Vine copula modeling of high-dimensional inputs in uncertainty quantification problems

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**First Order Reliability Method (FORM), generalized**

**Goal:** Find (small) failure probability $P_f$, i.e. probability of failure domain $D_f$.

**Steps:**
1. **New:** Model $F_X$ by marginals + vine copula: $F_X = C(F_1(X_1), \ldots, F_M(X_M))$.
2. **New:** Map $X \rightarrow X'$ is $T(X)$ with independent components and marginals $F_i$.
   Bring to classical FORM settings.
3. Transform marginally $X_i \rightarrow U_i$ standard normal: $U_i = \Phi^{-1}(F_i(X_i))$.
4. Find numerically the design point $U^* \in D_f$ closest to 0.
   By successive runs (few, usually $O(100)$) of the computational model $M$.
5. Approximate $D_f$ by hyperplane tangent to it in $U^*$.
   Easy to extend to second order approximation: SORM.
   Correction factor by importance sampling.

**Reliability analysis of a truss structure with generalized FORM**

**Truss structure (e.g. bridge):**
- Subject to uncertain loads (e.g. traffic, snow, ...) $P_1, \ldots, P_6$.
- Works well for close-to-linear responses.
- Vine of $X$ known (solid purple) or fitted to $(\hat{X}^j)_{j=1}^M$.

<table>
<thead>
<tr>
<th>Copula assumed</th>
<th>Method (# runs)</th>
<th>$P_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>True Vine</td>
<td>MC (10^4)</td>
<td>$4.8 \times 10^{-4}$</td>
</tr>
<tr>
<td>Gaussian</td>
<td>MC (10^4)</td>
<td>$3.1 \times 10^{-5}$</td>
</tr>
<tr>
<td>Indep</td>
<td>MC (10^4)</td>
<td>$1.3 \times 10^{-5}$</td>
</tr>
<tr>
<td>True Vine</td>
<td>FORM (108)</td>
<td>$4.9 \times 10^{-4}$</td>
</tr>
<tr>
<td>Fitted Vine</td>
<td>FORM (148)</td>
<td>$2.5 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

**Conclusions:**
- Tail dependence matters (the independent and jointly Gaussian models fail).
- The vine representation captures such complex dependence, and ...
- It combines well with UQ techniques such as FORM.
- The vine is properly inferred from observations, yielding a close estimate $P_f$.

**Outlook:**
We are now ready to:
- Apply the novel analysis framework to real world engineering problems.
- Integrate the code in a Vine Copulas module for UQLab, the Matlab-based software for Uncertainty Quantification (www.uqlab.com)

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**Conclusions:**
We developed a framework suitable for the use of vine copulas in UQ:
- Enables the use of advanced UQ methods when input $X$ exhibits complex dependencies.
- Works well in combination with inference.
- Demonstrated application on reliability analysis of a truss structure.
- Not shown: used vine representations in combination with metamodeling techniques (polynomial chaos expansions).