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Ambühl, Lukas; Loder, Allister; Bliemer, Michiel; Menendez, Monica; Axhausen, Kay W. 

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Lukas Ambühl
IVT, ETH Zürich,
CH 8093 Zürich, Switzerland
phone: +41 44 633 32 51
fax: +41 44 633 10 57
lukas.ambuehl@ivt.baug.ethz.ch

Allister Loder
IVT, ETH Zürich,
CH 8093 Zürich, Switzerland
phone: +41 44 633 62 58
fax: +41 44 633 10 57
allister.loder@ivt.baug.ethz.ch

Michiel C.J. Bliemer
Institute of Transport and Logistics Studies,
University of Sydney,
NSW 2006, Australia
phone: +61 2 9114 1840
fax: +61 2 9114 1863
michiel.bliemer@sydney.edu.au

Monica Menendez
IVT, ETH Zürich, Switzerland,
CH 8093 Zürich
phone: +41 44 633 66 95
fax: +41 44 633 10 57
menendez@ivt.baug.ethz.ch

Kay W. Axhausen
IVT, ETH Zürich,
CH 8093 Zürich, Switzerland
phone: +41 44 633 39 43
fax: +41 44 633 10 57
axhausen@ivt.baug.ethz.ch

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1 ABSTRACT

2 The uncertainty in the estimation of the macroscopic fundamental diagram (MFD) under real-
3 world traffic conditions and urban dynamics, might result in an inaccurate estimation of the
4 MFD parameters - especially if congestion is rarely observed network-wide. For example, as
5 data normally comes from punctual observations out of the whole network, it is unclear how
6 representative these observations might be (i.e. how much is the observed capacity affected
7 by the network's inhomogeneity). Similarly, if the observed data does not exhibit a distinct
8 congested branch, it is hard to determine the network capacity and critical density. This, in turn,
9 also leads to uncertainties and errors in the parametrization of the MFD for applications, e.g.
10 traffic control.

11 In this paper we introduce a novel methodology to estimate (i) the level of inhomogeneity in
12 the network, and (ii) the critical density of the MFD, even when no congested branch is observed.
13 The methodology is based on the idea of re-sampling the empirical data set. Using an extensive
14 data set from Lucerne, Switzerland, and London, UK, we provide insights on the performance
15 and the application of the proposed methodology. We use the proposed methodology to illustrate
16 how the level of inhomogeneity is lower in Lucerne than in the three areas of the network of
17 London that we investigate. The proposed measure of the level of inhomogeneity gives city
18 planners the possibility to analyze and investigate how efficiently their road network is utilized.
19 Additionally, we show that, for the network of Lucerne, the proposed methodology allows us
20 to accurately estimate the critical density up to 16 times more often than it would be possible
21 otherwise. This simple and robust estimation of the critical density is crucial for the application
22 of many traffic control algorithms.

1 INTRODUCTION

2 The macroscopic fundamental diagram (MFD) is the upper bound in the macroscopic flow-density
 3 (q - k) relationship for vehicular traffic in an urban road network. Similar to a fundamental
 4 diagram of a single road, MFDs are characterized by an uncongested branch and a congested
 5 branch. In the uncongested branch, flow increases with density, whereas in the congested branch,
 6 flow decreases with increasing density until gridlock is reached, and speeds drop to zero. Thus,
 7 MFDs can be described by a combination of some of the following parameters: free flow speed,
 8 backward wave speed, capacity, jam density, critical density, and a smoothing parameter that
 9 describes the change in the slope of the average speed. Notice that for control purposes, the
 10 critical density is an important parameter, as it marks the difference between the congested
 11 and the uncongested branch. The shape of the MFD is determined by the urban road structure,
 12 traffic control (1) and the level of inhomogeneity in the distribution of traffic (2–4), but more
 13 or less independent of the demand (assuming trip length remains more or less constant). New
 14 applications have been developed exploiting the elegance of the MFD to model traffic dynamics,
 15 predict travel behavior, and control traffic in urban networks (5–16), all relying on an accurate
 16 estimation of the MFD.

17 The MFD can be either analytically derived or approximated from empirical data (17).
 18 Daganzo and Geroliminis (1) proposed a semi-analytical formulation for urban networks.
 19 However, their method relies on technical information (green times, number of links that can be
 20 passed by a fast vehicle without stopping, etc.), which might be highly variant or not available at
 21 all. The MFD can also be directly estimated from empirical data leading to a concave and to some
 22 extent well-defined and reproducible relationship. In real road networks, however, homogeneity
 23 is rarely found, and network-wide congestion is not always apparent. In the case where only a few
 24 roads show a different behavior than all the other roads, the observed flow-density relationship
 25 lies slightly below the upper bound (2). In general, empirical MFDs exhibit scatter that leads to a
 26 range of observed flows for any given accumulation, which leads to uncertainty in the estimation
 27 of the descriptive parameters (18–21). Seminal studies have identified the inhomogeneous spatial
 28 distribution of vehicle densities as the reason for this scatter (22, 2, 23, 4, 24). A partial solution,
 29 in case of contiguous and homogeneous sub-regions, is partitioning (25, 26).

30 This uncertainty in the estimation of the MFD under real-world traffic conditions and urban
 31 dynamics, might result in an inaccurate estimation of the MFD parameters - especially if
 32 congestion is rarely observed network-wide (27). For example, if the observed data does not
 33 exhibit a distinct congested branch, it is hard to determine the network capacity and critical density.
 34 Similarly, as the data normally comes from punctual observations out of the whole network,
 35 it is unclear how representative these observations might be (i.e. how much is the observed
 36 capacity affected by the network's inhomogeneity). This, in turn, also leads to uncertainties and
 37 errors in the parametrization of the MFD for applications (e.g. traffic control), reducing the
 38 effectiveness of such applications. This leads to the question addressed in this paper: Given
 39 day-to-day fluctuations from limited and probably biased empirical data, how can the shape of
 40 the MFD, and especially the critical density for traffic control purposes, as well as the level of
 41 inhomogeneity, be accurately estimated?

42 This paper proposes a novel re-sampling methodology to estimate the level of inhomogeneity
 43 and the critical density in urban networks under uncertain traffic conditions, even when only
 44 limited empirical data is available. We provide empirical evidence on the performance of the
 45 proposed methodology using two large data sets for the cities of Lucerne, Switzerland (1 year)
 46 and London, UK (3 weeks).

The results show that the level of inhomogeneity is lower in Lucerne than in the three areas of the network of London that we investigate. Moreover, we show that in the case of London, the capacity of the system could be increased by around 20%, under the assumption that all links behave in a similar manner as the 50% best links. Additionally, we illustrate how the proposed methodology estimates the critical density accurately for Lucerne, even when no congested branch of the MFD is available. All things considered, this new methodology promotes the concept and the use of the MFD in real world applications.

The remainder of this paper is organized as follows. The next section introduces the re-sampling approach, then we present and discuss the two available data sets in detail. Thereafter we show the empirical results. Conclusions are given at the end.

METHODOLOGY

Re-sampling the MFD

This section introduces the re-sampling method, which later will be used to estimate the critical density and the level of inhomogeneity in an urban network. The idea of the re-sampling method is to identify the most homogeneous sub-samples of all roads by first creating many random sub-samples of the network, estimating for each an MFD, and extracting the smooth upper bound from the superposition of all MFDs. When the re-sampling parameters are chosen appropriately, all points on the upper bound represent the most homogeneous traffic states.

As an illustration, consider an urban road network in and around a central business district (CBD) with many roads and vehicles. During the morning commute, most roads are congested, but not all of them, e.g. arterial roads leading into the CBD are more congested than those leading out. The resulting averages provide an estimate on the actual mean performance of the road network, but not the potential performance if all links were similarly congested. Choosing a certain sample size and repeating the sampling many times enables to find a homogeneous set of roads without the need to filter the most homogeneous links manually or apply a more complex partitioning (which usually assumes spatial contiguity of the partitioned regions).

The urban road network of a city is given by N directed links, where N refers to all the monitored links (i.e. N might be any number covering between 0 and 100% of the links in the network). The length of each link i is known as l_i . Following Geroliminis and Daganzo (18), the MFD is given by the length-weighted means of flow q , and density k , $q = \sum_i q_i l_i / \sum_i l_i$ and $k = \sum_i k_i l_i / \sum_i l_i$, respectively. As previously discussed, under real conditions a network might not be homogeneously congested, and some links might be more congested than others. If these form a connected subgraph in the road network, we could partition the network as for example in (23). However, if these links are randomly distributed across the network and the sample varies with time, we can estimate the MFD as follows in order to reduce the influence of inhomogeneity in the measurements. We randomly sample Ω -times without replacement $N_s \subset N$, where the ratio N_s/N denotes the sample size as a proportion of all observations. The maximum number of combinations is given by the binomial coefficient $C = \binom{N}{N_s}$. Notice that sometimes, the number of possible combinations that can be considered might be limited by the computational resources, leading to $\Omega \ll C$.

The intuition behind this statistical sampling is simple. In the case of a network with homogeneous roads (i.e. all exhibiting identical fundamental diagrams) and a homogeneous distribution of congestion, the proof of the equivalence between the upper bound of the full sample

and the re-sampled upper bound is trivial. For a more realistic and therefore inhomogeneous network, a representative sub-sample with $N_s \gg 0$ links will never exceed the theoretical upper bound by Daganzo and Geroliminis (1); the sub-sample will be equal to the upper bound in case of perfect homogeneous congestion, and below the upper bound in all other cases. In other words, estimating average flows and densities for all combinations C (or a subset of Ω combinations) of a representative subsets of links, increases the chance of obtaining for some combinations and time intervals a homogeneous distribution of congestion, leading to an MFD estimate that is less susceptible to inhomogeneities.

Estimating the level of inhomogeneity

Following the re-sampling methodology, we propose a measure to estimate inhomogeneity in a network based on the observed capacities in the re-sampled MFD without explicitly considering spatial and temporal effects. This approach is different from the approaches by (22) and (23), which explicitly capture temporal and spatial effects. The idea is to obtain the highest capacity from all investigated combinations in each sample size, and calculate the relative difference in reference to the observed capacity of the full sample. We define this relative difference as the additional capacity that the network could handle if all links were to behave similarly to the best sub-sample of the respective sample size.

As an illustration, imagine a perfectly homogeneous road network, where all roads carry the same level of traffic. It is clear that for such a case the additional capacity will be zero for all sample sizes. In all other cases, where roads and traffic states are inhomogeneous, we will observe non-zero additional capacities, as we expect that some combinations from lower sample sizes will exclude the constraining roads from the sample. In other words, we estimate the additional capacity as a function of the sample size, and propose to calculate the area below this function - the larger this area is, the less homogeneous the urban network is based on its currently observed state. Notice that the level of inhomogeneity, as defined here, is a measure which is *relative* to the reference case.

Identifying the critical density and capacity for empirical MFDs

Since the proposed sampling method is repeated Ω times (i.e. we take Ω sub-samples of the same sample size), it significantly increases the scatter and range of flow-density relationships, but exhibits a smooth upper bound. To identify such a stable upper bound for the re-sampled MFD, we use the median of the top M flow values per density bin, where M depends on the number of observations in each bin. We chose the median to avoid any bias due to outliers. After a number of empirical trials, we have defined for most cases $M = 50$, but it can decrease down to 5 when the number of observations is small. From this upper bound we define the capacity as the 97.5th percentile of flow to avoid also the influence of outliers. The critical density is then the mean density corresponding to this capacity.

DATA

This section presents the available data sets for the cities of Lucerne, Switzerland and London, UK. Both cities differ significantly in size, population, and network topology. Lucerne has around 80'000 inhabitants, whereas London has around 100 times as many, with a population density which is around twice as large. The traffic authorities of Lucerne and London operate an extensive network of loop detectors in their cities. Such loop detectors are mainly installed for traffic control and congestion identification purposes. They measure traffic flow (number

of vehicles passing the detector) and occupancy (fraction of time the detector is occupied by a vehicle) during a certain observation interval T . Due to the system design, the loop detectors in London are located more frequently at the upstream end of the link, while in Lucerne the detectors are located further downstream. For London, we use three different regions, one around Whitechapel (253 loop detectors), one around Fulham (93 loop detectors), and one around Chelsea (102 loop detectors). For Lucerne, we consider the entire downtown area (158 loop detectors). Figure 1 provides an overview of the two cities, and Table 1 provides some relevant statistics. In the following, we discuss in more detail how the data sets were prepared.

In order to locate each detector, we geo-referenced all spatial information of the loop detector positions in reference to the whole road network. Connecting the network and the loops has several advantages. First, MFDs for arbitrary shaped perimeters can be estimated, and potential partitioning can be carried out. Second, attributes of the road network, e.g. road type and speed limit, can be linked to the loop detectors, allowing further filtering, e.g. removing residential roads (20) that generally have no connecting character. Third, we enriched the data with information on the driving direction of each lane covered by a loop detector. This allowed us to identify multiple detectors per lane, and remove some to avoid any duplicity. For this analysis, we queried the road network from OpenStreetMap. At the end, we selected for this study only loops located on trunk, primary, secondary, and tertiary roads.

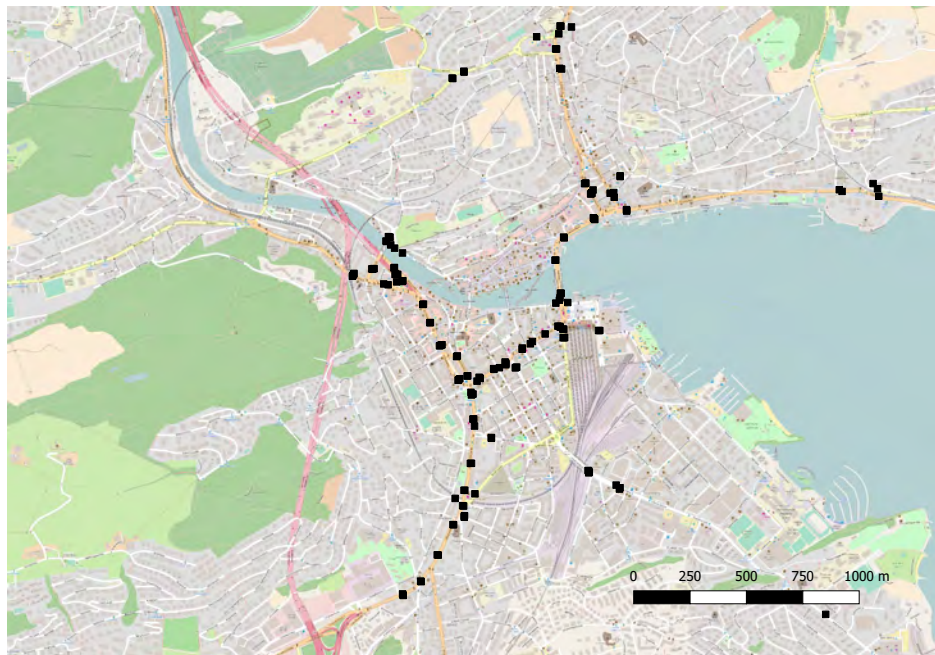
It is clear that empirical datasets are error prone. Both Lucerne and London have an internal system that detects malfunctioning loops. Nevertheless, we defined a set of rules, whose objective is to identify potentially malfunctioning loop detectors as in (28). We automatically identified measurements as false when no variation of the values were registered during a full day, 80 % of the values were zero, either flow or occupancy were zero while the other was not (as long as the occupancy was smaller than 0.95), and obviously when the internal system itself reported an error. Additionally, we inspected each loop detector's scatter plot and verified the results of the filtering. The error rate is 30% (23% internal system, 7% verification) for Lucerne and 31% (30% internal system, 1% verification) for London. Finally, we further scrutinized the data by identifying outliers (29) and by reducing noise with a moving average technique (30) and removed time slots where less than 85 % of all loop detectors provide valid measurements.

Occupancy-density conversion

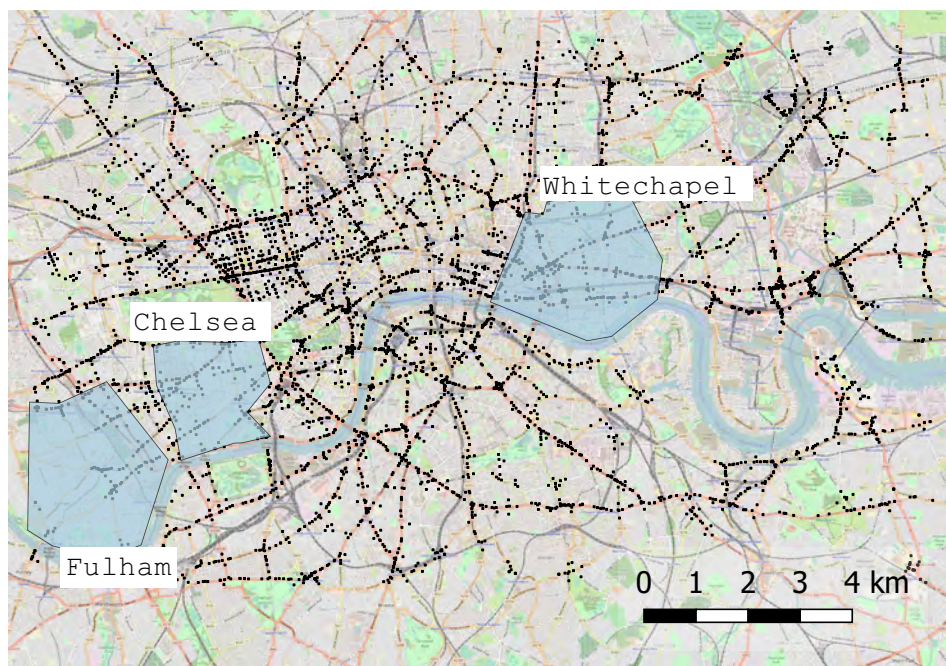
Loop detectors measure flow and occupancy. Traffic density can be approximated from occupancy using a scalar conversion (31, 32, 28). The scalar corresponds to the mean vehicle length as seen by the detector, i.e. the sum of vehicle length and detector length (33). Hall and Persaud (32) and Coifman (33) showed that this scalar conversion is a good approximation with small errors in uncongested traffic. Even though previous research predominantly focused on highways, based on recent findings we assume that this scalar conversion also holds for urban networks (20). In the context of MFD research, values around 0.005 km have been identified for the mean vehicle length: 0.0053 km for Yokohama (18) and 0.0063 km for Zurich (20). We estimate the mean effective vehicle length in Lucerne to be 0.0063 km and for London to be 0.006 km.

RESULTS

In this section, we present the results of the re-sampling methodology using the data described above.



(a) Lucerne.



(b) London.

FIGURE 1 Regions in Lucerne and London which are analyzed. Loop detectors are represented as black squares. Both maps are oriented towards north.

TABLE 1 Overview of the two cities and the respective data set.

City	Lucerne (CH)	London (UK)
Total population	81'000	8'500'000
Total number of detectors	158	5'719
Total lane-km covered [km]	26	1'298
Aggregation interval [min]	3	5
Number of working days	258	15

1 Empirical MFDs

2 First, let us show the full-sample MFD for all regions in London and Lucerne. Figure 2 displays
3 the MFD for 258 working days (i.e. one full year) for the city of Lucerne, and 15 working days for
4 Whitechapel, Chelsea, and Fulham (London) all in the respective aggregation interval (see Table
5 1). Every point represents a macroscopic traffic state in terms of vehicle density and vehicle flow
6 for every aggregation interval during the observation period, indicated in Table 1. All regions
7 show an uncongested branch and the beginning of the congested branch. It is not surprising
8 that Lucerne shows a higher level of scatter, as there are significantly more days included in
9 the dataset. In addition, in a more detailed analysis, we found that some of the working days
10 (less than 6%) showed a slight hysteresis. The reasons for the hysteresis loops could not be yet
11 investigated, especially due to the fact that we do not have access to signal timing, nor signal
12 plans. We have added the stable upper bound, as defined previously, in white squares, as well as
13 a line indicating the critical density. We observe that the highest capacity is found in Fulham,
14 whereas the lowest is in Whitechapel. The free flow speeds of Lucerne and Fulham are similar
15 and higher than the speeds observed in the other two regions. We attribute this to the difference
16 in the network topology (road hierarchy) and traffic control. More details can be found in (34).

17 Remarkably, all MFDs exhibit a smooth upper bound, which supports the general theory of
18 the MFD, defined as a tight upper bound relatively independent of demand. Notice also that,
19 to the best of our knowledge, this is the first time that a full year MFD is estimated. It shows
20 relatively little variation which indicates that empirical MFDs can indeed be used for control
21 schemes and other applications demanding a long-term invariance.

22 Estimating the level of inhomogeneity

23 We will now discuss the first application of our re-sampling method. We follow the approach
24 as outlined in the previous section. For the re-sampled MFDs we choose a total of 10 sample
25 sizes (10% - 100% of the full sample), and the number of draws without replacement Ω is set to
26 500. As an example, we randomly select 20% of the loop detectors of a region, create the MFDs
27 thereof, and repeat this 500 times re-selecting another 20% of loop detectors each time. We then
28 find the stable upper bound from the joint set of the 500 samples and derive its capacity.

29 Figure 3 shows the results of measuring the level of inhomogeneity in a city. We show the
30 relative capacity increases as a function of the sample size for Lucerne and the 3 different regions
31 in London introduced in Figure 1. As expected, the lower the sample size, the higher the relative
32 difference is. We attribute this to the fact that with smaller sample sizes, it is easier to identify
33 combinations of roads that exhibit a behavior exceeding the average in terms of vehicle flow, e.g.

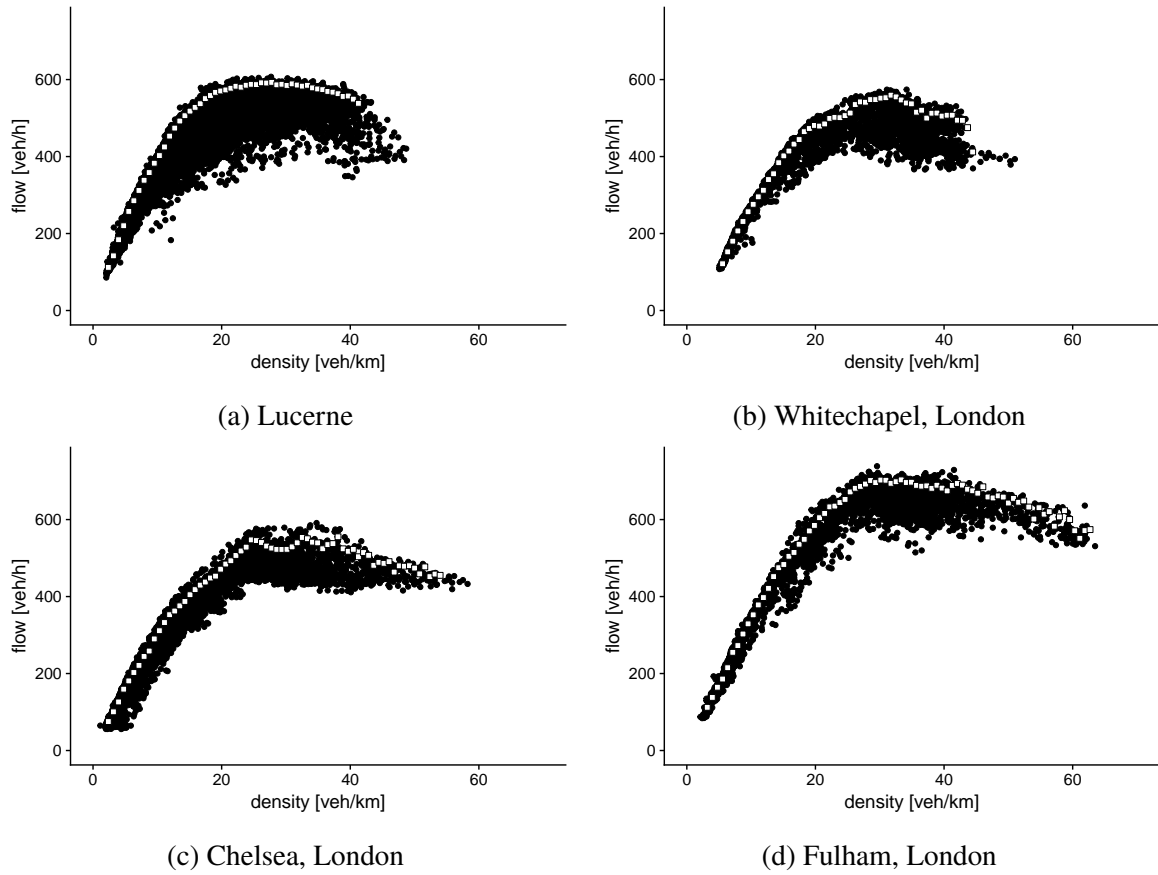


FIGURE 2 MFDs from all days in which data is available and the upper bound. Estimated critical density is indicated by the vertical line.

pockets of congestion. Two issues, however, deserve some consideration: First, it is clear that a certain number of roads need to be included in order to have a representative and meaningful sub-sample for the investigated network. We set the minimal sample size to 10%. Second, the increases in capacity are of theoretical nature, because they would only be observed if all roads were to behave like the best roads in the sub-sample. For example, we see that the region around Whitechapel in London would have a 19% higher capacity if all roads were to behave like the 50% best roads.

Regarding the differences between the different regions in Figure 3, we notice that Lucerne has the lowest curve and therefore the smallest area below the curve (i.e. integral of the curve) compared to the three regions in London. Thus, we conclude that Lucerne exhibits the least amount of inhomogeneity from the investigated sample. We attribute this to (i) the size of the network, which offers only a few higher level roads, and (ii) to the limited feasible route alternatives that are possible in Lucerne in combination with the very limited number of OD-pairs within this small city. Within the investigated perimeter, the south and the north of the city are only connected by two bridges (see Figure 1(a)); all other bridges are pedestrian bridges. In other words, for the few OD combinations that exist, there are only 1 or 2 routes available, constraining route choice. Conversely, for the regions of the city of London, we see from the map in Figure 1(b) that the network is connected with more higher level roads and thus, the number of feasible routes is larger. In other words, many route options combined with many OD-pairs increases the likelihood of an inhomogeneous distribution of traffic. When we calculate the area below the curves as an indicator for the level of inhomogeneity, we find that the areas in London are around

1 1.5 to 2 times larger than the one in Lucerne. Interestingly, the three curves for the London area
 2 follow the same trend, including a relatively sharp increase at the full sample size (100% sample
 3 size). This indicates that a few roads exist in the sample, which have a significant impact on the
 4 average capacity of the network. In the best case, removing the 10% worst roads (in terms of
 5 capacity) from the sample, theoretically increases the network capacity by around 15%.

6 Notice that this approach explicitly quantifies the level of inhomogeneity from the shape of
 7 the MFD, without considering spatial nor temporal variations in traffic volumes and densities.
 8 However, the method does have some clear advantages: it is easy to use, requires very little
 9 inputs, and is robust to the placement bias of loop detectors (19, 20). Link-based partitioning
 10 methods might include such a bias if they are based on traffic densities. For the re-sampling
 11 method, on the other hand, we consider the spatial mean of the densities, as multiple detectors
 12 are included in every sample. Hence, this method represents the first of its kind for assessing
 13 and identifying the level of inhomogeneity in an urban road network.

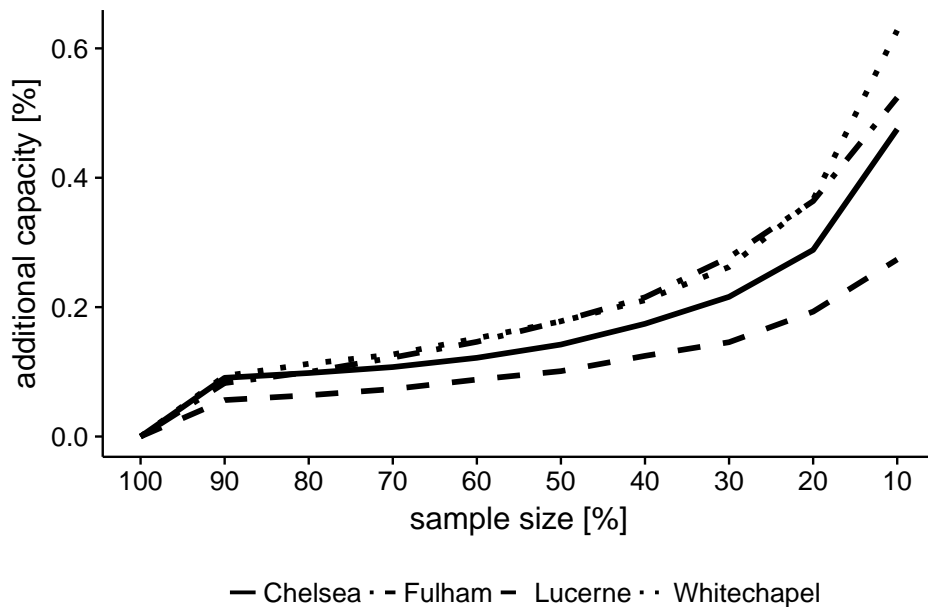


FIGURE 3 Results of the analysis on the level of inhomogeneity. All results cover a period of 15 consecutive weekdays.

14 Estimating the critical density

15 The few existing MFD empirical studies show that in reality, data availability is very often limited
 16 and only few days can be used for analysis. Empirical studies rely on observation periods that
 17 may cover only several days including weekends, which show little or no congestion making an
 18 estimation of the critical density hard or even impossible (*e.g.* 18, 19, 35, 20). In the following,
 19 we investigate how the proposed re-sampling method performs in the estimation of the critical
 20 density when only limited data is available, *e.g.* if we have only data from 1 day, 3 consecutive
 21 days, or 6 consecutive days, and not all exhibit a distinct congested branch.

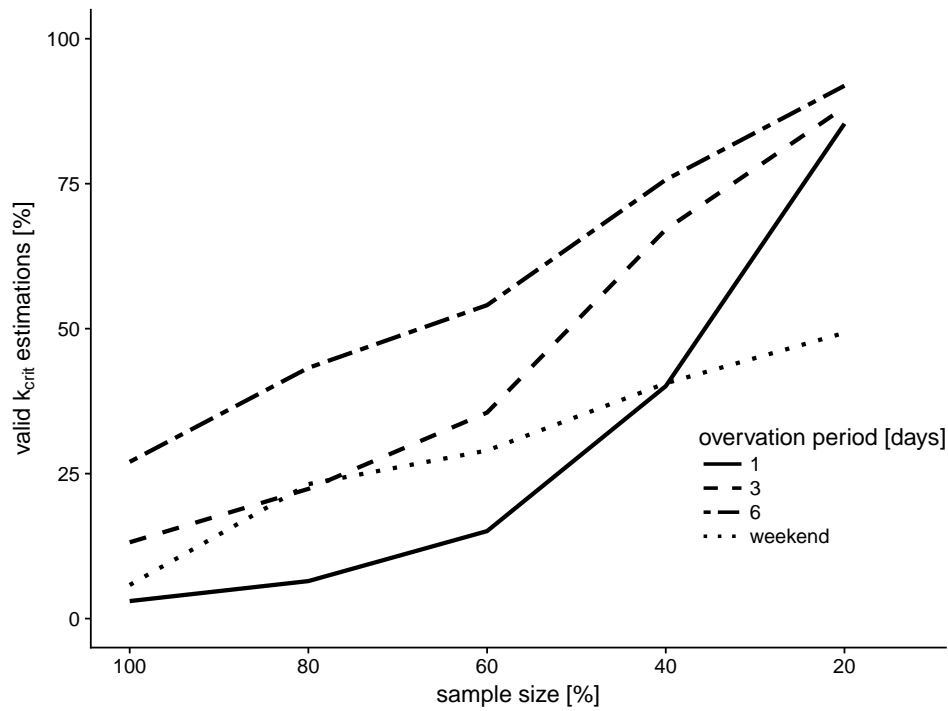
22 Here, we focus only on the city of Lucerne and refrain from showing the same results for
 23 London, as the MFD for the 3 regions in London exhibits a congested branch every day. Hence,
 24 there is no need to apply our method for identifying the critical density. For this analysis, the
 25 MFD in Figure 2(a) serves as the reference MFD. The experimental set-up is similar to the

previous section, but now we focus on the critical density instead of the capacity.

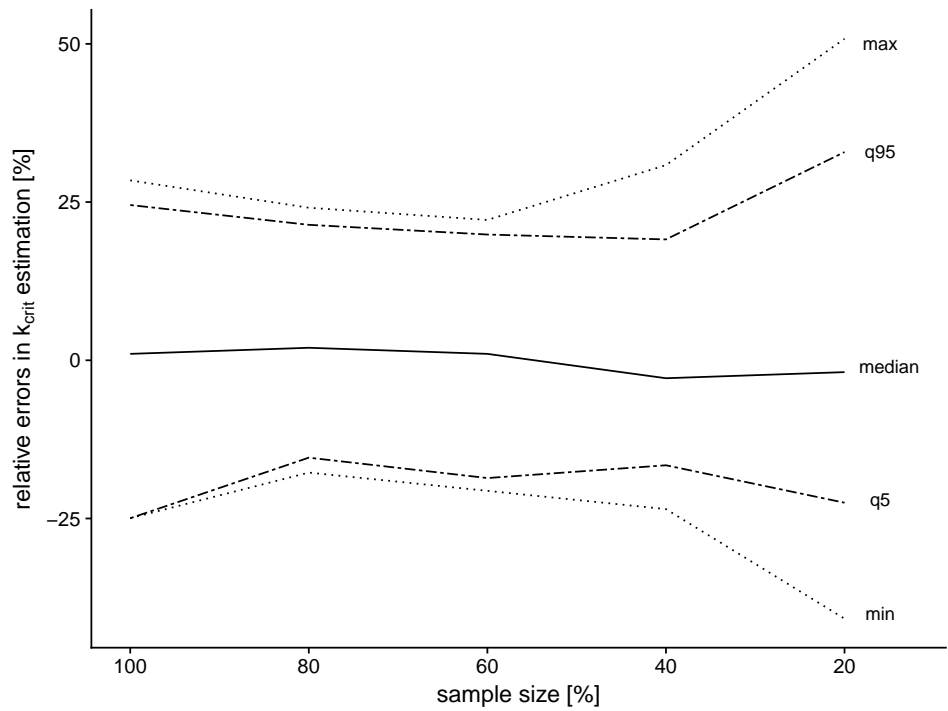
We choose a total of 4 sample sizes (20%, 40%, 60% and 80% of the full sample), and the number of draws without replacement Ω is set to 500. For a sensitivity analysis we also vary the observation period for each MFD estimation using either 1, 3, or 6 consecutive day(s). These observation periods do not include weekends. In order to apply the methodology to days where congestion is highly unlikely, we further included weekend days (Saturday and Sunday) as an observation period. We then try to determine the critical density for each of the estimated MFDs. As an example, we randomly select 20% of Lucerne's loop detectors, create the MFDs thereof, and repeat this 500 times re-selecting another 20% of loop detectors each time for every set of 1, 3 and 6 consecutive day(s) and the weekends. Notice that not every MFD shows a decreasing branch, thus it is not possible to properly determine a critical density for every estimated MFD. We assume that the determined value of the critical density is valid only when the MFD shows a decrease in flow of at least 30 veh/h (around 5% of the capacity in the observed cases) for densities higher than the determined value of the critical density. Figure 4 shows the results of the critical density estimation for the MFD of Lucerne. Figure 4(a) shows the percentage of successful critical density estimations, for each combination of sample size and observation period. In case of an observation period of 1 day, the maximum number of estimations of critical densities is 258, in case of 3 days that number is 86, in case of 6 days the number is 43, and in case of weekends 40. From the figure we first see that as we reduce the sample size, the percentage of days where we can properly determine the critical density increases significantly. As a matter of fact, we can effectively determine the critical density over 75% of the time when the sample size is 20% or lower and the number of available days is 1, 3 or 6 workdays. In addition, and not surprisingly, the more days we include in the estimation of the MFD, the more likely we are to observe a valid critical density. For example, if we had only 1 day of data, then an estimation of the critical density would only be possible during 5% of all days (13 days) using the full sample (100% sample size), whereas if we had 3 days of available data, we could estimate the critical density in 17% of all cases. For the weekend days as observation period, we find that it is possible to increase the fraction of valid estimations from 2% up to around 45% when we consider a sample size of 20%. Some of the weekend days do not show any signs of congestion, even at a link level. Thus, it is not possible to estimate the critical density for such days.

These results show how our re-sampling methodology can effectively increase our ability to determine the critical density, even when only limited data is available, e.g. when no clearly congested branch is apparent in the MFD. The analysis of the weekend days, however, show that the network must be at least loaded with traffic to some extent, in order for our methodology to work.

Given that it is possible to determine a critical density for a given day, Figure 4(b) shows the range of the relative errors of such densities compared to the reference critical density, identified from the 1-year MFD. For brevity, we only show the results for the 1 day estimates; the results for the 3 and 6 days look similar. In the figure, an overestimation of the critical density has a positive sign. Interestingly, the accuracy remains within a similar range for all sample sizes (the different sample sizes include the number of observations given in Figure 4(a)). In other words, even as we decrease the sample size to increase the number of days where we can determine the critical density, the error in the estimation remains approximately the same. As a matter of fact, we can see that for the lowest two sample sizes (20% and 40%) we only underestimate the value of the critical density by less than 2% in the median. To conclude, this clearly indicates that the re-sampling method does not only increase our ability to estimate the critical density, but also maintains the level of accuracy of such estimation.



(a) Number of valid critical density estimations.



(b) Errors in the critical density estimation using the re-sampling methodology for an observation period of 1 day.

FIGURE 4 Results for the critical density estimation.

As a further validation, we compared the error distributions from the full sample and the re-sampling method using a Kolmogorow-Smirnov test. We found a high p-value (0.4) for all sample sizes indicating that they all come from the same distribution of errors.

DISCUSSION AND CONCLUSIONS

In this paper we introduced a novel methodology that allows us to estimate the critical density of the MFD, even when no congested branch is observed, and to estimate the level of inhomogeneity within the network. The proposed methodology is based on the idea of re-sampling the empirical data set. Using an extensive data set, which included the cities of Lucerne, Switzerland, (1 year) and London, UK, (3 weeks) we provided insights on the performance and the application of the re-sampling method. First, using the re-sampling method, we quantified the level of inhomogeneity in urban networks. For London, we find that the capacity of the system could be increased by around 20%, under the assumption that all links behave in a similar manner as the 50% best links. For Lucerne this value is around 10%. We propose to estimate the level of inhomogeneity as the area below the curve relating the sample size and the capacity increase. Second, we estimated the critical density of the MFD using limited data and found that we can drastically increase the number of days on which an estimation is possible - even if the network does not show network-wide congestion every day. This is important, as very often, access is only given to data of a very limited time period. We showed that for the network of Lucerne, we can increase the fraction of days that allow an estimation of the critical density, based on data of 1 day, from 5% to 80% while keeping a similar level of accuracy. Such robust estimation of the empirical critical density is crucial for many control algorithms. In summary, the re-sampling method presented in this paper is very promising for different application purposes of the MFD. It is easy to use, requires only very few inputs, and is robust against a potential placement bias in loop detector data.

The proposed measure of the level of inhomogeneity gives city planners the possibility to analyze and quantify how efficiently their road network is utilized. This line of questions was started over 50 years ago by Smeed (36), but can now be evaluated in more detail with the availability of big data. There have been numerous studies investigating the performance of the network when implementing a perimeter control scheme based on the MFD. However, the estimation of the required critical density has not yet been discussed in an empirical context and only few of the cities investigated up to now, exhibit a strong congested branch. The re-sampling methodology proposed in this paper provides a promising approach to estimate the critical density empirically at large urban scale.

Future research will concentrate on how the re-sampling method and the theoretical upper bound from analytical approaches relate to each other. In other words, it should address the question of whether we can approach the theoretical upper bound with our re-sampling methodology based on the findings of (2) and (22). A more detailed investigation on the spatial and temporal distribution of congestion could generate further interesting findings. Furthermore, additional insights could be gained from an application of the methodology to floating car data, or the fusion of multiple data sources (37). On-going studies are also evaluating the possibility to estimate an infrastructure potential using the MFD, and thereby deepen the understanding of the effects of inhomogeneous traffic in urban networks.

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