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Corrigendum to "On the approximability and hardness of minimum topic connected overlay and its special instances" [Theoret. Comput. Sci. 429 (2012) 144–154]

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Erratum to "On the Approximability and Hardness of Minimum Topic Connected Overlay and Its Special Instances" [Theoret. Comput. Sci. 429 (2012) 144–154]*

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Abstract

As was pointed out in [1], Theorem 8 of the paper *On the Approximability and Hardness of Minimum Topic Connected Overlay and Its Special Instances*[2] is incorrect. This erratum proves a slightly weaker version of this theorem.

In Minimum Topic Connected Overlay (Min-TCO), we are given a set T of topics and a collection U of users. Each user is interested in a set of topics. This relation is expressed by the user interest function $\text{INT} : U \to 2^T$. Our goal is to find a minimum set of edges between users so that, for each topic, the subgraph determined by users interested in this topic is connected, i. e., users interested in the same topic are connected in a network. Although the general problem is $\mathcal{LOGAPX}$-complete, we show in the following theorem a class of instances on which the problem can be solved yet in polynomial time.

Theorem 1. *An optimal solution of* Min-TCO *can be computed in polynomial time if* $|T| \le (1 + \varepsilon(|U|))^{-1} \cdot \log_8 \log_8 |U|$, *for a function*

$$\varepsilon(n) \ge \frac{3 \log_8 \log_8 \log_8 n}{\log_8 \log_8 n - 3 \log_8 \log_8 \log_8 n}.$$

In other words, Min-TCO *can be computed in polynomial time if* $|T|$ *can be*

bounded from above by a function $f(|U|) \leq \log_8 \log_8 |U| - 3 \log_8 \log_8 \log_8 |U|$ *(for all sufficiently large instances).*

PROOF. Let (U, T, INT) be an instance of Min-TCO such that $|T| \leq (1 + \varepsilon(|U|))^{-1} \cdot \log_8 \log_8 |U|$. Moreover, $|T| > 2$, otherwise the problem is solvable in polynomial time. We shorten the notation by setting $t = |T|$ and $n = |U|$.

We reduce our instance (U, T, INT) using the reduction from [1] (Theorem 3) to an instance of size no larger than $m := tc \cdot 8^t$, for some constant $c \in \mathbb{N}$ which is fixed by Theorem 3 of [1].

number of users in this instance cannot be larger than m as well. Furthermore the reduction never adds topics and hence the number of topics in the reduced instance cannot be larger than t. On this smaller instance, we exhaustively search over all possible solutions and we pick the one which is minimal.

Observe that the optimal solution of our reduced instance cannot have more than $t(m-1)$ edges—this many edges has a feasible solution which merges together a spanning tree of each topic. Hence, in our exhaustive search, we try all possible sets of $1 \leq i \leq t(m-1)$ edges and we verify the topic-connectivity requirements for such sets. The verification of the topic-connectivity property can be done in polynomial time per set. Hence, the proof that the exhaustive search is polynomial boils down to a proof that the number of checked sets is polynomial.

The number of sets the search checks can be bounded as follows:

$$
\begin{aligned}
\sum_{i=1}^{t(m-1)} \binom{\binom{m}{2}}{i} &\leq \sum_{i=1}^{tm} \binom{m^2}{i} \\
&\leq tm \cdot \binom{m^2}{tm} \\
&\leq tm \cdot m^{tm}
\end{aligned}
$$

(The second inequality easily follows from $m \geq 2t$. To prove that $\binom{m^2}{tm} \leq m^{tm}$ modify the binomial coefficient to a multiplication of factorial numbers and use the fact that $n! > n^n e^{-n}$ and $t \geq 3$.)

If the factor m^{tm} would be polynomial, then also $tm \cdot m^{tm}$ would be polynomial and our statement holds. Thus, in what follows further, we focus on proving that, under the given assumptions, m^{tm} can be bounded by a polynomial of n.

We continue with the calculation:

$$
m^{tm} = (8^t tc)^{t^2 8^t c} = c^{t^2 8^t c} \cdot t^{t^2 8^t c} \cdot (8^t)^{t^2 8^t c}.
$$

From the above factors, if the factor $(8^t)^{t^2 8^t}$ is bounded by a polynomial factor, then we can be sure that all the three above factors are polynomial and hence their multiplication produces just another polynomial.

Note that up to this point we have intentionally used simpler notation: Instead of considering functions $t(n)$ (for the number of used topics) and $m(n)$ (for the size of the reduced instance), we used constants t and m. However, since we bound from above, each step in our argumentation is true also once t

and m are replaced by $t(n)$ and $m(n)$, respectively. (Note that $t(n) \geq 2$ and $t(n) \leq 2m(n)$.)

To finish the proof we use the fact that $\log_8 \log_8 n - 3\log_8 \log_8 \log_8 n \leq \log_8 \log_8 n$ and the assumption that $t(n) \leq \log_8 \log_8 n - \log_8 \log_8 \log_8 n$ to estimate $t(n)^3 8^{t(n)}$:

$$
\begin{aligned}
t(n)^3 8^{t(n)} &\leq & (\log_8 \log_8 n - 3\log_8 \log_8 \log_8 n)^3 \cdot 8^{\log_8 \log_8 n - 3\log_8 \log_8 \log_8 n} \\
&\leq & (\log_8 \log_8 n)^3 \cdot 8^{\log_8 \log_8 n - 3\log_8 \log_8 \log_8 n} \\
&= & 8^{3\log_8 \log_8 \log_8 n} \cdot 8^{\log_8 \log_8 n - 3\log_8 \log_8 \log_8 n} \\
&= & 8^{\log_8 \log_8 n} \\
&= & \log_8 n.
\end{aligned}
$$

Thus, $8^{t(n)^3 8^{t(n)}} \leq n$, which finishes the proof. $\qquad\square$

References

[1] Jiehua Chen, Christian Komusiewicz, Rolf Niedermeier, Manuel Sorge, Ondrej Suchý and Mathias Weller. Effective and Efficient Data Reduction for the Subset Interconnection Design Problem. In *Proc. of the 24th International Symposium on Algorithms and Computation (ISAAC 2013)*, volume 8283 of *Lecture Notes in Computer Science*, pages 361–371, Springer-Verlag, 2013.

[2] Jun Hosoda, Juraj Hromkovič, Taisuke Izumi, Hirotaka Ono, Monika Steinová and Koichi Wada. On the approximability and hardness of minimum topic connected overlay and its special instances. In *Theoretical Computer Science*, volume 429, pages 144–154, Elsevier, 2012.