d \in D$, where $d \neq d^*$, which implies $m_{(a,d^*)} = 0$. Then there exists a matching $m' \in \{0, 1\}^{A_R}$ such that $m'_{(a,d)} = 0$ and $m'_{(a,d^*)} = 1$ and $x^T m' \leq x^T m$.*

Proof First, observe that the tension of the fastest transition between a and D can be expressed without knowing the event-times and without knowing which transition satisfies the minimal transition time, by exploiting the local periodicity $\frac{T}{f_2}$ of the events in D . Since the departures in D happen every $\frac{T}{f_2}$ minutes, the shortest tension between a and some $d \in D$ can take at most $s + \frac{T}{f_2} - 1$, where s is the lower bound on the tension. It follows that the value of the shortest tension can be obtained with the following constraint:

$$s \leq \pi_d - \pi_a + z \frac{T}{f_2} \leq s + \frac{T}{f_2} - 1 \quad (5.3)$$

Let $d^* \in D$ be the departure for which the tension x_{ad^*} achieves the minimal transition time, i.e.

$$s \leq x_{ad^*} \leq s + \frac{T}{f_2} - 1 \quad (5.4)$$

as in constraint (5.3).

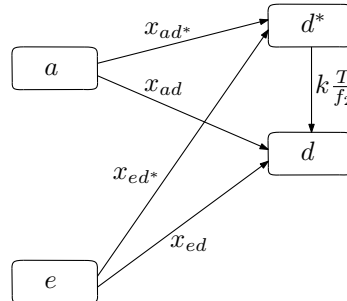


Figure 5.2: Matching a with a departure d , other than d^* is at least as expensive as matching a with d^* .

Now consider the matching $m \in \{0, 1\}^{A_R}$ with $m_{(a,d)} = 1$ for some $d \in D$ and assume that $m_{ad^*} = 0$, i.e. $d \neq d^*$. Then we have that $m_{(e,d^*)} = 1$ for some other event e , which corresponds to an arrival in s . (Note that e may be the arrival of a line other than l_1 , i.e. $e \notin A$ is possible.) See Figure 5.1 for an illustration of the events and tensions. Since d^* and d are both departures of l_2 , the events are separated by $k \frac{T}{f_2}$ for some $k \in \{1, \dots, f_2 - 1\}$. $k \frac{T}{f_2}$ is a fixed duration, which corresponds to the tension x_{d^*d} . It follows that $0 \leq x_{d^*d} \leq T$, which enables us to apply cycle periodicity (2.6) and inequalities (2.8).

Consider the two cycles that are depicted in Figure 5.1, where

$$C_1 = a \rightarrow d \rightarrow d^* \quad \text{and} \quad C_2 = e \rightarrow d^* \rightarrow d .$$

Cycle periodicity (2.6) yields

$$q_1 T = x_{ad} - k \frac{T}{f_2} - x_{ad^*} \quad (5.5a)$$

$$q_2 T = x_{ed^*} + k \frac{T}{f_2} - x_{ed} , \quad (5.5b)$$

where $q_1 \in \mathbb{Z}$ and $q_2 \in \mathbb{Z}$ are cycle-integers for C_1 and C_2 respectively. The cutting planes for cycle-integers (2.8) imply that q_1 and q_2 are binary:

$$\begin{aligned} \left\lceil \frac{s+0-(s+T-1)}{T} \right\rceil &= \left\lceil \frac{-T+1}{T} \right\rceil = 0 \\ &\leq q_i \leq \\ \left\lfloor \frac{s+T-1+T-s}{T} \right\rfloor &= \left\lfloor \frac{2T-1}{T} \right\rfloor = 1 , \end{aligned}$$

for $i \in \{1, 2\}$. Adding equations (5.5) gives:

$$q_1 T + q_2 T = x_{ad} - k \frac{T}{f_2} - x_{ad^*} + x_{ed^*} + k \frac{T}{f_2} - x_{ed} \quad (5.6a)$$

$$x_{ad^*} + x_{ed} + (q_1 + q_2)T = x_{ad} + x_{ed^*} \quad (5.6b)$$

$$x_{ad^*} + x_{ed} \leq x_{ad} + x_{ed^*} . \quad (5.6c)$$

The last inequality holds, since $q_1 + q_2 \in \{0, 1, 2\}$. It follows that the matching m with $m_{ad} = m_{ed^*} = 1$ and $m_{ad^*} = m_{ed} = 0$ is at least as expensive as a matching m' , where m and m' are equal except $m'_{ad} = m'_{ed^*} = 0$ and $m'_{ad^*} = m'_{ed} = 1$. \square

Proposition 5.1 can be used to formulate constraints, rendering infeasible the matchings, which do not fulfill the proposition. However, since it is not known apriori which event in D will be the one with the shortest transition time, the desired constraint can only be achieved by limiting the cost contributed by the matching variables $M_{aD} = \{m_{ad} \mid d \in D\}$. Let c_{aD} denote

the shortest transition time given by (5.4). Then the total cost contribution of the matching variables M_{aD} , as well as the individual cost contribution of each of those variables can be bounded by c_{aD} , as expressed by the following constraints.

$$\begin{aligned} \sum_{d \in D} m_{ad} \cdot x_{ad} &\leq c_{aD} \\ m_{ad} \cdot x_{ad} &\leq c_{aD} \quad \forall d \in D \end{aligned}$$

Using the linearization technique from Section 4.5.2, the following equivalent constraints are obtained:

$$\sum_{d \in D} w_{ad} \leq c_{aD} \tag{5.7a}$$

$$w_{ad} \leq c_{aD} \quad \forall d \in D. \tag{5.7b}$$

Post-processing

6.1 Extracting RS cycles from a solution

After a feasible solution for the RS-PESP, given by a timetable and a perfect matching in the transition graph $R = (N_{\text{terminal}}, A_R)$ has been found, a more detailed version of the rolling stock schedule can be computed as post-processing step. The rolling stock cycles, i.e. the itineraries for the rolling stock compositions are obtained by traversing each cycle along RS transitions and trip arcs. Listing 6.1 describes the procedure in pseudo-code.

```

1  procedure extractRollingStockCycles( $R, m$ )
2       $M \leftarrow \{a \in A_R \mid m_a = 1\}$ 
3       $\mathcal{C} \leftarrow \emptyset$ 
4      while ( $M \neq \emptyset$ )
5           $a_{\text{init}} \leftarrow$  arbitrary from  $M$ 
6           $C \leftarrow [ ]$ 
7           $a_c \leftarrow a_{\text{init}}$ 
8          do
9               $l \leftarrow$  departing line from  $a_c$ 
10              $a_T \leftarrow$  trip activities of  $l$ 
11              $C \leftarrow C + [a_c, a_T]$ 
12              $M \leftarrow M \setminus \{a_c\}$ 
13              $\text{arr}_l \leftarrow$  terminal arrival event of  $l$ 
14              $a_c \leftarrow$  outgoing from  $\text{arr}_l$  s.t.  $m_a = 1$ 
15             while ( $a_c \neq a_{\text{init}}$ )
16                  $C \leftarrow C \cup \{C\}$ 
17         end
18     return  $\mathcal{C}$ 

```

Listing 6.1: Pseudocode for extracting the rolling stock cycles from a given matching

The sum of all tensions along a cycle, which are given by the timetable, corresponds to the number of rolling stock compositions that are required to operate the cycle.

6.2 Maximizing the number of RS cycles

Once a timetable requiring few rolling stock compositions has been found, and the rolling stock cycles have been extracted (see Section 6.1), it may become evident that the solution uses only few and thus relatively long cycles. Typically, solutions with more and shorter rolling stock cycles are preferred, because they are more robust against delay propagation. The question arises as to whether it is possible to operate the timetable with more rolling stock cycles, still using the same number of rolling stock compositions.

A facet of the perfect matching polytope Since the number of rolling stock cycles is determined by a perfect matching in the transition graph, which represents a rolling stock schedule, the goal is to maximize the number of cycles by optimizing over the perfect matchings. However, we want to ensure that any perfect matching considered in the post-processing does not increase the number of required rolling stock compositions by adding an additional constraint.

For the rest of this section let $x \in \mathbb{R}^{A_R}$ denote the tensions of the transition activities for a fixed timetable and let $c_{\mathcal{R}} \in \mathbb{R}$ be the minimum cost of a perfect matching w.r.t. x . Furthermore, let \mathcal{P} denote the perfect matching polytope for $R = (N_{\text{terminal}}, A_R)$ and let $\mathcal{P}_{\min} \subseteq \mathcal{P}$ be the matching polytope with an additional constraint requiring optimality to the timetable, i.e. $\mathcal{P}_{\min} = \{m \in \mathcal{P} \mid x^T m = c_{\mathcal{R}}\}$. Since \mathcal{P} is given by equations (3.4b)–(3.4d) we have the following description of \mathcal{P}_{\min} :

$$\mathcal{P}_{\min} = \left\{ \begin{array}{ll} \sum_{a \in A_R} x_a m_a = c_{\mathcal{R}} & \forall a \in A_R \\ \sum_{a \in \delta^+(e)} m_a = 1 & \forall e \in N_{\text{arr}} \\ \sum_{a \in \delta^-(e)} m_a = 1 & \forall e \in N_{\text{dep}} \\ m_a \geq 0 & \forall a \in A_R \end{array} \right\}. \quad (6.1)$$

We show that despite adding a constraint to the perfect matching polytope \mathcal{P}_{\min} is still an integral polytope.

Proposition 6.1 *Let \mathcal{PM} denote the set of all perfect matchings in R and let $\mathcal{PM}_{\min} \subseteq \mathcal{PM}$ denote the set of min-cost perfect matchings with respect to the cost function x and characteristic vectors $\chi^M \in \{0, 1\}^{A_R}$. Then*

$$\mathcal{P}_{\min} = \text{conv}(\{\chi^M \mid M \in \mathcal{PM}_{\min}\}).$$

Proof Direction “ \supseteq ” of the claim holds, because every perfect matching fulfills the perfect matching constraints (3.4b)–(3.4d) and constraint $x^T m = c_{\mathcal{R}}$

holds for min-cost perfect matchings by definition.

To show “ \subseteq ”, assume towards contradiction that there exists an $m \in \mathcal{P}$, such that $m \notin \text{conv}(\{\chi^M \mid M \in \mathcal{PM}_{\min}\})$. Since m is in the perfect matching polytope \mathcal{P} it can be represented as convex combination of perfect matchings:

$$m = \sum_{i=1}^n \lambda_i \chi^{M_i}, \text{ where } \sum_{i=1}^n \lambda_i = 1 \quad (6.2)$$

and $M_i \in \mathcal{PM}$ for $i = 1, \dots, n$. Given the assumption of $m \notin \text{conv}(\{\chi^M \mid M \in \mathcal{PM}_{\min}\})$ one of the perfect matchings of the convex combination has to be non-optimal, i.e.

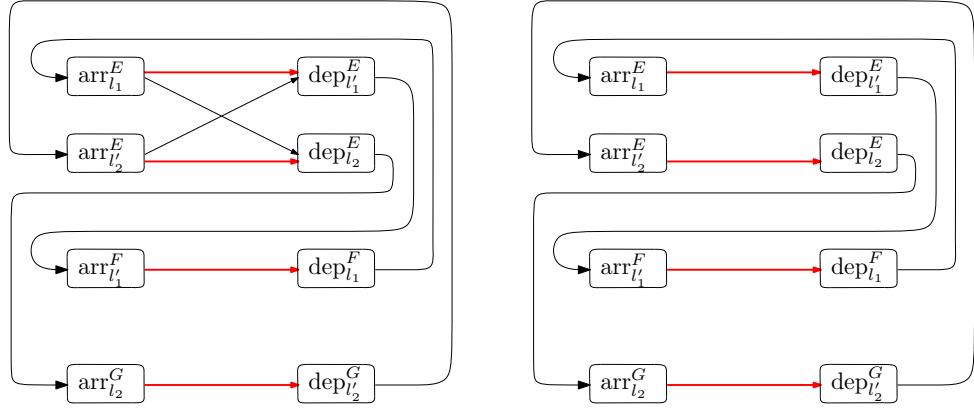
$$\exists i \in \{1, \dots, n\} \text{ s.t. } x^T \chi^{M_i} > c_R.$$

But then in order for $x^T m = c_R$ to hold there also has to exist $j \in \{1, \dots, n\}$ such that $x^T \chi^{M_j} < c_R$, which is a contradiction according to the definition of c_R . \square

In fact, since the inequality $x^T m \leq c_R$ only holds for min-cost perfect matchings and it has to be fulfilled also by a vertex, it defines a face of the perfect matching polytope, i.e. \mathcal{P}_{\min} is a face of \mathcal{P} .

The number of cycles as optimization problem A given matching induces a set of rolling stock cycles that can be counted e.g. with the algorithm in Listing 6.1. Our goal is to optimize the number of cycles over the polytope \mathcal{P}_{\min} , so we have to count the number of cycles using an optimization problem. For this chapter we redefine the set of trip activities A_T by merging each path of alternating trips and dwells that corresponds to the itinerary of a service line into one activity. We call this activity a service activity. Then the two events of each such activity are an arrival and a departure at a terminal station, i.e. $e_i \in N_{\text{dep}}$ and $e_j \in N_{\text{arr}}$ for all $(e_i, e_j) \in A_T$. Augmenting the transition graph R with the activities A_T yields a graph $R_C = (N_{\text{terminal}}, A_R \cup A_T)$, which will serve as input for counting the number of rolling stock cycles. An example is given in Figure 6.1(a).

Let $M \subseteq A_R$ denote a perfect matching in $R = (N_{\text{terminal}}, A_R)$. Note that the graph $(N_{\text{terminal}}, A_T \cup M)$ is a collection of cycles since every event is incident to exactly one service activity in A_T and exactly one transition in M , e.g. as in Figure 6.1(b). We will count the number of cycles in $(N_{\text{terminal}}, A_T \cup M)$ using a multi-commodity circulation. Since every arrival has to be in exactly one cycle we have one commodity s for every $s \in N_{\text{arr}}$. Then a multi-commodity



(a) $R_C = (N_{\text{terminal}}, A_R \cup A_T)$ and a perfect matching $M \subseteq A_R$ indicated by the red transitions.

(b) The graph $(N_{\text{terminal}}, A_T \cup M)$ consists of two cycles.

Figure 6.1: An example of $R_C = (N_{\text{terminal}}, A_R \cup A_T)$ for four services. The arcs A_T going from a departure to an arrival represent the itineraries of the services. The other arcs A_R going from arrivals to departures are the transition activities, as in the transition graph in Figure 3.1.

circulation is given by the following constraints:

$$\sum_{a \in \delta^-(e)} f_s(a) = \sum_{a \in \delta^+(e)} f_s(a) \quad \forall e \in N_{\text{terminal}} \quad \forall s \in N_{\text{arr}} \quad (6.3a)$$

$$\sum_{a \in \delta^-(s)} f_s(a) = y_s \quad \forall s \in N_{\text{arr}} \quad (6.3b)$$

$$\sum_{s \in N_{\text{arr}}} f_s(a) = 1 \quad \forall a \in A_T \quad (6.3c)$$

$$\sum_{s \in N_{\text{arr}}} f_s(a) = m_a \quad \forall a \in A_R \quad (6.3d)$$

$$y_s \in \{0, 1\} \quad \forall s \in N_{\text{arr}} \quad (6.3e)$$

$$f_s \in \{0, 1\}^{A_T \cup A_R} \quad \forall s \in N_{\text{arr}}. \quad (6.3f)$$

The variables for a circulation of commodity $s \in N_{\text{arr}}$ are denoted by $f_s \in \{0, 1\}^{A_T \cup A_R}$. Constraints (6.3a) ensure conservation of each circulation at each event. For each commodity an indicator variable $y_s \in \{0, 1\}$ determines the value of the respective circulation in constraints (6.3b). Furthermore constraints (6.3c)–(6.3d) impose a demand on the circulations aggregated over all commodities. This demand is one for every service arc and given by the matching value for the transition arcs. It follows that for a matching M , only the arcs in M require a circulation of value one and all other transitions must have a circulation of value zero, so the multi-commodity circulation in fact circulates on the network $(N_{\text{terminal}}, A_T \cup M)$. Note that constraint (6.3c)

is implied by (6.3d), circulation conservation (6.3a) and the fact that the sum of matching values incident to one event is one, i.e. $\sum_{a \in \delta^+(e)} m_a = 1$ for all $\forall e \in N_{\text{arr}}$. Since for every cycle in $(N_{\text{terminal}}, A_T \cup M)$ the circulation may only consist of exactly one commodity, the number of cycles is given by the sum of indicators of the commodities $\sum_{s \in N_{\text{arr}}} y_s$.

In conclusion, we have the following exact formulation for maximizing the number of cycles.

$$\text{MAX} \sum_{s \in N_{\text{arr}}} y_s \quad (6.4a)$$

s.t.

$$\sum_{a \in A_R} x_a m_a = c_R \quad (6.4b)$$

$$\sum_{a \in \delta^+(e)} m_a = 1 \quad \forall e \in N_{\text{arr}} \quad (6.4c)$$

$$\sum_{a \in \delta^-(e)} m_a = 1 \quad \forall e \in N_{\text{dep}} \quad (6.4d)$$

$$\sum_{a \in \delta^-(e)} f_s(a) = \sum_{a \in \delta^+(e)} f_s(a) \quad \forall e \in N_{\text{terminal}} \quad \forall s \in N_{\text{arr}} \quad (6.4e)$$

$$\sum_{a \in \delta^-(s)} f_s(a) = y_s \quad \forall s \in N_{\text{arr}} \quad (6.4f)$$

$$\sum_{s \in N_{\text{arr}}} f_s(a) = m_a \quad \forall a \in A_R \quad (6.4g)$$

$$m_a \in \{0, 1\} \quad \forall a \in A_R \quad (6.4h)$$

$$y_s \in \{0, 1\} \quad \forall s \in N_{\text{arr}} \quad (6.4i)$$

$$f_s \in \{0, 1\}^{A_T \cup A_R} \quad \forall s \in N_{\text{arr}} \quad (6.4j)$$

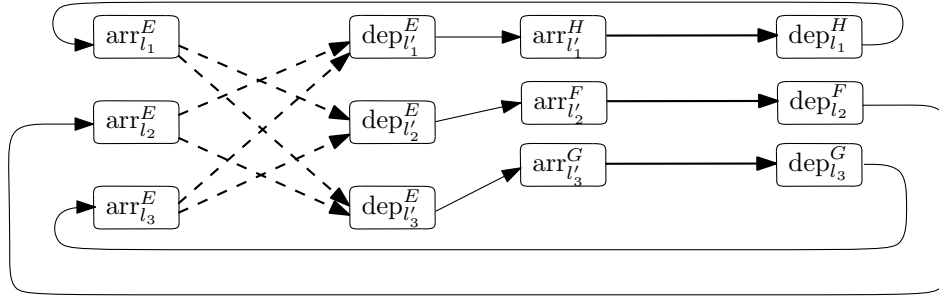
Proposition 6.2 *The domain of the indicators (6.4i) and flow variables (6.4j) can be replaced with*

$$y_s \geq 0 \quad \forall s \in N_{\text{arr}} \quad (6.5a)$$

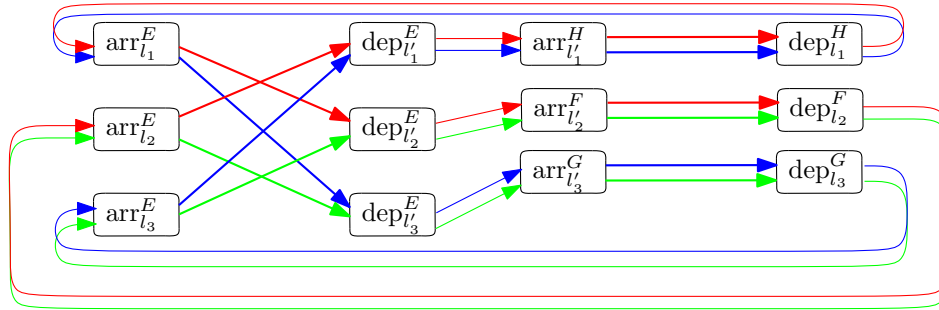
$$f_s \geq \mathbb{R}_{\geq 0}^{A_T \cup A_R} \quad \forall s \in N_{\text{arr}} \quad (6.5b)$$

Proof Let $M \subseteq A_R$ be an arbitrary but fixed perfect matching in $R = (N_{\text{terminal}}, A_R)$. Consider optimization problem (6.4) with m_a for $a \in A_R$ fixed according to M . Then the maximum value for $\sum_{s \in N_{\text{arr}}} y_s$ is the number of cycles in $(N_{\text{terminal}}, A_T \cup M)$, which is integral. It can be achieved by setting $y_s = 1$ for exactly one arrival $s \in N_{\text{arr}}$ in every cycle. It follows that for every fractional solution there exists an integral solution, which is at least as good. \square

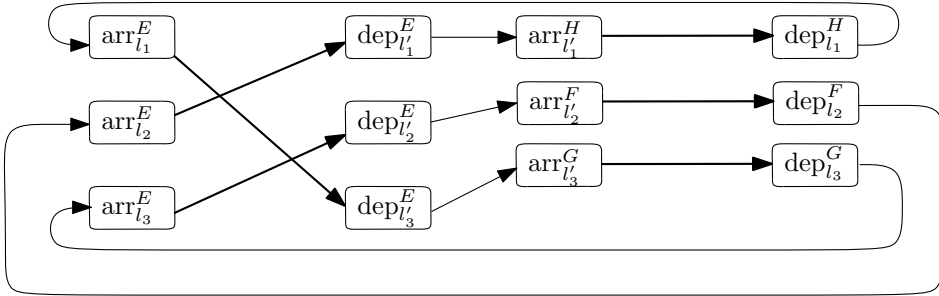
As the example in Figure 6.2 shows, the domain of the matching variables (6.4h) cannot be relaxed. However, Proposition 6.1 suggests that the formulation has a good LP-relaxation.



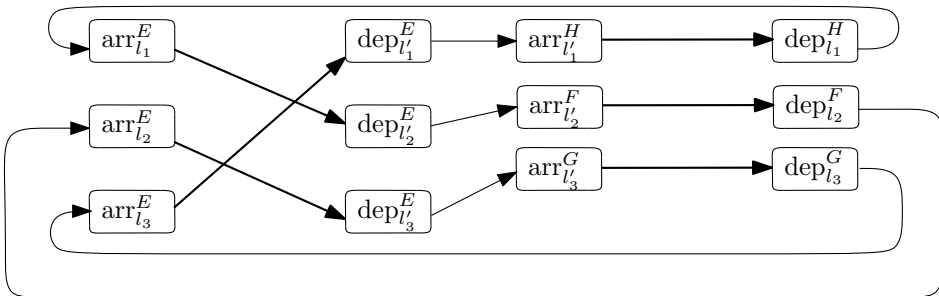
(a) Let m be a perfect matching with a fractional value of 0.5 for the dashed matching arcs in station E and a value of 1 for the solid matching arcs in stations F, G and H .



(b) Then for m , maximizing $\sum_{s \in N_{\text{arr}}} y_s$ yields a circulation in which three commodities have a value 0.5, implying an objective value of 1.5. Since the only way of representing m as a convex combination of integral matchings is $m = \frac{1}{2}(m_1 + m_2)$, where m_1 and m_2 are depicted in Figure 6.2(c) and Figure 6.2(d) respectively, and for both the maximum objective value is 1, the example shows the existence of a fractional feasible solution that cannot be expressed as convex combination of integral feasible solutions. Thus, formulation (6.4) has a fractional vertex in this example.



(c) The integral matching m_1 .



(d) The integral matching m_2 .

Figure 6.2: An example showing that formulation (6.4) is not integral for relaxed matching variables.

In fact, the problem of finding the perfect matching that maximizes the number of cycles is NP-complete.

Theorem 6.3 *Let $R = (N_{\text{terminal}}, A_R)$ be a transition graph and A_T a set of trip arcs. Then finding a perfect matching in R that maximizes the number of cycles in $(N_{\text{terminal}}, A_R \cup A_T)$ is NP-complete.*

Proof We show the result by polynomial reduction from partitioning a 3-partite graph into triangles. Let *partition into triangles* (PIT) denote the following problem:

Instance PIT: Let $G = (X \dot{\cup} Y \dot{\cup} Z, E)$ be a 3-partite graph, where $|X| = |Y| = |Z| = k$. Let $V = X \dot{\cup} Y \dot{\cup} Z$ denote the set of vertices of G .

Question: Can G be partitioned into triangles, i.e. is there a set of disjoint triangles \mathcal{T} , where $|\mathcal{T}| = k$ and for all $T \in \mathcal{T}$ we have $T \subseteq V$ and $|T| = 3$, such that $\dot{\bigcup}_{T \in \mathcal{T}} T = V$ and for all $T = \{a, b, c\} \in \mathcal{T}$ we have that $\{\{a, b\}, \{a, c\}, \{b, c\}\} \subseteq E$?

The problem PIT was shown to be NP-complete even for 6-regular 3-partite graphs in [CKW15]. We give the following polynomial reduction from an instance of PIT, given by $G = (X \dot{\cup} Y \dot{\cup} Z, E)$, to a transition graph with service arcs $R_C = (N_{\text{terminal}}, A_T \cup A_R)$:

We copy every vertex of G . Let the copies be denoted by $V' = \{v' \mid v \in V\}$. Semantically this means that every vertex is split into a departure and arrival, i.e. $N_{\text{terminal}} = V \cup V'$. Every copy is connected to its original by an arc (v, v') . These arcs can be understood as the service arcs, so we define the set of service arcs $A_T = \{(v, v') \mid v \in V\}$. The transition arcs are derived from the edges E of G in the following way:

$$\begin{aligned} A_R = & \{(x', y) \mid \{x, y\} \in E, x \in X, y \in Y\} \\ & \cup \{(x', z) \mid \{x, z\} \in E, x \in X, z \in Z\} \\ & \cup \{(y', z) \mid \{y, z\} \in E, y \in Y, z \in Z\} \end{aligned}$$

Note that trip arcs (resp. transitions) always go from V to V' (resp. from V' to V), so V and V' correspond to the departures and arrivals respectively. In this way we have obtained a transition graph with service arcs $R_C = (N_{\text{terminal}}, A_T \cup A_R)$. An example for the reduction is given in Figure 6.3

We complete the reduction by proving the following claim:

Claim: $G = (X \dot{\cup} Y \dot{\cup} Z, E)$ is a YES-instance for PIT, if and only if the optimal value of formulation (6.4) for $R_C = (N_{\text{terminal}}, A_T \cup A_R)$ is k , where $k = |X| = |Y| = |Z|$.

Proof of claim: Clearly, G cannot be partitioned into triangles, if there is no perfect matching in $(N_{\text{terminal}}, A_R)$.

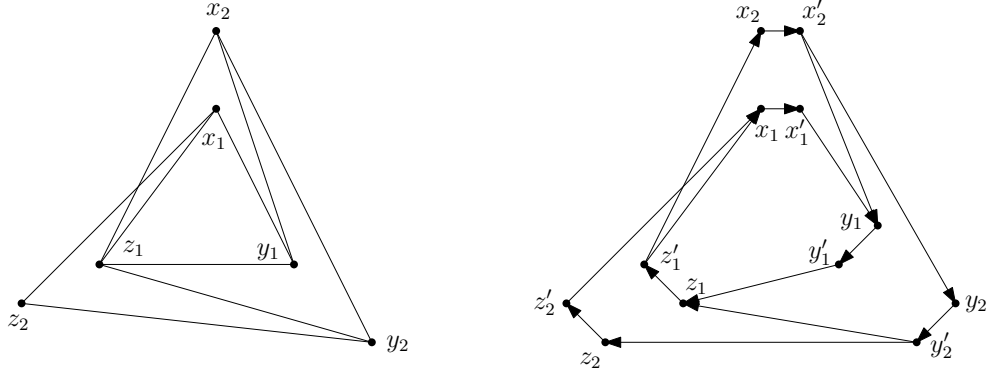


Figure 6.3: Polynomial reduction from $G = (X \dot{\cup} Y \dot{\cup} Z, E)$ to $R_C = (N_{\text{terminal}}, A_T \cup A_R)$

Let M be a perfect matching in the transition graph $(N_{\text{terminal}}, A_R)$, then every cycle in $(N_{\text{terminal}}, A_T \cup M)$, which is a collection of cycles as mentioned previously, corresponds to a cycle in G that can be obtained by contracting the trip arcs. A cycle in $(N_{\text{terminal}}, A_T \cup M)$ has to consist of at least 6 vertices, and it corresponds to a triangle in G if and only if it consists of exactly 6 vertices. It follows that G is partitioned into triangles if and only if every cycle in $(N_{\text{terminal}}, A_T \cup M)$ has exactly 6 vertices. Since $|N_{\text{terminal}}| = 6k$ this is only the case if $(N_{\text{terminal}}, A_T \cup M)$ is a collection of k cycles. Note that $(N_{\text{terminal}}, A_T \cup M)$ can consist of at most k cycles, since every cycle has to consist of at least 6 vertices, which establishes the claim.

The transition graph with service activities $R_C = (N_{\text{terminal}}, A_T \cup A_R)$ can be constructed, using $G = (X \dot{\cup} Y \dot{\cup} Z, E)$ as input, in polynomial time according to its definition. The decision problem whether an instance for formulation (6.4) has a matching that induces at least j cycles is in NP, since for a given matching the number of cycles can be counted in polynomial time. It follows that also the optimization problem in equations (6.4) is in NP. Together with the reduction from PRT this shows that optimization problem (6.4) is NP-complete. \square

A trade-off The question arises as to whether a rolling stock schedule with even more rolling stock cycles can be achieved by utilizing more rolling stock compositions. Let the number of additional RS compositions be denoted by $n_{\text{add}\mathcal{R}}$. Then replacing constraint (6.4b) by

$$\sum_{a \in A_R} x_a m_a \leq c_R + n_{\text{add}\mathcal{R}} T \quad (6.6)$$

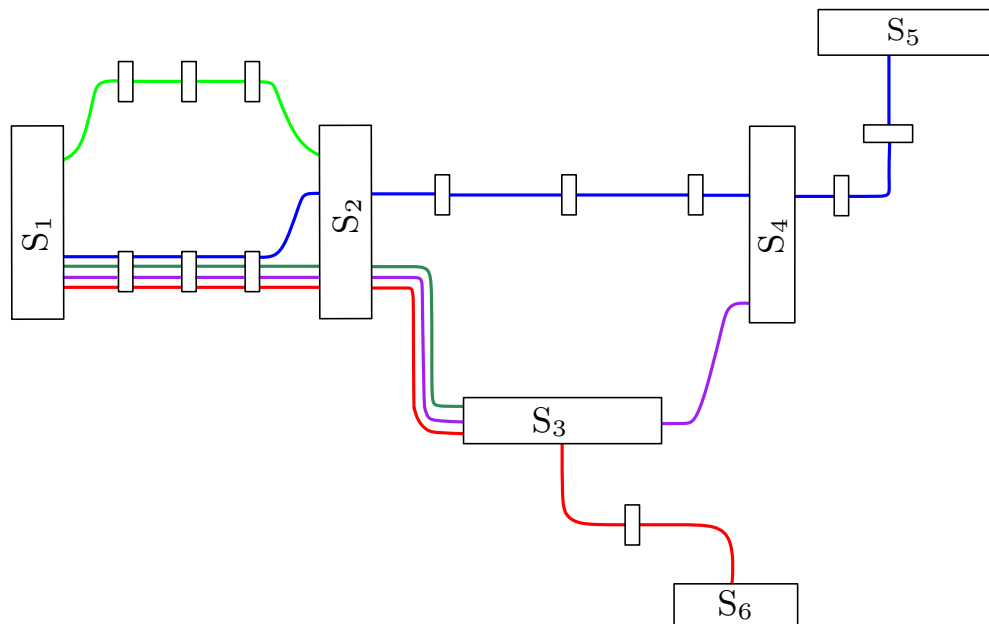
yields a model that maximizes the number of rolling stock cycles for a given timetable and allows for $n_{\text{add}\mathcal{R}}$ rolling stock compositions in addition to the minimum number of rolling stock compositions required by the timetable.

Computational Studies

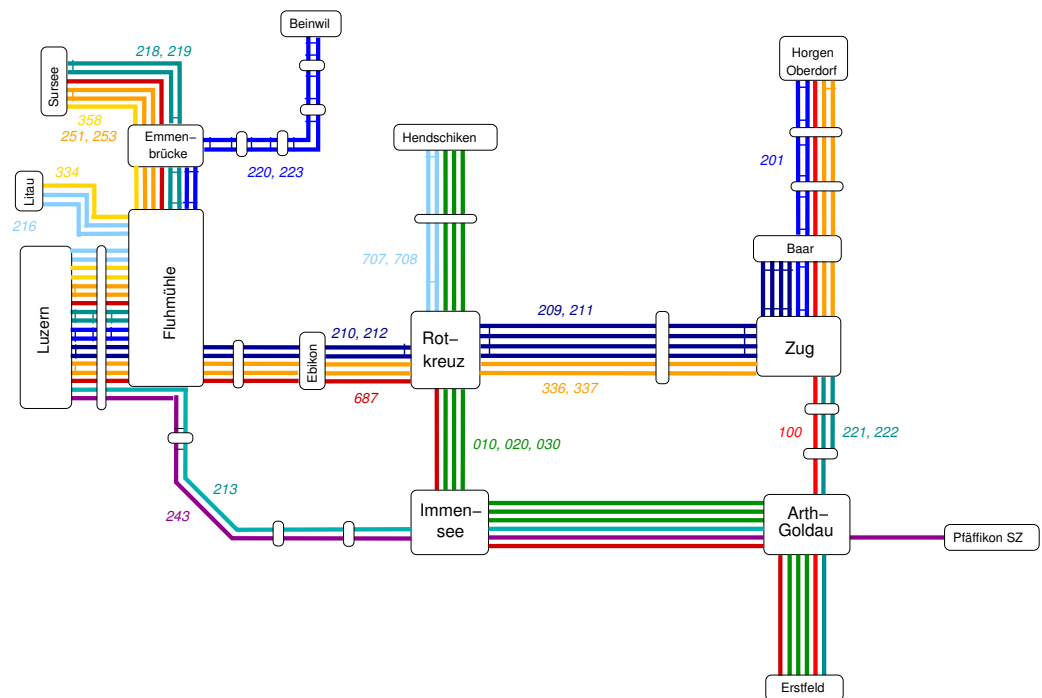
In this chapter we present the results of computational studies in order to evaluate the performance in practice of the models and methods proposed. Two test instances consisting of real world data were used for this thesis. First, the morning rush-hour of a small timetabling instance¹, where the input consists of 5 service lines, which are served in both directions. A special property is that one of the terminal stations is one of the two terminal stations for every service line, so the terminal stations of the services have a star topology. (However, the services still intersect and a terminal station of one service can also be an intermediate station of another service.) The second instance was the regional railway network of Luzern with 42 service lines, exhibiting a more general topology. Figure 7.1 shows the line-plans of the two railway networks. For both inputs, it was necessary to use the model for service lines with different frequencies, described in Section 5.1.

The RS-PESP, the linearization of the RS-PESP and the iterative method for solving it, presented in Chapter 4, have been implemented on the basis of a code by IBM, which among other functionality for railway scheduling implements the PESP. In order to improve the performance in solving the linearized RS-PESP the effectiveness of the alternative objective formulation from Section 4.6 and the additional constraints given in Section 5.2 have been evaluated. Furthermore, the post-processing steps introduced in Chapter 6 were implemented. Algorithm 6.1 was useful to present a solution, while the implementation of model (6.4) was used in computational experiments to maximize the number of rolling stock cycles.

¹In this version of the thesis, the information for this instance is pseudonymized due to confidentiality. We will call the instance I.SMALL.



(a) Line-plan of the small instance LSMALL. Only the names of the terminal stations are shown. The ten service lines are: $S_1 \leftrightarrow S_2$, $S_1 \leftrightarrow S_5$, $S_1 \leftrightarrow S_4$, $S_1 \leftrightarrow S_3$, $S_1 \leftrightarrow S_6$. $T = 90$



(b) The regional network in the area of Luzern, Zug and Arth-Goldau. $T = 60$
Source: Sabrina Herrigel (ETH Zürich)

Figure 7.1: Illustrations of the line-plans of the two real-world test instances.

All implementations were done in C++, and, for the computational experiments the MIP-solvers of IBM ILOG CPLEX² Optimizer versions 12.6.1, 12.6.2 and 12.6.3 were used. In addition, a brief comparison with the MIP-solver GUROBI version 6.5.1 has been made for the linearized RS-PESP. We have split the report on each computational experiment into four sections: goal of the experiment, setup, i.e. how it was implemented, results and the conclusion of the experiment.

7.1 Tractability of the RS-PESP

Goal. The RS-PESP (see Figure 4.2) is stated to be intractable in practice in [LP02]. In light of progress in the development MIP-solvers, we attempt to solve the RS-PESP for the two test-instances and investigate whether relaxing the matching variables as described in Section 4.3.1 is beneficial.

Setup. Based on an implementation of the PESP, the RS-PESP was implemented in C++ and tested on the two instances LSMALL and LUZERN. Since the service lines of the input data are operated with different frequencies, the approach for modeling frequencies described in Section 5.1 has been used.

Result. The RS-PESP could not be solved using CPLEX 12.6.1 and 12.6.2. The MIP-solvers of these CPLEX versions could not be invoked, because the matrix representing the quadratic term in the objective of the RS-PESP is not positive semi-definite. Since CPLEX version 12.6.3 the MIP-solver also supports non-convex MIQPs. Therefore, the RS-PESP could be tested with this solver. While a few small test-instances could be solved, no feasible solution was obtained for LSMALL or LUZERN. The relaxation of the matching variables did not show any improvements.

Conclusion. The computational experiment confirms that the RS-PESP is still intractable in practice. It can be shown in the following way that the quadratic term in the objective is not positive semi-definite:

Consider the submatrix for one of the quadratic terms $x_a \cdot m_a$, where x_a is given as linear expression of event-times and a shift-integer in the implementation used for the PESP, i.e. $x = \pi_j - \pi_i + zT$. Given a non-zero vector $v = (0, b, 0, 0)^T$ and Q_a , the matrix for the quadratic term $x_a \cdot m_a$ with the variables π_i, π_j, z and m , the term $v^T Q_a v$ is not necessarily non-negative, due

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to Q_a having only zeroes on the diagonal and a negative entry.

$$v^T Q_a v = (0 \ b \ 0 \ 0) \cdot \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & T \\ 1 & -1 & T & 0 \end{bmatrix} \cdot \begin{pmatrix} 0 \\ b \\ 0 \\ 0 \end{pmatrix} = -b^2.$$

The fact that relaxing the matching variables showed no benefit indicates that the main difficulty of the model lies in the quadratic objective function and not the addition of the binary matching variables.

7.2 Performance evaluation of the linearized RS-PESP

Goal. In this experiment we investigate whether the linearized formulation of the RS-PESP is suitable to generate a periodic timetable, which requires a minimum number of rolling stock compositions in practice.

Setup. The linearization of the RS-PESP (Figure 4.8) could be implemented by extending the implementation of RS-PESP. Then the performance of the linearized RS-PESP was tested on the LSMALL and LUZERN instances.

Result. For both instances an optimal solution could not be found within 8 hours of computation time on a machine with 32 Intel Xeon 2.70 GHz CPUs and 256 GB RAM, using CPLEX version 12.6.0. It was observed that the optimality gap hardly changed during the optimization after the best integral solution was found. All further experiments were done on a machine with 4 Intel Core i5 2.60 GHz CPUs and 8 GB RAM, with a timelimit for CPLEX such that the previously seen best integral solution could be achieved with CPLEX version 12.6.1. Table 7.1 shows the performance of the linearized RS-PESP. In order to avoid too solver-specific results, we attempted to solve the linearized RS-PESP with GUROBI version 6.5.1. For LSMALL the same best integral solution as with CPLEX and a similar optimality gap of 6.36% could be obtained in 100 seconds on a machine with two Intel Xeon 3.00 GHz CPUs and 2 GB RAM. However, for LUZERN no feasible solution could be found using 3000 seconds computation time.

Furthermore, as additional experiment the services $S_1 \leftrightarrow S_2$ and $S_1 \leftrightarrow S_5$ were separated from the rest of the rolling stock circulation, applying the concept of Section 3.4. An objective value of 2610 could still be achieved for the constrained rolling stock circulation.

Conclusion. Although the optimal solution could not be obtained, the experiment shows that the linearized RS-PESP can provide valuable solutions for small and medium sized instances, since for many timetabling instances

7.3. Performance evaluation of the Iterative Method

	I.SMALL	LUZERN
t_{first}	8.2 s	300.3 s
t_{best}	1086.8 s	4893.1 s
first $n_{\mathcal{R}}T$	3510	3000
best $n_{\mathcal{R}}T$	2610	2400
first LB	2421.0	1850.0
best LB	2486.0	1938.5
first Gap	31.03%	35.38%
best Gap	4.75%	19.23%

Table 7.1: Computational results for the linearized RS-PESP. t_{first} (resp. t_{best}) denotes the time until the first (resp. best) feasible solution was found. These times correspond to the aggregated time used by all processes. The global CPLEX timelimit was set to 3000s. First $n_{\mathcal{R}}T$ and best $n_{\mathcal{R}}T$ denote the objective values of first and best feasible solution respectively. LB denotes the lower bound for the objective value.

it is already difficult to solve the PESP itself. Especially for the small instance I.SMALL the solution was near to optimal, because the lower bound only allows for a decrease of $T = 90$ in the objective value, which corresponds to one required rolling stock composition. This shows that the linearized RS-PESP, which is equivalent to the intractable RS-PESP according to Theorem 4.6, can be useful in practice. We conjecture that the weak LP-relaxation of the big- M constraints, mentioned in Section 4.5.2, plays a major role in causing the remaining optimality gap.

7.3 Performance evaluation of the Iterative Method

Goal. Furthermore, we compare the performance and quality of solutions of the linearized RS-PESP and the iterative method, which was proposed in Section 4.5.1.

Setup. For the two steps of the iterative method, first, in the RS-PESP the matching variables were replaced with fixed values for the timetabling step. Secondly, the rolling stock scheduling step was implemented as a perfect matching in the transition graph. The iterative method was invoked for the two case studies with the heuristic fractional matching value in (4.4) as initial matching m_{init} . In addition, in order to gain some understanding of the impact of the chosen m_{init} , random fractional matchings were sampled and used as initial matching.

Result. For the heuristic fractional value of m_{init} the best solution from the iterative method and the linearized RS-PESP for I.SMALL were equal, i.e. a value of 2610 was achieved. For the LUZERN instance, the iterative method found an integral solution with objective value 2340, which is better than the

value of the RS-PESP by one period of $T = 60$. However, this value could also be obtained with a performance improvement for the RS-PESP (see Section 7.4.2). When using a random initial matching the iterative method achieved a range of objective values for both test instances, where the best value in the range matched that of heuristic initialization. The results are summarized in Figure 7.2.

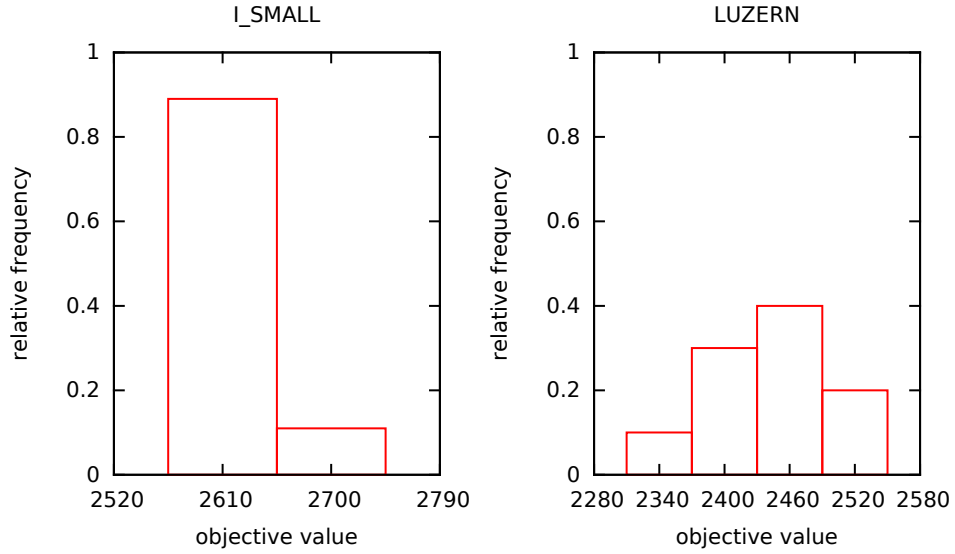


Figure 7.2: The figure shows the relative frequencies of the objective values achieved with the iterative method for random initialization. However, this only shows that the initialization matters, but not in which way since the fractional matchings were not chosen uniformly at random and the sample sizes were only 100 and 10 for I_SMALL and LUZERN respectively.

The two instances exhibit a significant difference in run-time. When solving the LUZERN instance, almost all time is used in the timetabling steps, i.e. solving the RS-PESP for a fixed matching, which can be done comparably fast for I_SMALL. We believe that this is due to the fact, that I_SMALL is a smaller instance than LUZERN, but other reasons such as the topology of the specific network may also play a role. For practical reasons, we even imposed a timelimit of 3000s for the first timetabling step for LUZERN. Subsequent timetabling steps for integral matching values were carried out without timelimit.

Conclusion. The iterative method proves to be a good heuristic in order to find feasible solutions of the RS-PESP. The bottleneck is the step of solving the RS-PESP for fixed matching values, which is an augmented PESP. It follows that the iterative method may benefit from future improvements in the PESP. In practical use, feasible solutions and lower bounds obtained

using the linearized RS-PESP can help to judge a feasible solution.

7.4 Practical performance improvements and heuristics

In order improve the performance of the MIP-solver for the RS-PESP, and also with the goal to narrow the optimality gap, we implemented the branch-and-cut rule from Section 4.6 and the additional constraints from Section 5.2.

7.4.1 Warm-start

Using the heuristical solution obtained with the iterative method (see Section 4.5.1) as a warm-start for the linearized RS-PESP (Figure 4.8) did not show any benefits. The MIP-solver started with the given solution and objective value, but the optimality gap could not be closed further than without warm-start. Furthermore, for both of the studied instances, introducing an upper bound of the best obtained integral objective value lowered by one period T , did not help CPLEX to find a better feasible solution or prove infeasibility.

7.4.2 Branch-and-cut rule

The branch-and-cut rule described in Section 4.6 was implemented and tested by using CPLEX Callbacks. The additional condition allowed to prune branches, but only when using the “traditional branch-and-cut mode” of CPLEX, which performed slower than the previously used “dynamic search” for branch-and-cut. Even in comparison to the “traditional branch-and-cut mode” without pruning, no improvements could be observed when pruning branches.

An improvement could be achieved by reformulating the objective function, such that CPLEX would recognize and exploit the special structure of the objective without the use of callbacks, while using the “dynamic search” mode. The new objective corresponds to the number of rolling stock compositions $n_{\mathcal{R}}$, modeled by an integer variable and lower bounded by the durations of all rolling stock cycles divided by T :

$$\text{MIN } n_{\mathcal{R}} \tag{7.1a}$$

$$\text{s.t. } n_{\mathcal{R}} \geq \frac{1}{T} \left(\sum_{a \in A_T} x_a + \sum_{a \in A_R} x_a m_a \right) \tag{7.1b}$$

$$n_{\mathcal{R}} \in \mathbb{Z} \tag{7.1c}$$

and subject to RS-PESP constraints.

As depicted in Figure 7.3, the best known integral solution for the LSMALL instance was found roughly five times faster with the reformulation. When

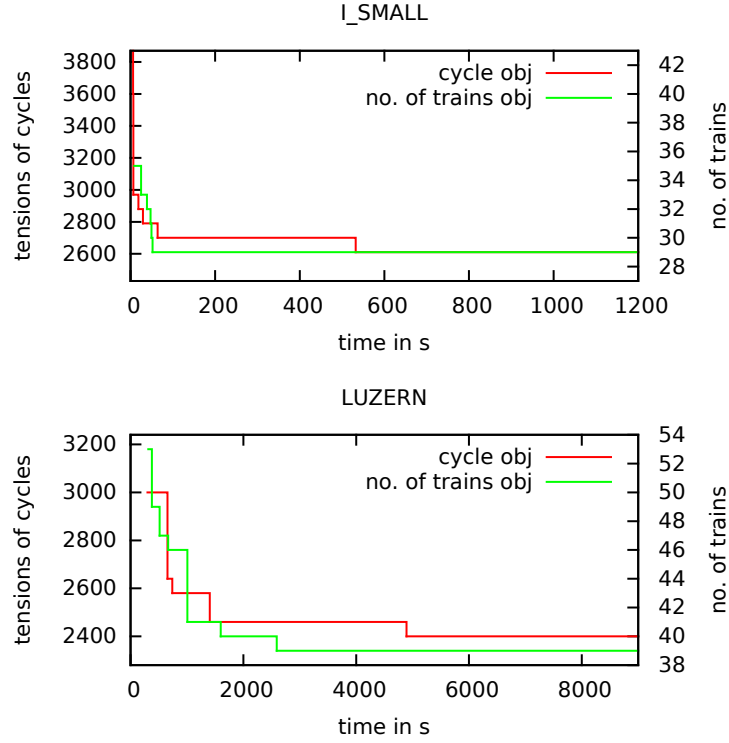


Figure 7.3: Comparison of different objective formulations with CPLEX 12.6.1.

solving the LUZERN instance, the previously best know solution was obtained about three times faster. In addition, a better integral solution was obtained in the same time limit when using the reformulated objective function.

In conclusion, the experiment shows that exploiting the properties of the objective function can be beneficial for some MIP-solvers, e.g. CPLEX 12.6.1. However, using CPLEX 12.6.2 with the default CPLEX settings for the same experiment did not yield the same improvements.

7.4.3 Additional constraints for frequencies

The additional constraints (5.7) were implemented, extending the linearization of the RS-PESP, with the purpose of excluding some non-optimal matchings and narrow the search-space of the MIP-solver.

In the computational experiment it became evident that the additional constraints, albeit excluding feasible solutions, do not improve the performance of the MIP-solver. With additional constraints less feasible solutions with a relatively high objective value were found, but the exact same best integral solutions were obtained as without additional constraints. For objective

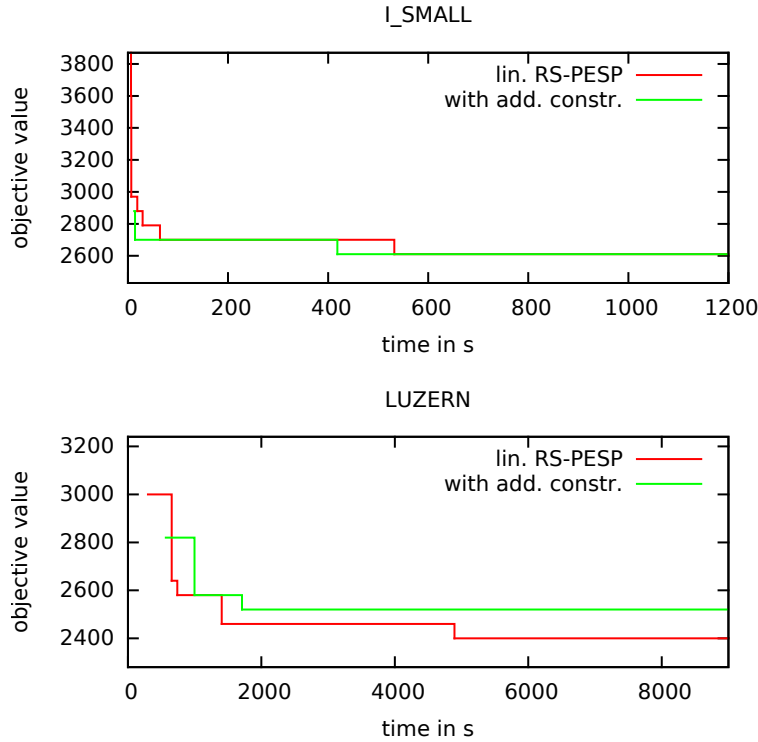


Figure 7.4: Decrease of objective value over time when solving the RS-PESP with CPLEX.

values close to the best known value adding constraints (5.7) even seems to complicate the optimization. While only being slightly slower for LUZERN, obtaining the best integral solution took more than three times longer for I_SMALL, when the additional constraints were used. This behavior can be observed in Figure 7.4.

7.5 Increasing the number of RS cycles

Goal. As discussed in Section 6.2, solutions of the RS-PESP that use more rolling stock cycles are preferred, e.g. for robustness reasons. In this computational experiment it was investigated, if the number of rolling stock cycles can be increased for the two test instances, since the best obtained solutions for I_SMALL and LUZERN consisted only of three and eight rolling stock cycles respectively. Furthermore, the trade-off between using more rolling stock compositions and obtaining a schedule with more rolling stock cycles was examined.

Setup. Model (6.4) was implemented based on the implementation of a perfect matching in the transition graph and used as a post-processing step,

after an integral solution for the linearized RS-PESP was obtained. The trade-off was investigated by replacing constraint (6.4b) with constraint (6.6) and using different values for the number of additional rolling stock compositions $n_{\text{add}\mathcal{R}}$.

Result. Solving model (6.4) increased the number of rolling stock cycles compared to the originally obtained solution for both test instances. The MILP could be solved using CPLEX in less than a second. Furthermore, the LP-relaxation of the MILP was solved, yielding the same integral optimal solution. The results of the experiment investigating the trade-off are shown in Figure 7.5.

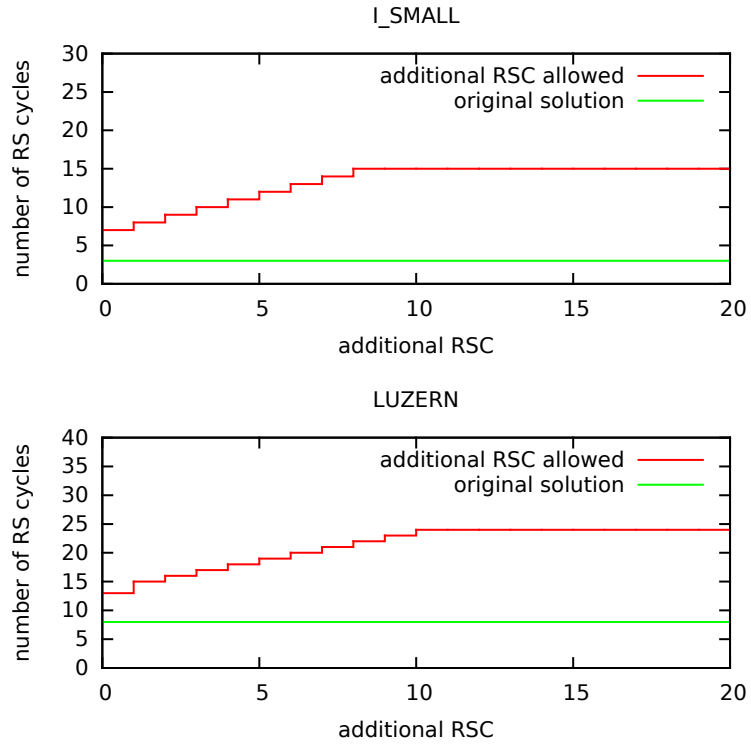


Figure 7.5: Allowing the utilization of more rolling stock compositions makes it possible to obtain rolling stock schedules, which consist of more rolling stock cycles. At a certain number of rolling stock compositions it is possible to operate a rolling stock schedule with the maximum number of cycles, where each cycle consists of two trip activities and two transition activities. The green horizontal line shows the number of rolling stock cycles in the original solution.

Conclusion. The post-processing step is suitable to obtain feasible solutions of the RS-PESP that use more rolling stock cycles than an initial solution, without increasing the number of required rolling stock compositions. If

more rolling stock compositions are available, it can also be used to find a schedule with the maximum number of cycles, utilizing a certain number of additional rolling stock compositions. The assumption that the optimization problem (6.4) has a strong LP-relaxation (see Section 6.2) is supported by the fast running times of the MIP-solver and the LP-relaxation having the same integral objective value.

Conclusion

In this thesis the integration of periodic timetabling and rolling stock scheduling has been investigated, since performing these railway planning steps sequentially may lead to expensive timetables and suboptimal rolling stock utilization. A periodic timetable determines the number of required rolling stock compositions. Therefore, the goal was to obtain timetables that minimize this number. An exact formulation of this problem is given by the RS-PESP, which is intractable in practice. With the linearization of the RS-PESP we introduce a way to obtain feasible solutions for the RS-PESP using a MIP-solver. The linearization yields a MILP, which is still too difficult to solve to optimality for a small and a medium sized instances, questioning the applicability of the linearized RS-PESP for general timetabling instances. On the other hand, obtaining feasible solutions for small and medium sized instances with a MIP-solver can be considered as progress in comparison to an intractable model, since the formulation might benefit from future improvements in the development of MIP-solvers. Especially, the results for the relatively small test-instance in the computational study were promising, since the optimal solution may require at most one rolling stock composition less than the best feasible solution obtained. Further research with regard to the linearized RS-PESP could concentrate on strengthening its weak LP-relaxation, e.g. with problem-specific cutting planes.

A second approach to obtain solutions for the RS-PESP is given by an iterative heuristic, which alternates between timetabling and rolling stock scheduling steps. In the computational study, the iterative method generated results of similar quality as the MILP. An option to improve the heuristic could be to exploit some structure in the perfect matching polytope for the rolling stock schedule, as was observed in this thesis.

In addition, we introduced a model to maximize the number of cycles in a rolling stock circulation for a given timetable without increasing the number of required rolling stock compositions. It can be used as a post-processing

8. CONCLUSION

step, in order to increase the robustness of the rolling stock circulation, because a greater number of independent cycles in terms of resources might help to prevent delay propagation. Furthermore, we investigate the relationship between a number of additionally used rolling stock compositions and the number of rolling stock cycles. The models were easy to solve in the computational experiments and provided a good increase in the number of rolling stock cycles.

Besides improving the approaches for solving the RS-PESP, the integration of timetabling and rolling stock scheduling could also benefit from further modeling extensions, e.g. more freedom in the rolling stock schedule by allowing deadheading, i.e. empty rolling stock movements.

List of symbols and abbreviations

Abbreviations

CPF	Cycle periodicity formulation of the PESP
EAN	Event activity network
FA	Frequency as attribute
FM	Frequency as multiplicity
PESP	Periodic event scheduling problem
RSCP	Rolling stock circulation problem
RS-PESP	Integrated model for a rolling stock schedule and PESP

Symbols

T	The period length in a periodic timetable, e.g. $T = 60$ minutes
$D = (N, A)$	Event activity network, a directed graph with vertices N , which are the events, and arcs A , which are the linking activities.
N_{terminal}	Set of arrival and departure events of all service lines at their terminal stations.
$N_{\text{arr}}, N_{\text{dep}}$	Partition of N_{terminal} into arrivals and departures, i.e. $N_{\text{terminal}} = N_{\text{arr}} \cup N_{\text{dep}}$
$R = (N_{\text{terminal}}, A_R)$	Transition graph with transition activities A_R . The graph is bipartite, where N_{arr} and N_{dep} are the two classes of vertices.
$n_{\mathcal{R}}$	Number of rolling stock compositions, which are required to operate a timetable

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