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Neural Tree for Estimating the Uniaxial Compressive Strength of Rock Materials

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Abstract. Uniaxial Compressive Strength (UCS) is the most important parameter that quantifies the rock strength. However, determination of the UCS in laboratory is very expensive and time-consuming. Therefore, common **index tests** like point load (Is-50), ultrasonic velocity test (V_p), block punch index (BPI) test, rebound hardness (SRH) test, physical properties have been used to predict the UCS. The objective of this work is to develop a predictive model using a **neural tree** predictor that estimates the UCS with high accuracy and assess the effectiveness of different index tests in predicting the UCS of rock materials. UCS and indices such as BPI, Is-50, SRH, V_p, effective porosity and density were determined for the granite, schist, and sandstone. The constructed model predicted the UCS with a high accuracy and in a quick time (9 seconds). Additionally, the destructive mechanical rock indices BPI and Is-50 proved to be the best index tests to estimate the UCS.

Keywords: Uniaxial compressive strength; index tests; rock materials; heterogeneous flexible neural tree; feature analysis

1 Introduction

Uniaxial compressive strength (UCS) is the most familiar parameter used in rock engineering projects. In *rock mass rating*, proposed by Bieniawski [1], UCS is the only parameter to assess the strength of rock material. However, high quality machined specimens are required for determining UCS in the laboratory [2]. This makes the test expensive and time-consuming. Therefore, different empirical predictive models are being used to estimate the UCS of rock materials indirectly from the *index tests*, which require little or no specimen preparation and are less expensive than the uniaxial compression test. Amongst different predictive models, regression analysis is the most commonly used. Lists of such relationships between the index tests and UCS obtained by different researchers have been given in [3].

In the last decade, however, the use of computational intelligence (CI) methods to establish predictive models has gained much interest in the areas of rock mechanics. A comprehensive list of such works is presented in Table 1. These research works indicate the requirement of improvement in the CI methods to estimate UCS from the index test results and to assess the effectiveness of individual index tests for predicting the UCS of rock materials.

Table 1. Previous studies on the use of computational intelligence (CI) techniques in estimating UCS and other rock engineering parameters from index tests.

Ref.	Rock types	Inputs	Output/s	CI model/s	r/r^2
[3]	Granite; Schist; Sandstone	BPI; Is-50; SRH; V_p	UCS	FIS (M)	$r = 0.99$
[4]	Sandstone; Granodiorite	Hardness; Density; Porosity; Grain size	UCS	NN	$r = 0.96$
[5]	Greywacke	V_p; Is-50; BPI; TS	UCS; EM	FIS (M)	$r = 0.8$
[6]	Greywacke; Agglomerate	UCS; Unit weight	EM	NN	$r = 0.82$
[7]	Sandstone	Packing Density; Concavity-convexity; Quartz %	UCS	NN	$r = 0.87$
[8]	Granite	Porosity; V_p; UCS	WG	NN; FIS	$r = 0.96$; $r = 0.93$
[9]	Gypsum	Water content; Is-50; V_p	UCS; EM	NN; ANFIS	$r^2 = 0.88$; $r^2 = 0.94$
[10]	Travertine	V_p; Is-50; SRH; porosity	UCS; EM	NN	$r^2 = 0.64$
[11]	Sandstone; Shale; Limestone	Porosity; Bulk density; Water saturation	UCS	NN	$r = 0.91$
[12]	Granite	TS; BPI; Is(50); V_p	UCS	NN; ANFIS	$r = 0.6$; $r = 0.69$
[13]	Granite	Quartz %; Plagioclase %; Orthoclase %	UCS	ANFIS	$r = 0.87$
[14]	Shale	Dry density; Is-50; BTS; V_p; SRH	TFA	PSO–NN	$r = 0.94$
[15]	Granite	Dry density, V_p; Qtz content; Plg content	UCS; EM	NN	$r = 0.91$
[16]	Granite; Schist; Sandstone	BPI; Is-50; SRH; V_p	UCS	FIS (TSK); NN	$r = 0.98$ $r = 0.94$

Note: FIS - Fuzzy Inference System; ANFIS - Adoptive Neuro-Fuzzy Inference System; M - Mamdani, TSK – Takagi-Sugeno-Kang; PSO Particle Swamp Optimization; BPI - Block Punch Index; Is-50 - Pointload Strength; SRH - Rebound Hardness; V_p - Ultrasonic P-wave Velocity; TS -Tensile Strength; EM - Elastic Modulus; WG -Weathering Grade; TFA - Internal Friction Angle.

Although several predictive models have been proposed to estimate the UCS from index tests, the influence of various index tests in predicting UCS is not entirely understood. More specifically, so far, to the best of our literature survey, no study has been done on assessing the effectiveness of different index tests. Moreover, it is always advantageous if development time and effort of the predictive modeling can be reduced. To achieve these goals, the use of the neural network

(NN) or similar CI methods on a limited set of experimental data for predictive modeling can be a great benefit. Hence, the aim of this work is to develop a predictive model that can estimate the UCS with a high accuracy and assess the effectiveness of different index tests in predicting UCS of rock materials.

The multiobjective heterogeneous flexible neural tree (HFNT$^{\mathrm{M}}$) produces a tree-like model [17], in which the nodes of the tree are similar to the NN nodes was used in this study. Interestingly, HFNT$^{\mathrm{M}}$ differs from NN [18] in its structural configuration, and it differs from the commonly used regression tree [19] in its node type. Moreover, the tree-like structure in the HFNT$^{\mathrm{M}}$ is created by using multiobjective genetic programming (MOGP) [17,20]. Therefore, the primary advantages of using HFNT$^{\mathrm{M}}$ over other CI techniques lie in its ability of the automatic adaptation into the structure and the input feature selection.

Such an ability like HFNT$^{\mathrm{M}}$ is necessary and has been successfully used in several real-life applications [21]. However, this method has not been exploited in estimating the UCS from index tests. In this work, index tests namely block punch index (BPI), point load strength (Is-50), Schmidt rebound hardness (SRH), ultrasonic P-wave velocity ($\mathrm{V_p}$), and physical properties namely effective porosity (η_e) and density (ρ) are determined and used for estimating the UCS of granite, schist, and sandstone. An improved CI technique, HFNT$^{\mathrm{M}}$, was employed for this purpose and it was used to determine the effectiveness of index tests BPI, Is-50, SRH, $\mathrm{V_p}$, η_e, and ρ in the prediction of UCS of three rocks. Such kind of individual test *effectiveness assessment* using HFNT$^{\mathrm{M}}$ has not been studied in the past, and it is the noble contribution of this research along with the predictive modeling. The evaluation of individual tests provides a detailed insight of UCS prediction of rock materials.

2 Materials and Methodology

Three completely different rock types, granite, schist, and sandstone were consciously investigated in the laboratory to capture a broad scenario in evaluating UCS. Core samples of granite, schist, and sandstone were collected from *Malanjkhand* Copper Project, *Malanjkhand*, India; UCIL mine at *Jaduguda*, India; and SCCL, *Kothagudem*, India respectively. Each core sample (20 from each rock type) was cut into four pieces as required for uniaxial compression, point load, block punch, and Schmidt rebound hardness tests (as mentioned in [2]). Results of the entire laboratory investigation were taken from [16]. During the point load and block punch tests, few granite and sandstone specimens failed in invalid failure modes. For these specimens, BPI and Is-50 were indirectly calculated using simple regression equations (developed from same rock types of the same locality as this study) presented in [3]. A total *60 samples* (20 from each rock types) were used for the predictive modeling.

3 Predictive Models

3.1 Computational Intelligence Techniques

Discovering knowledge contained in data and developing predictive models are vital tasks performed by the CI methods. Moreover, predictive modeling identifies the underlying relationship between an input variable $\mathbf{X} = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N\}$ and an output variable $\mathbf{d} = \{d_1, d_2, \ldots, d_N\}$ through the learning parameter $\mathbf{w}$, which defines the said relationship. A CI method finds the learning parameter w by usually reducing the root mean square error (RMSE) e as per Eq. (1) between the predicted output $\mathbf{y} = \{y_1, y_2, \ldots, y_N\}$ and the desired output $\mathbf{d}$. Hence, the learning of a CI method indicate the search for a proper learning parameter $\mathbf{w}$.

In this study, the RMSE e was evaluated as:

$$e = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i - d_i)^2}, \tag{1}$$

where N denotes the total training/test samples. Additionally, correlation coefficient r as per Eq. (2) was also used in this study. The correlation coefficient r computes the correlation between the predicted output $\mathbf{y}$ and the desired output $\mathbf{d}$, which is written as:

$$r = \frac{\sum_{i=1}^{N}(y_i - \bar{\mathbf{y}})(d_i - \bar{\mathbf{d}})}{\sum_{i=1}^{N}(y_i - \bar{\mathbf{y}})^2 \sum_{i=1}^{N}(d_i - \bar{\mathbf{d}})^2} \tag{2}$$

where $\bar{\mathbf{y}}$ and $\bar{\mathbf{d}}$ are the average of predicted and desired outputs. The correlation coefficient ranges from -1 to 1, where a value 1 indicates the best performance.

In this study, CI methods such as fuzzy inference system (FIS), multilayer perceptron (MLP), and adaptive neuro-fuzzy inference system (ANFIS) were considered apart from the proposed HFNT$^{\mathrm{M}}$. The FIS, MLP, and ANFIS are most widely used CI methods for modeling industrial and engineering problems [22]. The brief introduction to the mentioned CI methods are as follow:

- An FIS is a rule-based system, where a set of IF-THEN rules are designed from the given input variables. Subsequently, the rules are inferred (conclusion drawn from the rules set) to predict the output of the system for any given input [23].
- An MLP is an NN model, which is an imitation of human-like learning. An MLP is a layered network of neural nodes (computational node) arranged in a layered structure. The nodes in an MLP are connected with synaptic links. The primary form of MLP training for a given dataset is the discovery of appropriate values for the synaptic connections [18].
- An ANFIS is a combination both FIS and NN-based systems, where an FIS is designed as an NN-like model. ANFIS is typically a six-layered system in which the layers indicate the IF and THEN parts of an FIS [24].

3.2 Multiobjective Heterogeneous Flexible Neural Tree (HFNT$^{\mathrm{M}}$)

An HFNT$^{\mathrm{M}}$ similar to any other CI methods often tries to minimize the error e given in Eq. 1 for the given dataset by optimizing its parameter using some learning algorithms (HFNT$^{\mathrm{M}}$ employ multiobjective genetic programming and differential evolution for this purpose). The structure of HFNT$^{\mathrm{M}}$ is a tree-like that has internal nodes as the computational nodes (analogous to MLP neurons), branches (similar to MLP synaptic links), and leaf nodes (to represent input variables).

Mathematically, an HFNT$^{\mathrm{M}}$, denoted as G, is a union of internal node V and the leaf node T. The internal node V is a set of internal (computational) nodes and, the leaf node T is a set of inputs [17]. Hence, an HFNT$^{\mathrm{M}}$ G can be expressed as:

$$G = V \cup T = \{v_2^{U(k)}, v_3^{U(k)}, \ldots, v_{tn}^{U(k)}\} \cup \{x_1, x_2, \ldots, x_d\} \tag{3}$$

where $v_i^{U(k)}$ $(i \in \{2, 3, \ldots, tn\})$ indicates an internal (computational) node that takes two or more arguments; whereas, the leaf node takes no argument. The function $U(k)$ randomly invoke an activation function at a computational node from a set of activation function: {Gaussian, tangent hyperbolic, bipolar sigmoidal, unipolar sigmoidal, and Fermi}. HFNT$^{\mathrm{M}}$ training has two aspects. Firstly, discovering appropriate tree structure and secondly, the optimization of tree parameters. These two HFNT$^{\mathrm{M}}$ training parts are performed in two phases:

- Phase 1: Tree structure training using multiobjective genetic programming.
- Phase 2: Tree parameter training using differential evolution (DE) [25].

A detailed description of the aforementioned two phases are described in [17].

3.3 Input Feature Analysis

Feature analysis was conducted to assess the effectiveness of different index tests in predicting UCS of rock materials. To perform such feature analysis, 20 HFNT$^{\mathrm{M}}$ models were created. Each HFNT$^{\mathrm{M}}$ models provided information selected inputs and models prediction strength (in terms of RMSE). Hence, a list of 20 models with the account of their chosen inputs and RMSE was prepared.

To analyze the selected input features and their predictability, two performance indicators feature selection rate R and predictability score P were used [21]. Feature selection rate R is the measure of the total number of times a specific set of input feature was occurred in the prepared list $M = [m_1, \ldots, m_{20}]$ of 20 models. Here, $|M|$ indicating the size of the list was 20. Therefore, the input-feature selection rate is computed as:

$$R_j = \frac{1}{|M|} \sum_{i=1}^{|M|} \mathbb{I}(m_i = \mathbf{A}_j) \tag{4}$$

where R_j is the selection rate of j-th input feature set $\mathbf{A}_j \in \mathcal{P}(\{$BPI, Is-50, SRH, V_p, η_e, ρ $\})$, and function $\mathbb{I}(m_i = \mathbf{A}_j)$ is a function that returns "1" if j-th input-feature set $\mathbf{A}_j$ is selected by the i-th model m_i, otherwise, it returns "0." Feature selection rate $R_j = 1$ is the highest (i.e., all the models selected the input-feature set ($\mathbf{A}_j$) and $R_j = 0$ is the lowest (i.e., no model selected the input-feature set $\mathbf{A}_j$).

The predictability score P_j of an input feature set $\mathbf{A}_j$ is on the other hand is necessary to determine along with the selection rate R_j because the models in the list may not be equal in their performances. To determine the predictability score P_j of j-th input-feature set $\mathbf{A}_j$, the performance F_j (typically the RMSE) of the corresponding input-feature set $\mathbf{A}_j$ was at first computed as per Eq. (5):

$$F_j = \begin{cases} \sum_{i=1}^{|M|} e_i \times \mathbb{I}(m_i = \mathbf{A}_j) & \text{if} |\mathbf{A}_j| = 1 \\ \sum_{i=1}^{|M|} e_i \times \mathbb{I}(m_i = \mathbf{A}_j) / \sum_{i=1}^{|M|} \mathbb{I}(m_i = \mathbf{A}_j) & \text{if} |\mathbf{A}_j| > 1 \end{cases} \tag{5}$$

where e_i indicates the RMSE of i-th model. The performance F_j for $|\mathbf{A}_j| = 1$ is the sum of RMSEs and F_j for $|\mathbf{A}_j| > 1$ is the average RMSEs of all models that selected a subset $\mathbf{A}_j$. Accordingly, the predictability score P_j corresponding to an input-feature set $\mathbf{A}_j$ was computed by normalizing the performance as [21]:

$$P_j = \frac{F_j}{\max_{j=1 \text{ to } z}(F_j)} \tag{6}$$

where function max(.) evaluate the maximum performance value among all F_j. Analogous to the selection rate R_j, the predictability score $P_j = 1$ for an input-feature set $\mathbf{A}_j$ describes the heights impact on the model's predictability and the score $P_j = 0$ describes the least impact on the model's predictability.

4 Result and Discussion

4.1 Models Prediction

The developed best HFNT$^{\mathrm{M}}$ model using the parameter setting mentioned in [17] and two-fold training is shown in Fig. 1, where the leaf nodes indicate the input features and the root node gives the predicted output UCS of the model. UCS values estimated from the model are plotted against their corresponding experimentally determined UCS values both for training and test data (Fig. 2). The predictive performance of the developed model was examined through e, and r computed as per Eq. 1 and Eq. 2 and *complexity* χ (total number of parameters in the developed models). The e, r, r^2, and χ obtained in this study was compared with the e, r, r^2, and χ obtained from FIS (TSK), MLP and ANFIS models as presented in [16] for the same dataset and same setting of training and test set-up. The obtained results are presented in Table 2.

From the Fig. 2 and Table 2, it can be said that correlation coefficient r of the developed HFNT$^{\mathrm{M}}$ model is similar to that of the FIS (TSK), MLP and ANFIS models. However, based on the RMSE of test data, it can be said that the

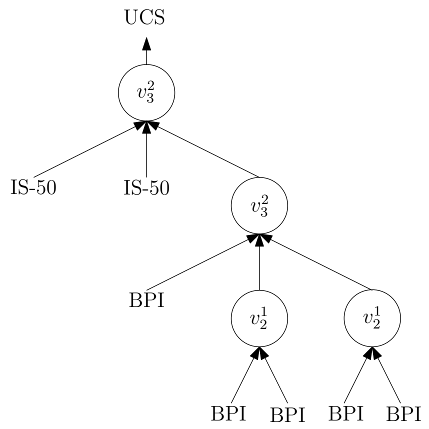

Fig. 1. Developed multiobjective heterogeneous flexible neural tree.

HFNT$^{\mathrm{M}}$ model is better than other models. Besides the statistical parameters, *computational time* of the HFNT$^{\mathrm{M}}$ model was 9 seconds, and the complexity was 11 parameters for such a wide range of data obtained for three completely different rock types. Considering all these factors, RMSE, r, r^2, and computation time, it can be said the HFNT$^{\mathrm{M}}$ is a very effective tool for determining the UCS from index tests. Additionally, the HFNT$^{\mathrm{M}}$ is a less complex model than the other CI model. As shown in Table 2, the total parameters in the HFNT$^{\mathrm{M}}$ were smaller than the other CI models. Hence, HFNT$^{\mathrm{M}}$ is a better model in both implementation and computational points of view.

Table 2. Performance of the models for the selected training and test datasets.

	FIS[a]	MLP[a]		ANFIS[a]		HFNT$^{\mathrm{M}}$	
		Train	Test	Train	Test	Train[c]	Test[c]
e	9.54	14.33	16.9	7.87	13.72	10.92	5.87
r	0.98	0.96	0.94	0.99	0.97	0.98	0.99
r^2	0.97	0.92	0.89	0.98	0.94	0.97	0.99
χ	5800[b]	25		44[b]		11	

Note: [a]results of FIS, ANN, and ANFIS were taken from [16]; [b]approximate calculation; [c]results of the proposed method.

4.2 Feature Analysis Results

A total 20 models were created using HFNT$^{\mathrm{M}}$ for feature analysis. Since the models were created by employing an evolutionary algorithm, the inputs that contributed most towards UCS prediction were primarily selected. Hence, a list

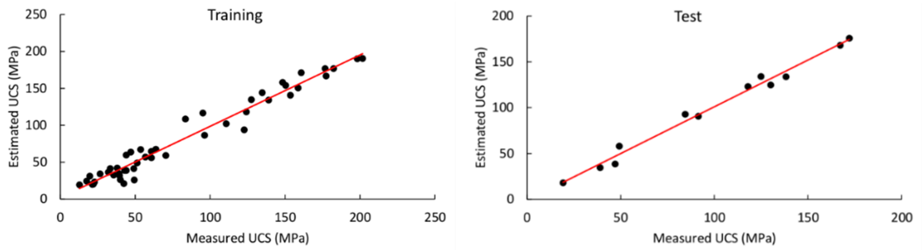

Fig. 2. Estimated UCS plotted against measured UCS: training (left) and test (right).

of selected inputs by each model was prepared with the account of their RMSEs. Afterward, a comprehensive feature analysis of all 6 input features was performed. For the input feature analysis, two performance indicators, feature selection rate R as defined in Eq. (4) and feature predictability score P as defined in Eq. (5), were adopted.

The effectiveness of individual index tests BPI, Is-50, SRH, $\mathrm{V_p}$, η_e and ρ in estimating the UCS were examined through R and P and setting $|\mathbf{A}_j| = 1$. Table 3 presents the results of feature analysis carried out for the all 6 individual inputs. The input features BPI and Is-50 represent the *destructive mechanical indices*; whereas, SRH and $\mathrm{V_p}$ represent the *non-destructive rock indices*. Effective porosity (η_e) and density (ρ) are the determined physical properties of the concerned rocks. It can be observed that R and P of destructive mechanical indices BPI and Is-50 were much higher than that of R and P values of non-destructive and physical rock indices.

Table 3. Significance of individual input features.

#	Input Features	Selection Rate (R)	Predictability Score (P)
1	$\mathbf{A}_1 = \{\mathrm{BPI}\}$	0.862	0.899
2	$\mathbf{A}_2 = \{\text{Is-50}\}$	0.959	1
3	$\mathbf{A}_3 = \{\mathrm{SRH}\}$	0.525	0.547
4	$\mathbf{A}_4 = \{\mathrm{V_p}\}$	0.147	0.154
5	$\mathbf{A}_5 = \{\eta_e\}$	0.339	0.354
6	$\mathbf{A}_6 = \{\rho\}$	0.574	0.599

Among the individual input features, R and P of Is-50 are 0.959 and 1 respectively. Next in the list is BPI with R and P of 0.862 and 0.899 respectively. From the performed feature analysis, it can be said that Is-50 is the best index to predict the UCS of rock materials. Since BPI also has significantly high R and P, we can say that destructive mechanical rock indices are the best proxy for estimating the UCS of the rock material. This finding is also in accordance with the experimental findings of [26].

5 Conclusions

The experimental program included the determination UCS, BPI, Is-50, SRH, V_p, porosity and density of three different rock types (granite, schist, and sandstone). To estimate the UCS from the index tests, a multiobjective heterogeneous flexible neural tree ($HFNT^M$) model was proposed. The inputs and outputs for the developed $HFNT^M$ model were used from the rock materials experimental results. The constructed model efficiently estimated the UCS based on the information gathered from the experimental data in a very quick time (9 seconds). Additionally, the $HFNT^M$ was a less complex model than the other CI model. Therefore, this model can be effectively used in estimating the UCS. Developed $HFNT^M$ also assess the effectiveness of different index tests in predicting UCS of rock materials with the help of feature selection rate R and predictability score P. From R and P values of the individual input features (index test results), it is found that Is-50 is the best proxy for the UCS. Among the different types of index tests—destructive indices, non-destructive indices, and physical properties—the destructive mechanical rock indices BPI and Is-50 are found to be the best index tests to estimate the UCS.

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