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Ambient seismic source inversion

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Welcome to reality, baby.

— S. Bolay

To Tobelhof.
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Abstract

Ocean waves and other phenomena occurring at the Earth’s surface interact with the Earth’s crust and cause faint seismic signals that can be measured at great distance. The sources of these ambient vibrations are of long-standing interest in seismology, both in their own right, as they carry information about environmental processes and conditions, and because they persistently probe the Earth’s interior, providing signals for nearly continuous imaging and monitoring of subsurface structure even in areas of low seismicity.

Here, we present the first application of an iterative inversion method for the sources of ambient seismic noise with a three-dimensional Earth model. In a step leading up to inversion, we investigate how robust information about noise source properties can be derived from cross-correlations of continuously recorded ambient noise. Signal energy ratios of ambient signals traveling in opposite directions can be used to rapidly elaborate first-order estimates of ambient noise source distribution at a regional and global scale. At the regional scale, windowed signal energy measurements taken on the cross-correlation reflect the rapidly changing ambient noise field excited by passing storms.

Based on a Green’s function database approach, we numerically model cross-correlations of the ambient seismic noise in a three-dimensional Earth model with laterally varying seismic structure. This allows us to construct a gradient-based, non-linear iterative inversion for the time-, location- and frequency dependent source power spectral density of ambient noise that honours the three-dimensional structure of the Earth’s interior.

We apply this inversion to ten-year averaged observations of vertical-component ambient noise recorded in North and South hemisphere winter, in order to image the sources of the Earth’s hum, which is the long-periodic background seismic signal. The results reveal seasonally varying, narrowly delineated areas of high hum excitation, predominantly located at Pacific shelves or coasts during North Hemisphere winter, and at Southern Ocean locations of shallow bathymetry, as well as South Pacific shelves or coasts during austral winter.

The investigated inversion method contributes to the development of full-waveform inversion with ambient noise cross-correlations. Future extension to horizontal- and mixed-component cross-correlations, as well as applications to ambient noise at the regional scale, may help advance our understanding of ambient noise excitation processes.
Zusammenfassung

Durch die Wirkung von Ozeanwellen und anderen an der Erdoberfläche auftretenden Phäno
momenen auf die Erdkruste werden schwache seismische Wellen ausgelöst, die über weite
Distanzen hinweg gemessen werden können. Die Quellen dieser seismischen Bodenunruhe
werden seit langem seismologisch untersucht, zum einen, da ihre Anregung selbst Infor-
mationen über Umweltprozesse und -bedingungen widerspiegelt, zum anderen, da sie das
Erdinnere ständig durchlaufen und somit nahezu kontinuierlich Signale zu dessen Beobach-
tung generieren, was besonders in Regionen mit geringer Erdbebenrate von großem Interesse
ist.

In der folgenden Arbeit wird die erste Anwendung einer iterativen Inversion nach den Quellen
der seismischen Bodenunruhe mit einem dreidimensionalen Erdmodell vorgestellt. In einem
vorbereitenden Schritt wird untersucht, wie man aus Kreuzkorrelationen der kontinuierlich
aufgenommenen Bodenunruhe eine robuste Abschätzung erster Ordnung für deren Quellei-
genschaften ermitteln kann. Es zeigt sich, dass das Verhältnis der Signalenergien gegenläufig
propagierender Wellen der Bodenunruhe genutzt werden kann, um unmittelbar auf die un-
gefähre Verteilung von sowohl regional als auch global verteilter Bodenunruhequellen zu
schließen. In einer regionalen Studie zeigt die Signalenergie der Kreuzkorrelation in ausgewähl-
ten Laufzeitfenstern das schnell variierende Bodenunruhefeld an, das durch vorbeiziehende
Stürme ausgelöst wird.

Basierend auf einer Datenbank Green'scher Funktionen können Kreuzkorrelationen der Bo-
denunruhe numerisch modelliert werden, wobei auch die dreidimensionale Struktur der
Erde mit lateral variierenden seismischen Geschwindigkeiten berücksichtigt wird. Anhand
dieses Modells wird ein gradientenbasiertes, iteratives Inversionsverfahren für die orts- und
frequenzabhängige spektrale Leistungsdichte der Bodenunruhequellen aufgestellt.

Dieses Inversionsverfahren wird auf Kreuzkorrelationen der Vertikalkomponente der seis-
mischen Bodenunruhe, gestapelt über einen Beobachtungszeitraum von je drei Monaten
aus zehn Jahren, angewandt, um Karten von den Quellen des seismischen "Hum", des lang-
periodischen seismischen Hintergrundrauschens der Erde, zu erstellen. Die resultierenden
Karten zeigen jahreszeitlich schwankende, eng begrenzte Gebiete hoher Anregung, die sich im
nördlichen Winter hauptsächlich an den Pazifischen Kontinentschelfrändern oder Küsten,
und im südlichen Winter in Gegenen des Südlichen Ozeans mit hoher Bathymetrie, sowie im
Schelfbereich und an den Küsten des Südpazifiks befinden.

Das untersuchte Inversionsverfahren ist ein wichtiger Schritt zur Weiterentwicklung der tomog-
graphischen Wellenforminversion von Kreuzkorrelationen der Bodenunruhe. Die zukünftige
Ausweitung der Methode auf die Kreuzkorrelationen horizontal polarisierter Bodenunruhe, sowie die Anwendung auf regionaler Skala, können darüber hinaus neue Erkenntnisse zum Verständnis der physikalischen Anregung der Bodenunruhe beitragen.
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From the earliest days of instrumental seismology, seismologists not only studied earthquakes, signals of obvious relevance to many societies, but also took an interest in anything else that their increasingly sensitive instruments recorded (Bertelli, 1872; Gutenberg, 1912; Omori, 1913). They found a plethora of signals, including ground motions they associated with meteorological low pressure systems, ocean waves, wind, weather conditions, and human activity. Early studies on these motions were focused on their origin and propagation, and on exploring their meteorological meaning.

Early authors, like Omori (1913) and Gutenberg (1931), already noted that these signals carry information about the medium of propagation. Specifically, they remarked upon the local subsurface at the recording site, and regional "barriers" to microseism propagation. The difficulty of quantitatively extracting this information lay in the stochastic nature of the signal: Clear phase onsets are not visible, and spectral characteristics had to be evaluated manually from analog recordings, which allowed the observation of characteristic amplitudes and periods, but made the computation of cross-spectra or other statistic treatment nearly impossible.

Building on earlier work by K. Akamatsu, Aki (1957), using a specially built computer, succeeded in analyzing spatial cross-spectra of microseisms over short ranges for the inference of shallow subsurface shear wave velocities. Almost fifty years later, the idea of retrieving coherently propagating waves from the ambient seismic field by cross-correlation was applied to microseisms over long ranges - more than 1000 km - and it was found successful (Shapiro and Campillo, 2004). This led to the development of ambient noise tomography (Sabra et al., 2005; Shapiro et al., 2005), which has evolved into a standard method in seismology, with a multitude of applications in various regions and at various scales, during the last decade.

At the same time, interest in the sources of ambient signals also rose again, not least due to the increasing availability of high-quality continuous recordings and of the capacity to digitally store and process them (Larose et al., 2015). This gave rise to a new field of environmental seismology, which links seismology with other geoscientific disciplines, most notably hydrology,
glaciology and oceanography. However, as in the case of earthquake signals, the imprint of both source and propagation medium are intimately linked in the recorded ground motions. To study one always requires studying, or making assumptions about, the other. The work presented here is part of a larger collaborative effort to drop assumptions and, sequentially or jointly, account for both. Therefore, a programmatic sentence from the paper of Aki (1957) summarizes the fundamental motivation of this thesis:

"The object [...] is to develop a method for dealing with those complicated waves in order that the nature of the waves, as well as the nature of the medium of propagation may be revealed."

In the following introduction, the state of our knowledge about ambient signals will first be briefly sketched, including the current state of research on ambient noise source inversion. After describing current limitations of ambient noise tomography, the motivation of the project will then be described in more detail.

1.1 Overview of the ambient seismic field

Throughout this thesis, ambient seismic waves are defined as continuously occurring signals due to sources at the surface of the solid Earth, and without relation to earthquakes. Traditionally, they are referred to as ambient 'noise', or, specifically for the two strongest noise peaks, as microseisms, and we will mostly use these designations. Reviews of different aspects of the ambient seismic field are found in Gutenberg (1912, 1958), Bonnefoy-Claudet et al. (2006) and Ebeling (2012), as well as Tanimoto et al. (2015).

An impression of the rich variety of ambient signals recorded at seismic stations is provided by the review of Díaz (2016). On average, however, power spectral densities of the noise worldwide show similar characteristic features. These are summarized in the New Low and High Noise Models by Peterson (1993) and give rise to a broad classification of the noise into different frequency ranges. The three characteristic features which are relevant to applications presented here are the double-frequency or secondary and the primary microseism, as well as Earth’s hum. These are illustrated in Fig. 1.1 and will be discussed in more detail below.

Noise at periods shorter than 1 second (not shown in Fig. 1.1) is generally referred to as high-frequency noise (e.g. Díaz, 2016), and is thought to be dominated by anthropogenic noise and local meteorological conditions (e.g. Bonnefoy-Claudet et al., 2006; Díaz, 2016), although many of the signals considered in environmental seismology, such as river discharge (Burtin et al., 2008; Gimbert et al., 2014; Díaz, 2016), rock falls (Gualtieri and Ekström, 2017) or glacial moulin tremors (Röösli et al., 2016) equally fall into this frequency range. Recently, there has also been increasing focus on observations (Zhang et al., 2010; Pyle et al., 2015; Möllhoff and Bean, 2016), and modeling (Gimbert and Tsai, 2015) of short-period ocean-generated noise at frequencies above 1 Hz. Although the method development shown below in principle holds
1.1. Overview of the ambient seismic field

Figure 1.1 – Probabilistic power spectral density plot illustrating the characteristic features of the noise spectrum. Gray lines are Peterson’s New Low and High Noise model (see text for reference). The hum period band is loosely defined. Some studies include all signals at periods longer than those of the primary microseism, while others focus on the Earth’s normal mode peaks observable in the hum between approx. 2 and 7 mHz.

for high-frequency noise sources, we have not explicitly considered them in our applications.

Before turning to the characteristic peaks of the ambient seismic field, one should mention the existence of particular regional signals, which due to their emergent character and persistence are sometimes considered part of the noise. Two examples are the Gulf of Guinea signal that has a dominant period of 26 s and can be persistently observed at stations worldwide (Shapiro et al., 2006; Gaudot et al., 2016), and the Kyushu microseism, which has a dominant period of approx. 11 s (Zheng et al., 2011). Since the Gulf of Guinea signal is tentatively (Xia et al., 2013) and the Kyushu signal conclusively (Zeng and Ni, 2011) linked to volcanic activity, they do not fall under our rather restrictive definition of ambient sources. However, their near-surface location means they may be misinterpreted as such if one does not take into account the results of Shapiro et al. (2006); Gaudot et al. (2016); Xia et al. (2013); Yamamoto et al. (1999).

1.1.1 Ocean microseisms

In Fig. 1.1, the two highest spectral peaks are the secondary or double-frequency, and the primary microseisms. Although their physical excitation mechanisms differ, and we will therefore discuss them separately below, they also share a number of characteristics: Ubiquitous occurrence (e.g. Peterson, 1993), oceanic excitation (e.g. Haubrich and McCamy, 1969; Cessaro, 1994), seasonality (e.g. Stehly et al., 2006; Gerstoft and Tanimoto, 2007; Stutzmann et al., 2009;
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Traer et al., 2012; Tian and Ritzwoller, 2015; Juretzek and Hadziioannou, 2016), and the possibility to link their observation on seismological records to the occurrence of particular storms (e.g. Gerstoft et al., 2006).

Secondary microseism

The secondary microseism peak ranging from 1 to 10 s period consists predominantly of fundamental mode Rayleigh surface waves, with further contributions of Love waves (e.g. Haubrich and McCamy, 1969; Friedrich et al., 1998; Juretzek and Hadziioannou, 2016), of compressional body waves (e.g. Vinnik, 1973; Gerstoft et al., 2008; Landès et al., 2010), and, as has been reported recently, of weak shear body waves (Liu et al., 2016; Nishida and Takagi, 2016). Higher mode surface waves are less readily observed (examples include Nishida et al., 2008a; Kimman et al., 2012, for Love and Rayleigh higher modes), and appear to occur more commonly at the high-frequency end of the peak (Gal et al., 2015).

It was hypothesized early on that secondary microseisms are excited by ocean waves (Wiechert, 1904). Their oceanic origin has been demonstrated by a range of studies identifying their predominant source locations and global-scale seasonality linked to hemispheral storm patterns (e.g. Stehly et al., 2006; Landès et al., 2010; Traer et al., 2012). Observations have further revealed source locations related to particular storms (e.g. Gerstoft et al., 2006; Zhang et al., 2010; Davy et al., 2014; Gualtieri et al., 2014; Lin et al., 2014; Farra et al., 2016; Nishida and Takagi, 2016). Secondary microseisms have been studied extensively, not only to reveal their source locations, but also to use them as a proxy for ocean wave states, present and past wave climate (e.g. Aster et al., 2008; Stutzmann et al., 2009; Aster et al., 2010; Ardhuin et al., 2011; Donne et al., 2014), sea ice variability (e.g. Pedersen et al., 2007; Yao and van der Hilst, 2009; Harmon et al., 2010; Kimman et al., 2012; Sadeghisorkhani et al., 2016). Their physical excitation mechanism was elucidated by Longuet-Higgins (1950) and Hasselmann (1963). While pressure fluctuations due to ocean waves to first order decay exponentially with water depth, and therefore do not reach the deep ocean floor, the interaction of ocean wave trains of similar frequency traveling in nearly opposite directions creates a second order pressure term that does not decay, and therefore acts as a pressure source on the ocean floor. The frequency of this source is the sum of frequencies of the interacting waves, hence the term double-frequency. Kedar et al. (2008) first modeled microseismic sources based on this mechanism using ocean wave hind-casts, and showed that the resulting seismic displacements due to Rayleigh surface waves compared well to observed ones. Hillers et al. (2012) compared a similarly elaborated source model to source locations inferred from observed microseismic P-waves by Landès et al. (2010). Ardhuin et al. (2011, 2012) included coastal reflection coefficients into the modeling. With the aim to use seismological observations as proxy for ocean wave states, they also provided a detailed discussion of the different classes of wave states that produce the opposing wave trains, required for the double-frequency mechanism. Stutzmann et al. (2012) modelled microseismic noise power spectra in various environments. Their study illustrated the trade-off, within
models of microseism excitation, between the anelastic attenuation of seismic waves and the coastal reflection coefficient needed to quantify the level of the microseism sources. At many locations, an additional parameter was required to take up the (unmodelled) effects of 3-D heterogeneous Earth structure, which can for example affect seismic amplitudes by focusing. While all previous models had approached the problem mainly from the oceanographic side, and had treated elastic wave propagation of the seismic waves by strongly simplified models such as homogeneous half-spaces, Gualtieri et al. (2013) introduced the use of a spherically symmetric Earth model, with the layered structure from PREM (Dziewoński and Anderson, 1981) to account for seismic wave propagation. The use of PREM with included ocean layer permitted them to replace the excitation coefficients from Longuet-Higgins’ original work by modelled amplification factors.

To understand the relation of microseisms and ocean wave states on a global scale, an important question is whether excitation is dominated by coastal or deep-water sources. While some observational studies found evidence of predominant excitation in shallow water (e.g. Bromirski, 2001; Bromirski and Duennebier, 2002; Schulte-Pelkum et al., 2004; Bromirski et al., 2005; Traer et al., 2012), others reported a predominance of pelagic sources (Stehly et al., 2006; Landès et al., 2010; Obrebski et al., 2013; Euler et al., 2014; Tian and Ritzwoller, 2015; Liu et al., 2016). Numerical models (Gualtieri et al., 2015) suggest that the ocean-to-land propagation is frequency-dependent, and that pelagic sources form an important contribution to the lower half of the secondary microseism peak. Further studies (Gualtieri et al., 2013; Sergeant et al., 2013; Gualtieri et al., 2014; Nishida and Takagi, 2016) underlined the importance of site-specific conditions, in particular bathymetry and the presence of sedimentary deposits, on the excitation and propagation of both surface and body wave secondary microseisms, and therefore on our ability to observe them on seismological records.

While vertical-component secondary microseisms due to both Rayleigh waves and compressional body waves are well understood through the models of Longuet-Higgins (1950); Hasselmann (1963) and Ardhuin et al. (2011), the excitation mechanism of their horizontally polarized counterparts remain unclear (Juretzek and Hadziioannou, 2016), although both Love waves (Haubrich and McCamy, 1969; Nishida et al., 2008b; Stehly et al., 2009; Juretzek and Hadziioannou, 2016) and SH body waves (Liu et al., 2016; Nishida and Takagi, 2016) have been observed. One hypothesis for their generation is P-to-S conversion due to scattering by sedimentary layers and bathymetric irregularities (Nishida and Takagi, 2016; Liu et al., 2016).

**Primary microseism**

The less prominent primary microseismic peak occurs at the frequency of ocean gravity waves. Hasselmann (1963) derived the pressure sources induced by shoaling of ocean waves as they run up to the shore. Ardhuin et al. (2015) modeled this excitation mechanism using realistic ocean wave spectra, and showed that it explains the power spectrum of noise at periods longer than approx. 13 s. Excitation by shoaling can occur at monotonous bathymetric slopes, but is amplified by small-scale bathymetric variations (Ardhuin et al., 2015). An analogy to the
Chapter 1. Introduction

double-frequency mechanism can be made by regarding the undulating bathymetry of the ocean bottom as a ‘frozen wave train’ of zero frequency.

The mechanism by Hasselmann (1963) and Arduhn et al. (2015) requires primary microseism to be generated in near-coastal shallow water, and several studies have localized its sources there. In particular, Cessaro (1994) and Friedrich et al. (1998) have used triangulation from several arrays to demonstrate near-coastal excitation; this is supported by the results of Juretzek and Hadziioannou (2016), although they do not explicitly perform triangulation. Traer et al. (2012) and Möllhoff and Bean (2016) identify near-coastal areas as most probable source locations by comparison of observed primary microseism levels and significant wave height maps. In many other studies, directions of incidence estimated from surface waves point towards regional (e.g. Pedersen et al., 2007; Kimman et al., 2012; Möllhoff and Bean, 2016) or both regional and remote coastlines (e.g. Schulte-Pelkum et al., 2004; Tian and Ritzwoller, 2015), while the distance to source is not constrained.

As in the case of secondary microseisms, the characteristics of the observed primary microseism field depend on the geographic location of observation. For example, Sadeghisorkhani et al. (2016) show that the Swedish National Seismic Network reports dominant incidence from the West Pacific coast, contrary to all other studied arrays in Europe (Chevrot et al., 2007; Friedrich et al., 1998; Pedersen et al., 2007; Juretzek and Hadziioannou, 2016; Möllhoff and Bean, 2016). Another example is the primary microseism variability due to sea ice cover changes in Antarctica (Stutzmann et al., 2009).

Observational studies from various regions find that, contrary to the secondary microseism, the most energetic signal in the primary microseism are Love surface waves (Haubrich and McCamy, 1969; Friedrich et al., 1998; Nishida et al., 2008b; Juretzek and Hadziioannou, 2016). The mechanism by Arduhn et al. (2015) appears to extend naturally to horizontal component excitation, because a vertical pressure source acting on a sloping sea floor decomposes into a normal and a tangential component. A mechanism by which wave pressure interacts with sea floor irregularity to create a traction source, was hypothesized by Nishida et al. (2008b) and formulated by Saito (2010). (Fukao et al., 2010, presented a related study, but for the long-period noise, see below; their study does not account for noise in the primary microseism band). As yet, there is no study published investigating the shear components of the primary mechanism in a quantitative manner for realistic ocean wave and seismic wave propagation models in the primary microseism frequency range analog to the vertical-component model by Arduhn et al. (2015).

Tentative evidence of open-ocean sources of primary microseism body waves in the Indian Ocean was found by Landès et al. (2010) and Liu et al. (2016). To be consistent with the mechanism of Arduhn et al. (2015), these would have to be linked to shallow bathymetry, e.g. around islands. However, primary microseism body waves are not readily observed, as they are usually masked by primary microseism surface waves, which experience little attenuation due to their long periods. The presented results therefore suffer rather large observational
1.1. Overview of the ambient seismic field

uncertainty and their precise source regions are not well constrained (Landès et al., 2010).

Quintessentially, the synopsis of modeling and observational results from global-scale and regional studies of both primary and secondary microseisms underlines that microseismic noise, far from being globally uniform, reflects the influence of both regional and remote geographic and oceanographic conditions. Questions remain about the relative contribution of coastal and pelagic sources to the secondary microseism, as well as about the excitation of microseismic Love waves.

1.1.2 Earth's hum

The term ‘Earth's hum’ was first coined to describe the modal peaks of the Earth's free oscillations excited by ambient sources (Nishida et al., 2000). Here, we use it to refer to the low broad peak of Peterson's New Low Noise Model stretching from roughly 2 to 15 mHz (see Fig. 1.1), although isolated normal mode peaks are only observed between approximately 2 and 7 mHz (Tanimoto et al., 2015).

Hum signals are far less energetic than the microseismic noise, and they therefore could not be detected in the studies of Gutenberg (1912); Omori (1913) and other early authors. Benioff et al. (1959) hypothesized that Earth's normal modes must be excited by ambient sources such as the ocean, and searched for spectral peaks below 1 mHz with the aim of using them for inferences of Earth structure. This search was unsuccessful due to the limited sensitivity of their instruments. The first report of continuously excited spheroidal normal modes was finally made by Nawa et al. (1998) from a quiet gravimeter in Antarctica (although Tanimoto et al. (2015) questions their result, citing Suda et al. (1998) as first reliable detection of the signal). Further detections at quiet gravimeter and seismometer sites followed (Tanimoto et al., 1998; Kobayashi and Nishida, 1998). Kurrle and Widmer-Schnidrig (2008) succeeded in observing also continuously excited toroidal modes, which they reported to be of similar amplitude as the spheroidal ones. There is little evidence of observable body waves in the hum. Although long-period cross-correlations showing body wave phases have been presented by Nishida (2013), these body wave phases appear to be dominated by the shorter periods included in their study (around 30 s; a similar analysis by Boué et al. (2014) also focuses on periods of less than 100 s).

Early studies established that the incessant modal peaks of the hum could not be explained solely by the cumulative effect of small earthquakes (Suda et al., 1998). This was corroborated when seismic observations based on time series analysis (Ekström, 2001), beamforming (Rhie and Romanowicz, 2004), and linearized inversion of cross-correlations (Nishida and Fukao, 2007) revealed seasonal modulation of the signal. Therefore, an atmospheric or oceanic source was suggested; Rhie and Romanowicz (2004, 2006) invoked coupling of ocean infragravity waves to the ocean floor as the most plausible mechanism. This view is now generally accepted, although questions remain about the possible contribution of atmospheric turbulences by resonant coupling (Nishida, 2014).
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However, there has been considerable debate by what precise physical mechanism infragravity waves couple to the seismic waves that propagate at far higher wave speeds than the ocean waves. Webb (2007, 2008) proposed to extend the double-frequency mechanism of Longuet-Higgins (1950) to longer periods. Fukao et al. (2010) emphasized the importance of shear waves, based on the findings of toroidal modes by Kurrle and Widmer-Schnidrig (2008). They proposed a source mechanism of shear traction due to direct coupling of ocean waves and topographic irregularities of the same wavelength. Traer and Gerstoft (2014) suggested a double-frequency mechanism in which the difference term of ocean wave frequencies, rather than their sum, determines the frequency of the seismic waves (see also Gerstoft and Bromirski, 2016). This would allow for hum excitation by oblique wave trains, and would only occur in shallow water depths on the order of 100 m. They had previously reported that beam power time series of the hum correlate with significant wave height time series in coastal, but not deep-water regions, indicating near-coastal areas as preferential source locations (Bromirski and Gerstoft, 2009; Traer et al., 2012).

Unconvinced, Ardhuin et al. (2015) demonstrated that observed hum power spectra could instead be modeled by excitation akin to the primary microseism (Hasselmann, 1963), with long-wavelength infragravity waves coupling to the sea floor due to bathymetric undulations or slopes. An important prediction of their model is that locations of steep bathymetric gradients like shelf breaks form narrow areas of preferential hum excitation. Hence, a current point of debate is the relative importance of coastal, shelf and pelagic hum sources. Bromirski and Gerstoft (2009); Traer et al. (2012) and Traer and Gerstoft (2014) emphasized the dominance of coastal source regions, Ardhuin et al. (2015) linked hum excitation to shelf breaks, and Nishida and Fukao (2007), based on their inversion results, argued that pelagic sources contribute as well. Furthermore, the above described mechanisms (Webb, 2007; Traer and Gerstoft, 2014; Ardhuin et al., 2015) model pressure only and do not explicitly take the excitation of horizontal-component hum into account. Fukao et al. (2010) and Nishida (2014) have emphasized that shear traction sources, which may be caused by the interaction of the pressure of infragravity waves with sloping bathymetry, play an important role. Analog to the primary microseism, it is conceivable that the model by Ardhuin et al. (2015) ‘automatically’ accounts for the required horizontal forces. At this moment, to quantitatively model the excitation of the horizontal components of the Earth’s hum is a matter of ongoing research.

1.1.3 Current methods for ambient source imaging and inversion

With microseisms having intrigued seismologists for more than a century, it is not surprising that various methods have been applied and developed for locating their origin. These methods are partly related to those developed for earthquakes (e.g. back projection used by Xu et al., 2009; Liu et al., 2016), partly they are designed differently, due to the inability to the lack of phase onsets in the noise, as mentioned by Aki (1957).
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In the simplest case, a single three-component station may be available. Early studies have investigated microseism characteristics from time-varying power spectral densities and have identified likely source areas through high correlation between the time series of noise power at seismic stations and significant wave height at buoys (e.g. Bromirski, 2001; Bromirski and Duennebier, 2002; Essen et al., 2003). Directions of arrival can furthermore be estimated by particle motion polarization analysis under the assumption that the noise is primarily composed of fundamental mode surface waves (Schulte-Pelkum et al., 2004; Chevrot et al., 2007; Schimmel et al., 2011).

In the next better case, one may have two stations or a sparse network at one’s disposal. A number of noise studies have utilized noise cross-correlations for inferences about the sources (Stehly et al., 2006; Yang and Ritzwoller, 2008; Tian and Ritzwoller, 2015; Delaney et al., 2017, and chapter 4 of this thesis). Asymmetric noise cross-correlation functions, where either the causal or the anti-causal branch contain a higher-amplitude signal, are observed when sources are dominantly located behind one station only. Stehly et al. (2006) inferred the noise source direction directly from the normalized noise amplitudes of Rayleigh wave arrivals, whereas Yang and Ritzwoller (2008) and Tian and Ritzwoller (2015) used the signal-to-noise ratio of both branches. An important motivation for studying noise sources using noise cross-correlations (besides having sparse station coverage) is, as Tian and Ritzwoller (2015) mention, that ambient noise tomography is done on stacks of noise cross-correlation functions. The processing and stacking involved to obtain them can change the way the cross-correlations ‘see’ the noise sources, which is hinted at by the seemingly paradoxical results of Stehly et al. (2006) and Yang and Ritzwoller (2008), who find contradictory results about the coastal vs. pelagic origin of microseism sources, due to their different processing strategies. Performing source imaging on a set of pre-processed cross-correlations yields an effective source distribution which includes this effect, as is discussed by Fichtner et al. (2017a).

In the best case, one may have a dense array or network available. A widely used method for locating noise sources is beamforming (e.g. Schulte-Pelkum et al., 2004; Bromirski and Gerstoft, 2009; Kurrle and Widmer-Schnidrig, 2006; Rhie and Romanowicz, 2004; Traer et al., 2012; Reading et al., 2014; Gal et al., 2015). Azimuth and propagation speed which yield the largest beam power at an array are interpreted as directions of arrival of the main noise component. Recent advances in beamforming (Gal et al., 2016) and other array processing techniques (Craig et al., 2016) allow the separation of contributions from multiple different sources. Beamforming is used on all scales: Rhie and Romanowicz (2004) estimated incidence angles of seismic hum using a fixed propagation velocity, and thereby succeeded to map large-scale hum excitation areas. Using beamforming with variable propagation velocity, Schulte-Pelkum et al. (2004) found evidence for the strong directionality of ambient noise, for the importance of wave propagation effects on noise recording, and for the excitation of microseismic noise by distant storms. Most commonly, although not necessarily, beamforming is performed based on the assumption of plane waves incident from sources at a large distance, and the subsurface below the array is assumed homogeneous.
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The above techniques - polarization analysis, cross-correlation signal energies and beamforming - provide measurements that indicate directions of arrival; to obtain estimates of the source regions themselves, an additional step such as back-projection is necessary. Depending on the scope of studies, back projection can be performed with simple or elaborate Earth models (Liu et al., 2016, use spherically symmetric ak135 (Kennett et al., 1995), for example). Nishida and Takagi (2016) emphasize the importance of station corrections for back-projection of microseismic body waves.

Schulte-Pelkum et al. (2004) and Chevrot et al. (2007) compared results from polarization analysis to those from beamforming. Directions of arrival estimated were approximately consistent, with directions from polarization analysis showing somewhat more scatter. Chevrot et al. (2007) furthermore attempted to combine direction-of-arrival estimates from array and polarization analysis with attenuation estimates to locate sources precisely. However, numerous empirical parameters needed to be fit to model absolute noise amplitudes, including site response and source energy. In addition, they allowed for one location along the great-circle propagation path between source and array only. Therefore, they identified very sharply determined source regions, but could not quantify the degree of uncertainty of these results. Finite wave propagation was taken into account by Brzak et al. (2009), who used a migration technique to locate sources in the Mediterranean. (A similar reasoning lies behind the approaches of Gaudot et al. (2016); Retailleau et al. (2016)). More recent studies have developed systematic inversion techniques for the azimuthal distribution of noise sources. Harmon et al. (2010) and Sadeghisorkhani et al. (2016) modeled the cross-correlation by a superposition of plane waves, and thus parametrize noise sources by a limited number of azimuthal weights. Sadeghisorkhani et al. (2016) optimized the fit of cross-correlation envelopes, which has the advantage of capturing cross-correlation signals propagating at high apparent velocities. Their inversions allow quantification of the source-induced travel time bias, but they do not resolve distance to sources, and their plane wave assumption may become unrealistic at close distance from sources. Nishida and Fukao (2007) inverted for the seasonal sources of the Earth’s hum using cross-correlation amplitudes and a forward model of cross-correlations based on normal modes. While one may account for realistic wave propagation with such a model, it is most commonly applied to long-wavelength variations and using a spherically symmetric Earth model.

Existing methods have provided great insight into the generation areas of ambient noise, but the major limitation which has not yet been addressed is to account for laterally heterogeneous Earth structure during ambient source inversion. Based on the source location methods of Nishida and Fukao (2007) and Brzak et al. (2009), this appears as the logical next step. It is expected to contribute to our understanding of microseism excitation by resolving source locations with higher accuracy and in more detail.
1.2 Waveform-based tomography with ambient noise

We now flip the coin from the influence that 3-D heterogeneous Earth structure has on the localization of noise sources, to the influence of noise sources on our understanding of heterogeneous seismic structure.

Since the developments of Shapiro and Campillo (2004); Sabra et al. (2005) and Shapiro et al. (2005), surface wave tomography based on ambient noise correlations has developed into a standard seismological technique (e.g. Yang et al., 2007; Lin et al., 2008; Nishida et al., 2008a; Stehly et al., 2009; Nishida et al., 2009; Behr et al., 2010; Zheng et al., 2011; Saygin and Kennett, 2012; Mordret et al., 2013, 2015; Korostelev et al., 2015; Green et al., 2017; Lehujeur et al., 2017). Besides the widely used method of regionalizing phase and group velocity measurements into maps and inverting for local depth profiles (e.g. Stehly et al., 2009), direct inversion methods have been proposed (Chen et al., 2014; Fang et al., 2015), large-N array data have been used for mapping phase speed by wavefront tracking (Lin et al., 2009), and time-lapse noise tomographies based on subtle waveform changes have been performed with the aim of subsurface monitoring, making use of the continuous availability of the ambient signal (de Ridder et al., 2014; Mordret et al., 2014). A range of further seismological techniques have been elaborated, which equally rely on ambient noise cross-correlations. These include coda wave interferometry for monitoring of small subsurface changes (e.g. Sens-Schönfelder and Wegler, 2006; Brenguier et al., 2008; Obermann et al., 2013), body wave tomography (e.g. Nakata et al., 2015) and the measurement of attenuation and ground motion amplification from cross-correlations (e.g. Prieto and Beroza, 2008; Denolle et al., 2013, 2014a; Viens et al., 2015; Bowden et al., 2015).

The idea of retrieving approximate inter-receiver Green’s functions from cross-correlations of ambient seismic noise is conceptually elegant, contributing to its immense popularity in the seismological community. A number of studies have therefore investigated theoretical derivations of the equivalence of noise cross-correlations and Green’s functions under different assumptions on the ambient noise wavefield. These assumptions include a favorable distribution of noise sources (Snieder, 2004; Roux et al., 2005), equipartitioning (Weaver and Lobkis, 2004; Sánchez-Sesma and Campillo, 2006; Gouédard et al., 2008), and excitation by uncorrelated mono- and dipolar point sources on a contour surrounding the receivers (Wapenaar and Fokkema, 2006). Reviews of these derivations can be found in Wapenaar et al. (2010); Boschi and Weemstra (2015).

As was discussed above, the characteristics of the ambient noise field in any location are determined by a complex interplay between excitation and propagation; therefore, these assumptions may be fulfilled in certain settings, but not in others. For example, some locations show a strongly directional noise wavefield (e.g. Kimman et al., 2012) while others show a more homogeneous, and therefore more favorable, distribution of incidence azimuths (e.g. Nishida et al., 2008a); regional geology may equally influence the noise wavefield, e.g. by scattering which enhances equipartition. The considerations of Snieder (2004); Roux et al.
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(2005); Sánchez-Sesma and Campillo (2006); Wapenaar and Fokkema (2006) aim to provide the theoretical underpinning to the concept of Green's function retrieval without explicitly modeling the heterogeneous noise field. While convenient, this approach prevents us from addressing a number of limitations on current ambient noise tomography.

Spatially varying noise excitation biases observed travel times (Tsai, 2009; Froment et al., 2010). Depending on the study region and the application, this effect may be negligible compared to the travelt ime residuals caused by structural heterogeneity (Yang and Ritzwoller, 2008; Harmon et al., 2010), or it may be important, e.g. if structural heterogeneity is weak or minute time-lapse travelt ime changes are to be imaged (Yao and van der Hilst, 2009; Delaney et al., 2017; Sadeghisorkhani et al., 2017). To mitigate this, it has been proposed to derive a model of the azimuthal source distribution, estimate the travel time bias based on a plane wave model of the cross-correlation, and remove it (e.g. Yao and van der Hilst, 2009; Sadeghisorkhani et al., 2017). This approach is adequate for dispersion-curve based tomographies. More generally, however, a heterogeneous source distribution introduces variations into the cross-correlation waveform. If these are interpreted by the tomographic inversion procedure as an effect of subsurface properties, artefacts may appear in the imaged subsurface structure (e.g. Delaney et al., 2017).

Non-linear pre-processing is often applied to the continuous seismic recordings in order to retrieve broad-band cross-correlations, diminishing the spectral signature of the sources (Bensen et al., 2007). This can affect the relative contribution of different noise sources to the cross-correlation (Cupillard and Capdeville, 2010; Chen et al., 2016; Gaudot et al., 2016), and it alters the cross-correlation waveform. It then cannot be straightforwardly interpreted in terms of seismic structure or ambient source distribution anymore (Fichtner et al., 2017a). A changing source distribution additionally changes the finite-frequency sensitivity of cross-correlations to seismic structure, i.e. the illumination of the subsurface by observed cross-correlations, which again affects tomographic results (Fichtner, 2014). Therefore, the application of full waveform inversion to noise cross-correlation waveforms requires taking into account a model of space-dependent noise sources. Although Chen et al. (2014) model cross-correlation waveforms as Green's functions to a localized point force source, the work of Hanasoge (2013a) demonstrates by synthetic inversion that the sensitivity of the measurement to subsurface structure is different for cross-correlations than for Green's functions. In the particular example he choses, the effect of source distribution is minor compared to that of using a consistent forward model. The effect of source distribution on illumination of the subsurface is more easily visible in the synthetic study of Fichtner (2014), and particularly in the sensitivity study by Basini et al. (2013). Recently, Sager et al. (2017) have demonstrated how to systematically quantify resulting trade-offs. To provide these insights, all three models worked with differently shaped assumed source distributions.

When directly interpreting cross-correlations as Green's functions, some studies use rigorous data selection to exclude unsuitable receiver-receiver azimuths or frequency bands (e.g. Pedersen et al., 2007; Stehly et al., 2009, 2011; Mordret et al., 2013). While this selection adds
reliability to the tomographic results, it may result in discarding large parts (as much as 75% of traces, Stehly et al., 2009) of the correlation datasets. Waveform features which are entirely undesired in an empirical Green's function, such as asymmetry or spurious arrivals with an apparent velocity much higher than the P-wave velocity, can be interpreted in a cross-correlation as an effect of source distribution (e.g. Hanasoge, 2013b). Recent studies have shown how coherent information about persistent sources can be retrieved in this way (e.g. Brzak et al., 2009; Tian and Ritzwoller, 2015; Gaudot et al., 2016; Sadeghisorkhani et al., 2016; Retailleau et al., 2016). This indicates that they constitute a potentially useful source of information also for tomography.

For applications other than ambient noise tomography, the effects of non-stationary source distribution and pre-processing have not been studied as extensively. To exploit amplitude information of noise cross-correlations, it appears necessary to constrain noise source distribution, although different workarounds have been invoked such as array stacking and working with deconvolution instead of cross-correlation. Research on the question of amplitude retrieval from cross-correlations is ongoing (Prieto et al., 2009; Harmon et al., 2010; Tsai, 2011; Denolle et al., 2013; Hanasoge, 2013b; Bowden et al., 2015; Viens et al., 2015; Stehly and Boué, 2017). For coda wave monitoring, it was long assumed that the cross-correlation coda, which is by definition scattered, has lost the imprint of seismic sources to a sufficient degree. However, Zhan et al. (2013) and Daskalakis et al. (2016) find that this is not always the case.

In summary, when studying noise cross-correlations, it is desirable to give up the Green's function assumption in a number of scenarios, namely when even the smallest travel-time bias has to be avoided (e.g. for time lapse tomographies aimed at monitoring), when full-waveform misfits of the cross-correlation, which include ‘spurious’ arrivals or asymmetry, are to be utilized in tomographic inversion, or when amplitude information is considered. The inversion of body wave cross-correlations from specific ambient source events as suggested by Nishida and Takagi (2016) is another such scenario.

Accounting for the noise sources can be achieved by modeling cross-correlations instead of Green's functions, and inverting them by gradient-based iterative inversion, as suggested among others by Tromp et al. (2010), Hanasoge (2013a) and Sager et al. (2017). Several previous studies have introduced ambient noise source inversion techniques aimed at reducing bias in ambient noise tomography, in particular Harmon et al. (2010) and Sadeghisorkhani et al. (2016). However, to provide a viable starting model of the source distribution for noise cross-correlation waveform inversion, we seek to account for 3-D wave propagation through the Earth's laterally heterogeneous structure so as to include any prior knowledge on the structure of the source region, and to drop the plane wave assumption that previous studies employed for the construction of cross-correlation forward models.
1.3 Motivation and outline

The motivation of this thesis is now apparent. We require a model of noise source distribution that allows us to drop the Green's function assumption of ambient noise tomography and apply the method of noise cross-correlation waveform inversion, which has been proposed and studied synthetically by Tromp et al. (2010); Hanasoge (2013a); Fichtner (2014); Sager et al. (2017), to observed data. This source model should be optimized to fit observed cross-correlations, should not require assumptions of plane wave propagation, and should be able to account for heterogeneous structure, e.g. from a long-wavelength starting model for the tomographic inversion. Providing the method to derive such a model may also benefit the studies of noise sources themselves, in particular those of the oceanic microseisms, which due to their higher frequencies are particularly sensitive to the complex and strongly heterogeneous structure of Earth's crust and upper mantle.

The following thesis presents my best attempt at developing such a method: Chapter 2 will introduce the theory, which is based on the work of Tromp et al. (2010) and Fichtner (2014), and inspired by the work of Stehly and Boué (2017) and Sager et al. (2017). It also describes the computational aspect of using stored Green's function databases for source inversion, which renders the problem more efficient. Chapter 3 summarizes processing procedures for ambient noise cross-correlations. Although we have not actively investigated these, this chapter is a tribute to how large an influence processing has on the results of ambient noise studies, and is intended to serve future users of the processing tools that we developed for the applications that follow as a comfortable introduction. Applications of the method are presented in chapters 4, 5 and 6. The first of these applications is a study of a suitable observable, which is intended to gauge the potential for source inversion. The second is a global-scale inversion for the sources of Earth's hum in North and South hemisphere winter. Finally, preliminary results are presented from a study of primary microseism sources in the Sea of Japan, elaborated during my time as guest researcher at the Earthquake Research Institute, Tokyo, in collaboration with Prof. Kiwamu Nishida.

The inverted hum source models may provide a starting source model for long-period ambient noise tomography with waveform inversion according to the strategy presented by Sager et al. (2017), expanding upon previous global tomographic models using the long-period background signal known as the Earth's hum by Nishida et al. (2009) and Haned et al. (2016).
2 Modeling and inversion of noise cross-correlations

2.1 Theoretical development

The aim of this thesis is the development and application of an iterative optimization procedure to obtain a best-fit model for the sources of ambient seismic noise on the basis of ambient noise cross-correlation wave forms. In the following, we will first discuss the forward model of ambient noise cross-correlations used throughout this work, subsequently introduce the optimization procedure, and finally describe the computational approach that we developed in order to tackle the challenges associated with modeling and inverting ambient noise cross-correlations.

2.1.1 Definitions

We define the continuous cross-correlation of two time series of ground velocity $v_i(t), v_j(t)$ as

$$C_{i j}(\tau) = \int v_i(t) \cdot v_j(t + \tau) \, dt$$  \hspace{1cm} (2.1)

where $\tau$ is the cross-correlation lag. We use the symbol $\star$ to denote cross-correlation in time domain

$$C_{i j}(\tau) = v_i(t) \star v_j(t)(\tau)$$  \hspace{1cm} (2.2)

In frequency domain, $C_{i j}$ is given by

$$\tilde{C}_{i j}(\omega) = \tilde{v}_i(\omega)^* \cdot \tilde{v}_j(\omega)$$  \hspace{1cm} (2.3)

where $\omega$ is angular frequency and $u^*$ denotes the complex conjugate of $u$. The developments in this chapter are given for continuous functions. In chapter 3, we will require the discrete
cross-correlation for the discussion of the actual observations. It is given by

\[ C_{ij}(\tau) = \sum_{k=1}^{n_t} v_i(t_k) \cdot v_j(t_k + \tau), \] (2.4)

which we also denote by the \( \ast \) for brevity. It should be apparent from the context when discrete correlation is intended.

### 2.1.2 Forward model

The forward model of cross-correlations excited by noise sources goes back to the work of Woodard (1997) in helioseismology and was adapted to terrestrial seismology by various authors (e.g. Tromp et al., 2010; Hanasoge, 2013b,a; Fichtner, 2014). The studies by Nishida et al. (2008b) and Nishida (2014) are independent formulations of a similar approach, and contain a range of representations for the cross-correlation of ambient noise under different assumptions about the noise sources. Following this general approach, we regard the cross-correlation itself as seismological quantity which we can model, based on prior knowledge of the elastic medium, and subsequently invert to improve upon the initial source distribution. We do not assume that the cross-correlation converges to an approximation of the inter-station Green’s function.

The \( i \)th component of the seismic displacement field, \( u_i(x, t) \), recorded at a receiver located at \( x \), can be expressed by representation through a Green’s function, due to the linearity of seismic wave equations (Einstein’s summation convention applies):

\[ u_i(x, t) = \int_\mathbb{S} [\hat{G}_{in}(x, \xi, t') \ast N_n(\xi, t')] (t) d\xi, \] (2.5)

where we denote convolution by \( \ast \) and the \( n \)th component of the forcing term by \( N_n \). In the case of the studies presented here, this is the field of sources of the ambient seismic signal. The Green’s function \( \hat{G}_{in}(x, \xi, t') \) denotes the displacement impulse response to a point-localized force source and the spatial integral is over the volume of the Earth \( \mathbb{S} \). (Several examples of representation theorems for elastic media are found in Aki and Richards (2002)).

Seismic broadband receivers are designed to record seismic velocity. For convenient use with the observed data, we therefore change the notation of the theory to correlations of seismic velocity. To obtain velocity rather than displacement, we write

\[ v_i(x, t) = \frac{\partial}{\partial t} u_i(x, t) = \frac{\partial}{\partial t} \left[ \int_\mathbb{S} [\hat{G}_{in}(x, \xi, t') \ast N_n(\xi, t')] (t) d\xi \right]. \] (2.6)

We take the time derivative of the system’s impulse response for obtaining seismic velocity:

\[ v_i(x, t) = \frac{\partial}{\partial t} u_i(x, t) = \int_\mathbb{S} \left[ \frac{\partial}{\partial t'} \hat{G}_{in}(x, \xi, t') \ast N_n(\xi, t') \right] (t) d\xi. \] (2.7)
after which the representation is in terms of a velocity Green's function $G_{in}(x, \xi, t)$. The idea of a velocity Green's function is familiar from integrating velocity to displacement seismograms, or obtaining a numerically modeled velocity seismogram directly from a wave propagation code which models displacement by replacing the source time function with its time derivative. The non-normalized cross-correlation function of two noise recordings $v_i(x_1)$ and $v_j(x_2)$ is given by

$$C_{ij}(x_1, x_2, \tau) = v_i(x_1, t) \ast v_j(x_2, t) = v_i(x_1, -t) \ast v_j(x_2, t)$$

(2.8)

$$= \int_{\mathbb{R}} \int_{\mathbb{R}} \left[ G_{in}(x_1, \xi_1) \ast N_n(\xi_1) \right] (-t) \ast \left[ G_{jm}(x_2, \xi_2) \ast N_m(\xi_2) \right] (t) \, d\xi_1 \, d\xi_2.$$ 

(2.9)

Where $\tau$ is the cross-correlation time lag, defined through the correlation $\ast$ according to eqs. 2.1 and 2.2. In this cross-correlation model, the noise sources $N_n, N_m$ are continuously or intermittently acting sources with random phase; if their average power was distributed uniformly, the surface waves propagating between the receivers at $x_1$ and $x_2$ would emerge after several realizations of the random source signals (e.g. Snieder, 2004). Modeling $C_{ij}$ as given by equation 2.9 with spatially varying source power is conceivable, but it is numerically very costly due to the necessity of modeling long realizations for the noise fields from both sources $N_n(\xi_1)$ and $N_m(\xi_2)$, (e.g. 20 days of noise in van Driel et al., 2015). Such models have only been computed for a small number of exemplary station pairs, and/or in simplified wave propagation scenarios (Cupillard and Capdeville, 2010; Kimman and Trampert, 2010; van Driel et al., 2015; Fichtner et al., 2017a). For the purpose of inverting a large number of observations in a realistic Earth model, however, we simplify the model by the following assumptions. First, we define ambient seismic sources as those located at the Earth’s surface $\partial \mathbb{R}$, such as oceanic, wind, or traffic-generated sources. Second, in practice one usually considers a stack of windowed correlations of continuous seismic data to reach better signal to noise ratios (Bensen et al., 2007; Seats et al., 2012). Therefore, we may consider the long-term average of $C_{ij}$ formed by the stack.

At this point, it is important that we briefly turn our attention to the stacking process. In most ambient noise studies, cross-correlations are stacked in order to retrieve a signal that has converged to an approximate Green's function. For a coherent surface wave between the stations to emerge at the correct inter-station travel time, it is necessary that the stationary phase region described by Snieder (2004) has been sufficiently covered by sources during the stacked time intervals. Since the ocean-generated ambient sources are highly dynamic, a longer stack spanning all seasons usually fulfills this better. In the study presented here, we make no such requirement, as we take the sources into account directly and permit cross-correlations to have an appearance that differs from inter-station (surface wave) Green's functions. However, stacking is required to average out incoherent noise caused by coincidental correlation of instrumental and local (e.g. tilt) noise, as well as by coincidental constructive interference of signals from sources in different locations $\xi$, so that the stack retrieves only signals from coherently propagating seismic waves. If we assume that incoherent noise is white, its expected
value approaches zero for a large number of stacked windows. The assumption that incoherent noise tends to zero can be summarized by writing the expected value of the correlation of the noise source time series as

\[
\langle N_n(\xi_1, -t') \ast N_m(\xi_2, t') \rangle = S_{nm}(\xi_1, t) \cdot \delta(\xi_1 - \xi_2)
\]  

(2.10)

with auto-correlation \( S_{nm}(t) \). It is common to apply this assumption (e.g. Snieder, 2004; Tromp et al., 2010; Hanasoge, 2013a), both to keep the problem of noise cross-correlations tractable, and because one expects the correlation length of ocean-generated noise sources to be well below the seismic wavelength (Nishida and Fukao, 2007). It appears parsimonious to keep it, but we will discuss in chapter 5 how the implementation of the method that we propose below allows for an extension to sources with finite spatial correlation length.

To apply the argument of spatially uncorrelated sources by stacking, we consider the expected value \( \langle \cdot \rangle \) and rearrange the convolutions

\[
C_{ij}(x_1, x_2, \tau) = \left( \langle \mathcal{E}_{ij}(x_1, x_2, \tau) \rangle \right)
\]

\[
= \int \int \left[ G_{jn}(x_1, \xi, -t') \ast G_{im}(x_2, \xi, t') \right] \ldots \ast [N_n(\xi_1, -t') \ast N_m(\xi_2, t')] (\tau) \, d\xi_1 \, d\xi_2
\]

(2.11)

Again, \( \tau \) is the cross-correlation lag. Both Green’s functions can be considered time-invariant for the duration of the observation period, and consequently the expected value is applied only to the multiple realizations of the noise sources \( N_n, N_m \). With eq. 2.10, eq. 2.9 thus condenses to

\[
C_{ij}(x_1, x_2, \tau) = \left( \langle \mathcal{E}_{ij}(x_1, x_2, \tau) \rangle \right) = \int \left[ G_{jm}(x_2, \xi) \ast G_{in}(x_1, \xi, -t) \ast S_{nm}(\xi, t) \right] (\tau) \, d\xi.
\]  

(2.12)

Using source-receiver reciprocity, we can also rewrite the correlation wavefield as

\[
C_{ij}(x_1, x_2, \tau) = \int \left( G_{mj}(\xi, x_2, t) \ast G_{ni}(\xi, x_1, -t) \ast S_{nm}(\xi, t) \right) (\tau) \, d\xi.
\]  

(2.13)

a representation which will be useful in the practical implementation of the forward model: the sources are now located where the stations are placed at \( x_1, x_2 \) and the source locations \( \xi \) have become receivers. This integral can be discretized and evaluated numerically if the Green’s functions due to point sources injected at the receiver locations, and recorded at all source locations, are provided.

Observed ambient noise cross-correlations are usually obtained from raw data by applying a variety of pre-processing steps which will be the subject of chapter 3. Clearly, the above model equation 2.12 does not account for such processing steps except those, one might argue, which recover the true signals of ground velocity \( v_i \) by correcting for instrument
response or diminishing meaningless noise such as recording glitches. To keep the synthetic cross-correlations consistent with the pre-processed observed ones, the main challenge lies in the application of the ensemble average, which enables convenient modeling, but precludes non-linear operations on the raw signals $v_i$. Accounting correctly for non-linear pre-processing steps in the forward model is possible from a theoretical point of view (Fichtner et al., 2017a), but its practical application is the subject of current research. Therefore, minimal pre-processing is applied in all the presented applications.

2.1. Theory

2.1.3 Inversion

Eq. 2.13 describes how noise cross-correlations can be modeled with a space- and time-dependent source distribution. Consequently, we can use it to invert observed ambient noise cross-correlations for a model of the source distribution, $S_{nm}$. In eq. 2.13, $S_{nm}$ denotes the location-dependent source autocorrelation, but its frequency domain equivalent is the location-dependent source power spectral density (PSD). We will therefore refer to the model parameter as source PSD even though we may sometimes imply the time domain. In the following section, we first introduce the inversion procedure using a general misfit function, before discussing particular misfit functions suitable for the particular problem of noise source inversion. Finally, we will link the inversion procedure to the derivations by Sager et al. (2017).

The discrepancy between synthetic and observed cross-correlations is quantified by a misfit function $\chi$:

$$\chi = \chi \left( C_{ij}(S_{nm}), C_{ij}^o, \tau \right), \quad (2.14)$$

where $S_{nm}$ in turn depends on location and correlation lag. Due to the assumption of delta-correlated sources in equation 2.10, phase information of the single ambient noise signals is removed. However, $S_{nm}$ contains the spectral amplitude information of the source PSD.

Given a set of observed noise cross-correlation functions $C_{ij}^o$, the aim of the optimization problem is then to find a space-dependent source PSD $S_{nm}$ that minimizes the total misfit $\mathcal{X}$ according to

$$S_{nm,\text{opt}}(\tau, \xi) = \arg\min_{S_{nm}} \left\{ \mathcal{X}(S_{nm}) \right\} = \arg\min_{S_{nm}} \left[ \sum_r \chi(S_{nm}) \right]. \quad (2.15)$$

The sum over $r$ runs over all station pairs of the data set. Concrete examples for the misfit functions $\chi$ will be introduced below. In general, these are based on measurements that are non-linear in $S_{nm}$, for example cross-correlation signal energy. Therefore, an iterative minimization procedure such as the steepest descent or conjugate gradient scheme is required to conduct the optimization 2.15. To perform the iterative inversion, we must first determine the (negative) gradient of the misfit function with respect to $S_{nm}$. (In the following paragraphs, we drop the subscripts of $S_{nm}$, which refer to the components of the acting noise sources, in the interest of a more concise notation, but they are implied throughout.)
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A small perturbation $\delta S$ will induce a variation of the misfit $\chi$, given by $\delta \chi$:

$$\delta \chi = \nabla_S \chi \delta S$$  \hspace{1cm} (2.16)

The misfit between the synthetic and observed cross-correlations with respect to the source model $S(\xi, \tau)$ may be influenced by source perturbations in the entire model domain and at any frequency of the source PSD. We therefore seek a way to express the misfit variation by two integrals, one over space $\xi \in \partial \oplus \subset \mathbb{R}^2$ and the other either over frequency, or equivalently over lag $\tau \in [-\tau_{max}, \tau_{max}]$. We express the misfit function in this form so as to be able to evaluate the sensitivity or the misfit variation to perturbations applied in any source location $\xi$, and at any lag $\tau$, regardless of the actual misfit function chosen, by a source sensitivity kernel $K$:

$$\nabla_S \chi \delta S = \int K(\xi, \tau) \ast \delta S(\xi, \tau) \, d\xi \, d\tau.$$  \hspace{1cm} (2.17)

Rather than varying all model parameters one by one to test each $\delta S$, which would be an impracticable exercise, $K$ will allow us to directly evaluate the negative gradient of misfit for each linearized optimization step. We determine the shape of $K$ for a general misfit function $\chi$. Applying the chain rule, we obtain for the directional derivative of $\chi$ with respect to $S$:

$$\nabla_S \chi \delta S = \nabla_c \chi \delta S = \nabla_c \chi \nabla_S C \delta S$$  \hspace{1cm} (2.18)

In the above equation, both $\nabla_c \chi$ and $\nabla_S C$ have to be understood as operators acting on the expressions to their right, rather than factors. Determining $\nabla_S \chi$ thus requires determining both $\nabla_c \chi$ and $\nabla_S C \delta S$. The former term depends on the specific misfit function that is used to compare the data and synthetics, and will be discussed below. The latter term reflects the first-order change of the correlation function $C$ with respect to $S$ due to a small variation $\delta S$. To evaluate this first-order change, we make use of the linearity of the cross-correlation $C(\tau)$ in $S$ (see eq. 2.13) and write down its Fréchet derivative directly, yielding

$$\nabla_S C(\tau) \delta S = \int_{\delta \theta} \left( G_{mj}(\xi, x_2, t) \ast G_{ni}(\xi, x_1, -t) \ast \delta S(\xi, t) \right) (\tau) \, d\xi$$  \hspace{1cm} (2.19)

where we see that the operator $\nabla_S C$ acts on $\delta S$ by convolution with the correlation of two Green’s functions, and integration over all source locations $\xi$. Comparing equation 2.17 to equations 2.18 and 2.19, we find that the general form of the sensitivity kernel for cross-correlation source inversion can be expressed as

$$K(\xi, \tau) = f(\tau) \cdot \{ G_{jm}(x, \xi, t) \ast G_{in}(x_1, -\xi, -t) \} (\tau),$$  \hspace{1cm} (2.20)

where we anticipate that $\nabla_c \chi$ contributes the integral over lag $\tau$ in equation 2.17, i.e. that

$$\nabla_c \chi = \int f(\tau) \cdot \circ \, d\tau$$  \hspace{1cm} (2.21)

The placeholder $\circ$ is used to denote how $\nabla_c \chi$ will be applied to the following term. Equation
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2.20 conveniently shows that the general shape of the sensitivity kernel is determined by the cross-correlation of the Green's functions between a source location and both receivers, regardless of the actual form of the misfit function $\chi$. In the last paragraph of this section we briefly show that in the special case of measuring the cross-correlation peak amplitude, the cross-correlation of the Green's functions can be directly interpreted as sensitivity kernel, as suggested by Stehly and Boué (2017).

The kernel shows the sensitivity of the total misfit to a unit change in source energy depending on source location and correlation lag. The negative kernel provides a descent direction in which the model has to be updated in order to decrease and finally minimize the misfit function. We will now introduce a parametrization for the source PSD, describe different measurements, derive the corresponding sensitivity kernels needed for iterative inversion, and finally make the link with Sager et al. (2017).

**Parametrization with basis functions**

Given the noisy nature of observed ambient seismic data, we do not expect to resolve the PSD at each single frequency or each single lag $\tau$; besides, an inversion for the fully resolved spectrum would require the construction of misfit gradients with dimensions of the number of source locations by the number of frequency samples, which is impractical from a computational point of view. Instead, the source PSD at each location can be conveniently decomposed into a superposition of a limited number of basic spectra $s_k$. We express this parametrization in frequency domain for better readability:

$$S_{nm}(\omega, \xi) = \frac{1}{2\pi} \int S_{nm}(\tau, \xi) \cdot \exp(-i\omega\tau) \, d\tau = \sum_k s_k(\omega) \cdot S_{k,nm}(\xi) \quad (2.22)$$

where $\omega$ is angular frequency. With a fixed set of basis functions $s_k$, the optimization procedure would then adjust the weights $S_k$ to find the optimal model. In this parametrization, the forward model becomes

$$C_{ij}(x_1, x_2, \tau) = \frac{1}{2\pi} \sum_k \int \hat{c}_k(\xi, \tau) \cdot S_{k,nm}(\xi) \, d\xi \quad (2.23)$$

where $\hat{G}$ denotes the frequency domain equivalent of $G$ etc, and $c_k$ a 'raw' correlation pertaining to one basis function. The space-dependent source sensitivity kernel for one particular spectral basis is then

$$K_k(\xi) = \nabla_{c} \chi c_k(\xi, \tau), \quad (2.24)$$
including the integral over \( \tau \) from eq. 2.21. This approach could naturally be extended to the spatial domain, e.g. by using spherical harmonic basis functions to parametrize the spatial distribution of sources on global scale (cf. Nishida and Fukao, 2007). However, in this work we follow a more intuitive smoothing approach for the spatial domain instead, which will be described in the application chapter 5, and limit the use of basis functions to the spectral domain.

**Misfit functions**

Source PSD is not expected to exert a strong influence on cross-correlation travel times (e.g. Tsai, 2009; Froment et al., 2010), while seismic structure does (as evidenced by many ambient noise tomography studies, e.g. Shapiro et al., 2005; Sabra et al., 2005). Contrarily, the influence of source PSD on cross-correlation amplitudes is expected to be strong in all but the most scattering seismic media. We are inverting for source PSD without permitting updates of the seismic structure. Therefore, we rely on choosing misfit functions based on a measure of amplitude rather than cross-correlation phase in order to avoid source-structure trade-offs.

The signal energy of the recording of seismic waves traveling between two stations \( A \) and \( B \), both from station \( A \) to station \( B \) and vice versa, has already been used for source imaging in earlier studies in the form of peak envelopes and signal-to-noise ratios (Stehly et al., 2006; Tian and Ritzwoller, 2015). In addition, Sadeghisorkhani et al. (2016) inverted cross-correlation envelopes for azimuthal amplitude of noise sources using a plane-wave approach, Stehly and Boué (2017) presented an investigation of noise sources on the basis of peak amplitude with a spherically symmetric Earth model, and Nishida and Fukao (2007) conducted a linearized inversion for the cross-spectrum of effective pressure sources of the Earth’s hum using again correlation peak amplitudes. To introduce the measurements used in this work, we omit the dependence of the misfits \( \chi \) and of the forward cross-correlation model \( C_{ij} \) on the source PSD \( S_{nm} \) for readability. Instead of picking a single peak amplitude, we use signal energy \( E \) in a selected window of the cross-correlation function, since we expect this to be a more robust measurement:

\[
E = \int [w(\tau)C(x_1, x_2, \tau)]^2 d\tau, \tag{2.25}
\]

where \( w(\tau) \) denotes the signal window. Depending on the application, this window can be centered, for example, on the arrival of a surface wave packet.

While the misfit function \( \chi \) in its most general form depends on both source location and frequency of the PSD, the measurement \( E \) has no explicit frequency-dependence due to the integral over \( \tau \) which removes all phase information. An indirect frequency dependence of \( E \) is only given by the band-limited nature of the observations and simulations and by the choice of signal window for dispersive surface waves. We therefore modify the measurement to include spectral information. A convenient choice is to equip it with a pre-filter \( H(\tau) \), and subsequently combine a sequence of pre-filters \( H_l(\tau) \) covering separate frequency bands by
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Figure 2.1 – Illustration of both measurements. A window $w(\tau)$ is selected, in which signal energy is determined ($E$); alternatively, the signal energy of this window is compared to the one placed at exactly the opposite lag by logarithmic ratio ($A$). In the example shown here, the expected arrival time of the Rayleigh wave group is used to select the window; however, different arrival times can be selected, for example to target body waves or surface waves arriving at offset times due to persistent localized sources (cf. Gaudot et al., 2016).

![Graph showing correlation between stations 1 and 2 with data C(\tau), signal window w_+(\tau), and acausal window w_-(\tau).](image)

The calculation of summation; the filtered measurement is:

$$E_l = \int [w(\tau) \cdot H_l(\tau) \ast C(x_1, x_2, \tau)]^2 d\tau$$  \hspace{1cm} (2.26)

For practical purposes, this filter should be zero-phase; otherwise the correlation, interpreted for processing as one single time-series, would experience opposite phase shifts on the causal and a-causal branch. A more elaborate treatment, taking the acausal part of the cross-correlation into account separately, would then be necessary. To avoid this, we decided to choose zero-phase filters for $H_l(\tau)$ throughout.

We evaluate the discrepancy between observed and synthetic cross-correlations in terms of windowed energy using the $L_2$-norm, where we denote the measurement on observed filtered cross-correlations as $E_l^o$. For the total misfit, we consider measurements for all filter bands:

$$\chi(\xi, \tau) = \sum_l \frac{1}{2} |E_l - E_l^o|^2$$  \hspace{1cm} (2.27)

where the sum over $l$ implies dependence on $\tau$ via $H_l(\tau)$. With $\chi(\xi, \tau)$ given, we can evaluate its derivative with respect to the modelled cross-correlation $C$. Assuming that $H(\tau)$ is zero-phase
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for the reasons mentioned above and writing the filtered windowed cross-correlation as

\[ C_{l,+} = [w_+ \cdot H_l(\tau) \ast C(\tau)] \] (2.28)

and \( C_{l,-} \) analogously, we obtain:

\[
\nabla c \chi_l(\xi, \tau) = [E_l - E_o^l] \cdot \nabla C E = [E_l - E_o^l] \cdot \int w(\tau) \cdot H_l(\tau) \ast C_{l,+}(x_1, x_2, \tau) \cdot \circ d\tau,
\]

\[(2.29)\]

where the placeholder \( \circ \) marks that this operator will be applied to the term following to the right. Thus, comparing again equations 2.20, 2.19 and 2.21, the sensitivity kernel can be written as:

\[
K_l(\xi, \tau) = [E_l - E_o^l] \cdot [w(\tau) \cdot H_l(\tau) \ast C_{l,+}(x_1, x_2, \tau) \ast G_{mj}(x_2, \xi, t) \ast G_{mj}(x_2, \xi, -t)] (\tau)
\]

(2.30)

This sensitivity kernel is composed of three parts. The first term in square brackets is a weighting determined by the difference between the measurement on observed and synthetic data. The larger the discrepancy, the higher the contribution of this kernel to the overall misfit gradient will be. The second term accounts for the filtering and windowing of the correlation. The shape of this term is determined by the measurement function, and in the last paragraph of this section we briefly show that this term reduces to a delta function for the peak amplitude misfit presented by Stehly and Boué (2017). The third term in square brackets is determined by the derivative of the correlation eq. 2.13 with respect to the model parameters and contains the cross-correlation of two Green’s functions linking the two receiver locations to one source location.

With the filter \( H(\tau) \) selecting specific frequency bands, a parametrization with spectral basis functions suggests itself. The sensitivity kernel for one particular filtered measurement \( E_l \) and for one particular spectral basis function \( s_k \) becomes:

\[
K_{k,l}(\xi, \tau) = [E_l - E_o^l] \cdot [c_k(\xi, \tau) \cdot w(\tau) \cdot H_l(\tau) \ast C_{l,+}(x_1, x_2, \tau)] d\tau,
\]

(2.31)

with \( c_k(\xi, \tau) \) defined by eqs. 2.23. The variation of the total misfit for one station pair is found by a double sum over all filters and basis functions:

\[
\nabla S \delta S = \sum_k \sum_l \int_{\partial \Omega} K_{k,l}(\xi) \cdot \delta S_k(\xi) d\xi
\]

(2.32)

where the \( \delta S_k \) have become location-dependent scalar weighting factors for their respective spectral basis. The double sum over \( k, l \) indicates that each filtered measurement \( E_l \) could produce an update of the weights for each basis function \( s_k \). For practical purposes, one can reduce the effort involved in the inversion by making assumptions about the exchange of energy between different basis functions: A matched sequence of basis functions and filters, for example, should ensure that each measurement contributes predominantly to the update.
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of one spectral weight $S_k(\xi)$ only. One can then simplify equation 2.32 as

$$\nabla S \delta S = \sum_k \int_{\delta \Theta} K_{k,l=k}(\xi, \tau) \cdot \delta S_k d\xi. \quad (2.33)$$

Alternatively, one might only let neighboring frequency bands exchange energy. Measurements that are more sensitive to variations of the source power spectrum, such as waveform differences, are too sensitive to potentially unknown Earth structure to be used in an isolated source inversion (Sager et al., 2017). Future research may open up new possibilities to use these measurements in joint inversion of noise sources and Earth structure.

The above measurement $E$ is very intuitive, but absolute amplitudes and, in consequence, signal energies are affected by Earth structure as well as by the ambient source distribution. Anelastic attenuation diminishes the signal energy, and elastic effects such as scattering, focusing and resonance also influence it, especially in the case of surface waves propagating through the Earth’s complex crust. To circumvent the ensuing tradeoffs, we introduced a misfit function based on the signal energy on the causal, compared to the signal energy on the a-causal correlation branch. This has the advantage that inter-station structural effects cancel due to source-receiver reciprocity, so that minimal tradeoff with respect to Earth structure is expected. The synthetic misfit study by Sager et al. (2017) demonstrates this in a 2-D setting.

The measurement $A$ is given by

$$A = \ln \left( \frac{\int [w_+(\tau) C(\tau)]^2 d\tau}{\int [w_-(\tau) C(\tau)]^2 d\tau} \right) \quad (2.34)$$

where $w_+, w_-$ are symmetrically placed signal windows on the causal and acausal branch, respectively; after introduction of a pre-filter,

$$A_l = \ln \left( \frac{\int [w_+(\tau) \cdot H_l(\tau) * C(\tau)]^2 d\tau}{\int [w_-(\tau) \cdot H_l(\tau) * C(\tau)]^2 d\tau} \right). \quad (2.35)$$

The discrepancy between synthetic and observed cross-correlations is evaluated by the $L_2$ norm

$$\chi_l = \frac{1}{2} [A_l - A_l^0]^2. \quad (2.36)$$

For simplicity, we imply the correct filtering of the synthetic cross-correlation and denote the filtered version by $C_l$ so that, combining eqs. 2.35 and 2.36, the misfit derivative with respect to the cross-correlation becomes

$$\nabla \chi_l = [A_l - A_l^0] \cdot \int \left[ \frac{w_+^2(\tau) C_l(\tau)}{E_{l,+}} - \frac{w_-^2(\tau) C_l(\tau)}{E_{l,-}} \right] \cdot d\tau \quad (2.37)$$

where

$$E_{l,+} = \int [w_+(\tau) H_l(\tau) * C(\tau)]^2 d\tau, \quad E_{l,-} = \int [w_-(\tau) H_l(\tau) * C(\tau)]^2 d\tau, \quad (2.38)$$
are evaluations of the synthetic windowed signal energy. In analogy to eq. 2.31, the sensitivity kernel for one particular frequency band and spectral basis function becomes

$$K_{k,l}(\xi) = \nabla_c \chi_l c_k(\xi, \tau)$$  \hspace{1cm} (2.39)

with $c_k(\xi, \tau)$ defined in eqs. 2.23 and $\nabla_c \chi_l$ given by equation 2.37 here; arguments similar to the above ones apply for simplifying the inversion for a combination of frequency bands.

Based on the two presented misfit variations, we can now run iterative inversions for $S_{nm}$. The filters $H(\tau)$ are fixed prior to inversion; if one single filter is used, that leads to an internally frequency-independent inversion for the spatial distribution of one particular frequency band. This approach will be employed in the study presented in chapter 5. Obviously, the fixed frequency content needs to be taken into consideration for the interpretation of such an inversion.

Alternatively, measurements taken with a sequence of different filters can be combined to one misfit function. In this case, it is useful to match the sequence of filters by a sequence of spectral basis functions in the cross-correlation model for convenient updating of the resulting frequency-dependent model. Combining a sequence of filtered measurements with a superposition of spectral basis functions corresponding to the filtered bands leads to the following workflow:

1. Forward model with the entire spectrum (linear combination of basic spectra)

2. Measurement with pre-filtering sequence

3. Updates of separate frequency bands to a common new model

4. Repeat

**Optimization**

Once the misfit gradient is computed, we use a conjugate gradient scheme for iterative inversion (e.g. Fichtner, 2010). Both measurements $E$ and $A$ can require updates that introduce negative values in the source PSD models. This has to be avoided, as the PSD is non-negative by definition. Therefore, we introduce a non-negativity constraint. Evaluating the projected descent direction yields the update for step $s + 1$:

$$S_{s+1}(\xi) = S_s(\xi) + l_s \cdot \text{max}(h_s, 0).$$  \hspace{1cm} (2.40)

where $l_s$ is optimal step length and $h_s$ is the update for step $s$. 26
Remarks on the optimal model

Because the described inversion method for ambient seismic sources is not a direct measurement, but based on inferring the model that fits the data optimally, any retrieved model comes with similar limitations to a tomographic one. With limited coverage and noisy observations, the solution to the optimal problem is non-unique; as a simple example, one can consider that viewed from a small, continent-based network, a close-by, weaker source and a distant, strong source of microseismic noise may appear indistinguishable. Recovery tests can inform us about the potential of the network to retrieve structures in various locations, although they cannot provide a comprehensive resolution analysis. In addition to the non-unique character of the retrieved model, it may be biased by trade-offs of the source model with badly constrained parameters of the forward model - such as anelastic attenuation. These limitations are inevitable, and should be kept in mind when interpreting the results of noise source inversion. An elegant way to investigate resolution and trade-offs in a quantitative manner is provided by the use of Hessian-vector products as described by Sager et al. (2017).

Link to adjoint formulation of Sager et al. (2017)

In the above derivations, we have worked without explicitly invoking an adjoint field (e.g. Fichtner, 2010). For the purpose of noise source inversion, where the governing equation is linear in the noise sources $N_n$, the approach presented here appears more intuitive, and simpler to link to the computational approach introduced below. However, the interested reader is encouraged to put this derivation side by side with the adjoint formulation of noise source sensitivity kernels laid out by Sager et al. (2017) under the following considerations:

Here, we regard the Green's functions of the Earth as time series that are known and that have obtained prior to the modeling of the cross-correlation. In contrast, Sager et al. (2017) formulate the cross-correlation as a wave field with a source given by the time-reversed generating Green's function wave field, convolved by the noise source auto-correlation. This can be illustrated by expressing the cross-correlation (omitting $\tau$ for clarity) as:

$$C_{ij}(x_1, x_r) = \int G_{in}(x_r, \xi, t) \ast \left[ G_{mj}(\xi, x_1, t') * S_{nm}(\xi, t') \right] (-t) \, d\xi. \quad (2.41)$$

Here, we have replaced $x_2$ by $x_r$ since the correlation wave field can be sampled at any receiver in the model. Equation 2.41 can be expressed with the help of two linear operators, one for the correlation wave field proper and one for the generating wave field. This formulation permits to treat the cross-correlation as seismic wave field like any other (albeit with a peculiar source). The adjoint method (e.g. Tromp et al., 2005) can then be used to determine $\nabla_s C$. This means that the sensitivity kernel is obtained by propagating an adjoint wave field from an adjoint
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source localized at the seismic receivers, essentially re-arranging our equation 2.20 as

$$K(\xi, \tau) = \int_{\partial \Omega} \nabla_c \chi(\tau, x) * G_{in}(\xi, x, -t) \cdot \delta(x - x_2) \, d x \cdot G_{jn}(x, \xi, t) \tau.$$

The benefits of using the adjoint formulation are clearly that it is very general and applies in a similar way to the inversion for both noise sources and Earth structure. Is is also more compatible with the numerical approach described by Tromp et al. (2010), based on correlation wave field due to a distributed source (which was not used in this work, as will be discussed below). In addition, the second linear operator (describing the generating wave field) can be extended conveniently in order to include further linear operations, such as filters, on the cross-correlation. However, the approach presented above is sufficient for many applications, more approachable and allows for relatively straightforward relaxation of the assumption of delta-correlated sources (eq. 2.10), as will be described in chapter 5.

Link to the kernel formulation of Stehly and Boué (2017)

It may be illustrative to link our derivation of source sensitivity kernels to the one presented by Stehly and Boué (2017). They present an approach to the inversion for noise sources that uses a simple misfit function, is very intuitive and connects naturally to approaches using more elaborate misfit functions such as the ones introduced above. Here we outline this connection.

In their description of the source kernel, they explain that the measurement is taken through comparison of two cross-correlation by taking the difference of their value at the peak ampli-
tude of the first. Their measurement can thus be described as

$$C_p = C_{ij}(\tau) \cdot \delta(\tau - \tau_{max})$$

and the lag-dependent $L_2$ misfit function as

$$\chi(\tau) = \frac{1}{2} \left[ C_p - C_p^o \right]^2,$$

The derivative of this misfit function with respect to the model $C_{ij}$ is

$$\nabla_c \chi = \left[ C_p - C_p^o \right] \cdot \delta(\tau - \tau_{max})$$

and consequently, the misfit variation due to a change of source PSD becomes according to eqs. 2.18 and 2.19

$$\nabla_s \delta S = \left[ C_p - C_p^o \right] \cdot \int_{\partial \Omega} \delta(\tau - \tau_{max}) \cdot \left[ G_{jn}(x, \xi, t) * G_{in}(x_1, \xi, -t) * \delta S(\xi, t) \right] (\tau) \, d \xi \, d \tau$$

$$= \left[ C_p - C_p^o \right] \cdot \int_{\partial \Omega} \left[ G_{jn}(x, \xi, t) * G_{in}(x_1, \xi, -t) * \delta S(\xi, t) \right] (\tau_{max}) \, d \xi$$
leaving us with a space-dependent sensitivity kernel that is exactly the evaluation of the synthetic cross-correlation itself at \( \tau_{\text{max}} \), scaled by the difference between peak amplitudes as described by Stehly and Boué (2017). Returning to other misfit functions simply requires replacing eqs. 2.44 and 2.45 by the corresponding expressions for \( \chi \) and \( \nabla c \chi \).

### 2.2 Computational strategy

#### 2.2.1 Precomputed wave field approach

In the previous section, we formulated an inversion method for noise source distribution that is able to account for full 3-D wave propagation through the seismic structure of (a subregion of) the Earth. Current 3-D tomographic models summarize our best present knowledge of the Earth’s interior seismic structure. However, computing synthetic seismograms numerically using 3-D solvers of visco-elastic wave propagation such as specfem3d_globe (Komatitsch and Tromp, 2002a) is a computationally intensive task requiring, depending on the scope of the model, large supercomputing resources. Since both equations 2.6 and 2.13 are linear in the source and PSD term, respectively, there is no need for an inversion of time- and space-dependent source PSD \( S_{nm} \), to update the Green’s functions. Therefore, we implemented the noise source inversion using pre-computed wave fields, in order to save computational cost. The Green’s functions \( G_{mj}, G_{ni} \) of eq. 2.13 can be approximated in band-limited form by letting an impulsive point-force source act on a 3-D tomographic model. Equation 2.13, which is related by source-receiver reciprocity to the correlation forward model equation 2.12, allows us to calculate the required Green’s functions efficiently by placing such sources at the locations of seismic receivers, and recording the resulting wave field at a dense grid of possible source locations. Fig. 2.2 shows a schematic illustration. Once the wave field has been computed and stored for all source locations, it can be used to construct both the forward model and the source sensitivity kernels (eq. 2.20) by approximating the spatial integral with a sum over the integrand for discrete source locations.

While this approach cannot be extended to structural inversion (since the Green’s functions are updated during the latter), it still serves several purposes. The first is the inversion for noise source distribution in regions where Earth structure is adequately represented by tomographic models constrained by earthquake data (For a discussion of source-structure trade-offs, see Fichtner, 2015; Sager et al., 2017). The second is to obtain starting models of source PSD, as well as source sensitivity kernels for joint source-PSD/structure inversion of noise cross-correlations (see Sager et al., 2017). Finally, it may be used for exploratory forward modeling of cross-correlations using various source distributions.
Figure 2.2 – An illustration of the precomputed wavefield approach. The map on the left shows a time snapshot of the wavefield emanating from a point source at Eskaldemuir, Scotland (indicated by red triangle). The gray dots show an illustrative 5x5 degree grid of source locations (sampling on the real model has a far finer grid); the seismograms on the right are individual Green's functions at the indicated locations. The vertical line indicates the timing of the snapshot.
Many 3-D numerical wave propagation models which are currently used in the computational seismology community are time-domain solvers that determine the wave field throughout the modeling domain at each time step (e.g. Komatitsch and Tromp, 2002a; Gokhberg and Fichtner, 2016). The cost of writing seismograms to disc is negligible in comparison to the cost of the simulations themselves. Therefore, these codes are well suited to simulating impulse responses to sources at a dense grid of discrete locations on the Earth's surface. Restricting the source locations to the Earth's surface considerably eases storage of the wavefield. This assumption is not particularly restrictive, and equally necessary for practical implementation of the approach described by Tromp et al. (2010).

Important considerations on such a precomputed wave field are in what format to store this data, and whether storage of the surface wave field can be accommodated for a sufficient number of receivers. Traditionally, wave propagation solvers for regional and global seismology output the data as one file per receiver in formats such as ascii text or binary SAC format (Goldstein et al., 2003). However, with the large number of individual seismograms necessary for storing the discretized wave field at an adequate spatial sampling (typically on the order of \(10^4\) to \(10^5\) time series), this approach becomes impracticable. Formats allowing the storage of multiple seismograms per file, and non-sequential access, are needed. Therefore, we stored the precomputed wave fields in hdf5 files after conversion from the formats provided by the respective solver. The hdf5 file format is an open-source, flexible hierarchical format for data objects of various types and shapes\(^1\). Figure 2.3 shows the required file setup we use for storing the discretized Green's function wavefield relating to one reference station in one hdf5 file.

More recently, specfem3d_globe has been updated to use the ASDF format with very large numbers of seismic receivers (Krischer et al., 2016). Moreover, the recently developed spectral element solver for wave propagation Salvus (http://salvus.io) offers the possibility to store surface wavefields at the GLL points (i.e. the complete modeled surface wavefield). Adopting such recently emerged features will greatly improve the usability of the methods discussed here.

Figure 2.4 shows an approximate estimate for storage requirements depending on number of source locations and number of samples per seismogram. The size of wave field files for realistic scenarios ranges between orders of magnitude of 1 and 10 GB per receiver. This is under the assumption that on larger modeling domains, only longer seismic wavelengths are considered, so that the wavefield can be downsampled in both time and space accordingly. During the preparation of the pre-computed wave field, a large amount of temporary storage is needed, as the wave field simulation is strongly over-sampled in time due to the CFL-criterion of numerical wave propagation. In the case of the presented studies, this large temporary storage requirement could be accommodated by the scratch filesystem of the Piz Daint cluster at the Swiss National Supercomputing Center (CSCS). On smaller clusters it may be necessary to run the pre-computation step in batches, down-sampling after each batch.

\(^1\)see https://www.hdfgroup.org
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Figure 2.3 – Setup of the hdf5 file required for use with the Python noise cross-correlation forward modeling / inversion module.

```
Group /

Dataset 'sourcegrid'
data: array 2 x Nr. sources floats
attributes: None

Coordinates of the source locations
x1, x2, .....       xN
y1, y2, .....       yN

Dataset 'data'
data: array 2 x Nr. sources floats
attributes: None

Values of the seismograms
v11, v21, .....      vn1
v21, v22, .....      vn2
....
vm1, vm2, .....      vmn
```

Dataset 'stats'
data: None
attributes:
Fs (int)
Sampling rate
complex (bool)
wave field stored as Fourier spectrum
data_quantity (string)
seismograms stored as displacement (dis), velocity (vel) or acceleration (acc)
n (int)
number of samples
ntraces (int)
number of sources (= number of seismograms)
reference station (string)
station code of the station at which the point source was placed

Figure 2.4 – Approximate file size for each pre-computed wave field, assuming single-precision arrays. The green and blue stars mark the approximate requirements of precomputed wave-field files used for the applications in chapters 5 and 6, respectively. The range of numbers of source locations and seismograms can be considered reasonable for a large range of applications, because as applications move to larger spatial scale, longer seismic periods will most likely be considered, reducing the sampling frequency on both temporal and spatial scales. The (comparatively very small) overhead for meta data and the file format itself is not considered.
2.2. Computational strategy

By using the fixed scheme shown in Figure 2.3 for the storage of the Green’s function wavefields inside hdf5 files, wavefields computed by any wave propagation solver can be used after format conversion by a dedicated Python tool for noise correlation and source inversion. This tool uses the h5py library for accessing the wave field files. It can thus model cross-correlations on the basis of stored Green’s functions of any provenance, from very simplified Earth models (such as in chapter 6) to complex, laterally varying ones including oceans and topography (such as in chapter 5).

The Python tool evaluates equations 2.12 and 2.20 by approximating the spatial integrals as Riemann sums. Correlations for each source location are performed in the frequency domain using the SciPy library. For convenience, the source grid is defined such as to ensure a uniform spatial sampling. This is achieved simply by subdividing the elliptic Earth in a grid of equal distances, first along latitude, and then along longitude. Topography is neglected for this purpose.

Both tasks - evaluating cross-correlations between all pairs of receivers, and evaluating the corresponding sensitivity kernels - are embarrassingly parallel for a large number of station pairs (e.g. approx. 20 000 correlations / kernels for a data set of 200 seismic stations), and because a single correlation or kernel can be computed within short time (typically, a few minutes) on a single CPU, we parallelized the tasks simply by distributing the station pairs to a number of CPUs. The mpi4py library is used for parallelization. Each CPU computes a number of correlations or kernels and writes them independently.

The described approach makes computational cost scale with the number of station pairs. In contrast, using the approach of Tromp et al. (2010), computational cost scales with the number of single stations in the data set (because the correlation wave field and kernel can be simulated by one time run per ‘reference station’). Hence, with a very large number of stations, the latter approach will finally become less costly despite requiring large resource for the wave field simulation. Figure 2.5 shows an approximate comparison of computational cost per iteration based on the example of global surface wave noise source inversion. Such an estimate should be made to pick the most adequate strategy. The estimate of how much computation time is saved at low to moderate station numbers by the precomputed wavefield approach is rather conservative, and in practice the cost of full simulation may be somewhat higher: This is because the runtime of the spectral element solver is estimated from a point force source simulation. For a simulation of cross-correlations, the generating wavefield has to be read in, which is neglected here. Furthermore, it was assumed that the structural model is not updated in between iterations. Updates of the structure require to recompute the generating wavefield (see eq. 2.41), and therefore increase computational cost per iteration by \( \frac{1}{3} \). Optimizations are conceivable to improve the current Python modeling tool. A possible starting point for optimizing the pre-computed wave field approach in both respects would be to exploit the relatively sparse nature of the global-scale wave field, e.g. not to store any arrival prior to the first-arriving phase, and to correlate only those time steps that contain amplitudes above a

\( \text{https://github.com/lermert/noisi} \)
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Figure 2.5 – Approximate computational cost per iteration of noise source inversion using the precomputed wavefield approach and the approach presented in Tromp et al. (2010). The example is based on the global-scale application shown in chapter 5. For the full simulation approach, it was assumed that the Green's functions are not updated during the source inversion, and the duration of the I/O operation for reading in the generating wavefield was neglected. The number of required simulations in the approach by Tromp et al. (2010) scales with the number of seismic stations in the dataset, while the number of required evaluations of the cross-correlation and kernel in the precomputed approach scales with the number of station pairs, i.e. approximately with the square of the number of stations. Therefore, the full simulation approach becomes more convenient at very large numbers of station pairs.
minimum threshold, or only those that will fall into a predefined measurement window.
3 Processing continuous seismic data

Due to the stochastic nature of continuous seismic recordings and the generally rather low signal amplitude of coherent waves in the noise, a large number of pre-processing approaches exist. The choice of processing steps should be tailored to the particular data set and application. In this chapter we will provide a non-exhaustive overview of the existing processing steps and outline the aims of their application, as basis for choosing an adequate procedure. We focus on the processing aimed at obtaining cross-correlations that for tomographic or noise source inversion at regional to global scale. We do not discuss processing for coda-wave monitoring studies which may follow different rationales. The sequence of described processing steps approximately illustrates the usual processing flow.

3.1 Preparatory steps

Regardless of the ambient noise application, instrumental and otherwise undesired effects related to the acquisition of seismic data should be removed prior to any further analysis. Preparatory steps usually include the removal of mean, linear trends, spikes and glitches from the data, and instrument correction. For convenient application of these preparatory steps, the continuous data are cut into segments. In regional-scale studies, a common segment length for convenient data preparation is 24 hours (e.g. Bensen et al., 2007). From the segments of raw data, one first **subtracts the mean and linear trends**. Trends are caused, for example, by the effect of thermal variations on the instrument. Depending on the type of data and length of each segment processed, it may be reasonable to fit more than one linear trend (Beyreuther et al., 2010).

In many applications, the **instrument response** of the seismometer has to be accounted for to obtain measurements of ground velocity (or acceleration) rather than digital counts. Correction is necessary if: (a) the data set contains recordings from different instrument types, and/or (b) the considered frequency band lies outside the flat part of the instrument's response spectrum. In other cases, this relatively time-consuming step is often skipped. The correction is usually applied by deconvolution of the transfer function of the seismometer (e.g.
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Scherbaum, 1996). During deconvolution, those parts of the spectrum where the amplitude of the transfer function is close to zero are strongly emphasized. An undesired effect is that instrumental and other meaningless noise can be considerably amplified. To avoid this, mainly two strategies are used: The first is to apply a so-called waterlevel, a low constant amplitude cutoff above the actual instrument sensitivity. The second is to remove unwanted noise by bandpass pre-filtering the data in a frequency range where the transfer function amplitude is sufficiently large. In the case where the instrument response is not corrected for, a bandpass filter is usually applied to select the usable frequency range. In the applications described in the following chapters, we use prefilters throughout.

High-amplitude spikes and short-duration glitches can occur in continuous seismic data due to a variety of reasons. In particular, human activity close to the station and the release of mechanical stresses by ageing components of the seismometer (Bormann et al., 2002, chapter 5) may produce high spikes. Glitches are artificial jumps in the continuous wave form record caused by faulty transmission of digital data. Their occurrence mainly depends on the installation and the communication mode of a specific station or network. To cause spurious signals in the correlation, spikes and glitches would have to randomly coincide between different instruments, which may seem unlikely. However, non-normalized cross-correlations are highly amplitude-sensitive, so that the occurrence of one such event could severely degrade the observation. Conservative processing schemes therefore attempt to avoid a degradation from spikes and glitches by amplitude clipping at several times the standard deviation of a segment. For example, Mordret et al. (2015) remove all signals exceeding 10 times the daily standard deviations after bandpass filtering, and Zigone et al. (2015) clip signals at 15 times the daily standard deviation after highpass filtering.

In some studies, amplitude clipping is also applied to reduce the effect of earthquake signals. While this may be effective (a systematic comparison is often performed, but rarely published; Bensen et al. (2007) show one example where such clipping does not work), it bears the disadvantage that the clipping operation is non-linear and may introduce undesired effects on the cross-correlation wave form. While clipping spikes and glitches is a pragmatic necessity, earthquake signals can be removed, if necessary, with more elaborate processing schemes as described below and in chapter 6.

Long-period seismic stations suffer from the influence of barometric pressure variations. Besides direct effects of atmospheric pressure change, such as change of the buoyancy of the seismometer mass, sensor tilt causes high noise levels at low frequency (Bormann et al., 2002). Tilt is induced through the deflection of the ground in a range of several kilometers around the seismic station. For the analysis of very long-period recordings in the mHz range, in particular of horizontal components that suffer higher tilt noise, subtraction of barometric tilt is desirable. Such a barometric correction can be applied through the method of Beauduin et al. (1996) if a co-located barometric measurement is available. In practice, this is rarely performed for noise cross-correlation studies due to limited availability of barometric data and the effort of such an approach. Instead, tilt noise effects are mitigated by data selection at the stacking stage (e.g. Nishida, 2014, see below).
3.2 Pre-processing steps

Before the pre-processing specific to ambient noise and the cross-correlation itself are applied, the segments used for pre-processing are frequently cut into shorter windows, the cross-correlations of which will finally be used to form a stack. In the case of raw cross-correlation without any pre-processing and weighting, this stack would be a mere convenience and its results approximately equal to a single cross-correlation over the entire time series; in fact, cross-correlating the entire traces would be preferable, as cutting windows results in loss of information beyond the windows unless they overlap. However, in practice, a stack always serves to apply convenient weights through various strategies that will be discussed below. The window length depends on the application. The maximum lag has to be long enough to allow the slowest waves to propagate through the studied region, and the window length should be at least twice the maximum cross-correlation lag to give the cross-correlation a reasonable ‘region of support’ (e.g. Beyreuther et al., 2010). Windows may be chosen to overlap, in order to mitigate loss of information by windowing (e.g. Seats et al., 2012).

3.2 Pre-processing steps

For completeness, we give an overview of pre-processing steps that are very commonly applied prior to correlation to obtain ambient-noise based surface wave dispersion curves for ‘classic’ ambient noise tomography. They aim at retrieving cross-correlations that contain a broadband empirical surface wave Green’s function that would be observed by one receiver after an impulsive point source acted at the other. These steps have been established as de facto standard of pre-processing by Bensen et al. (2007). They are included in the Python software tool, but have not been used in our applications to noise sources described in the next chapters. The first reason is that we are not aiming at Green’s function retrieval, but instead model cross-correlations directly. Therefore, effects of non-homogeneously distributed sources, including phase bias, amplitude asymmetries and ‘non-physical’ early arrivals, are all acceptable features of the observed cross-correlation. The second reason is that all of the pre-processing steps described below constitute non-linear operations on the raw data. As described in chapter 2, accounting for them in a forward model of ambient noise cross-correlations is difficult and the subject of current research. Therefore we have not made use of them so far.

One of the simplest and earliest ideas was to completely discard the amplitude of the seismic signals and keeping only their sign, so-called one-bit normalization. It was used by Aki (1957) for the first application of spatial autocorrelation (SPAC) using traffic noise, although mostly out of the necessity to build a simple computational device for processing measured noise in real time. Aki (1957) justified the use of this processing by applying a narrow-band filter prior to cross-correlation. Observations by Pedersen et al. (2007) confirmed that cross-correlations of one-bit normalized data correspond well to those of broadband continuous data only if the data are filtered prior to the one-bit processing (which adds considerable processing effort). Shen et al. (2012) formalized this approach as time-frequency normalization, essentially applying the one-bit normalization in very narrow frequency bands. Although Hanasoge and Branicki (2013), based on the work of Van Vleck and Middleton (1966), came to the
conclusion that the ambient noise cross-correlation signal could be recovered well from one-bit processed data, numerical experiments (Cupillard and Capdeville, 2010) as well as observed cross-correlations (Fichtner et al., 2017a, Figure 1) show that this conclusion is not generally valid for observed broad-band ambient noise. The disparity must be due to the non-Gaussian statistics of the broad-band noise, in which case the results from Hanasoge and Branicki (2013) have to be viewed cautiously. Rather than recovering the ‘true’ signal from random noise, broad-band one-bitting selectively emphasizes distant coherent sources over more local ones, which in some regions may lead to a better signal-to-noise ratio (Stehly and Boué, 2017). Although still popular due to its simplicity, broad-band one-bitting should therefore not be applied blindly; its waveform-changing characteristics have to be taken into consideration. The time-frequency approach of Shen et al. (2012) may overcome the problem, but it appears to be hardly used, probably due to the added computational effort.

A further step is the frequency-domain normalization of either raw noise trace prior to correlation by their (smoothed) amplitude, so-called spectral whitening (Bensen et al., 2007). It serves to “broaden the band of the ambient noise signal in cross-correlations and also [to combat] degradation caused by persistent mochromatic sources such as the Gulf of Guinea source” (Bensen et al., 2007). Since the ambient noise spectrum shows strong peaks at the primary and secondary microseism frequencies, raw correlations are dominated by these signals. In addition, the 26-seconds Gulf of Guinea source (Shapiro et al., 2006) is a persistent quasi-monochromatic source in the period range of interest to many tomographic studies. Such a persistent and localized source can strongly bias the dispersion curves and therefore has to be down-weighted. To extract broad-band dispersion curves, spectral whitening is an effective processing step, and consequently it is pervasively used for classic ambient noise tomography (e.g. Lin et al., 2008; Stehly et al., 2009; Fry et al., 2010; Zheng et al., 2011; Gaite et al., 2012; Mordret et al., 2013; Chen et al., 2014; Zigone et al., 2015; Obermann et al., 2016; Lehujeur et al., 2017, to name some examples). In practice, it can be carried out conveniently on the Fourier transformed segments of continuous data by simply discarding the amplitude (Lecocq et al., 2014):

$$U_1(\omega) = a_1 \exp(i\phi_1), \quad U_{1,\text{white}}(\omega) = \exp(i\phi_1), \quad (3.1)$$
$$U_2(\omega) = a_2 \exp(i\phi_2), \quad U_{2,\text{white}}(\omega) = \exp(i\phi_2), \quad (3.2)$$

rather than by dividing the spectra by a smoothed version of their amplitudes which may be unstable. Fichtner (2014) showed that the application of spectral whitening introduces subtle changes into the cross-correlation wave form, which change the imaged medium into an effective medium, an undesired effect for high-precision tomography applications, in particular cross-correlation full waveform inversion. Some studies (e.g. Saygin and Kennett, 2010) therefore avoid spectral whitening altogether. Importantly, spectral whitening is not intended to remove the influence of earthquakes. The issue of balancing earthquake signals is addressed by time domain moving-average normalization (described next).
In the moving average normalization, raw data are weighted according to a running-absolute-mean defined on the filtered data. This strongly down-weights the time windows with high energy level compared to their surroundings. To be effective for down-weighting earthquakes, raw data on which the moving average weights are defined must first be filtered in the period band containing the strongest earthquake signals (Bensen et al., 2007). This step was recommended by Bensen et al. (2007) as the principal step for mitigating the effect of earthquake signals on surface wave dispersion curves from noise correlations. However, it is not used frequently. One reason for this is that it introduces phase shifts into the processed data, as argued by Seats et al. (2012).

### 3.3 Cross-correlation, stacking, and data selection

The ‘raw’ discrete cross-correlation function is (compare equation 2.9 for symbols)

\[
C_{ij}(x_1, x_2, \tau) = \sum_{t=t_1}^{t_2} v_i(x_1, t) \cdot v_j(x_2, t + \tau) = v_i(x_1, t) \star v_j(x_2, t)
\]  

(3.3)

where we use \( \star \) to denote the discrete cross-correlation over a finite time interval. It can be equivalently expressed as cross-spectrum in the frequency domain after discrete Fourier transform,

\[
\hat{C}_{ij}(x_1, x_2, \omega) = \hat{v}_i(x_1, \omega)^* \cdot \hat{v}_j(x_2, \omega).
\]

(3.4)

The sign convention of the cross-correlation is chosen such that a wave traveling coherently from location \( x_1 \) to \( x_2 \) will appear as causal, or positive-lag signal after cross-correlation. If performed on essentially unprocessed data, the cross-correlation is highly amplitude-sensitive and dominated mostly by the contribution from local sources close to the seismic stations. Problematically, due to its amplitude sensitivity, it may also be dominated by incoherent noise such as tilt and spikes, as well as by any occurring large-amplitude earthquake signals. However, raw cross-correlation is linear and can thus be adequately modeled by the equations presented in chapter 2 under the respective assumptions.

For more robustness with respect to large amplitudes, geometrical normalization may be applied (e.g. Schimmel et al., 2011), which weights the cross-correlation by the product of the root mean square amplitude of the two original windows:

\[
C_{\text{norm}, ij}(x_1, x_2, \tau) = \frac{v_i(x_1, t) \star v_j(x_2, t)}{\sqrt{\sum [v_i(x_1, t)^2] \cdot \sum [v_j(x_2, t)^2]}}
\]

(3.5)

This scalar weighting factor results in a down-weighting of all windows affected by large amplitude signals such as spikes or earthquakes. This does not necessarily provide a safe means to entirely remove earthquakes from the cross-correlation stack, because their signal is usually highly coherent over large inter-station distances.
Normalization of the cross-spectrum by amplitude spectra, rather than scalar root mean square amplitude, leads to cross-coherency (Prieto et al., 2009; Seats et al., 2012)

\[
\hat{c}_{coh,i,j}(x_1,x_2,\tau) = \frac{\tilde{v}_i(x_1,\omega)^* \cdot \tilde{v}_j(x_2,\omega)}{|\tilde{v}_i(x_1,\omega)| \cdot |\tilde{v}_j(x_2,\omega)|}
\]  

which is equivalent to cross-correlation after spectral whitening. The only subtle difference is that spectral whitening is usually limited to a particular frequency band.

Exploiting phase coherency provides another alternative to the raw cross-correlation of equation 3.3. Rather than discarding the amplitude of the continuous data entirely by one-bit normalization, Schimmel et al. (2011) introduced the phase cross-correlation for ambient noise processing, in which the correlation is formed between the instantaneous complex phases of the two original time series, rather than their amplitudes. Constructive and destructive interference lead to the retrieval of weak, coherent signals; long-lasting signals are favoured over short ones. The instantaneous phase of a signal is given by the complex phase \( \phi \) of its Hilbert transform. The phase cross-correlation is then

\[
C_{pcc}(\tau) = \frac{1}{2N} \sum_{t=t_1}^{t} \left[ |e^{i\phi(t)} + e^{i\phi(t+\tau)}| - |e^{i\phi(t)} - e^{i\phi(t+\tau)}| \right]^\nu
\]  

where || denotes the absolute value of the resulting complex correlations and the exponent \( \nu \) may be used to ‘sharpen’ the signal (Schimmel et al., 2011). The second term in square brackets ensures that equal signals of opposite phase yield a phase correlation of -1. Although conceptually elegant, the phase correlation, similarly to the processing steps summarized by Bensen et al. (2007), is hard to model, because its non-linearity introduces variable weights on the contributions from different sources, and therefore precludes the use of an ensemble cross-correlation argument. However, phase coherency techniques produce observed cross-correlations of very high signal to noise ratio for surface wave dispersion analysis, especially at global scale (Haned et al., 2016). On the other hand, Gaudot et al. (2016) caution that using phase coherency approaches enhances persistent signals such as the Gulf of Guinea microseism.

For computational purposes cross-correlation is usually implemented in the frequency domain. Using the Fast Fourier Transform algorithm for discrete Fourier transform (Cooley and Tukey, 1965) makes the computation of cross-correlation functions as cross-spectra in Fourier domain far faster than the computation in time domain. For example, computing a cross-correlation of two time-series with \( 10^4 \) samples, zero-padded to a convenient length for FFT, up to a lag of 1500 samples is approximately ten times faster via the Fourier domain than in time domain using two comparable implementations of Python-wrapped C code.

For retrieving adequate signal levels from the weak coherent noise, several single correlation
3.3. Cross-correlation, stacking, and data selection

Windows have to be averaged, or stacked. Stacks were well-known in seismology before the emergence of ambient noise cross-correlation techniques, for example from array seismology (Rost and Thomas, 2002, and references therein). The goal of stacking is to enhance coherent signals over incoherent noise by averaging in space (e.g. in the case of seismic arrays) or time (e.g. in the case of windowed cross-correlations). For noise cross-correlation stacks, linear stacking is most commonly used; however, several of the pre-processing strategies described earlier, as well as geometrical normalization and selection procedures discussed below, can essentially be regarded as applying varying weights to the stack.

A special case of stacking used in ambient noise techniques is symmetrizing the causal and a-causal parts of the cross-correlation (e.g. Lin et al., 2008). Provided that both branches contain a surface wave phase of good quality, this may increase signal to noise ratio and reduce effects of non-homogeneously distributed sources. However, it may also deteriorate the results if noise source distribution favours one branch of the correlation.

A concept related to the phase cross-correlation is the phase-weighted stack (Schimmel and Paulssen, 1997), in which the instantaneous phases of all single correlation windows are stacked to obtain a lag-dependent weighting function

\[
 w(\tau) = \frac{1}{N} \left| \sum_{j=1}^{N} e^{i\phi_j(\tau)} \right| \tag{3.8}
\]

which enhances re-occurring phases and is applied as a weight to the linear stack by multiplication of the two, resulting in the phase-weighted stack. When applied to surface wave dispersion analysis, the phase weighted stack in time-frequency domain should be used, an extension of the above time-domain phase-weighted stack introduced by Schimmel and Gallart (2007). Generally, the time-frequency phase weighted stack should be used in preference to the time domain phase weighted stack. However, if the latter is used, the resulting phase weight should be smoothed over several samples to prevent loss of broad-band information (T. Lecocq, pers. comm., 2016).

A second strategy is to exclude data segments of sudden high energy as described for example by Nishida et al. (2008a). The principle of detecting earthquake signals by sudden high energy compared to the long term average is conceptually similar to the application of short-term-average to long-term-average (STA/LTA) triggers (Vanderkulk et al., 1965), but in the case of noise processing, longer data segments are chosen for the short-term energy in the interest of lower computational cost and at the expense of (usually unnecessary) accuracy. This second strategy has the advantage of being entirely data-driven. It can therefore be tuned to exclude small local earthquakes which are difficult to exclude by catalog, and other unwanted high-energy signals. However, the tuning requires setting a number of parameters, a task that is time-consuming and somewhat arbitrary. We will discuss this problem in more detail and describe one data selection algorithm in chapter 6.
3.4 Further processing techniques

There are multiple approaches to further improve the characteristics of the correlations of continuous passive signals for interferometry, which can be roughly collected under the term post-processing, since they are usually applied after a cross-correlation stack has been elaborated. In particular, there are a number of techniques developed with regard to exploration-scale problems that can also be utilized on regional scale, such as **directional balancing** (Curtis and Halliday, 2010) and **multi-dimensional deconvolution** (Wapenaar et al., 2008; Weemstra et al., 2017). They require specific receiver geometries that at larger scale may not be available; however, the deployment of dense arrays such as USArray\(^2\), IberArray\(^3\) and AlpArray\(^4\) makes their application to regional-scale tomographic problems more realistic. Other array-based strategies include the correlation of the coda of correlations, **C3** (Stehly et al., 2008), and azimuthal sub-array averaging reminiscent of the SPAC method (Lawrence and Prieto, 2011).

3.5 Processing strategies

Depending on the ambient-noise based application, the goals of processing may differ. Once the processing goals are set, the chosen strategy still depends on the characteristics of the data, such as geographic location and the investigated frequency range.

For any application, the seismicity and structural characteristics of the study region, as well as geographical and seasonal noise level variability should be considered in order to assess the expected behaviour of observed cross-correlations, and potential biases by earthquakes and earthquake coda. Particular attention to biases should be paid for studies with time-dependent observations based on shorter continuous time series (e.g. time lapse tomography), because the shorter the time series, the higher the weight of transient events.

Adequate quality control should be designed for the elaborated cross-correlation stacks. An example for the quality control of cross-correlations for dispersion analysis is the symmetry of causal and a-causal arrival times used by Stehly et al. (2009). For cross-correlation waveforms, it is not easy to define what a ‘good’ waveform should contain. During the presented studies, two arguments were used: Signal-to-noise ratios of the surface wave window with respect to trailing noise (chapter 4) and consistency of neighbouring time windows (chapter 6).

**Cross-correlations as empirical Green's functions**

Most ambient noise tomographies are based on the analysis of fundamental (and rarely higher) mode surface wave dispersion curves. The aim of processing in these studies is to retrieve a broad-band cross-correlation that resembles the surface wave Green's function. To achieve that, the spectrum of noise needs to be equalized and normalizations should contribute to exclude earthquakes and favour sources distributed across the ‘stationary phase regions’

\(^2\)http://www.usarray.org

\(^3\)http://iberarray.ictja.csic.es

\(^4\)http://www.alparray.ethz.ch/home/
3.5. Processing strategies

Most of these studies follow the processing scheme by Bensen et al. (2007), whose time-domain normalization mitigates the effect of earthquakes, and whose spectral whitening broadens the noise spectrum. Both steps have been found effective for cross-correlations in the microseismic frequency range, although quality control is usually required, which is applied after stacking, for example by checking the consistency of a-causal and causal surface wave arrival times (Stehly et al., 2009). Additional considerations are necessary if periods longer than 40 seconds are targeted, because Yanovskaya et al. (2016) observed that in this period range, earthquake coda has a significant effect on the dispersion curves that must be excluded. Running average normalization is no longer sufficient due to the lower noise level at these periods. Yanovskaya et al. (2016) suggest to resolve this problem by averaging over long time periods and excluding earthquake clusters such as long aftershock series.

Contrary to surface waves, which dominate the ambient seismic field, ambient body waves are weaker, and more easily masked by earthquakes and earthquake coda. The caveat therefore applies for them at even shorter periods. If observations of ambient noise teleseismic body wave phases at $\gtrsim 10$ seconds are to be directly interpreted in terms of Earth structure, their correct retrieval requires careful selection of ‘low coherence’ windows (Boué et al., 2014), since otherwise earthquake coda dominates and yields cross-correlations with different characteristics than those from well-distributed ambient sources. This observation complements the results by Lin and Tsai (2013), who showed that teleseismic body waves in cross-correlations at periods of 10 seconds and more are dominated by earthquake coda even after running average normalization and spectral whitening. Since these processing steps do not remove earthquake coda effectively enough, additional processing or data selection steps have to be taken for body wave retrieval at any but the shortest periods (Boué et al., 2014, report that the influence of coda becomes negligible at periods of less than 10 seconds).

**Noise cross-correlation wave forms**

Contrary to surface wave dispersion studies with ambient noise, the retrieval of approximate Green’s functions is not necessary for cross-correlation waveform studies, whereas linearity of the processing is important, at least until strategies for generalized interferometry are developed further, to ensure consistency with the forward model. Excluding non-linear processing means that one cannot use the standard tools for down-weighting dominant transient signals.

The work presented here aims to invert for the source characteristics or cross-correlation wave forms in order to (a) use them for cross-correlation full waveform inversion and (b) to investigate ambient source characteristics such as hum sources and oceanic microseisms. For (a), even though an ‘effective source’ can be defined which may include earthquakes, it is undesirable that earthquake and coda signals dominate this effective source. Such a situation is addressed more adequately by specifically targeting the earthquake coda (Lin and Tsai, 2013) instead of processing large amounts of continuous data, and by applying double-difference tomography (Yuan et al., 2016), than by cross-correlation full-waveform inversion with a distributed source. For (b) the exclusion of any earthquake effects is necessary, and becomes particularly important for highly resolved time-dependent observations (see
chapter 6). Therefore, a method to exclude earthquake and coda signals is required. Due to the current lack of other, more sophisticated techniques, we implemented a multiple-threshold data selection for earthquake exclusion similar to the one described by Nishida et al. (2008a). This will be described in more detail in chapter 6.

One may argue that time-domain filtering to remove earthquakes or other transients (i.e. multiplying by a time series that is 1 or 0 depending on whether an earthquake is recorded or not) is a non-linear operation. The choice to apply it depends on the targeted signals, and the resulting modification of the observed data should be taken into consideration.

**Noise cross-correlation amplitudes**

Much attention has been devoted to the recovery of amplitude information from ambient noise cross-correlations (e.g. Prieto et al., 2009; Lawrence and Prieto, 2011; Tsai, 2011; Hanasoge, 2013b). Aside from the fact that the notion of retrieving attenuation information from cross-correlation amplitudes has been called into question (Cupillard and Capdeville, 2010; Stehly and Boué, 2017) due to the inhomogeneous, non-stationary noise sources, most standard pre-processing is unsuitable for those applications attempting to conserve amplitude information. At the time of writing, there were two strategies for retrieving noise cross-correlation amplitudes: (a) To follow an empirical approach using deconvolutions, rather than correlations, of continuous recordings as described by Denolle et al. (2014b); however, so far no forward models of deconvolution exist and its rigorous inversion is therefore currently not possible. (b) to extract amplitude information from cross-correlations for which a forward model exists, but which are affected by the limitations of incoherent noise and affected by earthquake signals as described above.

### 3.6 Noise processing and cross-correlation tool

Software to carry out the preprocessing and correlation is an essential requirement for the presented work. However, since the processing steps themselves have already been described in detail, here we will only provide a very brief overview of the processing tool ants\(^5\) and its functionality.

The tool is set up in four parts: Data download, preparation, pre-processing and correlation and finally post-processing routines including measurements on the cross-correlations, plotting, etc. The first three parts are parallelized using mpi4py. The tool works with a rigid directory structure that is automatically set up upon initializing a new project. For each of the first three parts (download, preparation, correlation), the user may fill in a configuration file, and then start the processing from the command line. The post-processing (plotting and measurements) is entirely command-line based.

For data download, the tool only provides a Python wrapper for the IRIS FetchData scripts\(^6\). The user has to specify a list of channels as text file and can then run the download in parallel

\(^5\)https://github.com/lermert/ants_2
\(^6\)https://seiscode.iris.washington.edu/projects/ws-fetch-scripts
using \texttt{mpirun}. The IRIS FetchData scripts may skip requests upon server timeouts, leading to data gaps. Alternatives to procure raw continuous data include the IRIS breqfast data request which provides a temporary folder for FTP download; the obspy data center clients (Beyreuther \textit{et al.}, 2010); and the recently published obspyDMT tool suite (Hosseini and Sigloch, 2017).

Data preparation is focused on reducing the bookkeeping effort to a minimum for the user. This means that the continuous data, which often contain gaps with random start and end times, can be entered as a collection of arbitrary length files of any data format read by obspy (including mseed, sac and many others), into the preparation directly without the need to organize them into daily or similar segments and directories before. The preparation routines such as bandpass filtering and deconvolution are provided by obspy, which also ensures compatibility with a large number of input formats. Again, preparation can be run in parallel.

The pre-processing and correlation stages are set up so as to minimize the number of input and output operations, while keeping the memory requirement at a reasonable level. The user may specify a number of stations to be processed in one block by a single process. In a loop over the time axis of the continuous data, each process will read in all files available for the stations of its block and relevant for the current start time, pre-process them, correlate each with respect to each other, and stack the resulting cross-correlation windows, while discarding from memory the segments no longer used, and loading the next required ones. If requested by the user, each single window correlation will be written to one container file per station pair along with the root mean square energy of the original traces. These can then be evaluated at the stacking stage as criteria for acceptance or rejection of the cross-correlation window. The flexibility of the format allows to add diagnostic criteria other than root means square energy very easily, paving the way for more elaborate data selection strategies.

Postprocessing routines are provided to plot station maps, single cross-correlation traces and record sections of cross-correlations; to take measurements of logarithmic amplitude ratio (eq. 2.34); and finally to evaluate these measurements by creating a preliminary source map which we will discuss in chapter 4. All of the steps performed by the \texttt{ants} tool can be viewed as preparatory steps for noise cross-correlation inversion; it was used to prepare the data used in the following chapters 4, 5 and 6. The first version of this processing and cross-correlation tool has been used as prototype of a high performance computing tool for noise cross-correlation calculation (Fichtner \textit{et al.}, 2017b).
4 Cross-correlation imaging of ambient noise sources


We have developed the fast imaging technique for noise source distribution presented here so as to be able to assess whether measurements on observed cross-correlations, $A_l$ (eq. 2.35) in the case of this application and the one in chapter 5, and $E_l$ (eq. 2.26) in chapter 6, contain sufficient information about the noise sources to warrant an inversion. It can be regarded as an analogue step to the assessment of travel time residuals prior to travel-time tomography.

4.1 Motivation and Outline: Ray theory imaging of noise sources using cross-correlation asymmetry

We propose an improved cross-correlation-based noise source imaging technique. We require this technique to: (i) Operate on noise correlation data sets directly, and (ii) use an observable that can be measured robustly, modeled numerically, and related directly to variations in the power-spectral density distribution of the noise sources.

This chapter is organized as follows. In section 4.2, we briefly review the modeling of noise cross-correlations and the computation of noise source sensitivity kernels. We propose to use the logarithmic energy ratio of the causal and anti-causal cross-correlation branches as measurement to infer the noise source distribution. Sensitivity kernels for this measurement with respect to the power-spectral density distribution of noise sources in space and frequency are introduced in the same section. As an intermediate step towards a full 3-D inversion for noise sources, we derive a ray-theoretical simplification of the noise source kernels. This theoretical part forms the foundation of a fast method for noise source imaging. Applications to real data can be found in section 4.3, where we illustrate the proposed ray-theory imaging of noise sources with examples of globally recorded hum, and microseismic noise in the Western Mediterranean and the Swiss Digital Seismic Network. Finally, in section 4.4 we discuss the benefits and limitations of our method, and we place it in the context of the ongoing transition
from ray-based to finite-frequency tomography.

4.2 Theory for noise source imaging

For the purpose of this thesis, the theoretical development needed for the first application of noise source imaging has been slightly condensed to the essential results, because it closely follows the one introduced in chapter 2. It is presented in frequency, rather than time, domain. Note that a different sign convention is used for the cross-correlation.

A non-normalized noise cross-correlation under the assumption of spatially uncorrelated noise sources can be expressed as:

\[ C_{ij}(x_1, x_2) = \int G_{in}(x_1, y) G_{jm}^*(x_2, y) S_{nm}(y) \, dy. \]  \hspace{1cm} (4.1)

The approximation of spatially uncorrelated sources means that only sources that are exactly collocated contribute to the stacked correlation. In practice noise sources may be extended, and consequently the approximation made above may limit the applicability of the method in cases where noise sources are correlated over distances that are large compared to the seismic wavelength. The Hermitian matrix \( S_{nm} \) is the power-spectral density (PSD) of the noise sources as a function of position and frequency. The PSD is real-valued when the off-diagonal elements of \( S_{nm} \) are zero, that is when different noise source components are uncorrelated. Equation (4.1) provides a model for the forward calculation of inter-station correlation functions for arbitrary distributions of the noise PSD in both space and frequency. The forward model is therefore free of assumptions on wavefield equipartitioning or isotropic source distributions needed to ensure equality of correlations and Green’s functions (e.g. Lobkis and Weaver, 2001; Wapenaar, 2004; Weaver and Lobkis, 2004; Wapenaar and Fokkema, 2006; Sánchez-Sesma and Campillo, 2006).

The forward model in equation (4.1) provides synthetic correlation functions \( C_{ij}(x_1, x_2) \) that can be compared to observed correlation functions \( C_{ij}^0(x_1, x_2) \) with the help of a suitably chosen misfit functional \( \chi \). An infinitesimal perturbation of the PSD, \( \delta S_{nm} \), induces an infinitesimal perturbation of the misfit, \( \delta \chi \). The misfit variation \( \delta \chi \) can generally be written in the form

\[ \delta \chi = 2 \text{Re} \int_0^\infty \delta C_{ij}(x_1, x_2, \omega) f(\omega) \, d\omega, \]  \hspace{1cm} (4.2)

The term \( f(\omega) \), which we refer to as the adjoint source, depends on the specific choice of the measurement (such as, for example, traveltime or amplitude difference) and the misfit functional \( \chi \) (such as, for example, the \( L_2 \)-misfit). In this paragraph, we keep \( f(\omega) \) general, and specify the measurement and misfit later on.

The misfit variation \( \delta \chi \) depends on the perturbation of the correlation \( \delta C_{ij} \), with respect to the model parameters, i.e., the source PSD. Writing the perturbation as a function of the source
4.2. Theory for noise source imaging

PSD perturbation (assuming that the structure does not vary in the time interval we consider), yields:

\[ \delta C_{ij}(x_1, x_2, \omega) = \int_0^\infty G_{in}(x_1, y, \omega) G^*_{jm}(x_2, y, \omega) \delta S_{nm}(y, \omega) dy, \]  

(4.3)

Introducing this perturbation of the correlation into equation (4.2), gives

\[ \delta \chi = 2 \Re \int_0^\infty \int_0^\infty f(\omega) G_{in}(x_1, y, \omega) G^*_{jm}(x_2, y, \omega) \delta S_{nm}(y, \omega) d\omega dy \]

\[ = \int_0^\infty \int K_{nm}(y, \omega) \delta S_{nm}(y, \omega) d\omega dy. \]  

(4.4)

The sensitivity kernel

\[ K_{nm}(y, \omega) = 2 \Re f(\omega) G_{in}(x_1, y, \omega) G^*_{jm}(x_2, y, \omega) \]  

(4.5)

represents the first-order change of the measured misfit in response to a perturbation of the noise source PSD. We can now briefly exemplify why \( f(\omega) \) is referred to as adjoint source. In equation (4.5), the term

\[ f(\omega) G_{in}(x_1, y, \omega) = \int_0^\infty f(\omega) G_{in}(y, x_1, \omega) \delta(x_1) dx_1 \]  

(4.6)

may be interpreted in terms of an adjoint wavefield excited by \( f(\omega) \) at receiver position \( x_1 \) (and can be calculated numerically as such). This establishes an analogy with the adjoint-based computation of sensitivity kernels for Earth structure (e.g. Tarantola, 1988; Tromp et al., 2005; Fichtner et al., 2006). If \( \delta S_{nm} \) is a unit perturbation to the source PSD, then according to equation 4.4 the kernel informs us about the spatial shape of the resulting misfit perturbation. Equation (4.5) is valid for all differentiable misfit functionals \( \chi \). Now, we chose a specific measurement and a specific misfit, \( \chi \). This determines a specific adjoint source \( f(\omega) \), and therefore the shape and amplitude of the sensitivity kernel \( K_{nm} \). We first explain why we chose the measurement \( A \), which is the causal/anti-causal asymmetry. We then derive the sensitivity kernel corresponding to this measurement, and to an \( L_2 \)-misfit. In the following sections, the kernel will be strongly simplified for application examples.

To image the geographic distribution of noise sources \( S_{nm} \), we require \( \chi \) to be insensitive, at least to first order, to the presence of unmodeled 3-D Earth structure. While the amplitudes of correlation waveforms are primarily sensitive to the geographic distribution of noise sources (e.g. Hanasoge, 2013b,a), they are also affected by focusing and defocusing in the presence of 3-D structure, as well as by visco-elastic attenuation (e.g. Lawrence and Prieto, 2011). To circumvent the difficulty of interpreting amplitudes themselves, we quantify noise correlation asymmetry in terms of the logarithmic energy ratio of the causal and anti-causal parts. For this we first consider a time window \( w_+(t) \), centered around a synthetically computed
correlation waveform on the positive lag axis. Its mirrored counterpart on the negative lag axis is denoted by \( w_-(t) \). An exemplary window selection is shown in Fig. 2.1. The correlation asymmetry is then:

\[
A = \ln \frac{\int_{-\infty}^{\infty} \left[ w_+(t)C_{ij}(t) \right]^2 dt}{\int_{-\infty}^{\infty} \left[ w_-(t)C_{ij}(t) \right]^2 dt}.
\] (4.7)

The asymmetry of observed correlation functions \( A^0 \) is defined analogously to equation (4.7), which allows us to quantify the discrepancy between observations and synthetics in terms of the total \( L_2 \) misfit:

\[
\chi_{tot} = \sum_{n=1}^{N} \frac{1}{2} \left[ A_n - A^0_n \right]^2.
\] (4.8)

where the sum is over all pairs of receivers, and \( A_n \) and \( A^0_n \) are the modeled and observed logarithmic asymmetry at the n'th receiver pair. This is the summed misfit for all measurements; the sum is now omitted, and one measurement only is considered, for notational convenience:

\[
\chi = \frac{1}{2} \left[ A - A^0 \right]^2.
\] (4.9)

A homogeneous reference noise source distribution is assumed to produce perfectly symmetric synthetic correlation functions with \( A = 0 \). Higher energy on the causal side corresponds to positive asymmetry (\( A \) or \( A^0 > 0 \)), and vice versa.

The measurement (4.7) and misfit definition (4.9) generate the following adjoint source \( f(\omega) \), needed for the computation of noise source sensitivity kernels according to equation (4.5):

\[
f(\omega) = (A - A^0) \left[ \frac{1}{\pi E_+} \left( w^2_+(\omega) \ast C_{ij}(\omega) \right)^* - \frac{1}{\pi E_-} \left( w^2_-(\omega) \ast C_{ij}(\omega) \right)^* \right],
\] (4.10)

where \( E_+ \) and \( E_- \) denote the energies in the causal and anti-causal windows, respectively:

\[
E_+ = \int_{-\infty}^{\infty} \left[ w_+(t)C_{ij}(t) \right]^2 dt, \quad E_- = \int_{-\infty}^{\infty} \left[ w_-(t)C_{ij}(t) \right]^2 dt.
\] (4.11)

The symbol * denotes convolution, and \( w^2_+(\omega) \) is the Fourier transform of the squared time window \( w_+^2(t) \). Combining equations (4.5) and (4.11), gives an explicit expression for the noise source kernel \( K_{nm}(y, \omega) \):

\[
K_{nm}(y, \omega) = \left( A - A^0 \right) \left[ \frac{1}{\pi E_+} \left( w^2_+(\omega) \ast C_{ij}(\omega) \right)^* - \frac{1}{\pi E_-} \left( w^2_-(\omega) \ast C_{ij}(\omega) \right)^* \right] \ldots \ G_{in}(x_1, y, \omega) \ G_{jm}^*(x_2, y, \omega).
\] (4.12)
4.2. Theory for noise source imaging

While the term \( G_{in}(x_1, y, \omega) G_{jm}^*(x_2, y, \omega) \) determines the regions where sensitivity can be non-zero, the term in square brackets controls the sign and amplitude of sensitivity within these regions as a function of frequency.

4.2.1 Simplified kernels

Equation (4.12) for the noise source kernel \( K_{nm} \), based on the logarithmic energy ratio \( A \), is valid in 3-D visco-elastic media where both single- and mixed-component correlations are considered. To develop a computationally inexpensive tool for rapid estimation of the noise source distributions prior to a detailed inversion, we adopt two simplifications: (i) We limit the starting model to a homogeneous noise source distribution within a 2-D structurally homogeneous medium, and (ii) we reduce spatially extended sensitivity kernels to rays. These simplifications are detailed below.

2-D homogeneous media

In the first stage of simplifications, we choose a homogeneous noise source distribution as reference, and a homogeneous medium of wave propagation. Given this homogeneous source distribution and homogeneous medium, it follows that \( E_+ = E_- = E \) and \( A = 0 \). We limit the analysis to vertical-component correlations. Furthermore, we use 2-D wave propagation as an analogue for single-mode surface wave propagation. A similar approach was taken by Hanasoge (2013b,a) to study noise correlation amplitude kernels. With the noise source distribution \( S(\omega) \) being constant in space, equations (4.1) and (4.12) are simplified to

\[
C(x_1, x_2) = S \int G(x_1, x) G^*(x_2, x) \, dy \quad \text{(4.13)}
\]

and

\[
K(x, \omega) = -\frac{A^0}{\pi E} \left[ \left( w^2(\omega) * C(\omega) \right)^* - \left( w^2(\omega) * C(\omega) \right)^* \right] G(x_1, x, \omega) G^*(x_2, x, \omega). \quad \text{(4.14)}
\]

In an unbounded 2-D medium with mass density \( \rho \) and shear modulus \( \mu \), the far-field Green’s function is given by

\[
G(x, y, \omega) = -i \frac{1}{4\rho v^2} \sqrt{\frac{2v}{\pi \omega r}} e^{-i\frac{\omega}{v Q} r} e^{-i\frac{\mu}{\rho} r}, \quad \text{(4.15)}
\]

In equation (4.15), \( v = \sqrt{\mu/\rho} \) denotes the phase velocity, \( r = |x - y| \) the source-receiver distance, and \( Q \) the quality factor or inverse attenuation. The far-field approximation has no effect on the synthetic correlation functions and the sensitivity kernels at distances of more than a wavelength from each of the receivers, and it greatly simplifies the subsequent analysis. Substituting the Green’s function (4.15) into expression (4.5) for the sensitivity kernel \( K(x) \),
Figure 4.1 – Gallery of noise source kernels for $Q = 100$ (top row) and $Q = 300$ (bottom row) at periods of 20, 50 and 100 s (from left to right). Red triangles mark the receiver positions. Plotted below each kernel is its integral in $y$-direction, which more clearly shows the decay of the kernel amplitude with increasing distance from the receiver pair. With increasing frequencies, Fresnel zones become narrower and decay more quickly. For $Q = 100$ and $T = 20$ s, the kernel extends approximately 2500 km behind both receivers (top left panel). Sensitivity is practically zero in between the receivers. The longer-period line kernels show oscillations that are a ringing effect caused by the integration of kernels from monochromatic synthetics.

\[
K(x) = \text{Re} \frac{1}{4\pi \omega \rho^2 v^3} \int \frac{e^{i\frac{\pi}{Q}\left(r_2 - r_1\right)}}{r_1 r_2} e^{-\frac{\omega}{2vQ}\left(r_1 + r_2\right)} dx, \quad (4.16)
\]

where we defined $r_1 = |x - x_1|$ and $r_2 = |x - x_2|$. A gallery of noise source kernels for different values of $Q$ and different periods, is shown in Fig. 4.1. All kernels have a characteristic hyperbolic shape with almost no sensitivity in between the pair of receivers. With increasing frequency, Fresnel zones become narrower, and kernel amplitudes decay more rapidly – as predicted by equation (4.16). For instance, at 20 s period and $Q = 100$, the kernel extends approximately 2,500 km on either side of the receiver pair. This is in contrast to the long-period kernel at 100 s that extends further than 20,000 km for $Q = 300$. More examples of noise source
4.2. Theory for noise source imaging

Kernels for different types of measurements may be found in Tromp et al. (2010) and Hanasoge (2013b,a); more detail on source-structure tradeoffs and attenuation-dependent kernels can be found in Fichtner (2015).

Reduction to ray theory and the imaging concept

To simplify the noise source kernels geometrically, we collapse sensitivity into an infinitesimally thin ray with amplitude equal to the integral perpendicular to the great-circle connecting the two receivers. In the specific context of the previous 2-D examples where both receivers were located along the x-axis, we thus replace \( K(x) = K(x, y) \) by

\[
K_{\text{ray}}(x) = \int K(x, y) \, dy. \tag{4.17}
\]

To derive the decay of the ray-theoretical sensitivity along the great circle through the receiver pair, we let both receivers be positioned on the x-axis at equal distances from the origin, as shown in Fig. 4.2. Furthermore, we work under the assumptions used for the computation of the 2-D examples in section 4.2.1. These assumptions include a homogeneous reference distribution of noise sources and waves propagating through a 2-D homogeneous, unbounded medium. In order to preserve the total sensitivity, we ascribe to each x-coordinate along the ray the sensitivity integrated along the ray-perpendicular direction, that is

\[
K_{\text{ray}}(x) = \int_{-\infty}^{\infty} K(x, y) \, dy. \tag{4.18}
\]

Recalling equation (4.16), the 2-D sensitivity kernel is given by

\[
K(x) = \text{Re} \frac{1}{4\pi \omega \rho^2 \nu^3} \frac{f}{\sqrt{r_1 r_2}} e^{i \frac{\mu}{\nu} (r_2 - r_1)} e^{-\frac{\omega}{\nu Q} (r_1 + r_2)}. \tag{4.19}
\]

The 2-D kernel \( K(x) \) is constant along the hyperbolae \( r_2 - r_1 = \text{const.} \), which are asymptotically close to straight lines crossing the origin. Furthermore, \( K(x) \) is an oscillatory function along the y-axis for constant x. We can obtain an approximation of the integral (4.18) by considering integrals over just one oscillation for a fixed x:

\[
K_{\text{ray}}^{[a,b]}(x) = \int_{a}^{b} K(x, y) \, dy. \tag{4.20}
\]

Assuming that x is located at sufficient distance from the receiver pair, we observe the following when changing x to \( \gamma x \), where \( \gamma \) is some real number (see also Fig. 4.2): (i) The same oscillation is now approximately located within the interval \( \gamma a \leq y \leq \gamma b \), meaning that the oscillation period increases by the factor \( \gamma \). (ii) The scaling factor \( (r_1 r_2)^{-1/2} \approx |x|^{-1} \) in equation (4.19) changes to \( \gamma^{-1} |x|^{-1} \). (iii) The attenuation term \( \frac{\omega}{\nu Q} (r_1 + r_2) \approx \frac{\omega}{\nu Q} |x| \) is modified to \( \frac{\gamma \omega}{\nu Q} |x| \).

Effects (i) and (ii), that is stretching and scaling of the oscillation, cancel each other exactly;
Figure 4.2 – Illustration for the computation of a ray theoretical noise source kernel. In the absence of attenuation, the integral of $K(x)$ over $y \in [a, b]$ is equal to the integral over $y \in [\gamma a, \gamma b]$ because stretching and scaling of one oscillation in the kernel compensate each other.

meaning that the ray-perpendicular integral (4.18) is constant with respect to $x$ when $Q$ is infinite. Finite $Q$ adds the attenuation factor $e^{-\frac{1}{Q} |x|}$ to the integrand. Since $e^{-\frac{1}{Q} |x|} \leq e^{-\frac{\gamma}{Q} x}$, we obtain the following upper bound for the ray-theoretical kernel amplitude:

$$K_{\text{ray}}(x) \approx \text{const.} \cdot e^{-\frac{\gamma}{Q} x}. \quad (4.21)$$

The constant in equation (4.21) absorbs the scaling factors from the 2-D kernel in equation (4.19), as well as the sum of the integrals over all individual oscillations. The fact that $K_{\text{ray}}(x)$ is constant in the absence of attenuation highlights the global nature of noise propagation. Noise sources at arbitrary distance from the receiver pair can have the same effect as noise sources close to the receivers. Only the presence of attenuation reduces the impact of distant noise sources. In contrast to the amplitudes of traveling waves that decay as $e^{-\frac{\gamma}{Q} x}$, the amplitude of the noise source kernel decays more quickly as $e^{-\frac{\gamma}{Q} x}$. We finally note that our analysis makes no assumptions on the properties of the adjoint source $f$. It is therefore valid for all types of measurements. Thus, the ray theory kernel $K_{\text{ray}}$ is given by

$$K_{\text{ray}}(x) \approx \text{const.} \cdot e^{-\frac{\gamma}{Q} x}. \quad (4.22)$$

In media without attenuation, that is infinite $Q$, the kernel amplitude along the ray is constant. It follows that the inversion for ambient noise sources in the hypothetical absence of attenuation is an inherently global problem, even when a small local array of receivers is used. The presence of attenuation effectively localizes the kernels closer to the receiver pair. When attenuation is large, perturbations of noise sources at great distance will have little effect on
4.3 Applications: Imaging noise sources from regional to global scales

the correlation asymmetry.

A particularly noteworthy aspect of equation (4.22) is the fact that the kernel amplitude decays as $e^{-1/Q}$ whereas the amplitude of the wavefield decays less rapidly as $e^{-1/2Q}$. The kernels decay comparatively rapidly because a product of two wavefields and not a wavefield itself enters the computation of the correlation function.

Based on the ray-theoretical kernels, we can introduce a source imaging concept. This is illustrated in Fig. 4.3 using the example of a noise correlation between the North American stations GLA and WCI. The noise correlation is strongly asymmetric with larger surface-wave amplitudes on the anti-causal branch, thus indicating that energy travels predominantly from west to east. The corresponding noise source kernel multiplied by $-1$ designates a descent direction, that is the direction of a first update that would place stronger sources to the west of the receiver pair, and weaker sources to the east - assuming a homogeneous reference. The ray-theoretical kernel encodes the same information but in a simplified fashion that enables the rapid estimation of noise source distributions for a large number of receiver pairs, without the need to compute extended finite-frequency kernels for all of them.

For the imaging of noise sources, we scale the ray theoretical kernels such that the kernel value 1 corresponds to the maximum observed asymmetry for the complete noise correlation data set. The ensemble of kernels for all receiver pairs then provides an image of the normalized PSD variation

$$\delta S_{\text{norm}} = \frac{\Delta S(x)}{\max(\Delta S(x))}. \quad (4.23)$$

This image is a scaled version of the first update in a gradient-based optimization of the source PSD. Put in more colloquial terms, the image shows in what direction an optimization of source PSD to fit the amplitude-ratio of the data would look like. It is the first step towards running an inversion for source PSD: the image - similar to a color-coded travel time misfit plot in a tomographic inversion - enables us to decide whether running an inversion is worthwhile, and it provides a valuable first estimate of the results. However, these must be cautiously interpreted as first-order estimates, which are additionally limited by the approximations taken and by shortcomings of similar nature to those of tomographic images, such as the effect of receiver coverage.

Determining the normalization constant would require additional forward modeling steps to determine the optimal step length in the first iteration of a gradient-based optimization; which would be the logic next step beyond the imaging proposed here. In section 4.3, we illustrate this imaging concept using a variety of regional and global examples.

4.3 Applications: Imaging noise sources from regional to global scales

To illustrate the proposed ray-theory imaging of noise sources on the basis of asymmetry measurements, we present three real-data examples on different scales. These include (i) noise correlations for globally distributed stations in the hum frequency band, (ii) noise correlations in the microseismic band for stations deployed in the Western Mediterranean,
Figure 4.3 – Schematic illustration of the noise source imaging concept. The noise correlation between stations GLA and WCI is strongly asymmetric, indicating that seismic energy propagates predominantly from west to east. The negative noise source kernel, shown in the lower left, constitutes a descent direction, that is an update to the homogeneous reference that would increase the noise source power-spectral density to the west of the receiver pair and decrease power-spectral density to the east. Shown in the lower right is the ray theoretical kernel that encodes the same information, though in a simplified way that avoids the computation of finite-frequency kernels for a potentially large dataset of noise correlations.
and (iii) correlations of microseismic noise recorded at broadband stations within Switzerland. The Western Mediterranean data originate from permanent stations in Spain, Portugal and France, as well as from temporary deployments in the Iberian Peninsula (IberArray), northern Africa (PICASSO, Universities of Münster and Bristol) and Southern France (PYROPE). More details on the data can be found in Díaz et al. (2009), Chevrot et al. (2014) and Thurner et al. (2014). Station maps for all three examples are shown in Fig. 4.4. The processing and measurement details for all datasets are described in section 4.3.1. This will be followed in sections 4.3.2 to 4.3.4 by the presentation and discussion of the individual noise source images.

### 4.3.1 Processing, measurements and visualization

Data provenance and processing are summarized in Table 4.1. During the processing, we tried to avoid any non-linear operations. We use the classical cross-correlation with geometric normalization. All correlation windows are then stacked linearly. Where necessary, we removed windows containing earthquakes using an sta/ltb network coincidence trigger (this is discussed in more detail for each application example).

Given what strong effects the pre-processing scheme applied to ambient noise correlations can have on the results, and how much effort has been spent to investigate different processing strategies (e.g. Bensen et al., 2007; Lawrence and Prieto, 2011; Schimmel et al., 2011; Groos et al., 2012; Shen et al., 2012; Boué et al., 2014), this approach may appear unconventional at first sight. However, we do not aim to obtain an empirical Green’s function from the noise correlations, but rather a correlation per se that can be forward modeled. The less pre-processing is applied, the more closely the forward model corresponds to the observations.

Moreover, pre-processing usually seeks to isotropize the ambient noise field. This is not conducive to our goal of mapping ambient noise sources. Finally, processing can influence whether local sources or distant sources dominate the correlation (Stehly et al., 2006; Cupillard and Capdeville, 2010). We take a simple approach of correlating and stacking. We potentially sacrifice good signal to noise ratios, and weak coherent signals from distant sources, at the benefit of being able to derive the above sensitivity kernels, and to construct forward models of ambient noise correlations.

Processing such as instrument correction, and correlation was undertaken with a parallel processing tool which is based on Python and the obspy seismological toolkit (Beyreuther et al., 2010). Our parallel processing/correlation toolbox is available at https://github.com/echolite/ANTS.
Figure 4.4 – Distribution of seismic stations used in the noise source imaging examples. 
**Top:** Global distribution of STS-1 stations used for correlations in the hum frequency band. 
**Bottom:** Seismic stations in the Western Mediterranean (left), and from the Swiss broadband network (right).
<table>
<thead>
<tr>
<th>Data set</th>
<th>globally recorded hum</th>
<th>Iber Array</th>
<th>Swiss Nat. Network</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time covered</strong></td>
<td>2004 to 2013</td>
<td>2008 to 2011</td>
<td>January and July 2014</td>
</tr>
<tr>
<td><strong>Average stack length</strong></td>
<td>290 days (Fig. 4.5), 520 days (Fig. 4.6)</td>
<td>434 days</td>
<td>24 days</td>
</tr>
<tr>
<td><strong>Network</strong></td>
<td>IRIS virtual network _STS-1</td>
<td>IberArray, PICASSO, PYROPE</td>
<td>CH (SDSNet)</td>
</tr>
</tbody>
</table>

| Processing steps         | zero-pad gaps ≤ 60 sec Decimate to 1 Hz Correct to ground velocity Windows 32768 s | Fill gaps of ≤ 5 min Decimate to 5 Hz Correct to ground velocity Windows 14400 s Spectral whitening | zero-pad gaps ≤ 2 sec Decimate to 5 Hz Correct to ground velocity Windows 4096 s Removal of earthquakes |
|                         | Decimate to 0.1 Hz Bandpass filter 0.002 to 0.05 Hz Correlation -12000 to 12000 s | Correlation -3000 to 3000 s | Correlation -300 to 300 s |

| Analyzed frequency bands | 0.004 - 0.02 Hz          | 0.05 - 0.1 Hz and 0.1-0.2 Hz | 0.05 - 0.1 Hz and 0.1-0.2 Hz |

Table 4.1 – Summary of data provenance and processing for the three real-data examples presented in the following paragraphs.
Chapter 4. Cross-correlation imaging of ambient noise sources

To investigate sources in different frequency bands, we bandpass filter correlations, as summarized in table 4.1. With the processing used here, the differences between correlations filtered before or after correlation are negligible. In the interest of greater flexibility, we therefore apply the bandpass filter after correlation and stacking.

To measure correlation asymmetry, we empirically determine Rayleigh wave group traveltimes on record sections of the noise correlations. We then measure energy ratios for Hann windows centered on the empirical arrival times on the causal and anti-causal branches. To ensure that the main surface wave signal is indeed captured, we visually inspect a subset of the correlations.

We assess measurement quality by comparing the energy in the measurement windows to the energy in two noise windows centered at times where no major arrival is expected. An example is shown in Fig. 2.1. Measurements where the signal-to-noise ratio, defined here as the energy ratio of the measurement and noise windows, drops below a pre-defined threshold are excluded. We choose a threshold of 10 for the microseismic regional data and a threshold of 7.5 for long-period global data.

Adopting the imaging concept introduced in section 4.2.1, we plot all correlation asymmetry measurements on the great-circle major arc connecting the relevant pairs of receivers. While we determined the group speed of the Rayleigh surface waves empirically from record sections of the cross-correlations, values of $Q$ were approximated in the global case by using the quality factors from PREM for the fundamental spheroidal mode closest in frequency to the center frequency of the observations, while for the regional cases we used a quality factor of 120. Varying $Q$ from 100 to 120 has little effect on the resulting maps. To avoid a visual bias in regions where many rays cross, we bin rays into $2^\circ \times 2^\circ$ bins for the long-period global correlations and into $0.5^\circ \times 0.5^\circ$ bins for the regional correlations. This is illustrated in Fig. 4.5.

In the following sections, we show maps of noise source distributions for the global hum, and the microseismic noise in the Western Mediterranean and in the Swiss broadband network. It is important to keep in mind that these maps show the deviation from a homogeneous reference distribution of sources rather than an absolute PSD distribution, for which the normalization term of equation 4.23 would have to be determined.

4.3.2 Case I: The sources of globally recorded hum

Images of seismic hum sources are shown in Fig. 4.5 and 4.6. Fig. 4.5 shows hum sources in a broader frequency band (4-20 mHz) for two different years (2005 and 2008). Fig. 4.6 shows hum sources in a narrow frequency band of 6-8 mHz for four different months. Each monthly plot contains data from ten times the same month during the years 2004 to 2013, to improve signal-to-noise ratios and detect ‘typical’ seasonal patterns.

Earthquake signals were not cut out during processing in this case; the resulting maps therefore strictly speaking show effective sources of the correlation wave field including anything recorded. We propose that the large-scale features of these maps can still be interpreted in terms of ambient seismic sources, because (i) a large number of time windows was stacked for each correlation, reducing the influence of any comparatively short signals; (ii) station
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distances are generally large and preclude highly coherent earthquake signals as would be expected at a single array; an exception are the Berkeley Digital Seismic Network and Southern California Earthquake Center arrays, but excluding these does not change overall map patterns (see supplementary Fig. S1); (iii) observed source patterns in Fig. 4.6 are clearly seasonal and (iv) results obtained with the phase cross-correlation (Schimmel, 1999), which emphasizes weak coherent signals, are consistent with the results presented here except for minor details (see supplementary Fig. S2).

The images for the years 2005 and 2008 are similar on length scales of $\sim 1000$ km in both the patterns and their amplitudes. This indicates that the sources of hum averaged over one year are stable over time, without being dominated by non-repeatable events. We observe a clear dominance of the Pacific hemisphere, with the strongest sources located in the North Pacific. The sources of hum mapped here are also mostly weaker on the continents than in the oceans. Notable exceptions are continental East Asia and Australia where hum sources are stronger than average, as well as the Atlantic Ocean where sources are relatively weak. Coverage in these regions is good, as shown in the hit count map in the bottom row of Fig. 4.5. The polar regions should be excluded from interpretations, because the geographic bins are too small to provide stable results.

The hit count map in Fig. 4.5 as well as the ray image reveal a strong contribution of the Berkeley Digital Seismic Network and the Southern California Seismic Network that contributed a comparatively large number of STS-1 recordings for the study period. However, excluding both networks does not significantly change the hum source images (see supplementary Fig. S1).

In Fig. 4.6, the images for January and July show a strong difference. In January the strongest increase in source power is located in the North Pacific. A weaker region of increased source power is observed in the North Atlantic. In July, increased source power is mapped in the South Pacific, Indian Ocean and Southern Ocean. April and October show intermediate patterns, which are similar to each other. Our observations are in accord with previous studies on the origin of the Earth's hum. Yang and Ritzwoller (2008) found that the direction of arrival of the strongest European noise sources at periods $> 50$ s invariably points towards sources in the Pacific, as on our yearly images. The North Atlantic, on our source images, also shows elevated source power in January, but comparatively weaker than the North Pacific.

In their global study, Rhie and Romanowicz (2004) found that hum sources are primarily located in the Northern Pacific during northern hemisphere winter, and in the Southern Ocean during northern hemisphere summer. Similarly, the inversion of cross-spectra by Nishida and Fukao (2007) revealed that from November to April a spherical harmonic degree-1 pattern with a maximum in the Northern Pacific dominates the hum source distribution. Furthermore, they found that during the northern summer months hum sources are primarily located along the Pacific rims as well as in the Indian Ocean.

Our data for the Earth's hum suggest that the North Atlantic is, during most seasons, a region of weaker than global average noise excitation. This result may initially appear surprising considering the work of Essen et al. (2003) and Kedar et al. (2008) which suggests strong microseismic
Chapter 4. Cross-correlation imaging of ambient noise sources

Figure 4.5 – Source images for globally recorded hum in the frequency band from 4-20 mHz. **Top:** Maximum-normalized power-spectral density of noise sources for year 2005 (left) and year 2008 (right), as defined in equation (4.22). The maximum observed asymmetry, $A_{\text{MAX}}$ is indicated above the colour bar. Ray paths trace the great-circle major-arc between each receiver pair. **Middle:** As above, but with rays binned and normalized into $2^\circ \times 2^\circ$ cells. **Bottom:** Number of rays crossing the $2^\circ \times 2^\circ$ cells.

noise sources in the North Atlantic. However, the excitation mechanism of the Earth’s hum and its bathymetric requirements are different from the one of secondary microseisms (Ardhuin et al., 2015). Moreover, predominance of Pacific hum sources is consistent with earlier studies (Yang and Ritzwoller, 2008; Rhie and Romanowicz, 2004; Nishida and Fukao, 2007). Our results indicate that while North Atlantic hum sources do contribute to the excitation of the Earth’s hum in northern hemisphere winter, their contribution is less energetic than that from the Pacific.

4.3.3 Case II: Microseismic noise in the Western Mediterranean

Noise source images for the Western Mediterranean correlations are displayed in Fig. 4.7 for the single-frequency microseismic band (0.05-0.10 Hz) and the double-frequency microseismic band (0.10-0.20 Hz). We observe a distinctly two-sided pattern in both frequency bands, pointing to weaker than average sources in the Western Mediterranean, and stronger than
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Figure 4.6 – Source images for globally recorded hum in the frequency band 6 - 8 mHz and for the months January (including all Januarys from 2004 to 2013), April, July, October. A distinct seasonal change between January and July is observed where in January, sources power is elevated in the northern hemisphere and in July, source power is elevated in the Southern hemisphere, corresponding to the respective winter months.

average sources in the Atlantic and along the coast of the Bay of Biscay in particular. In the double-frequency band the maximum observed asymmetry of ±4.6 is ~30% larger than for the single frequency band. This translates to maximum causal/anti-causal energy ratios of ~33 and ~100, respectively.

The slight differences in the pattern between the single- and double-frequency images mostly result from the faster decay of sensitivity at higher frequencies, already discussed in section 4.2.1. Attenuation leads to a stronger localization of potential noise sources in the vicinity of the receiver pair. Small differences not related to frequency-dependent sensitivity are limited to the region north-east of the Canary Islands. In the single-frequency band the Canary Island stations require sources towards south, whereas the double-frequency microseisms require a weak patch of stronger than average sources along the Moroccan coast.

The Western Mediterranean results presented here were elaborated on the basis of a pre-existing correlation dataset, which was computed using spectral whitening. It is therefore not entirely evident that they can be interpreted in terms of noise source power, although the Atlantic coast is a plausible source region for microseisms. What the results do provide, however, is an image of the ‘effective source’ of this correlation dataset, indicating clearly, for example, that the signal energy in the correlations is not isotropic even after the application of spectral whitening. Further research is needed to assess how spectral whitening affects the source imaging method used here, and related source imaging methods.
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Figure 4.7 – Maximum-normalized power-spectral density distribution of noise sources for ambient noise correlations between Western Mediterranean stations (Díaz et al., 2009) in the single-frequency microseismic band (0.05-0.10 Hz, left) and the double-frequency microseismic band (0.10-0.20 Hz, right). The maximum observed asymmetries are given in brackets above the colour bars.

4.3.4 Case III: Microseismic noise at the Swiss Digital Seismic Network

For the small Swiss dataset of ~25 stations, we analyzed correlations for two different months, January and July 2014. The corresponding noise source images for the single- and double-frequency microseismic bands are displayed in Fig. 4.8. For this dataset, we removed windows containing earthquake signals if they were detected by an sta/lta trigger at more than 20 stations.

In the double-frequency band, we observe a two-sided pattern. While the pattern is similar for both January and July, the observed asymmetries are ~50% stronger in January, that is during the northern winter when storm activity in the North Atlantic is high. Moreover, the directionality of noise sources tips more towards the Mediterranean in summer. With the given geometry of the Swiss network, distance to the dominant sources is badly constrained. Considering this, the closest coast is a plausible dominant source area, but more removed, stronger sources can fit the observations equally well. Therefore, tipping of the double-frequency source map towards south during summer can also indicate a more prominent contribution from the South Atlantic or Southern Ocean during that season.

The single-frequency band is distinguished by an even more pronounced seasonal dependence of the noise sources. In January, strong single-frequency noise sources are located west to north-west of the array in the North Atlantic, whereas in July, source power is elevated towards various directions. The source directions found here are broadly consistent with those of Yang and Ritzwoller (2008), who show the azimuthal dependence of noise correlation signal-to-noise ratios for northern summer and northern winter at a European array.

In general, microseismic noise incident from the Atlantic Ocean dominates the noise wavefield in Switzerland. Although contributions from the Mediterranean are certainly present, they are consistently smaller when viewed on the monthly average maps. It remains to be investigated
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Figure 4.8 – Maximum-normalized power-spectral density distribution for the Swiss Digital Seismic Network. **Top:** Images for the single-frequency band from 0.05-0.10 Hz for January (left) and July 2014 (right). **Bottom:** As above but for the double-frequency band from 0.10-0.20 Hz.

whether on shorter timescales - such as one day - the Mediterranean may produce equally strong or stronger signals at the Swiss network.

4.3.5 Synthetic test of the imaging procedure

To understand how well the imaging procedure retrieves an input model, we constructed a synthetic test for the case of the Earth’s hum. Compared to their microseism counterparts, the long-period surface waves of the hum propagate over very long distances before being attenuated. As a result, constraining the sources of hum is expected to be challenging, as sensitivity to sources drops off slowly and kernels may ‘smear out’ the imaged source PSD over large areas.

To assess this effect, we use a spherically symmetric Earth model (PREM, (Dziewoński and Anderson, 1981)) and constructed synthetic correlations as follows:

To subdivide the Earth’s surface into approx. 8000 source locations on a regular grid. We then define a source PSD mask, $S(x)$, that prescribes a location-dependent source PSD factor.
Figure 4.9 – Synthetic test for the retrieval of two Gaussian-shaped source distributions. The upper two panels show the input, the spatial distribution of a source PSD which is assumed to be constant with respect to frequency (supposing that the analysis is restricted to a narrow frequency band, as is the case here). The lower panels show the source maps retrieved from the synthetics by the approach presented here.

We restrict the analysis to a frequency-independent $S(x)$, as our observational analysis treats narrow frequency bands. With this source mask, and under the assumption that sources are spatially uncorrelated, we construct the synthetic correlation function between stations $a, b$ as

$$C(x_a, x_b) = \sum_{n=1}^{N} G(y_n, y_a) G^*(y_n, y_b) S(y_n) \, dy.$$  \hspace{1cm} (4.24)

which is a discrete approximation to equation 4.1, for vertical components only. $N$ is the number of source locations. Single Green’s functions are constructed using vertical point force sources in INSTASEIS (van Driel et al., 2015). We evaluate the synthetics that we constructed in exactly the same way as the observed correlations, except that no signal-to-noise threshold is used. We thus recover an imaged $\tilde{S}(x)$.

A comparison of original $S(x)$ and retrieved images $\tilde{S}(x)$ is shown in Fig. 4.9. Two cases are presented. Two narrow Gaussian sources, shown by the upper two plots, are placed in the North Atlantic and Indian Ocean, respectively. These are used as an input to the synthetic correlations constructed according to equation 4.24. The lower two plots show the retrieved images.

The test demonstrates that the method is, in principle, effective. The narrow sources are imaged broadly across the ocean into which they are placed; accuracy is limited, most probably, by the array configuration. While it is desirable to constrain single sources more narrowly than is achieved here, the synthetic approach enables us to evaluate future improvements of the method.
4.4 Discussion

We developed and applied a new method for the imaging of ambient noise sources based on the measurement of cross-correlation asymmetry. The presented method has the advantages that (i) Computational requirements are negligible, and it can be applied directly to existing noise correlation datasets without the need for additional processing. One caveat applies: The method is only strictly valid for linearly processed data. Results such as those from the Western Mediterranean show an effective source distribution after pre-processing. Therefore, interpretations in terms of physical noise sources should be treated very cautiously; further research is needed to quantify the effects of nonlinear processing on waveforms, and consequently on the measurement performed here. (ii) In contrast to beamforming techniques, neither a dense array nor the plane-wave assumption are required. (iii) The chosen measurement is independent of absolute noise correlation amplitudes.

In the next paragraphs we discuss further details of our method, including the underlying assumptions and limitations, the nature of the imaged noise sources, and possible future directions of research. Furthermore, we place our method in the context of the ongoing transition from ray-based to finite-frequency tomography.

4.4.1 Assumptions and limitations

The presented imaging method rests on the assumption that noise sources are spatially uncorrelated in the sense of equation (4.3). This is a crucial assumption to make large numbers of synthetic correlations computable and to keep the theory tractable. Kimman and Trampert (2010) constructed numerical experiments in which noise sources overlapped in time, and thus sources that were not collocated could be correlated. Comparing the resulting correlations to those produced with uncorrelated sources showed that the fundamental mode surface wave converged, given a sufficient number of sources. Convergence was harder to achieve for higher mode surface waves (which are not subject of the analysis here). The result by Kimman and Trampert (2010) means that correlations are consistent with spatially uncorrelated sources if enough contributions are averaged. Whether an observational stack is long enough to have converged past the effect of spatially correlated sources is hard to judge. For simplicity, we assumed convergence here. To include a criterion for convergence is a possible improvement to the method.

A minor limitation of the method as presented here stems from the simple window selection procedure, where windows are centered on predicted Rayleigh wave group arrivals. This selection may not always match the arriving wave group perfectly, especially in the microseismic range where Rayleigh waves sample complex crustal structure. However, tests with group velocities varying between 2.5 and 3.1 km/s on the Swiss dataset show that the resulting noise source images are not significantly affected, thereby demonstrating the robustness of the method.

In section 4.2 we proposed to measure logarithmic amplitude ratios between the causal and anti-causal parts of the correlation function in order to infer the distribution of noise sources.
This measurement is expected to show negligible sensitivity to Earth structure when the refer-
ence noise source distribution is isotropic. Strictly speaking, laterally varying Earth structure
(in particular, variable attenuation) can exert small effects on the correlation asymmetry even
in the presence of isotropically distributed sources (Liu and Ben-Zion, 2013). A rigorous quan-
tification of possible tradeoffs is a topic for further research; in the case of a homogeneous
reference source distribution, we assume that these effects are small. A full inversion would
allow to account for biases induced by Earth structure. For arbitrary noise source distributions,
including the first update to an initially isotropic distribution, the argument of negligible
influence of Earth structure on our measurement does not hold. Once the reference isotropic
source distribution is perturbed towards a more realistic one, perturbations to the structural
model may affect the asymmetry measurement as well. It follows that an iterative inversion
scheme for the noise source distribution should ideally account for 3-D Earth structure in all
iterations.

All current methods to image the sources of ambient seismic noise, including the one pre-
sented here, implicitly operate under the assumption that the structure of the Earth is known.
Since tomographic resolution is finite, this assumption can clearly not be perfectly met even
when elaborate Earth models are used to solve the forward problem and compute sensitivity
kernels. Consequently, small-scale Earth structure that acts as scatterer does have the poten-
tial to act as apparent source of noise. While such trade-offs can never be completely avoided,
they may be minimized by future joint inversions for noise sources and Earth structure. The
work presented here is intended to contribute to their development.

4.4.2 Finite-frequency ambient noise inversions?

Following the work of Yomogida (1992) and Friederich (1999), seismic inversion for Earth
structure started to transition from ray theory to methods based on spatially extended finite-
frequency kernels (e.g. Dahlen et al., 2000; Dahlen and Baig, 2002; Yoshizawa and Kennett,
2004, 2005; Zhou et al., 2004; de Vos et al., 2013). These developments appeared when seismic
tomography – first applied in the 1970’s (Aki and Lee, 1976; Aki et al., 1976; Dziewoński et al.,
1977) – had already reached a mature stage.

The inversion for seismic noise sources based on the rigorous computation of sensitivity
kernels is emerging and still far from a similar state of maturity. So far, only synthetic inversions
have been studied (Hanasoge, 2013b,a), and experience with real data is still very scarce (Basini
et al., 2013). Our approach must be seen in this context as a first step in a developing field. It
is intended to serve as a data analysis step prior to a complete iterative inversion for noise
sources. It should provide insight into the usefulness of a noise correlation dataset for the
imaging of noise sources, and it should train the physical intuition that is generally needed
to produce meaningful solutions of ill-posed inverse problems. In this regard, our imaging
of noise sources resembles the ray theory imaging of traveltine delays frequently used to
anticipate the outcome of a tomographic traveltime inversion (see, for instance, Fig. 2 of
Ritsema et al. (2007)).
4.4.3 Beyond asymmetry

While we limited our measurements to the easily observable asymmetry of the main surface wave arrivals, additional information on the noise source distribution is certainly contained in other parts of ambient noise correlations. In this context, precursory arrivals that appear prior to the P-wave may be particularly valuable (Landès et al., 2010). The study of such signals is, however, beyond the scope of this work. One challenge in analyzing such signals is that it is not immediately clear which type of coherent wave gives rise to the correlation here: It may be surface waves incident at a high angle to the receiver-receiver line, or highly coherent body waves incident subvertically at an array, in which case the kernels turn 3-D. Distinguishing the two cases requires analyzing several correlations from an array jointly. Investigating into such signals may be a promising future direction of research.

4.5 Conclusions

We present a method for the imaging of ambient noise sources based on measurements of cross-correlation asymmetry and the computation of ray-theoretical sensitivity kernels. Advantages of the method include the following: (i) The measurement is robust and independent of absolute noise correlation amplitudes that are affected by attenuation and elastic focusing. (ii) Neither a dense array nor a plane-wave assumption are needed. (iii) The method properly accounts for visco-elastic attenuation and images an actual physical quantity, that is the normalized PSD of noise sources as a function of space and frequency. (iv) Furthermore, the imaging technique operates directly on noise correlation data sets. No additional processing is required, which makes the method fast and computationally inexpensive. The method may serve as a tool to image ambient noise sources on correlation datasets carefully elaborated without nonlinear processing steps, and may serve as a tool to image the 'effective source distribution' seen by correlation datasets that were processed using, for example, spectral whitening or one-bit normalization.

Applied to globally recoded hum in the period range from 4-20 mHz, our method indicates sources that are stronger in the oceans than on the continents, in accord with previous studies. However, well-covered exceptions appear in continental East Asia and Australia (stronger sources), and in the Atlantic Ocean (weaker sources). For microseismic noise recorded at a regional-scale array in Switzerland, our method reveals almost unidirectional noise propagation except for primary microseismic noise in the summer months. The dominant directions are thus functions of both frequency and time.

Our method is intended as a step towards joint inversions for 3-D Earth structure and ambient noise sources. It will serve, in this context, as a fast tool to assess the information on noise sources contained in ambient noise correlations – and thus to gain the physical intuition needed to solve any ill-posed inverse problem. An extension to finite frequency kernels and other measurements is a topic of further research.
5 Global sources of the Earth’s hum

This chapter has been modified from:

As case study for the first application of the described 3-D ambient source inversion method, we chose the vertical-component sources of the long-period background signal known as the Earth’s hum (Nawa et al., 1998; Suda et al., 1998; Tanimoto et al., 1998; Nishida et al., 2000; Kurrle and Widmer-Schnidrig, 2006, and others). There are several motivations for this choice: First, at long periods, the Earth’s seismic structure appears smooth, and the seismic wave field including surface waves is predicted rather well by laterally varying tomographic models such as S40RTS (Ritsema et al., 2011). This allows us to invert for ambient sources of the hum without updating the structure of the Earth model, under the assumption that long-periodic surface waves are sufficiently well modelled so as not to bias our results. Second, the study of Stehly et al. (2006) and others have shown that ambient noise can be observed coherently over very long distances, which makes noise source inversion an inherently global problem. A global inversion spares us from having to define an arbitrary limit to our model domain. Third, the seasonally varying source distribution of the hum has been studied previously (Rhie and Romanowicz, 2004; Nishida and Fukao, 2007; Bromirski and Gerstoft, 2009), although with methods resting on more assumptions, and only in the case of Nishida and Fukao (2007) with a global inversion. Nevertheless, these previous results provide us with points of comparison to evaluate the proposed inversion procedure. There are several open questions regarding the excitation mechanism of the hum, as described in section 1. While in the present study our main focus is on the applicability of the inversion method to observed data, we will report on our findings, and discuss in section 5.3 what and how cross-correlation inversion can contribute to the understanding of seismic hum.
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5.1 Description of dataset and inversion setup

5.1.1 Dataset

We aim to observe a very low amplitude signal, with typical ground displacements not exceeding a few micrometers, coherently at distant locations. We focus on the broad noise peak between 120 and 330 seconds period; sensor characteristics make the observation of long-period signals more susceptible to noise, and noise due to barometric variations and tilt mostly affects long periods. Therefore, Kurrle and Widmer-Schnidrig (2006) argued that propagating hum waves could only be observed at the quietest stations; indeed Nishida and Fukao (2007) used only the 54 quietest very-high gain broadband stations of the GSN network. Although this has been put into perspective by the successful use of temporary stations for array analysis of the hum (Bromirski and Gerstoft, 2009), at the long propagation distances we intend to investigate here, we expect lower coherency than through a regional array. In order to retrieve a meaningful cross-correlation signal despite these limitations, we compiled a global dataset from all permanent stations that operate STS-1 broadband seismometers, which have a frequency response that is flat up until 360 seconds (Wielandt and Streckeisen, 1982), summarized under network code _STS-1 at IRIS data management center (ds.iris.edu/mda/_STS-1). We retrieved data for 148 stations during the years 2004-2013. A map of the station locations is shown in Fig. 5.1. A detailed description of the data compilation and processing is provided in Ermert et al. (2016) where we used the data for a ray-based map of global hum sources. Instead of explicitly excluding earthquake signals, we normalized each cross-correlation window by the energy of both traces, down-weighting the contribution of any large-amplitude signals. The maximum correlation lag was set to 3 hours and 20 minutes, to capture both minor- and major-arc Rayleigh waves. Rather than forming stacks by month (e.g. stacking each December through years 2004 to 2013) as in Ermert et al. (2016), here we compiled two seasonal cross-correlation stacks for improved signal-to-noise ratio (SNR), one for North Hemisphere (NH) winter and one for South Hemisphere (SH) winter. Each of them spans three months out of the ten years (e.g. December, January, February 2004 - 2013 for NH winter). For the SH winter inversion, we excluded 9 stations from the cluster of STS-1 stations in California as these are located at short distances. A map comparing the network sensitivity with and without these stations is shown in supplementary Fig. B.1.

5.1.2 Data quality assessment

Due to the described observational challenges, a careful assessment of the data quality is important before iterative optimization of the source model, particularly as we analyze relative amplitudes of the cross-correlations, rather than their phases. This assessment is done in order to avoid overfitting our data during the iterative inversion. Most selection criteria employed by ambient noise tomography, such as symmetry of the cross-correlation function (Stehly et al., 2009), are designed to select cross-correlations with a favourable source distribution, and are therefore unusable for us. Instead, we invoke a consistency criterion for our assessment
of data quality, namely the comparison of cross-correlations at neighbouring stations with respect to a distant reference station. We utilized the four broad-band stations of the Canadian Yellowknife array YKW1 to YKW4 to investigate the reliability of the measurement $A_{filt}$. Since these stations are nearly co-located with a maximum distance of less than 10 km ($<<$ the investigated seismic wave-lengths that are on the order of 100 to 1000 km), cross-correlations of data from the four sensors spanning the approximate same time range (with only very few short outages at each station) with a distant reference must produce equal values of $A_{filt}$ except for effects caused by: 1) instrumental noise, and 2) highly site-specific effects, for example tilt induced by barometric pressure changes in a distance of few km around the station. Figure 5.1 illustrates the quality assessment. It shows the correlations of the four Yellowknife broad-band stations with respect to reference station G.TAM in Algeria, stacked over the years 2006-2008 which contain few data gaps and band-pass filtered between 330 and 100 seconds. The Rayleigh wave phases are highly consistent, which is shown by the right column. The top and bottom plot of the right column are zoomed onto the acausal and causal R1 arrival, respectively. While phases align well, absolute amplitudes show large variability. The relative amplitudes compared between causal and acausal Rayleigh wave are more consistent, but differences appear in the measurement of logarithmic signal energy ratio $A_{filt}$. For the example shown in Fig. 5.1, $A_{filt}$ ranges between $-0.15$ to $-0.34$, showing a consistently higher excitation of the acausal Rayleigh wave, but varying levels of asymmetry. In addition, the correlations show varying SNRs.

To assess these differences somewhat more quantitatively, we measured $A_{filt}$ for YKW1-4 with respect to all available reference stations in the dataset, this time on the seasonal stacks used for the inversion. We then computed the variance of the four measurements and defined their median over all reference stations as the overall observational variance. This observational variance is 0.33 for the NH winter and 0.46 for the austral (SH) winter. (The measurement $A_{filt}$ has no unit).

The correlation traces in Fig. 5.1 would have passed our visual inspection. In addition, their phases show good consistency. From this we conclude that amplitude deviations such as these are to be expected as observational error.

For the limited comparisons available, we found hardly any correlation between high SNR and high consistency between the four measurements. Therefore, we did not use an SNR criterion for data selection. Data gaps (be they sensor outages, archive gaps or download gaps) are a rather common problem of the ten-year, continuous global dataset. Therefore, we applied a selection criterion of a minimal number of stacked windows. In addition, we discarded all data where the inter-station distance was so short as to cause overlap of the causal and a-causal windows (effectively removing most Californian-Californian station paths from the data set).

### 5.1.3 Inversion setup

Based on the developments in section 2 and the observed cross-correlation data set, we invert for the optimal source distribution $S_{km}(\xi)$ both during NH and SH winter using the filtered logarithmic amplitude measurement $A_{f}$ (eq. 2.35). We restrict the problem to the vertical-
Figure 5.1 – Observational error was assessed by comparing correlations from the 4 broadband stations of the Yellowknife seismic array (top left map) with reference stations worldwide. This example shows correlations with respect to G.TAM in Algeria, at a distance of approx. 9000 km (marked by green circle, top right map). The left and center columns show the correlation traces for each of the four Yellowknife stations. The right column shows an overlay of the four traces, centered on the causal and a-causal R1 wave group. While phases are rather consistent, amplitudes and relative amplitudes vary more, indicating that a high variability of the measurement has to be expected. The a-causal correlation branch shows however consistently higher amplitudes.
vertical component of the cross-correlation tensor. We use one broad period range from approximately 130 to 400 seconds, and parametrize the spectrum of \( S_{nm}(\xi, \tau) \) by a Gaussian spectrum approximately matching the filter \( H(\tau) \) that selects this period band, obtaining the optimal space-dependent hum source PSD \( S_{k,nm}(\xi) \) without varying the shape of the spectrum. We assume that the influence of horizontally acting noise sources on the vertical-vertical cross-correlation is negligible, i.e. that off-diagonal elements of the Green’s tensors are negligible. Consequently, there is no need to consider cross-terms of the source PSD tensor. Hence, we invert for \( S_{k,zz}(\xi) \) using the \( C_{zz} \)-component. Choosing to investigate \( C_{zz} \)-components is not least dictated by the data themselves. We did not retrieve satisfactory SNRs for \( C_{rr} \) or \( C_{tt} \)-components. To our knowledge only one study has successfully investigated propagating long-range horizontal-component cross-correlations of the hum, using an elaborate data selection procedure (Nishida, 2014). Through the choice of filter and window in eq. 2.35, we target minor-arc Rayleigh surface waves (\( R_1 \)). Considering major-arc Rayleigh waves, body waves, or \( R_1 \)-waves arriving at spurious lag times is beyond the scope of this study. In accord with eq. 2.13, we only permit sources to act at the Earth’s surface. We define the source locations at the Earth’s surface by constructing an equal-distance grid of 30 km step length; this step approximately corresponds to the size of one spectral element in the simulation. For simplicity, receivers are located at sea level. For the simulation of the pre-computed impulse response wave fields, we use specfem3d_globe (Komatitsch and Tromp, 2002a,b). We choose S40RTS (Ritsema et al., 2011) as structural model, in combination with the attenuation model from PREM (Dziewoński and Anderson, 1981) and the crustal model CRUST2.0 (Bassin et al., 2000). The simulations include gravity, rotation, ellipticity, ocean load and topography.

We place a point force source at the location of each receiver, run one simulation for each of the 146 seismic stations, for a duration of 8 hours, and record it at all noise source locations. The long simulation duration is chosen in order to obtain cross-correlations with an overlap of at least half a window that matched the duration of the observed cross-correlations. Another important consideration of the simulation duration is to retain several surface wave trains. Major-arc Rayleigh waves are observed in the cross-correlations of seismic hum (e.g. Schimmel et al., 2011), which shows that even low-amplitude long-period ambient surface waves may propagate coherently over very long distances, i.e. at least a full Earth orbit. Multiple orbit surface waves all contribute to the cross-correlation and the time- (or frequency-) integrated source sensitivity kernel \( K(\xi) \) when global scales are considered (e.g. when an \( R_2 \) wave from an ambient sources correlates with the later arrival of this wave at the second station, producing an \( R_1 \) wave in the cross-correlation).

After concluding the numerical simulation, seismograms were collected in one file per receiver, prefiltered, downsamped to 0.1 Hz sampling rate, and corrected to seismic velocity.

On the basis of the resulting Green’s function database, we forward model all observed cross-correlations. As starting model for the source PSD, we choose a spatially homogeneous distribution of sources with a Gaussian spectrum:

\[
S_{k,zz}(\xi) = \text{const}, \quad s_k(f) = e^{-\left(f-f_c\right)^2/(2\sigma_f)^2} \cdot \frac{1}{s}, \quad f = \frac{\omega}{2\pi}
\]  

(5.1)
with a Gaussian spectrum centered at $f_c = 5 \text{ mHz}$ and a standard deviation of $\sigma_f = 1 \text{ mHz}$. The weights $S_{k,zz}$ are updated relative to a constant reference level during the inversion, because the measurement used is insensitive to absolute amplitude. The assumption of a homogeneous initial source distribution was taken so as to be impartial with respect to previous findings. However, investigating other starting models, such as one where sources are restricted to the ocean or even to coastal and shelf areas, may provide questions for future study.

The phases of observed cross-correlations are reasonably well fit by cross-correlations modelled with the starting model. Examples are shown in Figure 5.2. At each iteration, we evaluate the misfit of the logarithmic signal energy ratio between synthetic and observed cross-correlation $A_{filt}$, and use it to compute the source sensitivity kernels, an example for which is shown in Fig. 5.3. After compiling the gradient from all sensitivity kernels and measurement, we apply a 95-percentile clipping and a Gaussian smoothing filter to the negative gradients before updating. During the first three iterations, the standard deviation of the smoothing filter was set to 500 km, and during subsequent iterations to 250 km. After each gradient is obtained, we conduct a line search for optimal step length using a subset of approx. 20% of the observed data. Updates are performed using the conjugate gradient algorithm. We terminate the inversions once the misfit change of the current iteration is less than 1% of the total misfit reduction. Finally we compare the remaining residual to the data error estimated in section 5.1.2.

### 5.2 Results

The hum source models resulting from the inversion are summarized in Fig. 5.4, and show strong seasonal variations. Here, we briefly describe both models. We then present two synthetic inversions that were run to test how well point-like sources in various locations are recovered by the inversion procedure.
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Figure 5.3 – An example for the source sensitivity kernel associated with the measurement \( A_{filt} \) for stations located in California and Algeria (marked by red triangles) and a period range of 120 to 330 seconds. For the chosen measurement window, a Hann window centered on surface waves travelling with a group velocity of \( 3.7 \text{ km/s} \), sensitivity is concentrated in the stationary phase regions behind the stations. A particularity of the global model and the long-period surface waves is that sensitivity refocuses at the antipodes of the stations and extends globally, with weak sensitivity also in between the stations due to multiple-orbit surface wave propagation.

5.2.1 Inversion 1: North hemisphere winter

The stopping criterion (a step with misfit reduction of less than 1% of the overall misfit reduction) was reached after 6 iterations. The mean misfit per observed cross-correlation was reduced by 18% by the inversion. Compared to the data uncertainty, which was estimated at 0.3 for this seasonal stack, the mean distance between data and synthetics of

\[
\frac{1}{N} \cdot \sum_{i=1}^{n_{corr}} [A_{filt} - A_{filt}^0]^2 = 0.31
\]  

is sufficiently high to indicate we did not overfit the data. In Fig. 5.5, we show cross-correlation envelopes of observed and synthetic (starting and final) cross-correlations for NH winter. In most cases, relative amplitudes of the final model approach those of the observed data. For most station pairs, the Rayleigh wave group emerges clearly. However, for a number of cross-correlation envelopes, in this example from station pairs G.CAN–G.TRIS and G.CAN–G.SCZ, show signals that appear at short cross-correlation lags, equivalent to high apparent inter-station velocities. These occur at both stations of comparatively high and low local noise levels. From their emergence in long-term stacks, we conclude they are coherently propagating surface waves from sources located outside the stationary phase region of the station pair. The cross-correlations in question appear of good quality as evidenced by the observation of the expected great-circle path Rayleigh wave phases at the same locations (for example at G.CAN–G.SCZ in Figs. 5.5.5.2). Similar high apparent velocity arrivals are commonly observed at higher frequencies in ambient noise applications, and have been linked to persistent localized sources (Gaudot et al., 2016; Retailleau et al., 2016). Our final source model does not reproduce these features, which will be discussed in section 5.3.
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The source distribution model after 6 iterations is shown in the top part of Fig. 5.4. The models are normalized, because the measurement $A_{filt}$ due to the reciprocity of the energy ratio has no sensitivity to absolute noise PSD. Locations of dominant source PSD are found in the oceans. The most salient feature of the model is a rim of strong source PSD around the Pacific. It extends from the North American Pacific coast via the Alëutian arc, Kamtchatka, Japan and Izu-Bonin to Fiji and Samoa. Additional strong sources appear in the North Atlantic extending southwards from Greenland and westwards from the European coast. The Gulf of Mexico also shows elevated source PSD. Finally, an elevated PSD level is visible across the central Pacific. Anticipating the results of the recovery tests and discussion, we suggest to pay no heed to any features beside these ones which are the most prominent and the largest.
5.2. Results

Figure 5.4 – Results of the two seasonal stack inversions. The top two maps show two views of the normalized source PSD model for North Hemisphere winter, the two bottom ones for austral winter. See text for details.
5.2.2 Inversion 2: South hemisphere winter

In the inversion for SH winter, we interrupted the inversion after five iterations as the average misfit was dropping below the estimated data uncertainty. Although we retrieved a lower value of data uncertainty for SH winter than for NH winter, this estimate may be biased by the fact that it was determined with an array in the North hemisphere. To be conservative, we use the value of 0.3. The model we present is the intermediate model of iteration 5, which reaches a misfit reduction of 15%. We kept iterating and the model after iteration 11, which is likely overfitted, is presented in supplementary Fig. B.2.

The model for SH winter shows mostly different patterns from NH winter. During austral winter, source PSD is clearly dominated by the South hemisphere, and distributed in several localized patches. The largest one is located off the Chilean coast. There is a chain of patches extending from the South Sandwich islands region towards the Indian Ocean, two south of Australia, one between Fiji and Samoa and several in the larger Philippine Sea region. Similarities to the NH winter model are the strong excitation off the Chilean coast, and the sources in the Philippine Sea and the Izu-Bonin region.

5.2.3 Synthetic recovery test

Given the relatively sharp appearance of the Pacific rim feature on the NH winter model, but also observation of an elevated source PSD in the Central Pacific, we were interested in an assessment how well localized sources in various locations are resolved, and how much smearing occurs from the land-based receivers towards the ocean, where receiver coverage is rather sparse. The global array of STS-1s that we used here is rather irregularly distributed. In particular, there are 102 stations in the Northern and only 46 in the Southern Hemisphere. In order to probe the resolution capabilities of the inversion, which we expect to be largely controlled by the receiver distribution, we set up two recovery tests with sources shaped as positive Gaussian anomalies on a source-free background (similar to Nishida and Fukao, 2007). We chose anomalies with standard deviations of 350, 500 and 1000 km and located them in shelf regions as well as pelagic regions of both hemispheres. The target models are shown in first and third column of Fig. 5.6 (Note that the illustrations show two recovery tests; three different views centered on three anomalies are shown for each model). The target models were used to compute synthetic data, which then served to invert the recovered models. Since we were mainly interested in the influence of the array shape on the recovery, we did not add noise to the synthetic data. We ran an inversion with 4 iterations for each recovery test. As the synthetic data are noise-free, we pre-defined this number of iterations to avoid an overly optimistic estimate of resolution. An aspect of the recovery tests that intrigued us was that although sources are located in only few patches in our target models with zero source PSD throughout most of the domain, the resulting cross-correlations appear rather symmetrical and ‘spurious’ arrivals do not appear (see 5.6). As will be discussed in more detail below, we take this as an indication that those sources that give rise to such early arrivals in the observed
5.3. Discussion

Figure 5.5 – Normalized envelopes of observed correlations (gray solid line), synthetic correlations of the starting (blue dashed line) and final model (red dashed line) for NH winter. For many observed cross-correlations, the Rayleigh wave envelope emerges clearly. For certain station pairs, additional phases at small lags appear. Their persistent observation at quiet stations indicates they are coherently propagating surface waves caused by strong persistent sources located outside of the stationary phase region of the receiver pair.

cross-correlations must be more strongly localized.

The recovered models are shown in the second and fourth column of Fig. 5.6. The first observation on these two tests is that source located far from most stations are resolved better. In recovery test 2, source locations are recovered better than in recovery test 1, although they are located in the central Pacific and the Southern Ocean, far from most stations of the array. This appears counterintuitive, but is a particular feature of the global scale noise source inversion. The reason is that sensitivity is not concentrated in between station pairs, but behind them (along the major arc) as shown in Fig. 5.3. The second observation is that resolution depends on location and is on the order of 1000s of kms. Finally, we note that smearing from a near-coastal source to the ocean appears far more pronounced than from a pelagic source towards the coastline. The resolution tests provide us with the insight needed to discuss our results in the context of previous studies.

5.3 Discussion

With the work presented here, we have aimed to address a current challenge in ambient noise seismology, namely that of applying ambient source inversion while taking the complex propagation of seismic waves through Earth’s heterogeneous seismic structure into account. This serves the two aims of firstly exploring the feasibility of, and obtain source models for,
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Figure 5.6 – Top: Two recovery tests are shown, which are performed to investigate the capability of the network to recover localized sources. The respective left columns show the target model of each test in three different perspectives, each focusing on a different source region. The target models were used to simulate synthetic data. We inverted the synthetic data performing four iterations, departing from a homogeneous source starting model. The resulting recovered models are shown on the right. Bottom: Synthetic cross-correlations obtained from target source model 2. These show rather symmetric phases and zero-lag ‘spurious ‘arrivals are not prominently observed.

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ambient noise waveform tomography, and secondly of studying the sources themselves with a realistic wave propagation model. In the following, we will first compare the results of our application to the results of previous studies, and suggest implications for hum sources. We will then discuss the inversion method itself, both independently and in conjunction with cross-correlation waveform tomography, and conclude with suggestions for further research.

5.3.1 Interpretation of the hum source models

Our hum source models are in good qualitative agreement with those of previous studies (Rhie and Romanowicz, 2004, 2006; Nishida and Fukao, 2007; Bromirski and Gerstoft, 2009; Traer et al., 2012). The seasonal excitation pattern we observe is consistent with the one reported by Rhie and Romanowicz (2004), Nishida and Fukao (2007) and Traer et al. (2012). While we are investigating a longer observation period, our model captures the strongest source locations reported by Rhie and Romanowicz (2004, 2006) and Bromirski and Gerstoft (2009) during the relevant seasons, namely a location close to the South Sandwich islands during SH winter (Rhie and Romanowicz (2004); also reported by Traer et al. (2012)), the European Atlantic coast during NH winter (Bromirski and Gerstoft, 2009), and, in particular, the Pacific Northwest coast of which two separate events occurring in 2000 and 2006 have been reported by Rhie and Romanowicz (2006) and Bromirski and Gerstoft (2009), respectively. Detection by the ten-year average model indicates that these locations all contribute persistently (during the relevant season) to the excitation of hum.

We located a number of additional sources in the Southern hemisphere, both during NH winter and during SH winter. During NH winter, we find a rim of high source PSD around the Pacific that extends not only from California to Kamtchatka (similar locations identified during the year 2010/11 by Traer et al. (2012)) but also from the Philippine Sea towards Fiji and Samoa. During SH winter, we find source locations that are confined to localized patches in the Southern Ocean, southwest of New Zealand, southwest of Australia, southeast of Madagascar, as well as to the south and southwest of the South African coast. We again find high source PSD located in the Philippine Sea, as well as close to Fiji, during SH winter. We place some confidence in the locations of the strongest peaks of source PSD, without interpreting the outlines or shapes of the features surrounding them. The following paragraphs outline why we regard these results as reliable.

Comparison to published results

The results of dominant source regions with high relative source PSD which we retrieved are consistent with results from the two previous longer-term studies. The broad pattern of source locations obtained here agrees excellently with the results by Nishida and Fukao (2007). At closer comparison, the features of our models are more localized, and in particular, our NH winter model shows a sharp rim around the central Pacific and drops to a lower level of excitation within, while the model of Nishida and Fukao (2007) shows high effective pressure throughout the Northern Central Pacific. We attribute the shorter scale features of our model
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to the higher number of stations we used, the shorter periods (we used 100 to 300 seconds, and Nishida and Fukao (2007) 100 to 500 seconds), to the iterative inversion during which smaller scale features emerge gradually, and to the parametrization of the source as an arbitrary function on a fine grid, rather than spherical harmonics. Based on recovery tests in both studies, we suggest that our model improves upon the resolution of features insofar as it does not tend to introduce spurious maxima at some distance from the target source location. We also believe to have avoided overfitting, because the level of misfit after the last iteration still exceeds the level of measurement uncertainty that we estimated. The hum excitation features stretching West and South from the American West coast may be explained by smearing of sources located close to the Californian coast, according to our recovery test. This prevents us from drawing conclusions on the contribution of pelagic sources in this region, except that we can exclude with some confidence a dominance of pelagic sources.

Comparing to the source locations identified by Traer et al. (2012), our NH winter model equally shows high source PSD along the North Pacific coasts, in the North Atlantic between Greenland and Spain, and along the Chilean coast (although they report low correlation with significant wave height here). In SH winter, our model, like the observations from Traer et al. (2012), also shows a source region off the Chilean coast and a source region close to South Sandwich Islands. We observe additional source locations in accord with Nishida and Fukao (2007). There are two possible explanations why they are not observed by Traer et al. (2012); the first is the shorter duration of their study (one year); the second is that the shape of the two arrays used in their study may be unfavourable to signals incident along an East-West axis, which would explain that strong sources in the West Pacific are not observed.

Network geometry and smoothing

The models we obtain are robust with regard to network geometry and smoothing applied during the inversion. We analyzed the shape of the network sensitivity to exclude that localized prominent source areas such as the source in the Pacific Northwest are due to a clustering of sensitivity, e.g. caused by a cluster of quiet STS-1 stations in California. To judge this, we summed the unweighted sensitivity kernels resulting from the homogeneous starting model (compare eq. 2.39):

\[ K_l(\xi, \tau) = \left[ \frac{w_+(\tau) \cdot H_l(\tau) \cdot C_l}{E_{l,+}} - \frac{w_-(\tau) \cdot H_l(\tau) \cdot C_l}{E_{l,-}} \right] \cdot \left[ G_{jm}(x, \xi, t) * G_{ln}(x_1, \xi, -t) \right] (\tau). \]

This is the sensitivity kernel before multiplication by the data residual. Summing the absolute value of the respective unweighted kernels for all station pairs provides a map of network sensitivity akin to a ray coverage plot, but extended to finite frequencies. We examined the absolute value of this sensitivity map (provided in supplementary Fig. B.3) after clipping it at the 95th percentile in agreement with the inversion procedure. We found no indication that spots of high sensitivity of the network coincide with the strongest features of our models. On the contrary, sensitivity of the network is low south of South Africa and Australia, where we observe strong sources in the SH winter, and comparatively low along the Alaskan coast.
with much higher sensitivity due to the Californian stations extending south- and westwards from their location. The sensitivity maps emphasize that sensitivity and resolution are both limited North of 30° N. Importantly, they also demonstrate that the locations of many station antipodes provide many small scale, high-sensitivity locations in the Southern ocean (sensitivity refocuses at the station antipodes, cf. Fig. 5.3).

We further compared the results for two SH winter models elaborated using two different smoothing strategies, one in which we employed weaker smoothing ($\sigma = 250$ km) throughout, and one in which smoothing was decreased from $\sigma = 500$ km to $\sigma = 250$ km after the first three iterations (presented in Fig. 5.4). Although the shapes of the discussed features change, and the weakly smoothed model appears clearly noisy, the location of the strongest peaks remain stable (see supplementary Fig. B.4).

**Cumulative effect of earthquakes**

In the time interval spanned by our continuous dataset, more than 5500 earthquakes with magnitude $> 5.5$ occurred (USGS website\(^1\)). *Suda et al.* (1998) show that these contribute to the incessant normal mode peaks of the hum, although they are not sufficient to explain it. While the largest events occur infrequently enough to be outweighed during stacking by persistent hum signals due to our normalization procedure, less strong ones frequently occurring in similar areas may have contributed to our models; it was shown recently that various normalization procedures are insufficient to remove their signature, at least from data of shorter periods (*Seydoux et al*., 2016; *Yanovskaya et al*., 2016). Most features of our models shift seasonally, but there are three regions which show particularly strongly non-seasonal hum excitation, prompting us to look into a possible contribution of seismicity: The region off the coast of Chile, the Izu-Ogasawara and Mariana region, and the Fiji basin (we identified these by comparison of NH and SH models, see supplementary Fig. B.5). Three arguments suggest that the cross-correlation stacks we analyse are dominated by ambient signals: First, our results are consistent with *Nishida and Fukao* (2007), who used a wholly different procedure of earthquake removal by data selection. Second, several areas of strong seismicity do not appear in our model, among them the location of the 2004 Sumatra-Andaman event with a high number of aftershocks. Third, a hum source off the coast of Chile is also observed by *Traer et al.* (2012), who additionally show that its temporal evolution correlates with ocean wave activity during SH winter.

**Hypotheses for hum sources**

Based on these careful considerations, we propose the following interpretations of our models. Due to the caveat of limited resolution (Fig. 5.6) and possible contributions from seismic events, these should be regarded as working hypotheses.

The observations that source locations stretch westwards from the European coast in NH

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\(^1\)https://earthquake.usgs.gov/earthquakes/search/
winter, and westwards from the Chilean coast, are compatible with the generation of hum by ocean infragravity waves, which are most dominantly excited at the eastern margins of ocean basins (e.g. Rawat et al., 2014). Bromirski and Gerstoft (2009), Traer et al. (2012) and Traer and Gerstoft (2014) argue for coastal excitation of the hum in shallow water, while Ardhuin et al. (2015) has demonstrated a mechanism that requires hum to be excited at shelf breaks. Most of our results can be plausibly explained by either mechanism. Our models clearly support the near-coastal excitation areas previously reported, without being able to distinguish if these are linked to shallow water or the sloping shelf, due to insufficient resolution and smearing from the coastline towards the open ocean in some cases, e.g. off the Chilean coast. In addition, we find a number of features during SH winter for which occurrence on continental coastlines is not plausible. In particular, these are the source located close to the South Sandwich islands, the source near Fiji and the sources in the Marianas and Izu-Ogasawara region. They may be linked to regions of high and complex bathymetry, including islands.

The strong hum PSD features observed in our models are comparatively narrow, while Gaussian-shaped sources in the recovery test are laterally smeared. We interpret this as an indication that hum sources are more localized features than those in the target models of the recovery test, i.e. that they are on the order of 100 km. An observation that lends tentative support to this interpretation is the appearance of arrivals at near-zero lag on cross-correlations at several quiet stations (Fig. 5.5). Such phases are an indicator of localized sources. If the sources were more extended, these signals would cancel out due to the oscillating nature of the sensitivity kernel outside the stationary phase region (Fig. 5.3), which is also reflected in our observation that the synthetic models of the recovery tests show no such early arrivals. If our interpretation holds, it corroborates the finding by Nishida and Kobayashi (1999), based on statistical features of the hum, that hum sources must have a spatial correlation length of less than 600 km. It is furthermore consistent with the estimate by Tanimoto (2005) that infragravity waves over small areas on the order of 100 by 100 km are sufficient to account for the observed seismic signal levels. Our results moreover suggest that the same source areas must be active repeatedly, indicating localized, preferential source spots for the hum such as the Pacific Northwest coast.

5.3.2 Discussion of the method

The application of iterative cross-correlation source inversion with 3-D finite frequency kernels to image the sources of Earth's hum has produced physically reasonable results that are in agreement with previous studies, and in some cases show more detail than those. The practical application of the method was considerably eased through the use of precomputed wavefields. The method rests on a number of assumptions, which we will briefly discuss in the following together with suggestions for future improvements.
5.3. Discussion

Data uncertainty

The variability of the measurement, which we evaluated by comparing measurements on cross-correlations from two closely located stations of the Yellowknife array with respect to stations elsewhere, was found to be rather large. While this is partly due to local noise such as ground tilt, we also found that asynchronous observations appear to result in larger scatter. While stations YKW1 and YKW3 were active throughout the period covered by our dataset, YKW2 and YKW4 were discontinued in 2009. The consistency is comparable between each set of stations covering the same recording period, despite the shorter stack length of YKW2, YKW4, while it drops if all four are compared. Data gaps occur commonly in a long-duration, global-scale dataset. Decreasing measurement variability and obtaining a better fit between final cross-correlation model and data therefore probably requires carefully selecting recordings that are as synchronous throughout the network, as possible.

Furthermore, we were not able to retrieve horizontal component cross-correlations of sufficient signal-to-noise ratio. To our knowledge, only one study has successfully investigated propagating long-range horizontal-component cross-correlations of the hum, using an elaborate data selection procedure (Nishida, 2014) and considering frequency-wavenumber spectra rather than cross-correlation waveforms.

Useability for cross-correlation waveform inversion

To evaluate the viability of the source inversion for a future global tomography based on cross-correlation waveforms, we determined the phase misfit of observed cross-correlations with starting and final synthetics. We found an improvement of negligible magnitude (∼ 1‰). This is not surprising, as we are evaluating the phases of stationary-phase Rayleigh waves, which were already reasonably well fit by our starting model (Fig. 5.2); phase shifts due to smooth variations in source distribution are expected to be low (Froment et al., 2010), in particular due to the global propagation of long-period Rayleigh waves. What remains to be investigated is a comparison of illumination of the structure by a homogeneous, and by the inverted source models. Importantly, however, the application shown here demonstrates that it is feasible to obtain models of noise source distribution which may be used for noise cross-correlation waveform inversion. With the precomputed wavefield approach, the method can more or less seamlessly be extended to regional scale.

We furthermore compared the absolute signal energies level predicted by our final model to those of observed cross-correlations. Since we worked with geometrical normalization (normalization by trace energy), there is little correlation (<0.5) between modelled and observed root mean square (rms) amplitudes in the signal window. This may also partially be due to the high measurement uncertainty. This prevents us from finding a model that is in sufficiently good agreement with the rms amplitudes of observed data. For tomographic inversion, it is therefore necessary to use an amplitude independent misfit, such as the time-frequency phase misfit, in keeping with common tomographic practice (e.g. Fichtner, 2010). An alternative
would be to obtain cross-correlations without normalization, which can only be done after selecting data which are not affected by high amplitude transients.

**Effects of parametrization**

We used a fixed, spatially uniform spectral basis function between $\sim 125$-330 s. Hence, our model does not resolve possible differences in the shape of the spectrum of different hum source regions. A direction of future research is therefore to extend the presented approach to a sequential inversion for different frequency bands. The synthetic study by Sager et al. (2017) indicates that joint frequency-dependent noise source inversion for several spectral bases is difficult to conduct, because measurements that are sufficiently sensitive to the spectral content of the cross-correlation functions show large trade-offs with unmodelled Earth structure.

For the spatial parametrization, we used a regular grid with a sub-wavelength grid step. The choice of spatial parametrization influences the inversion results. The gridded parametrization allows us to retrieve more detail than the spherical harmonics parametrization by Nishida and Fukao (2007), but is also more prone to overfitting. We have sought to avoid overfitting by stopping the inversions before the average misfit dropped below the estimated data uncertainty. A gridded parametrization for determining the likely excitation area of the Pacific Northwest source was also used in the study of Rhie and Romanowicz (2006). It is the most opportune choice at the moment, because the extent or correlation length of particular sources is poorly known, and sources may be confined to narrow areas such as shelf breaks (Ardhuin et al., 2015).

**Effects of processing, synthetics duration and attenuation on amplitudes**

Several components of our forward modelling affect the amplitudes in a way that is not fully consistent with observed data. While our measurement $A_{\text{filt}}$ is entirely insensitive to absolute amplitudes, these discrepancies have implications for the misfit gradient with respect to the source PSD. We briefly discuss these here for completeness and possible future refinement, while we expect their effect to be insignificant for the current models.

Any study on long-range ambient cross-correlations employs some form of data processing (e.g. Stehly et al., 2006; Nishida, 2013; Boué et al., 2014). A notable exception is found in Nishida and Fukao (2007), who, however, use an elaborate rejection filter instead. Here, we have used normalization by trace energy, which is expected to alter cross-correlation amplitudes. Our forward model does not include such a normalization. The problem of processing effects is addressed in Fichtner (2014) and Fichtner et al. (2017a); in the present case, conceptually we expect that sources close to the stations themselves and sources close to the station antipodes may be overemphasized during inversion due to processing effects, because the raw cross-correlation is more sensitive to these locations than the normalized one.
We think that this has no significant effect on our current models, because we would otherwise expect a higher correlation of the final models with the network sensitivity (see supplementary Fig. B.2). However, for further refinement of the method it should be taken into account either by removing all processing steps (Nishida and Fukao, 2007), which, however, likely causes observations to become a-synchronous, or by the approach of Fichtner et al. (2017a).

The attenuation model used in forward modeling also affects amplitudes, and therefore influences sensitivity (see Fig. 2 of Ermert et al., 2016). Here, we used the spherically symmetric attenuation model from PREM (Dziewoński and Anderson, 1981). Since attenuation of long-periodic surface waves is generally weak, we expect its lateral variations to have only a minor effect. It may, however, be significant in studies on regional scale. It may be possible to make a virtue of necessity and extract attenuation estimates from long-range ambient correlations, similar to Prieto and Beroza (2008) on regional scale. The ambient source inversion method presented here is a key tool for this (Stehly and Boué, 2017). The large amplitude uncertainty of our current data prevents their immediate use for attenuation studies.

Finally, the duration of the synthetic Green’s functions has a subtle influence on the sensitivity kernels. An exemplary illustration of this effect is shown in supplementary Fig. B.6. Schimmel et al. (2011) showed that minor and major arc Rayleigh waves can be observed to propagate coherently between station pairs. Hence, the typical propagation distance of hum must be at least one orbit, and is possibly longer. We took a conservative approach and allowed for several multiple orbits.

Assumptions about hum sources

We have limited the modeling here to vertical point force sources. By obtaining precomputed wave fields also for horizontal point force sources, one can obtain a cross-correlation due to all components of the source correlation tensor. A challenge in this context is that we would move from a one-to-one mapping between source autocorrelation $S_{zz}$ and cross-correlation $C_{zz}$ to a nine-to-nine mapping from $S_{nm}$ to $G_{ij}$. This increases the number of model parameters by about an order of magnitude.

An inversion for the 3-D properties of the noise sources would, however, be highly interesting, since there are several open questions, in particular how important shear sources are compared to pressure sources, and how large the correlation between different source components is. Nishida (2014) reports on the importance of shear sources for fitting observed hum spectra, but little else is known in this regard.

In eq. 2.10, we have assumed that sources are spatially uncorrelated. Although this assumption is commonly used, its influence on the resulting forward model is not well known (theoretical considerations have been presented by Godin, 2009). The precomputed wavefield approach we use here can be extended in order to relax the assumption of spatially uncorrelated sources. There are two possible ways to do this: The first is to model noise time series by the approach
Chapter 5. Global sources of the Earth’s hum

of van Driel et al. (2015), but using a 3-D precomputed, rather than a spherically symmetric Green’s function database, and introducing a finite correlation length into the random phase source field. This approach appears well suited for exploratory forward models, in particular to simulate spatially varying temporal and spatial source spectra. It is probably too computationally intensive due to the long realizations of noise required for either trace giving rise to one correlation, to compute for a large number of station pairs, especially on global scale. The second is to introduce a band-limited spatial spectrum with a pre-defined correlation length $L$ into eq. 2.10. Instead of eq. 2.13, one would then evaluate

$$C_{ij}(x_1, x_2, \tau) = \int_{\partial\Omega} G_{ni}(\xi_1, x_1, -t) * \ldots$$

$$\ldots \int_{\Delta\xi = -L} G_{mj}(\xi_1 + \Delta\xi, x_2, t) * N_n(\xi_1, -t) * N_m(\xi_1 + \Delta\xi, t) d\Delta\xi d\xi_1.$$

(5.4)

In this case, the cross-correlation function of sources $N_n(\xi_1, -t)$ and $N_m(\xi_1 + \Delta\xi, t)$ would have to be computed, while our current assumption of a white spatial spectrum makes it sufficient to use the source autocorrelation. An open question is what phases to assign to the noise source time series $N_n(\xi_1, -t)$ and $N_m(\xi_1 + \Delta\xi, t)$. One might further assume some functional form of the cross-correlation between noise sources, which would allow to evaluate the cross-correlation as before, but with a distance-dependent term replacing the delta function in eq. 2.10. Since there are hardly any observational constraints on the spatial spectrum of the noise sources (an upper limit of the correlation length $L$ has been estimated by Nishida and Kobayashi, 1999), it appears arbitrary to include it in the cross-correlation modeling at this point.

Suggestions for further study

Based on the experiences from this study, we suggest two strategies to improve the cross-correlation based localization, so as to gain higher resolution for the study of hum sources. The first is to increase coverage. Although arguably only the quietest stations provide long-range cross-correlations of sufficient quality at hum periods, there is a number of stations operating STS-1 and Geotech borehole seismometers which are not yet included in our dataset.

The second is to utilize apparent high velocity arrivals described earlier. The sensitivity regions of such arrivals are narrow hyperbolas between the receivers, so that accounting for them during inversion adds a large amount of sensitivity to short-wavelength features (e.g. Retailleau et al., 2016). Including them into the inversion procedure could be achieved using an envelope misfit similar to Sadeghisorkhani et al. (2016). The envelope may contain surface and body wave phases of the cross-correlation, and our method would account for both. A reduction of the amplitude uncertainty of the cross-correlations would be necessary before using the envelope measurement. Finally, the best resolution we could achieve is on the order of several
100 km due to the seismic wavelengths of the hum.

A natural extension of the work presented here is to apply the presented inversion procedure to microseismic noise at regional scale, where crustal complexity has a large impact on the observed cross-correlations, modeling the regional wave propagation with a tomographic model. A challenge in this regard is to define a suitable model domain, since even microseismic noise can propagate globally (e.g. Stehly et al., 2006; Landès et al., 2010; Liu et al., 2016; Nishida and Takagi, 2016). It may be necessary to attempt a selection of locally occurring noise events by comparison of seismic noise time series to meteorological or oceanographic data (e.g. Bromirski, 2001), or to work with non-normalized cross-correlations, which greatly enhances sensitivity to local sources, but requires a meticulous data selection procedure to exclude earthquakes and other high-amplitude transients.

The presented inversion procedure can almost seamlessly be applied to obtain sensitivity kernels for body wave sources. Using fast wave propagation algorithms like Axisem (Nissen-Meyer et al., 2014) or Axisem_3D (Leng et al., 2016), a precomputed wavefield can be obtained that includes short-period body waves, which would then enable us to determine cross-correlation source sensitivity kernels. The procedure can equally be used to invert auto-correlations or power spectral density curves at single stations for their best-fit source model. This would complement forward modeling by Ardhuin et al. (2015) and related studies. As the auto-correlation is by definition symmetric, a different measurement from the causal-acausal signal energy ratio, which was used here, would have to be developed.

5.4 Conclusions

We conclude that the presented cross-correlation source inversion method is effective at locating the dominant ambient sources from cross-correlations and improving the fit of the ratio of cross-correlation signal energies. It provides hum source maps that are physically reasonable and consistent with previous studies, while offering a more general and slightly more detailed result than many of them. It is to our knowledge the first application of this inversion technique to an observed cross-correlation dataset after theoretical and synthetic studies (e.g. Tromp et al., 2010; Hanasoge, 2013b,a; Fichtner, 2014; Sager et al., 2017) and the sensitivity study by Basini et al. (2013), and constitutes a step towards the application of ambient cross-correlation waveform tomography as suggested in theoretical and synthetic studies by Tromp et al. (2010), Sager et al. (2017) and others.

Our hum source models for North and South hemisphere winter show narrowly concentrated areas of high excitation with seasonally shifting patterns. Source locations off the coast of Chile, close to Fiji and in the Marianas region show little seasonal variation, but we suggest that their sources are of ambient rather than seismic nature, based on comparisons to the results of Nishida and Fukao (2007) and Traer et al. (2012). Our results do not permit a direct distinction between excitation in shallow water and along shelf breaks (Traer and Gerstoft, 2014; Ardhuin et al., 2015) due to limited resolution. However, we find source locations which
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are more likely linked to islands and regions of complex bathymetry, than to continental coastlines, particularly in the Southern Ocean. Remarkably, the long-period cross-correlations which we have investigated here show tentative evidence for the occurrence of signals from sources outside the so-called stationary phase region of the inter-station correlations. Their observation is an indication of frequently repeatedly active, narrowly localized source spots for the seismic hum.
6 Microseismic noise in the Sea of Japan

This chapter was modified from a research report submitted to the Japanese Society for the Promotion of Science (JSPS) under the ETH-JSPS Strategic Exchange program.

6.1 Introduction

The use of ambient seismic noise has enabled a number of new seismological methods during the last decade. In particular, ambient noise cross-correlations constitute a measurement that can be obtained at any time and, given two seismic receivers, any location, independently of the occurrence of Earthquakes. It dramatically increases coverage in regions of low seismicity (e.g. Stehly et al., 2009). Measurements of ambient noise are repeatable, and each cross-correlation contains a wealth of information about the interior of the Earth. Thus, they are used for seismic imaging and tomography of the Earth's interior (e.g. Shapiro et al., 2005; Sabra et al., 2005; Yao et al., 2006; Lin et al., 2008; Stehly et al., 2009, and many others), as well as for monitoring of volcanoes, geothermal extraction sites and engineered structures (e.g. Sens-Schönfelder and Wegler, 2006; Brenguier et al., 2008; Michel et al., 2010; Obermann et al., 2015).

Observations clearly showed that in many regions noise, far from being diffuse, retains information about its sources. This can introduce biases in correlation-based measurements, depending on the properties of the sources and the type of application. To account for such biases, models of the noise source distribution are needed (e.g. Harmon et al., 2010; Sadeghisorkhani et al., 2016). In particular, such models would enable us to model cross-correlation waveforms and perform full-waveform tomography of noise (Tromp et al., 2010; Hanasoge, 2013b; Fichtner, 2014; Sager et al., 2017), providing an alternative method to the widely used ambient noise cross-correlation technique, which is based on the analysis of surface wave dispersion curves.

In addition to attempts to extend the scope of noise tomography, there is great interest in the study of noise sources in their own right (Rhie and Romanowicz, 2004; Gerstoft et al., 2006;
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Nishida and Fukao, 2007; Landès et al., 2010; Stutzmann et al., 2012; Traer et al., 2012; Gualtieri et al., 2013; Sergeant et al., 2013; Nishida, 2014; Tian and Ritzwoller, 2015; Arduin et al., 2015; Craig et al., 2016; Juretzek and Hadziioannou, 2016; Chen et al., 2016, and numerous others) and for applications that make use of the temporal variations of noise, such as studies of present or historic ocean wave climate (e.g. Stutzmann et al., 2009; Donne et al., 2014).

Certain aspects of noise sources are well understood, like the excitation of vertically polarized surface waves by interacting ocean wave trains (Arduin et al., 2011; Stutzmann et al., 2012; Gualtieri et al., 2013), others are a topic of current research, such as the cause of horizontally polarized surface waves at intermediate periods (Juretzek and Hadziioannou, 2016). Thus, detailed and reliable maps of the space-, time- and frequency-dependent seismic noise may provide important constraints to test hypotheses on the physical origins of seismic noise, to provide more detailed quantification of source-induced biases in studies of Earth structure (Sadeghisorkhani et al., 2017), and as a key ingredient for full-waveform inversion of noise correlations.

Current state-of-the-art methods to image noise sources include beamforming (e.g. Reading et al., 2014; Gal et al., 2015; Davy et al., 2015), polarization analysis (e.g. ?) and the backprojection of relative noise amplitudes (Tian and Ritzwoller, 2015). However, in the case of surface waves, which dominate microseismic noise (Haubrich and McCamy, 1969), these methods have only limited potential to retrieve information on distance to the sources. Moreover, beamforming and polarization analysis usually rest on the assumption of a plane-wave noise field, which becomes unrealistic in the case of larger arrays or close-by sources. Backprojection of relative amplitudes assumes that receiver pairs are sensitive only to sources along the great-circle linking the receiver pairs, which is a simplification that becomes problematic in the presence of persistent localized sources (Gaudot et al., 2016).

Few studies have performed inversions for the distribution of noise sources, i.e., adapting noise source distribution to fit observations using different numerical models. Using normal mode summation, Nishida and Fukao (2007) revealed the characteristics of the long-period background noise known as Earth’s hum (0.002-0.01 Hz) and confirmed its seasonal behavior. Harmon et al. (2010) and Sadeghisorkhani et al. (2016) used a plane-wave modeling approach to constrain source models of microseismic noise (approx. from 0.05 to 0.2 Hz).

While these methods provided important insights, their further application is limited by the use of simplified wave propagation models, namely a spherically symmetric model in the case of Nishida and Fukao (2007) and a homogeneous model with plane wave propagation in the two other studies. In light of these limitations, we proposed to extend upon previous studies by using a 3-D wave propagation model for a regional-scale inversion for the sources of primary microseisms. Seismic waves at periods of microseismic noise are sensitive to the complex structure of the Earth’s crust and upper mantle; to gain detailed insights about microseismic noise sources, it is therefore necessary to include the 3-dimensional structure of the Earth’s subsurface in the model. The goal of the proposed research is to invert for ambient noise source distribution using a 3-D forward model for noise cross-correlations, in the primary
microseism frequency range (approximately 0.05 to 0.1 Hz). Recent models elaborated with seismic tomography predict seismic wave propagation well (Simute et al., 2016). However, most models do not cover the period range of secondary microseisms (shorter than approx. 12 seconds). Therefore, cross-correlation models in the primary microseism range are likely more realistic and less affected by unknown Earth structure. In addition, the excitation of primary microseism Love waves is a current subject of study.

The Sea of Japan is an ideal location for this study due to several characteristics: It is a deep but almost closed ocean basin, with strong topographic variations and relatively large significant wave heights. Thus, strong microseismic signals are expected to occur and are indeed observed (Nishida et al., 2008a). With the deployment of NecessArray (Ni, 2009) and the availability of the Japanese broadband network F-net (Okada et al., 2004), sources within the Sea of Japan can be enclosed by receivers on either side, a far more advantageous situation for noise source observations than in the open oceans where observations by seismic receivers are typically one-sided. A map of the networks is shown in Fig. 6.1. In addition, Japan has a complex geological structure (e.g. Yoshizawa et al., 2010), suitable to study the effects of 3-D wave propagation on the observation of noise sources.
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6.2 Feasibility study

Here, we present a feasibility study consisting of three steps. To begin with, we select microseismic noise data of locally occurring noise events, and process them to obtain cross-correlations. Second, we use these cross-correlations to obtain noise source distribution maps by a simplified mapping procedure; and finally, we construct forward models of noise cross-correlations for homogeneous source distribution models, as well as for spatially varying source distribution models and investigate the capability of the network to recover these source distributions.

Selection of the microseismic data aims to choose periods of a few days to weeks when noise sources in the Sea of Japan dominate the land-based seismic recordings on the two arrays. This is done using previously published seismological results, as well as publicly available oceanographic data. Once selected, the ‘noise events’ are processed to obtain noise cross-correlations. During the processing stage, two aspects have to be taken into consideration: First, earthquakes need to be meticulously removed to avoid biasing the conclusions about ambient, i.e. non-earthquake sources. Second, since noise sources are best studied using the amplitude, rather than the phase, of noise cross-correlations, amplitudes have to be preserved during the processing. Therefore, a processing approach different to what is commonly used in ambient noise tomography, where amplitude removal is less problematic, has to be chosen.

The processing scheme by Bensen et al. (2007) has reliably yielded cross-correlation observations, whereas an entirely raw processing without normalization of the continuous traces is rarely used (examples include Pedersen et al., 2007; Nishida and Fukao, 2007). Therefore, a data review to ensure that this processing yields meaningful cross-correlations is needed and is presented in the first step.

In the second step, we elaborate approximate maps of the noise source distribution. To infer the noise source distribution from observed noise cross-correlations, we investigate several different measurements (e.g. Stehly et al., 2006, and chapter 4 of this work) rather than inverting observed amplitudes directly. The full source inversion procedure described in chapter 2 adapts the source distribution iteratively by a gradient descent method. For the approximate mapping, however, we are only interested in the coarse features of the source distribution, elaborated in a fast way prior to the iterative inversion. Thus, instead of running an optimization procedure, we only investigate the shape of the first update of the noise cross-correlations with respect to sources, \( \nabla S \chi \) (cf. eq. 2.17). This provides the first-order noise source distribution. To keep this step simple, 3-D structure is disregarded at first, and we elaborate \( \nabla S \chi \) based on simplified wave propagation in a 2-dimensional homogeneous medium, using analytic Green's functions. We perform this step in order to understand whether, using cross-correlations, we can extract a physically reasonable source distribution map.

Furthermore, we perform a synthetic test of the mapping procedure to understand how the irregular shape of the seismic networks influences the recovery of the noise source distribution. This synthetic test is again performed on the basis of analytic Green's functions for a
homogeneous, 2-D medium. As a step towards the future incorporation of 3-D Earth structure, we reviewed the openly available tomographic models and we discuss the selection of one of them. We create a mesh of this model so as to use it with the newly developed salvus wave propagation solver\(^1\).

### 6.3 Selection of regional noise events

As first step, we identify segments of continuous data of typically several days duration, when ambient noise is predominantly incident from the Sea of Japan. The noise field on the Japanese islands is commonly dominated by microseismic waves incident from the vast Pacific ocean \((\text{Nishida et al., 2008c})\). To select the noise ‘events’ occurring in the Sea of Japan, we proceed as follows: First, we review results from a previously published beamforming study \((\text{Nishida et al., 2008c})\). We analyze beamforming output from the West Japan part of Hi-Net stacked over 1 day, for each day in 2010-2011 (these years were selected because they are within the operational period of NecessArray). Beamforming results are only available in the frequency band 0.1-0.2 Hz. We interpret high microseism levels in this frequency band as a first proxy for high microseism levels at lower frequency, although microseisms of the two bands are not necessarily correlated. The criterion for selection is clear surface wave propagation incident from 300 degree to 360 degree azimuth. Doubtful events are excluded. After the selection based on beamforming, we compare the selected dates to significant wave heights at buoys at the Japanese coast from the website of Nationwide Ocean Wave information network for Ports and HArbourS \((\text{Nowphas}^2\)\) and to typhoon tracks published by Japan Aerospace Exploration Agency\(^3\). Mainly, strong noise events around the Sea of Japan occur in winter from December to January. In addition, a small number of events occur in springtime and very few in August / September, during typhoon season. We chose representative episodes for all three periods, as well as two episodes representative of Pacific ocean sources. All selected events are listed in Table 6.1. We processed data for all events. Two events are subsequently analyzed in more detail.

### 6.4 Data processing and data assessment

Before correlation, the data are downsampled to 1 Hz sampling rate and corrected for instrument response. We then cut the data in segments of 30 minutes and calculate non-normalized noise cross-correlations according to

\[
C_{ij}(x_1, x_2, \tau) = \frac{1}{T_0} \sum_{t=0}^{T_0} \left[ v_i(t, x_1) \cdot v_j(t + \tau, x_2) \right],
\]

\(^1\)http://salvus.io
\(^2\)http://www.pari.go.jp/unit/kaisy/en/nowphas/
\(^3\)http://sharaku.eorc.jaxa.jp/TYP_DB/index_e.shtml
where $v_i(x_1), v_j(x_2)$ are the i- and j-component recordings of seismic velocity at two receivers. We store all individual cross-correlation windows, and later stack them. We obtain cross-correlations for all components of the cross-correlation tensor for selected events, and vertical-vertical components for the rest. The selection of ‘good’ windows not affected by earthquakes and other transients is applied at the stacking stage (see below). This processing is deliberately kept simple. This is to ensure that the observed quantity is fully compatible with the forward-modeled cross-correlation eq. 2.13. Although Fichtner et al. (2017a) have introduced considerations on how to account correctly for nonlinear processing during forward modeling, this approach has not been validated on observed data so far, limiting our current approach to linear processing. As amplitudes are preserved by the non-normalized cross-correlation, it is crucial to select and stack only realizations containing microseismic noise, removing earthquakes and other high amplitude, highly coherent transient signals.

To exclude them, we use a multiple threshold selection strategy similar to the one used by Boué et al. (2014). The idea is to combine an absolute threshold on the root mean square (rms) of windows, chosen so as to eliminate strong earthquakes, with a relative rms threshold that compares windows in a moving window of several hours and eliminates those that pass a pre-defined multiple of the median rms. We determine which segments pass any of the two thresholds after pre-filtering the data in two bands - 0.01 to 0.05 and 0.05 to 0.1 Hz - and retain only segments that fall below the threshold in both frequency bands, and for both criteria. The second part of the procedure bears resemblance to an STA/LTA trigger algorithm (Vanderkulk et al., 1965), but is coarser and can therefore be evaluated faster. Finally, the moving window rms criterion is applied again to the correlation windows themselves. We test the exclusion strategy for windows contaminated by earthquakes and other impulsive events using a subset of stations. The results indicate that the thresholds have to be tuned to seasonal variations of noise levels. Tuning the parameters to achieve this result is time-consuming and somewhat arbitrary. It is desirable to automatize this step in future studies, e.g. using higher order statistics (Poiata et al., 2016). An example illustrating the effectiveness of the threshold approach is shown in Fig.s 6.2 and 6.3. In practice, we attach the rms values of both traces $v_i(x_1), v_j(x_2)$ to the dataset for each cross-correlation window, and later use them as diagnostic criterion to reject windows during stacking. This keeps the stacking flexible.

### Table 6.1 – List of selected noise episodes.

<table>
<thead>
<tr>
<th>Event</th>
<th>Dates</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>01.01. - 09.01.2010</td>
<td>Longer lasting strong storm</td>
</tr>
<tr>
<td>B</td>
<td>13.03. – 14.03.2010</td>
<td>Only North Japan</td>
</tr>
<tr>
<td>C</td>
<td>13.04. - 15.04.2010</td>
<td>See text</td>
</tr>
<tr>
<td>D</td>
<td>08.08. - 14.08.2010</td>
<td>Coincides with severe tropical storm Dianmu</td>
</tr>
<tr>
<td>E</td>
<td>31.08. - 09.09.2010</td>
<td>Coincides with tropical storms Kompasu, Malou</td>
</tr>
<tr>
<td>Y</td>
<td>31.07. - 02.08.2010</td>
<td>'No event’, calm, beamforming shows body waves</td>
</tr>
<tr>
<td>Z</td>
<td>24.09. - 30.09.2010</td>
<td>'No event’ Pacific coast sources</td>
</tr>
</tbody>
</table>
6.4. Data processing and data assessment

Figure 6.2 – Illustration of the data selection procedure on the example of a vertical-component time series recorded at station BO.IZH on Tsushima island during April 13-15, 2010. A number of transient signals of various scales are excluded based on a multiple-threshold algorithm (see text for details). The largest transients occurring after midnight on 14th April correspond to a series of earthquakes of up to magnitude 6.9 in southern China.

and offers us the possibility to refine the selection procedure as more elaborate detection and selection algorithms, for example based on the results of Poiata et al. (2016), become available. As an additional plausibility check that the cross-correlations of the chosen events show dominant incidence from the Sea of Japan, we stack the cross-correlations of the selected station subsets for the entire duration of the events, and plot them sorted by backazimuth. An example is shown in Fig. 6.4. The cross-correlation stack for an event in winter (upper panel) does not show clearly distinguishable Rayleigh waves, while for an event in summer (lower panel) surface waves are visible, and the incidence supports a dominant signal from the direction of the Sea of Japan. We interpret the result for winter as an effect of multiple strong sources acting close to both the Sea of Japan and the Pacific coast, rendering the cross-correlations incomprehensible. This interpretation is supported by comparison of selected events to significant wave height maps provided by the Japan Meteorological Agency (JMA) \(^4\). For events in spring and summer, the comparison to significant wave height maps shows consistency between wave height and incidence direction of microseismic energy.

\(^4\)http://www.data.jma.go.jp/gmd/kaiyou/db/wave/chart/daily/oceanwave.html
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Figure 6.3 – Spectrograms of the high-noise episode shown in Fig. 6.2, but here for station BO.HID on Hokkaido, North Japan. The upper and lower panel show the spectrogram before and after transients were muted. Data have been bandpass filtered between 0.05 and 0.08 Hz to render the contribution of the primary microseism visible. The increase in microseism signal amplitudes from the evening of April 13th coincides with high significant wave heights at the Northwest Japanese coast (shown in Fig. 6.8).
6.4. Data processing and data assessment

Figure 6.4 – Vertical-vertical cross-correlation stacks for noise events A and D, filtered between 0.05 and 0.1 Hz, and sorted by inter-station distance, as well as azimuth. Event A (top panel) is a longer-lasting winter storm with high beam power; despite high noise amplitudes, the cross-correlations show no clear Rayleigh wave propagation, which we interpret to be due to the simultaneous action of several proximal sources. Event D (bottom panel) shows far clearer Rayleigh waves. Propagation from azimuths of 200 to 360 degree (the direction of the Sea of Japan for most station pairs) appears on the acausal branch of the correlations in the lower panel, confirming preferential incidence of microseisms from the Sea of Japan for event D.
6.5 Preliminary synthetics and recovery tests

Tests carried out with the mapping procedure presented in chapter 4 suggested that the first-order maps of seismic noise sources might be biased by the shape of the arrays. The shape of F-Net gives rise to many inter-station paths that are oriented approximately Southwest – Northeast, while the large number of receivers in NecessArray (which are also deployed on a regular grid) gives rise to a large number of correlated inter-station paths towards the Sea of Japan.

To understand this effect, we set up a recovery test using a simplified model for the rapid calculation of cross-correlations. The Green’s function for a homogeneous, unbounded 2-D medium is:

$$G(x, y, \omega) = -\frac{1}{4\rho v^2} \sqrt{\frac{2v}{\pi \omega r}} e^{-i \frac{\omega r}{v}} e^{i \frac{\pi}{4}}, \quad (6.2)$$

which is non-dispersive, with constant velocity $v$ and quality factor $Q$. We used values of 3 km/s for Rayleigh waves and a quality factor of 100. As source spectrum, we choose a tapered Gaussian with a mean of 0.1 Hz. As starting model for the spatial distribution, we use a homogeneous source in the ocean. We simulate test cases with four different Gaussian-shaped localized sources in the Sea of Japan and the East China Sea (see the target distributions in Fig. 6.5). They are chosen to represent different source locations of interest, in particular at the Japanese and Chinese coastlines as well as in the middle of the Sea of Japan. We treat the resulting four sets of cross-correlations as synthetic ‘data’, and use the homogeneous starting model to calculate finite-frequency sensitivity kernels with respect to source distribution. We measure signal energy $E_l$ (eq. 2.26) in a frequency band from 0.05 to 0.1 Hz on both causal and acausal branch of the cross-correlation, and construct the misfit gradient with respect to source distribution using F-Net together with a subset of NecessArray stations facing the Sea of Japan, as well as F-Net only. In the recovered maps in 6.5, the negative gradients are shown. This imaging procedure is similar to the one presented in chapter 4; however, instead of collapsing the sensitivity kernels onto rays, here we evaluate the finite frequency kernels and use these to construct the gradient. This is more appropriate, since the characteristic inter-station distance of F-Net station pairs facing the Sea of Japan is short compared to the seismic wavelengths of primary microseism, and to the proximity of noise sources to the coast. The two record sections in the top row of Fig. 6.5 show a subset of simulated correlations from F-Net station pairs. The cross-correlations for sources distributed homogeneously in the ocean are shown on the left. The right plot shows correlations from test case Gaussian shaped source nr. 2. Although the source distribution of the starting model is direction-dependent, because no sources are located on land, surface waves propagating at constant velocity emerge from the correlation. In the case of a strongly localized source, the lag time of the maximum correlation is not proportional to inter-station distance but rather depends on inter-station azimuth with respect to the source (the inter-station azimuth is not considered in the plot). Such correlations would typically be rejected in an ambient noise tomography study, whereas
6.5. Preliminary synthetics and recovery tests

Figure 6.5 – Recovery test for different Gaussian-shaped sources in the Sea of Japan and the East China Sea. The top two panels show record sections of synthetic cross-correlations. On the left, the cross-correlations from the starting model, which corresponds to a homogeneous distribution of sources in the ocean and none on land, are shown. On the right, the 'synthetic data' cross-correlation section obtained from target model 2 is shown, demonstrating that complex cross-correlation waveforms result from a point-like source located outside the stationary phase region of certain station pairs. The maps in the second row show the target models. Below them, the recovered first-order source updates are displayed for a combination of F-Net and NecessArray stations, and F-net only, respectively.
Chapter 6. Microseismic noise in the Sea of Japan

here we attempt to utilize them for source imaging. We first attempted to recover the target distribution using F-Net and selected stations from NecessArray. Only a subset of stations from NecessArray closest to the coast was used. The maps presented in the first row of recovery tests. The last row shows recovery with F-Net stations only. We found that for the imaging procedure used here, adding station pairs from NecessArray increased smearing. Smearing towards both networks occurs, but is less severe towards the Japanese network, which is probably due to its curved geometry. On the basis of this simple test, we cannot conclude that this effect of array shape would persist during an iterative inversion. However, for the simplified imaging procedure used below, we restricted the analysis to F-net stations. In our recovery test, the geometry of F-Net appears well-suited to image sources in the Sea of Japan. As expected, sensitivity to sources is highest close to the Japanese coastline which may render more distant sources more difficult to image. In test case 4, the imaging procedure fails to ‘see’ a source located at the Chinese coast.

6.6 The April 13, 2010 storm

In the next step, we apply the same imaging procedure to observed data. We focus on one noise event between April 13 and 15, 2010. A time series and spectrogram of the event are shown in Figs. 6.2 and 6.3. On the spectrograms in Fig. 6.3 (bandpass filtered between 0.03 and 0.08 Hz to render the primary microseism visible), one can observe how noise increases in the primary microseism band (approx. 0.05 to 0.1 Hz) between the afternoon of 13th and the night from 14th to 15th April, 2010. This noise ‘event’ is more prominently observed at stations close to the Tohoku and Hokkaido coast than to the Southern Japanese coast lines. We choose it because of its comparatively high amplitude and because cross-correlation stacks of this event show clearer surface wave propagation than those from events in winter, providing a simpler starting point for our analysis. We attempt to extract short cross-correlation stacks, aiming for a time resolution of less than 12 hours. Incoherent noise can mask the properties of the coherently propagating seismic waves, and stacking a sufficient number of time windows is necessary to reduce the incoherent noise. Primary microseismic noise in particular has comparatively low amplitudes (Peterson, 1993). We obtain 6 hour stacks in order to assess their quality. As we do not make the assumption of retrieving an approximate Green’s function, we do not require the waveforms to appear as Rayleigh waves from a virtual source at one station to the other station; rather, we propose that signals reoccurring in several subsequent correlation windows are of physical origin and not caused by coincidental correlation of incoherent noise. Fig. 6.6 shows subsequent 6-hour correlation windows. Each window is normalized by its absolute amplitude for visibility; the signal energy in the arrival window of a Rayleigh wave of 2.9 km/s group velocity (marked by the green dashed lines) is shown on the plots on the right. For several short stacks, we observe reoccurring consistent phases, while for others, this consistency breaks down. We argue that the latter is an effect of rapidly moving noise sources, and proceed with the analysis of 6-hour stacks. High signal levels in one of the stacks may be contaminated by the series of earthquakes in southern China which also
The April 13, 2010 storm

Figure 6.6 – Subsequent 6-hour cross-correlation stacks (large left panels). Cross-correlation stacks are normalized to unity per-trace for better visibility. Green dashed lines mark approximate Rayleigh wave group window. While the variability of the cross-correlations is generally high, coherent phases in the Rayleigh wave window can be distinguished, which persist for a duration of several 6-hour windows, suggesting that a time-dependent investigation of the noise events listed in table 6.1 is possible. The smaller panels to the right show signal energy in the windows delineated by the dashed green lines. High signal levels in the fifth stack may be contaminated by a series of earthquakes. This stack is consequently excluded from the analysis.

appear on the time series and spectrograms in Figs. 6.2 and 6.3. This stack is consequently excluded from the analysis. An example of two record sections of 6-hour cross-correlations for vertical-vertical and transverse-transverse component is shown in Fig. 6.7. One can discern propagating signals up to about 700 km inter-station distance. Fig. 6.8 shows a summary of the mapping results in conjunction with significant wave height maps provided by JMA. Two days are shown (13th and 14th April, 2010); the top row shows the significant wave height maps (available at half-day intervals), and the bottom row the negative misfit gradients. The maps are all normalized with respect to the largest occurring amplitude (at April 14, 6 am). The map for the time window starting at midnight of April 14th is cut out due to the high amplitude transients, which are probably linked to a number of shallow earthquakes between magnitudes 5 and 6.1 in Southern China occurring between midnight and 4 am on April 14 (see the time series of Fig. 6.2). The homogeneous starting model had lower overall amplitudes than the observed data, so that all the first updates are positive. Their shapes inform us about
Figure 6.7 – Two record sections of randomly selected, six-hour stacked cross-correlations sorted by inter-station distance for the noise episode on April 13 and 14, 2010 (Fig. 6.3). While signal levels are low, propagating signals can be distinguished up to distances of approx. 700 km.
6.6. The April 13, 2010 storm

Figure 6.8 – Comparison of significant wave height maps from the Japan Meteorological Agency to first-order updates of source PSD from vertical-vertical cross-correlations during 6-hour windows on April 13 and 14, 2010. Individual first-order noise source maps are normalized by the amplitude of the largest negative gradient. Elevated levels of primary microseism excitation at the Tohoku and Hokkaido coast temporally coincide with large significant wave heights in the same region.

The sequence of updates shows coincidence with the development of significant wave height maps. The event is observed most strongly at the Tohoku coast. Although we cannot distinguish any details, we conclude from these maps that despite low signal levels in the primary microseism band and proximity of the noise sources, it is possible to extract a meaningful signal from the cross-correlations. Therefore, we intend to use the observational data for an iterative noise source inversion in a future study. Maps of a second event are shown in Fig. 6.9, together with significant wave height. They cover roughly two days in September 2010, during which two tropical storms passed the Sea of Japan. The storm tracks can be intuited from the significant wave height maps. The source distribution maps, elaborated in the same manner as those in Fig. 6.8 before, show the strongest sources on September 3rd towards the Japanese...
Chapter 6. Microseismic noise in the Sea of Japan

Figure 6.9 – Significant wave height maps from Japan Meteorological Agency (top row); first-order updates of source PSD from vertical-vertical cross-correlations during several 6-hour windows of noise episode E (middle and bottom row). Individual first-order noise source maps are normalized by the amplitude of the largest negative gradient. During the displayed time windows, two tropical storms reached the Japan Islands region. Their approximate locations coincide with large significant wave heights, which in turn coincide with high levels of primary microseismic noise.

East Coast and on September 4th towards South, both of which coincide with high significant wave heights. While the first-order mapping was also carried out for one noise event in winter, no conclusive results could be found. We interpret this as a consequence of the considerably more complicated noise source field in winter. Primary microseism noise levels are higher and microseisms are likely excited by more distributed sources. This is consistent with our observations on record sections of the cross-correlations themselves (Fig. 6.4).

6.7 Steps towards 3-D modeling of primary microseismic noise cross-correlations

Microseismic surface wave cross-correlations between 10 and 20 seconds are sensitive to lower crustal structure as well as the upper mantle, making it necessary to employ a 3-D model to
model them as accurately as possible. In preparation of 3-D modeling of cross-correlations in the Sea of Japan region, we reviewed available tomographic models. A range of models based on different observations, and elaborated with different tomographic techniques, have been published in recent years. Nishida et al. (2008a) elaborated a high-resolution S-wave tomography using ambient noise cross-correlation Love and Rayleigh waves, which, however, did not cover the Sea of Japan, but focused on the islands themselves. Ambient noise tomographies including the structure below the Sea of Japan were presented by Zheng et al. (2011) and Kim et al. (2016). The former model is an isotropic S-wave velocity model from classic dispersion analysis, and the latter a transdimensional ambient noise tomography. Earthquake tomographies were presented by Yoshizawa et al. (2010), who employed the two-station method, and by Chen et al. (2015) and by Simute et al. (2016), who both used adjoint full-waveform tomography. The model by Chen et al. (2015) includes extensive station coverage of the entire East Asian region.

A model for ambient noise cross-correlations should address the following requirements. First, it should fit observed surface waves well, since ambient noise cross-correlations are dominated by fundamental mode surface waves. The second is to account for the effect of the ocean layer. Guided surface waves in deep water (Yomogida et al., 2002) are likely negligible at periods of 10 seconds and more for the Sea of Japan with a maximum depth of approx. 3 km. However, the ocean load affects the dispersive characteristics of Rayleigh waves (e.g. Komatitsch and Tromp, 2002a) and the model should take this into account. Long-wavelength features of topography and bathymetry, such as the rise from the Sea of Japan basin to the Japanese Islands, or the deeper basins within the Sea of Japan such as the Yamato basin, should be represented. Since we are merely modeling wave propagation from a source that is due to any excitation mechanism and not the excitation mechanism itself, short-wavelength features such as small elevations of the sea floor can be neglected, as the seismic waves with wavelengths on the order of 10 km should have little sensitivity to these.

Given these requirements, we choose the model of Simute et al. (2016). The authors report that most of the utilized waveforms, which are selected for measurements by an automated window selection procedure, are performed on surface waves, which are the most prominent signal in the period range of their study, hence fulfilling our first requirement. The model was constructed by simultaneous inversion for compressional and anisotropic shear wave speeds $v_p$, $v_{sh}$, $v_{sv}$ and density, and it can therefore be used to model elastic waves without invoking any scaling relationships (e.g. between density and shear wave velocity). In addition, all iterations included the depth-dependent attenuation model QL6 (Durek and Ekström, 1996). As Simute et al. (2016) did not apply crustal or topographic corrections or explicitly account for ocean loading during the model construction, the uppermost part of the model includes a smooth effective layer that mimics the effects of ocean, crustal structure and topography. This makes it convenient to use with different wave propagation solvers, in particular as it is parametrized in blocks of spherical coordinates.

Fig. 6.10 shows the vertical-component displacement response to a vertical point force source.
Figure 6.10 – An example record section of displacement Green’s functions for the tomographic model of the Japanese Islands region from Simute et al. (2016).

at the location of station BO.FUK of the Japanese broadband network, giving an impression of the complex wave propagation. In order to utilize the model with the salvus solver\(^5\), which is currently under development at ETH Zurich, we interpolated the model onto an unstructured mesh from the salvus mesher\(^6\). The usage of Green’s functions obtained by this model for the calculation and inversion of cross-correlation waveforms awaits future work.

### 6.8 Conclusion

From the first-order mapping and comparison to significant wave height, we conclude that it is possible to elaborate approximate maps of primary microseism noise sources at short time intervals (here, six-hour intervals) as long as data coverage is sufficient (e.g. no large Earthquakes occur). We suggest that the selected events hold the potential to image surface wave noise sources linked to single storms in proximity to the coast, which makes an inversion using a 3-D model of the Earth’s crust and upper mantle worthwhile. Here, we have laid the basis for such an inversion. The processing part of the study - to obtain the cross-correlation tensors of selected events, and to tune the stacking parameters in order to exclude transient events - have been finalized. A framework for the calculation of synthetic cross-correlations and sensitivity kernels is available. The preparation of Green’s functions from a 3-D tomographic model, and the subsequent source inversion, await future study. There are many possibilities for further extension of this project. These range from the investigation of the horizontal component and mixed-component cross-correlations, to probabilistic inversion approaches for the noise sources, to the investigation of secondary microseisms, feasible if tomographic models that can predict surface waves at periods shorter than 10 seconds become available. A particular challenge is to account for the influence of laterally varying

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\(^5\)http://salvus.io

\(^6\)https://gitlab.com/Salvus/salvus_mesher
anelastic attenuation. Although models of the attenuation structure beneath the Japanese islands have been published (e.g. Hashida, 1989), the tomographic models that we intend to utilize contain a purely depth-dependent attenuation structure (Simute et al., 2016).
7 Conclusions and outlook

This work was begun with the objective of advancing ambient noise source inversion from theoretical considerations and synthetic inversions (Tromp et al., 2010; Hanasoge, 2013a; Sager et al., 2017) to the application stage with the two-fold motivation of imaging ambient source distributions more accurately compared to previous methods such as back-projection, and modeling more realistic noise cross-correlations for seismic tomography. We have now conducted the first application on vertical-vertical cross-correlations on global scale. We can draw conclusions which both regard the method itself, and results it has yielded so far.

1. Imaging ambient noise sources using cross-correlations and a causal-acausal logarithmic signal energy ratio yields a robust first-order estimate of dominant noise source regions. While details cannot be distinguished, large scale features show good qualitative agreement with results from a full inversion. It is therefore recommended to apply this fast source imaging procedure to determine the approximate outcome of an inversion.

2. Using precomputed wavefields for the forward modeling and inversion of noise cross-correlations renders the problem of ambient noise source inversion efficiently computable even in a three-dimensional Earth model. While this approach, relying on known response of the Earth's structure, cannot be used for ambient noise waveform tomography itself, its application can serve the latter greatly by providing a starting model for the source distribution constrained by the data, and by providing source sensitivity kernels at each iteration step.

3. We have applied ambient noise inversion to vertical-vertical cross-correlations on global scale to image the sources of the Earth's hum. The results, physically reasonable and in qualitative agreement with previous studies, show a new amount of detail, especially of hum sources in the Southern Hemisphere. These indicate that during austral winter, hum preferentially originates at the South Pacific coasts and at additional source locations, removed from continental coastlines, which are possibly associated with regions of high and complex bathymetry.
4. Processing crucially influences results of ambient noise cross-correlation studies, and we have therefore investigated the possibility of retrieving short stacks of a duration of several hours without applying any nonlinear processing steps other than a selection filter. Preliminary results indicate that the resulting cross-correlation stacks contain clearly observable signals from passing storms.

There is a number of directions for meaningful future research, some of which have already been mentioned in chapters 4 and 5.

First of all, the present limitations of the current method should be overcome. On global scale, our application was most strongly limited by station coverage and data quality. Station coverage can be improved upon, although the availability of permanent very broad-band stations in the South Hemisphere is limited. Data quality presents greater difficulties at long periods matching the Earth's hum. The reduction of tilt noise using the selection scheme of Nishida and Fukao (2007) or the barometric correction by Beauduin et al. (1996) could contribute, as well as ensuring synchronicity of the observed data as far as possible.

Data quality suffers from the loss of coherency over long distances. Therefore, it may be profitable to add auto-correlations to the inversion, i.e. power spectral densities at individual stations. The presented method can be used to determine the sensitivity of a measurement of energy at each station to the distribution of sources. Without the directional information of the cross-correlation, this sensitivity forms approximately radial patterns around each station, the symmetry of which is modified by the non-uniform elastic properties of the subsurface. In order to invert auto-correlations, observables different from the causal-acausal signal energy ratio need to be investigated; signal energy such as presented in chapters 2 and 6 may be a viable alternative. More generally, the exploration of additional misfit functions may permit to exploit larger amounts of information contained in the cross-correlation than is currently used.

We advanced the application of ambient source inversion with the aim to use it in ambient noise tomography. While the method was found to function well on global scale, the resulting source models neither improved nor worsened phase misfits of the windowed cross-correlation waveforms. Therefore, based on the current result, we cannot conclude that global-scale cross-correlation waveform tomography at long periods will significantly benefit from a space-dependent source power spectral density model. Future studies should therefore both apply the method to regional scales, where the source model is expected to have a more important effect, and revisit the global scale inversion with an improved dataset that is built with amplitude-preserving processing. In addition, it may be informative to evaluate how the hum source models found here change the illumination of subsurface structure compared to homogeneous models.

A challenge for the application of the method to regional scale is to define adequate limits for the modeling domain (since even secondary microseisms can be observed coherently
over long distances). One may consider to construct an effective source, e.g. given by a ring of distant sources as suggested by Hanasoge (2013a). However, this prevents the retrieval of information about the physical nature of the sources themselves. We have started to explore the feasibility of an alternative approach of specifically targeting independently constrained episodes of high-amplitude regionally acting sources. The Sea of Japan region presents a very suitable study region for the further development of both ambient source inversion and ambient noise cross-correlation waveform inversion due to its dense instrumentation and the availability of previous state-of-the-art waveform tomographies (Chen et al., 2015; Simute et al., 2016). Coverage in the western Sea of Japan (towards the Chinese coast) is limited in many previous studies due to the spatial distribution of seismicity (e.g. Yoshizawa et al., 2010; Simute et al., 2016). Ambient noise cross-correlations between stations located in Korea and northwest Japan, respectively, could add valuable observations to future tomographic studies.

Future developments of a more technical nature are the extension to nine cross-correlation components and three source components and the extension to a frequency-dependent inversion allowing to fit the weights of a limited number of spectral basis functions. Both are relevant extensions from a scientific point of view:

The extension to horizontal components ties back to the open questions raised in the introduction of where horizontal component microseisms originate, and how important traction sources are in the excitation of different periods of the ambient seismic fields. An inversion for all components of the source auto-correlation tensor may provide insights on the relative importance of vertically and horizontally acting sources, and it may give us a more detailed image of the respective locations of the sources of vertical-vertical, horizontal-horizontal and mixed cross-correlations.

Frequency-dependent inversion, if feasible, would be a clear improvement of the method. It should permit the separation of sources with different characteristic periods occurring at different locations and is therefore crucial for more detailed studies of microseism excitation. At present, our method only permits the sequential inversion of cross-correlations filtered in different frequency bands: the synthetic study by Sager et al. (2017) indicates that the misfit functions used for source inversion so far, namely logarithmic signal energy ratio and windowed signal energy, have a too limited sensitivity to the spectrum of the noise source to separate distinct sources of different characteristic period. This again underlines the importance to investigate further suitable observables.

With the presented work, I hope make a productive contribution to a new chapter in ambient noise seismology, one which a number of recent publications have opened up (e.g. Basini et al., 2013; Nishida and Takagi, 2016; Stehly and Boué, 2017; Sager et al., 2017) and which regards ambient sources as a quantity to work with, rather than a nuisance to correct for. This will hopefully, in the spirit of Aki (1957), further our understanding of both the “complicated waves” and their medium of propagation.
A Supplementary material to chapter 4

Supplementary Material

These figures are presented to support our hypothesis that hum source maps elaborated with simple processing – only relying on the downweighting included in the cross-correlation – show no large influence of earthquake signals. From the two plots below, we conclude that our processing approach is robust and suitable at a global scale to image ambient sources, as long as a large enough number of correlation windows are stacked.

**Figure S 1:** We argue that the strongest coherence of earthquake signals is achieved on relatively small arrays. The smallest arrays of STS-1 seismometers in our dataset are the Berkeley Digital Seismic Network and the Southern California Seismic Network. Excluding these arrays (right map) decreases the resolution of the resulting map, but the global pattern does not change.

**Figure S 2:** The second argument relies on a comparison to the phase cross-correlation. This amplitude-independent correlation function emphasizes weak, coherent signals. Again, no large-scale differences between the processing we used (left map) and the phase cross correlation, which we used for testing purposes, can be observed.
Appendix B. Supplementary material to chapter 5

Figure B.1 – Comparison of network sensitivity map including thirteen stations in California (STS-1 of BK and CI arrays), shown on the left; and sensitivity map including only four selected stations in California, shown on the right.

Figure B.2 – Model for SH winter, after 11 iterations. The low misfit (below estimated observational uncertainty) indicates that this model is overfit. We display it for completeness; the corresponding result after 5 iterations and before the misfit dropped below the estimated measurement uncertainty is presented in the main text. The model shown here shows similarity to the one obtained with low smoothing, shown below in Fig. B.4.
Figure B.3 – Map of network sensitivity (two views). Similar to a ray density plot, this map shows the sum of the absolute values of the sensitivity kernels, where data residuals have not been considered. The sensitivity kernels are clipped at the 95th percentile, but not yet smoothed.

Figure B.4 – Comparison of results from two inversions with different smoothing strategies. The left side shows a model which was smoothed with a Gaussian kernel of standard deviation 250 km, the right side a model resulting from an inversion during which the smoothing was decreased from 500 km to 250 km standard deviation after three iterations. While the result on the left appears very noisy, the peaks of the most prominent features of the model, marked by green circles, show locations consistent with the more strongly smoothed model.
Appendix B. Supplementary material to chapter 5

Figure B.5 – Map of features visible in both NH winter and SH winter season (two views). This map was constructed from the squared product of the NH winter and SH winter source distribution model (the squared value is shown for better visibility of dominant features). The most prominent locations of strong source PSD in both seasons are the West and South Pacific coasts including Fiji/Samoa, the Chilean coast and the Philippine Sea region.

Figure B.6 – An example source sensitivity kernel calculated with synthetics of three different durations. The kernel itself is normalized and the color scales are truncated at 0.05 and 0.01 respectively, to render the relatively low sensitivity visible. A synthetic Green's function of 5400 s length (left) yields a cross-correlation of 2700 seconds maximum lag, etc.


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Key tasks  During my PhD, I aim to make a contribution to ambient noise-based imaging on different scales. My goal is to map the spatial distribution of ambient noise sources with cross-correlation techniques. Resulting source maps can be used to model more realistic ambient noise correlations between various pairs of receivers. This allows using synthetic cross-correlations as forward model in a novel approach to noise tomography. However, the techniques I am developing may also be used to pose questions about the noise sources themselves. In collaboration with ERI Tokyo, we have started to investigate surface wave sources within the Sea of Japan.

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