Numerical simulations of phase sensitive X-ray imaging using Monte Carlo Methods

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Abstract

X-ray grating interferometry (GI) is a recently established phase sensitive imaging technique, which utilizes micro-structured gratings and the Talbot effect to gain access to the absorption, phase-shift and dark-field signal of the X-rays after a sample. In addition to these three contrasts, reciprocal space information of a sample can be obtained through the GI Ultra Small-Angle X-ray Scattering (GI-USAXS) analysis scheme. The physical processes occurring during the GI image formation process can be described using numerical simulations. The particle-wave duality of photons presents two distinctive ways to consider X-rays, that provide complementary insight into the interactions of X-rays with matter, both of which have to be considered in an accurate numerical simulation of GI.

In this thesis, a framework for numerical simulations of GI was developed, that combines both particle- and wave-like properties of X-rays. The framework was based on Monte Carlo methods (MC) and implemented using egss++, the C++ interface for EGSnrc, a well established code for the simulation of particle transport utilizing MC. Since MC simulations of X-rays treat X-rays as particles, the framework was modified to include the wave-like properties necessary for the simulation of GI by implementing Snell’s law and including an additional variable to account for the phase shift. The interference phenomena of X-rays relevant for the simulation of GI were included into the framework in two steps. The first step consisted of a combination of MC with wave-optics simulations where the X-rays were simulated as particles within MC for the source and sample part, then coherently summed up into a wave after the sample and further propagated using wave-optics simulations. This Hybrid-model was validated by comparing simulations to experimental results which showed good agreement (correlation coefficient > 0.92). Simulations performed with this Hybrid-model were used for the investigation of the connection between the obtained signal and the physical structure of the sample in GI-USAXS and are opening the possibility of a quantitative interpretation of the obtained image. In a second step, the imaging geometry was fully modeled within MC. To account for interference phenomena within the simulation, Huygens principle was implemented into MC. Comparisons of simulation results with experimental data were used for the validation of this full MC model and demonstrated good agreement with a correlation coefficient > 0.9.

The full MC model was used for the determination of the visibility of a GI setup with a polychromatic X-ray source. Since MC methods are a well established method for the determination of realistic radiation dose estimations, a routine was implemented that enables the calculation of dose distributions within a sample which was applied to determine relative dose distributions in-vivo X-ray lung imaging of small animals.

The framework developed in this work was thus shown to successfully combine both particle- and wave-like properties of X-rays and to accurately model the relevant physical interaction processes occurring in GI. Furthermore, different applications of the developed framework in contemporary research were presented.
Zusammenfassung


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Chapter 1

Introduction
1.1 Motivation and Outline

X-ray grating interferometry (GI) is a recently established phase-sensitive imaging technique [1, 2] that utilizes an X-ray source, micro-structured gratings and the Talbot effect to gain access to the attenuation, phase-shift and scattering signal of the X-rays after a sample [3, 4, 5]. Those three signals provide three complementary types of contrast. Attenuation contrast, which is also called absorption contrast, is related to the attenuation of X-rays in matter and a widely used technique for instance in material sciences, airport security or medical diagnostics. Phase contrast occurs due to the phase-shift of X-rays passing through matter and has a high sensitivity towards electron density variations and consequently can provide an improved contrast compared to absorption contrast for materials with low atomic number $Z$. It is thus well suited for instance for the imaging of soft tissue matter. Dark-field contrast is the change in visibility of the signal as a consequence of inhomogeneities in the sample, which affect its macroscopic refraction properties [3, 6]. Furthermore, it has been shown that reciprocal space information of a sample can be obtained through the GI Ultra Small-Angle X-ray Scattering (GI-USAXS) analysis scheme [7, 8]. For a parallel beam X-ray source, a GI setup typically consists of a phase grating, an absorption grating and a detector. For a phase grating with a phaseshift of $\pi$, the Talbot effect leads to a self image of the phase grating with half the period at the fractional Talbot distances. This self image is distorted if a sample is introduced into the beam in front of the phase-grating. If the pixel size of the detector is larger than the interference pattern period, the GI signals can be extracted using the absorption grating through a phase-stepping procedure [3, 4]. Even though GI requires a coherent X-ray source, it has been successfully realized not only at high coherent, parallel beam sources such as could be found at third generation synchrotrons, but through the use of a source grating, also at standard laboratory X-ray tubes [9, 10]. This leads to a wide range of possible applications of GI imaging, for instance in medical diagnostics, biomedical research or material science [11, 12, 13].

To obtain an accurate description of the imaging mechanism, the physical processes occurring during the image formation process in GI, all relevant behaviors and interactions of X-rays with matter have to be considered. One method to realize such a description is through numerical simulations. Such numerical simulations can for instance be utilized in the validation of theoretical models of the imaging process, or to predict the performance or determine optimal design of an experimental GI setup. The particle-wave duality of photons presents two distinctive ways to consider X-rays, which provide complementary insight into the interactions of X-rays with matter. On the one hand, the particle character is well suited to describe effects such as Compton scattering or photoelectric absorption, which can be implemented in numerical simulations using Monte Carlo methods (MC). However, wave-like behavior is generally not considered in
Chapter 1. Introduction

MC. On the other hand, the wave character offers a method to describe coherent effects such as diffraction or interference. Such wave-like behavior is commonly modeled utilizing wave-optics simulations, but those can not be used to model particle-like behavior such as incoherent scattering. In an exact description of GI imaging, both particle- and wave-like properties of X-rays have to be taken into account.

The goal of this work is to obtain an accurate description of the image formation process of phase-sensitive and more specifically GI imaging. For this purpose, a simulation framework that includes wave-like behavior into MC is developed. The framework is implemented using egs++ [14], the C++ interface for the well established EGSnrc MC software toolkit [15]. To include the wave-like properties necessary for the simulation of GI imaging into the framework, several modifications to the implementation are made. Snell’s law is implemented and an additional variable is included to account for the phase-shift. The interference phenomena necessary for the simulation of GI are included into the framework in a two step approach.

The first step is through a hybrid approach, which combines MC with wave-optics simulations. Here, the X-rays are simulated as particles within MC for the source and sample part, coherently summed up into a wave right after the sample and further propagated through the remaining imaging geometry containing the gratings in a wave-optics simulation by utilizing the projection approximation and the free space propagator. To verify the validity of this combination approach, simulation results obtained with this Hybrid-model are compared to experimental data. This comparison demonstrates good agreement (correlation coefficient > 0.92) between experimental and simulated data. This Hybrid-model and its validation are presented in Chapter 2.

In Chapter 3 an investigation of the connection between the GI Ultra Small-Angle X-ray Scattering (GI-USAXS) signal and the physical structure of the sample using the Hybrid-model is shown, since it has previously been shown that a moment analysis of the GI-USAXS scattering distribution has the potential to make subpixel size structures available [16]. For this purpose, numerical simulations of GI of samples with defined microstructural parameters are performed. The scattering distributions of the simulations are obtained and their moments calculated and used for describing the relationship between the moments and the simulation parameters. This relationship is then applied to experimental data of a sample with known sample parameters. The comparison of retrieved and known sample parameters shows an agreement within 25%. This demonstrates the possibility for this specific case to retrieve quantitative sub-pixel information about the sample structure through GI-USAXS from experimental data using the proposed model.

Chapter 4 describes the second step which is taken to include the interference phenomena necessary for the simulation of GI into MC. This is called the full MC model, because the GI imag-
1.2. X-ray imaging

X-ray imaging is based on the attenuation of X-rays in matter and has become a widely used imaging method for instance in non-destructive biomedical investigations and material sciences. Due to the penetrating ability of X-rays, they also found a wide field of applications in medical imaging where they are used for example in radiography, computer tomography or fluoroscopy. It has been shown that additional and complementary information can be obtained through phase-contrast and scattering based X-ray imaging [5, 23]. Phase contrast has a high sensitivity towards electron density variations. Compared to absorption contrast, it can therefore provide an increased contrast for materials with low atomic number \( Z \). Inhomogeneities in the sample due to different materials or densities within the sample affect its macroscopic refraction index. As a consequence, the scattering signal which occurs from X-rays scattering at these
inhomogeneities, can provide structural information about the sample.

### 1.2.1 X-ray interaction with matter

Phase-shift and attenuation of X-rays passing through a material are described by the real part $\delta$ and imaginary part $\beta$ of its refractive index,

$$n = 1 - \delta + i\beta$$

(1.1)

where $\delta$ describes the change in phase and $\beta$ describes the attenuation [24]. The wave function of X-rays described as a plane wave after passing through a material is given by

$$U(\vec{r}, t) = U_0 e^{i(\vec{k}\vec{r} - \omega t)}$$

(1.2)

with $U_0$ the amplitude, $\omega$ the frequency and $\vec{k}$ the wave-vector. Through the substitution

$$\frac{\omega}{k} = \frac{c}{n},$$

(1.3)

with $c$ the speed of light, the following description can be obtained

$$U(\vec{r}, t) = U_0 e^{-i\omega(t-r/c)} e^{-ikdr} e^{-k\beta r},$$

(1.4)

where the first exponential corresponds to the vacuum propagation, and the second and third to phase-shift and attenuation. For a material of density $\rho$, consisting only of one element with atomic mass $A$, $\beta$ is related to the attenuation cross section $\sigma_{tot}$ through [25]

$$\beta = \frac{\lambda}{2\pi} \mu = \frac{\lambda}{2\pi} \rho \frac{N_A}{A} \sigma_{tot}$$

(1.5)

where $\lambda$ is the wavelength of the X-rays, $\mu$ is the attenuation coefficient of the material and $N_A = 6.022140 \times 10^{23}$ mol$^{-1}$ is the Avogadro constant. For a material consisting of different elements, $\beta$ is the weighted sum over the individual $\beta_i$ of the different elements. The total cross section $\sigma_{tot}$ is the sum of the different interaction cross sections

$$\sigma_{tot} = \sigma_{Comp} + \sigma_{pe} + \sigma_{Rayleigh} + \sigma_{pair} + \sigma_{trip} + \sigma_{ph,n},$$

(1.6)

where $\sigma_{Comp}$ is the Compton scattering cross section, $\sigma_{pe}$ is the atomic photo effect cross section, $\sigma_{Rayleigh}$ is the Rayleigh scattering cross section, $\sigma_{pair}$ and $\sigma_{trip}$ are the cross sections for electron-positron production in the fields of the nucleus and of the atomic electrons and $\sigma_{ph,n}$ is the photo nuclear cross section. In the X-ray energy range of 1 to 160 keV, which is the range considered in this thesis, the last three interactions do not occur and thus $\sigma_{pair}$, $\sigma_{trip}$ and $\sigma_{ph,n}$ do
not contribute to the total cross section. The total cross section $\sigma_{tot}$ in this range can therefore be written as the sum of only the first three interaction cross sections

$$\sigma_{tot} = \sigma_{Comp} + \sigma_{pe} + \sigma_{Rayleigh}.$$  

(1.7)

The two main processes contributing to $\sigma_{tot}$, and thus responsible for the generation of absorption contrast in X-ray absorption imaging, are photoelectric absorption $\sigma_{pe}$ and the Compton scattering $\sigma_{Comp}$. Elastic or Rayleigh scattering $\sigma_{Rayleigh}$ describes the scattering of photons without any loss of energy and has only a negligible contribution to the attenuation of X-rays. For low energies, the photoelectric absorption is the dominant contributor to the attenuation of X-rays in matter, with increasing energy Compton scattering becomes the main contributor.

**Compton scattering**

Compton scattering is incoherent scattering of the photon with an outer electron of the atomic hull. The shift of wavelength of the scattered photon is connected to the scattering angle $\theta$ through:

$$\lambda' - \lambda = \frac{\hbar}{m_e c} (1 - \cos(\theta))$$  

(1.8)

where $\lambda'$ is the wavelength after scattering, $\lambda$ is the initial wavelength, $\hbar$ is Planck’s constant, $m_e$ is the electron rest mass and $c$ is the speed of light. The scattering angle $\theta$ is described by the Klein-Nishina formula [26], which describes the differential cross section of photons scattered from a single free electron. For an incident photon of energy $E_{\gamma}$ the differential cross section is

$$\frac{d\sigma_{Comp}}{d\Omega} = \alpha^2 r_c^2 P(E_{\gamma}, \theta)^2 \{P(E_{\gamma}, \theta) + P(E_{\gamma}, \theta)^{-1} - 1 + \cos^2(\theta)\}/2,$$  

(1.9)

where $\alpha \approx 1/137.04$ is the fine structure constant, $r_c = \hbar/m_e c$, and

$$P(E_{\gamma}, \theta) = \frac{1}{1 + (E_{\gamma}/m_e c^2)(1 - \cos(\theta))}$$  

(1.10)

is the ratio of photon Energy before and after the collision. In the energy range of X-ray imaging, the distribution of scattering angles from Compton scattering is thus very broad.

**Photoelectric absorption**

The photoelectric absorption is the absorption of a photon by the inner electrons of an atomic hull. The cross section for photoelectric absorption of a photon with energy $E_{\gamma}$ is given by:

$$\sigma_{pe} = a Z^n E_{\gamma}^{-3}$$  

(1.11)
where \( Z \) is the atomic number, \( n \) varies between 4 and 5 (depending on K, L and M edges of the atom), and \( a \) is a constant [27, 28].

**Phase shift**

The real part \( \delta \) of the decrement of the refractive index (see Equation 1.1) determines the phase-shift of the X-rays of wavelength \( \lambda \) passing through a material. For a material consisting of \( M \) different elements, \( \delta \) is given by [29]

\[
\delta = \frac{\lambda^2 r_e}{2\pi} \sum_{i=1}^{M} f_{1i} N_i
\]

(1.12)

where \( r_e = 2.82 \times 10^{-15} \) m denotes the classical electron radius, \( f_{1i} \) is the real part of the atomic scattering factor of element \( i \) and \( N_i \) is the number density of atoms. For energies sufficiently far away from an absorption edge, \( f_{1i} \) can be approximated with the atomic number \( Z_i \) of element \( i \), \( f_{1i} \approx Z_i \) [30]. For X-rays passing through a material along the path \( s \), the phase-shift \( \Phi \) of the wave front is then given by

\[
\Phi(x, y, z) = k \int \delta(x, y, z) ds.
\]

(1.13)

where \( k = 2\pi/\lambda \) is the wave number.

**1.2.2 Grating interferometry**

Phase-sensitive X-ray imaging has been an important topic of research in recent years, due to its higher sensitivity compared to absorption contrast in materials of low atomic number \( Z \) [11, 13, 31, 32, 33, 34, 35]. One technique for phase-sensitive X-ray imaging is Grating Interferometry (GI), which has the advantage of providing three complementary types of contrast: absorption, phase and dark-field contrast [1, 2]. Absorption contrast occurs due to the attenuation of X-rays in matter and has become a widely used method for instance in material sciences or medical diagnostics. The phase-shift of the X-ray passing through a material leads to the so-called phase-contrast, which has a high sensitivity towards electron density variations. Dark-field contrast results from the change in visibility of the signal due to the scattering of X-rays on macroscopic inhomogeneities in the sample and provides additional structural information about the sample [3, 6]. GI phase contrast and dark-field imaging have therefore a wide range of possible applications for instance in biomedical research, medical diagnostics or material sciences [36, 37, 38, 39, 40, 41, 42, 43, 44].
1.2. X-ray imaging

GI utilizes the Talbot effect to gain access to the phase-shift, absorption and scattering signal of a sample. The general setup for GI with a parallel beam source, consists of two gratings. Usually the first grating is a phase-grating with period $g_1$ that introduces a periodic phase-shift into an X-ray beam. Due to the Talbot effect, this leads to an interference pattern the so called Talbot carpet. A self image of the grating of half the period $g_1/2$ can be observed at the fractional Talbot distances, also called Lohmann distances. The fractional Talbot distances $d_m$ for a parallel beam and a phase-grating with a $\pi$ phase shift are given by

$$d_m = \frac{g_1^2}{4\lambda}(m - \frac{1}{2}) \text{ with } m \in \mathbb{N},$$

where $\lambda$ is the wavelength of the X-rays [45]. Due to the small wavelengths of X-rays (0.1-1 Å for 10-120 keV), the period of $g_1$ is usually set in the order of a few micrometers. In order to resolve the interference pattern with a detector of larger pixel size than the interference pattern period, a second, absorbing grating with a period that matches the interference pattern $g_2 = \frac{g_1}{2}$ is introduced in front of the detector. If one of the gratings is moved perpendicular to the pattern over the period $g_2$, a variation of the intensity $I$ on the detector as a function of the grating position can be observed. This variation in intensity is called the phase-stepping curve (PSC) [3]. If a sample is placed in the beam, an additional phase-shift is introduced into the X-ray beam and the interference pattern is changed, which leads to a shift $\Delta \phi$ in the phase-stepping curve. This is expressed in relation to the refraction angle $\alpha$ and using the small angle approximation ($\tan(\alpha) \approx \alpha$) through

$$\Delta \phi = 2\pi \frac{d_m}{g_2} \alpha$$

The relation of the refraction angle to the phase $\Phi$ of the incoming wave is given by

$$\alpha = \frac{1}{k} \frac{\partial \Phi}{\partial x},$$

where $x$ depicts the direction along which the grating is shifted and $k = 2\pi/\lambda$ is the wavenumber. This relation can be derived by considering a simple scenario of a plane wave $\Psi(x, z)$ propagating along direction $z$. The wave front $\Psi$ can be described by

$$\Psi(x, z) = \Psi_0 e^{ikz}.$$  

If a sample with phase shift $\Phi(x)$ is introduced into the beam, the wave function after the sample is given by

$$\Psi'(x, z) = \Psi_0 e^{i(kz+\Phi(x))}.$$  

An illustration of this scenario can be found in Figure 1.1. The wave after the sample $\Psi'$
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Figure 1.1: Illustration of a plane wave $\Psi$ traveling along direction $z$ impinging on a sample, resulting in the refracted wave $\Psi'$ with refraction angle $\alpha$ along direction $z'$.

The refracted wave $\Psi'$ propagates along direction $z'$, which is given by the normal of $\Psi(x, z)$ [24]:

$$\nabla \Psi'(x, z) = \left[ \frac{\partial \Psi'(x, z)}{\partial x}, \frac{\partial \Psi'(x, z)}{\partial z} \right]$$

$$= \left[ \Psi_0 e^{i(kz + \Phi(x))} \frac{i}{k} \frac{\partial \Phi(x)}{\partial x}, \Psi_0 e^{i(kz + \Phi(x))} i_k \right]$$

(1.19)

(1.20)

If the substitution $\nabla \Psi'(x, z) = [A, B]$ is used, the refractive angle $\alpha$, corresponding to the angle between $z$ and $z'$, is given by

$$\alpha = \arctan \left( \frac{A}{B} \right).$$

(1.21)

Using the small angle approximation $\arctan \left( \frac{A}{B} \right) \approx \frac{A}{B}$, the relation

$$\alpha = \frac{1}{k} \frac{\partial \Phi(x)}{\partial x}$$

(1.22)

is obtained. The shift $\Delta \phi$ of the phase-stepping curve is thus related to the phase of the wave through

$$\frac{\partial \Phi(x)}{\partial x} = \frac{g_2}{d_m \lambda} \Delta \phi.$$  

(1.23)

Figure 1.2 depicts a schematic image of the GI setup utilizing two gratings. If the sample is not a pure phase-object, but also attenuates the incoming X-ray beam, the mean value of the PSC will be reduced. Additionally, if there is also scattering present, the amplitude of the PSC will change. Hence, the three signals attenuation, phase-shift and dark-field, can be acquired from the PSCs. A schematic image of a PSC without a sample and a PSC with sample is shown in Figure 1.3. The three signals can be obtained from the phase-stepping curve through a Fourier component analysis [5, 46]. If $s(x)$ is the sample PSC and $f(x)$ is the PSC without a sample...
1.2. X-ray imaging

Figure 1.2: Schematic illustration of a GI setup, utilizing a phase grating with period \( g_1 \) and an absorption grating with period \( g_2 \). The absorption grating is stepped perpendicular to the interference pattern over a period of \( g_2 \). The intergrating distance \( d_m \) is given by the fractional Talbot distances described by Equation 1.14.

(flatt-field PSC), with \( x \) the lateral offset of the grating, the absorption signal \( A \) can be obtained through

\[
A = \frac{\hat{s}(q_0)}{\hat{f}(q_0)}
\]

(1.24)

where \( \hat{\cdot} \) denotes the Fourier transform with respect to \( x \) and \( q_0 \) corresponds to the 0th Fourier component. \( \Delta \phi \) can be obtained through

\[
\Delta \phi = \arg(\hat{s}(q_n)) - \arg(\hat{f}(q_n))
\]

(1.25)

with \( q_n \) the \( n \)th harmonic Fourier component and \( n \) corresponding to the number of measured periods in the PSC. The dark-field contrast \( B \), which corresponds to the reduction in visibility \( V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \), can be obtained through

\[
B = \frac{V_s}{V_f} = \frac{\hat{s}(q_n)\hat{f}(q_0)}{\hat{s}(q_0)\hat{f}(q_n)}
\]

(1.26)

An additional advantage of GI is that the setup does not require parallel beam sources with high spatial coherence such as may be found in a third generation synchrotron, but can also be implemented for an X-ray tube source. To achieve the necessary spatial coherence in these cases, a source grating is often introduced into the GI setup. This grating is an absorption grating and is placed in front of the source, thereby creating an array of small sources of width \( l_0 \) [9, 10].
Figure 1.3: Schematic image of two phase-stepping-curves (PSC), one without (f) and one with a sample (s). It depicts the intensity I on the detector as a function of the grating position and shows the difference in mean value M, the phase $\phi$ and the amplitude $a$, which lead to absorption, phase and dark-field contrast.

While this reduces the photon flux from the source, it also increases the spatial coherence of the system. The spatial coherence $\xi_s$ required for a GI setup is $\xi_s = \lambda \Delta \alpha / \alpha$ where $\Delta \alpha / \alpha$ is the angular acceptance [9]. Using a source grating $G_0$ of period $g_0$, the spatial coherence is given by $\xi_s = \lambda l / \gamma_0 g_0$, where $l$ is the distance between $G_0$ and $G_1$ and $\gamma_0 = l_0 / g_0$. The possibility of utilizing an X-ray tube source creates a high interest in GI for potential diagnostic applications in medicine, for instance in mammography [11, 36].

### 1.2.3 USAXS

Passing through a sample, X-rays are scattered through refraction and reflection by the small sample structures. In GI the distribution $g(\phi)$ of the scattering angles $\phi$ is connected to the phase-stepping curve PSC of the sample $s(\phi)$ and the flat PSC $f(\phi)$ through [16]:

$$ s(\phi) = f(\phi) \otimes g(\phi), $$  

(1.27)
1.2. X-ray imaging

where $\otimes$ denotes a convolution. The scattering distribution $g(\phi)$ can thus be obtained through a deconvolution of the sample PSC $s(\phi)$ with the flat PSC $f(\phi)$. The centralized statistical moments can be calculated from the obtained scattering distributions using:

$$ M_0 = \int g(\phi) d\phi $$

(1.28)

$$ M_1 = \int \phi g(\phi) d\phi / M_0 $$

(1.29)

and for $n \geq 2$

$$ M_n = \int (\phi - M_1)^n g(\phi) d\phi / M_0. $$

(1.30)

It was shown that the absorption, phase and dark-field signal are connected to the moments of the scattering distribution through [7, 8]:

$$ A \rightarrow M_0 = \int g(\phi) d\phi $$

(1.31)

$$ P \rightarrow M_1 = \int \phi g(\phi) d\phi / M_0 $$

(1.32)

$$ \sqrt{-2 \cdot \log(B)} \rightarrow \sigma = \sqrt{\int (\phi - M_1)^2 g(\phi) d\phi / M_0} = \sqrt{M_2} $$

(1.33)

The higher moments of the scattering distribution offer a description of the scattering behavior of the sample and therefore have the potential to provide additional information about its substructure [16]. For the deconvolution of the scattering distribution from the phase-stepping curves, a higher number of phase-steps is necessary, which can be a drawback since it can lead to a higher total exposure time and thus a higher radiation dose in the sample. However, for samples where the acquisition of a larger number of phase-steps is possible, GI-USAXS can offer more insight into the scattering of X-rays by the sample and thus potentially provide information about the sample structure on a sub-pixel level. There are also dedicated techniques to gain access to the reciprocal space information of a sample such as small angle X-ray scattering (SAXS) and ultra small angle X-ray scattering (USAXS). For this technique, the X-ray beam is focused on the sample. The intensity of the scattered X-rays is then acquired by an area detector which is positioned several meters downstream of the sample [47, 48]. Unlike GI-USAXS where a deconvolution of the phase-stepping curves is necessary, SAXS allows direct access to the reciprocal space information of a sample.
1.3 Numerical simulations

Numerical simulations can be used for a description of the physical processes occurring in the X-ray image formation process. Through the modeling of the interaction processes of X-rays with matter, numerical simulations can for instance be used to determine the influence of the different interactions on the obtained signal. They can further be used for modeling imaging setups and thus help to identify optimal setup parameters or for determining source characteristics. In theoretical descriptions of X-rays both particle- and wave-view are used and the same applies to numerical simulations. Wave-optics simulations can be used for the simulation of wave like behavior, but particle like behavior such as Compton scattering are not considered. Such particle properties can be modeled using Monte-Carlo methods, which in turn do not account for wave-like behavior. However, to obtain an accurate description of the image formation process in GI, both descriptions need to be considered.

1.3.1 Monte Carlo Methods

Numerical simulations of X-rays using Monte Carlo methods are based on a particle view of photons. The principle lies in obtaining a numerical result of a process using probability theory. In the case of X-ray transport through matter, the different interaction cross sections as mentioned above are implemented as probability distributions. Random numbers are used to determine if, where and by what interaction a particle interacts. Since this is a stochastic approach, the number of simulated particles the so called histories, has to be chosen in accordance with the desired statistical uncertainty of the result. There are many frameworks for MC particle transport available. The MC code considered in this work is egg++ [14], a C++ interface to EGSnrc (National Research Council Canada’s Electron Gamma Shower) [15]. EGSnrc is a well established framework for electron, positron and photon transport in the range of 1 keV to 1 GeV and has been used and validated in many applications in the field of medical physics. However, there are also many other MC codes for the simulation of particle transport through arbitrary media such as GEANT4 [49], developed at CERN that would also enable other particle transports for instance of muons or hadrons, FLUKA [50] which is best known for its hadron event generator, PENELOPE a MC simulation code of coupled electron-photon transport in arbitrary materials [51] or MCNPX, which is a general-purpose MC code for neutron, photon, electron, or coupled neutron/photon/electron transport [52].

Within an egg++ simulation, particles are created at a source and transported through the implemented geometry based on the interaction probabilities. These interaction probabilities are determined from the interaction cross sections of a material and the energy of a particle. EGSnrc
1.3. Numerical simulations offers a calculation routine for these cross sections for any material, which requires the elemental composition and the density of the material and the energy range for particle transport. Both particle source and geometry can be defined by the user. There is a wide range of predefined source types and basic geometries available, such as planes, boxes, spheres, cylinders, which can be combined to create more complicated simulation geometries [53]. The framework also allows to implement user defined geometries or sources. Each particle within a simulation has a set of ten variables, three for position (x,y,z), three for direction (u,v,w), one each for charge, statistical weight, energy and a variable called the latch. The latch is a 32 bit variable, which can be used to track the interaction of the particle. This can be achieved by associating different bits of the latch with different interaction types, such as Compton scattering, or locations in the geometry, such as the phantom that is imaged. A particle is transported from the source through the geometry until it reaches the outside of the defined geometry or its energy becomes smaller than a predefined energy cut-off. If the energy falls below the cut-off energy, the particle is considered to be absorbed in the local geometry element. This opens the possibility to calculate radiation dose distribution, since the dose is connected to the energy deposited by the ionizing radiation. The choice of the cut-off determines not only the accuracy of the simulation result, but is also a factor influencing the simulation time, since the number of interactions an electron is likely to undergo increases with decreasing energy. The cut-off can be set separately for photons and electrons, the lowest possible energy for particle transport in EGSnrc is 1 keV. Secondary particles, which are particles created through an interaction process, are treated in the same manner. A new particle history at the source is only started after the previous particle from the source and all the secondary particles from its interaction either are outside the simulation geometry or are considered absorbed. EGSnrc allows to interact with the simulation at every step through the ausgab() routine. This routine can be called at every particle interaction or step and can for instance be used for setting the latch or implementing a scoring function for the simulation result. Since particle histories are independent, they can be run in parallel, which makes MC simulations suited for parallel computing. MC simulations of X-rays allow the description of particle like behavior, such as absorption or incoherent scattering [17], but wave like behavior such as interference are not considered. For accurate simulation of GI imaging, certain modifications are necessary to consider the wave-like behaviors that occur during the GI image formation process.

1.3.2 Wave-optics simulations

Wave-optics simulations as the name suggests, consider X-rays as waves. The wave-optics simulations utilized in this work, use the so called projection approximation. The projection
Chapter 1. Introduction

approximation assumes that X-rays passing through an object are not diffracted, thus phase and intensity variations at the objects exit surface arise only from the projected complex refractive index. In this approximation the phase-shift and absorption of an object with refractive index \( n(x, y) \) as defined in Equation 1.1 and a thickness \( T \) along the beam direction \( z \) is given by the projection along \( z \)

\[
E(x, y, T) = E(x, y, 0) \exp(iknT)
\]  

(1.34)

where \( E(x, y, 0) \) is the incident electromagnetic wave with wavelength \( \lambda \) and \( k = \frac{2\pi}{\lambda} \) is the wave number. The resulting wave is thus given by the multiplication of the incident wave with the projection of projection of the object along the beam direction. This can be rewritten as

\[
E(x, y, T) = E(x, y, 0) \exp(ikT) \exp(-k\beta T) \exp(-ik\delta T)
\]  

(1.35)

where \( \delta \) and \( \beta \) are the decrements of the refractive index \( n \). For the modeling of the absorption and the phase-contrast signal, this approach is sufficiently accurate for large enough propagation distances such as are used in GI imaging [54], however this may not be the case for the scattering signal. In free space, wave functions are propagated along the beam direction \( z \) using Fresnel propagation [24]

\[
E(x, y, z) = E(x, y, 0) \otimes h(x, y, z)
\]  

(1.36)

where \( \otimes \) denotes a convolution and the free space propagator \( h(x, y, z) \) is defined as

\[
h(x, y, z) = \frac{e^{ikz}}{i\lambda z} e^{\frac{i}{2\lambda z}(x^2+y^2)}.
\]  

(1.37)

This is applicable under the paraxial approximation, which has the assumption that the angle \( \theta \) between wave-vector \( \vec{k} \) and the optical axis \( z \) is small, \( \theta \ll 1 \). That assumption holds true for the cases considered in this work. In numerical simulations, this convolution can be achieved by a multiplication in Fourier space:

\[
\mathcal{F}\{E(x, y, z)\} = \mathcal{F}\{h(x, y, z)\} \cdot \mathcal{F}\{E(x, y, 0)\}
\]  

(1.38)

where \( \mathcal{F} \) denotes a Fourier transform and \( \mathcal{F}\{h(x, y, z)\} \) is given by

\[
\mathcal{F}\{h(x, y, z)\} = \frac{e^{ikz}}{z} e^{i\frac{2\pi}{\lambda^2}(q^2+p^2)}
\]  

(1.39)

where \( p \) and \( q \) denote the coordinates in Fourier space. Equation 1.36 can thus be rewritten as

\[
E(x, y, z) = \frac{1}{i\lambda z} e^{\frac{i}{2\lambda z}(x^2+y^2)} \mathcal{F}\{E(x, y, 0)e^{\frac{i}{2\lambda z}(x^2+y^2)}\}.
\]  

(1.40)

In general this equation cannot be solved analytically, but has to be solved numerically. The Fourier transformation used to solved this equation numerically, is usually an implementation of
the Fast Fourier Transform, such as provided in MATLAB (MathWorks) [55], which was used in this work.

Wave-optics simulations can be utilized in the simulation of wave-like behavior of X-rays such as interference, however since they are based on the wave-view of X-rays, they do not account for particle-like behavior such as Compton scattering.
Chapter 2

Combining Monte Carlo methods with coherent wave optics for the simulation of phase sensitive X-ray imaging

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2.1 Abstract

Phase-sensitive X-ray imaging shows a high sensitivity towards electron density variations, making it well suited for imaging of soft tissue matter. However, there are still open questions about the details of the image formation process. We present a framework for numerical simulations of phase-sensitive X-ray imaging, which takes both particle- and wave-like properties of X-rays into consideration. We present a split approach where we combine a Monte Carlo method (MC) based sample part with a wave optics simulation based propagation part, leading to a framework that takes both particle- and wave-like properties into account. The framework can be adapted to different phase-sensitive imaging methods and has been validated through comparisons with experiments for grating interferometry and propagation based imaging. The validation of the framework shows that the combination of wave-optics and MC has been successfully implemented and yields good agreement between measurements and simulations. This demonstrates that the physical processes relevant for developing a deeper understanding of scattering in the context of phase-sensitive imaging are modelled in a sufficiently accurate manner. The framework can be used for the simulation of phase-sensitive X-ray imaging, for instance for the simulation of grating interferometry or propagation based imaging.

2.2 Introduction

In recent years, a wide variety of techniques for phase-sensitive X-ray imaging have been developed. Crystal interferometry [56, 57] has a high sensitivity to phase variations, but is limited with respect to field of view. Analyser-based imaging [58, 59, 60] has a larger field of view, but requires a monochromatic beam. Phase propagation imaging [61, 62] offers the advantage of a comparably simple experimental set-up and the possibility to acquire high-resolution images at high speed. A phase-sensitive imaging technique which exploits absorption, phase and dark-field contrast is Grating-based hard X-ray Interferometry (GI) [2, 1]. GI has been shown to have a particularly high sensitivity to electron density variations, making it well suited for biological imaging [12, 31]. An additional advantage of GI is the comparatively low coherence requirement, which allows not only the use of synchrotron sources but, utilizing a third grating, also of standard laboratory X-ray tubes [9]. This makes GI of special interest for medical applications and in recent years a lot of effort has been put into making this technique available for clinical applications [11, 32, 33]. The particle-wave duality of photons suggests two distinctive ways to consider X-rays, which offer complementary insight into the interactions of X-rays with matter. On the one hand, the
Chapter 2. The Hybrid Model

particle character is well suited to describe effects such as Compton or Rayleigh scattering or photoelectric absorption. On the other hand, the wave character offers a convenient way to describe coherent effects such as phase shifts. The different interactions of X-rays with matter lead to different kinds of contrasts in X-ray imaging. A grating interferometer delivers three kinds of contrast (absorption, differential phase and dark-field), corresponding to three different X-ray interaction processes. Specifically, the dark-field contrast is related to scattering and phase contrast relies on beam coherence. This indicates that for realistic investigations of the image formation process it is essential that both particle- and wave-like properties of X-rays are taken into account.

We present a simulation framework that takes both behaviours in that sense into account by combining wave-optics with Monte Carlo methods (MC). Wave optics simulations treat X-rays as an electromagnetic wave, which opens the possibility to model interference [63]. In MC the path of individual particles through matter is modelled based on probabilities of scattering and absorption obtained from the physical cross-sections of the material. MC methods are widely used for the deep investigation of X-ray imaging techniques, particularly for dose estimations (in a CT scanner for instance) or scattering corrections in cone-beam CT systems. Through the combination of wave-optics and MC, absorption, phase-shift, interference and scattering can be modelled within one framework. Recently a number of publications have been made that investigate the dark-field contrast formation mechanism [64, 65, 6, 23, 66, 67] and different efforts to investigate GI using MC have already been published [68, 69], but so far in the investigation of X-ray scattering, the particle and wave approach have been considered separately. However our approach should provide new insights into the matter, since it combines MC with wave-optics within the same framework. This should provide an ideal tool for the investigation of the scattering process within a sample that can for instance be used to investigate the physical interpretation as well as the potential application of the ultra-small x-ray angle scattering with GI [7]. Due to the combination of MC and wave-optics, our framework can be used for different phase-sensitive X-ray imaging methods, it would also be applicable for the exploration of the parameter space or optimization of different set-ups. An additional advantage of using MC is the possibility to calculate dose distributions, which is of interest for future medical applications of phase-sensitive X-ray imaging.
2.3 Materials and methods

2.3.1 The simulation Framework

Phase-shift and absorption of a material are described by the real part $\delta$ and imaginary part $\beta$ of its refractive index $n = 1 - \delta + i\beta$. $\beta$ is related to the attenuation coefficient $\mu$ through $\mu = \frac{2\pi}{\lambda}\beta$, where $\lambda$ is the wavelength. The attenuation coefficient of a material consisting of only one element with atomic mass $A$ is related to the total cross section $\sigma_{\text{tot}}$ through [25]

$$\frac{\mu}{\rho} = \frac{N_A}{A} \sigma_{\text{tot}}$$

(2.1)

where $\rho$ denotes the mass density of the material and $N_A$ is Avogadros number. The total cross-section $\sigma_{\text{tot}}$ can be written as the sum of the cross sections of contributing photon interactions in the material. In the considered X-ray energy range, between 1 and 25 keV, the relevant contributions to the cross-sections are due to the photoelectric absorption $\sigma_{\text{pe}}$, Compton scattering $\sigma_{\text{Compton}}$ and Rayleigh scattering $\sigma_{\text{Rayleigh}}$. The total cross section can thus be written as

$$\sigma_{\text{tot}} = (\sigma_{\text{pe}} + \sigma_{\text{Compton}} + \sigma_{\text{Rayleigh}}).$$

(2.2)

In case of a material consisting of different elements, the mass attenuation coefficient $\frac{\mu}{\rho}$ is the weighted sum over the individual mass attenuation coefficients $\left(\frac{\mu}{\rho}\right)_i$

$$\frac{\mu}{\rho} = \sum_i w_i \left(\frac{\mu}{\rho}\right)_i$$

(2.3)

where $w_i$ denotes the fraction of element $i$ per weight. The real decrement $\delta$ for a wavelength $\lambda$ is connected to the composition of a material through [29]

$$\delta = \frac{\lambda^2 r_e}{2\pi} \sum_{i=1}^M f_{1i} N_i,$$

(2.4)

where the sum runs over all elements within the material, $f_{1i}$ is the real part of the atomic scattering factor of element $i$, $N_i$ stands for the number of atoms of element $i$ per unit volume and $r_e = 2.82 \cdot 10^{-15}$ m denotes the classical electron radius. For energies sufficiently far away from an absorption edge, the scattering factor is approximately equal to the atomic number $f_{1i} \approx Z_i$ [30].

Since the scattering processes, which are relevant for the contrast formation, take place in the sample and the interference occurs in front of the detector, the simulation package was split into two parts: the MC based sample part and the wave-optics based propagation part. In between
the two parts there is a transition part, which transforms particles into a wavefront. A sketch of the framework can be found in Figure 2.1. In the MC part of our simulation framework, particles are created at the source and transported through the predefined geometry, consisting of different materials. The initial position, direction and energy of the particles created at the source are sampled based on the probability distributions of the implemented source. The simulation of a particle history is terminated when the particle either leaves the geometry or its energy is reduced by scattering to a value below a predefined cut-off energy. In the second case it is assumed that the particle deposits its remaining energy locally and is considered to be absorbed. Within the simulation of the transport of the particle through the geometry, scattering and absorption occur based on probability distributions obtained from the physical scattering and absorption cross-sections for the respective material. By simulating a large number of particle histories, precise results can be obtained for the quantities of interest, such as fluence or characteristics of transmitted and scattered particles. In our case, the simulations return the phase space containing variables for location, direction, energy, charge and statistical weight of each particle. The phase-space is then transformed into a complex wave-amplitude which is passed on to the wave-optics part. The wave-optics part of our simulation framework models propagation through convolution with a propagator, while phase-shifts are obtained through multiplication with transmission functions.

The separation of the sample part and the propagation part makes it possible to have a sample specific but imaging method independent sample part and a sample independent but imaging
2.3. Materials and methods

method specific wave-optics part. The MC simulation of a sample can therefore be used for different kinds of phase sensitive imaging techniques such as GI and propagation based imaging. The different parts are described in more detail below.

Due to the separation of the imaging method part from the sample part, the scattering which occurs due to the imaging method is neglected. In the case of GI this means that the scattering which occurs in the gratings is neglected. However for an energy of 25 keV, which was used for GI, the noise introduced by scattering within the grating can be neglected due to the small scattering cross section and the wide scattering angle of Compton and Rayleigh scattering at this energy. To accurately account for the noise an additional simulation could be run where the gratings are simulated within MC and the intensity resulting from scattering could then be added as noise to the signal of the combination simulation.

MC part

The MC part of the framework containing the sample and the X-ray source is based on egss++, a c++ implementation of the well established EGSnrc code [53, 15]. For the considered energy range EGSnrc includes Compton and Rayleigh scattering as well as photoelectric absorption. For those scattering events the default EGSnrc form factors and cross-sections were used [15]. For phase sensitive X-ray Imaging it is necessary to take the phase shift of the X-rays passing through a sample into account. Therefore the MC code was extended by introducing the optical path length as an additional variable for each particle. The optical path length is the path integral of the real part of the decrement of the refractive index $\delta$ multiplied with the wave-number $k$ and accounts for the phase shift $\Phi$ of the particle $p$ passing through a material along the path $s$

$$\Phi(p) = k \int \delta(r) ds.$$

The particle transport occurs step-wise from one part of the geometry or interaction site to the next, thus in each step the path is determined and the respective phase-shift is added to the optical path-length of the photon. In the MC part, the coherence of the source can be considered through the initial values of the optical path length, energy, direction and position of the photons. A perfectly coherent plane-wave source for instance, can be obtained if all photons are starting from a plane perpendicular to the beam direction and have the same initial energy, direction and optical path length. Thus at any distance from the source, all particles that did not undergo any interactions will have the same optical path length. An other example would be a perfectly

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1This is only applicable for imaging methods where the phase modulation occurs after the sample. For other methods such as a GI setup where the sample is placed after the phase grating the approach as it is presented here would not be applicable.
coherent point source, which can be generated by using the same starting point, initial energy and optical path length for all particles, while the initial direction of each particle is sampled uniformly over all directions. To obtain a partially spatially incoherent source within our model, photons can be assigned an initial position and direction or an initial phase, where phase and direction are sampled according to the source distribution that was for instance experimentally determined. If the direction for particles starting in one point is sampled according to a distribution, two particles arriving in the same place will not necessarily have originated in the same position. If this is the case, their paths will be different, as will their optical path lengths and with it their phases. Thus the particles will not be coherent anymore. Illustrations of some of the possible sources within the model are shown in Figure 2.2. With this, it is possible to account for the effects on the imaging process caused by finite coherence of the source. In principle, it would be possible to also include the electrons generating the photons within the simulation process of the source. This would allow for an intrinsic simulation of the photon source from the electron beam within the model, but it would lead to a simulation cost that is orders of magnitude higher. In addition to the phase shift, the change of the particle direction due to the refraction process at surfaces was included through a new subroutine. It is called at each transition of a photon from one material into another, similar to the one described by Wang et al. [70]. In this subroutine the direction vector of a particle passing from one material with refractive index $n_i$ into another material with refractive index $n_r$, with an entrance angle $\Theta_i$, is changed by determining the angle of refraction $\Theta_r$ according to Snell’s law [24]

$$\frac{\sin \Theta_i}{\sin \Theta_r} = \frac{n_r}{n_i}$$

and changing the direction vector of the particle accordingly. The angle of refraction depends only on the incident angle and the refractive indices of the materials. For large angles of incidence with respect to the surface normal Snell’s law predicts that the sine of the angle of refraction to be larger than 1. In this case total external reflection occurs and the photon is reflected at the surface which is also taken into account within the routine. As seen in Figure 2.1, including refraction implies the optical path length to be equal to the actual path integral (see Equation 2.5) and not to the more intuitive but simple line integral. The latter would be equivalent to the so-called projection approximation (PA), often used in wave-optics, which assumes a thin sample, such that the angular deviation due to the refraction is negligible. It has been shown that the projection approximation is valid for large propagation distances compared to the object size [54, 71], therefore modelling the change in particle direction due to the refraction may not be relevant for the phase and absorption signal. However, to accurately model the small angle scattering within the sample, and thus to realistically simulate the dark-field signal, the inclusion of the refraction is essential. Both the refraction subroutine and the optical path length require
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Figure 2.2: Sketch of implementation of different source types within the framework. a) shows a fully coherent plane wave, where all particles have the same initial direction and are uniformly sampled over the beam area. b) shows the situation of a fully coherent spherical wave where all particle start in same point and the direction is uniformly sampled. c) shows a partially coherent divergent beam with a uniform distribution of the initial position over the source. d) shows a partially coherent divergent beam with a gaussian distribution of the initial positions. The directions in case of c) and d) are sampled according to the beam divergence.

The input of the real decrements of the refractive indices of all simulated materials. Those can be obtained using Equation 4.3.

The transition from MC to wave-optics

For the wave-optics part, the resulting phase-space of the MC part needs to be converted into a complex wave amplitude. A complex amplitude is obtained through associating each particle with a wave:

\[ p \iff \sqrt{E(p)}e^{-i\Phi(p)}, \]  

(2.7)

where \(E(p)\) is the energy of the particle and \(\Phi(p)\) the optical path length as defined in Eq 2.5. Within the MC simulation, the plane behind the sample is divided into a grid of \(N_x \times N_y\) areas \(u(x, y)\) of size \(\Delta x \times \Delta y\). All waves corresponding to particles which fall into the same area are summed up under consideration of their phase. This is repeated for all areas \(u(x)\) which results
in a wave-front $D_s(x)$ after the sample $s$ with

$$D_s(x) = \sum_{p \in u(x)} \sqrt{E(p)} e^{-i\Phi(p)}, \quad (2.8)$$

where

$$p \in u(x) \iff x - \frac{\Delta x}{2} < x(p) < x + \frac{\Delta x}{2} \quad (2.9)$$

and $x(p)$ is the position of particle $p$ within the area $u(x)$. To reduce the computational effort only one dimension was considered by setting $N_y = 1$. For a sample which is translation invariant in $y$ direction, this is equivalent to considering only one slice of the tomogram, or one line of the projection image. In case of a parallel beam set-up, the scattered particles that scatter into the slice from outside the direct projection direction can then be accounted for by setting the size of the source sufficiently larger than $\Delta y$. The wave-front $D_s(x)$ is then passed on to the wave-optics part.

**Wave-optics part**

As previously stated, the wave optics part is adaptable to the specific imaging method. In general the intensity obtained at the detector $I_{det}$ is a function of the amplitude obtained from the transition

$$I_{det}(x) = |F\{D_s(x)\}|^2, \quad (2.10)$$

where the function $F$ is defined by the imaging method. We will provide the function $F$ for two methods in the following section: propagation based imaging and GI. Due to the combination of MC and wave-optics, the number of simulated photons determines the accuracy of the simulated interference pattern but does not correspond to the photon statistics related noise in the final image.

### 2.3.2 Experimental validation

**Propagation based imaging**

In the case of propagation based imaging the complex wave front from the transition of the phase-space file of the MC simulation is propagated to a plane at the distance $d$ from the sample through convolution with the free space propagator $P_d(x)$

$$D_{sd}(x) = D_s(x) \otimes P_d(x) \quad \text{with} \quad P_d(x) = \frac{e^{ikd}}{i\lambda d} \exp(i \frac{k}{2d} x^2). \quad (2.11)$$
2.3. Materials and methods

To obtain the intensity of the signal on the detector at distance \( d \), the square of the absolute value of \( D_{sd}(x) \) is taken

\[
I_{det}(x) = |D_{sd}(x)|^2. \tag{2.12}
\]

To validate the simulation framework for propagation-based imaging we compared data simulated with our approach with data obtained with the projection approximation as well as with measured data. The signal of a X-ray beam impinging on a hollow cylinder with outer radius of 5.5 mm and an inner radius of 4.5 mm consisting of polypropylene (PP) was simulated and measured. A sketch of the experimental set-up is shown in Figure 2.3. The MC signal was created using the MC part of the framework as described above, the finite source size and small beam divergence were accounted for within the simulation of the source. The PA signal was obtained using the transmission function of the sample. Both signals were propagated to the detector plane through convolution with the free space propagator using Eq. 2.11. The measurements were performed at the TOMCAT beamline [18] at the Swiss Light Source (Villigen, Switzerland) with a source to sample distance of 25 m and at a photon energy of 10 keV. Comparisons were made for the following three different sample detector distances (SDD): 1.5 mm, 3 mm and 10 mm.

![Figure 2.3](image)

**Figure 2.3:** Sketch of the experimental set-up, the source sample distance was 25 m at an energy of 10 keV. The shaded area shows the area of the cylinder within the beam.

using the MC part of the framework as described above, the finite source size and small beam divergence were accounted for within the simulation of the source. The PA signal was obtained using the transmission function of the sample. Both signals were propagated to the detector plane through convolution with the free space propagator using Eq. 2.11. The measurements were performed at the TOMCAT beamline [18] at the Swiss Light Source (Villigen, Switzerland) with a source to sample distance of 25 m and at a photon energy of 10 keV. Comparisons were made for the following three different sample detector distances (SDD): 1.5 mm, 3 mm and 10 mm.

**Grating interferometry**

The experimental set-up of a grating interferometer is shown in Figure 2.4. It consists of a X-ray source, a sample and two gratings. The first grating, located right behind the sample, is usually a phase grating with period \( g_1 \), which generates an interference pattern of period \( g_1/2 \) at the
so called Lohmann distances [45]. For a phase grating with a phase shift of $\pi$, the Lohmann distances for a parallel beam are given by

$$d_m = \frac{g_1^2}{4\lambda} (m - \frac{1}{2}) \text{ with } m \in \mathbb{N},$$

where $\lambda$ is the wavelength of the X-rays. If a sample is placed between source and grating the interference pattern is shifted due to the phase-shift introduced by the sample. Since the period of the interference pattern is usually smaller than the pixel size, a second grating is required to detect this shift. The second grating is an absorption grating with the same period $g_2$ as the interference pattern. By scanning either the first or second grating in multiple steps over the period $g_2$, a phase stepping curve can be obtained for each pixel [4]. The shift $\Delta\phi$ of the interference pattern introduced by the sample is proportional to the refraction angle $\alpha$ through

$$\Delta\phi = \frac{2\pi d_m}{g_2} \alpha.$$  

The first derivative of the phase shift $\Phi$ of the wave front introduced by the sample is proportional to the phase shift of the fringes $\Delta\phi$ through[4]

$$\frac{\partial\Phi}{\partial x} = \frac{g_2}{\lambda d} \Delta\phi.$$  

The total phase shift $\Phi$ of the sample can then be obtained through integration and the refractive index decrement $\delta$ can be calculated using Eq. 2.5.

For the simulation of GI, the signal $D_s(x)$, obtained from the MC part of the framework, is propagated through the two gratings by first multiplying the amplitude with the phase-shift function

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**Figure 2.4:** Sketch of the Experimental set-up of the grating interferometer.
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\(G_1\) of the phase grating

\[ D_{s1}(x) = D_s(x) \cdot G_1, \quad (2.16) \]

neglecting the absorption of the phase grating. To propagate the signal to the absorption grating located at distance \(d\) from the phase grating, a convolution with the free space propagator \(P_d\) is performed:

\[ D_{s1d}(x) = D_{s1}(x) \otimes P_d(x). \quad (2.17) \]

The resulting amplitude is multiplied with the transmission function \(G_2(x_p)\) of the absorption grating at position \(x_p\)

\[ D_{s1d2}(x, x_p) = D_{s1d}(x) \cdot G_2(x - x_p). \quad (2.18) \]

The intensity \(I(i, x_p)\) in the detector pixel \(i\) at phase-step position \(x_p\), is the integral over the whole area of the pixel of the square of the absolute value of \(D_{s1d2}(x, x_p)\)

\[ I(i, x_p) = \int_{\text{pixel}} |D_{s1d2}(x, x_p)|^2 dx. \quad (2.19) \]

This procedure is repeated for all phase step positions \(x_p\). The projection images of the three different contrast modalities are then obtained from the intensity using a Fourier based approach [3, 9].

For tomography, these steps are repeated for different rotation angles of the sample and the resulting projection images are reconstructed using the reconstruction algorithm gridrec [72].

To validate the framework for GI, simulations and measurements of two phantoms were compared.

The phantom used for the validation of the absorption and phase contrast signal consists of a Polystyrene (PS) cylinder with 5 cylindrical holes as shown in Figure 2.5 (left). The holes were filled with five different concentrations of a water and ethanol mixture. The liquids were pure ethanol, pure water, and mixtures of water and ethanol with mass ratio 1:1, 1:2 and 2:1. The theoretical values for \(\delta\) were calculated using Eq. 4.3, the theoretical values for \(\mu\) were obtained from the NIST database. The theoretically calculated delta and beta values for all liquids are shown in Table 2.1. To avoid artifacts from high phase-gradients between sample and surrounding air, the sample was placed in a water filled aquarium.

The measurements were carried out at the TOMCAT beamline [18] at the Swiss Light Source. 1081 projections over 180° with five phase-steps and a pixel size of 7.4 \(\mu m\) were acquired at an energy of 25 keV and with the absorption grating placed at the 2\(^{nd}\) Lohmann distance, which is close optimal imaging conditions [46]. Further details about the experimental arrangement can be found in [12]. For the simulation, the same parameters as for the measurement were chosen. Per projection \(2 \cdot 10^9\) histories were calculated with an energy cut-off of 10 keV. The cut-off was
Chapter 2. The Hybrid Model

Figure 2.5: Left: Sketch of the phantom for the absorption and phase signal, the innermost cylinder was filled with water, the outer cylinders were filled counter-clockwise with liquids of increasing ethanol concentration. Right: sketch of the second phantom, used to compare the dark-field signal. The cylinder on the left is the full PMMA cylinder, the area on the right on top contains 381 cylinders, the area on the bottom contains 795 cylinders.

chosen to obtain a certain simulation efficiency and the cut-off level was set under consideration of the mean free path of photons in water, which is 2 mm at 10 keV. To obtain the same degree of coherence in simulation and experiments, the finite source size was modeled by sampling the initial position and direction of the photons within an area with the same second moment as the source at TOMCAT beamline, which is 53 \( \mu \text{m} \). Since for grating interferometry only the spatial coherence in the horizontal direction, perpendicular to the gratings, is relevant, this was only done for the horizontal direction.

To investigate the effects of small structures, the simulation of a phantom with small substructures is considered. The phantom consists of three circular areas each with a radius of 0.75 mm. The first area contains a full polymethyl methacrylate (PMMA) cylinder, the other two are filled with small PMMA cylinders each with radius of 1 \( \mu \text{m} \). The second area contains 381 of these cylinders, the third contains 795. The PMMA cylinders are distributed randomly within the two areas. A sketch of the phantom is illustrated in Figure 2.5 (right). For these simulations 541 projections were taken over 180°, the pixel size of the projections was set to 4 \( \mu \text{m} \), the photon energy was set to 25 keV and the absorption grating positioned at the second Lohman distance.
2.4 Results and Discussion

2.4.1 Validation for propagation based imaging

Figure 2.6 shows the absorption signal of the inner edge of the hollow cylinder as shown in Figure 2.3, for three different sample detector distances.

Figure 2.6: Top Left: Projection image obtained at 10 keV with a sample detector distance of 3 mm. The dashed line indicates where the profiles for the comparison were taken. Top Right to bottom left: Comparison of the line profile for MC and projection approximation (PA) to a measurement in a plane 1.5, 3 and 10 mm behind the object.
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The images show a comparison of line profiles from the measurement signal, the MC signal and the PA signal. The fringes observed in the experiment are much better approximated by the MC signal than the PA signal for the smaller sample detector distances. For larger sample detector distances i.e. longer propagation distances the PA signal approaches the MC signal as would be expected. The difference between MC and PA for the small propagation distances is due to the inclusion of the refraction in the particle transport within the sample, which is one of the main aspects of this framework. Due to the refraction, the fringes are already included in the MC signal. However they are not considered in the PA signal, since there the sample is assumed to be thin and thus angular deflection is neglected. The differences between MC signal and measurement can be explained by uncertainties in the composition, density and surface roughness of the cylinder. The surface roughness, which was not considered in the simulations, has a high influence on the signal, since small substructures in the surface will lead to a broadening of the fringe coming from the inner edge of the cylinder. This is also indicated by the fact that the right fringe is better approximated than the left fringe. This result shows that the refraction is modelled accurately within this framework and thus it can be used for accurate simulations of phase propagation imaging.

2.4.2 Validation for GI

The reconstructed phase images for simulation and measurement are shown in Figure 2.7. The qualitative agreement of the two images is excellent which shows that the phase signal is simulated in a realistic way. The correlation coefficient for the two images is 0.96 and the normalized

![Figure 2.7: Reconstructed phase image of the measurement (left) and MC simulation (right) of an ethanol-water phantom. The dashed lines indicate the line profiles shown in Figure 2.8](image-url)
mean square error is 0.0006. In Figure 2.8 the profiles along the two lines shown in Figure 2.7 are depicted. They show that the values agree well for the water-ethanol mixtures and the PS, but not for the water in the middle. There is also a peak artifact visible in the middle. This may be due to water impurities in the measurement. Although Milli-Q water used in both phantom and the aquarium, contamination cannot be completely excluded. Further possibilities for this effect might be drift of the beam in the experiment, which is a known effect that can only be partially corrected for in the postprocessing. Beam drift was not considered in the simulation. The correlation coefficient for the profiles are 0.99 for the horizontal profiles and 0.98 for the vertical profiles.

The reconstructed absorption images for simulation and measurement are shown in Figure 2.9. The two images show a good qualitative agreement, in both absorption images the edge enhancement can clearly be observed. Due to the beam coherence, the edge enhancement occurs at the interfaces of the liquids with high ethanol concentration, which is where the gradient of the phase is largest. It can be seen that the edge-enhancement is more emphasized in the simulation than in the measurement, even though the finite source size, which determines the coherence was considered in the simulation. This is due to the detector response, which would lead to a smoothing of the edge and has not been considered. The simulation image shows ring artifacts which are due to the statistical noise from the MC part which is amplified through the wave-propagation. The correlation coefficient for the two images is 0.92 and the normalized mean squared error for the two images is 0.0061. Figure 2.10 shows the profiles through the absorption images indicated in Figure 2.9. To reduce the noise the profiles were averaged over 10 pixels in the direction perpendicular to the profile. The agreement found is good, the correlation

![Figure 2.8: The line profile through measured and simulated phase image along the horizontal (left) and vertical (right) line indicated in figure 2.7.](image-url)
Figure 2.9: Comparison of absorption measurement (left) and MC simulation (right) of an ethanol-water phantom. The dashed line indicate the lines along which the profiles were taken.

coefficient for the horizontal profiles is 0.85 and 0.89 for the vertical profiles.

Figure 2.11 shows a scatterplot of the $\mu$ and $\delta$ values. The values for $\delta$ and $\mu$ values were obtained from the reconstructed images by averaging the grey values over several pixels and using Eqs (2.15) and (2.5). To reduce the noise, the measured images were averaged over 30 slices.

The values were compared to the theoretical values given in Table 2.1. The output of the reconstruction, the measured and simulated values were calibrated to the theoretically calculated values for water and ethanol.

The errorbars show the standard deviation of the values within the averaged area. The overall

Figure 2.10: The line profile through the absorption image for measurement and simulation along the horizontal (left) and vertical (right) line indicated in Figure 2.9.
Table 2.1: Theoretical (A), simulated (B) and measured (C) values for δ and µ at 25 keV. The measured and simulated values were calibrated to the theoretically calculated values using the values ethanol and water.

<table>
<thead>
<tr>
<th>material</th>
<th>density [g/cm³]</th>
<th>δ [10⁻⁷]</th>
<th>µ [cm⁻¹]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>100% water*</td>
<td>0.9982</td>
<td>3.687</td>
<td>3.69 ± 0.01</td>
</tr>
<tr>
<td>33% ethanol</td>
<td>0.9487</td>
<td>3.510</td>
<td>3.523 ± 0.004</td>
</tr>
<tr>
<td>50% ethanol</td>
<td>0.9143</td>
<td>3.400</td>
<td>3.421 ± 0.004</td>
</tr>
<tr>
<td>66% ethanol</td>
<td>0.8749</td>
<td>3.281</td>
<td>3.277 ± 0.003</td>
</tr>
<tr>
<td>100% ethanol*</td>
<td>0.7893</td>
<td>2.960</td>
<td>2.96 ± 0.04</td>
</tr>
<tr>
<td>polystyrene</td>
<td>0.7893</td>
<td>3.715</td>
<td>3.746 ± 0.005</td>
</tr>
</tbody>
</table>

Figure 2.11: Scatter plot of measured, simulated, and theoretical values for δ and µ. The values were calibrated for ethanol and water.

Agreement of the values for both µ and δ is very good. The standard deviations for the attenuation coefficients µ are much higher than for the δ values due to the much weaker absorption signal. The differences for the water-ethanol mixtures may be explained by uncertainties in the exact mixture ration of the two liquids that was measured. The PS cylinder also displays differences between the theoretical and simulated values for µ which may be due to the fact that the exact composition and density of the PS cylinder was unknown, so for the theoretical calculation and the simulation the composition was assumed to be the pure polymer C₈H₈ with a density of
\( \rho = 1.04 \, g \, cm^3 \), according to values found in literature. While this results in a good agreement for \( \delta \), there is a visible difference in \( \mu \), suggesting that these assumptions may not be completely accurate.

The simulated and reconstructed dark-field image of the second sample are shown in Figure 2.12 (right). For comparison the sketch of the phantom is shown in Figure 2.12 (left). It can clearly be seen that three areas give a different dark-field signal. As expected, the homogeneous cylinder area yields a dark-field signal only at the edges, while the areas filled with less and more cylinders yield respectively a weak and strong dark-field signal from within the whole circular area. The area with the higher number of cylinders gives a stronger signal than the area with less cylinders, as would be expected. The observable streak artifacts are most likely due to violations of the basic assumptions about the scattering in the model used for tomographic reconstruction of the dark-field [73]. Our results indicate that simulation of the dark-field signal can be obtained qualitatively with our model, provided that all substructures of a sample are known and included in the geometry of the sample. This offers the possibility to obtain a deeper understanding of the dark-field contrast formation process, which is closely related to the scattering and sub-pixel refraction properties of the sample. The framework allows for accessing both the scattering as well as the distributions of refraction directions in one voxel, which can be used for future investigation into the nature of dark-field as well as ultra-small angle X-ray scattering.

In general, exact numerical simulation of coherent effects could be achieved by utilizing Huygens’ principle in the MC part of the developed framework. Treating each particle at each spatial step as a new point source by particle splitting, a fully coherent simulation would be accomplished. While this opens the possibility to simulate interference and to include the grat-

![Figure 2.12: Sketch and reconstructed dark-field images of the artificial sample, same as Figure 2.5 right. The cylinder on the left is the full PMMA cylinder, the area on the right on top contains 381 cylinders, the area on the bottom contains 795 cylinders.](image-url)
ings within the MC part, it would be computationally extremely expensive. The MC part of the model we present in this paper constitutes a first order approximation to the fully coherent imaging formation process. The results suggest that our approach can be used for the reliable simulation of coherent X-ray imaging.

2.5 Conclusion

We developed a framework for the simulation of phase sensitive X-ray imaging which takes into account both particle- and wave-like properties of the X-rays by combining MC with wave-optics simulation. The combination was achieved by splitting the simulation into a sample dependent MC part and an imaging method dependent wave-optics part. To take into account coherent effects, the MC part was extended by including refraction and the optical path length. As a validation of the framework, comparisons between measurements and simulations of a phantom were carried out. A comparison between simulated and measured propagation images was performed which showed that the proposed MC model accounts for the edge enhancement in the simulation of propagation based imaging. A second comparison was performed for the case of GI where a plastic cylinder phantom with holes filled with different ethanol water mixtures and a plastic phantom were used. The comparisons showed excellent agreement (correlation coefficient $> 0.925$) between measured, simulated and theoretically calculated values for both the attenuation coefficient $\mu$ and decrement of the refractive index $\delta$. This shows that the combination of wave-optics with MC was successful and the relevant physical processes were modeled accurately within the simulation. Future applications of this framework could now be investigations into X-ray phase contrast formation mechanisms through simulation of different sample parameters. Since the framework can also be used for different phase-sensitive X-ray imaging methods it would also be applicable for the optimization of different set-ups, for instance in investigation of high-energy set-ups for GI.
Chapter 3

Quantification of subpixel structural information using Monte Carlo simulations of GI-USAXS

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\textsuperscript{c}Previously Centre d’Imagerie BioMedicale, Ecole Polytechnique Federale de Lausanne, Switzerland, and Swiss Light Source, Paul Scherrer Institut, CH-5232 Villigen, Switzerland, now at Department of Medical Physics and Bioengineering, University College London, Gower Street, WC1E 6BT London, United Kingdom
\textsuperscript{d}\textit{Division of Medical Radiation Physics and Department of Radiation Oncology, Inselspital, University Hospital Bern and University of Bern, CH-3010 Bern, Switzerland}

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3.1 Abstract

X-ray Grating Interferometry (GI) is a phase-sensitive imaging technique with the advantage of providing three complimentary types of contrast, absorption, phase and dark-field contrast. Furthermore, it has been shown that through a recently presented analysis scheme, GI Ultra Small Angle X-ray Scattering (GI-USAXS), reciprocal space information of a sample can be obtained. This offers the possibility to reveal subpixel structural information about a sample. Numerical simulations of GI-USAXS of samples with defined micro structural parameters were performed using a simulation framework for phase-sensitive X-ray imaging. The framework combines wave-optics with Monte Carlo methods and is based on egs++, a C++ interface for the well established EGSnrc framework. The scattering distributions of the simulations were obtained and their moments calculated and used for describing the relationship between the moments and the sample parameters. This relationship was then applied to experimental data of a sample with known sample parameters. The comparison between known and retrieved sample parameters showed an agreement within 25%. This demonstrated the possibility to retrieve quantitative sub-pixel information through GI-USAXS both with simulated and experimental data for the considered sample.

3.2 Introduction

Phase-sensitive X-ray imaging using grating interferometry (GI) is a recently established imaging technique with the advantage to provide three different types of contrast: absorption, differential phase and dark-field [1, 2]. Phase contrast has a high sensitivity towards electron density variations and consequently can provide a superior contrast compared to absorption for materials with low atomic number Z. Thus it found a wide range of applications in a variety of fields such as material sciences, biology and medicine [13, 44]. Dark-field contrast is related to the scattering of X-rays within the sample, thereby containing information about the substructure, and has for instance been indicated to have the potential to improve diagnostic predictions of breast cancer in mammography [11, 36].

GI utilizes two or more gratings to gain access to the absorption, differential phase and dark-field signal of X-rays passing through a sample [1, 2]. In this work, a setup with two gratings is considered, a phase grating with period \( g_1 \) that introduces a phase modulation in the coherent X-ray beam, which leads to an intensity interference pattern at specific distances downstream [45]. These so called fractional Talbot distances for X-rays of wavelength \( \lambda \) and a phase grating
with a phase-shift of $\pi$ are given by

$$z_m = \frac{g_1^2}{4\lambda}(m - \frac{1}{2}) \text{ with } m \in \mathbb{N}. \quad (3.1)$$

The interference pattern at these distances has half the period of the phase grating $g_1/2$. If a sample is placed in front of the phase grating, the refraction and absorption within the sample will lead to a distortion of the interference pattern. Since the pattern itself is in general smaller than the pixel size of the detector and thus too small to be resolved by the detector, an absorption grating with the same period as the interference pattern is placed in front of the detector. If the absorption grating is stepped along the interference pattern in several steps over one period, a periodic oscillation of the intensity in each pixel, the phase-stepping curve (PSC), can be observed [4]. The signals for absorption, differential phase and dark-field can be obtained through a Fourier component analysis of the PSC without a sample and the PSC with a sample in the beam [3, 5].

It has recently been shown that the amount of information obtainable from a GI measurement can be increased through the use of the GI Ultra Small Angle X-ray Scattering (GI-USAXS) analysis scheme, which provides reciprocal space information about the sample [8, 7]. Previous work [16] furthermore indicated, that a moment analysis of the GI-USAXS scattering distribution has the potential to make subpixel structures available. For a wide number of applications quantitative imaging of sub-pixel structures would be of interest, such as in lung imaging where the structures of interest can be smaller than the pixel size [37].

In this work, a first step towards investigating the relationship between the statistical moments of the scattering distributions and certain structural sample parameters in case of a well defined sample is presented, which opens the possibility to obtain quantitative sub-pixel structural information about a sample through GI-USAXS. Thus, numerical simulations of GI-USAXS of a set of well defined samples were performed using a recently published framework for the numerical simulation of GI [74]. This framework combines Monte Carlo methods (MC) with wave optics for numerical simulations of phase-sensitive X-ray imaging. The scattering distributions of the simulations of these samples were obtained through a deconvolution method and the statistical moments obtained from these scattering distributions were fitted with a function containing structural parameters. In order to demonstrate consistency, this fitting function was then used to obtain sample parameters from the moments. It was further applied to experimental data, to retrieve sample parameters from the measured data which were then compared to the known sample parameters.
3.3 Methods and Materials

Passing through a sample, X-rays are coherently scattered through refraction and reflection by the small sample structures. In GI the distribution $g(\phi)$ of those scattering angles $\phi$ is connected to the PSC of the sample $s(\phi)$ and the flat PSC $f(\phi)$ through [16]:

$$s(\phi) = f(\phi) \otimes g(\phi),$$

where $\otimes$ denotes a convolution. The scattering distribution $g(\phi)$ can thus be obtained through a deconvolution of the sample PSC $s(\phi)$ with the flat PSC $f(\phi)$.

Previous work indicated a connection between the moments of the scattering distribution and sample structures [16]. To investigate this connection between different moments and sample parameters, numerical simulations of projection images of several well defined samples were performed. The simulations of the sample were conducted using a previously presented framework for the numerical simulations of phase-sensitive X-ray imaging [74]. This framework combines MC methods with wave-optics simulations and is based on egs++ [14, 53], the C++ interface for the well established EGSnrc [15], a MC code for the simulation photon and electron transport through arbitrary media. Through the combination of wave-optics simulations with MC methods, the numerical simulation of phase-sensitive X-ray imaging such as GI is possible within this framework.

The simulated samples consist of a Polymethylmethacrylat (PMMA) slab with randomly distributed holes that were filled with air. Two parameters were varied, one was the minimal distance $d$ between the holes which varied from 4 to 11 $\mu$m and the second was the radius $r$ of the holes, which varied between 0.5 and 4.5 $\mu$m. The samples have a length of 1.04 mm and a thickness of 52 $\mu$m and 100 $\mu$m. The different combinations of radii and distances as well as the corresponding fill ratio, defined as the volume ratio between air and PMMA, for all the samples that were used in the simulations is displayed in Table 3.1. Due to the random distribution of the cylinders, there can be a small mismatch between the fill ratios for the two thicknesses. The ratio was calculated using the radii and the number of cylinders per sample. A monoenergetic, parallel beam setup was simulated with an energy of 25 keV. The period $g_1$ of the phase grating was set to 4 $\mu$m and the absorption grating $G_2$ with a period $g_2 = 2$ $\mu$m was placed at the third fractional Talbot distance (121 mm). The size of the pixels was set to 52 $\mu$m to ensure sufficient cylinders per pixels, leading to 20 pixels. For each sample, 32 phase-steps over one period of $G_2$ were simulated. A schematic image of the setup is depicted in Figure 3.1.

The scattering distribution was obtained through deconvolution of the simulated flat and sample phase-stepping curves using a Lucy-Richardson deconvolution method [16, 75, 76]. The second
Figure 3.1: Schematic view of the simulated and experimental setup (not to scale), showing the sample, the phase grating with a period of 4 \( \mu \text{m} \) and the absorption grating with 2 \( \mu \text{m} \) period. The X-ray energy was 25 keV, the integrating distance 121 mm, which corresponds to the third fractional Talbot distance. The samples consist of PMMA slabs with cylindrical holes of different radii and distances.

<table>
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<th>Fill ratio</th>
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<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
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<td>0.008</td>
<td>0.006</td>
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<td>0.43</td>
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</table>

Table 3.1: Fill ratio defined as the ratio of the air volume and the total sample volume for the different samples used in the simulations. The ratio was calculated using the radii and the number of cylinders per sample. Two sample thicknesses were simulated: 52 \( \mu \text{m} \) and 100 \( \mu \text{m} \). Due to the random distribution of the cylinders, there can be a small mismatch between the fill ratios for the two thicknesses.
$M_2$ and fourth moment $M_4$ were determined from the scattering distributions using:

$$M_n = \int (\phi - M_1)^n g(\phi) d\phi / M_0,$$

(3.3) with $M_0$ and $M_1$ as defined below. It was previously shown that the absorption (A), phase (P) and dark-field (B) signal can be described by the moments of the scattering distribution [7]:

$$A \rightarrow M_0 = \int g(\phi) d\phi$$

(3.4)

$$P \rightarrow M_1 = \int \phi g(\phi) d\phi / M_0$$

(3.5)

$$\sqrt{-2 \cdot \log(B)} \rightarrow \sqrt{M_2} = \sqrt{\int (\phi - M_1)^2 g(\phi) d\phi / M_0}$$

(3.6)

Through the deconvolution of the sample PSC with the flat PSC, also higher moments of the scattering distributions can be determined. In this work, the second and fourth moment are considered, since previous work indicated a connection between these moments and sample parameters for minimal distance and radius of the holes [16]. In this work, the parameter $\kappa'$ was defined as the 4th root of the kurtosis $M_4 / M_2^2$:

$$\kappa' = M_4^{1/4} / \sqrt{M_2},$$

(3.7) and used to investigate the connection. This parameter $\kappa'$ that was obtained from the simulation

Figure 3.2: Illustration of the process; the phase-stepping curves of the samples are simulated using a framework for numerical simulations of GI that combines MC with wave-optics. The scattering distributions are obtained from the simulated phase-stepping curves through a deconvolution. The ratio of fourth and second moment calculated from these scattering distributions were then fitted as a function of radius, minimal distance and thickness.

results, was then fitted as a function of the hole radius and the hole distance using MATLAB (MathWorks), resulting in a fitting function for $\kappa'$ which was called $\kappa'_\text{mod}$. This fitting function
can then be used to extract the sample hole distance or radius from \( \kappa' \). An illustration of the process is shown in Figure 3.2.

In a second step, the function \( \kappa'_{\text{mod}} \) was utilized to extract sample parameters of similar samples of USAXS data experimentally obtained at the beamline for TOmographic Microscopy and Coherent rAdiology experimenTs (TOMCAT) of the Swiss Light Source (Paul Scherrer Institut, Villigen, Switzerland) [12, 18]. The samples consist of PMMA cylinders with four circular areas of radius 1 mm, each filled with holes of different radii and minimal distances. Projection images with 32 phase-steps were obtained at an energy of 25 keV. The period of the phase grating was \( g_1 = 4 \, \mu m \), the intergrating distance 121 mm and a pixel size of 6.5 \( \mu m \) that was binned 8 times to 52 \( \mu m \) [75, 16]. The combination of radii and distances for the different samples measured in the experiment is displayed in Table 3.2.

<table>
<thead>
<tr>
<th>sample number</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<th>7</th>
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<tbody>
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<td>1.86</td>
<td>1.92</td>
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<tr>
<td>minimal distance [( \mu m )]</td>
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<td>5.05</td>
<td>6.14</td>
<td>7.03</td>
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<tr>
<td>fill ratio</td>
<td>0.35</td>
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<td>0.23</td>
<td>0.18</td>
<td>0.26</td>
<td>0.24</td>
<td>0.21</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Table 3.2: Parameters of the sample used in the experiment, the sample consists of PMMA with cylindrical air-filled holes with different radii and minimal distances.

### 3.4 Results and Discussion

The values for \( \kappa' \) obtained from the simulation are displayed for both thicknesses in Figure 3.3 as a function of the fill ratio. It can be observed that for both thicknesses, \( \kappa' \) flattens with increasing fill ratio, but the value is different for the two thicknesses. The data points were fitted with the model function:

\[
\kappa'_{\text{mod}} = 1.35 + \left( \frac{T}{20} \right)^2 \cdot \left( \frac{r}{d} \right)^2 \cdot 4.5 \cdot r^{1.25} \cdot \sqrt{\frac{r}{d}} - \frac{1}{4} 
\]

where \( T \) is the thickness of the sample, \( d \) is the minimal distance between the cylinders and \( r \) is the radius of the holes (all in \( \mu m \)). Since only the air-PMMA material combination was considered and this is a purely observational fitting function, this model only implicitly depends on the real decrement \( \delta \) of the refractive index \( n = 1 - \delta - i\beta \) of PMMA. The fitted values for \( \kappa'_{\text{mod}} \) are displayed for both sample thicknesses in Figure 3.3. The correlation coefficient for the model fitting function \( \kappa'_{\text{mod}} \) and the simulated values of \( \kappa' \) is 0.98 for both sample thicknesses. To show how the model fitting function can be used as a predictor of sample parameters and to show consistency, Equation 3.8 was then inverted to obtain the radius \( r \) from \( \kappa' \) for known
3.4. Results and Discussion

Figure 3.3: Values for $\kappa'$ obtained from simulation and the fitting function $\kappa'_\text{mod}$ as a function of the fill ratio, defined as the air volume in the sample divided by the total sample volume. The errorbars correspond to the statistical uncertainty as one standard deviation of the simulated values. The correlation coefficient for the simulation curve and the fitting curve is 0.98 for both thicknesses, i.e., 52 $\mu$m and 100 $\mu$m.

values of distance $d$ and thickness $T$. This inverted function was then applied to the values for $\kappa'$ obtained from the simulation of the sample with 100 $\mu$m thickness. The resulting values for the radius $r_{\text{mod}}(\kappa', d, T)$ are compared to the input radius $r_{\text{sim}}$ of the simulation and are shown in Figure 3.4.

The radii retrieved from the fitting function model $r_{\text{mod}}$ lie within one standard deviation of the actual radii used in the simulations, though the agreement is better for smaller radii than for the larger ones. One explanation for the higher discrepancy of the larger radii is, that for the larger radii, the fill factor is higher which puts them in the flatter region of $\kappa'_\text{mod}$ as shown in Figure 3.3. This leads to a higher uncertainty in the inversion of the fitting function.

The fitting function in Equation 3.8 was then applied to the values of $\kappa'$ obtained from the experimental data to retrieve the sample parameters, which were plotted versus the known sample parameters. First the radius $r_{\text{mod}}(\kappa', d_{\text{exp}}, T)$ was calculated using Equation 3.8 and the known sample distances $d_{\text{exp}}$. The so obtained values for $r_{\text{mod}}$ are plotted against the known sample radius $r_{\text{exp}}$ in Figure 3.5. The same was done for the sample distance, the calculated distance $d_{\text{mod}}(\kappa', r_{\text{exp}}, T)$ was calculated using $r_{\text{exp}}$ and plotted versus the known distance $d_{\text{exp}}$ in Figure 3.6. The agreement between values obtained from the fitting function and sample parameter is
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Figure 3.4: Values for the radii obtained from model versus the simulated radii. The blue line corresponds to the ideal case $r_{mod} = r_{sim}$. The errorbars correspond to one standard deviation.

Figure 3.5: Values for the radii obtained from model versus the known radii from the sample used in the experiment. The black line corresponds to the ideal $r_{mod} = r_{exp}$ and the errorbars correspond to one standard deviation.

quite good for both radius and distance. Except for the smallest radius which is off by 20%, the modeled radii $r_{mod}$ lie within 12% of the known sample radii $r_{exp}$. For the distance, all modeled
3.4. Results and Discussion

Figure 3.6: Values for the distances obtained from model versus the known distances from the sample used in the experiment. The black line corresponds to the ideal $d_{mod} = d_{exp}$ and the errorbars correspond to one standard deviation.

values $d_{mod}$ lie within 25% of the sample distances $d_{exp}$. The discrepancy can be explained by the fact that there is an uncertainty in the actual radii of the holes and distances between the holes of the experimental sample, which are due to fabrication and were not taken into account [75]. A Scanning electron microscope (SEM) image of one of the samples which illustrates this can be found in Figure 3.7.

Since the sample parameter for $r_{exp}$ is used to determine the modeled value for $d_{mod}$, and the sample parameter for $d_{exp}$ is used to determine the modeled value for $r_{mod}$, this will lead to an additional error. Furthermore, the sample in the experiment may not have been perfectly aligned, making the cylinder axes not perfectly perpendicular to the beam axis. This may explain the remaining discrepancies. The Fill ratio of the sample is connected to the average distance and radius of the holes. Thus knowing the radius of the holes and the thickness of the sample, the average distance could be extracted from the absorption image or $M_0$, if the attenuation properties of the sample are also known. However, the aim of this work was to investigate the relationship of the higher statistical moments and the structural sample parameters. The results indicate that in the case of the well defined sample presented here, using the proposed model fitting function, sub pixel parameters can be extracted from the GI-USAXS scattering distribution within an accuracy of 25%.
Figure 3.7: Scanning electron microscope (SEM) image of the cross section of a small area of one of the samples used in the experiment (radius $r = 1.92 \, \mu$m and minimal distance $d = 7.03 \, \mu$m). This image illustrates the imperfections in the sample used in the experiment.

## 3.5 Conclusion

Simulations of GI-USAXS for a PMMA slab with cylindrical holes of different radii and hole distances were performed using a simulation framework that combines MC methods with wave-optics for the simulation of GI imaging. The framework was used for the simulation of phase-stepping curves of the sample. From these simulated phase-stepping curves, scattering distribution was obtained through a deconvolution and the moments of this distribution were calculated. A fitting function was proposed to fit the fourth root of the kurtosis as a function of the sample radii and distances. This function was then used to extract sample parameters from the moments of the obtained scattering distribution, which were compared to the parameters used in the simulation. Further, the model was applied to experimental results obtained at TOMCAT beamline and the parameters retrieved from the simulation compared to the actual sample parameter. The so obtained sample parameters agree within 20% the known sample parameters for the radius and within 25% for the distance. The discrepancies can be explained by uncertainties in the experimental sample parameters such as the actual hole radii and alignment issues within the experiment. These results show that the moments of the scattering distribution contain sub-pixel structural parameters of the sample, which in the case of the well defined samples considered
here, can be modeled. This opens the possibility to retrieve quantitative structural information about a sample through GI-USAXS. A next step for developing a more general model for the connection between the moments of the scattering distribution and the sample structure, would be to apply the approach presented here to different types of samples consisting of different types of materials and having different types of sample structures.
Chapter 4

Simulations of grating interferometry using Monte Carlo methods

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4.1 Abstract

X-ray Grating interferometry (GI) is a well established technique for phase-sensitive X-ray imaging, with the advantage of providing three complimentary types of contrast: absorption, phase and dark-field contrast. A framework for numerical simulations of phase-sensitive X-ray imaging using Monte Carlo methods (MC) has been developed and applied to GI. The framework is based on EGS++, the C++ interface of the well established EGSnrc MC particle transport code. It was extended to account for the refraction and diffraction phenomena which are relevant for phase-sensitive X-ray imaging. To include interference phenomena relevant for GI within this particle model, Huygens principle was implemented into the transport code using a particle splitting method.

The framework was used for the simulation of both parallel beam and fan beam GI setups and showed good agreement both with the theoretical expectations and measurements (correlation coefficient >0.9). It was further used for the determination of the visibility of a GI setup with a polychromatic X-ray source. The framework was thus shown to provide reliable simulation of the GI image formation process and to be a useful tool for the optimization of experimental setups both in case of synchrotron radiation sources as well as for conventional X-ray tube setups.

4.2 Introduction

Phase-sensitive X-ray imaging using grating interferometry (GI) is a recently established imaging method [1, 2] with applications in a wide variety of fields such as material sciences, biology and medicine [31, 13, 40]. Compared to other phase-sensitive X-ray imaging techniques, GI has the advantage of producing three complementary types of contrasts from the same set of measurement: absorption, differential phase and dark-field contrast. Absorption contrast is the change of the intensity of the X-ray beam due to the sample and is described by the imaginary part $\beta$ of the decrement of the refractive index $n = 1 - \delta + i\beta$. Phase-contrast occurs due to the phase-shift in the X-ray wave-front due to the sample and is connected to the real part $\delta$ of the decrement of the refractive index $n$. Dark-field contrast is related to scattering within the sample originating from subpixel size inhomogeneities that affect the refraction properties, thereby revealing information about the substructure of the sample. This is of high interest for instance in mammography [11, 36].

The general setup for GI with a parallel beam X-ray source consists of a sample, a phase grating, an absorption grating and a detector. Using the Talbot effect, access to the phase-shift, absorp-
Chapter 4. The full MC model

tion and scattering signal of a sample can be obtained. If the first grating $G_1$ in the GI setup is a phase-grating with period $p_1$, a periodic phase-shift is introduced into the incoming X-ray beam. If the phase-grating introduces a $\pi$ phase-shift, this leads to an interference pattern of half the period $p_1/2$ of the phase-grating at the so called fractional Talbot distances $d_m$. For a parallel beam and a phase-grating with a phase shift of $\pi$, these distances are given by $d_m = \frac{p_1^2}{4\lambda}(m - \frac{1}{2})$ with $m \in \mathbb{N}$, where $\lambda$ is the wavelength of the X-rays [45]. The period $p_1$ of the phase-grating is usually set in the order of a few micrometers. Thus to resolve the interference pattern with a detector of larger pixel size than the interference pattern period, a second, absorbing grating $G_2$ with a period $p_2 = \frac{p_1}{2}$ that matches the interference pattern period is introduced into the beam directly in front of the detector. If one of the gratings is moved perpendicular to the interference pattern over the period $p_2$, a variation of the intensity on the detector as a function of the grating position can be observed. This is the so called phase-stepping curve [3]. When a sample is placed in the beam in front of the phase-grating, the attenuation and refraction of the beam by the sample leads to an altered interference pattern, which leads to a modified phase-stepping curve. The absorption, differential phase (DPC) and dark-field signal can be obtained from the two phase-stepping curves through a Fourier component analysis [5, 46]. In case of an X-ray tube source, an additional absorption grating is often placed directly in front of the source. This source grating creates an array of small sources and thereby increases the spatial coherence of the system [9, 10].

Numerical simulations are a convenient tool in gaining a deeper understanding of the image formation process and also for optimizing setup parameters of experimental GI setups. To obtain an accurate description of the physics occurring during the image formation process, all relevant effects need to be considered. In numerical modeling of the contrast formation processes occurring in X-ray GI imaging, both particle- and wave-view offer their benefits and drawbacks. The particle-view for instance is well suited for the description of scattering events, but neglects coherent effects such as interference. The wave-view on the other hand, is a convenient way to simulate interference and phase-shifts, but incoherent scattering such as Compton scattering cannot be modeled in a straightforward way. Thus, realistic numerical simulations of phase-sensitive X-ray imaging need to take both aspects into account. In a previous model [74] this challenge was addressed by splitting the model into two parts. The sample and the physical interactions therein were simulated using Monte Carlo methods (MC), and the interferometer consisting of two gratings was modeled using wave-optics. However, this approach has the drawback that information of the individual particle after the sample is lost due to the transformation of the X-rays into a wave. An other approach is to separately do MC and wave-optics simulation and combine the results of the two at the detector [77, 78]. In this work, the whole imaging process was modeled using MC by including Huygens principle into the MC simulation. This approach
4.3 Methods and Materials

4.3.1 The simulation framework

A framework for numerical simulations of phase-sensitive X-ray imaging has been implemented into egs++ and applied to GI imaging. Since the refraction of X-rays is not taken into account by EGSnrc, but is necessary for the GI image formation process, the particle transport code had to be extended. This was done according to the following principle: The photons are created at the source with initial values for position, direction, energy and initial phase. These values can be set by the user, which allows for the simulation of parallel beam sources with high spatial coherence, as for instance, can be found in synchrotrons, but also allows the simulation of less coherent sources such as conventional X-ray tubes. From the source, the particles are transported through the simulation geometry using the egs++ geometry package and the EGSnrc MC transport routines. Particle transport within egs++ occurs step-wise from one part of the simulation geometry or interaction site to the next. The phase-shift $\Phi(p)$ that occurs when X-rays are passing through a material is considered through a variable corresponding to the optical path length. This is achieved by determining the path $s_i$ at each step $i$ and adding up the corresponding phase shift

$$\Phi(p) = k \sum_i \delta_i s_i,$$

(4.1)

where $k$ is the wavenumber, and $\delta_i$ the real part $\delta$ of the decrement of the refractive index $n$ of the material through which the particle is transported in step $i$.

The refraction that occurs at interfaces of different materials has also been implemented through using Snell’s law

$$\frac{\sin \Theta_i}{\sin \Theta_r} = \frac{n_r}{n_i}$$

(4.2)

where $\Theta_i$ is the angle of incidence, $\Theta_r$ is the angle of refraction and $n_i$ and $n_r$ are the refractive indices of the two materials. For a particle arriving at the surface between two materials, the angle of refraction is determined using Equation 4.2 and the direction of the particle is adjusted accordingly. A similar method of implementing Snell’s law into MC has also been described by
Wang et al [6]. To adequately consider the interference phenomena that occur in GI, within this framework, a particle splitting method at a specific plane was implemented. The plane for the particle splitting was set directly behind the phase grating. Each particle arriving at this plane is considered to be the source of a spherical wave, in accordance with the Huygens principle. This was implemented by copying each particle arriving at the plane $n_p$ times and assigning each copy a new direction. Each of the copies has then exactly the same parameters as the original particle except for the direction. The directions of the copies are sampled uniformly over the space angle, thereby creating a point source from each particle arriving at this plane. The particles are then transported through the remaining geometry to the detector, according to their new direction. This is the difference in the approach presented here from the one proposed by Cipiccia et al. [79], where only a new angle is assigned to the particle without the copying, which makes it necessary to simulate a larger number of incident particles for sufficient statistics. Thus the approach proposed here is more efficient with respect to simulation time. In the plane of the detector, the particles are binned according to their position and summed up in each bin under consideration of their phase to obtain the intensity. Thus contribution to the signal on the detector of an individual photon can be calculated. A sketch of the framework can be found in Figure 4.1.

Within the model, source size and coherence can be taken into account through the definition of the source, which determines the initial position, direction and phase of a particle in the
simulation. It further is possible to simulate particle sources with a polychromatic initial energy spectrum such as for instance an X-ray tube. Thus the particle energy has to be taken into account for the determination of the refraction angle as well as for the calculation of the phase-shift. Both refraction angle (Equation 4.2) and phase-shift (Equation 4.1) are described by the real part of refractive index \( n = 1 - \delta + i\beta \) of the considered material. The Energy \( E \) of the photons is connected to the wavelength \( \lambda \) through \( E = \frac{hc}{\lambda} \), where \( c \) is the speed of light and \( h \) corresponds to the Planck constant. The real decrement \( \delta \) for a material consisting of \( M \) different elements and X-rays with wavelength \( \lambda \) is given by [29]

\[
\delta = \frac{\lambda^2 r_e^2}{2\pi} \sum_{i=1}^{M} f_{1i} N_i,
\]

where the sum runs over all elements within the material, \( f_{1i} \) is the real part of the atomic scattering factor of element \( i \), \( N_i \) stands for the number of atoms of element \( i \) per unit volume and \( r_e = 2.82 \cdot 10^{-15} m \) denotes the classical electron radius. For energies sufficiently far away from an absorption edge, the scattering factor is approximately equal to the atomic number \( f_{1i} \approx Z_i \) [30]. Energy resolution at the detector can be achieved through additionally to their position, also binning the particle according to their energy. This leads to an energy resolved simulation result and thus allows to analyze the influence of the spectrum on the obtained signal.

### 4.3.2 The polychromatic X-ray simulation

The MC model was used to simulate the interference pattern of a parallel beam setup with a design energy of 25 keV and an energy spectrum as could be found in an X-ray tube. The period \( p_1 \) of the phase grating was set to 4 \( \mu m \) and the intensity was calculated at 121 mm corresponding to the third fractional Talbot distance. In a first simulation, the intensity at the detector without an absorption grating \( G_2 \) was simulated to observe the interference pattern. In a next step, a phase-stepping curve with 8 steps was simulated. This was done by including the absorption grating \( G_2 \) with a period \( p_2 \) of 2 \( \mu m \) and simulating 8 different positions, i.e. 8 phase-steps for \( G_2 \) over \( p_2 \). From the so obtained phase-stepping curve, the visibility was calculated as a function of the energy. The field of view (FOV) was set to 200 \( \mu m \). Additionally, a second set of simulations was performed including a PMMA cylinder of 100 \( \mu m \) diameter. 8 phase-steps over one period were simulated and the DPC image obtained using a Fourier component analysis. The pixel size was set to 8 \( \mu m \). For each phase step \( 8 \cdot 10^6 \) particles were simulated. In all cases the cut-off energy of photons and electrons in the simulation was set to 10 keV.
4.3.3 Comparison with experimental data

To validate the results obtained with this MC model, a comparison of simulation results with experimental data was performed. A plastic phantom was measured at the beamline for Tomographic Microscopy and Coherent Radiology experiments (TOMCAT) of the Swiss Light Source [18] and compared to simulation results using the same setup. The phantom consists of a polystyrene (PS) cylinder with five holes filled with different mixtures of water and ethanol. The ratio of water and ethanol of these mixtures are: 1:0, 2:1, 1:1, 1:2 and 0:1. A schematic depiction of the phantom can be found in Figure 4.4 (left). To avoid phase-wrapping, the phantom was placed in a water filled aquarium[12]. For the comparison, one DPC projection image was acquired at an energy of 25 keV. The pitch of the phase grating was $p_1 = 4 \mu m$ and the pitch of the absorption grating was $p_2 = p_1/2 = 2 \mu m$. The inter grating distance $z$ was set to 121 mm, which corresponds to the third fractional Talbot order for 25 keV, $z = 3p_2^2/8\lambda$, where $\lambda$ is the wavelength [45]. Five phase-steps were acquired over one period. The projection image of the measurement consists of 1622 pixel with a size of 7.4 $\mu m$. To compare the measurements and simulations the measured projection was binned to 162 pixels.

4.3.4 The X-ray tube simulation

In addition to the above mentioned parallel beam setup, the MC framework was used to simulate the interference pattern from a GI X-ray tube setup with a design energy of 28.3 keV. This setup consists of a 38 kVp X-ray tube, a source grating $G_0$ with period $p_0 = 14 \mu m$ and a phase grating $G_1$ with period $p_1 = 3.5 \mu m$. The total setup length was set to 1.64 m, with a distance between $G_1$ and detector plane of 230 mm and a distance between $G_0$ and $G_1$ of 1.4 m. The distance from source to $G_0$ was set to 10 mm. To investigate the effect of the source size on the interference pattern, two simulations were performed. The first simulation was performed with a point source and for the second simulation, the source size was set to $800 \times 400 \mu m^2$. The simulations were conducted with an energy spectrum of an X-ray tube with a tungsten anode and a tube voltage of 38 kVp, which was calculated using SpekCalc [80, 81]. In this case the FOV was set to 12 $\mu m$ with a pixel size of 0.1 $\mu m$. The cut-off energy in the simulations was set to 10 keV to reduce simulation time.

4.3.5 Dependence of the visibility on the height of the absorption grating

Grating fabrication for GI is a challenging task, especially in case of the absorption gratings, where the height of the structure is often required to be much larger than the pitch [82, 83, 84] to
ensure sufficient attenuation of the radiation. Insufficient attenuation of the radiation leads to a reduced visibility of the setup and thus lowers setup performance. In this section, the influence of the height of the absorption grating on a polychromatic system is investigated using the presented model. For this investigation, a parallel beam setup with a uniform energy spectrum with a range from 10 to 40 keV was simulated. The design energy for the system is 24 keV, the phase grating consists of silicon and has a period \( p_1 = 2.4 \mu m \) and height \( h_1 = 30 \mu m \) for a \( \pi \) phase-shift. The inter-grating distance \( z \) was set to 9.78 cm, which corresponds to the seventh fractional Talbot distance for 24 keV. The energy range, grating parameters and inter-grating distance correspond to those of a setup that could be used for mammography using GI [11, 36].

To investigate the influence of the height on the visibility, in a second set of simulations a gold absorption grating \( G_2 \) with a period \( p_2 = 1.2 \mu m \) was included into the simulation and three different heights \( h_2 \) for \( G_2 \) were simulated: 25 \( \mu m \), 50 \( \mu m \) and 100 \( \mu m \). The absorption at the design energy 24 keV for these heights are 90%, 99% and 99.99% [30]. For each grating height \( h_2 \) five phase-steps were simulated and the visibility determined [3, 9].

<table>
<thead>
<tr>
<th>phase grating period</th>
<th>( p_1 = 2.4 \mu m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>inter-grating distance</td>
<td>( z = 9.78 \text{ cm} )</td>
</tr>
<tr>
<td>absorption grating period</td>
<td>( p_2 = 1.2 \mu m )</td>
</tr>
<tr>
<td>absorption grating height</td>
<td>( h_2 = 25 \mu m, 50 \mu m, 100 \mu m )</td>
</tr>
<tr>
<td>design energy</td>
<td>24 keV</td>
</tr>
<tr>
<td>cut-off energy</td>
<td>10 keV</td>
</tr>
<tr>
<td>number of phase-steps</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 4.1: Simulation parameters for the simulations of the different heights of the absorption grating \( G_2 \).

The simulation parameters are also listed in Table 4.1.

### 4.4 Results and discussion

#### 4.4.1 The polychromatic X-ray simulation

As described in section 4.3.2 the framework was used to simulate an interference pattern for a parallel beam setup with an X-ray tube spectrum and without \( G_2 \). The FOV at the detector was set to 20 \( \mu m \). The resulting intensity at the detector is shown in the following figure. Figure 4.2
(left) shows the intensity as a function of the energy. It can be seen that the interference pattern has the desired period of $2 \mu m$ at 25 keV, while for higher and lower energies the interference pattern becomes less ideal. Figure 4.2 (right) shows the intensity at the detector for the design energy of 25 keV and the normalized intensity of the total energy spectrum. The figure clearly shows that for 25 keV an almost perfect intensity pattern is obtained, as would be expected. For the whole spectrum, an interference pattern is still present, but the maxima vary and the minima do not reach zero. This occurs because there is no full destructive interference, which reduces visibility. The double structure, meaning a maximum at 100% followed by a maximum that only reaches 80% is due to the fact that the setup meets the condition of third fractional Talbot distance and $\pi$ phase-shift only for the design energy. For the other energies both phase-shift and inter-grating distance are off. The same simulation was also performed using wave-optics simulations to check the accuracy of the implemented MC model. The comparison in Figure 4.2 (right) shows that the wave-optics simulation matches the MC simulation very well. These results demonstrate that the implemented MC model works for a polychromatic energy spectrum.

In a next step, $G_2$ was included to simulate a phase-stepping curve with 8 phase-steps. Figure 4.3 shows the visibility of the PSC at the detector as a function of energy. The visibility reaches the maximum at the design energy of 25 keV and a minimum at 19 keV where the inter-grating distance corresponds to an even Talbot order. A second set of simulations was performed including a PMMA cylinder of 100 $\mu m$ diameter. Figure 4.3 (right) shows the differential phase
4.4. Results and discussion

Figure 4.3: Visibility obtained at the detector as a function of energy (left) for a parallel beam setup with an energy spectrum from a 38 kVp X-ray tube, a design energy of 25 keV at 121 mm (the third fractional Talbot distance). The right shows the DPC signal of a 100 $\mu$m diameter PMMA cylinder with a pixel size of 8 $\mu$m for the same setup, integrated over all energies.

contrast (DPC) signal of the cylinder. The DPC signal shows the first derivative of a circle, which would be expected from a cylindrical sample.

4.4.2 Comparison to experimental data

To validate the presented MC framework, simulations of a projection image of a phantom obtained with the framework were compared to a projection of the same phantom obtained at the TOMCAT beamline. For this purpose the same projection of the phantom that was experimentally obtained, was simulated. The phantom consists of a polystyrene cylinder with five holes filled with different water ethanol mixtures. A sketch of this phantom can be found in Figure 4.4 (left). Five phase-steps were simulated with $1.5 \cdot 10^9$ particles per phase-step. Figure 4.4 (right) shows the comparison of the DPC signal for both simulated and measured projection. It can be observed that the noise is considerably higher in the simulation, but the two signals agree with a correlation coefficient of 0.91.
4.4.3 The X-ray tube simulation

The model was further used to simulate an interference pattern generated by an X-ray tube setup. In one situation a point source was assumed, in the second situation a source of $400 \times 800 \mu m$ was simulated. The resulting interference patterns as a function of the energy are shown in Figure 4.5. It can clearly be seen that for the point source the interference pattern is much more uniform than for the finite source.

To further demonstrate the differences between the two situations, a comparison of the total intensity obtained at the detector is shown in Figure 4.6 (left). The design energy for this setup was 28.3 keV. Unlike in the case of the parallel beam setup, the interference pattern is less rectangular due to the beam divergence. For the finite source there is a bigger variation in height of the maxima, which is due to the smaller coherence (coherence length of 9 $\mu m$ at 28 keV [9]) and decrease of intensity towards the edges can be observed due to the fan geometry. The latter can also be observed for the point source. The normalized intensity at the detector is shown for the design energy and the total energy for the case of the finite source in Figure 4.6 (right). In this case it is visible that the differences between maxima and minima of the interference pattern, and thereby the visibility, is considerably larger for the design energy than for the total signal. This is to be expected since, as in case of the parallel beam setup, the phase-shift of the grating and the inter-grating distance do not match for the other energies. Unlike the parallel beam setup, the design energy also does not produce an ideal interference pattern. This is on one
4.4. Results and discussion

Figure 4.5: Normalized intensity along the grating direction at the detector as a function of energy for a point source (left) and for a source size of $800 \times 400 \, \mu m$ (right).

Figure 4.6: Total intensity along the grating direction at the detector for a source size of $400 \times 800 \, \mu m$ and for a point source (left). And the comparison of total intensity and the intensity for the design energy for the finite source size (right). For comparison, all signals are normalized to the maximum.

hand due to the beam divergence, and on the other hand due to decreased spatial coherence due to the finite source size. The visibility of the system is 35\% in case of the point source. The finite source leads to a decreased coherence, which leads to a visibility of 17\%. This corresponds to the expected outcome.
4.4.4 Dependence of the visibility on the height of the absorption grating

Figure 4.7 shows the simulated interference pattern along the detector as a function of the energy without absorption grating. The field of view in the simulation was set to 20 $\mu$m, and the cut-off energy was set to 10 keV for both photons and electrons. The matching pitch of the interference pattern at the design energy with half the grating period, $p_1/2 = 1.2 \mu$m, is clearly visible. It can also be observed that the interference pattern almost vanishes at 21 keV and at 28 keV. This is due to the fact that the inter-grating distance of 9.78 cm corresponds to an even Talbot distance for a $\pi$ phase-shift grating for those two energies.

To investigate the influence of the height on the visibility, in a second set of simulations a gold absorption grating was included into the simulation and three different heights $h_2$ for $G_2$ were simulated and the visibility determined. Figure 4.8 shows the visibility as a function of the energy for the three different heights. In all cases the visibility is at a global maximum at the design energy. As can be expected [85], the visibility oscillates as a function of the energy, the minima are at the energies for which the inter-grating distance corresponds to an even fractional Talbot order, the maxima lie at the energies for which the fractional Talbot order is odd. The main differences between the visibility for the different heights $h_2$ are at the higher energies above 28 keV. This is due to the fact that attenuation decreases with increasing energy (see
4.5 Conclusion and Outlook

A framework for the numerical simulation of phase-sensitive X-ray imaging using GI based on Monte Carlo methods has been presented. To account for the diffraction phenomena that need to be considered in the simulation of GI, Huygens principle has been implemented into an exist-
ing MC framework. This was achieved using a particle splitting method to obtain a wave front generated by point sources. Furthermore phase-shift and refraction were included to obtain the necessary wave properties for simulating interference effects. To validate the framework different situations were simulated, both for parallel beam and fan beam setup. It was shown that the simulated results match the theoretical expectations and show a good agreement with measured data. The framework was further used to predict the visibility of an experimental GI setup as a function of the absorption grating height. This shows that the relevant physical processes are considered adequately within the presented framework and validates the framework as a tool for setup performance prediction and optimization.

In a next step, this framework can be used for the optimization of different setups and the prediction of image quality for different scenarios. One possibility would be to use the framework to predict the expected visibility of an X-ray tube setup for different parameters such as grating pitch, Talbot order or energy spectra, which could help in optimizing an experimental system. Another application would be the determination of contrast that can be expected for a specific sample on a specific setup. In principle, the framework presented here is not limited to GI, but could further be applied to other imaging methods such as inline phase-contrast imaging or coded aperture imaging.
Chapter 5

A Monte Carlo model for dose calculations at TOMCAT beamline

In this Chapter a Monte Carlo model for radiation dose calculations at the beamline for TOmographic Microscopy and Coherent rAdiology experimenTs (TOMCAT) [18] is presented. The model was applied to the case of in-vivo tomographic lung imaging in small animals, which is a contemporary area of research [19]. In in-vivo tomographic lung imaging, precise knowledge of the applied radiation dose is important for the determination of the health risk to the animal. The dose calculation routine presented here is implemented using egs++ [14], a C++ interface for the EGSnrc MC framework [15]. It is furthermore compatible with both the Hybrid model as well as the full MC model presented in Chapters 2 and 4. Thus, the same dose calculation routine can for example also be applied to Grating Interferometry (GI) setups utilizing X-ray tube sources as shown in related work [86, 87]. This is of increasing interest for instance in mammography using GI [11, 36], where accurate knowledge about the dose delivered to the patient or specimen imaged is of high interest.

Sections 5.1 to 5.6 of this chapter were also submitted under the title ”Implementation of a Monte Carlo model for dose calculation at TOMCAT beamline” as a master thesis for the Master of Advanced Studies in Medical Physics at ETH, that was part of this PhD project. The last Section, 5.7, contains an expansion of the model presented in the master thesis to include dose calculations based on a Computer Tomography (CT) data set of a small rat obtained at TOMCAT beamline.
5.1 Abstract

In-vivo tomographic lung imaging is an important contemporary topic of research at the beamline for TOmographic Microscopy and Coherent rAdiology experimenTs (TOMCAT). An important aspect of such in-vivo measurements is the dose delivered to the animal. In this work a Monte Carlo (MC) model of TOMCAT beamline has been implemented that allows the estimation of the dose accumulated by a small animal phantom. All beam defining elements in the beam path were included in the simulation. To estimate the dose distributions two phantoms were implemented: a water phantom and a small animal phantom. For both phantoms, calculations for different setups were performed and analyzed. Three energy settings were considered: a 21 keV monoenergetic beam, a 5% filtered white-beam and a 0.5% filtered white-beam. The results show that the maximal relative dose was accumulated in the region of interest (ROI) for the water sample and in the bones for the small animal phantom. This model can now be used to help determine the optimal setup for in-vivo tomography.

5.2 Introduction

High-resolution, in-vivo imaging of small animals is an important contemporary topic in research at TOMCAT (beamline for TOmographic Microscopy and Coherent rAdiology experimenTs) beamline [18]. One example is functional three dimensional lung imaging. So far in-vivo lung imaging has only been achieved in 2D or with low spatial and temporal resolution, in-vivo, high-resolution tomographic imaging therefore remains an open topic [88]. The high spatial and temporal resolution requires a high photon flux to obtain a reasonable contrast and can typically be found at a synchrotron facility, which provide the ability to obtain submicro-meter resolution images in a very short time. However a high flux as it can be found in a synchrotron facility, makes accurate dosimetry a challenging task. Nevertheless, accurate knowledge of the applied radiological dose is important for the determination of the health risk to the animal. Due to the health hazard of high radiological doses this may be a limiting factor. The dose accumulated by the animal is therefore an important factor in determination of the optimal setup with respect to size of the field of view (FOV), the used energy spectrum and the number of projections. In an inhomogeneous imaging sample, such as a rat or a mouse, not only the total absorbed dose is relevant for a health risk estimation, but also the distribution within the volume. Accurate dose distributions within an animal are difficult to measure. A convenient method to obtain realistic dose distributions in 3 dimensions are therefore Monte Carlo methods (MC). The advantage of MC is the possibility to calculate accurate three dimensional
dose distribution under consideration of the of scattering of both electrons and photons within an object. MC methods are a common way to obtain radiation dose distributions in radiotherapy and imaging [17] and in recent years several MC models of synchrotron beamlines have been published [89, 90].

In this work a MC model of TOMCAT beamline was implemented and used for the estimation of the dose distribution of two different phantoms for different energy settings. One of the phantoms was used to model the dose distribution within the torso of a small animal, to obtain a dose distribution such as it may occur in in-vivo lung imaging.

### 5.3 The MC model of the beamline

The MC model of the beamline was implemented using egs++, a C++ interface for the well established EGSnrc MC code [15, 53]. EGSnrc enables the simulation of particle transport of photons, electrons and positrons with energies between 1 keV and 1 GeV. It provides a source package and a geometry package to implement a particle source and a geometry through which the particles are transported. The advantage of egs++ compared to other frameworks such as BEAMnrc [91] is the flexibility of its geometry modules, which allows the simulation of a great variety of shapes and furthermore is compatible with the in house developed MC model for phase-sensitive x-ray imaging.

Within an egs++ simulation particles are created at the source and transported through the implemented geometry based on scattering probabilities. Each particle within the simulation has variables for position (x,y,z), direction (u,v,w), charge, energy, statistical weight and a variable called the latch, which can be used for the tracking of particle interactions. The locations of the interactions of a particle are determined based on probability distributions using random numbers. At the location where scattering occurs the direction, energy and latch are changed based on the probabilities of the scattering process. For the considered energy range which is below 110 keV, the relevant particle interactions in material are Rayleigh scattering, Compton scattering and photoelectric absorption, with the latter being the dominant interaction. All three processes are included in EGSnrc. The absorption cross-section for the photoelectric effect is dependent on the energy $E$ of the incoming particle, the density $\rho$ of the material and the atomic number $Z$ through[27]

$$\sigma_{pe} \propto \rho \frac{Z^n}{E^3}$$  \hspace{1cm} (5.1)
where $n = 3.5$ for low $Z$ and $n = 3$ for high $Z$. The cross-section for Compton scattering is given by [27]

$$
\sigma_C \propto \rho \frac{Z}{A \cdot E}
$$

where $A$ is the atomic mass number. For the cross-section of the Rayleigh scattering the relationship

$$
\sigma_R \propto \rho \frac{Z^{1.5}}{E^2}
$$

is given [27]. The total attenuation cross-section can then be obtained through

$$
\sigma_{tot} = \sigma_{pe} + \sigma_C + \sigma_R
$$

and is related to the attenuation coefficient $\mu$ through [25]

$$
\frac{\mu}{\rho} = \frac{N_A}{A} \sigma_{tot}
$$

where $N_A$ is Avogadros number. For the egs++ simulations, the cross-sections are stored in the so called PEGS files which are user generated and need to be available for any simulated material. The generation of a PEGS file for any material requires the input of the elemental composition, the density and the particle energy transport range for electrons and photons. The transport range defines the range within which particles are transported. If the energy of a particle falls below the lower limit, it is discarded and its energy is locally deposited.

TOMCAT beamline at SLS [18] was modelled in the egs++ framework for dose calculations in small animal imaging. For the simulations only elements that are in direct contact with the beam were considered. Elements with no direct contact with the beam such as the walls of the vacuum tube were neglected, assuming that contributions to the beam originating from reflections on these elements can be neglected. The elements that were implemented in the MC model are described in more detail below. The model consists of a vacuum part including different elements and the air part including the sample. The different elements are described in more detail below. The two parts are separated by a thin kapton foil which has a negligible effect on the spectrum and therefore was not included in the simulation. A sketch of the model is shown in Figure 5.1. For further discussions, the coordinate system of the model was set such that the $z$-axis is parallel to the beam axis. The orientation of the coordinate system is also shown in Figure 5.1. For dose calculations the deposited energy can be scored in any geometric region of the model. At the end of a simulation the dose in those regions is stored in the dose file. In addition to scoring the deposited energy, planes can be defined where all particles that pass through are written out into a so called phase-space file. A phase-space file is a file containing a list for all variables of each particle crossing that plane. These files can be analysed using the object oriented data
analysis framework root [92]. To analyse a phase-space file with root, the phase-space file has first to be converted into a root-file. Within a root-file the variables of the particles are stored within a tree structure, making it possible to efficiently access the data during analysis. This makes root a very useful tool for instance to evaluate energy spectra or particle distributions.

**The different elements of the simulation**

**The source**

The photon source of the TOMCAT beamline is a 2.9 T superbend magnet with a critical energy of 11.1 keV [18]. The photon source has a size of $53 \times 16 \, \mu m^2$ with a divergence of 2 mrad and 0.6 mrad in horizontal and vertical direction\(^1\). There are two possible modes for imaging: monochromatic or white-beam. The white-beam mode has a very high photon flux and an energy spectrum ranging from 1 up to 100 keV and is shown in Figure 5.2. The formula for the calculation of the photon spectrum from a bending magnet source is provided in [94, 95].

For the monochromatic mode a double crystal multilayer monochromator is available [18], which covers an energy range from 8 to 45 keV with a relative bandwidth of a few percent. The photon flux decreases with increasing energy. In order to obtain a sufficient high flux but also an energy which is sufficiently high for imaging, the energy was set to 21 keV. Since the reflection of photons on a surface is not included within egsp++, the monochromator was not implemented within the MC model.

The source was implemented in the following way: the initial position was sampled from a

---

\(^1\)New measurements [93] show the source size to have a horizontal and vertical full-width-at-half-maximum FWHM of $125 \times 46 \, \mu m^2$, however, this should not have a big influence on the dosimetry results.
Figure 5.2: Initial spectrum used for the simulation of the white-beam. The spectrum was obtained from theoretical calculations [94, 95].

Gaussian distribution with a width corresponding to the source size. For the simulation of the monoenergetic beam, the energy was set to 21 keV monoenergetic, neglecting the bandwidth of the monochromator. For the white beam modus, the source was implemented with the spectrum obtained from theoretical calculations$^2$. The beam divergence was implemented by setting the area of the cone at the plane in the middle of the sample slightly bigger than the field of view (FOV), such as to keep the simulation time reasonably low. The direction is then sampled uniformly, by picking the vector from the source to a randomly selected point in the plane.

**The front end slits**

The front end slits are the first pair of slits after the source and define the area of the beam that will be used for imaging. One of the functions of the slits is to limit the amount of the beam that reaches the monochromator as to not cause overheating. They are implemented in the simulations as two sets of planes the first with a slit in x-direction and the second with a slit in y-direction, forming a rectangular aperture. The size of the x- and y-slit can be set individually to any desired size.

---

$^2$For the simulations, the energy spectrum was assumed to be independent of position within the source. In reality this is not the case, and will have an influence on the spectra obtained after the filter.
The CVD-window

The Chemical Vapour Deposited (CVD) diamond window separates the ultra high vacuum section ($10^{-10}$ bar) from the high vacuum section ($10^{-7}$ bar). It consists of Chemical Vapour Deposited diamond window of 100 $\mu$m thickness and was implemented in the simulation as an infinite plane in the x-y-plane.

The white beam filters

There are three different filters available to reduce the white beam flux and harden the spectrum: a 50%, 5% and 0.5% filter. The white-beam filters limit the power flux density to 50%, 5% and 0.5% of its value at the CVD-window. The materials and thicknesses of each filter is listed in table 5.1. The filters are implemented as infinite planes in x- and y-direction consisting of the corresponding materials and thicknesses. Sigradur which is used in all filters is a high purity glassy carbon with high temperature resistance and little surface roughness. The spectra after the different filters are shown in the results section.

<table>
<thead>
<tr>
<th>Filter</th>
<th>50%</th>
<th>5%</th>
<th>0.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sigradur</td>
<td>5 mm</td>
<td>20 mm</td>
<td>20 mm</td>
</tr>
<tr>
<td>Molybdenum</td>
<td>0</td>
<td>75 $\mu$m</td>
<td>350 $\mu$m</td>
</tr>
</tbody>
</table>

Table 5.1: The different filter thicknesses and materials for the white-beam filters.

Optical filters

A second battery of filter exists which are used to increase beam quality of the monoenergetic beam by filtering out residual photons of other energies. It is implemented in the same fashion as the white-beam filters. The materials and thickness of these filters are shown in table 5.2. Unlike the white-beam filters, the optical filters can be used in any desired combination.
5.3. The MC model of the beamline

<table>
<thead>
<tr>
<th></th>
<th>filter 1</th>
<th>filter 2</th>
<th>filter 3</th>
<th>filter 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>first</td>
<td>100 µm Al</td>
<td>50 µm Al</td>
<td>10 µm Fe</td>
<td>-</td>
</tr>
<tr>
<td>second</td>
<td>200 µm Al</td>
<td>10 µm Cu</td>
<td>50 µm Fe</td>
<td>10 µm Cu</td>
</tr>
<tr>
<td>third</td>
<td>400 µm Al</td>
<td>40 µm Cu</td>
<td>10 µm Cu</td>
<td>100 µm Al</td>
</tr>
<tr>
<td>fourth</td>
<td>20 µm Cu</td>
<td>50 µm Cu</td>
<td>100 µm Al</td>
<td>200 µm Al</td>
</tr>
</tbody>
</table>

Table 5.2: The different filter thickness and materials for the optical filters.

The last slits

The last slits are the last set of slits before the sample and can be used to define the field of view (FOV), although this can also be done by the front end slits. However, the MS slits block the scattered radiation outside the FOV and due to the beam divergence define a sharper edge of the FOV. They are implemented in the same manner as the front-end slits but are outside of the vacuum.

The phantoms

Two different phantoms are implemented for the dose calculations and positioned at 25 m from the source. The first phantom is a water cylinder with 7.5 mm radius. The axis of the cylinder is parallel to the y-axis (see Figure 5.1). To get a spatial resolution of the dose deposited in the water the volume within the cylinder has to be divided into different areas through a grid. For this a cube with a length of 7.5 mm was divided into a grid of $200 \times 200 \times 200$ voxels in x, y and z direction. The volumes in the cube belonging to the cylinder were filled with water. Since the phantom is invariant along the y-axis only one voxel slice was considered, creating a grid with $200 \times 1 \times 200$ voxels. The energy deposited in each volume by a particle can then be scored throughout the simulation using a special energy scoring class [53]. For the small animal phantom the material of the cylinder was set to soft tissue instead of water with inserted areas for air and bone. The air is inserted to simulate the lungs and the bones are used to represent the ribs and the spine. An image of the small animal phantom is shown in Figure 5.3.

Table 5.3 shows the elemental compositions for the materials used in the phantom. The compositions of the tissue and bone were taken from the ICRU report 44 [96], which were assumed to
Figure 5.3: Image of the small animal phantom, the cylinder consists of soft tissue which is displayed in yellow and has inserts for bones, displayed in red and air displayed in blue.

<table>
<thead>
<tr>
<th>Density $[g/cm^3]$</th>
<th>Tissue</th>
<th>Bone</th>
<th>Air</th>
<th>Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0.102</td>
<td>0.047</td>
<td>-</td>
<td>0.1119</td>
</tr>
<tr>
<td>C</td>
<td>0.143</td>
<td>0.144</td>
<td>0.000142</td>
<td>-</td>
</tr>
<tr>
<td>N</td>
<td>0.034</td>
<td>0.042</td>
<td>0.7552</td>
<td>-</td>
</tr>
<tr>
<td>O</td>
<td>0.708</td>
<td>0.446</td>
<td>0.2318</td>
<td>0.8881</td>
</tr>
<tr>
<td>Na</td>
<td>0.002</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Mg</td>
<td>-</td>
<td>0.0022</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P</td>
<td>0.003</td>
<td>0.105</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>S</td>
<td>0.003</td>
<td>0.00315</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Cl</td>
<td>0.002</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ag</td>
<td>-</td>
<td>-</td>
<td>0.00128</td>
<td>-</td>
</tr>
<tr>
<td>K</td>
<td>0.003</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ca</td>
<td>-</td>
<td>0.21</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Zn</td>
<td>-</td>
<td>0.0001</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5.3: The elemental composition of the different materials used in the small animal phantom. The ratios are given per weight.

be sufficiently close to the compositions of small animal tissue. However, due to the $Z^3$ dependence of the photoelectric absorption (see Eq. 5.1) small variations in composition, especially of high $Z$ elements can have a high impact on the absorbed energy and therefore the total dose. The cut-off energy for photons of all materials was set to 1 keV to account for all energies, the cut-off energy for electrons was set to 10 keV, to keep the simulation time reasonably low. To
simulate local tomography, a region of interest (ROI) within the phantom can be selected and the phantom is then rotated around a fixed axis in the middle of the ROI\(^3\).

**Implementation of the tomography routine**

The simulation of the MC model was running on a cluster where in average 150 nodes are available for calculation.

For tomography, projection images are obtained at different beam angles over \(180^\circ\) or \(360^\circ\) and then reconstructed. There are two possibilities to acquire tomograms. One is to use a shutter and only irradiate the sample when a projection is taken. The other possibility is to leave the beam on continuously and take projections during rotation. In the simulation this difference can be made by either only simulating the discrete angles at which projections are taken or to simulate also angles in between. Therefore two possibilities are implemented. The first is to simulate a number of fixed angles over \(180^\circ\) or \(360^\circ\), in which case the simulation is split into one job per discrete angle and submitted to the nodes. The second method is to simulate a high number of randomly picked angles over the same range which are submitted to the nodes.

The total number of particle histories \(N_{\text{tot}}\) calculated in a full simulation can then be calculated by

\[
N_{\text{tot}} = N_p \cdot N_s \cdot N_a
\]

where \(N_p\) is the number of particles per angle, \(N_s\) is the number of simulations per angle and \(N_a\) is the number of angles. For a full tomographic simulation the user is required to provide the initial input-file for the simulation. To keep the handling user friendly, the default input-file contains all possible elements and they can be activated by changing only one line within the file.

**Simulation parameters**

The beamline model was used for the calculation of the energy spectra of the different white-beam settings and for dose distributions for the two phantoms for different settings. The number of histories simulated for the energy spectra was \(2 \cdot 10^6\) for the 50% filter, \(2 \cdot 10^7\) for the 5% filter and \(2 \cdot 10^8\) for the 0.5% filter. Dose distributions are simulated for different FOVs and

---

\(^3\)Here, the region of interest (ROI) is defined as the area within the phantom that is imaged. This means that the ROI corresponds to the area of the sample that appears within the tomographic image. The field of view (FOV) defines the size of the projection images used to reconstruct the tomogramm. The size of the ROI is determined by the selection of the field of view in the projections, where the diameter of the ROI (i.e. the length of the tomographic image) corresponds to the length of the FOV perpendicular to the rotation axis.
energy spectra that may occur in an experiment. Three energy settings are considered, the 21 keV monoenergetic beam, the 5% white-beam and the 50% white-beam. The FOVs were chose as \(848 \times 882 \, \mu m^2\) and \(445 \times 375 \, \mu m^2\). For the dose distributions 2000 beams were simulated over 180°. The relative statistical uncertainty for all simulations was below 3% for 21 keV and below 8% for the white-beam. The orientation of the phantom and the lines along which the profiles were taken are shown in Figure 5.4.

For the small animal phantom and the water phantom in the 5% and 0.5% white-beam case the ROI\(^4\) was set 2 mm of the middle z-axis of the phantom as shown in Figure 5.4. For the water cylinder the profiles were taken along the lines through the ROI, for the small animal phantom the profiles were taken through the middle of the phantom as shown in Figure 5.4. Since the goal are relative dose distributions, all dose distributions were normalized to the maximal value of the distribution unless otherwise specified.

### 5.4 Determination of the dose

**Definition of dose**

In radiation physics the dose absorbed by a medium \(D_m\) is defined as the energy \(E_{ion}\) deposited in a medium by ionizing radiation divided by its mass \(M_m\) [27, 97]

\[
D_m = \frac{E_{ion}}{M_m}. \tag{5.7}
\]

\(^4\)The size of the ROI is determined by the FOV, the diameter of the ROI corresponds to the length of the FOV perpendicular to the rotation axis.

![Figure 5.4: Sketch of the orientation of the small animal phantom in the beam. The profiles were taken along the dashed line shown. The phantom was rotated along the axis through the middle of the ROI.](image)
The unit of the absorbed dose is Gray [Gy] with

\[ 1 \text{ Gy} = \frac{1 \text{ J}}{1 \text{ kg}}. \] (5.8)

The total dose \( D_T \) in an inhomogeneous volume can be calculated through the weighted sum of the dose deposited in the subvolumes \( D_i \)

\[ D_T = \frac{\sum D_i m_i}{\sum m_i}. \] (5.9)

### Calculation of the dose

Each simulation results in a binary file where the energy deposited in each subvolume of the phantom is stored. These energies can be evaluated using a Matlab routine where the energies for each volume of all projection are summed up and displayed in a 2 dimensional dose distribution. To obtain the estimated dose accumulated by the phantom, the energy which is stored in MeV must first be converted to J by

\[ 1 \text{ MeV} = 1.602 \cdot 10^{-13} \text{ J}. \] (5.10)

To calculate the dose, the energy \( E_p(x, y) \) in each voxel \((x,y)\) is first divided by the number of particles simulated from the source \( N_{tot} \)

\[ E_p(x, y) = \frac{E(x, y)}{N_{tot}}. \] (5.11)

This produces an independence of the result from the number of initial particles.

To determine the dose in each voxel deposited per particle \( D_p(x, y) \) the energy is divided by the mass of the voxel containing the point \((x,y)\)

\[ D_p(x, y) = \frac{E_p(x, y, t)}{M(x, y)}, \] (5.12)

where the mass can be calculated using the volume of the voxel \( dV = dx \times dy \times dz \)

\[ M(x, y) = \rho(x, y) \cdot dV, \] (5.13)

where \( \rho \) is the density of the material within a voxel. For the water cylinder the density is the same for all voxels, for the small animal phantom the densities range from 0.00123 g/cm\(^3\) to 1.6 g/cm\(^3\). The above determination of the dose was implemented in a MATLAB routine.

To make a connection to the dose accumulated by the phantom during an experiment, the dose
rate can be calculated. To obtain the dose rate, the dose per particle can be multiplied by the flux of the bending magnet source integrated over the area of the beam $F$ resulting in the energy deposition per time

$$D(x, y, t) = D_p(x, y) \cdot F. \tag{5.14}$$

The dose rate can be multiplied with the total acquisition time of the images to obtain the dose per tomographic scan. This was however not considered in the simulations in this work.

**Determination of statistical uncertainty**

To determine the statistical uncertainty in the dose calculations, the standard deviation of the energy deposited in each voxel was calculated. This was achieved, in a similar fashion as in [53], by additionally scoring the square of the energy deposited in each voxel of the phantom during a simulation. The standard deviation of the deposited energy can then be obtained through

$$s_E(x, y) = \frac{1}{N} \sqrt{\sum_i E_i(x, y)^2 - (\sum_i E_i(x, y))^2}, \tag{5.15}$$

where the sums run over all particle histories and $N$ is the total number of initial particles. The standard deviation for the dose can then be obtained through division of the mass. This process was implemented in the same Matlab routine as the dose calculation above.
5.5 Results

The white-beam spectra after the different filters

The energy spectra after each of the white-beam filters were simulated and are shown in Figures 5.5 to 5.7. The spectra were extracted from the simulation using a root-analysis file. The spectrum after the 50\% filter shown in Figure 5.5 has a similar shape as the spectrum at the source but is shifted to higher energies due to the stronger relative absorption of the lower energies in the filter. The mean energy of the spectrum is changed from 14 keV to 24 keV. The 50\% filter is not well suited for small animal imaging due to the high amount of low energy particles ($< 15$ keV). Low energy particles are mostly absorbed by the sample. This means that to obtain a good contrast to noise ration a high number of particles are needed, which leads to a high amount of dose.

The spectra for the 5\% filter is shown in Figure 5.6 and has a mean energy of 38.4 keV. The 0.5\% filter is shown in Figure 5.7 with a mean energy of 59 keV. Both spectra show a sharp peak close to 20 keV which originates from the absorption edge of the K-shell of molybdenum which is just below 20 keV. The peak is much lower for the 0.5\% filter due to the thicker molybdenum which leads to a stronger attenuation of the lower energies.

As mentioned previously, the energy spectrum for the simulations was assumed to be independent of position within the source. In reality this is not the case, and will have an influence on the spectra obtained after the filter.

![Figure 5.5: Spectrum after the 50\% filter. The number of initial particles was $2 \cdot 10^6$.](image-url)
Figure 5.6: Energy spectrum after the 5% filter. The 5% filter consists of 20 mm sigradur and 75 $\mu$m molybdenum. The number of initial particles was $2 \cdot 10^7$.

Figure 5.7: Spectrum after the 0.5% filter. The 0.5% filter consists of 20 mm sigradur and 350 $\mu$m molybdenum. The number of initial particles was $2 \cdot 10^8$.

The FOV in the monoenergetic case

To investigate the influence of the size of the FOV on the dose distributions, dose distributions for two different FOVs were made for the case of a monoenergetic beam using the homogeneous water cylinder phantom. Figures 5.8 and 5.9 show different dose distributions and profiles
through the distributions for a water phantom. It can be seen that the dose is highest in the center which corresponds to the imaged region of interest. The profiles were taken along the lines through the middle of the phantom as indicated in Figure 5.4.

The difference in the dose distributions occurs in ROI. The high dose area is narrower for the smaller FOV which is expected. For both cases the dose in the ROI is above 97% of the maximal dose and falls off steeply to all sides. The gradient is steeper for the smaller FOV, due to the smaller scattering volume. In the second case all particles outside a 25 pixel radius (1.9 mm) obtain less than 10% of the maximal dose. The figures also show that the dose in the volume to the left, which points towards the beam, is higher than the other side. This is due to the exponential nature of Beer-Lambert’s law of attenuation [27].

To compare the absorbed dose of the cylinder, the relative quantities for the total dose absorbed, the maximal dose and the mean dose for the two different field of views was normalized to the dose for the larger field of view and the values are compared in Table 5.4. The doses for the larger field of view are higher due to the larger scattering volume. The simulations were performed with the same number of initial particles \(1.4 \cdot 10^{10}\) for both FOVs, resulting in a higher number of particles per area for the smaller field. This assumption holds not true in the experiment. For absolute dosimetry this should be corrected by using either accurate numbers for the flux, or setting the angle of the cone-beam identical and setting the field of view using the slits. The second method will however result in a much higher simulation time for a reasonable
Figure 5.9: The dose distribution in a homogeneous water phantom or radius 7.5 mm for a beam size of $445 \times 375 \, \mu m^2$ at an energy of 21 keV. The scale was normalized to the maximal dose value of the distribution. The profiles on the left were taken along the lines through the middle of the phantom shown on the left.

statistics, since many particles for the smaller FOV will get caught in the slits.

<table>
<thead>
<tr>
<th>FOV</th>
<th>$D_{\text{max}}$</th>
<th>$D_{\text{mean}}$</th>
<th>$D_{\text{tot}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$842 \times 882 \mu m$</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$445 \times 375 \mu m$</td>
<td>0.745</td>
<td>0.399</td>
<td>0.399</td>
</tr>
</tbody>
</table>

Table 5.4: The different values for the maximal, the mean and the total dose for different FOVs. The numbers were normalized to the respective values of the largest FOV. All values were calculated with $1.4 \cdot 10^{10}$ initial particles per field of view.

The small animal phantom for the monoenergetic case

To investigate the dose distribution within a small animal, the simulations were carried out for the small animal phantom. In this case the ROI was chosen in the right lung of the phantom, which corresponds to an experimental setup for lung imaging. The resulting dose distributions for two different FOVs are shown in Figure 5.10. The dose distributions look similar, but is smeared out in case of the larger FOV. This is due to the larger scattering volume. The maximal dose is accumulated in the bones due to the larger cross-section of the photoelectric absorption. For both FOVs the bones on the right side accumulate the highest dose due to the fact that the images are acquired over $180^{\circ}$ and these bones are close to the beam entry surface. Furthermore,
Figure 5.10: The dose distribution in small animal phantom for a 21 keV beam. The FOV was $848 \times 882$ for the left image and $445 \times 375 \, \mu m^2$ in the right image. Both distributions were normalized to the maximal value.

for the same reason, the tissue on the right side accumulates more dose than the left side which can be clearly seen in the profiles which are shown in Figure 5.11.

The two peaks on both ends of the $z$-profile are the dose depositions in the bones. The bones show a gradient in dose, the dose is highest on the right side, which is the side that faces the source. The smaller peak in the middle originates from the proximity to the FOV area. This can also be seen in the $x$-profile, where a slope in the dose from the left to the right is visible. There it can also be seen that the dose in the ROI is much broader for the bigger FOV as was already

Figure 5.11: Profile through the dose distribution in small animal phantom for a FOV of $848 \times 882 \, \mu m^2$ (left) and for a FOV of $445 \times 375 \, \mu m^2$ (right) at an energy of 21 keV.
observed in the water cylinder.

5% white-beam and 0.5% white-beam

Figure 5.12 shows the dose distributions for the 5% and the 0.5% for a field of view of $445 \times 375 \mu m^2$ in the water cylinder. The location of the field of view was chosen 2 mm off-axis at the same location as for the small animal phantom.

The dose distributions for the two cases look similar, the relative dose on the right side of the

![Figure 5.12: Dose distribution in the water phantom for a FOV of $445 \times 375 \mu m^2$ for the 5% white-beam and the 0.5% white-beam. The distributions are normalized to the maximal value of the distribution.](image)

phantom is slightly lower in case of the 0.5% white-beam compared to the 5% white-beam. For the same number of initial particles, the total dose of the 0.5% white-beam case is 4.6% of the one obtained for the 5% white-beam case. This can be explained by the fact that for the 0.5% white-beam the number of particles that reach the phantom is roughly a factor 10 smaller and the particles have a higher energy, which makes them less likely to be absorbed. With the same number of particles reaching the phantom for both cases, the dose for the 0.5% white-beam case would only be 46% of the dose for the 5% white-beam case. Figure 5.13 shows the profiles in x- and z-direction through the dose maximum for both cases. The profiles in z-direction look the same for the 5% white-beam case and the 0.5% white-beam case, while for the x-direction the relative dose of right side is lower for the 0.5% case compared to the 5% case. This can be explained by the harder energy spectrum of the 0.5% beam, which will lead to more penetration of the sample and due to the normalisation the curve is lower on the right side.
5.5. Results

Figure 5.13: Profile through the dose distribution in the water phantom for a FOV of $445 \times 375 \, \mu\text{m}^2$ for the 5% white-beam and the 0.5% white-beam. Each of the curves is normalized to the maximal value of the dose distribution.

5% white-beam vs. Mono beam

To compare dose distributions between the 21 keV beam and the 5% white-beam both were simulated for the small animal phantom for a FOV of $848 \times 882 \, \mu\text{m}^2$. The two dose distributions were normalized for the maximal value of the 21 keV distribution and are shown in Figure 5.14. A comparison between the 21 keV mono beam and the 5% white-beam distributions show a

Figure 5.14: The dose distribution in small animal phantom for a monoenergetic beam of 21 keV (left) and the 5% white-beam (right). The two distributions were normalized to the maximal dose value of the 21 keV distribution.
difference in the maximal dose of approximately a factor 60. This can be expected since in case of the 5\% white-beam only about 4\% of the initial particles reach the phantom and the mean energy of the beam is higher. The actual flux for the mono beam is in the order of $10^{14}$ at the monochromator, while the flux for the white-beam is in order of $10^{17}$ at the source. For the same acquisition time this will lead to a much higher dose in case of the white-beam. However, for the same number of particles reaching the phantom the dose in the 5\% white-beam case is only 30\% of the 21 keV case. There is also a difference in the relative dose deposited in the ROI area. The dose in the monoenergetic case goes up to about 45\%, in case of the 5\% white beam it goes up to 85\% of the maximal value. Most particles that reach the phantom in the white-beam case have higher energy than 21 keV, which makes the probability for photoelectric absorption lower. This leads to a smaller dependence of the absorption on the atomic number Z and leads to higher relative dose in the ROI, since all beams cross there.

Figure 5.15 shows profiles through the dose distributions for both the 21 keV beam and the 5\% white-beam. The profiles were taken along the x- and z-axes through the middle of the phantom. To compare the shape, both curves are normalized to the maximal dose value of each distribution. The shape of the two curves is similar to the z-profile. There are small differences in the height of the dose peak in the soft tissue and the shape of the slope in the bones. The differences can be explained by the different energy spectra of the beams which lead to a difference in absorbed dose which is more pronounced in bone due to the higher photoelectric absorption cross-section. The x-profiles show a large difference in the ROI which was also observed and discussed in the 2D distribution above.

![Graph showing profiles through the dose distribution for both the 21 keV beam and the 5\% white-beam.](image_url)

Figure 5.15: Comparison of the profiles through the dose distribution in small animal phantom for a FOV of $445 \times 375 \mu m^2$ for 21 keV (blue) and the 5\% white-beam (red).
5.6 Conclusion and outlook

In this work a MC model for TOMCAT beamline was created for the purpose of dose estimations in small animal imaging. The model contains all relevant elements of the beamline within the beam path. It was used for the calculation of dose distributions in a water cylinder and in a small animal phantom. There were three energy settings considered: a 21 keV monoenergetic beam, a 5% white-beam and a 0.5% white-beam. The water phantom showed a dose maximum in the ROI for all energy settings. For the small animal phantom the highest dose were accumulated in the bones close to the surface of the beam entry side for both the 21 keV case and the 5% white-beam case. However the relative dose in ROI for the white beam case was significantly higher. The simulations also showed that the total dose for the same initial flux in the 5% white-beam case is about 2% of the monoenergetic case. The 0.5% white-beam calculation for the water phantom shows that the relative dose outside the ROI is lower than for the 5% case. These observations lead to the conclusion that the composition of the phantom and the energy spectrum of the beam have an influence on the dose distribution within the animal.

Further steps in this project would be the implementation of a routine for reading in CT data. Within this model it would be possible to replace the phantom by a density matrix generated from a CT scan and the respective conversion of density to material. This would enable the calculation of a dose distribution on the actual anatomic structure of a small animal. Furthermore the model could be used to calculate absolute dose distributions in a phantom through the comparison and calibration of simulations with measurements. The model can then be applied for the determination of the ideal setup with respect to dose through a parameter study of the different elements in the simulation such as a combination of filters, FOV and acquisition time. For accurate dose estimations it will also be necessary to experimentally determine the relationship between image quality and dose, which is an aspect that was neglected so far.
5.7 Further development of the model

The model presented in the sections above, was further expanded by implementing a routine that allows the dose calculation based on a Computer Tomography (CT) data set. In this routine, the gray values of a CT image are first associated with a material such as bone or tissue and the corresponding density [96]. The absorbed energy is then calculated for each pixel, and from this the relative radiation dose can be determined. As a proof of concept, a slice of a CT data set of a small rat that was previously obtained at TOMCAT beamline\(^6\) for the purpose of lung imaging in small animals, was used to calculate the dose distribution. One slice of the CT data set was used, corresponding to \(920 \times 920 \times 1\) voxel of size 0.0163 mm\(^3\).

Figure 5.16: CT scan (left) of a small rat obtained at TOMCAT beamline for the purpose of lung imaging in small animals and calculated dose (right) for the slice, with a beam size of 2.2 \(\times\) 2.2 mm\(^2\) at an energy of 21 keV.

For the simulation, the MC model from the beamline presented in sections 5.1 to 5.6 was used. The X-ray energy was set to 21 keV and the beam size set to 2.2 \(\times\) 2.2 mm\(^2\) corresponding to a field of view that would be used in a local tomography scan for lung imaging [19]. The dose for 180 projections over 180\(^\circ\) were calculated and combined. For each projection \(10^7\) particle histories were simulated. The beam intensity profile in the simulation was assumed to be uniform, which is not the case for an actual in-vivo CT at the beamline. This assumption is sufficient for

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\(^6\)Courtesy of David Haberthur.
the proof of concept shown here. However, for a realistic prediction of the dose to the animal, the X-ray beam intensity profile would need to be considered.

Figure 5.16 (left) shows the CT image used in the calculation and Figure 5.16 (right) shows the obtained relative dose for a beam size of $2.2 \times 2.2 \text{ mm}^2$ at an energy of 21 keV. The distribution of the relative dose is shown in Figure 5.16. As expected, the dose is highest in the area of the field of view, and lower in the areas outside. The only structures outside that also obtain a significant amount of dose are the bones. This is due to the fact that the main attenuation process at this energy is the photoelectric absorption, which is proportional to the third power of the atomic number $Z$ [25], which is much higher for bone than for the surrounding soft tissue. The reason that not all the areas of the bone obtain a high dose is due to the incoming beam which only covers $180^\circ$. Thus only the bones that are close to the surface of the incoming beam receive a high dose. There are visible artifacts in the dose distribution that are also visible in the CT image, for instance the streaking from the bones. However the dose distribution is visible and corresponds to the expected result and thus is sufficient as a proof of concept. This shows that the dose calculation model can also be applied to CT-data based scans, which allows for more realistic dose distributions for in-vivo applications.
Chapter 6

Conclusion and Outlook
6.1 Conclusion

The aim of this thesis was to obtain an accurate description of the physical process occurring during the Grating Interferometry (GI) imaging process. Such a description requires that all relevant physical interactions of X-rays with matter are considered. In this work, a successful implementation of a simulation framework for phase-sensitive X-ray imaging that takes both particle- and wave-like properties of X-rays into account, was presented. The simulation framework is based on egs++, a C++ interface for the well established EGSnrc Monte Carlo methods (MC) Code for the simulation of particle transport. Since MC simulations of X-rays are based on the particle view of photons, wave-like behavior of X-rays is generally not considered within MC. Therefore, the wave-like properties relevant for the GI imaging process were included through several modifications of the transport code. To take the phase-shift of X-rays passing through material into account, an additional variable for each particle was included into the MC framework. Refraction of X-rays at surfaces was considered through the implementation of Snell’s law and the diffraction and interference phenomena of X-rays necessary for the simulation of GI imaging, were implemented into the framework in two steps.

The first step was a combination of MC methods with wave-optics simulations, which was called the Hybrid-model. In this approach, the sample part of the GI setup, including source and sample, was implemented in MC. Directly after the sample, the particles were coherently summed up into a wave-front with respect to their phase and position, thus transferring within the simulation from the particle view of X-rays into the wave view. The so obtained wave was then propagated through the remaining imaging geometry using wave-optics, by utilizing the projection approximation and the free space propagator. This model was validated through comparison with experimental data and demonstrated good agreement (correlation coefficient > 0.925). The agreement between experimental and simulated data showed that the combination of wave-optics simulation and MC was successful and all the relevant physical processes were modeled accurately within the framework. This establishes the Hybrid-model as a reliable simulation tool for phase-sensitive imaging.

The Hybrid-model was then used to investigate the connection between the GI-USAXS signal and the sample structure. Simulations of a GI-USAXS setup were performed by simulating the phase-stepping curves of well defined samples, consisting of Polymethylmethacrylat slabs with air-holes of different radii and distances. Through a deconvolution of the phase-stepping curves, the corresponding scattering distributions of the samples were obtained and their statistical moments calculated and used for describing the relationship between the moments and the simulation parameters, i.e. radii and minimal distances of the holes. A fitting function was proposed, that links the ratio of the fourth and second moment of the scattering distribution to
the radii and distances of the air holes. The fitting function was then applied to experimental data of a sample with known sample parameters and the comparison of retrieved and known sample parameter showed an agreement within 25%. This showed that in the case of the sample considered in this work, sample parameters can be extracted from the moments of the scattering distributions, thus opening up the possibility of quantitative GI-USAXS imaging.

In a second step, interference phenomena were fully included into MC through the implementation of Huygens principle into the MC framework. This implementation was achieved by considering the particles after the phase-grating as point sources and implementing a particle splitting routine. The particles reaching the splitting plane located directly after the phase-grating are copied and each of the copies is assigned a random angle of direction. All the copies are then transported to the detector plane, resulting in a point source for each particle arriving at the splitting plane. At the detector plane the particles are binned according to position and summed up with respect to their phase. This so called full MC model allows the simulation of interference without losing the particle aspect of the X-rays. Thus, photons can be traced as particles from the source through the geometry to the detector. This allows to connect the signal of the photons obtained at the detector to the processes they underwent during the imaging process. Validation of this model was achieved through comparisons of simulations with experimental data. The good agreement of this comparison (correlation coefficient $> 0.9$) confirmed that the inclusion of wave-like behavior into the MC framework was successful and showed that the physical processes relevant for the simulation of GI are considered adequately within the model. Furthermore, the model was shown to be suitable for numerical simulations of GI utilizing polychromatic X-ray sources. To highlight a possible application of the framework, the full MC model was utilized for the calculation of the visibility of a polychromatic GI setup, such as could be used for GI mammography. The results show that the influence of the height of the absorption grating depends on the energy. This application shows a case in which this framework can be beneficial to contemporary research performed in the area of X-ray tube based GI imaging.

Additionally, due to the fact that MC methods are a common and well suited tool for the calculation of radiation dose estimation in X-ray imaging, a radiation dose calculation routine was implemented into the sample part of the framework. This routine was applied to the calculation of relative dose distributions within different phantoms for an in-vivo setup for lung imaging in small animals at TOMCAT beamline. The model was moreover successfully expanded to include a routine enabling the dose calculation based on CT data sets, which allows the calculation of realistic dose estimations in in-vivo imaging. The results from these simulations show the influence of sample composition and energy spectrum on the radiation dose received by the sample.
In conclusion, it was shown that the framework for the simulation of GI based on MC that was presented in this thesis, successfully considers both wave- and particle-like behavior in both of the two described steps. Through the comparisons with experimental data, it was further demonstrated that the relevant physical processes for the simulation of GI were considered adequately within the simulation framework. Furthermore, different applications of the framework in contemporary research were shown.
6.2 Outlook

In Chapters 3, 4 and 5 of this work, selected applications for the presented simulation framework were shown, such as the investigation of the GI-USAXS signal, the application to polychromatic GI setups and a radiation dose calculation model. However, there are many other cases and areas, where the utilization of the simulation framework presented in this thesis could be of use. In this section, I would like to briefly present two of these areas, where I believe the use of the framework could be beneficial.

Scattering signal

The first potential application of the framework discussed here, is the investigation of scattering signal in GI imaging. Theoretical models of the dark-field signal are a contemporary area of research and can be used to link sample structures to the obtained image thus allowing quantitative dark-field imaging. A generally applicable theoretical model of dark-field imaging would benefit many areas such as lung imaging, were it could be used to obtain structural information on the micro-structure of the lung. This could potentially improve diagnostics in respiratory disease or further advance the understanding of the air exchange process in the lung. Another area that could benefit from quantitative dark-field imaging is mammography. There, it can be applied to characterize and classify microcalcification, which could benefit a better diagnostic prediction of breast cancer through GI mammography. The framework presented in this thesis could be used in the theoretical modeling of the dark field signal as a method to develop, validate or benchmark such theoretical models. This can be achieved by comparing the results obtained from simulations of well defined samples to the dark-field values obtained from the theoretical models. Such a comparison can used to determine if and under which conditions the two values agree, and thereby determine the range of applicability of such theoretical models. The framework could thus be a useful tool for the investigation of quantitative dark-field imaging.

Dosimetry and image quality

The second potential application of the presented framework I would like to discuss here, is the investigation of the connection between image quality and radiation dose in GI imaging. As previously stated, accurate knowledge of the radiation dose and its volumetric distribution within a specimen or patient is an important aspect of any in-vivo application of X-ray imaging with GI, since it is indicative of the health risk from the radiation exposure to the specimen or patient. For a defined imaging setup and sample, the framework can be used to compare the image acquired
6.2. Outlook

at the detector to the dose obtained by the sample. This could potentially benefit setup design, by comparing the maximal achievable image quality with respect to resolution and contrast, to the maximal dose allowed. The parameters of the setup, such as X-ray source spectrum or Talbot order can then be adapted to the optimal configuration with respect to image quality and dose. In case of a phantom, this can be achieved in a straightforward way, since all necessary information about the phantom such as geometry and material composition are known and can be implemented into the simulation framework directly.

The second step would be to connect the image quality to the dose based on a Computer Tomography (CT) data set instead of a phantom. This results in a more realistic dose distributions for an experimental setup, but is also more challenging. Phase-shift and scattering based on a CT data set could be achieved similarly to the dose calculation model presented in Chapter 5, through associating each gray value of the CT image with a material and a corresponding density. This association defines the absorption and phase-shift of the sample (see Chapter 1). However, the scattering signal is more challenging to implement into the framework in this case, since the sub-pixel structures causing the scattering are not directly obtainable from the CT image. Thus a method would need to be implemented to translate the obtained dark-field values from the dark-field CT image into the simulation.

The framework could further be used for optimizing the GI imaging setup with respect to other parameters such as grating quality (with respect to duty cycle, uniformity) or material, energy spectrum of the X-ray source or source size, through associating image quality obtained through simulations with the parameters used and then adapt the parameters in order to meet the desired image quality.
Bibliography


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List of Publications

Conference Talks

- S. Peter et al., *Numerical simulation of X-ray grating interferometry imaging using Monte Carlo methods*, Swiss Society of Radiobiology and Medical Physics (SGSMP) annual scientific meeting, Fribourg (CH), 2015

- S. Peter et al., *Combining Monte Carlo and wave-optics methods for the simulation of phase-sensitive X-ray imaging*, American Association of Physicists in Medicine (AAPM) 55th Annual meeting, Indianapolis (USA), 2013

- S. Peter et al., *Implementation of a Monte Carlo model for dose calculations in small animal imaging at TOMCAT beamline*, Swiss Society of Radiobiology and Medical Physics (SGSMP) annual scientific meeting, Biel (CH), 2012

- S. Peter et al., *Combining wave-optics and Monte Carlo methods for the simulation of grating based hard X-ray interferometry*, Swiss Society of Radiobiology and Medical Physics (SGSMP) annual scientific meeting, Biel (CH), 2012

- S. Peter et al., *Combination of Monte Carlo methods and wave-optics for the simulation of grating based hard X-ray Interferometry*, Third European workshop on Monte Carlo treatment planning of the European Workgroup on MCTP, Seville (ESP), 2012

Poster presentations


- S. Peter et al., *Numerical Simulations of quantitative GI-USAXS using Monte Carlo Methods*, XTOP 12th Biennial Conference on High-Resolution X-Ray Diffraction and Imaging, Grenoble and Villars-de-lans (FR), 2014, awarded the Poster Prize

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- S. Peter et al., *Combining wave-optics with Monte Carlo Methods for the simulation of grating based hard X-ray Interferometry*, XTOP 11th Biennial Conference on High-Resolution X-Ray Diffraction and Imaging, St. Petersburg (RUS), 2012
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