Doctoral Thesis

Transport Experiments Towards the Quanum Spin Hall Effect in InAs/GaSb

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Transport experiments towards the quantum spin Hall effect in InAs/GaSb

A thesis submitted to attain the degree of DOCTOR OF SCIENCES of ETH ZURICH

(Dr. sc. ETH Zurich)

presented by

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Abstract

A suitable layer combination of indium arsenide (InAs) and gallium antimonide (GaSb) has been proposed as a candidate for a two-dimensional (2D) topological insulator. This implies the existence of perfectly conducting (ballistic), counter-propagating and spin-polarized edge states in combination with an insulating bulk. These signatures of the quantum spin Hall effect are absent in a conventional insulator, suggesting that topological insulators represent a novel state of matter. Magnetoconductance measurements at cryogenic temperatures serve to test the transport properties of InAs/GaSb and thus to inquire about the nature of its insulating bulk phase. The aim of this work is to study different ingredients necessary to build a 2D topological insulator based on InAs/GaSb and to investigate the characteristic edge states. Single InAs quantum wells with normal (conventional) band alignment are taken as a reference to the inverted (topological) InAs/GaSb system.

First, we investigate the importance of the spin-orbit interaction. In particular, the Rashba contribution leads to gate tunable spin-splitting in the electron regime of standard InAs/GaSb and InAs Hall bars. The splitting properly normalized by considering the densities of the two spin-split subbands is independent of device size, can be changed by a factor of three and allows to deduce the electric field present in these heterostructures. The reduced confinement due to the additional GaSb quantum well decreases the electric field by a factor of two compared to pure InAs quantum wells. These results could verify theoretical predictions about the electric field.

An insulating phase requires an energy gap in-between the conduction and the valence band. Strain is known to effect the band structure in other materials. We implement strain as a tunable parameter in Hall bar devices along the crystallographic orientations [011] and [001] by mounting them on top of piezo stacks. Our measurements not only demonstrate strain causing a transition from an insulator to semimetal but also its effect on transport properties. For instance, strain along the [011] direction of the zinc-blende unit cell results in a density modulation, which is absent for Hall bars along [001]. The high symmetry axis makes it less susceptible to symmetry breaking strain. Tight-binding calculations are consistent with these findings highlighting that strain in InAs caused by the epitaxial growth on GaSb substantially influences the band structure. We conclude that strain may be crucial in optimizing the bulk insulator.
A finite bulk conduction in inverted InAs/GaSb systems often masks edge transport. Optimizing the amount of impurities leads to the desired result of increased bulk resistance. We confirm edge conduction in mesoscopic InAs/GaSb Hall bars with nonlocal measurements together with pronounced plateau resistances around charge-neutrality. Plateau values above and below the expected quantization can be explained with the device size and the remaining bulk conduction one order of magnitude lower than the edge contribution, respectively. The plateau resistances scale according to the expectations for helical edge states independent of edge segment lengths suggesting ballistic transport properties. These results are in contrast to InAs devices of the same geometry, showing in a consistent picture with Corbino and finger gate samples diffusive edges of 1-2 kΩ/µm. The topological insulator candidate InAs/GaSb with the InAs quantum well as an inherent building block is suspected to carry these diffusive edge states as well. This is supported additionally by diffusive edges of the same resistivity found in InAs/GaSb finger gate samples. We discuss this discrepancy critically with the help of an edge state anisotropy determined by tilted magnetic field experiments, and develop ideas of how the different edge current contributions could be disentangled in future device concepts.
Zusammenfassung

Eine geeignete Schichtkombination von Indiumarsenid (InAs) und Galliumantimonid (GaSb) wurde als Kandidat für einen zwei-dimensionalen (2D) topologischen Isolator vorgeschlagen. Das impliziert die Präsenz von perfekt leitenden (ballistischen), entgegengesetzt und spin-polarisierten Randkanälen in Kombination mit einem isolierendem Volumen. Diese Merkmale des Quanten Spin Hall Effekts kommen bei einem konventionellen Isolator nicht vor, was auf die Entdeckung eines neuen Zustands der Materie deutet. Leitfähigkeitsmessungen im Einfluss von Magnetfeldern bei kryogenen Temperaturen ermöglichen das Testen von Transporteigenschaften von InAs/GaSb und somit das Erforschen der Art der isolierenden Volumenphase. Die Absicht dieser Arbeit ist das Studieren unterschiedlichster Bestandteile, die es für einen 2D topologischen Isolator basierend auf InAs/GaSb braucht und dessen charakteristische Randleitfähigkeit zu untersuchen. Einzelne InAs Quantentöpfe mit normaler (konventioneller) Bandreihenfolge werden als Referenz zu den invertierten (topologischen) InAs/GaSb Systemen angeschaut.

Als erstes untersuchen wir die Bedeutung der starken Spin-Bahn Wechselwirkung. Speziell der Einfluss des Rashba Effekts führt zu Gatter-abhängiger Spinauflösung von Elektronen in gebräuchlichen InAs/GaSb und InAs Hallbars. Die Auflösung der beiden Spinzustände normiert durch die Messung der Besetzung der jeweiligen Spin-Subbänder ist unabhängig von der Probengröße, kann um einen Faktor drei modifiziert werden und erlaubt auf das elektrische Feld in der Heterostruktur zu schliessen. Die reduzierte Einschränkung durch den zusätzlichen GaSb Quantumtopf reduziert das elektrische Feld um einen Faktor zwei gegenüber reinen InAs Proben. Diese Resultate können zur Überprüfung theoretischer Vorhersagen des elektrischen Feldes nützlich sein.

entlang [001]. Die Hochsymmetriearchse macht sie weniger empfindlich für Symmetriebrechende Dehnung. Tight-Binding Berechnungen sind konsistent mit diesen Erkenntnissen und heben hervor, dass die Dehnung in InAs verursacht durch das epitaxiale Wachstum auf GaSb bereits erheblichen Einfluss auf die Bandstruktur hat. Wir kommen zum Schluss, dass Dehnung entscheidend sein kann beim Optimieren des Volumenisolators.

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<td>ac</td>
<td>alternating current</td>
</tr>
<tr>
<td>BIA</td>
<td>bulk inversion asymmetry</td>
</tr>
<tr>
<td>dc</td>
<td>direct current</td>
</tr>
<tr>
<td>DOS</td>
<td>density of states</td>
</tr>
<tr>
<td>CB</td>
<td>conduction band</td>
</tr>
<tr>
<td>CNP</td>
<td>charge-neutrality point</td>
</tr>
<tr>
<td>HH</td>
<td>heavy hole</td>
</tr>
<tr>
<td>HSE</td>
<td>Heyd-Scuseria-Ernzerhof</td>
</tr>
<tr>
<td>LH</td>
<td>light hole</td>
</tr>
<tr>
<td>L-R config.</td>
<td>local resistance configuration</td>
</tr>
<tr>
<td>MBE</td>
<td>molecular beam epitaxy</td>
</tr>
<tr>
<td>NI</td>
<td>normal insulator</td>
</tr>
<tr>
<td>NL-R config.</td>
<td>nonlocal resistance configuration</td>
</tr>
<tr>
<td>PAW</td>
<td>projector augmented waves</td>
</tr>
<tr>
<td>PBE</td>
<td>Perdew-Burke-Ernzerhof</td>
</tr>
<tr>
<td>QHE</td>
<td>quantum Hall effect</td>
</tr>
<tr>
<td>QSHE</td>
<td>quantum spin Hall effect</td>
</tr>
<tr>
<td>QW</td>
<td>quantum well</td>
</tr>
<tr>
<td>SdH</td>
<td>Shubnikov de-Haas</td>
</tr>
<tr>
<td>SIA</td>
<td>structure inversion asymmetry</td>
</tr>
<tr>
<td>SM</td>
<td>semi metal</td>
</tr>
<tr>
<td>SOC</td>
<td>spin-orbit coupling</td>
</tr>
<tr>
<td>TB</td>
<td>tight-binding</td>
</tr>
<tr>
<td>TI</td>
<td>topological insulator</td>
</tr>
<tr>
<td>VB</td>
<td>valence band</td>
</tr>
<tr>
<td>XRD</td>
<td>x-ray diffraction</td>
</tr>
<tr>
<td>2D</td>
<td>two dimensional</td>
</tr>
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<td>3D</td>
<td>three dimensional</td>
</tr>
<tr>
<td>2DEG</td>
<td>two dimensional electron gas</td>
</tr>
<tr>
<td>2DHG</td>
<td>two dimensional hole gas</td>
</tr>
<tr>
<td>Symbol</td>
<td>Explanation</td>
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<td>-------------</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Rashba material coefficient</td>
</tr>
<tr>
<td>$\alpha_R$</td>
<td>Rashba coefficient</td>
</tr>
<tr>
<td>$a, a_1, a_2$</td>
<td>lattice constants</td>
</tr>
<tr>
<td>$\vec{a}_i, \vec{a}_i^s$</td>
<td>basis vectors without and with strain</td>
</tr>
<tr>
<td>$A_j$</td>
<td>areas</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Dresselhaus material coefficient</td>
</tr>
<tr>
<td>$\beta_D$</td>
<td>Dresselhaus coefficient</td>
</tr>
<tr>
<td>$B$</td>
<td>magnetic field</td>
</tr>
<tr>
<td>$B_\parallel$</td>
<td>parallel magnetic field</td>
</tr>
<tr>
<td>$B_\perp$</td>
<td>perpendicular magnetic field</td>
</tr>
<tr>
<td>$C$</td>
<td>capacitance</td>
</tr>
<tr>
<td>$C_{ijkl}$</td>
<td>stiffness tensor</td>
</tr>
<tr>
<td>$C_{\text{zinc-blende}}$</td>
<td>stiffness tensor for zinc-blende materials</td>
</tr>
<tr>
<td>$d_{33}$</td>
<td>piezoelectric coefficient</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>hybridization gap</td>
</tr>
<tr>
<td>$\Delta \varepsilon$</td>
<td>strain range</td>
</tr>
<tr>
<td>$\Delta k$</td>
<td>momentum shift</td>
</tr>
<tr>
<td>$\Delta n$</td>
<td>density splitting</td>
</tr>
<tr>
<td>$\Delta n_e, \Delta n_h$</td>
<td>electron resp. hole density change</td>
</tr>
<tr>
<td>$\Delta \rho_{xx}$</td>
<td>amplitude decay of the SdH oscillations</td>
</tr>
<tr>
<td>$\Delta V$</td>
<td>hysteresis</td>
</tr>
<tr>
<td>$\Delta V_{\text{piezo}}$</td>
<td>change in piezo voltage</td>
</tr>
<tr>
<td>$-e &lt; 0$</td>
<td>electron charge</td>
</tr>
<tr>
<td>$E$</td>
<td>energy</td>
</tr>
<tr>
<td>$E_F$</td>
<td>Fermi energy</td>
</tr>
<tr>
<td>$E_{g_0}$</td>
<td>band overlap</td>
</tr>
<tr>
<td>$E_{g_1}$</td>
<td>energy difference between CNP and conduction band minima</td>
</tr>
<tr>
<td>$E_{g_2}$</td>
<td>energy difference between CNP and valence band maxima</td>
</tr>
<tr>
<td>$E_z$</td>
<td>electric field in growth direction</td>
</tr>
<tr>
<td>$\langle E_{z_{\text{ext}}} \rangle$</td>
<td>external electric field</td>
</tr>
<tr>
<td>$E_1$</td>
<td>first electron sub-band</td>
</tr>
<tr>
<td>$E_\uparrow, E_\downarrow$ or $E_1, E_2$</td>
<td>energy of the resp. spin state</td>
</tr>
<tr>
<td>$\eta_i$</td>
<td>reflected edge channels</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>dielectric permittivity</td>
</tr>
<tr>
<td>$\epsilon_0$</td>
<td>vacuum dielectric constant</td>
</tr>
<tr>
<td>$\epsilon, \epsilon_{\text{InAs}}, \epsilon_{\text{GaSb}}$</td>
<td>internal or external strain, strain in the InAs or GaSb QW</td>
</tr>
<tr>
<td>$\epsilon_{kl}$</td>
<td>strain vector</td>
</tr>
<tr>
<td>$F, F_i$</td>
<td>forces</td>
</tr>
<tr>
<td>$f, f_i$</td>
<td>frequencies</td>
</tr>
<tr>
<td>$G$</td>
<td>conductance matrix</td>
</tr>
<tr>
<td>Symbol</td>
<td>Explanation</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$h = h/2\pi$</td>
<td>reduced Planck's constant</td>
</tr>
<tr>
<td>$H$</td>
<td>Hamiltonian</td>
</tr>
<tr>
<td>$H_D$</td>
<td>Dresselhaus contribution to Hamiltonian</td>
</tr>
<tr>
<td>$H_{\text{kin}}$</td>
<td>kinetic energy term in Hamiltonian</td>
</tr>
<tr>
<td>$H_R$</td>
<td>Rashba contribution to Hamiltonian</td>
</tr>
<tr>
<td>$H_{\text{SO}}$</td>
<td>spin-orbit term in Hamiltonian</td>
</tr>
<tr>
<td>$H_0$</td>
<td>Hamiltonian of a 2DEG</td>
</tr>
<tr>
<td>$H_{1}$</td>
<td>first hole sub-band</td>
</tr>
<tr>
<td>$I$</td>
<td>current</td>
</tr>
<tr>
<td>$I_{\text{bulk}}$</td>
<td>bulk current</td>
</tr>
<tr>
<td>$I_{\text{edge}}$</td>
<td>edge current</td>
</tr>
<tr>
<td>$I_i$</td>
<td>current through lead $i$ or edge segment $i$</td>
</tr>
<tr>
<td>$I_{\text{noise}}$</td>
<td>current noise</td>
</tr>
<tr>
<td>$\vec{k}, k_x, k_y, k_z, k_\parallel$</td>
<td>wave vectors</td>
</tr>
<tr>
<td>$k_B$</td>
<td>Boltzmann constant</td>
</tr>
<tr>
<td>$\kappa_i$</td>
<td>incoming edge channels</td>
</tr>
<tr>
<td>$l$</td>
<td>separation of 2DEG and 2DHG</td>
</tr>
<tr>
<td>$L, W$</td>
<td>system size (length, width)</td>
</tr>
<tr>
<td>$L_{\text{edge}}$</td>
<td>edge length</td>
</tr>
<tr>
<td>$L_{\text{gate}}, W_{\text{gate}}$</td>
<td>gate length, width</td>
</tr>
<tr>
<td>$L_{\text{gated}}^{(i)}$</td>
<td>length of gated edge $i$</td>
</tr>
<tr>
<td>$L_i$</td>
<td>edge segment length</td>
</tr>
<tr>
<td>$L_{\text{tg}}^{(i)}$</td>
<td>width of finger gate $i$</td>
</tr>
<tr>
<td>$L_{\text{total}}$</td>
<td>total length</td>
</tr>
<tr>
<td>$m_0$</td>
<td>free electron mass</td>
</tr>
<tr>
<td>$m^*$</td>
<td>effective electron mass</td>
</tr>
<tr>
<td>$m^<em>_e$, $m^</em>_h$</td>
<td>effective electron mass of conduction and valence band</td>
</tr>
<tr>
<td>$\mu$</td>
<td>charge carrier mobility or chemical potential</td>
</tr>
<tr>
<td>$\mu_{\text{tg}}$</td>
<td>chemical potential of the top gate</td>
</tr>
<tr>
<td>$\mu_{\text{2DEG}}$</td>
<td>chemical potential of 2DEG</td>
</tr>
<tr>
<td>$n$</td>
<td>total charge carrier density</td>
</tr>
<tr>
<td>$n_e$</td>
<td>electron density</td>
</tr>
<tr>
<td>$n_h$</td>
<td>hole density</td>
</tr>
<tr>
<td>$n_{\text{Hall}}$</td>
<td>charge carrier density calculated with Hall-data</td>
</tr>
<tr>
<td>$n_L$</td>
<td>Landau level degeneracy</td>
</tr>
<tr>
<td>$n_{\text{SdH}}$</td>
<td>charge carrier density calculated with SdH-data</td>
</tr>
<tr>
<td>$n_1, n_2$</td>
<td>spin split densities</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson ratio or Landau level filling factor</td>
</tr>
<tr>
<td>$\nu_{\text{piezo}}$</td>
<td>Poisson ratio of piezo stack</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>cyclotron frequency</td>
</tr>
<tr>
<td>$p, \vec{p}$</td>
<td>momentum</td>
</tr>
<tr>
<td>Symbol</td>
<td>Explanation</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\bar{R}$</td>
<td>resistance</td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>space vector</td>
</tr>
<tr>
<td>$R_B$</td>
<td>bulk resistance</td>
</tr>
<tr>
<td>$R_C$</td>
<td>bulk leakage resistance</td>
</tr>
<tr>
<td>$R_E$</td>
<td>edge resistance</td>
</tr>
<tr>
<td>$\rho_{\text{edge}}$</td>
<td>edge resistivity</td>
</tr>
<tr>
<td>$R_{\text{gated}}, R_{\text{ungated}}$</td>
<td>resistance of gated and ungated section</td>
</tr>
<tr>
<td>$\rho_{\text{gated}}, \rho_{\text{ungated}}$</td>
<td>resistivity (only normalized by length) of gated and ungated section</td>
</tr>
<tr>
<td>$R_i$</td>
<td>resistance of edge segment $i$</td>
</tr>
<tr>
<td>$R_{\text{nl}}$</td>
<td>nonlocal resistance</td>
</tr>
<tr>
<td>$R_{\text{NL} - \text{R type 1}}$</td>
<td>nonlocal resistance of type 1</td>
</tr>
<tr>
<td>$R_{\text{NL} - \text{R type 2}}$</td>
<td>nonlocal resistance of type 2</td>
</tr>
<tr>
<td>$R_{\text{pinch-off}}$</td>
<td>pinch-off resistance</td>
</tr>
<tr>
<td>$R_{\text{SD}}$</td>
<td>two-terminal resistance between source and drain</td>
</tr>
<tr>
<td>$R_{xx}$</td>
<td>longitudinal resistance</td>
</tr>
<tr>
<td>$\rho_{xx}$</td>
<td>longitudinal resistivity</td>
</tr>
<tr>
<td>$R_{xy}$</td>
<td>Hall resistance</td>
</tr>
<tr>
<td>$\rho_{xy}$</td>
<td>Hall resistivity</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>Drude resistivity</td>
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<tr>
<td>$s, \bar{s}$</td>
<td>electron spin</td>
</tr>
<tr>
<td>$S_{ijkl}$</td>
<td>compliance tensor</td>
</tr>
<tr>
<td>$s_p$</td>
<td>strain in percent</td>
</tr>
<tr>
<td>$\Sigma_c$</td>
<td>band offset of the conduction band</td>
</tr>
<tr>
<td>$\Sigma_v$</td>
<td>band offset of the valence band</td>
</tr>
<tr>
<td>$\sigma_{ij}$</td>
<td>stress vector</td>
</tr>
<tr>
<td>$\sigma_x, \sigma_y$</td>
<td>Pauli spinor components in the 2DEG plane</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature</td>
</tr>
<tr>
<td>$t$</td>
<td>QW thickness</td>
</tr>
<tr>
<td>$t_c$</td>
<td>critical QW thickness</td>
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<td>$t_{\text{GaSb}}$</td>
<td>QW thickness of GaSb</td>
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<tr>
<td>$t_{\text{InAs}}$</td>
<td>QW thickness of InAs</td>
</tr>
<tr>
<td>$\tau_q$</td>
<td>quantum scattering time</td>
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<td>$\tau_0$</td>
<td>transport scattering time</td>
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<td>$U$</td>
<td>potential in Hamiltonian</td>
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<tr>
<td>$u, u_i$</td>
<td>displacements</td>
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<td>Symbol</td>
<td>Explanation</td>
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<td>----------</td>
<td>--------------------------------------------</td>
</tr>
<tr>
<td>$V$</td>
<td>voltage</td>
</tr>
<tr>
<td>$V_{bg}$</td>
<td>back gate voltage</td>
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<tr>
<td>$v_G$</td>
<td>group velocity</td>
</tr>
<tr>
<td>$V_i$</td>
<td>electric potential of lead $i$</td>
</tr>
<tr>
<td>$V_{nl}$</td>
<td>nonlocal voltage</td>
</tr>
<tr>
<td>$V_{piezo}$</td>
<td>piezo voltage</td>
</tr>
<tr>
<td>$V_{tg}$, $V^{(i)}_{tg}$</td>
<td>top gate voltage</td>
</tr>
<tr>
<td>$V_{CB}^{tg}$, $V_{HB}^{tg}$, $V_{CNP}^{tg}$</td>
<td>top gate voltage of electron and hole band extrema and CNP</td>
</tr>
<tr>
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<td>longitudinal voltage</td>
</tr>
<tr>
<td>$V_{xy}$</td>
<td>transverse voltage</td>
</tr>
<tr>
<td>$x, y$</td>
<td>spatial coordinates</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

A new quantum phenomena was proposed in 2005 by Kane and Mele [1] and opened a broad research field. Theoretical and experimental physicists were fascinated by the properties of the quantum spin Hall effect. The insulating bulk is combined with quantized counter-propagating edge channels, which carry spin polarized electrons. A source of spin polarized charges could find use in many spintronic experiments. The combination with superconductivity could support Majorana bound states [2]. Braiding of these fermions, which are their own antiparticle, are named as possible building blocks in the next generation of computers, the quantum computers [3]. Only two years later, first experimental achievements on the HgTe quantum well system were reported [4]. Even though, the confirmation of the spin-polarization and therefore the unquestionable confirmation of the quantum spin Hall effect was still outstanding, it boosted the confidence that the numerous theoretical concepts have a solid experimental basis.

The double quantum well system InAs/GaSb was predicted as an additional host for the quantum spin Hall effect based on the convenient combination of established semiconductor fabrication and a gate tunable topological phase transition [5]. The latter is absent in HgTe devices. For coupled InAs/GaSb quantum wells one single device is expected to be tunable by gates between two distinct topological phases, switching the edge conduction on and off. This is a smoking gun experiment for the quantum spin Hall effect and anticipates a fast verification of the quantum effect.

Such a fast and smooth chain of research results is an enormous source of motivation, but it can also bear a risk: Namely to conclude from one experimental result about a general physical effect. This is often an unspoken challenge for researchers, not less significant than any technical obstacle. A. M. Worthington (1852-1916) faced this difficulty in his own research, but can still be taken as role model for nowadays scientists due to his exemplary capability to revise his findings. Attracted by the mark caused by drops falling on smoked glass plates, he started to study the symmetries of “splashes” notably before the invention of high speed photographs [6]. In a clever experimental setup he used an electric spark of three or

---

1Inspired by the lecture “Empiricism of Emotions”, autumn semester 2008 at ETH Zurich
four thousandths of a second to illuminate the drop at the instant of hitting a liquid or solid surface. The electric flash irritated the retina of his eyes used to the dark room and let the splash be visible for some time. Many repetitions allowed him to draw his observations. Twenty years later he returned to this research topic, eager to study the drops with the latest technology of photographic plates, sufficiently sensitive for these ultra short exposures. He stated: “I think the first comment that any one would make is that the photographs, while they bear out the drawings in many details, show greater irregularity than the drawings would have let one to expect [7].” Not downhearted by these results, but eager to learn most from the images, he went over the whole research again [8]. His capability of describing the experimental setup in a meticulously systematic way, explaining the results appealingly and instructive, and to revise previous findings set the base that his research results are of importance up to date finding diverse applications for example in spray coating or soil erosion studies due to rain droplets [9].

We learn that experimental research outcomes are not always consistent with their prediction. But this is not a drawback, rather it is a chance to bring our knowledge to a next level. This insight had an impact on the presented work. The remaining bulk conduction masking possible edge transport in the pioneering experiments on the quantum spin Hall insulator candidate InAs/GaSb [10], was a first hint that the experimental verification of the topological phase would unexpectedly require some more effort. In the framework of this thesis we also learned that one single measurement showing a quantized resistance of the expected magnitude for quantum spin Hall edge states can be misleading. Only a set of transport experiments allows a full understanding of the device. Acknowledging A. M. Worthington’s systematic research, this thesis studies first the necessary ingredients to observe the quantum spin Hall phase in mesoscopic Hall bars with the mean of transport, before exploring the edge conduction. The reader will not just find bare measurement results, but also further explanation on how to interpret transport results cautiously, not neglecting possible other influences. Upon completion of this work a definite confirmation of the quantum spin Hall effect in InAs/GaSb coupled quantum wells is still missing. The thesis concludes therefore with a critical discussion, summarizes the state of knowledge, and identifies further investigations needed.

The thesis is structured as follows:

- This thesis starts with a general introduction to the theoretical concept of the quantum spin Hall effect, chapter 2, setting a common basis of vocabulary.

- The strong spin-orbit coupling, chapter 3, and the inverted band alignment, chapter 4, are necessary ingredients for the quantum spin Hall phase. Fingerprint experiments are introduced to verify these properties.

- Chapter 5 highlights the impact of strain on the band structure, concluding that already little strain can cause a bulk transition from an insulator to a
semimetal. To improve the bulk insulator additionally, facilitating the study of edge transport, also the interplay of quantum well design, gating, and disorder has to be optimized, as explained in chapter 6.

- Ballistic edge transport in mesoscopic InAs/GaSb Hall bars are confirmed with the mean of nonlocal measurements in chapter 7. This chapter illustrates with experimental data that perfect quantization can arise due to the interplay of bulk and edge, stressing that one perfectly quantized level is not enough to confirm the quantum spin Hall phase.

- Diffusive edges unexpectedly observed in InAs quantum wells are investigated in various device geometries in chapter 8. This system is an integral part of the double quantum well system InAs/GaSb were in contrast to the mesoscopic devices, this diffusive edge characteristic could be found in finger gate samples.

- Chapter 9 critically discusses the contradicting ballistic and diffusive edges found in InAs/GaSb devices.

- A summary together with possible follow-up investigations are listed in chapter 10.
Chapter 2

Quantum spin Hall effect

Studying different phases as well as phase transitions has driven the condensed matter community for a long time. Previously, in the context of classical physics and nowadays also for quantum states of condensed matter, symmetry breaking is often used for classification (Landau theory). But there are also phases which lack such an explanation. For example the topological states of matter are based on a fundamentally different concept.

This chapter starts with the first known topological state, the integer quantum Hall effect (QHE). The latter is present at large magnetic fields. Eager to find a topological phase in the absence of magnetic field led to the theoretical prediction of the quantum spin Hall effect (QSHE) \[11\]. This is the main focus of this chapter, explaining the necessary ingredients for such a phase to occur as well as possible material realizations. This chapter intends to give a general introduction to the QSHE for a broad audience. The following chapters will relate this concept to the experimental studies and extend these explanations with further theoretical details if needed.

2.1 Gedankenexperiment

A two dimensional electron gas (2DEG) in the presence of a strong magnetic field and low temperature shows a quantization of the classical Hall conductance \[12\], the so-called integer quantum Hall effect. Klaus von Klitzing received the Nobel prize for these findings in 1985. In a magnetic field the density of states redistributes into Landau levels. The longitudinal resistivity on a standard Hall bar is zero whenever the Fermi energy resides between two consecutive Landau levels (\(\nu\) Landau levels are completely filled) and is finite otherwise. Concomitantly the Hall resistivity shows a plateau formation with values \(\rho_{xy} = \frac{h}{(\nu e^2)}\). This phase state is characterized by the robustness of the Hall plateau against small variations in density.

For a fixed number of completely filled Landau levels \(\nu\), the Fermi energy is in a gap of the density of states in the bulk of the sample, but \(\nu\) ballistic chiral edge channels are present. The charge carrier transport is therefore in opposite
2.2. General concept of the QSHE

The necessary ingredients for a 2D topological insulator (2D material system with QSH phase) are inverted band alignment, strong spin orbit coupling and time reversal symmetry, which will be apparent in the following discussion.

**Interface between topologically distinct phases**

A normal insulator has a band structure as schematically explained in Fig. 2.2 on the left with a conduction band (in blue) spaced by an energy gap from the energetically lower valence band (in red). The Fermi energy \( E_F \) is situated within this energy gap. In case of an inverted band alignment as shown in Fig. 2.2 on the right conduction
Chapter 2. Quantum spin Hall effect

Figure 2.2: Schematic band structure of a trivial (left) resp. an inverted system (right). Conduction band in blue, valence band in red. Conduction in a standard Hall bar sample can split up into an edge $R_E$ and bulk $R_B$ resistance (middle). The bulk conduction can be electronic or hole like symbolized by red/blue colors. Theoretically it is expected that when the Fermi energy $E_F$ is situated in the gap, the bulk is insulating and only the inverted system shows edge conduction (edge channels in yellow).

Figure 2.3: Schematic energy versus real space illustration of the lowest electron (conduction band) states in blue and highest hole (valence band) states in red of a cross-section through a normal (a) and inverted (b) insulator. The Fermi energy $E_F$ resides within the bulk energy gap for both materials and crosses bands at the edge of the topological (inverted) insulator only. This is a simplified illustration of the edge states expected in the QSH phase. Inspired by [14].

and valence band overlap. Due to electron-hole coupling a hybridization gap opens. An insulating state is expected in the bulk of the sample, if the Fermi energy is within this minigap.

Band continuity forces conduction and valence band to cross the Fermi energy at the interface between topologically distinct phases, i.e. at the edge of a topological insulator (vacuum has a normal band order), forming edge states. This intuitive explanation is sketched in Fig. 2.3, where the bottom of the conduction and top of
the valence band of a normal as well as inverted band alignment is shown on a cross section of a sample in real space. To study the number of Kramer’s pairs of the edge states is a more direct way to identify the topological phase. An odd (even) number of Kramer’s pairs is related to the edge of a QSH (normal) insulator [5].

The linear dispersion relation of the edge states is a consequence of the continuity of the Einstein mass. The Einstein’s equivalence principle relates the energy to a mass, which is positive (negative) in the sample exterior (interior) due to the band alignment. At the edge the mass has to be zero and therefore obeys a Dirac like dispersion.

**Helical edge conduction**

The spin polarized, helical nature of the edge states is a direct consequence of time-reversal symmetry in combination with strong spin-orbit coupling.

The system of interest can be described with a Hamiltonian of the form $H = H_{\text{kin}} + H_{\text{SO}} + U$ consisting of three terms: The kinetic energy $H_{\text{kin}} = H_{\text{kin}}(\vec{p}^2)$, the spin-orbit coupling $H_{\text{SO}} = H_{\text{SO}}(\vec{s} \cdot \vec{p})$ and a general potential $U = U(\vec{r})$ [15]. The Hamiltonian is therefore time-reversal symmetric at zero magnetic field since both $\vec{s}$ and $\vec{p}$ are odd, but $\vec{r}$ is even under this symmetry.

Kramer’s theorem states that in quantum mechanical systems each energy state of a half-integer spin is at least double degenerate, if the system obeys time-reversal symmetry: $E_\uparrow(\vec{k}) = E_\downarrow(-\vec{k})$. As a consequence, at the time-reversal point $\vec{k} = 0$ (Γ point) there must be at least one Kramer’s pair. In other words, the edge states do intersect at the Γ point.

The two edge states are counter-propagating and spin polarized since spin-orbit coupling locks spin and momentum and the effective magnetic fields of the spin directions feel the opposite spin-orbit coupling forces [16, 17] An intuitive explanation of the propagation direction can be found with the definition of the group velocity $\vec{v}_G = \frac{i}{\hbar} \nabla E(\vec{k})$ in combination with the above defined conditions for the dispersion relation of the edge states.

**Edge conduction protected against backscattering**

In the assumption of an insulating bulk (no bulk states at the Fermi energy) and all sample dimensions exceeding the edge state width, scattering could only occur within the two counter-propagating spin polarized edge states. But backscattering would require a spin flip, which is suppressed in time-reversal systems since magnetic impurities, external magnetic field or the like are absent.

The dissipationless edge states carry one resistance quantum $\hbar/e^2$ each and the expected transport resistances can be calculated within the Landauer-Büttiker framework. Further explanations to these calculations can be found in section 7.6.
2.3 Material realization in 2D

The first prediction of the QSH phase by Kane and Mele was based on graphene [1]. An experimental realization is not expected though due to small bulk gap of the order of $10^{-3}$ meV in absence of an electric field perpendicular to the sample plane [18, 19] in this material. Also the prediction of a conventional semiconductor under the influence of a strain gradient as host of the QSH phase [17] still lacks an experimental realization.

At the moment two material systems are investigated experimentally with the scope to study the QSH phase in time-reversal two-dimensional systems: HgTe/(Hg,Cd)Te and InAs/GaSb quantum wells.

2.3.1 HgTe/(Hg,Cd)Te

Experimentally the quantum spin Hall effect was first realized in an inverted HgTe/(Hg,Cd)Te QW [20]. While a nonlocal measurement proved the existence of edge transport [4], the confirmation of spin-polarized transport demanded more complex transport experiments [21]. The inverted band alignment is realized by the proper choice of the QW thickness, a discrete sample parameter. For an individual sample either the normal or topological insulating regime can be studied.

2.3.2 InAs/GaSb

The combination of a 2DEG in InAs and a 2DHG in GaSb in a double quantum well structure confined by AlSb was predicted in 2008 by Liu et al. [5] as a host for the QSHE. This material system combines experience in semiconductor sample processing with the possibility of tuning between normal and inverted insulator regime as will be explained in the following.

Inverted band alignment

The InAs/GaSb/AlSb heterostructure combines semiconductors of the so-called 6.1 Å-family. Therefore, they have a similar unit cell resp. lattice constant, facilitating the epitaxial growth. Despite their similar crystal structure, they have a large variation in energy gap ranging from 0.36 eV of InAs to 1.61 eV of AlSb. The band lineups are explained in Fig. 2.4. AlSb with its large band gap is ideal as a barrier material. It confines electrons (but no holes) of a InAs QW and holes of a GaSb QW. Combining all three semiconductors allows to realize the inverted band alignment at the interface of InAs (conduction band) and GaSb (valence band). The exact value of the band overlap depends on quantum well thicknesses and electric field.

Hybridization gap

The QW thicknesses also influence the coupling of the electron and hole states. For rather large QW thicknesses it can be expected that the charge carrier states
2.3. Material realization in 2D

Figure 2.4: Semiconductors of the 6.1 Å-family with their energy gaps (shaded area) and band lineups. All energies are in eV. Reprinted from [22], with permission from Elsevier.

mainly reside within their respective QWs. For reduced QWs their wave function start to overlap and couple, causing a hybridization of the two bands [5, 23]. The hybridization gap is much smaller than the band overlap and is the crucial bulk gap of a few meV.

Double gating
Due to the spatial separation of the QWs hosting electrons and holes, double gating to tune density and band lineup independently was predicted [5] and experimentally verified [25]. Top and back gates implemented as schematically explained in Fig. 2.5(a) are the tuning knobs to reach metallic as well as normal and inverted insulator regimes. The rich phase diagram is schematically explained in Fig. 2.5(b), where the sum of the voltages of the two gates $V_{tg} + V_{bg}$ determines the Fermi level and therefore the density and the difference $V_{tg} - V_{bg}$ modifies the perpendicular electric field and therefore the band alignment. The gate accessible phase boundary of the normal and inverted insulator is the major advantage compared to previous QSH insulator proposals. It is expected that the QSH edge states can be turned “on” or “off” with such a dual-gate geometry. The latter can let us dream of a QSH field effect transistor. But a room temperature realization would require a larger
Figure 2.5: a) Schematic explanation of the conduction (blue) and valence (red) band edge in growth direction of an inverted InAs/GaSb/AlSb system. Inspired by [24]. b) Schematic phase diagram of dual-gating with top $V_{tg}$ and back $V_{bg}$ gate. Metallic regimes (white) with inverted band alignment and non-inverted band alignment can be distinguished from the QSH and normal insulating (NI) states (light blue). Inspired by [5].

insulating gap.
Chapter 3

Strong spin-orbit coupling

The spin-orbit coupling can be understood as a magnetic field interaction to the electron spin $s$ dependent on the momentum $k$. It is a necessary ingredient for topological insulator materials. The first system studied with the objective to find the helical edge states expected for 2D topological insulators were HgTe/(Hg,Cd)Te quantum wells (QWs). For most semiconductors the p-band (s-band) hosts holes (electrons) and therefore builds the valence (conduction) band. The spin-orbit coupling strength varies with an increased QW thickness $t$ leading to the inversion of s- and p-bands as soon as $t$ is larger than a critical thickness $t_c$. In this topological band order the s-band (p-band) builds the valence (conduction) band [26]. In the InAs/GaSb 2D topological insulator candidate the spin-orbit coupling is not needed to realize the inverted band ordering. But it is still an essential ingredient to open a hybridization gap, as it mixes light and heavy hole bands [5]. This chapter will start with a theoretical background on spin-orbit interaction with a strong focus on spin splitting of the energy bands. Afterwards the chapter will focus on two experimental signatures of the strong spin-orbit coupling: First, a beating pattern in Shubnikov de Haas (SdH) oscillations of an electron gas in InAs or InAs/GaSb QWs is analyzed. Second, an unconventional Landau fan spectrum in inverted InAs/GaSb QWs due to the influence of spin-orbit coupling on the crossings of electron and hole type Landau levels is discussed.

Spin-orbit coupling is not only of interest in the context of topological insulators, but also for spintronic devices [27], especially if the spin-orbit coupling can be controlled by gating (Rashba spin-orbit coupling). The combination of strong-spin orbit coupling, large g-factor and induced superconductivity raised interest in the Majorana community in the context of the topological superconducting phase [28].
3.1 Spin-orbit interaction: A theoretical introduction

At zero magnetic field, a material with spatial inversion symmetry and time reversal symmetry is spin-degenerate: \( E_{\uparrow}(\vec{k}) = E_{\downarrow}(\vec{k}) \). This double degeneracy can be lifted with an asymmetry in the potential \([29]\). The latter is caused by either a bulk inversion asymmetry (BIA, Dresselhaus spin-orbit contribution) for example in zinc-blende materials \([30]\) or a structure inversion asymmetry (SIA, Bychkov and Rashba spin-orbit contribution) \([31, 32]\). This results in a system left with only time reversal symmetry still following Kramer’s degeneracy: \( E_{\uparrow}(\vec{k}) = E_{\downarrow}(−\vec{k}) \).

A two-dimensional electron gas in a zinc-blende heterostructure grown on a [100] substrate shall be described with \( H_0 \), if spin-orbit coupling is neglected. Taking both the Rashba \( H_R \) and Dresselhaus \( H_D \) contribution in lowest order in \( \vec{k} \) into account, then the Hamiltonian \( H_0 \) modifies to:

\[
H = H_0 + H_R + H_D \\
= H_0 + \alpha_R(\sigma_x k_y - \sigma_y k_x) + \beta_D(\sigma_x k_x - \sigma_y k_y)
\]

where \( \sigma_x \) and \( \sigma_y \) denote the Pauli spinor components in the 2DEG plane.

The Dresselhaus effect is influenced both by the band structure of the bulk material and by the confinement of the electron gas. If only terms linear in \( \vec{k} \) are taken into account the Dresselhaus coefficient simplifies to:

\[
\beta_D = \beta \langle k_z^2 \rangle
\]

where \( \langle k_z^2 \rangle \propto 1/t^2 \) and therefore \( \beta_D \) depends on the quantum well width \( t \). An enhanced confinement (smaller \( t \)) will lead to a stronger Dresselhaus spin-orbit contribution.

The electric field in growth direction \( \langle E_z \rangle \) influences the Rashba component. This is especially interesting because it allows gate tunable spin-orbit interaction, as will be discussed later in this chapter.

\[
\alpha_R = \alpha \langle E_z \rangle
\]

3.2 Spin-orbit interaction in InAs

Literature values on the spin-orbit coupling in InAs 2DEGs assume a much stronger Rashba than Dresselhaus influence (theoretically: \( \alpha = 117.1 \text{ eÅ}^2 \) and \( \beta = 27.18 \text{ eVÅ}^3 \) \([29]\); experimentally: \( \alpha = 170 \text{ eÅ}^2 \) \([33]\)). In a simplified approximation we can neglect the Dresselhaus term in the Hamiltonian and get the eigenenergies:
3.2. Spin-orbit interaction in InAs

The energy dispersions are sketched in Fig. 3.1. They are still parabolic, but the energy bands related to the two spins are not degenerate and split in $k$-direction by $2m^*\alpha_R/\hbar^2$.

The calculations were done with great support by Lun-Hui Hu from the group of Prof. Zhang at Zhejiang University.

The energy bands $E_{1,2}(k)$ can be used to calculate the density of states (DOS), and also the total density related to each band:

$$
\text{DOS}(E) = \frac{1}{2\pi} |k||\frac{dk}{dE}|
$$

$$
n = \int \text{DOS}(E)dE
$$

Now applied to the specific energy bands $E_{1,2}$, taking the integration boundaries into account as explained in Fig. 3.1 (in blue).

$$
\text{DOS}_1(E) = \frac{m^*}{2\pi\hbar^2} \left(1 + \frac{1}{\sqrt{1 + \frac{2\hbar^2 E}{m^*\alpha_R^2}}}ight)
$$
\[ n_1 = \int_{-\frac{\alpha_R^2}{2\mu}}^{0} 2 \cdot \text{DOS}_1(E)dE + \int_0^\mu \text{DOS}_1(E)dE \]
\[ = \frac{m^*}{2\pi\hbar^2} \left[ \mu + \frac{m^*\alpha_R^2}{\hbar^2} \sqrt{1 + \frac{2\hbar^2\mu}{m^*\alpha_R^2} + \frac{m^*\alpha_R^2}{\hbar^2}} \right] \]

and

\[ \text{DOS}_2(E) = \frac{m^*}{2\pi\hbar^2} (1 - \frac{1}{\sqrt{1 + \frac{2\hbar^2E}{m^*\alpha_R^2}}} \]
\[ n_2 = \int_0^\mu \text{DOS}_2(E)dE \]
\[ = \frac{m^*}{2\pi\hbar^2} \left[ \mu - \frac{m^*\alpha_R^2}{\hbar^2} \sqrt{1 + \frac{2\hbar^2\mu}{m^*\alpha_R^2} + \frac{m^*\alpha_R^2}{\hbar^2}} \right] \]

Then the total density is

\[ n = n_1 + n_2 = \frac{m^*}{2\pi\hbar^2} \left[ 2\mu + \frac{2m^*\alpha_R^2}{\hbar^2} \right] \quad (3.3) \]

Therefore,

\[ \frac{\Delta n}{n} = \frac{n_1 - n_2}{n_1 + n_2} = \frac{2}{\sqrt{1 + \frac{2\hbar^2\mu}{m^*\alpha_R^2} + \frac{1}{\sqrt{1 + \frac{2\hbar^2\mu}{m^*\alpha_R^2}}}} \approx \frac{2}{\sqrt{1 + \frac{2\hbar^2\mu}{m^*\alpha_R^2}}} \quad (3.4) \]

This mathematical derivation shows that Rashba spin-orbit coupling leads to a lifting of the spin degeneracy. It is visible as a density splitting \( \Delta n \).

### 3.3 Beating in the SdH oscillations

A 2DEG in perpendicular magnetic field shows the Shubnikov-de Haas (SdH) effect, meaning periodic oscillations in the longitudinal resistivity \( \rho_{xx} \) as a function of the inverse magnetic field \( 1/B \). The periodicity of these oscillations is a direct link to the density \( n = fe/\hbar \). If two densities \( n_i \) are present, two oscillations with different frequencies \( f_i \) overlap leading to a beating pattern. This chapter first introduces the samples which are used to study beating in SdH oscillation in InAs 2DEGs. Then the standard analysis of the transport experiments is explained and the density splitting is calculated in the whole gating range. The last part concentrates on a combined discussion of theory and experiment to derive the gate tunable Rashba coefficient \( \alpha_R \).
3.3. Beating in the SdH oscillations

<table>
<thead>
<tr>
<th>Device name</th>
<th>Wafer</th>
<th>Hall bar dimensions ($W \times L$)</th>
<th>Etching</th>
<th>Beating?</th>
<th>$m^*$ calculation?</th>
</tr>
</thead>
<tbody>
<tr>
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<td>2x8 µm</td>
<td>dry</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>InAs/GaSb-2</td>
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<td>4x8 µm</td>
<td>dry</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>InAs/GaSb-3</td>
<td>#1</td>
<td>25x50 µm</td>
<td>dry</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>InAs/GaSb-4</td>
<td>#1</td>
<td>25x50 µm</td>
<td>dry</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>InAs/GaSb-5</td>
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<td>2x11.5 µm</td>
<td>dry</td>
<td>no trend</td>
<td>no</td>
</tr>
<tr>
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<td>wet</td>
<td>no trend</td>
<td>no</td>
</tr>
<tr>
<td>InAs/GaSb-7</td>
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<td>wet</td>
<td>no trend</td>
<td>no</td>
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<tr>
<td>InAs/GaSb-8</td>
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</tr>
<tr>
<td>InAs-2</td>
<td>#4</td>
<td>25x50 µm</td>
<td>dry</td>
<td>no</td>
<td>yes</td>
</tr>
</tbody>
</table>

Table 3.1: Overview of InAs/GaSb double QWs and InAs single QW samples to study beating in SdH oscillations of the 2DEG.

3.3.1 Sample details for beating analysis

Here we present data from eleven standard Hall bars (dimensions summarized in table 3.1), fabricated with four different epitaxial grown wafer materials. Wafer #1, #2 and #3 contain a 15 nm/8 nm InAs/GaSb double QW confined by AlSb barriers with nominally identical growth sequence. Nevertheless, wafer #1 shows a higher mobility ($\mu \approx 300’000$ cm$^2$/Vs at $n = 1 \cdot 10^{12} \text{cm}^{-2}$) than wafer #2 and #3 ($\mu \approx 75 – 100’000$ cm$^2$/Vs at $n = 1 \cdot 10^{12} \text{cm}^{-2}$). The discrepancy can be explained by a different amount of inhomogeneities in these wafers. Wafer #4 differs only by the missing GaSb layer. This wafer contains solely a confined 8 nm InAs QW. (Wafer notations used in the groups of Prof. Ensslin and Prof. Wegscheider are summarized in appendix B.)

The Hall bar structures were defined by optical lithography and wet or dry etching (etching process used for each sample is indicated in the table 3.1) as described in [34] or in appendix A. Au/Ge/Ni eutectics serve as Ohmic contacts. The gate is implemented by Ti/Au spaced by 200 nm Si$_3$N$_4$ from the capping layer of the wafer material.

3.3.2 Typical beating measurement analysis

Data were obtained with a standard four-terminal ac measurement technique. A constant current of $I = 10$ nA is applied along the Hall bar and the longitudinal $V_{xx}$ and transverse $V_{xy}$ voltages are recorded.

Figure 3.2 shows the typical result exemplary on the InAs/GaSb-2 sample. The longitudinal resistivity $\rho_{xx} = W/L \cdot V_{xx}/I$ at different gate voltages $V_{tg} = 1.5$ V to
Figure 3.2: The longitudinal resistivity $\rho_{xx}$ of the InAs/GaSb-2 sample as a function of perpendicular magnetic field $B$ in (a) shows beating pattern. The normalized Fourier spectrum in (b) analyzed at low field (resp. high field) shows the density splitting $n_1$ (red cross) and $n_2$ (red x) (resp. the total density $n_{\text{SdH}}$ (blue circle)). The density $n = n_1 + n_2$ agrees well with the total density extracted from the high field SdH data $n_{\text{SdH}}$ and the Hall data $n_{\text{Hall}}$ shown in (c).

5 V versus the perpendicular magnetic field B is plotted in Fig. 3.2(a). The beating pattern is visible by eye. The SdH oscillations have not a continuously increasing amplitude as expected if only one frequency was present, but beating knots are visible (a regular suppression of the oscillations).

The longitudinal resistivities $\rho_{xx}$ as a function of $1/B$ are analyzed with the objective to extract the different overlapping frequencies with a standard Fourier transform algorithm. The normalized power spectrum shown in Fig. 3.2(b) is calculated as described in the supplementary material of [35] by subtracting a slowly varying background, multiply them with a smooth window function and zero-padding in order to increase the number of points in the spectrum. If we apply this calculation procedure in the magnetic field range $B < 2 \text{T}$, the power spectrum shows two split peaks in frequency $f_i$ proportional to a density $n_i = f_i e / h$ (shown in Fig. 3.2(b) left). The same procedure at high magnetic field displays only one peak in the Fourier
3.3. Beating in the SdH oscillations

Figure 3.3: The relative density splitting $\Delta n/n$ increases with decreasing total density $n$ for Hall bar samples of wafer #1 and #4 independent of device size.

spectrum. The corresponding frequency can be used to calculate the total density extracted from SdH oscillations $n_{\text{SdH}}$ (shown in Fig. 3.2(b) right). The extracted densities are summarized in Fig. 3.2(c). The density $n = n_1 + n_2$ agrees well with the total density calculated from the high field SdH data $n_{\text{SdH}}$ and the Hall data $n_{\text{Hall}}$, supporting the previous assumption of a density splitting of the two spin carrier bands due to Rashba spin-orbit coupling.

3.3.3 Density splitting of different spin carriers

This section will summarize the density splitting extracted from the standard analyzing procedure as described above. Wafer differences, especially between single InAs QW and double InAs/GaSb QWs, and the influence of sample size will be highlighted.

Figure 3.3 shows the relative density splitting as a function of total density for InAs (in brown) resp. InAs/GaSb (yellow to red) Hall bars fabricated from wafers #1 and #4. In the accessible gate range a change in $\Delta n/n$ up to a factor of three can be realized, with an enhanced splitting below $n = 4 \cdot 10^{15} \text{ m}^{-2}$ for the InAs-1 device. The InAs/GaSb-1,-2,-3, and -4 samples show the same trend, but their relative density splitting is already increasing below $n = 6 \cdot 10^{15} \text{ m}^{-2}$. The independence on sample dimensions is demonstrated by the latter device series ranging from 2 $\mu$m up to 25 $\mu$m in width.
Chapter 3. Strong spin-orbit coupling

Figure 3.4: Summary of the relative density splitting $\Delta n/n$ as a function of the total density $n$ for all Hall bar samples listed in table 3.1. Samples fabricated with the same wafer material are plotted with the same color, but different symbols.

This systematic dependence of the relative density splitting on total density can not be found with Hall bars fabricated with wafer #2 and #3, even though a double peak in the power spectrum could be confirmed for certain densities. The summary is shown in Fig. 3.4, samples from different wafers are color coded. A possible reason could be the higher inhomogeneity suspected for wafers #2 and #3.

3.3.4 Effective mass $m^*$ in InAs and InAs/GaSb

Theoretical explanation of the density splitting due to Rashba spin-orbit interaction, as shown in equation 3.4, depends on the effective mass $m^*$. This section explains first, how the effective mass can be found in transport experiments, before discussing the difference of the 2DEG effective mass in single InAs QW and InAs/GaSb double QWs.

Energy states of a free electron gas subject to an external uniform magnetic field are quantized in so-called Landau levels. The Landau-levels are separated in energy by $\hbar \omega_c = \hbar |eB| / m^*$ with $\omega_c$ the cyclotron frequency. The Landau levels are broadened due to disorder and scattering with an energy uncertainty described by the quantum scattering time $\tau_q$. Each Landau level has a degeneracy of $n_L = |eB| / \hbar$ and Landau levels up to the filling factor $\nu = n/n_L$ are filled. It is apparent that both magnetic field as well as gate voltage (and therefore density) influence the
3.3. Beating in the SdH oscillations

The number of occupied Landau levels. The longitudinal resistivity $\rho_{xx}$ depends on the position of the Fermi energy in the band structure, i.e. within a Landau level or in the gap between consecutive Landau levels. Therefore, keeping the magnetic field constant and changing the charge carrier density continuously leads to oscillations in $\rho_{xx}$ named as de Haas-van Alphen effect. Changing B-field at a fixed electron density leads to oscillations in $\rho_{xx}$ called Shubnikov-de Haas oscillations, which can be described with the Ando formula in the single subband case:

$$\rho_{xx} = \frac{m^*}{n e^2 \tau_0} \left( 1 - 2e^{-\pi/\omega_c \tau_0} \right) \frac{2\pi^2 k_B T/h\omega_c}{\sinh(2\pi^2 k_B T/h\omega_c)} \cos\left( 2\pi \frac{h n}{2eB} \right)$$

(3.5)

where the longitudinal resistivity $\rho_{xx}$ oscillates around the classical Drude resistivity $\rho_0$ with the term number (3), which was used above to extract the total density $n = n_{SdH}$. The amplitude of the oscillations increases with increasing magnetic field $B$ described by the Dingle factor, number (1). The term (2) is a function $f(B, T)$ of magnetic field $B$ and temperature $T$ and is taking account for the thermal damping of the $\rho_{xx}$ oscillates with enhanced temperature. Based on equation 3.5 the relative amplitude decay $\Delta \rho_{xx}/\rho_{xx}$ at a specific magnetic field $B$ at different temperatures can be used to derive the electronic effective mass $m^*$ by fitting:

$$\frac{\Delta \rho_{xx}}{\rho_{xx}} = 2e^{-\pi/\omega_c \tau_0} f(B, T)$$

(3.6)

where the quantum scattering time $\tau_q$ and the effective mass $m^*$ are fitting parameters.

The effective mass was calculated for the InAs-2 and InAs/GaSb-4 samples at different densities showing a linear dependence on the magnetic field. Extrapolating to zero field gives almost density independent effective masses of $m^*_{\text{InAs}} \approx 0.021 \cdot m_0$ and $m^*_{\text{InAs/GaSb}} \approx 0.029 \cdot m_0$, see Fig. 3.5. The independence on density suggest almost perfect parabolic bands. The enhanced effective mass of the double quantum
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Figure 3.6: Normalized density splitting $\Delta n/n$ as a function of the total density $n$ calculated with the equation 3.4, assuming an electric field of $\langle E_z \rangle = 4 \text{mV/nm}$, a material coefficient $\alpha = 117.1 \text{eÅ}^2$ and an effective mass of $m^* = 0.023 \cdot m_0$.

well structure InAs/GaSb compared to the single InAs QW could be explained by a weaker confinement due to the neighboring GaSb QW instead of the AlSb barrier. The values are of the same order of magnitude as literature values. In a review by Nakwaski [36] $m^* = 0.023 \cdot m_0 \pm 5\%$ is suggested as the most reliable value. But he also reports a wide range of literature values, namely from $m^* = 0.018 \cdot m_0$ up to $m^* = 0.136 \cdot m_0$.

### 3.3.5 Rashba coefficient

In the theoretical introduction to this chapter we derived the equation 3.4 here shown again:

$$
\Delta n \approx \frac{2}{\sqrt{1 + \frac{2\hbar^2 \mu}{m^* \alpha_R^2}}}
$$

with the chemical potential $\mu$ calculated from 3.3:

$$
\mu = \frac{\pi \hbar^2 n}{m^*} - \frac{m^* \alpha_R^2}{\hbar^2}
$$

The function $\frac{\Delta n}{n}(n)$ depends therefore only on the effective mass $m^*$, which was derived in the previous section, and the Rashba coefficient $\alpha_R = \alpha \langle E_z \rangle$. The fit parameter $\alpha$ can be determined, if the density dependence of the mean electric field $\langle E_z \rangle$ is known.

In the most simplified approach we assume a constant electric field as for example in Ref. [37]. The normalized density splitting $\Delta n/n$ calculated with the equation
3.3. Beating in the SdH oscillations

Figure 3.7: Normalized density splitting $\Delta n/n$ as a function of the total density $n$ calculated with the equation 3.4, assuming a density dependence of the electric field of $\langle E_z \rangle = -en/\varepsilon_0$, a Rashba coefficient $\alpha = 117.1 \, \text{eÅ}^2$ and an effective mass of $m^* = 0.023 \cdot m_0$

3.4, an electric field of $\langle E_z \rangle = 4 \, \text{mV/nm}$ and a material coefficient $\alpha = 117.1 \, \text{eÅ}^2$ is plotted in Fig. 3.6. This calculation reproduces the trend of enhanced density splitting at lower density as shown by the experimental data in Fig. 3.3. But the assumption that the electric field stays constant, even though the density is modified by almost a factor of four, seems to oversimplify the present situation.

As a first approach to include the influence of a modified electric field, we start with [29]

$$\langle E_z \rangle = \Sigma_c + \Sigma_v \langle E_{z,\text{ext}} \rangle.$$  \hspace{1cm} (3.7)

The prefactor including the band offsets $\Sigma_c, \Sigma_v$ of the conduction and valence bands is approximately equal to 1. The internal electric field depends therefore mainly on the external field $\langle E_{z,\text{ext}} \rangle$, which can be calculated with the 2DEG density $n$:

$$\langle E_{z,\text{ext}} \rangle = -\frac{en}{\varepsilon_0}$$  \hspace{1cm} (3.8)

with the relative dielectric constant of InAs $\varepsilon$. The trend of the normalized density splitting $\Delta n/n$ changes completely with this approach as can be seen in Fig. 3.7, not reproducing the experimental data shown in Fig. 3.3.

In conclusion, the electric field is probably more complex and might even be different for InAs and InAs/GaSb. Up to now there is no theoretical derivation, which allows for determining the self-consistent Hartree-Potential for InAs/GaSb [38] and therefore to predict the density dependence of the Rashba coefficient $\alpha_R$.

In the following we will concentrate on the experimental results to get some insights on the electric field. We fit the density splitting of Fig 3.3 by the approach of a constant electric field, and therefore also a constant Rashba coefficient $\alpha_R$, using
Figure 3.8: (a) The density splitting $\Delta n/n$ as a function of $n$ with the assumption of a constant Rashba coefficient $\alpha_R$ as a fitting parameter. InAs/GaSb data of wafer #1 in red and InAs data of wafer #4 in brown. (b) The electric field $\langle E_z \rangle$ calculated with the same data keeping $\alpha = 117.1 \text{ eÅ}^2$ constant.

The respective effective masses $m_*$ for the two material systems shown in Fig. 3.8(a). For InAs/GaSb resp. InAs the best fit can be achieved with $\alpha_R = 1.7 \cdot 10^{-11} \text{ eV m}$ resp. $\alpha_R = 3.3 \cdot 10^{-11} \text{ eV m}$. The fit agrees with the trend of the experimental data, but limitations of this approach are visible. The fit values have the expected order of magnitude, if we assume an inner electric field of 10 mV/nm.

On the other hand we can assume a material constant $\alpha = 117.1 \text{ eÅ}^2$ and calculate the effective field with the density splitting data shown in Fig. 3.8(b). The electric field $\langle E_z \rangle$ ranges between 10 and 35 mV/nm depending on density. We find higher values for the more confined InAs QW system, differing by about a factor of 2 to the InAs/GaSb double QWs system. A slight trend towards higher electric fields for smaller densities can be seen.
Beating pattern were already studied previously in InAs 2DEGs. Luo et al. [39, 40] stated already in 1988 Rashba dominated spin-orbit in InAs 2DEGs by analyzing beating pattern. In the following top gate enhanced Rashba spin-orbit coupling came into focus [37], where a density modulation of 1.6 to 2.4 × 10^{12} cm$^{-2}$ could be achieved. For the analysis a constant electric field $\langle E_z \rangle$ was assumed, which is reasonable for this relatively small change in density.

In the course of these investigations, there were also inconsistencies found with the theoretical expectations. For example Heida et al. [41] could not find a gate tunable Rashba coefficient $\alpha_R$, even though the density could be modified by almost a factor of two. Brosig et al. [42] reported on the absence of a beating pattern, even though Shubnikov-de-Haas oscillations down to very low field confirm a good quality material. Also Rowe et al. [43] could not find beating in InAs/AlSb before the second subband is populated (magneto-intersubband oscillations). The observed InAs/GaSb quantum wells showed beating only for lowest densities, even though the carrier concentration could be modulated between $4.8 \times 10^{11}$ and $3.6 \times 10^{12}$ cm$^{-2}$. A tunable Rashba coefficient could be realized without changing the charge carrier density, just by modifying the penetration of the electron wave function into the barrier [44].

In summary, all these experimental investigations helped to improve the understanding of the density splitting, but a full understanding of the Rashba coefficient $\alpha_R$ with top gate voltage tuning is missing. The top gate modifies not just the 2DEG density, but also the electric field, which modifies itself the Rashba coefficient $\alpha_R = \alpha \langle E_z \rangle$. This makes the data fitting difficult as described in the section above.

The density dependence of the electric field $\langle E_z \rangle$ could be predicted with self-consistent Hartree potentials. The latter is not sufficiently modeled up to now [38]. The previously described method of predicting the electric field with the assumption of a material coefficient $\alpha$ could help to improve these calculations. Especially to identify differences between heterostructures as here for AlSb/InAs/AlSb and AlSb/InAs/GaSb/AlSb. We learn that the reduced confinement due to the additional 8 nm GaSb QW reduces the electric field in the 15 nm InAs QW by about a factor of 2.

The recent improvements in sample fabrication allows double gating of InAs [33] or InAs/GaSb [45] QW structures. This allows for modifying the spin-splitting while keeping the 2DEG charge density constant. On the other hand, one can also use this double gating technique to keep the electric field constant and study the density dependence of the Rashba spin splitting. With this method a Rashba coefficient $\alpha_R = 75$ to 53 meVÅ could be extracted [45]. The latter agrees well with the experimental results found in [33].

3.3.6 Interpretation and conclusion to beating in InAs 2DEGs
Chapter 3. Strong spin-orbit coupling

3.4 Unconventional Landau fan spectrum

The strong spin orbit-coupling of the InAs/GaSb double quantum well system influences also the Landau fan spectrum. This section reports on two unconventional features in the Landau fan spectrum, one in an extended density range [46], the other one close to charge neutrality [47].

Parts of this chapter have been published in the article:

**Experimental signatures of the inverted phase in InAs/GaSb coupled quantum wells**


In contrast to the normal band alignment, the Landau fans of electron and hole charge carriers overlap in case of an inverted band alignment. The minima in longitudinal resistivity $\rho_{xx}$ corresponding to integer filling factors $\nu$ as a function of top gate voltage (density) and perpendicular magnetic field show a modulation in strength. Thermal activation experiments confirmed a correlation between energy gap and pronouncement of $\rho_{xx}$ minima. The energy gaps do not alter in parallel magnetic field. This is taken as an evidence for a Landau-level coupling dominated by the spin-orbit interaction.

If the band alignment is normal, the modulation in $\rho_{xx}$ is absent. Such an unconventional Landau fan structure can therefore identify the inverted band alignment regime (more details in section 4.3).

In our experiments we could not identify a spin-orbit polarization larger than 15%, see Fig. 3.3. The largest polarization observed are found at smallest densities, limited by the simultaneously reduced mobility. Recently Nichele et al. [47] also reported an increasing spin-orbit polarization with reduced density, but reaching a polarization of 100% near charge-neutrality. This is explained by a spin-orbit splitting larger than the Landau level separation resulting also in an unconventional Landau fan spectrum at low densities.
Chapter 4
Experimental signatures of the inverted band structure

The inverted band structure is a necessary ingredient for the TI phase as explained in chapter 2. An optimal choice of the InAs and GaSb QW thicknesses together with a proper internal electric field allows an inverted band alignment at the interface of the double QW system. This chapter will list different experimental techniques to identify the inverted band structure in transport experiments. Parts of this chapter have been published in the article:

Experimental signatures of the inverted phase in InAs/GaSb coupled quantum wells

4.1 Co-existence of electron and holes around charge neutrality

Top gate tuning allows to modify the charge carrier density. For very positive (negative) gate voltage $V_{tg}$ the Fermi energy is deep in the conduction (valence) band and we expect electron (hole) transport only. At the CNP though we know that the electron density $n_e$ is equal to the hole density $n_h$. There is a finite $V_{tg}$ range around the CNP, where both charge carriers co-exist, if the band structure is inverted. At more positive (negative) gate voltages, but in proximity to the CNP, the electrons (holes) are the majority charge carriers, where the holes (electrons) are in minority.

The low-field transverse resistivity $\rho_{xy}$ as a function of the perpendicular magnetic field $B_\perp$ with respect to the sample plane gives information about charge carrier mixing. A representative example is shown in Fig 4.1. At $V_{tg} = 0$ V and
Figure 4.1: Transverse resistivity $\rho_{xy}$ of a sample with inverted band alignment at small perpendicular field $|B_\perp| \leq 1\, \text{T}$ and different top gate voltages $V_{tg}$. Adapted from [46].

$V_{tg} = -9\, \text{V}$ we observe a completely linear dependence and can extract the charge carrier density with:

$$\rho_{xy} = \frac{B}{en}$$

where the calculated charge carrier density is electron like ($n > 0$) for $V_{tg} = 0\, \text{V}$ and hole like ($n < 0$) for $V_{tg} = -9\, \text{V}$.

For the intermediate gate voltages $V_{tg} = -5.5\, \text{V}$ and $V_{tg} = -6.5\, \text{V}$ the $\rho_{xy}$ is not linear around $B_\perp = 0\, \text{T}$, which can be explained by the simultaneous presence of electron and hole charge carriers. This confirms the inverted band alignment.

4.2 Longitudinal resistivity in parallel magnetic field

Also an in-plane magnetic field $B_\parallel$ can be used to identify the inverted band structure.

Let’s assume $B_\parallel$ is applied along the y-axis. The two charge carriers (electrons and holes) in the x,y-plane will experience a relative momentum shift along the x-axis of $\Delta k = eB_\parallel l/h$. $l$ is the separation of 2DEG and 2DHG. Therefore, a parallel field component can force a phase transition from a topological insulator to a semimetal as soon as the distortion of the dispersion relation due to the momentum shift is larger than the energy gap [48]. The experimental signature is a reduced charge neutrality peak in $\rho_{xx}$ with increasing $B_\parallel$ [46, 48].

4.3 Landau fan spectrum

The longitudinal resistivity $\rho_{xx}$ of a sheet of charge carriers in the influence of perpendicular magnetic field $B_\perp$ and density modulation shows the peculiar Landau
4.3. Landau fan spectrum

The InAs/GaSb systems show Landau fans of the 2DEG in InAs and 2DHG in GaSb simultaneously, while the density modulation can be experimentally realized by a top gate $V_{tg}$. Such a spectrum is shown in Fig. 4.2(left) for inverted band alignment realized with a QW thickness combination of 12/8 nm InAs/GaSb and (right) for a normal band alignment with 10/8 nm InAs/GaSb. In white the Landau level filling factors $\nu$ are indicated (positive for electrons, negative for holes). With the dashed box an unconventional modulation of the Landau levels are highlighted, which only appears for inverted band structures. The modulations in the resistivity are related to crossings and anti-crossings of Landau levels. Numerical simulations based on $k \cdot p$-calculations relate this to the influence of the spin-orbit coupling on crossings or anti-crossings of the overlapping Landau level spectra [46].

Only wafer materials of highest quality show the unconventional Landau level spectrum. But also in absence of the modulation, the Landau fan can be used to verify the band alignment by calculating the band overlap $E_{g0}$. $E_{g0}$ is schematically explained in Fig. 6.2. There are two ways to estimate $E_{g0}$:

- Extrapolating the Landau levels linearly to $B_\perp = 0$ T can be used to find the extrema of electron $V_{tg}^{+CB}$ and hole $V_{tg}^{HB}$ bands. An inverted band alignment leads to $V_{tg}^{+CB} < V_{tg}^{VB}$.

- Often the electronic Landau fan is clear, whereas the low hole-mobility hinders
Chapter 4. Experimental signatures of the inverted band structure

to see pronounced hole Landau levels. In this case we can use the assumption of parabolic dispersions for electron \(m^* = m_e^*\) and hole \(m^* = -m_h^*\) bands \(DOS = \frac{2m^*}{\pi\hbar^2}\) to compensate for the missing valence band extrema. Additionally, the top gate voltage \(V_{\text{tg}}^{\text{CNP}}\) at charge neutrality is needed to estimate the band overlap:

\[
E_{g_1} = (V_{\text{tg}}^{\text{CNP}} - V_{\text{tg}}^{\text{CB}}) \frac{\Delta n}{\Delta V_{\text{tg}}^{\text{CNP}}} \frac{1}{DOS} = C
\]

where \(C\) is the capacitance of the top gate and \(E_{g_0} = E_{g_1} + E_{g_2}\) with \(E_{g_1}\) \((E_{g_2})\) is the energy difference from conduction (valence) band extrema to the CNP.

\[
\frac{E_{g_1}}{m_e^*} = \frac{m_h^*}{\pi\hbar^2}
\]

\[
E_{g_0} = E_{g_1}(1 + \frac{m_h^*}{m_e^*})
\]

4.4 Conclusion to the identification of the inverted insulator regime

This chapter explained three fingerprint experiments to verify the inverted band alignment even if the full phase diagram including metallic, normal and inverted insulator regimes can not be studied due limited top and/or back gate tunability. We can nail down the inverted insulator regime with

- The co-existence of electrons and holes at charge neutrality.
- A vanishing CNP peak with increasing parallel magnetic field.
- The observation of an unconventional Landau fan spectrum.

This might be an incomplete list. But all these indicators could be investigated on standard Hall bar devices along with other transport properties discussed in this thesis.
Chapter 5

The influence of strain

Strain results from a deviation of the crystal lattice constants from their equilibrium values and directly influences the band structure and the electronic properties of a semiconductor.

In this chapter we first explain the influence of strain in a general mathematical approach for which the compliance tensor is introduced. The latter will be used in a study that demonstrates the influence of strain on the electronic properties of the InAs/GaSb double quantum well system. This work combines band structure calculations that incorporate strain and experiments with artificially strained InAs/GaSb systems mounted onto piezo elements. The chapter will be concluded by a review on optimizing the hybridization gap with the means of growth technologies.

Parts of this chapter have been published in the article:

Impact of strain on the electronic properties of InAs/GaSb quantum well systems
Physical Review B 95, 115108 (2017)

*The first two authors state that they contributed equally to this work.

5.1 Strain tensor

The mathematical introduction follows the book "Fundamentals of Semiconductors" by Peter Y. Yu and Manuel Cardona [49] and the lecture notes of E. Y. Tsymbal [50].

Every physicist remembers from introductory classes in solid state physics that due to energy minimization atoms arrange into periodic crystals. Forces acting on such crystalline materials will lead to deformations. Following Hooke’s law, a force
Chapter 5. The influence of strain

Figure 5.1: Mathematical explanation of compression and shear stress with a schematic illustrations.

$F$, which leads to a small deformation, is proportional to the resulting displacement $u$ of the equilibrium position of the atoms. It is more common to name the force $F$ per unit area $A$ the stress vector $\sigma_{ij}$ and to define the deformation of the crystal as a relative displacement with a dimensionless vector called strain $\varepsilon_{kl}$.

$$\sigma_{ij} = \sum_{ijkl} C_{ijkl} \varepsilon_{kl} \quad (5.1)$$

where the proportionality is described by the stiffness tensor $C_{ijkl}$ and each index can represent an orientation (e.g., in three dimensions $i, j, k$ and $l$ stand for the coordinates $x, y$ or $z$).

The components of the stress vector $\sigma_{ij}$ can be divided into compression and shear stress as explained in Fig. 5.1 and are defined as:

$$\sigma_{ij} = \frac{F_i}{A_j}$$

The strain is defined in an analogous way:

$$\varepsilon_{ij} = \frac{d u_i}{d x_j}$$

There are six independent components for the strain as well as for the stress vector since angular acceleration inside the crystal has to be compensated. That is to say that components with permuted indices are equal. Note the commonly used short form for the indices: $1 = xx$, $2 = yy$, $3 = zz$ and $4 = yz$, $5 = zx$, $6 = xy$. The general stress-strain relation reads therefore:
5.2 Interest of strain in InAs/GaSb double quantum well systems

The stiffness tensor \( C_{ijkl} \) has 36 components. If they are known, the crystal’s behavior under external stress can be calculated. As a first approach it is helpful to take the crystal symmetries into account to further reduce the number of independent components of this tensor. For example it can be shown that the compliance tensor of a cubic crystal (e.g. zinc-blende structure) can be represented by only three independent variables [49, 50]: \( C_{11}, C_{12} \) and \( C_{44} \).

\[
C_{\text{zinc-blende}} = \begin{bmatrix}
C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\
C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\
C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{44}
\end{bmatrix}
\] (5.2)

It is trivial to see that an analogue of equation 5.1 can be written as

\[
\varepsilon_{ij} = \sum_{ijkl} S_{ijkl} \sigma_{kl}
\] (5.3)

The inverse of the stiffness tensor \( C_{ijkl} \) is the compliance tensor \( S_{ijkl} \).

5.2 Interest of strain in InAs/GaSb double quantum well systems

Strain influences the band structure and the electronic properties of a semiconductor. The application of external strain can lift the valley degeneracy as observed in AlAs quantum well systems [51], or change the spin decoherence times through the manipulation of the quadrupolar splitting in GaAs [52, 53]. Epitaxial growth of alternating materials with different lattice constants results in intrinsically strained systems with modified electronic properties, which is, for example, exploited in modern high electron mobility field effect transistors.

Here we study the influence of strain on the electronic properties of the InAs/GaSb double quantum well system. In a heterostructure consisting of an InAs and a GaSb quantum well (QW), a hybridization gap opens up when the respective QW widths are fine-tuned to bring the lowest conduction band state in the InAs QW energetically below the highest valence band state in the GaSb QW because of hybridization of the electron and hole wave functions [22, 23, 54]. Experimentally,
however, InAs/GaSb samples generally do not become insulating when the carrier density is tuned across the hybridization gap [10, 24, 34, 55–58] but exhibit a finite resistance instead. This semimetallic behavior has been attributed to a variety of mechanisms such as to the anisotropy of the heavy hole band [10] or a perturbation of hybridization by scattering [23, 59]. In two recent works, pseudomorphically-strained InAs/In$_x$Ga$_{1-x}$Sb samples were used to study the impact of constant strain on this material system and its property as a quantum spin Hall insulator [60, 61]. For more information on strain induced by epitaxial growth see section 5.7.

We take a more direct approach that exploits strain as a tunable parameter. By using a combination of band calculations that incorporate strain and experiments with artificially strained InAs/GaSb systems mounted to piezo elements, we demonstrate that strain can not only be responsible for the semimetallic behavior but also strongly influences other transport properties of this material system. More specifically, our data show that InAs/GaSb can generally be a semimetal with a finite conductance at the charge neutrality point due to intrinsic strain originating from the lattice mismatch between InAs and GaSb and the buffer material underneath. We use external strain applied via a piezo ceramic to further modify the electronic properties, including the charge carrier density and observe changes in magneto-transport. Since the hybridization gap only persists in a limited range of InAs and GaSb thicknesses, semimetallic behavior will also arise as a result of inadequate quantum well thicknesses. Here, we limit our investigation to a single system that is not fully insulating. This facilitates the study and detection of the changes in resistance near the charge neutrality point due to strain.

### 5.3 Sample details for strain engineering

We have investigated four samples from two different wafers to guarantee the reliability of our data and the validity of our conclusions. Samples 1, 2 and 4 were fabricated from wafer #3, whereas sample 3 was fabricated from wafer #1. The heterostructures (see inset of Fig. 5.2(a)) are nominally the same for both wafers and were grown on GaAs substrates. The heterostructures consist of an 8 nm GaSb layer on top of 15 nm InAs [24]. Beneath, 50 nm AlSb, a 50 nm superlattice of AlSb/GaSb, 500 nm GaSb, a layer of Al$_x$Ga$_{1-x}$Sb ($x = 0.65$) and additional buffer materials are used to ease the lattice mismatch with the GaAs substrate.

The sample preparation begins with a standard optical lithography process and a chemical etching following the process detailed in Ref. [34] or appendix A to define a Hall bar structure. Ohmic contacts were obtained by optical lithography and the evaporation of Au, Ge and Ni. The devices were then passivated with 200 nm of Silicon Nitride Si$_3$N$_4$ deposited by Plasma Enhanced Chemical Vapor Deposition (PECVD) at a temperature that also acts as an annealing step for the Ohmic contacts. Si$_3$N$_4$ serves as the dielectric for a Ti/Au top gate, produced by evaporation through a shadow mask. The top gate is used to tune the system
5.4 Strain magnitude and direction

from the electron transport regime in the conduction band (InAs) through charge neutrality point into hole transport in the valence band (GaSb). The sample surfaces were then protected with a thick layer of optical photo resist and thinned from the back side in a chemical-mechanical bromine methanol etching process to 50-70 µm. To screen the large electric fields from these piezos which are driven with up to 200 volts, a 100 nm thick layer of Cr/Au was evaporated on the back side of each sample. This screening layer is grounded during the experiments. Then, the samples were transferred to a ceramic chip carrier with a removable base plate for wire bonding.

We use lead-zirconium-titane piezo-ceramic stacks which are specifically designed for use in low-temperature and vacuum conditions (Piezomechanik GmbH Munich, Germany). Its piezoelectric coefficient $d_{33}$ is positive, i.e. positive piezo voltages will lead to an elongation while negative voltages lead to its contraction along the poling direction. Each piezo stack has a nickel-chromium strain gauge (Vishay Precision Group, linear pattern type) glued to one of its insulated sides. The glue is a two-component epoxy with a thermal characteristic specified down to 4 kelvin (M-BOND 610, Vishay Precision Group). The resistance of the gauge changes in response to length variations of the piezo. Figure 5.2(a) shows a simplified schematic. The rear face of the piezo stack is mounted to a sample holder with the strain gauge facing down. To use standard bonding techniques, the sample was first bonded in a conventional chip carrier with removable base plate (piezo stacks are sensitive to pressure and therefore bonding on the piezo has to be avoided). After removing the base plate of the chip carrier, the sample hangs freely on its bonding wires and it can be glued to the piezo stack.

5.4 Strain magnitude and direction

The physics of the InAs/GaSb double QW system is governed by the hybridization of the lowest subband in the InAs QW and the highest subband in the GaSb QW, whose energies are determined by the respective well widths. When the piezo elongates (contracts) the sample along [001] or [011] (Figs. 5.2(a) and (b)), it will shrink (expand) in growth direction, i.e. along [100]. This will slightly modulate the quantum well widths, their energy levels and the resulting hybridization.

In addition to the external strain that we apply with our piezos, the double quantum well system is subject to intrinsic biaxial strain due to the epitaxial pseudomorphic growth of different semiconductor materials with different bulk lattice constants (AlSb 6.1355 Å, GaSb 6.0959 Å, InAs 6.0583 Å at 300 K). For simplicity we denote both intrinsic and external strain as $\varepsilon$. In section 5.4.1, we estimate the intrinsic strain for the ideal situation of a complete relaxation of all materials to the GaSb lattice constant. In this case, only the InAs quantum well is strained ($\varepsilon_{\text{InAs}} \approx 54 \times 10^{-4}$), whereas the GaSb quantum well is unstrained ($\varepsilon_{\text{GaSb}} = 0$). The tunable external strain, which we estimate to be in the range $-1.3 \times 10^{-4} < \varepsilon < +1.7 \times 10^{-4}$ ($\Delta \varepsilon \approx 3 \times 10^{-4}$) at low temperatures, is added to the intrinsic strain and will modify
Figure 5.2: (a) Schematics of the piezo stack with the strain gauge underneath and the sample mounted on top. The Hall bars are always aligned along the piezo’s direction of compression (for negative voltages) and elongation (for positive voltages). The inset shows the heterostructure. (b) The Hall bars were either patterned to be orientated along the [011] or the [001] direction. Shown is a top view of the sample’s (100) surface. (c) Demonstration of the perfect linearity of our piezo ceramics, exemplarily showing the strain gauge resistance change at a temperature of 30 mK for an upsweep (red curve) and downsweep (blue curve) of the piezo voltage.

or even enhance the lattice distortion due to the lattice mismatch at the InAs/GaSb interface [62]. Please note that the piezo adds strain in a non-trivial way, e.g. when the piezo elongates in one direction, it will shrink in its perpendicular direction just like the sample which is glued to its surface. That means that the piezo strain is not uniaxial. These effects have been carefully included into our band structure model by calculating how the piezo’s perpendicular dimension will change [63] and how this is transferred to the sample. For further information on this calculation see section 5.4.2.

We will demonstrate below that despite the smallness of the external strain exerted by the piezo electric elements, the resistivity at the charge neutrality point and other transport properties can be modified.

5.4.1 Internal strain estimate

We estimate the intrinsic strain $\varepsilon$ due to the pseudomorphic growth of materials with different lattice constants following Ref. [64], where:

$$\varepsilon = \frac{a_{\parallel} - a_1}{a_{\parallel}}$$
5.4. Strain magnitude and direction

with

\[ a_\parallel = \frac{a_1 d_1 + a_2 d_2}{d_1 + d_2}, \]

while ignoring the elastic compliance constants. Here \( a_1 \) is the lattice constant of the material 1 to be grown on to the material 2 with the lattice constant \( a_2 \). The constants \( d_1 \) and \( d_2 \) are the corresponding layer thicknesses. The lattice constants we use are AlSb 6.1355 Å, GaSb 6.0959 Å, and InAs 6.0583 Å. In a first approximation, we can assume that zinc-blende AlSb, GaSb and InAs exhibit comparable thermal expansion so that thermal effects on the lattice constants are canceled out in the estimate of \( \varepsilon \).

The double quantum well structure consists of an 8 nm GaSb layer on top of 15 nm InAs. Beneath, 50 nm AlSb, a 50 nm superlattice of AlSb/GaSb, 500 nm GaSb, a 1100 nm thick layer of Al\(_x\)Ga\(_{1-x}\)Sb (\( x = 0.65 \), lattice constant ca. 6.12 Å) and additional buffer materials are used to ease the lattice mismatch with the GaAs substrate. We assume that InAs grows pseudomorphically on the first two buffer layers (50 nm AlSb and 50 nm total thickness of a AlSb/GaSb superlattice), which are thin and have adapted to the lattice constant of the GaSb underneath, i.e. also InAs is intrinsically strained. This assumption is based on estimates for the critical thickness for relaxation given in Refs. [65, 66]. For 15 nm InAs on a 100 nm thick layer with a lattice constant of 6.0959 Å this results in a strain of approximately \( \varepsilon \approx 54 \times 10^{-4} \). If the InAs layer has adopted the lattice constant of GaSb, then the 8 nm GaSb quantum well will grow unstrained (no strain).

The estimate for the external strain exerted via the piezo elements is based on the manufacturer’s specifications of strain/volt efficiency, the maximal stroke (change in length) at room temperature. At room temperature, the maximal strain from the piezo is about \(-3.1 \times 10^{-4} < \varepsilon < +15.5 \times 10^{-4}\) (based on the specifications by the manufacturer). At low temperatures, the strain/volt efficiency drops to a value not specified by the manufacturer. At the same time, however, the piezo allows bipolar operations over much larger voltages to compensate for the reduction in strain/volt efficiency. Since we measure the strain gauge resistance at room temperature and low temperature, we can compare both measurements to find the absolute reduction in strain/volt efficiency, while taking into account the gauge’s gauge factor and thermal output, i.e. the correction due to thermal changes [67]. We estimate the strain range exerted by the piezos at 20 mK to be of the order of

\[-1.07 \times 10^{-4} < \varepsilon < +1.83 \times 10^{-4} \text{ (} \Delta \varepsilon = 2.90 \times 10^{-4}\) for sample 1\]
\[-1.32 \times 10^{-4} < \varepsilon < +1.75 \times 10^{-4} \text{ (} \Delta \varepsilon = 3.07 \times 10^{-4}\) for sample 2\]
\[-1.41 \times 10^{-4} < \varepsilon < +1.89 \times 10^{-4} \text{ (} \Delta \varepsilon = 3.30 \times 10^{-4}\) for sample 3\]
\[-1.25 \times 10^{-4} < \varepsilon < +1.65 \times 10^{-4} \text{ (} \Delta \varepsilon = 2.90 \times 10^{-4}\) for sample 4\]

Our values are in good agreement with Ref. [63], but we want to stress that strain measured by the gauges are influenced by the thermal properties of the glue, which differs from the one used in the cited reference.
5.4.2 Pseudo uniaxial strain caused by a piezo

As already mentioned above the piezo adds strain in a non-trivial way, e.g. when the piezo elongates in one direction, it will shrink in its perpendicular direction just like the sample which is glued to its surface. That means that the piezo strain is not uniaxial. This chapter will explain, how these effects have been taken into account for the band structure calculations.

The piezo stack as well as the used sample heterostructure consist of III-V materials. A useful starting point is therefore the stiffness tensor of cubic crystals as explained in section 5.1, equation 5.2. Here shown again:

\[
C_{\text{zinc-blende}} = \begin{bmatrix}
C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\
C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\
C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{44}
\end{bmatrix}
\]

The Poisson’s ratio is defined by the negative ratio of shear and compression strain.

\[
\nu := -\frac{S_{12}}{S_{11}}
\]

With the compliance tensor \( S_{\text{zinc-blende}} = C_{\text{zinc-blende}}^{-1} \) the Poisson’s ratio \( \nu \) can be calculated:

\[
\nu = \frac{C_{12}}{C_{11} + C_{12}}
\]

We use the same piezo stacks as tested in Ref. [63], which finds a Poisson’s ratio of \( \nu_{\text{piezo}} = \frac{1}{2\pi} \). Therefore, the piezo extends along the poling direction and at the same time contracts perpendicularly. We can conclude for the InAs/GaSb sample on top of the piezo, that the strain in the xy-plane is determined by the piezo, and the z component of the strain is material dependent. The latter findings will set the number of independent variables for the strain vector \( \varepsilon \).

For the [001] direction:

\[
\varepsilon = \begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{zz} \\
\varepsilon_{yz} \\
\varepsilon_{xz} \\
\varepsilon_{xy}
\end{bmatrix} = \begin{bmatrix}
\varepsilon_{xx} \\
-\varepsilon_{xx} \cdot \nu_{\text{piezo}} \\
\varepsilon_{zz} \\
0 \\
0 \\
0
\end{bmatrix}
\]
5.4. Strain magnitude and direction

Additionally, stress \( \sigma \) applied by the piezo on the sample can be described by:

\[
\sigma = \begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\sigma_{yz} \\
\sigma_{xz} \\
\sigma_{xy}
\end{bmatrix} = \begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

With the relation \( \sigma = C_{\text{zinc-blende}} \times \varepsilon \), we find the strain tensor in the general form:

\[
\varepsilon = \begin{bmatrix}
\varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\
\varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz} \\
\varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz}
\end{bmatrix} = \begin{bmatrix}
\varepsilon_{xx} & 0 & 0 \\
0 & -\varepsilon_{xx} \cdot \nu_{\text{piezo}} & 0 \\
0 & 0 & \frac{C_{12} \varepsilon_{xx} (-1 + \nu_{\text{piezo}})}{C_{11}}
\end{bmatrix}
\] (5.4)

The stiffness constants for GaSb at low temperature (here 0 K taken for the calculation) [68] are

\[
C_{11} = 9.07; C_{12} = 4.13; C_{44} = 4.45
\]

With the strain gauge mounted underneath the crystal, we will be able to measure the amount of strain in percent \( s_p \) along the pulling direction, which will fix the only free parameter \( \varepsilon_{xx} \).

For the numerical simulation of the band structure, as explained in section 5.6, we need the changed basis vectors due to strain. The original basis vectors for the zinc-blende material are:

\[
\vec{a}_1 = \begin{pmatrix} 0 \\ 1/2 \\ 1/2 \end{pmatrix}; \vec{a}_2 = \begin{pmatrix} 1/2 \\ 0 \\ 1/2 \end{pmatrix}; \vec{a}_3 = \begin{pmatrix} 1/2 \\ 0 \end{pmatrix}
\]

Multiplied by the strain tensor (5.4) derived above, we find

\[
\vec{a}_1^\prime = \vec{a}_1 + \frac{s_p/100}{2} \begin{pmatrix} 0 \\ -\nu_{\text{piezo}} \\ \frac{-\nu_{\text{piezo}}}{C_{12} \varepsilon_{xx} (-1 + \nu_{\text{piezo}})} \end{pmatrix}
\]

\[
\vec{a}_2^\prime = \vec{a}_2 + \frac{s_p/100}{2} \begin{pmatrix} 1 \\ 0 \\ \frac{C_{12} \varepsilon_{xx} (-1 + \nu_{\text{piezo}})}{C_{11}} \end{pmatrix}
\]

\[
\vec{a}_3^\prime = \vec{a}_3 + \frac{s_p/100}{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\]
Chapter 5. The influence of strain

For the [011] direction:

The strain calculation for the [011] direction follows the same idea as for [001]. Whereas it is intuitively clear that now stress is not just applied in compression ($\sigma_{xx}$ and $\sigma_{yy}$) but also as a shear component $\sigma_{xy}$, the strain vector needs some further explanation. The piezo will lead to a superposition of an elongation along [011] with $\varepsilon_{11}$ and a contraction perpendicular to [011] with $-\varepsilon_{11}\nu_{\text{piezo}}$. As shown in Fig. 5.3, a simple geometric consideration explains

$$
\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{xy} = \varepsilon_{11} - \varepsilon_{11}\nu_{\text{piezo}} = \varepsilon_{11}(1 - \nu_{\text{piezo}})
$$

(5.5)

Figure 5.3: Schematic to explain the strain component along the $x$-axis $\varepsilon_{xx}$, if a cubic crystal is strained along [011] with $\varepsilon$. With this geometrical calculation it is clear that $\varepsilon = \varepsilon_{xx}$.

With this explanation we find

$$
\sigma = \begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
0 \\
0 \\
\sigma_{xy}
\end{bmatrix} ; \varepsilon = \begin{bmatrix}
\varepsilon_{11}(1 - \nu_{\text{piezo}}) \\
\varepsilon_{11}(1 - \nu_{\text{piezo}}) \\
\varepsilon_{zz} \\
\varepsilon_{yz} \\
\varepsilon_{zx} \\
\varepsilon_{11}(1 - \nu_{\text{piezo}})
\end{bmatrix} ; \sigma = C_{\text{zinc-blende}} \times \varepsilon
$$

And therefore the strain tensor in the general form for pulling with a piezo along [011] reads
\[ \varepsilon = \varepsilon_{11}(1 - \nu_{\text{piezo}}) \begin{bmatrix} 1 & 1/2 & 0 \\ 1/2 & 1 & 0 \\ 0 & 0 & -2C_{12}/C_{11} \end{bmatrix} \] (5.6)

Resulting in strained basis vectors of the zinc-blende crystal structure, if elongated along the [011] direction by \( s_p \) (strain in percent):

\[
\vec{a}_1^s = \vec{a}_1 + \frac{s_p}{100} \cdot (1 - \nu_{\text{piezo}}) \begin{bmatrix} 1/4 \\ 1/2 \\ -\frac{C_{12}}{C_{11}} \end{bmatrix}
\]

\[
\vec{a}_2^s = \vec{a}_2 + \frac{s_p}{100} \cdot (1 - \nu_{\text{piezo}}) \begin{bmatrix} 1/2 \\ 1/4 \\ -\frac{C_{12}}{C_{11}} \end{bmatrix}
\]

\[
\vec{a}_3^s = \vec{a}_3 + \frac{s_p}{100} \cdot (1 - \nu_{\text{piezo}}) \begin{bmatrix} 3/4 \\ 3/4 \\ 0 \end{bmatrix}
\]

5.5 Transport measurements under the influence of strain

The samples were cooled down with a 10 MΩ resistor between the piezo connectors to avoid thermal voltage build-up and a possible damage of the piezo stacks. At room temperature and prior to the measurements, the piezo voltages were swept several times to test their performance by monitoring the strain gauge. While piezo mechanical stacks show hysteresis and non-linear behavior at room temperature, at low temperatures these effects vanish and linear bipolar operation over large voltage ranges is possible [63].

Figure 5.2(c) illustrates the 4-terminal resistances of a strain gauge versus piezo voltage at 30 mK. We would like to stress that the strain gauge can obviously only detect relative changes due to the elongation or contraction of the piezo stack; it is insensitive to the intrinsic strain resulting from the lattice mismatch in the sample.

Figure 5.4(a) shows the longitudinal resistances \( R_{xx} \) versus the top gate voltage \( V_{tg} \) at zero magnetic field \( B \) and Fig. 5.4(b) at 7 T for the two maximal piezo voltages that were applied, i.e. for the maximal elongation and compression of the piezo. In this sample (sample 4) the Hall bar is oriented along the [011] direction. For each piezo voltage, we have performed two consecutive gate voltage sweeps in the same direction and observe perfect reproducibility (see Fig. 5.5).

The variation of the top gate voltage changes the carrier concentration and carrier type by shifting the Fermi energy from the conduction band through the hybridization gap into the valence band. At finite magnetic fields, the density of states splits into dispersionless Landau levels and integer quantum Hall effect emerges [69], see
Figure 5.4: The longitudinal resistance $R_{xx} = V_{xx}/I$ versus gate voltage at two exemplary magnetic fields at $B = 0$ T (a) and $B = 7$ T (b) at 20 mK for the maximal piezo voltages, i.e. for maximal compressive strain and maximal tensile strain (sample 4, Hall bar aligned in the [011] direction). For $\Delta V_{\text{piezo}} = 350$ V, we estimated the strain range to be of the order of $3.3 \times 10^{-4}$ (see section 5.4.1). Each sweep was repeated to demonstrate the stability (Fig. 5.5). (c) and (d) Blow-ups of the shifts in the Landau level positions. The insets show $R_{xx}$ vs. piezo voltage at a fixed gate voltage (indicated by the nearby arrow). (e) Illustration on how the carrier densities (filling factors) respond to the strain. Changing the piezo voltage from -150 V to +200 V at a fixed gate voltage results in an increase in the electron density but a decrease in the hole density.

Fig. 5.4(b). Varying the top gate voltage at constant magnetic field of 7 T empties or fills the Landau levels while their degeneracy remains constant. For positive voltages, $V_{tg} > 0$ V, at $B = 7$ T we observe Landau level quantization for the electrons in the InAs QW, which is signaled by the well-known Shubnikov-de-Haas oscillations. Close to the resistance maximum around $V_{tg} \approx -2.5$ V the Fermi energy is expected to reside in the hybridization gap. However, we do not find the sample to become...
5.5. Transport measurements under the influence of strain

Figure 5.5: Additional exemplary measurements at 2 T (sample 4). At each magnetic field and each piezo voltage, we have performed two consecutive gate voltage sweeps to ensure the stability/reliability of the measurements. The right panel shows a blow-up of the marked area.

insulating. For $V_{tg} < -4$ V, the onset of the integer quantum Hall effect of holes is seen, which is less pronounced than for electrons. This difference originates from the higher effective mass of the holes in the GaSb quantum well, resulting in smaller mobilities and cyclotron energies for the holes. At very low densities, i.e. for gate voltages near the charge neutrality point, these differences become very pronounced in magneto-transport.

We find two distinct effects that result from changes in the in-plane strain: (1) a dependence of the peak height at the charge neutrality point (Figs. 5.4(a) and (b)), and (2) shifts in the position of the resistivity minima (blow-ups for $B = 7$ T in Figs. 5.4(c) and (d) indicate these shifts). The impact on the resistance at the charge neutrality point is directly seen in Figs. 5.4(a) and (b), while the shifts in the minima is only seen upon closer inspection in Figs. 5.4(c) and (d). The inset in Fig. 5.4(b) demonstrates that the resistance at the charge neutrality point scales linearly with the piezo voltage when the gate voltage is fixed to a value marked by the arrow. Also the shift in the longitudinal resistance displays a linear dependence on piezo voltage (insets in Figs. 5.4(c) and (d)). At the charge neutrality point, we observe some hysteretic behavior between up- and downsweeps of the piezo voltage. These two effects, i.e. the variations of the resistance near the charge neutrality point and the aforementioned shift, appear consistently within the range of magnetic fields between 0 T and 12 T and in all samples, however, at magnet fields exceeding 9 T, strong “noise” is overlaid to resistance at the charge neutrality point, see Fig. 5.6. Both effects result from the band distortion upon strain which we will address in the next section.

To ensure that these effects are really the result of strain and not an artifact arising from stray electric fields from the piezo for example, a fifth sample was
prepared, which rested on the piezo stack but was only glued to it at one of its edges. In this test sample, which was not influenced by strain, we did not observe any effect on the transport properties of the sample when the piezo was driven with large positive or negative voltages. This demonstrates that the observations described above are purely strain-related.

The significance of the resistivity minima shift in gate voltage is illustrated in Fig. 5.4(e). Generally, sweeping the gate voltage results in a linear response of the carrier density. The piezo-strain induces variations of the (intrinsic) carrier density in the sample, which in turn results in a shift of the density-voltage curve, i.e. under strain different gate voltages are needed to access the same filling factors. For electrons, the density increases upon sample elongation and decreases upon its compression, while for the holes, we observe the opposite behavior. From these shifts and from additional measurements of the Hall resistance we have extracted the change in carrier density arising from strain.

Figure 5.7 summarizes the measurements performed on all four samples, including measurements on two Hall bars that are aligned along the [001] direction. The density change, $\Delta n$, only depends on the piezo strain and the sample orientation, with samples aligned along [011] displaying the strongest response to strain. As expected, $\Delta n$ is independent of the magnetic field (Figure 5.7(a)). Within our measurement uncertainty, we find that electron and hole densities change by approximately the same absolute value. The [011] direction of the zinc-blende unit cell is not a high symmetry axis, which makes it more susceptible to symmetry breaking strain; a behavior we observe in the experiments. This susceptibility to strain is reflected in the larger number of terms in the tight-binding model that we use to numerically calculate the band structures below.

Figure 5.7(b) shows the change in “peak” resistance around the charge neutral-
5.5. Transport measurements under the influence of strain

Figure 5.7: (a) Absolute density change for electrons and holes between maximal tensile and compressive strain versus magnetic field. $\Delta n$ is positive (negative) for electrons (holes). The maximal external strain for each sample, $\Delta \varepsilon = (3.04 \pm 0.16) \times 10^{-4}$ (a dimensionless number), varies only slightly between samples. Each color corresponds to one sample. Samples 1 & 2 (3 & 4) with the Hall bar aligned in the [001] ([011]) direction. (b) $(R_{150V} - R_{200V})/R_{0V}$, i.e, the normalized change in the peak resistance close to the charge neutrality point between maximal tensile strain and maximal compressive strain. The peak resistance slightly shifts with the magnetic field, $B$. At large $B$, strong fluctuations in the resistance appear (see Fig. 5.6).
ity point between maximal tensile and maximal compressive strain, normalized to its resistance when no external strain was applied. Although the density does not depend on the magnetic field, we found that the top gate voltage corresponding to the peak resistance slightly shifts with $B$, which results in a very different magnetic field dependence with respect to Fig. 5.7(a). We have no consistent picture to explain this behavior. However, in this very low density regime, the field-dependence of the Coulomb energy may induce fluctuations in the localization and screening, which we observed as the behavior shown in Fig. 5.7(b).

To understand and interpret these results, we have performed numerical band structure calculations for a strained InAs/GaSb system.

5.6 Band structure calculations and analysis

Numerical simulations of strained and unstrained quantum wells were performed using symmetrized Wannier-based tight-binding (TB) models. First, symmetrized TB models with long-range hoppings are constructed for bulk InAs and GaSb [70]. To approximate the exchange correlations, a first-principles simulation of a bulk material is carried out using Heyd-Scuseria-Ernzerhof (HSE) hybrid functional without spin-orbit coupling (SOC). The resultant spectrum within a specially chosen energy window is then projected onto the In (Ga) s- and p-states and As (Sb) p-states to construct a $7\times7$ Wannier-based TB model [71, 72]. This model is symmetrized to satisfy all the spatial symmetries of the zinc-blende $T_d$ crystal structure. Local SOC is added separately to this model in such a way that the spectrum of the resultant TB model coincides with a HSE simulation with SOC [70]. We carried out the HSE hybrid functional [73] simulations using the VASP software package [74]. We took the 300 K lattice constants and used the energy gap measured at 4 K to fit a screening parameter in the hybridization functional method [75]. The screening parameters used in the HSE method were adjusted to 0.2 and 0.15 for InAs and GaSb, respectively, such that the experimental band gap is fitted [76]. This method accounts for the thermal effects on the lattice constants upon cooling. The Perdew-Burke-Ernzerhof (PBE) functional for exchange-correlation [77] and projector augmented waves (PAW) [78, 79] pseudopotentials were employed, with SOC implemented in the pseudopotentials. The energy cut-off was 380 eV for both InAs and GaSb. We used a $6 \times 6 \times 6$ $\Gamma$-centered $k$-mesh and Gaussian smearing of width 50 meV for Brillouin zone integrations. To simulate the interface between the two quantum well materials, we interpolated [80] the hopping parameters. This procedure provides reasonable agreement with experiment for InAs/GaSb quantum wells [80].

The underlying strain model maps the crystal structure of an idealized InAs/GaSb system, in which the two quantum wells were grown pseudomorphically, with InAs having adopted the lattice constant of GaSb. Comparison of the experimental data with theory will later show that in our samples also the GaSb quantum well is subject
to a small amount of growth-related strain. Additive external strain is incorporated into our model by calculating the response of the primitive lattice vectors to (uniaxial) strain in the [001] and [011] directions and by defining the strain tensor with the elastic stiffness constants for InAs and GaSb. Symmetrized TB models were then constructed for the strained bulk materials first, following the procedure described above. After interpolation to simulate the interface, the resultant TB models are used to obtain the band structure and density of states (DOS) for various values of strain for the InAs/GaSb quantum wells.

![Diagram](image)

Figure 5.8: (c) Band structure without external strain. (a) and (b) Under compression by -0.5% along [001] and [011]. (d) and (e) Under elongation by +0.5% along [001] and [011]. Note that the in-plane wave vector is only displayed with its two in-plane directions.

We have performed TB band structure calculations for several amounts of compression and elongation along [001] and [011] by the piezo and for the same heterostructure as in our experiments. Figure 5.8 shows a selection of exemplary results. Our band structure calculations use the $C_{2v}$-symmetry for the zinc-blende structure, which does not display radial symmetry. For that reason, the band structure in the two orthogonal directions in $k$-space will differ. The center figure (c), represent the growth condition without external strain. Figures 5.8(a) and (b) show the band structures when the unit cell is compressed by -0.5% along [011] or [001], while Figs. 5.8(d) and (e) show the band structures for a corresponding elongation by +0.5%. Because the intrinsic strain in the InAs layer surpasses the external strain, $\varepsilon_{\text{InAs}}$ is always positive, whereas for GaSb, $\varepsilon_{\text{GaSb}}$ can be both positive or neg-
ate. Even without any detailed analysis, the strong band distortion for only 0.5\% strain implies a dramatic impact on the transport properties of this material system. Figure 5.8(c) implies that an 8 nm GaSb / 15 nm InAs double quantum well system [10, 24] is essentially semimetallic, even when no external strain is applied. For future experiments, we suggest an 8 nm GaSb / 12 nm InAs double quantum well system instead which by contrast displays a well-developed mini-gap, see Fig. 5.9.

![Figure 5.9: Results of TB band calculations for an 8 nm GaSb / 12 nm InAs heterostructure using the same strain model as for our regular structure, i.e. the GaSb and the InAs quantum wells grow pseudomorphically with the same GaSb lattice constant. In contrast to the 8 nm / 15 nm structure, we now observe a clear mini band gap.](image)

From the band structures, we calculated the respective DOS, from which, in turn, we extracted the electron and hole densities. To extract the density changes from the density of states (DOS), we start by determining the onsets of electron and hole conduction, located at the parabolic minimum of the electron-like band (denoted as $E_1$) and the parabolic maximum of the hole-like band (denoted as $H_1$). Next, we determine the effective mass of the electrons directly from the constant part of the density of states. Integration of the DOS yields the total density of electrons and holes. With the previously determined onsets $H_1$ and $E_1$ we can separate the total density into the respective electron and hole densities and thus determine the charge neutrality point (CNP). The latter determines the zero density point and the offset for the DOS integration. By doing these calculations for the DOS at all strain values, we determine how the charge carrier density changes due to strain.

Figures 5.10(a) and (b) illustrate the theoretically expected densities as a function of the Fermi energy for different values of strain along [001] and [011]. At Fermi energies exceeding $\approx 0.47$ eV, the InAs QW is populated by electrons (while the GaSb QW is depleted) and at lower energies the GaSb QW is populated by holes (while the InAs is depleted). Figures 5.10(c) and (d) represent cross sections at two
5.6. Band structure calculations and analysis

Figure 5.10: (a) and (b) Electron and hole densities as a function of the Fermi energy for all calculated strain values and directions. The dashed lines at 0.46 eV and 0.52 eV are cross sections whose results are shown in subfigures (c) and (d). (c) Electron density change $\Delta n_e$ as a function of strain along [011] and [001]. (d) Hole density change $\Delta n_h$ as a function of strain along [011] and [001]. The magenta-colored vertical bar indicates the amount of strain $\Delta \varepsilon$ which is actually exerted by the piezo; its position is subject to an uncertainty (see main text for details).

fixed Fermi energies, i.e. they show how electron and hole densities change when strain is applied (at a fixed energy). While we calculated the band structures for strain ranging from -1% to +1%, the strain we can exert with our piezo electric elements, $\Delta \varepsilon$, is comparatively small, as indicated by the magenta-colored vertical bars.

Our band structure calculations and the subsequent analysis exhibit the same general response and directional dependence to strain as the experimental data, which demonstrates the overall reliability of our model. In particular, the stronger response shown for the [011] crystal direction is reproduced. For example, Fig. 5.10(c) shows the electron regime at 0.52 eV, where we find that strain along [011] yield a stronger increase in the electron density, or a much larger shift in the SdH minima, respectively. This is indeed what we observe experimentally. The picture is ambiguous for the holes since it is much more dependent on the energy at which we
Chapter 5. The influence of strain

scrutinize the system. Experimentally, we are only able to observe the onset of hole transport near the CNP, where electrons may still co-exist. Figure 5.10(d) depicts the hole density as a function of strain at 0.46 eV. For strain > 0 %, the model anticipates a decreasing hole density with a stronger response for strain along [011]. Also this is in agreement with experiment. Our piezo generates a ≈ 1.5 - 1.75 % electron density increase for [011], while theoretically at a Fermi energy of 0.52 eV we find 3 %. For the [001]-direction, the experimental values are ≈ 0.15 %, while theory predicts 1.4 % (at $E_F = 0.52$ eV). For much larger values of strain, which we cannot generate with our piezos, the model anticipates that the hole density should change more rapidly with $\varepsilon$ than the electron density. The experiments, however, do not reproduce the non-monotonic behavior of Fig. 5.10(d) around 0 % strain. Experimentally, we had observed a linear dependence in the hole regime on the piezo voltage instead (Figure 5.4(c)). We believe that the reason for this discrepancy lies in the AlSb-rich buffer material, whose lattice constant exceeds that of GaSb so that it already adds additional (intrinsic) strain to the InAs and GaSb QWs. Thus, when we apply external strain to our samples, we operate within the range indicated by the error bar (based on an estimate using the AlSb lattice constant, similar to section 5.4.1).

5.7 Strain with growth

As mentioned above, the epitaxial grown wafers have an AlSb-rich buffer material, which acts as a back gate dielectric due to its large band gap. But because of the lattice mismatch between AlSb and GaSb, this brings also the risk of internal strain. This section will elaborate on this challenge and demonstrates with optical microscope pictures and XRD scans, how to detect a surpass of the critical thickness of AlSb.

A typical heterostructure of an InAs/GaSb double QW grown on a GaSb substrate is schematically explained in the left inset of Figure 5.11 (not the same wafer as used for the samples on piezo stacks). All epitaxial grown materials belong to the so-called 6 Å family, meaning that they have a lattice constant close to 6 Å in common. This fact facilitates the ideal pseudomorphos epitaxial growth starting from a GaSb substrate and keeping this lattice constant for all materials. InAs as well as AlSb will therefore exhibit strain. For a certain critical thickness $t_c$ these materials can tolerate the assembled tension. Exceeding $t_c$ leads to dislocations, while the tension is relaxed [81]. The latter is sometimes visible as dislocation lines on the surface in optical microscope pictures (see Fig. 5.11, right inset) with polarizing filter. To study the growth quality in more detail, the material was examined with X-ray diffraction (XRD). This standard method enables among many other applications the study of the crystal structure. The importance of this technique is also rewarded by two Nobel prices, the first for the initial work to Max von Laue.
5.7. Strain with growth

Figure 5.11: Measurement and simulation of an InAs/GaSb double QW system with an AlSb-rich buffer conducted by Dr. Emilio Gini, FIRST Center for Micro- and Nanoscience, ETH Zürich. Growth sequence of the analyzed heterostructure schematically explained in the left inset. Both the dislocation lines as seen in the optical microscope picture (right inset, square size 50 × 50 µm) as well as the broad substrate peaks in the XRD measurement indicate that the critical thickness for AlSb on GaSb substrate was exceeded, resulting in crystal relaxations.

in 1914 and the second to William Henry Bragg and William Lawrence Bragg in 1915 for the improvement of the structural analysis of crystals. Figure 5.11 shows the XRD measurement on top of its respective material simulation. As expected, the GaSb substrate as well as the buffer materials AlSb and GaSb lead to the most pronounced peaks (indicated in figure with arrows), whereas the InAs QW is hard to see. Even though simulation and measurement agree qualitatively (except of the AlSb peak shift), the diffraction peaks are much broader then the theoretical expectation, a first hint for dislocations. Several facts indicate that the critical thickness of AlSb was passed. First, all diffraction peaks are broad suggesting a relaxation starting already in the buffer material (consisting of GaSb and AlSb only). Second, the only discrepancy between theory and measurement is the AlSb peak, suspecting a relaxed lattice constant. Third, the GaSb substrate wafer as well as the initial overgrowth of the substrate wafer with GaSb could be excluded as a possible reason with similar XRD analysis.

It shall be pointed out that as long as the critical thicknesses of each material is respected, the buffer layer might be strained, but the ideal pseudomorphos growth with the lattice constant of GaSb is realized for the double QW InAs/GaSb. As soon as the crystal starts to relax, the lattice constant deviates and dislocation can
facilitate leakage paths for the back gate.

The work presented above demonstrates that strain can cause a transition from an insulating state to a semi metallic regime. Additionally, recent work by the group of Prof. Du [61] and Prof. Muraki [60] studied the influence of strain in the GaSb QW, stressing that a good understanding of the influence of strain could help to improve the hybridization gap by a factor of five. Further studies could therefore not only eliminate the disadvantages of strain, but even improve the material system.

5.8 Interpretation and conclusion to strain

By using InAs/GaSb samples mounted to piezoelectric elements, we have studied the influence of strain as a variable parameter on this material system and found a susceptibility of the resistance in the vicinity of the charge neutrality point. The strain exerted by our piezos is very small as compared to the intrinsic strain from the pseudomorphic growth, which highlights the relevance of the growth parameters for the semimetallic behavior. Complementary band structure calculations revealed that larger strain can have a considerable impact on the band edge positions of the InAs/GaSb system and the carrier concentrations. We therefore conclude that in this material system strain plays a major role for the semimetallic behavior which is regularly observed in InAs/GaSb quantum well samples.

In this context, Zakharova et al. [82] have discussed the possibility of a strain-induced semimetal-semiconductor phase transition in InAs/GaSb quantum wells which would emerge on InAs substrates from the resulting lattice mismatch. In addition to inducing the semimetallic behavior, lattice-mismatched strain has been identified theoretically to influence the Landau level structure [83]. We indeed found that we can study the changes in the positions of the SdH minima resulting from variations in the carrier density which qualitatively agree with those density changes extracted from our band simulations. The density change shows a clear direction-dependence in favor of the [011] direction. Hall bars oriented along [001] do not exhibit large density variations but are still semimetallic because the intrinsic lattice mismatch is strong and ubiquitous.

The results of our theoretical and experimental study have shown that a true minigap does not only depend on the choice of the quantum well thickness but also on the strain the quantum wells are exposed to. Strain engineering is therefore an additional tool to improve the insulating bulk properties in InAs/GaSb.
Chapter 6

Attempts to reduce the bulk conduction

Historically the inverted InAs/GaSb system was expected to be semi-metallic [84] due to the overlap of the conduction and valence band. Later, Altarelli [85] mentioned for the first time the possibility of a gap formation due to mixing of electron- and hole-like bands. The expected hybridization gap of less than 10 meV was later also confirmed in experiments [48, 86]. Subsequently, the prediction of the QSHE in this material system by Liu et al. [5] put the focus of transport experiments on edge physics. It was expected that edge transport dominates as soon as the bulk is insulating. But, contrary to the expectations of an insulating state, transport experiments on standard Hall bar devices often show a finite resistivity at charge-neutrality in the kΩ-range. Transport measurements on such devices are found to be always bulk dominated, masking edge physics completely. In recent years, theoretical as well as experimental work demonstrated that the band structure around the hybridization gap is affected by slight differences in quantum well thickness, electric field, strain, or disorder.

This chapter focuses on different aspects of material engineering to optimize the hybridization gap or to suppress the bulk conduction. Unfortunately within the scope of this doctoral thesis it was not possible to test all options experimentally. The overview presented here is therefore a mixture of theoretical expectations, literature review, gedankenexperiments and experimental experience with the goal to facilitate future material growth planing. This chapter closes with an explanation to the disordered low mobility material used for the following edge studies.

6.1 Quantum well thicknesses

The first 2D material system proposed as a topological insulator is HgTe/(Hg,Cd)Te quantum wells. It’s characteristic band inversion between the s-like (conduction) band $\Gamma_6$ and the p-like (valence) band $\Gamma_8$ is a result of the spin-orbit coupling and therefore a bulk effect. With spin-orbit coupling the degenerate $\Gamma_8$ splits into
Chapter 6. Attempts to reduce the bulk conduction

Figure 6.1: Energy gap $\Delta$ dependence on the InAs quantum well thickness at a fixed GaSb quantum well width of 6 nm showing three different regimes: Normal insulator (NI), inverted insulator (QSHE) and semi-metallic regime (SM). Schematic explanations of the band structure on top, solid and dotted lines indicate different directions in momentum $k$. Unpublished work by Q.-S. Wu, A. A. Soluyanov, and M. Troyer, ETH Zurich.

heavy hole (HH) and light hole (LH) band, whereas the latter gets inverted and acts as a conduction band [11]. Without confinement the system is gapless with the Fermi energy between the LH and HH band. To open a gap, the material can be strained (3D TI) [87, 88] or embedded between HgCdTe barriers (2D TI) [26]. The confinement splits each band into sub-bands while the HgTe quantum well thickness is reduced. Below the critical thickness $t_c$, though, the first electron and first hole sub-band invert in energy. The material turns into a normal state.

The quantum well thicknesses in the InAs/GaSb system play a completely different role. Both the InAs and the GaSb quantum well (QW) stay always in the state with normal band order. The quantum well thicknesses influence two aspects:

- The alignment of the conduction band (of the InAs QW) and the valence band (of the GaSb QW) can be controlled via the quantum well thicknesses.
- The quantum well thicknesses also effect the wave function overlap of electrons and holes. The latter influences the electron-hole coupling and therefore the size of the hybridization gap.

Tight-binding calculations by the group of Prof. Troyer at the ETH Zurich focus on the band gap size as a function of quantum well thicknesses. Whereas the variation of the GaSb QW thickness has a relatively small effect, the thickness of the InAs QW has a considerable effect, see Fig. 6.1. A normal band alignment (NI) is expected for a QW thickness smaller than 12.4 nm. An optimized inverted gap could be calculated for about 14 nm. Above an InAs QW thickness of 20 nm the material turns into a semi-metallic state (SM). The hybridization gap is not at the same energy in all $k$-directions (symbolized with solid and dotted lines in the schematic energy diagrams in Fig. 6.1). This so-called band anisotropy is discussed in more detail in the next section. Note that these calculations did not assume that
all semiconductors are ideally grown epitaxially on a crystal with a GaSb lattice constant. In these calculations all materials of the heterostructure are considered with their natural bulk lattice constant. For more details it is referred to the section 6.5 about the effect of strain on the bulk insulator. Even though this aspect might alter the QW thickness values for which the gap closes between the NI and QSHE regimes or for which there is a transition between the QSHE and SM regimes, the general trend stays unchanged. This three phase dependence on the InAs QW thickness could also be experimentally confirmed by Suzuki et al. [56]. Three wafer materials all containing a 10 nm GaSb quantum well showed normal insulator (for InAs QW width of $t = 10$ nm), inverted band alignment with intermediate resistance (for $t = 12$ nm) and metallic transport (for $t = 14$ nm).

### 6.2 Band anisotropy

More advanced tight-binding calculations taking the full zinc-blende crystal structure into account pointed out that the energy gap in inverted InAs/GaSb system could be smaller than expected from preliminary simplified models. The hybridization gap is not always found at the same energy for all directions of momentum $k$ due to band anisotropy. De-Leon et al. [89] proposed even a gapless state calculated by taking a more complicated, anisotropic, and nonparabolic valence band into account. There is enhanced anisotropy for larger InAs QW widths resulting in a transition from a gapped to a SM state, consistent with the calculations shown in Fig. 6.1.

### 6.3 Double gating

Due to the spatial separation of InAs and GaSb QWs, density and band overlap can be tuned independently by means of a double gating technique. The separated tuning of front- and back-gate shifts the Fermi energy through the band structure. Different bias polarity on the two gates modifies the perpendicular electric field $E_z$, affecting the alignment at the interface by $E_{g0} = eE_z \langle z \rangle$ ($\langle z \rangle$: average separation between electron hole gases). Within the phase diagram of both front- and back-gate we expect to find metallic regimes, when the Fermi energy resides either in the conduction or valence band, and two distinct insulator phases, one with normal and the other with inverted band alignment. This rich phase diagram was predicted theoretically [5, 23] and already shown experimentally by Qu et al. [25].

The perpendicular electric field $E_z$ influences directly the band overlap $E_{g0}$, see schematic explanation in Fig. 6.2. An increase of band overlap $E_{g0}$ enhances the material gap in case of normal alignment of conduction and valence band (a), but not necessarily in the inverted band ordering (b). The topological insulator behavior depends on the hybridization gap $\Delta$ and not on the band overlap $E_{g0}$. In fact, Suzuki et al. [57] found an increased resistivity in transport around charge-neutrality for
Chapter 6. Attempts to reduce the bulk conduction

Figure 6.2: The conduction band in blue and the valence band in red for (a) a normal band alignment and (b) an inverted band alignment is shown schematically, explaining the terms band overlap $E_{g0}$ and hybridization gap $\Delta$.

...a reduced band overlap. The additional perpendicular electric field $E_z$ has two effects, which both can be used to optimize the hybridization gap $\Delta$. First, the anti-crossing of the inverted conduction and valence band shifts to smaller momentum $k$. Second, the change from flat-band to triangular shaped QWs effects the center of the electron and hole wave functions. Depending on the order of the QWs, the two wave functions can either have a smaller or larger overlap, the latter is favorable for a large hybridization gap $\Delta$. This aspect will be explained in the following section 6.4 in more detail. In a subsequent theoretical study based on the eight-band Kane model by Hu et al. [62] two topological insulator regimes could be identified. If the band overlap $E_{g0}$ increases in both regimes, then one is characterized by an enhanced hybridization gap $\Delta$, while the other has a diminished gap. This result motivates that a simple QW thickness consideration is not sufficient to optimize the bulk insulator, but also the perpendicular electric field plays a role. The latter can be modified by doping, for example.

6.4 Quantum well stacking order

In this section we will concentrate on the difference between an InAs/GaSb and a GaSb/InAs double QWs systems. To understand the difference, we first consider the schematics in Fig. 6.3. In the flat-band condition of (a) the wave functions (dashed) are expected to be centered to their respective QW. In the presented situation, the bands are inverted, meaning that the hole band maxima (red) is above the electron band minima (blue). The applied top gate voltage will not only modify the position of the Fermi energy $E_F$, but also influence the inner electric field, symbolized with the tilt of the band extrema (solid blue and red lines) (b). Here, the GaSb layer is grown epitaxially subsequently to InAs. We consider exemplarily the tilt of the conduction band by the top gate voltage $V_{tg}$. The chemical potential of the 2DEG $\mu_{2DEG}$ decreases with respect to the chemical potential of the gate $\mu_{tg}$ for positive top gate voltage $V_{tg} > 0$ V by

$$-|e|V_{tg} = \mu_{tg} - \mu_{2DEG} \quad (6.1)$$

Analogously we can understand the tilt for positive and negative top gate volt-
6.5 Strain

Preliminary theoretical studies by Zakharova et al. [82] pointed out the influence of the substrate wafer on the bulk insulator of the InAs/GaSb system. The small difference of the lattice constant of $a = 6.0583 \text{Å}$ for an InAs substrate and $a = 6.0959 \text{Å}$ for a GaSb substrate modifies not only the position but also the magnitude of the hybridization gap. According to their calculations lattice-mismatched strain could even lead to a semimetal-semiconductor transition in inverted InAs/GaSb double quantum wells. The effect of strain in transport experiments is reported in chapter 5. To summarize, even though all semiconductors used for the heterostructure belong to the so-called 6Å-family and therefore have a similar lattice constant, strain can not be neglected in this material system. Strain is not only a reason for a reduced band gap, but could also be used to optimize the bulk insulator. A
Chapter 6. Attempts to reduce the bulk conduction

Figure 6.4: Hybridization gap $\Delta$ (blue shaded area in the upper inset) of the strained InAs/In$_x$Ga$_{1-x}$Sb as a function of the QW thickness of InAs $t_{\text{InAs}}$. The $k \cdot p$-calculation for pseudomorphic growth on AlSb (resp. GaSb) shown with solid (resp. dashed) red and black lines for different In concentrations $x$ causing enhanced compressive strain in the 6 nm thick In$_x$Ga$_{1-x}$Sb layer. Reprinted from [60], with the permission of AIP Publishing.

bulk resistivity two orders of magnitude higher than with relaxed InAs/GaSb samples could be measured recently in compressively strained InAs/In$_x$Ga$_{1-x}$Sb systems [60]. $k \cdot p$-calculations predict an improved hybridization gap $\Delta$ of a factor 5 compared to unstrained InAs/GaSb [60], see Fig. 6.4. Note, that the prediction requires a combined modification of strain and InAs QW thickness $t_{\text{InAs}}$.

6.6 Disorder

To optimize the wafer growth with respect to the hybridization gap not just QW thicknesses have to be considered, but also band anisotropy, strain, and the coupling of electron and hole wave functions, which depends on the internal perpendicular electric field. Despite the recent efforts, up to now an optimal wafer growth could not be achieved in our collaboration. There is always a residual bulk conduction limiting the bulk resistivity to the kΩ-range. Experimentally it could be shown that disorder helps to improve the bulk insulator, even if the same InAs/GaSb structure grown without disorder has residual bulk conductivity at charge-neutrality.

- **Silicon dopants at the interface of InAs and GaSb:**
  A sheet concentration of $1 \cdot 10^{11}$ cm$^{-2}$ of silicon impurity dopants at the interface of InAs and GaSb lead to a suppression of the residual bulk conduction. This disorder material has an activation behavior with a gap of $\Delta = 26$ K [58]. It is expected that the silicon dopants act as donors and acceptors at the same time and therefore are able to localize in-gap states [90].

- **Impure Ga source:**
  The impure Ga source used for the growth enhances the degree of disorder in the material with the goal to reduce the mobility of the charge carriers. The latter should suppress scattering channels inside the bulk [24]. Indeed transport measurements on two wafers fabricated with an identical growth protocol including a 15 nm InAs QW and a 8 nm GaSb QW, but a different
Figure 6.5: The longitudinal resistivity $\rho_{xx}$ as a function of the top gate voltage $V_{TG}$ at zero magnetic field and 1.3 K is shown for 15 nm InAs/8 nm GaSb double QW structure grown with a pure (red) and a Ga source of reduced purity (blue), effecting also the electron mobilities ($\mu = 300,000 \text{cm}^2/\text{Vs}$ resp. $\mu = 80,000 \text{cm}^2/\text{Vs}$ for wafer material grown with pure resp. impure Ga source at a density of $n = 8 \cdot 10^{11} \text{cm}^{-2}$). Reprinted from [24], with the permission of AIP Publishing.

Ga source have a large difference in charge carrier mobility. At a density of $n = 8 \cdot 10^{11} \text{cm}^{-2}$ the material grown with a pure Ga source has a mobility of $\mu = 300,000 \text{cm}^2/\text{Vs}$ with a finite bulk resistivity smaller than $< 10 \text{k}\Omega$, see Fig. 6.5. The resistivity maximum of the material grown with a Ga source of reduced purity is more than four orders of magnitude higher. The increased disorder reflects in a charge carrier mobility of $\mu = 80,000 \text{cm}^2/\text{Vs}$. 
Chapter 7

Ballistic edge conduction in InAs/GaSb

This chapter reports on low-temperature electronic transport measurements of InAs/GaSb double quantum wells with reduced bulk conduction. After an overview of samples and measurement technique, we start with introducing the concept of nonlocal measurements and the theoretical expectations for helical edges states according to Landauer-Büttiker, subsequently followed by the transport results. Hall bars of 50 µm in length down to a few µm gradually develop a pronounced resistance plateau near charge-neutrality, which comes along with distinct nonlocal transport along the edges. Plateau resistances are found to be above or below the quantized value expected for helical edge channels. We will conclude with discussing these results based on the interplay between imperfect edges and residual local bulk conductivity.

Parts of this chapter have been published in the article:

**Nonlocal transport via edge states in InAs/GaSb coupled quantum wells**

### 7.1 Background to edge studies in the InAs/GaSb system

After the prediction [26] and the observation [20] of the quantum spin Hall (QSH) phase in the two-dimensional topological insulator HgTe/(Hg,Cd)Te quantum well (QW) system, there is increased interest in the double QW structure InAs/GaSb sandwiched between AlSb barriers. In this system the overlap between the electron dispersions in the InAs conduction band and in the GaSb valence band is gate tunable [91] due to the spatial separation of the two wells. Hybridization of these two
bands at zero magnetic field has been theoretically predicted [23, 85, 89, 92] and pioneering experiments aimed at verifying this prediction [48, 55, 86, 93–95]. More recently a phase diagram was suggested [5] covering metallic, normal insulator and QSH phases. In the latter counter propagating topologically protected helical edge states are expected to dominate transport properties at zero magnetic field close to the charge-neutrality point (CNP) in devices with structure sizes smaller than typical scattering lengths. This scenario requires edge state scattering to be sufficiently reduced while the bulk is insulating. First studies aiming at the observation of the QSH phase used samples with the highest available material quality [10, 25, 96, 97], which still showed substantial residual bulk conductivity. In subsequent devices disorder was intentionally introduced [24, 58] based on the hope of suppressing the bulk conductivity without affecting the topologically protected edge states. In these samples the four-terminal resistance peak at charge-neutrality shrinks when the device dimension is reduced, gradually forming a plateau [56, 58, 98, 99]. The expected quantized edge-resistance value of $\frac{h}{e^2}$ between neighboring voltage contacts should be reached on samples smaller than the spin-relaxation length of the helical edge states. It is expected to remain insensitive to further reduction of device size as long as edge channels at opposite sample edges do not overlap.

Here we report edge-dominated transport where the resistance at charge-neutrality measured between neighboring contacts along the sample edge falls below the expected quantized value even though a bulk resistance of the order of 10 MΩ is seen in large area devices. This observation contrasts with the expectation based on ideal helical edge channels, which predicts a quantized minimum resistance in fully spin-coherent devices. The presence of nonlocal edge conduction is detected in measurement configurations, where a local conductivity model delivers a voltage drop between contacts too small to be experimentally detectable. Mesoscopic samples show pronounced nonlocal resistances scaling according to the expectations for helical edge modes without giving the precisely quantized values expected from theory. Based on a resistor network model we separate qualitatively edge and bulk resistances explaining the deviation of the experimentally observed plateau height from the theoretical prediction.

## 7.2 InAs/GaSb devices with reduced bulk conduction

Our devices were fabricated on MBE grown wafer #6 containing a 8 nm GaSb QW on top of a 15 nm InAs QW embedded between AlSb barriers. This wafer was grown using a source with reduced Ga-purity as described in Ref. [24] and section 6.6. AlGaAs/GaAs heterostructures grown with the same Ga source displayed significantly reduced mobilities leading us to conclude that additional unintentional impurities are present in this source. For this InAs/GaSb layer sequence the InAs conduction band overlaps with the GaSb valence band leading to a hybridization gap at finite
wave vector. Results from six Hall bars (denoted device A to F, see Table 7.1) with different lengths \( L \) between contacts and widths \( W \) (for an exemplary image including device dimension notations see Fig. 7.1(b)) are discussed in this chapter. Analogous experiments (not shown) as presented here but performed on a sample of similar dimensions fabricated on a low-disorder wafer did not display nonlocal transport effects or plateau-like features in the charge-neutrality regime. The latter can be explained by the low bulk resistance even close to charge neutrality, which makes the detection of edge states impossible. The bulk effects always dominate in these devices. Argon plasma etching was used for the large area devices E and F to fabricate the Hall bar mesa. The smaller devices A to D were patterned by wet etching as reported in Ref. [34] and appendix A. All devices were covered with a 200 nm thick Si\(_3\)N\(_4\) dielectric. Tunability is implemented with a Ti/Au top gate of width \( W_{\text{gate}} \) and length \( L_{\text{gate}} \) listed in Table 7.1, defining the area of the sample where the density is modulated. The top gate of device F was fabricated with a shadow mask, resulting in a gated area of about 500 \( \mu \)m in diameter.

### 7.3 The dc measurement technique

The experiments were performed at a temperature of 1.5 K and devices A and B were additionally tested at 125 mK. Reducing the temperature by an order of magnitude

<table>
<thead>
<tr>
<th>Device name</th>
<th>( W ) ([\mu\text{m}])</th>
<th>( L ) ([\mu\text{m}])</th>
<th>( W_{\text{gate}} ) ([\mu\text{m}])</th>
<th>( L_{\text{gate}} ) ([\mu\text{m}])</th>
<th>( L_{\text{edge}} ) ([\mu\text{m}])</th>
<th>Hysteresis ( \Delta V ) ([\text{V}])</th>
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<tr>
<td>A</td>
<td>2.1</td>
<td>3.3</td>
<td>9.3</td>
<td>11.3</td>
<td>40.3</td>
<td>6.6</td>
</tr>
<tr>
<td>B</td>
<td>2.2</td>
<td>5.1</td>
<td>9.5</td>
<td>13.2</td>
<td>41.8</td>
<td>4.6</td>
</tr>
<tr>
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<td>4.4</td>
<td>6.5</td>
<td>7.4</td>
<td>12.9</td>
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<tr>
<td>D</td>
<td>3.5</td>
<td>5.9</td>
<td>6.2</td>
<td>9.9</td>
<td>19.5</td>
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<tr>
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Table 7.1: Summary of device details as explained in the main text. The meaning of the length scales and the hysteresis \( \Delta V \) are indicated in Fig. 7.1(b) and Fig. 7.1(e), respectively.
7.3. The dc measurement technique

Figure 7.1: Comparison of two-terminal resistance vs. top gate voltage of a large device ($L = 50 \mu m$, device F) (a) with the four-terminal resistances of a medium sized device ($L = 10 \mu m$, device E) (c) and a small device (typical dimension $L = 5 \mu m$, device B) (e). The resistance peak around the CNP decreases with reduced structure size and forms a plateau. Black arrows indicate the top gate sweep direction. The optical microscope image of device B in (b) indicates the length $L$, width $W$, gate length $L_{\text{gate}}$, gate width $W_{\text{gate}}$ and the total length of the gated edge $L_{\text{edge}}$ used in Table 7.1. Four-terminal resistances of devices E and B aiming at the measurement of nonlocal resistances are shown in (d) and (f), respectively. The schematics in each panel indicate how the current source and the voltmeters were connected for the measurements shown. The theoretically expected resistance quantization values of 12.9 k$\Omega$ in case of the local configuration (a,c,e) or 4.3 k$\Omega$ for the nonlocal measurements (d,f) are indicated with horizontal dashed lines.
resulted in an insignificant enhancement of the longitudinal resistance by less than 5% (see appendix D.2 for details) similar as in Ref. [99]. Four-terminal resistance measurements were performed by applying a dc current of 5-10 nA between two contacts and by measuring the dc voltage between two different contacts using a low-noise (30 nV/√Hz) voltage preamplifier. The resistance of device F, which was highly insulating at charge-neutrality, was measured in a two-terminal voltage-biased configuration with an IV-converter. A dc measurement technique is required for these samples because of the high lead resistance. A large area of the contacts is ungated wafer material, which is highly resistive.

In all measurements shown in Figs. 7.1 and 7.4 we start the top-gate sweep at $V_{tg} = +6 \text{ V}$, where the resistance $R$ is governed by transport in the conduction band of the InAs QW. Upon lowering the top gate voltage, the resistance increases indicating that the electron density is reduced and the system approaches the CNP. Below $V_{tg} = -2 \text{ V}$ the behavior of the resistance depends strongly on device size as seen in Figs. 7.1(a,c,e).

### 7.4 Real plateau in local transport

Before we look into these differences in detail, we discuss the hysteresis of the resistance $R$ between down- and up-sweep of the gate voltage shown in Fig. 7.1(a,c,e) (black arrows indicate sweep direction). The hysteresis is probably due to the accumulation of charge in the gate insulator or at the insulator-semiconductor interface. The approximate gate voltage shifts $\Delta V$ between up- and down-sweeps are summarized for all devices in Table 7.1. The gate hysteresis of device B shown in Figs. 7.1(e) represents one of the worst cases ($\Delta V = 4.6 \text{ V}$) whereas in the best case $\Delta V<1 \text{ V}$ (device D, shown in the appendix D.1). For large values of gate hysteresis experiment-related plateau-like features may arise, since the resistance becomes independent of gate voltage.

Three reasons make us confident that the plateaus observed in Figs. 7.1(e,f) are still related to the topological properties of the material: First, there is a continuous evolution from the resistance maximum in the largest device F (Fig. 7.1(a)) via the intermediate device E (Fig. 7.1(c)) showing a clear plateau-like maximum, to the smallest devices. Second, the observed conductance behavior is well conceivable, if the hole mobility is rather low compared to the electron mobility, and if the contribution of the edge to the conductance scales with edge length. Third, the plateau comes along with a distinct nonlocal edge conductance, as shown below. Many publications on two-dimensional topological insulators [4, 20, 58, 100] show a marginal decrease in resistance for decreasing gate voltage beyond the CNP. Difficulties to tune deeply into the hole regime were explicitly reported [100]. A good gate tunability in the hole regime is shown in Refs. [4, 56]. We took great care that the plateaus which we discuss in the following are not related to experimental artifacts by following a consistent measurement protocol. It turned out that measurements
7.5 Nonlocal measurement technique

are stable and reproducible, if the gate voltage is repeatedly swept in the same range at constant rate. Data presented here are discussed and analyzed only for down-sweeps of the voltage, and the plateaus seen below \( V_{tg} = -2 \) V are considered to be reliable only down to \( V_{tg} = -5.5 \) V in case of device B (replication of plateau during up-sweep).

In Fig. 7.1(a) the two-terminal resistance \( R \) in device F as a function of top gate \( V_{tg} \) shows a profound charge-neutrality resistance-peak of about 90 MΩ. A continuous transition between the electron and the hole regime with a sign change of the Hall slope in four-terminal magneto-transport measurements (not shown) is observed while crossing the CNP. The peaked resistance transforms into a plateau at 131 kΩ for a reduced structure size (device E, shown in Fig. 7.1(c)), well above the quantization value of \( h/2e^2 \) expected for helical edge states. An even lower and well defined resistance plateau is found in all the small area devices (device A to D, device B is shown exemplarily in Fig. 7.1(e)). Contrary to the large area device F, a sign change of the Hall slope could not be experimentally detected for the intermediate and smallest samples, an observation whose origin remains to be investigated.

The resistance plateau in Fig. 7.1(e) stays below the expected quantization value indicated with a dashed black line, reaching about 97% of its anticipated height. The plateau heights for the other devices A, C and D are even lower and can be found in Table 7.1, column “local resistance configuration” (L-R config.). This is in contrast to the expectation that the quantized edge-resistance value is reached as soon as the edge length is smaller than the spin-relaxation length.

7.5 Nonlocal measurement technique

As explained above, there is a strong plateau in the local measurement configuration close to, but a little bit below the expected quantization value for helical edge states. This raises the question, if edge conduction can be confirmed in the gate voltage range of this resistance plateau. In this section it will be explained how nonlocal measurement configurations can affirm edge states.

So far we have concentrated on the local measurement configuration of a standard Hall bar. In this four-terminal measurement a voltage drop \( V \) parallel (perpendicular) to the constant current flow \( I \) is recorded. The plotted resistance \( R = V/I \) is referred to as the longitudinal (Hall) resistance. A standard Hall bar is a six terminal device and therefore allows for many more four-terminal measurements, the so-called nonlocal measurement configurations. One example is presented in the schematic of Fig. 7.2. The current is injected at the corner of the device, whereas the voltage drop is measured far away of the current flow at the opposite device corner. A simple bulk conduction contribution is sketched for a Hall bar with realistic aspect ratio in the right panel of Fig. 7.2. The current density (stream lines in gray) together with the electric potential (red) distribution is shown for this
Figure 7.2: Transport result (middle panel) of a Hall bar sample (device B as an example) measured in a nonlocal configuration as schematically explained in the upper right corner. For positive gate voltages, the Fermi energy is deep in the conduction band and a simple bulk pictures (right panel) leads to a negligibly small signal for this measurement configuration. For negative gate voltages, the bulk is highly resistive and transport is dominated by edge conduction (left panel).

Specific nonlocal measurement configuration. It is expected that the voltage drop measurement far away of the current flow will pick-up a negligibly small signal. A four-terminal measurement signal in which an entirely local resistivity model would give a vanishingly small resistance is called truly nonlocal. Indeed, here such a situation is present, when the Fermi energy resides deep in the conduction band, for $V_{tg} > 0.5 \text{ V}$. But a strong signal appears for a gate sweep towards charge neutrality $V_{tg} < -1 \text{ V}$. The latter can only be explained with a superimposed edge conduction as schematically explained in the left panel of Fig. 7.2. A strong signal in a truly nonlocal measurement configuration unambiguously confirms edge conduction.

At this point it shall be pointed out that in order to detect a truly nonlocal signal, the bulk needs to be sufficiently depleted. The injected current $I$ will always split into an edge $I_{\text{edge}}$ and a bulk $I_{\text{bulk}}$ contribution. As long as most of the current is flowing through the bulk, the voltage drop over an edge segment far away of the bulk current will be small.

### 7.6 Local and nonlocal resistance expectations for helical edge states

In the previous section it was shown how we can confirm the existence of edge states in a Hall bar device with nonlocal transport. In this section we focus on the properties of this edge conduction, which we can extract from a set of nonlocal measurements on one single device.

Ohm’s law for a n-terminal device with ballistic edge channels and insulating bulk can be described with the Landauer-Büttiker formalism [101]. Each lead $i$ has an electric potential $V_i$ and an outgoing current $I_i$. $\kappa_i$ resp. $\eta_i$ denote the incoming resp. the reflected channels in lead $i$ and $T_{ij}$ is the transmission probability from
leads i to j. Therefore, the general form can be written as:

\[ I_i = c \cdot \left[ (\kappa_i - \eta_i) V_i - \sum_{j \neq i} T_{ij} V_j \right] \]

Figure 7.3: The Landauer-Büttiker transmission matrix is shown next to a schematic explanation of the edge states in the QSH regime (a), the QH regime (b) and for one ballistic channel (c). The red arrows indicated the direction of the electron flow in the respective situations.

The quantum spin Hall effect (QSHE) is expected to have two counter-propagating edge states, which are spin polarized and quantized each carrying a conductance quantum of \( c = e^2/h \). The standard six terminal Hall bar in the QSH regime can therefore be described with the matrix form shown in Fig. 7.3(a). A schematic of a Hall bar with the two counterpropagating edges can also be found in figure 7.3(a), where the direction of electron flow is indicated by red arrows.

**Local configuration:**
In the local configuration, current \( I \) is forced to flow only from terminal 1 to 4, meaning \( I_1 = I \) and \( I_4 = -I \), otherwise \( I_i = 0 \). Resulting in longitudinal and Hall
resistance of:

\[ R_{xx} = \frac{V_2 - V_3}{I} = \frac{V_6 - V_5}{I} = \frac{h}{2e^2} \]
\[ R_{xy} = \frac{V_2 - V_6}{I} = \frac{V_3 - V_5}{I} = 0 \]

We can perform any four-terminal measurement of all possible contact combinations of current and voltage leads. The Onsager symmetry relations [101] reduce the number of independent measurement configurations in our six-terminal devices to ten, which classify into conventional (local) configurations, nonlocal resistance configurations (NL-R config.) of type 1, where the current is driven between neighboring contacts, and nonlocal resistance configurations of type 2, where the current flows between next nearest neighboring contacts.

**Nonlocal resistance configurations of type 1:**
Exemplary for a nonlocal resistance configuration of type 1, we consider current flowing from terminal 2 \((I_2 = I)\) to terminal 1 \((I_1 = -I)\), where we don’t allow a current flow through any other lead. The nonlocal resistance can be calculated to:

\[ R_{NL-R \, type \, 1} = \frac{V_3 - V_4}{I} = \frac{V_4 - V_5}{I} = \frac{V_5 - V_6}{I} = \frac{h}{6e^2} \]

Any cyclic permutation of the contact labeling will obviously not change this result. Therefore, the above nonlocal resistance \( R_{NL-R \, type \, 1} \) will be found for any current flow between neighboring contacts.

**Nonlocal resistance configurations of type 2:**
In analogy we find the result of a nonlocal resistance configuration of type 2 by assuming a current flow between next nearest neighbors, e.g. \( I_2 = I \) and \( I_6 = -I \) otherwise \( I_i = 0 \).

\[ R_{NL-R \, type \, 2} = \frac{V_3 - V_4}{I} = \frac{V_4 - V_5}{I} = \frac{h}{3e^2} \]

The Landauer-Büttiker formalism was first used to predict the transport behavior of samples in the quantum Hall regime (QHE) at finite magnetic field [102]. Figure 7.3 shows the transmission matrix for the QSHE (in (a)) and for the QHE (in (b)) (exemplary with only one Landau level) side by side. In the latter the direction of the electron flow is defined by the magnetic field direction via the Lorentz force and is indicated with a red arrow. The different transmission matrices will also lead to different nonlocal resistances.

If one can show the expected local and nonlocal resistances of type 1 and type 2 on one single InAs/GaSb device in the inverted regime, it is a strong indication for the QSHE expected in this material system. Nevertheless it is not a definite...
7.7 Nonlocal resistances in transport measurements

The predicted transport along helical edge channels is governed by the expectation of a strongly nonlocal resistance close to charge-neutrality. Nonlocality is best probed in four-terminal measurement configurations in which an entirely local resistivity model would give a vanishingly small resistance (see e.g. the inset of Fig. 7.1(d)). Device E shows a profound nonlocal plateau of 22.7 kΩ when the Fermi energy is tuned to the CNP as shown in Fig. 7.1(d). Device B (see Fig. 7.1(f)) shows a reduced plateau height below the expected quantization value of $h/6e^2$ (black dashed line). Since measurements on the high-resistance device F are rather cumbersome, so far no convincing nonlocal data could be detected.

For a detailed nonlocality discussion we focus on device B and the data shown in Fig. 7.4. In Fig. 7.4(a) an example of a measured vanishing local response at $V_{tg} > 0.5$ V (red and yellow traces) is shown, where the Fermi energy is deep in

Figure 7.4: Four-terminal nonlocal configurations on device B are shown and schematically explained in the respective upper right corners. For current flow between neighboring contacts (a,b) 4.3 kΩ, respective for a flow between next nearest neighbors (c,d) 8.6 kΩ is theoretically expected, indicated with black dashed lines.

proof, since other scenarios can lead to the same transmission matrix. For example in Fig. 7.3 (c) the transmission matrix of a single ballistic channel is shown, which is indistinguishable to the one for helical edge states. Note that this ballistic channel allows current flow in both directions, indicated by the red arrow.

7.7 Nonlocal resistances in transport measurements

The predicted transport along helical edge channels is governed by the expectation of a strongly nonlocal resistance close to charge-neutrality. Nonlocality is best probed in four-terminal measurement configurations in which an entirely local resistivity model would give a vanishingly small resistance (see e.g. the inset of Fig. 7.1(d)). Device E shows a profound nonlocal plateau of 22.7 kΩ when the Fermi energy is tuned to the CNP as shown in Fig. 7.1(d). Device B (see Fig. 7.1(f)) shows a reduced plateau height below the expected quantization value of $h/6e^2$ (black dashed line). Since measurements on the high-resistance device F are rather cumbersome, so far no convincing nonlocal data could be detected.

For a detailed nonlocality discussion we focus on device B and the data shown in Fig. 7.4. In Fig. 7.4(a) an example of a measured vanishing local response at $V_{tg} > 0.5$ V (red and yellow traces) is shown, where the Fermi energy is deep in

67
the conduction band and transport is well described by a local resistivity model. Comparing the red and yellow traces to the brown trace in the same voltage range, we realize that the measured four-terminal resistance grows when the pair of voltage probes is closer to the current carrying contacts. This corresponds to the fact that the current density flowing through the bulk of the sample decreases strongly with increasing distance from the current contacts. Note also that within a homogeneous local resistivity model, the bulk current density distribution does not depend on the value of the resistivity.

When the gate voltage is tuned close to the CNP \( (V_{tg} < -1 \text{ V}) \) in Fig. 7.4(a), the measured resistance shows a plateau. The red and yellow traces saturate at the same plateau value, whereas the brown trace shows its plateau at a larger resistance. In the picture of edge transport, the same current flows along the sample edges between all four involved voltage contacts, and we would therefore expect the same plateau resistance to be measured in all three configurations. The higher plateau resistance of the brown curve is in agreement with the notion of a bulk contribution to the current leading together with the edge current to a larger voltage drop along the sample edge. We draw the preliminary conclusion, that in our samples there is strong evidence for nonlocal transport along the edge, which is, however, still influenced by a finite residual bulk conductivity with a local character. We may call the red and yellow traces truly nonlocal, because the contribution of the local bulk conductivity vanishes for the corresponding voltage contacts, as confirmed by the zero resistance value deep in the electron regime.

This concept has to be stated more precisely, if we compare the plateau values of the three traces in Fig. 7.4(a) quantitatively to the theoretical expectation for transport in ideal helical edge channels (black dashed line). The experimental fact that the red and yellow traces show a plateau at 67% of the expected value seems to indicate that the current in the corresponding edge segments is reduced below its ideal value, probably because the finite bulk conductivity diverts some current away from these edge segments. For the brown trace, however, which results from an edge segment closer to the current contacts, it seems that the additional bulk current just about compensates for this reduced edge current leading to a close to ideal plateau value. Within this picture the ideal plateau value appears rather like an accidental coincidence than a cogent effect.

We have performed similar four-terminal measurements of all possible contact combinations of current and voltage leads. In general we find that different contact configurations related by the generalized Onsager symmetry relations \[101\] always give consistent results, as expected. This reduces the set of independent four-terminal measurements in our six-terminal devices to ten, which classify into conventional configurations (Fig. 7.1(e)), nonlocal resistance configurations (NL-R config.) of type 1 (Figs. 7.4(a,b)), where the current is driven between neighboring contacts, and nonlocal resistance configurations of type 2 (Figs. 7.4(c,d)), where the current flows between next nearest neighboring contacts.

Additionally, geometric symmetries of the samples (e.g. reflections at the Hall
bar axis, or inversions at the Hall bar center) always gave consistent results. This observation and the vanishing zero magnetic field Hall resistance (see Figs. 7.1(c,e)) give evidence for a homogeneous bulk and excellent contact properties witnessing the high quality of our devices.

Due to the Onsager symmetries the sketched schematics in Fig. 7.1(e) and Figs. 7.4(a-c), represent all the possible configurations. In Fig. 7.1(e) and Fig. 7.4 the expected quantization values for helical edge states of $\hbar/2e^2$ (Fig. 1(e)), $\hbar/6e^2$ (Figs. 7.4a,b) and $\hbar/3e^2$ (Figs. 7.4c,d) are shown as dashed black lines. In all configurations the experimental plateau values are systematically below the expectation. All voltage drops, which are far away from the current leads and have a vanishing local resistance in the electron regime (see Fig. 7.4a red and yellow, and Fig. 7.4e) are called truly nonlocal. The ratios of the truly nonlocal plateau values of pairs of different configurations are exactly given by the ratios of the expected plateau values in an ideal topological insulator without residual bulk conductivity. Device B, for example, reaches about 66% of the expected plateau values in all truly nonlocal configurations. The mean plateau values together with their uncertainties and these scaling factors are summarized in Table 7.1 for all devices and all types of configurations (all measurement configurations of sample A, B, C and D can be seen in appendix D.3). The clear nonlocal signals together with this consistent scaling are in agreement with the theoretically proposed current carrying helical states along the edge.

Device C has an off-centered gate resulting in different gated edge segment lengths $L_{es}$ between consecutive contacts (optical microscope picture in appendix D.3). Even though the length of these edge segments varies by a factor of 4, the corresponding nonlocal plateau resistances remain constant, as shown exemplarily in Fig. 7.5a in color for the configuration schematically described in the inset and with black circles for the corresponding measurements related by symmetry (corresponding measurements shown in appendix D.3). This independence on edge length excludes trivial edge conduction characterized by a resistance per length as indicated with the black dashed line strongly deviating from the measurements. Quasi-ballistic edge states arising for example from electrostatic edge confinement [103] might be compatible with the observations but are considered unlikely. Further insights on nonlocal measurements on this material system can be found in the article by Suzuki et al. [57].

7.8 Resistor network model

The independence of plateau resistances on edge length suggests a resistor network model for the description of transport in the plateau region of the devices similar as in Ref. [56] consisting of a series of resistors $R_E$ along the edge as schematically shown in Fig. 7.5b in red. The observed finite bulk conductance is accounted for by adding bulk leakage resistors $R_C$. Although this model oversimplifies the real
Figure 7.5: (a) Value of the nonlocal four-terminal plateau resistance as a function of the edge length \( L_i \). Black dashed line: Linear length dependence of resistive edge states. (b) Resistor network: Here \( R_C \) symbolizes a residual bulk resistance, \( R_E \) the edge-resistance and \( V_1 \) through \( V_6 \) the voltage probes. The bar graphs in (c)-(f) allow a comparison of samples A through D between the measured four-terminal resistances (filled bars), as schematically shown in the respective top right corners, and the calculated prediction with the model (dotted bars).

In this situation it may serve as a tool to compare the relative relevance of edge and bulk conduction. The conductance matrix of this resistor network model is

\[
G = \begin{bmatrix}
2/R_E & -1/R_E & 0 & 0 & 0 & -1/R_E \\
-1/R_E & 2/R_E + 1/R_C & -1/R_E & 0 & 0 & -1/R_C \\
0 & -1/R_E & 2/R_E + 1/R_C & -1/R_E & -1/R_C & 0 \\
0 & 0 & -1/R_E & 2/R_E & -1/R_E & 0 \\
0 & 0 & -1/R_C & -1/R_E & 2/R_E + 1/R_C & -1/R_E \\
-1/R_E & -1/R_C & 0 & 0 & -1/R_E & 2/R_E + 1/R_C
\end{bmatrix}
\]

If we assume that current \( I \) flows from contact one to six (the latter grounded, there-
fore $V_6 = 0 \text{ V}$, we have to solve the following equation for the unknown variables $V_1$ to $V_5$ and $I$ (contact labeling can be found in Fig. 7.4(b)).

$$
\begin{bmatrix}
I \\
0 \\
0 \\
0
\end{bmatrix} = 
\begin{bmatrix}
2/R_E & -1/R_E & 0 & 0 & 0 \\
-1/R_E & 2/R_E + 1/R_C & -1/R_E & 0 & 0 \\
0 & -1/R_E & 2/R_E + 1/R_C & -1/R_E & -1/R_C \\
0 & 0 & -1/R_C & -1/R_E & 2/R_E + 1/R_C \\
0 & 0 & 0 & 0 & -1/R_C \\
0 & 0 & 0 & -1/R_E & 2/R_E + 1/R_C \\
0 & 0 & 0 & 0 & 2/R_E + 1/R_C
\end{bmatrix} \times 
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
V_5
\end{bmatrix}
$$

we find

$$
\frac{V_4 - V_5}{I} = \frac{V_3 - V_4}{I} = \frac{R_C^2 R_E}{6R_C^2 + 16R_C R_E + 8R_E^2}
$$

and

$$
\frac{V_2 - V_3}{I} = \frac{R_C R_E}{6R_C + 4R_E}
$$

This measurement configuration is therefore sufficient to calculate the two unknown resistors in the network model. Exemplarily we show the calculations on device B with the measurement results

$$
\frac{V_4 - V_5}{I} = \frac{V_3 - V_4}{I} = 2.9 \text{ kΩ}
$$

and

$$
\frac{V_2 - V_3}{I} = 3.6 \text{ kΩ}
$$

resulting in $R_E = 23.3 \text{ kΩ}$ and $R_C = 193.4 \text{ kΩ}$ as also listed in Table 7.1.

The result for all devices is a bulk coupling $R_C$ between 65.4 kΩ (device D) and 339.5 kΩ (device A), which is an order of magnitude larger than the edge-resistance $R_E$ ranging from 15.3 kΩ (device D) to 21.9 kΩ (device A) (for more details see Table 7.1), confirming the dominant edge conduction and the model assumption. Even though $R_C$ simplifies the description of the bulk contribution, the calculated predictions of all possible measurement configurations based on the configuration in Fig. 7.4(a) pass the test of being compared with the measurements as illustrated with the bar graphs in Figs. 7.5(c-f). Our analysis demonstrates that one set of $R_E$ and $R_C$ is sufficient to describe all measurement configurations.

7.9 Conclusion to ballistic edge conduction in InAs/GaSb

To conclude, we have investigated edge conduction in standard Hall bar samples of InAs/GaSb QWs. At the CNP a pronounced nonlocal resistance below the expected
quantization value was observed in all devices with structure size below 6 µm together with a consistent scaling according to Landauer-Büttiker theory, suggesting edge conduction with helical character. The deviation from the theoretical prediction could be described qualitatively in a resistor network model.
Chapter 8

Diffusive edge conduction in InAs and InAs/GaSb quantum wells

We investigate low-temperature transport through single InAs QWs and broken-gap InAs/GaSb double QWs. Nonlocal measurements beyond bulk pinch-off confirm the presence of theoretically unexpected edge conduction in InAs QWs. The edge resistivity of 1-2 kΩ/µm is of the same order of magnitude as edge resistivities measured in the InAs/GaSb double QW system. Finger gate samples on both material systems shine light on the length dependence of the edge resistance.

This chapter is structured as follows: After an introduction to wafer material, sample fabrication and measurement setup in section 8.2, we present the results on InAs devices of different geometries in section 8.3. In section 8.4 we then compare finger gate samples of InAs to measurements on InAs/GaSb double QWs.

Parts of this chapter have been published in the article:

Edge transport in InAs and InAs/GaSb quantum wells

8.1 Introduction

Topological insulators have been predicted [1, 26] to show the quantum spin Hall effect based on dissipationless transport in edge states separated by an insulating bulk. Experimentally such a situation was first realized in an inverted HgTe/(Hg,Cd)Te QW by the proper choice of QW thickness [20]. While a nonlocal measurement proved the existence of edge transport [4], the confirmation of spin-polarized transport demanded more complex transport experiments [21]. For the coupled QW system InAs/GaSb, double gating was predicted to tune density and band alignment independently [5, 25], resulting in a tunable two-dimensional topological insulator.
According to theory, conductance in helical edge states is switched on or off when crossing the boundary between the topological and trivial insulator by a appropriate change of front- and back-gate voltage.

In a series of pioneering experiments the group of R. R. Du reported evidence for edge modes in inverted InAs/GaSb QWs [96, 99]. In addition, quantized conductance close to charge neutrality was reported [58]. Subsequently, edge conduction was confirmed by a number of groups in the regime of inverted band alignment by nonlocal transport measurements [56, 99, 104], by scanning SQUID microscopy [105], and via the detection of edge-mode superconductivity [106]. So far there is no experimental report in literature directly demonstrating the helical nature of these edge states. Several publications [56, 96, 104] reported on the relevance of bulk conduction and limited gate tunability which prohibit to study edge conduction in the full phase diagram and hamper, for example, a detailed survey of the length dependence of the edge conductance.

Recent findings of edge conduction in the non-inverted regime [107, 108] raised additional questions. The physical origin of these edge states is under debate. In addition, such trivial edge states may possibly co-exist with helical edges in the inverted regime and it is unclear how such edge states of different origin could be distinguished by transport experiments.

8.2 Wafer material, sample fabrication and measurement setup

Three different wafers were grown with molecular beam epitaxy. Wafers #4 and #5 host a two-dimensional electron gas in an InAs QW. Wafer #6 contains a two-dimensional electron and hole gas in a InAs/GaSb double QW. Wafer #4 is grown on a GaAs substrate and contains a 15 nm InAs QW confined by AlSb barriers. The layer sequence of wafer #4 was also used for wafer #6, differing only by the additional 8 nm GaSb QW on top of the InAs QW and by the use of a Ga-source with reduced purity as described in Ref. [24] and explained in section 6.6. Wafer #5 is grown on a GaAs substrate and contains a 15 nm InAs QW confined by AlSb barriers. The layer sequence of wafer #5 was also used for wafer #6.

Hall bar structures were patterned by optical lithography combined with wet chemical etching deep into the lower barrier material as described in Ref. [34] and appendix A. Ti/Au pads separated from the wafer surface by a 200-nm-thick Si₃N₄ dielectric are used as gates. Ohmic contacts were made by a Au/Ge/Ni eutectic.

If not mentioned otherwise, the measurements were conducted at 1.5 K. Four-terminal resistance measurements on wafer #4 and #5 were performed by applying an ac current of 10 nA with a frequency of 31 Hz. On devices from wafer #6, four-terminal dc measurements were conducted because of high contact resistances (of
8.3 Transport in InAs devices of different geometries

<table>
<thead>
<tr>
<th>Wafer name</th>
<th>QW material(s)</th>
<th>Well thicknesses</th>
<th>Barrier materials</th>
<th>Substrate</th>
<th>Ga-source purity</th>
</tr>
</thead>
<tbody>
<tr>
<td>#4</td>
<td>InAs</td>
<td>15 nm</td>
<td>AlSb, AlSb</td>
<td>GaAs</td>
<td>high</td>
</tr>
<tr>
<td>#5</td>
<td>InAs</td>
<td>24 nm</td>
<td>Al$<em>x$Ga$</em>{1-x}$Sb, AlSb</td>
<td>GaSb</td>
<td>high</td>
</tr>
<tr>
<td>#6</td>
<td>InAs/GaSb</td>
<td>15 nm/8 nm</td>
<td>AlSb, AlSb</td>
<td>GaAs</td>
<td>low</td>
</tr>
</tbody>
</table>

Table 8.1: Wafer details including information on layer sequence, substrate wafer and Ga-source purity used during the epitaxial growth.

the order of 10 kΩ) in these devices. The Corbino device (explanation in the next section) was measured by applying an ac voltage and measuring the ac current with an IV-converter.

8.3 Transport in InAs devices of different geometries

We start by discussing the results obtained on InAs 2DEGs. By applying a suitable top gate voltage $V_{tg}$ the Fermi energy falls within the band gap of this semiconductor and the 2DEG is depleted. One expects a high bulk resistance in the so-called pinch-off regime. In theory, no edge states are expected. To separate bulk and edge transport, we start with Corbino measurements. Afterwards edge properties are included with measurements on Hall bar devices.

8.3.1 Corbino measurements on InAs 2DEGs

Figure 8.1 shows the top-gate voltage $V_{tg}$ dependent resistance at zero magnetic field of different device geometries fabricated on wafer #4. The Corbino device (Fig. 8.1(a), inset) allows us to deduce the bulk resistivity from the measured conductance, in the absence of any edges connecting the two ohmic contacts. For positive gate voltages $V_{tg}$ the longitudinal resistivity is $\rho_{xx} \approx 30 \Omega$ indicating that the Fermi energy is deep in the conduction band. Around $V_{tg} \approx -2.5$ V the resistivity $\rho_{xx}$ increases rapidly signaling the depletion of the two-dimensional electron gas. It reaches approximately 10 MΩ for $V_{tg} < -3.6$ V, which is the measurement limit due to parasitic cable capacitances parallel to the sample.

The pinch-off resistance in this device can be further analyzed. An I/V converter with a feedback resistor of 100 MΩ is used to apply an ac voltage of $V = 100 \mu$V with variable frequency. The measured current flow $I$ through this two terminal device at pinch-off is:

- Measured with a frequency $f = 34.41$ Hz: $I = 1'210 \pm 20$ fA
- Measured with a frequency $f = 15.7$ Hz: $I = 550 \pm 25$ fA
Chapter 8. Diffusive edge conduction in InAs and InAs/GaSb quantum wells

Figure 8.1: Transport results on InAs two-dimensional electron gases, Wafer #4: The longitudinal resistivity $\rho_{xx}$ as a function of top gate voltage $V_{tg}$ of a Corbino device (a) and of an asymmetric Hall bar (b). Arrows indicate the gate-voltage sweep-direction. The optical microscope pictures together with the measurement configuration are shown as insets. (c) Finite nonlocal resistances $R$ measured in the configuration indicated in the inset. (d) Measurement results from (c) normalized by the respective edge segment length $L_i$ (dotted lines in the lower inset of (d)). The resistor network model in the upper inset is explained in the text.

- Measured with a frequency $f = 7.85$ Hz: $I = 280 \pm 20$ fA

Therefore, the current $I$ varies proportionally to the frequency $f$, an indication of a purely capacitive component. The resistance at pinch-off can therefore be estimated with the noise of the I/V converter ($I_{\text{noise}} = 20$ fA) to $R_{\text{pinch-off}} > V/I_{\text{noise}} = 5$ GΩ.

The same device under the influence of a perpendicular field is shown in Fig. 8.2. The pinch-off at $V_{tg} < -3.4$ V (already discussed at $B = 0$ T) is visible together with the expected spin-split Landau levels, which depend both on gate voltage $V_{tg}$ and magnetic field $B$. The filling factors $\nu$ are indicated in white. Additionally there are modulations of the Landau levels, which are gate independent (horizontal), which can be associated with the filling factors labeled in blue. Note that equal filling factors (blue and white) cross at $V_{tg} = 0$ V. Therefore, both 2DEGs, the one that is modified by gate voltage and the one that is only influenced by the magnetic field,
8.3. Transport in InAs devices of different geometries

Figure 8.2: Landau fan of a two dimensional electron gas (2DEG) in InAs QW of width $t_{\text{InAs}} = 15 \text{ nm}$ (Corbino device structure, Wafer #4). Filling factors are indicated in white (gate tunable) and blue (gate independent).

have the same density. Since both parts of the 2DEG can lead to zero resistance $R_{\text{SD}} = 0 \Omega$, they are expected to be in series. We conclude that there is a ring like area in the Corbino device, which can not be gated. This could be observed in two independent samples on the same wafer.

A Corbino device was made with the same fabrication technique on a wafer with wider InAs QW ($t_{\text{InAs}} = 21 \text{ nm}$, wafer #7), confined by AlSb upper and Al$_x$Ga$_{1-x}$Sb lower barrier. The corresponding Landau fan is shown in Fig. 8.3. Note the absence of gate independent modulation of the Landau level spectrum. The gate independent ring area of the 2DEG is therefore not a generic feature of this device fabrication. Due to the wider QW, the second subband can be populated in the accessible gate voltage range, seen for $V_{\text{tg}} > 4 \text{ V}$. This wafer can not be pinched off, instead the lower Al$_x$Ga$_{1-x}$Sb barrier gets populated with holes for gate voltages below $V_{\text{tg}} \approx -6 \text{ V}$. Therefore, this wafer material was not used for further edge studies.
Chapter 8. Diffusive edge conduction in InAs and InAs/GaSb quantum wells

8.3.2 Hall bar measurements on InAs 2DEGs

The transport behavior on a Hall bar device fabricated on wafer #4 displays different characteristics from the Corbino devices shown in Fig. 8.1(b). The resistivity reaches a maximum of $\rho_{xx} \approx 5 \, \text{kΩ}$, rather than the 10 MΩ expected from the results of the Corbino geometry. This Hall bar device cannot be pinched-off even at gate voltages well below $V_{tg} < -4 \, \text{V}$. As a preliminary observation, we state that an additional conductive channel must be present in a device with edges as compared to the Corbino device without edge contributions.

The measurements were conducted on the Hall bar presented in the inset of Fig. 8.1(b). On this device, two longitudinal voltages can be recorded between leads with different separation. Current and voltage probes are indicated in the inset. The two deduced resistivities agree with each other for $V_{tg} > -2.8 \, \text{V}$, but deviate for $V_{tg} < -2.8 \, \text{V}$. This shows that in the low gate-voltage regime the resistance has a geometry dependence different from the usual length/width-scaling in diffusive two-dimensional systems.

In Fig. 8.1(b) the sweep direction of the top gate $V_{tg}$ is indicated with black arrows. The hysteresis, i.e. the difference in resistivities between the two sweep direc-
8.3. Transport in InAs devices of different geometries

tions, depends strongly on the gate voltage range. Measurements above \( V_{tg} = -3.4 \text{ V} \)
hardly suffer from hysteresis effects. For down sweeps the resistivity is almost gate
voltage independent below \( V_{tg} < -3.6 \text{ V} \), showing a plateau-like resistivity. However,
as soon as the sweep direction is reversed, the resistivity drops. The absence
of hysteresis in the Corbino device at all gate voltages supports the interpretation
that the hysteresis in Hall bar devices is related to the sample edges.

Nonlocal measurements on the same device shown in Fig. 8.1(c) reveal further
insights into the properties of the additional conductance. An exemplary measure-
ment configuration is shown in the inset. At voltages \( V_{tg} > 0 \text{ V} \) the nonlocal voltage
is vanishingly small, compatible with bulk-dominated two-dimensional diffusive con-
duction. Based on the Corbino-results, we expect the bulk to be depleted at gate
voltages \( V_{tg} < -3.6 \text{ V} \), where the nonlocal resistance in Fig. 8.1(c) appears to be fi-
nite. We interpret this finite nonlocal conductance as evidence for conduction along
the sample edge.

Edge conduction in these InAs two-dimensional electron gas devices becomes
dominant in transport regimes where the bulk resistivity exceeds the edge resistivity.
The bulk resistivity can be tuned only below the gate seen as the yellow shaded
square in the inset of Fig. 8.1(c). We normalize the two nonlocal resistances \( R = V/I \)
shown in Fig. 8.1(c) (red and blues traces) by the respective gated edge lengths \( L_i \)
between the measurement contacts (see inset of Fig. 8.1(d)) to demonstrate the
linear length dependence. This scaling suggests that the sample can be modeled
with a resistor network as shown in the upper inset of Fig. 8.1(d). All possible four-
terminal measurement configurations on this device are consistent when scaled with
respective edge segment lengths \( L_i \) leading to the edge resistivity \( \rho_{edge} = R_i/L_i =
V/I_iL_i = 2.5 \pm 0.2 \text{ k}\Omega/\mu\text{m} \) (current flow along the edge \( I_i \) calculated with the model).
This consistency confirms in retrospect the resistor network model used for the
analysis.

Similar measurements and analysis were conducted on six Hall bar devices fab-
ricated by wet etching [34] on the two different wafers #4 and #5. All measure-
ments agree with the above findings and result in edge resistivities in the range
of \( \rho_{edge} = 1.3 - 2.5 \text{ k}\Omega/\mu\text{m} \). The wafers used have different QW thicknesses, are
confined by different lower barriers and were grown on different substrates, each
requiring a different growth procedure for the buffer. The observed edge resistivity
was independent of all these parameters. Two Hall bars were fabricated with a dry
etching technique (also described in Ref. [34]). These devices showed edge resist-
tivities of the same order of magnitude as well. The proportionality of resistance
to edge length was not found. Still, these measurements support the existence of
trivial edge conduction in InAs QWs fabricated with state-of-the-art techniques.

Edge states, or more generally, conducting surfaces, were already suspected pre-
viously to reduce the efficiency of IR detectors based on InAs/GaSb superlattices
[110] and therefore were studied optically. Fermi level pinning in the conduction
band of InAs is often mentioned as a possible reason for the enhanced electron
density at the edge, especially because the effect is robust against changes of layer
Figure 8.4: Finger gate samples (optical microscope picture of a device in the inset of (a)) to study the longitudinal resistance $R_{xx}$ as a function of all eight gates with gate lengths reaching $L_{tg}^{(i)} = 0.5$ to 49 $\mu$m for an InAs 2DEG, wafer #4 (a) and for a coupled QW InAs/GaSb, wafer #6 (c). The longitudinal resistance $R_{xx}$ at $V_{tg}^{(i)} = -4$ V is plotted versus the gate length $L_{tg}^{(i)}$ together with a linear fit (black dashed line) in (b) resp. (d).

8.3.3 Finger gate measurements on InAs 2DEGs

We continue with an investigation of the length dependence of the edge resistance inspired by the experiments of Nichele et al. [107]. The device is shown in the inset of Fig. 8.4(a). Eight gates, 0.5 to 49 $\mu$m in width, cross a long Hall bar (width $W = 4 \mu$m, length $L = 162 \mu$m). The longitudinal resistance $R_{xx}$ plotted in sequence and fabrication [103, 111–113]. Others also suggest the effect of electric field line concentration, present when gates overlap the sample edges. Edge conduction may also be a side effect of sample processing, either after long exposure to air [114] or because of Sb residues on the surface after etching [115]. Our experiments add to these results that edge conduction can occur in pure InAs QW samples and that it may dominate transport if the bulk is insulating. Its physical origin cannot be assessed by the present experiments.
Fig. 8.4(a) is the sum of a gate-voltage independent resistance due to the ungated sections of the Hall bar and the resistance caused by the gated section below one of the finger gates biased with $V_{tg}^{(i)}$ (note that the constant resistance also depends on the length of the biased gate), i.e.,

$$R_{xx}(V_{tg}^{(i)}) = \rho_{gated}(V_{tg}^{(i)}) L_{tg}^{(i)} + \rho_{ungated}(L_{total} - L_{tg}^{(i)}).$$  \hspace{1cm} (8.1)

A systematic length dependence is seen in Fig. 8.4(a), where only $V_{tg}^{(i)}$ down-sweeps are shown for simplicity.

The longitudinal resistance at $V_{tg}^{(i)} = -4$ V is plotted versus the respective gate length $L_{tg}^{(i)}$ in Fig. 8.4(b). The black dashed line is a linear fit to the data according to eq. (8.1) using $\rho_{ungated}$ and $\rho_{gated}$ as fitting parameters, resulting in an edge resistivity $\rho_{edge} = 2\rho_{gated}(-4V) \approx 1.7 \text{k}\Omega/\mu\text{m}$ (the factor of 2 accounts for the two edges). Further details to the fitting can be found in section 8.5. The fit line also serves as a guide to the eye to demonstrate the obvious proportionality between longitudinal resistance $R_{xx}$ and the gated edge length.

### 8.4 Comparison to transport in InAs/GaSb

With these insights about InAs in mind we now turn to the discussion of InAs/GaSb double QW structures, which contain a hybridized electron-hole system [23, 55, 91, 92, 97, 116]. Despite multiple affirmations of edge conduction in the inverted regime of InAs/GaSb [56, 58, 96, 99, 104–106], a careful analysis of the various possible contributions to edge conduction is missing.

Figures 8.4(c) and (d) show measurements on an InAs/GaSb finger gate sample (wafer #6, same sample dimensions as the InAs sample) analogous to Figs. 8.4(a) and (b). Also in this device the resistance depends linearly on gate segment length, in agreement with Ref. [58]. Analogous results could be found on a second device with six gates, 0.5 to 20 \(\mu\text{m}\) in width, crossing a 2 \(\mu\text{m} \times 76 \mu\text{m}\) Hall bar (data shown in appendix, Fig. C.4(a) and (b)). For the finger gate samples of both material systems we find an edge resistivity $\rho_{edge} = 2\rho_{gated}(-4V) = 1.5 - 1.8 \text{k}\Omega/\mu\text{m}$. Note that the slope of the fit (black dashed line) indicated in Fig. 8.4(b) and (d) is not the edge resistivity $\rho_{edge}$. Further details to the fitting can be found in section 8.5.

### 8.5 Calculation details to edge resistivity of finger gate samples

A finger gate sample is a long Hall bar of length $L_{total}$ covered by multiple gates of different width $L_{tg}^{(i)}$. Such a device structure is used to study the length dependence of effects modified by gates. For both the InAs as well as the InAs/GaSb system, we know from other experiments that we expect both a bulk as well as an edge
contribution. The latter dominates transport as soon as the bulk is sufficiently depleted and is of trivial nature for the length scales studied by the finger gate samples. This section will explain how we can use the finger gate samples to extract the edge resistivity of the InAs as well as of the InAs/GaSb system.

**InAs/GaSb QW system:**

The measured longitudinal resistance $R_{xx}$ is the sum of the resistance of the ungated part $R_{ungated}$ of the Hall bar $(L_{total} - L_{tg}^{(i)}$ in length) plus the resistance modified by the gate $R_{gated}$. The gate voltage $V_{tg}^{(i)}$ modifies the density of the material underneath the gate, whereas the density in the ungated area stays constant. If we assume that both resistances scale linearly in length, we can define an ungated resistivity $\rho_{ungated}$ as well as a gated resistivity $\rho_{gated}$. Therefore, the longitudinal resistance modifies with the top gate voltage $V_{tg}^{(i)}$ on gate $i$ as:

$$R_{xx}(V_{tg}^{(i)}) = \rho_{gated}(V_{tg}^{(i)})L_{tg}^{(i)} + \rho_{ungated}(L_{total} - L_{tg}^{(i)}). \quad (8.2)$$

The above equation can be rearranged to:

$$R_{xx}(V_{tg}^{(i)}) = (\rho_{gated}(V_{tg}^{(i)}) - \rho_{ungated})L_{tg}^{(i)} + \rho_{ungated}L_{total} \quad (8.3)$$

$$= \Delta \rho(V_{tg}^{(i)})L_{tg}^{(i)} + \rho_{ungated}L_{total} \quad (8.4)$$

If we therefore plot the longitudinal resistance $R_{xx}$ at a specific top gate voltage $V_{tg}^{(i)} = \text{const}$ versus the gate width $L_{tg}^{(i)}$, we expect a linear dependence as shown in Fig. 8.4(b) and (d). The slope as well as the offset divided by $L_{total}$ of the fit at each gate voltage $V_{tg}^{(i)}$ are shown in Fig. 8.5(a) and (b), blue and black refer to two different finger gate samples. Both samples were fabricated in the same way, differentiating only in Hall bar width (black: $W = 2.1 \mu m$, blue: $W = 3.8 \mu m$) and in the number of finger gates (black: 8 gates ranging from 0.5 to 49 $\mu m$ in length, blue: 6 gates ranging from 0.5 to 20 $\mu m$ in length). The slope of the fit, $\Delta \rho(V_{tg}^{(i)})$, is not zero for $V_{tg}^{(i)} = 0 V$. Apparently, the simple presence of the gates, even kept at ground, already modifies the density. No gating effect is observed at a finite gate voltage, i.e. $V_{tg}^{(i)} = 1.3 V$ for the fit results in black (marked with a yellow dashed line). The bulk resistance $\rho_{ungated}$ at the same gate voltage is now used to find the resistivity modified by the gate:

$$\rho_{gated}(V_{tg}^{(i)}) = \Delta \rho(V_{tg}^{(i)}) + \rho_{ungated} \quad (8.5)$$

The result is plotted in Fig. 8.5(c). Starting at $V_{tg}^{(i)} = 6 V$ the resistivity $\rho_{gated}$ increases for decreasing gate voltage, consistent with the expectation of the depletion of the 2DEG. Note that the depletion is similar in both finger gate devices, but offset, indicating a different intrinsic density underneath. The latter is also confirmed by a
8.5. Calculation details to edge resistivity of finger gate samples

Figure 8.5: Explanation of fit results: The resistance $R_{xx}$ of two InAs/GaSb finger gate samples (blue: finger gate sample with six gates of 0.5 to 20$\mu$m in width, crossing a 2.1$\mu$m $\times$ 76$\mu$m Hall bar, black: finger gate sample with eight gates of 0.5 to 49$\mu$m in width, crossing a 3.8$\mu$m $\times$ 162$\mu$m Hall bar) at a fixed gate voltage $V_{tg}^{(i)}$ depends linearly on the finger gate width $L_{tg}^{(i)}$. The slope ($\Delta \rho$) and offset ($\rho_{ungated} \cdot L_{total}$) of the linear fit is shown in (a) and (b) as a function of the top gate voltage $V_{tg}^{(i)}$. These two data sets can be used to calculate the resistivity $\rho_{gated}$ (c) (calculation explained in the text). The edge resistivity is twice the saturation values at negative top gate voltages $V_{tg}^{(i)} < 0$ V (also explained in the text).

At negative gate voltages $V_{tg}^{(i)} < 0$ V the resistivity $\rho_{gated}$ saturates. A schematic of the resistivity $R_{xx}$ of a standard InAs/GaSb Hall bar as a function of the gate voltage is plotted in Fig. 8.6. A conduction (valence) band can be depleted with more negative (more positive) gate voltage $V_{tg}$. In case of an inverted band structure we expect that edge states co-exist and become dominant in transport as soon as both conduction and valence band are sufficiently depleted. Therefore the resistance $R_{xx}$ is always finite, either limited by bulk or by edge conduction. If we can see a transition from edge to hole dominated transport (indicated with a yellow 1) or not (yellow 2) depends on the 2DHG resistance compared to the edge resistance. Here, the hole mobility is low due to the large effective mass of the holes, resulting in a high 2DHG resistance. For relatively short edges, the expected edge resistance is therefore smaller, we expect to find scenario 2. The latter scenario is consistent with the findings in Fig. 8.5(c). From the saturation value we can directly read the edge resistivity (note that each finger gate crosses two edges): $\rho_{edge} = 2 \cdot \rho_{gated} \approx 1.8$ k$\Omega$/µm (black) and $\rho_{edge} \approx 1.5$ k$\Omega$/µm (blue).
Figure 8.6: Schematic expectation of the resistance $R_{xx}$ in an inverted InAs/GaSb system. Transport is always dominated by the smallest resistance, either by the bulk (conduction band in blue, valence band in red) or by the edge resistance (in yellow). Depending on the 2DHG (valence band) resistance compared to the edge resistance, hole transport can be observed for negative top gate voltages (situation marked with a yellow 1) or edge transport stays dominating (situation 2).

InAs QW system:

We have analyzed a finger gate sample fabricated from an InAs QW, consisting of 8 gates ranging from 0.5 to 49 $\mu$m in length. The edge resistivity is much easier to calculate than in the disordered InAs/GaSb system because of two reasons: First, the bulk of InAs can be depleted. Therefore, we can expect that for very negative top gate voltages (here $V_{tg}^{(i)} < -4$ V) there is only transport along the edges. Second, the longitudinal resistivity at $V_{tg}^{(all)} = 0$ V is of the order of 100 $\Omega/\mu$m, therefore:

$$\rho_{gated}(V_{tg}^{(i)}) = \Delta \rho(V_{tg}^{(i)}) + \rho_{ungated} \approx \Delta \rho(V_{tg}^{(i)})$$

(8.6)

In case of the InAs QW system, we can therefore extract the edge resistivity directly from the slope of the fit in Fig. 8.4(b). Again taking into account that each finger gate covers two edges, the edge resistivity is $\rho_{edge} = 1.7$ k$\Omega/\mu$m.

8.6 Conclusion to diffusive edges

Edge transport observed in finger gate samples of both the InAs and the InAs/GaSb system are indistinguishable. Both show an edge resistance of $\approx 1.7$ k$\Omega/\mu$m with a linear dependence on edge length down to at least 1 $\mu$m. These results are in contradiction to the edge independent nonlocal resistances observed in standard mesoscopic InAs/GaSb Hall bars (discussed in chapter 7). The next chapter attempts to find a consistent interpretation of all the presented data and critically discusses the conclusions that can be drawn.
Chapter 9

Discussion of edge conduction in InAs/GaSb

Measurements on low mobility InAs/GaSb showed indications of both ballistic (in mesoscopic Hall bars) and diffusive (in finger gate samples) edge transport. This chapter starts with asymmetric Hall bars with the intent to unravel the question about the nature of edge conduction in InAs/GaSb. Unfortunately this device geometry was not as successful as expected. Measurements in tilted magnetic field suggest an anisotropy of the conducting regions at the edges with a larger extent in the plane of the sample than normal to it. This is found for InAs as well as for InAs/GaSb. These insights helped to find a consistent interpretation of all the presented data and critically discuss the conclusions.

Parts of this chapter have been published in the article:

**Edge transport in InAs and InAs/GaSb quantum wells**


9.1 Asymmetric Hall bar of low mobility InAs/GaSb

This section summarizes the measurement results of asymmetric Hall bars fabricated with the low mobility InAs/GaSb double QW system. Two factors raised interest in studying this device geometry for this specific material:

- Measurements on Hall bars fabricated with this material system show nonlocal resistances, which scale according to the expectations for helical edge states. Details can be found in chapter 7. Even though the latter is a promising indication for the QSHE, the only evidence for edge length independence can be
Chapter 9. Discussion of edge conduction in InAs/GaSb

Figure 9.1: Summary of measurement results on an asymmetric Hall bar fabricated with the low mobility InAs/GaSb wafer (wafer #6). An optical microscope picture of the device in (a) and (c) also shows the measurement configuration for (b) and (d). In (b) and (d) the nonlocal resistance as a function of gate voltage shows a plateau. A zoom on the positive gate voltage regime, where the Fermi energy is expected to reside deep in the conduction band, can be used to verify true nonlocality (explanation in the text).

found in Fig. 7.5(a). The investigated sample has an off-centered gate resulting in different edge segment lengths between consecutive contacts. Nevertheless, all nonlocal resistances are independent on the measurement configuration. A more detailed study on edge length dependence is therefore missing. Finger gate samples showed an edge length dependence of diffusive nature for edge lengths \( L_{\text{tg}}^{(i)} > 1 \mu m \). An asymmetric Hall bar is a possibility to combine mesoscopic device characteristics with intentionally fabricated edge length differences.

- Asymmetric Hall bars were already successfully used to demonstrate the diffusive nature of the edges states in InAs QWs, see Fig. 8.4.

Figure 9.1 summarizes the measurements on asymmetric Hall bars fabricated with low mobility InAs/GaSb. An optical microscope picture of the device is shown in Fig. 9.1(a). We added a nonlocal measurement configuration, where the current is passed over one corner and the voltage difference is measured on opposite corners. The two voltage probes test a nonlocal resistance over edge segments with a large difference in length, defined by fabrication and gate area. In Fig. 9.1(b) the measured resistance \( R = V/I \) is plotted as a function of the top gate voltage \( V_{\text{tg}} \). The overall gate voltage dependence is similar to the one explained in chapter 7. At
high gate voltages, the Fermi energy resides in the conduction band, when tuning towards more negative gate voltages the resistance increases, before it saturates at a gate voltage of $V_{tg} < -1 \text{V}$. The plateau is reproduced for a back sweep and therefore assumed to be real and not due to a simple gate independence. For more information about hysteresis in this material system, refer to section 7.4. At first glance the two plateaus do not have the same resistance value, suggesting an edge length dependence for these nonlocal signals. But the latter statement has to be revised: In this material system it is not possible to pinch-off the bulk completely. Therefore we can only compare truly nonlocal signals in order to make a statement about edge state properties. Truly nonlocal signals have a vanishingly small signal in a simple bulk picture. If a signal can be detected in such a truly nonlocal measurement configuration, it has to be caused by edge properties. For additional explanations see section 7.5. To check these configurations for true nonlocality, the zoom at high gate voltages, when the Fermi energy is deep in the conduction band, is considered. As can be seen in the zoom of Fig. 9.1(b) (marked with a yellow dashed box) only the blue measurement configuration can be called truly nonlocal. The result of the red measurement configuration will always show a superposition of bulk and edge contribution. In conclusion, the statement that all nonlocal resistances scale according to the expectations for helical edge states in mesoscopic low mobility InAs/GaSb devices, is not disproved by these measurement results. This statement is additionally supported by the fact that all truly nonlocal signals on this device do show the same plateau resistance. In Fig. 9.1 the blue measurement configuration in (a,b) and both measurement configurations (blue and red) in (c,d) are truly nonlocal, all have a plateau resistance of $R = 2.2 \text{k} \Omega$. The truly nonlocal measurement configurations shown here are of type 1. The plateau resistance of truly nonlocal measurements of type 2 in this sample are limited to one with a resistance of $R = 4.4 \text{k} \Omega$. The scaling according to expectations based on Landauer-Büttiker for helical edge states is therefore also fulfilled for this asymmetric Hall bar device. Even though the measurements on asymmetric Hall bars do not contradict the findings on standard Hall bars, they are unfortunately not useful to study the mesoscopic edge length dependence in more detail.

9.2 Transport in tilted magnetic fields

In order to obtain information about the spatial extent of orbital states in the conducting channels along the edges we measure the resistance of micron-sized Hall bars in magnetic fields tilted by an angle $\alpha$ with respect to the normal of the sample plane (inset of Fig. 9.2(a)). Figure 9.2(a) displays the resistivity of an InAs sample (wafer #4, Hall bar width 2 $\mu\text{m}$, length 3 $\mu\text{m}$) measured at $V_{tg} = -5.5 \text{V}$, i.e., in the regime dominated by edge conductance, as a function of the magnetic field component $B_\perp$ for different angles $\alpha$. We find that all curves scale on top of each other when plotted as a function of $B_\perp$. This suggests an anisotropy of the conducting
regions at the edges with a larger extent in the plane of the sample than normal to it. Furthermore, the resistance decreases with increasing magnetic field. Theories describing such a trend in narrow channels of two-dimensional systems are found, for example, in Ref. [117].

The measurements shown in Fig. 9.2(b) were obtained on InAs/GaSb (wafer #6, Hall bar dimensions 2 $\mu$m $\times$ 5 $\mu$m and 5 $\mu$m $\times$ 10 $\mu$m, respectively), again in the regime dominated by edge conductance at $V_{tg} = -4$ V (c.f. Fig. 7.1). Resistivities taken at different tilt angles fall on top of each other when plotted versus $B_\perp$, like in InAs. Again, we interpret this finding with a larger extent of the conducting edge-region within the plane of the sample than in growth direction.

In InAs/GaSb the trend of the edge resistivity with increasing field depends on the Hall bar size. The edge resistivity of the larger device is suppressed with field like for InAs Hall bars (of all sizes, tested on the finger gate sample with edge lengths ranging from 0.5 to 49 $\mu$m). For small devices the trend in magnetic field is opposite. The reason for such a dependence on device size remains to be explained.

For InAs/GaSb samples in the inverted regime Du et al. [58] find the same dependence on magnetic field for mesoscopic four-terminal devices. Nichele et al. [118] showed an increase in resistivity with rising field for large devices, which is not in agreement with the findings here, but due to large bulk conductivity their measurement is not in an edge dominated regime. The same applies to magnetic field dependent measurements in Ref. [46]. The decrease in resistance around charge
neutrality was attributed to the enhanced anisotropy of the band structure in parallel field. The latter could not be observed for the disordered material presented here. The magnetic field dependence on InAs/GaSb samples in the trivial regime is measured in Ref. [108], but hard to extract from color plots. It can therefore not be compared with the results presented here.

9.3 Critical discussion of edge conduction in InAs/GaSb double QWs

Based on the new insights obtained from the measurements presented in this chapter we now critically discuss edge conduction and its length-dependence in InAs/GaSb devices. In this endeavor, we take the data presented in Fig. C.4 and data from chapter 7 into account, which were all measured on devices from the same wafer #6. Figure 9.3 presents a summary of all the data measured in our group. The blue and black stars in this figure represent the data points from Fig. C.4. Plotted is the resistance \( R_{\text{gated}} = 2\rho_{\text{gated}}(-4V)L^{(i)}_{tg} \) (c.f. Eq. (8.1)) versus the gate length \( L^{(i)}_{tg} \). All the other colored symbols are nonlocal resistances \( R_{\text{nl}} \) from the different devices of chapter 7 (in chapter 7 referred as truly nonlocal resistances of type 1).

In Fig. 7.5(a), only the data points of device C were explicitly shown, but all devices were analyzed and it was concluded that the edge resistance is independent of edge length, in apparent contrast to the data in Fig. C.4.

First we note that the resistance \( R_{\text{gated}} \) in Fig. 9.3 is only a lower bound for the true edge resistance. The measurements on the finger gate sample do not distinguish bulk- and edge contributions to the total current. If there was a bulk current in the InAs/GaSb finger gate devices, then \( R_{\text{gated}} \) would underestimate the true edge resistance.

The resistances for devices A–E from chapter 7 plotted in Fig. 9.3 are bare nonlocal resistances \( R^{(i)}_{\text{nl}} \) obtained by dividing particular measured nonlocal voltages \( V^{(i)}_{\text{nl}} \) by \( I_{\text{tot}} \), the total current applied. These resistances are plotted in Fig. 9.3 against the lengths \( L^{(i)}_{\text{gated}} \) of the gated edge \( i \) between the respective pair of voltage probes. Here, the striking phenomenon is that the bare nonlocal resistance is independent of \( i \), and therefore of \( L^{(i)}_{\text{gated}} \). For this reason it was concluded in chapter 7 that the edge transport is ballistic on the investigated length scales. We see in Fig. 9.3 that this holds true for the small devices A–D, but is no longer found for the larger device E. This behavior of InAs/GaSb is in stark contrast to the nonlocal resistances of the investigated InAs devices (c.f. Fig. 8.1(c,d)), where we found scaling with \( L^{(i)}_{\text{gated}} \).

However, the \( R^{(i)}_{\text{nl}} \) cannot be directly interpreted as edge resistances because \( I_{\text{tot}} \) is the sum of two edge currents of possibly unequal magnitude running along the two Hall bar edges of unequal lengths between the current contacts, plus a possible bulk current. Assuming completely ballistic edge conduction and zero bulk current we find that the edge resistance is larger than \( R^{(i)}_{\text{nl}} \) by a factor of two. Diffusive edge
Figure 9.3: Summary of InAs/GaSb devices from Fig. C.4(d) in this thesis and from chapter 7 measured in the edge dominated regime. The black and blue stars represent the calculated edge resistances $R_{\text{gated}}$ (derivation explained in text) versus the respective gate length $L_{\text{gated}}$ of the finger gate sample in Fig. C.4(d). The other symbols refer to the Hall bar devices presented in chapter 7. Plotted is the four terminal nonlocal resistance $R_{\text{nl}}$ against the gated edge between the respective pair of voltage probes (details in text).

Conduction and also a finite bulk current would raise this factor further. This would move all data points of devices A–E above the lower bound of the edge resistance given by the black and blue stars of the finger gate sample in Fig. 9.3. Based on these considerations, we may state that the edge resistances of all the investigated devices are of the same order of magnitude and consistent with each other.

The question still remains, why the nonlocal measurements on Hall bars give length independent $R_{\text{nl}}^{(i)}$, whereas the finger gate sample exhibits a linear length dependence down to at least 1 µm. In order to find possible sources of misinterpretations here, we take a critical look at the edge lengths $L_{\text{gated}}^{(i)}$ extracted for the devices in chapter 7. These lengths were taken from optical microscope images assuming that the width of conductive edge regions is much smaller than any lithographic width of the samples. However, considering the finding of a finite extent of the edge conducting regions in the plane of the sample (c.f. Fig. 9.2), it is conceivable that the conducting regions cannot enter the narrow voltage probes without
9.4 Conclusion to ballistic and diffusive edge conduction in InAs/GaSb

coupling so strongly that the gated edge length within these voltage probes does not contribute to the relevant edge length. This scenario would reduce the spread of the true $L_{\text{gated}}^{(i)}$ in Fig. 9.3 so strongly that a length-independent resistance could no longer be deduced from the data with sufficient confidence.

Similarly, one could find reasons why the linear dependence of $R_{\text{gated}}$ on gate length $L_{\text{tg}}^{(i)}$ arises in spite of the presence of helical edge modes in the finger gate sample of this paper. One possible scenario is the presence of bulk conductance shunting the significantly lower conductance of the helical edge modes, which is expected to be $e^2/h$.

Summarizing this critical discussion of our own measurements, we have to state that, first, the linear length dependence in the finger gate sample only gives a lower bound for the possible edge resistances which is well below the value expected for helical edge states at least up to lengths of 10 $\mu$m. Second, the nonlocal resistances of Hall-bar devices do not give a robust estimate of edge resistances either, because the relevant current along a particular sample edge is not known. Third, the edge-length estimates for these samples are based on the assumption of narrow edge channels that may well be violated, which would render the length-independent edge resistance an illusion. We believe that this discussion bears importance also beyond our data for the interpretation of related work on transport in InAs/GaSb double QWs by other authors.

9.4 Conclusion to ballistic and diffusive edge conduction in InAs/GaSb

Our experiments show edge conduction in InAs two-dimensional electron gases where no topological effects are expected. An edge resistivity of $\rho_{\text{edge}} = 1.3 - 2.5$ k$\Omega$/µm could be confirmed for standard as well as asymmetric Hall bars and finger gate samples. These results have to be compared to investigations in the InAs/GaSb double QW system, a topological insulator candidate. The latter also shows a resistance with linear dependence on edge length of the same order of magnitude for edge lengths as small as 1 $\mu$m. Additionally, both systems show a magnetoresistance in tilted field that is independent of the parallel magnetic field component with respect to the sample plane. Even though standard InAs/GaSb Hall bar samples have indications of edge length independent nonlocal resistances, an alternative, trivial explanation can not be excluded with the latter results in mind. The presented investigations motivate us to optimize sample processing in order to suppress trivial edge conduction or to enhance the spin-relaxation length of the topological edges. The precise length dependence of the trivial edge conduction could be an important aspect in view of the clearcut identification of the QSH phase in InAs/GaSb systems.
Chapter 10

Summary and outlook

The verification of the quantum spin Hall properties in the double quantum well system InAs/GaSb is the focus of this thesis. The necessary conditions to observe the quantized and spin-polarized edge states build a basis for a profound understanding and were studied as carefully as the edge states itself. A definite proof was experimentally challenged by remaining bulk conduction, limited back gate tunability and the co-existence of trivial edges. Two different material systems were chosen to compare the topological phase of InAs/GaSb with the conventional phase of InAs. Both material systems contain a two-dimensional electron gas (2DEG) in the InAs quantum well, but differentiate in the band alignment order. Transport measurements with magnetic field in different directions turned out to be a suitable technique to study characteristics of different device geometries. It allowed for example to determine four-terminal resistances and to compare them to expectations for helical edge states. It is a drawback that a direct measurement of the spin-polarization is not possible with this measurement technique. As a preliminary conclusion we can state, that a single transport measurement is not enough evidence for the quantum spin Hall phase.

10.1 Spin-orbit coupling

The strong spin-orbit coupling present in this material system displayed itself in a splitting of the spin subbands due to the Rashba effect. We observed a factor of three in gate tunability of $\Delta n/n$ in higher mobility samples ($\mu \approx 300'000$ cm$^2$/Vs at $n = 1 \cdot 10^{12}$ 1/cm$^2$), which could be used to derive the gate dependent electric field in these heterostructures. The normalized spin-density splitting is independent on device size. But samples with lower mobility ($\mu \approx 70'000$ – 100'000 cm$^2$/Vs at $n = 1 \cdot 10^{12}$ 1/cm$^2$) showed no systematic beating dependence. Indeed, spin-orbit related phenomena are better visible in good quality samples. Epitaxial growth on GaSb substrate wafers boosted the field, resulting in the discovery of unconventional Landau fan modulations [46] and full spin-orbit polarization [47]. The access to fully spin-polarized states, without the need of ferromagnetic contacts, opens a large field
of applications, as for example implemented by Karalic et al. [119] with p-n junctions in InAs/GaSb.

10.2 Bulk insulator improvements

Remaining bulk conduction often masked possible edge transport. Introducing impure Ga to localize the bulk and therefore to achieve the necessary increase in bulk resistance allowed us to start with edge studies. The drawback is a significant decrease in mobility. Our studies with a tunable strain parameter in Hall bar devices mounted on top of piezo stacks demonstrated that strain might be a crucial parameter in optimizing the bulk insulator. Our studies highlighted that strain can cause an insulator to semimetal transition and that the strain in InAs caused by the epitaxial growth on GaSb substantially influences the band structure. Therefore, our theoretical and experimental study have shown that a true minigap does not only depend on the choice of the quantum well thickness but also on the strain the quantum wells are exposed to. Strain engineering is therefore an additional tool to improve the insulating bulk properties in future InAs/GaSb systems.

10.3 Edge conduction properties

We confirmed edge conduction in the inverted InAs/GaSb system with localized bulk as well as in the conventional InAs system with nonlocal measurements. We found diffusive edge properties of 1-2 kΩ/µm in InAs samples of various geometries. These measurement results are in line with diffusive edges found in the trivial regime of InAs/GaSb [107, 108]. Both studies are in conflict with ballistic edges expected in the quantum spin Hall phase. To reveal the nature of the edge state properties in the inverted regime of InAs/GaSb, we presented a set of measurements on different device geometries together with a critical discussion. Our Hall bar study showed a plateau formation from large to small devices, with a pronounced plateau close to the expectation values for helical edge states. All truly nonlocal measurement configurations in mesoscopic Hall bar samples showed a convincing scaling as expected for helical edges. This could also be found with off-centered gates resulting in different edge segment lengths, suggesting ballistic edge properties. In contrast, we measured diffusive edges properties in InAs/GaSb finger gate samples of the same magnitude as in the single InAs quantum well samples. We discussed these conflicting results carefully, and could neither exclude ballistic nor diffusive properties.

If we want to proceed with the unambiguous verification of the quantum spin Hall phase in InAs/GaSb, we either have to find additional indications for ballistic properties or find a way around the masking, diffusive edge states. Measurements as schematically explained in Fig. 10.1 could deepen the discussion on the ballistic edges found in mesoscopic InAs/GaSb Hall bars by inducing an additional modular contact or equilibration point for the edge states symbolized by red stars. In
Figure 10.1: Measurement proposals for an additional modular contact. (a) A standard Hall bar resp. in (b) a $\pi$-shaped Hall bar with a back gate is combined with a scanning gate tip. The tip potential (yellow area) shifts the Fermi energy locally into the electron or hole regime acting as a contact, whereas the back gate (not shown) tunes the rest of the Hall bar into the quantum spin Hall regime. Alternatives for samples without back gate are shown in (c) and (d), where a double gate layer sequence on the sample surface act as global ($V_{tg}^{(1)}$) and local gates ($V_{tg}^{(2)}$). The red stars symbolize the equilibration of the edge states.

Fig. 10.1(a) a standard Hall bar resp. in (b) a $\pi$-shaped Hall bar are gate-tuned by back gate to bring the bulk in the inverted insulator regime. The potential of a scanning gate tip (yellow circle) is then used to locally pin the Fermi energy into the valence or conduction band, allowing the edge states to scatter. This additional contact will change the expectation values for the local as well as the nonlocal measurement configurations. Such an experiment is inspired by the theoretical proposal of Väyrynen et al. [120], who studied the influence of charge puddles on helical edges. A standard Hall bar (Fig. 10.1(a)) offers more nonlocal measurement configurations than a $\pi$-shaped Hall bar (Fig. 10.1(b)). But the latter is compatible also with a fairly large tip potential diameter compared to the sample dimension. It could even be possible to approach the sample only with the edge of the tip potential and therefore influence an area which is smaller than the tip itself. The area needed for a full equilibration of the edges states is not known at the moment. A scanning
10.3. Edge conduction properties

gate tip with variable influence area is an ideal tool to find a suitable regime. But the need for a large back gate tunability might hamper a fast realization of these proposals. Fig. 10.1(c) and (d) show alternatives, which can be realized with the to-date fabrication possibilities using a double gate layer sequence. The global gate $V_{tg}^{(1)}$ is combined with a second gate $V_{tg}^{(2)}$, acting as the perturbing potential. The device in Fig. 10.1(d) could profit from the experience of p-n junctions, but it might be difficult to find a regime, where the bulk is still insulating enough to hinder equilibration of the edge states at opposite sample edges, but to allow equilibration at a single edge. Therefore, the proposal in Fig. 10.1(c) with a single equilibration point and a guaranteed bulk insulator is probably easier to realize. A scaling in agreement with the expectations for helical edges with and without the modular contact would strengthen the observations of ballistic properties in InAs/GaSb Hall bar samples.

The existing results on mesoscopic Hall bars suggesting ballistic properties could be misleading, if the edge state width is comparable to the smallest sample dimension. In such a situation the edge length would not be determined by the gate area, but could be much shorter to the point that an edge length independence is not conclusive anymore. A further study on the edge state extent in the sample plane is crucial to substantiate the ballistic properties suggested by the mesoscopic Hall bar devices.

Most likely, diffusive edge states are always present and can mask possible ballistic edges due to the low edge resistivity. To tackle the problem of diffusive edges directly, we can improve the fabrication or modify the sample design. We can try to increase the diffusive edge resistivity with passivation until the expected ballistic edges dominate transport. Preliminary experiments demonstrated the possibility to modify the edge resistivity, but optimizing the fabrication recipe is laborious and will need further effort [121].

Up to now the origin of the diffusive edges is unknown. Fermi level pinning into the conduction band at the surface, electric field line concentration, present when gates overlap the sample edges or a consequence of fabrication residues are possible. These reasons are all related to a physical edge of the sample. Therefore, a split-gate technique to induce an insulator interface of different topology could be favorable. A possible split-gate sample is shown in the next section. Since the fabrication is cumbersome, it is worth to test if the diffusive edges can be influenced by gating. The edge resistivity depends slightly on gate voltage in the existing Hall bar devices, but this could also be related to an increased effective gate area by applying more negative gate voltages. Fig. 10.2 shows a sample proposal, where a global top gate $V_{tg}^{(1)}$ depletes the bulk of an InAs quantum well sample, that transport is dominated by edge conduction. The suggested nonlocal measurement configuration compares the voltage drops $V_1$ and $V_2$, which measure both over the same etched edge length, but only $V_2$ would be influenced by the gate voltage $V_{tg}^{(2)}$, without changing the effective gating area.
Chapter 10. Summary and outlook

Figure 10.2: Measurement proposal to test the diffusive edge states found in InAs devices in the influence of a gate. A global gate $V_{tg}^{(1)}$ depletes the bulk and transport is dominated by the diffusive edges. A nonlocal measurement configuration is shown, where the potential drops $V_1$ and $V_2$ test the same etched edge length, but only $V_2$ is influenced by the local gate $V_{tg}^{(2)}$.

10.4 Back gate tunability

One of the major fabrication challenges is the improvement of the back gate tunability. During this work a fabrication recipe could be developed for ohmic contacts without annealing. Ti/Au contact pads were evaporated as a last fabrication step and after a selective etching to the InAs quantum well. This guarantees no metal diffusion, which could create a short to the back gate. Additionally, we learned that their might be an optimal AlSb thickness in the buffer layer, building a good barrier due to the large band gap, but not exceeding a critical thickness resulting in crystal defects. The latter can act as possible leakage paths to the back gate. These are important steps towards a good back gate tunability, but optimization in both MBE growth and sample fabrication are needed. Having a double gate tunability helps not just to find an inverted band regime with a well developed minigap, but also opens possibilities in sample design.

Nanostructure designs bringing normal and topological insulators in close vicinity without a physical edge, but by gating, will require patterned back gates. This becomes clear by considering the schematic phase diagram of InAs/GaSb in the influence of two gates $V_{tg}$ and $V_{bg}$, shown in Fig. 10.3(a). For a fixed top gate voltage $V_{tg}$ only one of the insulator regimes (topological insulator TI or normal insulator NI) can be reached by a proper choice of the back gate voltage $V_{bg}$. The other insulator regime can only be realized with a combined adjustment of both top and back gates. A quantum point contact (QPC) is shown as an exemplarily nanostructure design in Fig. 10.3(b). If we start with a material in the inverted band
10.4. Back gate tunability

Figure 10.3: (a) A schematic phase diagram of InAs/GaSb in the influence of top $V_{tg}$ and back $V_{bg}$ gates. The phase diagram shows regimes of inverted as well as non inverted band alignment, and the normal (NI) and topological insulator (TI) regimes are indicated with blue areas. (b) Sample proposal for the realization of a quantum point contact in InAs/GaSb with a split-gate technique. In minimum three gate layers are needed, here shown in yellow, orange and red. The gate voltage settings for all three gate layers are marked with colored dashed lines in the phase diagram of (a).

alignment regime, we need one global top gate (Fig. 10.3(b) yellow) to bring the bulk material to charge-neutrality with an insulating bulk and counter-propagating edges. The split-gates, which define normal insulating regimes are realized with top (orange) and back (red) gate pairs. Also the sample edge has to be defined by a set of gates in order to exclude any physical edge carrying the diffusive edges. Such a device asks for a demanding fabrication. Recent work on structured back gates with ion implantation in GaAs/AlGaAs heterostructures by Berl et al. [122] could be a promising starting point.

The constriction of a QPC allows to study helical edge state interactions and is the key elements in the proposals of a spin-filter with near-perfect efficiency [123], a corner junction as a probe for the helical nature of the edge states [124], interferometers [125, 126], and many more. It also opens doors for many other nanostructure designs.

The basis for future studies including quantum spin Hall properties in InAs/GaSb are set by a profound understanding of the ingredients together with a careful interpretation of the edge conduction. Here, we present only a selection of further experimental efforts resulting directly from the findings presented in this thesis, but we are curious to see how this versatile research field will develop. Also A. M. Worthington (see introduction) could probably not have imagined that his studies would have such a diversity of applications honored by the statement: “He was the first to make the experiment of the falling milk drop, which has since become the visual icon of fluid dynamics [127].”
Chapter 12
Appendices

A  Fabrication

A.1 Standard recipe: Ohmic contacts fabricated first, Hall bar by dry etching technology

Fabrication recipe for InAs/GaSb devices: Standard recipe for optical lithography with Ohmic contacts as a first fabrication step and mesa defined by dry etching.

- Ohmic Contacts:

1. Cut wafer into 5.6 mm × 6 mm pieces according to wafer map, see Fig. A.1. Each piece has space for four individual samples. The following recipes are adjusted to process each sample separately, in other words to process wafer material of 2.8 mm × 3 mm in size.

2. Clean the wafers in acetone and IPA.
   If resist does not stick, use O2 Plasma Asher for 30 s at 75 W.

3. Pre-bake: At 900°C for 120 s

4. Photo-resist coating: AZ 5214E (reversible resist)
   (off-centered spinning, see section A.4)
   Spinning: 3000 rpm 3 s (ramp time) 3 s (spinning time);
   5000 rpm 5 s (ramp time) 60 s (spinning time)
   (⇒ ≈ 1.25 µm resist height)
   Baking at 90°C for 120 s

5. Exposure with optical mask for Ohmic contacts: Soft contact, 3.8 s
   Post baking: 120°C for 120 s
   Float exposure of whole chip without any mask: Soft contact, 18 s

6. Develop: MF 319 for 30 s, rinsing with DI water for 20 s

7. Plasma Asher Cleaning: O2, 200 W, 120 s
Figure A.1: Wafer map for 2 inch wafers. Cut lines and chip labeling are indicated.

8. Deposition of Ge/Au/Ni/Au (18 nm/50 nm/18 nm/100 nm) Ohmic contacts in Plasys II

9. Lift-off in warm (45 °C) acetone for 20-30 min.

• Optical Lithography to define the Hall bar structure:

1. Clean the wafers in acetone and IPA.
   If resist does not stick, use O₂ Plasma Asher for 30 s at 75 W.

2. Pre-bake: At 120 °C for 120 s

3. Photo-resist coating: AZ 1505 (positive resist)
   (off-centered spinning, see section A.4)
   Spinning: 3000 rpm 3 s (ramp time) 3 s (spinning time);
   5000 rpm 5 s (ramp time) 60 s (spinning time)
   (⇒ ≈ 0.45 μm resist height)
   Baking at 120 °C for 120 s

4. Border removal:
   Exposure: Soft contact, 10 s.
Develop: MF 319 for 15-20 s.
Rinse for 20 s in DI water.

5. **Hall bar/mesa:**
   Exposure: Soft contact, 3.2 s.
   Develop: MF 319 for 20 s.
   Rinse for 20 s in DI water.

- **Dry etching:**
  1. Fill LN\(_2\) dewar for the ICP.
  2. Take the sample to the ICP chamber as soon as possible. MF 319 etches the wafer surfaces and therefore the sample starts to oxidize as soon as it is developed. Use “ICP carrier wafer” as a substrate and fix samples with vacuum grease.
  3. Argon plasma etching for 8 min at -100°C.
  4. Resist lift-off in warm (45°C) acetone for \(\approx 30\) min. Use 10 s of ultrasonication at lowest power, if the sample can tolerate. Followed by IPA cleaning and N\(_2\) drying.
  5. Check the edges of the Hall bar carefully such that there are no resist residues.

- **Passivation with SiN\(_x\):**
  1. Continue immediately with PECVD SiN\(_x\) deposition for 10 min at 300°C.
     \(\Rightarrow \approx 200\) nm resist height

- **Top gate:**
  1. Define the top gate either by optical or e-beam lithography. For optical lithography use the same recipe as for ohmic contacts. The e-beam lithography recipe can be found in section A.5. Alternatively use a shadow mask.
  2. Deposit Ti/Au (10 nm/70-100 nm) with tilted deposition (\(\approx 20^\circ\) tilting). If shadow mask is used no tilt.

- **Mount sample on chip carrier:**
  1. Fix sample on chip carrier with a drop of PMMA and dry for 10 min at 180°C.
  2. Or use silver epoxy.

- **Bonding:**
1. Contacts are bonded or glued. Gate always glued!!
2. Bonding parameters: Ultrasonic power 340/340, time 100/40, force high/high

A.2 Wet etching instead of dry etching

The standard recipe can be used as described above, just replacing the dry etching step with the following.

- **Wet etching with the following chemicals:**

  (1) H$_2$O: Distilled water
  (2) C$_6$H$_8$O$_7$: Citric acid solution 50%
  (3) H$_3$PO$_4$: Ortho-phosphoric acid 85%
  (4) H$_2$O$_2$: Hydrogen peroxide 30%

- **How to mix the chemicals:**

  Add chemicals in the following order to a Teflon beaker.
  1) 110 ml (1)
  2) 55 ml (2)
  3) 1.5 ml (3)
  4) 2.5 ml (4)

  Mix thoroughly and let the mixture stand for about 10-20 min. The etching solution extenuates over times. It’s not measured, but experience assumes exponential damping. Therefore, the etching rate in the first 10 min is high and the etching depth is hard to control. Additionally, if the mixture is more than 2 hours old, the etching rate is too slow to work with.

- **Etching procedure:**

  Wear two layers of gloves for protection. Dip sample in mixture with plastic tweezers, no metal tweezers and rinse with IPA. Control etching depth optically (color change of sample) or with the Dektak profilometer (measure resist height before etching).

- **Wet versus dry etching**

  For large Hall bars of 25 x 50 µm fabricated with both etching technologies do not show any difference in transport properties. But mesoscopic devices have a decreasing electronic width $W_{\text{electronic}}$ with decreasing density for dry etched samples, whereas the latter stays constant for wet etched devices.

  Further details can be found in:
A.3 Ohmic contacts as a last fabrication step

Fabrication recipe for InAs/GaSb devices: Recipe for optical lithography with Ohmic contacts as a last fabrication step, including a selective etching to the InAs quantum well.

This fabrication recipe is favorable for devices with back gate. Due to the selective etching to the InAs quantum well before metal deposition, no annealing is needed. Metal diffusion during ohmic contact annealing is hard to control and therefore enhances the chance to short the back gate with the double quantum well structure. Additionally, room temperature silver epoxy is used for bonding, to exclude any enhanced temperatures after metal deposition. Combined with a bonding technique, where the bond wires are glued to the ohmic contacts without any pressure, is the optimized method to reduce back gate leakage caused by fabrication.

- Hall bar:
  1. Follow the standard recipe above to define the Hall bar structures with optical lithography and to etch the mesa.
  2. Passivation with SiNx: Don’t forget the Si-dummy during the PECVD process for SiNx deposition. The Si-dummy will be used to calibrate the SiNx etching process later.

- Gate:
  1. Use standard gate recipe.

- SiNx etching (preparation for Ohmic contacts):
  1. Blow dry with N2
  2. Pre-bake: At 90°C for 120 s
  3. Photo-resist coating: 5214E (reversible resist)
     (off-centered spinning, see section A.4)
     Spinning: 3000 rpm 3 s (ramp time) 3 s (spinning time);
               5000 rpm 5 s (ramp time) 60 s (spinning time)
     (⇒ ≈ 1.25 µm resist height)
     Baking at 90°C for 120 s
4. Exposure: Soft contact, 3.8 s.
   Mask: Use circles, which are smaller than the ohmic contacts. Make sure that the circles do not overlap the boarders of the mesa.
   Take care not to scratch the gate.
   Post bake: 120 °C for 120 s
   Flood exposure of whole chip without any mask: Soft contact, 18 s

5. Develop: MF 319 for 35 s (a bit longer than in standard recipe for ohmic contacts), rinsing with DI water for 20 s.

- **SiN$_x$ etching with RIE80**

1. Cleaning run of RIE80 (≈ 30 min)
2. Preconditioning: Let the CF4 SiN$_x$ etching recipe run with an etch step of 1 min duration.
3. Use an end point detection with the laser focused on the Si-dummy. The following instructions are specific for the end point detection system of the nanofabrication center “FIRST”.
   a. Plug in the end point detection system.
   b. Align the laser point to the middle of the window of the etching chamber with gray screws
   c. Start software by following the “Endpoint Detection System Manual”.
   d. Go to settings ⇒ Measurement settings ⇒ machine type and set “layer thickness measurement”.
   e. Set “cycle time”: 1 s, “material settings”: SiN/Si, “layer thickness”: 200 nm
   f. Load samples plus Si-dummy (place dummy sample in center of laser point).
   g. Close chamber, use black screws to adjust the reflection point to the middle of the endpoint detector sensor.
   h. Check for the reflection pattern in “measurement” of the software. Place a green box with shift and mouse around the reflection pattern and start Endpoint measurement with blue point
   i. Run CF4 SiN etching recipe with a step length of 5 min.
   j. Press “set zero” as soon as etching step starts. Observe interference pattern, you will see a sin-curve. When it flattens (typically after 100-200 s), let it etch for another 30 s.
4. Lift-off in warm acetone for 20-30 min, then 5 times of short sonication pulses at lowest frequency.
Selective etching to InAs quantum well and metal deposition to define ohmic contacts:

1. Blow dry with $N_2$
2. Pre bake: At 90 °C for 120 s
3. Photo-resist coating: 5214E (reversible resist)
   (off-centered spinning, see section A.4)
   Spinning: 3000 rpm 3 s (ramp time) 3 s (spinning time);
   5000 rpm 5 s (ramp time) 60 s (spinning time)
   ($\Rightarrow \approx 1.25 \mu m$ resist height)
   Baking at 90 °C for 120 s
4. Exposure: Soft contact, 3.8 s.
   Mask: Use circles, which are larger the the etched holes in the SiNx dielectric.
   Post bake: 120 °C for 120 s
   Flood exposure of whole chip without any mask: Soft contact, 18 s
5. Develop and etch selectively in one step (!!): MF 319 for 5-10 min, rinsing with DI water for 20 s. The surface should be homogeneous yellow-white.
   Check with optical microscope. Etch only directly before evaporating metal.
6. Evaporate 50 nm Ti (to fill up with Ti to SiNx, maybe 70 nm would be on the safe side) and 200 nm Au.
7. Lift-off in warm acetone for 20-30 min, sonication in short pulses at lowest frequency.

Mount sample on chip carrier:

1. Fix sample on chip carrier with room temperature silver epoxy, let it dry for 3 days.

Bonding:

1. Contacts and gates are glued with room temperature silver epoxy, let it dry for 3 days.

A.4 Off-centered spinning

We cleave a 2 inch wafer into 5.6 mm × 6 mm pieces, each offering the space for four samples. Due to limited material, we started to process each sample individually. The latter made it necessary to adapt the recipe to process 2.8 mm × 3 mm samples. Normally, samples are mounted in the center of a resist spinner with low-pressure. The spinning speed depends of course on the distance to the center of the chuck,
especially the center point is at rest. If the samples are too small, then the spinning is not sufficient to result in a uniform and reproducible resist layer. To overcome that issue samples can be spun off-centered. Samples can for example be mounted on glass slides with double-sided carbon tapes. The glass slide can then be mounted on the chuck of the spinner, a picture can be seen in A.2. This is a fast solution, with equipment available in a standard clean room. To avoid residues of the carbon tape on the back side of the sample, a spinner adapter was constructed made out of aluminum and springs to fix the samples.

A.5 E-beam lithography recipes

- General procedure:
  1. E-beam lithography for Hall bars with a SiN$_x$/PMMA/HSQ hard mask.
  2. Top gate fabrication by e-beam fabrication.
  3. Ohmic contacts fabrication and bonding according to the recipe “Ohmic contacts as a last fabrication step”, explained in section A.3.

- E-beam lithography for Hall bars with a SiN$_x$/PMMA/HSQ hard mask
  1. SiN$_x$ deposition for 1 min, together with an additional Si-dummy for the calibration of the later SiN$_x$ etching step.
  2. Cleaning:
     Acetone/IPA
     Plasma Asher: 30 s, 75 W, O$_2$
  3. Pre-bake: 120 s at 175 °C
  4. Spin coat 495 PMMA A4 in Annisol
     (off-centered spinning, see section A.4)
     Spinning: 1000 rpm 1 s (ramp time) 1 s (spinning time);
     4000 rpm 3 s (ramp time) 60 s (spinning time)
     Baking at 175 °C for 15 min
  5. Spin coat HSQ (XR1541-006)
     (off-centered spinning, see section A.4)
Spinning: 1000 rpm 1 s (ramp time) 1 s (spinning time);
8000 rpm 7 s (ramp time) 60 s (spinning time)
Baking at 90°C for 5 min

6. E-beam writing. Dose varies between 1-3 × 100 µC/cm². A dose test (spinning of dose test sample and real sample at the same time) needed every time.

7. Development in TMAH(25%):H₂O (ratio: 1:1) for 10 s, rinse in H₂O for 30 s and afterwards in IPA for 30 s.

8. Etch PMMA with RIE80 (PMMA is not removed during the previous development with TMAH):
RIE80 recipe: O₂ plasma, 60 s (20 sccm, 60 W, 40 µbar, 300 V dc bias, reflected power 0 W)

9. SiNx etching: Etching according to the recipe “SiNx etching (preparation for Ohmic contacts)” described above. The HSQ/PMMA mask patterns the SiNx during this etching process.

- **Top gate fabricated with e-beam lithography.**

1. Spin layer 1: PMMA 50K (CB) pure
   (off-centered spinning, see section A.4)
   Spinning: 3000 rpm 3 s (ramp time) 3 s (spinning time);
   5000 rpm 5 s (ramp time) 45 s (spinning time)
   Baking at 180°C for 4 min

2. Spin layer 2: Repetition of layer 1

3. Spin layer 3: PMMA 950K (Anisole)
   (off-centered spinning, see section A.4)
   Spinning: 3000 rpm 3 s (ramp time) 3 s (spinning time);
   5000 rpm 5 s (ramp time) 45 s (spinning time)
   Baking at 180°C for 4 min

4. E-beam writing, with a dose of 3 × 100 µC/cm².

5. Development in MIBK-IPA 1:3 for 30 s, rinsing with IPA for 15 s.

Fig. A.3 shows an optical microscope picture of a device fabricated with the above recipe together with a Landau fan measurement, proving the successful sample fabrication technique.

The following sample fabrication techniques were not successful (listed together with the fabrication issues):

- PMMA/HSQ mask: The PMMA layer is under-etched. Therefore, the combination of PMMA/HSQ without the SiNx hard mask is not sufficient to define small structure. The resolution could not be improved with this recipe compared to optical lithography.
Figure A.3: a) Optical microscope picture of a device fabricated with e-beam lithography and the recipe for the SiN$_x$/PMMA/HSQ mask. b) zoom to the Hall bar structure. c) Landau fan of the same device, proving the successful fabrication recipe.

- PMMA mask with reversed exposure (e-beam exposure of the whole area around the Hall bar structure): The resist did not stick well during wet etching. This caused the surface of the Hall bar to be etched slightly.

- UVOCS cleaning followed by the recipe of the PMMA mask with reversed exposure: PMMA resist does now stick well during the wet etching process. But the resolution is still limited due to proximity effect.

B  Wafer labeling

The wafers used for this thesis are labeled with numbers. In the groups of Prof. Wegscheider and Prof. Ensslin these wafers have different names. For more details it is referred to table B.1.

<table>
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<td>#4</td>
<td>IN3</td>
<td>E131205B</td>
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<td>#5</td>
<td>–</td>
<td>E160128A</td>
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<tr>
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<td>E150924A</td>
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</tbody>
</table>

Table B.1: Summary of wafer labeling.
**C Supplementary information: Diffusive edge conduction in InAs/GaSb**

Chapter 8.4 shows analysis of two finger gate samples 1 and 2 fabricated with wafer #6, whereas the raw data are only shown for the finger gate sample 2. To complete the raw data of both finger gate samples are shown in Fig. C.4.

![Figure C.4: Transport data of two finger gate samples](image)

**D Supplementary information: Ballistic edge conduction in InAs/GaSb**

**D.1 Hysteresis**

The hysteresis of the four-terminal resistance $R$ between down- and up-sweep of the top gate voltage $V_{tg}$ (black arrows indicate sweep direction) is characterized
in section 7.4 with $\Delta V$ (approximate gate voltage shifts between up- and down-sweeps). The hysteresis is probably due to the accumulation of charges in the gate insulator, or at the insulator-semiconductor interface. The gate hysteresis of device B, shown in Fig. 7.1(e), represents one of the worst cases ($\Delta V = 4.6\, \text{V}$), whereas in the best case (Device D) $\Delta V < 1\, \text{V}$, see Fig. D.5 (further information to the devices and there labeling in chapter 7).

D.2 Temperature dependence

Reducing the temperature by an order of magnitude resulted in an insignificant enhancement of the longitudinal resistance by less than 5%. The longitudinal resistance $R_{xx}$ of Device B measured at 125 mK and 1.5 K is shown in Fig. D.6

D.3 Nonlocal measurement configurations of mesoscopic Hall bars

In the following article transport results of four mesoscopic samples A to D are summarized.
Figure D.7: Optical microscopic pictures of the devices A to D of chapter 7. The length scale is indicated in the lower right corner.

Nonlocal transport via edge states in InAs/GaSb coupled quantum wells

Optical microscope pictures of these devices are shown in Fig. D.7. In the following figures the complete set of all nonlocal measurement configurations is shown.
Figure D.8: All four-terminal nonlocal measurement configurations of the conventional type on device A are shown and schematically explained in the upper right corners. The theoretical expected resistance of 12.9 kΩ for these measurement configurations is indicated with a dashed line.

Figure D.9: All four-terminal nonlocal measurement configurations of type 1 on device A are shown and schematically explained in the upper right corners. Here current flows between neighboring contacts. The theoretical expected resistance of 4.3 kΩ for these measurement configurations is indicated with a dashed line.
Figure D.10: All four-terminal nonlocal measurement configurations of type 2 on device A are shown and schematically explained in the upper right corners. Here current flows between next nearest neighboring contacts. The theoretical expected resistance of 8.6 kΩ for these measurement configurations is indicated with a dashed line.

Figure D.11: All four-terminal nonlocal measurement configurations of the conventional type on device B are shown and schematically explained in the upper right corners. The theoretical expected resistance of 12.9 kΩ for these measurement configurations is indicated with a dashed line.
Figure D.12: All four-terminal nonlocal measurement configurations of type 1 on device B are shown and schematically explained in the upper right corners. Here current flows between neighboring contacts. The theoretical expected resistance of 4.3 kΩ for these measurement configurations is indicated with a dashed line.

Figure D.13: All four-terminal nonlocal measurement configurations of type 2 on device B are shown and schematically explained in the upper right corners. Here current flows between next nearest neighboring contacts. The theoretical expected resistance of 8.6 kΩ for these measurement configurations is indicated with a dashed line.
Figure D.14: All four-terminal nonlocal measurement configurations of the conventional type on device C are shown and schematically explained in the upper right corners. The theoretical expected resistance of 12.9 kΩ for these measurement configurations is indicated with a dashed line.

Figure D.15: All four-terminal nonlocal measurement configurations of type 1 on device C are shown and schematically explained in the upper right corners. Here current flows between neighboring contacts. The theoretical expected resistance of 4.3 kΩ for these measurement configurations is indicated with a dashed line.
Figure D.16: All four-terminal nonlocal measurement configurations of type 2 on device C are shown and schematically explained in the upper right corners. Here current flows between next nearest neighboring contacts. The theoretical expected resistance of 8.6 kΩ for these measurement configurations is indicated with a dashed line.

Figure D.17: All four-terminal nonlocal measurement configurations of the conventional type on device D are shown and schematically explained in the upper right corners. The theoretical expected resistance of 12.9 kΩ for these measurement configurations is indicated with a dashed line.
Figure D.18: All four-terminal nonlocal measurement configurations of type 1 on device $D$ are shown and schematically explained in the upper right corners. Here current flows between neighboring contacts. The theoretical expected resistance of 4.3 kΩ for these measurement configurations is indicated with a dashed line.

Figure D.19: All four-terminal nonlocal measurement configurations of type 2 on device $D$ are shown and schematically explained in the upper right corners. Here current flows between next nearest neighboring contacts. The theoretical expected resistance of 8.6 kΩ for these measurement configurations is indicated with a dashed line.
Publications

Photoinduced modification of surface states in nanoporous InP
J. Lloyd-Hughes, S. Müller, G. Scalari, H. Bishop, A. Crossley, M. Enachi, L. Sirbu, and I.M. Tiginyanu

Infrared attenuated total reflection (IR-ATR) spectroscopy for detecting drugs in human saliva
K.M.-C. Hans, S. Müller, and M.W. Sigrist
Drug Test. Analysis 4, 420 (2012)

Influence of etching processes on electronic transport in mesoscopic InAs/GaSb quantum well devices
A.N. Pal, S. Müller, T. Ihn, K. Ensslin, T. Tschirky, C. Charpentier and W. Wegscheider
AIP Advances 5, 077106 (2015)

Nonlocal transport via edge states in InAs/GaSb coupled quantum wells

Tunnel barrier design in donor nanostructures defined by hydrogen-resist lithography
N. Pascher, S. Hennel, S. Mueller, and A. Fuhrer

Experimental signatures of the inverted phase in InAs/GaSb coupled quantum wells
On the impact of strain on the electronic properties of InAs/GaSb quantum well systems
*equal contribution

Scattering mechanisms of highest-mobility InAs/Al_{x}Ga_{1-x}Sb quantum wells
T. Tschirky, S. Mueller, Ch. A. Lehner, S. Fält, T. Ihn, K. Ensslin, W. Wegscheider

Edge transport in InAs and InAs/GaSb quantum wells

Passivation of edge states in etched InAs sidewalls
C. Mittag, M. Karalic, S. Mueller, T. Tschirky, W. Wegscheider, O. Nazarenko, M.V. Kovalenko, T. Ihn, K. Ensslin
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