Working Paper

Value of travel time savings incorporating the value of access

Author(s):
Kato, Hironori; Axhausen, Kay W.

Publication Date:
2009

Permanent Link:
https://doi.org/10.3929/ethz-a-005729289

Rights / License:
In Copyright - Non-Commercial Use Permitted
Value of Travel Time Savings Incorporating the Value of Access

Hironori Kato\textsuperscript{a} and Kay W. Axhausen\textsuperscript{b}

\textsuperscript{a}Department of Civil Engineering, University of Tokyo
7-3-1 Hongo, Bunkyo-ku, Tokyo, Japan, kato@civil.t.u-tokyo.ac.jp

\textsuperscript{b}IVT, ETH
CH - 8093 Zürich, axhausen@ivt.baug.ethz.ch

Abstract:
Value of travel time savings (VTTSs) is widely used in the evaluation of transportation projects. There have been a number of researches regarding the formulation and application of VTTSs. Using empirical data on conserved travel time in Britain, Metz (2008) points out that an improvement in transportation infrastructure does not contribute to the time saving but impacts accessibility. He also shows that the value of access must be considered to evaluate the improvement in transportation infrastructure. This paper examines the meanings of the value of access and proposes the VTTSs incorporating the value of access. This value includes not only the time saved in a given single OD trip but also the changes in the travel patterns of various trips. First, the paper formulates the time allocation model and derives the VTTS from the model. Then, it presents the econometric analysis with the diary activity data collected in Tokyo. Finally, it discusses the implications of the proposed VTTSs on the transportation modeling and policy.
1. Introduction

Typically, the dominant benefit component of a transportation investment is the travel time saving. The value of travel time saving (VTTS) is widely used to evaluate the benefit of travel time saving. There have been many empirical and theoretical studies on the VTTS after the economic theory of time allocation was introduced in the 1960s. It was Becker (1965) who first suggested that a consumer gains utility only from the consumption of time and not from the goods consumed directly. After Becker’s work, several researchers such as Oort (1969), DeSerpa (1971), and Evans (1972) have developed the time allocation model in which the consumer’s utility is maximized with respect to time and goods consumption under the constraints of the available time and money budgets.

In recent times, Metz (2008) has pointed out that improvement in transportation infrastructure does not contribute to time saving but impacts the accessibility. It shows that the empirical data on average travel time in Britain offers no obvious support to the idea that travel time savings comprise the dominant element of the benefits from investment in the transportation system. Then, he claims that the value of access (VOA) must be considered to evaluate the improvement in transportation infrastructure. VOA depends on the extent to which an individual can reach to his/her destination in a given travel time. In other words, VOA is determined based on the size of the choice set of available destinations in a given travel time. This reflects the notion that given that travel
time is constrained, the further it is feasible to travel, the more the choice of destinations of any particular type.

The current paper proposes the VTTSs incorporating the VOA and estimates them using empirical data. I refer to the VTTS incorporating VOA as “VTSSA”. The VTSSA is defined as the traveler’s willingness to pay to improve his/her welfare from the do-nothing case to the do-something case. This value reflects not only the saving time of a given single OD trip but also the changes in travel patterns of various trips.

The paper is organized as follows. The next section examines the meaning of VOA and formulates the VTSSAs. Section 3 formulates the econometric analysis using the time allocation model by following Kato et al. (2008). Then, section 4 shows the empirical analysis using the data collected from the Tokyo metropolitan area. Finally, section 5 summarizes the results and discusses further research topics.
2. Value of Travel Time Savings Incorporating VOA

Metz (2008) stated that the “emphasis on travel time savings as a measure of the economic benefit of an improvement to the transportation infrastructure arose in a context in which trip origins and destinations were assumed unchanged” (p.326) and “(t)his meant that the values of activities at trip ends could be disregarded” (p.326). It is quite evident that many practical travel demand analyses and economic evaluations have assumed the fixed OD matrix to valuate travel time savings. However, this does not imply that the theory on the value of travel time savings cannot include VOA.

As Metz (2008) shows, the VTTS is often defined as the individual’s willingness to pay to save travel time. In addition, VTTS is usually regarded as the value of time saved on a single trip from a given place of origin to a given destination. However, as he points out, the impacts of improvement in transportation infrastructure may include choosing a new destination for the purpose of the same journey, increasing the frequency of journeys, and making entirely new journeys. These changes cannot be evaluated with the VTTS that is defined in a given OD trip. To evaluate the comprehensive impacts of the improvement in transportation infrastructure, we must consider the traveler’s general willingness to pay to improve his/her welfare from the do-nothing case to the do-something case. This includes not only time saved in a given single OD trip but also the changes in travel pattern including various types of related trips. Since the proposed VTTSA covers selecting a new destination, increasing
journey frequency, and making entirely new journeys in addition to changing mode/route and retiming journeys, it is regarded to partly include the VOA.

To explain the concept of VTTSA, I will formulate the VTTSA by using a simple utility-based behavioral model based on the time allocation model. First, suppose that an individual maximizes his/her utility by choosing the goods and allocating the time under the constraints of available time and monetary budgets. Assume that individual utility depends on the amount of consumption of a composite good, the frequency of trips, the leisure time, and the time taken for the journey. Let the utility function be \( U(X, T, N, t) \), where \( X \) denotes the composite good consumption excluding the consumption of transportation services; \( T \), the leisure time; \( N \), the vector of travel frequencies; and \( t \), the vector of time taken for the journey. The available monetary budget is formulated as \( PX + \sum_i c_i N_i = I \), while the available time budget is formulated as \( T + \sum_i t_i N_i = T^0 \), where \( P \) denotes the price of the composite good; \( c_i \), the travel cost of travel type \( i \); \( I \), the monetary budget; and \( T^0 \), the available time. The travel type \( i \) could imply the trip timing, purpose, mode, route, the pair of trip origin and destination, or their combination. For simplicity, let type \( i \) denote the destination at this point. By following DeSerpa (1971), the time consumption constraint regarding the travel time is also introduced as follows: \( t_i \geq \hat{t}_i \) for \( \forall i \).

Then, let the Lagrange function corresponding to the above problem be
\[
L = U(\xi, T, N, t) \geq \lambda \left( I - PX - \sum_{i} c_i N_i \right) + \mu \left( T^* - T - \sum_{i} t_i N_i \right) + \sum_{i} \kappa_i' \left( -i_i^* \right)
\]  \hspace{1cm} (1)

where $\lambda$, $\mu$, and $\kappa_i'$ denote the multipliers. We derive one of the first-order optimality conditions as

\[
\frac{\partial U}{\partial t_i} \Big|_{\nu^*} = \mu^* N_i^* - \kappa_i'^* \hspace{1cm} (2)
\]

Next, let $\nu(\xi, t_i, T^*)$ denote the indirect utility function regarding the abovementioned utility maximization. Then, the VTTS is defined as the individual’s marginal rate of substitution of the minimum travel time for money (DeSerpa, 1971). This implies the willingness-to-pay for recovering the utility into the original level when the individual faces a change in his/her minimum travel time. On the basis of the classical microeconomic theory, the marginal rate of substitution of the minimum travel time for money can be formulated as $-\frac{\nu/\partial t_i}{\nu/\partial \lambda}$.

We can derive the following equations from the abovementioned utility maximization using the Envelope Theorem (Varian, 1985):

\[
\frac{\partial \nu}{\partial t_i} = \frac{\partial U}{\partial t_i} + \kappa_i' \frac{\partial \xi}{\partial t_i} \hspace{1cm} (3)
\]

\[
\frac{\partial \nu}{\partial I} = \frac{\partial U}{\partial I} + \lambda \left( -PX - \sum_{i} c_i N_i \right) \hspace{1cm} (4)
\]

Thus, the VTTS is derived as equation (5) from equations (3) and (4):

\[
-\frac{\partial \nu/\partial t_i}{\partial \nu/\partial \lambda} = \frac{\kappa_i'^*}{\lambda^*}. \hspace{1cm} (5)
\]

Finally, we obtain equation (6) by substituting equation (2) into equation (5).

\[
VTTS_i = N_i^* \mu^* \lambda^* \left( 1 - \frac{\partial U/\partial t_i}{\partial U/\partial \lambda} \right). \hspace{1cm} (6)
\]

If the trip frequency is assumed to be one in equation (6), we can derive the
conventional VTTS as
\[ \text{VTTS}_i = \frac{\mu^*}{\lambda^*} - \frac{\partial U/\partial \lambda}{\lambda^*}. \] (7)

The difference between the VTTSA and VTTS is whether or not \( N_i^+ \) is multiplied with \( \mu^*/\lambda^* \). This can be interpreted as follows. First, the first term of equations (6) or (7) is regarded as the value of leisure time. Note that the leisure time in the abovementioned model is defined as the free time whose time consumption constraint is ineffective (DeSerpa, 1971). Lesser travel time implies more leisure time under a given time budget constraint. Then, suppose that the minimum travel time of a given OD pair is reduced. This saves the travel time in a single journey of the given OD pair. As the travel frequency of the OD pair becomes higher, more travel time is saved for all the journeys of the corresponding OD pair, and this implies that more time is allocated to leisure. As the leisure time increases in proportion to the travel frequency, the value of leisure time is also in proportion to \( N_i^+ \). VTTS follows the special case in which the travel frequency is fixed to be one in VTTSA. Second, the second term of equations (6) or (7), \( \left( \frac{\partial U/\partial \lambda}{\lambda^*} \right) \), denotes the value of time allocated to the travel type \( i \). This value generates from the journey itself. Since both VTTSA and VTTS represent the value for a single journey, the value of time allocated to the travel type \( i \) should be the same between VTTSA and VTTS.

VTTSA considers the changes in OD travel patterns. For example, reduction in travel time may increase the demand for traveling to a destination. The demand
function is described as \( N_i^* (t_i, T^*) \), and we expect \( \partial N_i^* (t_i, T^*) / \partial t_i < 0 \). Note that \( N_i^* (t_i, T^*) \) follows Roy’s Identity with respect to travel time and travel cost since it is the individual demand function. As \( N_i^* (t_i, T^*) \) is included in the VTTSA formula (equation (6)), the VOA or the extent to which a traveler can reach his destination is incorporated into VTTSA.

In general, the VTTSA can consider the impacts of travel time savings on not only OD travel patterns but also on travel purposes, travel modes, routes, travel scheduling, etc. For example, suppose a transportation network includes links and nodes. Assume that the reduction of travel time in one route in the transportation network affects the flow of other routes in the network. Since the cross-elasticity among routes is nonzero, \( \partial N_i^* (t_i, T^*) / \partial t_j \neq 0 \) is satisfied, where \( i \) and \( j \) denote the routes. VTTSA with the demand function \( N_i^* (t_i, T^*) \) can be used for the VTTSA formula.

Note that both VTTS and VTTSA are derived from the marginal rate of substitution of the minimum travel time for money. However, the VTTS shown by DeSerpa (1971) is defined as the marginal travel time saved from the total travel time during a given period, while the VTTSA is defined as the marginal travel time saved in a single travel time. VTTS can be derived as follows. First, let the utility function be defined as \( U (t, T, t_i) \), where \( t_i \) is a vector of the total travel time during a given period, and \( t_i \), the total travel of travel type \( i \). Moreover, let the minimum total travel time be \( \tilde{t}_i \). Then, the VTTS of travel type \( i \) is derived as
VTTS \_i = \mu^* \lambda^* - \left( \frac{\partial U}{\partial \mu} \right)_{\lambda^*} \lambda^* . This is the same as equation (7). DeSerpa (1971) names the first term on the right hand-side of the equation as “the value of time as a resource” and the second term as “the value of time as a commodity.”

3. Formulation of Empirical Models

3.1 Basic Structure

The basic assumption is that an individual allocates his or her time and expenditure for discretionary activities in order to maximize his/her utility under the constraints of the available time and money budgets. Then, it is assumed that the individual allocates his/her time to either in-home or after-work leisure on a working day, while the individual allocates his/her time to either to in-home or out-of-home leisure on a nonworking day. The formulated time allocation model has a nested structure. It consists of two sub-models: a one-day time allocation sub-model and a weekly time allocation sub-model, both of which are formulated as constrained utility maximization problems. The former sub-model is a Becker-type (1965) time allocation model that allocates time and expenditure based on a given day’s time and income. The weekly time allocation model is an Evans-type (1972) time allocation model that determines the frequency of engaging in leisure activities at specific places in a given week by allocating time and expenditure under the constraints of time and money budgets. The weekly time allocation model can be also regarded as a combination of a classical demand model and DeSerpa’s model,
because the utility is derived from both the frequencies of out-of-home or after-work activities at specific places and in-home leisure time. In a broad sense, we can regard the weekly time allocation model covering trip generation and destination choice simultaneously. As Kockelman (1998, 2001, 2004) shows, this type of time allocation model retains the properties of the neoclassical microeconomic consumer demand model such as Roy’s Identity even with respect to time. The expected time and cost at a specific place, which are used as input data for the weekly time allocation model, are simulated using the one-day time allocation model. The abovementioned models consider the heterogeneity of individual preferences by introducing sociodemographic variables and error components into the marginal utility in the utility functions. The parameter estimation is based on the Tobit model (Tobin, 1958) because the model includes inequality and equality constraints.

3.2 One-day time allocation model

Suppose an individual allocates fixed, positive amounts of time and expenditure to in-home leisure and to out-of-home leisure that he/she engages in at place $k$ on a given nonworking day. In the same manner, suppose the individual allocates fixed, positive amounts of time and expenditure to in-home leisure and to after-work leisure that he/she engages in at place $k$ on a given working day. Here, it is assumed that the individual engages in out-of-home or after-work leisure at just one place, $k$, on that day. We assume that out-of-home leisure is not undertaken on a working day.
Then, let the utility of the individual on the given day be

$$U_{\text{day}}^{\text{day}}(T_{\text{day}}, Z_{\text{day}}, T_{\text{home}}^{\text{day}}, Z_{\text{home}}^{\text{day}})$$

where $Z_{\text{day}}^{\text{day}}$ denotes the amount of expenditure of an individual $n$ allocated to out-of-home leisure or after-work leisure that he/she engages in at place $k$; $T_{\text{day}}^{\text{day}}$, the amount of time allocated to the out-of-home leisure or to the after-work leisure engaged in at place $k$; $Z_{\text{home}}^{\text{day}}$, the amount of expenditure allocated to in-home leisure; and $T_{\text{home}}^{\text{day}}$, the amount of time allocated to in-home leisure.

The individual’s time allocation as an optimization problem is formulated as

$$\text{Maximize} \ U_{\text{day}}^{\text{day}}(T_{\text{day}}, Z_{\text{day}}, T_{\text{home}}^{\text{day}}, Z_{\text{home}}^{\text{day}})$$

subject to

$$Z_{\text{day}}^{\text{day}} + Z_{\text{home}}^{\text{day}} = I_n^{\text{day}}, \ T_{\text{day}}^{\text{day}} + T_{\text{home}}^{\text{day}} = T_\text{on}^{\text{day}}$$

$$Z_{\text{day}}^{\text{day}} > 0, \ T_{\text{day}}^{\text{day}} > 0, \ Z_{\text{home}}^{\text{day}} > 0, \ T_{\text{home}}^{\text{day}} > 0$$

where $I_n^{\text{day}}$ represents the total amount of budget available for discretionary activities on that day, and $T_\text{on}^{\text{day}}$, the total amount of time available for discretionary activities on that day. The allocated time and expenditure are assumed to be positive, because the one-day model expresses the time allocation of an individual under the condition that the individual engages in out-of-home leisure on that day. In our formulation, we assume the work and maintenance activity time as given and fixed.

Assume the utility of any activity to be the sum of two parts stemming from the consumption of time and expenditure corresponding to the activity. Then,
following Kato and Matsumoto (2007), let the total daily utility be the sum of in-home leisure and out-of-home leisure on a nonworking day, and let the total daily utility be the sum of in-home leisure and after-home leisure on a working day. The total daily utility can be expressed as

\[
U_{\text{day}}(Z_{k,n}^{\text{day}}, T_{k,n}^{\text{day}}, Z_{\text{homen}}^{\text{day}}, T_{\text{homen}}^{\text{day}}) = U_{Zk}^{\text{day}}(Z_{k,n}^{\text{day}}) + U_{Tk}^{\text{day}}(T_{k,n}^{\text{day}}) + U_{Zh}^{\text{day}}(Z_{\text{homen}}^{\text{day}}) + U_{Th}^{\text{day}}(T_{\text{homen}}^{\text{day}}). \tag{10}
\]

Then, let

\[
U_{Zk}^{\text{day}}(Z_{k,n}^{\text{day}}) = \alpha_{Zk,n}^{\text{day}} \ln(Z_{k,n}^{\text{day}}) \tag{11a}
\]

\[
U_{Tk}^{\text{day}}(T_{k,n}^{\text{day}}) = \alpha_{Tk,n}^{\text{day}} \ln(T_{k,n}^{\text{day}}) \tag{11b}
\]

\[
U_{Zh}^{\text{day}}(Z_{\text{homen}}^{\text{day}}) = \alpha_{Zh,n}^{\text{day}} \ln(Z_{\text{homen}}^{\text{day}}) \tag{11c}
\]

\[
U_{Th}^{\text{day}}(T_{\text{homen}}^{\text{day}}) = \alpha_{Th,n}^{\text{day}} \ln(T_{\text{homen}}^{\text{day}}) \tag{11d}
\]

be the functional form of each utility term. For \(\alpha_{Zk,n}^{\text{day}}, \alpha_{Tk,n}^{\text{day}}, \alpha_{Zh,n}^{\text{day}}, \alpha_{Th,n}^{\text{day}}\) in equations (11a), (11b), (11c), and (11d), we specify the following functions to allow for heterogeneity among locations and individuals as

\[
\alpha_{Zk,n}^{\text{day}} = \exp(A_{Z}^{\text{day}} X_{Z,k}^{\text{day}} + B_{Z}^{\text{day}} Z_{n}^{\text{day}} + \varepsilon_{Z,n}^{\text{day}}) \tag{12a}
\]

\[
\alpha_{Tk,n}^{\text{day}} = \exp(A_{T}^{\text{day}} X_{T,k}^{\text{day}} + B_{T}^{\text{day}} T_{n}^{\text{day}} + \varepsilon_{T,n}^{\text{day}}) \tag{12b}
\]

\[
\alpha_{Zh,n}^{\text{day}} = \exp(B_{Zh}^{\text{day}} Y_{Zh,n}^{\text{day}}) \tag{12c}
\]

\[
\alpha_{Th,n}^{\text{day}} = \exp(B_{Th}^{\text{day}} Y_{Th,n}^{\text{day}}) \tag{12d}
\]

where \(A_{Z}^{\text{day}}, A_{T}^{\text{day}}, B_{Z}^{\text{day}}, B_{T}^{\text{day}}, B_{Zh}^{\text{day}}, B_{Th}^{\text{day}}\) represent the vectors of unknown parameters; \(X_{Z,k}^{\text{day}}\) and \(X_{T,k}^{\text{day}}\), the vectors of exogenous variables corresponding to
place $k$; $y_{Z,n}^{day}$, $y_{T,n}^{day}$, $y_{Zh,n}^{day}$, and $y_{Th,n}^{day}$, the vectors of exogenous variables corresponding to individual attributes; and $\epsilon_{Z,n}^{day}$, $\epsilon_{T,n}^{day}$, the normal random components varying independently with a mean of zero and with the standard deviations $\sigma_{Z}^{day}$, $\sigma_{T}^{day}$, respectively. First, we use the exponential function because we can expect the marginal utility with respect to time and expenditure allocated to activities to be positive. Second, the error components are introduced in equations (12a) and (12b) because the heterogeneity in the individual preference stems from not only the individual attributes but also the other unknown factors. Although it may be ideal that the error components include the correlation between time and expenditure, we assume that they are independently distributed. This is one of the limitations of our analysis, and the assumption may be relaxed in future research. Third, we do not introduce error components into equations (12c) and (12d). Even if we introduce the error components into theses equations, it is possible to estimate the parameters through a likelihood maximization process shown later. However, we avoid doing so because this makes it difficult to estimate the expected allocated time and expenditure, which will be shown in equations (15a) and (15b) in the weekly time allocation model.

By applying Kuhn-Tucker’s theorem to the optimization problem of equation (9), the first-order conditions of optimality are derived as

$$\frac{\partial U_{day}}{\partial y_{k,n}^{day}} = \frac{\partial U_{day}}{\partial y_{home}^{day}}$$

$$\frac{\partial Z_{k,n}^{day}}{\partial y_{k,n}^{day}} = \frac{\partial Z_{home}^{day}}{\partial y_{home}^{day}}$$

(13a)

$$\frac{\partial U_{day}}{\partial T_{k,n}^{day}} = \frac{\partial U_{day}}{\partial T_{home}^{day}}$$

$$\frac{\partial Z_{k,n}^{day}}{\partial T_{k,n}^{day}} = \frac{\partial Z_{home}^{day}}{\partial T_{home}^{day}}$$

(13b)
\[ I_{n}^{\text{day}} - Z_{k,n}^{\text{day}} + T_{\text{home}n}^{\text{day}} = 0, \quad T_{\text{home}n}^{\text{day}} - T_{k,n}^{\text{day}} + T_{\text{home}n}^{\text{day}} = 0 \]  
(13c, d)

\[ Z_{k,n}^{\text{day}} > 0, \quad T_{k,n}^{\text{day}} > 0, \quad Z_{\text{home}n}^{\text{day}} > 0, \quad T_{\text{home}n}^{\text{day}} > 0 \]  
(13e, f, g, h)

where the asterisks in the superscripts of variables indicate the corresponding variables at their optimum values. Then, the two error components are derived from the first-order optimality conditions and the assumptions on the utility function as follows:

\[ \varepsilon_{Z,n}^{\text{day}} = \ln( Z_{k,n}^{\text{day}} ) - \ln( Z_{\text{home}n}^{\text{day}} ) + B_{k,n}^{\text{day}} Y_{\text{Z,n}}^{\text{day}} - A_{Z}^{\text{day}} X_{Z,k}^{\text{day}} - B_{Z}^{\text{day}} Y_{Z,n}^{\text{day}} \]  
(14a)

\[ \varepsilon_{T,n}^{\text{day}} = \ln( T_{k,n}^{\text{day}} ) - \ln( T_{\text{home}n}^{\text{day}} ) + B_{k,n}^{\text{day}} Y_{\text{T,n}}^{\text{day}} - A_{T}^{\text{day}} X_{T,k}^{\text{day}} - B_{T}^{\text{day}} Y_{T,n}^{\text{day}} \]  
(14b)

Since we cannot estimate \( B_{Z}^{\text{day}} \) and \( B_{\text{Z,Th}}^{\text{day}} \) in equation (14a) or \( B_{T}^{\text{day}} \) and \( B_{\text{T,Th}}^{\text{day}} \) in equation (14b) separately, we define \( C_{Z}^{\text{day}} W_{Z,n}^{\text{day}} = B_{Z}^{\text{day}} Y_{\text{Z,n}}^{\text{day}} - B_{Z}^{\text{day}} Y_{Z,n}^{\text{day}} \) and \( C_{T}^{\text{day}} W_{T,n}^{\text{day}} = B_{T}^{\text{day}} Y_{\text{T,n}}^{\text{day}} - B_{T}^{\text{day}} Y_{T,n}^{\text{day}} \). Finally, the following likelihood functions are obtained because of the assumptions of normally distributed error terms:

\[ L_{z,n}^{\text{day}} = \frac{1}{\sigma_{Z}^{\text{day}} Z_{k,n}^{\text{day}}} \left[ \frac{\ln( Z_{k,n}^{\text{day}} ) - \ln( Z_{\text{home}n}^{\text{day}} ) + C_{Z}^{\text{day}} W_{Z,n}^{\text{day}} - A_{Z}^{\text{day}} X_{Z,k}^{\text{day}}}{\sigma_{Z}^{\text{day}}} \right] \]  
(15a)

\[ L_{T,n}^{\text{day}} = \frac{1}{\sigma_{T}^{\text{day}} T_{k,n}^{\text{day}}} \left[ \frac{\ln( T_{k,n}^{\text{day}} ) - \ln( T_{\text{home}n}^{\text{day}} ) + C_{T}^{\text{day}} W_{T,n}^{\text{day}} - A_{T}^{\text{day}} X_{T,k}^{\text{day}}}{\sigma_{T}^{\text{day}}} \right] \]  
(15b)

where \( \phi(C) \) represents the probability density function of the standard normal distribution. The abovementioned likelihood functions include \( A_{Z}^{\text{day}} , A_{T}^{\text{day}} , C_{Z}^{\text{day}} , C_{T}^{\text{day}} \) and \( \sigma_{Z}^{\text{day}} , \sigma_{T}^{\text{day}} \) as the unknown parameters. They can be estimated by the maximization of the forms of the likelihoods mentioned above as follows:
\[ LL_{day}^{\text{day}} = \sum_{n} L_{Z,n}^{\text{day}} + \ln L_{T,n}^{\text{day}} \]  

(16)

3.3 Weekly time allocation model

Suppose an individual allocates his/her time and expenditure to in-home and out-of-home leisure by deciding on the frequency of visiting place \( k \) for out-of-home leisure on nonworking days. In the same manner, suppose the individual allocates his/her time and expenditure to in-home and after-work leisure by deciding on the frequency of visiting place \( k \) for after-work leisure on working days. We assume that the unit time and expenditure required for each leisure activity are constant and that the individual can allocate time and expenditure through the decision on the frequency of engagement in each leisure activity.

Let the total utility of the individual in a given week be

\[ U^{\text{week}}(n^W, n^H, t_n, Z_{\text{home}}^{\text{week}}, T_{\text{home}}^{\text{week}}) \]  

(17)

where \( n^W, n^H \) represent the vectors of individual \( n \)'s frequencies of visiting places on working and nonworking days, respectively; \( t_n \), a vector of travel time; and \( Z_{\text{home}}^{\text{week}} \) and \( T_{\text{home}}^{\text{week}} \), the time and expenditure allocated to in-home leisure, respectively. If we assume that an individual always chooses the shortest travel time path from his origin to a destination, we can fix the travel time as the minimum travel time from the origin to the destination. Since the minimum travel time is given and fixed, the utility maximization problem with time and budget constraints for a week can be expressed as follows:
Maximize \( \sum_{n} U_{\text{week}} \mathbf{w}_{n}, \mathbf{h}_{n}, \mathbf{z}_{\text{home}_n}, \mathbf{t}_{\text{home}_n} \) subject to
\[
\sum_{k} I_{k,n}^{W} (c_{k,n}^{W} + c_{k,n}^{H}) + \sum_{k} I_{k,n}^{H} (c_{k,n}^{W} + c_{k,n}^{H}) + Z_{\text{home}_n} = I_{n}^{W} \quad (18b)
\]
\[
\sum_{k} I_{k,n}^{W} (c_{k,n}^{W} + c_{k,n}^{H}) + \sum_{k} I_{k,n}^{H} (c_{k,n}^{W} + c_{k,n}^{H}) + T_{\text{home}_n} = T_{o,n} \quad (18c)
\]
\[
N_{k,n}^{W} \geq 0, N_{k,n}^{H} \geq 0 \quad (\forall k), \quad Z_{\text{home}_n} > 0, \quad T_{\text{home}_n} > 0 \quad (18d, e, f, g)
\]
where \( N_{k,n}^{W}, N_{k,n}^{H} \) denote frequencies of visiting place \( k \) on working and nonworking days, respectively; \( Z_{k.n}^{W}, Z_{k.n}^{H} \), the expected unit expenditures of the leisure engaged in at place \( k \) on a working and nonworking day, respectively; \( T_{k,n}^{W}, T_{k,n}^{H} \), the expected unit time allocated to the leisure engaged in at place \( k \) on a working and nonworking day, respectively; \( c_{k,n}^{W}, c_{k,n}^{H} \), the unit expenditure associated with the activities on a working and nonworking day, respectively; \( I_{k,n}^{W}, I_{k,n}^{H} \), the unit time consumed by activities on a working and nonworking day, respectively; and \( I_{n}^{W}, I_{n}^{H} \), the amount of income and time available for the week.

In the same manner as in the one-day time allocation model shown earlier, let the total weekly utility be the sum of the parts stemming from the in-home leisure, the out-of-home leisure on nonworking days, and the after-work leisure on working days.

\[
U_{\text{week}} \mathbf{w}_{n}, \mathbf{h}_{n}, \mathbf{z}_{\text{home}_n}, \mathbf{t}_{\text{home}_n} = U_{\text{week}} (Z_{\text{home}_n}) + U_{\text{day}} (T_{\text{home}_n}) + \sum_{k} U_{k,n}^{W} (N_{k,n}^{W}) + \sum_{k} U_{k,n}^{H} (N_{k,n}^{H}) \quad (19)
\]
Let the in-home leisure utility be the sum of the consumption of time and money, which are expressed as follows:

\[ U_{zh}^{\text{week}}(Z_{\text{home}}) = \exp \left( \mathbf{y}_{zh,n}^{\text{week}} \mathbf{d}_{zh}^{\text{week}}(Z_{\text{home}}) \right) \]  \hspace{1cm} (20a)

\[ U_{th}^{\text{week}}(T_{\text{home}}) = \exp \left( \mathbf{y}_{th,n}^{\text{week}} \mathbf{d}_{th}^{\text{week}}(T_{\text{home}}) \right) \]  \hspace{1cm} (20b)

where \( \mathbf{d}_{zh}^{\text{week}}, \mathbf{d}_{th}^{\text{week}} \) denote the vectors of unknown parameters, and \( \mathbf{y}_{zh,n}^{\text{week}}, \mathbf{y}_{th,n}^{\text{week}} \), the vectors of individual attributes.

Let the utilities for the out-of-home and after-work leisure be

\[ U_{k,n}^{W}(N_{k,n}^{W}) = \alpha_{k,n}^{W} \ln(N_{k,n}^{W} + 1) \]  \hspace{1cm} (21a)

\[ U_{k,n}^{H}(N_{k,n}^{H}) = \alpha_{k,n}^{H} \ln(N_{k,n}^{H} + 1) \]  \hspace{1cm} (21b)

respectively, where \( \alpha_{k,n}^{W}, \alpha_{k,n}^{H} \) denote the location factors that are assumed to possess the following functional forms:

\[ \alpha_{k,n}^{W} = \exp \left( \mathbf{w}_{k}^{W} \mathbf{x}_{k}^{W} + \mathbf{f}_{k}^{W} \mathbf{y}_{k}^{W} + \beta^{W}(c_{k,n} - c_{k,n}) + \beta^{W}(t_{k,n} - t_{k,n}) + \mathbf{e}_{k,n}^{W} \right) \]  \hspace{1cm} (22a)

\[ \alpha_{k,n}^{H} = \exp \left( \mathbf{w}_{k}^{H} \mathbf{x}_{k}^{H} + \mathbf{f}_{k}^{H} \mathbf{y}_{k}^{H} + \beta^{H}c_{k,n} + \beta^{H}t_{k,n} + \mathbf{e}_{k,n}^{H} \right) \]  \hspace{1cm} (22b)

where \( \mathbf{w}_{k}^{W}, \mathbf{w}_{k}^{H}, \mathbf{f}_{k}^{W}, \mathbf{f}_{k}^{H} \) represent the vectors of unknown parameters; \( \mathbf{x}_{k}^{W}, \mathbf{x}_{k}^{H} \), the vectors of the exogenous variables corresponding to the place \( k \); \( \mathbf{y}_{k}^{W}, \mathbf{y}_{k}^{H} \), the vectors of individual attributes; and \( \mathbf{e}_{k,n}^{W}, \mathbf{e}_{k,n}^{H} \), the error components, both of which follow the independent normal distribution with zero mean and the standard deviations \( \sigma^{W}, \sigma^{H} \), respectively.

The expected unit time and expenditure consumed are estimated using the one-day time allocation model. The expected unit time and expenditure are derived as follows:
where \( f(\varepsilon_{Z,n}^{day}) \) and \( g(\varepsilon_{T,n}^{day}) \) denote the probability density functions of the error terms \( \varepsilon_{Z,n}^{day} \) and \( \varepsilon_{T,n}^{day} \) respectively.

Employing the assumed functional forms of the utility function given here in conjunction with the first-order optimality conditions, the likelihood functions are derived as follows:

\[
L^{W}_{k,n} = \begin{cases} 
\Phi \left[ \frac{\ln(N_{k,n}^{W} + 1) + \ln S_{k,n}^{W}}{\sigma_{n}^{W}} \right] & \text{(if } N_{k,n}^{W} > 0) \\
\Phi \left[ \frac{\ln(N_{k,n}^{W} + 1) + \ln S_{k,n}^{W}}{\sigma_{n}^{W}} \right] & \text{(if } N_{k,n}^{W} = 0) 
\end{cases} 
(24a)
\]

\[
L^{H}_{k,n} = \begin{cases} 
\Phi \left[ \frac{\ln(N_{k,n}^{H} + 1) + \ln S_{k,n}^{H}}{\sigma_{n}^{H}} \right] & \text{(if } N_{k,n}^{H} > 0) \\
\Phi \left[ \frac{\ln(N_{k,n}^{H} + 1) + \ln S_{k,n}^{H}}{\sigma_{n}^{H}} \right] & \text{(if } N_{k,n}^{H} = 0) 
\end{cases} 
(24b)
\]

where

\[
S_{k}^{W} = \left( c_{R,n}^{W} - c_{R,n}^{W} \right) \exp \left( \sum_{Z \in \text{week~} Z_{n}} \theta_{k,n}^{Z} \right) - \exp \left( \sum_{T \in \text{week~} T_{n}} \theta_{k,n}^{Z} \right) \\
+ \left( c_{R,n}^{W} - c_{R,n}^{W} \right) \exp \left( \sum_{Z \in \text{week~} Z_{n}} \theta_{k,n}^{Z} \right) - \exp \left( \sum_{T \in \text{week~} T_{n}} \theta_{k,n}^{Z} \right) 
(25a)
\]
\[ S_k^H = \sum_{k,n} \left[ \exp \left( \sum_{\text{weeks}}^{\text{hours}} Y_{2h,n}^{\text{week}} - E^H X_k^H - F^H Y_n^H - \beta_{c, k,n}^H - \beta_{x, k,n}^H \right) \right] \]

\[ + \sum_{k,n} \left[ \exp \left( \sum_{\text{hours}}^{\text{weeks}} Y_{2h,n}^{\text{hour}} - E^H X_k^H - F^H Y_n^H - \beta_{c, k,n}^H - \beta_{x, k,n}^H \right) \right] \]

(25b)

and \( \phi(\cdot) \) denotes a probability density function of the normal distribution, and \( \Phi(\cdot) \), a cumulative probability function corresponding to \( \phi(\cdot) \). In equations (25a) and (25b), we need to prepare the data for the location wherein the individuals are not engaged in any activity. For this purpose, we define 29 zones in Tokyo. The travel time for all zones is estimated for each individual. Finally, the unknown parameters are estimated by the maximization of the following likelihood function:

\[ LL_{\text{week}} = \sum_n \sum_k \ln L_{k,n}^W + \sum_n \sum_k \ln L_{k,n}^H \]

4. Empirical Analysis

4.1 Data used

The data sources used for the empirical analyses are the 2001 Tokyo metropolitan area activity-travel survey data, which was designed and conducted by East Japan Marketing & Communication, Inc. for the Tokyo metropolitan area in 2001. The survey collected information on all activity episodes undertaken by 2,900 respondents over a week (see East Japan Marketing & Communication, Inc., (2001) for details on the survey and the sampling). The information collected on the activity episodes includes the type
of activities and the duration for the activities, travel time/cost, and individual and household sociodemographics.

We generated the analysis sample using the following steps. First, only data pertaining to individuals 18 years or older are used. Second, only workers were selected. This is because the workers may have stronger incentives to participate in leisure activities during weekends than nonworkers. Third, we selected the data pertaining to rail commuters. There are two reasons for selecting rail commuters. First, because of the intentions of the original client, the survey originally focused on the behavior of railway users. Second, the modal share of using railways for commuting is very high in Tokyo, for example, as of 2003, over 70% of commuters working in central Tokyo used railways for commuting. We can therefore expect the survey to represent the population of the sample as a whole.
Then, we calculate the individual constraints. First, the time constraint was calculated by subtracting the necessary time from a day or week under the assumption of nine hours per day of obligatory duties. The assumption on the duration of obligatory duties is simple because no in-home time usage data is available for those surveys. We assume the budget to be one-fourth of the monthly disposable income reported by the individuals, which excludes regular expenditure such as rent, insurance, commuting, and education costs. Since the data on the unit expenditure of purchasing goods in leisure activities is not available in the original survey data, the study team in Tokyo conducted an additional survey on consumer purchase behavior in November 2002 and collected the relevant data. The final sample for analysis includes the information on usage of time and expenditure of 389 individuals.
Table 1 shows the respondents’ mean sociodemographics, allocations of time and expenditure toward leisure activities and travel, and the time and budget constraints.

4.2 Estimation results of the models

The estimation results for the one-day and the weekly models are presented in Tables 2 and 3, respectively. Note that we eliminate the travel cost in the estimation process, although we have considered both travel time and cost in the utility function in the original formulation. This is because we found it highly correlated with the travel time.

For the one-day model, two models are specified independently for working and nonworking days, because individual behavior is expected to be different in these days. In Table 2, note that the density of retailers (number of retailers per square km) is derived from the official commercial statistics in Tokyo. For the estimation of the weekly model, as discussed earlier, it is necessary to use the expected unit time and expenditure. Although they can be obtained by the integrals shown in equations (23a) and (23b), they cannot be obtained analytically. The expected unit time and expenditure were simulated for all sample individuals by applying the Simpson method to the integral of each individual.
### Table 2 Estimation results of one-day models

<table>
<thead>
<tr>
<th>Explanatory vectors</th>
<th>Explanatory variables</th>
<th>Work day</th>
<th>Non-work day</th>
<th>Work day</th>
<th>Non-work day</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Parameter</td>
<td>T-statistic</td>
<td>Parameter</td>
<td>T-statistic</td>
<td>Parameter</td>
</tr>
<tr>
<td>$A_Z$</td>
<td>Dummy variable of car-ownership (1 if owning a car and 0 otherwise)</td>
<td>1.07</td>
<td>1.13</td>
<td>1.21</td>
<td>2.22</td>
</tr>
<tr>
<td>$A_T$</td>
<td>Number of retailers per km$^2$</td>
<td>0.000663</td>
<td>2.11</td>
<td>-0.226</td>
<td>-1.67</td>
</tr>
<tr>
<td></td>
<td>Dummy variable of car-ownership (1 if owning a car and 0 otherwise)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_Z$</td>
<td>Constant</td>
<td>-2.56</td>
<td>-10.1</td>
<td>-1.44</td>
<td>-2.92</td>
</tr>
<tr>
<td></td>
<td>Dummy variable of female (1 if female and 0 otherwise)</td>
<td></td>
<td></td>
<td>-3.07</td>
<td>-4.93</td>
</tr>
<tr>
<td></td>
<td>Dummy variable of marriage status (1 if married and 0 otherwise)</td>
<td></td>
<td></td>
<td>-1.37</td>
<td>-2.18</td>
</tr>
<tr>
<td></td>
<td>Dummy variable of age in 40s or 50s (1 if in his/her 40s or 50s and 0 otherwise)</td>
<td></td>
<td></td>
<td>1.59</td>
<td>2.55</td>
</tr>
<tr>
<td>$C_T$</td>
<td>Constant</td>
<td>0.885</td>
<td>4.74</td>
<td>1.20</td>
<td>4.74</td>
</tr>
<tr>
<td></td>
<td>Dummy variable of married woman (1 if married woman and 0 otherwise)</td>
<td>1.68</td>
<td>4.42</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Dummy variable of age in 30s (1 if in his/her 30s and 0 otherwise)</td>
<td></td>
<td></td>
<td>-0.477</td>
<td>-1.94</td>
</tr>
<tr>
<td></td>
<td>Dummy variable of age in 40s (1 if in his/her 40s and 0 otherwise)</td>
<td></td>
<td></td>
<td>-0.390</td>
<td>-3.17</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>Variance with respect to time</td>
<td>1.81</td>
<td>24.9</td>
<td>1.18</td>
<td>25.5</td>
</tr>
<tr>
<td></td>
<td>Variance with respect to expenditure</td>
<td>4.30</td>
<td>24.8</td>
<td>4.47</td>
<td>25.5</td>
</tr>
<tr>
<td></td>
<td>Initial log-likelihood</td>
<td>-8957.1</td>
<td>-10377.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Final log-likelihood</td>
<td>-5336.8</td>
<td>-5704.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Number of observations</td>
<td>290</td>
<td>287</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 3 Estimation results of weekly model

<table>
<thead>
<tr>
<th>Explanatory vectors</th>
<th>Explanatory variables</th>
<th>Parameter</th>
<th>T-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^W$</td>
<td>Dummy variable of age in 30s or 40s (1 if in his/her 30s or 40s and 0 otherwise)</td>
<td>-0.390</td>
<td>-3.17</td>
</tr>
<tr>
<td>$F^W$</td>
<td>Number of retailers per km$^2$</td>
<td>0.00158</td>
<td>7.93</td>
</tr>
<tr>
<td>$\beta^W_1$</td>
<td>Travel time by urban rail</td>
<td>-0.0129</td>
<td>-2.46</td>
</tr>
<tr>
<td>$E^H$</td>
<td>Dummy variable of age 40s or 50s (1 if in his/her 40s or 50s and 0 otherwise)</td>
<td>-0.398</td>
<td>-3.49</td>
</tr>
<tr>
<td>$F^H$</td>
<td>Number of retailers per km2</td>
<td>0.00156</td>
<td>6.74</td>
</tr>
<tr>
<td></td>
<td>Dummy variable of car-ownership (1 if owning a car and 0 otherwise)</td>
<td>2.50</td>
<td>10.5</td>
</tr>
<tr>
<td>$\beta^H_1$</td>
<td>Travel time by urban rail</td>
<td>-0.00024</td>
<td>-0.140</td>
</tr>
<tr>
<td></td>
<td>Travel time by automobile</td>
<td>0.00536</td>
<td>3.39</td>
</tr>
<tr>
<td>$D_T$</td>
<td>Constant</td>
<td>3.94</td>
<td>20.6</td>
</tr>
<tr>
<td>$D_Z$</td>
<td>Constant</td>
<td>0.762</td>
<td>2.41</td>
</tr>
<tr>
<td></td>
<td>Dummy variable of female (1 if female and 0 otherwise)</td>
<td>-1.15</td>
<td>-2.79</td>
</tr>
<tr>
<td></td>
<td>Dummy variable of marriage status (1 if married and 0 otherwise)</td>
<td>0.309</td>
<td>1.21</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>Variance with respect to work day</td>
<td>1.20</td>
<td>17.9</td>
</tr>
<tr>
<td></td>
<td>Variance with respect to non-work day</td>
<td>1.02</td>
<td>17.5</td>
</tr>
<tr>
<td></td>
<td>Initial log-likelihood</td>
<td>-2056.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Final log-likelihood</td>
<td>-1139.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Number of observations</td>
<td>389</td>
<td></td>
</tr>
</tbody>
</table>
4.3 Estimation results of VTTSAs

We calculate the VTTSAs for the observed activities of the individuals using the estimated models. We assume that on a given day, the frequency of visiting the observed place by each individual is one. The average VTTSAs of after-work-time and out-of-home leisure are shown in Table 4. The average VTTSAs of after-work leisure are higher than the average VTTSAs of out-of-home leisure. This appears to be a natural result because the opportunity cost on a working day is higher than that on a nonworking day. Second, the average

<table>
<thead>
<tr>
<th></th>
<th>After-work leisure on a work day</th>
<th>Out-of-home leisure on a non-work day</th>
<th>Average wage rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Means (Yen/hour)</td>
<td>18895.8</td>
<td>5541.5</td>
<td>3655.3</td>
</tr>
<tr>
<td>Ratios to average wage rate</td>
<td>5.17</td>
<td>1.52</td>
<td>1.00</td>
</tr>
<tr>
<td>Medians (Yen/hour)</td>
<td>7627.9</td>
<td>4154.5</td>
<td>3540.3</td>
</tr>
<tr>
<td>Ratios to average wage rate</td>
<td>2.09</td>
<td>1.14</td>
<td>0.97</td>
</tr>
<tr>
<td>Standard deviations (Yen/hour)</td>
<td>31896.4</td>
<td>8642.1</td>
<td>1894.50</td>
</tr>
</tbody>
</table>

Figure 1 Distributions of estimated GVTTSs and value of time as a resource
VTTSAs of both leisure types are higher than the average wage rate. This is because the average expenditure of the unit leisure time during one visit is quite high in Tokyo. Third, the variation of VTTSAs of after-work leisure on a working day is larger than the variation of VTTSAs of out-of-home leisure on a nonworking day. This may reflect that there is more variation in the available time during the after-work time on a working day than that on a nonworking day. This implies that on a working day, some work overtime until late evening, while others finish their work by closing time. For example, individuals with little available time after work hours due to the overtime work have higher willingness to pay for saving travel time for after-work leisure.

In order to compare the VTTSAs of the two types of leisure, we calculate the individual ratios. The results are shown in Figure 1. Note that the value of time as a resource implies \( N_1^* \xi / \hat{\lambda} \) in equation (6). Theoretically speaking, the VTR should be smaller than the VTTS, and the differences between the two values imply the consideration of the value of time as a commodity (VTC). Note that VTC implies \( \left( \frac{U}{\partial I_{1}} \right) \hat{\lambda} \) in equation (6). We can observe that the shares of the VTC of a trip for after-work leisure are larger than the VTC of a trip for out-of-home leisure. This may be reasonable because the traffic in the evening on a working day is more congested than that on a nonworking day.
5. Conclusions

This paper proposes the VTTSs incorporating the value of access and formulates VTTSA with the traditional time allocation framework. In general, the VTTSA can consider the impacts of travel time savings on not only OD travel patterns but also on travel purposes, travel modes, routes, travel scheduling, etc. Then, the VTTSAs are estimated empirically using the weekly activity diary data in the Tokyo metropolitan area.

Finally, I add the following three notes regarding the VTTSAs. First, the conventional VTTS is equal to VTTSA when the total travel time is equal to the single travel time. For example, when a specific type of journey is undertaken only once during a given period, VTTS is equal to VTTSA because the travel frequency is one. This may occur in the case of a typical discrete modal choice model. This is because the modal choice model analyzes the conditional choice assuming that a traveler engages in a single trip. Thus, the VTTS estimated using the conditional indirect utility function is the conditional VTTSA in which the travel frequency is given to be one. Second, the stated preference (SP) data may bias the estimation of the VTTSAs. The stated preference survey often requests the interviewees to answer their preference in the several given situations. For example, they are asked to choose one of two or more options including different travel time, travel cost, and the quality of the travel service. In most cases, the SP questions implicitly assume or
explicitly suppose that the travel frequency is fixed. However, in reality, the travel frequency may vary in different situations depending on different travel timings, travel cost, and so on. This implies that the values of saved time estimated using the SP data should be the VTTS with the given travel frequency. Third, Metz (2008) shows that VOA includes the changes in the land-use patterns. Since the time allocation model formulated in this paper does not include land consumption, the VTTSA derived from the model does not include the impacts from the changes in land-use pattern. However, land consumption may be included by using the time allocation model incorporating the location choice model framework. Kato et al. (2007) formulate the time allocation model incorporating the residential location choice and derive the VTTSA from the model. This covers the VOA including the changes in the land-use patterns.

References


