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PCE-BASED IMPRECISE SOBOL’ INDICES

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PROBLEM STATEMENT & CONTEXT

A computational model is defined as a mapping:

\[ \mathbf{x} \in \mathbb{D} \subset \mathbb{R}^N \rightarrow y = M(\mathbf{x}) \in \mathbb{R} \]

• \( \mathbf{x} \) is modelled by an imprecise random vector \( \mathbf{X} \), which accounts for both aleatory uncertainty (natural variability) and epistemic uncertainty (lack of knowledge).
• The elements of \( \mathbf{X} \) are assumed statistically independent.
• The computational model is considered as a black-box.

Goal: Sensitivity analysis – estimate the influence of each component \( X_i \in \mathbf{X} \) on the random response \( Y = M(\mathbf{X}) \).

PCE-BASED SOBOL’ INDICES

Considering a probabilistic input vector \( \mathbf{X} \), then a Polynomial Chaos Expansion (PCE) meta-model surrogates \( M \):

\[ Y = M(\mathbf{X}) \approx \sum_{\alpha \in A} a_\alpha \psi_\alpha(\mathbf{X}) \]

• \( \psi_\alpha(\mathbf{X}) \): multivariate orthonormal polynomials with respect to \( \mathbf{X} \).
• \( a_\alpha \): coefficients of the polynomials.
• \( A \): set of \( \alpha \) indices determined by an appropriate truncation scheme.
• Sparse PCE: obtained with least-angle regression (LARS).

Then, PCE-based Sobol’ indices read:

\[ S_{\alpha i}^{(PCE)} = \sum_{\alpha \neq \alpha_i} a_\alpha / \sum_{\alpha \neq \alpha_i} a_\alpha^2 \]

• \( I_{\alpha \neq \alpha_i} = \{ \alpha \in A : \alpha_i = 0 \} \)

⇒ Extension to imprecise probabilities?

PARAMETRIC P-BOX

Definition: CDF \( F_X \) (aleatory uncertainty) with interval-valued distribution parameters \( \Theta \) (epistemic uncertainty).

e.g. an imprecise Gaussian variable

\[ X \sim N(\mu_{\Theta}, \sigma_{\Theta}^2) \]

\[ \Theta = \{ \mu_{\Theta}, \sigma_{\Theta} \} \]


IMPRECISE SOBOL’ INDICES

Idea: Separation of sources of uncertainty within Sobol’ indices:

• Aleatory uncertainty ⇒ value of conventional Sobol’ indices
• Epistemic uncertainty ⇒ interval-valued indices

EXAMPLE: SIMPLY SUPPORTED TRUSS

Problem: assess deflection \( u_x(p) \) of truss (Hurtado, 2013):

• Loads \( P_i, i = 1, \ldots, 7 \) independent.
• \( \mu P_i \in [55, 105] \) kN, \( \sigma P_i \in [13, 17] \) kN.

Augmented PCE: \( N \geq 100 \) LHS samples.

Results:

• Computation of first order indices
• High accuracy in estimates of Sobol’ indices

CONCLUSIONS

• The augmented input space allows for a distinction between aleatory and epistemic uncertainty in \( X \).
• Imprecise Sobol’ indices allow for a distinction between aleatory and epistemic uncertainty in sensitivity analysis.
• Augmented PCE makes sensitivity analysis tractable for expensive-to-evaluate models with random input described by parametric p-boxes.