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Author(s):
Torre, Emiliano; Marelli, Stefano; Embrechts, Paul; Sudret, Bruno

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Vine copula modeling of high-dimensional inputs in uncertainty quantification problems

E. Torre¹,² [torre@ibx.baug.ethz.ch], S. Marelli¹, P. Embrechts²,³, B. Sudret¹,²

ETH Zürich: ¹Chair of Risk, Safety and Uncertainty Quantification, ²Risk Center, ³RiskLab

Problem statement & context

A computational model is defined as a map:

\[ \mathcal{M} : \mathbb{R}^m \to \mathbb{R} : x \mapsto y \]

- \( x \) is uncertain, due to aleatory + epistemic uncertainty
- \( y \) is modelled by a random variable \( Y = \mathcal{M}(X) \)
- \( \mathcal{M} \) is considered as a black-box

Goal of Uncertainty Quantification (UQ): Estimate statistics of \( Y \), e.g. \( E[Y] \), \( \text{Var}[Y] \), PDF \( P_Y \), P(\( Y < \text{threshold} \)) \.

UQ methods approximate statistics of \( Y \) avoiding expensive Monte-Carlo runs of the computational model. They usually rely upon:
- The knowledge of \( F_X \)
- Often, a transform \( T : X \to X' \) with independent components

COPULAS & SKLAR’S THEOREM

Joint CDFs are often difficult to model: they must capture marginal and joint properties. Copula theory allows one to model these separately.

Theorem (Sklar, 1959). Let \( X = (X_1, \ldots, X_M) \) be a random vector with continuous joint CDF \( F_X \) and marginal CDFs \( F_1, \ldots, F_M \). Then, under very general conditions, a joint CDF \( C : [0,1]^M \to [0,1] \) with uniform margins \( (M \text{-copula}) \) exists, such that:

\[ F_X(x_1, \ldots, x_M) = C(F_1(x_1), \ldots, F_M(x_M)) \]

C is unique if \( F_i \) are continuous: \( C(u) = F_i^{-1}(F_1^{-1}(u_1), \ldots, F_M^{-1}(u_M)) \).

Besides, for any \( M \)-copula \( C \) and any \( M \)-univariate CDFs \( F_1, \ldots, F_M \), the function \( C(F_1, \ldots, F_M) \) is a joint CDF with marginals \( F_1, \ldots, F_M \).

VINE COPULAS

Building suitable copula models in dimension \( M > 2 \) may be difficult.

Theorem (Vine copula construction; Bedford and Cooke, 2002). Under general conditions, an \( M \)-copula \( C \) can be factorized into \( \binom{M}{2} \) conditional pair copulas.

Vine copulas:
- Provide highly flexible models of joint dependencies
- Can be easily fit to data
- Provide the Rosenblatt transform \( T : X \to X' \) (and \( T^{-1} \))

Novel workflow proposed:
- Model \( F_X \) by its margins \( F_i \) and vine \( C \)
- Compute the Rosenblatt transform \( T \)
- Apply UQ methods to the composite model \( \mathcal{M}' = \mathcal{M} \circ T : X \to X' \to Y \)