Grid Referencing of Buildings

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1 Introduction

There are many different ways to reference locations within a building, such as the use of building numbers, room numbers, floor levels, latitude and longitude, and simple XYZ Cartesian coordinates [1, 4]. Some of these rely on semantic definitions, others on physical measurements, while yet others on a combination of both. When trying to integrate these various different systems, it can sometimes help to use a single intermediary system. This paper looks at the use of a grid-based intermediate CRS established using only two anchor points for producing the required transformation matrices between these various systems.

1.1 Coordinate Referencing of Buildings

The two main approaches to referencing locations inside of buildings include the use of identifiers and the use of coordinates [4, 6]. Identifiers use natural language descriptors — such as landmarks, building numbers, room numbers, and floors — and represent the most intuitive and natural approach, not only for communicating with people but also for topological modeling. On the other hand, coordinates use numerical measurements that can provide very high levels of precision but in a form that makes no sense to the uninitiated. When used in mapping, identifiers will often have coordinates to give objects geometric form.
1.2 Uncertainties In Location

The natural language that makes identifiers so easy to use can also make them problematic when it comes to geometry [1]. Identifiers can suffer from semantic and locational ambiguities, with the former representing problems such as duplicate or non-unique names (i.e., same name for different locations or different names for the same location) and the latter representing problems of geometry (e.g., point versus area representation, fuzzy boundaries, etc.).

Coordinates themselves can also have a certain degree of uncertainty stemming from their use of arbitrary datums and map transformations (e.g., map projections). For instance, a single pair of geographic lat-lon coordinates can point to different locations on Earth when used without specifying their coordinate system and datum, i.e., the “coordinate reference system” — a common oversight. Figure 1 shows identical geographic coordinates for “Room 1720” plotted in three different CRSs — WGS 84, NAD 27, and NAD 83 — resulting in three different locations on Earth’s surface.

![Fig. 1](image-url) Identical sets of coordinates for “Room 1720” plotted using three different CRSs. The top-left corner has coordinates 34°24′55.6″N, 119°50′43.1″W. a North American Datum of 1927 (NAD 27) versus World Geodetic Datum of 1984 (WGS 84). b North American Datum of 1983 (NAD 83) versus WGS 84.

1.3 Map Grids

Map grids partition maps using orthogonal lines with equal interval spacing (see Fig. 2a), and they work especially well with buildings that almost always use the Cartesian coordinate system. When used with outdoor maps, map grids serve as independent overlays to map projections that help to transform cryptic lat-lon coor-
coordinates into linear values of meters, feet, etc. [2, 5, 7]. This property of outdoor grid maps makes it possible to easily convert between indoor and outdoor coordinates.

While geographic lat-lon coordinates may also appear to run along straight orthogonal lines, they actually run along curved lines of parallels (latitude) and meridians (longitude) that together form graticules — rather than grids — which have a grid-like appearance in projected 2D form (see Figs. 2b and 2c) [7]. Graticules present two problems when used for ground measurements: first, at least three different methods exist for measuring a degree of latitude and second, meridian lines converge at the poles. Both properties result in a degree (1°) having varying ground distances; additionally, a single point at either pole can have an infinite number of longitude values due to convergence of the meridians. A map grid overcomes these problems by discretizing the projected map into regular linear units as illustrated in Fig. 2c, and it can accommodate different projections optimized for different areas, such as at the problematic poles.

(a) Grid Graticule (b) Grid Graticule (c) Grid overlaid on projected graticules

Fig. 2 Grids and graticules. a Grid. b Graticules. c Grid overlaid on projected graticules.

2 Grid Referencing of Buildings

The building grid proposed in this paper is nothing other than a conventional CRS based on the Cartesian coordinate system, with axis orientations, a unit of measurement, and a false origin. However, this paper is not concerned with CRS specifications; rather, its focus centers on the process for establishing the building grid, which can then serve as an intermediary for other building data as well as for integrating with outdoor reference systems as illustrated in Fig. 3. Once established, the building grid remains stable and does not change with time allowing for the integration of data that vary in referencing approaches or with time. Consolidating all CRSS into a single intermediate system can mitigate some of the uncertainties mentioned earlier; prevent the unintentional propagation of blunder-induced errors, e.g., use of the incorrect datum; and provide a way to cross-check data using multiple sources.

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1 i.e., geocentric, geodetic, and astronomic.
2.1 Specifying the Coordinate System

The first step in establishing the building grid involves defining the grid’s coordinate system, which includes defining the axis directions, the unit of measurement, and an optional operation to establish a false origin. The building grid must use a Cartesian coordinate system with the z-axis oriented in the up direction and can use any suitable unit of measurement. While use of meters should meet most requirements, millimeters can be used for applications that require all integer values. A false origin can also be established for applications that require all-positive coordinate values.

2.2 Placing the Building Model

Establishing a grid for a leveled building model\(^2\) requires selecting just two anchor points, \(b_1\) and \(b_2\), from the model and specifying a unit conversion factor \(s\). Here, point \(b_1\) represents an arbitrary datum point (i.e., origin) and a line from \(b_1\) to \(b_2\) points in the direction of local north (i.e., the \(y\)-axis on the grid), with both points using coordinates from the model’s original CRS. Equations 1 through 8 outline the entire process, which involves translating \(b_1\) to the origin, rotating the model so that the vector from \(b_1\) to \(b_2\) lines up with local north (i.e., \(y\)-axis), scaling the model to match the grid’s unit of measurement, and applying the optional false origin. Here, upper case notation represents the homogeneous form of the transformation matrices [3]. This process is repeated for every set of building data that uses a different CRS, as illustrated in Fig. 3, to transform them into the common building grid CRS.

\(^2\) I.e., all horizontal surfaces are aligned with the \(xy\) plane.
• Translate the model from \( b_1 \) to \((0, 0, 0)\).

\[
t_1 = -b_1 = \begin{bmatrix} -x_1 \\ -y_1 \\ -z_1 \end{bmatrix}; \quad T_1 = \begin{bmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

\(1\)

• Convert the vector pointing from \( b_1 \) to \( b_2 \) into a unit vector, \( \hat{v}_{12} \).

\[
b_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \quad b_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} \quad v_{12} = b_2 - b_1 = \begin{bmatrix} x_2 \\ y_2 \\ 0 \end{bmatrix} - \begin{bmatrix} x_1 \\ y_1 \\ 0 \end{bmatrix}
\]

\[
\hat{v}_{12} = \frac{v_{12}}{\|v_{12}\|} = \begin{bmatrix} \hat{v}_x \\ \hat{v}_y \\ 0 \end{bmatrix}
\]

\(2\)

\[
\hat{v}_{12} = \frac{v_{12}}{\|v_{12}\|} = \begin{bmatrix} \hat{v}_x \\ \hat{v}_y \\ 0 \end{bmatrix}
\]

\(3\)

• Rotate the model so that \( \hat{v}_{12} \) aligns with grid north, where \( \gamma \) is the angle between the grid north vector \((0, 1, 0)\) and \( \hat{v}_{12} \).

\[
r_z = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \hat{v}_y & -\hat{v}_x & 0 \\ \hat{v}_x & \hat{v}_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_z = \begin{bmatrix} \hat{v}_y & -\hat{v}_x & 0 & 0 \\ \hat{v}_x & \hat{v}_y & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

\(4\)

• Scale the model to match the target unit of measurement, where \( s \) is the conversion factor from source to target units.

\[
s = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} \quad S = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

\(5\)

• Translate the model to false easting (\(x_f\)), false northing (\(y_f\)), and false up (\(z_f\)) locations relative to the false origin (optional). \( T_2 = I \) for no false origin.

\[
t_f = \begin{bmatrix} x_f \\ y_f \\ z_f \end{bmatrix} \quad T_2 = \begin{bmatrix} 1 & 0 & 0 & x_f \\ 0 & 1 & 0 & y_f \\ 0 & 0 & 1 & z_f \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

\(6\)

• Assemble the 4x4 master transformation matrix, \( M \), for moving the model or individual points on the model from the original CRS to the grid CRS.

\[
M = T_2 \cdot S \cdot R_z \cdot T_1
\]

\(7\)

\(3\) Note that the \( z \)-coordinates are ignored since this process assumes grid up is aligned with the vertical direction.
• Transforming from grid coordinates to the building’s original coordinates involves applying the reverse transformation.

\[ M_{\text{inverse}} = T_1^{-1} \cdot R_2^\top \cdot S^{-1} \cdot T_2^{-1} \]  

(8)

3 Discussion

This paper covered a process for establishing a grid-based CRS for leveled building models that requires the use of only two anchor points and a unit conversion factor, and it can be further extended to non-level buildings using three anchor points and three rotations. The value of such a grid-based system lies in its ability to bridge and integrate indoor data sources that use disparate CRSs. However, the two- and three-point approaches have shortcomings, especially when applied to point clouds. Future work includes investigating best-fit optimization techniques for imperfect data; using the building grid to bridge indoor and outdoor CRSs as illustrated in Fig. 3; and applying the grid in future work on feature extraction using indoor point clouds and visualization of indoor models using augmented and virtual reality systems.

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References