Mathematical knowledge and the interplay of practices, by José Ferreirós

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Mathematical Knowledge and the Interplay of Practices studies the epistemology of mathematics from cognitive and historical points of view. It is naturalist in the sense of rejecting "first philosophy" as hopeless, but not in the sense of scientistic reduction. It is pragmatist in the sense of focusing on mathematical practice rather than on abstract mathematical entities. Instead of evaluating mathematics' referential truth, it shows how practical constraints transform intersubjective agreement into objective knowledge.

The first four chapters form a theoretical exposition. They place the book in the context of anti-foundationalist philosophy of mathematics and the emergent "philosophy of mathematical practice". The author defines mathematical practice as: "what the community of mathematicians do when they employ resources … on the basis of their cognitive abilities to solve problems, prove theorems, shape theories, and (sometimes) to elaborate new frameworks" (p. 33). These practices are not unified, but do interconnect and constrain each other. They are also agent based, but the book's agents are abstracted from "how much time and effort an agent may have taken to learn, to find out about something, and so on" (p. 62), and are considered as "so to speak, interchangeable" (p. 100). According to the book, the complementarity between practices and interpretive agents gives rise to mathematical meaning. Mathematics is thus not *essentially* different from other forms of knowledge, although it allows theory an exceptional degree of autonomy with respect to application and experimentation.

Chapters 5 and 7 and 8 concern Euclidean geometry, natural numbers, and real numbers respectively from a historico-philosophical point of view. Chapters 6, 9 and 10 provide the epistemological analysis, building on an "if-then" approach: mathematics consists of valid inferences from uncertain constitutive hypotheses (rejecting without argument critiques against the purported certainty of modern mathematical formal derivations). The main philosophical contribution of the book – which I find convincing and important – lies in showing how practice constrains the choice of these hypotheses. Mathematics enjoys some contingency (e.g. constructive vs. classical approaches), but strongly leans towards uniform choices.

Euclidean geometry is marked as important for marking the transition between the practical, everyday mathematics (striving for "reliability and robustness") and a hypothesis-based advanced mathematics (guided by "precision and consistency"). The Euclidean diagram is concretely perceived *and* conceptually interpreted, spanning a space of interaction between agent and representation. Euclidean geometry teaches us to "see" *in* the concrete diagram what is not *materially* there, which is precisely where the distinction between practice and theory blurs.

Counting is then presented as a cultural invariant (which does not reduce it to "nature") that obeys the Peano axioms. This left me confused, as the practical and theoretical (counting and axioms) superimpose in this kind of presentation, rather than enter the dialectic of Euclidean geometry or higher number theory. Perhaps my uneasiness stems from the book's ignoring of finitist critiques, which highlight the discrepancy between practical counting and Peano's axioms.

A particularly engaging chapter is the historical narrative behind the modern canonical understanding of the real number line as a continuum consisting wholly of points. It insists on the contingency of this choice, while highlighting the practices that led most mathematicians to endorse it. The explanation relies, among other things, on the practical-scientific (e.g. astronomical) roots of the continuous-atomistic number line. But such practical roots could also lead us to wrongly expect that mathematicians favor finitism, because calculations are always finite and require exhaustible resources. A more refined analysis could counter this wrong expectation and make the argument even more compelling.

The "contingency cum objectivity" or "invention cum discovery" (p. 248) of the real number line extends to higher infinities as well. Here, a less sturdy web of practices led mathematicians to a weaker consensus, but the book argues that accepting the infinity of natural numbers and of the continuum of real points constrains most mathematicians to endorse arbitrary infinities. The book then explains how mathematical practice yields standard models and axioms, even though we cannot formally individuate standard models, and new axioms (such as choice) are formally independent. This clearly demonstrates the reach of the book's pragmatist view beyond that of an "if-then" formalism.

The historical details (especially those that concern earlier history) sometimes struck me as dangerously oversimplified, and the critiques of some thinkers (especially Wittgenstein and Cavaillès) felt off the mark, but not in ways that undermine the book's main theses. I would also complement some of the historical narratives (e.g. relate the expanding universe of mathematical functions and series to the 19th century real number line), but this would only support the epistemological picture of the book. The book also ignores sociological-institutional factors, which is grudgingly excusable on grounds of scope.

I conclude with a speculative remark. Early in his career, Claude Lévi-Strauss discovered a "scandal": incest prohibition is obviously a cultural phenomenon, but also universal, and must therefore be natural. The same kind of scandal emerges in Ferreiros' book: counting is described as a universal (essentially the same despite variations and underdevelopment in some cultures), yet precise counting cannot exist independently of the contingencies of symbolic systems, which are not naturally innate (pp. 69-70, 185).

What was a scandal in the fifties is no longer a scandal today. The nature/culture binary has been problematized by many disciplines and theoretical approaches. Yet the practical response seems to have persisted. Derrida, in his seminal *Structure, Sign, Play*, noted that all too often, those who agree to let go of the lost natural grounding, mourn its loss with a nostalgia for its promise of stability. Often enough, in the philosophy of mathematical practice too, one finds nostalgia for the security of a mathematical knowledge that has lost its originary ontology. What's missing, as Derrida noted, is an affirmation of the freedom and possibilities that loss of ground opens.

My only substantial critique of Ferreiros' book is thus purely ideological: the book provides us with an epistemological justification of mathematics as we know it, instead of nudging us to explore what else mathematics could be. But in so far as one seeks epistemological justifications, Ferreiros' are rich, compelling and masterful.