FORECASTING AT THE CURRENT EDGE - REVISIONS
AND MIXED-FREQUENCY DATA

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Preface

This thesis was written while I was researcher and expert for the European economy at the KOF Swiss Economic Institute of the Swiss Federal Institute of Technology (ETH Zurich). I am indebted to my advisor, Prof. Dr. Jan-Egbert Sturm, who gave me the opportunity to work on this dissertation and made numerous helpful comments on earlier versions of the papers. Furthermore, I am grateful to Prof. Dr. Peter Egger who agreed to be my co-supervisor and who also made many helpful comments on earlier versions of the chapters of the dissertation.

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Stefan Neuwirth
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Summary of the thesis

This PhD thesis aims to contribute to dealing with problems that arise when forecasting at the current edge. Chapter 2 analyses revisions to German GDP and their predictability. Chapter 3 develops a distributed lag mixed-frequency VAR model for conveniently joining variables with mixed-frequencies in a VAR. In chapter 4 we propose a way how to deal with volatile and noisy high-frequency data and long lags. Chapter 5 analyses the usefulness of time-varying parameters for forecasting and if they can improve forecast accuracy in general and in times of crisis.

After an introductory chapter, chapter 2, which is joint work with Jens Boysen-Hogrefe, examines the predictability of German GDP revisions using forecast rationality tests. Previous studies on German GDP covering data until 1997 find that revisions of real seasonally adjusted GDP are predictable. This paper uses a real-time data set to analyze the revisions of real seasonal adjusted GDP, nominal unadjusted GDP, the seasonal adjustment and the GDP deflator for the period between 1995 and 2015. We find that the revisions of nominal unadjusted GDP are unpredictable, but that the revisions of the GDP deflator and the seasonal adjustment are predictable, especially at the current edge. The high predictability of the adjustments does not relate to a higher predictability of revisions of real seasonally adjusted GDP. Nevertheless, we find that revisions of German GDP can be predicted to some degree by using business cycle indicators.

Chapter 3, co-authored with Heiner Mikosch, develops a distributed lag mixed-frequency VAR model drawing from early literature on distributed lag models. The
distributed lag approach is a convenient way to join variables of mixed frequencies into a VAR, like quarterly GDP, monthly inflation and daily interest rates. Our approach contrasts with other mixed-frequency VAR models which solve the mixed-frequency challenge by reformulating lower frequency series as latent high frequency series. While the latent variable approach is straightforward theoretically, it shifts the burden to the empirical model estimation. Our contribution is to show analytically how to recast the distributed lag mixed-frequency VAR model into a simple linear equation system. This system can then be easily estimated and used in empirical applications. An out-of-sample forecasting exercise with US real-time data yields that the distributed lag mixed-frequency VAR substantially improves predictive accuracy upon a standard VAR. Forecast errors for, e.g., GDP growth decrease by 30 to 60 percent for forecast horizons up to six months and by around 20 percent for a forecast horizon of one year.

Chapter 4, written together with Dirk Drechsel, proposes a Bayesian optimal filtering setup for improving out-of-sample forecasting performance when using volatile high frequency data with long lag structure for forecasting low-frequency data. We test this setup by using real-time Swiss construction investment and construction permit data. We compare our approach to different filtering techniques and show that our proposed filter outperforms various commonly used filtering techniques in terms of extracting the more relevant signal of the indicator series for forecasting.

Chapter 5 tests the usefulness of time-varying parameters when forecasting with mixed-frequency data. For this I compare the forecast performance of bridge equations and unrestricted MIDAS models with constant and time-varying parameters. An out-of-sample forecasting exercise with US real-time data shows that the use of time-varying parameters does not improve forecasts significantly over all vintages. However, since the Great Recession, forecast errors are smaller when forecasting with bridge equations due to the ability of time-varying parameters to incorporate gradual structural changes faster.
Zusammenfassung der Dissertation


Kapitel 5 prüft die Nutzlichkeit von zeitvarierenden Parametern für die Prognose mit Daten unterschiedlicher Frequenz. Dafür wird die Prognosegüte von Brückengleichungen und unrestringierten MIDAS-Modellen mit jeweils konstanten und zeitvarierenden Parametern verglichen. Ein Test mit US-Echtzeit-Daten zeigt, dass die
Chapter 1

Introduction

1.1 Forecasting at the current edge

Economic forecasting is an important decision-making tool for governments, businesses and central banks in order to formulate financial and monetary policy and strategies. It helps reducing uncertainty about future outcomes and provides a foundation for planning. But when forecasting at the current edge of the data several problems can arise. One of them is revisions of the underlying data and the question of how reliable the currently available data are and if they will be revised considerably? Another problem are the different frequencies at which macroeconomic data are sampled. This leads to the questions of how to incorporate the available higher-frequency data into econometric models for forecasting lower-frequency data and how to deal with different publication lags and thus ragged edges of the data. A third problem when forecasting are potential structural breaks or temporal instabilities in the relationship between macroeconomic variables, which are difficult to detect at the current edge of the data with common break tests.
Revisions

The problem that policy makers and researchers face is that they have to use currently available data which can be preliminary, partly revised, or final. The quality of preliminary and partly revised data is very crucial, as important decisions rely on that data. Croushore & Stark (2003) show that data revisions can have significant effects on the empirical results of macroeconomic models. The impact of revisions on economic and policy analysis is the topic of several papers for example Bernanke & Boivin (2003), Clausen & Meier (2005), and Gerberding et al. (2005). Orphanides (2003) concludes that policy makers should make their decisions more passively and cautiously the larger the noise in the real time data is. Therefore decision makers need a way to quantify the noise in the available real time data and, if possible, enhance the preliminary data to conduct an optimal policy. The concept of news and noise was introduced by Mankiw et al. (1984) and Mankiw & Shapiro (1986). Noise is a pollution of preliminary estimates of GDP by errors or measurement deficiencies by the statistical agency which are corrected in subsequent revisions. Those measurement errors are uncorrelated with the final or true value of GDP. News in contrast is the arrival of previously not available data which is then included in the estimates of the statistical agency. Revisions which are due to news would thus not be predictable while the correction of previous errors or noise could be predictable. The rationality of GDP announcements and their statistical properties have been studied intensively in the literature, e.g. Faust et al. (2005), Fixler & Grimm (2006), Swanson & van Dijk (2006) or Jacobs & van Norden (2011).

Mixed-frequency data

Macroeconomic variables are usually sampled at different frequencies. For instance, GDP comes at a quarterly frequency whereas inflation is published monthly and short term interest rates are quoted at a daily or even higher frequency. The traditional solution is to simply time-aggregate all higher frequency series to the frequency of the lowest frequency series in the sample. An econometric model including GDP,
inflation and an interest rate will then be at quarterly frequency. Such time aggregation comes at costs. First, any new data release, which occurs within the lowest frequency, can only be taken into account after the end of each lowest frequency period. The delayed processing of information impairs forecasts. Second, the time-aggregation implies peculiar parameter constraints. For instance, a monthly flow variable, like industrial production, gets aggregated to the quarterly frequency by weighting the three monthly observations of a quarter equally with one third. The equal weights constraint on the parameters attached to higher frequency variables renders the parameter estimates inconsistent and inefficient (Andreou et al., 2010).

The problem of how to incorporate data with different frequency sampling in econometric models has been addressed in the literature in the last decade. A large number of studies have been published looking at the benefits of employing both high and low frequency data simultaneously in the context of single-equation approaches. One of those are bridge equations, which have been used for quite some time and are common in policy organizations due to their simple method and transparency. The general idea behind bridge equations is to explain a low-frequency variable by time-aggregated low-frequency lags of a high frequency variable. Early applications of bridge equations in the literature can be found for example in Ingenito & Trehan (1996) or Baffigi et al. (2004) as well at central banks like ECB (2008) or Bundesbank (2013).

Another single equation approach to handle time series with different frequencies that is also able to address the problem that arises when accounting for a long lag structure is mixed data sampling (MIDAS) proposed by Ghysels et al. (2004), building on Almon (1965). In this approach the high-frequency variable is not time-aggregated but directly related to the low-frequency variable. As this can lead to a high number of parameters to be estimated, lag polynomials can be used to decrease the necessary number of parameters and then be estimated by non-linear least squares (Ghysels et al. (2007)). Early applications of this method were mostly with financial data where sampling differences are quite big when using daily data,
for example in Ghysels et al. (2006). More recently MIDAS has also been used on macroeconomic data for example in Clements & Galvão (2008) and Clements & Galvão (2009a) or Armesto et al. (2010) and Andreou et al. (2011). Foroni et al. (2015b) have shown, that if differences in frequencies are small, for instance for a mixture of quarterly and monthly data, an unrestricted MIDAS setup (U-MIDAS) is equivalent or even superior compared with standard MIDAS setups. An unrestricted MIDAS setup requires less computational and modelling efforts compared with standard MIDAS setups.

For vector autoregression (VAR) models, the literature has taken a latent variable approach. The general idea is to conceptually assume the mixed frequencies away by reformulating each lower frequency series as a partially latent high frequency series in a state space framework. The Kalman filter or, in a Bayesian context, the Gibbs sampler then provide the possibility to estimate the partially latent VAR process. Zadrozny (1988, 1990), Kuzin et al. (2011), Bai et al. (2013), and Foroni & Marcellino (2014) develop state space type mixed-frequency VAR (MFVAR) models using a non-Bayesian version of the Kalman filter. Mariano & Murasawa (2010) present a state space type MFVAR using the EM algorithm, and Chiu et al. (2012) and Schorfheide & Song (2015) develop state space type MFVAR models using the Gibbs sampler. Another approach is Ghysels (2012a), building on Ghysels et al. (2004), which propose a stacked vector type mixed-frequency VAR (MFVAR) model. Based on this approach, a VAR process of any desired order can then be modelled including different frequencies. Further, in order to achieve parsimony, distributed lag or MIDAS style polynomial structures can then be imposed on the parameter space of the stacked vector type MFVAR. The MIDAS approach aims to reduce the parameter space while keeping models flexible.

**Temporal instabilities at the current edge**

Another problem when using up-to-date data are temporal instabilities of the parameters which are difficult to detect at the current edge of the data. This difficulty
can be addressed to some degree by using time-varying parameters where the parameters follow a random-walk process. A few approaches have been tested in the literature so far. Carriero et al. (2015) use a Bayesian mixed-frequency regression model with stochastic volatility and find some usefulness of using stochastic volatility for forecasting US GDP but not when using time-varying parameters. Galvão (2013) uses a transition function that governs for some parameters the change in parameters in MIDAS regressions. Guerin & Marcellino (2013) propose a Markov-Switching MIDAS approach which allows for switches between a small number of regimes. Schumacher (2014a) analyses MIDAS regressions with time-varying parameters for Euro area GDP and corporate bonds spreads by using a particle filter to deal with non-linearities in the MIDAS equation.

1.2 Objectives and outline of the thesis

This PhD thesis aims to contribute to dealing with those problems. Chapter 2 analyses revisions to German GDP and their predictability. Chapter 3 develops a distributed lag mixed-frequency VAR model for conveniently joining variables with mixed-frequencies in a VAR. In chapter 4 we propose a way how to deal with volatile and noisy high-frequency data and long lags. For this we develop a Bayesian beta filter for bridge models. Chapter 5 analyses the usefulness of time-varying parameters for forecasting and if they can improve forecast accuracy in general and in times of crisis.

Chapter 2 examines the predictability of German GDP revisions using forecast rationality tests. This study contributes to the literature in three ways. Firstly, it assesses a new data set on GDP revisions provided by the Deutsche Bundesbank and thus provides an update compared to earlier studies regarding the predictability of revisions of real seasonally adjusted GDP growth rates. Secondly, instead of looking at different revisions intervals (i.e. short- and long-term revisions), we analyze the
whole revision process. Thirdly, revisions of unadjusted GDP, revisions of price adjustments, and revisions of seasonal adjustments are assessed for the first time. For this purpose the real time data set of the Deutsche Bundesbank is enhanced correspondingly. Thus, this study provides a new real time data set on price and seasonal adjustments. It analyzes the predictability of their revisions and assesses the impact of adjustments on the predictability of revisions of real seasonally adjusted GDP for the period between 1995 and 2015. Previous studies on German GDP covering data until 1997 find that revisions of real seasonally adjusted GDP are predictable. We find that the revisions of nominal unadjusted GDP are unpredictable, but that the revisions of the GDP deflator and the seasonal adjustment are predictable, especially at the current edge. The high predictability of the adjustments does not relate to a higher predictability of revisions of real seasonally adjusted GDP. Nevertheless, we find that revisions of German GDP can be predicted to some degree by using business cycle indicators.

In Chapter 3 we develop a distributed lag mixed-frequency VAR model drawing from early literature on distributed lag models. The distributed lag approach is a convenient way to join variables of mixed frequencies into a VAR, like quarterly GDP, monthly inflation and daily interest rates. Our approach contrasts with other mixed-frequency VAR models which solve the mixed-frequency challenge by reformulating lower frequency series as latent high frequency series. While the latent variable approach is straightforward theoretically, it shifts the burden to the empirical model estimation. Our contribution is to show analytically how to recast the distributed lag mixed-frequency VAR model into a simple linear equation system. This system can then be easily estimated and used in empirical applications. An out-of-sample forecasting exercise with US real-time data yields that the distributed lag mixed-frequency VAR substantially improves predictive accuracy upon a standard VAR. Forecast errors for, e.g., GDP growth decrease by 30 to 60 percent for forecast horizons up to six months and by around 20 percent for a forecast horizon of one year.
In Chapter 4 we propose a filtering approach for bridge equations when forecasting with volatile high frequency data and a long lag structure. Specifically, we augment the bridge equations approach with a Bayesian beta filter which is designed to extract all informative signals from the data and to handle very long lags while keeping parsimony. We test our approach compared to a standard bridge model with unfiltered data and a bridge model with smoothed data by using established filtering techniques. We apply our approach to forecasting Swiss construction investments with construction permits using a real-time data set from 1993 to 2014. The real-time nature of the data allows us to account for data revisions and ragged edges. We find that our Bayesian filtering approach for bridge models clearly beats unrestricted MIDAS, standard MIDAS, traditional bridge equations and autoregressive (AR) models in terms of out-of-sample performance. This result holds irrespectively of whether the four benchmark approaches employ unfiltered data or data which is smoothed with standard techniques.

Chapter 5 tests the usefulness of time-varying parameters when forecasting with mixed-frequency data. This study extends the literature by using time-varying parameters in both bridge equations and unrestricted MIDAS models. Additionally, we compare forecast performance of the different models and methods for a long forecast horizon. An out-of-sample forecasting exercise with US real-time data shows that the use of time-varying parameters does not improve forecasts significantly over all vintages. However, since the Great Recession, forecast errors are smaller when forecasting with bridge equations. It could be that economic relationships between variables have changed since the Great Recession. Thus, the ability to incorporate gradual structural changes faster by using time-varying parameters can be beneficial for forecasting, at least in times of turmoil.
Chapter 2

The impact of seasonal and price adjustments on the predictability of German GDP revisions

2.1 Introduction

The problem that policy makers and researchers face is that they have to use currently available data which can be preliminary, partly revised, or final. The quality of preliminary and partly revised data is very crucial, as important decisions rely on these data. Croushore & Stark (2003) show that data revisions can have significant effects on the empirical results of macroeconomic models. The impact of revisions on economic and policy analysis is the topic of several papers for example Bernanke & Boivin (2003), Clausen & Meier (2005), and Gerberding et al. (2005). Orphanides (2003) concludes that policy makers should make their decisions more passively and cautiously the larger the noise in the real time data is. Therefore decision makers need a way to quantify the noise in the available real time data and, if possible, enhance the preliminary data to conduct an optimal policy. The concept of news and noise was introduced by Mankiw et al. (1984) and Mankiw & Shapiro (1986).

¹This chapter is based on Bojesen-Hogrefe & Neuwirth (2012)
Noise is a pollution of preliminary estimates of GDP by errors or measurement deficiencies by the statistical agency which are corrected in subsequent revisions. Those measurement errors are uncorrelated with the final or true value of GDP. News in contrast is the arrival of previously not available data which is then included in the estimates of the statistical agency. Revisions which are due to news would thus not be predictable while the correction of previous errors or noise could be predictable.

In their seminal study on the nature of errors in preliminary GNP data, Mankiw & Shapiro (1986) analyze the revisions of real and nominal seasonally adjusted US GNP during the period between Q1 1976 and Q4 1982. They check the correlations and variance of the subsequent revisions and try to predict those using data that was available at the time of the preliminary announcement. Their main finding is that revisions of GNP data cannot be predicted, which implies that preliminary estimates of the GNP are rational estimates that use efficiently all available information. This finding is challenged by Faust et al. (2005) and Fixler & Grimm (2006), who find predictability of US GDP revisions between 1983 and 2000. They analyze real and nominal seasonally adjusted GDP and find it is possible to predict future revisions to some degree by using real-time vintage data. Considering economic variables does not improve predictability substantially. Hogrefe (2008) analyzes US GDP between 1985 and 2003 and finds, like Fixler and Grimm, predictability of revisions. The predictability of revisions increases, in comparison to Fixler and Grimm, by using a mixed frequency approach.

Studies for German data are among other presented by Faust et al. (2005). They analyze the predictability of revisions of real seasonally adjusted GDP for the G7 countries between Q4 1979 and Q4 1997. They reject the hypothesis that the first estimates are rational for all 7 countries for long-term revisions and short-term revisions except for US short-term revisions. They find that at least one quarter of the variance in short-term revisions can be predicted by using information that was available at the time of the preliminary announcement. The preliminary announcement itself has the biggest impact in their regressions. This is an indication that it
is mostly noise that drive the revisions. Liedo & Carstensen (2005) provide a model to decompose revisions into pure news and noise components and analyze US, German and euro area real seasonally adjusted GDP/GNP revisions. Their data for West Germany covers Q1 1962 to 1994 Q4. They find that the news component is the dominating reason, but not the only one for revisions. The news component is still significant after two and three years, which means that even after such a long time new data becomes available that is relevant for GDP measurement. They conclude that it is possible to improve the releases of the statistical agency, as data revisions are partly subject to noise. All the studies for German GDP deal with real seasonally adjusted data only. However, Kavajecz & Collins (1995) show that seasonal adjustment at the current edge can induce noise. The same might be true for price adjustment. Thus, adjustments may play an important role in assessing the predictability of data revisions. Until now, studies on the predictability of German GDP revisions did not incorporate an analysis on the impact of adjustments on the predictability.

This study contributes to the literature in three ways. Firstly, it assesses a new data set on GDP revisions provided by the Deutsche Bundesbank and thus provides an update compared to earlier studies regarding the predictability of revisions of real seasonally adjusted GDP growth rates. Secondly, instead of looking at different revisions intervals (i.e. short- and long-term revisions), we analyze the whole revision process. Thirdly, revisions of unadjusted GDP, revisions of price adjustments, and revisions of seasonal adjustments are assessed for the first time. For this purpose the real time data set of the Deutsche Bundesbank is enhanced correspondingly. Thus, this study provides a new real time data set on price and seasonal adjustments. It analyzes the predictability of their revisions and assesses the impact of adjustments on the predictability of revisions of real seasonally adjusted GDP.

We find that the revisions of the original series, nominal unadjusted GDP, are not predictable. We cannot reject the null hypothesis of forecast rationality in Mincer-Zarnowitz tests for the whole revision process. Revisions of the seasonal
adjustments and of the price adjustments are to some degree predictable. However, 
the predictability is not transferred to the revisions of real seasonally adjusted GDP 
growth due to a high negative correlation between revisions to seasonal and price 
adjustments. Thus, the predictability of revisions of real seasonal adjusted GDP 
growth due to noise is diminished compared to studies on earlier data. The loss of 
predictability is also true for the revisions of the seasonal adjustments. This change 
in predictability may be due to the changeover from X-11 to X-12 ARIMA that took 
place within our sample. We conclude that more accurate procedures for adjusting 
data are crucial for the rationality of data announcements as revisions of the original 
data are not predictable and predictability of the revisions of real seasonally 
adjusted GDP seems to hinge on the predictability of revisions of the seasonal ad-
justments and the price adjustment. Furthermore, we cannot confirm the usefulness 
of survey data for forecasting revisions, as was noticed by Matheson et al. (2010). 
The application of the ifo index following Jacobs & Sturm (2005), who predict re-
visions of industrial production seems at least for the analysis of German GDP not 
to add additional information. But we find that the predictability of revisions to 
real seasonally adjusted GDP can be enhanced by using non-survey business cycle 
indicators.

The rest of the paper is structured as follows. Section 2.2 discusses the process of 
data revision in Germany and describes our real time data set. Section 2.3 presents 
a preliminary analysis of the data and the further course of the analysis. Section 
2.4 presents the results of our analysis and section 2.5 concludes.

2.2 Data revision process in Germany and data description

The German statistical agency (Statistisches Bundesamt) revises GDP estimates 
quite frequently. They differentiate between ongoing revisions and benchmark revi-
sions. The process of ongoing revisions takes about four years. During this time, the 
preliminary estimate of GDP is revised several times as new data becomes available.
While calculating the first estimate of GDP, which is announced in February, May, August, and November, the data for the other quarterly GDP figures of that year are also revised. In August, the preceding sixteen quarters are analyzed and revised if necessary. After a total of four years the data are considered final and will only be changed when benchmark revisions are made. For earlier estimates the statistical agency has to use output indicators in order to produce GDP figures. Later during the revision process, it is possible for the agency to access complete corporate statistics especially cost structure and turnover tax statistics.

Benchmark revisions are done approximately every five years to take new methods, data and statistics into account. Three benchmark revisions took place during the time period covered by our sample. The last benchmark revision in our sample was at the beginning of 2005, when the statistical agency introduced a chain price index that changes every year. Before then, the base year was fixed for several years. Additionally, from 2005 on, the real GDP has been reported as an index and the method of measuring banking services (FISIM) was changed to harmonize with EU methods. In 2000, the Deutsche Bundesbank started to use the Census X-12-ARIMA method for seasonal adjustment instead of Census X-11. This was done in order to obtain more reliable results using state-of-the-art mathematical methods. This should allow the statistical agency to reduce the size of revisions. The main changes in the benchmark revision of 1999 were the switch to a new year as price base, in this case 1995, and the application of the rules of the European System of Accounts (ESA). This was done to have common and comparable statistics in Europe.

In this paper we analyze the revisions to German real seasonally adjusted GDP. Additionally, we also look at the revisions to nominal unadjusted GDP, the GDP deflator and the seasonal adjustment. For this we built a real-time dataset containing the nominal unadjusted GDP, the real unadjusted GDP and the real seasonally adjusted GDP. We use data from the real time data base of the Deutsche Bundesbank and other sources. The Deutsche Bundesbank supplies real time data sets for
national accounts, prices, and short-term business cycle and labor markets indicators. The data for real seasonally adjusted GDP as well as for real unadjusted GDP for the period between Q2 1995 and Q1 2015 are from the database of the Deutsche Bundesbank. Due to the benchmark revision in 2005 mentioned above, real GDP is only reported as a chain index since then. This makes it more feasible to analyze the revisions in growth rates instead of levels, which is consistent with most of the work in this field. As Mankiw & Shapiro (1986) point out, using growth rates instead of level variables has the advantage that growth rates are mostly stationary. The vintage data for the nominal unadjusted GDP are also from the data base of the Deutsche Bundesbank from 2005 onwards. Vintages for the time between 2000 and 2004 were supplied by the German statistical agency. The previous vintage data are from the Destatis publication “Fachserie 18 Reihe 3”. Due to missing data, some assumptions for the real unadjusted GDP had to be made, in order to analyze the revisions for longer horizons. For Q4 1995, the 17th vintage is assumed to contain no revision compared to the previous vintage. The same holds for the two last vintages of Q1 1996, Q3 1996, Q4 1996, Q1 1997, Q4 1997 and Q1 1998. These assumptions should be reasonable as there were no major revisions in the vintages for the other data.

From these series we can derive the GDP deflator and the seasonal adjustment. The price index of the GDP deflator is calculated by dividing the levels of nominal and real unadjusted GDP. As we use only growth rates for our analysis due to the data issues mentioned above, we calculate quarter-on-quarter growth rates of the GDP deflator price index. The seasonal adjustment is obtained by subtracting the quarter-on-quarter growth rates of real seasonally adjusted GDP from the growth rates of real unadjusted GDP.

These assumptions allow us to analyze revisions for 17 vintages but do not drive the results. Similar results can be achieved when looking at fewer vintages without those assumptions.
2.3 Preliminary analysis and model description

The summary statistics of the estimates are presented in table 2.2 to table 2.5. They show that the mean for nominal seasonal unadjusted GDP, the seasonal adjustment and the GDP deflator are statistically not distinguishable from zero due to very high variances. For real seasonal adjusted GDP the mean is positive and significant. Following Mankiw & Shapiro (1986), if a provisional estimate is an efficient forecast of the subsequent estimates and revisions are only due to new information, then the variance of the subsequent estimates should increase. In contrast, if revisions are mostly due to measurement errors which are corrected over the course of the revision process then the variance of subsequent estimates should be decreasing. In our sample variances decrease slightly during the revisions process for all components except real seasonal adjusted GDP, but the differences are rather small and not significant.

Another possibility to analyse the rationality of revisions would be to look at the revisions themselves. A revision is calculated as the difference between the growth rate in the 17th vintage, which we consider the "final" value, and in the previous provisional estimates for the period $t$ and the estimates at time $h$. In addition to the revisions of nominal unadjusted GDP ($u$) and real seasonal-adjusted GDP ($r$), we also calculate those of price adjustments ($p$) and the seasonal adjustments ($s$). The summary statistics for the revisions between the subsequent provisional estimated are reported in tables 2.6 to 2.9. They show that the means of the revisions are statistically not different from zero. A mean different from zero would imply a bias in the revision process (i.e. a positive mean equals a biased under-estimation of GDP). A zero mean thus implies that there is no bias in the revisions of the four time series.

Further, as a preliminary analysis, we check the correlations between the estimates and the revisions. As Mankiw & Shapiro (1986) state, a correlation between a provisional estimate and a revision could point to previous measurement errors while a correlation between the revised estimate and the previous revision would be evidence for estimates being rational forecasts. When the correlations in the lower
triangle are zero, this could be an indication for the revisions occurring due to news. If, in contrast, the correlations in the upper triangle would be zero, this would point to measurement errors. The correlations for nominal unadjusted GDP are shown in table 2.10. The results are not clear for both cases. While some revisions seem to be correlated to all previous subsequent estimates, most are not significantly correlated. Also for the seasonal adjustment, the GDP deflator and the real seasonally adjusted GDP the implications of the correlations are inconclusive.\textsuperscript{3}

The preliminary analysis of revisions and the provisional estimates does not give a clear picture whether revisions of German GDP are due to news or noise. While the size and variation of estimates and revisions point to revisions not being due to measurement errors and indicate no systematic bias, the correlation analysis does not support this. For a clearer picture we conduct forecast rationality tests which can be used to determine whether the revisions of German GDP are subject to news or noise. The test used here is the Mincer-Zarnowitz forecast rationality test. In order to assess the predictability of revisions over time we analyse the subsequent cumulative revisions between the first estimate and the "final" data. This way we can analyze how long potential measurement errors are contained in the revisions. The corresponding model is given by the equation below:

\[ r_{f,h,t} = y_{f,t} - y_{h,t} = \alpha + \beta y_{h,t} + u_t, \]  

(2.1)

where \( r_{f,h,t} \) denotes the cumulative revision between the estimate for \( y_t \) in vintage \( h \) (\( y_{h,t} \)) and the "final" estimate for \( y_t \) (\( y_{f,t} \)), in our case the 17th vintage. The null hypothesis of rational estimates corresponds to the restriction on the parameters \( \alpha = \beta = 0 \). A rejection of this parameter restriction indicates that revisions are predictable. However, we consider an extended equation that includes additional information \( X_t \) available at the time of the \( h \)-th vintage but corresponding to the time of the first estimate:

\textsuperscript{3}As the correlations for the other components do not offer clear insights and due to the large amount of tables, the correlations can be found in the appendix.
\[ r_{f,h,t} = \alpha + \beta y_{h,t} + \gamma X_{t,h} + u_t. \] (2.2)

The first model that we estimate (model I) is the simple Mincer-Zarnowitz forecast rationality test. For model II we include two variables which according to the literature have potential information for the revision process. The first is the Ifo index which according to Jacobs & Sturm (2005) has some information for the revision process of German industrial production. We use the sub-index for the current business situation instead of the full Ifo index which also includes a forward looking component. Following Swanson & van Dijk (2006), we also include the previous revision history between the first and the \( h \)-th release \( (r_{h,1,t} = y_{h,t} - y_{1,t}) \) in our analysis. In this way we can also examine whether inefficiency arises via information available in the revision history for a given release of data as well as through other sources. In a third round (model III) we include other business cycle indicators that are generally in line with prior work. We use, like Mankiw & Shapiro (1986), an equity index, in this case German DAX performance. As supplemental business cycle indicators, we use the the first estimate of the German CPI, the oil price (BRENT), short-term and long-term interest rates, the unemployment rate and monetary aggregates M0 to M3. These time-series are not subject to large revisions themselves. The variables used in model III for GDP and the adjustments were selected via Bayesian information criteria (BIC). The selection of variables was done for the longer revision horizons \( (r_{17,1} - r_{17,4}) \) and the corresponding selected variables were used for the shorter horizons. In order to control for the benchmark revisions explained in section 2.2 we use a dummy variable in all aforementioned models.

We apply these models for all revision horizons from \( h = 1, \ldots, 16 \) quarters to assess the predictability over time. In addition to the growth rates of real seasonal adjusted GDP, we also examine revisions of the growth rates of nominal unadjusted GDP, the growth rates of the price adjustments, and the seasonal adjustments. Thus, we examine not only the aggregate typically analyzed in the literature (s.a., real GDP) but also its components. All the model estimations are done using OLS.
2.4 Results

2.4.1 Nominal unadjusted GDP

The forecast rationality test according to model I for the nominal unadjusted GDP is shown in table 2.11. It confirms forecast rationality for all revision horizons with all coefficients being insignificant. The explanatory power of this model is with a maximum $R^2$ of 5% rather low. In summary, the first estimate of the nominal unadjusted GDP by the statistical agency does seem to be rational.

The inclusion of the ifo index and the previous revision history in model II do not change the results significantly. All coefficients still stay insignificant. The additional variables do not allow us to reject the null hypothesis of nominal unadjusted GDP revisions being due to news. The inclusion of another variable (model III), in this case the quarterly growth rate of the monetary aggregate M1, changes the results somewhat. M1 seems to contain additional information for the revision process for up to 12 quarters ahead with small but significant coefficients (Table 2.12). This might confirm previous literature that finds M1 to be a good indicator for forecasting GDP like Brand et al. (2003). In conclusion, even though narrow money seems to contain some information for the revision process of nominal unadjusted GDP, the revisions seem to be rational and mostly due to new information and not due to initial measurement errors.

2.4.2 Seasonal adjustment

The Mincer-Zarnowitz test based on model I rejects the null hypothesis of forecast rationality for revisions of seasonal adjustments (Table 2.13). The provisional estimate $y_{h,t}$ is significant for more than two years after the first estimate. A high provisional estimate seems to be revised down by around a third. Even this simple model can explain 20% of the variation. The additional variables (model II) do not

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4Due to a large amount of results only tables are shown if they change compared to the previous model. All tables can be found in the appendix.
contain much information about the future revisions of seasonal adjustments. We were still able to reject forecast rationality for the first 9 revisions and the provisional estimate was still significant. By adding further information that was available at the time of the provisional estimate (model III) the results change slightly. In addition to the ifo business situation and the previous revision history we add the quarterly growth rate of the monetary aggregate M1. In contrast to nominal un-adjusted GDP, narrow money does not contain much information. The coefficients for the provisional estimate turn a bit more negative but the coefficients for the monetary aggregate are insignificant. In conclusion, the results for the revisions of the seasonal adjustment do not give a clear indication whether revisions are due to new information or early measurement errors.

One explanation for the inconclusive results could be a structural break in the data. As noted above, the Deutsche Bundesbank has been using the Census X-12-ARIMA method for seasonal adjustment since the first quarter of 2000. The purpose of this changeover was to have a more reliable method and to reduce the size of revisions. If the new method is successful in reducing the noise in the first estimate, we should see a lower variance in the estimates and revisions that were made for the estimates since 2000. In order to analyze the effect of the changeover closer, we split our sample at Q4 1999 and look only at estimates and revisions done before or after the potential break. The mean of both the estimates as well as the revisions is in both sub-samples not significantly different from zero. But the variance in the second sub-sample is less than half than the variance in the first sub-sample. This indicates that the changeover to Census X-12-ARIMA fulfilled its purpose by reducing the variability of revisions. This could introduce a break in our data. We apply the Chow break test on the general Mincer-Zarnowitz specification to check for a possible structural break at the first quarter 2000 in our sample. However, we find that the changeover from Census X-11 to Census X-12-ARIMA has basically no impact on the mean regressions.
2.4.3 Price adjustment

Using model I the Mincer-Zarnowitz forecast rationality test rejects the null hypothesis clearly for the longer revision horizons (Table 2.14). Even with model I it is possible to predict a considerable fraction of future revisions. The $R^2$ for the longest revision horizon ($p_{17,1}$) is around 20%. This result indicates that revisions of the provisional estimates by the statistical agency for the GDP deflator are to some degree driven by noise and the provisional estimate at the time $h$ of the revision has information about the future revision. This is especially the case for early estimates for which the coefficient is rather big and comparable to the revisions of the seasonal adjustment.

In model II, we enhance the regression equation by introducing the quarterly change of the ifo Business Situation and the previous revision history. The test still rejects forecast rationality for longer forecast horizons from $p_{17,1}$ to $p_{17,12}$ but the additional variables do not seem to contain information about the future revision process. A selection by BIC for additional potential variables did not yield any results. In conclusion, the revisions of the GDP deflator seem to contain a certain amount of noise, which is indicated by the significant coefficient for the provisional estimate in the Mincer-Zarnowitz forecast rationality test, and is reduced over the revision horizon.

2.4.4 Real seasonally adjusted GDP

A revision of the real seasonally adjusted GDP equals the revision of the nominal unadjusted GDP minus the revisions of the seasonal adjustment and the price adjustment. As two of the three components showed some amount of predictability, the real adjusted GDP revisions should also be predictable, unless the price adjustment revisions and seasonal adjustment revisions cancel each other out.
The Mincer-Zarnowitz forecast rationality test in model I cannot reject the null hypothesis of forecast rationality for all revision horizons and the $R^2$ is, compared to the results presented above, quite low (Table 2.15). These results seem to contradict the previous results. We continue by adding the ifo Business Situation and the previous revision history (model II). This enhanced the results only slightly. Both the ifo Business Situation and the revision history seem to contain no information on future revisions of real seasonally adjusted GDP. Only the $R^2$ increases slightly. In model III, we use the unemployment rate, the year-on-year growth rate of M1 and the quarter-on-quarter growth rate of the DAX as additional indicators. Especially the unemployment rate and the narrow money aggregate seem to contain information on the revision process and are significant for most of the revision horizon. The results are shown on table 2.16. Additionally, the constant turned significant as well. The explanatory power of the model increased considerably up to an $R^2$ of 33%. In summary, it can be concluded, that the first estimate of the real adjusted GDP does not contain measurement errors, as the Mincer-Zarnowitz forecast rationality test in model I could not reject the null hypothesis. But it seems that the provisional estimates are not efficient estimates, as subsequent revisions can be explained to some degree by additional variables that were available at the time of the first estimate.

Still, it is surprising that even though the estimates for the seasonal adjustment and the GDP deflator contain noise, indicated by the significant coefficients in the Mincer-Zarnowitz regressions, the estimates of the real seasonal-adjusted GDP do not. An explanation could be that the revisions of the seasonal adjustment and the GDP deflator are highly negatively correlated. In table 2.1, we find that the correlation coefficient of -0.86 is highly significant. Thus the initial noise in the estimates most likely nullify each other, at least until they are not detectable any more by the Mincer-Zarnowitz test.
Table 2.1: Correlation between revisions

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Note: * significant at 10 percent level, ** significant at 5 percent level, *** significant at 1 percent level.

2.5 Conclusion

This paper analyzes the predictability of revisions of German GDP between 1995 and 2015, focusing on the impact of adjustment procedures, namely price adjustment and seasonal adjustment. For this reason, we not only assess revisions of real seasonally adjusted GDP but also revisions of the price adjustment, the seasonal adjustment, and the nominal unadjusted GDP. We conduct forecast rationality tests on those series in order to distinguish whether the revisions are due to actual news, which is unpredictable, or noise like measurement errors, which could be predictable.

We find that revisions of nominal unadjusted GDP are de facto unpredictable. Revisions of the two adjustments on the other hand are to some extent irrational and predictable. Both contain noise in the provisional estimates which take around two years to vanish in the case of seasonal adjustment and three years in case of the GDP deflator. Even simple regression models yield an $R^2$ of around 20%. As revisions to those two adjustments are highly negatively correlated they seem to cancel each other out when it comes to revisions to real seasonally adjusted GDP. For revisions to real seasonally adjusted GDP we cannot reject the null hypothesis of forecast rationality. But it is questionable whether the estimates are completely efficient. With the addition of different business cycle indicators we are able to explain around a third of the revisions to real seasonally adjusted GDP, especially at the current edge.
For practitioners, this study provides support for modelling revisions both to real seasonally adjusted GDP as well as the adjustment procedures, especially at the current edge of the data. As the in-sample analysis indicates quite some gains can be made by adding the provisional estimates or business cycle indicators.

2.6 Figures and tables
Table 2.2: Summary statistics: unadjusted GDP estimates

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Table 2.3: Summary statistics: seasonal adjustment estimates

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Table 2.4: Summary statistics: price adjustment estimates

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Table 2.5: Summary statistics: real adjusted GDP estimates

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## Table 2.7: Summary statistics: adjusted GDP revisions

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Table 2.7: Summary statistics: seasonal adjustment revisions
### Table 2.8: Summary statistics: price adjustment revisions

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### Table 2.9: Summary statistics: real adjusted GDP revisions

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Table 2.10: Correlations between estimates and revisions of nominal unadjusted GDP

Note: * significant at 10 percent level, ** significant at 5 percent level, *** significant at 1 percent level.

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Table 2.13: In-sample results: seasonal adjustment model I

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Table 2.14: In-sample results: price adjustment model I

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Table 2.16: In-sample results: real adjusted GDP – model III
Chapter 3

A distributed lag mixed-frequency vector autoregression model

3.1 Introduction

Vector autoregression (VAR) models are a standard tool for macroeconomic forecasting. They are easy of use, flexible and able to produce coherent forecasts for multiple variables. A challenge to joining multiple macroeconomic variables in a VAR is that macroeconomic variables are usually sampled at different frequencies. For instance, GDP comes at a quarterly frequency whereas inflation is published monthly and short term interest rates are quoted at a daily or even higher frequency. The traditional solution is to simply time-aggregate all higher frequency series to the frequency of the lowest frequency series in the sample. A VAR including GDP, inflation and an interest rate will then be a quarterly frequency VAR (QFVAR). Such time aggregation comes at costs. First, any new data release, which occurs within the lowest frequency, can only be taken into account after the end of each lowest frequency period. To stick to the above example, a QFVAR does not allow to consider inter-quarterly inflation and interest rate releases before the end of a quarter. The delayed processing of information impairs forecasts. Second, the

1This chapter is based on Mikosch & Neuwirth (2015)
time-aggregation implies peculiar parameter constraints. For instance, a monthly flow variable, like industrial production, gets aggregated to the quarterly frequency by weighting the three monthly observations of a quarter equally with one third. The equal weights constraint on the parameters attached to higher frequency variables renders the parameter estimates inconsistent and inefficient (Andreou et al., 2010).

To overcome the aforementioned drawbacks the literature has taken a latent variable approach. The general idea is to conceptually assume the mixed frequencies away by reformulating each lower frequency series as a partially latent high frequency series in a state space framework. The Kalman filter or, in a Bayesian context, the Gibbs sampler then provide the possibility to estimate the partially latent VAR process. Zadrožný (1988, 1990), Kuzin et al. (2011), Bai et al. (2013), and Foroni & Marcellino (2014) develop state space type mixed-frequency VAR (MFVAR) models using a non-Bayesian version of the Kalman filter. Mariano & Murasawa (2010) present a state space type MFVAR using the EM algorithm, and Chiu et al. (2012) and Schorfheide & Song (2015) develop state space type MFVAR models using the Gibbs sampler. While the latent variable approach is straightforward and transparent from a theoretical perspective, it shifts the burden to the model estimation. The estimation of a state space system with multiple latent time series processes is computationally challenging. The estimation of rich MFVAR specifications (multiple variables, multiple lags) is infeasible unless Bayesian prior and shrinkage techniques are employed. While nothing is to complain about Bayesian prior and shrinkage techniques themselves, they should be employed to improve the performance of models, rather than to make the models estimable in the first place.

In this paper, we shift the burden back to the analytical pencil and paper work and take a different approach to modelling multiple mixed-frequency variables in a VAR. In a first step, we propose a stacked vector MFVAR framework which is general (multiple mixed-frequency variables and multiple frequencies) but designed to still be compact and tractable. The MFVAR framework comes in a form that
makes it easily estimable and usable in empirical applications. In a second step, we augment the MFVAR with a non-linear distributed lag polynomial scheme. The distributed lag polynomial scheme prevents overparametrization even with long lag lengths while, at the same time, keeping the regression model flexible. In a third step, we show how to rewrite the augmented MFVAR into a linear equation system. While the rewriting of the MFVAR into linear form proves to be analytically demanding, the resulting linear equation system itself is transparent and easy to use in empirical applications. Multiple variables, multiple mixed-frequencies and long lag lengths prove no challenge. The model parameters can be easily estimated equation by equation by using standard ordinary least squares (OLS).

MFVARs with multiple high-frequency variables and/or long lag structures face the following trilemma. If they are left unrestricted they easily run into parameter proliferation. In contrast, if they are restricted with direct parameter restrictions or via Bayesian prior strategies they tend to get parametrically inflexible. Further, if they are restricted with highly non-linear and, hence, flexible polynomial schemes, estimation becomes cumbersome if not infeasible at all (provided no auxiliary restrictions and no fiddling with starting values). A system with multiple highly non-linear elements in it is too complex for proper estimation even when employing the most advanced estimation strategies. Against this background, the aforementioned transformation in linear form can be viewed as the main contribution of our paper. Due to this transformation the MFVAR (a) is both parametrically flexible and parsimonious, (b) is easy to estimate and to use in empirical applications even with multiple variables, multiple mixed-frequencies and long lag lengths, and (c) can be easily augmented and combined with other methods.

Our paper draws on an older and nowadays less cited literature on distributed lag models (e.g., Solow, 1960, Jorgenson, 1966, Tsurumi, 1971, Schmidt, 1974, Lütkepohl, 1981, Judge et al., 1985, Mitchell & Speaker, 1986, among many others). Distributed lag models relate a regressand variable to its own lags and/or the lagged values of one or several other regressor variables, where the regression parameters
are either unrestricted or restricted with potentially highly non-linear functions. The purpose of these functions is to reduce the parameter space while keeping the regression parametrically flexible at the same time. We adopt the Almon lag polynomial originally proposed by Shirley Montag Almon in her seminal Almon (1965) paper. The Almon lag figures among the most popular distributed lag functions in the literature. Comparisons between alternative distributed lag functions in a single equation context yielded that the Almon lag polynomial is a preferable choice (Mikosch & Zhang, 2014).

Ghysels (2012a) provides a rich exposition of a stacked vector type MFVAR model building on previous research on Mixed Data Sampling (MIDAS) for bivariate single equation models (e.g., Ghysels et al., 2004, Ghysels et al., 2007, Andreou et al., 2010, Bai et al., 2013, and Foroni et al., 2015b). Our model is similar to Ghysels' model in some respects and different in other respects. First, we present a general and still compact MFVAR framework (multiple variables and multiple frequencies), whereas Ghysels limits his exposition to two variables and two frequencies for reasons of tractability (a low-frequency variable and a high-frequency variable). Second, when it comes to applying the MFVAR models empirically, we employ alternative specifications with few variables or with multiple variables, with short lag structures or with long lag structures. Rich specifications pose no problem due to the aforementioned transformation in linear form. In contrast, Ghysels examines various important aspects of mixed-frequency models, but he does not present a solution to the aforementioned trilemma and, as a consequence, his MFVAR is limited to few variables with few lags.\(^2\) Third, Ghysels' and our framework fundamentally differ in the way they handle the contemporaneous relationships between variables with different frequencies. We propose to capture the contemporaneous relationships via a recursive form MFVAR scheme. Ghysels proposes a scheme that augments a

\(^2\)Götz et al. (2013) extend the model of Ghysels (2012a) to non-stationary time series. Francis et al. (2011), Götz et al. (2013), Chauvet et al. (2013), and Foroni et al. (2015a) apply the model in contexts with two variables.
reduced form MFVAR with a matrix structure derived from the Cholesky lower triangular decomposition. Fourth, while both papers adopt a stacking strategy, the stacking is done quite differently resulting in different model setups and solutions. Last, in the empirical application we focus on forecasting, whereas Ghysels mainly concentrates on impulse response analysis.

In order to test for the empirical usefulness of our MFVAR model, we conduct an out-of-sample forecast exercise with US real-time mixed-frequency data. The MFVAR substantially improves forecast accuracy upon a standard QFVAR for various different specifications. Root mean squared forecast errors for GDP growth get reduced by 30 to 50 percent for forecast horizons up to six months and by about 20 percent for a forecast horizon of one year. For inflation the gains are even bigger with improvements of up to 90 percent in the very short run. For forecasts of two years ahead our model improves forecasts upon a QFVAR by around 20 percent. Even bigger are the improvements when forecasting the short term interest rate where the MFVAR is constantly better than a QFAVR for all forecast horizons. Further, we find that the distributed lag augmentation has a distinct advantage for specifications with longer lag structures. For robustness, we compare our MFVAR model with a Bayesian quarterly-frequency VAR (BVAR). Even though results are less pronounced, we find that generally our MFVAR outperforms the BVAR benchmark model in terms of forecast accuracy.

The remainder of the paper is structured as follows: Section 3.2 presents our MFVAR model. In particular, we show how to recast the distributed lag augmented MFVAR into a linear equation system that is transparent and easy to use for empirical applications. Section 3.3 describes the out-of-sample forecast evaluation set up and the real-time data used. In Section 3.4, we analyze the empirical usefulness of the MFVAR model by means of a forecast exercise with US real-time data. Finally, Section 3.5 provides conclusions and possible directions of further research.
3.2 A general distributed lag MFVAR framework

Let there be a set of time series variables of different frequencies. Denote each variable by \( y_{i,t-1+\tau_i/T_i} \) where \( i \) is the variable subscript with \( i = 1, \ldots, I \) and \( t - 1 + \tau_i/T_i \) is the time period subscript.\(^3\) \( t = 1, \ldots, T \) denotes the lowest frequency period such that \( T_i \) is the frequency of the \( i \)-th variable in each period \( t \), and \( \tau_i = 1, \ldots, T_i \) denotes the subperiod of the \( i \)-th variable in each period \( t \). For instance, when the set of variables comprises quarterly, monthly and daily variables, \( t = 1, \ldots, T \) denotes the quarters, \( T_i = 1/3/90 \) is the frequency of any quarterly/monthly/daily variable in each quarter \( t \) and \( \tau_i = 1/1, 2/3/1, \ldots, 90 \) denotes the quarter/months/days in each quarter \( t \) (assuming that a quarter has 90 days).

For ease of exposition we assume in this section that each variable \( y_{i,t-1+\tau_i/T_i} \) with \( i \in \{1, \ldots, I\} \) is released simultaneously with each variable \( y_{j,t-1+\tau_j/T_j} \) with \( j \in \{1, \ldots, I\} \) and with \( t - 1 + \tau_j/T_j = t - 1 + \tau_i/T_i \). Further, we assume for ease of exposition that each variable \( y_{i,t-1+\tau_i/T_i} \) is released directly after the end of period \( t - 1 + \tau_i/T_i \) (no ragged edge). For instance, any quarterly variable is released directly after the end of a quarter simultaneously with any other quarterly variable and with any monthly variable observation on the third month of that quarter. Our empirical application presented in Section 3.4 explicitly models ragged edges and delayed releases.

The appendix illustrates the MFVAR framework with the simple case of just one monthly and one quarterly variable.

\(^3\)We model the time period subscript as \( t - 1 + \tau_i/T_i \) in order to let \( \tau_i \) denote the \( \tau_i \)-th subperiod in period \( t \). Clements & Galvão (2008, 2009b), e.g., model the time period subscript as \( t - \tau_i/T_i \) so that \( \tau_i \) denotes the \((T_i - \tau_i)\)-th subperiod. Both variants are equally possible.
The stacking approach

We stack all observations of the $i$-th variable in period $t - p$ with $p \in \mathbb{N}_0$ starting with the latest observation and ending with the earliest observation to get \(^4\)

$$
\begin{bmatrix}
y_{i,t-p} \\
y_{i,t-p+\frac{1}{T_i}} \\
\vdots \\
y_{i,t-p+\frac{P}{T_i}} 
\end{bmatrix}
$$

Doing this for all $I$ variables yields $I$ variable vectors for period $t - p$. Next, we stack these $I$ variable vectors to get

$$
\begin{bmatrix}
y_{1,t-p} \\
y_{2,t-p} \\
\vdots \\
y_{I,t-p}
\end{bmatrix}
$$

Further, we lag each element in the variable vector $y_{i,t-p}$ by the period $\frac{1}{T_i}$ to get

$$
\begin{bmatrix}
y_{i,t-p-\frac{1}{T_i}} \\
y_{i,t-p-\frac{1+T_i}{T_i}} \\
\vdots \\
y_{i,t-p-\frac{P-1+T_i}{T_i}}
\end{bmatrix}
$$

Doing this for all periods $t - p$ with $p = 0, 1, \ldots, P$ results in $P + 1$ vectors for the $i$-th variable. We stack these $P + 1$ vectors to get

$$
\begin{bmatrix}
y_{i,t-0-\frac{1}{T_i}} \\
y_{i,t-1-\frac{1}{T_i}} \\
\vdots \\
y_{i,t-P-\frac{1}{T_i}}
\end{bmatrix}
$$

\(^4\)In case of the lowest frequency variable, $y_{t-p}$ consists of just one observation.
\( x_{i,t} \) contains all past observations of the \( i \)-th variable until (and including) period \( t-P \). For ease of exposition we set \( P_i = P \) for all variables. Our empirical application presented in Section 3.4 allows \( P_i \) to be different for each variable. Iterating the last two stacking steps for all variables leaves us with \( I \) variable vectors \( x_{1,t}, \ldots, x_{I,t} \). In a last step, we stack these \( I \) variable vectors to get the data vector \(^5\)

\[
\begin{bmatrix}
x_{1,t} \\
\vdots \\
x_{I,t}
\end{bmatrix} \equiv \left[ \begin{bmatrix} x_{1,t} \\ \vdots \\ x_{I,t} \end{bmatrix} \right].
\]

**Stacked vector mixed frequency VAR**

The stacked vectors from the previous subsection build the basis for our stacked vector mixed frequency VAR framework. In particular, we model each element of vector \( y_{i,t} \), namely each \( y_{i,t-1+\tau_i} \), as a regression function of the data vector \( x_t \):

\[
y_{i,t-1+\frac{\tau_i}{T_i}} = a_{i,\tau_i} x_t + \epsilon_{i,t-1+\frac{\tau_i}{T_i}} \quad (3.1)
\]

\[
= \begin{bmatrix} a_{i,\tau_i,1} & \cdots & a_{i,\tau_i,I} \end{bmatrix} \begin{bmatrix} x_{1,t} \\ \vdots \\ x_{I,t} \end{bmatrix} + \epsilon_{i,t-1+\frac{\tau_i}{T_i}}.
\]

\( \epsilon_{i,t-1+\frac{\tau_i}{T_i}} \) is an error term, and \( a_{i,\tau_i,1}, \ldots, a_{i,\tau_i,I} \) are selection vectors defined as

\[
a_{i,\tau_i,j} \equiv \begin{bmatrix} 0_{i,\tau_i,j,1} & \alpha_{i,\tau_i,j} & 0_{i,\tau_i,j,2} \end{bmatrix}_{1 \times (P+1) \cdot T_j} \quad (3.2)
\]

\(^5\)Stacking of lagged variables into a vector \( x_t \) follows Hamilton (1994, p. 292) and Hayashi (2000, p. 397).
which relates period $\tau_i$-observation of the $i$-th variable to all observations of the $j$-th variable with $j = 1, \ldots, I$. The three parts of $a_{i,\tau_i,j}$ are explained in turn.

First, $0_{i,\tau_i,j,1}$ denotes a row vector of zero parameters with length $T_j - \left\lceil \frac{\tau_i}{T_i} T_j \right\rceil$. This vector of zero parameters relates $y_{i,t-1+\tau_i/T_i}$ to those observations of the $j$-th variable which are published simultaneously with or later than $y_{i,t-1+\tau_i/T_i}$, i.e. in periods $t-1+\tau_i/T_i, \ldots, t-1+(T_i-1)/T_i$. The reason for setting these relations to zero is as follows: Any observation which is published simultaneously or later than observation $y_{i,t-1+\tau_i/T_i}$ cannot be used for forecasting observation $y_{i,t-1+\tau_i/T_i}$. Hence, these simultaneous or future observations must be disregarded during the parameter estimation stage. This can be done by discarding these observations, or by setting their parameters to zero.

Second, $\alpha_{i,\tau_i,j}$ denotes a row vector of parameters with length $K_{i,j} \equiv P \cdot T_j + \left\lceil \frac{T_j}{T_i} \right\rceil$:

$$\alpha_{i,\tau_i,j} \equiv [\alpha_{i,\tau_i,j,1} \ldots \alpha_{i,\tau_i,j,K_{i,j}}]. \quad (3.3)$$

This parameter vector relates $y_{i,t-1+\tau_i/T_i}$ to past observations of the $j$-th variable from period $T_j - \left\lceil \frac{\tau_i}{T_i} T_j \right\rceil + 1$ until period $T_j - \left\lceil \frac{T_j}{T_i} \right\rceil + K_{i,j}$. $\alpha_{i,\tau_i,j}$ will be further discussed in the next subsection.

Third, $0_{i,\tau_i,j,2}$ denotes a row vector of zero parameters with length $\left\lceil \frac{T_j}{T_i} \right\rceil - \left\lceil \frac{T_j}{T_i} \right\rceil$. This vector of zero parameters relates $y_{i,t-1+\tau_i/T_i}$ to those past observations of the $j$-th variable which have been published earlier than period $T_j - \left\lceil \frac{T_j}{T_i} \right\rceil + K_{i,j}$. Given the choice of lag length $P$ these observations are too “old” to be considered for forecasting observation $y_{i,t-1+\tau_i/T_i}$. Hence, these observations must be disregarded during the parameter estimation stage. Again, this can be done by discarding these observations, or by setting their parameters to zero.

---

$^6_i = j$ or $i \neq j$.

$^7$The ceiling function $\lceil z \rceil \equiv \min\{n \in \mathbb{Z}|n \geq z\}$, where $z$ is a real number and $n$ is an integer from the set of all integers $\mathbb{Z}$. 

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In a next step, we stack the selection vectors \( a_{i,1,j}, \ldots, a_{i,T_j,j} \) to get

\[
A_{i,j} \equiv \begin{bmatrix} a_{i,T_j,j} \\ \vdots \\ a_{i,1,j} \end{bmatrix}.
\]

\( A_{i,j} \) contains the (zero and alpha) parameters which relate the \( I \) observations of the \( i \)-th variable in period \( t \) to \( x_{j,t} \), i.e. to all observations of the \( j \)-th variable from period \( t - P - 1 \) to period \( t - \frac{1}{T_j} \). Then, we collect all \( I \) times \( I \) matrices \( A_{i,j} \) in the big matrix

\[
A \equiv \begin{bmatrix} A_{1,1} & \ldots & A_{1,I} \\ \vdots & \ddots & \vdots \\ A_{I,1} & \ldots & A_{I,I} \end{bmatrix}.
\]

Iterating Equation (3.2) for all \( T_i \) observations of the \( i \)-th variable in period \( t \), namely for \( y_{i,t-1+1/T_i}, \ldots, y_{i,t-1+T_i/T_i} \), yields \( T_i \) error terms \( \epsilon_{i,t-1+1/T_i}, \ldots, \epsilon_{i,t-1+T_i/T_i} \). We stack these error terms to get

\[
\epsilon_{i,t} \equiv \begin{bmatrix} \epsilon_{i,t-1+1/T_i} \\ \vdots \\ \epsilon_{i,t-1+T_i/T_i} \end{bmatrix}.
\]

Further iterating the above error term stacking for all variables gives \( I \) error term vectors \( \epsilon_{1,t} \ldots \epsilon_{I,t} \). We again stack these error term vectors to get

\[
\epsilon_t \equiv \begin{bmatrix} \epsilon_{1,t} \\ \vdots \\ \epsilon_{I,t} \end{bmatrix}.
\]
The mixed frequency vector autoregressive (VAR) process then generally writes

\[
\begin{bmatrix}
    y_{1,t} \\
    \vdots \\
    y_{I,t}
\end{bmatrix}
= 
\begin{bmatrix}
    A_{1,1} & \ldots & A_{1,I} \\
    \vdots & \ddots & \vdots \\
    A_{I,1} & \ldots & A_{I,I}
\end{bmatrix}
\begin{bmatrix}
    x_{1,t} \\
    \vdots \\
    x_{I,t}
\end{bmatrix}
+ 
\begin{bmatrix}
    \epsilon_{1,t} \\
    \vdots \\
    \epsilon_{I,t}
\end{bmatrix},
\]

or more compactly

\[
y_t = A x_t + \epsilon_t. \tag{3.4}
\]

Unrestricted and Almon frequency VAR

The core building blocks of the big matrix \(A\) are the parameter vectors \(\alpha_{i,\tau_i,j}\). \(\alpha_{i,\tau_i,j}\) has been generally defined in Equation (3.3) and relates \(y_{i,t-1+\tau_i/T_i}\) to all past observations of the \(j\)-th variable from period \(T_j - \left[\frac{T_j}{T_i}\right] + 1\) until period \(T_j - \left[\frac{T_j}{T_i}\right] + K_{i,j}\). We model the elements of \(\alpha_{i,\tau_i,j}\) in two alternative ways.

First, an intuitive strategy is to regard each element of \(\alpha_{i,\tau_i,j}\) as an unrestricted parameter. Equation (3.2) is then a linear regression equation with \(\sum_{j=1}^{I} (T_j - \left[\frac{T_j}{T_i}\right])\) zero restrictions and with \(\sum_{j=1}^{I} K_{i,j}\) unknown unrestricted parameters. The unrestricted parameters in matrix \(A\) of Equation (3.4) can thus be easily estimated row by row via ordinary least squares (OLS). The resulting unrestricted MFVAR is very flexible in terms of parametrization, but gets overparameterized/looses parsimony as the number of employed lags grows. Foroni et al. (2012) originally proposed the unrestricted approach for single equation forecasting models.

Second, following Ghysels and co-authors (e.g. Ghysels et al., 2007, Andreou et al., 2010) one can regard \(\alpha_{i,\tau_i,j}\) as a vector of unknown weights \(\alpha_{i,\tau_i,j,1}, \ldots, \alpha_{i,\tau_i,j,K_{i,j}}\), where each weight is a function of the unknown parameter vector \(\theta_{i,\tau_i,j}\) and of the

\(^{8}\)In case \(j = i\) the relation is between observations of the same variable, in case \(j \neq i\) the relation is between observations of two different variables.
lag index \( k \) with \( k = 1, \ldots, K_{i,j} \). Accordingly, Equation (3.4) can be specified as

\[
y_t = A(\Theta, K)x_t + \epsilon_t,
\]

where \( \Theta \) denotes the space of Almon parameter vectors and \( K \) is the lag index space. We model each weight function as a Almon lag polynomial,

\[
\alpha_{i,\tau_i,j,k} = \alpha_{i,\tau_i,j,k}(\theta_{i,\tau_i,j},k) = \alpha_{i,\tau_i,j,k}(\theta_{i,\tau_i,j,0}, \ldots, \theta_{i,\tau_i,j,Q}, k) = \sum_{q=0}^{Q} \theta_q k^q,
\]

where \( Q \in \mathbb{N} \) denotes the polynomial order.\(^9\)

The Almon lag polynomial combines three features which makes it particularly attractive for a stacked vector mixed frequency VAR. The first feature concerns parameter parsimony: The polynomial order \( Q \) determines the number of \( \theta \)-parameters to be estimated. By setting \( Q \ll K_{i,j} \) the number of parameters to be estimated gets reduced. This prevents parameter proliferation or overfitting even for a long lag length \( K_{i,j} \). The second feature concerns model flexibility: Even when \( Q \) is a small number the form of the function \( \alpha_{i,\tau_i,j,k} \) with respect to \( k \) – i.e. the relative importance of any lagged observation as compared to any other lagged observation – is flexible and depends on the Almon parameter values \( \theta_{i,\tau_i,j} \). Figure 3.1 illustrates this graphically. In practice, \( \theta_{i,\tau_i,j} \) is estimated and, thus, the relative weight of each lagged observation is optimally chosen by the data.

The third feature concerns estimation: Since each Almon lag polynomial weight function \( \alpha_{i,\tau_i,j,k} \) is highly non-linear in its parameters \( \theta_{i,\tau_i,j} \), OLS is not feasible for a direct estimation of the \( \theta \)-parameters in matrix \( A \) in Equation (3.5). Further, since each row of \( A \) contains a multitude of Almon lag polynomial weight functions, \( \alpha_{i,\tau_i,1,1}, \ldots, \alpha_{i,\tau_i,1,K}; \alpha_{i,\tau_i,2,1}, \ldots, \alpha_{i,\tau_i,2,K_{i,j}}, \ldots, \alpha_{i,\tau_i,I,1}, \ldots, \alpha_{i,\tau_i,I,K} \), even non-linear optimization methods reach their limits in practice. Specifically, each

\(^9\)The literature knows yet other weight functions (see e.g. Ghysels et al., 2007)
Figure 3.1: Almon lag polynomial

$y$-axis: Almon lag polynomial weight of order $Q = 2$, $\alpha_{i,\tau, j, k}(\theta_{i,\tau, j, 0}, \theta_{i,\tau, j, 1}, \theta_{i,\tau, j, 2}, k)$, attached to observation $y_{j,t-1+\tau_{i}/T_{i}-k/T_{j}}$ conditional on different values of $\theta_{i,\tau, j, 0}$, $\theta_{i,\tau, j, 1}$ and $\theta_{i,\tau, j, 2}$. See Equation (3.6). $x$-axis: $k = 1, \ldots, 12$ (= first to twelfth lag). Sum of weights normalized to one for ease of exposition.

Row of $A$ contains $\sum_{j=1}^{J} K_{i,j}$ Almon lag polynomial weight functions. Fortunately, as we show in the next subsection, when the weight functions are modeled as Almon lag polynomials, Equation (3.5) can be recast such that OLS estimation of the $\theta$-parameters becomes feasible.

In sum, the use of the Almon lag polynomial brings together three goals which are often in a trade-off position to each other: parsimony in terms of parametrization, model flexibility and feasibility concerning estimation.
Recasting the Almon MFVAR in linear form

The Almon MFVAR model is both parsimonious and flexible. However, the model contains a multitude of highly non-linear terms which makes a direct estimation infeasible. In the following we show how to recast the Almon MFVAR model in a linear equation system which can be easily estimated via OLS.

Equation (3.6) has defined the Almon parameter vector as

$$\theta_{i,\tau_{i},j} \equiv \begin{bmatrix} \theta_{i,\tau_{i},j,0} & \cdots & \theta_{i,\tau_{i},j,Q} \end{bmatrix}. \quad (3.7)$$

We stack the series of Almon parameter vectors $\theta_{i,\tau_{i},1}, \ldots, \theta_{i,\tau_{i},I}$ into

$$\theta_{i,\tau_{i}} \equiv \begin{bmatrix} \theta_{i,\tau_{i},1} & \cdots & \theta_{i,\tau_{i},I} \end{bmatrix}. \quad (3.8)$$

Further, we define a transformation matrix

$$M_{i,\tau_{i},j} \equiv \begin{bmatrix} 0_{i,\tau_{i},j,1} & 1 & 1 & \ldots & 1 & 0_{i,\tau_{i},j,2} \\ 0_{i,\tau_{i},j,1} & 1 & 2 & 3 & \ldots & K_{i,j} & 0_{i,\tau_{i},j,2} \\ 0_{i,\tau_{i},j,1} & 1 & 4 & 9 & \ldots & K_{i,j}^2 & 0_{i,\tau_{i},j,2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0_{i,\tau_{i},j,1} & \cdots & \cdots & \cdots & \cdots & 0_{i,\tau_{i},j,2} \\ 0_{i,\tau_{i},j,1} & \cdots & \cdots & \cdots & \cdots & 0_{i,\tau_{i},j,2} \\ 0_{i,\tau_{i},j,1} & 1 & 2^Q & 3^Q & \ldots & K_{i,j}^Q & 0_{i,\tau_{i},j,2} \end{bmatrix},$$

and condense the series of transformation matrices $M_{i,\tau_{i},1}, \ldots, M_{i,\tau_{i},J}$ into the block matrix

$$M_{i,\tau_{i}} \equiv \text{diag}(M_{i,\tau_{i},1}, \ldots, M_{i,\tau_{i},J}). \quad (3.9)$$
Next, we premultiply the data vector $x_{i,t}$ with the block matrix to get a row vector of transformed data

$$x^*_{i,\tau_{i},t} \equiv M_{i,\tau_{i}} x_t, \quad (3.10)$$

Notably, $M_{i,\tau_{i}}$ and, hence, also $x^*_{i,\tau_{i},t}$ is specific to the subperiod $\tau_{i}$-observation of the $i$-th variable. Using Equation (3.10) and the fact that

$$a_{i,\tau_{i}} = \theta_{i,\tau_{i}} M_{i,\tau_{i}}$$

we can rewrite the basic regression equation (3.2) as

$$y_{i,t-1+\tau_{i} \bar{\tau}_{i}} = \theta_{i,\tau_{i}} x^*_{i,\tau_{i},t} + \epsilon_{i,t-1+\tau_{i} \bar{\tau}_{i}}. \quad (3.11)$$

We stack the Almon parameter vectors $\theta_{i,1}, \ldots, \theta_{i,T_i}$ defined in Equation (3.8) to get

$$\Theta_i \equiv \begin{bmatrix} \theta_{i,T_i} \\ \vdots \\ \theta_{i,1} \end{bmatrix}.$$  

Doing this for each variable index $i = 1, \ldots, I$ yields the series $\Theta_1, \ldots, \Theta_I$ which can be stacked to the big Almon parameter matrix

$$\sum_{i=1}^{I} \Theta_i \equiv \begin{bmatrix} \Theta_1 \\ \vdots \\ \Theta_I \end{bmatrix}. \quad (3.12)$$
Further, upon transposition we stack all transformed data vectors specific to the $i$-th variable in period $t$ starting with the latest subperiod and ending with the earliest subperiod to get

$$X^*_{i,t} \equiv \begin{bmatrix} x^*_{i,T_i,t} \\ \vdots \\ x^*_{i,1,t} \end{bmatrix}.$$  

Iterating the last stacking step for all variables leaves us with $I$ matrices $X^*_1,t, \ldots, X^*_I,t$. We stack these matrices again to get the transformed data matrix

$$X^*_t \equiv \begin{bmatrix} X^*_1,t \\ \vdots \\ X^*_I,t \end{bmatrix}.$$  

(3.13)

Next, we define a selection matrix

$$S_{L \times L \cdot L} \equiv (I_{(L)} * I_{(L)})',$$

where $I_{(L)}$ denotes the identity matrix of size $L \equiv \sum_{i=1}^I T_i$ and where "*" is the (column-wise) Khatri-Rao product. I. e.

$$I * I \equiv [I_l \otimes I_l]_l,$$  

(3.14)

where $l$ denotes the $l$-th column of $I$ with $l = 1, \ldots, L.$\(^{10}\)

As is easily seen, the stacked vector non-linear MFVAR from Equation (3.5) can then be reformulated as

$$y_t = S \text{vec} (\Theta X^*_t) + \epsilon_t.$$  

(3.15)

\(^{10}\)See Khatri & Rao (1968).
Using the fact that for any two matrices $B$ with size $R \times T$ and $C$ with size $U \times V$
\[ \text{vec}(BC) = (C' \otimes I(R)) \text{vec}(B), \]
we further rewrite Equation (3.15) as a linear equations systems
\[ y_t = S(X_i^* \otimes I(L)) \text{vec}(\Theta) + \epsilon_t. \quad (3.16) \]

$S(X_i^* \otimes I(L))$ with size $L \times I \cdot (Q + 1) \cdot L$ builds the right-hand side data matrix and
$\text{vec}(\Theta)$ with size $I \cdot (Q + 1) \cdot L \times 1$ is the parameter vector. Equation (3.16) conforms
to the standard VAR matrix notation in the sense that each element, $y_{i,t-1+\tau_i/T_i}$, in
the left-hand side variable vector, $y_t$, is a function of always the same right-hand side
data matrix, namely of $S(X_i^* \otimes I(L))$. On the other hand, Equation (3.16) deviates
from the standard VAR matrix notation in that the data come in matrix form (but
not in vector form) while the parameters come in vector form (but not in matrix
form).

Importantly, the $\theta$-parameters in Equation (3.16) can be easily estimated row by
row via OLS. As each row of $S(X_i^* \otimes I(L))$ contains only $I \cdot (Q + 1)$ non-zero elements,
a single row can only be used to estimate $I \cdot (Q + 1)$ $\theta$-parameters in $\text{vec}(\Theta)$. Hence,
all rows in Equation (3.16) are needed to fully estimate the $I \cdot (Q + 1) \cdot L$ parameters
in $\text{vec}(\Theta)$. Upon estimation of $\text{vec}(\Theta)$, the $\alpha$-weights in matrix $A$ of the general
stacked vector MFVAR equation (3.4) can be calculated using Equation (3.6).

### 3.3 Data and forecast evaluation setup

We apply the previously developed mixed-frequency VAR model to forecasting the
US economy on the basis of a real-time dataset.\(^{11}\) The real-time data used in this

\(^{11}\)Notwithstanding the limitation to forecasting in this application, the mixed-frequency VAR
model is equally suitable for impulse response analysis.
application comprise quarterly real seasonal adjusted GDP as well as the monthly consumer price index, industrial production and housing starts. Data sources are the Archival Federal Reserve Economic Data (ALFRED) published by the Federal Reserve Bank of St. Louis and the Federal Reserve Bank of Philadelphia Real-Time Data Set for Macroeconomists (RTDSM). In addition, a set of time series that are not subject to data revisions is employed: the ISM indices for manufacturing and supplier delivery times, the S&P 500 stock market index, the 3-month treasury bill yield, the 10-year treasury bond yield, and average weekly hours worked by production and supervisory workers. Table 3.1 provides a data overview. The real-time dataset comprises 383 vintages covering the time frame January 1970 to November 2016. The first vintage starts in January 1970 and includes all data available until the end of January 1985. In order to use a rolling window setup each following vintage is shifted by one month, i.e. the second vintage starts in February 1970 and includes all data available until the end of February 1985, and so on.

We forecast at the end of each month. In a first step, we estimate the model using the first vintage and forecast all variables 1 to 24 months ahead of release. We then re-estimate the model using the second vintage and generate again 1 to 24-months-ahead forecasts, and so on. This procedure results for each variable in a series of \( h \)-months ahead forecasts with \( h = 1, \ldots, 24 \) months ahead of the actual realization of the variable.\textsuperscript{12} Forecast errors are then calculated by subtracting each forecast from its actual realization. Finally, the forecast errors are used to calculate a root mean square forecast error (RMSFE) for each variable and each forecast horizon \( h \).

\textsuperscript{12}Monthly (inter-quarterly) forecasts are also generated for quarterly GDP.
Table 3.1: Data overview

<table>
<thead>
<tr>
<th>VARIABLE NAME</th>
<th>FREQUENCY</th>
<th>REAL TIME</th>
<th>SOURCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real seasonal adjusted GDP</td>
<td>quarterly</td>
<td>yes</td>
<td>RTDSM</td>
</tr>
<tr>
<td>Industrial production index</td>
<td>monthly</td>
<td>yes</td>
<td>RTDSM</td>
</tr>
<tr>
<td>Consumer price index</td>
<td>monthly</td>
<td>yes</td>
<td>ALFRED</td>
</tr>
<tr>
<td>Housing starts</td>
<td>monthly</td>
<td>yes</td>
<td>RTDSM</td>
</tr>
<tr>
<td>ISM manufacturing index (= ISM total)</td>
<td>monthly</td>
<td>no</td>
<td>Institute for Supply Management</td>
</tr>
<tr>
<td>ISM supplier delivery times index</td>
<td>monthly</td>
<td>no</td>
<td>Institute for Supply Management</td>
</tr>
<tr>
<td>Production and supervisory workers: Average weekly hours</td>
<td>monthly</td>
<td>no</td>
<td>US Bureau of Labor Statistics</td>
</tr>
<tr>
<td>S&amp;P 500 stock market index</td>
<td>monthly</td>
<td>no</td>
<td>Thomson Reuters</td>
</tr>
<tr>
<td>3-month treasury bill yield</td>
<td>monthly</td>
<td>no</td>
<td>Thomson Reuters</td>
</tr>
<tr>
<td>10-year treasury bond yield</td>
<td>monthly</td>
<td>no</td>
<td>Thomson Reuters</td>
</tr>
</tbody>
</table>

- Note on the consumer price index: For the following periods two vintages were available: 2000M9, 2005M2, 2006M2, 2007M2, 2008M2, 2009M2, 2010M2, 2011M2 and 2012M2. For these cases we use the later vintage publications.
- Note on housing starts: No observations were available for the first publication of 1995M11 and 1995M12. We make the assumption that in these cases no revision took place and use the same values as in the second publication.
The choice of actual realizations is a delicate issue in real time data contexts (cf. the discussions in Croushore, 2006, Romer & Romer, 2000, and Sims, 2002). There have been several benchmark revisions in the time series we use; the latest revision for GDP occurred in mid-2014 and included a substantial redefinition of gross fixed capital formation which accounts for 20 percent of GDP. A forecaster in, e.g., 1985 could not have predicted such a definition change. Therefore, we follow Romer & Romer (2000), Faust & Wright (2009) and Carriero et al. (2015) and use the second release of each variable as the actual realization when calculating the forecast error. As a robustness test we also calculate the forecast errors using either the first release or the third release of the variables leaving the results virtually unchanged.

In order to reproduce the available information a forecaster would have had at each forecast date, i.e. at the end of each month $m$, we need to take differing publication lags — so called ragged edges — into account. For most monthly variables the (first) release for each month is not available directly at the end of that month, but is published with a time lag of up to one month. We call these variables lag variables. Hence, when forecasting at the end of a month $m$ the aforementioned releases cannot be used for forecasting but must be backcasted themselves using all information available until the end of $m$. Only the S&P 500 index, the 10-year treasury bond yield and the 3-month treasury bill yield are available directly at the end of each month and, hence, can be used for back-, now- and forecasting. The latter three variables are released daily and are never revised. As stated above, we time-aggregate the variables to monthly frequency. Equally, for the quarterly variable, namely GDP, the (first) release for each quarter $q$ is published with a time lag of up to one month. Consequently, when forecasting at the end of a quarter $q$ the releases for quarter $q$ cannot be used for forecasting but must be backcasted using all information available until the end of $q$.

The forecast performance of the mixed-frequency VAR model is always measured in terms of forecast error reduction relative to a benchmark model. Specifically, the
forecast performance for the $i$-th variable at forecast horizon $h$ is measured by the relative RMSFE ratio, i.e. the difference between the RMSFE of the mixed-frequency VAR model and the RMSFE of a benchmark model in percent of the RMSFE of the benchmark model:

$$100 \times \left( \frac{RMSFE_{i,h}^{MFVAR} - RMSFE_{i,h}^{BM}}{RMSFE_{i,h}^{BM}} \right).$$

The more negative the relative RMSFE ratio is, the better performs the mixed-frequency VAR model relative to the benchmark model in terms of predictive power.

In addition to the RMSFE comparisons the Giacomini & White (2006, ch. 3.4) test of unconditional equal predictive ability is employed to test whether the forecasts stemming from the mixed-frequency VAR model can be considered as significantly more accurate than the forecasts from the benchmark model. This is indicated in the figures in the results section. Solid lines show significantly different forecast errors while dashed lines indicate insignificant differences.

### 3.4 Empirical results

The following sections present the results of the forecast evaluation outlined in Section 3.3. We compare our MFVARs with a standard quarterly frequency VAR (QFVAR) and a Bayesian VAR (BVAR) by Koop & Korobilis (2010), where all higher than quarterly frequency series are time-aggregated to quarterly frequency. We look at four alternative specification setups: few variables and few lags, few variables and many lags, higher number of variables and few lags, and higher number of variables and many lags. Competing MFVARs and QFVARs always have the same amount of lagged information available. Thus, differences in forecast performance between competing models solely accrue from two sources: how flexible – or parsimonious – the models are in terms of parametrization, and whether they can incorporate higher frequency data updates (new releases or revisions).
**3-variable MFVAR with 3-month memory**

In a first step it might be interesting to see whether our MFVAR models improve forecasts upon a standard VAR when model specifications are kept minimal. Following common practice in macroeconomics our minimum VAR specification includes three variables only: GDP growth, consumer price inflation and a (3-month) short-term interest rate (treasury bill rate).

Figure 3.2 presents results for an Almon MFVAR of order 1 and an unrestricted MFVAR both with one quarterly GDP growth lag, three monthly inflation lags and three monthly interest rate lags.\(^{13}\) The benchmark model is a quarterly frequency VAR (QFVAR) with one quarterly lag of GDP growth, inflation and interest rate, respectively. As regards forecasting GDP growth, the two MFVAR specifications reduce the RMSFE significantly by more than 30 percent compared to the QFVAR for forecast horizons of one to six months. The RMSFE improvement gradually declines as the forecast horizon increases, but is still substantial for longer horizons: One year before publication of GDP the MFVAR forecasts are still more than 20 percent better than the QFVAR forecasts. For forecast horizons of 22 months or longer the MFVARs yield no improvement over the QFVAR anymore. The zig-zag pattern of the relative RMSFE is due to the mixed frequency structure of our setup: the MFVAR gets new information every month whereas the data of the QFVAR are only updated every third month when new quarterly information is available.

For inflation, the Almon MFVAR and the unrestricted MFVAR yield very high RMSFE reductions for short forecast horizons (more than 65 percent for months 1 to 3). On the other hand, RMSFE improvements quickly vanish as the forecast horizon grows. And for forecast horizons of one year onwards the relative change in the RMSFE decreases again. The reason for this pattern is that the QFVAR forecast errors initially decrease with an increase in the forecast horizon, but then increase

---

\(^{13}\)When three lags are employed an Almon MFVAR of order 2 delivers identical results as an unrestricted MFVAR.
again from a forecast horizon of one year onwards. In contrast, the forecast errors of the MFVAR slowly and steadily increase as the forecast horizon becomes bigger (which is what one would expect). When it comes to forecasting the interest rate, the Almon MFVAR and the unrestricted MFVAR improve forecasts substantially and significantly for a forecast horizon of up to one year compared to the QFVAR. The RMSFE reductions reach more than the 75 percent for horizons of one to three months. Forecast improvements gradually decline with an increase in the forecast horizon. All previous findings remain robust when we iterate the forecast evaluation for the first or third GDP estimate instead of the second estimate (see Section 3.3).

Figure 3.2: **MFVAR with 3 variables and 3 months of lagged information**

$x$-axis: Forecast horizon, i.e. months prior to GDP growth/inflation/interest rate release. $y$-axis: relative RMSFE ratio.

In contrast to our MFVARs, a QFVAR cannot use inter-quarterly data updates. So during a quarter, a MFVAR has more information than a QFVAR. Thus, it is necessary to differentiate between the ability to incorporate inter-quarterly data updates and the ability to have a flexible weighting scheme for higher-frequency data. For this we abstract from ragged edges for now and only look at those vintages when our mixed-frequency VAR and the QFVAR have the same amount of information, namely the first month of each quarter, when all monthly variables that exhibit a publication lag are available. For this exercise we use the unrestricted MFVAR specification presented above, which has the most flexibility. Figure 3.3 shows that a lot of the improvement in predictive ability is due to the flexible weighting scheme.
that our MFVAR can employ. This is especially true for high-frequency variables like in this case the short-term interest rate and the inflation rate. But also for GDP results are slightly better. This flexible weighting scheme is likely to be even more important when more high-frequency business cycle indicators are used.

Figure 3.3: Full model and model with only flexible weighting scheme

\[ x \text{-axis: Forecast horizon, i.e. months prior to GDP growth/inflation/interest rate release.} \]
\[ y \text{-axis: relative RMSFE ratio.} \]

3-variable MFVAR with 6- or 12-month memory

VAR models of higher order than the ones presented in the previous section arguably achieve better forecasting performance for longer forecast horizons. Thus, it is important to know whether MFVARs still improve forecasts upon a standard VAR for specifications with longer time series memory. Figure 3.4 shows the results for an Almon MFVAR of order 1 and an unrestricted MFVAR both with two quarterly GDP growth lags, six monthly inflation lags and six monthly interest rate lags. The benchmark model here is a QFVAR with two quarterly lags of GDP growth, inflation and interest rate, respectively, where two lags are chosen in order to provide each competing model with the same amount of lagged information (see Section 3.3).

To allow for even longer memory, Figure 3.5 depicts results for an Almon MFVAR of order 1 and an unrestricted MFVAR both with four quarterly GDP growth lags,
twelve monthly inflation lags and twelve monthly interest rate lags. In accordance
with the above reasoning about the appropriate information set, the benchmark
model now is a QFVAR with four quarterly lags of GDP growth, inflation and inter-
est rate, respectively. Generally, the RMSFE patterns in Figures 3.4 and 3.5
strongly resemble the patterns in Figure 3.2: the two MFVAR specifications yield
substantial improvements in forecast accuracy upon the QFVAR for GDP growth,
inflation as well as the interest rate. That said, for the 12-month memory specifi-
cation the unrestricted MFVAR performs clearly worse than the Almon MFVAR when
it comes to forecasting GDP growth. When the forecast horizon exceeds 13 months
the unrestricted MFVAR forecasts are even less accurate, albeit insignificantly, than
the QFVAR forecasts. This finding is not surprising: unrestricted specifications
easily become overparametrized when the lag order grows. As a consequence, the
unrestricted MFVAR is less suitable for – and is actually not meant for – forecasting
with longer lags.

We iterated the analysis with an Almon MFVAR of order 2 (not shown in the
figures). The RMSFE patterns very closely resemble the patterns of the order 1
Almon MFVAR.
Figure 3.4: MFVAR with 3 variables and 6 months of lagged information

Figure 3.5: MFVAR with 3 variables and 12 months of lagged information

x-axis: Forecast horizon, i.e. months prior to GDP growth/inflation/interest rate release. y-axis: relative RMSFE ratio.

6-variable MFVAR with 3-month memory

The previous minimum scale VAR specifications including only GDP growth, inflation and an short-term interest rate provide us with first evidence on the potential of MFVAR models for forecasting. However, in order to improve predictive accuracy forecasters usually employ VAR models with more than just the aforementioned variables. So, do MFVARs still improve forecasts upon standard VARs when more variables are included? To answer this question we include three additional variables which are commonly considered to be helpful for now- or forecasting GDP growth: industrial production in month-on-month growth rates, housing starts in
year-on-year growth rates and the Standard & Poor’s 500 stock market index in month-on-month growth rates.\textsuperscript{14}

Figure 3.6 depicts results for an Almon MFVAR of order 1 and an unrestricted MFVAR both with one quarterly GDP growth lag and three monthly lags of the five monthly variables. The benchmark model here is a QFVAR with one quarterly lag of each of the aforementioned six variables. Thus, as before each competing model has the same amount of lagged information such that differences in forecast performance cannot accrue from differences in the (lagged) information set. The RMSFE patterns in Figure 3.6 strongly resemble the patterns presented in the previous subsections: the two MFVAR specifications substantially improve predictive accuracy upon the QFVAR for GDP growth, inflation and the interest rate.

Figure 3.6: MFVAR with 6 variables and 3 months of lagged information

- **x-axis**: Forecast horizon, i.e. months prior to GDP growth/inflation/interest rate release.
- **y-axis**: Relative RMSFE ratio.

\textsuperscript{14}Indeed, we find that a small scale MFVAR (with industrial production, housing starts, S&P 500, GDP growth, inflation and the interest rate) improves forecast accuracy upon a minimum MFVAR (with only the latter three variables). Equally, the corresponding small scale benchmark QFVAR performs better than the corresponding minimum scale benchmark QFVAR. A rigorous variable selection procedure might deliver small scale specifications with still greater forecasting ability. That said, it is not the goal of this paper to find the best model specification. Rather, we compare MFVARs and standard VARs for several sensible, yet alternative model specifications in order to see whether MFVARs robustly outperform standard VARs.
6-variable MFVAR with 6- or 12-month memory

The previous section has shown that small scale MFVARs substantially improve predictive accuracy upon a standard small scale VAR when only very short time series memory is taken into account (one quarterly lag or three monthly lags). Is this gain robust to allowing for longer time series memory (which increases the risk of overparametrization)?

Figure 3.7 (or Figure 3.8) shows the results for an Almon MFVAR of order 1 with two (or four) quarterly GDP growth lags and six (or twelve) monthly lags of inflation, the short-term interest rate, industrial production, housing starts and the S&P 500 stock market index. The benchmark model is a QFVAR with two (or four) quarterly lags for each of the aforementioned six variables such that each competing model has again the same amount of lagged information (see Section 3.3). The Almon MFVAR still largely outperforms the benchmark QFVAR in terms of forecast accuracy at least for shorter horizons. A comparison between the RMSFE improvements resulting from the longer memory Almon MFVARs in Figures 3.7 and 3.8 with the RMSFE improvements from the 3-month memory Almon MFVAR in Figure 3.6 yields the following differences: Regarding GDP growth, the forecast improvement vanishes earlier (at a forecast horizon of 16 instead of 19 months) and turns into a deterioration for higher forecast horizons. In contrast, for inflation and the interest rate, the longer horizon forecast improvement is substantially larger.

The predictive accuracy of the unrestricted MFVAR substantially deteriorates for longer memory small scale specifications. Figure 3.7 shows that for a specification with two quarterly and six monthly lags the unrestricted MFVAR is generally outperformed by the Almon MFVAR when it comes to forecasting GDP growth, inflation or the short-term interest rate (an exception being the very short-term forecasts for inflation and the interest rate). For the specification with four quarterly lags and twelve monthly lags the forecast performance of the unrestricted MFVAR is so poor that we do not show it in Figure 3.8. The unrestricted MFVAR does not help for – and is actually not meant for – forecasting with longer memory because
of overparametrization. In contrast, the Almon lag polynomial keeps models parsimonious despite long lags.

Again, iterating the analysis with an Almon MFVAR of order 2 yields RMSFE patterns that closely resemble the patterns of the order 1 Almon MFVAR (not shown in the figures).

Figure 3.7: MFVAR with 6 variables and 6 months of lagged information

Figure 3.8: MFVAR with 6 variables and 12 months of lagged information

\( x \)-axis: Forecast horizon, i.e. months prior to GDP growth/inflation/interest rate release. \( y \)-axis: relative RMSFE ratio.

12-variable MFVAR with 3-month memory

Using an Almon lag polynomial in a MFVAR setup does not only give an advantage when using a model with longer memory but also when using a bigger amount of
variables, both when short or long memories are considered. To show this we increase our variable set by six additional variables, namely the (10-year) long-term interest rate (treasury bond yield), hours worked by production and supervisory workers in 3-month growth rates, the ISM index for manufacturing (= ISM total) in levels, the ISM index for supplier delivery times both in levels and year-on-year growth rates and the month-on-month growth in housing starts. In order to improve precision of the parameter estimates we increase the estimation sample by 5 years. The forecast comparison now starts in January 1990 only instead of January 1985.

Figure 3.9 depicts results for an Almon MFVAR of order 1 and an unrestricted MFVAR both with one quarterly GDP growth lag and three monthly lags of the eleven monthly variables. The benchmark model here is a QFVAR with one quarterly lag of each of the aforementioned twelve variables giving again each competing model the same amount of lagged information. The two MFVAR specifications substantially improve predictive accuracy upon the QFVAR for GDP growth, inflation and the interest rate over all forecast horizon.

Figure 3.9: MFVAR with 12 variables and 3 months of lagged information

$x$-axis: Forecast horizon, i.e. months prior to GDP growth/inflation/interest rate release. $y$-axis: relative RMSFE ratio.
12-variable MFVAR with 6- or 12-month memory

As shown in the previous section the 12-variable unrestricted and Almon MFVARs deliver a forecast improvement over a 12-variable QFVAR when considering only a short memory of 1 quarter (3 months). Does this result hold when taking into account longer time series memories? Figure 3.10 (3.11) shows the relative RMSFE changes of the Almon MFVAR of order 1 with two (four) quarterly lags of GDP growth and six (twelve) monthly lags of the aforementioned eleven variables as compared to a QFVAR with two (four) quarterly lags for each of the twelve variables. The improvement in forecast accuracy of the Almon MFVAR relative to the QFVAR benchmark is even higher than for the short memory specification shown in Figure 3.9. Apparently, the Almon MFVAR is rather robust against overparametrization, while the QFVAR tends to get overparametrized with both a large number of variables and a large number of lags. Not surprisingly, the unrestricted MFVAR suffers even more from overparametrization. We do not show the unrestricted MFVAR in Figures 3.10 and 3.11 because of its very poor relative forecast performance.

Figure 3.10: MFVAR with 12 variables and 6 months of lagged information

x-axis: Forecast horizon, i.e. months prior to GDP growth/inflation/interest rate release. y-axis: relative RMSFE ratio.

12-variable MFVAR in comparison to quarterly Bayesian VAR

In the previous sections we have shown that our MFVAR significantly outperforms a standard QFVAR in terms of forecast accuracy. While this is the case for the
Almon MFVAR for all specifications, the unrestricted MFVAR only outperforms the QFVAR for small specifications with few variables and lags. The comparison of the Almon MFVAR with the QFVAR might be considered as unfair as our model uses the Almon lag polynomial to reduce the number of parameters while the QFVAR possesses no shrinkage. For robustness we compare the Almon MFVAR with the Bayesian quarterly-frequency VAR (BVAR) from Koop & Korobilis (2010). For shrinkage we use a Minnesota prior.

We find that the shrinkage provided by the Minnesota prior is helpful in terms of producing better forecasts. Still, the Almon MFVAR generally outperforms the BVAR benchmark model in terms of forecast accuracy. For brevity, we only show the results for the specification with 12 variables and 6 lags (Figure 3.12).\textsuperscript{15} Forecasts for GDP by the MFVAR are still better than forecasts by the BVAR benchmark, but only between 15 to 20 percent in contrast to around 55 percent in case of the QFVAR benchmark. For the short-term interest rate and the inflation rate the Almon MFVAR outperforms the BVAR, especially for very short forecast horizons.

\textsuperscript{15}Further results are available upon request.
The previous finding that the Minnesota prior improves forecasts of the quarterly frequency VAR indicates that it could be useful to enhance our MFVAR with Bayesian prior and shrinkage techniques. For mixed-frequencies it is common to use the latent variable mixed-frequency VAR models cited in the introduction. While this approach is straightforward and transparent from a theoretical perspective, it shifts the burden to the model estimation. Estimation of state space systems with multiple latent time series processes is computationally cumbersome and not feasible for higher frequencies. In contrast, the linearly transformed stacked vector MFVAR approach offers an analytical closed form solution and the equation-by-equation estimation with OLS can be easily augmented with Bayesian prior and shrinkage techniques. We leave these steps for further research.

Figure 3.12: MFVAR vs BVAR with 12 variables and 6 months of lagged information

![Graphs of GDP growth, 2-month treasury bill yield, and Consumer price inflation](image)

- **x-axis:** Forecast horizon, i.e. months prior to GDP growth/inflation/interest rate release.
- **y-axis:** Relative RMSFE ratio.

### 3.5 Conclusion

VAR models are widely used for macroeconomic forecasting and policy analysis (e.g. Stock & Watson, 2001 and Karlsson, 2013). An initial challenge for VARs is that the variables they include must have the same frequency, whereas macroeconomic time series usually come at different frequencies. For instance, GDP is published
every quarter, inflation is a monthly variable and interest rates come at a daily or even higher frequency. The traditional solution is to time-aggregate all variables to a common frequency. A VAR including GDP, inflation and an interest rates series will then be a quarterly frequency VAR. However, the time-aggregation comes with inconveniences: First, higher frequency data releases can only be taken into account with a delay. For instance, a quarterly frequency VAR can only consider inter-quarterly inflation or other indicator releases after the end of a quarter. Second, the time aggregation implies a peculiar constraint on the parameters attached to higher frequency variables which is suboptimal. We find that a forecast performance can be improved by using flexible parameters for the higher-frequency variables.

Against this backdrop, a number of authors have developed VAR models which can deal with variables of differing frequencies. Zadrozny (1988, 1990), Mariano & Murasawa (2010), Kuzin et al. (2011), Chiu et al. (2012), Bai et al. (2013) and Schorfheide & Song (2015) propose mixed frequency VAR (MFVAR) models using a state space approach. Ghysels (2012a) develops a MFVAR using a stacked vector approach (cf. applications in Francis et al., 2011 and Foroni et al., 2015a). Both approaches have their merits and promises (Kuzin et al., 2011, Bai et al., 2013 and Foroni & Marcellino, 2014). We make the following contributions to this emerging research on MFVARs. First, we propose a general and yet tractable stacked vector MFVAR framework. Previous expositions are limited to VARs with only two different frequencies for reasons of tractability. Second, we augment the stacked vector MFVAR with a non-linear Almon lag polynomial scheme (Almon, 1965) which is designed to prevent overparametrization even with long lag lengths while, at the same time, keeping the VAR model flexible. In turn, we show how to transform the resulting stacked vector non-linear MFVAR into a linear equation system which can be easily estimated via OLS. Due to the linear transformation the stacked vector MFVAR becomes feasible for estimation with multiple variables. Previous stacked vector MFVARs were limited to only few variables. The reason is that, in absence of a linear transformation, the non-linear MFVARs had to be estimated directly which

See a discussion in Judge et al. (1985).
is cumbersome, if not infeasible, in a VAR context involving multiple variables (provided no auxiliary restrictions).

Using quarterly and higher frequency US real-time data we test the forecast performance of our MFVAR against a quarterly frequency VAR for various different specifications (three, six or twelve variables, three, six or twelve months of lagged information). The MFVAR yields root mean squared forecast error reductions of 30 to 50 percent for forecast horizons up to six months and of about 20 percent for a forecast horizon of one year. For inflation and interest rates, forecast error reductions are even bigger. According to our results, augmentation of a stacked vector MFVAR with an Almon lag polynomial scheme has a distinct advantage for specifications with longer lag structures. While a MFVAR with fully unrestricted parameters still yields considerable forecast improvements over a standard QFVAR when using few lags, these improvements vanish almost completely for longer lags. An Almon augmented MFVAR instead still yields high improvements over a QF-VAR for longer lags.

The VAR specifications in our empirical application are intentionally kept simple. For instance, each variable’s own lags are kept as flexible concerning polynomial order as its other variables’ lags. An optimal model specification procedure with respect to polynomial flexibility and lag length will boost the predictive power of the MFVAR. Further, the linearly transformed stacked vector MFVAR could be estimated using Bayesian methods instead of OLS. This would allow to employ Bayesian VAR specifications with priors that are known to deliver a better forecast performance than non-Bayesian or flat prior Bayesian VAR specifications (see e.g. Giannone et al., 2015). We leave these steps for future research.
Chapter 4

Taming volatile high frequency data with long lag structure: An optimal filtering approach

4.1 Introduction

Policy makers need up-to-date information on the state of the economy in order to timely implement policy actions. Often, publication lags complicate this task, calling for inclusion of readily available high frequency data into forecasting models. Bridge models and Mixed Data Sampling (MIDAS) models tackle this issue by combining high-frequency and low-frequency data for estimation and forecasting.

In this application we investigate how standard MIDAS, unrestricted MIDAS and bridge equations perform if the required lag length of the high frequency variable is very long and if the data are volatile and noisy. Long lags pose problems for unrestricted MIDAS (U-MIDAS) models and bridge equations as many parameters have to be estimated (e.g. Ghysels et al. (2004)). Further, volatile and noisy data challenge all of the aforementioned model approaches: Restricted MIDAS runs into

\[^{1}\text{This chapter is based on Drehsel \\& Neuwirth (2016)}\]
the problem of forming a proper weighting scheme. Bridge equations suffer from the noise introduced into the autoregressive forecast of the high-frequency variable.

We contribute to the literature on mixed-frequency data models by proposing a filtering approach for bridge equations to circumvent the aforementioned problems. Specifically, we augment the bridge equations approach with a Bayesian beta filter which is designed to extract all informative signals from the data and to handle very long lags while keeping parsimony. We test our approach compared to standard bridging with unfiltered data as well as bridging with smoothed data by using established filtering techniques.

We apply our approach to forecasting Swiss construction investments (low frequency) with construction permits (high frequency) using a real-time data set from 1993 to 2014. The real-time nature of the data allows us to account for data revisions and ragged edges. We find that our Bayesian filtering approach for bridge models clearly beats U-MIDAS, standard MIDAS, traditional bridge equations and autoregressive (AR) models in terms of out-of-sample performance. This result holds irrespectively of whether the four benchmark approaches employ unfiltered data or data which are smoothed with standard techniques.

The paper proceeds as follows: Section 4.2 gives an overview on the relevant literature. In section 4.3 the used data are explained. The formal model is presented in section 4.4. In section 4.5 the real-time estimates are analysed. The out-of-sample forecasting exercise and the results are presented in section 4.6. Section 4.7 concludes.

4.2 Literature review

The problem of how to incorporate data with different frequency sampling in econometric models has been addressed in the literature in the last decade. A large number of studies have been published looking at the benefits of employing both
high and low frequency data simultaneously in the context of single-equation approaches. One of those are bridge equations, which have been used for quite some time and are common in policy organizations due to their simple method and transparency. The general idea behind bridge equations is to explain a low-frequency variable by time-aggregated low-frequency lags of a high frequency variable. First, forecasts of the high frequency variable are generated by using an additional model, normally an autoregressive process, which are then time-aggregated to the lower frequency. The estimation of both equations can be easily done by ordinary least squares (OLS). Forecasts are then done iteratively by using the previously obtained forecasts. Early applications of bridge equations in the literature can be found for example in Ingenito & Trehan (1996) or Baffigi et al. (2004) as well at central banks like ECB (2008) or Bundesbank (2013).

Another single equation approach to handle time series with different frequencies that is also able to address the problem that arises when accounting a long lag structure is mixed data sampling (MIDAS) proposed by Ghysels et al. (2004), building on Almon (1965). In this approach the high-frequency variable is not time-aggregated but directly related to the low-frequency variable. As this can lead to a high number of parameters to be estimated lag polynomials can be used to decrease the necessary number of parameters and then be estimated by non-linear least squares (Ghysels et al. (2007)). Early applications of this method were mostly with financial data where sampling differences are quite big when using daily data, for example in Ghysels et al. (2006). More recently MIDAS has also been used on macroeconomic data for example in Clements & Galvão (2008) and Clements & Galvão (2009a) or Armesto et al. (2010) and Andreou et al. (2011). More recently Foroni et al. (2015b) have shown, that if differences in frequencies are small, for instance for a mixture of quarterly and monthly data, an unrestricted MIDAS setup (U-MIDAS) is equivalent or even superior compared with standard MIDAS setups. An unrestricted MIDAS setup requires less computational and modelling efforts compared with standard MIDAS setups.
Schumacher (2014a) proposed an iterative MIDAS (MIDAS-IT) as a combination between those two approaches. It differs from bridge equations by using the MIDAS weighting scheme on the right-hand side of the equation instead of a time-aggregated high frequency variable. The high frequency variable has still to be forecasted by using a separate model. The difference to the MIDAS approach is the iterative forecasting method instead of a direct forecast. In his application he finds no systematic advantage of any of the three methods proposed methods.

A not finally clarified question when handling volatile high-frequency data in mixed frequency models is if MIDAS models or bridge equations could benefit from the inclusion of filtering techniques to tackle volatile high-frequency data. Established methods for dealing with volatile data are for instance a one-sided simple moving average or the Hodrick-Prescott-Filter (Hodrick & Prescott (1997)). The decomposition of the data into trend and cyclical components is one of the workhorse filters in economics, but especially for forecasting it leads to some problems due to being a two-sided filter (Baxter & King (1999)), but some workarounds have been suggested in the literature, mainly extending the current edge of the data with forecasts (European Commission (1995)). This filtered data can be used in the standard mixed-frequency models as a benchmark for comparison with more advanced filtering techniques (in this application a Bayesian beta filtering approach).
4.3 Data

Construction permits issued by municipalities offer information about upcoming construction activity as well as its volume. Since most construction activity in Switzerland requires a permit, these permissions can be utilized to forecast Swiss construction investments.\(^2\)

In our analysis we use quarterly nominal, non-seasonal adjusted construction investment as low frequency series and monthly construction permits as high frequency series. For the data on construction investment volumes the sample range from Q1 1993 to Q4 2014 and is taken from the Swiss State Secretariat for Economic Affairs (SECO).\(^3\) The data on construction permits range from January 1993 to December 2014 and is available from Dokumedia Baublatt (www.doku.ch). Each permit contains information on the expected construction volume measured in Swiss Francs. The series used in our analysis is the sum of nominal construction permits approved by municipal authorities within Switzerland in the respective month. As can be seen in Figure 4.1 the data on construction permits are highly volatile with many spikes. These spikes are caused by large construction permits projects which drive up the permit series in a month by several hundred million CHF and drop out the next month again.

The data for construction investment in Switzerland are revised sometimes considerably, as can be seen in Figure 4.2. Thus, we conduct a real-time forecasting...
exercise for testing the forecast performance of our model. For this we construct a real-time data set for vintages starting in January 2005 until December 2014 taking both into account the state of revisions as well as the publication lag of both time series. Construction permits are normally available within one month while the publication lag for construction investment is 3 months. In our first vintage for January 2005 we would thus be able to use the construction permits up to December 2004 as well as construction investment until the third quarter 2004.

\footnote{Quarterly investment data are published by SECO at the beginning of March, June, September, December. Construction permission data are published by Documedia Baublatt on the 1st every month.}
4.4 A beta filter approach for forecasting

4.4.1 Formal model description

In order to use a volatile high frequency variable such as construction permits to forecast a low frequency variable such as construction investments a couple of latent variables have to be estimated: The way how a single permit is mapped into a construction investment is unknown. Volumes will not be spent entirely at the initiation of the construction project - the volumes will be spread over an extended time frame as it takes several months and up to many years to finish a construction project. Furthermore, the relationship between independent variable (permit volumes) and dependent variable (construction investment) is unknown because a)
permit holders are not required to realize their permits and to build and b) most, but not all construction activity require a permit. Therefore construction permits are only a rough indicator of later realized construction investments.

The combination of unknown distributions and parameters poses problems. The estimation of distributions becomes computationally feasible if one is willing to select distributions out of the set of known statistical distributions. Such distributions are characterized by their moments which can be chosen in such a fashion that the overall fit of model and data is optimized. By using the beta distribution, the estimation strategy reduces itself to retrieve these moments and is made feasible by reducing the parameter space. This is represented by the weighting equation:

\[
\tilde{X}_{HF_i} = \sum_{k=0}^{n-1} B(k/n; p, q) \tilde{L}_{HF}^k X_{HF_i} \tag{4.1}
\]

where \(X_{HF_i}\) is a monthly high-frequency variable, \(\tilde{L}_{HF}^k\) a lag operator, \(\tilde{X}_{HF}\) the redistributed high-frequency variable and \(n\) the number of desired lags. \(B\) represents a beta distribution, depending on the number of discrete density steps \(k\) and the shape parameters \(p, q\). The Beta distribution has the convenient property that values cannot be negative (which would be implausible in terms of construction volumes) and can be written as follows:

\[
B(k/n; p, q) = \frac{1}{B((k+1)/n, k/n; p, q)} \left( x - \frac{k+1}{n} \right)^{p-1} \left( \frac{k}{n} - x \right)^{q-1} \tag{4.2}
\]

Outside of the interval \(x \in [0, 1]\) the function values of \(f(x)\) are set to zero. Shape parameters \(p, q\) are strictly positive \(> 0\). The term \(B \left( \frac{k+1}{n}, \frac{k}{n}, p, q \right)\) indicates a restricted Beta function with the upper and lower boundaries \(\frac{k+1}{n}\) and \(\frac{k}{n}\):

\[
B \left( \frac{k+1}{n}, \frac{k}{n}, p, q \right) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \left( \frac{k}{n} - \frac{k+1}{n} \right)^{p+q+1} \tag{4.3}
\]
where $\Gamma$ represents a Gamma function:

$$\Gamma(z) = \int_0^\infty u^{z-1}e^{-u}du$$

(4.4)

The redistributed high-frequency variable $\tilde{X}_{HF}$ is then time-aggregated to the lower frequency and can then be plugged into the standard bridge equation as $\tilde{X}_{HF}$:

$$\Delta Y_t = c + \sum_{i=1}^{p} \beta_i L^i \Delta Y_t + \gamma \Delta \tilde{X}_{HF} + \epsilon_t.$$  

(4.5)

$\Delta Y_t$ are observed quarterly year-over-year construction investment growth rates, $c$ is a constant, $p$ is the desired lag length of the autoregressive term with the coefficients $\beta_i$ to be estimated, $L^i$ is a lag operator, $\gamma$ is the coefficient associated to the redistributed construction permit series and $\tilde{X}_{HF}$ are the time-aggregated quarterly redistributed construction permits at time $t$ which are used as year-on-year growth rates, indicated by $\Delta$.

4.4.2 Estimation strategy

By utilizing a Gibbs sampler the bridge equation parameters $\psi = (c, \beta_1, ..., \beta_p, \gamma)$ are drawn from a Normal-Gamma distribution given the Beta distribution shape parameters $p, q$. Proposals for $p, q$ are drawn in a Metropolis-within-Gibbs sampler step. Random walk candidate draws for $\theta = (p, q)$ are generated by

$\text{A Bayesian Metropolis-within-Gibbs sampler setup for an estimation of a MIDAS regression has for instance been employed by Ghysels (2012b), Ghysels & Owyang (2011) and Rodríguez & Puggioni (2010). In contrast to Rodríguez & Puggioni (2010), we do not impose prior values on the lag structure of beta distributions. In contrast to Ghysels (2012b), we employ a Random-Walk Metropolis Hastings step, while they draw from candidate densities utilizing importance sampling. While Rodríguez & Puggioni (2010) employ Bayesian model selection to identify the model fitting the data best based on marginal likelihoods, we consider the joint distribution of prior and likelihood over all models visited by the algorithm to find the expected mean parameters values fitting the data.}$
\[ \theta^* = \theta^{(s-1)} + z \quad (4.6) \]

with \( s \) being the number of the current draw, \( s - 1 \) indicating the previous accepted draw, and \( \theta^* \) the new draw. Innovations \( z \) are determined by drawing random numbers from a Normal density with mean \( \theta^{(s-1)} \) and covariance-matrix \( \Sigma_\theta \), which is chosen to yield acceptance ratios between 0.2 and 0.6 for each shape parameter. The shape parameters \( p, q \) are drawn independently of each other, but are jointly accepted or discarded in each iteration. Draws will be accepted based on their acceptance probability:

\[
\alpha(\theta^{(s-1)}, \theta^*) = \min \left[ \frac{p(\theta = \theta^* | y, \psi)}{p(\theta = \theta^{(s-1)} | y, \psi)}, 1 \right] \quad (4.7)
\]

Thus if the posterior of the new draw given the data \( y \) and the parameters \( \psi \) is higher than the posterior of the previous draw the new draw will be accepted with probability 1. If the posterior of the new draw is lower than the posterior of the previous draw the new draw will only be kept if the ratio of the new posterior and the posterior evaluated at the previous step is higher than a random number drawn from a uniform distribution between 0 and 1. I.e. even if the posterior declines and the new draw has a lower probability it will be kept if it is not too unlikely. This ensures, that the Metropolis-Hastings algorithm walks over the entire parameter space even into regions with lower probability while taking more draws in regions with high probability, i.e. where the posterior is highest.

### 4.4.3 Forecasting strategy

Once the beta shape parameters have been drawn, the coefficients of the bridge equation regression can be drawn. For each set of coefficients a forecast \( \hat{Y}_s \) is calculated. The median forecast for each forecasting horizon is the result of
\( \hat{Y} = \text{median} \hat{Y}_s(\theta^{(s)}, \psi^{(s)}) \)  

(4.8)

where \( S \) is the number of draws.\(^6\) Forecasts will be generated for a nowcast and up to four quarters ahead. A nowcast for the unknown \( y_t \) (due to the publication lag) is estimated by using the drawn parameters and relying on observed investment values \( y_{t-1}, \ldots, y_{t-4} \) and redistributed construction permits for \( t.\(^7\) The forecasts are done iteratively, so the forecast for \( \hat{Y}_{t+1} \) uses the drawn parameters with both the nowcast \( \hat{Y}_t \) as well as the past observed values \( y_{t-1}, y_{t-2}, y_{t-3} \) and the redistributed construction permits for period \( t+1 \):

\[
\Delta Y_{t+1} = c + \sum_{i=1}^{P} \beta_i L^i \Delta Y_{t+1} + \gamma \Delta \hat{X}_{HF_{t+1}}^L.
\]

(4.9)

In contrast to bridge models or the MIDAS-IT approach without leads by Schumacher (2014a), which generate high frequency variable forecasts by employing a simple AR model, the forecasts by the beta filtering approach for the high frequency variable \( \hat{X}_{HF_i} \) are done by rescaling the truncated density function to correct for missing probability mass and re-weighting the remaining high-frequency observations:

\[
\hat{X}_{HF_{i+h}} = \frac{1}{\sum_{k=h}^{n-1} \mathcal{B}(k/n; p, q)} \sum_{k=h}^{n-1} \mathcal{B}(k/n; p, q) \hat{L}_{HF}^k X_{HF_i}.
\]

(4.10)

4.5 Real-time parameter estimates

In order to check if the estimation results are stable over time we analyse the distribution and estimation parameters of the real-time estimation. The mean coefficients

\(^6\)In our setting 200,000 draws minus 160,000 burn in, divided by 10 to adjust for autocorrelation in the draws.

\(^7\)In January, February, April, May, July, August, October, November the previous quarter investment values have not been published yet. Therefore also a backcast for the previous quarter has to be estimated.
for the shape parameters $p, q$ over all vintages can been seen in figure 4.3. The shape parameters were mostly stable over all vintages between 4.5 and 5 for the first parameter $p$ and between 2.5 and 2.8 for the second parameter $q$.

Figure 4.3: Real-time coefficients of beta distribution

In June 2014 the beta shape parameters dropped considerably, probably to a very high first estimate of construction investment for the first quarter 2014 of 12.4% year-on-year growth which was shortly afterwards revised down to 3.7% during an extensive revision of the whole history of construction investment. This led to a shift of the distribution mean to the right, i.e. extending the time until construction permits are actually effective for construction investment.

The mean shape coefficients can be displayed as Beta distribution for example with $p = 5$ and $q = 2.7$ in figure 4.4. The estimated beta distribution gives little weight to the first observations but rises rapidly thereafter, reaching a climax after
five years and quickly reducing its weight structure thereafter. Such a weight structure might be the consequence of the long time it takes for construction to start after the actual permit was issued. Furthermore, the lag structure of large projects is very long, implying a longer duration of construction projects.

Using the beta-filtered real-time series to depict the estimated coefficients for the bridge equation shows a varying picture over the real-time vintages (see figure 4.5). This figure shows that the constant as well as the coefficient for the redistributed permit data are more or less stable over all vintages. The autoregressive terms of the bridge equation exhibit more variation, which could be attributed to major revisions or surprising releases of construction investments. Before June 2012 the coefficients for Lag 3 and Lag 4 are basically not statistically distinguishable from zero. The story changes for vintages until September 2014. In June 2012 the first release of negative construction investment growth rates was published for Q1/2012 (a drop
by more than -10%). Until September 2014 negative growth rates for construction investments were published by the statistical office (see also figure 4.2). In October 2014 the construction investment growth rate for Q1/2012 was revised to +1.5% - an upward revision by more than 11 percentage points compared to the first estimate which now indicates more a boom than a slump. This could also explain the slight decrease in the coefficient for the redistributed permits and the sudden increase of the coefficient in October 2014.

Figure 4.5: **Real-time mean coefficients of the bridge equation**
4.6 Out-of-sample tests

4.6.1 Benchmark model setup

We employ four different benchmark models for the forecast comparison with the bridged MIDAS model. Namely we use a simple autoregressive model, a bridge model, a MIDAS regression with an Almon lag polynomial for parameter reduction and an unrestricted MIDAS regression to compute forecasts, which will be covered in more detail in the following sub-chapters.

**Autoregressive model**

We use a simple autoregressive model as benchmark model for the construction investment. An autoregressive model is specified using the formula

\[ Y_t = c + \sum_{i=1}^{p} \beta_i L^i Y_t + \epsilon_t, \]

where \( L^i \) indicates a lag function ranging from 1 to the desired lag length \( p \) and \( \beta_i \) the parameter for each \( i \)th lag of variable \( Y_t \). \( \epsilon_t \) is an error term. Forecasts of \( \hat{Y}_{t+1}, \hat{Y}_{t+2}, ..., \hat{Y}_{t+h} \) are done iteratively by plugging in the previously forecast values, i.e \( \hat{Y}_{t+2} = c + \beta_1 \hat{Y}_{t+1} + \beta_2 Y_t + ... + \beta_p Y_{t-p+2} \).

**Bridge model**

Additionally we also use a simple bridge model, which enables us to enhance the autoregressive model with monthly information. This approach is separated into three steps. Firstly, we specify a model using the own lags of the quarterly variable \( Y_t \) as well as the time aggregated quarterly values of the monthly variable \( X_t^q \). This is defined as follows:

\[ Y_t = c + \sum_{i=1}^{p} \beta_i L^i Y_t + \sum_{j=0}^{n} \gamma_j L^j X_t^q + \epsilon_t \]
In a second step, using only a monthly variable and an autoregressive model, forecasts of the monthly variables are computed, as specified in the previous subsection. The monthly variable and the respective forecasts are time-aggregated to quarterly frequency and plugged into the above specification in order to compute forecasts of the quarterly variable. The forecasts are again computed iteratively.

By aggregating the high-frequency variable to the lower frequency potential information is lost because time averaging assumes that each high-frequency observation $X_t$ inside a low frequency period receives the same weight. It could be possible that for instance the first observation of $X_t$ should get a higher weight in the time aggregation than the others. Thus, potential information is lost due to this process.

**MIDAS**

Ghysels et al. (2004) introduced the MIDAS approach. This approach circumvents the problem of time averaging by approximating the parameters of each high-frequency observation of the high frequency variable $X_t$ with a polynomial function $\theta(k; \omega)$. As benchmark model we use a MIDAS model with a non-exponential Almon lag polynomial, which is quite flexible but at the same time still maintains parsimony. Additionally the non-linear weighting function can be transformed back into linear form, which allows the model to be easily estimated by using OLS. The approximation of the high-frequency observations is done for $K = n \times m$ lags, which depend both on the number of high frequency periods inside the low frequency period $m$ as well as the number of low frequency period lags $n$. For instance, if $X_t$ were a monthly variable and $Y_t$ a quarterly variable, then $m = 3$. The model is specified with respect to the desired forecast horizon $h$. The aim is to find the specification that would have predicted $Y_t$ $h$ quarters before. Thus the MIDAS equation for growth $Y_{t+h}$ in period $t + h$ would be defined as:

$$Y_{t+h} = c + \sum_{i=0}^{p} \beta_i L^i Y_t + \gamma \sum_{k=0}^{K} \theta(k; \omega) L^{k/3} X_{t+i} + \epsilon_{t+h}$$
\( L^{k/3} \) is a lag operator for monthly variables which is defined as \( X_{t-1/3} = L^{1/3}X_t \) and \( l \) an is defined as the lead of the high-frequency variable on the low-frequency variable. Thus, the forecasts are done directly by plugging the most recent data into to the formula using the estimated parameters for each forecast horizon \( h \).

**U-MIDAS**

The unrestricted MIDAS approach was promoted by Foroni et al. (2015b). Instead of approximating the parameters of each high-frequency observation of the high frequency variable \( X_t \), this approach estimates the weights as unrestricted parameters. This allows for even more flexibility than the MIDAS approach. But in contrast U-MIDAS approach does not maintain parsimony. Depending on the frequency of the high-frequency variable, this approach can easily be over-parametrized. But when frequency differences are small as with macroeconomic data the results of the U-MIDAS are similar or even slightly superior to the MIDAS approach. The functional form of the U-MIDAS is defined as:

\[
Y_{t+h} = c + \sum_{i=0}^{p} \beta_i L^i Y_t + \sum_{k=0}^{K} \gamma_k L^{k/3} X_{t+l} + \epsilon_{t+h}
\]

**4.6.2 Benchmark results**

Our aim is to forecast the quarterly year-on-year growth rate of nominal construction investments up to one year ahead using the construction permits and investment data that were available at the moment of the forecast. In order to test the forecast performance of our model we conduct a real-time experiment using Vintages from January 2005 until December 2014. For each of the vintages we conduct a nowcast for the quarter of the vintage as well as one to four step-ahead forecasts.
In order to evaluate the forecast performance of our model we take several steps. First we compare those forecasts with a simple autoregressive model, which uses the same amount of information as the bridge model with our redistributed data, namely 4 quarterly lags. In a second step we produce forecasts for Swiss construction investment using Swiss building permit data as additional information, but in its original form. For this we use different benchmark models, namely a bridge model, a standard MIDAS setup and an unrestricted MIDAS (U-MIDAS) as explained in section 4.6.1. All models have the same quarterly information as our model, but as the bridge model and especially the U-MIDAS tend to become over-parametrized quite fast, we also restrict the lagged information of the high frequency variable to one year. The Almon-MIDAS can use more lagged information due to the reduction of the parameter space by using an Almon lag polynomial. Thus we use the same lag length as in our baseline model (84 monthly lags). In a third step we use a Hodrick-Prescott filter for smoothing the high-frequency data, both in its simple form and with an adjustment for endpoint problems. In a final step we compare the forecast performance of our setup with a moving average for the high-frequency data.

To compare the forecast performance of the different models we calculate the relative root mean squared forecast error (RMSFE) for each model and forecast horizon $h$ which is defined as:

$$\Delta RMSFE_h = 100 \times \left(\frac{RMSFE_{h}^{\text{Beta filtered}} - RMSFE_{h}^{\text{Benchmark}}}{RMSFE_{h}^{\text{Benchmark}}}\right).$$

We refer to $\Delta RMSFE_h$ as the relative change in the RMSFE. The more negative $\Delta RMSFE_h$ is, the better performs our setup relative to the respective benchmark model in terms of predictive power.

The following results are robust for different lag specifications. When using more monthly lags for the bridge model and the U-MIDAS the models will be over-parametrized and their forecast performance thus decrease further. Fewer monthly
lags do not improve the forecast performance significantly. The results are also robust when using 72 or 96 monthly lags for beta-filtered data, moving average and Almon MIDAS. Additionally, the results don’t change significantly when using different growth rates for the high frequency variable, i.e. month-on-month growth instead of year-on-year growth.

**Autoregressive process**

In order to test if the inclusion of the high-frequency building permit data actually delivers any additional information we first test the forecast performance of a bridge model including the beta-filtered data compared to a simple autoregressive process with 4 lags (AR(4)). As can be seen in figure 4.6 the filtered data improve upon the AR(4) model for the whole forecast horizon. The relative RMSFE actually decreases for a longer forecast horizon, which means the forecast errors of the model including the beta-filtered data are even smaller compared to the AR(4) for forecasts up to one year ahead.

*Figure 4.6: Relative RMSFE to autoregressive model*
Unfiltered Data

In this section we test if the filtering of the data using our method leads to an improved forecast performance compared to using the original data. For this we look at three standard mixed-frequency forecasting models. As indicated above bridge models and the unrestricted MIDAS are not able to cope with long lags which is why we restrict the lags for those models to 4 quarterly and 12 monthly lags. The Almon MIDAS in contrast uses the same amount of information as the bridge model with the filtered data, i.e. 84 monthly lags. As can be seen in figure 4.7 the addition of the original permit data seems to give at least some additional information compared to AR(4) model in figure 4.6. The relative RMSFE of the bridge model, which includes the permit data, aggregated to quarterly frequency, yields a slightly lower forecast error than the AR(4) model, i.e. the relative RMSFE is slightly better when both are compared to the bridge model using the beta-filtered data. In contrast, both the unrestricted MIDAS model as well as the Almon MIDAS model do not seem to be able to get a good signal from the original data. While the relative RMSFE of the UMIDAS model compared to the bridge model using the beta-filtered data is constantly below zero, the relative RMSFE of the Almon MIDAS model is at least for the nowcast horizon close to zero but drops rapidly afterwards. In conclusion, the addition of the original permit data can lead to small increases in forecast performance as long as the model is able to get a useful signal from the data.

HP-filtered Data

A method to extract a useful signal from volatile data is filtering the data. One of the most used filters is the Hodrick-Prescott Filter (Hodrick & Prescott (1997)). This allows a decomposition of the data into a trend and a cyclical component. For the use of the HP-filter with monthly data we use a $\lambda$ of 129,600 as suggested by Ravn & Uhlig (2002). We use the trend component as high frequency variable for forecasting.

As can be seen in figure 4.8 the forecast performance of the bridge model and the Almon MIDAS improve slightly when compared to the usage of the unfiltered data.
in the previous subsection. The relative RMSFE of those two benchmark models is nevertheless still clearly below zero and thus performs worse than the bridge model using the beta-filtered data. The UMIDAS seems to have severe problems dealing with the HP-filtered data. This is due to very high collinearity due to the low variation in the trend component of the filtered data between monthly observations.

The HP-filter has well known endpoint problems ((Baxter & King (1999))). This stems from the fact that the smoothed series at the beginning and the end of the time series tends to be close to the observed data. A general workaround is to use forecasts for several observations ahead in order to produce a better smoothing at the current edge of the data. In our case the mediocre results of the HP-filtered data could be related to this problem. Thus, we use a simple autoregressive process to forecast the high frequency data in order to get more reliable smoothing results. The outcome can be seen in figure 4.9. The forecast performance actually worsens when compared to the HP-filter without correction for endpoint problems. The relative
RMSFE is more negative for bridge models and the Almon MIDAS. It seems that autoregressive forecasts when using very volatile data are not advantageous.

**Moving average**

Another standard method for smoothing volatile data is to use a moving average of the data, which is closest to our method. But instead of having a separate weight for each high frequency observation a moving average gives all past observations the same weight. The basic analysis uses seven years or 84 months of past data, thus each monthly observation would get a weight of $1/84$. The forecast is done in the same manner as with the beta filter by re-weighted the remaining observations.

The result can be seen in figure 4.10 which shows the relative RMSFE of a bridge model using the moving average data and the bridge model using the beta filtered data. When nowcasting using moving average data does seem to give slightly smaller forecast errors, for longer forecast horizons the model using the beta-filtered data
clearly outperforms the moving-average data. It seems that the weighting scheme of the beta-filtering enables us to make better forecasts by correctly re-weighting the past observations.

4.7 Conclusion

To study the usefulness of a Bayesian beta filtering approach for a setup with a long lag structure of a volatile high frequency variable to forecast a low frequency we apply beta filtering approach on construction permit and investment data. Quarterly estimates of construction activity are subject to a substantial publication lag, thus, timely available construction permits can give policy makers an early indication on the state of the construction sector.
As construction permits can contain information for several years ahead and display a volatile structure, they can pose a problem for traditional mixed-frequency approaches even when using standard filtering methods. To deal with the special structure of the data we construct a Bayesian beta filtering setup which allows us to use both long lags and is at the same time able to filter a usable signal from the data.

To test the forecast performance of our model we conduct a real-time experiment using vintages from January 2005 until December 2014. We compare the out-of-sample forecast performance of our model with different benchmark models, namely a simple AR-process, a bridge model, an unrestricted MIDAS and a restricted MIDAS using an Almon polynomial for the reduction of the parameter space. The beta filtering approach clearly improves the forecast accuracy upon an AR-model from 10% for shorter forecast horizons to more than 25% for longer horizons. While the inclusion of permit data in standard mixed-frequency models leads to small improvements in the forecast performance compared to the AR-model, they are still clearly outperformed by our beta filtering approach, especially for longer

Figure 4.10: Relative RMSFE to moving average
forecasting horizons. This result also holds when pre-filtering the data with a HP-filter, although the forecast performance of the benchmark models slightly increases. Also the application of a one-sided moving average yields a weaker forecast performance, especially for longer forecast horizons. The specific structure of construction permit data seems to contain mostly noise for traditional models while the Bayesian beta filter can still use the contained information.
Chapter 5

Time-varying mixed frequency forecasting: A real-time experiment

5.1 Introduction

Economic forecasting is an important decision-making tool for governments, businesses and central banks in order to formulate financial and monetary policy and strategies. It helps reducing uncertainty about future outcomes and provides a foundation for planning. But when forecasting at the current edge of the data forecasters face the challenge of using macroeconomic data which are sampled at different frequencies. This lead to the emergence of models that can incorporate readily available up-to-date high frequency data into econometric models. Another problem when using up-to-date data are temporal instabilities of the parameters which are difficult to detect at the current edge of the data. This difficulty can be addressed to some degree by using time-varying parameters where the parameters follow a random-walk process. The aim of this paper is to analyse whether the use of time-varying parameters gives an advantage when forecasting with mixed frequency data compared to established methods, in this case ordinary least squares (OLS).

\(^1\)This chapter is based on Neuwirth (2017)
A few approaches have been tested in the literature so far. Carriero et al. (2015) use a Bayesian mixed-frequency regression model with stochastic volatility and find some usefulness of using stochastic volatility for forecasting US GDP but not when using time-varying parameters. Galvão (2013) uses a transition function that governs for some parameters the change in parameters in MIDAS regressions. Guerin & Marcellino (2013) propose a Markov-Switching MIDAS approach which allows for switches between a small number of regimes. Schumacher (2014b) analyses MIDAS regressions with time-varying parameters for Euro area GDP and corporate bonds spreads by using a particle filter to deal with non-linearities in the MIDAS equation.

This paper extends the literature by using time-varying parameters in both bridge equations and unrestricted MIDAS models. Additionally, we compare forecast performance of the different models and methods for a long forecast horizon. For this analysis we employ a large real-time data set for the US.

The real-time data set of US data contains both quarterly data (GDP) as well as monthly data. We use 11 monthly standard business cycle indicators and their growth rates (month-on-month, 3-month change, year-on-year) to predict quarter on quarter GDP growth. The real-time data set ranges from 1970 until mid-2013.

Due to technical restrictions we can only incorporate few lags, as unrestricted MIDAS models tend to get over-parametrized fast. Still, even a minimum specification includes enough information for now- and short-term forecasting. Albeit forecasts are made with the use of a single variable, we also look at forecast combinations of the individual models, both as an unweighted average and as weighted average based on the past forecast performance. We find that the use of time-varying parameters does not significantly improve forecast performance of bridge equations over all vintages. But the possibility to incorporate gradual structural changes can help when forecasting recessions and especially the phase since the Great Recession. Economic relationships between variables have changed since the Great Recession.
This is the reason why forecasting with bridge models using time-varying parameters is superior to forecasting with OLS. The results are also robust when estimating with a rolling window instead of an expanding window.

The paper is structured as follows: Section 5.2 explains the method and the estimation strategy. Section 5.3 discusses the used data set and Section 5.4 presents an analysis of the real-time parameter estimates of the used models and methods over all vintages. The results of the real-time experiment are shown in Section 5.5. Section 5.6 concludes.

5.2 Mixed-frequency models with time-varying parameters

5.2.1 Model setup

We employ two standard single equation mixed frequency models for forecasting, namely bridge equations and unrestricted MIDAS models. Due to their simplicity and transparency bridge equations are common in central banks and other policy institutions (e.g. ECB (2008) or Bundesbank (2013)). Early applications in the literature can be found for example in Ingenito & Trehan (1996) or Baffigi et al. (2004). The intuition of bridge equations is to use time-aggregated contemporary high frequency variables for explaining low-frequency variables. For forecasting the lower-frequency variable, the high-frequency variables are forecast using simple models, like an autoregressive process and afterwards time-aggregated to the lower frequency. These forecasts are plugged into the previously estimated model explaining the low-frequency variable. Forecasts are done iteratively, using the previous forecasts for the low-frequency variable.

Mixed data sampling (MIDAS) is another single equation approach and was proposed by Ghysels et al. (2004). This approach is able to combine time series with different frequencies in the same model and also deal with a long lag structure. In
contrast to bridge equations the high-frequency variable is not time-aggregated but
directly related to the low-frequency variable by estimating a single parameter for
each high-frequency lag. Depending on the differences in frequency this method can
lead to a high number of parameters which have to be estimated. To circumvent this
problem, lag polynomials which decrease the necessary amount of parameters can
be used. Estimation can be done by non-linear least squares of more sophisticated
algorithms. Early applications for financial data, where frequency differences are
big, can be found in Ghysels et al. (2007). MIDAS models have also been used for
macroeconomic data more recently for example in Clements & Galvão (2008) and
Clements & Galvão (2009a) or Armesto et al. (2010) and Andreou et al. (2011).
Additionally, for macroeconomic data, where the differences between frequencies of
the used data are small, for instance when using quarterly and monthly data, an
unrestricted MIDAS model can be used without risking overparameterization. In
such cases an unrestricted setup is equivalent or even superior compared with stan-
dard MIDAS setups, as has been shown by Foroni et al. (2015b). Another advantage
of an unrestricted MIDAS setup is the reduced requirement for computational and
modelling efforts compared with standard MIDAS setups. As the U-MIDAS ap-
proach represents a compromise between parsimony, simplicity and accuracy it is
often used for nowcasting (Aprigliano et al., 2017).

In order to test the usefulness of time-varying parameters for forecasting both
models are estimated in their standard form by using ordinary least squares as well
as in a Bayesian state-space framework with time-varying coefficients. The time-
varying approaches are presented in the next sub-chapters.

**Time-varying bridge equations**

We specify the bridge equation model using lags of a quarterly variable $y_t$ as
well as time aggregated quarterly values of a monthly variable $x_t^q$. This is defined
as follows:
\[ y_t = c_t + \sum_{i=1}^{p} \beta_{i,t} L^i y_t + \sum_{j=0}^{n} \gamma_{j,t} L^j x_t^q + \epsilon_t \]  \hspace{1cm} (5.1)

\( L^i \) and \( L^j \) indicate the lag operator for lag lengths \( p \) and \( n \) respectively. The parameter \( \beta_{i,t} \) for each \( i \)th lag of variable \( y_t \) and the parameter \( \gamma_{j,t} \) for the \( j \)th lag of the time-aggregated high-frequency variable \( x_t^q \) as well as the constant \( c_t \) are time-varying and follow a random walk. With the bridge equations parameters as \( a_t = [c_t \beta_{1,t} ... \beta_{p,t} \gamma_{1,t} ... \gamma_{n,t}] \) the measurement equation can be written as:

\[
\begin{bmatrix}
1 \\
y_{t-1} \\
\vdots \\
y_{t-p} \\
x_t^q \\
\vdots \\
x_{t-n}^q
\end{bmatrix}
\begin{bmatrix}
0 \\
c_t \\
\beta_{1,t} \\
\vdots \\
\beta_{p,t} \\
\gamma_{1,t} \\
\vdots \\
\gamma_{n,t}
\end{bmatrix} + \epsilon_t
\]

The state equation models the random walk behaviour of the time-varying parameters:

\[ a_t = a_{t-1} + \nu_t \]  \hspace{1cm} (5.3)

The variance-covariance matrix of the innovations is block-diagonal:

\[
\begin{pmatrix}
\epsilon_t \\
\nu_t
\end{pmatrix} \sim N(0, V), V = \begin{pmatrix}
\sigma^2 & 0 \\
0 & Q
\end{pmatrix}
\]  \hspace{1cm} (5.4)

The forecast procedure for bridge equations consists of three steps. Firstly, the forecasts for the high-frequency variable are generated. This is normally done by
using an autoregressive model. The model is estimated by using OLS and the lag length is optimized according to the Bayesian Information Criteria (BIC). In a second step, the high-frequency forecasts are time-aggregated to the lower frequency. Thirdly, the forecasts are plugged into the above specification in order to compute forecasts of the quarterly variable. The forecasts are computed iteratively. In this application, we analyse both bridge models using OLS and time-varying parameters. In both cases the high-frequency forecasts are done by OLS as described in the first step. When forecasting using OLS, the estimated parameters are used for forecasting. In the case of time-varying parameters only the parameters of the last quarter in each vintage are used. The real-time estimates are described and analysed in section 5.4.

Time-varying U-MIDAS model

The unrestricted MIDAS approach was promoted by Foroni et al. (2015b). In contrast to the standard MIDAS approach where parameters of each high-frequency observation of the high frequency variable \( x_t \) is approximated with the help of polynomials, the unrestricted approach estimates the weights as unrestricted parameters. The number of parameters depends both on the number of high frequency periods \( m \) inside the low frequency period as well as the number of desired low frequency period lags \( n \). When frequency differences and thus \( m \) are big or a large number of lags are desired, this approach can easily be over-parametrized. But when frequency differences are small as with macroeconomic data, the results of the U-MIDAS are similar or even slightly superior to the MIDAS approach. For instance, when \( x_t \) is a monthly variable and \( y_t \) a quarterly variable, then \( m = 3 \). Forecasts in U-MIDAS models are done directly, thus the model is specified with respect to the desired forecast horizon \( h \). The aim is to find the specification that would have predicted \( y_t \) \( h \) quarters ahead. The functional form of the U-MIDAS can be written as:

\[
y_t = c_t + \sum_{i=h}^{p+h} \beta_{i,t} L^i y_t + \sum_{k=mh}^{K} \gamma_{k,t} L^{k/m} x_{t+l} + \epsilon_t
\]  

(5.5)
\( L^{k/3} \) is a lag operator for monthly variables which is defined as \( x_{t-1/3} = L^{1/3}x_t \) and \( l \) is defined as the lead of the high-frequency variable on the low-frequency variable. As the models is specified for each forecast horizon \( h \), \( K = mnh \). The forecasts are done directly by plugging the most recent data into the formula using the estimated parameters for each forecast horizon \( h \). Thus, with the U-MIDAS parameters as \( a_t = [\epsilon_t, \beta_{h,t}, \ldots, \beta_{p+h,t}, \gamma_{nh/3,t}, \ldots, \gamma_{K/3,t}] \) the measurement equation can be written as

\[
y_t = a_t \begin{pmatrix}
1 \\
y_{t-h} \\
\vdots \\
y_{t-(p+h)} \\
x_{t+1-nh/3} \\
\vdots \\
x_{t+l-K/3}
\end{pmatrix} + \epsilon_t \tag{5.6}
\]

and the state equation as

\[
a_t = a_{t-1} + \nu_t \tag{5.7}
\]

with the variance-covariance matrix:

\[
\begin{pmatrix}
\epsilon_t \\
\nu_t
\end{pmatrix} \sim N(0, V), V = \begin{pmatrix}
1 & 0 \\
0 & Q
\end{pmatrix} \tag{5.8}
\]

### 5.2.2 Estimation

Due to the complexity of the estimation of time-varying parameters, classical methods lead to problems when encountering peaks in regions of low probability and thus can lead to unreasonable results. A Bayesian framework offers the tools to circumvent these problems. We adopt a Bayesian approach and use the Markov
Chain Monte Carlo (MCMC) method for the estimation of the time-varying model following Cogley & Sargent (2001), Cogley & Sargent (2005), Primiceri (2005). More precisely we employ a Gibbs sampling algorithm which involves the following steps. Firstly, the matrix $V$ is initialized. The initial values in this step are set to $\sigma_0^2$ being 0.01 and $Q_0 \sim IW(k_Q^2 I_s, T_Q)$ with $s$ being the number of states, the scaling factor $k_Q$ set to 0.05 and the shape parameter $T_Q$ set to $\dim(Q) + 2$. As suggested by DeJong (1991), for the initial conditions $a_0$ we chose an uninformative prior centered at zero with a high variance $p_0$ of 1000. In a second step $a^T$ is sampled from the conditional probability $p(a^T|y^T, x^T, V)$, given the previous results for the variance-covariance matrix $V$ as well as the actual data $y^T$ and $x^T$. In a third step, conditional on the data $y^T$ and $x^T$ as well as the previous draws for the parameter vector $a^T$ the innovations of $\nu_t$ are treated as observable. Thus, $V$ can be sampled by sampling $Q$ from $p(Q|y^T, x^T, a^T)$. The second and third step are then repeated for a number of iterations. In this case we set the number of iterations to 50,000 while discarding the first 80% as burn-in.

5.3 Data

The real-time data used in our analysis consist of quarterly real seasonally adjusted GDP as well as the following monthly data: the consumer price index, industrial production, housing starts and the unemployment rate. The data were acquired from the Archival Federal Reserve Economic Data (ALFRED) published by the Federal Reserve Bank of St. Louis and the Federal Reserve Bank of Philadelphia Real-Time Data Set for Macroeconomists (RTDSM). Additionally, we also use a set of time series that are not subject to data revisions, namely the ISM indices for manufacturing, supplier delivery times and orders, the S&P 500 stock market index, the 3-month treasury bill yield, the 10-year treasury bond yield and average weekly hours worked by production and supervisory workers. Following common practice we time-aggregate all variables with a higher than monthly frequency by using their
end-of-month values (Carriero et al., 2015 and Schorfheide & Song, 2015, e.g.) Table 5.1 provides precise data definitions. We use different data transformations for our forecast evaluation, namely year-on-year growth rates, 3-month growth rates as well as month-on-month growth rates. The real-time dataset comprises 344 vintages covering the time frame January 1970 to August 2013. The first vintage starts in January 1970 and includes all data available until the end of January 1985. In this application we use an expanding window setup, so each following vintage one month is added.

In a real time data context, the choice of the benchmark series for the calculation of the forecast errors is an often discussed topic (cf. the discussions in Croushore, 2006, Romer & Romer, 2000, and Sims, 2002). Our quarterly time series, namely real seasonal adjusted GDP, was subject to several benchmark revisions; the latest revision for GDP occurred in mid-2014 and included a substantial redefinition of gross fixed capital formation which accounts for 20 percent of GDP. Such a benchmark revision could not have been predicted by a forecaster in, e.g. 1985. Thus we follow Romer & Romer (2000), Faust & Wright (2009) and Carriero et al. (2015) and use the second estimate of quarterly GDP as comparison to our forecasts.

We take differing publication lags into account in order to reproduce the available information a forecaster would have had at each forecast date. Most monthly variables are subject to a publication lag, thus the (first) release for each month is not available directly at the end of the month, but with a delay of up to one month. Only the financial variables, namely the S&P 500 index, the 10-year treasury bond yield and the 3-month treasury bill yield are available directly at the end of each month. Additionally, those variables are released daily and not subject to revisions. Also the first release of the quarterly real GDP is only available with a time lag of one month.
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<td>Industrial production index: manufacturing</td>
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<td>For some months two vintages were available. In those cases we used the later publications. This was the case for 2000M9, 2005M2, 2006M2, 2007M2, 2008M2, 2009M2, 2010M2, 2011M2 and 2012M2.</td>
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<td>Housing starts</td>
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<td>yes</td>
<td>RTDSM</td>
<td>There were no observations available for the first publication of 1995M11 and 1995M12. We make the assumption that in those cases no revision took place and use the same values as in the second publication.</td>
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<td>10-year treasury bond yield</td>
<td>monthly</td>
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5.4 Real-time parameter estimates

To start our analysis we look at the real-time estimates of the parameters that are used for the forecasts, starting with the estimates of the bridge equations. In total, over all vintages we have time series with over 300 observations. They contain the parameters of the bridge equations for each variable both for the estimates using OLS and the TVP parameters. In the TVP case we only show the parameter values of the last quarter of each vintage (which are the ones used for the calculation of the forecast). Figure 5.1 shows the OLS and TVP parameters of the bridge equations using the month-on-month growth rate of industrial production and the 3-month growth rate of housing starts.\(^2\) As can be seen from the figure, the OLS parameters that are used to produce the forecasts are relatively stable over the vintages and exhibit a much lower variance than the time-varying parameters. Nonetheless the estimated time-varying parameters do not show a lot of sudden jumps and do not, at least at first glance seem to introduce much noise.

The same analysis is done for the parameters of the U-MIDAS equations. As forecasts with the U-MIDAS method are done directly in contrast to the iterative approach of bridge equations, for each forecast horizon \(h\) and each variable we have a set of \(1, \ldots, H\) estimated parameters both for OLS and TVP. Additionally, parameters change with each month of each quarter. Thus, for sake of simplicity, in figure 5.2 we show only the estimates of the first month \(m1\) of each quarter for horizons \(h = 1, \ldots, 5\). The figure shows the constant of the U-MIDAS specification when using the 3-month growth rate of housing starts as high-frequency variable. In case of U-MIDAS, the estimated parameters are a bit more volatile compared to the parameters for the bridge equations in figure 5.1 which could introduce more noise into the forecasts. In the next section we will analyse whether this parameter volatility is actually a problem for the forecast performance of the different models.

\(^2\)An overview over all used parameters can be found in Appendix C.
Figure 5.1: Parameters for bridge equation

Figure 5.2: Parameters for U-MIDAS model
5.5 Real-time out-of-sample results

5.5.1 Forecast comparison

In this section the forecast results of the different models and methods are presented. Namely, these are bridge equations and U-MIDAS models estimated by either OLS or time-varying parameters in a Bayesian state space setup (TVP). In order to coherently compare the forecasts of the models and methods we have to make sure that always the same information is used, so that differences in forecast performance result only from the different models / methods. As U-MIDAS models tend to get overparametrized quickly we restrict the data used in this forecasting exercise to one lagged quarter of low-frequency data and three months of high-frequency data. For the comparison of the forecast performance of the different models we calculate the relative change in the root mean squared forecast error (RMSFE) for each model and forecast horizon $h$ which is defined as:

$$\Delta RMSFE_h = 100 \times \left( \frac{RMSFE_{Model 1}^h - RMSFE_{Model 2}^h}{RMSFE_{Model 2}^h} \right).$$

The more negative $\Delta RMSFE_h$ is, the better performs model 1 compared to the respective benchmark model in terms of forecast accuracy.

In order to test if the two forecasts are actually different from each other we employ the Diebold-Mariano test for equality of forecast accuracy (Diebold & Mariano, 1995). For this we use a quadratic loss function differential $d$ between the forecast errors $\epsilon$ of the models 1 and 2 at time $t$ for the period $t + h$:

$$d_t = (\epsilon^1_{(t+h|t)})^2 - (\epsilon^2_{(t+h|t)})^2$$

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The Diebold-Mariano test statistic $DM$ is defined as

$$DM = \frac{\bar{d}}{\sqrt{(1/T) \ast LRV(d)}}$$

with $\bar{d}$ being the sample mean of $d_t$, $T$ being the number of forecasts and the estimated long run variance (LRV) which corrects for possible serial correlation of the loss differentials $d$ for forecast horizons $h > 1$:

$$LRV(d) = \text{Var}(d) + 2 \sum_{k=1}^{h-1} \text{Cov}(d_t, d_{t-k})$$

Under the null hypothesis of equal forecast accuracy the test statistic is asymptotically $N(0,1)$ distributed, thus the null hypothesis will be rejected if the test statistic falls outside the range of -1.96 and 1.96 at the 5 percent significance level. The significance is incorporated in the following subsections in the graphs for the relative RMSFE. In case the null hypothesis is not rejected the relative RMSFE will be in a dashed line. If the Diebold-Mariano test indicates a significant difference in forecast accuracy the lines will be solid. In the following sections the different combinations of models and methods will be tested separately.

### 5.5.2 Performance over all vintages

When estimating with OLS, the forecast performance of bridge equations and U-MIDAS models over all vintages turns out to be basically equal. This is in line with previous work by Marcellino et al. (2006) who find that direct forecasting can be at least as accurate as indirect forecasting. Bridge models have a slightly smaller RMSFE for some of the used variables like the ISM total or hours worked, whereas for some other variables the U-MIDAS models have smaller forecast errors. Figure 5.3 shows the relative RMSFE on average over all variables as well as the results for a forecast combination using forecast errors of the previous 4 quarters of each
vintage to weight forecasts.\textsuperscript{3} The figure shows a slightly lower RMSFE for bridge equations up to a maximum of 9 percent. But the results are hardly significant. Only for forecast horizons from 7 to 9 months ahead the Diebold-Mariano test indicates actual differences in forecast accuracy (pointed out by the solid line for those months). But even in those cases the difference in forecast accuracy is very small.

![Figure 5.3: Rel. RMSFE: Bridge vs U-MIDAS using OLS](image)

Note: The dashed line shows the relative RMSFE of bridge equations compared to the benchmark model U-MIDAS. Values below zero indicate the smaller percentage forecast error of bridge equations compared to U-MIDAS. The solid line indicates significant results according to the Diebold-Mariano test.

The picture changes somewhat when using time-varying parameters to compare those two models: In most cases the forecast errors of bridge models are smaller compared to the U-MIDAS specification. Still, when using short term interest rates or industrial production the RMSFE is somewhat smaller for the U-MIDAS setup compared to the bridge equations. Over all variables, as can be seen in figure 5.4 the RMSFE is clearly – and for shorter forecast horizons significantly – smaller for bridge equations than for U-MIDAS models. This result could be due to an introduction of noise when using time-varying parameters with U-MIDAS as shown in chapter 5.4.

\textsuperscript{3}The graphs for all variables and all forecast comparisons can be seen in Appendix C.
Figure 5.4: **Rel. RMSFE: Bridge vs U-MIDAS using TVP**

Note: The dashed line shows the relative RMSFE of bridge equations compared to the benchmark model U-MIDAS. Values below zero indicate the smaller percentage forecast error of bridge equations compared to U-MIDAS. The solid line indicates significant results according to the Diebold-Mariano test.

This result can also be seen in the direct comparison of OLS and TVP in the case of U-MIDAS models: For almost all variables the forecast errors are smaller when using OLS and also the average and combination forecasts show a clear out-performance by OLS over TVP (figure 5.5). The differences in forecast accuracy are also significant for shorter forecast horizons. In the case of bridge equations, the results are not that distinct. The forecasts errors for almost all variables are smaller when using OLS instead of TVP. But, as figure 5.6 shows, the differences in forecast accuracy are not statistically different, except for a forecast horizon of 7 to 9 months when using a combined forecast instead of an average over all variables. But even in this case the differences in forecast performance are rather small.
Figure 5.5: Rel. RMSFE: TVP vs OLS using U-MIDAS

![Graph showing relative RMSFE for TVP vs OLS using U-MIDAS](image)

Note: The dashed line shows the relative RMSFE of TVP compared to the benchmark method OLS. Values below zero indicate the smaller percentage forecast error of TVP compared to OLS. The solid line indicates significant results according to the Diebold-Mariano test.

Figure 5.6: Rel. RMSFE: TVP vs OLS using Bridge

![Graph showing relative RMSFE for TVP vs OLS using Bridge](image)

Note: The dashed line shows the relative RMSFE of TVP compared to the benchmark method OLS. Values below zero indicate the smaller percentage forecast error of TVP compared to OLS. The solid line indicates significant results according to the Diebold-Mariano test.
In summary, over all vintages, the use of time-varying parameters does not give an advantage when forecasting with mixed-frequency models. The estimation of U-MIDAS models with time-varying parameters mostly introduces noise due to the more volatile monthly data. Forecast errors of U-MIDAS models when using TVP are clearly worse. So when deciding between bridge equations and U-MIDAS models, it might be slightly beneficial using bridge equations when using TVP. But bridge equations when using TVP are, over all vintages, also not clearly better compared to bridge equations estimated with OLS.

5.5.3 Performance in times of structural change

The higher flexibility of time-varying parameters and thus the possibility to incorporate gradual structural changes could lead to a better performance of models using this technique in some special phases. In order to test this conjecture a sub-sample of our data is created. Specifically, we look at the Great Recession and the subsequent upswing. It can be argued that with the Great Recession economic relationships between GDP and different indicators changed. In this case, a model using time-varying parameters should be able to react faster to such changes than a simple OLS model with an expanding window.

In the following, for the sake of presentation, only the average forecast results of the previous analysis are presented. Figure 5.7 shows that since the Great Recession bridge equations are slightly superior than U-MIDAS models both when using OLS or TVP. When forecasting with bridge equations it was advantageous to use TVP instead of OLS. The relative RMSFE lies now mostly below zero and is, at least for longer forecast horizons, significant. This result can be explained by the change in the time-varying parameters in 2009. Figure 5.8 shows as example the time-varying parameter of the autoregressive term in bridge models when using the short-term interest rate as high-frequency variable over several vintages. Beginning

\footnote{All results can be found in Appendix C}
in 2009 the parameters shift upwards and show a very different dynamic also in 2010 and 2011. Similar reactions can be found using other high-frequency variables. This higher flexibility allows for better forecasts after the Great Recession.

Figure 5.7: Rel. RMSFEs for Great Recession

Figure 5.8: Time-varying parameters over different vintages
In order to check the robustness of the results for the full sample we split the sample at the beginning of the Great Recession. As can be seen in figure 5.9 most results do not differ too much between the sample before the Great Recession and afterwards. Bridge models still are preferable compared to U-MIDAS when using OLS (albeit not significantly anymore) and when using TVP. Also when forecasting with U-MIDAS models OLS should be the preferred estimation method. The biggest differences between the samples occur when using bridge models for forecasting. Before the Great Recession the estimation with OLS was slightly superior. During and after the Great Recession the use of TVP is slightly superior, increasingly so with longer forecast horizons. In order to check if this result stays robust we estimate the OLS models using a rolling window instead of an expanding window. This allows the parameters to be more flexible and to adapt faster to such structural changes. While the OLS parameters are more flexible they do not adapt as fast
time-varying parameters and even with rolling windows of 36, 48 and 60 quarters
the results are robust. In summary, the use of time-varying parameters can be ben-
eficial when using bridge models for forecasting especially since the Great Recession.

5.6 Conclusion

To study the usefulness of time-varying parameters for forecasting with mixed-
frequency data we compared different forecasting models and estimation methods.
We used two standard mixed-frequency forecasting models, namely bridge equation
and unrestricted MIDAS (U-MIDAS) models. These models were estimated with
standard ordinary least squares (OLS) and in a Bayesian state space framework that
allows for the estimation of time-varying parameters.

Time-varying parameters offer the possibility to address potential temporal in-
stabilities which are hard to detect, especially when working at the current edge of
the data. By using the estimated parameters for the latest quarter, forecasts could
be improved compared to using OLS parameters that do not include potential shifts
in the correlation of different variables.

To test the forecast performance of bridge equations and U-MIDAS models es-
timated with OLS and time-varying parameters (TVP) we conduct a real-time ex-
periment using US data with vintages from January 1985 until August 2013. We
compare the out-of-sample forecasts of all models and methods separately. We
find that when using OLS, bridge equations and U-MIDAS models perform almost
equally over all vintages. When using TVP, bridge equations perform significantly
better. Due to the more volatile high-frequency data used in U-MIDAS models in
contrast to the time-aggregated data in bridge equations the time-varying param-
eters seem to introduce noise compared to the estimation with OLS. When using
bridge equations the estimation method does not seem to matter much: the fore-
cast performance is roughly the same when using OLS or TVP and the differences
are hardly significant for most forecast horizons. For U-MIDAS models the classic estimation method is clearly and significantly better for forecasting.

We also analyse if the higher flexibility of time-varying parameters and thus the possibility to incorporate gradual structural changes could lead to a better performance of models using this technique in times of structural changes. We check a sub-sample of our data, namely the period since the start of the Great Recession. While most results change only slightly in the sub-sample, the results for bridge models stand out. Forecast errors are smaller when using time-varying parameters compared to OLS and get smaller for longer forecast horizons. The results are robust even when estimating the bridge models with a rolling window. Even though models with a rolling window can adapt faster to structural changes compared to the use of an expanding window, the time-varying parameters are able to react faster to the changes after the Great Recession. In summary, overall vintages the use of time-varying parameters did not improve forecast results significantly. However, since the Great Recession it was advantageous to use time-varying parameters when forecasting with bridge models due to the higher flexibility of those models.
Bibliography


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Appendix A

Appendix Chapter 2
## Correlations between estimates and revisions of nominal unadjusted GDP

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*Note: * significant at 10 percent level, ** significant at 5 percent level, *** significant at 1 percent level.

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Correlations between estimates and revisions of seasonal adjustment

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Note: * significant at 10 percent level, ** significant at 5 percent level, *** significant at 1 percent level.
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Note: * significant at 10 percent level, ** significant at 5 percent level, *** significant at 1 percent level.
## Correlations between estimates and revisions of real adjusted GDP

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*Note: * significant at 10 percent level, ** significant at 5 percent level, *** significant at 1 percent level.*
### In-sample results: unadjusted GDP model I

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### In-sample results: unadjusted GDP model II

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\( y_{h,t} \)

-0.37 -0.36 -0.36 -0.34 -0.30 -0.28 -0.29 -0.22 -0.17 -0.12 -0.11 -0.06 -0.06 -0.02 -0.01 0.01

(-3.76) (-3.69) (-3.73) (-3.69) (-3.38) (-3.15) (-3.36) (-2.65) (-2.26) (-1.72) (-1.70) (-1.31) (-1.56) (-0.27) (-0.18) (0.59)

IfoBusi.Sit.QoQ

-0.01 0.00 0.00 0.00 0.00 0.00 0.00 -0.01 -0.01 0.00 0.00 -0.01 0.00 0.00 0.00 0.00

(-0.33) (-0.22) (-0.21) (-0.24) (-0.13) (-0.21) (0.03) (-0.70) (-0.58) (-0.34) (0.09) (-0.84) (-0.61) (0.15) (0.30) (-0.07)

\( y_{h,t} - y_{1,t} \)

0.04 -0.01 0.02 0.11 0.11 0.11 0.00 0.03 0.03 0.04 0.03 0.02 -0.02 -0.02 0.00

(0.07) (-0.02) (0.08) (0.44) (0.46) (0.53) (0.02) (0.25) (0.28) (0.41) (0.48) (0.42) (-0.36) (-0.43) (-0.10) 0.00

Dummy

-0.02 0.14 -0.06 -0.26 -0.16 -0.06 0.07 -0.43 -0.25 -0.13 0.14 -0.52 -0.43 -0.19 0.09 -0.56

(-0.05) (0.37) (-0.16) (-0.75) (-0.47) (-0.17) (0.21) (-1.38) (-0.84) (-0.42) (0.44) (-2.14) (-1.98) (-0.95) (0.43) (-5.54)

\( R^2 \)

0.20 0.19 0.21 0.22 0.17 0.15 0.16 0.15 0.10 0.05 0.05 0.10 0.10 0.03 0.01 0.35

In-sample results: seasonal adjustment - model II

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\( y_{h,t} \)

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(-3.76) (-3.69) (-3.73) (-3.69) (-3.38) (-3.15) (-3.36) (-2.65) (-2.26) (-1.72) (-1.70) (-1.31) (-1.56) (-0.27) (-0.18) (0.59)

IfoBusi.Sit.QoQ

-0.01 0.00 0.00 0.00 0.00 0.00 0.00 -0.01 -0.01 0.00 0.00 -0.01 0.00 0.00 0.00 0.00

(-0.33) (-0.22) (-0.21) (-0.24) (-0.13) (-0.21) (0.03) (-0.70) (-0.58) (-0.34) (0.09) (-0.84) (-0.61) (0.15) (0.30) (-0.07)

\( y_{h,t} - y_{1,t} \)

0.04 -0.01 0.02 0.11 0.11 0.11 0.00 0.03 0.03 0.04 0.03 0.02 -0.02 -0.02 0.00

(0.07) (-0.02) (0.08) (0.44) (0.46) (0.53) (0.02) (0.25) (0.28) (0.41) (0.48) (0.42) (-0.36) (-0.43) (-0.10) 0.00

Dummy

-0.02 0.14 -0.06 -0.26 -0.16 -0.06 0.07 -0.43 -0.25 -0.13 0.14 -0.52 -0.43 -0.19 0.09 -0.56

(-0.05) (0.37) (-0.16) (-0.75) (-0.47) (-0.17) (0.21) (-1.38) (-0.84) (-0.42) (0.44) (-2.14) (-1.98) (-0.95) (0.43) (-5.54)

\( R^2 \)

0.20 0.19 0.21 0.22 0.18 0.15 0.17 0.16 0.10 0.06 0.05 0.12 0.11 0.03 0.01 0.35
In-sample results: seasonal adjustment – model III

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<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(2.52)</td>
<td>(2.64)</td>
<td>(3.26)</td>
<td>(2.70)</td>
<td>(2.24)</td>
<td>(1.72)</td>
<td>(1.36)</td>
<td>(1.53)</td>
<td>(1.5)</td>
<td>(1.14)</td>
<td>(1.86)</td>
<td>(2.92)</td>
<td>(2.54)</td>
<td>(2.27)</td>
<td>(1.75)</td>
<td>(1.3)</td>
</tr>
<tr>
<td>Dummy</td>
<td>-0.08</td>
<td>-0.08</td>
<td>-0.03</td>
<td>-0.02</td>
<td>-0.04</td>
<td>-0.01</td>
<td>-0.06</td>
<td>-0.03</td>
<td>-0.05</td>
<td>-0.03</td>
<td>-0.06</td>
<td>0.07</td>
<td>0.09</td>
<td>0.06</td>
<td>0.16</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>(-0.84)</td>
<td>(-1.01)</td>
<td>(-0.43)</td>
<td>(-0.24)</td>
<td>(-0.56)</td>
<td>(-0.2)</td>
<td>(-0.84)</td>
<td>(-0.47)</td>
<td>(-0.71)</td>
<td>(-0.41)</td>
<td>(-0.92)</td>
<td>(1.23)</td>
<td>(1.68)</td>
<td>(1.03)</td>
<td>(3.71)</td>
<td>(2.04)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.33</td>
<td>0.32</td>
<td>0.33</td>
<td>0.28</td>
<td>0.25</td>
<td>0.20</td>
<td>0.23</td>
<td>0.21</td>
<td>0.17</td>
<td>0.14</td>
<td>0.19</td>
<td>0.27</td>
<td>0.26</td>
<td>0.16</td>
<td>0.38</td>
<td>0.24</td>
</tr>
</tbody>
</table>

In-sample results: real adjusted GDP – model III
Appendix B

Appendix Chapter 3

MIDAS VAR example for one quarterly and one monthly variable

In this section we give a simple illustrative example of the MIDAS VAR framework presented in Section 3.2. Let there be one monthly variable, $y_{1,t-1+\tau_1/3}$ and one quarterly variable, $y_{2,t-1+\tau_2/1}$, with $t = 1, \ldots, T$, $\tau_1 = 1, 2, 3$ and $\tau_2 = 1$. Further let $P = 1$. In this case,

$$y_{t-p} = \begin{bmatrix}
y_{1,t-p} \\
y_{1,t-p-\frac{1}{3}} \\
y_{1,t-p-\frac{2}{3}} \\
y_{2,t-p}
\end{bmatrix}^\top,$$
and

\[
x_t = \begin{bmatrix}
y_{1,t-\frac{3}{2}} \\
y_{1,t-\frac{3}{3}} \\
y_{1,t-1} \\
y_{1,t-1\frac{3}{2}} \\
y_{1,t-2} \\
y_{1,t-2\frac{3}{2}} \\
y_{2,t-1} \\
y_{2,t-2}
\end{bmatrix}.
\]

The mixed-frequency VAR process from Equation (3.4) then writes

\[
\begin{bmatrix}
y_{1,t-\frac{3}{2}} \\
y_{1,t-\frac{3}{3}} \\
y_{1,t-1} \\
y_{1,t-1\frac{3}{2}} \\
y_{1,t-2} \\
y_{1,t-2\frac{3}{2}} \\
y_{2,t-1} \\
y_{2,t-2}
\end{bmatrix} = \begin{bmatrix}
\alpha_{1,1,3} \\
\alpha_{1,2,1} \\
\alpha_{1,2,2} \\
\alpha_{1,2,3} \\
\alpha_{1,2,4} \\
\alpha_{1,2,5} \\
\alpha_{1,2,6} \\
\alpha_{1,2,7}
\end{bmatrix} + \begin{bmatrix}
y_{1,t-\frac{3}{2}} \\
y_{1,t-\frac{3}{3}} \\
y_{1,t-1} \\
y_{1,t-1\frac{3}{2}} \\
y_{1,t-2} \\
y_{1,t-2\frac{3}{2}} \\
y_{1,t-3} \\
y_{1,t-3\frac{3}{2}}
\end{bmatrix} + \begin{bmatrix}
\epsilon_{1,t} \\
\epsilon_{1,t-\frac{3}{3}} \\
\epsilon_{1,t-1} \\
\epsilon_{1,t-1\frac{3}{2}} \\
\epsilon_{1,t-2} \\
\epsilon_{1,t-2\frac{3}{2}} \\
\epsilon_{1,t-3} \\
\epsilon_{1,t-3\frac{3}{2}}
\end{bmatrix}.
\]

To keep the example simple we assume that all lags are modeled as Almon lag polynomials of order \(Q = 1\). To exemplify the data transformation, consider the block transformation matrix of Equation (3.9) for the third observation of the first variable \((i = 1, \tau_1 = 3)\)

\[
M_{1,3} = \text{diag}(M_{1,3,1}, M_{1,3,2}) = \begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 2 & 3 & 4 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 2
\end{bmatrix}.
\]

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As shown in Equation (3.10) this matrix can be used to build the transformed data vector

\[
\mathbf{x}^{*}_{1,3,t} = \begin{bmatrix}
\mathbf{x}^{*}_{1,3,1,0,t} \\
\mathbf{x}^{*}_{1,3,1,1,t} \\
\mathbf{x}^{*}_{1,3,2,0,t} \\
\mathbf{x}^{*}_{1,3,2,1,t}
\end{bmatrix} = \begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 2 & 3 & 4 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 2
\end{bmatrix} \begin{bmatrix}
y_{1,t-\frac{1}{3}} \\
y_{1,t-\frac{2}{3}} \\
y_{1,t-1} \\
y_{1,t-\frac{4}{3}} \\
y_{1,t-\frac{5}{3}} \\
y_{1,t-2} \\
y_{2,t-1} \\
y_{2,t-2}
\end{bmatrix}
\]

Further, we construct the big parameter matrix of Equation (3.12) which contains all Almon parameters of the MFVAR:

\[
\Theta = \begin{bmatrix}
\Theta_1 \\
\Theta_2
\end{bmatrix} = \begin{bmatrix}
\theta_{1,3} \\
\theta_{1,2} \\
\theta_{1,1} \\
\theta_{2,1}
\end{bmatrix} = \begin{bmatrix}
\theta_{1,3,1,0} & \theta_{1,3,1,1} & \theta_{1,3,2,0} & \theta_{1,3,2,1} \\
\theta_{1,2,1,0} & \theta_{1,2,1,1} & \theta_{1,2,2,0} & \theta_{1,2,2,1} \\
\theta_{1,1,1,0} & \theta_{1,1,1,1} & \theta_{1,1,2,0} & \theta_{1,1,2,1} \\
\theta_{2,1,1,0} & \theta_{2,1,1,1} & \theta_{2,1,2,0} & \theta_{2,1,2,1}
\end{bmatrix}
\]

Accordingly, the big data matrix of Equation (3.13), which contains all transformed data, writes

\[
X^{*}_t = \begin{bmatrix}
X^{*}_{1,t} \\
X^{*}_{2,t}
\end{bmatrix} = \begin{bmatrix}
\mathbf{x}^{*}_{1,3,t} \\
\mathbf{x}^{*}_{1,2,t} \\
\mathbf{x}^{*}_{1,1,t} \\
\mathbf{x}^{*}_{2,1,t}
\end{bmatrix} = \begin{bmatrix}
\mathbf{x}^{*}_{1,3,1,0,t} & \mathbf{x}^{*}_{1,3,1,1,t} & \mathbf{x}^{*}_{1,3,2,0,t} & \mathbf{x}^{*}_{1,3,2,1,t} \\
\mathbf{x}^{*}_{1,2,1,0,t} & \mathbf{x}^{*}_{1,2,1,1,t} & \mathbf{x}^{*}_{1,2,2,0,t} & \mathbf{x}^{*}_{1,2,2,1,t} \\
\mathbf{x}^{*}_{1,1,1,0,t} & \mathbf{x}^{*}_{1,1,1,1,t} & \mathbf{x}^{*}_{1,1,2,0,t} & \mathbf{x}^{*}_{1,1,2,1,t} \\
\mathbf{x}^{*}_{2,1,1,0,t} & \mathbf{x}^{*}_{2,1,1,1,t} & \mathbf{x}^{*}_{2,1,2,0,t} & \mathbf{x}^{*}_{2,1,2,1,t}
\end{bmatrix}
\]
With \( L = \sum_{i=1}^{T} T_i = 4 \) the selection matrix of Equation (3.14) writes

\[
S_{4 \times 16} = (I_4 \ast I_4)' = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 
\end{bmatrix}.
\]

The above matrices can then be used to rewrite the Almon MIDAS type MFVAR from Equation (3.5) into the representation given in Equation (3.15). As shown in Section 3.2 we can further transform the MFVAR into the linear system of equations displayed in Equation (3.16), namely \( y_t = S(X_t \otimes I_L) \text{vec}(\Theta) + \epsilon_t \). In case of this example with one monthly and one quarterly variable,

\[
S(X_t \otimes I_4) = \begin{bmatrix}
x_{1,3,1,0,t}^* & 0 & 0 & 0 & x_{1,3,1,1,t}^* & 0 & 0 & 0 & \ldots \\
0 & x_{1,2,1,0,t}^* & 0 & 0 & 0 & x_{1,2,1,1,t}^* & 0 & 0 & \ldots \\
0 & 0 & x_{1,1,1,0,t}^* & 0 & 0 & 0 & x_{1,1,1,1,t}^* & 0 & \ldots \\
0 & 0 & 0 & x_{2,1,1,0,t}^* & 0 & 0 & 0 & x_{2,1,1,1,t}^* & \ldots \\
\ldots & x_{1,3,2,0,t}^* & 0 & 0 & 0 & x_{1,3,2,1,t}^* & 0 & 0 & 0 \\
\ldots & 0 & x_{1,2,2,0,t}^* & 0 & 0 & 0 & x_{1,2,2,1,t}^* & 0 & 0 \\
\ldots & 0 & 0 & x_{1,1,2,0,t}^* & 0 & 0 & 0 & x_{1,1,2,1,t}^* & 0 \\
\ldots & 0 & 0 & 0 & x_{2,1,2,0,t}^* & 0 & 0 & 0 & x_{2,1,2,1,t}^* 
\end{bmatrix}
\]
and

$$\text{vec}(\Theta) = \begin{bmatrix}
\theta_{1,3,1,0} \\
\theta_{1,2,1,0} \\
\theta_{1,1,1,1} \\
\theta_{2,1,1,0} \\
\theta_{1,3,1,1} \\
\theta_{1,2,1,1} \\
\theta_{1,1,1,1} \\
\theta_{2,2,1,0} \\
\theta_{1,2,2,0} \\
\theta_{1,3,2,1} \\
\theta_{1,2,2,1} \\
\theta_{1,1,2,1} \\
\theta_{2,1,2,1}
\end{bmatrix},$$

so that the linear system of equations of Equation (3.16) actually writes

$$\begin{bmatrix}
y_{1,t} \\
y_{1,t-\frac{1}{2}} \\
y_{1,t-\frac{3}{2}} \\
y_{2,t}
\end{bmatrix} = \begin{bmatrix}
\theta_{1,3,1,0} \cdot x_{1,3,1,0,t} + \theta_{1,3,1,1} \cdot x_{1,3,1,1,t} + \theta_{1,3,2,0} \cdot x_{1,3,2,0,t} + \theta_{1,3,2,1} \cdot x_{1,3,2,1,t} \\
\theta_{1,2,1,0} \cdot x_{1,2,1,0,t} + \theta_{1,2,1,1} \cdot x_{1,2,1,1,t} + \theta_{1,2,2,0} \cdot x_{1,2,2,0,t} + \theta_{1,2,2,1} \cdot x_{1,2,2,1,t} \\
\theta_{1,1,1,0} \cdot x_{1,1,1,0,t} + \theta_{1,1,1,1} \cdot x_{1,1,1,1,t} + \theta_{1,1,2,0} \cdot x_{1,1,2,0,t} + \theta_{1,1,2,1} \cdot x_{1,1,2,1,t} \\
\theta_{2,1,1,0} \cdot x_{2,1,1,0,t} + \theta_{2,1,1,1} \cdot x_{2,1,1,1,t} + \theta_{2,1,2,0} \cdot x_{2,1,2,0,t} + \theta_{2,1,2,1} \cdot x_{2,1,2,1,t}
\end{bmatrix} + \begin{bmatrix}
\epsilon_{1,t} \\
\epsilon_{1,t-\frac{1}{2}} \\
\epsilon_{1,t-\frac{3}{2}} \\
\epsilon_{2,t}
\end{bmatrix}.$$
Appendix C

Appendix Chapter 5

Parameter estimates for all models and forecast horizons

In this section the parameter estimates for all models and forecast horizons are presented. In order to save space in the graphs the used variables are abbreviated. An overview can be found in the following table. If the variable name has no ending, it is used in its original form. Three different transformations are also used, when appropriate, namely year-on-year growth rates, 3-month growth rates as well as month-on-month growth rates. These are labelled by the addition of 1y, 3m or 1m respectively as ending of the variable name.
## Abbreviations of variables

<table>
<thead>
<tr>
<th>VARIABLE NAME</th>
<th>ABBREVIATION</th>
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<tbody>
<tr>
<td>Industrial production index: manufacturing</td>
<td>hfindpro</td>
</tr>
<tr>
<td>Consumer price index</td>
<td>hfcpi</td>
</tr>
<tr>
<td>Housing starts</td>
<td>hfhstarts</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>hfunemp</td>
</tr>
<tr>
<td>ISM index for manufacturing</td>
<td>hflSMTot</td>
</tr>
<tr>
<td>ISM index for supplier delivery times</td>
<td>hflSMsupply</td>
</tr>
<tr>
<td>ISM index for orders</td>
<td>hflSMorder</td>
</tr>
<tr>
<td>Average weekly hours of production and supervisory workers</td>
<td>hflhworked</td>
</tr>
<tr>
<td>S&amp;P 500 stock market index</td>
<td>hfsp500</td>
</tr>
<tr>
<td>3-month treasury bill yield</td>
<td>hfi3m</td>
</tr>
<tr>
<td>10-year treasury bond yield</td>
<td>hfi10y</td>
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</tbody>
</table>
Parameters for bridge equation
Parameters for U-MIDAS model with $h = 1$
Parameters for U-MIDAS model with $h = 2$
Parameters for U-MIDAS model with $h = 3$
Parameters for U-MIDAS model with $h = 4$
Parameters for U-MIDAS model with $h = 5$
Results for full sample for all models

Rel. RMSFE: U-MIDAS vs Bridge using OLS

Note: The dashed line shows the relative RMSFE of bridge equations compared to the benchmark model U-MIDAS. Values below zero indicate the smaller percentage forecast error of bridge equations compared to U-MIDAS. The solid line indicates significance of results according to the Diebold-Mariano test.
Rel. RMSFE: U-MIDAS vs Bridge using TVP

Note: The dashed line shows the relative RMSFE of bridge equations compared to the benchmark model U-MIDAS. Values below zero indicate the smaller percentage forecast error of bridge equations compared to U-MIDAS. The solid line indicates significant results according to the Diebold-Mariano test.

Rel. RMSFE: OLS vs TVP using Bridge

Note: The dashed line shows the relative RMSFE of TVP compared to the benchmark method OLS. Values below zero indicate the smaller percentage forecast error of TVP compared to OLS. The solid line indicates significant results according to the Diebold-Mariano test.
Rel. RMSFE: OLS vs TVP using U-MIDAS

Note: The dashed line shows the relative RMSFE of TVP compared to the benchmark method OLS. Values below zero indicate the smaller percentage forecast error of TVP compared to OLS. The solid line indicates significant results according to the Diebold-Mariano test.
Results for Great Recession

Rel. RMSFE: U-MIDAS vs Bridge using OLS

Note: The dashed line shows the relative RMSFE of bridge equations compared to the benchmark model U-MIDAS. Values below zero indicate the smaller percentage forecast error of bridge equations compared to U-MIDAS. The solid line indicates significant results according to the Diebold-Mariano test.
Note: The dashed line shows the relative RMSFE of bridge equations compared to the benchmark model U-MIDAS. Values below zero indicate the smaller percentage forecast error of bridge equations compared to U-MIDAS. The solid line indicates significant results according to the Diebold-Mariano test.
Rel. RMSFE: OLS vs TVP using Bridge

Note: The dashed line shows the relative RMSFE of TVP compared to the benchmark method OLS. Values below zero indicate the smaller percentage forecast error of TVP compared to OLS. The solid line indicates significant results according to the Diebold-Mariano test.
Rel. RMSFE: OLS vs TVP using U-MIDAS

Note: The dashed line shows the relative RMSFE of TVP compared to the benchmark method OLS. Values below zero indicate the smaller percentage forecast error of TVP compared to OLS. The solid line indicates significant results according to the Diebold-Mariano test.
Curriculum Vitae

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