Analog Source Coding and Robust Frames

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Publication Date:
2018-02

Permanent Link:
https://doi.org/10.3929/ethz-b-000245060

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***Abstract*—Analog coding is a low-complexity method to combat erasures, based on linear redundancy in the signal space domain. Previous work examined "bandlimited discrete Fourier transform (DFT)" codes for Gaussian channels with erasures or impulses. We extend this concept to source coding with "erasure side-information" at the encoder. Furthermore, we show that the performance of bandlimited DFT can be significantly improved using irregular spectrum, and more generally, using equiangular tight frames.**

Key words: Distortion side information, erasures, equiangular tight frames, Welch bound, random matrix theory.

I. Distortion side information at the encoder

Consider encoding a source X under a side-information dependent distortion measure $d(x, \hat{x}, s)$, where the side information S is statistically independent of the source X and is available at the encoder. It is shown in [5] that if an optimal conditional distribution $p(\hat{x}|x, s)$ satisfies $I(S; \hat{X}) = 0$, then the rate-distortion performance is the same as if S was available also at the decoder. Specifically, this condition holds for the case of an "erasure distortion measure" $d(x, \hat{x}, s) = s \cdot d(x, \hat{x})$, for $s \in \{0, 1\}$, where only source samples for which $S = 1$ are "important".

II. Analog coding of a source with erasures

Analog coding decouples the tasks of protecting against erasures and noise [7]. For erasure correction, it creates an analog redundancy by means of band-limited discrete Fourier transform (DFT) interpolation, or more generally, by an overcomplete expansion based on a frame. In [2] we examine the analog coding paradigm for the dual setup of a source with erasure side-information (SI) at the encoder [5]. The excess rate of analog coding above the rate-distortion function (RDF) is associated with the energy of the inverse of submatrices of the frame, where each submatrix corresponds to a possible erasure pattern. We show that by selecting the DFT frequencies from a *difference set*, or more generally, by using equiangular tight frames (ETF), we minimize the excess rate over all possible frames (although do not achieve the RDF); see Section III below.

III. Random subsets of deterministic frames

Suppose we draw a random subset of k rows from a frame with n rows (vectors) and m columns (dimensions), where k and m are proportional to n. Consider the distribution of singular values of the k-subset matrix. For a variety of important ETFs and tight non-ETFs, we observe in [3] that, for large n, the singular values can be precisely described by a known probability distribution: Wachter's MANOVA (multivariate ANOVA) spectral distribution, a phenomenon that was previously known only for two types of random frames [1]. In terms of convergence to this limit, the k-subset matrix from all of these frames is shown to be empirically indistinguishable from the classical MANOVA (Jacobi) random matrix ensemble. Thus, empirically, the MANOVA ensemble offers a universal description of the spectra of randomly selected k subframes, even those taken from deterministic frames.

IV. Welch Bounds with Erasures

The Welch Bound [6] is a lower bound on the root mean square cross correlation between n unit-norm vectors $f_1, \ldots, f_n$ in the m dimensional space (R^m or C^m), for $n > m$. Letting $F = [f_1| \ldots |f_n]$ denote the m-by-n matrix (frame) composed of the n vectors, the Welch bound can be viewed as a lower bound on the second moment of F, namely on the trace of the squared Gram matrix $(F'F)^2$. In [4] we extend the Welch Bound to a random selection of a subset from F, as well as to higher order moments of F. The extended lower bound holds with equality if and only if F is an ETF. Thus, it provides an analytical support for the results in [2], and sheds light on the superiority of ETFs for a variety of applications, such as spread spectrum communications, compressed sensing and analog coding [2].

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