Calculating benefits of infrastructural investment

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Author(s):
Zöllig, Christof; Axhausen, Kay W.

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Calculating Benefits of an Infrastructural Investment

Christof Zöllig
Kay W. Axhausen
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Christof Zöllig
Institute of Transport Planning and Systems (IVT)
ETH Zürich (Hönggerberg)
CH-8093 Zürich
Telephone: +41-44-633 27 19
Fax: +41-44-633 10 57
christof.zoellig@ivt.baug.ethz.ch

Kay W. Axhausen
Institute of Transport Planning and Systems (IVT)
ETH Zürich (Hönggerberg)
CH-8093 Zürich
Telephone: +41-44-633 39 43
Fax: +41-44-633 10 57
axhausen@ivt.baug.ethz.ch

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Abstract

This paper shows that models with more degrees of freedom produce different results in terms of traffic distribution, volumes and consequently of utility gains. If long term decisions are considered, short term travel time savings do not capture utility gains appropriately. In this context we discuss the expected maximum utility as more comprehensive indicator, which has also the advantage of being highly consistent with discrete choice theory. The errors made when disregarding relevant dimensions are substantial and might influence the decision on whether to build a transportation infrastructure or not.

Key words

Random utility maximisation, travel time savings, expected maximum utility, micro-simulation, land use

1 Introduction

1.1 Problem

To evaluate infrastructural projects it is common practice to perform a cost-benefit analysis ex ante. To assess the benefits of the investment, guidelines usually propose to predict travel time savings and to monetise them using values of travel time savings. Travel time savings originate due to reduced traffic volume on congested links. To predict the expected reductions standard transport models usually allow for changes in route and mode choice as the relevant behavioural dimensions of travellers. However, it is pretty obvious that in reality travellers might also alter departure time or destination/location (if there are alternatives available) to adapt to a new transport infrastructure. Consequently, a model that does not consider departure time and destination/location choice might not include all consequences (processes) arising from an adapted transport infrastructure, especially in a long term perspective. A model that neglects relevant degrees of freedom, might produce misleading results in terms of calculated traffic volumes leading to misjudged utility gains. This issue was recently discussed by Metz (2008).

The integration of other dimensions of decision making to calculate future traffic flows raises the question whether short term travel time savings are still an appropriate indicator to judge an infrastructural investment. When we assume that travellers do not only change their connections but also locations and departure time, we should integrate utility components influenced by such decisions into our models to maintain consistency. Therefore we want to show how it is relevant for evaluation whether we only consider travel time savings or further decision dimensions.

However we do not want to discuss the question of what utility components to integrate in an evaluation in detail. The assumption is that the explainable utility of an alternative can be calculated by considering the travel time on a given connection, the time of late or early arrival and the price of the activity at its location. Other aspects determining the utility of the traveller like comfort, risk or monetary costs are not considered for simplicity. We recognise that the integration of such aspects will be crucial for a judgement but the focus of this paper is on the influence of additional decision dimensions.

We want to concentrate on the calculation of utilities produced through a transport infrastructure as mentioned above. The hypothesis is that expected gains in utility do not only depend on connection choice (allowing to minimise travel time), but also on location choice (allowing to visit cheaper locations) and departure time choice (allowing to avoid congestion and delays). Consequently a model
that is not considering departure time and location choice might not be able to calculate the gains produced by an improved transport infrastructure correctly. Part of this hypothesis is that a model neglecting decision dimensions is not able to locate the origin of benefits correctly.

This paper addresses the following research questions:

1. What differences in simulating the impacts of an improved transport system occur when we allow for more decision dimensions than route choice in the transport model?
2. Is it sufficient to invest in the benefits of an infrastructural investment in terms of travel time savings?
3. Would it be more consistent to evaluate an investment using the expected maximised utility (EMU) from the underlying choice model?

Accordingly the research objectives are formulated:

1. Show that traditional models of transport assignment cannot show all effects of behavioural changes occurring after an infrastructural investment.
2. Investigate the difference between calculating the benefits in terms of travel time savings only and an approach considering utilities from departure time choice and location choice as well.
3. Compare the estimated utility gains derived by different indicators, namely travel time savings and the EMU.

To investigate these issues a proof of concept simulation is implemented using Python programming language. The agent-based simulation approximates for a minimal urban system the stochastic user equilibrium of commuting inhabitants using discrete choice theory (Ben-Akiva and Lerman, 1985). The simulation of the agents’ decisions results in traffic volumes on links and population densities at locations. The agent-based approach has been chosen for reasons of consistency and intuitiveness.

In this research the concept of the simulation is subject of investigation. We want to know the effects of different assumptions made in our simulations. We do not compare the simulated data with observed data but with data out of different simulations. This allows us to show supposed errors produced by different assumptions entering the simulations.

The experiments simulate the reaction of the agents due to an improved transport infrastructure depending on how many decision dimensions are free. Altering the transportation system modifies the set of alternatives. The agents are reacting to the new set of alternatives by selecting different options at free decision dimensions. This results in different link loads, location occupancies and utilities for the agents. We analyse the results in respect to demand shifts and donated utilities.

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1 See chapter 2.5.2 for explanation.
We do not assume costs for the infrastructural investments because it is not necessary to calculate the profitability in terms of a cost-benefit ratio to answer the research questions. We focus on differences between calculated utility gains applying different approaches evaluating the same investment. The costs are constant as long as the same investment is observed and will therefore not affect the results. The paper is organised in four main sections. The first section explains the theoretical background of the simulation. The following section describes the implemented algorithm and the simulated experiments in detail. We show and interpret the results in section 4. The fifth section contains conclusions and an outlook.
2 Assessment of Infrastructural Investments

In this chapter we present the ideas and concepts usually used to assess infrastructural investments a priori. We recapitulate aggregate and disaggregate approaches in transport modelling, discrete choice theory, assignment, simulation as research method, its function in the assessment of infrastructural investments and two types of utility indicators. This will provide the background of the simulation presented in section 3.

2.1 Transport models

If we want to evaluate the investment a priori, we need a transport model to predict the future state of the system. Two main approaches exist.

2.1.1 Aggregate Approach

The traditional aggregate transport models (first generation) typically estimate traffic flows on relations between zones based on the analysis of traveller groups. This is done in four sequential modelling steps:

1. Trip generation modelling
2. Trip distribution modelling
3. Modal split modelling
4. Assignment

For every step individual models and estimation techniques exist. The fist step calculates the number of trips expected to origin or end in a specified location. This gives the outflows and inflows for a zone.

In the second step the amount of trips on a given OD-relation is calculated by using matrix techniques. Several models have been presented to perform this task like growth factor models, synthetic or gravity models or intervening-opportunities models.

Growth factor models are the simplest models. They just consider a uniform growth factor which is multiplied by each observed flow on a relation. It is obvious that such a model will not be able to capture demand shift when introducing a new link. In gravity models transportation demand is derived directly from attributes of locations and transportations services. This qualifies them as synthetic, because they do not vary an observed flow. Such distribution models are actually very similar to discrete choice models as they model shares of travellers for a specific relation. This share can be
interpreted as an approximation of choice probability. Another synthetic model is the intervening-opportunities model. The basic concept is that the probability of a trip to a destination \( j \) depends on the closer, intervening opportunities (destinations) which also allow to satisfy the objective of the trip. All these distribution models are not able to consider characteristics of travellers, unless they are separating for each segment.

In the third step the modal split is estimated for a given relation using empirically defined curves which determine the fraction of a mode depending on its characteristics like generalised costs. It is possible to integrate the models for trip generation, trip distribution and modal split. If the models are calibrated simultaneously we have a direct demand model.

The assignment step distributes a known OD-matrix on the existing network. The question answered in this step is: Which path do the travellers take? With the assumption that travellers take the path with minimal generalised costs, the problem is to find a state in which all travellers use a cost minimal path. This problem cannot be solved analytically for realistic networks because of the relation between link loads and generalised costs. Therefore we have to apply a numerical method which approximates the solution in iterations. The relation between traffic load and generalised costs is cast in capacity restraint functions.

2.1.2 Disaggregate Approach

The second family of transport models are disaggregate because they calculate the traffic flow in a transportation infrastructure due to the analysis of individual travellers decisions (second generation). They are based on the random utility theory pioneered by Daniel McFadden (1974). The economic framework was applied and specialised for the transportation context by Domenchich and McFadden (1975) formulating discrete choice models to forecast transportation demand. Ben-Akiva and Lerman (1985), Ortuzar and Willumsen (2001) and Train (2003) give comprehensive overviews and good introductions. The last book focuses especially on the use of the model framework in simulations.

2.2 Discrete Choice Modelling

Discrete choice models assume that individual actors process information in a rational way when they face a decision situation. A decision situation is given when an individual \( q \) has to select an alternative out of a set of discrete alternatives \( A = \{a_1, a_2, ..., a_N\} \) at hand. The individual is assigning a utility value to each one of the alternatives \( a_n \). The rational decision is then to pick the alternative with the highest utility. This utility maximising approach is in line with the economic concept of the homo oeconomicus. The choice set \( A \) may vary from situation to situation.
The utility of an alternative has two basic parts: An explainable component $V$ and an unexplainable component $\epsilon$.

$$U_a = V_a \ast \epsilon$$  \hspace{1cm} (2.1)

The unexplainable part, or residual, contains all utility an individual assigns to an alternative that is not coming from an observed variable. The residuals are supposed to be random numbers distributed in some way. This makes the composed utility $U$ stochastic.

The deterministic utility $V$ of an alternative is calculated with a utility function which is usually a linear combination of (functional) variables.

$$V = \sum_k \Phi_{kj} X_{jk}$$  \hspace{1cm} (2.2)

Assumptions about the number of individuals that travel and the choices of these individuals gives us directly the demand of a certain transport service. Still demand depends on the decisions of other individuals, which requires an iterative calculation. A trip generation model could also be formulated as a discrete choice model. Appropriately formulated choice models can therefore simultaneously perform the tasks of trip generation modelling, trip distribution modelling, modal split modelling and assignment.

### 2.2.1 Specification

Instead of actually deciding whether an individual is selecting a specific alternative, it is also possible to calculate the probability of an alternative to be chosen according to its utility. To calculate the probability usually either a logit or a probit model is used. This mathematical model relates the utility of an alternative to other alternatives' utility. The utility is transformed into a probability.

The formulation of the discrete choice model depends on the assumptions we make. If we assume that the residuals are independent and identically Gumbel distributed (IID) we have a multinomial logit model (MNL). In an MNL the probability of an alternative, its logit, takes the form:

$$P_{ai} = \exp(\beta V_{ai}) / \sum_{A} \exp(\beta V_{aj})$$  \hspace{1cm} (2.3)

The model just holds when we assume that the alternatives are in fact independent. The model also satisfies the condition that calculated probabilities are independent from irrelevant alternatives (IIA-
condition). The problem in this respect lies in similar alternatives like in the red bus-blue bus problem (McFadden, 1974). The argument is that very similar alternatives have error correlation.

If it is possible to identify nests of correlated alternatives, it is possible to formulate a hierarchical logit (HL) or nested logit model. Within these nests the alternatives must again be independent which allows the assumption that residuals are IID.

If the correlations between the alternatives are unstructured a probit model can be used. The probit model is derived from a multivariate Normal distribution and can handle totally arbitrary covariance matrices. A major drawback of such models is that in cases of more than three alternatives the estimation is very complex.

Further specification concerns the utility function and the identification of the choice set. In respect to the utility function it has to be decided which explanatory variables to integrate. If the variables itself are found, an appropriate functional form has to be specified. Variables enter the utility function in a convenient functional form. The expression (2.2) would more precisely be formulated as:

\[ V = \sum_k \Phi_{kj} f_k(x_{ik}) \]  

(2.4)

Often a linear formulation is adequate. However, for destination choice non-linear functions have been found more appropriate (Foerster, 1981; Daly, 1982). The functional form is important for parameter estimation because estimation routines do not always converge to a unique value. The functional form also has effects on trade-offs, elasticities and explanatory power. To find the appropriate functional form it is recommended to go back to economic theory (Ortuzar and Willumsen, 2001, p. 252).

Both formulations (2.2) and (2.4) are linear in parameters.

### 2.2.2 Estimation

If once the discrete choice model is properly formulated the parameters are estimated. This is normally done by applying the maximum likelihood method. The estimates are then tested in terms of their sign and significance. The modeller then has to decide whether or not to integrate the variable in question. The variable usually remains if the explanatory power of the model is increased. As this study is not based on observed data and therefore no model estimation is done, we will not explain model estimation in detail.
2.3 Assignment

Discrete choice theory allows us to see the assignment problem in a comprehensive economic framework. Classic economic theory postulates that an equilibrium mechanism exists. This mechanism will adjust supply and demand to that point when the cost of an additionally produced unit meets the revenue of selling it. The mechanism is caused by individuals searching their optimal alternative with maximum utility. Therefore balancing of supply and demand is an emergent process in a socio-economic system. The assignment models this mechanism and wants to find the equilibrium between supply and demand. In context of transportation, supply is given by transport infrastructure and demand arises from the decisions of travellers.

2.3.1 Equilibrium

In traffic assignment we usually want to calculate the user equilibrium (UE). It is defined as the state of a capacity restraint transport system in which no traveller can find a better travel alternative any more. This is also known as Wardrop's equilibrium (Wardrop, 1952). In this state each traveller has optimised its personal situation, what we normally assume as principle of behaviour in reality. In a more general economic system one would speak of a Walrasian equilibrium (Tesfatsion, 2006, p. 13).

A second state we could be interested in is the social optimum where the travel costs over the whole system are minimised. This is also referred to as Wardrop's second principle. Because one cannot expect the travellers to act altruistic, this criterion is more interesting for planers, who might want to design the transport network in such a way that the user equilibrium meets the before mentioned condition. If the discrete choice model includes a randomly distributed component, a stochastic user equilibrium (SUE) is computed.

2.3.2 Aspects of Assignment Methods

The following section introduces two important aspects which distinguish traffic assignment methods: consideration of stochastic effects and capacity restraints.

Integration of stochastic effects

Stochastic effects recognise that a model cannot predict the demand of an alternative with absolute certainty. This means that no exact solution exists, but a bundle of solutions with a certain probability. Methods to integrate stochastic effects are simulation-based or proportion-based. Simulation-based methods usually use the Monte Carlo technique to find the solution. This allows to incorporate stochastic effects, which originate in the individual perception of costs by each individual. The
assignment problem is solved numerically by simulating a series of random number experiments (Burrell, 1968). By analysing the outcome of these experiments the researcher is able to make statements about the variation in the simulation results. In the proportion-based method a loading algorithm is used that distributes trips arriving at a node to subsequent links. This distribution allows to integrate stochastics.

**Capacity restraints**

If no capacity restraints are considered the link costs are fixed. In this case the *all-or-nothing* assignment is used. In other cases, when link costs depend on calculated loads, an equilibrium must be found in an iterative calculation. This is especially important in congested networks within which link costs vary substantially. Starting from an initial state, which often uses travel costs found under free flow conditions, subsequent states are calculated until an equilibrium condition is satisfied.

Within the methods for congested networks two important ways of loading the network can be identified. Fractions of the whole trip matrix are loaded in subsequent iterations in an incremental assignment. A fraction once assigned will not be removed. This contains the disadvantage of assigning too much flow on a link, which will not be corrected afterwards. The found solution will then not meet the equilibrium conditions. A second way of loading a congested network is known as the method of successive averages. In each iteration the whole trip matrix is loaded or all-or-nothing with the link loads available form the preceding iteration. This yields new, auxiliary link loads $F$. The link loads of the current iteration are then calculated from the previous link loads and the auxiliary link loads with the following formula:

$$V_a^n = (1 - \Phi) V_a^{n-1} + \Phi F_a$$

with $\Phi = 1 / n$

With an aggregate model this assignment method guarantees to converge towards a user equilibrium, even though not most efficiently in most cases. In the Frank-Wolfe algorithm $\Phi$ is calculated in every iteration to optimise convergence (Frank and Wolfe, 1956).

**Capacity restraint functions**

Capacity restraint functions relate the traffic load of a link $s$ with the travel time needed to pass it. The functions are characterised by a minimum travel time and a steady increase until the capacity is reached. The capacity defines the maximal throughput of a link. There are several formulations of
calculating benefits of an infrastructural investment

capacity restraint functions, which can be classified as either hard or soft. Hard formulated capacity restraint functions do not allow loads over the capacity, which means that the curve is asymptotic towards capacity level. Soft formulations in contrast also yield cost values for loads exceeding capacity. For a soft capacity restraint function we give the example formulated by the Bureau of Public Roads in the USA (Ortuzar and Willumsen, 2001, p. 325):

\[ T_s = t_{0s} [1 + \alpha (V_s/Q_s)^\beta] \]  

(2.6)

t\(_{0s}\)...free flow travel time  
V\(_s\)...load on the link s  
Q\(_s\)...capacity of link s  
\alpha, \beta...parameters specific to road type of link s

A hard formulation is the one by Davidson (1966):

\[ T_s = t_0 [1 + \zeta (V_s/Q_s - V_s)] \]  

(2.7)

with \(\zeta\) ... parameter specific for road type of link s

These functions relate link load with travel time. To obtain travel costs associated with a link we define a cost factor (value of time) and multiply it with travel time. Such an approach neglects that travel time not only depends on the load of the link in question, but also on the loads of other elements in the network like subsequent or preceding nodes. Note that link loads vary from time interval to time interval.

2.4 Agent-based Simulation

Agent-based simulations represent a complex system by single components, which make up the system and interact with each other. We simulate the behaviour of agents to get to the system behaviour which makes it a bottom up approach. This feature makes agent-based simulation an interesting tool for the analysis of complex systems. The components are named agents if they have an autonomous behaviour.

The first step is to define the entities of the system, which shall be represented as agents. Usually we are looking for autonomous elements that influence the system. To choose the right entities we should
also have the process in mind we want to simulate. In a second step we have to define the behaviour of
the agents. The basic elements of the agents' behaviour are: Perception of information, processing of
information, decision rules, actions and their timing. An advantage of the agent-based simulation is
that we have the possibility to define various types of agents, which behave in different ways. It is also
possible to define other elements that are not agents itself, but influenced by them.
In addition to the individual behaviour we have to specify how the agents communicate or interact.
Through interaction, phenomena emerge on the macro level. Emergence is either unforeseen or
unpredictable (Gilbert and Terna, 2000). According to them unforeseen emergence is a phenomenon
we just did not think of but would have been detectable by analysing the structure of the simulation in
advance. In such a case the simulation helped us to think. Unpredictable emergence, on the other hand,
is not detectable in advance.

Another nice feature of agent-based models is that they are intuitively understandable and increase
therefore the credibility (Tesfatsion, 2006) of the modelling forecast. Agent-based simulation has been
recognised as a powerful research tool in various disciplines (Portugali, 2000). The agent-based
approach is especially suitable to analyse socio-economic systems because they consist of multiple
behaving agents, which interact with each other.
The method of simulation can be used to gain more insight into a process or to make predictions. It is
logical that the understanding of the process is the basis for predictions. This suggests that we should
first build simulations to improve our understanding. In a second step simulations may be refined up to
a level where useful predictions are possible. After the second step simulations may be applied in
practice. This is also the argument of starting with a simple model and extending it step by step.

2.4.1 Agent-based Simulation Used as Research Method

In research we focus more on gaining insight, even though we use the prediction property to do so.
Robert Axelrod (1997) names simulation as third way of doing research combining induction and
deduction. After making assumptions in deductive tradition we run the simulation to analyse the
generated data inductively. Gilbert and Troitzsch (2005, p. 15) define simulation as follows:

“Simulation means ‘running’ the model forward through (simulated) time and watching
what happens.”

The logic of simulation is very similar to statistical modelling. In statistical modelling we estimate
parameters for a postulated model based on observed data. This quality is admirable if we have
collected data at hand because it allows us to fit our model systematically to the observed data. The parameter estimates allow afterwards to make predictions.

In contrast, simulated (predicted) data is gained by simulation of a postulated model. Also the simulated data is compared to observed data if these are available. The better the similarity, the better the model. By building the simulation and comparing its output to observations we can check whether our assumptions about the functioning of the process are plausible or not. If we observe the expected behaviour we can assume our simulation to work correct. Simulation is especially useful if we want to study processes (Gilbert and Troitzsch, 2005).

In a very first step we might just want to check whether our idea of the process is plausible. In this stage we do a proof-of-concept simulation. In this stage it is possible to validate the simulations against expectations. This means that no explicit empirical data is needed. We just check whether the simulation is producing plausible output.

### 2.4.2 Used as Assignment Method in Assessment

Numerical methods can be regarded as simulations too. By iterative approximation of the solution we are kind of ‘...’running the model forward...’ until we (hopefully) reach the desired solution. Against the background of this interpretation simulation has a long tradition in the field of transportation research. Simulation can be used to solve the assignment problem\(^2\) which cannot be solved analytically. The function of the simulation is here to do the assignment.

To simulate the assignment we basically need to simulate demand and supply. As traffic shares a lot of the characteristics mentioned under section 2.4 it is not surprising that traffic is simulated agent-based. This is pursuing a disaggregate approach. The obvious system element to be modeled as agent is the traveller. The behaviour we are interested in can be reduced to decision processes which determine the selection of transport alternatives. These decisions can be simulated applying discrete choice theory as explained in section 2.2. Summing up all decisions results in the overall demand.

To apply discrete choice theory we need discrete alternatives. The determination of the alternatives is a crucial issue and depends on the decision(s) we want to simulate. If we have just one option it is not even possible to make a decision. This shows that it is very important to think of all options available in a decision situation!

In early transport models just routes were considered as alternatives. This assumes a decision situation in which neither departure time nor starting and end point are regarded as options. Nowadays this

\(^2\) See chapter 2.3.
scope is enlarged because we recognise that travellers not only select routes but also departure time and location.

When we think of congested networks departure time becomes a relevant option in terms of avoiding congestion and therefore saving travel time. This leads to the development of dynamic transport models which assign travel demand on routes and time slots.

By considering location choice we are not just simulating the distribution of the agents on the transport infrastructure but also the locations visited in space. This allows to study the consequences of an infrastructural adjustment for the spatial distribution of activities. This way land use is integrated into the modeling of the spatial socio-economic system.

We note that the integration of the aspects mentioned above requires a more detailed or enlarged set of alternatives. Depending on how detailed we are going to model the supply, more aspects of decision making can be considered. This can also be interpreted as expanding assignment to multiple dimensions.

To cover the afore mentioned aspects we have to model the relevant conditions of the transport infrastructure and possible locations over time. This represents the available alternatives, the supply. Usually it is conceptualised in a network formulation.

To reach the equilibrium state of a congested network we have to simulate the decisions of the agents multiple times, because of the interdependence of agents’ decisions and characteristics of available alternatives. The performance of the network elements is influenced by the decisions of the travellers. If we just consider one stage in advance, this is a first order Markov-Chain process. However, it is not obvious that the simulation will get to the expected UE. It is possible that a local maximum is calculated or that during the optimisation process oscillation occurs. To get to an equilibrium in the simulation the share of agents choosing has to be reduced. Similar to MSA. The reduction of choosing agents results in forced calming of the system.

2.5 Indicators of Utility Gain

To evaluate infrastructural investments various assessment schemes have been developed. They have in common that they suggest one or more indicators upon which decisions are taken. Usually these indicators are calculated on the basis of predicted traffic volumes using a transport model.

The projects have to be judged form a public point of view. The assessment has to answer the question whether the project increases welfare for society. To do so costs and benefits for society are compared.
The benefits are the reductions in *generalised costs* due to a certain investment. In the course of discrete choice model applications a new indicator has come into focus, the *expected maximum utility* (EMU). A comprehensive literature review and explanation can be found in De Jong (2005). We will discuss these two concepts briefly.

### 2.5.1 Generalised costs

In the most general sense generalised costs are calculated by summing up all costs which arise to society given a certain development path. The path producing less generalised costs is preferable, at least from an economic point of view.

Applying this concept in a discrete choice model means to sum up all costs (or utilities) of the chosen alternatives given a certain transport supply. We will name this quantity $\sum\text{realised utility}$ because the utility has actually been experienced by the traveller using an alternative. An improvement of the situation is achieved when the sum of all costs is reduced.

In the context of transportation we focus on costs given a infrastructural development. Some of theses costs depend more obviously on transport infrastructure than others. Usually the focus is the cost of a trip. They include costs directly associated with travelling such as travel time costs, ticket costs, costs occuring due to unpunctuality or costs of comfort. More precisely we could name them *perceived travel costs*. Already less obvious is to consider costs of season tickets, car ownership or even taxes which are paid to build transport infrastructures. Such costs my be named *not perceived travel costs*.

The third type of costs arise indirectly. They are often called *external costs* or *secondary costs*, such as emission costs, opportunity costs, maintenance costs, costs of accidents or land costs.

We note that not perceived and secondary costs are long run costs. This means that they become relevant in a long term perspective which we are dealing with when judging an infrastructural investment.

### 2.5.2 Expected Maximum Utility (EMU)

In contrast to the realised utility the EMU considers the utilities of all alternatives in the choice set of each traveller $n$. The formula for the EMU following Ben-Akiva and Lerman (1985) of a traveller $n$ is:

$$EMU_n = \frac{1}{\alpha_n} \ln(\sum_{A} e^{V_{na}})$$

(2.8)

with $A$ ... choice set of traveller $n$

$\alpha_n$ ... marginal utility in income of traveller $n$ (here set to 1.0)
The EMU describes the potentially available utility of an individual. The individual cannot make use of all this utility because it can just select one alternative. The EMU is therefore more a potential. We can interpret the EMU as welfare indicator (McFadden, 1981). We sum up the consumer surplus of all individuals. In the economic welfare concept we assume that welfare depends on the individual's possibilities in its life situation. In transport research the same formula is interpreted as accessibility (Ben-Akiva and Lerman, 1985). Also this concept is consistent with the welfare theory. An individual which has access to more possibilities is supposed to be wealthier.

If we want to calculate the increase in welfare of a whole population due to infrastructural investment, we simply compute the difference between the sums of all agents’ EMU before and after the intervention.

An important property of the EMU is that it grows steadily with the addition of new alternatives. This means that the indicator depends on the number of alternatives in the choice set. The logarithmic formulation is chosen to account for the law of decreasing marginal utility.
3 An Agent-based Simulation

3.1 Description of the Simulation *MiniStadt*

3.1.1 Purpose of the Model

The simulation shall help us to estimate the error when we neglect behavioural dimensions in the welfare assessment of an investment. The simulation presented allows us to study the differences due to omitted behavioural dimensions. This will give us indications whether the typically considered dimensions of behaviour are sufficient to assess an infrastructural investment. Here we have to point out that the model shall help us to make conclusions about modelling itself. We are interested in the degree of approximation. How far can we abstract the behaviour in transport demand models if we want to estimate the benefits in socio-economic system like a city?

To study this question we have to simulate the reaction of inhabitants to an infrastructural investment allowing for different degrees of freedom. The simulation should contain several decision dimensions, which can be switched on or off. It shall be possible to introduce infrastructural investments, of which the utility gains are calculated. The experiments have to be designed in a way that decision making is meaningful for the dimensions considered.

Primarily we do not want to uncover some real world phenomena. Therefore the simulation is abstract. Nevertheless the model should reveal some insight into the functioning of the system.

3.1.2 Model Components

*Locations*

The simulation considers a minimal representation of a city. The city space is represented by four locations A, B, C and D. A is the working location of all inhabitants. The houses where the agents live are located in B, C, or D. The locations have capacity to accommodate 600 agents each.
**Transport Infrastructure**

To get home from work the agents must make use of the transport infrastructure which consists of links connecting the locations. The links are either of type main road (S1, S2, S3), highway (S4, S5) or railway (S6). The number of links to peripheral locations is smaller because of the reduced number of agents travelling so far. This allows us to maintain comparison on all links.

In case of main roads and highways we use the BPR-function to describe the relation between link load and travel costs. To consider the fact that a train can be full at a certain departure time the Davidson-function is applied in case of the railway link. The parameters specifying the capacity restraint functions are in the appendix (see table 8). The links are the transport services which is the supply (see figure 1).

![Figure 1 MiniStadt: Initial transport infrastructure](image)

**Agents**

The city is populated by 1000 agents, all working in A and living in any location apart. We assume a constant population and that all agents have to make a home trip.

The agents want to optimise their utility by choosing from the choice set the alternative with maximum utility. The objective function of the agents is:

\[
\max (U) = \max (V(X) + \varepsilon_{ij})
\]  

(3.1)
According to discrete choice theory the stochastic utility $U$ is the sum of the deterministic utility given by a utility function and a Gumbel distributed random utility. In this simulation the random part $\varepsilon_{rtj}$ of utility is generated once per agent and specific alternative. The random part represents the unknown preferences of the agent. This preference for an alternative remains constant during the simulation (see also section 3.2).

The deterministic utility is an additive, linear combination of weighted utility components. The utility components are a function of the three choices connection choice ($r$), departure time choice ($t$) and destination choice ($j$).

$$V(r, t, j) = \beta_r * V_r(r, t, j) + \beta_t * V_t(r, t, j) + \beta_j * V_j(j)$$  \hspace{1cm} (3.2)

with $\beta_r$, $\beta_t$, $\beta_j$ ... weight parameters

The explanatory variables are options at the corresponding decision dimension. The options at each decision dimension can be described as sets.

We model mode choice in a comprehensive way by presenting connections as alternatives. We define connections as a sequence of links from origin to destination of a trip. For the link sequence it does not matter whether the link is assigned to public transport or individual transport. This means that an agent is allowed to continue his trip on a road even though he travelled on a rail way link before. The set of connections consists of all possible sequences of links from location A to one of the locations B, C or D. These locations are the options for destination choice. The initial network provides 15 connections.

Time is represented as a set of 24 possible departure time intervals. Each interval represents 5 minutes which qualifies the model as dynamic (Janson, 1991, p. 143). This means that the agents have a time span of 2 hours to leave from work. Note that in the simulation time is represented as a discrete quantity.

Each combination of the options makes up an alternative of the choice set. Not all combinations are actually valid alternatives. It is for example meaningless to choose a connection, which is not corresponding to the selected location. With other words we just consider possible alternatives in the choice set.

Travel time utility has the following functional form:

$$V_t(r, t, j) = \beta_g * T_t$$  \hspace{1cm} (3.3)
with \( T_r = \sum s T_s \cdot \delta_r^s \) (\( \delta_r^s = 0 \), if \( s \) is part of connection \( r \); \( \delta_r^s = 1 \), otherwise)

The formula shows that travel time utility depends on connection choice, departure time choice, location choice and an agent specific value of travel time savings (VTTS) \( \beta_g \). We just distinguish between agents which have a high VTTS (\( \beta_g = 2 \)) versus agents with a low VTTS (\( \beta_g = 1 \)). \( T_r \) is the sum over the link travel times \( T_s \) which are part of the connection. Travel time \( T_s \) is calculated with the capacity restraint function corresponding to link \( s \).

To model the utility originating in punctuality we use a formulation following Small (1982). Small introduces the arrival time \( \tau = t + T_s \) and calculates than the utility according to:

\[
V_t(\tau) = \zeta * SDE(\tau) + (\gamma * SDL(\tau) + \delta * d_L)
\]  

(3.4)

with \( SDE = \max(PAT - \tau, 0) \)  
\( SDL = \max(\tau - PAT, 0) \)  
\( d_L = 1, \text{ if } \tau > PAT, \text{ d}_L = 0, \text{ if } \tau \leq 0 \)  
\( PAT \ldots \text{prefered arrival time} \)  
\( \delta \ldots \text{penalty for being late} \)  
\( \zeta, \gamma \ldots \text{utility loss rates for SDE and SDL respectively} \)

The utility depends on the difference between arrival time \( \tau \) and the preferred arrival time (PAT) which is set to the beginning of time interval 24. Utility from punctuality depends on choices in all three dimensions because arrival time depends on travel time.

Utility from location choice is calculated with the following function:

\[
V(j) = \exp(\lambda * A_j / Q_j)
\]  

(3.5)

In a very rough approximation we assume that living costs at location \( j \) depend on the availability of living space, which is represented by the occupancy rate. The occupancy rate is the coefficient of capacity \( Q_j \) of location \( j \) and the number of agents \( A_j \) selecting location \( j \). We further assume that costs increase exponentially with the occupancy rate. We follow the common assumption about prices of limited goods responding to demand.
The parameter \( \lambda \) allows to make the simulation more sensitive with respect to occupancy rates of locations.

The utility functions are actually rather cost functions. Utility gains are visible in reduction of costs. We should keep this in mind for the interpretation of the results.

It is obvious that we are not claiming to integrate all important costs. We would have to model house costs, costs of mobility tools (fix costs of vehicles, season tickets etc.), transfer costs and so on.

### 3.2 Experiments

We simulate the reaction to an infrastructural investment. The experiment consists of calculating a SUE for the initial conditions (state 1), introducing an infrastructural investment and calculating four subsequent SUE as a reaction to the investment (states 2) with different degrees of freedom for the agents. Note that subsequent SUE depend on the degrees of freedom (available decision dimensions) and the infrastructural investment. The random components of the alternatives' utilities are generated once per experiment.

#### 3.2.1 Infrastructural Investment

We experimented with three types of investments. We introduced new links, modified existing links or deleted links. For the two first types we tested several variations.

In this paper we just discuss the results of one scenario. A new highway S7 shall connect the working location A directly with peripheral location C. The link has a free-flow travel time of 5.0, a capacity of 40 agents per time interval and is characterised by a BPR-function.
The investment leads to a new choice set. The choices of the agents will change if the new alternatives promise better utility. The new decisions will lead to a new equilibrium. The agents also have a preference for new alternatives. Therefore we have to generate a new stochastic component for each agent and new alternative. We further assume that the preferences towards the old alternatives stay the same.

### 3.2.2 Choice Spaces

The combination of decision dimensions we name *choice spaces*. We experiment with four choice spaces:

- RTJ
- RT
- R
- RJ

Each letter represents a decision dimension at which the agents find a discrete number of options. If a dimension is not present, all agents will remain with their earlier chosen option at this dimension. This means for example that the agents' departure time cannot change in choice space RJ. As a consequence the number of alternatives is very different according to applied choice space (see figure 3).

This affects the EMU, which is calculated applying formula 2.8 across all available alternatives of each agent's choice set. By applying choice space RT all alternatives with different options at decision dimension J than the current one are not considered. Therefore the choice set is smaller and
consequently the EMU. To isolate the effect of the infrastructural investment from the effect of more degrees of freedom we calculate the EMU before and after the investment with the same choice space. The EMU considers by definition only available alternatives. This is the reason why we do not sum across all alternatives in choice space RTJ when simulate e.g. RT.

We note that restricted choice spaces depend on the decisions made earlier by the agents. This means that restricted choice spaces are individual. The number of utility components of overall utility remains the same for all choice spaces.

The choice spaces in figure 3 reflect the initial transport infrastructure. If we consider the network with S7 we have 408, 17, 72 – 168, 3 – 7 combinations with choice spaces RTJ, RJ, RT and R respectively.

**Figure 3** Visualisation of Choice Spaces
As mentioned before we just want to have all possible alternatives in the choice set. Therefore we require that connection choice and location choice are consistent. This has implication for the number of possible of choice spaces. It is not possible to choose a new location without choosing a new connection as well. Therefore the combinations TJ and J are not considered.

The choice space has also to be appropriate for the investment, meaning that the agents should be able to react to the investment. For example, agents with a choice space T cannot react to a new link, because they cannot chose a new connection. If we had a investment spreading PAT of the agents, choice space T would make more sense. This also means that the differences between the results simulated with different choice spaces depends on the investment applied.

### 3.2.3 Calculation of SUE

Algorithms formulated to solve aggregate models (Fernandez and Friesz, 1983) are not applicable in a straight forward way because of the discrete nature of agent-based models. Unlike flows agents cannot be split apart. Hence, an infinitesimal approximation of the equilibrium is not possible. To meet theoretical equilibrium conditions is unlikely because the approximation of equilibrium conditions is only possible up to the “agent resolution”.

To find the SUE of state 1 and state 2 an iterative incremental assignment algorithm is implemented:

1. Load the initial conditions and set the number of iterations $n = 0$.
2. Calculate the number of choosing agents $M = \text{number of agents}/(n + 1)^2$.
3. Order the agents by descending maximal potential utility gains.
4. Select the first $M$ agents.
5. Randomise the order of these $M$ agents.
6. Let these agents make their decisions one after the other and update the network after each decision.
7. Update the utilities of all agents for the chosen and non chosen alternatives.
8. Calculate the maximal potential utility gain and other attributes for each agent.
9. Calculate statistics of the whole system.
10. Go back to step 2 as long as $n < 20$ or sum of potential utility gains $\neq$ minimum of potential utility gains in all preceding iterations. Also stop iterating if no agent finds a better alternative, oscillation occurs or the maximum number of iterations is reached.
Reasons for the formulation of the algorithm

The updating of the network after each decision is the extreme case of an incremental assignment. Extreme situations in which a lot of agents choose the same alternative are avoided. This leads to a pretty good approximation already in the first iteration.

A drawback of the incremental assignment is the information bias which handicaps early agents deciding. The first agent choosing does not know anything about later decisions until the utilities of the alternatives are updated at the end of an iteration. Therefore the agent might end up with a non-optimal alternative. By iterating several times over the population we overcome this problem.

To speed up equilibrium search we calculate the maximum potential utility gain for each agent considering the current utilities at the end of each iteration. Then we let these agents re-decide which have the highest potential utility gain.

The termination condition is rather complex because of possible oscillation and unsteady convergence respectively. Because of oscillation we cannot rely on the theoretical termination condition of no agent switching the selected alternative. Let us think of a single remaining agent to decide, which will choose the alternative with highest utility. This leads to a decrease of utility of this specific alternative because the loads are now higher. Possibly the previous alternative with new loads is again better for the agent and therefore switches back. This mechanism leads to oscillation.

One could argue that the agent choosing shall consider the effect of having chosen the alternative before. To find the actually best alternative in an iteration the agent would have to compare the before chosen alternative minus its demand for all other alternatives plus its demand. In each case new network loads would have to be calculated. Already in this small system storage requirements and computation time would increase substantially which makes doing one oscillation cycle the better option.

The unsteady convergence obliges us to set a minimum number of iterations. To be sure to stop iterating with a good approximation of the equilibrium we require that the average relative potential utility gain, an indicator for how close we are to the equilibrium, is equal to the minimum calculated value of this quantity during the iterations before.

Still this means that the algorithm cannot guarantee to find the exact equilibrium. If we hit a local minimum in the preceding iterations the algorithm will stop too early.

We define the reduction of agents deciding such that after the minimum number of iterations approximately 1% of all agents decide again. The minimum number of one choosing agent is reached after 22 iterations. The maximum is set as \( n = 40 \).
3.3 Parametrisation

Because the simulation is very abstract and not based on empirical data, we assume the parameters such that the average elasticity of demand $y_a$ of alternative $a$ with respect to travel time is approximately 0.6. Further we required that the elasticities of demand in respect to the utility components have the following relation: $E(y_a, V_r) > E(y_a, V_t) >> E(y_a, V_j)$. The idea behind this requirements is that it is easy to change connection, hard to change departure time because of fixed working hours$^3$ and even harder to change location. Table 1 shows that the requirement is met in this simulations.

<table>
<thead>
<tr>
<th>Averages of 15 Simulations</th>
<th>$V_r$</th>
<th>$V_t$</th>
<th>$V_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean elasticity</td>
<td>0.499</td>
<td>0.100</td>
<td>0.032</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>0.004</td>
<td>0.005</td>
<td>0.013</td>
</tr>
</tbody>
</table>

Furthermore we require that the agents are distributed over all links and locations. The distribution over all links ensures that the fastest links are congested in some time intervals and that changing to an earlier or later time slot is not a better alternative because of late or early arrival. Therefore utility loss of early or late arrival must be chosen high enough to make agents changing the link. This ensures that the agents make trade-offs between all decision dimensions. Further additional travel time shall be punished stronger than earlier arrival. The parameters of the utility function and the capacity restraint functions are set accordingly. They are shown in table 10.

The simulation at hand is on a performance level 1 according to the categorisation found in Gilbert’s paper (Gilbert and Terna, 2000) because it “is in qualitative agreement with empirical macro-structure, as established by plotting the distributions of some attributes of the agent population.”

$^3$ One could argue that departure time is easiest to alter. This is only true for people which determine their schedule themselves. Employed people, however, have an externally determined schedule which they cannot change that easily.
4 Results

In this section we present the differences between simulation results calculated with choice spaces RTJ, RT, R and RJ respectively. All the results refer to the scenario with the newly introduced highway S7. To have an idea of the robustness of the results due to stochasticities we simulated the scenarios 5 times each. The result tables usually contain the average of 5 simulations. The coefficients of variation in the appendix (table 12 to 17) show the variance of the results.

The difference in an indicator \( I \) between state 1 and state 2 (prediction) are the predicted changes due to the investment. These are computed as follows for each choice space \( X \):

\[
\Delta X = I_{2X} - I_1
\]  

(4.1)

We assume that the simulation with RTJ predicts more adequate results because more decision dimensions are considered. Therefore we compare predictions with choice space RTJ to predictions which neglect decision dimensions. The absolute error made with a choice space \( X \) are given by:

\[
F_X = \Delta X - \Delta RTJ
\]  

(4.2)

The relative error is given by:

\[
f_X = \frac{F_X}{\Delta RTJ}
\]  

(4.3)

The simulated data are compared on the basis of:

- link and locations loads
- total travel time (\( \sum \text{Travel time} \))
- total travelled distance (\( \sum \text{Travelled distance} \))
- total of realised utility (\( \sum \text{Realised utility} \))
- total of utility potentials (\( \sum \text{EMU} \))
- external costs

We start out with describing the simulated equilibria by means of link and locations loads. We then show the indicators of total travel time and total travelled distance. We proceed with presenting the
calculated utilities, utility gains and external costs. In section 4.1.4 we describe the accuracy of calculated equilibria.

4.1.1 Quantity Indicators

The simulated loads of locations and links depend on the choice space used for simulation. We show this in table 2 and figure 4. Table 2 shows different occupancy rates of agents residing in B, C or D according to choice space applied.

If location and departure time choice is possible, the agents react by exploiting the higher accessibility of location C and D. We note higher total occupancy of location D (increase of 3%) while occupancy at location B is lowered (4%). Surprisingly, the total occupancy in C remains the same. For explanation we have to have a closer look at the occupancy rates of agents with high and low time value respectively.

We see that the occupancy of agents with a high value of travel time increases in C by 6% while the occupancy of agents with a low one decreases by 5%. It is interesting to see this process of agents with low time value being crowded out from location C to location D. Looking at the values at location B we recognise that some agents with low time value take the opportunity to reside nearer to working location A (increase of 2%).

In a way we are also capturing a social process which is forming the environment we live in. The process of gathering agents with similar characteristics can be interpreted as self selection which forms social segregation. From this point of view the simulation shows a segregation phenomenon first simulated by Schelling with his famous segregation model (Schelling, 1969). The different shares of agents with different characteristics (in this case just low or high time value) points at the influence of transport infrastructures on the social structure at locations.

If agents can adapt connection and location only, the occupancies show a higher movement of agents with low time value to peripheral locations. The explanation is that agents choose their location such that they arrive punctually. They search for lower living costs and low costs because of unpunctuality. This way they actually benefit from travelling an equal duration but faster.

Obviously, simulations which neglect location choice will not predict occupancy rates different from state 1.

The rates in table 2 can also be interpreted as living costs at the corresponding locations. Therefore we can argue that simulations with location choice predict price variations at locations. In this case such models suggest that owners of living space in location D have benefits while owners of living space in location B have losses.

Location A is not shown because it cannot be chosen as destination to go home to.
Table 2: Occupancy Rate of Locations by Choice Spaces and Time Values of Agents

<table>
<thead>
<tr>
<th>Average of 5 Simulations (%)</th>
<th>B</th>
<th></th>
<th>C</th>
<th></th>
<th>D</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>High</td>
<td>Low</td>
<td>Total</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>RTJ</td>
<td>84</td>
<td>76</td>
<td>8</td>
<td>67</td>
<td>9</td>
<td>59</td>
</tr>
<tr>
<td>RT</td>
<td>88</td>
<td>82</td>
<td>6</td>
<td>67</td>
<td>3</td>
<td>64</td>
</tr>
<tr>
<td>R</td>
<td>88</td>
<td>82</td>
<td>6</td>
<td>67</td>
<td>3</td>
<td>64</td>
</tr>
<tr>
<td>RJ</td>
<td>84</td>
<td>79</td>
<td>5</td>
<td>66</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>State 1</td>
<td>88</td>
<td>82</td>
<td>6</td>
<td>67</td>
<td>3</td>
<td>64</td>
</tr>
</tbody>
</table>

Figure 4 shows that also the loads of links are different depending on choice space used. Clearly, the different loads will influence the utility of the alternatives and therefore produce different equilibrium states for the whole system.

In the link loads we observe as well the consequences of neglected location choice. More obvious, however, are the differences which arise due to departure time choice. With choice spaces RTJ and RT agents will depart later which is visible in the graphs of S1, S4 and S6.

The graph of S7 shows that with restricted departure time choice the number of predicted agents choosing the new link is smaller compared to simulations with departure time choice. The reason is the given distribution on the time axis which is like a departure regime. The regime brings about that the agents cannot catch up. Hence, it needs only a few agents changing connection to resolve congestion.
Figure 4   Link Loads
In table 3 we see that the total of travel time decreases with all choice spaces as expected. However, we note that the decrease is not the same for all choice spaces. It is plausible that choice space RT reveals a high reduction in travel time. Only with the possibility to change departure time the agents are able to profit fully from the investment as shown above. In case of choice space RJ the high travel time saving origins in the given distribution of departure times. When all three dimensions are available the travel time savings are not as high because more agents chose peripheral locations.

Travelled distances increase with location choice. The agents make use of the increased accessibility of location D where they can profit from low living costs (see table 2). This is reasonable. That \( \sum \text{Travelled distance} \) does not change with connection choice is an artefact of the equal distances assumed for the links.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Predicted Changes by Choice Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average of 5 Simulations</td>
<td>( \Delta \text{RTJ} )</td>
</tr>
<tr>
<td>( \sum \text{Travel time} ) [min]</td>
<td>-1187</td>
</tr>
<tr>
<td>( \sum \text{Travelled distance} ) [km]</td>
<td>874</td>
</tr>
</tbody>
</table>

Table 4 contains the absolute and relative error of predicted changes in travel time and travelled distance. The errors are generally very high. In respect of travel time savings we do overestimate the gains when decision dimensions are neglected.

The prediction of travel time reduction is overestimated up to 43%, if we neglect departure time choice. Neglecting departure time choice and location choice leads in this case to an overestimation of 30% and neglecting location choice of 41% respectively. This finding is consistent with the notion that travel time savings tend to disappear in a long term perspective in which location choice should be considered as well.

Without location choice no variation in travelled distance can occur. That is why the change of travelled distance is underestimated by 100% with such choice spaces. Neglecting departure time choice leads to an overestimation of 24%.

\(^4\)We focus on indicators which are describing the population in this section. To calculate the indicator for the whole population we sum up the indicators of the individual agents. Thus we write \( \sum \text{Travel time} \) for the sum of all agents travel time.
Table 4  Absolute and Relative Error of Predicted Change by Choice Space

<table>
<thead>
<tr>
<th>Average of 5 simulations</th>
<th>$F_{RT}$</th>
<th>$f_{RT}$</th>
<th>$F_{R}$</th>
<th>$f_{R}$</th>
<th>$F_{RJ}$</th>
<th>$f_{RJ}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma$ Travel time</td>
<td>-460</td>
<td>0.41</td>
<td>-318</td>
<td>0.30</td>
<td>-478</td>
<td>0.43</td>
</tr>
<tr>
<td>$\Sigma$ Traveled distance</td>
<td>-874</td>
<td>-1.00</td>
<td>-874</td>
<td>-1.00</td>
<td>211.6</td>
<td>0.24</td>
</tr>
</tbody>
</table>

4.1.2 Utilities

Table 5 shows the predicted variations of the utility indicators. $\Sigma EMU$ and $\Sigma Realised utility$ are indicators for overall utility. $\Sigma Realised utility$ is the sum of the utility components $V_r$, $V_t$ and $V_j$. The deltas of both over all indicators have a positive sign with any choice space. This shows the expected increases in utility because of the investment.

$\Sigma EMU$ and $\Sigma Realised utility$ show different utility levels. This is not very surprising because they measure different things.

$\Sigma Realised utility$ shows the expected increase in utility gains with additional decision dimensions. This demonstrates that neglecting decision dimensions leads to underestimation of utility gains. The reason is that agents cannot profit from all possible utility gains with reduced decision dimensions. We see this by analysing the composition of the realised utility.

The three utility components $V_r$, $V_t$ and $V_j$ show us the composition of $\Sigma Realised utility$. The results show that the compositions are quite different. This means that the agents gain their utility differently according to the choice space. More decision dimensions provide the agent with more possibilities to adapt to new circumstances. The agents can make better use of the available alternatives. However, we cannot allocate utility gained through an additional decision dimension to one utility component because a choice at one dimension influences all components.

$V_t$ is the utility arising from travel time savings. This indicator is not showing the same utility gains as the more comprehensive indicators of $\Sigma Realised utility$ and $\Sigma EMU$. Further we see that the indicator is not showing the same utility gains with respect to choice spaces. Neither do we find the expected increase in utility with more degrees of freedom. Contrasting the utility out of travel time savings with the realised utility shows that the utility is not lost but transferred to other utility components. This demonstrates that travel time savings do not show all utility gains. We conclude that utility gains from travel time savings do not capture all utility gains if we suppose decision dimensions such as departure time choice and location choice to exist.
Table 5  Predicted Utility Change by Choice Space

<table>
<thead>
<tr>
<th>Average of 5 simulations</th>
<th>ΔRTJ</th>
<th>ΔRT</th>
<th>ΔR</th>
<th>ΔRJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>∑EMU</td>
<td>302.91</td>
<td>166.95</td>
<td>158.95</td>
<td>437.70</td>
</tr>
<tr>
<td>∑V_t</td>
<td>68.79</td>
<td>111.50</td>
<td>102.74</td>
<td>129.83</td>
</tr>
<tr>
<td>∑V_t</td>
<td>73.76</td>
<td>53.02</td>
<td>-15.31</td>
<td>-27.87</td>
</tr>
<tr>
<td>∑V_j</td>
<td>133.34</td>
<td>0.00</td>
<td>0.00</td>
<td>157.82</td>
</tr>
<tr>
<td>∑Realised utility</td>
<td>275.88</td>
<td>164.52</td>
<td>87.43</td>
<td>259.78</td>
</tr>
</tbody>
</table>

As mentioned before we assume that the prediction with choice space RTJ is most accurate. In table 6 we list the errors in predicted utility gains compared to the prediction made with choice space RTJ. We note as well substantial errors. For over all utility indicators we generally see underestimation. This up to 68% for the indicator ∑Realised utility with choice space R. In case of choice space RJ ∑EMU overestimates the utility increase by 44%. This is due to the given “regime” of departure times as discussed above.

The components of the realised utility show even higher errors which reflects the suppressed trade-offs. Utility out of travel time savings is generally overestimated. The relative errors are very high with 65%, 54% and 95% respectively.

The utility component of punctuality ∑V_t is underestimated in all restricted choice spaces. This shows that all three decision dimensions need to be available to get maximum utility out of punctuality. The error is obviously smaller if departure time choice is available (only 28%).

Utility gains because of relocation are underestimated by 100% if location choice is absent. If the agents cannot adjust their departure time, utility from location choice is still overestimated by 18%.

Table 6  Absolute and Relative Error of Predicted Utility Change by Choice Space

<table>
<thead>
<tr>
<th>Average of 5 simulations</th>
<th>F_{RT}</th>
<th>f_{RT}</th>
<th>F_{R}</th>
<th>f_{R}</th>
<th>F_{RJ}</th>
<th>f_{RJ}</th>
</tr>
</thead>
<tbody>
<tr>
<td>∑EMU</td>
<td>-135.98</td>
<td>-0.45</td>
<td>-143.96</td>
<td>-0.47</td>
<td>134.79</td>
<td>0.44</td>
</tr>
<tr>
<td>∑V_t</td>
<td>42.72</td>
<td>0.65</td>
<td>33.95</td>
<td>0.54</td>
<td>61.05</td>
<td>0.95</td>
</tr>
<tr>
<td>∑V_t</td>
<td>-20.74</td>
<td>-0.28</td>
<td>-89.07</td>
<td>-1.21</td>
<td>-101.63</td>
<td>-1.38</td>
</tr>
<tr>
<td>∑V_j</td>
<td>-133.34</td>
<td>-1.00</td>
<td>-133.34</td>
<td>-1.00</td>
<td>24.48</td>
<td>0.18</td>
</tr>
<tr>
<td>∑Realised utility</td>
<td>-111.36</td>
<td>-0.40</td>
<td>-188.46</td>
<td>-0.68</td>
<td>-16.10</td>
<td>-0.06</td>
</tr>
</tbody>
</table>
We expect that the presented errors are quite high because the evaluated infrastructural investments alter the system substantially with respect to its size. In real world networks the errors might be smaller.

### 4.1.3 External Costs

It is clear that estimated external costs also depend on the choice space simulated given that omitting decision dimensions leads to different network loads (chapter 4.1.1). In table 7 we list the changes in predicted annual external costs calculated with the simplified approach of the Swiss Standard SN 641 828 (VSS, 2006).

The changes in accidents cost dominate. The accident costs decrease compared to the reference state because there is more traffic on highways. Highways have an accident cost rate per vehicle kilometre which is approximately ten times smaller compared to main roads. All other external costs rise because total travelled distance by car increases. The reasons are choice of peripheral locations and reduced usage of railway.

<table>
<thead>
<tr>
<th>Average of 5 simulations (CHF / a)</th>
<th>ΔRTJ</th>
<th>ΔRT</th>
<th>ΔR</th>
<th>ΔRJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accident costs</td>
<td>-479'103</td>
<td>-472676</td>
<td>-154'474</td>
<td>-148'352</td>
</tr>
<tr>
<td>Traffic noise costs</td>
<td>9'849</td>
<td>4584</td>
<td>2'421</td>
<td>9'990</td>
</tr>
<tr>
<td>Air pollution costs</td>
<td>26'462</td>
<td>13'633</td>
<td>7'201</td>
<td>25'501</td>
</tr>
<tr>
<td>Climate costs</td>
<td>5'706</td>
<td>2'655</td>
<td>1'403</td>
<td>5'787</td>
</tr>
<tr>
<td>Sum external costs</td>
<td>-437'087</td>
<td>-451'805</td>
<td>-143'449</td>
<td>-107'074</td>
</tr>
</tbody>
</table>

Table 8 shows the absolute and relative error of estimated external costs in respect of the estimations with choice space RTJ. The relative errors have absolute values within a range of 21 – 77%, which is substantial.

If we do not model location choice we underestimate the increase in costs of traffic noise, air pollution and climate change. Without location choice we do not consider the increase in total travelled distance.

In case of simulating connection and location choice we overestimate the number of agents travelling longer distances (see chapter 4.1.1). The result is overestimated external costs.
Neglecting departure time choice results in high underestimation of accident costs. The reason is the scarce usage of the new highway S7 (see figure 4).

Table 8 Absolute and Relative Error of Estimated External Costs by Choice Space

<table>
<thead>
<tr>
<th>Average of 5 simulations</th>
<th>$F_{RT}$</th>
<th>$F_{RT}$</th>
<th>$F_R$</th>
<th>$F_R$</th>
<th>$F_{RU}$</th>
<th>$F_{RU}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accident costs</td>
<td>6427</td>
<td>-0.01</td>
<td>324630</td>
<td>-0.67</td>
<td>330751</td>
<td>-0.69</td>
</tr>
<tr>
<td>Traffic noise costs</td>
<td>-5265</td>
<td>-0.54</td>
<td>-2163</td>
<td>-0.21</td>
<td>7569</td>
<td>0.77</td>
</tr>
<tr>
<td>Air pollution costs</td>
<td>-12829</td>
<td>-0.49</td>
<td>-6432</td>
<td>-0.24</td>
<td>18300</td>
<td>0.69</td>
</tr>
<tr>
<td>Climate costs</td>
<td>-3050</td>
<td>-0.54</td>
<td>-1253</td>
<td>-0.21</td>
<td>4385</td>
<td>0.77</td>
</tr>
<tr>
<td>Sum external costs</td>
<td>-14718</td>
<td>0.04</td>
<td>308356</td>
<td>-0.71</td>
<td>36375</td>
<td>-0.09</td>
</tr>
</tbody>
</table>

4.1.4 Approximation of Equilibrium

Several variations of the algorithm were tested. They vary the point in time of network updates, the function to reduce the number of choosing agents, the criteria to stop the simulation and the minimal number of agents remaining to choose. None of the algorithm's variations is able to ensure reaching the SUE in which potential utility gains in the next iteration would be 0.

Because convergence to the SUE is not reliable, we stop iterating when the best solution out of 20 iterations is exceeded. To see how close the solution is to the SUE, we compute the average relative potential utility gain per agent. This indicator shows that the agents find very good alternatives. In general an average improvement of no more than 2.1% is still possible (see table 9). We reason that this is an acceptable approximation because in reality people hardly perceive a difference of 2.1%.

We note that the calculated states are less accurate with more decision dimensions. With more degrees of freedom it is harder to find the optimal solution. The coefficients of variation, on the other hand, show that simulations with more degrees of freedom produce more stable results in terms of accuracy.
### Table 9  Statistics for the average potential utility gain per agent

<table>
<thead>
<tr>
<th>Statistic</th>
<th>RTJ</th>
<th>RT</th>
<th>R</th>
<th>RJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average [%]</td>
<td>2</td>
<td>1.5</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>Maximum [%]</td>
<td>2.1</td>
<td>1.7</td>
<td>0.6</td>
<td>0.5</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>0.045</td>
<td>0.119</td>
<td>0.541</td>
<td>0.410</td>
</tr>
</tbody>
</table>

---

5 The statistics are calculated on the basis of 5 simulations.
5 Conclusions

The simulation experiments let us conclude that neglecting decision dimensions in a transport model overlooks important effects occurring after an infrastructural investment. In this paper we show that traffic flows in the transport system vary according to choice space. Consequently utilities calculated depend as well on the number of decision dimensions considered. This concerns also the utility component of travel time savings. The results indicate that the utility from travel time savings is overestimated, if departure time and/or location choice are not considered. Different compositions of the realised utility suggest that the reason lies within trade-offs between utility components. With more decision dimensions more trade-offs are possible.

A lot depends on the considered decision situation. Traditional transport models focus on short term decisions like route or mode choice. It is only logical that such models omit costs arising due to long term decisions because they are assumed constant. We argue that transport models to judge long term perspectives should include long term decisions and therefore additional utility components like those arising from location choice. This leads to the conclusion that travel time savings alone do not capture the utility gains appropriately for the long run.

Modelling more decision dimensions reveals higher utility gains. The reason is the higher flexibility of the actors, which allows them to adapt their choices more comprehensively.

Neglecting decision dimensions also prevents us to some extent from knowing who is going to profit from the infrastructural investment. In this respect the simulations point at the fact that land prices are influenced by improvements of transportation infrastructure and that models without location choice cannot capture this effect.

If we suppose behavioural dimensions to exist, we should model them and consider utility components directly influenced by them. This suggests that connection choice, departure time choice and location choice should be integrated because people are free to choose in these dimensions especially from a long term perspective. Otherwise it is likely that we are missing some consequences of an infrastructural investment. The results of this work suggests that investments are misjudged in such cases. However, it depends also on the investment we want to evaluate which decision dimensions have to be considered.

When we model transport with discrete choice models, the indicator EMU allows us to capture all considered utility components and therefore all utility variations due to an infrastructural investment in a consistent way. Furthermore one can think of extending transport models to capture the
consequences for the whole economic system as influenced by the transport system. In this case all utility components should be integrated as comprehensively as possible. The more utility components are considered the EMU can become a very general indicator for welfare. Straatemeier and Bertolini (2008) therefore suggest the concept of the EMU as “common language” to be used by land use and transportation planners now.

To assess the errors made in real assessments we have to consider realistic situations. This requires more detailed modelling and estimation of parameters using empirical input data. The right weighting of the utility terms considered is crucial because over all utility and consequently decisions depend a lot on them. The parameters of the utility function should be estimated by stated preference or revealed preference surveys for sound results. However, the proof of concept model is able to show the qualitative differences in traffic flows and resulting utilities. It also shows what type of the errors we have to expect.

5.1 Critique of the Model

The approach with a simple MNL-Model does not account for the fact that two similar alternatives should have smaller joint probabilities. This shortcoming is critical in terms of connections with same links.

Further, it is unsatisfying to have discrete time options because the number of alternatives increases drastically when higher resolution in time is applied. Time resolution implemented becomes critical. It is troublesome to deal with continuous quantities as choice dimensions inside discrete choice theory. On one hand a higher resolution promises exacter results, on the other hand higher resolution means more alternatives to be calculated resulting in longer computation time. It will be necessary to develop heuristics for reasonable choice set generation if we want to simulate detailed real world scenarios. This means dropping the assumption of perfect information. The assumption that agents do not know about all alternatives is more realistic any way.

The algorithm for equilibrium calculation has some drawbacks. It cannot guarantee to find the SUE and it may get stuck in a local optimum. Furthermore it is not easily applicable for more complex networks.

5.2 Further Research

The purpose of the simulation is to find the SUE. Therefore it was not tried to simulate the processes at real life speeds in this attempt. This means that decisions in all dimension occur equally quickly. Changing location is as easy as changing departure time. Obviously this is not true because in reality
there are costs associated with location change and time has to be spent on preparing relocation. Therefore location choice is more burdensome and consequently less frequent than connection or departure time choice. These issues should be considered if we want to have a better understanding of development processes in space and time. We will have to model the appearance of decision situations over time to interrelate short- mid- and long-term decisions. To simulate the adoption process of an urban system adequate in time, we have to consider the frequencies of decisions and durations of actions in more detail.

To consider the fact that decision makers overlook some components, we would have to distinguish between perceived and unperceived utility components. It is possible to decide whether agents incorporate a certain utility component into their decision making or if they do not care about it. Think of utility produced by reducing emissions. Usually this utility component is unnoticed by the deciding traveller. In this case we would integrate it as unperceived component. If we want to simulate a scenario with an emission tax, we integrate the utility as perceived component. This way comprehensive models will also allow us to investigate situations in which the external costs are internalised. A main challenge will be to estimate the relevant costs correctly.

Furthermore, comprehensive models allow for a wider range of policies to be evaluated. A model with location choice, for example, allows to assess separately land use and transport policies with the same tool but – and more importantly – policy combinations. What is the effect of time dependent road pricing in combination with mixed land use?
6 References


Appendix

Table 10   Model parameters

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weighting parameter for travel time utility</td>
<td>$\beta_i$</td>
<td>-0.27</td>
</tr>
<tr>
<td>Weighting parameter for utility from punctuality</td>
<td>$\beta_i$</td>
<td>-0.06</td>
</tr>
<tr>
<td>Weighting parameter for location choice utility</td>
<td>$\beta_j$</td>
<td>-0.05</td>
</tr>
<tr>
<td>VTTS</td>
<td>$B_g$</td>
<td>1 or 2</td>
</tr>
<tr>
<td>BPR-parameter alpha main road link</td>
<td>$\alpha$</td>
<td>0.7</td>
</tr>
<tr>
<td>BPR-parameter beta main road link</td>
<td>$\beta$</td>
<td>5</td>
</tr>
<tr>
<td>Travel time under free flow on main road link</td>
<td>$T_0$</td>
<td>3</td>
</tr>
<tr>
<td>Capacity of main road link per time interval</td>
<td>$Q$</td>
<td>27</td>
</tr>
<tr>
<td>BPR-parameter alpha highway link</td>
<td>$\alpha$</td>
<td>0.4</td>
</tr>
<tr>
<td>BPR-parameter beta highway link</td>
<td>$\beta$</td>
<td>6</td>
</tr>
<tr>
<td>Travel time under free flow on highway link</td>
<td>$T_0$</td>
<td>2.5</td>
</tr>
<tr>
<td>Capacity of highway link per time interval</td>
<td>$Q$</td>
<td>40</td>
</tr>
<tr>
<td>Davidson-parameter jota</td>
<td>$\zeta$</td>
<td>0.4</td>
</tr>
<tr>
<td>Travel time under free flow on railway link</td>
<td>$T_0$</td>
<td>3.5</td>
</tr>
<tr>
<td>Capacity of railway link per time interval</td>
<td>$Q$</td>
<td>33</td>
</tr>
<tr>
<td>Utility loss rate for SDE</td>
<td>$\zeta$</td>
<td>$\beta_g - 0.1^6$</td>
</tr>
<tr>
<td>Utility loss rate for SDL</td>
<td>$\gamma$</td>
<td>20</td>
</tr>
<tr>
<td>penalty for being late</td>
<td>$\delta$</td>
<td>5</td>
</tr>
<tr>
<td>Location occupancy rate sensitivity</td>
<td>$\lambda$</td>
<td>4</td>
</tr>
</tbody>
</table>

6 To avoid that agents choose longer travel time in favour of better punctuality, we require parameter $\zeta$ for utility loss rate for SDE to be smaller than the value of travel time.
### Table 11  Calculated Indicators Respective to Choice Space

<table>
<thead>
<tr>
<th>Average of 5 simulations</th>
<th>RTJ</th>
<th>RT</th>
<th>R</th>
<th>RJ</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>∑EMU</td>
<td>1870.32</td>
<td>830.66</td>
<td>-1975.61</td>
<td>-1085.00</td>
<td>1567.40</td>
</tr>
<tr>
<td>∑V_r</td>
<td>-1859.09</td>
<td>-1816.37</td>
<td>-1825.13</td>
<td>-1798.04</td>
<td>-1927.87</td>
</tr>
<tr>
<td>∑V_t</td>
<td>-318.61</td>
<td>-339.35</td>
<td>-407.69</td>
<td>-420.24</td>
<td>-392.38</td>
</tr>
<tr>
<td>∑V_j</td>
<td>-1040.00</td>
<td>-1173.33</td>
<td>-1173.33</td>
<td>-1015.51</td>
<td>-1173.33</td>
</tr>
<tr>
<td>∑V_{rtj}</td>
<td>-3217.70</td>
<td>-3329.06</td>
<td>-3406.15</td>
<td>-3233.79</td>
<td>-3493.58</td>
</tr>
<tr>
<td>∑Travel time</td>
<td>24543.78</td>
<td>24083.77</td>
<td>24225.21</td>
<td>24065.41</td>
<td>25730.36</td>
</tr>
<tr>
<td>∑Travelled distance</td>
<td>36468.8</td>
<td>35594.8</td>
<td>35594.8</td>
<td>36680.4</td>
<td>35594.8</td>
</tr>
</tbody>
</table>

### Table 12  Coefficient of Variation of Occupancy rates

<table>
<thead>
<tr>
<th>Out of 5 Simulations</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>RTJ</td>
<td>0.004</td>
<td>0.004</td>
<td>0.072</td>
</tr>
<tr>
<td>RT</td>
<td>0.005</td>
<td>0.009</td>
<td>0.171</td>
</tr>
<tr>
<td>R</td>
<td>0.005</td>
<td>0.009</td>
<td>0.171</td>
</tr>
<tr>
<td>RJ</td>
<td>0.004</td>
<td>0.010</td>
<td>0.140</td>
</tr>
<tr>
<td>State 1</td>
<td>0.005</td>
<td>0.009</td>
<td>0.171</td>
</tr>
</tbody>
</table>
Table 13  Coefficients of Variation of Calculated indicators

<table>
<thead>
<tr>
<th>Average of 5 simulations</th>
<th>RTJ</th>
<th>RT</th>
<th>R</th>
<th>RJ</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΣEMU</td>
<td>0.00</td>
<td>0.02</td>
<td>-0.01</td>
<td>-0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>ΣV_r</td>
<td>-0.01</td>
<td>-0.01</td>
<td>0.00</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td>ΣV_t</td>
<td>-0.02</td>
<td>-0.02</td>
<td>-0.02</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td>ΣV_j</td>
<td>-0.01</td>
<td>-0.02</td>
<td>-0.02</td>
<td>-0.01</td>
<td>-0.02</td>
</tr>
<tr>
<td>ΣV_rj</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.01</td>
<td>0.00</td>
<td>-0.01</td>
</tr>
<tr>
<td>ΣTravel time</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>0.01</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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Table 14  Coefficients of Variation of Predicted Variations

<table>
<thead>
<tr>
<th>Average of 5 simulations</th>
<th>ΔRTJ</th>
<th>ΔRT</th>
<th>ΔR</th>
<th>ΔRJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΣEMU</td>
<td>0.05</td>
<td>0.06</td>
<td>0.05</td>
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</tr>
<tr>
<td>ΣV_r</td>
<td>0.22</td>
<td>0.17</td>
<td>0.10</td>
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<tr>
<td>ΣV_t</td>
<td>0.11</td>
<td>0.19</td>
<td>-0.44</td>
<td>-0.12</td>
</tr>
<tr>
<td>ΣV_j</td>
<td>0.08</td>
<td>0.00</td>
<td>0.00</td>
<td>0.16</td>
</tr>
<tr>
<td>ΣRealised utility</td>
<td>0.08</td>
<td>0.12</td>
<td>0.08</td>
<td>0.13</td>
</tr>
<tr>
<td>ΣTravel time</td>
<td>-0.20</td>
<td>-0.14</td>
<td>-0.13</td>
<td>-0.11</td>
</tr>
<tr>
<td>ΣTravelled distance</td>
<td>0.07</td>
<td>0.00</td>
<td>0.00</td>
<td>0.16</td>
</tr>
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</table>
### Table 15  Coefficients of Variation of Absolute and Relative Errors

<table>
<thead>
<tr>
<th>Average of 5 simulations</th>
<th>F_{RT}</th>
<th>f_{RT}</th>
<th>F_{R}</th>
<th>f_{R}</th>
<th>F_{RJ}</th>
<th>f_{RJ}</th>
</tr>
</thead>
<tbody>
<tr>
<td>\sum EMU</td>
<td>-0.07</td>
<td>-0.03</td>
<td>-0.10</td>
<td>-0.06</td>
<td>0.17</td>
<td>0.15</td>
</tr>
<tr>
<td>\sum V_t</td>
<td>0.31</td>
<td>0.42</td>
<td>0.41</td>
<td>0.57</td>
<td>0.21</td>
<td>0.41</td>
</tr>
<tr>
<td>\sum V_t</td>
<td>-0.34</td>
<td>-0.37</td>
<td>-0.08</td>
<td>-0.09</td>
<td>-0.08</td>
<td>-0.05</td>
</tr>
<tr>
<td>\sum V_j</td>
<td>-0.08</td>
<td>0.00</td>
<td>-0.08</td>
<td>0.00</td>
<td>0.61</td>
<td>0.59</td>
</tr>
<tr>
<td>\sum Realised utility</td>
<td>-0.11</td>
<td>-0.11</td>
<td>-0.11</td>
<td>-0.05</td>
<td>-0.93</td>
<td>-0.98</td>
</tr>
<tr>
<td>\sum Travel time</td>
<td>-0.35</td>
<td>0.48</td>
<td>-0.60</td>
<td>0.72</td>
<td>-0.36</td>
<td>0.52</td>
</tr>
<tr>
<td>\sum Travelled distance</td>
<td>-0.07</td>
<td>0.00</td>
<td>-0.07</td>
<td>0.00</td>
<td>0.63</td>
<td>0.61</td>
</tr>
</tbody>
</table>

### Table 16  Coefficients of Variation of Changes in Predicted External Costs

<table>
<thead>
<tr>
<th>Average of 5 simulations</th>
<th>\Delta RTJ</th>
<th>\Delta RT</th>
<th>\Delta R</th>
<th>\Delta RJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accident costs</td>
<td>-0.15</td>
<td>-0.09</td>
<td>-0.15</td>
<td>-0.24</td>
</tr>
<tr>
<td>Traffic noise costs</td>
<td>0.13</td>
<td>0.21</td>
<td>0.18</td>
<td>0.15</td>
</tr>
<tr>
<td>Air pollution costs</td>
<td>0.14</td>
<td>0.21</td>
<td>0.18</td>
<td>0.15</td>
</tr>
<tr>
<td>Climate costs</td>
<td>0.13</td>
<td>0.21</td>
<td>0.18</td>
<td>0.15</td>
</tr>
<tr>
<td>Sum external costs</td>
<td>-0.17</td>
<td>-0.10</td>
<td>-0.17</td>
<td>-0.37</td>
</tr>
</tbody>
</table>

### Table 17  Coefficients of Variation of Absolute and Relative Error of Estimated External Costs by Choice Space

<table>
<thead>
<tr>
<th>Average of 5 simulations</th>
<th>F_{RT}</th>
<th>f_{RT}</th>
<th>F_{R}</th>
<th>f_{R}</th>
<th>F_{RJ}</th>
<th>f_{RJ}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accident costs</td>
<td>4.98</td>
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<td>0.21</td>
<td>-0.09</td>
<td>0.22</td>
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</tr>
<tr>
<td>Traffic noise costs</td>
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<td>-0.09</td>
<td>-0.38</td>
<td>-0.27</td>
<td>0.18</td>
<td>0.07</td>
</tr>
<tr>
<td>Air pollution costs</td>
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<td>-0.10</td>
<td>-0.38</td>
<td>-0.26</td>
<td>0.18</td>
<td>0.06</td>
</tr>
<tr>
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<td>-0.09</td>
<td>-0.38</td>
<td>-0.27</td>
<td>0.18</td>
<td>0.07</td>
</tr>
<tr>
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<td>-0.09</td>
<td>0.51</td>
<td>-0.50</td>
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