Computational Framework for Online Estimation of Fatigue Damage using Vibration Measurements from a Limited Number of Sensors

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Abstract

This study proposes a computational framework for the online estimation of fatigue damage using operational vibration measurements from a limited number of sensors. To infer the stress response time histories required for fatigue prediction, the measured structural response is driven to a high fidelity finite element (FE) model, which is reconciled using appropriate model updating techniques that minimize the discrepancy between the experimental and analytical frequency response functions (FRFs). Fatigue is accordingly estimated via the Palmgren-Miner rule, while the available FE model allows for stress estimation at unmeasured spots. The method is successfully validated and assessed through an experimental study that pertains to a linear steel substructure supporting the entire body of a pre-beater assembly at a PPC power plant.

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1. Introduction

The main objective of the present work is to estimate the full strain time histories at critical locations of a complex mechanical subassembly using operational vibration measurements from a limited number of sensors. To achieve this, appropriate numerical and experimental methods are applied to the identification, updating and optimization of critical model parameters. During this process, issues related to the construction of the FE model, the experimental modal analysis procedure and the development of effective computational model updating techniques are considered. Recently, diverse output-only vibration measurements were proposed for the estimation of fatigue damage accumulation in metallic structures [1,2]. Predictions of fatigue damage accumulation at a point of a structure can be estimated using available damage accumulation models that analyze the actual stress time histories developed during operation [3,4]. To proceed with fatigue predictions, one has to infer the stress response time histories characteristics based on the monitoring information contained in vibration measurements collected from a limited number of sensors attached to a structure. Such predictions are possible if one combines the information in the measurements with information obtained from a high fidelity FE model of the structure. Such a model may be optimized with respect to the data, using model updating methods [5, 6, 14-16]. These are often formulated as weighted least-squares estimation problems, in which the objective function usually involves weighted, modal-based metrics.

2. Description of the proposed computational framework

2.1. FE model updating

Let \( D = \{ \omega_i, \phi_i \in \mathbb{R}^N, r = 1, \ldots, m \} \) be the measured modal data from a structure at \( N_o \) degrees of freedom (DOFs), where \( \omega_i \) and \( \phi_i \) denote the natural frequencies and the mode shapes, respectively, for a number \( m \) of observed modes. Consider a parameterized class of linear structural models used to model the dynamic behaviour of the structure and let \( \theta \in \mathbb{R}^{N_0} \) be the set of free structural model parameters to be identified using the measured modal data. The objective in a modal-based structural identification methodology is to estimate the values of the parameter set \( \hat{\theta} \) so that the modal data \( \{ \omega_i(\theta), \phi_i(\theta) \in \mathbb{R}^N, r = 1, \ldots, m \} \) predicted by the linear class of models at the corresponding \( N_0 \) measured DOFs best matches the experimentally obtained modal data in \( D \). To this end, define

\[
\varepsilon_{\omega}(\theta) = \frac{\omega_{\hat{\omega}}(\theta) - \omega_{\omega}}{\omega_{\omega}} \quad \text{and} \quad \varepsilon_{\phi}(\theta) = \left\| \beta(\theta) \phi(\theta) - \hat{\phi} \right\| \quad \text{(1)}
\]

for the \( r \)-th modal frequency and mode shape components, respectively, where \( \left\| z \right\|^2 = z^T z \) is the usual Euclidean norm and \( \beta(\theta) = \hat{\phi}^T(\theta) \phi(\theta) \left\| \phi(\theta) \right\|^2 \) is a normalization constant [5, 6]. A measure of fit of the form

\[
J_1(\theta) = \sum_{r=1}^{m} \varepsilon_{\omega}^2(\theta) \quad \text{and} \quad J_2(\theta) = \sum_{r=1}^{m} \varepsilon_{\phi}^2(\theta) = \sum_{r=1}^{m} [1 - MAC_r^2(\theta)]
\]

is then assigned to each component, where \( MAC_r(\theta) = \hat{\phi}_{\omega}^T(\theta) \phi(\theta) \left\| \phi(\theta) \right\| \) is the Modal Assurance Criterion (MAC) between the \( r \)-th experimental and numerical mode shape [7-11]. Using the MAC definition, a global correlation coefficient may be defined for any measured frequency point \( \omega_k \).
where \( \{H_x(\omega_k)\} \) and \( \{H_a(\omega_k)\} \) are the experimental (measured) and the analytical (predicted) response vectors at matching excitation - response locations, respectively [12, 13]. Similar to the MAC, \( x_a(\omega_k) \) is bounded between zero and unity, while it cannot detect scaling errors and it is only sensitive to discrepancies in the overall deflection shape of the structure. This feature does not allow the identification of identical FRFs, especially when a measurement and its prediction are correlated. To account for this problem a supplementary correlation coefficient is defined as

\[
x_a(\omega_k) = \frac{2 \{H_x(\omega_k)\}^H \{H_a(\omega_k)\}}{\{H_x(\omega_k)\}^H \{H_x(\omega_k)\} + \{H_a(\omega_k)\}^H \{H_a(\omega_k)\}}
\]

again bounded between zero and unity. From Eqs. 3-4 and similar to modal residuals, two additional measures of fit are proposed, corresponding to the identified resonant frequencies of the system,

\[
J_3(\hat{\omega}) = \sum_{r=1}^{m} \left[1 - x_r(\hat{\omega}, \hat{\theta})^2\right] \quad \text{and} \quad J_4(\hat{\omega}) = \sum_{r=1}^{m} \left[1 - x_r(\hat{\omega}, \hat{\theta})^2\right]
\]

Thus, the objective function is formulated by the four individual objectives as

\[
J(\theta, \omega) = w_1 J_1(\theta) + w_2 J_2(\theta) + w_3 J_3(\theta) + w_4 J_4(\theta)
\]

using the weighting factors \( w_i \geq 0, \ i = 1,2,3,4 \), with \( w_1 + w_2 + w_3 + w_4 = 1 \), [14-16].

2.2. Estimation of Fatigue Damage Accumulation using Stress Reconstruction

According to the Palmgren-Miner rule [2,3], fatigue damage accumulation at a point in the structure is defined as

\[
D = \sum_{i=1}^{n} \frac{n_i}{N_i}
\]

where \( n_i \) denotes the number of cycles at a stress level \( \sigma(i) \) of the stress time history \( \sigma_k \), \( N_i \) stands for the number of cycles required for failure at a stress level \( \sigma(i) \), estimated using the rainflow counting method, and \( \kappa \) is the number of stress levels identified in a stress time history at the corresponding structural point. As the number of cycles to fail depends also on the mean stress, the fatigue accumulation model is herein revised to account for a non-zero mean stress according to the Goodman relationship

\[
\Delta \sigma_{ri} = \Delta \sigma_R \left(1 - \frac{\sigma}{\sigma_u}\right)
\]

where \( \Delta \sigma_{ri} \) stands for the modified stress cycle range, \( \Delta \sigma_R \) signifies the original stress cycle range, \( \sigma \) denotes the mean stress, and is calculated by the cycle counting algorithm; and \( \sigma_u \) is the ultimate tensile strength of the material. Once the stress range spectrum for a structural member is obtained and the relevant detail category is determined, S-N curves are used for estimating fatigue strength. In this regard, Miner summation is employed, and the fatigue
damages pertinent to the stress ranges are linearly summed. The parametric representation of S-N curves reads

\[ D = \sum_{i \geq \Delta N_1} \frac{n_i}{N_D} \left( \frac{\Delta \sigma_i}{\Delta \sigma_D} \right)^m + \sum_{i < \Delta N_1 \leq \Delta N_2} \frac{n_i}{N_D} \left( \frac{\Delta \sigma_i}{\Delta \sigma_D} \right)^{m+2} \]  

(9)

where \( \Delta \sigma_D \) denotes fatigue limit for constant amplitude stress ranges at \( N_D = 5 \times 10^6 \) cycles; \( \Delta \sigma_i \) stands for the cut-off limit; \( \Delta \sigma_i \) and \( \Delta \sigma_j \) are the \( i^{th} \) and \( j^{th} \) stress ranges; \( n_i \) and \( n_j \) are the number of cycles in each \( \Delta \sigma_i \) and \( \Delta \sigma_j \) block; \( \kappa_i \) and \( \kappa_2 \) represent the number of different stress range blocks above or below the constant amplitude fatigue limit \( \Delta \sigma_D \), respectively.

3. Experimental Application

The method described in the previous section is now applied to a linear steel frame (secondary base) that supports a lignite grinder assembly of the PPC power plant displayed in Fig.1a. The geometry of the steel frame (Fig.1b) is discretized mainly by solid (tetrahedral) elements [17,18], leading to a detailed FE model with approximately 2,500,000 DOFs. In order to determine the actual dynamics of the frame an experimental modal analysis is performed, by approximating free-free boundary conditions for the test and focusing on the 0-100 Hz frequency range. Figure 1c shows the locations of the acceleration sensors used for the tests. Based on the measured FRFs, twelve vibration modes are successfully estimated [9,11] in the aforementioned range, the natural frequencies and the damping ratios of which are listed in Tab.1. It follows that the estimated differences between the experimental (columns 2-3) and the numerical modes (columns 4-5) are not insignificant, necessitating thus a FE model updating process.

Fig. 1. (a) The lignite grinder assembly; (b) FE model of the secondary base; (c) Accelerometer locations; (d) FE model parametrization parts.
Table 1. Experimental modes and their numerical counterparts before (columns) and after the FE model updating process.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Identified Frequency (Hz)</th>
<th>Identified Damping (%)</th>
<th>Numerical (before updating) Frequency (Hz)</th>
<th>Error (%)</th>
<th>Numerical (after updating) Frequency (Hz)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.10</td>
<td>0.08</td>
<td>8.53</td>
<td>16.81</td>
<td>7.29</td>
<td>2.60</td>
</tr>
<tr>
<td>2</td>
<td>41.71</td>
<td>0.02</td>
<td>44.01</td>
<td>5.23</td>
<td>41.36</td>
<td>0.86</td>
</tr>
<tr>
<td>3</td>
<td>42.70</td>
<td>0.56</td>
<td>47.09</td>
<td>9.31</td>
<td>42.12</td>
<td>1.39</td>
</tr>
<tr>
<td>4</td>
<td>43.82</td>
<td>0.02</td>
<td>47.63</td>
<td>8.00</td>
<td>42.55</td>
<td>2.98</td>
</tr>
<tr>
<td>5</td>
<td>45.46</td>
<td>0.37</td>
<td>49.72</td>
<td>8.58</td>
<td>44.80</td>
<td>1.46</td>
</tr>
<tr>
<td>6</td>
<td>59.71</td>
<td>0.09</td>
<td>64.03</td>
<td>6.75</td>
<td>59.57</td>
<td>0.23</td>
</tr>
<tr>
<td>7</td>
<td>64.23</td>
<td>0.05</td>
<td>70.19</td>
<td>8.49</td>
<td>63.96</td>
<td>0.42</td>
</tr>
<tr>
<td>8</td>
<td>72.36</td>
<td>0.22</td>
<td>77.96</td>
<td>7.18</td>
<td>71.45</td>
<td>1.28</td>
</tr>
<tr>
<td>9</td>
<td>73.72</td>
<td>0.04</td>
<td>81.86</td>
<td>9.94</td>
<td>72.72</td>
<td>1.37</td>
</tr>
<tr>
<td>10</td>
<td>75.02</td>
<td>0.31</td>
<td>84.51</td>
<td>11.24</td>
<td>75.22</td>
<td>0.26</td>
</tr>
<tr>
<td>11</td>
<td>77.37</td>
<td>0.08</td>
<td>90.13</td>
<td>14.15</td>
<td>78.30</td>
<td>1.19</td>
</tr>
<tr>
<td>12</td>
<td>78.09</td>
<td>0.12</td>
<td>90.59</td>
<td>13.79</td>
<td>78.69</td>
<td>0.75</td>
</tr>
</tbody>
</table>

The latter is conducted by parametrizing the FE model as shown in Fig.1(d). All associated parts are modeled with solid elements, while the Young’s modulus and the material density are used as design variables, leading to a total number of twenty-eight (28) design parameters with the ranges of Tab.2. The results from the FE model updating process are also listed in Tab.1 (columns 6-7), from where it is that the maximum percentage error between the experimental and the updated frequencies drops below 3%.

Table 2. Design variables and optimization design limits.

<table>
<thead>
<tr>
<th>Density (kg/m³)</th>
<th>Young’s modulus (GPa)</th>
<th>Move limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>Low/Upper Bound</td>
<td>Initial</td>
</tr>
<tr>
<td>7850</td>
<td>6050/9660</td>
<td>210</td>
</tr>
</tbody>
</table>

Having established an updated FE model of the steel frame, the Miner’s rule is applied to the estimation of the fatigue damage accumulation. To this end and accordance to Eurocode 3 [19] for detail category 36, a static strength $\sigma_u = 440$ MPa is assigned, while the associated parameters of the design S-N curves are selected as $m = 3$, $\Delta \sigma_D = 26.5$ MPa and $\Delta \sigma_L = 14.5$ MPa. The maximum stresses are calculated by incorporating the measured acceleration time histories and the results are displayed in Fig. 2(a). Four indicative points (ST1-ST4) that exhibit maximum stresses are considered, the calculated fatigue life for which is 35.64, 65.85, 58.96 and 269.35 days, respectively. These results are in very close agreement with the real structure. Indeed, the induced cracks of the experimental frame (Fig.8) are located exactly at the same positions (e.g. ST2-ST3) predicted by the numerical analysis. And appeared only after 60 days from the time that the base was put into operation. These findings indicate that the applied method can be a powerful tool in predicting the fatigue damage accumulation.

4. Conclusions

A computational framework is proposed for estimating fatigue damage accumulation in a linear steel substructure of the entire body of a lignite grinder assembly at a PPC power plant. This is accomplished by combining fatigue damage accumulation laws with stress predictions based on output only vibration measurements collected from a limited number of sensors. Methods for estimating strains by integrating high fidelity finite element model and estimation techniques were summarized. From the results is clear that fatigue damage is estimated, a fact that reveals that the methodology for estimating damage due to fatigue on the entire body of a structure by combining linear damage accumulation laws, S-N fatigue curves, rainflow cycle-counting algorithms, and acceleration measurements...
at a limited number of locations is a valuable tool for designing optimal fatigue-based maintenance strategies in a wide variety of structures.

**Fig. 2.** (a) Locations of the frame where maximum stresses appear; (b) Crack in the real frame.

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**References**


