COMPACT HIGH-FLUX SOLAR DISH SYSTEMS FOR CONCENTRATED PHOTOVOLTAICS

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Abstract

This work investigates ways to overcome the challenges associated with achieving a cost-effective and sustainable utilization of the solar resource with the ultimate goal of securing future energy supply. The pursuit of concentrated photovoltaics (CPV) represents a promising route towards attaining this target. High solar concentration benefits the efficiency of the photovoltaic conversion of electromagnetic radiation into electric power, while also creating the potential for an overall cost reduction. To exploit this potential, however, numerous aspects need to be addressed, such as finding ways to achieve high concentration with simple and practical designs, reducing the cost of the solar concentrator and overcoming the penalties caused by the non-uniform nature of concentrated radiation. The thesis focuses on these and several other key aspects and ultimately culminates in the final design proposal of a novel point-focus solar high-concentration photovoltaic-thermal (HCPVT) concentrator, performance predictions and recommendations for future research in the field.

As a starting point for finding practical high-concentration solar concentrators for CPV applications, a rigorous geometrical analysis of two-stage systems based on parabolic primary mirrors and flat receiver geometries is carried out in line-focus (2D). Specifically, asymmetric concentrators are compared to their symmetric counterparts and exemplary designs are examined in terms of key optical performance metrics. Notably, it is shown that asymmetric designs can achieve significantly higher overall concentrations and are always more compact than symmetric systems designed for the same concentration ratio. Using this analysis as a basis, novel asymmetric designs are developed, including two-wing and nested configurations, which surpass the optical performance of two-mirror aplanats and are comparable with the best reported 2D simultaneous multiple surface designs for both hollow and dielectric-filled secondary optics. These designs therefore have a high potential for trough applications with
concentration ratios below 100×, where less complex, extruded optics and a simpler system design are favored.

To follow the “third-generation photovoltaics approach” and utilize the latest high-efficiency multi-junction solar cells, however, higher concentration ratios are required, which can only be reached with point-focus (3D) concentrators. A crucial issue with such optics is their usually higher cost as a consequence of the manufacturing complexities associated with their non-extrudable shapes. To address this limitation, a solar dish concept based on elliptical vacuum-membrane facets, which have the promise to lower the mirror production cost, was proposed. To assess the technology’s potential, a prototype dish was optically characterized on-sun using two different types of silvered aluminum membranes. Characterization was carried out by a new absolute irradiance measurement system, the Lambertian Flat Plate Calorimeter (LFPC), which mitigates many of the limitations of the existing characterization methods, and of which the background, design and measurement methodology is presented in this work. During the on-sun-tests, a total focal plane radiative power of 4.1 kW, a peak concentration ratio of 3140 suns and an average concentration ratio of 897 suns on a 60 × 60 mm area were achieved, thus proving the proof-of-concept of the proposed vacuum-membrane design.

The theoretical findings for two-stage 2D concentrators and the experience gained with the 3D prototype were ultimately used for the design of an improved, full-scale, high-concentration, photovoltaic-thermal solar dish system. The advantages of asymmetric concentrators, namely high compactness and concentration ratio, are transferred to point-focus geometry via the introduction of the multi-focus solar dish concept, where the parabolic dish is divided into several modules, each comprising an individual central focal point, arranged symmetrically around the central axis. The concept was implemented on a dish having an inlet aperture of 38.7 m², consisting of six modules and achieving a geometric concentration ratio of 1733× at each of its six receivers. The majority of the dish components are modular and cast from ultra-high-performance concrete to leverage on-site mass production and reduce investment cost while offering improved rigidity in comparison to a conventional steel frame. The dish was used to continue the investigation of novel technologies for the generation
of 3D reflective surfaces, commenced with the earlier prototype. Ultimately, thin, back-surface glass mirrors, brought into shape by fixation to a cast concrete shell bearing the desired profile were retained for their advantage of allowing a high flexibility in the optical design and a high optical efficiency. In on-sun tests, the mirrors achieved an average concentration ratio of 1353 suns on a 61 × 61 mm area, resulting in 5.04 kW focal plane radiative power per module. Every receiver comprises 36 triple-junction CPV cells, connected in a unique hybrid parallel-serial interconnection scheme that mitigates mismatch losses with non-uniform irradiance distributions. Additionally, cogeneration of electricity and low grade heat for secondary thermal processes is enabled by using high-performance microchannel cooler chips, which allow the desired high coolant temperatures without drastically impacting the photovoltaic efficiency of the cells. In its present form, the system achieves a solar-to-electricity conversion efficiency of 23.2% in conditions optimized for electricity production. In cogeneration mode, the efficiency drops to 21.3%, while heat at 90 °C can be extracted. By optimizing several design parameters, the solar-to-electricity conversion efficiency with the presented concept is projected to increase to 28.5% in PV operation and 26.6% in cogeneration mode, with an extractable power of 12.1 kW_{el} and 11.3 kW_{el}/21.5 kW_{th}, respectively, matching the performance of state of the art CPV commercial systems.

Finally, the issues with non-uniform concentrated solar radiation of a PV receiver are addressed with a design methodology for nonimaging, single-reflection mirrors, consisting in adapting the primary mirror profile from the parabolic shape such as to achieve a uniform irradiation of the receiver. By allowing polygonal inlet and outlet apertures, it enables a multitude of applications within the domain of concentrated photovoltaics. Notably, exemplary single-mirror concentrators of square and hexagonal perimeter are presented, which achieve very high irradiance uniformity on a square receiver at concentrations ranging from 100 to 1000 suns. These optical designs can be assembled in compound concentrators with maximized active area fraction by leveraging tessellation. More advanced multi-mirror concentrators, where each mirror individually illuminates the whole area of the receiver, allow for an improved performance while permitting greater flexibility for the concentrator
shape and robustness against partial shading of the inlet aperture. The method thus represents a promising potential route for a future HCPVT system design.

The concepts and designs introduced in this thesis contribute to the continuous development of solar collectors and CPV systems towards lower cost and higher efficiency, while extending the range of potential applications.
Zusammenfassung


Als Ausgangspunkt für die Suche nach praktischen hochkonzentrierenden Solarkonzentratoren für CPV-Anwendungen wird eine rigorose geometrische Analyse von zweistufigen Systemen auf Basis parabolischer Primärspiegel und flacher Austrittsgeometrien im Linienfokus (2D) durchgeführt. Insbesondere werden asymmetrische Konzentratoren mit ihren symmetrischen Pendants verglichen und beispielhafte Designs in Bezug auf die entscheidenden optischen Leistungsmetriken untersucht. Hervorzuheben ist, dass asymmetrische Konfigurationen deutlich höhere Gesamtkonzentrationen erzielen können und immer kompakter sind als symmetrische Systeme, welche auf das selbe Konzentrationsverhältnis ausgelegt sind. Mit Hilfe dieser Analyse werden
Zusammenfassung

neuartige asymmetrische Designs entwickelt, darunter zweiflügelige und verschachtelte Konfigurationen, die die optische Leistung von Zweispiegelaplanaten übertreffen und mit den besten 2D Simultaneous Multiple Surface Designs, sowohl für hohle als auch dielektrisch gefüllte Sekundär-optiken, vergleichbar sind. Die aufgezeigten Konstruktionen haben daher ein hohes Potenzial für solare Troganwendungen mit Konzentrationsverhältnissen unter 100×, bei denen weniger komplexe, extrudierte Optiken und ein einfacheres Systemdesign bevorzugt sind.


Die gewonnenen Erkenntnisse aus der theoretischen geometrischen Analyse von 2D-Konzentratoren und der Praxiserfahrung mit dem 3D-Prototyp wurden letztlich für die Entwicklung eines verbesserten Hochkonzentrations-photovoltaikwärmesystems eingesetzt. Die Vorteile asymmetrischer Konzentratoren, nämlich die hohe Kompaktheit bei hohem Konzentrationsverhältnis,
dargestellten Konzept auf 28,5% im PV-Betrieb bzw. 26,6% im KWK-Betrieb, bei einer extrahierbaren Leistung von 12,1 kW\textsubscript{el} bzw. 11,3 kW\textsubscript{el}/21,5 kW\textsubscript{th}, gesteigert werden kann, was der Leistung einiger der besten aktuellen CPV-Designs entspricht.


Die in dieser Arbeit eingeführten Konzepte und Entwürfe tragen zur kontinuierlichen Entwicklung von Solarkollektoren und CPV-Systemen hin zu niedrigeren Kosten und höherer Effizienz bei und erweitern das Spektrum der Einsatzmöglichkeiten.
Résumé

Ce travail se destine à étudier des moyens de surmonter les défis liés à une utilisation de moindre coût et durable de la ressource solaire, avec le but ultime de sécuriser l’approvisionnement énergétique futur. La recherche dans la photovoltaïque concentrée (CPV) représente une voie prometteuse dans cette quête. La haute concentration solaire contribue à l’efficacité de la conversion photovoltaïque de la radiation électromagnétique en énergie électrique, tout en créant le potentiel pour une réduction globale des coûts. Cependant, afin d’exploiter ce potentiel, de nombreux aspects doivent être abordés, tels la recherche de voies d’aboutir à une haute concentration par des solutions simples et pratiques, réduire le coût du concentrateur solaire et surmonter les pénalités occasionnées par la nature non-uniforme de la radiation concentrée. La présente thèse cible, entre autres, ces divers aspects et culmine finalement dans la propose du design d’un nouveau concentrateur solaire à point focal pour applications photovoltaïques/thermiques à haute concentration (HCPVT), une prédiction de sa performance et des recommandations pour les champs de recherche futurs.

Comme point de départ pour la recherche de concentrateurs solaires à haute concentration pour applications CPV, une analyse géométrique rigoureuse de systèmes à deux étages, basés sur des miroirs paraboliques primaires et des géométries à receveurs plats, est exécutée en symétrie 2D. En particulier, des concentrateurs asymétriques sont comparés à leurs contreparties symétriques et des exemples de design sont examinés sur base des indicateurs significatifs de performance optique. Notamment, il est montré que les designs asymétriques savent aboutir à des concentrations totales supérieures et sont toujours plus compacts que des systèmes symétriques dimensionnés pour le même rapport de concentration. En utilisant cette analyse comme fondement, de nouveaux designs asymétriques sont développés, incluant des configurations à deux ailes ainsi que des configurations imbriquées, lesquels dépassent la performance optique
d’aplanats à deux miroirs, et qui sont comparables aux meilleurs designs *simultaneous multiple surface* 2D rapportés, et ce pour les optiques secondaires creuses aussi bien que diélectriques. En conséquence, ces designs ont un haut potentiel pour applications à collecteurs cylindro-paraboliques avec rapports de concentration inférieurs à 100×, pour lesquels des optiques extrudées moins complexes et des systèmes plus simples sont favorisés.

Pour suivre l’« approche de la photovoltaïque de la troisième génération » c’est-à-dire utiliser les cellules solaires multi-jonction à haute efficacité les plus récentes, des rapports de concentration plus élevés sont nécessaires, lesquels ne sont réalisables seulement à l’aide de concentrateurs à point focal (3D). Un aspect crucial de ce genre d’optiques est leur coût normalement plus élevé, dû à la complexité de fabrication associée à leur profil non-extrudable. Dans le but de surmonter ce problème, le concept d’un concentrateur solaire se composant de facettes elliptiques avec membranes sous vide, promettant de minimiser les coûts de production des miroirs, a été proposé. Afin d’évaluer le potentiel de cette technologie, un prototype fut optiquement caractérisé en deux configurations différentes quant au type de membrane d’aluminium argenté utilisé. La caractérisation fut effectuée par un nouveau système de mesure absolue d’irradiation, qui surmonte la plupart des limitations des méthodes de caractérisation existantes, et dont les fondements théoriques, le design et la méthodologie de mesure sont présentés dans ce travail. Lors de tests *on-sun*, une énergie radiative totale de 4,1 kW dans le plan focal, une concentration de pointe de 3140 soleils et une concentration moyenne de 897 soleils sur une surface de 60 × 60 mm ont pu être établis, prouvant ainsi la validité du concept du concentrateur à membranes sous vide.

Les recherches théoriques sur les collecteurs à deux étages en 2D et l’expérience obtenue par le prototype 3D furent finalement utilisés pour le design d’un système amélioré de concentrateur solaire à point focal pleine-échelle pour applications photovoltaïques/thermiques à haute concentration. Les avantages de concentrateurs asymétriques, à savoir la haute compacité pour des rapports de concentration élevés, sont transférés à la géométrie à point focal par l’intermédiaire de l’introduction du concept de concentrateur solaire multifocal, où le collecteur est divisé en plusieurs modules, dont un chacun possède un point
focal individuel et qui sont disposés symétriquement autour de l’axe central. Le concept en question fut réalisé sur un collecteur ayant une ouverture d’entrée de 38,7 m², constitué de six modules et réalisant un rapport de concentration géométrique de 1733× à chacun des six receveurs. La plupart des composants du collecteur sont modulaires et coulés dans du béton à haute performance, et ce dans le but d’établir une production de masse sur le site et de réduire ainsi les frais d’investissement, tout en assurant une rigidité améliorée comparé à la structure traditionnelle en fer. Le collecteur fut utilisé pour continuer la recherche, commencée avec le premier prototype, de nouvelles technologies pour la génération des surfaces réfléchissantes 3D. En fin de compte, des miroirs en verre plat, mis en forme par fixation sur une coque en béton au profil souhaité, furent retenus pour leurs avantages de permettre une haute flexibilité dans le design optique ainsi qu’une efficacité optique élevée. Lors de tests on-sun, les miroirs ont réalisé une concentration moyenne de 1353 soleils sur une surface de 61 × 61 mm, ce qui équivaut à une énergie radiative en plan focal de 5,04 kW par module. Chaque receveur comprend 36 cellules CPV triple jonction, reliés dans un unique circuit mixte parallèle-sériel qui minimise les pertes reliées à la répartition non-uniforme de la radiation concentrée. En outre, la cogénération d’électricité et de chaleur à basse température pour processus thermiques secondaires est rendue possible par l’utilisation de chips réfrigérants à micro-canaux, qui eux permettent les hautes températures du liquide de réfrigération nécessaires, sans impact considérable sur l’efficacité photovoltaïque des cellules. Dans sa forme actuelle, le système réalise un rendement de conversion énergétique soleil-électricité de 23,2% dans les conditions optimisées pour la production d’électricité. Dans le mode cogénération, le rendement est réduit à 21,3%, tandis que de la chaleur peut en être extraite à une température de 90 °C. En optimisant certains paramètres du design, le rendement soleil-électricité, dans des conditions identiques, est envisagé d’augmenter à 28,5% en mode PV et à 26,6% en mode cogénération, avec des énergies extractibles de 12,1 kWel et 11,3 kWel/21.5 kWth respectivement, ceci en adéquation avec les meilleurs designs CPV commerciaux.

Enfin, la problématique de la non-uniformité de la radiation solaire concentrée sur un capteur PV est abordée d’un point de vue différent, à savoir
par une méthodique de design de miroirs à réflexion unique non-imageurs, consistant dans l’adaptation du profil de miroir primaire à partir de la forme parabolique de façon à aboutir à une irradiation uniforme du capteur. Par le fait de permettre des ouvertures d’entrée et de sortie polygonales, une multitude d’applications dans le domaine de la photovoltaïque concentrée est rendue possible. Notamment sont présentés des concentrateurs à miroir unique exemplaires de périmètres rectangulaire et hexagonal, atteignant une très grande uniformité de radiation sur un receveur rectangulaire avec des rapports de concentration entre 100 et 1000 soleils. Ces designs optiques peuvent être assemblés en concentrateurs composites avec fraction de surface active maximale en exploitant la tessellation. Des concentrateurs plus avancés à miroirs multiples, où chaque miroir illumine individuellement toute la surface du receveur, occasionnent une performance améliorée tout en permettant une flexibilité accrue en ce qui concerne la forme du contour du collecteur et une résistance envers les conséquences d’un ombrage partiel de l’ouverture d’entrée. Cette méthode représente ainsi une voie potentielle prometteuse pour un design futur de système HCPVT.

Les concepts et designs introduits dans cette thèse contribuent au développement continu des collecteurs solaires et des systèmes CPV vers un coût réduit et une efficacité plus élevée, tout en étendant la gamme des applications potentielles.
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## Nomenclature

### Latin characters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
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<tr>
<td>$a$</td>
<td>calibration coefficient</td>
<td>-</td>
</tr>
<tr>
<td>$a_2$</td>
<td>scaled apothem of image (receiver)</td>
<td>m</td>
</tr>
<tr>
<td>$a_i$</td>
<td>inlet aperture width</td>
<td>m</td>
</tr>
<tr>
<td>$a_o$</td>
<td>outlet aperture width</td>
<td>m</td>
</tr>
<tr>
<td>$a_{proj}$</td>
<td>total projected width of the concentrator</td>
<td>m</td>
</tr>
<tr>
<td>$a_P$</td>
<td>polygon apothem</td>
<td>m</td>
</tr>
<tr>
<td>$A$</td>
<td>area</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$A_{act,rec}$</td>
<td>active receiver area</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$A_{cell}$</td>
<td>active cell area</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$A_i$</td>
<td>square area of width $w_i$ in the focal plane, used for the calculation of the intercept factor</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$A_{in}$</td>
<td>inlet aperture of a solar concentrator</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$A_{out}$</td>
<td>outlet aperture of a solar concentrator</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$A_{rec}$</td>
<td>receiver area</td>
<td>m$^2$</td>
</tr>
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<td>$A_{tot}$</td>
<td>total irradiated area in the focal plane</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$A'$</td>
<td>point</td>
<td>-</td>
</tr>
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<tr>
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<td>W</td>
</tr>
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<tr>
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<tr>
<td>$B''$</td>
<td>point</td>
<td>-</td>
</tr>
<tr>
<td>$c$</td>
<td>irradiance coefficient of the short-circuit current</td>
<td>A/W</td>
</tr>
<tr>
<td>$c_p$</td>
<td>mass-specific heat capacity of the cooling water</td>
<td>kJ/(kg·K)</td>
</tr>
<tr>
<td>$C$</td>
<td>solar irradiance (flux) concentration ratio</td>
<td>suns</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>$C_g$</td>
<td>geometric concentration ratio</td>
<td></td>
</tr>
<tr>
<td>$C_{g,\text{design}}$</td>
<td>geometric design concentration of the primary mirror</td>
<td></td>
</tr>
<tr>
<td>$C_{g,\text{max,2D}}$</td>
<td>thermodynamic limit of concentration in line-symmetry (2D)</td>
<td></td>
</tr>
<tr>
<td>$C_{g,\text{max,3D}}$</td>
<td>thermodynamic limit of concentration in point-symmetry (3D)</td>
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</tr>
<tr>
<td>$C'$</td>
<td>point</td>
<td></td>
</tr>
<tr>
<td>$\text{CAP}_{2D}$</td>
<td>concentration-acceptance product</td>
<td></td>
</tr>
<tr>
<td>$d_{\text{dish}}$</td>
<td>dish diameter</td>
<td></td>
</tr>
<tr>
<td>$d_m$</td>
<td>projected diameter of mirror surface</td>
<td></td>
</tr>
<tr>
<td>$D$</td>
<td>density of the cooling water</td>
<td></td>
</tr>
<tr>
<td>$D_{\text{disk with } A = 1}$</td>
<td>disk with $A = 1$</td>
<td></td>
</tr>
<tr>
<td>$\text{DNI}$</td>
<td>direct normal irradiance</td>
<td></td>
</tr>
<tr>
<td>$e$</td>
<td>edge connecting two nodes in a grid</td>
<td></td>
</tr>
<tr>
<td>$E$</td>
<td>irradiance</td>
<td></td>
</tr>
<tr>
<td>$\langle E \rangle$</td>
<td>mean irradiance on the image area</td>
<td></td>
</tr>
<tr>
<td>$\langle E_{\text{av}} \rangle$</td>
<td>mean PV array irradiance</td>
<td></td>
</tr>
<tr>
<td>$E_{\text{av,cell}}$</td>
<td>cell-averaged irradiance</td>
<td></td>
</tr>
<tr>
<td>$\langle E_{\text{av,cell}} \rangle$</td>
<td>mean cell-averaged irradiance</td>
<td></td>
</tr>
<tr>
<td>$E_{\text{av,cell,max}}$</td>
<td>maximum cell-averaged irradiance</td>
<td></td>
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<tr>
<td>$E_{\text{av,LFPC-rec}}$</td>
<td>average irradiance over the LFPC-receiver</td>
<td></td>
</tr>
<tr>
<td>$E_{\text{av,rec}}$</td>
<td>average irradiance over the PV receiver</td>
<td></td>
</tr>
<tr>
<td>$E_i$</td>
<td>average irradiance at the inlet aperture of a solar concentrator</td>
<td></td>
</tr>
<tr>
<td>$E_{\text{max}}$</td>
<td>peak irradiance</td>
<td></td>
</tr>
<tr>
<td>$E_o$</td>
<td>average irradiance at the outlet aperture of a solar concentrator</td>
<td></td>
</tr>
<tr>
<td>$f$</td>
<td>focal length</td>
<td></td>
</tr>
<tr>
<td>$f_{\text{obj}}$</td>
<td>objective function</td>
<td></td>
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<tr>
<td>$F$</td>
<td>irradiance scaling factor</td>
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<tr>
<td>$F_{\text{corr}}$</td>
<td>corrected irradiance scaling factor in the region outside of the ROI</td>
<td></td>
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<tr>
<td>$FF$</td>
<td>fill factor</td>
<td></td>
</tr>
<tr>
<td>$F$</td>
<td>focal point</td>
<td></td>
</tr>
<tr>
<td>$GV$</td>
<td>gray value</td>
<td></td>
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<tr>
<td>$h$</td>
<td>height of a rectangle enclosing the two-stage concentrator and its receiver</td>
<td></td>
</tr>
<tr>
<td>$h_{\text{conv}}$</td>
<td>convective heat transfer coefficient</td>
<td></td>
</tr>
<tr>
<td>$H_{1,2}$</td>
<td>étendue at secondary inlet</td>
<td></td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>---------------------------------------------------------------------------</td>
<td>------------</td>
</tr>
<tr>
<td>$H_{0,2}$</td>
<td>étendue at secondary outlet</td>
<td>m²·sr</td>
</tr>
<tr>
<td>$i$</td>
<td>index</td>
<td>-</td>
</tr>
<tr>
<td>$i_{Al2O3}$</td>
<td>photodetector signal with the sample</td>
<td>V</td>
</tr>
<tr>
<td>$i_{std}$</td>
<td>photodetector signal with the reflectance standard</td>
<td>V</td>
</tr>
<tr>
<td>$I$</td>
<td>current</td>
<td>A</td>
</tr>
<tr>
<td>$I_\lambda$</td>
<td>spectral irradiance</td>
<td>W/(m²·nm)</td>
</tr>
<tr>
<td>$j$</td>
<td>index</td>
<td>-</td>
</tr>
<tr>
<td>$J$</td>
<td>current density</td>
<td>A/cm²</td>
</tr>
<tr>
<td>$J_0$</td>
<td>diode saturation current density</td>
<td>A/cm²</td>
</tr>
<tr>
<td>$J_{ph}$</td>
<td>photocurrent density</td>
<td>A/cm²</td>
</tr>
<tr>
<td>$k$</td>
<td>number of nodes along one basis vector for the generation of a gird on a polygon</td>
<td>-</td>
</tr>
<tr>
<td>$k$</td>
<td>index</td>
<td>-</td>
</tr>
<tr>
<td>$k'$</td>
<td>number of nodes along the positive x-axis of a grid on a disk</td>
<td>-</td>
</tr>
<tr>
<td>$k_B$</td>
<td>Boltzmann constant</td>
<td>J/K</td>
</tr>
<tr>
<td>$l$</td>
<td>mirror length of a V-trough secondary optic</td>
<td>m</td>
</tr>
<tr>
<td>$l_{cond}$</td>
<td>effective length of the conductor</td>
<td>m</td>
</tr>
<tr>
<td>$m$</td>
<td>edge index in triangulated grid</td>
<td>-</td>
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<tr>
<td>$\dot{m}$</td>
<td>mass flow rate of the cooling water</td>
<td>kg/s</td>
</tr>
<tr>
<td>$M$</td>
<td>number of edges in triangulated grid</td>
<td>-</td>
</tr>
<tr>
<td>$n$</td>
<td>refractive index</td>
<td>-</td>
</tr>
<tr>
<td>$n$</td>
<td>node index in triangulated grid</td>
<td>-</td>
</tr>
<tr>
<td>$n_D$</td>
<td>diode ideality factor</td>
<td>-</td>
</tr>
<tr>
<td>$\langle n_r \rangle$</td>
<td>average number of reflections</td>
<td>-</td>
</tr>
<tr>
<td>$\hat{n}$</td>
<td>surface normal unit vector</td>
<td>-</td>
</tr>
<tr>
<td>$N$</td>
<td>number of nodes in triangulated grid</td>
<td>-</td>
</tr>
<tr>
<td>$NA$</td>
<td>numerical aperture</td>
<td>-</td>
</tr>
<tr>
<td>$N_{m}$</td>
<td>number of concentrator modules</td>
<td>-</td>
</tr>
<tr>
<td>$N_P$</td>
<td>number of sides of polygon $\mathcal{P}$</td>
<td>-</td>
</tr>
<tr>
<td>$O$</td>
<td>origin of coordinate system</td>
<td>-</td>
</tr>
<tr>
<td>$p$</td>
<td>node/point on polygon with $A = 1$</td>
<td>-</td>
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<tr>
<td>$P$</td>
<td>electrical power</td>
<td>W</td>
</tr>
<tr>
<td>$\mathbf{P}$</td>
<td>point on parabola</td>
<td>-</td>
</tr>
<tr>
<td>$\mathcal{P}$</td>
<td>polygon with $A = 1$</td>
<td>-</td>
</tr>
<tr>
<td>$q$</td>
<td>index indicating the region within a polygon</td>
<td>-</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>----------</td>
<td>------------------------------------------------------------------------------</td>
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</tr>
<tr>
<td>$q$</td>
<td>elementary charge</td>
<td>C</td>
</tr>
<tr>
<td>$\dot{Q}$</td>
<td>radiative power</td>
<td>W</td>
</tr>
<tr>
<td>$\dot{Q}_{\text{cells}}$</td>
<td>radiative power incident on the active area of the PV receiver</td>
<td>W</td>
</tr>
<tr>
<td>$\dot{Q}_{\text{el}}$</td>
<td>joule heat supplied to the ROI</td>
<td>W</td>
</tr>
<tr>
<td>$\dot{Q}_{\text{flow}}$</td>
<td>heat transferred to the cooling water</td>
<td>W</td>
</tr>
<tr>
<td>$\dot{Q}_i$</td>
<td>solar radiative power intercepted by the area $A_i$</td>
<td>W</td>
</tr>
<tr>
<td>$\dot{Q}_i$</td>
<td>total radiant power through the inlet aperture of a solar concentrator</td>
<td>W</td>
</tr>
<tr>
<td>$\dot{Q}_{\text{in}}$</td>
<td>solar radiative power incident on the ROI</td>
<td>W</td>
</tr>
<tr>
<td>$\dot{Q}_{\text{LFPC-rec}}$</td>
<td>total power incident on the LFPC-receiver</td>
<td>W</td>
</tr>
<tr>
<td>$\dot{Q}_{\text{loss}}$</td>
<td>heat losses from the ROI</td>
<td>W</td>
</tr>
<tr>
<td>$\dot{Q}_o$</td>
<td>total radiant power through the outlet aperture of a solar concentrator</td>
<td>W</td>
</tr>
<tr>
<td>$\dot{Q}_{\text{rec}}$</td>
<td>total power incident on the PV receiver</td>
<td>W</td>
</tr>
<tr>
<td>$\dot{Q}_{\text{refl.,1}}$</td>
<td>total power reflected by the primary mirrors</td>
<td>W</td>
</tr>
<tr>
<td>$\dot{Q}_{\text{refl.,ROI}}$</td>
<td>solar radiative power reflected by the ROI</td>
<td>W</td>
</tr>
<tr>
<td>$\dot{Q}_{\text{ROI}}$</td>
<td>solar radiative power absorbed in the ROI</td>
<td>W</td>
</tr>
<tr>
<td>$\dot{Q}_{\text{sol}}$</td>
<td>solar radiative power incident on the mirror aperture</td>
<td>W</td>
</tr>
<tr>
<td>$\dot{Q}_{\text{thermal}}$</td>
<td>total thermal power absorbed in the cells</td>
<td>W</td>
</tr>
<tr>
<td>$\dot{Q}_{\text{hot}}$</td>
<td>total solar radiative power reaching the focal plane</td>
<td>W</td>
</tr>
<tr>
<td>$r$</td>
<td>radial coordinate</td>
<td>m</td>
</tr>
<tr>
<td>$\Delta r$</td>
<td>radial distance between nodes along $y = 0, x &gt; 0$ on a disk</td>
<td>m</td>
</tr>
<tr>
<td>$\mathbf{\hat{r}}_i$</td>
<td>inverse incident ray direction, unit vector</td>
<td>-</td>
</tr>
<tr>
<td>$\mathbf{\hat{r}}_o$</td>
<td>reflected ray direction, unit vector</td>
<td>-</td>
</tr>
<tr>
<td>$R$</td>
<td>electrical resistance</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>$R_b$</td>
<td>electrical resistance of a bottom electrode segment</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>$R_{b,\text{cnt}}$</td>
<td>electrical contact resistance between the cell and submodule bottom electrodes</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>$R_{b,\text{pl}}$</td>
<td>electrical resistance of the bottom electrode plate</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>$R_{\text{bar}}$</td>
<td>electrical resistance of the submodule interconnection bars</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>$R_D$</td>
<td>disk radius</td>
<td>m</td>
</tr>
<tr>
<td>$R_f$</td>
<td>radial translation of the focal point</td>
<td>m</td>
</tr>
<tr>
<td>$R_i$</td>
<td>inner mirror perimeter</td>
<td>m</td>
</tr>
<tr>
<td>$R_o$</td>
<td>outer mirror perimeter</td>
<td>m</td>
</tr>
<tr>
<td>$R_P$</td>
<td>circumradius of polygon</td>
<td>m</td>
</tr>
<tr>
<td>$R_s$</td>
<td>lumped model series resistance</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
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</tr>
<tr>
<td>$R_{sh}$</td>
<td>lumped model shunt resistance $\Omega$</td>
<td></td>
</tr>
<tr>
<td>$R_{t,300}$</td>
<td>electrical resistance of a free grid finger segment of thickness 300 $\mu$m $\Omega$</td>
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</tr>
<tr>
<td>$R_{t,300}'$</td>
<td>electrical resistance of a cell-connected grid finger segment of thickness 300 $\mu$m $\Omega$</td>
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</tr>
<tr>
<td>$R_{t,400}$</td>
<td>electrical resistance of a free grid finger segment of thickness 400 $\mu$m $\Omega$</td>
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<tr>
<td>$R_{t,400}'$</td>
<td>electrical resistance of a cell-connected grid finger segment of thickness 400 $\mu$m $\Omega$</td>
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<tr>
<td>$R_{t,600}$</td>
<td>electrical resistance of a free grid finger segment of thickness 600 $\mu$m $\Omega$</td>
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<tr>
<td>$R_{t,600}'$</td>
<td>electrical resistance of a cell-connected grid finger segment of thickness 600 $\mu$m $\Omega$</td>
<td></td>
</tr>
<tr>
<td>$R_{t,cnt}$</td>
<td>electrical contact resistance between the cell and submodule top electrodes $\Omega$</td>
<td></td>
</tr>
<tr>
<td>$R_{t,pl}$</td>
<td>electrical resistance of the top electrode plate $\Omega$</td>
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</tr>
<tr>
<td>$R_{term}$</td>
<td>electrical resistance of the receiver terminals $\Omega$</td>
<td></td>
</tr>
<tr>
<td>$R_{th}$</td>
<td>thermal resistance of the high-performance cooler chip $\text{cm}^2 \cdot \text{K/W}$</td>
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</tr>
<tr>
<td>$\mathbf{R}$</td>
<td>rotation matrix in the $x$-$y$ plane $-$</td>
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<tr>
<td>$S$</td>
<td>square with $A = 1$ $-$</td>
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<tr>
<td>$t_{\text{cond}}$</td>
<td>thickness of the conductor $\text{m}$</td>
<td></td>
</tr>
<tr>
<td>$t_{\text{rel}}$</td>
<td>relative temperature coefficient of the photovoltaic efficiency $1/\text{K}$</td>
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</tr>
<tr>
<td>$\mathbf{i}$</td>
<td>tangent unit vector $-$</td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td>temperature $^\circ\text{C}$</td>
<td></td>
</tr>
<tr>
<td>$T_{\text{av,cell}}$</td>
<td>average cell temperature $^\circ\text{C}$</td>
<td></td>
</tr>
<tr>
<td>$T_{\text{av,rec}}$</td>
<td>average receiver temperature $^\circ\text{C}$</td>
<td></td>
</tr>
<tr>
<td>$T_{\text{in}}$</td>
<td>calorimeter inlet temperature $^\circ\text{C}$</td>
<td></td>
</tr>
<tr>
<td>$T_{\text{in}}$</td>
<td>receiver coolant inlet temperature $^\circ\text{C}$</td>
<td></td>
</tr>
<tr>
<td>$T_{\text{flow}}$</td>
<td>cooling water temperature $^\circ\text{C}$</td>
<td></td>
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<tr>
<td>$T_{\text{out}}$</td>
<td>calorimeter outlet temperature $^\circ\text{C}$</td>
<td></td>
</tr>
<tr>
<td>$T_{\text{out}}$</td>
<td>receiver coolant outlet temperature $^\circ\text{C}$</td>
<td></td>
</tr>
<tr>
<td>$T_{\text{ref}}$</td>
<td>reference temperature $^\circ\text{C}$</td>
<td></td>
</tr>
<tr>
<td>$\Delta T$</td>
<td>$= T_{\text{out}} - T_{\text{in}}$, water temperature rise over the calorimeter $^\circ\text{C}$</td>
<td></td>
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<tr>
<td>$\mathbf{u}$</td>
<td>basis vector for node generation on a polygon $-$</td>
<td></td>
</tr>
<tr>
<td>$U$</td>
<td>cell-to-cell irradiance uniformity $%$</td>
<td></td>
</tr>
<tr>
<td>$\mathbf{v}$</td>
<td>basis vector for node generation $-$</td>
<td></td>
</tr>
</tbody>
</table>
Nomenclature

\( V \)  
\[ \text{voltage} \]

\( \dot{V} \)  
\[ \text{volume flow rate of the LFPC cooling water} \quad \text{m}^3/\text{s} \]

\( \dot{V}_{\text{rec}} \)  
\[ \text{volume flow rate of the PV receiver coolant} \quad \text{m}^3/\text{s} \]

\( w \)  
\[ \text{width of a rectangle enclosing the two-stage concentrator and its receiver} \quad \text{m} \]

\( w_{\text{cond}} \)  
\[ \text{width of the conductor} \quad \text{m} \]

\( w_i \)  
\[ \text{width of central square in the focal plane} \quad \text{m} \]

\( w_{\text{fringe}} \)  
\[ \text{half-width of the fringe region} \quad \text{m} \]

\( x \)  
\[ \text{coordinate frame axis} \]

\( x \)  
\[ \text{spatial coordinate} \quad \text{m} \]

\( x \)  
\[ \text{node/point on the mirror/receiver in 3-dimensional space} \]

\( X \)  
\[ \text{matrix (M×N) containing the z-coordinates of the desired normal vectors for the surface optimization} \]

\( y \)  
\[ \text{coordinate frame axis} \]

\( y \)  
\[ \text{spatial coordinate} \quad \text{m} \]

\( z \)  
\[ \text{spatial coordinate} \quad \text{m} \]

\( \Delta z \)  
\[ = z_m - z_p, \text{profile difference between the mirror and an ideal parabola} \quad \text{m} \]

\( z \)  
\[ \text{column vector (N×1) containing the unknown z-coordinates of the nodes for the surface optimization} \]

Greek characters

\( \alpha \)  
\[ \text{absorptance} \quad \% \]

\( \alpha \)  
\[ \text{angle in circumferential direction, defining the position of a focal point} \quad \text{o} \]

\( \alpha_{\text{cell}} \)  
\[ \text{fraction of radiation incident on the active area that is not converted into electrical power} \quad \% \]

\( \alpha_{\text{te}} \)  
\[ \text{absorptance of the copper top electrode} \quad \% \]

\( \beta_{1-5} \)  
\[ \text{independent fitting parameters for the PV cell efficiency at MPP} \quad \text{-} \]

\( \gamma \)  
\[ \text{intercept factor} \quad \% \]

\( \gamma_{\text{LFPC-rec}} \)  
\[ \text{intercept factor on the LFPC-receiver} \quad \% \]

\( \Gamma \)  
\[ \text{mapping function} \quad \text{-} \]

\( \delta_{\alpha} \)  
\[ \text{error in absorptance} \quad \% \]

\( \delta_{ij} \)  
\[ \text{Kroneker delta} \quad \text{-} \]

\( \varepsilon \)  
\[ = (\Phi_{p1} - \Phi_{p1,m})/\Phi_{p1}, \text{offset of the minor rim angle of the mirror from an ideal parabola} \quad \text{o} \]
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$\varepsilon^*$</td>
<td>offset of the minor rim angle of the mirror from an ideal parabola</td>
</tr>
<tr>
<td>$\varepsilon_{\text{Voc}}$</td>
<td>error at open circuit of a $J-V$ curve fit</td>
</tr>
<tr>
<td>$\varepsilon_{\text{MPP}}$</td>
<td>error at MPP of a $J-V$ curve fit</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>factor accounting for the loss in active area fraction due to gaps between the glued mirror segments and for soiling of the mirrors</td>
</tr>
<tr>
<td>$\eta_{\text{acc}}$</td>
<td>acceptance efficiency</td>
</tr>
<tr>
<td>$\eta_{\text{el}}$</td>
<td>electrical receiver efficiency</td>
</tr>
<tr>
<td>$\eta_{\text{MPP,rec}}$</td>
<td>electrical efficiency at MPP of a PV receiver module</td>
</tr>
<tr>
<td>$\eta_{\text{opt}}$</td>
<td>optical efficiency</td>
</tr>
<tr>
<td>$\eta_{\text{opt,rec}}$</td>
<td>optical receiver efficiency</td>
</tr>
<tr>
<td>$\eta_{\text{SOE}}$</td>
<td>SOE efficiency</td>
</tr>
<tr>
<td>$\eta_{\text{sol.-to-el.}}$</td>
<td>full-system solar-to-electricity efficiency</td>
</tr>
<tr>
<td>$\eta_{\text{tot}}$</td>
<td>total system efficiency</td>
</tr>
<tr>
<td>$\theta$</td>
<td>zenith angle (in spherical coordinates)</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>extreme inlet angle, acceptance half-angle</td>
</tr>
<tr>
<td>$\theta_o$</td>
<td>extreme outlet angle</td>
</tr>
<tr>
<td>$\theta_{\text{sun}}$</td>
<td>cone-angle of incident solar radiation</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>wavelength</td>
</tr>
<tr>
<td>$\rho$</td>
<td>reflectance</td>
</tr>
<tr>
<td>$\rho_{\text{Al}_2\text{O}_3,\text{Al}}$</td>
<td>$8^\circ$/hemispherical reflectance of the $\text{Al}_2\text{O}_3$ coating on an aluminum substrate</td>
</tr>
<tr>
<td>$\rho_{\text{Al}_2\text{O}_3,\text{Cu}}$</td>
<td>$8^\circ$/hemispherical reflectance of the $\text{Al}_2\text{O}_3$ coating on a copper substrate</td>
</tr>
<tr>
<td>$\rho_{\text{mirror}}$</td>
<td>solar-averaged mirror reflectance</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>electrical conductivity</td>
</tr>
<tr>
<td>$\sigma_{\text{err,1}}$</td>
<td>angular dispersion error standard deviation in circumferential direction</td>
</tr>
<tr>
<td>$\sigma_{\text{err,2}}$</td>
<td>angular dispersion error standard deviation in radial direction</td>
</tr>
<tr>
<td>$\tau$</td>
<td>primary outlet aperture tilt</td>
</tr>
<tr>
<td>$\tau^*$</td>
<td>optimal primary outlet aperture tilt</td>
</tr>
</tbody>
</table>
Nomenclature

\( \tau_1 \) optimal primary outlet aperture tilt in Regime 1 °
\( \tau_2 \) optimal primary outlet aperture tilt in Regime 2 °
\( \varphi \) parametric rim angle °
\( \varphi \) circumferential polar coordinate rad
\( \Delta \varphi_i \) angular circumferential spacing between nodes on a disk rad
\( \phi \) azimuth angle (in spherical coordinates) rad
\( \Phi_1 \) inner rim angle of the primary mirror °
\( \Phi_1^* \) limiting inner rim angle between regimes °
\( \Phi_2 \) (outer) rim angle of the primary mirror °
\( \Phi_2^* \) limiting outer rim angle between regimes °
\( \Phi_{av} \) average rim angle, \( \Phi_{av} = (\Phi_1 + \Phi_2)/2 \) °
\( \Phi_D \) rim angle of a disk concentrator °
\( \Phi_{P1^-} \) minor rim angle of a polygonal concentrator °
\( \Phi_{P1^+} \) major rim angle of a polygonal concentrator °
\( \Delta \Phi \) rim span, \( \Delta \Phi = \Phi_2 - \Phi_1 \) °
\( \Omega \) solid angle sr
\( \psi \) vertex angle of a V-trough secondary optic °

Subscripts

+ primary outlet aperture tilt larger than \( \tau_1 \)
– primary outlet aperture tilt smaller than \( \tau_1 \)
1 pertaining to the primary concentrator
2 pertaining to the secondary concentrator
3D pertaining to 3D (point focus) concentrators
Al2O3 pertaining to the alumina coating
asym pertaining to an asymmetric design
av average (concentration ratio, irradiance)
c central node of the primary mirror
CPC pertaining to a CPC secondary concentrator
Fit indicating a fitted value
i pertaining to incident radiation
m quantity measured on the mirror
max maximum concentration for full-collection
meas indicating a measured value
oc open-circuit
p quantity measured on an ideal parabola
peak peak (concentration ratio)
r pertaining to reflected radiation
rec pertaining to the fictitious PV receiver with size 60 × 60 mm (only Chapter 4)
R1 regime 1 for primary outlet aperture tilt angle
R2 regime 2 for primary outlet aperture tilt angle
sc short-circuit
square pertaining to a square receiver geometry
sym pertaining to a symmetric design
tot pertaining to the overall (2-stage) system
\( \lambda \) indicating a spectral property

**Superscript**

0 indicating an initial parameter guess of the fitting algorithm

**Abbreviations**

3-J triple-junction (solar cell)
AgSheet0.2mm silvered aluminum sheet of thickness 0.2 mm
AgSheet0.3mm silvered aluminum sheet of thickness 0.3 mm
AM air mass
AR anti-reflection (coating)
CANCER Computer Analysis of Nonlinear Circuits, Excluding Radiation
CDU cooling distribution unit
CEC compound elliptical concentrator
CMOS complementary metal-oxide semiconductor (camera sensor)
CPC compound parabolic concentrator
CPV concentrating photovoltaics
CSP concentrating solar (thermal) power
DC direct current
DSMTS dielectric single-mirror two-stage concentrator
HCPVT high-concentration photovoltaic thermal
LFPC Lambertian Flat-Plate Calorimeter
LGH low-grade heat
MPP maximum power point
POM Polyoxymethylene
PV photovoltaics
PVT photovoltaic-thermal
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>rms</td>
<td>root mean square</td>
</tr>
<tr>
<td>ROI</td>
<td>region of interest</td>
</tr>
<tr>
<td>RTD</td>
<td>resistance temperature detector</td>
</tr>
<tr>
<td>SMTS</td>
<td>single-mirror two-stage concentrator</td>
</tr>
<tr>
<td>SMS</td>
<td>simultaneous multiple surface method</td>
</tr>
<tr>
<td>SOE</td>
<td>secondary optical element</td>
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<tr>
<td>SPICE</td>
<td>Simulation Program with Integrated Circuit Emphasis</td>
</tr>
<tr>
<td>TERC</td>
<td>tailored edge ray concentrator</td>
</tr>
<tr>
<td>UHPC</td>
<td>ultra-high-performance concrete</td>
</tr>
</tbody>
</table>
1 Introduction

With a global yearly energy demand currently estimated around $580 \cdot 10^{18}$ J and expected to increase to $860 \cdot 10^{18}$ J by the year 2040, mostly driven by a rise of the world’s population in combination with a rapid economic development in emerging countries [1], securing future energy supply is of foremost importance. A considerable and growing part of the energy demand can be associated to electricity production, which in 2013 amounted to $84 \cdot 10^{18}$ J, and which, despite gradual improvements, is still predominantly based on the combustion of fossil fuels (coal and peat 41%, natural gas 22%, oil 4%) [2].

The search for alternative, renewable sources for electricity production to bridge the gap between future demand and current production is primarily driven by two factors: (1) recurrent concerns about the long-term supply of fossil fuels; and (2) more recent efforts to constrain the emission of carbon dioxide and other greenhouse gases on grounds of their effect on climate change.

When considering the magnitude of the solar radiative power impinging on the Earth’s surface, estimated at $94 \cdot 10^{15}$ W [3], which amounts to a total energy of $3 \cdot 10^{24}$ J per year, a utilization of this free and clean resource appears evident. Nevertheless, currently only 0.5% of electricity production is accomplished by solar energy [1].

1.1 Solar concentration

This low adoption of solar energy is chiefly a result of the dilute nature of solar irradiance at the Earth’s surface, which under favorable conditions amounts to 1 kW/m$^2$. This dilute nature has several important consequences.

On the macroscopic scale, it signifies that a large collecting (land) area is required to capture a substantial total power. This can be illustrated by a

---

*The primary energy invested for the production of this secondary energy is considerably higher, estimated at $215 \cdot 10^{18}$ J in 2014 [1], i.e. over 1/3rd of the total energy demand.*
simplified calculation. Consider a large coal fired power plant that produces $P = 4000$ MW of electricity. If $\eta$ is the efficiency with which electromagnetic solar radiation can be converted to electricity, then the collecting area required to replace this power plant by solar collectors is at least

$$A_i = \frac{P}{\eta \cdot 1 \text{ kW/m}^2}.$$  \hspace{1cm} (1.1)

For current state-of-the-art PV panel systems ($\eta = 20\%$ [4]), this results in an area of $20 \text{ km}^2$ or over 2500 times the size of a regular football pitch.

Moreover, the diluted nature of solar irradiance has an intrinsic effect on the energy quality, i.e. the portion of useful extractable energy, as, for both idealized thermal and photovoltaic (PV) conversion, an increase of the power per unit receiver area, results in lower inherent losses and consequently a higher conversion efficiency when keeping the energetic potential (temperature or voltage respectively) constant [5–10].

Besides the direct effect on the energy quality, an increase in energy density of the radiation incident on the receiver also has the benefit of reducing the required collecting area via an increase in conversion efficiency, as evident from Eq. (1.1), which is of prime importance in a future where the competition over exploitable soil between different stakeholders can be expected to increase with growing population and economic development.

This increase in solar power per unit receiver area can be realized by a solar concentrator, i.e. an optical device that collects radiant energy at its inlet aperture $A_i$ and transports it to its outlet aperture $A_o$ with minimal losses and where $A_o$ is smaller than $A_i$. The ratio of the inlet and outlet apertures is denoted as geometrical concentration ratio

$$C_g = \frac{A_i}{A_o}.$$  \hspace{1cm} (1.2)

and serves as an important metric for the characterization of solar concentrators.

\hspace{1cm}

\begin{footnote}
This neglects i.a. the intermittency of solar radiation, when observed from a static position on the Earth’s surface, most importantly that solar radiation is only available during part of the day.
\end{footnote}
While the geometrical concentration ratio can be made arbitrarily large, e.g. by decreasing the size of the outlet aperture for a constant inlet aperture, the solar energy concentration that can be achieved at the outlet is limited. To emphasize this difference, we define the irradiance (or flux) concentration ratio, i.e. the ratio of the average irradiance at the outlet aperture to that of the inlet aperture,

$$C = \frac{E_o}{E_i}. \quad (1.3)$$

Geometrical and irradiance concentration ratios are linked through the optical performance of the concentrator. If $\dot{Q} = E \cdot A$ is the total power through an aperture, then

$$C = \frac{E_o}{E_i} = \frac{\dot{Q}_o \cdot A}{\dot{Q}_i \cdot A_i} = \frac{\dot{Q}_o}{\dot{Q}_i} \cdot C_g, \quad (1.4)$$

where we can denote the ratio of transmitted power, $\dot{Q}_o/\dot{Q}_i$, as the optical efficiency $\eta_{opt}$ of the concentrator, such that $C = \eta_{opt} \cdot C_g$. Together with the geometric concentration ratio, the optical efficiency of a concentrator is a measure for the quality of the energy concentration that it achieves.

### 1.2 The thermodynamic limit of concentration

The maximum concentration ratio achievable with an optical concentrator that collects all radiation within a cone of given angular aperture at its inlet and redirects it to its outlet, i.e. the maximum attainable value for $C$ irrespective of the magnitude of $C_g$, is fundamentally bound by the laws of thermodynamics and is consequently known as the thermodynamic limit of concentration. The same limit may be derived from first principles using the concept of conservation throughout the optical system of the quantity of étendue via several approaches [11–13]. Étendue is essentially a measure for the spatial and angular extent of radiation and its conservation dictates that a decrease of spatial extend (resulting from an increase in concentration) mandates a proportional increase in angular extent. As the maximum achievable angular extent is limited (to the hemisphere), so is the achievable concentration.
When considering the limit of concentration, a distinction needs to be made between two types of solar concentrators, exemplarily compared to a solar collector without irradiance concentration in Figure 1.1. With the first, referred to as line-focus concentrators or troughs, radiation is redirected to a focal line along which the receiver is placed. Their shape can be obtained by extrusion of a planar geometry, which explains their frequent designation as 2D concentrator despite their three-dimensional profile. Line-focus concentrating optics combine the advantages of reasonably high concentration and low cost due to the extruded nature of the mirror shape and the requirement for only one tracking axis. Furthermore, they can be easily scaled by increasing the axial length of the trough. With the second type, radiation is contrariwise directed to a point, from which stems their common denomination as point-focus or 3D concentrators. As we shall see, they offer considerably higher concentration ratios in exchange for a higher system complexity, e.g. in terms of requirement for 2-axis tracking and the manufacturing of 3-dimensional reflective surfaces. In general, no particular restriction is imposed with regards to their symmetry, although in practice a lot of designs possess rotational symmetry about the optical axis.

In line-focus, the thermodynamic limit of concentration is

\[ C_{g,\text{max},2D} = \frac{1}{\sin \theta_i}, \quad (1.5) \]

where \( \theta_i \) is the acceptance half-angle of the concentrator, i.e. the half-aperture of the cone within which incident radiation is accepted by the concentrator. E.g. for an acceptance angle equal to the apparent size of the sun when viewed from the surface of the Earth, \( \theta_i = \theta_{\text{sun}} = 4.65 \text{ mrad} \), the maximum achievable concentration is 215×. Conversely, for point-focus, the thermodynamic limit of concentration is

\[ C_{g,\text{max},3D} = \frac{1}{\sin^2 \theta_i}, \quad (1.6) \]

with a limit of 46248× for \( \theta_i = \theta_{\text{sun}} \).

Note that the given limits are for receivers not immersed in refractive media, which are the major focus of this work. For the concentration of radiant energy
from air to a target immersed in a medium of refractive index $n$, the respective limits in 2D and 3D geometry are increased to $\frac{n}{\sin \theta_i}$ and $\frac{n^2}{\sin^2 \theta_i}$ [14].

1.3 Concentrator optics

1.3.1 Image-forming concentrators

The fundamental property of traditional, image-forming (or imaging) optical devices such as lenses and parabolic concentrators, namely their ability to redirect a set of parallel rays to a single point, has been understood since the classical antiquity, with written references confirming knowledge about the underlying principles from as early as the 5th century B.C. [15]. Archeological evidence allows tracing back the use of *burning glass* (lenses) to the 7th century B.C., while mathematical descriptions of *burning mirrors* (parabolic concentrators) first appeared in the 2nd century B.C. [16, 17]. Research on both imaging systems was continued by Arabic scholars throughout the Middle Ages, most notably by Ibn Sahl [18, 19]. Despite the subsequent growth of the scientific branch of optics, which brought forth a deeper physical understanding of the underlying principles in the more recent modern history, it was not until the 1860s that the expectation of a future shortage of coal lead to research investigating alternative means of supplying power to a flourishing industrial sector, and ultimately to the first recorded successful use of parabolic collectors to exploit solar energy, viz. for steam generation [20].
If solar radiation were perfectly collimated, i.e. all rays were parallel, and diffraction effects are neglected then the irradiance concentration achievable with image-forming collectors would effectively be infinite. Due to the non-collimated nature of solar radiation however, this is not the case. Off-axis rays are significantly aberrated such that the achievable concentration to collect all radiation within an acceptance angle $\theta_i$ with these single-reflection concave mirrors is fundamentally limited. Harper [21] was the first to derive these limits, which are

\[
C_{g,1,max,sym,2D} = \frac{\sin 2\Phi_2}{\sin 2\theta_i} - 1 = \frac{\sin \Phi_2 \cos \Phi_2}{\sin \theta_i \cos \theta_i} - 1
\] (1.7)

and

\[
C_{g,1,max,sym,3D} = \left( \frac{\sin 2\Phi_2}{\sin 2\theta_i} \right)^2 - 1 = \left( \frac{\sin \Phi_2 \cos \Phi_2}{\sin \theta_i \cos \theta_i} \right)^2 - 1
\] (1.8)

in 2D and 3D geometry respectively [11, 21, 22], where $\Phi_2$ is the rim angle of the mirror and where the term $-1$ accounts for the obstruction of the inlet aperture by the receiver. The maximum achievable concentrations occur for $\Phi_2 = 45^\circ$, which, with $\theta_i = \theta_{sun}$ yields $107\times$ and $11561\times$ respectively, i.e. reductions in concentration with respect to the thermodynamic limit of approximately 2 and 4. Nevertheless, the achievable concentrations are relatively elevated and image-forming concentrators offer several practical advantages such as economical use of mirror surface area and a relatively high compactness.

1.3.2 Nonimaging optics

For a long time, no device capable of collecting non-collimated radiation efficiently with a higher concentration than suggested by Eqs. (1.7) and (1.8) was known. It was not until the mid-1960s and early 1970s, when several researchers [23–26] independently described the compound parabolic concentrator (CPC).

\footnote{In reality, in an imaging optical system, diffraction effects ultimately limit the maximum concentration. When using a system of numerical aperture $NA = n \sin \theta$ and without aberrations to form an image of a point source emitting radiation at wavelength $\lambda$, the image is a diffraction pattern mostly constraint within a circle of radius $0.61\cdot\lambda/NA$ [11].}
and gradually developed the theory required for its application to solar energy collection [27], that these limits were finally exceeded. By relaxing the constraint to form an image of the sun and instead focusing solely on the transport of radiant energy, a concentrator was designed that can achieve the thermodynamic limit of concentration in 2D geometry and perform very close to ideal in 3D symmetry. More fundamentally, the edge-ray principle was formulated, which became the cornerstone of the new field of nonimaging optics, allowing to design ideal or close-to-ideal concentrators for numerous sources of radiant energy [11, 27].

Several designs and applications based on the CPC were thereupon studied, constructed and tested for solar energy applications [28–31]. However, they generally suffer from poor compactness when designed for the small acceptance angles necessary to capture direct solar radiation at high concentrations, i.e. require a large mirror area for a relatively small inlet aperture and become unwieldy for tracking.

1.3.3 Practical high-concentration solar concentrators

The apparent trade-off between high concentration and practicality of designs capturing solar radiation can partially be overcome by the combination of an image-forming (e.g. parabolic) primary mirror with a nonimaging secondary optic. In these two-stage or tandem concentrators, the geometry of the nonimaging optic is derived using the edge-ray method by employing the exit aperture of the primary mirror as virtual source [32]. The resulting secondary optic, a compound elliptical concentrator (CEC), is consequently ideal in the nonimaging sense and corrects the majority of the optical aberrations and losses introduced by the compact primary mirror, resulting in concentrations approaching the thermodynamic limit.

This development lead to continued interest in two-stage systems for solar applications that are based on parabolic primary mirrors and to the discovery of several new types of ideal nonimaging secondary optics [33–40]. Eventually, analytical [41] and numerical [42] methods for the simultaneous design of primary and secondary optics were developed that allow a higher number of degrees of freedom, which can be used to mitigate several optical aberrations and increase concentration ratios [43, 44]. It is these multiple-stage, hybrid imaging-
nonimaging designs that allow the highest concentration ratios for practical solar energy applications\textsuperscript{d}.

When examining such designs, it is important to bring up their limitations in comparison to the concentration limits of incident solar radiation given in Section 1.1. On one hand, despite being able to approach the thermodynamic limit for specific configurations, practicality considerations often constrain the designs to concentration ratios below the limits of Eqs. (1.5) and (1.6). On the other hand, a relaxation of $\theta_i$ is mandated to have adequate robustness against tracking, surface and alignment inaccuracies. Consequently, concentrations in the range of $10\times$ to $100\times$ are typical for line-focus designs, while point-focus designs allow concentration ratios beyond several $1000\times$ to be readily attainable with practical designs.

1.4 Concentrating photovoltaics

In addition to the fundamental aspects briefly discussed earlier of increasing the quality of solar radiation, the primary motivation for using concentration with photovoltaic cells is of economical nature. In an admittedly oversimplified juxtaposition, the advantages of concentrating photovoltaic (CPV) systems in comparison to regular PV are twofold. On one hand, under the assumption of efficient optics and the same photovoltaic cell technology, a direct cost benefit is realized if expensive cell area is predominantly substituted by comparatively inexpensive optics. On the other hand, the offset of cost towards the optical system allows the utilization of cells with advanced structures, e.g. multi-junction solar cells, which offer a more than twofold increase in efficiency\textsuperscript{e} at the expense of a higher specific cost. The gain in overall conversion efficiency then indirectly translates into an economic benefit via a reduction of required inlet aperture (i.e. number of concentrators) to generate an equal amount of electrical power, as stated earlier.

\textsuperscript{d} An extensive overview of a large number of proposed optical concepts and realized systems, together with an in-depth survey of the theoretical background has been compiled by Chaves [12], Rabl [45] and Winston [11].

\textsuperscript{e} While the efficiency of single-junction cells is fundamentally limited to 30\% under 1 sun irradiance, multi-junction cells have the theoretical potential for 68\% (for an infinite stack of subcells) [46].
It is important to note that, while the use of concentration introduces several losses (e.g. attenuation of radiative energy by the optics, ability to only make use of direct normal irradiance) and additional system cost (e.g. tracking, tolerances), this can be partially offset by the increase of the achievable cell efficiencies under concentration\(^f\).

Following the first successful demonstration of a CPV system in the 1960s\(^{47}\), CPV technology has made considerable improvements, founded on hand on advancements in concentrator technology and on the other on a continuous increase in PV cell efficiencies. On the cell level, the sustained improvements from simple crystalline Si cells, via several innovations in terms of i.a. support for high irradiance, production techniques, band-gap materials, to, ultimately, the introduction of latest-generation multi-junction cells\(^{48, 49}\), lead to a continuous increase in efficiency, with a recently demonstrated record of 46.0%\(^{4}\), and promise a high potential for future progress\(^{46}\). On the system level, the variety of applications developed over the years and using these cells, ranging from stationary tracking nonimaging concentrators over parabolic troughs and dishes to Fresnel lenses, is substantial, with significant advances in efficiency and cost reduction\(^{5, 7, 50}\)\(^g\).

For the comparison of CPV systems, a classification according to their geometrical concentration ratio is often adopted. Besides the evident implications for the employed type of solar concentrator, the concentration ratio also impacts the majority of the remaining system components, most importantly the PV cell technology and cell cooling.

(1) Low-concentration systems\((C_g = 1-10\times)\) typically consist of conventional PV panels, augmented by simple adjacent optics able to collect part of the peripheral radiation. Accordingly, passive cooling is sufficient to keep the cells at safe operating conditions and tracking is usually omitted or performed seasonally.

\(^f\) Under a concentration close to\(C_g,\text{max,3D}\), the efficiency limits of single- and multi-junction cells are increased to 40\% (relative increase compared to the unconcentrated case of +33\%) and 86\% (+26\% rel.), respectively\(^{46}\).

\(^g\) A broad review of concentrator photovoltaics technologies, focusing on both cells and concentrators, can be found in the work of Luque and Andreev\(^{7}\).
Medium-concentration systems \((C_g = 10-100×)\) usually employ line-focus concentrators and one-axis tracking. The PV cells are typically enhanced single-junction cells, optimized for extraction of the higher current densities generated under the higher levels of irradiation. With suitable heat sinks, passive cell cooling is still feasible.

High-concentration systems \((C_g > 100×)\) conversely use point-focus concentrators and two-axis trackers with specialized, usually multiple-junction, PV cells and the high irradiance densities mandate the use of active cell cooling.

Despite the higher cell cost, the trend for the last decades has been the development of high-concentration systems, in accordance with the “third-generation photovoltaics” approach [51, 52], and as a direct consequence of the efficiency improvements with concentrator cells mentioned above.

### 1.5 Implications of irradiance uniformity

Aside from concentration, the second parameter of major importance in CPV is irradiance uniformity across the receiver. While direct solar radiation reaching the Earth has near-perfect spatial uniformity, this uniformity is reduced along the radiation’s path through the optical system, partly due to optical aberrations and nonidealities such as surface inaccuracies, but also fundamentally due to the very principles that allow concentration. When the concentration is increased, a uniform irradiance distribution is therefore increasingly difficult to uphold.

On the single cell, irradiance non-uniformities result in a gradient in the charge carrier density which leads to a drop in the open-circuit voltage and therefore a lower electrical cell efficiency [10, 53–55]. While the losses resulting from non-uniform irradiance across a cell are generally low in practical applications, another type of irradiance non-uniformity can result in more severe reduction of the electrical efficiency. In many CPV systems, the individual cells are collected together in dense-array receivers where a non-uniform irradiance creates a current mismatch between the cells [53, 56, 57]. The individual cells are often interconnected in series, such that the current through the array is limited by the cell with the lowest average irradiance.
Unfortunately, concentration, optical efficiency and irradiance uniformity are generally subject to a trade-off, which imposes particular care in the system design, especially for high-concentration systems, to maximize the overall efficiency.

1.6 Thermal management and broad-spectrum utilization

While a high concentration is a key driving factor in third-generation PV systems, it introduces the problem of thermal management of the cells [58]. To maintain safe operating conditions and elevated photovoltaic efficiencies, active cell cooling is required to evacuate the produced heat, which, even with the highest-efficiency cells, can be up to 50% of the total incident solar radiative power.

Conversely, if heat can be efficiently extracted at high enough temperatures using efficient photovoltaic-thermal (PVT) receivers (e.g. [59, 60]) this heat can then be used in secondary applications such as space heating, heat-driven cooling and water desalination, and increase the overall utilization of the solar spectrum [61–63].

1.7 Thesis outline

The principal goal of this thesis is the development of a novel point-focus solar high-concentration photovoltaic-thermal (HCPVT) concentrator that accomplishes a cost-effective utilization of the solar resource by combining advances in optical design to innovative materials, new production methods and full-spectrum utilization. The chapters of the thesis focus on several key aspects that ultimately culminate in the final design proposal, performance predictions and suggestions for future research.

Chapter 2 is devoted to the systematic geometrical analysis of two-stage concentrator designs based on parabolic primary mirrors in line-symmetry (2D). The theoretical concentration limits for asymmetric two-stage troughs are derived and a systematic comparison of the performance of symmetric and asymmetric designs is presented. It is shown how simple asymmetric configurations offer improvements in both compactness and concentration with regards to symmetric designs. Notably, previously unidentified high-
performance solar concentrator designs are developed, which are comparable with the best reported practical 2D designs for both hollow and dielectric-filled secondaries.

In Chapter 3, a new absolute irradiance measurement system is introduced, which mitigates many of the limitations of existing methods and is employed for the on-sun characterization and assessment of solar concentrators through the subsequent chapters of this work.

In Chapter 4, the first on-sun experimental demonstration and proof-of-concept of a novel solar dish concept based on elliptical vacuum-membrane facets is reported. The membrane-mirror technology is investigated for its potential to drastically reduce the manufacturing cost in comparison to systems based on conventional back-silvered glass mirror technology. The dish is further utilized as a prototype to investigate several technologies foreseen for the final dish design, notably, different types of silvered aluminum membranes and a prototype of the dense-array photovoltaic receiver.

A novel solar dish design that achieves high concentration and compactness through a combination of the theoretical and experimental findings of the preceding chapters, is introduced in Chapter 5. A practical implementation of this concept, which incorporates a multitude of innovations in terms of materials and production methods and is intended for high-concentration photovoltaic-thermal energy generation, is presented, modeled and experimentally characterized. It incorporates a receiver design able to mitigate losses caused by a high irradiance non-uniformity, which allows a less complicated optical design. Finally, performance predictions over a range of operating conditions are made using a detailed opto-electric model.

Chapter 6 addresses the issue of irradiance non-uniformity with point-focus concentrators from a different perspective by introducing a design methodology for nonimaging, single-reflection mirrors with polygonal inlet apertures that generate a uniform irradiance distribution on a polygonal outlet aperture, enabling a multitude of applications within the domain of concentrated photovoltaics, and notably much simpler receiver designs.

Chapter 7 finally gives an outlook and recommendations for future research activities on high-concentration solar energy systems.
2 Two-stage solar concentrators based on parabolic troughs: asymmetric vs. symmetric designs

This chapter is devoted to the systematic geometrical analysis of two-stage concentrator designs based on parabolic primary mirrors in line-symmetry (2D).

2.1 Introduction

As discussed in Chapter 1, concentrating optics offer a potential route to increase the efficiency of solar energy conversion by augmenting the flux of the otherwise dilute solar radiation. Line-focus (2D) concentrating optics combine the advantages of reasonably high concentration and low cost due to the extruded nature of the mirror shape and the requirement for only one tracking axis. Furthermore, they can be easily scaled by increasing the axial length of the trough. Nonimaging concentrators such as the commonly employed compound parabolic concentrator (CPC) can achieve the thermodynamic limit of concentration in line symmetry. However, they generally suffer from poor compactness when designed for small acceptance angles, e.g. to capture direct solar radiation [30, 32].

This can be overcome to some extent by symmetric two-stage systems employing an image forming primary mirror, e.g. parabolic trough, in tandem with a nonimaging secondary, e.g. compound elliptical concentrator (CEC) [32]. Figure 2.1 shows a schematic of the geometry of such a symmetric two-stage system. However, these symmetric systems are still limited by an inherent trade-off between compactness and concentration, with the latter ultimately being bound by shading due to the central obstruction caused by the secondary.

---

Substantial improvements in concentration and compactness can be achieved by considering systems having asymmetric cross-sections, that is having no reflection symmetry about the optical axis, as illustrated in Figure 2.2 [32, 64]. A number of specific designs of asymmetric two-stage concentrators employing parabolic primaries have recently been developed for concentrating photovoltaic (CPV) and concentrating solar thermal power (CSP) applications [33, 65, 66]. However, to date, a systematic comparison between asymmetric and symmetric two-stage concentrators has not yet been reported in the literature, and is therefore one focus of this work. Section 2.2 is devoted to the derivation of the theoretical concentration limits for asymmetric two-stage troughs. With the concentration limits derived, Section 2.3 presents a systematic comparison of the performance of symmetric and asymmetric designs. Section 2.4 focuses on novel compound asymmetric configurations, which offer improvements in both compactness and concentration. Importantly, this work shows how a systematic

Figure 2.1. Cross-sectional geometry of a generic symmetric two-stage solar concentrator utilizing a parabolic trough primary and a CEC secondary.
Two-stage solar concentrators based on parabolic troughs geometrical analysis can serve to bring to light previously unidentified high-performance solar concentrator designs.

2.2 Concentration limits for a parabolic trough with a nonimaging secondary

The geometrical concentration ratio in line symmetry (2D) is defined as

\[ C_g = \frac{a_i}{a_o}, \]  

(2.1)

where \( a_i \) is the unshaded inlet aperture width and \( a_o \) the outlet aperture width. The thermodynamic limit of concentration, set by conservation of étendue, is

\[ C_{g,\text{max},2D} = \frac{1}{\sin \theta_i}, \]  

(2.2)

Figure 2.2. Cross-sectional geometry of a generic asymmetric two-stage solar concentrator utilizing a parabolic trough primary and a CEC secondary.
where $\theta_i$ is the acceptance half-angle of the concentrator \[^b\]. Various design-specific geometric constraints will limit the achievable concentration to below this thermodynamic limit. Knowledge of concentration limits for a particular class of concentrator allows an optical designer to identify configurations which maximize concentration ratio for a given acceptance angle, or conversely maximize the acceptance angle for a given concentration. In this section, symmetric concentration limits are first reviewed in order to keep the definitions consistent and make a fair comparison with the asymmetric concentration limits subsequently derived. A one-sided receiver (unifacial solar cell, flat thermal receiver or cavity aperture) is considered, which can only collect rays at its lower surface. All concentrators presented here are designed for full collection, meaning that all radiation striking the inlet aperture within an acceptance angle $\pm \theta_i$ reaches the receiver.

2.2.1 Symmetric concentrators

*Primary concentration,* $C_{g,1} = a_{i,1}/a_{o,1}$

The geometric concentration limit for one-reflection mirrors, previously derived in 3D axis symmetry \([11, 21]\) and extended to the 2D line-focus case \([22]\) is given by

$$C_{g,1,max,sym} = \frac{\sin \Phi_2}{\sin 2\theta_i} - 1 = \frac{\sin \Phi_2 \cos \Phi_2}{\sin \theta_i \cos \theta_i} - 1$$

(2.3)

where $\Phi_2$ is the rim angle of the primary concentrator as indicated in Figure 2.1. To reach this concentration, the outlet aperture is placed at the intersection points of the right and left edge rays reflected from the rims of the mirror. To maintain full collection, the mirror is extended inward to the point where it crosses the edge ray that just passes the receiver \([22]\). Note that for the case of a parabolic mirror designed for maximum full-collection concentration, the outlet aperture does not lie at the paraxial focus but slightly below it \([12, 32]\). Such a concentrator reaches its highest concentration at $\Phi_2 = 45^\circ$ as shown in Figure 2.3.

\[^b\] For the remainder of this Chapter, the subscript 2D is omitted for brevity.
Secondary concentration, $C_{g,2} = \frac{a_{o,1}}{a_{o,2}}$

There are two main approaches to designing a secondary concentrator for a parabolic trough primary: (1) approximating the primary by a Lambertian source which results in a compound elliptical (CEC) secondary [32]; and (2) tailoring the secondary to the edge rays reflected by the actual parabolic primary which results in a tailored edge ray concentrator (TERC) secondary [39]. The first approach creates an étendue mismatch since the secondary is designed to accept rays from a fictitious Lambertian source whose étendue is larger than that of the rays actually reflected by the primary mirror. This étendue mismatch causes the concentration of the resulting two-stage parabola-CEC to be lower than the thermodynamic limit. On the other hand, the second approach of edge-ray tailoring conserves étendue everywhere in the optical system such that the thermodynamic limit can be reached in theory. However, the resulting TERC secondary wholly shades the primary mirror and must be either truncated or designed for an oversized receiver which significantly reduces the achievable

![Figure 2.3. Primary concentration ratio ($C_{g,1}$) and CAP$_{2D}$ for symmetric and asymmetric systems with acceptance angle $\theta_i = 1^\circ$ as a function of the rim angle of the primary $\Phi_2$. The markers indicate the concentrations of the designs compared in Section 2.3.2.](image)
concentration. When designed for full collection, even when the TERC is optimally truncated, the shading of the TERC is so severe that the Lambertian approach actually results in a higher concentration design, as evidenced by Figure 2.4. For this reason, we focus on two-stage systems having CEC-like secondaries designed according to the Lambertian approximation of the primary.

When approximating the parabolic trough from Figure 2.1 by a Lambertian source, the method of Hottel [32] can be used to express the étendue incident on the secondary concentrator as

\[
H_{1,2} = 2 \cdot \left( \llVert \mathbf{AB} \rrVert - \llVert \mathbf{A}'B' \rrVert \right) = 2a_{0,1} \frac{\sin \Phi_2}{\cos \theta_i},
\]

where \( \llVert \mathbf{AB} \rrVert \) denotes the 2-dimensional Euclidian norm of a vector from point A to B. The étendue at the outlet of the secondary concentrator is

\[\]

**Figure 2.4.** Overall concentration ratio \( C_{g,\text{tot}} \) and \( \text{CAP}_{2D} \) for symmetric and asymmetric systems with acceptance angle \( \theta_i = 1^\circ \) as a function of the rim angle of the primary \( \Phi_2 \). Solid lines are for a CEC secondary and dashed lines are for a CPC secondary. Asymmetric concentrators perform better than their symmetric counterparts for all rim angles and can achieve concentrations approaching the thermodynamic limit \( C_{g,\text{max},2D} \) for small rim spans. The markers indicate the concentrations of the designs compared in Section 2.3.2.
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\[ H_{o,2} = 2a_{o,2} \cdot \sin \theta_o, \]  

(2.5)

where \( \theta_o \) is the extreme outlet angle. Since an ideal concentrator conserves \( \text{étendue} \) \( (H_{i,2} = H_{o,2}) \), the maximum concentration ratio for a CEC secondary, occurring for \( \theta_o = \pi/2 \), is

\[ C_{g,2,\text{max},\text{sym}} = \frac{\cos \theta_1}{\sin \Phi_2}. \]  

(2.6)

If instead of a CEC the more common CPC (which is non-ideal for a finite source) is used as secondary concentrator, the maximum secondary concentration [27] after a parabolic trough primary becomes

\[ C_{g,2,\text{CPC},\text{sym}} = \frac{1}{\sin(\Phi_2 + \theta_1)}. \]  

(2.7)

2.2.2 Asymmetric concentrators

*Primary concentration, \( C_{g,1} = a_{i,1}/a_{o,1} \)*

The theoretical concentration limit for asymmetric parabolic troughs has been previously derived under the limiting assumption of a vanishing acceptance angle [33]. In order to provide a fair comparison to symmetric concentrators, the concentration limit of an asymmetric parabolic trough for finite acceptance angles is required. In most cases, one can achieve maximum concentration in a similar fashion to the symmetric case by placing the primary outlet aperture between the intersection points of the edge rays reflected from the inner and outer rim of the parabola respectively [32], as shown schematically in Figure 2.2.

However, the introduction of an additional degree of freedom, the inner rim angle \( \Phi_1 \), increases the complexity of the problem. Indeed, both the inner and outer rim angles have an influence on the tilt of the outlet aperture at which the maximum concentration is reached. To aid in the derivations, the average rim angle \( \Phi_{av} = (\Phi_1 + \Phi_2)/2 \) and the rim span \( \Delta \Phi = \Phi_2 - \Phi_1 \) [67] are introduced. Following the derivations of Appendix A.1.1, for maximum primary concentration the primary outlet aperture \( BB' \) must be tilted by an angle \( \tau_1 \), where
As $\Phi_1$ is increased, the optimal tilt angle $\tau_1$ increases. At the point where $\tau_1 > \Phi_2 - \theta_i$ it is interestingly no longer optimal to place the aperture at the intersection of the edge rays. Instead, the maximum concentration aperture must be tilted at an angle $\tau_2 = \Phi_2 - \theta_i$ such that it extends perpendicularly from the $+\theta_i$ edge ray reflected from the rim (as seen in Figure A.1). A detailed discussion is provided in Appendix A.1.2. Designs where $\tau_1$ is the optimal aperture tilt angle are denoted as Regime 1 designs, and those for $\tau_2$ as Regime 2 designs. Accounting for both regimes, the optimal primary outlet aperture tilt $\tau^*$ for maximum primary concentration is therefore

$$\tau^* = \begin{cases} 
\tau_1 & \text{for } \tau_1 \leq \Phi_2 - \theta_i \quad \text{(Regime 1)} \\
\tau_2 = \Phi_2 - \theta_i & \text{for } \Phi_2 - \theta_i < \tau_1 \quad \text{(Regime 2)}
\end{cases}$$

(2.9)

The geometric concentration limit of the primary mirror with optimal tilt then becomes

$$C_{g,1,\text{max,asym}} = \begin{cases} 
C_{g,1,\text{max,asym,R1}} & \text{for } \tau_1 \leq \Phi_2 - \theta_i \quad \text{(Regime 1)} \\
C_{g,1,\text{max,asym,R2}} & \text{for } \Phi_2 - \theta_i < \tau_1 \quad \text{(Regime 2)}
\end{cases}$$

(2.10)

with

$$C_{g,1,\text{max,asym,R1}} = \frac{2 \cos(\Phi_1/2) \cdot \cos(\Phi_2/2) \cdot \sin \Delta \Phi}{\sin 2\theta_i \cdot \sqrt{\cos \Phi_1 + \cos \Phi_2 + 3/2 + 1/2 \cos \Delta \Phi}},$$

$$C_{g,1,\text{max,asym,R2}} = \frac{\sin \Phi_2 - \sin \Phi_1 + \sin \Delta \Phi}{\sin 2\theta_i \cdot [\cos(\Phi_i + \theta_i) + \sec(\Delta \Phi/2) \cdot \cos(\Delta \Phi/2 - \theta_i)]},$$

(2.11)

as shown in Appendix A.1.

In some practical applications a choice $\tau \neq \tau^*$ might be inevitable (e.g. asymmetric primary with symmetric secondary). The primary concentration limits for non-optimal tilt angles are presented in Appendix A.1.3 for completeness. Importantly, for a two-stage system, a tilt angle of $\tau_1$ always results in a higher overall concentration, regardless of whether the primary
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Concentration is in Regime 1 or 2. In other words, tilting the primary outlet aperture by $\tau_2$ is only beneficial for highly-asymmetric parabolic concentrators without a secondary concentrator, while a tilt of $\tau_1$ should always be chosen for two-stage systems. Therefore, for all two-stage designs presented in this work, a tilt angle of $\tau_1$ is used.

**Secondary concentration, $C_{g,2} = a_{o,1}/a_{o,2}$**

As for symmetric concentrators, the method of Hotte l can be used to determine the maximum geometric concentration for asymmetric secondary concentrators [32]. Referring to Figure 2.2, the étendue incident on the asymmetric secondary concentrator from a Lambertian source approximating a parabolic primary (Regime 1) is

$$H_{i,2} = \|A'B'\| - \|A'B\| + \|A'B'\| - \|AB\|$$

$$= a_{o,1} \cdot \frac{\sin(\Phi_2 - \tau_1) + \sin(\tau_1 - \Phi_1)}{\cos \theta_i}, \quad (2.12)$$

while the étendue at the outlet is again $H_{o,2} = 2a_{o,2}\sin \theta_o$ [Eq. (2.5)]. Conservation of étendue then results in the concentration limit

$$C_{g,2,\text{max, asym}} = \frac{2\cos \theta_i}{\sin(\Phi_2 - \tau_1) + \sin(\tau_1 - \Phi_1)}. \quad (2.13)$$

If instead of a CEC an asymmetric CPC is used as secondary concentrator, the maximum concentration becomes [68, 69]

$$C_{g,2,\text{CPC, asym}} = \frac{2}{\sin(\Phi_2 - \tau_1 + \theta_i) + \sin(\tau_1 - \Phi_1 + \theta_i)}. \quad (2.14)$$

An interesting observation can be made when comparing the secondary concentration limits of the CEC to the CPC for both symmetric [compare Eqs. (2.6) and (2.7)] and asymmetric [compare Eqs. (2.13) and (2.14)] concentrators: the factor $\cos \theta_i$ disappears from the numerator, while $\theta_i$ is added inside the sine terms in the denominator. For vanishing acceptance angles, the shape of the CEC
converges to that of the CPC and the concentration of the secondary simplifies to

\[ C_{g,2,\text{asym}} = \frac{2}{\sin(\Phi_2 - \tau_1) + \sin(\tau_1 - \Phi_1)}. \]  

(2.15)

For both the symmetric and asymmetric two-stage system, the overall concentration is simply the product of the primary and secondary concentration

\[ C_{g,\text{tot}} = \frac{a_{g,2}}{a_{i,1}} C_{g,1} \cdot C_{g,2}. \]  

(2.16)

2.3 Comparison of symmetric and asymmetric concentrators

With the concentration limits parametrically defined in terms of the acceptance angle and rim angles of the primary, it is possible to compare the performance of symmetric and asymmetric designs over a wide range of possible configurations. For this purpose, it is of particular importance to define a set of metrics to judge concentrator performance. In this analysis, 6 performance metrics are considered:

1. the primary and overall (total) geometrical concentration ratios, \( C_{g,1} \) and \( C_{g,\text{tot}} \), which give an understanding of the concentration potential of a system under a given acceptance angle;
2. the 2D concentration-acceptance-product, defined as \( \text{CAP}_{2D} = C_g \cdot \sin \theta_i \) [70, 71] which quantifies how close the concentrator is to being ideal (for an ideal étendue-conserving optical system \( \text{CAP}_{2D} = n \), where \( n \) is the refractive index of the medium in which the receiver is immersed);
3. the active area fraction, AAF, defined as the ratio of the unshaded inlet aperture \( a_{i,1} \) to the total projected area of the concentrator \( a_{\text{proj}} \), where a larger number indicates both a larger fraction of the tracking structure is actively capturing radiation and a better ground coverage ratio;
4. the aspect ratio, AR, defined as the ratio of the height \( h \) to the width \( w = a_{\text{proj}} \) of a rectangle enclosing the two-stage concentrator and its receiver [67],
where a smaller AR indicates a more compact design and suggests the potential practical benefits of increased rigidity and lower center of gravity;

(5) the average number of reflections, \( \langle n_r \rangle \), of the system which is an indication of how much energy will be lost due to the imperfect reflectance \( (\rho < 1) \) of real mirrors;

(6) the irradiance distribution at the outlet aperture of the secondary, where a uniform distribution generally indicates superior performance, especially for CPV applications.

It is impractical to develop an analytical solution for the average number of reflections and the irradiance distribution, as these are not defined by the edge rays alone. These quantities can conveniently be determined by Monte Carlo ray-tracing for which the in-house VeGaS code has been utilized [72]. The average number of reflections has been calculated from the ray-tracing results using the method presented in [73],

\[
\langle n_r \rangle \approx \frac{\eta(\rho = 1) - \eta(\rho = 1 - \Delta \rho)}{\Delta \rho \cdot \eta(\rho = 1)},
\]

where \( \eta(\rho) \) is the system efficiency as a function of the reflectance.

In Section 2.3.1 these performance metrics are used to compare symmetric and asymmetric concentrators over a wide range of rim angles. To afford a more intuitive comparison, Section 2.3.2 presents a direct comparison of five pairs of specific symmetric/asymmetric designs sharing certain geometric similarities. All concentrators are designed for full collection with an overall acceptance angle of the 2-stage system of \( \theta_i = 1^\circ \). This acceptance half-angle accounts for tracking errors and mirror surface inaccuracies in practical concentrators and allows for a relaxation of the manufacturing tolerances of the primary [32]. The full collection condition has been verified by checking that for perfect reflectivity \( (\rho = 1) \), the geometrical concentration \( C_g \) equals the flux concentration (radiant power incident on the receiver per unit area). All designs presented in this section further have an inner rim angle \( \Phi_1 = 2\theta_i \). This choice yields the best inlet area utilization while eliminating shading of the primary mirror by the secondary
concentrator, such that primary concentration and active area fraction are maximized.

2.3.1 Performance comparison

Figures 2.3 and 2.4 show the primary ($C_{g,1}$) and overall ($C_{g,tot}$) concentrations as a function of the rim angle $\Phi_2$ of the primary mirror for full collection at $\theta_i = 1^\circ$ and the corresponding CAP_{2D} achievable with symmetric and asymmetric concentrators. Note that the CAP_{2D} is only weakly dependent on the acceptance angle such that the plots in this section can be considered valid for a broad range of $\theta_i$.

The primary concentration of a symmetric parabolic trough is maximized at $\Phi_2 = 45^\circ$ with $C_{g,1} = 27.7$, while the maximum primary concentration of an asymmetric parabolic trough is $C_{g,1} = 25.8$ at $\Phi_2 = 82.61^\circ$. $C_{g,tot}$ of a symmetric two-stage system is maximized at a much smaller rim angle than the primary alone ($C_{g,tot} = 51.5$ at $\Phi_2 = 14.86^\circ$), and falls short of the thermodynamic limit due to increased shading as the rim angle decreases. In contrast, $C_{g,tot}$ of a two-stage asymmetric trough is higher over the whole rim angle range despite having lower $C_{g,1}$, and approaches the thermodynamic limit for small rim spans $\Delta \Phi = \Phi_2 - \Phi_1$. This can be attributed to both the absence of shading of the primary and the higher secondary concentration. While in this section only results with fixed $\Phi_1$ and varying $\Phi_2$ are presented, Appendix A.2 gives results for fixed $\Phi_2$ and varying $\Phi_1$.

Also plotted in Figure 2.4 are the concentration limits achievable with two-stage symmetric and asymmetric concentrators when a CPC is employed instead of a CEC. Since a CPC is not ideal for a source at a finite distance, it reduces the maximum concentration achievable with a symmetric design to $C_{g,tot} = 48.7$ and shifts it to a higher rim angle of $\Phi_2 = 18.35^\circ$. In case of an asymmetric concentrator, the thermodynamic limit is no longer approached for small rim angles but instead the concentration is maximized at $\Phi_2 = 30.69^\circ$ with $C_{g,tot} = 52.0$.

For comparison, Figure 2.4 additionally shows the concentration limit of a symmetric parabolic primary mirror with a symmetric TERC secondary (tailored edge ray concentrator, [39]), designed for full collection and optimally truncated
Two-stage solar concentrators based on parabolic troughs

Two-stage solar concentrators based on parabolic troughs for maximum concentration as discussed in Section 2.2.1. Even though the TERC is ideal in theory, the concentration achieved when using such a concentrator lies below that reached with both two-stage parabola-CEC concentrators for all rim angles, due to shading of the primary. It is worth noting however that designs using TERC secondaries can achieve a very good performance if full collection is not required, especially in point-focus systems where the central shaded area accounts only for a small portion of the total inlet aperture.

To prevent complete shading of the primary inlet aperture by the TERC secondary, the TERC can either be truncated or its outlet aperture can be oversized [39, 40]. The first approach violates the full-collection criterion and will therefore be disregarded here. The second approach conserves full collection but decreases the geometric concentration. For each $\Phi_2$, an optimally truncated TERC is designed to maximize $C_{g,\text{tot}}$ by determining the best trade-off between oversizing of the outlet aperture and minimizing shading of the primary. At low primary rim angles the oversizing required to counteract the shading losses reduces the concentration considerably. With increasing $\Phi_2$, $C_{g,\text{tot}}$ approaches that of the symmetric two-stage concentrator. Starting at $\Phi_2 = 33.5^\circ$ however, the optimal receiver oversizing exceeds the maximum allowed to keep the caustics behind the receiver, which is required for avoiding singularities in the TERC profile [12]. This causes the shading losses to increase dramatically up to $\Phi_2 = 36.4^\circ$, where no oversizing of the outlet is allowed by design as the TERC completely shades the primary.

![Figure 2.5.](image)

**Figure 2.5.** Active area fraction (AAF) of two-stage symmetric and asymmetric concentrators with acceptance angle $\theta_i = 1^\circ$ as a function of the primary rim angle $\Phi_2$. The markers indicate the concentrations of the designs compared in Section 2.3.2.
Figures 2.5 and 2.6 show the active area fraction AAF and the aspect ratio AR, respectively, of two-stage symmetric and asymmetric concentrators as a function of their rim angle $\Phi_2$. In symmetric systems, AAF is governed by the size of the secondary inlet aperture that shades the primary. It is therefore reduced for small $\Phi_2$ where the required outlet aperture, prescribed by the size of the acceptance angle, is large relative to the small primary inlet aperture. It is also reduced for large $\Phi_2$ where the edge rays intersect almost horizontally, resulting in a large secondary inlet aperture. In between there is a broad range of rim angles where symmetric systems have AAF > 0.9. For asymmetric concentrators on the other hand, AAF is controlled by the size of the horizontal projection of the secondary relative to the inlet aperture. The secondary length decreases with increasing $\Phi_2$ due to the higher secondary acceptance angle but at the same time the tilt of the secondary increases. The combination of these two effects causes the projected area of the secondary to remain almost constant and results in an increasing AAF with $\Phi_2$ due to the growing secondary inlet aperture. In general, AAF of symmetric concentrators is superior because the secondary is not tilted.
In regard to AR, symmetric concentrators are more compact compared to asymmetric concentrators having the same $\Phi_2$. The compactness of both designs increases with $\Phi_2$. However, asymmetric systems have a higher concentration in this rim angle range. Therefore, for a fair comparison, AAF and AR are also plotted as a function of the overall concentration $C_{g,tot}$ in Figures 2.7 and 2.8. It becomes clear that when comparing designs of equal concentration, asymmetric concentrators are more compact over the complete concentration range while symmetric concentrators generally have a better area usage.

### 2.3.2 Comparison of exemplary symmetric and asymmetric concentrator designs

The advantages of asymmetric two-stage concentrators can more readily be seen by comparing a few exemplary designs that have certain geometrical similarities (e.g. equal $\Phi_2$, $C_{g,tot}$, or AR). Five pairs of symmetric and asymmetric systems
are shown in Figure 2.9. Table 2.1 provides a summary of the results, which are also indicated by the markers in Figures 2.3-2.8. The five cases are:

**Equal rim angles, Φ₂ = 45°**

As a first comparison, consider symmetric and asymmetric systems based on a parabolic trough primary with Φ₂ = 45° (Θ), as it produces the maximum \( C_{g,1} = 27.7 \) and is reasonably compact in the symmetric case. While \( C_{g,1} \) of the asymmetric concentrator is much lower (19.1×), its \( C_{g,2} \) is almost doubled due to the reduction of the acceptance angle required by the CEC secondary (2.78× for asymmetric vs. 1.41× for symmetric). This leads to a much higher overall \( C_{g,tot} \) for the asymmetric case (53.2× for asymmetric vs. 39.1× for symmetric).
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Equal rim angles, $\Phi_2 = 65^\circ$

As noted in Section 2.3.1, the superiority of asymmetric concentrators vis-à-vis their symmetric counterparts becomes most evident for large $\Phi_2$. Consider symmetric and asymmetric systems based on a parabolic trough primary with $\Phi_2 = 65^\circ$ ($\nabla$). The symmetric system has a lower $C_{g,1}$ ($24.3\times$ for asymmetric vs. $21.0\times$ for symmetric) due to the coma of the parabola and a lower $C_{g,2}$ ($1.99\times$ for asymmetric vs. $1.10\times$ for symmetric) due to the high acceptance angle required for the secondary. This results in a much lower $C_{g,tot}$ for the symmetric system as
compared to the asymmetric one (48.2× for asymmetric vs. 23.1× for symmetric). While the compactness of the asymmetric concentrator is worse (AR = 0.80 for asymmetric vs. AR = 0.41 for symmetric), it can be improved using a compound configuration as detailed in Section 2.4.1.

**Equal overall concentration,** $C_{g,\text{tot,asym}} = C_{g,\text{tot,sym}} (\Phi_2 = 45^\circ)$

As mentioned in Section 2.3.1, AR of symmetric and asymmetric concentrators are best compared between systems of identical $C_{g,\text{tot}}$. The same concentration of $C_{g,\text{tot}} = 39.1$ as the symmetric concentrator from Case 1 at $\Phi_{2,\text{sym}} = 45^\circ$ can be obtained by an asymmetric concentrator at $\Phi_{2,\text{asym}} = 89.45^\circ$. While $C_{g,1}$ of the asymmetric concentrator is lower (25.6× for asymmetric vs. 27.7× for symmetric), $C_{g,2}$ compensates for this difference (1.53× for asymmetric vs. 1.41× for symmetric). The notable advantage of the asymmetric design is a significantly improved compactness (AR = 0.52 for asymmetric vs. AR = 0.63 for symmetric).

**Equal overall concentration,** $C_{g,\text{tot,asym}} = C_{g,\text{tot,max,sym}}$

A symmetric two-stage concentrator with acceptance angle $\theta_i = 1^\circ$ achieves its maximum $C_{g,\text{tot}} = 51.5$ at $\Phi_2 = 14.86^\circ$. An asymmetric system with the same $C_{g,\text{tot}}$ is more than twice as compact (AR = 1.04 for asymmetric vs. AR = 2.10
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for symmetric). Its compactness can further be 2-fold improved by a two-wing configuration, as outlined in Section 2.4.1.

Equal aspect ratio, $\text{AR} = \text{AR}_{\text{asym}} (\Phi_2 = 70^\circ)$

An asymmetric concentrator at $\Phi_2 = 70^\circ$ has the same AR as a symmetric one at $\Phi_{2,\text{sym}} = 39.55^\circ$, lower $C_{g,1}$ (25.0× for asymmetric vs. 27.1× for symmetric) but higher $C_{g,2}$ (1.86× for asymmetric vs. 1.57× for symmetric) which results in a higher $C_{g,\text{tot}}$ (46.6× for asymmetric vs. 42.6× for symmetric) (◇).

Table 2.1 summarizes the performance metrics discussed above, and also lists the average number of reflections $\langle n_r \rangle$ for each exemplary design. As all rays undergo strictly one reflection on the primary mirror, the difference in $\langle n_r \rangle$ is related solely to the design of the secondary concentrator and increases in general with increasing $C_{g,2}$. For this reason, $\langle n_r \rangle$ tends to be slightly higher for asymmetric systems, with the notable exception of Case 4 where $C_{g,2}$ of the asymmetric design and hence $\langle n_r \rangle$ are lower than those for the symmetric system.
Figure 2.10 compares the irradiance distribution at the outlet of the secondary concentrators of a symmetric system at $\Phi_{2,\text{sym}} = 45^\circ$ ($C_{g,\text{tot,sym}} = 39.1$) with an asymmetric concentrator of equal rim angle ($\Phi_{2,\text{asym}} = 45^\circ$; Case 1) and with an asymmetric concentrator of equal concentration ($C_{g,\text{tot,asym}} = 39.1$; Case 3). Monte-Carlo ray tracing simulations were performed with $10^9$ rays, $\theta_i = 1^\circ$, $\rho = 100\%$ and DNI = 1 kW/m$^2$. All systems are scaled to have an inlet aperture width of 1 m. The higher $C_{g,\text{tot}}$ with the asymmetric concentrator of Case 1 is evident from the smaller width of the irradiance distribution and the higher irradiance peak. The asymmetric concentrator of Case 3, reaching the same concentration as the symmetric counterpart, produces a slightly less uniform irradiance distribution.

2.4 Compound asymmetric designs

While the basic asymmetric systems discussed in the previous sections exhibit considerable advantages in both concentration and compactness compared to their symmetric counterparts, there exist several practical compound asymmetric configurations that can further boost their performance. In this section, we explore two such configurations which achieve notably higher concentrations and lower aspect ratios.

2.4.1 Two-wing asymmetric concentrators

A possible practical configuration for asymmetric single- or two-stage systems can be realized by combining two concentrators side-by-side on a single tracker [66, 74] as shown in Figure 2.11. Such a two-wing asymmetric design can essentially double the compactness of the asymmetric concentrators presented in Section 2.3 while reaching the same high concentrations. The aspect ratio for two-wing asymmetric designs is indicated by the dashed lines in Figures 2.6 and 2.8. Figure 2.11 shows the application of the two-wing concept to the asymmetric concentrators of Case 2 and Case 4 presented in Section 2.3.2 and compares them to the respective symmetric counterparts scaled to have the same inlet aperture width. In Case 2 both concentrators have $\Phi_2 = 65^\circ$ and, as shown before, the asymmetric one reaches a much higher $C_{g,\text{tot}}$ (48.2× for asymmetric vs. 23.1× for symmetric) but would be less compact in the single-wing
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A two-wing asymmetric design conserves the higher concentration advantage while being of same compactness as the symmetric one (AR\textsubscript{asym,2-wing} = 0.40 vs. AR\textsubscript{sym} = 0.41). In Case 4 both concentrators achieve \( C_{g,\text{tot}} = 51.5 \), but using a 2-wing configuration the asymmetric concentrator is 4 times more compact (AR\textsubscript{asym,2-wing} = 0.52 vs. AR\textsubscript{sym} = 2.10).

Figure 2.11. Two-wing asymmetric concentrators compared to the corresponding symmetric concentrators for Case 2 (\( \Phi_2 = 65^{\circ} \)) and Case 4 (\( C_{g,\text{tot,asym}} = C_{g,\text{tot,max,sym}} \)) scaled to have the same inlet aperture. The asymmetric concentrator of Case 2 reaches a much higher concentration (48.2× asym. vs. 23.1× sym.) with same compactness. The asymmetric concentrator of Case 4 reaches the same concentration of 51.5× while being 4 times more compact (AR\textsubscript{asym,2-wing} = 0.52 vs. AR\textsubscript{sym} = 2.10).

configuration. A two-wing asymmetric design conserves the higher concentration advantage while being of same compactness as the symmetric one (AR\textsubscript{asym,2-wing} = 0.40 vs. AR\textsubscript{sym} = 0.41). In Case 4 both concentrators achieve \( C_{g,\text{tot}} = 51.5 \), but using a 2-wing configuration the asymmetric concentrator is 4
times more compact than the symmetric one ($AR_{\text{asym,2-wing}} = 0.52$ vs. $AR_{\text{sym}} = 2.10$).

2.4.2 Nested asymmetric concentrators

Another interesting concentrator emerged from the realization that the concentration limit of asymmetric concentrators of given rim angle $\Phi_2$ increases with increasing inner rim angle $\Phi_1$. In fact for a fixed $\Phi_2$, $C_{g,\text{tot}}$ approaches the thermodynamic limit as the rim span $\Delta \Phi = \Phi_2 - \Phi_1$ is reduced, as shown in Appendix A.2.

An exemplary concentrator with $\Phi_1 = 60^\circ$ and $\Phi_2 = 90^\circ$, designed for $\theta_i = 1^\circ$ is shown in Figure 2.12(a). This design achieves $C_{g,\text{tot}} = 54.23$, falling only 5\% short of the thermodynamic limit ($CAP_{2D} = 0.95$), but suffers from a poor area utilization ($AAF = 0.39$) due to the inactive region between the inner rim angle and the optical axis.

Figure 2.12. (a) Asymmetric concentrator with $\Phi_1 = 60^\circ$ and $\Phi_2 = 90^\circ$ ($CAP_{2D} = 0.95$, $AAF = 0.39$); (b) similar geometry as in (a) but with the secondary folded to the other side by a vertical mirror extending up from the inner rim, thus improving $AAF$ ($\Phi_1 = 52.56^\circ$, $\Phi_2 = 89^\circ$, $CAP_{2D} = 0.93$, $AAF = 0.88$, $AR = 1.07$).
Two-stage solar concentrators based on parabolic troughs

To increase AAF while maintaining a high CAP$_{2D}$, one could add a vertical mirror extending up from the inner rim to effectively fold the secondary to the other size. It is convenient to set $\Phi_2 = 90^\circ - \theta_i$ such that the secondary is directly attached to the rim of the primary and to set $\Phi_1$ such that the primary extends over half of the unshaded width from the optical axis to the rim, as shown in Figure 2.12(b). The resulting concentrator with $\Phi_1 = 52.56^\circ$ and $\Phi_2 = 89^\circ$ achieves AAF = 0.88 while retaining a high concentration ($C_{g,tot} = 53.03$, CAP$_{2D} = 0.93$). However, this design is not very compact (AR = 1.07) and suffers from the additional reflection on the vertical mirror.

Both these issues are resolved if the vertical mirror is replaced by the mirror image of the primary and secondary concentrators as illustrated in Figure 2.13. (a) similar geometry as in Figure 2.12(b) but replacing the vertical mirror by the mirror image of the primary and secondary concentrators, thus improving compactness (CAP$_{2D} = 0.93$, AAF = 0.88, AR = 0.37); (b) similar geometry as in (a) but replacing the reflective secondary by a dielectric-filled CEC, thus increasing concentration (CAP$_{2D} = 1.39$, AAF = 0.81, AR = 0.33). The nested single-mirror two-stage design shown in (a) has a higher concentration and better compactness than the best reported reflective two-mirror aplanatic or 2D SMS designs with a lower average number of reflections $\langle n_r \rangle = 1.79$. 

Figure 2.13. (a) similar geometry as in Figure 2.12(b) but replacing the vertical mirror by the mirror image of the primary and secondary concentrators, thus improving compactness (CAP$_{2D} = 0.93$, AAF = 0.88, AR = 0.37); (b) similar geometry as in (a) but replacing the reflective secondary by a dielectric-filled CEC, thus increasing concentration (CAP$_{2D} = 1.39$, AAF = 0.81, AR = 0.33). The nested single-mirror two-stage design shown in (a) has a higher concentration and better compactness than the best reported reflective two-mirror aplanatic or 2D SMS designs with a lower average number of reflections $\langle n_r \rangle = 1.79$. 

To increase AAF while maintaining a high CAP$_{2D}$, one could add a vertical mirror extending up from the inner rim to effectively fold the secondary to the other size. It is convenient to set $\Phi_2 = 90^\circ - \theta_i$ such that the secondary is directly attached to the rim of the primary and to set $\Phi_1$ such that the primary extends over half of the unshaded width from the optical axis to the rim, as shown in Figure 2.12(b). The resulting concentrator with $\Phi_1 = 52.56^\circ$ and $\Phi_2 = 89^\circ$ achieves AAF = 0.88 while retaining a high concentration ($C_{g,tot} = 53.03$, CAP$_{2D} = 0.93$). However, this design is not very compact (AR = 1.07) and suffers from the additional reflection on the vertical mirror.

Both these issues are resolved if the vertical mirror is replaced by the mirror image of the primary and secondary concentrators as illustrated in Figure
2.13(a). The resulting concentrator is then composed out of two equal asymmetric two-stage concentrators, nested by reflection symmetry about a line extending up from the inner rim angle. This design achieves a very good compactness and concentration (for $\theta_i = 1^\circ$: AR = 0.37, $C_{g,\text{tot}} = 53.03\times$, $\text{CAP}_{2D} = 0.93$) with $\langle n_r \rangle = 1.79$. It performs well compared to concentrators designed with more advanced design methods. For example, the simultaneous multiple surface (SMS) method [42] has been used to design the SMTS concentrator for $\theta_i = 1.07^\circ$, having AR = 0.44, $\text{CAP}_{2D} = 0.84$ and $\langle n_r \rangle = 2.02$ [43].

Another class of two-stage concentrators achieving good compactness and high concentration by eliminating spherical and comatic aberrations are the two-mirror aplanats. A detailed investigation of the optical performance has been carried out for the 3D Case [41]. For the 2D Case, the best reported designs, while achieving good compactness, suffer from large intrinsic losses due to shading of the primary by the secondary mirror$^d$. They achieve, at best, a $\text{CAP}_{2D}$ of 0.85 and, due to their imaging design, all rays reaching the receiver undergo exactly two reflections such that $\langle n_r \rangle = 2.00$.

A similar parabola-CEC system can be designed using a dielectric-filled CEC secondary concentrator, e.g. a glass with refractive index $n = 1.5$, as shown in Figure 2.13(b). Such a design can either boost $C_{g,\text{tot}}$ by a factor of $n$ or enables a higher acceptance angle for the same concentration. Fresnel reflection losses at the inlet of the dielectric secondary are partially offset by the fact that all rays undergo total internal reflection within the secondary. Designed for $\theta_i = 1^\circ$, such a concentrator achieves $C_{g,\text{tot}} = 79.54$ (CAP$_{2D} = 1.39$) and AR = 0.33. The comparable dielectric SMS design, the DSMTS concentrator, achieves $\text{CAP}_{2D} = 1.25$ for $\theta_i = 0.72^\circ$ and $\text{CAP}_{2D} = 1.33$ for $\theta_i = 2.55^\circ$ [44].

---

$^d$ Aplanats are nearly ideal as they do not suffer from spherical aberration or coma. If shading is neglected, they approach the thermodynamic limit of concentration $C_{g,\text{max},2D}$. However, for practical (compact) 2D designs, the shading considerably impacts the achievable concentration. In contrast, in our asymmetric two-mirror configuration, shading is diminished while the losses in concentration are primarily due to the coma of the parabolic primary. This coma is precisely what is minimized by taking an asymmetric design with a reduced rim span, increasing the contribution by the nonimaging secondary concentrator.
2.5 Summary and conclusions

A detailed investigation of asymmetric line-focus two-stage concentrating systems, comprising an image-forming parabolic primary in tandem with a nonimaging CEC secondary has been carried out. The concentrators are designed for a flat receiver geometry and are relevant for cavity absorber and concentrating photovoltaic applications.

Exemplary symmetric and asymmetric concentrators were examined in terms of the key optical performance metrics: concentration, acceptance angle, concentration-acceptance-product (CAP$_{2D}$), compactness, active area fraction and average number of reflections. The irradiance distributions at the concentrator outlet apertures were simulated using Monte-Carlo ray tracing.

It was shown that asymmetric two-stage concentrators can reach higher concentrations than their symmetric counterparts, approaching the thermodynamic limit for small rim angle spans. The increase in concentration is primarily due to the reduction of shading due to the removal of the obstruction of the inlet aperture caused by the secondary, allowing smaller rim spans to be utilized. While symmetric concentrators can in general have a better active area fraction and aspect ratio, they are bound by an inherent trade-off between concentration and compactness. Asymmetric designs are therefore always more compact than symmetric systems designed for the same concentration. Five exemplary pairs of symmetric and asymmetric systems were compared to better illustrate the advantages of asymmetric two-stage concentrators. In terms of practical considerations, the asymmetric designs may suffer from a more complex structural design and off-axis alignment due to their asymmetry.

Two practical and compact configurations that further increase the performance of asymmetric systems were presented. Firstly, the two-wing configuration was introduced, which comprises two asymmetric concentrators side by side on a single tracker. Two-wing designs can double the compactness of asymmetric concentrators and further leverage their benefits compared to symmetric concentrators. Thus, from a practical point of view, the disadvantages of the added complexity due to the asymmetric structure are alleviated. In particular, an asymmetric design that achieves a CAP$_{2D}$ of 0.90 while being 4 times as compact as a comparable symmetric design was introduced. Secondly,
the nested concentrator, composed out of two asymmetric two-stage concentrators with high inner rim angle, nested by reflection symmetry about a line extending up from the inner rim was presented. Exemplary nested designs reaching a $\text{CAP}_{2D}$ of 0.93 (using a hollow reflective secondary) and 1.39 (using a dielectric-filled secondary), higher than the best reported two-mirror aplanatic or 2D SMS designs and with very good compactness, were presented. The designs presented here offer the additional advantages of: (1) having analytical solutions for the mirror geometries; and (2) of being based on commonly employed parabolic primaries.
3 A new device for absolute irradiance mapping

For the characterization of the optical performance of solar concentrators, the irradiance distribution attained in the focal plane under on-sun conditions is of major importance. This chapter presents the background, design and measurement methodology of a new absolute irradiance measurement system, which mitigates many of the limitations of the existing methods and is employed for the assessment of solar concentrators through the subsequent chapters of this work.

3.1 Theory and background

Commonly, this spatial distribution is recorded by a system similar to the one depicted in Figure 3.1 wherein a camera images the irradiance distribution diffusely reflected by a Lambertian reflector installed in the focal plane of the concentrator. As shown in Appendix B.1, the radiance diffusely reflected by the Lambertian target is directly proportional to the incident irradiation.

The grayscale value, \( GV(x,y) \), of each pixel recorded by the camera sensor therefore provides a relative spatial measurement of the irradiance distribution. While obtaining this relative, i.e. arbitrarily scaled, distribution is straightforward, the challenge consists in determining the scaling factor \( F \) between the relative distribution and the desired absolute irradiance distribution \( E(x,y) \) such that

\[
E(x, y) = F \cdot GV(x, y). \quad (3.1)
\]

Two different techniques to determine \( F \) are commonly employed: (1) by means of a cavity calorimeter; and (2) using a circular foil radiometer.

---

The first method consists in performing a separate measurement by placing a cavity-type calorimeter in the center of the focal plane in lieu of the Lambertian reflector and then computing $F$ by equating the measured radiative power with the integral of the recorded grayscale map over the region of the calorimeter aperture [75, 76]. The advantages of this method are: (1) a very low directional and spectral dependence of the measured radiative power due to the near blackbody behavior of the cavity; and (2) low thermal losses from the cavity since it is kept at a low temperature and can be well insulated. The downside is the temporal offset between the calorimeter measurement and the image.

Figure 3.1. (a) Schematic of the Lambertian Flat-Plate Calorimeter (LFPC) absolute irradiance mapping system; (b) photograph of the Al$_2$O$_3$-coated Lambertian target mounted in the focal plane; and (c) photograph of the CMOS camera mounted at the base of the dish. The camera images the radiation diffusely reflected by the Lambertian target, providing a relative distribution of the irradiance at the focal plane, while the calorimeter continuously measures the radiative power in the central region (ROI) of the target, allowing the absolute scale of the irradiance map to be determined.
recording as well as the large thermal inertia of the system that cannot resolve the time scale of the changing direct normal irradiance (DNI) during on-sun measurements.

The second and most common method used to scale the relative irradiance map is by incorporating a circular foil radiometer, or Gardon gauge [77], with a small active area into the Lambertian reflector. This allows for the simultaneous recording of the full relative irradiance map and of the absolute irradiance at a discrete point on that map that can be used to compute $F$. Additionally, foil radiometers offer the advantage of a fast response time (< 1 s) allowing them to react well to fluctuations in DNI. However, this method has several drawbacks, particularly when used in systems that exhibit large spatial gradients of the irradiance. Firstly, the Gardon gauge is effectively a point radiometer as the active foil diameter is typically on the order of a few millimeters, and therefore cannot provide the same accuracy in the total radiative power measurement as the larger-aperture calorimeter. Secondly, the irradiance map recorded by the camera is disturbed by the radiometer due to its low reflectance coating in contrast to the white target coating. The part of the irradiance map obscured by the radiometer is precisely the region that must be accurately known to compute the scaling factor. For reasonably smooth irradiance distributions, the irradiance map can be interpolated over the gauge area, but the disturbance causes an unknown source of error that is difficult to quantify. Thirdly, circular foil radiometers are susceptible to spectral and directional errors, since they use an absorbing surface rather than a blackbody cavity. Specifically, special care must be taken to relate the gauge calibration, typically performed for radiation from a blackbody at 850 °C, to the spectral and directional conditions at which the gauge is being used. For example, the gauges designed for high irradiance (> 3500 kW/m²) are often coated with colloidal graphite and can exhibit a measurement error up to 30% over-prediction caused by the spectral dependency of the coating [78]. Finally, at high irradiance, the temperature at the center of the Gardon gauge can exceed temperatures 100 °C higher than ambient, making the measurement sensitive to convection losses, especially in windy conditions.

A simple, less common method [79] relies on the assumption that all radiation reflected from the concentrator hits the target surface. The sum of the gray values
over the target can then be related to the total radiative power reaching the target. However, the method introduces critical uncertainties. For the calculation of the total radiative power on the target, the geometry and reflectance of the concentrator need to be accurately known. The DNI is needed to measure absolute irradiance, in contrast to the methods presented above, where it is only needed to calculate flux concentration based on the absolute irradiance. Further, the method places a major importance on the low-flux/large-area peripheral regions of the irradiance map, which inevitably leads to significant errors. Lastly, the method is also prone to larger errors caused by changes in the spectral composition of the incident irradiation such that the calibration factor for each measurement needs to be calculated separately.

In this chapter, we propose an instrument that mitigates many of the aforementioned limitations of the existing approaches. A schematic of the system, which we denote the Lambertian Flat-Plate Calorimeter (LFPC), is shown in Figure 3.1. If the cooled Lambertian target is designed such that the average incident irradiance is high enough, then it can effectively be employed as a flat-plate calorimeter able to perform an accurate absolute measurement of the total incident radiative power while measuring the irradiance distribution at the same time. Figure 3.2(a) gives a representative schematic of the heat flow through the system. The major part of the radiation incident on the target is diffusely reflected and recorded by the camera. The remaining radiation is absorbed in the target and can be calorimetrically measured.

A preliminary evaluation indicates that accurate measurements are possible with the proposed system. Consider a target with a front surface reflectance of $\rho = 80\%$, a perfectly insulated back surface, and assume that the water enters the calorimeter at ambient temperature and that its flow rate through the calorimeter is chosen such that a water temperature rise of $\Delta T = T_{\text{out}} - T_{\text{in}} = 20 \, ^\circ\text{C}$ occurs across the calorimeter inlet and outlet. The calorimeter will measure the net heat captured by its front surface, i.e. absorbed radiation minus convection to the environment. Therefore, to correctly act as a radiometer, the convection should be small compared to the absorbed radiation. Considering a convective heat transfer coefficient of $h_{\text{conv}} = 50 \, \text{W/(m}^2\cdot\text{K)}$, which is representative of moderate
forced convection, and assuming the front surface temperature is equal to the calorimeter outlet temperature, the convection losses amount to 1 kW/m². If the system is used to measure radiation from a solar concentrator having an average irradiance of 500 kW/m², i.e. 500 suns, then the convective losses will amount to just 1% of the absorbed radiative power (100 kW/m² for $1 - \rho = 0.2$). This calculation indicates that measurement accuracies on the order of 1% or better should be reachable with the LFPC.

In a typical irradiance mapping system for point-focus concentrators, the target is designed such that the distribution on a large area around the central “hot
spot” can be measured. This allows the full characterization of the peripheral regions, which provide important information on the specularity of the concentrator. However, as a consequence of this large target size compared to the hot spot, the average irradiance over the target becomes relatively small, leading to higher relative convection losses compared to the illustrative calculation above. For example, if the average irradiance drops to 50 kW/m² over the target, then the convection losses would increase to 10% of the absorbed radiation under the same conditions as above. A simple solution to avoid this issue is to divide the Lambertian target into two concentric parts, separated by a small air gap and each having an independent cooling circuit, as detailed in Figure 3.1(a). The central part can then be dimensioned such that it covers an area slightly larger than the hot spot, i.e. small enough to ensure that the average irradiance over its surface is high, but still large enough that a good integral radiative power measurement is achieved. The outer part then only serves as an extension of the Lambertian target for the recording of the full irradiance map. If both parts are covered with the same Lambertian coating, \( F \) determined for the central part can be applied to the full target. In this way, an accurate calorimetric measurement is achieved without disturbing the image in the region of interest. As an additional benefit, this concentric design allowed the installation of a kaleidoscope optical mixer [80, 81] in the Lambertian target with the purpose of homogenizing the irradiance distribution for the on-sun characterization of the first HCPVT receiver prototype [59] (Section 4.3.2).

### 3.2 Experimental setup

Figure 3.1 shows the experimental setup of the proposed absolute irradiance-mapping device. A CMOS camera (Basler acA2500-14gm, resolution 2592 × 1944 pixels) is mounted at the base of the dish to capture the relative irradiance distribution (8-bit) on the Lambertian target in the focal plane. A neutral density filter (Thorlabs NE230B, OD3.0) is used to prevent an overloading of the camera detector.
A new device for absolute irradiance mapping

A schematic of the target with relevant details is given in Figure 3.3. The outer part of the Lambertian target consists of two bolted aluminum plates with external dimensions 300 × 300 mm. Cooling channels milled into the front plate evenly distribute the water flow between the inlet and outlet and keep the target at safe temperatures. The inner calorimetric plate fits into a 53 × 57 mm opening in the center of the outer aluminum part. The calorimetric part is made of copper to increase heat conduction and, similar to the outer part, is comprised of two bolted plates with cooling channels. It has front surface dimensions of 52 × 56

Figure 3.3. Schematic of the absolute irradiance mapping system with the calorimetric part of the Lambertian Flat-Plate Calorimeter (LFPC) installed (a) in the focal plane; and (b) at the exit of a homogenizing kaleidoscope secondary optic.
mm, leaving a 0.5 mm air gap between the two target bodies. To diminish heat losses from the inner to the outer part, the air gap is increased to 1 mm behind the front surface and the calorimeter is held in place solely by low-conductivity Polyoxymethylene (POM) beams mounted at the rear of the target. Additionally, the backside of the calorimetric part is thermally insulated to reduce heat losses to the surroundings. Both the copper calorimeter and aluminum target are coated with Al$_2$O$_3$ (Aluminum oxide aerosol refractory paint, Alfa Aesar) to achieve the uniform and highly Lambertian surface required for the irradiance mapping.

In an alternative setup, the calorimeter can be placed at the outlet of a kaleidoscope secondary optic, as shown in Figure 3.3(b). The refrigerated kaleidoscope has a length of 130 mm, allowing for a good homogenization of the incident irradiation. It consists of 4 water-cooled aluminum walls with highly reflective mirror foils (Solar Mirror Film 1100 3M™ [82]) glued to their inner surfaces. This setup was used for the measurement of the homogenized irradiance acting as input for the HCPVT receiver characterization [59]. Four-wire resistance temperature detectors (RTD, Omega RTDM12-1/8NPT-3MM-24MM-A) monitor the inlet ($T_{\text{in}}$) and outlet ($T_{\text{out}}$) temperatures of the cooling water through the calorimeter, and a turbine flow meter (Omega FTB601B-T) measures the volumetric flow rate $\dot{V}$. An additional RTD (Omega RTDM12-1/8NPT-3MM-13MM-A) records the water temperature at the exit of the flow meter ($T_{\text{flow}}$) allowing the computation of the mass flow $\dot{m} = \dot{V} \cdot D(T_{\text{flow}})$, where $D$ is the density of the water. The rate of heat absorbed in the water flow is then given by

$$\dot{Q}_{\text{flow}} = \dot{m} \cdot c_p \cdot (T_{\text{out}} - T_{\text{in}}),$$  

(3.2)

where $c_p$ is the mass-specific heat capacity of the cooling water. The DNI is continuously recorded using an on-site pyrheliometer. Finally, the ambient temperature and the wind speed are measured to log the external conditions during the experiment. An error analysis performed for the described setup indicates that the expected uncertainty of the irradiance measurement [Eq. (3.1)] is ±3.0% (Appendix B.4). The choice of the reflectance of the Lambertian surface is of importance for the experimental precision. A high reflectance amplifies the calorimeter inaccuracies, while a low reflectance reduces the
intensity of the irradiance distribution, leads to larger radiative and convective energy losses, and can cause issues with the cooling of the target.

3.3 Device calibration

In the ideal steady-state case without convective or conductive losses, $Q_{\text{flow}}$ would be equal to the incident solar radiative power absorbed by the calorimeter $\dot{Q}_{\text{ROI}} = (1 - \rho_{\text{Al}_2\text{O}_3}) \cdot \dot{Q}_{\text{in}}$, where $\dot{Q}_{\text{in}}$ is the incident radiative power and $\rho_{\text{Al}_2\text{O}_3}$ is the reflectance of the target coating. However, in practice, it is not possible to completely eliminate thermal losses from the system. Thus, to correct for these losses, the system is designed such that it can be electrically calibrated off-sun. Using electrical cartridge heaters inserted into the front surface (Figure 3.3) a known thermal power input $\dot{Q}_{\text{el}} = V \cdot I$, measured by four-terminal sensing, substituting the absorbed solar power $\dot{Q}_{\text{ROI}}$, can be supplied to the system. The heat fluxes in this configuration are indicated in Figure 3.2(b). The key to the calibration is to keep the conditions similar between calibration and experiment such that $\dot{Q}_{\text{el}} = \dot{Q}_{\text{ROI}}$. It was verified using CFD simulations that the effect on the thermal losses resulting from the differences in surface and volumetric temperature distribution between calibration and experiment is negligible. It is therefore sufficient to equate absolute heat supplied to the ROI. Calibration can be performed in situ before or after an on-sun measurement, such that the impact of the external conditions (i.e. ambient temperature, cooling water inlet temperature and flow rate, wind speed) on the thermal losses are correctly accounted for. The response of $\dot{Q}_{\text{flow}}$ to $\dot{Q}_{\text{el}}$ can then be measured and consequently the associated losses $\dot{Q}_{\text{loss}}$ can be determined.

Figure 3.4 presents a representative calibration measurement using a cooling water flow rate around 0.15 l/min and stepwise increasing input radiative powers $\dot{Q}_{\text{el}}$ of 10 W, 100 W, 200 W, 300 W and 400 W. The response of $\dot{Q}_{\text{flow}}$ is recorded during the duration of the measurement. At each step, the rate of heat absorbed in the flow reaches a steady state after a transient phase of 100-200 s. In practice, the values can be averaged over a particular steady-state time interval to decrease the influence of small fluctuations. The steady-state intervals are shaded in gray. Figure 3.5 shows the steady-state response of $\dot{Q}_{\text{flow}}$ at each of the steps in supplied $\dot{Q}_{\text{el}}$. The thermal losses of the system increase linearly with the input
radiative power and are in the range of 7% of the absorbed radiative power. A linear calibration curve

\[ \dot{Q}_{\text{flow}} = \frac{1}{a} \dot{Q}_{\text{el}} + \frac{-b}{a} \]  

(3.3)

can therefore be fitted to the measurement data. Knowing the calibration parameters \( a \) and \( b \), and assuming an equivalent system response during on-sun measurements, the radiative power absorbed in the ROI can then be corrected for thermal losses using

\[ \dot{Q}_{\text{ROI}} = a \cdot \dot{Q}_{\text{flow}} + b. \]  

(3.4)

### 3.4 On-sun measurement

Figure 3.6 shows the procedure of a representative on-sun absolute irradiance measurement. In a first step, (a), the radiative power absorbed by the ROI, \( \dot{Q}_{\text{ROI}} \),
is continuously measured, while the relative irradiance distribution on the Lambertian target is recorded by the CMOS camera. A perspective correction using the target corners as reference points is applied to the camera image before it is cropped to the exact size of the target. The irradiance scaling factor $F$ can then be computed by dividing the absorbed radiative power in steady-state, $\dot{Q}_{ROI}$, by the absorptance of the target coating, $\alpha_{Al2O3} = 1 - \rho_{Al2O3}$, and the pixel grayscale value $GV(x,y)$ integrated over this area,

$$ F = \frac{\dot{Q}_{ROI}}{(1 - \rho_{Al2O3}) \cdot \int_{ROI} GV(x,y) dx dy}. \quad (3.5) $$

The solar-averaged hemispherical reflectance of an alumina-coated copper sample, $\rho_{Al2O3}$, was determined by solar weighting of the spectral hemispherical reflectance $\rho_{Al2O3,\lambda}$ which was experimentally measured using UV-Vis-NIR spectroscopy (Appendix B.2). In a second step, (b), this scaling factor, having units of W/(m$^2$·GV), can then be used to evolve the absolute irradiance distribution on the entire image using Eq. (3.1).
Figures 3.7 and 3.8 show a representative optical characterization measurement using the concentrator introduced in Chapter 4. The 18 mirrors of the concentrator are focused gradually in groups of 6 (cf. Figure 4.3; mirrors nr. 1-6, 7-12, 13-18). In Figure 3.7 the flow rate ($\dot{V}$) and calorimeter inlet and outlet temperatures ($T_{in}$, $T_{out}$) are plotted as a function of the measurement time. After a flow rate adjustment causing fluctuations in the measurement, $\dot{V}$ is kept constant around 0.15 l/min. While the water inlet temperature slightly rises proportionally to the ambient temperature during the course of the measurement, the outlet temperature shows a strong response to the increase in incident solar radiation, culminating in a final steady-state temperature difference $\Delta T = 53 \, ^\circ\text{C}$.
A new device for absolute irradiance mapping

For the fully focused concentrator. It was the aim in this measurement to accurately meter the incident radiative power, i.e. with large enough $\Delta T$, over the full range of incident radiative power and using a constant water flow rate such that the calibration correction (Section 3.3) can be readily applied. However, the flow rate could be increased under high-irradiance conditions to lower the $\Delta T$, e.g. to 20 °C, and reduce the thermal losses accordingly.

In Figure 3.8, the evolution of the measured radiative power absorbed in the cooling water, $\dot{Q}_{\text{flow}}$, and of the corrected radiative power absorbed in the ROI, $\dot{Q}_{\text{ROI}}$, are plotted. Together with the measured solar-averaged reflectance $\rho_{\text{Al2O3}}$ of the Lambertian coating and the recorded relative irradiance distribution image $G\nu(x,y)$, this data allows the determination of the scaling factor $F$ and ultimately the generation of the irradiance distribution over the complete target in the focal

Figure 3.7. Measured flow rate ($\dot{V}; \text{l/min}$) and inlet and outlet temperatures ($T_{\text{in}}, T_{\text{out}}; ^{\circ} \text{C}$) of a representative on-sun optical characterization measurement. The radiative power absorbed in the calorimeter, $\dot{Q}_{\text{ROI}}$, is continuously measured while images of the Lambertian target are periodically taken with the CMOS camera. In the shown measurement the 18 mirrors of the concentrator were focused gradually in groups of 6, resulting in the stepwise increase of measured radiative power.
plane using Eqs. (3.5) and (3.1). Also shown is the DNI, which varies between 865 kW/m$^2$ and 940 kW/m$^2$ during the course of the measurement.

### 3.5 Measurement uncertainty

It is essential to determine the accuracy of the irradiance measurement with the described system as a function of the uncertainties of the individual measured entities and their propagation through the system.

An error analysis with uncertainty propagation [83] was therefore performed for the LFPC, where the absolute uncertainty of a multivariate function $f(x_1, x_2, \ldots, x_n)$ with a set of individual absolute uncertainties $\{\sigma_{x_1}, \sigma_{x_2}, \ldots, \sigma_{x_n}\}$ in the variables is obtained from

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x_1}\right)^2 \sigma_{x_1}^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 \sigma_{x_2}^2 + \ldots + \left(\frac{\partial f}{\partial x_n}\right)^2 \sigma_{x_n}^2}.$$  

(3.6)
E.g. the absolute uncertainty in irradiance \( E = F \cdot GV \); Eq. (3.1)) decomposes into

\[
\sigma_E = \sqrt{\left(\frac{\partial E}{\partial F}\right)^2 \sigma_F^2 + \left(\frac{\partial E}{\partial GV}\right)^2 \sigma_{GV}^2}
\]

\( = \sqrt{GV^2 \sigma_F^2 + F^2 \sigma_{GV}^2} \),

which yields the relative error in irradiance

\[
\frac{\sigma_E}{E} = \sqrt{\left(\frac{\sigma_F}{F}\right)^2 + \left(\frac{\sigma_{GV}}{GV}\right)^2}.
\]

By continuing this decomposition for the constituting terms, down to the uncertainties of the measured entities, the propagation of all the errors through the system can be evolved.

The entity-specific uncertainties are summarized in Table 3.1 and lead to a total relative error of the irradiance of ±3.0%, which compares well with alternative irradiance measurement methods, e.g. [79].

### 3.6 Summary and conclusions

The background and design of a newly developed absolute irradiance measurement system, referred to as the Lambertian flat plate calorimeter (LFPC), was presented. Using a representative characterization measurement of a solar concentrator, the measurement methodology and calibration is described in detail. The device, which mitigates many of the limitations of the previously existing methods, is employed for the characterization of solar concentrators in the following chapters.
Chapter 3

Table 3.1. Collection of the specified uncertainties of the measured entities feeding into the calculation of the irradiance distribution with the LFPC.

<table>
<thead>
<tr>
<th>Measured entity</th>
<th>Uncertainty</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{\text{flow}}$</td>
<td>±0.15°C</td>
<td>[84, 85]</td>
</tr>
<tr>
<td>$T_{\text{in}}$</td>
<td>±0.15°C</td>
<td>[84, 85]</td>
</tr>
<tr>
<td>$T_{\text{out}}$</td>
<td>±0.15°C</td>
<td>[84, 85]</td>
</tr>
<tr>
<td>$\dot{V}$</td>
<td>±0.31%$^b$</td>
<td>[86–88]</td>
</tr>
<tr>
<td>$I$</td>
<td>±0.01%</td>
<td>[86]</td>
</tr>
<tr>
<td>$V$</td>
<td>±0.01%</td>
<td>[86]</td>
</tr>
<tr>
<td>$c_p$</td>
<td>±0.20%$^c$</td>
<td>[89]</td>
</tr>
<tr>
<td>$\rho_{\text{Al}_2\text{O}_3}$</td>
<td>±2.10%$^d$</td>
<td>Appendix B.2</td>
</tr>
<tr>
<td>$GV$</td>
<td>±0.75%$^e$</td>
<td>[79, 90]</td>
</tr>
<tr>
<td>DNI</td>
<td>±0.20%</td>
<td>[91]</td>
</tr>
</tbody>
</table>

$^b$ The uncertainty of the volumetric flow comprises the uncertainties of the flow meter after calibration (±0.1%), the pulse-to-voltage signal converter (±0.2%), and the voltage accuracy of the data acquisition system (±0.01%).

$^c$ The uncertainty of the specific heat capacity of water accounts for the maximum variation of $c_p$ as a function of temperature during the heat-up process from $T_{\text{in}}$ to $T_{\text{out}}$ with respect to the mean temperature.

$^d$ The uncertainty of the solar-averaged reflectance includes the error of the measurement of the spectral reflectance (±1.0%; see Appendix B.2) and a variation of the incident solar spectrum between AM1.0 and AM2.0 (±1.1%; see Appendix B.2).

$^e$ The uncertainty of the recorded image includes linearity uncertainty (±0.5%) and spectral error (±0.25%).
4 Experimental investigation of the elliptical vacuum membrane concept\textsuperscript{ab}

In this chapter, the first on-sun experimental demonstration and proof-of-concept of a novel solar dish concept based on elliptical vacuum-membrane facets is reported. The membrane-mirror technology is investigated for its potential to drastically reduce the manufacturing cost in comparison to systems based on conventional silvered-glass mirror technology. The dish is further utilized as a prototype to investigate several technologies foreseen for the final dish design, notably, different types of silvered aluminum membranes and a prototype of the dense-array photovoltaic receiver.

4.1 Introduction

As discussed in Chapter 1, point focus solar concentrators have the advantage of achieving the high solar concentration ratios required for concentrating solar power (CSP) and concentrating photovoltaics (CPV). For CSP applications, e.g. Stirling engines, dish concentrators allow elevated operating temperatures leading to high heat engine efficiencies [8, 9]. For CPV systems, the prominent cost of high-efficiency solar cells can be offset by the low ratio of cell area to inlet aperture, leading to high electrical efficiency and cost-effectiveness [5–7]. Especially in the solar power market dominated by inexpensive flat-plate PV, the added cost of a concentrator constitutes a major bottleneck in making the aforementioned dish technologies viable. To reduce the concentrator cost, several dish designs using inexpensive stretched membrane mirrors have been


\textsuperscript{b} The work presented in this chapter has been performed in cooperation with Airlight Energy and IBM Research Zurich.
developed. Most commonly, a single steel membrane is used to support thin second-surface glass or metallized polymer membrane mirrors [92, 93].

However, vacuum membrane concentrators are limited in their capability to approximate the parabolic shape, especially for the large rim angles required for achieving high concentration ratios and a compact design [11]. Possibilities for improving the shape of such mirrors have been presented for metallized polymer [94] and elasto-plastic [95] membrane mirrors. Ultimately, the maximum rim angle that can be achieved for a single mirror membrane is around 20 degrees, above which material limitations emerge and/or the deviation from the desired parabolic shape becomes severe. To achieve high-concentration compact designs, it is therefore necessary to use multiple, commonly circular, vacuum-membrane facets. This concept has been implemented with thin glass mirrors on steel support membranes [96] and silvered polymer membrane mirrors [92, 97, 98].

A promising new concept relies on multiple inflated elliptical membrane facets to approximate a parabolic dish with lightweight, inexpensive mirrors, resulting in an improvement, compared to multiple circular facets, of both the flux concentration ratio and the active area fraction, as shown by Zanganeh et al. [99], especially for large mirror curvatures. A prototype dish concentrator based on this design has been constructed and tested on-sun in the context of a research project aiming to develop a high-concentration photovoltaic-thermal (HCPVT) system that maximizes exergy efficiency [59, 100] (Chapter 5). This chapter discusses the optical characterization of this dish using two different mirror membrane configurations. Section 4.2 describes the design of the prototype vacuum-facet solar dish concentrator. Section 4.3 presents the results

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Higher flux concentration ratio is achieved as a consequence of the elliptical facet frames (which represent the boundary condition for the deformation of the mirror membrane) lying exactly on the ideal paraboloidal shape by design (intersection of vertical cylinder with paraboloid of revolution), which is not the case with circular facets. Higher active area fraction is achieved as the projected inlet areas of the elliptical facets are hexagonally densely packed circles, while the projected inlet apertures of appropriately tilted circular facets are ellipses where the major axis is the radius of the circular facet (and the minor axis is smaller, hence leaving larger gaps). For both metrics, the difference between elliptical and circular facets increases with the dish rim angle. A potential avenue for increasing the active area fraction with both elliptical and circular facets is the overlapping of individual facets, at the cost of partial mutual shading.
Experimental investigation of the elliptical vacuum membrane concept

4.2 Prototype design

The prototype vacuum-facet solar dish concentrator, constructed and installed in Biasca, Switzerland is shown in Figure 4.1. It is based on a previously reported design [99] using 18 mirror facets to approximate the parabolic dish shape. The shapes of the mirror frames are derived by intersecting a paraboloid with hexagonally densely packed cylinders with their axes parallel to the axis of revolution of the paraboloid. The resulting intersections, which trace out the shape of each membrane frame, are planar ellipses, which makes the frames

\[ \text{Figure 4.1. Photograph of the vacuum-facet solar dish prototype installed in Biasca, Switzerland, during an on-sun optical characterization measurement. The engineering design and fabrication was carried out by Airlight Energy.} \]

of the optical measurements and identifies potential improvements to the dish design.

\[ \text{The engineering design and fabrication of the dish system was carried out by Airlight Energy.} \]
particularly easy to construct as hoops. Each hoop is assembled to a steel back-plate for added support and fastening to the dish framework.

The basic geometry is shown in Figure 4.2. The dish has a focal length $f$ of 3 m, an outer diameter $d_{\text{dish}}$ of 3 m, a focal ratio $f/d_{\text{dish}}$ of 1.0, and a rim angle $\Phi_2$ of $28^\circ$, which minimizes spillage around the irradiance hot spot and facilitates manufacturing. While the mirrors that make up the dish have the same projected area, the semi-major axis of their elliptical frame varies with the radial distance from the dish center.

This results in 3 sets of identical facets, as indicated in Figure 4.3(a). The dimensions of a single facet are shown in detail in Figure 4.3(b). The projected diameter of the active mirror surface is $d_m = 0.55$ m, leading to a total projected mirror area (dish inlet aperture) of $4.28$ m$^2$. Figure 4.3(c) shows a cross-section of a mirror membrane, clamped inside the facet frame. During operation, a slight vacuum is applied in the facet cavity, leading to the deformation of the membrane. The pressure difference across the membrane in each facet is controlled by measuring the vertical displacement of the mirror center using

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Figure 4.2. Cross-sectional schematic of the vacuum-facet solar dish (Section A-A in Figure 4.3(a)).
4.3 Results and discussion

Utilizing the LFPC presented in Chapter 3, vacuum membrane facets employing two different silvered aluminum sheets were tested on the dish presented in Section 4.2:

1. AgSheet0.3mm, having a thickness of 0.3 mm and solar-averaged specular reflectance at 15° incidence angle of 95.4% [101, 102]; and

2. AgSheet0.2mm, with a thickness of 0.2 mm and a solar-weighted specular reflectance at 15° incidence angle of 93.9% [101, 102].
Both membranes are front-surface silvered aluminum sheets. The silver reflective layer applied to the aluminum sheet is covered by a dielectric reflectance enhancing layer. It is expected that the difference in membrane thickness will have an effect on the elasto-plastic membrane deformation under vacuum pressure [95, 103] which will have the largest influence on performance.

Figure 4.4. Contour map of the solar flux concentration ratio at the focal plane for the AgSheet0.3mm membranes: (a) close-up on the central 60 × 60 mm section. The average solar concentration on this area is $C_{av, \text{rec}} = 762$ suns; the peak solar concentration is $C_{\text{peak}} = 2912$ suns. The dashed lines indicate the dimensions of the calorimeter (ROI); (b) Contour map of the solar flux concentration ratio on the full target with emphasis on concentrations below 250 suns to show the peripheral regions. The solid lines indicate the 60 × 60 mm receiver area detailed in (a).
difference between the two membranes. The higher specular reflectance of AgSheet0.3mm is likely to play only a secondary role. An overview of the main measurement results with the two membranes is provided in Table 4.1.

4.3.1 Focal plane irradiance

The recorded irradiance distributions were normalized to the DNI during the respective measurements, resulting in a map of the solar flux concentration ratio \( C \),

\[
C(x, y) = \frac{E(x, y)}{\text{DNI}}.
\]

Figure 4.4 shows the contour map of the measured \( C \) achieved with the AgSheet0.3mm membranes. A close-up of the central \( 60 \times 60 \) mm section is given in Figure 4.4(a). It represents the area that was foreseen for the \( 6 \times 6 \) cell CPV and thermal receiver, discussed in Chapter 5. The average solar concentration over this area is \( C_{\text{av,rec}} = 762 \) suns, while the peak solar concentration is \( C_{\text{peak}} = 2912 \) suns. The dashed lines indicate the dimensions of the ROI (calorimeter area and kaleidoscope inlet aperture, \( 53 \times 57 \) mm).

Figure 4.4(b) shows the concentration map on the full \( 300 \times 300 \) mm target, with emphasis on concentrations below 250 suns to reveal the peripheral regions of the concentration distribution. The solid lines indicate the dimensions of the central receiver area, shown in detail in Figure 4.4(a).

Figure 4.5 shows the analogous results for the AgSheet0.2mm membranes. The average solar concentration in the region shown in Figure 4.5(a) has increased compared to the AgSheet0.3mm membranes to \( C_{\text{av,rec}} = 897 \) suns (+18%), while the peak solar concentration is boosted to \( C_{\text{peak}} = 3140 \) suns (+8%). Figure 4.5(b) shows the contour map of \( C \) on the full \( 300 \times 300 \) mm target, where a total focal plane radiative power of 4.1 kW was achieved. A smaller portion of the incident radiation is spilled outside of the central receiver area compared to the AgSheet0.3mm membranes, further indicating an improvement in performance.
A quantitative comparison of the optical performance of the primary mirrors is provided by the intercept factor, defined as the ratio of the solar radiative power intercepted by a central, square area $A_i = w_i^2$ and the total solar radiative power reaching the focal plane.

**Figure 4.5.** Contour map of the solar flux concentration ratio at the focal plane for the AgSheet0.2mm membranes: (a) close-up on the central $60 \times 60$ mm section. The average solar concentration on this area is $C_{av, rec} = 897$ suns; the peak solar concentration is $C_{peak} = 3140$ suns. The dashed lines indicate the dimensions of the calorimeter (ROI); (b) Contour map of the solar flux concentration ratio on the full target with emphasis on concentrations below 250 suns to show the peripheral regions. The solid lines indicate the $60 \times 60$ mm receiver area detailed in (a). A smaller portion of the incident solar radiation is spilled outside of this area compared to the AgSheet0.3mm membranes.
Experimental investigation of the elliptical vacuum membrane concept

In Figure 4.6 the intercept factor for the two membranes is plotted versus the width $w_i$. The improvement of concentrator performance achieved with the AgSheet0.2mm membranes becomes apparent. 78.3% of radiation are directly collected within the foreseen $60 \times 60$ mm receiver area whereas only 68.2% are collected with the AgSheet0.3mm membranes. While a fraction of spilled radiation above 20% can appear high, it is worth noting that the presented intercept factor merely serves as a means of comparing the two membranes. In the finalized HCPVT system, the major part of this radiation can be collected and redirected to the receiver by an appropriate secondary concentrator.

$$\gamma_i = \frac{\dot{Q}_i}{\dot{Q}_{tot}} = \frac{\int_{A_i} E(x, y) \, dx \, dy}{\int_{A_{tot}} E(x, y) \, dx \, dy}.$$ (4.2)
The main difference in performance between the two membrane types is attributed to the stiffness of the membrane causing an unwanted bending near the facet rim, where the exact replication of the parabola would require a sharp edge. The membrane stiffness is higher in the thicker membrane, causing the bending to have a larger radius and ultimately resulting in a less ideal shape. The extent of this effect is largely due to the small facet radius used on the prototype concentrator and is expected to be of lesser importance in a larger system.

### 4.3.2 Irradiance homogenization

An important role of the vacuum-facet solar dish concentrator described in this work was the on-sun characterization of the prototype HCPVT electrical/thermal receiver [59], which was conducted with the AgSheet0.3mm mirrors. The prototype receiver has an aperture of 53 × 57 mm and a simplified electrical design with an array of 5 × 5 PV cells, where the cells of each row are connected in series. In order to maximize the electrical performance of the receiver it was therefore necessary to spatially homogenize the incident irradiance. For this purpose, the receiver was installed at the exit of the kaleidoscope irradiance distributor introduced in Section 3.2.

Crucial to an accurate characterization of the electrical performance is the knowledge of the irradiation incident on the PV array. This irradiance distribution was determined in a separate measurement where the calorimetric part of the target was installed at the outlet of the kaleidoscope (at the same location the prototype receiver was later placed), as illustrated in Figure 3.3(b).

<table>
<thead>
<tr>
<th></th>
<th>AgSheet0.3mm</th>
<th>AgSheet0.2mm</th>
<th>Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{av,rec}$</td>
<td>762 suns</td>
<td>897 suns</td>
<td>+18%</td>
</tr>
<tr>
<td>$C_{peak}$</td>
<td>2912 suns</td>
<td>3140 suns</td>
<td>+8%</td>
</tr>
<tr>
<td>$\gamma_{rec}$</td>
<td>68.2%</td>
<td>78.3%</td>
<td>+15%</td>
</tr>
</tbody>
</table>

Table 4.1. Main results of the on-sun optical characterization of the solar dish. The AgSheet0.2mm membranes show a considerable improvement, with respect to the AgSheet0.3mm membranes, of the average solar concentration on the receiver (+18%), the peak solar concentration (+8%) and the intercept factor at the 60 × 60 mm receiver aperture (+15%).
The contour map of the solar flux concentration ratio measured in this location is shown in Figure 4.7. As an effect of the reflections in the kaleidoscope, the homogeneity of the irradiance distribution has been considerably increased from the focal plane [cf. Figure 4.4(a)] to the kaleidoscope outlet. The peak solar concentration was reduced from 2912 to 1143 suns and the minimum solar concentration was increased to above 500 suns. The cell-to-cell uniformity, defined as $U = \min(E_{av})/\langle E_{av} \rangle$, where $E_{av}$ are the irradiances averaged over each cell, gives a good indication of the performance of a series cell array [56]. On the 5 × 5 cell array of the prototype receiver, $U$ is improved from 35% at the focal plane to 89% at the kaleidoscope exit. The optical efficiency of the kaleidoscope, defined as the fraction of the outlet to the inlet radiative power, is 93.5%.
4.3.3 Improvements to the dish

The results presented above and the on-sun experience gained with the vacuum membrane dish suggest several improvements to the solar concentrator design. By comparing the two mirror types, we have shown that the bending stiffness of the membranes has a major effect on the concentrator performance by inhibiting a sharp edge at the facet rim, in accordance with the findings by Dähler et al. [95] for circular membrane mirrors of similar dimension. Better results can be achieved by decreasing the ratio of the membrane thickness to the facet diameter, i.e. employing thinner membranes and/or scaling up the mirror facets. Another important design parameter, especially when scaling up the dish, is the rim angle. A larger rim angle results in a more compact concentrator, which has structural as well as tracking advantages. On the other hand, it imposes a stronger surface curvature to each facet if the number of facets is kept constant. However, it is specifically at higher rim angles that the elliptic facets outperform the more common circular facets in terms of active area fraction and concentration [99]. Simulations and tests with high-curvature mirror facets show a good approximation to the parabolic shape and high solar concentrations.

4.4 Summary and conclusions

The first on-sun demonstration of a novel solar dish concept based on multiple vacuum-membrane facets with an elliptical perimeter was carried out. The optical performance of the dish with two different types of silvered aluminum membranes was characterized using the Lambertian Flat Plate Calorimeter introduced in Chapter 3. The best performance was achieved with the thinner, 0.2 mm-thick membranes and is attributed to the smaller bending stiffness, resulting in a sharper edge at the facet rim and hence a shape closer to that of a parabolic dish. The solar dish reached a peak solar flux concentration ratio of 3140 suns and an average of 897 suns on a 60 × 60 mm area.

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[95] Ultimately, the facet size is limited by the width of the available membrane materials.
5 A 6-focus high-concentration photovoltaic-thermal dish system\textsuperscript{ab}

In this chapter, the theoretical findings discovered and the experimental experience gained in the preceding chapters are leveraged to design a high-concentration photovoltaic-thermal solar dish system in pursuit of a cost-effective and sustainable solution for solar energy utilization.

A novel solar dish concept that achieves high concentration and compactness through transferring the advantages of asymmetric two-stage designs (Chapter 2) to point focus geometry is presented. A practical implementation of this concept, expanding on the prototype introduced in Chapter 4 and optimized for mass production, is characterized using the absolute irradiance measurement device described in Chapter 3. Broad-spectrum utilization is achieved with a photovoltaic-thermal receiver enabling polygeneration of electricity and heat, while the issue of irradiance non-uniformity is tackled by a cell array capable of mitigating mismatch losses. Using a meticulous opto-electric model, performance predictions over a range of operating conditions are made.

5.1 Introduction

As elaborated in Chapter 1, the fundamental principle behind concentrating photovoltaics (CPV) is the substitution of expensive cell area with inexpensive optics [6, 7]. As a consequence, high-efficiency multi-junction cells, with a recently demonstrated record photovoltaic efficiency of 46.0% [4], can be employed. However, in order for an economic benefit to be upheld, several requirements need to be met. The cost per active area of the optical concentrator


\textsuperscript{b} The work presented in this chapter has been performed in cooperation with Airlight Energy/DSolar and IBM Research Zurich.
has to be substantially lower than the cost per area of the cells to offset the additional system costs [104, 105], mandating the use of inexpensive materials, efficient fabrication and assembly, high concentrations, and suggesting the use of large concentrator apertures to reduce the specific number of system components. While a high concentration – beyond several 100 suns – is a key driving factor for cost decrease due to the reduced cell area, it introduces the problem of thermal management of the cells [58]. To maintain safe operating conditions at high photovoltaic efficiencies, active cell cooling is required to evacuate the produced heat, which, even with the highest-efficiency cells, is typically more than 50% of the total incident solar radiation, from the dense-arrays. Conversely, if heat can be efficiently extracted at high enough temperatures using photovoltaic-thermal (PVT) receivers [59, 60], this heat can then be used further in applications such as space heating, cooling, and water desalination, and increase the overall solar resource utilization [61–63].

In this chapter, we address the aforementioned challenges in an effort to develop an energy-efficient and cost-effective viable solution for solar energy utilization relying on polygeneration of electricity and heat. Section 5.2 presents the design concept and implementation of a novel modular, multi-focus solar dish for CPV applications. The first step towards predicting the performance achievable with the presented system lies in the characterization and subsequent modeling of its optical components (Section 5.3). The results of this optical modeling can then be coupled to a receiver model (Section 5.4) to forecast the on-sun behavior under various operating conditions and allow feedback for the design of an improved system (Section 5.5).

5.2 A modular 6-focus solar dish concentrator

5.2.1 Multi-module asymmetric solar dish

Line focus (2D) designs can in theory reach a geometrical concentration of $215\times$ for a half-acceptance angle $\theta_i = 4.65 \text{ mrad}$ [Eq. (1.5)]. However, practical designs having reasonable compactness, e.g. two-stage solar troughs using parabolic primary mirrors, fall short of this limit [106]. Since the concentrator should additionally have adequate robustness against tracking, surface and alignment
inaccuracies, the acceptance angle has to be relaxed, resulting in lower concentrations in the range of medium concentration PV, typically defined as 10× to 100× [74, 107]. The analogous theoretical maximum for point focus (3D) concentrators is 46 248× [Eq. (1.6)], where the aforementioned limitations decrease the concentration achievable with practical designs in comparable fashion, yet, due to the high margin, concentration ratios beyond multiple 1000× (high concentration PV; HCPV) remain readily attainable.

Meanwhile, it has been shown that, in line focus geometry, asymmetric concave concentrators based on parabolic primary mirrors can be superior to their symmetric counterparts (Chapter 2) [11, 106]. Asymmetric one-stage designs, if arranged in compound two-wing configuration, where the concentrator is mirrored at the receiver edge, achieve much higher compactness for the same concentration ratio, which in practice allows designs with increased rigidity and lower center of gravity, and ultimately larger inlet apertures. Besides conserving the concentration ratio, a low acceptance angle for the receiver is preserved, which is crucial for the design of the secondary optics and the performance of the PV cells, whose response breaks down for incidence angles above 50° to 60° due to high surface reflection and shading by the grid fingers. With two-stage concentrators, where an ideal nonimaging secondary optic is added to the primary outlet aperture, the optical performance of the asymmetric designs surpasses that of the symmetric designs and can approach the thermodynamic limit of concentration $C_{g,max,2D}$ [106].

This raises the question whether both the advantages of point focus concentrators and of asymmetric designs can be combined. While transferring a symmetric line focus design to point focus is straightforward, the asymmetric case is more complex. In a symmetric design, the mirror and receiver surfaces are generated by revolving the 2D geometry about the axis of symmetry, which is equal to the optical axis, as indicated in Figure 5.1(a). In the asymmetric case, due to the tilted receiver, the axis of symmetry of the compound design is different from the optical axes of each mirror, as shown in Figure 5.1(b). In the limit case, it intersects the bottom edge of the receiver such that both receivers meet in this point. As a result, an asymmetric design generated by rotational symmetry about this axis would no longer possess point focus. Instead, the focus
would be distributed around a circular perimeter and the hypothetical concentrator would require a conical receiver, resulting in a much lower concentration than with the symmetric design.

This issue is avoided if instead of rotating the entire 2D design about the axis of symmetry, the dish is circumferentially divided into \( N_m \geq 2 \) concentrator modules, each comprising an individual focal point, i.e. generated by rotation about the individual optical axes, as indicated in Figure 5.1(d) for \( N_m = 6 \). Each module is consequently a section of a regular parabolic dish where the focal point is translated radially outside from the center by the distance \( R_f \). Each focal point is then given by \( \mathbf{F}_k = [R_f \cos \alpha_k; R_f \sin \alpha_k; f] \), where \( f \) is the focal length, \( \alpha_k \) defines the circumferential position of the focal points, i.e. \( \alpha_k = 2\pi/N_m(k-1) \), and \( k = 1, \ldots, N_m \). The mirror surface is then obtained as

\[
z_k(r, \phi) = \frac{1}{4f} \left[ (r \cos \phi - R_f \cos \alpha_k)^2 + (r \sin \phi - R_f \sin \alpha_k)^2 \right], \quad (5.1)
\]
A 6-focus high-concentration photovoltaic-thermal dish system

\[ R_i(\phi) \leq r \leq R_o(\phi) \text{ and } \frac{\pi}{N_m(2k-3)} \leq \phi_k \leq \frac{\pi}{N_m(2k-1)} \], and where \( R_i \) and \( R_o \) describe arbitrary inner and outer mirror perimeters.

The design can be extended to additionally include radial subdivisions between modules, as indicated in Figure 5.1(e). The individual focal points can further be rearranged by adjusting \( R_t \), \( \alpha_k \) and their vertical coordinate. The focal length of the individual modules can then be used to control the vertical gap between mirrors of neighboring modules to produce a quasi-continuous primary mirror.

5.2.2 Practical implementation

Figure 5.2 shows a practical implementation of the presented multi-module dish design, used to investigate various design innovations and mirror materials towards a commercial application and installed in Biasca, Switzerland.

To leverage mass production and thereby reduce investment cost, the majority of the dish components are modular and cast from ultra-high-performance concrete (UHPC). The concrete is optimized for high tensile strength and offers improved rigidity in addition to wind and vibration resistance in comparison to a steel frame. Meanwhile, elaborate structural design restricts

\[ \text{Figure 5.2. (a) Rendering of the multi-module HCPVT dish prototype indicating its overall dimensions; (b) and (c) photographs of the dish, installed in Biasca, Switzerland. The engineering design and fabrication was carried out by Airlight Energy/DSolar.} \]

\[ \text{The engineering design and fabrication of the dish system was carried out by Airlight Energy/DSolar.} \]
the weight of the tracked components to a minimum and facilitates the dish assembly. This production technique allows the manufacturing of the structural components on site, sourcing of local material and labor, and reducing transportation cost.

The system cost may be lowered by reducing the specific number of system components per energy produced. Thus, the concentrator is designed for a large inlet aperture (outer dish diameter of 8.6 m and accordingly total height of 10.5 m). With a focal length $f = 2.6$ m and radial offset $R_f = 155$ mm, this yields a rim angle $\Phi_2 = 77^\circ$ and results in a compactness unmatched with comparably large solar dish concentrators [9, 108]. The inner edge of the mirror is defined by the inner rim angle $\Phi_1 = 10^\circ$ such that no part of the mirror is shadowed by the receiver assembly. The resulting rim span $\Delta \Phi = \Phi_2 - \Phi_1 = 67^\circ$ indicates the approximate acceptance angle in vertical direction of subsequent optics required to accept all rays. To achieve similar angular span in circumferential direction in foresight of a symmetrical secondary optical element (SOE), the dish is divided into $N_m = 6$ modules (resulting in $360^\circ / N_m = 60^\circ$ circumferential angular span).

The primary mirror design is based on a concept where the reflector is made up of an array of elliptical facets, whose perimeters are generated by intersecting the paraboloid with hexagonally densely packed cylinders with their axes parallel to its optical axis. This allows the use of vacuum membranes as mirrors [99, 109], at the expense of the dish active area fraction. Figure 5.3(a) schematically shows the resulting facet layout\(^d\). Each of the six identical modules comprises 6 facets with a projected active area diameter $d_m = 1.17$ m, leading to an area of $6.5$ m\(^2\) per module or a total of $38.7$ m\(^2\) for the entire dish. For the protection from environmental impacts, an optional transparent protective top membrane can be inflated over the entire inlet aperture, as demonstrated for similar solar trough designs [105, 110, 111].

Three mirror technologies that exploit silvered polyester and aluminum membranes, and ultimately thin back-surface glass mirrors were investigated. Membrane mirrors have the general advantages of being inexpensive, lightweight and shippable in rolls, which can be a significant advantage for

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\(^d\) The engineering design and fabrication of the dish system was carried out by Airlight Energy/DSolar.
remote sites (as all other major components of the proposed concentrator can be cast on site). Inflated mirrors provide an additional safety feature to solar dishes that consists in the potential to rapidly defocus by membrane depressurization whereas dishes using traditional mirrors need to use the slower tracking mechanism. An important disadvantage of membrane mirrors is that, due to the front surface metallization, protection of the specular surface from environmental impacts is not ensured and hence a protected environment is required, i.e. by covering the inlet aperture with a transparent inflated membrane, which introduces additional complexity and transmission losses. The need for this protected environment is eliminated with back-surface glass mirrors. The three mirror technologies, schematically shown in Figure 5.3(b-d), are described in detail below.

**Technology 1: Improved controlled vacuum membranes** [Figure 5.3(b)] – Mirror membranes are clamped inside an elliptical facet frame and a slight vacuum is applied at their lower surface, leading to the desired deformation of the membrane. The pressure difference across the membrane in each facet is adjusted such that the membrane surface best approximates the desired parabolic shape. In contrast to an earlier design where the vacuum pressure needed to be
continuously monitored [99, 109], the improved design features inherent shape control with an air outlet nozzle that automatically closes when the membrane reaches the desired shape. Thus, a single vacuum pressure can control all mirrors with minimal electronic control. This concept was tested with silvered aluminum membranes with thicknesses of 0.3 mm and 0.2 mm. Disadvantages of the method include potential membrane creep in the central region close to the air outlet, pressure loss in the whole concentrator if a single membrane is damaged, and the general inability of vacuum membrane mirrors to reproduce a perfect paraboloidal shape [99, 109].

Technology 2: Vacuum membranes in contact [Figure 5.3(c)] – To gain additional degrees of freedom in the mirror profile design while retaining the aforementioned advantages of membrane mirrors, the membrane is pulled into direct contact with a prescribed (e.g. parabolic) profile shape by vacuum pressure. To allow fast and simple reproduction, the shell structure bearing the profile is produced by concrete moulding with high dimensional precision. This concept was tested with silvered aluminum (thickness 0.3 mm) and polyester (thickness 0.05 mm) membranes. To ensure a uniform adherence of the membrane to the underlying profile, i.e. avoid air pockets and wrinkles, air evacuation pathways were introduced in the mold. In addition to the attenuation caused by these channels, thinner membranes tend to convey small-scale surface inaccuracies from the profile to the mirror surface, which can lead to significant small-angle scattering.

Technology 3: Glued mirror [Figure 5.3(d)] – The same advantages of prescribing the surface profile can be leveraged by permanently attaching the mirror to the shell structure. This resolves the issues of achieving a uniform adherence of the mirror while eliminating the need for vacuum generation (pump) and reducing the number of system components. The downside is the loss of the ability to rapidly defocus the facets via pressurization of the facet cavity. This concept was tested with silvered aluminum membranes (thickness 0.3 mm) and thin, back-surface glass mirrors (thickness 1 mm). While aluminum membranes can be shipped in rolls, glass mirrors eliminate the requirement for a protective top membrane and generally exhibit both higher reflectance and better specularity with a solar-averaged reflectance $\rho_{\text{mirror}} = 94.2\%$ [101, 102]. Finally,
this concept, no longer relying on stretched membranes, permits arbitrary perimeter shapes for the facets, which ultimately allows a concentrator with a higher active area fraction of the inlet aperture.

While relatively good performance was achieved with all listed technologies, system simplicity in combination with the highest primary optical efficiency led to the choice of glued back-surface glass mirrors for the ultimate full-system concentrator. A detailed view normal to the front surface of one facet is schematically shown in Figure 5.3(e). While the low thickness of the glass has a high elasticity for 1-D bending, 2-D shaping requires the mirror to be segmented into multiple sections separated by small gaps, which allows the relief of tangential stresses. The reduction of the active mirror surface as a result of the gaps is $\leq 1\%$. Figure 5.3(f) shows a photograph of the backside of the UHPC mirror facet shell. Its structural design allows the facet to be light ($< 30$kg), without compromising on rigidity.

The concentrator uses a two-axis, 45° exocentric gimbal tracking system, similar to the one developed by Advanco Corporation for the Vanguard dish/Stirling system [8, 112, 113]. It has the advantage of maintaining the location of the center of gravity of the dish close to the tracking axes. While the first axis of rotation remains vertical, the second axis, rotated with respect to the first axis by 45°, rotates rather than lifts the center of mass, minimizing torque. As a result, the concentrator can be tracked with high accuracy and low power consumption.

5.2.3 Solar receiver

The radiation reflected from the primary mirrors of the six modules is collected in the central receiver assembly shown in Figure 5.4. Its main structure is a single piece of cast concrete that holds the PV receivers and the connected array of power electronics (DC-DC converters). For each module, a dedicated SOE channels the radiation to the receiver at its outlet aperture. Its chief objective is to shield the receiver assembly and its internal components from fringe radiation by redirecting it to the receiver, thereby increasing the total collected radiative

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*The engineering design and fabrication of the dish system was carried out by Airlight Energy/DSolar.*
power. Each SOE consists of four main flat mirror surfaces protruding from each side of the receiver. Receiver orientation and wall tilt angles are optimized to achieve the best trade-off between a high acceptance and a low number of added reflections. Two additional small mirrors truncate the lateral corners of the inlet aperture to avoid mutual intersection of neighboring SOEs. To tolerate the high irradiance close to the outlet aperture and precisely match the outlet to the receiver, the bottom 25 mm of the secondary optic are made from silver-coated aluminum, actively cooled and directly attached to the receiver. The same thin-walled back-surface coated glass mirrors used for the primary concentrator are used for the remaining specular surfaces and are directly attached to the concrete receiver assembly without active cooling. Additionally, a 1 mm thick, anti-reflection (AR) coated borosilicate glass window is added in front of the SOE inlet aperture to protect the electrical components from the environment.

Each PV receiver holds 36 high-efficiency triple-junction (3-J) concentrator cells [114, 115], arranged in a 6 × 6 dense-array. As the receiver was originally designed for the larger acceptance angle required with vacuum-membrane...
mirrors compared to glued back-surface glass mirrors, it was necessary to relocate it out of focus in order to distribute the radiation over a larger area and avoid a too high irradiance on the cells. A translation of 35 mm from the focal point towards the primary mirror in the direction normal to its surface was sufficient to keep the maximum cell irradiance below 2500 kW/m² without significant loss in optical efficiency.

While a non-uniform irradiance on the aperture of a single PV cell can lead to a drop in the open-circuit voltage and therefore a lower electrical cell efficiency [10, 53–55], this effect is relatively small and negligible for modest irradiance gradients [116, 117]. The repercussions from non-uniform irradiance between cells connected within the same dense-array depend on the way the cells are interconnected. If the cells are interconnected in series, the current mismatch between differently irradiated cells causes an important drop in efficiency [53, 56, 57]. In contrast, mismatch losses created by connecting the cells in parallel are insignificant, since, unlike the current, the cell voltage is only marginally affected by irradiance intensity. However, the array current increases linearly with the number of cells connected in parallel and rapidly requires large conductor cross-sections to avoid resistance losses through the array and downstream conductors.

Irrespective of the extent of the irradiance non-uniformity, a favorable trade-off between mitigating current mismatch losses and preventing resistance losses can be realized by considering square dense arrays (1) having an even number of cells per side, and (2) where the irradiance distribution is uniformly distributed over the four quadrants (e.g. centered and rotationally symmetric). In the simplest case, depicted in Figure 5.5(a), where the array consists of 2 × 2 cells, each of the cells receives the same average irradiance due to the symmetry of the irradiance distribution. The cells therefore produce the same current and can be connected in series without mismatch losses. The same principle can be applied to a dense array with a larger number of cells by dividing the array into four equal quadrants or sub-modules. The situation for a 6 × 6 cell array is schematically shown in Figure 5.5(b). Within the submodules (composed of 3 × 3 cells), the individual, differently irradiated cells produce dissimilar currents. By connecting them in parallel, mismatch losses are mitigated. The cumulative current and
voltage of the 4 submodules are identical due to symmetry, similar to the 4-cell case in Figure 5.5(a). As a result, they can be series-connected without mismatch losses. Compared to an all-parallel connected array, the concept essentially reduces the current (and required conductor cross-sections) by 75%, while increasing the voltage by 300%. If the irradiance is not equally distributed between the submodules, e.g. due to malfunctioning or inappropriately designed optics, current mismatch occurs, reducing the receiver efficiency. Contrariwise, in the case of appropriate and functioning optics, the information of the individual submodule currents or voltages can be used for tracking adjustment, much like with a four-quadrant photodiode, which represents the most common type of sun trackers [118, 119].

The practical implementation of this concept is detailed in Figure 5.6\(^\text{f}\). In each submodule, 9 PV cells are connected in parallel using common top and bottom electrodes, as indicated in Figure 5.6(a). The 350 \(\mu\)m thick, four-fingered top electrode is made from a copper-invar-copper (CIC) sheet and allows

\(^{f}\)The electrical design of the PV receiver has been developed by IBM.
A 6-focus high-concentration photovoltaic-thermal dish system

soldering onto both busbars of each cell without requiring a large cell spacing and thus unnecessarily compromising active receiver area. The width of the fingers varies from 300 µm and 400 µm for the exterior fingers only connected to cells on one side to 600 µm for the interior fingers connected to a cell on either side. The 50 µm thick, common bottom electrode is directly electro-plated to the high-performance cooler chip and made from 99.99% pure copper. Electrical interconnection of the cells is performed using Ag-based sinter paste (ASP043-04) [120] and Indalloy® 63Sn/37Pb ribbon solder preform [121] for the top and bottom interfaces, respectively. The contacts of both electrodes exit the submodules below the SOE mirrors, at a 90° angle, allowing the neighboring submodules to be easily series-connected using copper bars, as shown in Figure 5.6(b). The individual cells are spaced by 200 µm in both directions, resulting in dense array outer dimensions of 61 × 61 mm. Of this area, 33.5 cm² (90.1%) are

Figure 5.6. Schematic showing the design and different components of the electrical receiver. (a) Submodule, where 3 × 3 PV cells are connected in parallel using a bottom electrode directly plated to the cooler chip and a four-fingered top electrode that allows reduction of inter-cell gaps to a minimum; (b) receiver, where four submodules are series-connected using copper bars. The design of the electrical receiver has been developed by IBM.
active cell surface, the remaining surface being taken up - in decreasing order of importance - by the cell busbars, inter-cell gaps (both predominantly covered by the top electrode fingers; 8.1%) and inactive outer perimeter (1.8%). The relatively large fraction of receiver area lost due to the surface area required for the cell busbars is a common design issue of PV dense arrays and difficult to avoid without major redesign of the receiver, e.g. by breaking up the array into individual cells [122] or linear “semi-dense arrays”, where the busbars and cell interconnections can be covered by optical elements [56, 123], with higher penalties in terms of complexity and efficiency of the concentrator optics.

The voltage from the receiver terminals is stepped up from nominal 12 V/200 A to nominal 400 V/5 A using a dedicated DC-DC converter for each receiver (EFORE, efficiency >94%) to minimize transmission losses, before the outputs from the receivers are connected in parallel in groups of 3 (2× nominal 400 V/15 A) and sent to a grid-connected inverter.

The high-performance cooler chip relies on silicon (Si) micro-channels to minimize the temperature gradient within the cooled area and guarantee a high heat-exchange efficiency [59, 60]. Within its total thickness of 1.5 mm, a first manifold system distributes the coolant to 1200 50-µm-wide and 300-µm-high channels, such as to ensure equal channel inlet temperature, flow rate and pressure drop. A channel length of 1 mm reduces the fluid residence time, which, together with the small distance from the bottom surface of the bottom electrode (250 µm), is key to achieving a low thermal resistance. The cooler chip is fed by a lower manifold that connects the receiver to a cooling distribution unit (CDU) installed in the base of the dish, which, in turn, connects the dish to the external low-grade heat applications through a heat exchanger. It has been shown that, with the utilized type of microchannel heat exchanger, the coolant temperature has a negligible effect on the thermal resistance in the encountered conditions [60]. Figure 5.7 shows the measured thermal resistance $R_{th}$ with the final high-performance cooler chip design, at a coolant temperature of 50 °C, versus the coolant flow rate $\dot{V}_{rec}$. The receiver ensures active cooling of the full cell area with a measured thermal resistance of 0.105 cm²·K/W and pressure drop <60

---

8 The measurement of the thermal resistance has been performed by IBM.
mbar at 10 l/min. Due to the low thermal resistance, the receiver can be operated with relatively high coolant inlet temperatures (up to 85 °C) without causing an overheating of the cells. It consequently allows the investigation of different operation modes, balancing between electrical and thermal performance, where low grade heat (LGH) extracted from the receiver is used in secondary applications such as space heating, heat-driven cooling and water desalination [61–63].

5.3 Optical characterization and modelling

5.3.1 Optical characterization

Optical on-sun characterization of a single concentrator module, comprised of 6 facets with glued glass mirrors, was carried out using the LFPC method first presented in [109]. Compared to the original design, several improvements were made to improve the measurement accuracy, as detailed in Appendix B.3. Further, the temperature difference across the LFPC was kept below 20 °C throughout the characterization to guarantee lower convection and conduction losses, which, considering the larger inlet aperture and the higher quality of the

Figure 5.7. Measured thermal resistance $R_{th}$ with the high-performance cooler chip at coolant temperature 50 °C versus the coolant flow rate $\dot{V}_{rec}$. The measurement has been performed by IBM.
mirrors, mandated an increase of the cooling water flow rate to 1.5 l/min. **Figure 5.8(a)** shows a photograph of the LFPC mounted in the focal point of the characterized concentrator module and parallel to the receiver. For practical reasons, it is oriented such that its bottom edge is perpendicular to the optical axis. The difference in orientation between the receiver and the LFPC is schematically shown in **Figure 5.8(b)**.

With ideal parabolic reflectors, the irradiance distribution in the focal plane would exhibit a small central region, i.e. a “hot spot”, of uniform irradiance around 8000 kW/m², contoured by a narrow fringe region of decreasing irradiance, originating from the superposition of the focal images of the sun from
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Each location on the mirrors [124]. Compared to the circular shape of the sun the shape of the hot spot is slightly deformed due to the orientation of the LFPC relative to the concentrator module and the fringe region is to some extent enlarged due to the interaction of the solar beam with atmospheric particles [125, 126].

In contrast, the measured focal irradiance distribution on a central 61 × 61 mm area of the LFPC, representing the receiver size and henceforth referred to as “LFPC-receiver” [cf. Figure 5.8(b)], normalized to an incident direct normal irradiance (DNI) of 1000 W/m$^2$ (= 1 sun), is shown in Figure 5.9. Deviations of the mirror profile from the ideal parabolic shape cause the irradiance distribution to be blurred out. Three types of imperfections with respect to the ideal shape can be identified: (1) macroscopic slope errors in the profile of the cast concrete shape; (2) macroscopic slope errors caused by the inaccurate reproduction of the concrete profile with the glued mirrors; and (3) narrow-angle scattering of the reflective surface. While narrow-angle scattering is negligible with the employed back-silvered glass mirror (standard deviation below 0.05 mrad at incident angles below 45° [101, 102]), the two remaining surface errors have a considerable impact on the irradiance distribution. An additional error distorting the irradiance

Figure 5.9. Contour plot of the irradiance distributions measured on the LFPC. The irradiance is normalized to a DNI of 1000 W/m$^2$ (= 1 sun).
distribution resulted from inaccurate tracking but this tracking error was mostly mitigated by aligning each mirror facet to produce an irradiance peak at the center of the LFPC at the expense of a more elliptical distribution.

The accumulation of these errors resulted in the elliptical Gaussian distribution shown in Figure 5.9, with a major axis tilted by ~15° with respect to the LFPC vertical. It has a central peak $E_{\text{max}} = 3698$ kW/m² and its average magnitude over the LFPC-receiver area is $E_{\text{av,LFPC-rec}} = 1353$ kW/m². The radiative power incident on this area,

$$
\dot{Q}_{\text{LFPC-rec}} = E_{\text{av,LFPC-rec}} \cdot A_{\text{rec}} = 5.04 \text{ kW},
$$

represents $\gamma_{\text{LFPC-rec}} = 84.4\%$ of the total power incident on the entire LFPC area. Considering the large scale of the LFPC with respect to the irradiance distribution, the latter is a reasonably accurate measure of the total power reflected by the mirrors and is obtained as

$$
\dot{Q}_{\text{refl.,1}} = \dot{Q}_{\text{LFPC-rec}} / \gamma_{\text{LFPC-rec}}.
$$

The factor accounting for the loss in active area fraction due to gaps between the glued mirrors and soiling of the mirrors can then be obtained as

$$
\zeta = 1 / \rho_{\text{mirror}} \cdot \dot{Q}_{\text{refl.,1}} / \dot{Q}_{\text{solar}} = 98.2\%,
$$

where $\rho_{\text{mirror}}$ is the solar-weighted specular reflectance of the mirrors (94.2%) and $\dot{Q}_{\text{solar}} = 6 \cdot \pi (d_{m}/2)^2 \cdot \text{DNI}$ is the solar radiative power incident on the mirror aperture.

### 5.3.2 Optical model

Optical modeling of the concentrator is performed using the Monte Carlo ray-tracing technique, carried out with the in-house VeGaS code [72]. The incident radiation is modelled by the ASTM AM1.5d (G173-03) reference spectrum for direct and circumsolar irradiance [127] and the sun-shape model of Buie,
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Monger, and Dey (Buie et al., 2003) with a circumsolar ratio (CSR) of 5%. Simulations are performed with a DNI of 1000 W/m$^2$ and $10^7$ rays.

The primary mirrors are modeled as paraboloidal, using the experimentally determined spectral reflectance of the mirror material [101, 102], attenuated by the correction factor $\zeta$ that accounts for the active area lost in the gaps between the individual glued mirror segments and mirror surface soiling. Fresnel reflection at the mirror front surface and absorption within the glass are modeled using data from the manufacturer [128]. The cumulated effects of the various macroscopic mirror surface inaccuracies are modeled by a normally distributed angular dispersion error. To model the elliptical shape of the distribution caused by the tracking error, angular dispersion is modeled as orthotropic, with different standard deviations in the circumferential ($\sigma_{err,1}$) and radial directions ($\sigma_{err,2}$). The fitted standard deviations, minimizing the root mean square (rms) error between the measured and simulated irradiance distribution on the LFPC while matching the integral power over an area with dimensions $100 \times 100$ mm, representative of the SOE inlet aperture, are $\sigma_{err,1} = 1.17$ mrad and $\sigma_{err,2} = 1.69$ mrad. The resulting simulated distribution on the LFPC-receiver, shown in Figure 5.10, is in good agreement with the measurement, having a peak irradiance $E_{\text{max}} = 3634$

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.10}
\caption{Contour plot of the irradiance distributions simulated with LFPC geometry. The irradiance is normalized to a DNI of 1000 W/m$^2$ (= 1 sun).}
\end{figure}
kW/m² (−1.7% compared to the measured distribution) and average irradiance on the LFPC-receiver $E_{av,LFPC-rec} = 1330$ kW/m² (−1.7%). The rms error of 138 kW/m² is due to the inability of the orthotropic dispersion angle error to predict the tilt of the elliptical irradiance distribution, caused by the tracking error but incorporating this tracking error in the model was not possible. Nevertheless, it provides a good representation of the performance of the primary mirrors.

The SOE geometry is modeled with flat specular surfaces that reflect the design described in Section 5.2.2. The glass mirrors have been modeled with the same properties as the primary mirrors barring the macroscopic mirror surface inaccuracies caused by the mirror curvature. Since the mirrors are flat, only the experimentally determined narrow-angle scattering [101, 102] is used. For the inner silver-coated aluminum mirrors, literature values of the specular reflectance are used [129] and the glass cover is modeled with experimentally measured spectral transmission [101, 102]. Figure 5.11 shows the final

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$h$ For an accurate reproduction of the measured distribution, the tracking error (i.e. off-axis incident rays with unknown normal direction) would need to be modeled, in addition to a 2-directional realignment of each mirror facet such that the distribution center is centered with the LFPC. The lack of information on the extent of the tracking error at the time of measurement and the increased number of unknowns in the fitting renders such an approach highly impractical.
simulated irradiance distribution at the receiver. As a result of the out-of-focus placement of the receiver, the irradiance is more uniformly distributed over the receiver area, with a lower peak \( E_{\text{max}} = 2549 \text{ kW/m}^2 \) (−29.9\% compared to the irradiance in the focal plane of the primary mirrors; Figure 5.10). The difference in orientation between the LFPC and the receiver (cf. Figure 5.8) is the reason for the apparent rotation of the elliptical distribution. Despite the off-axis placement, absorption in the borosilicate window, and attenuation by the SOE mirror walls, the average irradiance on the receiver has slightly increased (\( E_{\text{av,rec}} = 1374 \text{ kW/m}^2, +3.3\% \)), indicating a good collection of fringe radiation by the SOE. The radiative power absorbed by the receiver then becomes

\[
\dot{Q}_{\text{rec}} = E_{\text{av,rec}} \cdot A_{\text{rec}} = 5.11 \text{ kW},
\]

yielding an optical solar-to-receiver efficiency

\[
\eta_{\text{opt}} = \frac{\dot{Q}_{\text{rec}}}{\dot{Q}_{\text{solar}}} = 79.3\%.
\]

The optical efficiency decomposes into

\[
\eta_{\text{opt}} = \zeta \cdot \rho_{\text{mirror}} \cdot \eta_{\text{window}} \cdot \eta_{\text{SOE}},
\]

where \( \eta_{\text{window}} = 91.3\% \) accounts for the losses associated to the window (reflection and transmission) and \( \eta_{\text{SOE}} = 93.8\% \) is the efficiency of the SOE. Besides absorption in the mirrors, \( \eta_{\text{SOE}} \) accounts for the radiation that is either not reaching the SOE inlet aperture or being rejected. This explains why \( \eta_{\text{SOE}} \approx \rho_{\text{mirror}} \) although the average number of reflections on the SOE is \( \ll 1 \).

### 5.4 Receiver model

The complexity of the electrical design of the receiver and the large number of components do not allow for a purely analytical modelling of the receiver. Thus, the numerical “Simulation Program with Integrated Circuit Emphasis” (SPICE) code, first published as CANCER [130], in its current version by Linear Technologies Corporation [131] is used.

Figure 5.12 shows a schematic of the electric circuit representing the receiver. The first level (a) defines the series-connection of the four submodules,
where the resistances $R_{\text{bar}}$ and $R_{\text{term}}$ represent the conductor resistances of the interconnecting copper bars and terminals respectively. Within the submodules
(b), a realistic representation of the implemented parallel interconnection scheme mandates a more complex circuit model. The top electrode is decomposed into an array of resistances to account for the changing geometry and division of current flow (the current through the grid fingers decreases from the plate to the end of each finger), i.e. the plate interconnecting the four grid fingers \((R_{t,\text{pl}})\), the free (unsoldered) segments of the grid fingers of respective thickness \((R_{t,300}'; R_{t,400}'; R_{t,600}')\) and the cell-connected grid finger sections, each discretized into three segments covering the length of one cell \((R_{t,300}; R_{t,400}; R_{t,600})\). The bottom electrode is modeled in the same manner, divided into interconnecting plate \((R_{b,\text{pl}})\) and discretized finger resistances \((R_b)\). The contact resistances of the cell electrodes to the submodule top \((R_{t,\text{cnt}})\) and to the bottom electrodes \((R_{b,\text{cnt}})\) account for the conductivity losses within the solder. Finally, the PV cells (c), which represent the most important components of the array, are modeled by a single-diode, lumped equivalent circuit model \([56, 132]\), consisting of a current source representing the generated photocurrent (with photocurrent density \(J_{\text{ph}}\)), an exponential diode modeling recombination (with ideality factor \(n_D\) and saturation current density \(J_0\)), a shunt resistance representative of current leakage \((R_{\text{sh}})\), and a series resistance accounting for conduction losses across the cell \((R_s)\).

By lumping the three subcells of the 3-J cell into a single equivalent circuit, the model combines the advantage of a relatively low number of unknown fitting parameters to a high accuracy in predicting \(J-V\) behavior. However, as a consequence, it is only able to predict performance for the incident spectrum at which the cells were measured \([56]\), ASTM AM1.5d (G173-03) \([127]\).

The performance of the receiver and its components is essentially governed by two factors: (1) the incident solar radiation (i.e. irradiance intensity and distribution) and (2) the coolant characteristics (i.e. coolant inlet temperature and flow rate). As a result, each location on the receiver is subjected to a local combination of irradiance intensity and temperature. For simplification, their distribution can be discretized. The conductor resistances are independent of

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\(^1\) Note that this discretization is conservative as the entire current of each cell is modeled as being conducted through a resistance representing the entire portion of the top electrode finger connected to that cell. In reality, only the current generated at the outer cell edge travels through the entire length of the electrode while the average travel length of the current from the electrode-attached cell is only half the cell length.
incident irradiance and have only marginal temperature dependence, and can therefore be modeled as temperature dependent with the average receiver temperature \( T_{av,rec} \). In contrast, the PV cells have a strong dependence on both irradiance and temperature. As discussed in Section 5.2.2, local irradiance non-uniformities on the cells (of the extent encountered in the present application) have a negligible effect on the cell performance, allowing the cells to be modeled as uniformly irradiated with the area-averaged irradiance incident on their inlet aperture \( (E_{av,cell}) \) and average cell temperature \( (T_{av,cell}) \) using the aforementioned single-diode lumped equivalent circuit model [Figure 5.12(c)]. The parameters \( J_{ph}, n_D, J_0, R_{sh} \) and \( R_s \) can then be determined for each cell as a function of \( E_{av,cell} \) and \( T_{av,cell} \).

### 5.4.1 Optical performance

Figure 5.13 shows the discretized average cell irradiance \( E_{av,cell} \) for each cell in the receiver, with a peak cell irradiance \( E_{av,cell,max} = 2463 \text{ kW/m}^2 \) and average cell irradiance \( \langle E_{av,cell} \rangle = 1390 \text{ kW/m}^2 \). Also visible (shaded in black) are the inactive receiver areas (top electrode, inter-cell gaps and cell busbars). Due to the non-uniform nature of the irradiance distribution, the unexploited fraction of the radiation incident on the receiver is not necessarily equal to the fraction of inactive receiver area (equal to 9.9%; cf. Section 5.2.2). The useful radiative power incident on the active area of the cells \( A_{act,rec} \) is

\[
\hat{Q}_{cells} = \int_{A_{act,rec}} EdA = \sum_{i=1}^{36} A_{cell} E_{av,cell,i} = 36 \cdot A_{cell} \langle E_{av,cell} \rangle. \tag{5.8}
\]

This yields an electrically useful fraction of incident radiative power, or optical receiver efficiency \( \eta_{opt,rec} = \hat{Q}_{cells}/\hat{Q}_{rec} = 90.6\% \). Thus \( 1 - \eta_{opt,rec} = 9.4\% \) of the incident radiation cannot be utilized electrically, however this energy, \( (1 - \eta_{opt,rec}) \cdot \hat{Q}_{rec} \), contributes to the thermally utilized power.

### 5.4.2 Thermal performance

The temperature of each cell can be expressed as function of the average incident irradiance by
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\[ T_{av,cell,i} = T_{in} + \alpha_{cell,i} \cdot E_{av,cell,i} \cdot R_{th} (\dot{V}_{rec}), \]  

(5.9)

where \( T_{in} \) is the coolant inlet temperature, equal for each cell as a result of the cooling chip architecture, the factor \( \alpha_{cell,i} \cdot E_{av,cell} \) represents the thermal power absorbed by the cell, \( R_{th} \) is the thermal resistance of the microchannel heat exchanger and \( \dot{V}_{rec} \) is the coolant flow rate. As previously discussed, \( R_{th} \) can be modeled as a function of only the volume flow rate, with the correlation measured at a coolant flow rate of 50 °C (Figure 5.7). For each cell, \( \alpha_{cell,i} \) is the fraction of radiation incident on the active area that is not converted into electrical power. To avoid an implicit dependence on the actual, temperature-dependent electrical cell efficiency, the conservative value \( \alpha_{cell} = 0.65 \) is assumed for all cells (i.e. representing an electrical cell efficiency of 35%). Figure 5.14 shows the resulting discretized temperature distribution across the cell array for a coolant inlet temperature \( T_{in} = 20^\circ \) and volume flow rate \( \dot{V}_{rec} = 10 \) l/min, where the cell temperatures vary between 22 °C and 36 °C.

The total thermal absorbed power in the cells and the inactive area is
\[ Q = \alpha_\text{te} \alpha_\eta \eta_\text{opt,rec} \cdot \alpha_\text{te} \cdot (1 - \eta_\text{opt,rec}) \cdot \dot{Q}_\text{rec}, \]  

(5.10)

where \( \alpha_\text{te} = 0.15 \) is the absorptance of the copper top electrode (solar-averaged with spectral reflectance from Palik et al. [133]). A good estimate of the average receiver surface temperature, required for the modeling of the conductor resistances, is the mean of the cell temperatures, \( T_{\text{av,rec}} = \text{mean}(T_{\text{av,cell,i}}) \). The coolant outlet temperature is

\[ T_{\text{out}} = T_{\text{in}} + \dot{Q}_\text{thermal} / (\dot{V}_\text{rec} \cdot D \cdot c_p), \]  

(5.11)

where \( D \) and \( c_p \) are the density and the heat capacity of the coolant respectively.

With the temperature distribution in Figure 5.14, the average receiver surface temperature is 28.9 °C, while the coolant outlet temperature is 24.6 °C.

**Figure 5.14.** Average cell temperature resulting from the irradiance distribution in (a), a coolant inlet temperature \( T_{\text{in}} = 20^\circ\text{C} \) and volume flow rate \( \dot{V}_\text{rec} = 10 \text{ l/min} \). The cell temperatures range between 22°C and 36°C.
5.4.3 Electrical component modeling

Conductor resistances

The conductor resistances of the receiver and submodules are modeled as temperature dependent with the average receiver surface temperature. Resistance values are calculated as a function of electrode geometry and using literature values for the electrical resistivity of the conductor material [134]. The electrical resistance then becomes

\[ R = \frac{1}{\sigma(T_{av,rec})} \cdot \frac{l_{cond}}{w_{cond} \cdot t_{cond}}, \]  

(5.12)

where \( \sigma \) is the electrical conductivity\(^{1} \) and \( l_{cond} \), \( w_{cond} \) and \( t_{cond} \) respectively represent the effective length, the width and the thickness of the conductor. The contact resistances at the soldered interfaces of the cells with the top and bottom electrodes respectively are modeled as small with resistance \( 1.0 \cdot 10^{-6} \, \Omega \) due to the small cross-section and large contacting surface area. The values of the electrical resistances obtained for exemplary temperatures of 20 °C and 100 °C, representing the temperature boundaries in the modeled system, are summarized in Table 5.1.

PV cell model

To determine the lumped model parameters for each cell in the receiver array as a function of the operating conditions and based on current-voltage measurements at discrete irradiance and temperature, two pathways can be envisioned.

The first option (Method 1) consists in fitting lumped equivalent circuit model parameters to each measured current-voltage curve, followed by determining temperature and irradiance correlations for these parameters, using which the cell performance can then be extrapolated. An underdetermined fitting problem, caused by the relatively large number of fitting parameters for each measurement, can be avoided by modeling \( J_0, n_D, R_s \) and \( R_{sh} \) as independent of

\(^{1}\) The electrical resistivity is written as \( 1/\sigma \), where \( \sigma \) is the electrical conductivity, to prevent confusion between the resistivity and the optical reflectance, both usually denoted with \( \rho \).
injection level and therefore only dependent of temperature [135]. Fitting can then be performed over a range of irradiances at equal temperature, which considerably increases confidence in the fits. The advantage of this method is a fast and simple extrapolation to an arbitrary number of desired operating conditions once the measurement data is fitted and the correlations established. Besides the uncertainty introduced by the simplification of irradiance independence of the fitting parameters, the main disadvantage of the method is that a large number of measurement points (especially with regard to temperature) are required to establish confident temperature correlations and allow consistent extrapolation. The scarce available literature on the temperature dependence of lumped model parameters is an indication of the difficulty associated with this task.

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<th>$w_{\text{cond}}$ [m]</th>
<th>$t_{\text{cond}}$ [m]</th>
<th>$R_{@20^\circ\text{C}}$ [Ω]</th>
<th>$R_{@100^\circ\text{C}}$ [Ω]</th>
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<td>$10.05\cdot10^{-3}$</td>
<td>$0.30\cdot10^{-3}$</td>
<td>$0.35\cdot10^{-3}$</td>
<td>$1.93\cdot10^{-3}$</td>
<td>$2.55\cdot10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>$R_{\text{t,400}}$</td>
<td>$10.05\cdot10^{-3}$</td>
<td>$0.40\cdot10^{-3}$</td>
<td>$0.35\cdot10^{-3}$</td>
<td>$1.45\cdot10^{-3}$</td>
<td>$1.91\cdot10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>$R_{\text{t,600}}$</td>
<td>$10.05\cdot10^{-3}$</td>
<td>$0.60\cdot10^{-3}$</td>
<td>$0.35\cdot10^{-3}$</td>
<td>$9.64\cdot10^{-4}$</td>
<td>$1.28\cdot10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>$R_{\text{t,300'}}$</td>
<td>$4.30\cdot10^{-3}$</td>
<td>$0.30\cdot10^{-3}$</td>
<td>$0.35\cdot10^{-3}$</td>
<td>$8.25\cdot10^{-4}$</td>
<td>$1.09\cdot10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>$R_{\text{t,400'}}$</td>
<td>$4.30\cdot10^{-3}$</td>
<td>$0.40\cdot10^{-3}$</td>
<td>$0.35\cdot10^{-3}$</td>
<td>$6.19\cdot10^{-4}$</td>
<td>$8.18\cdot10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>$R_{\text{t,600'}}$</td>
<td>$4.30\cdot10^{-3}$</td>
<td>$0.60\cdot10^{-3}$</td>
<td>$0.35\cdot10^{-3}$</td>
<td>$4.13\cdot10^{-4}$</td>
<td>$5.46\cdot10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>$R_{\text{t,pl}}$</td>
<td>$22.30\cdot10^{-3}$</td>
<td>$30.25\cdot10^{-3}$</td>
<td>$0.35\cdot10^{-3}$</td>
<td>$4.24\cdot10^{-5}$</td>
<td>$5.61\cdot10^{-5}$</td>
</tr>
</tbody>
</table>
The second option (Method 2) consists in first establishing the temperature and irradiance dependencies of the current-voltage behavior using the measurements, followed by subsequent fitting of a lumped equivalent circuit model to each extrapolated current-voltage curve. While having the disadvantage of requiring fitting for each desired combination of irradiance and temperature and thus potentially higher computational effort, this method allows accurate predictions in a large range of operating conditions based on established literature relations and with a smaller number of required measurements.

**Measured cell data** - The HCPV receiver uses “Improved Third Generation CPV Technology” (C3MJ+) triple-junction cells, consisting of GaInP, GaInAs and Ge subcells on a Ge substrate, having a rated efficiency of 39.2% and recommended operating temperature <110 °C [114, 115]. Detailed measurement data was provided by the manufacturer for irradiances $E_i = \{315 \, \text{kW/m}^2, 500 \, \text{kW/m}^2, 630 \, \text{kW/m}^2, 810 \, \text{kW/m}^2\}$ and temperatures $T_j = \{10 \, \text{°C}, 25 \, \text{°C}, 80 \, \text{°C}, 95 \, \text{°C}, 110 \, \text{°C}\}$ in the form of $I$-$V$ curves. Throughout this Section, the current-voltage behavior of a single cell is expressed by its current density $J = I/A_{\text{cell}}$, where $A_{\text{cell}} = 0.99 \, \text{cm}^2$ is the active cell aperture area.

To filter out the stochastic variation between different cell batches and allow the extraction of average irradiance and temperature trends, the $J$-$V$ curves are pre-processed and all curves with equal temperature and irradiance are jointly averaged, as discussed in Appendix C.1. In the range of available measurement data, interpolation between the most important performance metrics, i.e.

1. short-circuit current density $J_{sc}$;
2. open-circuit voltage $V_{oc}$;
3. fill factor $FF = J_{MPP} \cdot V_{MPP} / (J_{sc} \cdot V_{oc})$; and
4. efficiency at maximum power point (MPP) $\eta_{MPP} = J_{MPP} \cdot V_{MPP} / E$,

where $J_{MPP}$ and $V_{MPP}$ are the current density and voltage at the MPP, provides a reasonably good estimate of the cell performance. The extracted characteristics of the average cells and a comparison to the best-performing cells is provided in Appendix C.2. However, an extrapolation of the cell performance beyond the measurement conditions (especially to higher irradiance) requires modelling as
discussed above. Due to the limited availability of measurement points, the determination of the lumped model parameters is carried out with Method 2 to guarantee a high degree of confidence in the results.

**Extrapolation of current-voltage behavior** - The efficiencies at MPP ($\eta_{MPP,i,j}$), the short-circuit current densities ($J_{sc,i,j}$) and the open-circuit voltages ($V_{oc,i,j}$) of the average cells are listed in Table C.1 (Appendix C.2) for every combination of irradiance and temperature \( \{i,j\} \). For each of these three performance metrics, a model predicting the behavior as a function of irradiance and temperature is established.

The efficiency at MPP is modeled based on Helmers et al. [136] as

\[
\eta_{MPP}(E,T) = \eta_{MPP}(E,T_{ref}) \cdot \left[1 + t_{ref}(E) \cdot (T - T_{ref})\right],
\]

where

\[
\eta_{MPP}(E,T_{ref}) = \beta_1 \ln(\beta_2 E) \cdot \frac{\ln(\beta_2 E) - \ln[\ln(\beta_2 E) + 0.72]}{\ln(\beta_2 E) + 1} - \beta_3 E
\]

![Figure 5.15. Measured (×) and modeled (--) efficiency at MPP ($\eta_{MPP}$) as a function of the irradiance for the measured temperatures and an exemplarily prediction for $T = 60^\circ$C. The circular markers (○) and dashed line show the change with temperature of the maximum of the $\eta_{MPP}$-$E$ curves. As the temperature is increased, the efficiency-maximum shifts to a higher irradiance.](image-url)
Table 5.2. Fitted parameters of the MPP efficiency model.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>$5.400 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>$2.219 \cdot 10^{305}$</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>$4.893 \cdot 10^{-6}$</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>$-2.258 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>$\beta_5$</td>
<td>$1.475 \cdot 10^{-4}$</td>
</tr>
</tbody>
</table>

is the irradiance-dependent efficiency at reference temperature $T_{\text{ref}} = 25 \, ^\circ\text{C}$,

$$
t_{\text{rel}}(E) = \beta_4 + \beta_5 \ln(E)
$$

(5.15)
is the relative temperature coefficient of the photovoltaic efficiency and $\beta_1 > 0$, $\beta_2 > 0$, $\beta_3 > 0$, $\beta_4 < 0$ and $\beta_5 > 0$ are independent fitting parameters. Figure 5.15 shows the measured and simulated efficiency with the fitting parameters listed in Table 5.2 for $E = 100,\ldots,5000 \, \text{kW/m}^2$ and $T = \{10 \, ^\circ\text{C}, 25 \, ^\circ\text{C}, 80 \, ^\circ\text{C}, 95 \, ^\circ\text{C}, 110 \, ^\circ\text{C}\}$. The five optimized parameters accurately fit the 17 measurement points with an rms error of 0.16%. Two important observations can be made from the results: (1) with increasing irradiance, the cells’ temperature sensitivity decreases (i.e. the $\eta_{\text{MPP}}-E$ curves converge) in accordance to results by [136–138]; and (2) as the temperature increases, the efficiency maximum shifts to a higher irradiance.

The short-circuit current density can be modeled using

$$
J_{\text{sc}}(E,T) = c(T) \cdot E,
$$

(5.16)

where $c(T)$ is a temperature-dependent irradiance coefficient. To determine $c(T)$, the discrete irradiance coefficients $c_j$ can be fitted to $J_{\text{sc},i,j} = c_j \cdot E_i$ for each measured temperature $T_j$. The best-fit $c_j$ are given in Table 5.3, where an increase with temperature can be observed. This increase is associated to a reduction of the bandgap energy for increasing temperatures, ultimately leading to higher currents [139]. The temperature dependence of the coefficients is found to be effectively linear in the measured range, and can be accurately described with
the correlation \( c(T) = 1.337 \times 10^{-1} + 8.243 \times 10^{-5} \cdot T[^\circ C] \) (rms error 0.15\%). Using this correlation and Eq. (5.16), the measured \( J_{sc} \) are predicted with an rms error of 0.46\%, which indicates a good fit over the whole range. Figure 5.16 shows the corresponding comparison of the measured and simulated short-circuit current densities, where the inlay details the agreement of the simulation to the measurements. Short-circuit current densities around 35 A/cm\(^2\) are predicted for 2500 kW/m\(^2\) (the highest-illuminated cell in the array) with slight dispersion due to operating temperature.

The open-circuit voltage is modeled based on Helmers et al. [136] as

\[
V_{oc}(E,T) = \frac{n_D(T) \cdot k_B \cdot T}{q} \cdot \ln \left( \frac{J_{sc}(E,T)}{J_0(T)} \right),
\]

(5.17)

where \( n_D(T) \) and \( J_0(T) \) represent the temperature-dependent ideality factor and saturation current density of the diode in the equivalent circuit respectively, \( k_B \) is the Boltzmann constant and \( q \) is the elementary charge. With the limited amount

\[^k \text{Alternatively, linear interpolation between } c_j \text{ could be performed to determine } c(T). \text{ Fitting of the measured } J_{sc} \text{ with Eq. (5.16) would then result in a slightly smaller rms error of 0.43\%.} \]
of available measurements at each temperature, finding valid models for $n_D(T)$ and $J_0(T)$ is not as straightforward as for $C(T)$. When fitting the single parameter $c_j$ to $J_{sc,i,j} = c_j E_i$ for each temperature, the results for $c_j$ are nonambiguous despite the scarce measurement data. Fitting two parameters $n_D,j$ and $J_0,j$ at each temperature to the same scarce data $V_{oc,i,j} = n_D,j \cdot k_B \cdot T_j/q \cdot \ln(J_{sc,i,j}/J_0,j)$ however, produces flat optima of the objective function. I.e. many combinations of $n_D,j$ and $J_0,j$ fit the measurements with similarly small rms error. The resulting best-fit parameters then do not necessarily follow a well-defined temperature trend. While the fitted $n_D,j$ and $J_0,j$ are perfectly suitable for representing the cell performance at the measured temperatures, it can cause issues when interpolating between the best-fit parameters to predict performance at other temperatures, i.e. the combined effect of the two interpolated parameters doesn’t necessarily represent the desired intermediate performance with high accuracy. To address this issue, the objective function for the parameter fit is extended from solely minimizing the rms error between the simulated and independent measured $V_{oc}$ data to also include a desired alignment of the parameters across the temperature range based on literature correlations [140]: (1) a linear temperature dependence of the ideality factor and (2) an exponential temperature dependence of the

<table>
<thead>
<tr>
<th>$T_j$ [°C]</th>
<th>$c_j$ [A/W]</th>
<th>$n_D,j$ [-]</th>
<th>$J_0,j$ [A/cm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.135</td>
<td>3.28</td>
<td>1.16·10$^{-17}$</td>
</tr>
<tr>
<td>25</td>
<td>0.136</td>
<td>3.28</td>
<td>1.73·10$^{-16}$</td>
</tr>
<tr>
<td>80</td>
<td>0.140</td>
<td>3.22</td>
<td>4.91·10$^{-13}$</td>
</tr>
<tr>
<td>95</td>
<td>0.142</td>
<td>3.21</td>
<td>2.93·10$^{-12}$</td>
</tr>
<tr>
<td>110</td>
<td>0.143</td>
<td>3.19</td>
<td>1.44·10$^{-11}$</td>
</tr>
</tbody>
</table>
saturation current density (linear temperature dependence of \( \ln[J_0] \)). Table 5.3 lists the best-fit \( n_D \) and \( J_0 \), determined with this method, which exhibit a good alignment to the desired temperature dependences. \( n_D(T) \) and \( J_0(T) \) for Eq. (5.17) can then be confidently determined by linear interpolation between \( n_{D,j} \) and \( \ln[J_{0,j}] \). Figure 5.17 shows the comparison of the measured and simulated open-circuit voltage. The measurements are predicted accurately, with an rms error of 0.83%. Without the additional objectives on the temperature-dependence of \( n_{D,j} \) and \( J_{0,j} \) the rms error for predicting the measurement data would only be slightly smaller (0.78%) while risking a lower consistency for interpolated conditions.

Eqs. (5.13), (5.16) and (5.17) can be used together to extrapolate the performance of average cells at other operating conditions, as exemplarily shown in Figures 5.15 to 5.17 for a simulated cell at \( T = 60 \, ^\circ C \).

**Equivalent circuit model** - To translate the extrapolated cell performance into the corresponding \( J-V \) curves, i.e. to determine the parameters \( J_{ph} \), \( n_D \), \( J_0 \), \( R_{sh} \) and \( l \).

---

1 Note that these additional objectives would not be required if more measurement data (specifically for a larger irradiance range) was available at each temperature, as the parameters would then be inherently linked through the cell behavior, expressed through the measured \( V_{oc} \). The added objectives merely aid in linking the cell behavior over all measurements.
describing each cell within the array, a lumped equivalent circuit model is analytically fitted. The $J$-$V$ curve based on a lumped model is described by

$$J = J_{\text{ph}} - J_0 \left[ \exp \left( \frac{q(V + J \cdot A_{\text{cell}} \cdot R_s)}{n_D \cdot k_B \cdot T} \right) - 1 \right] - \frac{V + J \cdot A_{\text{cell}} \cdot R_s}{R_{\text{sh}}}.$$  

(5.18)

By construction, $J_{\text{ph}}$ is equal to the current density at short circuit ($J_{\text{sc}}$) and is directly obtained from Eq. (5.16), while the remaining parameters need to be obtained numerically. An essential part in the fitting of a lumped model is the objective function, for which several approaches have been studied in literature [56, 132, 141]. Good results have been achieved with the “three-point fitting” procedure, which only requires fitting against short-circuit, open-circuit, and the maximum power point, i.e. the performance metrics that can be extrapolated with Eqs. (5.13), (5.16) and (5.17), and which can thus be readily applied here. With $J_{\text{ph}} = J_{\text{sc}}$, fitting at short circuit is superfluous. Hence, the objective function $f_{\text{obj}}$ to be minimized only needs to contain the errors at open circuit ($\varepsilon_{\text{Voc}}$) and MPP ($\varepsilon_\eta$) [56],

$$f_{\text{obj}} = \frac{1}{2} \left( \varepsilon_{\text{Voc}} + \varepsilon_\eta \right),$$

(5.19)

with

$$\varepsilon_{\text{Voc}} = \left( \frac{V_{\text{oc,fit}} - V_{\text{oc,meas}}}{V_{\text{oc,meas}}} \right)^2 \quad \text{and} \quad \varepsilon_\eta = \left( \frac{\eta_{\text{MPP,fit}} - \eta_{\text{MPP,meas}}}{\eta_{\text{MPP,meas}}} \right)^2.$$  

(5.20)

Parameter fitting is performed using unconstrained non-linear optimization with the Nelder-Mead simplex algorithm [142, 143], as detailed in Appendix C.3. It is important to note that, although the fitting problem for each cell is in theory underdetermined, as four parameters ($n_D$, $J_0$, $R_{\text{sh}}$ and $R_s$) are fitted to only three inputs ($J_{\text{sc}}$, $V_{\text{oc}}$ and $\eta_{\text{MPP}}$), the result from the fitting is a valid representation of the PV cell at the prescribed operating conditions. In Appendix C.4, this is verified by fitting a known $J$-$V$ curve (average measurement at $E = 500 \text{ kW/m}^2$ and $T = 25 \degree \text{C}$) by using only the extracted $J_{\text{sc}}$, $V_{\text{oc}}$ and $\eta_{\text{MPP}}$ and the described method.
5.5 Results

5.5.1 Present system performance

Using the model described in the previous Section, we analyze the performance achievable with the present system, subsequently referred to as Design A, under two distinct operating conditions governed by the coolant inlet temperature:

1. PV-only operation with $T_{in} = 20^\circ C$, where the PV cells are kept at low temperatures to maximize photovoltaic conversion; and
2. PV-T operation with $T_{in} = 85^\circ C$, where the hot coolant outlet can be used to drive downstream LGH processes, as discussed earlier.

In both modes, a coolant flow rate of 10 l/min is employed.

As discussed in Section 5.4.2, a coolant inlet temperature of 20 °C then results in cell temperatures between 22 °C and 36 °C [cf. Figure 5.14], which promises a high PV cell efficiency. The solid lines in Figure 5.18 show the current-voltage

![Figure 5.18. Single receiver module current-voltage and power-voltage curves for the irradiance distribution from Figure 5.13 and with coolant inlet temperatures $T_{in} = 20^\circ C$ (solid lines) and $T_{in} = 85^\circ C$ (dashed lines). The elevated coolant temperature reduces the power at MPP from 1498 W to 1370 W.](image)
A 6-focus high-concentration photovoltaic-thermal dish system and power-voltage behavior achieved in a single receiver module for this operation mode. An electrical power $P_{\text{MPP}} = 1498$ W can be extracted at a current $I_{\text{MPP}} = 132.9$ A and voltage $V_{\text{MPP}} = 11.27$ V, yielding an electrical efficiency at MPP of the PV receiver module of $\eta_{\text{MPP,rec}} = 32.33\%$. The total electrical power extracted from the six modules of the dish is 8.986 kW. The characteristic temperatures and receiver performance under the two operating conditions are summarized in Table 5.4.

The higher coolant temperature ($T_{\text{in}} = 85^\circ$) in PV-T mode causes the cell temperatures to vary between 87 °C and 100 °C, guaranteeing safe operating conditions ($< 110$ °C; [114]) while allowing low grade heat for downstream applications to be extracted at a maximum temperature $T_{\text{out}} = 89.8$ °C. The average receiver surface temperature, used for the modeling of the temperature of the conductor resistances, is $T_{\text{av,rec}} = 93.85$ °C. As a consequence of the higher cell temperatures (and to a smaller extent the higher conductor temperatures), the voltage is considerably decreased to $V_{\text{MPP}} = 9.96$ V, while the current is slightly increased to $I_{\text{MPP}} = 137.6$ A (see the discussion of the temperature effect on PV cell performance in Appendix C.2). This causes a drop in produced power per receiver to $P_{\text{MPP}} = 1370$ W (8.219 kW for the entire dish), signifying a decrease of $-8.54\%$ compared to the PV-only operation mode. Accordingly, the electrical efficiency at MPP is reduced to 29.57%. The thermal power absorbed in the coolant and available as LGH is $\dot{Q}_{\text{thermal}} = 3335$ W (20.01 kW for the full system).

From these results, the electrical receiver efficiency is obtained as

$$\eta_{\text{el,rec}} = \eta_{\text{opt,rec}} \cdot \eta_{\text{MPP,rec}},$$

(5.21)

where the optical receiver efficiency ($\eta_{\text{opt,rec}} = 90.6\%$) accounts for the solar radiation impinging on the inactive receiver area and lost for photovoltaic conversion, as discussed in Section 5.4. The electrical receiver efficiencies with the PV-only and PV-T operation modes then become 29.3% and 26.8% respectively. Finally, the full-system, solar-to-electricity efficiency is obtained as

$$\eta_{\text{sol-to-el.}} = \eta_{\text{opt}} \cdot \eta_{\text{el,rec}},$$

(5.22)
where $\eta_{\text{opt}}$ is the optical efficiency of the concentrator (= 79.3% for the current system, as discussed in Section 5.3). The solar-to-electricity efficiencies finally are 23.2% and 21.3% respectively for the two operation modes.

A breakdown of the efficiencies associated to the different components of the present system is provided in Table 5.5. The components of the optical concentrator efficiency and the optical receiver efficiency were already discussed in Sections 5.3.2 and 5.4.1, respectively. Of the listed values, the electrical efficiency at MPP of the PV receiver module, $\eta_{\text{MPP,rec}}$, exhibits a discrepancy when compared to the cell efficiencies in the same irradiance and temperature range (cf. Figure 5.15). Two potential causes for this divergence are identified: (1) ohmic losses through the conductor resistances; and (2) losses due to current mismatch. As discussed in Section 5.2.2, the receiver with its hybrid parallel-serial cell-interconnections is designed such that it can avoid current-mismatch losses despite high cell-to-cell irradiance non-uniformities. However, this only holds if each submodule (each quadrant of the receiver) collects the same average irradiance, i.e. the irradiance distribution is symmetric with respect to the submodules. When observing the average cell irradiance obtained with the current system (Figure 5.13), it becomes evident that this requirement is not entirely met because of the elliptical irradiance distribution, the center of which is slightly displaced by the attenuation of the SOE, as discussed in Section 5.3.2. As a consequence, the power incident on the active cell area varies from 916 W in the least irradiated submodule to 1501 W in the most irradiated submodule.

To assess the impact of these mismatch losses, the performance of the receiver under a symmetric irradiance distribution but having the same degree of non-uniformity is simulated. The distribution is generated by averaging the cell

<table>
<thead>
<tr>
<th>$T_{\text{in}}$ [°C]</th>
<th>$T_{\text{av,rec}}$ [°C]</th>
<th>$T_{\text{out}}$ [°C]</th>
<th>$P_{\text{MPP}}$ [W]</th>
<th>$I_{\text{MPP}}$ [A]</th>
<th>$V_{\text{MPP}}$ [V]</th>
<th>$FF$ [%]</th>
<th>$\eta_{\text{MPP,rec}}$ [%]</th>
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<tr>
<td>20.0</td>
<td>28.9</td>
<td>24.6</td>
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<td>32.33</td>
</tr>
<tr>
<td>85.0</td>
<td>93.9</td>
<td>89.8</td>
<td>1370</td>
<td>137.6</td>
<td>9.96</td>
<td>81.7</td>
<td>29.57</td>
</tr>
</tbody>
</table>

Table 5.4. Characteristics of PV-only operation versus PV-T operation, per receiver module, of the presented system.
irradiance from Figure 5.13 using reflective symmetry along the lines
\( y = x \cdot \tan(k \cdot \pi/4) \) (\( k = 0, \ldots, 3 \)) and is shown in Figure 5.19. The electrical efficiency at MPP of the PV receiver module achieved with this irradiance distribution is considerably higher at 37.66% than with the non-symmetric distribution. A relative loss in efficiency of \(-14.2\%\) can be attributed to current mismatch in the present system.

5.5.2 Potential for improvements

Design A, described and assessed in the previous sections, is compared to two hypothetical systems: Design B, which includes immediate improvements to the system by correcting malfunctioning components of the presented first prototype, and Design C, which additionally considers enhancements and design changes that can be implemented in the medium-term.

For Design B, the chief improvement results from fixing the malfunctioning tracking system, discussed in detail in Section 5.3.1, followed by a realignment of the individual mirror facets. As a consequence, the apparent mirror slope error is reduced and the resulting irradiance distribution in the focal plane is more
circular. When considering that the apparent slope error ($\sigma_{\text{err},1} = 1.17$ mrad in circumferential direction and $\sigma_{\text{err},2} = 1.69$ mrad in radial direction) is composed of both the actual slope inaccuracies in the mirrors and the overlapping tracking error effects of the individual facets, a conservative assumption is that without the tracking error the apparent slope error is at most equal to the smallest of both dispersion errors [$= \min(\sigma_{\text{err},1}, \sigma_{\text{err},2})$], i.e. assuming that the tracking error only affected the direction of the large slope error and the actual slope inaccuracy of the mirror is isotropic. The concentrator of Design B was therefore modeled with an isotropic angular dispersion error $\sigma_{\text{err},1} = \sigma_{\text{err},2} = 1.17$ mrad. A summary of the expected efficiencies with Design B is given in Table 5.5. The first effect of the improved irradiance distribution is noticeable on the efficiency of the SOE. As more radiation directly impinges on the receiver without attenuation by the SOE, less energy is absorbed in the SOE mirrors, resulting in $\eta_{\text{SOE}} = 97.1\%$ (relative improvement of 8.3%) and consequently a higher optical concentrator efficiency $\eta_{\text{opt}} = 82.0\%$ (relative improvement of 3.4%). As a result of the higher optical efficiency and narrower irradiance distribution, the temperatures of the central PV cells increase. Due to the efficient cooling they can still be kept within safe operating conditions (maximum cell temperature 109 °C with $T_{\text{in}} = 85 \degree$C). The second, more important effect is a considerably reduced current mismatch between the submodules, leading to an electrical receiver efficiency at MPP of $\eta_{\text{MPP,rec}} = 37.0\%$ (+14.4% rel.) and $\eta_{\text{MPP,rec}} = 34.5\%$ (+16.7% rel.) for PV and PV-T modes respectively, and accordingly to higher electrical receiver efficiencies [$\eta_{\text{el,rec}} = 33.8\%$ (+15.2% rel.) and 31.5% (+17.4% rel.)]. Ultimately, all effects combined, the total solar-to-electricity efficiency is improved to $\eta_{\text{sol-to-el.}} = 27.7\%$ (+19.2% rel.) and $\eta_{\text{sol-to-el.}} = 25.8\%$ (+21.5% rel.) for both operation modes. The extractable power for the entire dish then becomes $P_{\text{MPP}} = 10.72 \text{ kW}_\text{el}$ for PV-only operation and $P_{\text{MPP}} = 9.99 \text{ kW}_\text{el}$ and $\dot{Q}_{\text{thermal}} = 19.39 \text{ kW}_\text{th}$ for PV-T operation.

For Design C, we look at how each constituent of the total solar-to-electricity efficiency can be improved by rigorous enhancement of the system components towards a more mature future concentrator. At 98.2%, $\zeta$ is already high in Designs A and B. With further development and optimization of the size, shape and arrangement of the mirror segments on each facet, a small increase in active
A 6-focus high-concentration photovoltaic-thermal dish system is possible, leading to a slight improvement of $\zeta$ to 99.0%. As the employed glass mirror with solar-averaged reflectance of $\rho_{\text{mirror}} = 94.2\%$ is among the best commercially available mirrors, the potential for improvements is negligible. While the losses introduced by the window protecting the SOE inlet are relatively high ($\eta_{\text{window}} = 91.3\%$), the protection of the PV cells from ambient conditions is essential and cannot be omitted. As discussed in Section 5.2.2, an alternative to the window is a protective polymer top membrane covering the entire dish inlet aperture. However, suited materials, e.g. ETFE or FEP membranes, offer similar or inferior performance [101, 102], while introducing additional system complexity by requiring an inflation system. An improvement of the SOE efficiency compared to Design B is also not likely, as it is mostly governed by the quality of the beam reflected from the primary mirrors, which dictates the average number of reflections on the SEO mirror walls. However, design changes to the SOE can have a positive impact on the optical receiver efficiency and the electrical receiver efficiency at MPP. As mentioned in Section 5.2.2, while most of the active area loss of the receiver can be attributed to the fingers of the top electrode and cannot be reduced without radical receiver redesign, a smaller part at the perimeter of the receiver aperture is taken up by top electrode conductors that can be covered beneath the SOE mirrors by reducing the size of the SOE outlet aperture, with the potential to increase $\eta_{\text{opt,rec}}$ to 92%. Furthermore, the current SOE design was optimized for optical concentrator

Table 5.5. Characteristics of PV-only operation versus PV-T operation, per receiver module, of the presented system.

<table>
<thead>
<tr>
<th>$T_{\text{in}}$ [°C]</th>
<th>$\zeta$ [%]</th>
<th>$\rho_{\text{mirror}}$ [%]</th>
<th>$\eta_{\text{window}}$ [%]</th>
<th>$\eta_{\text{SOE}}$ [%]</th>
<th>$\eta_{\text{opt}}$ [%]</th>
<th>$\eta_{\text{opt,rec}}$ [%]</th>
<th>$\eta_{\text{MPP,rec}}$ [%]</th>
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efficiency; the optimization did however not take into account the requirement of a maximally equal distribution of the irradiance with respect to the four submodules. By tweaking the tilt of the SOE walls, a reduction of the mismatch loss leading to an improvement of $\eta_{\text{MPP,rec}}$ of 0.5% is expected. These incremental improvements eventually lead to total solar-to-electricity efficiencies of 28.5% and 26.6% for the two operation modes respectively. A final crucial improvement can be achieved by changing the shape of the mirror facets. As mentioned in Section 5.2.2, the current elliptical facet design is an artifact of the flexibility required to investigate vacuum membranes as primary mirrors. With the ultimate decision to use thin back-surface glass mirrors, glued on concrete shell structures, this requirement is no longer necessary. The mirror facets can then be designed with arbitrary perimeter shape. For example, if the current facets are replaced by circumscribed hexagonal mirrors in the same arrangement, the active area fraction can be maximized and the inlet aperture is increased by a factor $6 \cdot \tan(\pi/6)/\pi = 1.1$ without changes to the outer dimensions of the dish. Accordingly, the extractable power for the entire dish then becomes $P_{\text{MPP}} = 12.13$ kW$\text{el}$ for PV-only operation and $P_{\text{MPP}} = 11.33$ kW$\text{el}$ and $\dot{Q}_{\text{thermal}} = 21.50$ kW$\text{th}$ for PV-T operation. Additional potential improvements not yet considered are tertiary optics within the receiver, placed on top of the top electrode fingers, allowing a further increase of $\eta_{\text{opt,rec}}$ at the expense of added cost, and utilization of dedicated PV cells that are optimized for the high concentrations present within the receiver (i.e. with larger grid finger cross-sections), which would result in an increase in $\eta_{\text{MPP,rec}}$.

Designs B and C reveal the short- and medium-term potential of the presented CPV-T system design, with full-system solar-to-electricity efficiencies matching the best-reported CPV designs [4, 144] and the potential to efficiently extract LGH for downstream thermal applications. Depending on the local boundary conditions such as climate, yearly insolation, applications for LGH, and even momentary fluctuations in the electricity market, an operation in either PV-only or PV-T mode can be favorable.
5.6 Summary and conclusions

In this chapter, the design, optical measurement, and performance modeling of a modular, multi-focus, high-concentration photovoltaic-thermal cogeneration system that combines high-performance components with various design innovations in pursuit of a cost-effective and sustainable solution for solar energy utilization is presented.

At the core of the system lays a novel solar dish design that achieves high concentration and compactness using a subdivision of a parabolic dish into identical modules, each comprising an individual focal point, and arranged symmetrically around the central axis.

A practical implementation of such a concentrator, having an inlet aperture of 38.7 m$^2$ and consisting of six modules, was constructed and tested. It predominantly uses prefabricated high-performance concrete components to improve the structural rigidity and scalability compared to traditional alternatives while reducing investment cost by leveraging mass-production. This prototype concentrator is employed to investigate and optically characterize different mirror solutions that combine the advantages of low cost with a high optical performance. A configuration with mirrors made from thin back-surface glass, glued onto precast shells bearing a parabolic shape, was selected for detailed characterization and system modeling.

In its present composition, it has a geometric concentration ratio at the receivers of 1733× and achieves an average concentration of 1353 suns at each of the six receivers. Every receiver comprises 36 triple-junction CPV cells, connected in a unique hybrid parallel-serial interconnection scheme that mitigates mismatch losses with non-uniform irradiance distributions. Cell cooling is carried out using high-performance micro-channel cooler chips with low thermal resistance and pressure drop, which leads to a low cooling power consumption. This allows the use of high coolant inlet temperatures and opens up the opportunity for efficient polygeneration of electricity and low-grade heat.

To assess the effect of different operation modes on the full system performance, a receiver model was developed to predict the performance of the various receiver components under specific irradiance and temperature conditions. Using this model, two representative operating modes were studied:
(1) PV-only operation with a coolant inlet temperature of 20 °C and where the cells operate under ideal conditions for photovoltaic conversion; and (2) PV-T operation with a coolant inlet temperature of 85 °C, and where, under the penalty of a slightly lower photovoltaic performance, low grade heat can be extracted at 89.8 °C. The full-system solar-to-electricity efficiencies are 23.23% and 21.25% for the two modes, respectively. A tracking malfunction in the characterized system, leading to a sizable current mismatch between the cells, is the chief reason for a relatively poor electrical performance in both situations. Correcting this malfunction has a positive impact on both the optical and electrical efficiencies, ultimately leading to an expected short-term improvement of the solar-to-electricity efficiencies to 27.7% and 25.8% and extracted power of 10.72 kW\textsubscript{el} and 9.99 kW\textsubscript{el}/19.39 kW\textsubscript{th} respectively. Finally, by adapting the design of several system components using the experience gained with the present system, an increase to 28.5% and 26.6% solar-to-electricity efficiency and extracted power of 12.13 kW\textsubscript{el} and 11.33 kW\textsubscript{el}/21.50 kW\textsubscript{th} is predicted for the two modes, respectively, matching the performance of some of the best-reported CPV commercial systems [6].
6 Nonimaging polygonal mirrors achieving uniform irradiance distributions

After tackling the issue of non-uniform irradiance on a PV dense-array by an elaborate electrical receiver design in an effort to reduce optical losses and constraints on the concentrator, this chapter proposes an alternative approach where the optical system is adapted such as to provide ideal conditions for a simple high-efficiency receiver design, representing a potential route for a future HCPVT system design.

6.1 Introduction

As noted in Chapter 1, the high concentration needed to directly and indirectly increase the efficiency with CPV systems, both in terms of generated power and cost, generally induces a high irradiance non-uniformity across the receiver. This non-uniformity can severely reduce the electrical efficiency as a result of the current mismatch induced between the cells, especially if series-connected, in the dense-array [53, 56, 57].

Various nonimaging secondary optics have been developed to partially counteract the apparent trade-off between concentration and irradiance uniformity, i.e. improve the irradiance uniformity while matching the square aperture of CPV cells/arrays without major penalties in concentration. Examples include kaleidoscope flux homogenizers [80, 81, 145, 146], Köhler integrators [147–150], and other dielectric total internal reflection based optics [151]. Alternative approaches use concurrent tailoring, e.g. with the simultaneous multi-surface (SMS) method [42], of the primary and secondary optics. The advantage of tailoring both surfaces is that certain optical aberrations inherent to

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point focus concentrators can be eliminated, which leads to higher concentrations and thus to a somewhat relaxed trade-off between concentration and irradiance uniformity. In each of these approaches, however, a certain amount of optical efficiency with respect to the single-stage concentrator is sacrificed to improve the irradiance uniformity, e.g., by absorption from additional reflections on a secondary mirror or within a dielectric optic, decrease of the acceptance angle (ray rejection), reduction of the intercept factor, or a combination of the above. At the same time, the system complexity and cost are increased.

Alternatively, to avoid these disadvantages, a single reflecting surface can be tailored such that it produces a uniform irradiance distribution with only one attenuation. The absence of dielectric optics (lenses) mitigates issues with cooling as well as chromatic aberrations. The single reflection ensures a high optical efficiency. However, one-reflection optics tailored for uniformity are not common for solar energy applications, due in part to the inherently lower achievable concentrations\(^\text{b}\). A notable exception is a design where a controlled combination of surface deformations, described using Zernike polynomials, is employed for the shaping of the mirror [152], but it is limited to circular mirrors. Other concepts relying on an array of carefully arranged small, flat mirror segments to produce uniform distributions have been successfully demonstrated [153, 154].

In this chapter, we present an alternative design method for single-reflection mirrors producing a uniform irradiance distribution that in contrast to previous designs uses freeform continuous mirrors with polygonal inlet apertures, which, as will be shown in the analysis that follows, has several intriguing advantages. Section 6.2 introduces the proposed method, Section 6.3 discusses various applications of the method for medium- and high-concentration PV and Section 6.4 presents results with exemplary single- and multi-mirror concentrators.

\(^\text{b}\) The geometric concentration limit for full collection with a one-reflection 3D mirror such as a parabolic dish is given by 
\[ C_{\text{g,1,max,3D}} = \sin(2\Phi)^2 / \sin(2\theta_{\text{sun}})^2 - 1 \]
where \( \Phi \) is the mirror rim angle [11], with a maximum for \( \Phi = 45^\circ \) of 11561× with \( \theta_{\text{sun}} = 4.65 \) mrad. The geometric concentration limit of a two-stage system is
\[ C_{\text{g,tot,max,3D}} = C_{\text{g,1,max,3D}} \cdot C_{\text{g,2,max,3D}} \]
with \( C_{\text{g,2,max,3D}} = \cos(\Phi)^2 / \sin(\theta_{\text{sun}})^2 \) [106] (see Chapter 2). It is maximized for \( \Phi = 14.86^\circ \) at a value of 43191×, which is close to the theoretical limit 
\[ C_{\text{g,max,3D}} = 1 / \sin(\theta_{\text{sun}})^2 - 1 = 46247\times \]. While practical designs, especially if designed for high irradiance uniformity, fall short of these theoretical limits by a considerable margin, the comparison of the limits provides a good concept of the fundamental difference in achievable concentration.
6.2 Methods

Achieving a prescribed irradiance distribution with a single reflection amounts to redirecting incident rays to different locations in the focal plane, which are determined as a function of the rays’ point of intersection with the mirror. The crucial task is to find a mapping technique that defines the relationship between a point on the mirror and a point on the receiver such that a continuous mirror and the desired irradiance distribution are produced. The proposed method allows to achieve a uniform irradiance distribution on a polygonal receiver by single reflection on a primary mirror having a polygonal perimeter. It comprises the following steps: (i) 2D mapping is performed for a regular nodal grid between the two polygons while preserving the fractional area between points; (ii) the mapped points are subsequently arranged in 3D space via linear transformations; (iii) the final mirror shape is numerically optimized; (iv) final evaluation is carried out using Monte-Carlo ray-tracing techniques.

6.2.1 Area-conserving mapping between two polygons

The presented mapping method $\Gamma_{P_1 \rightarrow P_2}$ between two polygons $P_1$ and $P_2$ is based on a technique for low-distortion mapping between a disk $D$ and a square $S$ ($\Gamma_{D \rightarrow S}$), first published in [155]. The mapping method is bi-continuous, i.e. mapping can be performed from $S$ to $D$ and vice-versa, and conserves the fractional area between points. Both these properties are essential for the mapping method presented here, as a disk serves as intermediate geometry to map between two polygons. For this purpose, the original method is extended to the more general case of mapping between a disk and a regular polygon with $N_P$ sides. Importantly, the method is further adapted such as to conserve the total area between the mapped geometries (i.e. the areas of the polygon and the disk are equal; $A_P = A_D$) as this simplifies its usage within the context of geometric optics.

In a first step, points on $P_1$ are mapped to $D$ ($\Gamma_{P_1 \rightarrow D}$). Using the inverse mapping ($\Gamma_{D \rightarrow P_2}$) the points are then mapped from $D$ to $P_2$. The property of the disk of having rotational symmetry around its center for every angle allows the transition between two polygons with different $N_{P_1}$ and $N_{P_2}$. The mapping $\Gamma_{P_1 \rightarrow D}$ from a regular polygon to a disk is illustrated by means of a hexagon ($N_{P_1} = 6$).
in the top of Figure 6.1. For convenience, the polygon and the disk both have an area $A_P = A_D = 1$, unlike in the original method where the polygon apothem and the disk radius are equal. The polygon can be divided into $N_{P1}$ rotationally symmetric triangular regions, delimited by the lines connecting the center and the corners, $y = x \cdot \tan[(2j - 1) \cdot \pi/N_{P1}]$ (or $\varphi = (2j - 1) \cdot \pi/N_{P1}$ in polar coordinates; and where $j = 1, \ldots, N_{P1}/2$), and the perimeter of the polygon, defined by the apothem

$$a_{P1} = \frac{1}{\sqrt{N_{P1} \cdot \tan(\pi/N_{P1})}}$$

and circumradius
These regions are mapped into the corresponding sectors within the disk with radius

\[ R_D = \frac{1}{\sqrt{\pi}}. \]  

\[ \Gamma_{p_1 \rightarrow p_2} \] is expressed in the following for a point \( p_{p_1} \), belonging to the first region on the polygon, which is defined by \( \varphi \in [-\pi/N_{p_1}, \pi/N_{p_1}] \) and is shaded grey in Figure 6.1. The mapping of a point belonging to a different region can be easily obtained by rotational symmetry around the origin. For \( p_{p_1} = [x_1; y_1] \), the coordinates of \( p_D \) on the disk become

\[
\begin{align*}
p_D &= \Gamma_{p_1 \rightarrow D} (p_{p_1}) = \begin{bmatrix} x_D \\ y_D \end{bmatrix} = \begin{bmatrix} r_D \\ \sin(\varphi_D) \end{bmatrix}, \\
&\text{with } \begin{cases} r_D = \left( \frac{R_D}{a_{p_1}} \right)x_1 \\ \varphi_D = \left( \frac{a_{p_1}^2}{R_D^2} \right) \left( \frac{y_1}{x_1} \right). \end{cases}
\end{align*}
\]  

(6.4)

Since the mapping method is bi-continuous, its inverse \( \Gamma_{D \rightarrow p_2} \) is straightforward. It is illustrated by means of a square \( (N_{p_2} = 4) \), representative of the most common receiver geometry, in the bottom of Figure 6.1. By reversing the coordinate transformation from Eq. (6.4), the Cartesian coordinates of \( p_{p_2} \) can be determined from \( p_D \)

\[
\begin{align*}
p_{p_2} &= \Gamma_{D \rightarrow p_2} (p_D) = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} \left( \frac{a_{p_2}}{R_D} \right)r_D \\ \left( \frac{R_D}{a_{p_2}} \right)r_D \varphi_D \end{bmatrix}.
\end{align*}
\]  

(6.5)

The complete mapping \( \Gamma_{p_1 \rightarrow p_2} \) between the two polygons then yields

\[
\begin{align*}
p_{p_2} &= \Gamma_{p_1 \rightarrow p_2} (p_{p_1}) = \Gamma_{D \rightarrow p_2} \left( \Gamma_{p_1 \rightarrow D} (p_{p_1}) \right) \\
&= \begin{bmatrix} \left( \frac{a_{p_2}}{a_{p_1}} \right)x_1 \\ \left( \frac{a_{p_1}}{a_{p_2}} \right)y_1 \end{bmatrix}.
\end{align*}
\]  

(6.6)
This elegant expression, however, only holds true for points that are located in the first region in both polygons, i.e. \( \phi \in [-\pi/\max\{N_{P1}, N_{P2}\}, \pi/\max\{N_{P1}, N_{P2}\}] \). For points that lie outside of the first region in at least one of the polygons, intermediate rotations around the origin are necessary between the two steps of the mapping. If mirror and receiver have the same perimeter shape \((N_{P1} = N_{P2})\), then Eq. (6.6) reduces to the obvious result \( p_{P1} = p_{P2} \), i.e. the initial shape remains undistorted. While such concentrators have certain advantages, it is a crucial benefit of the presented method that it allows designs with \( N_{P1} \neq N_{P2} \), which have several interesting applications, as outlined in Section 6.3. Throughout a major part of this work, designs with \( N_{P1} = 6 \) and \( N_{P2} = 4 \) are used as examples to illustrate a general case and showcase the flexibility of the method.

6.2.2 Grid generation

For the implementation of the mapping technique, \( p_{P1} \) needs to be discretized into a grid of \( N \) nodes \( p_{P1,n} \). For the subsequent least mean square optimization of the mirror surface it is advantageous if all surface elements within the grid, i.e. the areas defined by connecting 3 neighboring nodes, have equal area, as all nodes can then be weighted the same. Appendix D.1 describes a convenient way of generating a regular grid having these properties on \( P_1 \), which was applied in this work.

6.2.3 From 2D to 3D: scaling and translation

Having established the relationship between nodes on \( P_1 \) and their image on \( P_2 \), the nodes now have to be assigned their final location in 3D space such as to make up the desired solar concentrator. This procedure is illustrated by Figure 6.2. In addition to \( N_{P1} \) and \( N_{P2} \), the required design parameters of the concentrator are (1) the geometric design concentration \( C_{g,\text{design}} = A_1/A_2 \), that defines the size of the image relative to the inlet aperture (\( A_1 \) and \( A_2 \) are the respective projected areas of the mirror and the image); and (2) the focal length \( f \) of the concentrator. For the purpose of demonstration in this work, concentrators are normalized to unity inlet area, i.e. \( A_1 = A_{P1} = 1 \). Each node on the primary mirror is defined as
where \([x_{1,n}; y_{1,n}] = p_{P1,n}\) is a node on \(P_1\), \(z_{1,n}\) is a vertical coordinate to be determined, and \([x_{1,c}; y_{1,c}; z_{1,c}]\) are the coordinates of the mirror center, i.e. the vector by which the mirror is translated with respect to the origin. The mirror center is positioned onto a paraboloid of revolution with focal length \(f\) and centered at the origin, such that \(z_{1,c} = 1/(4f)(x_{1,c}^2 + y_{1,c}^2)\). This assures that multi-mirror concentrators (cf. Figure 6.6, Section 6.4.3), where individual mirrors are located off-axis, retain the correct focal length and don’t mutually overlap. For a single-mirror, on-axis concentrator (Section 6.4.2) on the other hand, no translation is performed and the mirror center is the origin, much like with an on-axis parabolic dish. The procedure for finding \(z_{1,n}\) of the remaining nodes is outlined in Section 6.2.4.

**Figure 6.2.** Schematic illustrating the scaling and translation of the mirror and the receiver.
The area of the image \( p_{2,n} = [x_{2,n}, y_{2,n}] \) is reduced by the factor \( C_{g,\text{design}}^{-1/2} \) (the apothem of the scaled image is \( a_2 = C_{g,\text{design}}^{-1/2}a_P \)) and the nodes are elevated to the focal plane, thus creating the receiver

\[
x_{2,n} = \begin{bmatrix} C_{g,\text{design}}^{-1/2} x_{2,n} \\ C_{g,\text{design}}^{-1/2} y_{2,n} \\ f \end{bmatrix}.
\]

(6.8)

### 6.2.4 Mirror surface optimization

The next task then consists in finding \( z_{1,n} \) of each node on the primary mirror such that (1) an on-axis ray impinging on that node is reflected to the corresponding node on the image; and (2) the resulting mirror surface is continuous. The optimization is thus based on on-axis rays only. Figure 6.3(a) depicts the initial situation after scaling and translation of the nodes, where \( z_{1,n} = 0 \). The (inverse) incident ray direction is \( \mathbf{r}_i = [0; 0; 1] \) for all nodes. At each node \( x_{1,n} \) on the mirror the desired reflected ray direction is

\[
\hat{\mathbf{r}}_{0,n} = \frac{x_{2,n} - x_{1,n}}{\|x_{2,n} - x_{1,n}\|}.
\]

(6.9)

If the ray is specularly reflected, the required surface normals at \( x_{1,n} \) to fulfill condition (1) are

\[
\hat{\mathbf{n}}_n = \frac{\hat{\mathbf{r}}_i + \hat{\mathbf{r}}_{0,n}}{\|\hat{\mathbf{r}}_i + \hat{\mathbf{r}}_{0,n}\|}.
\]

(6.10)

To simultaneously fulfill condition (2), \( z_{1,n} \) need to be determined such that the tangent vector of the surface \( \hat{\mathbf{t}}_n \) is perpendicular to \( \hat{\mathbf{n}}_n \), i.e.

\[
\hat{\mathbf{t}}_n \cdot \hat{\mathbf{n}}_n = 0.
\]

(6.11)

The required boundary condition is the fixed position of the central node, as mentioned in Section 6.2.3. The detailed optimization procedure applied to fulfill Eq. (6.11) is outlined in Appendix D.2. At the end of the optimization, on-axis rays impinging at each node on the primary mirror are reflected to the
Monte-Carlo ray tracing

The optimized surfaces are evaluated using Monte-Carlo ray tracing. An in-house code [72] has been extended with $C^0$-continuous Nagata patches that are ideal for the use with a mesh where the node positions ($x_{1,i}$) and normal vectors ($\hat{n}_i$) are known, and are commonly employed for ray tracing problems [156, 157]. For reasonably smooth surfaces, such as the ones encountered in this work, a mesh made up of $C^0$-continuous Nagata patches has $C^1$ continuity. All ray tracing simulations were performed using a diffuse (equi-cosine) source of cone-angle $\theta_{\text{sun}} = 4.65$ mrad. A spatially uniform flux distribution over the inlet aperture was
assumed throughout. Additionally, the following assumptions apply: the shape is ideal (unless stated otherwise); the reflectivity is uniform, independent of incident and outgoing directions, and perfectly specular; and geometric optics applies. Unless otherwise stated, simulations were performed with $10^8$ rays. For the generation of smooth irradiance distributions, grids with approximately 2500 nodes are used. Otherwise, a lower resolution is sufficient, as shown in the grid size study in Appendix D.3. Irradiance intensities are expressed for a direct normal irradiance (DNI) of 1000 W/m$^2$.

6.3 Applications

Several CPV applications can benefit from concentrators that on one hand have mirrors with a polygonal perimeter and on the other hand produce a uniform irradiance distribution on a polygonal receiver. The most straight-forward example is a dish-type concentrator consisting of a single on-axis mirror and a square PV receiver. Exemplary designs with square, hexagonal and circular$^c$ perimeter are shown in Figure 6.4. Due to their simplicity, these exemplary concentrators are ideally suited to analyze the fundamental behavior and performance of the design method (Section 6.4.2) before it is employed to construct more elaborate designs (Section 6.4.3). They are however also imaginable as standalone concentrators for decentralized, small-scale energy production as well as for large-scale applications in fields. Comparable concentrator form factors (square continuous mirror) have recently emerged in commercial CPV applications, albeit with a parabolic profile and secondary optics [148].

The facilitated imaging to a square PV cell array makes square concentrators particularly attractive. One of the benefits of the presented method is that it is not limited to square perimeters but instead enables simple imaging between arbitrary polygons with minimal distortion. Hexagonal concentrators have the advantage compared to square concentrators of having a more compact mirror (smaller height difference between mirror center and corner) for the same focal

$^c$ For a concentrator design with a circular mirror, the first mapping step ($\Gamma_{P_1\rightarrow D}$) can simply be omitted. A nodal grid can be generated directly on the disk and $\Gamma_{D\rightarrow P_2}$ directly produces the image on the receiver. The technique for regular grid generation on a disk used in this work is outlined in Appendix D.
Polygonal mirrors achieving uniform irradiance distributions

length, especially at high rim angles. Consequently, they produce a more uniform angular distribution of the irradiance on the receiver. Meanwhile, they still allow tessellation, which has several practical advantages. The circular design is equivalent to the limit case of a polygon with \( N_{P1} \to \infty \) and the most compact mirror design. Additionally, circular dishes of parabolic and close-to-parabolic profile are state of the art in the solar energy and antenna industries. The well-established fabrication processes could readily be adapted to the presented designs. Recently, shaping of square glass mirrors for solar dish concentrators by radiative heating has been successfully demonstrated with small slope errors (rms < 1.5 mrad) [158], representing a potential avenue for the fabrication of the presented mirrors, regardless of perimeter shape, in addition to the method introduced in Chapter 5 of attaching thin-walled back-surface glass mirrors to a freeform precast concrete shell.

The most interesting advantage of polygonal mirrors is their potential for tessellation, which allows various compound designs with high active area fraction (AAF). This potential can optimally be leveraged with CPV concentrator modules, where several on-axis mirrors, each with its own receiver, are mounted together on the same tracker to reduce complexity, e.g. with square concentrators [153, 159], as exemplarily shown in Figure 6.5(a). An additional advantage of such systems is that they can be designed with rectangular outer perimeters, which has a positive impact on the field shading efficiency [160]. The same concept can be applied to hexagonal concentrators, as depicted in Figure 6.5(b),
allowing different (e.g. circular) outer perimeter, while still maximizing the AAF and uniformly illuminating square receivers. Circular perimeters notably allow a rotationally symmetric mirror support structure, which has practical advantages.

When increasing the number of mirrors, smoother and more versatile perimeters can be obtained. Concentrator panels, where an array of small, densely packed mirrors each concentrate onto a single CPV cell held in place by a transparent cover, as suggested in Figure 6.5(c), are an extension of this concept. To facilitate the installation of electrical and thermal sinks for the cells,
such concentrators are often designed as Cassegrain systems as shown in Figure 6.5(d) [161]. By using a flat secondary mirror, the ray redirection introduced in Section 6.2 is conserved. Here, the more compact hexagonal mirrors have the additional advantage of allowing the fabrication of a more compact CPV module with less steep gradients between the individual mirrors.

In addition to these compound designs made up of individual on-axis concentrators, the presented design method also allows more elaborate concentrators composed of multiple off-axis mirrors. Within these multi-mirror concentrators, each mirror individually distributes the incident radiation uniformly over the entire area of a single common receiver. Exemplary concentrators based on square mirrors are shown in Figure 6.6. As with the compound designs, tessellation of the polygonal mirrors can be exploited to

Figure 6.6. Exemplary multi-mirror designs based on square ($N_1 = 4$) mirrors, where each mirror is designed such that it uniformly distributes incident rays over the complete receiver (image area). (a) Square concentrator with a similar perimeter to Figure 6.4(a) but composed of individual 4 mirrors; (b) same design with 9 mirrors; (c) rectangular concentrator composed out of 8 mirrors and having an aspect ratio of 2, which can be beneficial for the shading efficiency in a field of dishes; (d) circular dish composed out of 37 square mirrors e.g. allowing very large inlet apertures and having the structural advantages of circular perimeters.
create a continuous reflective surface with high AAF and versatile perimeter shape.

The simplest multi-mirror designs can be created by subdividing the reflective surface of the square single-mirror concentrator from Figure 6.4(a) into multiple mirrors of equal size. Exemplary designs with 4 and 9 mirrors are shown Figure 6.6(a)-(b). Despite their similarity to the single-mirror concentrator, the multi-mirror designs have several additional advantages, as will be discussed in Section 6.4.3. Figure 6.6(c)-(d) show other examples of multi-mirror designs, having essentially the same properties as the simpler designs from Figure 6.6(a)-(b) but with more complex perimeter shapes. This, notably, allows the same advantages as for the designs from Figure 6.5, e.g. rectangular perimeters to improve the field-shading efficiency or circular perimeters for a rotationally symmetric structure, but with the potential advantage of having only 1 single receiver.

Finally, by leveraging the tessellating nature of polygons not only for the reflective surface but also for the receiver, a design is conceivable where multiple mirrors redirect solar radiation on multiple adjacent receivers, as suggested in Figure 6.7. A potential advantage of such a design is that mirrors can be slightly offset radially such that shading by the receiver can be eliminated. On the downside, this reduces the AAF of the concentrator and produces slightly asymmetric irradiance distributions.
6.4 Results and discussion

6.4.1 Mapping method

Figure 6.8 shows the results of the mapping of a uniform grid of equally-spaced nodes between two polygons by the method outlined in Section 6.2. The nodes needed for the design of a hexagonal concentrator with square receiver (Figure 6.4(b); \( N_{P1} = 6, N_{P2} = 4 \)) are shown as example. All geometries have an equal projected area \( A_{P1} = A_D = A_{P2} = 1 \). Figure 6.8(a) shows the regular nodal grid \( p_{P1,n} \) generated on the hexagon. The node colors indicate their initial location within the hexagon to track the transformations. All triangular elements spanned between 3 neighboring nodes are equilateral and have equal area. The result of the first part of the mapping (\( \Gamma_{P1 \rightarrow D} \)) that transforms the initial grid into \( p_{D,n} \) with minimal distortion can be seen in Figure 6.8(b). The nodes are mapped into the sector within the disk corresponding to their initial region on the polygon. The triangular elements are no longer equilateral. However, their area is conserved. Figure 6.8(c) shows the result of the complete mapping from the hexagon to the square after the 2nd step, \( \Gamma_{D \rightarrow P2} \), that transforms the intermediate nodes on the disk into the final image \( p_{P2,n} \). Because the intermediate grid on the disk does not
respect the rotational symmetry of the square, small irregularities can be observed along the diagonals of the square. However, the areas of the grid elements are still conserved and the irregularities become less important with a larger number of nodes than used for illustration here.

6.4.2 Single-mirror concentrators

The hexagonal concentrator from Figure 6.4(b) \((N_{p1} = 6, N_{p2} = 4, A_1 = 1, f = 1)\) at \(C_{g,\text{design}} = 500\times\) is used as benchmark throughout this section, as the effects of various design parameters on the concentrator shape and performance are examined.

_Mirror shape_

_Figures 6.9 and 6.10_ show the profile difference \(\Delta z = z_m - z_p\) between hexagonal concentrators with \(C_{g,\text{design}} = 100\times, 500\times\) and \(1000\times\), representative of medium- to high-concentration PV applications, and a parabolic dish, normalized by the apothem of the primary mirror, \(a_1 = a_{p1}\).

_Figure 6.9_ displays \(\Delta z\) as a contour plot over the whole mirror aperture. The profile deviation increases with increasing radial distance from the mirror center (due to the positioning of the mirror center on a parabolic dish, the deviation in the center is 0). It is however not rotationally symmetric but instead exhibits a combination of the symmetries of both mirror and receiver shapes.

This results in the lowest deviation appearing along \(y = 0\), while the highest deviation appears along the lines \(y = \tan(\pm\pi/6)\cdot x\). As \(C_{g,\text{design}}\) is increased, the mirror profile approaches that of the parabolic dish. _Figure 6.10_ shows the radial difference along a direction with minimal offset \((x > 0, y = 0;\) solid lines\) compared with a direction of maximal offset \((x > 0, y = \tan(\pi/6)\cdot x;\) dashed lines\), which underlines the asymmetric concentrator profile and its convergence to a parabolic dish for increasing \(C_{g,\text{design}}\). In fact, in the limit case \(C_{g,\text{design}} \to \infty\), all nodes on the image would merge into the focal point \([0; 0; f]\) and hence \(\Delta z \to 0\) for the whole mirror. Essentially, for all other designs \((C_{g,\text{design}} < \infty)\) is a parabolic dish that is non-uniformly defocused, i.e. the extent of the divergence from the
parabolic shape varies both in radial and circumferential direction. Due to the deviation from the parabolic shape, the rim angle of the mirror, which can be of

![Figure 6.9. Contour plot of the mirror profile difference between optimized hexagonal concentrators with $C_{\text{g,design}} = 100\times$, $500\times$ and $1000\times$ and a parabolic dish, normalized with the mirror apothem $a_1$. With increasing $C_{\text{g,design}}$, the mirror shape approaches that of the parabola as the nodes on the image merge into a single focal point.](image)
interest for the receiver design, is affected. Appendix D.4 addresses the matter of the rim angle and shows that its variation with $C_{g,\text{design}}$ is marginal.

**Irradiance distribution at the receiver**

**Figure 6.11** compares the irradiance distribution on a square receiver ($N_{p2} = 4$) with square, hexagonal and circular primary mirrors designed with $C_{g,\text{design}} = 100\times, 500\times$ and $1000\times$. The irradiance distribution is plotted versus the receiver coordinate, normalized by the apothem of the scaled image, $a_2$. The square concentrator ($N_{p1} = 4 = N_{p2}$), represents the simplest design case where mirror and receiver have the same perimeter shape. As noted in Section 6.2.1, the mapping between the nodes on the primary mirror and their image introduces no distortions. The irradiance distribution on the receiver is very uniform for all $C_{g,\text{design}}$, with a large central part where the irradiance is equal to $C_{g,\text{design}}$, and a fringe region of gradually decreasing irradiance, which is centered at the border of the image area.

The size of the area encompassing the entire fringe region is prescribed by the edge rays reflected from the outer rim of the primary mirror. To collect all rays, the perimeter of $A_2$ has to be increased in either direction by the half-width
Polygonal mirrors achieving uniform irradiance distributions

of the fringe region (half-width of the focal image of the sun, produced by reflection from a point on the outer rim) [11],

\[ w_{\text{fringe}} = R_{p1} \cdot \sin(2\theta_{\text{sun}}) / \sin(2\Phi^+_{p1}), \]  

(6.12)

where \( \Phi^+_{p1} \) is the major rim angle of the primary mirror and a function of its circumradius and the focal length (Appendix D.4). As a consequence, the full-collection concentration ratio is lower than \( C_{g,\text{design}} \). In general, it is limited to values below the fundamental limit for convex single-reflection concentrators with axial symmetry [11, 21].

**Figure 6.11.** Irradiance distribution on a square receiver \((N_{p2} = 4)\) produced with a square \((N_{p1} = 4)\), hexagonal \((N_{p1} = 6)\) and disk \((N_{p1} \to \infty)\) primary mirror with \( C_{g,\text{design}} = 100\times \) (top row); \( 500\times \) (middle row); and \( 1000\times \) (bottom row). The receiver coordinates are normalized by the apothem of the scaled image, \( a_2 \). While the square primary produces a very uniform distribution for all \( C_{g,\text{design}} \), artifacts of the mapping are visible for the circular and hexagonal primary at low concentrations.
\[ C_{g,1,max,3D} = \frac{\sin^2(2\Phi_1^+)}{\sin^2(2\theta_{sun})}. \]

(6.13)

In particular, for a square receiver \((N_{P2} = 4)\), such as used throughout this work, the full-collection geometrical concentration limit yields

\[ C_{g,1,max,square} = \frac{C_{g,design}}{(2w_{fringe}\sqrt{C_{g,design}+1})^2}, \]

(6.14)

with a maximum of \(1/(4w_{fringe}^2) = C_{g,1,max,3D}/2\) for \(C_{g,design} \rightarrow \infty\), i.e. the smallest receiver collecting all radiation is a square circumscribing the ideal image of the sun in the focal plane. For \(N_{P2} \rightarrow \infty\), the maximum of the full-collection geometrical concentration limit converges to \(C_{g,1,max,3D}\). As \(w_{fringe}\) is independent of \(C_{g,design}\), the fringe region occupies a larger portion of the image area with increasing \(C_{g,design}\). These results demonstrate the high performance achievable with the presented design method over a wide range of concentrations.

The examples of the hexagonal and disk concentrators \((N_{P1} \neq N_{P2})\) are more complex as the mapping leads to distortions in the image of the nodal grid. These distortions are more severe the larger the difference between \(N_{P1}\) and \(N_{P2}\) becomes (a disk is equivalent to a polygon with \(N_{P1} \rightarrow \infty\)) and subsequently cause increasingly important non-uniformities in the irradiance distributions. Nevertheless, these non-uniformities are within reasonable bounds and become less severe at high \(C_{g,design}\). At least the hexagonal concentrator, which allows several interesting applications and produces good uniformity, remains of interest.

Table 6.1 summarizes the average irradiance within the image area, \(\langle E \rangle\), obtained with these concentrators. \(\langle E \rangle\) is in the vicinity of \(C_{g,design}\), but diminished by the increasing relative size of the fringe region for high \(C_{g,design}\). If the receiver is chosen smaller than the image area, an average irradiance close to \(C_{g,design}\) can

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\(d\) Obstruction by the receiver is neglected throughout this section. Otherwise, the fundamental limit for convex single-reflection concentrators with axial symmetry is \(C_{g,1,max,3D} = \sin^2(2\Phi_1^+)/\sin^2(2\theta_{sun}) - 1 \) [11, 21].

\(e\) When accounting for obstruction by the receiver, the full-collection concentration ratio with a square receiver is \(C_{g,1,max,\text{square}} = C_{g,design}/(2w_{fringe}(C_{g,design})^{1/2} + 1)^2 - 1\).
be maintained. A more quantitative evaluation of the performance taking this into consideration is provided in the following section for the hexagonal concentrator.

**Intercept factor and uniformity**

Of major importance for the design of a solar concentrator is the total system efficiency $\eta_{\text{tot}}$, mainly made up of the optical efficiency $\eta_{\text{opt}}$ (fraction of incident rays that reach the receiver after traveling through the optical system) and receiver efficiency $\eta_{\text{el}}$ (electrical efficiency of a PV array): $\eta_{\text{tot}} \propto \eta_{\text{opt}} \cdot \eta_{\text{el}}$. In an ideal optical system (mirror reflectance $\rho = 1$, receiver absorbance $\alpha = 1$), $\eta_{\text{opt}}$ is equal to the intercept factor, defined here as the ratio of the radiative power intercepted by a central, square receiver of area $A_i$ and the total radiative power reaching the focal plane

$$\gamma_i = \frac{Q_i}{Q_{\text{tot}}} = \frac{\int_{A_i} E(x, y) \, dx\,dy}{\int_{A_{\text{tot}}} E(x, y) \, dx\,dy}. \quad (6.15)$$

The electrical efficiency of a PV array on the other hand is closely linked to the irradiance uniformity on the receiver [56]. Commonly in CPV, $\eta_{\text{el}}$ is related to the cell-to-cell uniformity, as it is physically proportional to the mismatch losses with series-connected PV cells. It is defined as

$$U = \frac{\min(E_{\text{av},i})}{\langle E_{\text{av}} \rangle}, \quad (6.16)$$
where $E_{av,i}$ are the cell-averaged irradiances of each cell within the array and $\langle E_{av} \rangle$ is the mean array irradiance. $U$ varies between $U = 1$ (perfectly uniform) and $U = 0$ (highly non-uniform) and is highly dependent on the number of cells in the array.

In an optical system, high intercept factor and irradiance uniformity (high optical and receiver efficiencies) are subject to a fundamental trade-off. In the system presented here, the remaining tunable parameter for a chosen design ($N_{P1}, N_{P2}, C_{g,design}$) that has an impact on both $\gamma$ and $U$ is the size of the receiver compared to the image area. To find a configuration that maximizes total efficiency it is therefore important to know the relationship between intercept factor and uniformity, which is shown in Figure 6.12 for a 6×6 cell array with

**Figure 6.12.** Cell-to-cell irradiance uniformity on a 6×6 cell array in the receiver plane plotted vs. the intercept factor achieved within this array, for hexagonal mirrors with $C_{g,design} = 100\times, 500\times$ and $1000\times$. The circles (●) indicate the intercept factor and uniformity for a receiver having the size of the image area. The downward pointing triangles (▼) show the influence on the intercept factor and uniformity when taking shading of incident radiation by the receiver into account and upward pointing triangles (▲) indicate the improvement that is possible by using a simple secondary concentrator design. The dotted lines indicate irrational receiver designs to the left of the uniformity peak. For every such receiver there exists larger receiver on the opposite side of the peak that achieves the same uniformity with a higher intercept factor.
Table 6.2. Intercept factor, cell-to-cell uniformity and standard deviation with a receiver having the size of the image area, for hexagonal concentrators with $C_{\text{g,design}} = 100\times$, $500\times$ and $1000\times$.

<table>
<thead>
<tr>
<th>$C_{\text{g,design}}$ [×]</th>
<th>Without shading</th>
<th>With shading</th>
<th>With shading and secondary</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 γ [-]</td>
<td>0.965</td>
<td>0.955</td>
<td>0.979</td>
</tr>
<tr>
<td>U [-]</td>
<td>0.903</td>
<td>0.900</td>
<td>0.790</td>
</tr>
<tr>
<td>500 γ [-]</td>
<td>0.908</td>
<td>0.906</td>
<td>0.988</td>
</tr>
<tr>
<td>U [-]</td>
<td>0.774</td>
<td>0.776</td>
<td>0.921</td>
</tr>
<tr>
<td>100 γ [-]</td>
<td>0.868</td>
<td>0.867</td>
<td>0.988</td>
</tr>
<tr>
<td>U [-]</td>
<td>0.678</td>
<td>0.680</td>
<td>0.904</td>
</tr>
</tbody>
</table>

the $3 \ C_{\text{g,design}}$. The circles (●) indicate the intercept factor and uniformity for a receiver having the size of the image area. The corresponding values are also summarized in the left column of Table 6.2. With increasing $C_{\text{g,design}}$, the trade-off between intercept factor and uniformity becomes more severe due to the fringe region becoming larger relative to the image area.

If the receiver size is chosen smaller than the image area (towards the top left of Figure 6.12), $U$ increases up to the point where the fringe region lies completely outside of the receiver. If the receiver size is decreased further, the uniformity decreases slightly due to the larger impact of the irradiance peaks at the receiver edges before it increases again as these peaks come to lie outside the receiver. However, due to the quickly decreasing intercept factor at these receiver sizes, a design to the left of the uniformity peak (dotted lines) is irrational. If the receiver size is chosen larger than the image area (towards the bottom right of Figure 6.12), the uniformity drops quickly with only marginal increase in intercept factor. A receiver the size of the image area therefore offers a relatively good trade-off between high uniformity and high intercept factor. The optimum design point for a specific application might deviate slightly from this point.
Mirror shape error

As mentioned in Section 6.2.5, simulations have been performed with ideally shaped mirrors. If a realistic mirror shape error, i.e., an isotropic angular dispersion error with standard deviation 1.5 mrad, is introduced, the absolute size of the fringe region is slightly increased and the sharp corners of the irradiance distribution are rounded off, while the irradiance within a major part of the aiming area remains uniform. Figure 6.13 shows the resulting irradiance distributions as a function of $C_{g,\text{design}}$ and primary mirror perimeter. Compared to the values given in Table 6.1, the intercept factor with the hexagonal mirror then decreases to 0.941, 0.863 and 0.806 for $C_{g,\text{design}} = 100\times$, 500× and 1000× respectively.

Systems with secondary optics

Up until this point, shading of incident rays by the receiver has not been accounted for to show where spatially uniform rays reflected by the mirror would impinge on a receiver under ideal conditions. Figure 6.14 (left) shows the effect on the local and cell-averaged irradiance distribution caused by receiver shading for the hexagonal concentrator with $C_{g,\text{design}} = 500\times$ and a 6×6 cell array that has the exact dimensions of the image area ($C_g = C_{g,\text{design}}$). The performance metrics for all hexagonal concentrators ($C_{g,\text{design}} = 100\times$, 500×, 1000×) are summarized in Table 6.2 (middle column) and marked by the downward pointing triangles (▼) in Figure 6.12. The decrease in intercept factor and uniformity as a result of shading is negligible for the high-concentration designs $C_{g,\text{design}} = 500\times$ and 1000×, while a small drop in performance can be observed for $C_{g,\text{design}} = 100\times$.

A much more important change can be observed if a secondary optic is added in front of the receiver. If designed correctly, it can serve the double purpose of (1) collecting the radiation that would otherwise impinge on the outside of the receiver, thereby increasing the optical efficiency of the concentrator (fraction of radiation intercepted within the receiver area); and (2) redirecting this radiation predominantly to the outer region of the receiver where irradiance would otherwise be low, thereby increasing the uniformity.

\[\text{The shaded fraction of the inlet area is } 1/C_g.\]
Perhaps the most simple nonimaging concentrator can be constructed by orthogonally intersecting two V-troughs [69, 162, 163], a design usually referred to as crossed V-trough or pyramid concentrator, e.g. [164]. If dimensioned such that its outlet aperture is the same size as the receiver (the geometric concentration of the system remains unchanged; \( C_g = C_{g,\text{design}} \)), it can predominantly collect fringe radiation while not interfering with the irradiance directed towards the receiver center, and therefore has a low average number of
reflections and high optical efficiency. When designing the crossed V-trough, mirror size and vertex angle have to be selected such that the best overall performance is achieved. If the vertex angle $\psi$ of the V-troughs is chosen such that $\psi > \Phi^+ + \theta_{\text{sun}}$ (where $\Phi^+ = 34.47^\circ$ is the major rim angle of the hexagonal concentrator, as discussed in Appendix D.4), then the secondary optic will

![Figure 6.14. Irradiance distributions without (left) and with secondary concentrator (right) and including shading effects, for a hexagonal concentrator with $C_{g,\text{design}} = 500\times$. The receiver (secondary outlet aperture) has the exact dimensions of the image area; the irradiance outside this area is disregarded. (top) Local irradiance distributions. Most of the fringe radiation that would otherwise impinge outside of the receiver can be redirected to the receiver edge. As a result, both the average irradiance and the uniformity are substantially increased. While in the case without secondary stage the shading by the receiver can be neglected, the shading produced in the receiver center, by obstruction of the receiver and secondary optics is significant; (bottom) cell-averaged irradiance on a 6×6 cell array. As the low central irradiance caused by shading is distributed over 4 cells, the average irradiance is only marginally affected.](image-url)
Polygonal mirrors achieving uniform irradiance distributions

theoretically accept all incident rays, provided the mirror length $l$ is large enough to collect all the rays reflected to the outside of the image area and $C_{g,\text{design}} << 1/\sin(\theta_{\text{sun}})^2$. A small fraction of the incident radiation is however lost due to rejection caused by the 3D design (for rays that are reflected on two perpendicular walls, the ideal properties of the V-trough no longer apply). On the other hand, long walls increase the shaded area in the center of the mirror. Thus, they result in a decrease of the irradiance in the center of the receiver, and, more importantly, as a consequence, a decrease of the irradiance uniformity. A design with $\psi = 35^\circ$ and $l = 0.3$ provides a good tradeoff and overall performance.

**Figure 6.14** (right) shows the resulting local and cell-averaged irradiance distributions for the hexagonal concentrator with $C_{g,\text{design}} = 500\times$ with the secondary optic installed\(^8\). The corresponding performance metrics are plotted with the upward pointing triangles (▲) in **Figure 6.12** and summarized in the right column in **Table 6.2**. For the concentrator with $C_{g,\text{design}} = 100\times$ the additional shading of the incident radiation caused by the secondary further decreases the intercept factor and, to a larger extent, the irradiance uniformity achieved on the receiver. A secondary optic is therefore not advantageous in this case. For the higher concentrations however, the secondary optic is highly beneficial. While the additional shading of the incident rays by the secondary does result in a lower irradiance in the receiver center, the cell-averaged irradiance in the center is only marginally affected. The overall uniformity is considerably improved as an effect of the redirection of part of the fringe radiation to the edge of the receiver (e.g. from 77.6% to 92.1% for $C_{g,\text{design}} = 500\times$). The intercept factor is also significantly increased (from 90.6% to 98.8%) such that almost all radiation is collected by the receiver. In conclusion, a small addition to the design can considerably improve the concentrator performance at high concentrations without doing a lot of work, by only attenuating the rays at the outer region of the image area.

\(^8\) The secondary optic was modeled as ideal (wall reflectance $\rho = 100\%$) to provide results independent of mirror quality. The attenuation by the secondary mirror however is negligible. The average number of reflections of rays through the secondary optics was determined to be 9% with the method outlined in [73]. E.g. if the mirror reflectance was 90%, then only $1 - 0.9^{0.09} = 1\%$ of the rays would be absorbed.
6.4.3 Multi-mirror concentrators

As discussed in Section 6.3, the presented method also allows the design of multi-mirror concentrators, where each mirror is individually imaged onto the whole area of the receiver, allowing greater flexibility for the concentrator shape. For assessment of the properties of such multi-mirror designs, the 3 simplest concentrators based on square mirrors are compared. Inspecting these simple designs will allow an extrapolation to complex concentrators having a higher number of mirrors. Square mirrors are the ideal choice, as they achieve the best performance with a square receiver, due to the lack of distortions introduced during the mapping. As the number of mirrors is increased, the flexibility for the perimeter shape of the multi-mirror concentrator is increased such that the shape of the individual mirrors becomes less important, provided they allow tessellation such that the AAF can be maximized.

Mirror shape

Figure 6.15 compares the normalized mirror profile difference to a parabolic dish, \( \Delta z/a_1 \), of the 3 concentrators as a contour plot: (a) a square, single-mirror, on-axis concentrator [cf. Figure 6.4(a)]; (b) a square, four-mirror concentrator [cf. Figure 6.6(a)]; and (c) a square, nine-mirror concentrator [cf. Figure 6.6(b)], all with \( C_{g, \text{design}} = 500 \times \). As expected, the shape of the single mirror concentrator, shown in Figure 6.15(a), is similar to the hexagonal design shown in Figures 6.9 and 6.10, with radially increasing divergence from the parabola shape. However, as the circumradius of the square is larger than that of the hexagon, \( \Delta z/a_1 \) for the same \( C_{g, \text{design}} \) is larger in the corners. Another variation is that the profile difference is rotationally symmetric as the mirror and the receiver have the same perimeter shape \( (N_{P2} = N_{P1}) \). The dissimilar mirror profiles of the multi-mirror designs, plotted in Figure 6.15(b)-(c), become immediately apparent. By design, the centers of each individual mirror are placed on an on-axis paraboloid of revolution. The divergence from this paraboloid increases radially with respect to each mirror center. However, due to the off-axis positioning of the mirrors, \( \Delta z \) is not rotationally symmetric. Additionally, the maximum divergence becomes considerably smaller with increasing number of mirrors. Naturally, the profile
slope is not continuous between mirrors, i.e. the concentrator does not have $C^1$-continuity. In the four-mirror design, where the individual mirrors have line symmetry about the $x$- and $y$-axis, the concentrator has $C^0$-continuity. However, this is not entirely the case for the nine-mirror design, although differences are almost negligible.

In theory, the number of mirrors can be increased to infinity. In this limit case, each point on the concentrator images to the entire receiver area, and hence the individual infinitesimal mirrors are convex. In between, there has to exist a design where the mirrors change curvature from concave to convex, i.e. the mirrors become approximately flat$^h$, and which has an interesting potential for applications. The number of mirrors that results in such a design varies with focal length and concentration. This special case bears resemblance to similar designs, where a large number of flat mirrors is distributed over a concave surface with the goal of producing a uniform square irradiance distribution [153, 

$^h$ In reality, with the presented method, it is not possible that all mirrors are perfectly flat simultaneously. Mirrors in the concentrator center will always be slightly less concave than mirrors on the concentrator edge if the same area in the focal plane is to be illuminated. This is due to the inherent coma of focusing concave concentrators, i.e. off-axis rays reflected from the concentrator edge intersect the focal plane further away from the optical axis than rays reflected at the concentrator center. When increasing the number of mirrors, there is a first design point where the innermost mirror is flat while all other mirrors are still concave. Conversely, for a slightly higher number of mirrors, if the outermost mirror becomes flat, all remaining mirrors are convex. However, for an intermediate design, all mirrors can reasonably well be approximated as flat.
However, these designs have subtle differences such as a different arrangement and outer shape of the individual mirrors.

**Irradiance distribution at the receiver**

Figure 6.16 compares the irradiance distribution on the receiver produced by the four- and nine-mirror concentrators to that produced by the single-mirror design (cf. Figure 6.11). At the first glance, it appears as though there is no difference between the distributions produced by the different concentrators. However, if the local irradiance difference $\Delta E = E_{\text{single}} - E_{\text{multi}}$ is plotted, dissimilarities in the irradiance distribution become apparent. The single-mirror concentrator reflects a larger amount of radiation to the outside and the central region of the image area, whereas the multi-mirror concentrators tend to focus more to the inner edge of the image. The difference between both multi-mirror designs is relatively small.

Despite the seemingly important difference in radiation at the outside of the image, the intercept factor within the image area is only slightly improved from 0.89 with the single-mirror design to 0.90 with the four- and nine-mirror designs. Considering that, in addition, a large part of the radiation outside of the image area can be collected by a secondary optic such as the one described in Section 6.4.2, the intercept factor improvement alone might not justify the added complexity of using multiple mirrors with square designs. However, with other concentrator perimeters, multi-mirror designs based on square mirrors remain of significant interest, even if only taking into account the above results. When contemplating the comparatively poor irradiance uniformity produced by a circular, single-mirror concentrator (cf. Section 6.4.2), especially at low $C_{g,\text{design}}$, it is suggested that using a multi-mirror concentrator with a near-circular overall perimeter but composed of square individual mirrors, such as exemplarily shown in Figure 6.6(d), might be the better solution to achieve a uniform irradiance with a circular perimeter.

Importantly, an additional advantage of multi-mirror concentrators is their robustness to partial shading of the inlet aperture, e.g. in a field of solar dishes at low solar elevation angles. Figure 6.17 illustrates the reason for this robustness using the simple designs from Figure 6.15. It compares the irradiance
Figure 6.16. Comparison of the irradiance distributions produced by the four- and nine-mirror concentrators to the distribution produced by the single-mirror concentrator. (top) local irradiance; (bottom) irradiance difference with respect to the single-mirror irradiance, normalized by the design concentration. The multi-mirror designs redirect more rays towards the inside of the image area.

distributions produced by the 3 concentrators if a square area in the bottom right corner corresponding to respectively one-ninth (top row) and one-fourth (bottom row) of the inlet aperture is obstructed. Table 6.3 provides an overview of the corresponding cell-to-cell uniformities on a 6×6 cell array. The conclusions that can be drawn here extend to designs with a higher number of mirrors.

With the single-mirror concentrator [Figure 6.17(a)], the obstructed portion of the inlet aperture is directly imaged to the same location at the receiver. This results in zero irradiance in the affected location, while the remaining part of the receiver sees full irradiance (500 kW/m²), as indicated by the fractions overlaid
on the plot. The result is a very poor irradiance uniformity under shaded conditions ($U = 0\%$). With the multi-mirror designs however, the total irradiance distribution is the sum of the irradiances received by the individual mirrors. The shading therefore only affects the irradiance distributions produced by the individual shaded mirrors, while the unshaded mirrors still illuminate the entire image.

In the ideal case, the shaded inlet area exactly corresponds to the surface of one or multiple individual mirrors. This is the case for the four-mirror design when one-fourth of the inlet is shaded [Figure 6.17(b), bottom row] and the nine-mirror design with one-ninth shading [Figure 6.17(c), top row]. The unshaded mirrors then each distribute the irradiance over the whole receiver while the shaded mirror is entirely inactive. The resulting irradiance distribution therefore has the same uniformity as in the unshaded case ($U = 77.2\%$ resp. $77.6\%$) with an average intensity proportional to the fraction of the unshaded mirrors ($3/4 \times 500 \text{ kW/m}^2 = 375 \text{ kW/m}^2$ for the four-mirror design with one-fourth shading, and $8/9 \times 500 \text{ kW/m}^2 = 444 \text{ kW/m}^2$ for the nine-mirror design with the one-ninth shading respectively).

In the general case, when the perimeter of the shaded area does not exactly overlap with perimeters of the individual mirrors, some mirrors are partly shaded. As a result, the corresponding fractional area on the receiver does not receive radiation from that part of the mirror. Accordingly, with the four-mirror concentrator where $1/9^{th}$ of the inlet is shaded [Figure 6.17(b), top row], 3 mirrors are unshaded and illuminate the full receiver. An area corresponding to four-ninths of the bottom right mirror however is shaded. Consequently, five-ninths of the receiver sees full power ($4/4 \times 500 \text{ kW/m}^2 = 500 \text{ kW/m}^2$), while four-ninths of the receiver receives only power from 3 mirrors ($3/4 \times 500 \text{ kW/m}^2 = 375 \text{ kW/m}^2$). It can easily be verified that these numbers add up to a total of eight-ninths of radiation collected by the receiver, corresponding to the unshaded area fraction as expected ($5/9 \times 500 \text{ kW/m}^2 + 4/9 \times 375 \text{ kW/m}^2 = 444 \text{ kW/m}^2 = 8/9 \times 500 \text{ kW/m}^2$). Due to the fragmentation of the reflector into several mirrors the uniformity is still acceptable despite the considerable shading ($U = 66.6\%$).
Figure 6.17. Comparison of the irradiance distribution on the receiver produced by square (a) single-, (b) four- and (c) nine-mirror concentrators where a corner corresponding to one-ninth (top row) and one-fourth (bottom row) of the inlet aperture is obstructed. The fractions indicate the portion of the unshaded irradiance received by the different areas on the receiver.
Although the situation is slightly more complex, the distribution with the 9-mirror concentrator where one-fourth of the inlet is shaded [Figure 6.17(c), bottom row] can be explained in the exact same way: 5 mirrors remain unshaded, on 1 mirror the bottom right quarter is shaded, 2 mirrors are half shaded (bottom half and right half respectively), and 1 mirror is entirely shaded. By summing up the individual distributions of the mirrors on the receiver, the specific pattern from Figure 6.17(c) emerges with average distributions of $5/9 \times 500 \text{ kW/m}^2 = 278 \text{ kW/m}^2$, $7/9 \times 500 \text{ kW/m}^2 = 389 \text{ kW/m}^2$, and $8/9 \times 500 \text{ kW/m}^2 = 444 \text{ kW/m}^2$ in the different sectors of the receiver. Overall, the receiver collects $1/4 \times 278 \text{ kW/m}^2 + 2/4 \times 389 \text{ kW/m}^2 + 1/4 \times 444 \text{ kW/m}^2 = 375 \text{ kW/m}^2 = 3/4 \times 500 \text{ kW/m}^2$, accurately corresponding to the unshaded area with comparatively good uniformity for the large amount of shading ($U = 59.0\%$).

From these findings it becomes apparent that, as a general rule, with increasing number of mirrors, an increasingly larger fraction of mirrors is either fully shaded or fully unshaded and hence has no negative impact on the uniformity. In theory, the best performance would therefore be achieved by a concentrator with an infinite number of mirrors, i.e. where each point on the concentrator images to the entire receiver area. However, there is a practical limit to the number of mirror because of the difficulties associated with accurately reproducing the slopes, the penalty from small gaps in-between individual mirrors, and mirror alignment.

<table>
<thead>
<tr>
<th>$U$ [%]</th>
<th>single mirror</th>
<th>4 mirrors</th>
<th>9 mirrors</th>
</tr>
</thead>
<tbody>
<tr>
<td>no shading</td>
<td>77.4</td>
<td>77.4</td>
<td>77.4</td>
</tr>
<tr>
<td>1/9$^\text{th}$ shading</td>
<td>0.0</td>
<td>77.2</td>
<td>59.0</td>
</tr>
<tr>
<td>1/4$^\text{th}$ shading</td>
<td>0.0</td>
<td>66.6</td>
<td>77.6</td>
</tr>
</tbody>
</table>

Table 6.3. Offset of actual rim angles on a hexagonal concentrator compared to the parabolic dish.
6.5 Summary and conclusions

In this chapter, a design method for nonimaging mirrors with polygonal inlet apertures that produce a uniform irradiance distribution on a polygonal receiver with a single reflection was presented. The method uses low-distortion, fractional-area-conserving mapping between two polygons, followed by numerical optimization of the 3D mirror shape. Potential CPV applications for the presented method are manifold, ranging from simple on-axis mirror designs, via compound concentrators that leverage tessellation to maximize the active area fraction, to advanced multi-mirror concentrators with additional advantages.

The performance of single-mirror concentrators with square, hexagonal and circular perimeter and concentrations of 100×, 500× and 1000×, representative of medium- to high-concentration PV applications, was evaluated. For concentrators with a square inlet aperture, the achieved irradiance distribution on the receiver is highly uniform for all concentrations because no distortions are introduced in the mapping procedure. It is a significant strength of the presented method that it also allows the design of mirrors with other polygonal apertures. A good performance is noticeably achieved with hexagonal concentrators. Shading of the mirror by the receiver has almost no effect on the intercept factor and uniformity, except for the low-concentration designs. The addition to the design of simple secondary optics however can considerably improve the concentrator performance (optical efficiency and irradiance uniformity) at high concentrations by only attenuating the rays in the fringe region at the outer edge of the receiver.

Also assessed was the performance of multi-mirror concentrators, where each mirror is individually imaged onto the whole area of the receiver, allowing greater flexibility for the concentrator shape and increased robustness against shading of large parts of the inlet aperture.
7  Outlook and research recommendations

This thesis has introduced several novel concepts and practical solutions that contribute to achieving a more cost-effective and sustainable utilization of the solar resource. This chapter discusses the outlook and recommendations for future research activities in the field of high-concentration solar energy systems.

7.1  Compact, practical high-concentration solar concentrators in line focus geometry

In Chapter 2 it was shown that a rigorous geometrical analysis can still bring forth some new interesting designs with the potential to improve existing solar energy applications. Most notable are the presented two-wing and nested designs, both based on asymmetric parabolic primary mirrors and ideal nonimaging secondaries, which can reach concentrations close to the thermodynamic limit while conserving a good compactness. Both concepts have the potential for implementation in medium-concentration CPV systems.

While two-wing concentrators have already been examined for CSP before this work [33], a similar adaptation of the nested design might be of interest, in part due to the slightly higher achievable concentration ratios that come as a result of the rim span reduction, but more importantly because the concept allows for an elimination of a central receiver support structure. In comparison to the design shown in Chapter 2, the flat receiver geometry would need to be swapped for a circular cross-section and the shape of the secondary adapted accordingly, with similar methods as described in [33]. It has to be noted that the change to a circular absorber in this special case, notably due to the fact that the secondary inlet aperture is attached to the opposing primary mirror and cannot freely be truncated, might result in certain design complications that would have to be addressed in detail, such as the size and placement of the glass envelope surrounding the secondary optic. A particularity of the nested design is that, when
considering a single collector, the receivers are placed on its outside. While this does not cause any issues with unifacial PV receivers, in the case of CSP one wants to illuminate the receiver tube from all sides. To achieve this with the nested collector, a possible solution is to place multiple such collectors together on the same tracker and let each of them irradiate half of the receiver tube.

### 7.2 Lambertian Flat Plate Calorimeter

Chapter 3 introduced the concept and first implementation of a new absolute irradiance measurement system, the Lambertian Flat Plate Calorimeter (FLPC), which mitigates many of the limitations of existing methods. When taking into consideration the practical experience gained with the described device during the measurements, detailed in Chapters 4 and 5, certain enhancements in terms of a reduction of the thermal heat loss of the system are suggested for a potential future revision of the device.

The first improvement can be realized by modifying the flow configuration in the calorimeter such that the temperature difference between the two parts of the target (cf. Figures 3.1 and 3.3) is minimized at the boundary and the heat transfer reduced, i.e. such that the coolant in the inner part flows radially inside towards the center, while it flows radially outside towards the outer edge in the outer part.

One might also think about moving away from the traditional alumina coating to a diffuse material with a higher absorptance. Provided the material has a good stability under high temperature, a better performance is expected due to the higher fraction of radiation absorbed in the target. The temperature difference across the calorimeter can be kept the same as with the current coating by increasing the flow rate of the cooling water. The lower amount of radiation reflected to the camera can be compensated for by increasing the transmittance of the neutral density filter and/or increasing the exposure time of the recorded images.

Finally, the insulation of the backside of the target can be improved such that the convective losses are further reduced.
7.3 Vacuum-membrane concentrators

The vacuum-membrane concept, of which the first on-sun test on a prototype was presented Chapter 4 and which was further investigated after various conceptual improvements with the full-scale system, as briefly discussed in Chapter 5, has shown some potential provided some important aspects are considered.

As was noted, the major disadvantage with vacuum membranes is their need for shielding from environmental conditions as a consequence of their front-surface metallization. This requirement introduces a penalty on the optical efficiency in the order of 90% due to transmission losses in the transparent top membrane providing the shielding. Additionally, the requirement of a vacuum pump might be considered as introducing a too high degree of complexity.

However, the experimental campaigns conducted throughout this work have demonstrated good focusing abilities of vacuum mirrors, with a performance comparable to several traditional solar mirror concepts and which could be improved by (1) using elasto-plastic (i.e. aluminum) membranes and (2) decreasing the thickness-to-diameter ratio of the facets. Facet size in the used concentrators was merely limited by the commercial availability of metallized membrane material (1.8 m for Mylar; 1.2 m for aluminum). Yet, a simple scale-up of the mirror aperture is specifically a core advantage of membrane-mirror technologies, as their low weight solves many of the otherwise inherent problems with large systems and it is easy to realize a scale-up of the membrane production provided an appropriate demand. When considering the potential cost advantage that can be achieved with such systems, the small penalties in optical efficiency might become insignificant for the total price-performance ratio.

7.4 Multi-focus solar dishes

An important innovation presented in this work is the multi-focus solar dish concept, introduced in Chapter 5 and implemented in the form of a 6-focus mirror. Important advantages of the concept are the high concentration ratios possible while conserving a good collector compactness. As briefly mentioned, the concept is extendable to multiple radial as well as circumferential segmentations, which presents interesting avenues for several applications.
The presented multi-focus concept can be seen as an extension to point focus geometry of the two-wing asymmetric concentrators from Chapter 2, which showed superior performance compared to symmetric systems. An interesting opportunity might lie in the transfer to point focus of the nested asymmetric designs, which showed even more potential in terms of concentration ratio, compactness and not requiring a central receiver support structure.

### 7.5 Application of nonimaging polygonal mirrors

The design method for nonimaging mirrors with polygonal apertures generating a uniform focal irradiance distribution, presented in Chapter 6, could finally be used in a redesign of the multi-focus system that was the main focus of this work.

As was demonstrated, the employed method of bringing a thin back-surface glass mirror into shape by fixation onto a precast shell bearing the desired profile was very successful in the reproduction of the parabolic profiles for the mirrors used in Chapter 5 (measured slope error < 1.5 mrad, i.e. a performance equivalent to the industry standard). It is evident that nonimaging profiles generated with the method from Chapter 6 can be implemented with the same technique and accuracy, as their deviation from the parabolic shape is relatively small and the polygonal perimeters do not add any additional complications to their manufacturing process.

One can imagine a concentrator similar to the one from Chapter 5, but where the elliptical facets are replaced e.g. by hexagonal nonimaging mirrors. Two exemplary configurations of such a layout are shown in Figure 7.1. By leveraging tessellation, the active inlet aperture of the system can be increased while keeping the same outer dimensions. Evidently, the number of mirrors per module can be adapted from the examples presented here to find the desired trade-off between design flexibility, cost and size.

In addition to maximizing the irradiance uniformity on the receiver, the average concentration of such a system could be reduced from the very high values in Chapter 5 to e.g. 500×, for which good results are predicted in terms of uniformity in Chapter 6 and which coincides with the optimal operation irradiance for current state-of-the-art multi-junction cells. This could have interesting implications for the PV receiver design. On one hand, the electrical
To realize such designs, the method presented in Chapter 6 simply needs to be extended such as to allow a tilted and rotated receiver, which can easily be achieved by adapting Eq. (6.8) to incorporate the desired linear transformations of the receiver nodes. In preliminary simulations, it has been found that a tilt of the receiver can lead to a more complicated minimization problem, i.e. an increased difficulty to fulfill Eq. (6.11) for all nodes with a continuous surface, which tends to cause additional distortions in the final irradiance distributions. These effects could however be reduced by inserting an additional transformation of the disk nodes in-between the transformations \( \Gamma_{P_1 \rightarrow D} \) and \( \Gamma_{D \rightarrow P_2} \), where the nodes are rotated around the origin. By optimizing the angle of rotation, the distortions can be mitigated, albeit not entirely eliminated.

**Figure 7.1.** Exemplary configurations of a multi-focus solar dish with hexagonal non-imaging facets (a) using the same pattern as the dish from Chapter 5; (b) where the pattern is changed such as to create a more symmetric irradiance.
Appendix A

Derivations for asymmetric parabolic troughs

A.1 Derivation of the primary concentration of asymmetric parabolic troughs

In Section 2.2.2 the theoretical concentration limit for asymmetric parabolic troughs assuming a finite acceptance angle were presented. Section A.1.1 shows the derivation of the general case where the maximum concentration is achieved by placing the primary outlet aperture between the intersection points of the edge rays reflected from the inner and outer rim of the parabola respectively (Regime 1). Section A.1.2 investigates the case where a large inner rim angle of the parabola causes a primary outlet aperture tilted by \( \tau_2 = \Phi_2 - \theta_i \) to be optimal (Regime 2). Section A.1.3 generalizes the influence of nonoptimal tilt angle on the primary concentration. Besides bridging the gap between the special cases presented in Sections A.1.1 and A.1.2, the derived equations can be applied for the design of systems where the tilt of the primary outlet aperture is required to be different from the optimum in order to accommodate a specific secondary concentrator design [67].

A.1.1 Small inner rim angle

Referring to Figure 2.2, the curve of a parabola with vertical axis, focus at \( \mathbf{O}(0,0) \) and focal length \( f \) is given by

\[
P = \frac{2f}{1 + \cos \phi} \begin{bmatrix} \sin \phi \\ -\cos \phi \end{bmatrix}
\]

(A.1)

where \( \phi \) is the parametric angle measured between the z-axis and a line connecting a point on the curve to the focus. A parabola spanning between points

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Appendix A

A’ and A (defined by the parametric angles $\Phi_1$ and $\Phi_2$; Figure 2.2) therefore has the inlet aperture

$$a_{i,1} = 2f \cdot \left[ \sin \Phi_2 / (1 + \cos \Phi_2) - \sin \Phi_1 / (1 + \cos \Phi_1) \right]. \quad (A.2)$$

The points of intersection of the edge rays reflected from points A and A’ can be expressed as

$$B = A + \left[ -\sin (\Phi_2 - \theta_i) \right] \cdot \|AB\| = A' + \left[ -\sin (\Phi_1 - \theta_i) \right] \cdot \|A'B\| \quad (A.3)$$

$$B' = A + \left[ -\sin (\Phi_2 + \theta_i) \right] \cdot \|AB'\| = A' + \left[ -\sin (\Phi_1 + \theta_i) \right] \cdot \|A'B'\| \quad (A.4)$$

Eqs. (A.3) and (A.4) can be solved for the distances $\|AB\|$ and $\|AB'\|$. By applying the law of cosines in the triangle $ABB'$ and subbing the solutions for $\|AB\|$ and $\|AB'\|$, the outlet aperture becomes

$$a_{o,1} = \sqrt{\|AB\|^2 + \|AB'\|^2 - 2\|AB\|\|AB'\| \cos \theta_i}$$

$$= \frac{f \cdot \sin 2\theta_i \cdot \sqrt{\cos \Phi_1 + \cos \Phi_2 + 3/2 + 1/2 \cos \Delta\Phi}}{2 \cos^2 (\Phi_1/2) \cdot \cos^2 (\Phi_2/2) \cdot \cos (\Delta\Phi/2)}. \quad (A.5)$$

The geometric concentration limit of the primary mirror then becomes

$$C_{g,1,\text{max,asym,R1}} = \frac{2 \cos (\Phi_1/2) \cdot \cos (\Phi_2/2) \cdot \sin \Delta\Phi}{\sin 2\theta_i \cdot \sqrt{\cos \Phi_1 + \cos \Phi_2 + 3/2 + 1/2 \cos \Delta\Phi}}. \quad (A.6)$$

The ideal tilt of the outlet aperture, $\tau_1$, may then be found by calculating the slope of the segment $BB'$

$$\tan \tau_1 = \frac{2 \sin \Phi_{av} + \sin (\Phi_1 + \Phi_{av}) + \sin (\Phi_2 + \Phi_{av})}{2 \cos \Phi_{av} + \cos (\Phi_1 + \Phi_{av}) + \cos (\Phi_2 + \Phi_{av})}. \quad (A.7)$$

Interestingly, this expression of the tilt angle is independent of $\theta_i$ and equivalent to the formulations derived in [67] and [33] for the approximation of a small acceptance angle. An important finding is that, for non-zero acceptance
angles, placing the outlet aperture between the intersection points of the left and right edge rays reflected from A and A' only produces the maximum achievable primary concentration of such a system in a certain regime where $0 < \tau_1 < \Phi_2 - \theta_i$ (Regime 1).

A.1.2 Large inner rim angle

As $\Phi_1$ is increased, the tilt angle $\tau_1$ for maximum concentration increases. At the point where $\tau_1 > \Phi_2 - \theta_i$ (Regime 2), it is no longer optimal to place the aperture at the intersection of the edge rays. The maximum concentration aperture must instead be tilted at $\tau_2 = \Phi_2 - \theta_i$ such that it extends perpendicularly from the $+\theta_i$ edge ray reflected from the rim, as shown in Figure A.1. Point $B^*$ is defined as the orthogonal projection of $B'$ onto $AB$. The outlet aperture can then be found by solving for $||B'B||$

$$a_{o,1} = f \cdot \frac{1/2 \cdot \sec(\Phi_1/2)^2 \cdot \sec(\Phi_2/2)^2 \cdot \sin(2\theta_i) \cdot \sqrt{\cos(\Phi_1 + \theta_i) + \cos(\Delta\Phi/2 - \theta_i) \cdot \sec(\Delta\Phi/2)^2}}{\cdot}{(A.8)}.$$
With Eq. (A.5) and after simplification, the primary concentration in Regime 2 becomes

\[
C_{g.1,\text{max, asymmetric},R2} = \frac{\sin \phi_2 - \sin \phi_1 + \sin \Delta \phi}{\sin 2\theta_i \cdot [\cos(\phi_1 + \theta_i) + \cos(\Delta \phi/2 - \theta_i) \cdot \sec(\Delta \phi/2)]}.
\]  

(A.9)

Let $\phi_1^*$ be the limiting inner rim angle between regimes, obtained by solving the equation $\tau_1(\phi_1, \phi_2) = \phi_2 - \phi_i$ for $\phi_1$. Figure A.2 shows $\phi_1^*$ vs. $\phi_2$ for $\theta_i = 1^\circ$. It becomes apparent that $\phi_1^*$ is relatively low for all rim angles. Regime 2 might therefore only be practical in certain asymmetric trough concentrators where the design inhibits the lower rim of the parabola segment to extend all the way to the bottom. For example, for $\theta_i = 1^\circ$ and a common $\phi_2 = 45^\circ$ the limiting lower rim angle $\phi_1^*$ is $14.0^\circ$. Only if $\phi_1 > \phi_1^*$, the concentrator lies in Regime 2. For $\phi_2 = 90^\circ$, $\phi_1^* = 31.7^\circ$.

Conversely, when looking only at concentrators designed for maximum primary concentration and AAF ($\phi_1 = 2\theta_i$) then $\phi_2^*$, obtained by solving the equation $\tau_1(\phi_1, \phi_2) = \phi_2 - \phi_i$ for $\phi_2$, defines the minimum rim angle needed for a
concentrator to belong to Regime 1. For example, a concentrator designed with acceptance angle $\theta_i = 1^\circ$ ($\Phi_1 = 2^\circ$) will be in Regime 1 for $\Phi_2 > 10^\circ$. Most common, simple parabolic trough systems therefore lie in Regime 1.

A.1.3 Non-ideal tilt of the outlet aperture

If the tilt of the outlet aperture is lower than $\tau_1$ then, referring to Figure A.3, the outlet aperture for maximum concentration at full collection extends from $B'$ to $B$. Similarly, if the tilt of the outlet aperture is greater than $\tau_1$ then the outlet aperture for maximum concentration at full collection extends from $B^*$ to $B$. The concentration then is

$$C_{g,1,\text{asym}} = \begin{cases} 
C_{g,1,\text{asym},-} & \text{for } \frac{\pi}{2} - \Phi_i - \theta_1 < \tau < \tau_1 \\
C_{g,1,\text{asym},+} & \text{for } \tau_1 < \tau < \frac{\pi}{2} + \Phi_i - \theta_1
\end{cases} \quad (A.10)$$

with

**Figure A.3.** Cross-sectional geometry of the focal region of a generic asymmetric parabolic trough with non-ideal outlet aperture tilt.
Figure A.4. Concentration and CAP\textsubscript{2D} of an asymmetric primary concentrator versus the tilt $\tau$ of its outlet aperture for $\theta_i = 1^\circ$, $\Phi_1 = 10^\circ$ to $80^\circ$ and $\Phi_2 = 90^\circ$. The maximum concentration achievable for each $\Phi_1$ is marked with $\times$. For systems that lie in Regime 1 ($\Phi_1 = 10^\circ$ to $30^\circ$) this maximum concentration is $C_{g,1,\text{max,asym,R1}}$ and is reached at $\tau = \tau_1$ (marked with $+$) while for systems that lie in Regime 2 ($\Phi_1 = 40^\circ$ to $80^\circ$) the maximum concentration is $C_{g,1,\text{max,asym,R2}}$ and is reached at $\tau = \tau_2$. For comparison, the concentrations reached with the average tilt ($\tau = \Phi_{av}$) are marked with $\circ$ and lie below the maximum achievable concentration for all $\Phi_1$.

$C_{g,1,\text{asym,\textminus}} = \frac{\csc(2\theta) \cdot \cos(\Phi_2 - \tau - \theta) \cdot [\sin \Phi_2 - \sin \Phi_1 + \sin \Delta \Phi]}{\cos(\Phi_1 + \tau) + \sec(\Delta \Phi/2) \cdot \cos(\Delta \Phi/2 - \tau)}$

$C_{g,1,\text{asym,\text{\textplus}}} = \frac{4 \csc(2\theta) \cdot \cos(\Phi_1/2) \cdot \cos(\Phi_2/2) \cdot \cos(\tau - \Phi_1 - \theta) \cdot \sin(\Delta \Phi)}{\cos(\Phi_2 + \Delta \Phi/2 - \theta) + \cos(\Phi_{av} - \theta) + 2 \cos(\Delta \Phi/2 - \theta)} \tag{A.11}$

Note that, independent of the regime, the limiting tilt angle between $C_{g,1,\text{asym,\textminus}}$ and $C_{g,1,\text{asym,\text{\textplus}}}$ is defined by the intersection of the edge rays ($\tau = \tau_1$). The maximum concentration for ideal tilt in Regime 2, $C_{g,1,\text{max,asym,R2}}$, is equivalent to $C_{g,1,\text{asym,\textminus}}$ for the case where $\tau = \tau_2 = \Phi_2 - \theta_i$.

Figure A.4 shows $C_{g,1}$ vs. the tilt $\tau$ of the outlet aperture for $\Phi_2 = 90^\circ$ and $\Phi_1$ ranging from $10^\circ$ to $80^\circ$. The maximum achievable concentration for each concentrator is marked with crosses ($\times$). The switch from $C_{g,1,\text{asym,\textminus}}$ to $C_{g,1,\text{asym,\text{\textplus}}}$, occurring at $\tau = \tau_1$, is marked with plus signs ($+$). In Section A.1.2 it was shown
that the limit between regimes, $\Phi_1^*$, is $31.7^\circ$ for $\Phi_2 = 90^\circ$ and $\theta_i = 1^\circ$. Accordingly, the concentrators having $\Phi_1 = 10^\circ$, $20^\circ$ and $30^\circ$ lie in Regime 1 and the maximum concentration is reached for a tilt angle $\tau = \tau_1$. For concentrators having $\Phi_1 > \Phi_1^*$ the maximum concentration is reached for $\tau = \tau_2$. Plotted as circles (○) is $C_{g,1}$, achieved by tilting the outlet aperture by $\Phi_{av} = (\Phi_1 + \Phi_2)/2$, as required when designing a symmetric secondary concentrator (e.g. symmetric CPC) for an asymmetric trough. This case results in reduced $C_{g,1}$ and is further aggravated by a forfeit in secondary concentration due to the required oversizing of the symmetric secondary.

A.2 Effect of the inner rim angle of asymmetric parabolic troughs on the overall concentration

For the most part of Chapter 2, AAF (and with it $C_{g,1}$) of asymmetric systems has been maximized by designing all concentrators with $\Phi_1 = 2\theta_i$ and varying $\Phi_2$. The question of how the overall concentrations change for a fixed rim angle $\Phi_2$ and varying $\Phi_1$ is also of interest. Figure A.5 shows $C_{g,1}$ and $C_{g,tot}$ as a function of $\Phi_1$ varying from $2^\circ$ ($\Phi_1 = 2\theta_i$) to $80^\circ$ for asymmetric two-stage concentrators with $\Phi_2 = 90^\circ$, designed for $\theta_i = 1^\circ$. The primary concentration reached by placing the outlet aperture at the intersection of the edge rays reflected from the rims of the primary [Eq. (A.6)] decreases with increasing $\Phi_1$. Additionally, the concentration achieved with a tilt $\tau_2 = \Phi_2 - \theta_i$ [Eq. (A.9)] is plotted in Regime 2 ($\Phi_1 > \Phi_1^* = 31.7^\circ$), confirming the result from Section A.1.2 that such a design can be superior for a single-stage asymmetric concentrator. For a two-stage system, however, a secondary inlet aperture placed at the intersection of the edge rays (i.e. tilted by $\tau_1$) always results in a higher $C_{g,tot}$.

Also plotted is $C_{g,tot}$ for systems using CEC and CPC secondaries. The CEC-based systems approach the thermodynamic concentration limit for small rim angle spans $\Delta\Phi = \Phi_2 - \Phi_1$ where the nonimaging secondary does a greater part of the concentration work, despite decreasing $C_{g,1}$. In contrast, the CPC-based concentrators have a limited peak concentration due to the fact that a CPC is an ideal concentrator for a source at an infinite distance only. It is important to note however that by choosing $\Phi_1 > 2\theta_i$ to increase concentration results in a decrease of AAF and may therefore lead to a poorer overall performance.
Figure A.5. Comparison of primary ($C_{g,1}$) and overall ($C_{g,\text{tot}}$) geometrical concentration ratios and $\text{CAP}_{2D}$ of two-stage asymmetric designs with $\Phi_2 = 90^\circ$ and $\theta_i = 1^\circ$ versus the inner rim angle $\Phi_1$. The overall concentration achievable when using a CEC secondary concentrator approaches the thermodynamic limit $C_{g,\text{max,2D}}$ for small rim spans $\Delta \Phi = \Phi_2 - \Phi_1$ in spite of the lowering primary concentration $C_{g,1,\text{asym,R1}}$. When using the more common CPC secondary concentrator, the overall concentration is lower and does not approach the thermodynamic limit. Also plotted is the limit $\Phi_1 = \Phi_1^\ast$ between Regimes 1 and 2 of a single-stage primary concentrator. For $\Phi_1 > \Phi_1^\ast$ a primary outlet aperture tilted by $\tau_2 = \Phi_2 - \theta_i$ results in a higher primary concentration.
Appendix B

Lambertian Flat Plate Calorimeter

B.1 Relation between incident and reflected radiation from a Lambertian surface

The relation between the radiation incident on a surface from the zenith angle $\theta_i$ and azimuth angle $\phi_i$, $I_{\lambda,i}(\theta_i,\phi_i) \cdot \cos \theta_i \cdot d\Omega_i$, and its contribution to the reflected intensity in the direction described by $\theta_r$ and $\phi_r$, $dI_{\lambda,r}(\theta_r,\phi_r)$, is given by the spectral, bidirectional reflection function [165] or bidirectional spectral reflectivity [166] $\rho''_{\lambda}(\theta_r,\phi_r,\theta_i,\phi_i)$ as

$$dI_{\lambda,r}(\theta_r,\phi_r,\theta_i,\phi_i) = \rho''_{\lambda}(\theta_r,\phi_r,\theta_i,\phi_i) \cdot I_{\lambda,i}(\theta_i,\phi_i) \cdot \cos \theta_i \cdot d\Omega_i,$$

(B.1)

where $d\Omega_i = \sin \theta_i \cdot d\theta_i \cdot d\phi_i$ is the infinitesimal source solid angle. The radiation reflected in the $(\theta_r,\phi_r)$ direction from all incident directions $(\theta_i,\phi_i)$ can be found by integrating over the hemisphere $\cap$

$$I_{\lambda,i}(\theta_r,\phi_r) = \int_{\cap} dI_{\lambda,r}(\theta_r,\phi_r,\theta_i,\phi_i)$$

$$= \int_{\cap} \rho''_{\lambda}(\theta_r,\phi_r,\theta_i,\phi_i) \cdot I_{\lambda,i}(\theta_i,\phi_i) \cdot \cos \theta_i \cdot d\Omega_i.$$

(B.2)

If the surface is diffuse (Lambertian), the reflectance is the same for all viewing angles $(\theta_r,\phi_r,\theta_i,\phi_i)$. Eq. (B.2) then simplifies to

$$I_{\lambda,i}(\theta_r,\phi_r) = \rho''_{\lambda} \cdot \int_{\cap} I_{\lambda,i}(\theta_i,\phi_i) \cdot \cos \theta_i \cdot d\Omega_i$$

$$= \rho''_{\lambda} \cdot E,$$

(B.3)

---

which implies that the radiance diffusely reflected by the Lambertian target is directly proportional to the incident irradiation.

B.2 Characterization of the calorimeter coating

For an accurate measurement of the solar radiation incident on the flat plate calorimeter it is essential to know the reflectance of its Al₂O₃ coating. For this purpose, the hemispherical spectral reflectance \( \rho_{\lambda,Al_2O_3} \) of the coating, applied to a copper substrate, was measured using a spectroscopic goniometry system similar to the one described in [101] but adapted to measure \( 8^\circ/\text{hemispherical} \) reflectance in the range 300-2500 nm. The procedure uses an integrating sphere (Labsphere RT-060-SF) to perform a relative reflectance measurement using the comparison method [167] with respect to a standard reflective sample with solar-averaged reflectance \( \rho_{\lambda,\text{std}} = 99\% \) (Labsphere AS-01161-060),

\[
\rho_{\lambda,Al_2O_3}(\lambda) = \frac{i_{\lambda,Al_2O_3}(\lambda)}{i_{\lambda,\text{std}}(\lambda)} \rho_{\lambda,\text{std}}(\lambda), \tag{B.4}
\]

where \( i_{\lambda,Al_2O_3} \) and \( i_{\lambda,\text{std}} \) are the photodetector signals with the sample and the reflectance standard respectively. The accuracy of the measurement procedure was assessed using a second standard reflective sample with a solar-averaged reflectance of 50\% (SphereOptics Zenith Polymer Diffuse Reflectance Standard – 50\%) and found to be within 1\%. \( \rho_{\lambda,Al_2O_3} \) was then weighted with the spectral solar irradiance \( E_\lambda \) using the weighted ordinates method [168]

\[
\rho_{Al_2O_3} = \frac{\int_{300 \text{ nm}}^{2500 \text{ nm}} \rho_{\lambda,Al_2O_3}(\lambda) \cdot E_\lambda(\lambda) d\lambda}{\int_{300 \text{ nm}}^{2500 \text{ nm}} E_\lambda(\lambda) d\lambda}. \tag{B.5}
\]

ASTM AM1.5d [127] was used as representative spectral direct normal solar irradiance. Figure B.1 shows the measured spectral \( 8^\circ/\text{hemispherical} \) reflectance \( \rho_{\lambda,Al_2O_3} \) and normalized solar spectrum \( E_\lambda \), plotted versus the wavelength \( \lambda \) in the range 300-2500 nm. The solar-weighted reflectance of the calorimeter surface, \( \rho_{Al_2O_3} \), is 79.49\%.

During an on-sun measurement on the dish, the solar spectrum is attenuated upon reflection on the primary mirrors. To determine the effect of this attenuation
on the post-mirror solar-averaged reflectance, the ASTM AM1.5d spectrum was weighted with the spectral reflectance $\rho_{\lambda,1}$ of the mirrors determined by [101, 102]

$$E'_\lambda(\lambda) = \rho_{\lambda,1}(\lambda) \cdot E_\lambda(\lambda). \quad (B.6)$$

For both AgSheet0.3mm and AgSheet0.2mm mirrors the solar-averaged reflectance $\rho_{Al2O3}$ changes to 79.59%, which is used for the computation of the scaling factor in this Chapter 4. As the measured radiant power incident on the calorimeter is directly proportional to the absorptance $\alpha = 1 - \rho$, the error induced by neglecting the difference in incident spectrum with respect to ASTM AM1.5d can be quantified by

$$\delta_\alpha = \frac{\alpha - \alpha_{AM1.5d}}{\alpha_{AM1.5d}} = \frac{1 - \rho - (1 - \rho_{AM1.5d})}{1 - \rho_{AM1.5d}} = \frac{\rho_{AM1.5d} - \rho}{1 - \rho_{AM1.5d}}. \quad (B.7)$$
With an error of $-0.49\%$ the effect disregarding the spectral attenuation by the mirrors is relatively small, with a slight under-prediction of the effective absorptance. Larger changes in the incident spectrum can occur due to seasonal and climatic effects. To show the dependence of the solar-averaged reflectance to a changing solar spectrum more generally, results are further calculated for Air Mass 1 to 5. The corresponding solar spectra were modeled with the open-source Simple Model of the Atmospheric Transfer of Sunshine (SMARTS) [169] that also serves as a basis for the ASTM AM1.5d standard. Representative results of the solar-averaged reflectance and the associated $\delta_\alpha$ are shown in Table B.1 and are plotted in Figure B.2. Even for AM5 the offset is still relatively small when compared to other methods, with an error $\delta_\alpha = -5.27\%$.

### B.3 Improvements to the system

In comparison to the previously presented measurement device [109] (Chapter 3, Section B.2), the coating was replaced by plasma-coated $\text{Al}_2\text{O}_3$ with a
thickness of 75±25 μm for the optical characterization of the 6-focus high-concentration dish (Chapter 5). This coating process offers (1) a favorable heat conduction through the ceramic (avoiding heat accumulation on the front surface); and (2) a higher reliability in the recorded grayscale images.

The 8°/hemispherical spectral reflectance $\rho_{\lambda,Al_2O_3,Cu}$ of the coating on a copper substrate representative of the calorimeter surface was therefore re-determined for the new coating with the method described in Section B.2 and is plotted versus the wavelength $\lambda$ in Figure B.3, together with the normalized solar spectrum $E_\lambda$ (ASTM AM1.5d). The resulting solar-weighted reflectance ($\rho_{\lambda,Al_2O_3,Cu}$), determined with Eq. (B.5) is 68.37%. For further improvement of the measurement accuracy at the outside of the region of interest (ROI), where the Lambertian target is made of aluminum, the reflectance measurement was repeated with a coated aluminum sample. It can be seen in Figure B.3 that the measured spectral reflectance ($\rho_{\lambda,Al_2O_3,Al}$) deviates slightly from $\rho_{\lambda,Al_2O_3,Cu}$ since, as a result of the thin Al$_2$O$_3$ layer, the reflective properties of both substrates partially influence the apparent reflectance. The solar-weighted reflectance of the coated aluminum, $\rho_{Al_2O_3,Al}$, is 67.16%.

Table B.1. Sensitivity of the solar-weighted reflectance of the Lambertian target coating to solar spectrum change.

<table>
<thead>
<tr>
<th>Solar spectrum</th>
<th>$\rho_{Al_2O_3}$ [%]</th>
<th>$\delta_\omega$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM1</td>
<td>79.22</td>
<td>+1.32</td>
</tr>
<tr>
<td>ASTM AM1.5d</td>
<td>79.49</td>
<td>0.00</td>
</tr>
<tr>
<td>ASTM AM1.5d + AgSheet0.3mm</td>
<td>79.59</td>
<td>-0.49</td>
</tr>
<tr>
<td>ASTM AM1.5d + AgSheet0.2mm</td>
<td>79.59</td>
<td>-0.49</td>
</tr>
<tr>
<td>AM2</td>
<td>79.71</td>
<td>-1.07</td>
</tr>
<tr>
<td>AM3</td>
<td>80.07</td>
<td>-2.83</td>
</tr>
<tr>
<td>AM4</td>
<td>80.34</td>
<td>-4.14</td>
</tr>
<tr>
<td>AM5</td>
<td>80.57</td>
<td>-5.27</td>
</tr>
</tbody>
</table>
Using these measurements, the scaling factor between the relative and absolute irradiance distributions can be corrected in the region outside of the ROI by

$$F_{corr} = \frac{\rho_{Al2O3,Al}}{\rho_{Al2O3,Cu}} \cdot F = 0.982 \cdot F,$$

(B.8)

where $F$ is the calorimetrically determined scaling factor within the ROI [Eq. (3.5)]. Finally, the absolute irradiance distribution can be obtained with

$$E(x, y) = \begin{cases} \frac{F \cdot GV(x, y)}{F_{corr} \cdot GV(x, y)}, & \text{if } (x, y) \in \text{ROI} \\ F_{corr} \cdot GV(x, y), & \text{otherwise} \end{cases}.$$  

(B.9)
Appendix C

Evaluation and modeling of PV cell performance\textsuperscript{a}

C.1 Processing of \textit{I-V} curves

A total of 502 measured \textit{I-V} curves, of cells originating from various production batches were obtained from the manufacturer [114, 115]. The individual curves were pre-processed to eliminate measurement artifacts and facilitate the subsequent analysis: (1) a noisy measurement (bump) in the \textit{I-V} curve slightly below \textit{V}_{oc} was removed by interpolation in the affected region; (2) non-unique data points were filtered out by averaging points having equal voltage; and (3) the curves were smoothed by a 3-point moving average filter to reduce high-

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{C1.png}
\caption{(a) The 502 pre-processed \textit{J-V} curves provided by the manufacturer; (b) detail showing the averaging of the measured \textit{J-V} curves of equal irradiance and temperature.}
\end{figure}

frequency measurement noise. The totality of the processed curves are shown in Figure C.1(a), expressed in terms of current density $J = I/A_{cell}$.

To visualize temperature and irradiance trends, filter out the stochastic variation between different cell batches and obtain the expected response of a realistic, average PV cell, all curves with equal temperature and irradiance are averaged together. The inlay Figure C.1(b) details the averaging of the $J$-$V$ curves obtained with $E = 810$ kW/m$^2$ and $T = 110$ °C. First, each curve is interpolated to a common regular grid of $V$, where a grid refinement is applied in the vicinity of the MPP (right of the dashed line) for an accurate representation of the slopes. The interpolated curves are subsequently averaged at each grid point, to obtain the averaged curves (marked with $\times$). The averaged $J$-$V$ curves and the corresponding $\eta$-$V$ curves (where $\eta = J \cdot V/E$) for each set \{i,j\} of irradiance and temperature are shown in Figure C.2(a) and Figure C.2(b) respectively, giving a good qualitative overview of the irradiance and temperature trends.

Figure C.2. (a) Averaged $J$-$V$ curves for each combination of temperature and irradiance; and (d) corresponding $\eta$-$V$ curves.
C.2 Irradiance and temperature dependency

A qualitative insight into the irradiance and temperature dependency is provided by the commonly used performance metrics open-circuit voltage \( (V_{oc}) \), short-circuit current density \( (J_{sc}) \), voltage at MPP \( (V_{MPP}) \), current density at MPP \( (J_{MPP}) \), fill factor \( (FF) \), power at MPP \( (P_{MPP}) \) and efficiency at MPP \( (\eta_{MPP}) \). The performance metrics of an average cell in the measured range are summarized in Table C.1. They confirm the expected behavior described in literature, i.e. (1) linear increase of \( J_{sc} \) and \( J_{MPP} \) with increasing irradiance due to the linearly increased generated photocurrent [135, 170] and slight increase with temperature.

Table C.1. Performance metrics of the average cells at each measurement condition.

<table>
<thead>
<tr>
<th>( E ) [kW/m(^2)]</th>
<th>( T ) [°C]</th>
<th>( V_{oc} ) [V]</th>
<th>( J_{sc} ) [A/cm(^2)]</th>
<th>( V_{MPP} ) [V]</th>
<th>( J_{MPP} ) [A/cm(^2)]</th>
<th>( FF ) [%]</th>
<th>( P_{MPP} ) [W]</th>
<th>( \eta_{MPP} ) [%]</th>
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due to a reduced band gap energy [139]; (2) band-gap related marginal increase of $V_{oc}$ and $V_{MPP}$ with irradiance and decrease with temperature [58, 139]; (3) decrease of $FF$ with irradiance and with temperature [58, 136, 171]; and (4) a decrease of $\eta_{MPP}$ ($P_{MPP}$) with increasing temperature, where the slope is steeper for lower concentrations [136].

To allow a comparison of the average cells used throughout this work to the best-performing cells from the provided measured batches, Table C.2 shows the corresponding performance metrics for the cells with the highest measured efficiency for each combination of irradiance and temperature. Compared to the

Table C.2. Performance metrics of the best-performing cells at each measurement condition.

<table>
<thead>
<tr>
<th>$E$ [kW/m²]</th>
<th>$T$ [°C]</th>
<th>$V_{oc}$ [V]</th>
<th>$J_{sc}$ [A/cm²]</th>
<th>$V_{MPP}$ [V]</th>
<th>$J_{MPP}$ [A/cm²]</th>
<th>$FF$ [%]</th>
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highest-efficiency cells at each temperature and irradiance, the average cells have, on average, an efficiency lower by 0.56 abs.%, with a maximum difference of 0.84 abs.% and a minimum difference of 0.34 abs.% . Accordingly, the characteristic voltages, current densities and fill factors of the average cells are slightly lower than that of the best-performing cells.

C.3 Equivalent circuit fitting procedure

The fitting of the lumped equivalent circuit model is performed using unconstrained non-linear optimization with the Nelder-Mead simplex algorithm as implemented in Matlab® as the built-in function fminsearch [142, 143], with the objective function $f_{obj}$ [Eq. (5.19)]. Both the relative convergence tolerance of the fitted parameters and of the objective function were set to $10^{-9}$.

For good convergence, appropriate initial parameter guesses are required. If the full $J$-$V$ curve is known, such as is the case for the measured $J$-$V$ curves of the average cells, the following initial guesses, defined according to Ben Or and Appelbaum and Zhang et al. [132, 172] are employed

$$J_0^0 = \frac{J_{sc}}{\exp\left(\frac{q \cdot V_{oc}}{n_D \cdot k_B \cdot T}\right)}, \quad \text{(C.1)}$$

$$n_D^0 = 3, \quad \text{(C.2)}$$

$$R_{sh}^0 = -\left(\frac{A_{cell} \cdot \partial J}{\partial V}\bigg|_{J=J_{sc}}\right)^{-1}, \quad \text{(C.3)}$$

$$R_s^0 = -\frac{n_D \cdot k_B \cdot T}{A_{cell} \cdot J_{sc}} \cdot \left(\frac{A_{cell} \cdot \partial J}{\partial V}\bigg|_{V=V_{oc}}\right)^{-1}. \quad \text{(C.4)}$$

Especially Eq. (C.1) is important to ensure fast convergence due to the exponential dependence of $J_0$ on the temperature. The slopes $\partial J/\partial V|_{J=J_{sc}}$ and $\partial J/\partial V|_{V=V_{oc}}$ in Eqs. (C.3) and (C.4) are then determined from a linear regression of the first 5 data points with $V > 0$ and the last 3 data points with $J > 0$ of the $J$-$V$ curve, respectively. If the full $J$-$V$ curve is not known, such as is the case for
the extrapolated operating conditions (Section 5.4.3), these slopes cannot be computed. Eqs. (C.3) and (C.4) are then replaced by the initial guesses
\[ R_{sh}^0 = 10^6 \]
\[ R_s^0 = 10^{-2} \]
which represent the average values of \( R_{sh} \) and \( R_s \) of the fitted measured \( J-V \) curves, ensuring similarly good convergence.

**C.4 Exemplary fit of \( J-V \) curve**

To verify the validity of a lumped equivalent circuit model fit, as employed in Section 5.4.3, a known exemplary \( J-V \) curve (measurement at \( E = 500 \text{ kW/m}^2; T = 25 \text{ °C} \)) is fitted. Using only the performance metrics from Table C.1, i.e. \( J_{sc} = 6.80 \text{ A/cm}^2, V_{oc} = 3.22 \text{ V} \) and \( \eta_{MPP} = 37.7\% \), the lumped model parameters
\[ n_D = 2.88, J_0 = 9.62 \times 10^{-19} \text{ A/cm}^2, R_s = 0.0146 \Omega \text{ and } R_{sh} = 179 \Omega \]
are determined. **Figure C.3** shows the comparison between the measured and fitted \( J-V \) curves. For an objective function value of \( f_{obj} = 6.10 \times 10^{-15} \), the rms error over all the measurement points of the \( J-V \) curve is \( \varepsilon_{J-V} = 2.36\% \), confirming an accurate fit. The other measured \( J-V \) curves could be fitted with comparable accuracy.
Appendix D

Polygons concentrators

D.1 Generation of regular grids on a polygon and a disk

For the application of the concentrator design method outlined in Section 6.2, the object area (polygon or disk) needs to be discretized into a set of nodes before mapping. It is beneficial for the successful optimization of the concentrator that these nodes be regularly spaced within the grid, i.e. that the area of all triangular surface elements spanned by 3 neighboring nodes have equal (or as close as possible to equal) area.

Figure D.1(a) describes the procedure of creating \( N \) regularly spaced nodes \( p_{p1,n} \) on a polygon \( P_1 \) with \( N_{p1} \) sides, using the example of a hexagon \( (N_{p1} = 6) \). In the first region (shaded in gray), a basis \{u-v\} with

\[
\begin{align*}
\mathbf{u} &= a_{p1} \cdot [1; \tan(\pi/N_{p1})] \\
\mathbf{v} &= a_{p1} \cdot [1; -\tan(\pi/N_{p1})]
\end{align*}
\]  

\tag{D.1}

is created. Nodes are then generated such that

\[
p_{p1}(i, j) = \frac{i \cdot \mathbf{u} + j \cdot \mathbf{v}}{k-1},
\]  

\tag{D.2}

where \( k \) is the desired number of nodes along one basis vector, \( i = 0,\ldots,k \), and \( j = 0,\ldots,k-i-1 \). The nodes in the other regions \( q = 2,\ldots,N_{p1} \) are generated simply by rotation of \{u-v\} by \((q-1)2\pi/N_{p1}\) around the origin. The total number of nodes generated this way is

\[
N = 1 + N_{p1} \cdot k \cdot (k-1)/2.
\]  

\tag{D.3}

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\(^a\) Material in this chapter has been published in M. Schmitz, F. Dähler, F. Elvinger, A. Pedretti, A. Steinfeld, “Nonimaging polygonal mirrors achieving uniform irradiance distributions on concentrating photovoltaic cells”, Applied Optics, vol. 56, no. 11, pp. 3035-3052, 2017.
Delaunay triangulation [173] is subsequently performed on \( p_{p1} \) \((i,j,k)\) to create the triangular surface elements needed for the surface optimization and ray tracing. For \( N_{p1} = 3 \) and \( N_{p1} = 6 \) the elements are equilateral, while for \( N_{p1} = 4 \) and \( N_{p1} > 6 \) they are isosceles.

\[
\text{Figure D.1. (a) Schematic showing the generation of a uniform grid on a regular polygon with apothem } a_{p1}, \text{ illustrated for the example of a hexagon } (N_{p1} = 6) \text{ and } k = 6 \text{ nodes along each basis vector (total number of nodes } N = 91). \text{ The red nodes are created by the basis } \{u-v\} \text{ in the first region. The remaining nodes are obtained by rotation of } \{u-v\} \text{ around the origin; (b) schematic showing the generation of an approximately uniform grid on a disk with radius } R_D \text{ and } k' = 6 \text{ nodes along } x > 0 \text{ (marked in red). The total number of nodes is } N = 95. \text{ Using Delaunay triangulation of the nodes, triangular equal-area elements can be generated.}
\]
Figure D.1(b) describes the procedure of creating \( N \) uniformly spaced nodes on a disk, as required for the design of circular mirrors. \( k' \) is the desired number of nodes evenly spaced along the positive \( x \)-axis within the disk \((y = 0, 0 < x < R_D; \) nodes marked in red). The radial distance between these nodes then is \( \Delta r = R_D/k' \). The remaining nodes are generated by placing \( N_i \) nodes on each circle with radius \( i \cdot \Delta r \) \((i = 1, \ldots, k' - 1)\) such that their distance in circumferential direction is closest to \( \Delta r \), i.e.

\[
N_i = \text{round}\left[\frac{2\pi (i \cdot \Delta r)}{\Delta r}\right] = \text{round}(2\pi \cdot i). \tag{D.4}
\]

The angular circumferential spacing on that circle then yields

\[
\Delta \varphi_i = \frac{2\pi}{N_i} = \frac{2\pi}{\text{round}(2\pi \cdot i)} \tag{D.5}
\]

and the nodes follow from

\[
p_D(i, j) = i \cdot \Delta r \left[ \begin{array}{c} \cos((j - 1) \cdot \Delta \varphi_i) \\ \sin((j - 1) \cdot \Delta \varphi_i) \end{array} \right], \tag{D.6}
\]

where \( j = 1, \ldots, N_i \). The total number of nodes in the grid then is

\[
N = 1 + \sum_{i=1}^{k'-1} N_i = 1 + \sum_{i=1}^{k'-1} \text{round}(2\pi \cdot i). \tag{D.7}
\]

D.2 Mirror surface optimization procedure

When presented to the task of optimizing a surface defined by a grid with \( N \) nodes, the straight-forward approach is to simultaneously solve Eq. (6.11) at each node \( x_{1,n} = [x_n; y_n; z_n] \), with the desired surface normal at that node \( \hat{n}_n = [\hat{x}_n; \hat{y}_n; \hat{z}_n] \), for \( z_n \). However, it can be challenging to numerically determine the tangent vector to the surface in each node \( n \) based on its neighbors, especially if the grid is irregular (where the number of neighbors is not constant). In fact, if a node has more than 3 neighbors, the resulting system of equations is not linear.

These limitations can be mitigated if the optimization is instead performed on the mid-nodes of the edges connecting two nodes. The edges can be determined by Delaunay triangulation of the nodes, which maximizes the minimum angle of each triangular element spanned by 3 neighboring nodes.
In reasonably uniform grids, such as the one s encountered in this work, this is equivalent to minimizing the length of the edges, i.e. connecting two grid nodes by the shortest possible edge. The edges resulting from Delaunay triangulation are indicated in Figure D.1.

Having determined the edges $e_m$, i.e. the sets $\{i,j\}_m$ where $x_{1,i} = [x_i; y_i; z_i]$ and $x_{1,j} = [x_j; y_j; z_j]$ are connected nodes and $m = 1, \ldots, M$, the approximated tangent vectors to the surface are readily obtained as

$$
t_{ij} = x_{1,j} - x_{1,i}, \quad (D.8)
$$

as illustrated in Figure D.2. The corresponding approximated normal vectors are

$$
\hat{n}_{ij} = \frac{\hat{n}_i + \hat{n}_j}{\|\hat{n}_i + \hat{n}_j\|}, \quad (D.9)
$$

For a large enough nodal resolution this approximation leads to accurate results. Each mid-node must then fulfill Eq. (6.11), i.e.

$$
t_{ij} \cdot \hat{n}_{ij} = 0 \iff \left( z_{n,i} + z_{n,j} \right) \left( z_j - z_i \right) = \ldots \left( x_{n,i} + x_{n,j} \right) \left( x_j - x_i \right) \left( y_{n,i} + y_{n,j} \right) \left( y_j - y_i \right), \quad (D.10)
$$
which can be expressed as a system of linear equations

\[ \mathbf{X} \cdot \mathbf{z} = \mathbf{b}, \]

with

\[
\begin{align*}
(\mathbf{X})_{m,n} &= (z_{\hat{n},i} + z_{\hat{n},j})(\delta_{nj} - \delta_{ni}) \\
(\mathbf{z})_n &= z_n \\
(\mathbf{b})_m &= (x_{\hat{n},i} + x_{\hat{n},j})(x_i - x_j) + (y_{\hat{n},i} + y_{\hat{n},j})(y_i - y_j)
\end{align*}
\]

where \( \delta_{ij} = \{ \begin{array}{ll} 0 & \text{if } i \neq j; \\ 1 & \text{if } i = j \end{array} \} \). The matrix \( \mathbf{X} \) \((M \times N)\) contains the \( z \)-coordinates of the desired normal vectors, the column vector \( \mathbf{z} \) \((N \times 1)\) contains the unknown \( z \)-coordinates of the nodes, and the column vector \( \mathbf{b} \) \((M \times 1)\) contains the \( x \)- and \( y \)-coordinates of the desired normal vectors and of the nodes. Since the system is overdetermined \((N < M)\), Eq. (D.11) can be reformulated and solved as a least squares problem

\[ \mathbf{z} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{b}. \] (D.12)

Importantly, it is not possible to write Eq. (D.10) as a system of linear equations if the tangent vectors are normalized (otherwise the unknown \( z_n \) would appear in \( \mathbf{X} \)). However, if the tangent vectors are of different length, more weight is given to longer vectors in the least mean square solution. It is therefore essential that the nodal grid on the primary mirror is as regular as possible, i.e. all edges have equal length.

### D.3 Grid size study for Monte-Carlo ray tracing with Nagata patches

Figure D.3 shows the influence of the number of nodes \( N \) on the fraction of the irradiance \( \gamma \) intercepted within the image area of an exemplary hexagonal concentrator with \( C_{g,\text{design}} = 500 \times, A_1 = 1 \) and \( f = 1 \) [Figure 6.4(b)]. It can be seen that a grid with \( N = 500 \) provides reasonably good results for \( \gamma \). The same holds true for other performance metrics, such as average irradiance and efficiencies. For producing smooth, high-resolution irradiance maps however, a larger number of elements \((N \approx 2500)\) is required and used throughout this work.
Before the question of the variation of the concentrator rim angle with $C_{g,\text{design}}$ can be addressed, it is necessary to define the rim angle of a polygonal concentrator. Due to the concentrator’s noncircular perimeter, a line connecting the focal point to a point on the concentrator rim forms a different angle with the optical axis depending on the location of this point. The maximum rim angle is attained in the concentrator corners, while the minimum is reached in the middle of its edges.

The minor rim angle $\Phi_{p1}$ can therefore be defined as the angle between the optical axis and the mirror edge in the $x$-$z$ plane ($y = 0$) of a polygonal parabolic concentrator and can be determined by solving on a parabolic dish with focal length $f$ and radius $a_{p1}$.

**Figure D.3.** Influence of number of nodes $n$ on the irradiance intercepted within the image area, for a hexagonal primary mirror with $C_{g,\text{design}} = 500\times$, $A_1 = 1$ and $f = 1$. $N = 500$ provides reasonably good results for performance metrics, while $N \approx 2500$ is required for smooth, high-resolution irradiance maps.
Similarly, the major rim angle $\Phi_{P1}^+$ in one of the corners of the polygon can be determined in a plane rotated by $\pi/N_{P1}$ around $z$, and follows from

$$\tan(\Phi_{P1}^-) = \frac{a_{P1}}{f - \frac{1}{4f}a_{P1}^2} = \frac{4a_{P1}f}{4f^2 - a_{P1}^2}. \quad \text{(D.13)}$$

$$\tan(\Phi_{P1}^+) = \frac{4R_{P1}f}{4f^2 - R_{P1}^2}. \quad \text{(D.14)}$$

For the same inlet aperture and the same focal length, $a_{P1} < R_D < R_{P1}$, and hence $\Phi_{P1}^- < \Phi_D < \Phi_{P1}^+$, where $\Phi_D$ is the rim angle of a disk concentrator of radius $R_D$. The difference between $\Phi_{P1}^+$ and $\Phi_{P1}^-$ decreases for increasing $N_{P1}$ of the polygon. Ultimately, in the limit case $N_{P1} \to \infty$ (equivalent to a disk concentrator), $a_{P1}$ and $R_{P1}$ converge to $R_D$ and $\Phi_{P1}^- = \Phi_D = \Phi_{P1}^+$. Table D.1 summarizes $\Phi_{P1}^-$ and $\Phi_{P1}^+$ for the tessellating concentrators ($N_{P1} = \{3, 4, 6\}$), having $A_1 = 1$ and $f = 1$.

The fact that $\Phi_{P1}^-$ and $\Phi_{P1}^+$ are determined on a parabolic dish in Eqs. (D.13) and (D.14), while the actual concentrator profile is slightly defocused, introduces offsets of the actual rim angles $\Phi_{P1,m}^-$ and $\Phi_{P1,m}^+$ measured on the mirror profile, $\epsilon^- = (\Phi_{P1,m}^- - \Phi_{P1,m}^-)/\Phi_{P1}^-$ and $\epsilon^+ = (\Phi_{P1,m}^+ - \Phi_{P1,m}^+)/\Phi_{P1}^+$. Table D.2 shows the actual rim angles of a hexagonal concentrator for $C_{g,\text{design}} = 100\times$, $500\times$ and $1000\times$. The offset is negligible (below 1%) for reasonably high $C_{g,\text{design}}$, such as those encountered in this work.

<table>
<thead>
<tr>
<th>$N_{P1}$ [-]</th>
<th>$\Phi_{P1}^-$ [°]</th>
<th>$\Phi_{P1}^+$ [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>24.74</td>
<td>48.74</td>
</tr>
<tr>
<td>3</td>
<td>28.07</td>
<td>38.94</td>
</tr>
<tr>
<td>4</td>
<td>30.07</td>
<td>34.47</td>
</tr>
<tr>
<td>$\to \infty$ (disk)</td>
<td>31.51</td>
<td>31.51</td>
</tr>
</tbody>
</table>

Table D.1. Major and minor rim angles for different polygonal concentrators. For increasing $N_{P1}$, $\Phi_{P1}^- < \Phi_D$ and $\Phi_{P1}^+ > \Phi_D$ converge towards $\Phi_D$. The offsets $\epsilon^-$ and $\epsilon^+$ are negligible (below 1%) for $C_{g,\text{design}} = 100\times$, $500\times$ and $1000\times$. Table D.2 shows the actual rim angles of a hexagonal concentrator for $C_{g,\text{design}} = 100\times$, $500\times$ and $1000\times$. The offset is negligible (below 1%) for reasonably high $C_{g,\text{design}}$, such as those encountered in this work.
Table D.2. Offset of actual rim angles on a hexagonal concentrator compared to the parabolic dish.

<table>
<thead>
<tr>
<th></th>
<th>$\Phi_{P1}^-$ [°]</th>
<th>$\varepsilon^-$ [%]</th>
<th>$\Phi_{P1}^+$ [°]</th>
<th>$\varepsilon^+$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>on parabola</td>
<td>30.07</td>
<td>-</td>
<td>34.47</td>
<td>-</td>
</tr>
<tr>
<td>$C_{g,design} = 100\times$</td>
<td>29.90</td>
<td>0.58</td>
<td>34.19</td>
<td>0.82</td>
</tr>
<tr>
<td>$C_{g,design} = 500\times$</td>
<td>30.00</td>
<td>0.26</td>
<td>34.34</td>
<td>0.36</td>
</tr>
<tr>
<td>$C_{g,design} = 1000\times$</td>
<td>30.02</td>
<td>0.18</td>
<td>34.38</td>
<td>0.26</td>
</tr>
</tbody>
</table>
References


[61] G. Mittelman, a Kribus, and a Dayan, “Solar cooling with concentrating


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