The inter-temporal dimension to knowledge spillovers: any non-environmental reason to support clean innovation?

Christos Karydas

Working Paper 17/267
January 2017

Economics Working Paper Series
The inter-temporal dimension to knowledge spillovers:
any non-environmental reason to support clean innovation?

Christos Karydas\textsuperscript{a,*}

\textsuperscript{a}\textit{CER-ETH Center of Economic Research at ETH Zurich, ZUE-F14, Zuerichbergstrasse 18, 8032 Zurich, Switzerland}

Abstract
How should governments best allocate their budget to support private research activities? The consensus in the literature is that sector-specific R&D support policies should be increasing in the degree of compatibility of sectoral innovation with the practices of the wider economy. Using a multi-sector endogenous growth model with in-house R&D and knowledge spillovers, it is shown, that accounting for the time it takes for an innovation to diffuse modifies this widely-accepted result. Wide applicability of green innovations alone does not justify higher research subsidies.

Keywords: Climate Policy, Industrial Policy, Innovation Spillovers, Technology Diffusion, Endogenous Growth

\textit{JEL classification:} O31, O33, Q54, Q55, Q58, H23

1. Introduction

Recent empirical evidence suggests that governments in advanced economies should invest on average 40 percent more in R&D to account for the knowledge spillovers created by local firms (International Monetary Fund, 2016). Yet, for every public intervention that spurs entrepreneurial activity, there are many failed efforts that waste taxpayer money (Lerner, 2009). Hence, even though the level of subsidization of private R&D is of great importance, there is still a lot of room for improvement in the way governments allocate their budget among private research activities. Focused on the inter-sectoral aspect of knowledge spillovers the consensus in the literature is that R&D support policies should be sector-specific and increasing in the degree of compatibility of sectoral innovation with the practices of the wider economy. This paper shows that including the time it takes for knowledge to diffuse modifies this widely-accepted result. We use the example of green innovation to argue that wide applicability of green innovations alone does not justify higher research subsidies as commonly postulated in the literature, but only when coupled with the positive effects of a cleaner environment.

As a matter of fact not all ideas produced within a firm are useful to others. The compatibility of a firm-, or sector-specific innovation, with the practices and technologies of other sectors in an economy is a necessary condition for it to diffuse (Rogers, 2003). Moreover, the most widely-accepted approach is to assume that innovation spillovers increase proportionally to technology adoption. The fact that R&D-promoting policies should take the inter-sectoral compatibility aspect of knowledge spillovers into account has been widely supported by economists; see for example Goulder and Schneider (1999); Smulders and de Nooij (2003). In support of this, Dechezleprêtre et al. (2014) find strong evidence of larger spillovers

\*Email: karydasc@ethz.ch; Tel: +41 44 632 53 05

January 30, 2017
from clean technological innovation – electric cars, wind turbines, etc. – in comparison to dirty ones – coal power plants, combustion engines, etc. – and give clear policy recommendations that subsidies to clean R&D should be higher even if we abstract from the environmental externality. The same support towards environmentally beneficial technologies can be found in Jaffe et al. (2005).¹

The main contribution of this paper is to demonstrate the deviation from this result when including a second dimension to the analysis of knowledge spillovers when studying optimal R&D policies: time. Diffusion in this context is the process by which an invention is communicated through certain channels over time among the members of a social system (Rogers, 2003). Empirical data supports the thesis that research subsidies should differentiate not only on the basis of inter-sectoral compatibility but also on the time it takes for knowledge to diffuse. Related to this Sultan et al. (1990) calculate the time lag between invention and marketable application of medical technologies to be about 20 years, while Popp (2015) estimates for green technologies time lags in the range of 8-10 years.

To our end, we use endogenous growth theory in the spirit of Romer (1990) and Aghion and Howitt (1992), as modified in Peretto and Smulders (2002) to consider in-house R&D. The inter-temporal dimension is added in a similar fashion as in the analysis in Grossman and Helpman (1991), Chapter 3, with lags in knowledge dissemination.² One of the two key parameters of the model used is the speed of diffusion of research results; the rate at which a sector-specific invention creates positive innovation spillovers that increase the productivity of other sectors. The second important component is the degree of compatibility of sector-specific practices with the technological status of the wider economy; the more specific a firm’s practices within a sector are, the fewer spillovers it creates. Such a firm is able to appropriate higher returns to its research investment and, according to the general consensus in the literature, should be granted lower R&D support. It is shown, however, that once we account for the timing of knowledge diffusion, the optimal R&D policy responds non-monotonically to a change in the degree of compatibility, contradicting this common belief.

The next section presents the multi-sector R&D model. Section 3 provides a qualitative analysis in partial equilibrium and explains the main mechanism driving the results. This is then followed by section 4 with the general equilibrium version of the model and its numerical solution; this section closes the model by introducing the spillovers diffusion technology and solves for the optimal subsidy. Section 5 serves as an extension to the basic model and presents the optimal subsidy when information follows a more complex diffusion process with endogenous speed of diffusion. Finally, section 6 concludes with policy recommendations and a possible future research agenda.

¹Inter-sectoral spillovers become more severe as we move from applied to more basic research (Trajtenberg et al., 1992). However, basic research usually occurs within publicly funded institutes, is mostly driven by the deeper need for improving the understanding of how and why natural and social phenomena occur, and is most of the times publicly accessible. The framework used here is based on the profit seeking behavior of private firms and is therefore not suited for answering questions on basic research occurring in public institutes. A thorough discussion on guiding public investment policy in the area of basic research can be found in Gersbach et al. (2015) and Gersbach and Schneider (2015).

²The framework used here falls under the category of endogenous growth models exhibiting scale effects. The critique against the empirical relevance of the “linearity” that leads to endogenous sustained economic growth was raised in Jones (1995). Peretto (2015) using a model of endogenous growth without scale effects discusses the within-industry forces that could have led to the S-shaped historical path of economic growth from the industrial revolution: an initial phase of sluggish development, followed by rapid acceleration in economic growth, leading to a modern sustained growth rate. This paper serves as a mere exposition of the importance of taking into account the speed of diffusion of knowledge spillovers when designing R&D-support policies and is, therefore, in the interest of the author to use a model that, for all its criticisms, is both concise and widely understood.
2. The R&D model

2.1. General setup

Consider an infinite horizon economy in continuous time admitting a representative household with logarithmic preferences, i.e. \( U(C) = \log C \), with \( C \) being the flow of consumption in each time period. Variable \( t \) is the time index.\(^3\) The representative household is endowed with \( L \) units of labor, supplied inelastically to manufacturing and R&D; no population growth is considered. The unique consumption good \( (Y) \) is produced by combining labor \( (L_Y) \) and a continuum of intermediates \( (x_j) \), each available at a certain quality \( (q_j) \), in a Cobb-Douglas fashion. Total output is allocated to consumption \( (C) \) and production of intermediates. The representative firm of each manufacturing sector, \( j \), is responsible for its own research and improves upon its existing technological status by hiring scientific labor \( (L_{Sj}) \). In light of empirical evidence (see section 2.3), firm-specific research uses its own knowledge stock but benefits also from economy-wide knowledge spillovers. Higher quality of intermediate inputs translates into a higher labor productivity in the production of the final good and thus in a higher consumption growth. The externality associated with knowledge spillovers is corrected by a research subsidy raised by the government in a lump-sum way from the representative household.

2.2. Decentralized equilibrium

The consumption good is the numeraire; the representative household’s problem is standard and implies the usual Keynes-Ramsey rule for the growth rate of consumption, \( \frac{\dot{C}}{C} = r - \rho \); parameter \( \rho \) is the constant rate of time preference, and \( r \) the economy-wide interest rate. Production follows:

\[
Y_t = \frac{1}{\beta} L_Y^{1-\beta} \int_0^1 q_j x_j^\beta d j, \quad j \in [0, 1],
\]

where \( L_Y \) is the aggregate labor input in manufacturing, \( x_j \) the amount of an intermediate good from sector \( j \in [0, 1] \), used at time \( t \), and \( q_j \) the quality of that good. Each good is supplied by one firm and each firm produces one good. This production function supports monopolistic competition in the intermediates sector. Following the usual procedure we can derive the equilibrium in this economy.\(^4\) With \( Q = \int_0^1 q_j d j \) denoting the average technology level in the economy, the labor wage rate reads \( w = (1 - \beta)Q \), while \( Y = Q L_Y \) and \( C = (1 - \beta)Y \). The last equation implies that \( g \equiv \frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} \) is the growth rate of the economy. The monopoly profit flow from supplying good \( j \) (before R&D) reads

\[
\pi_j = (1 - \beta) q_j L_Y t.
\]

Labor market clears so that \( L = L_Y + L_S \), with \( L_S = \int_0^1 L_{Sj} d j \) being the aggregate level of scientific labor employed.

2.3. Inventive activity and innovation

Prescott and Visscher (1980) define a firm by its organizational capital: a firm-specific practice or technology is an asset that affects its production possibilities and can be accumulated through investment over time. As a matter of fact, a large strand of literature documents that research happens mostly in-house, and that established firms undertake incremental innovation improving their existing products (Malerba et al.,

---

\(^3\)Time index \( t \) will be dropped within the text when no confusion arises.

\(^4\)See for example Acemoglu (2008, Chapter 14).
A firm’s research also benefits from knowledge spillovers. Accordingly, each firm $j$ hires $L_{Sj}$ units of scientific labor and builds on the firm-specific knowledge base in order to improve upon quality $q_j$ with the following technology:

$$\dot{q}_{jt} = \eta q_j^\omega K_1^{1-\omega} L_{Sj}.$$  \hfill (2)

The elasticity parameter $\omega \in [0, 1]$ proxies the degree of compatibility of firm-specific practices with the technological status of the economy-wide knowledge spillovers, $K$. It is the extent to which a firm bases its research activity on its own firm-specific technology. A large value, $\omega \to 1$, means that this firm’s practices are very firm-specific; such a firm can appropriate the return to its research investment creating at the same time spillovers that are only minimally useful to the wider economy. Conversely, a model with an R&D technology fully compatible with the practices of other firms, $\omega \to 0$, implies a knowledge base that mainly depends on the aggregate pool of knowledge, i.e. the basic model of R&D-driven endogenous growth, e.g. Romer (1990). This crucial assumption can be justified if we think of firms clustering into technology classes in which they seek to improve. Coinciding interests between firms creates opportunity for exchange of knowledge (Peretto and Smulders, 2002); however, the more specific a firm’s technology is, the less it can benefit from or contribute to external knowledge.

So far the terms innovation and invention have been used interchangeably. However, according to the scope of this chapter, these terms need to be differentiated to reflect their true use in the economy. An invention is the product of research efforts, the creation of something new, while innovation introduces the concept of usefulness, the appreciation of an invention. A firm’s technology, $q_j$, creates innovative knowledge spillovers, $k_j$, the aggregation of which makes up the pool of knowledge, $K = \int_0^1 k_j \, dj$, accessible by all firms in the economy. We will allow for innovation, $k_j$, to lag behind invention $q_j$, following a distributed lag formulation (see section 4.1). With this specification research results enter the aggregate pool of knowledge upon announcement but their initial contribution as positive externality to innovation is small and increases with time and acceptance from the market. The standard assumption of immediate knowledge diffusion would be $k_j = q_j$ so that $K = Q$.

2.4. Dynamic labor allocation

The supplying firm of the $j$ good, taking into account a research subsidy $\phi$ that lowers the labor cost of R&D, employs $L_{Sj}$ units of scientific labor in order to maximize its discounted stream of profit (net of research expenditure) according to

$$\max_{L_{Sj}} V_\beta = \int_t^\infty \left[ \pi_{js} - w_s(1-\phi)L_{Sj} \right] e^{-\int_t^s r \, dv} \, ds,$$

subject to (2), and $\pi_j$ defined in (1). Assuming active research, the first order condition for scientific labor employment and the law of motion for the shadow price of the firm-specific technology, $\lambda_j$, read:

$$w_t(1-\phi) = \lambda_j \eta q_j^\omega K_1^{1-\omega},$$ \hfill (3)

$$\left(1 - \beta\right) L_{Yt} + \lambda_j \eta q_j^\omega K_1^{1-\omega} L_{Sj} + \dot{\lambda}_j = r_t \lambda_j.$$ \hfill (4)

Equation (3) states that scientific labor will be employed up to the point where the marginal cost from employing an additional unit of labor equals the marginal quality improvement that this unit can offer. Equation (4) is a no-arbitrage condition between investing in research and in a riskless asset at the market.
interest rate, \( r \). In the interest of tractability and in order to focus on the main mechanism of the model, we will assume symmetry across firms, i.e. \( q_j = Q \) and \( L_{Sj} = L_S \). Using the hat-notation to denote the growth rate of a variable we obtain (see Appendix for the derivation):

\[
\dot{L}_{Y_t} = \frac{\eta}{1 - \phi} \left( \frac{K_t}{Q_t} \right)^{1 - \omega} L_{Y_t} - (1 - \omega) \dot{K}_t - \rho. \tag{5}
\]

From (2), each firm’s research intensity depends on the scientific labor it employs. Accordingly, equation (5) is key to the results since it indicates in which way policy influences the labor allocation between manufacturing and R&D. We can first qualitatively identify the channels that affect labor allocation and the optimal policy response in partial equilibrium, i.e. keeping \( K/Q \) and \( \dot{K} \) exogenous. This is done in the next section. These variables shall be subsequently endogenized in section 4, in order to analyze how the optimal policy depends on the inter-sectoral and inter-temporal dimension of knowledge spillovers.

3. Qualitative results in partial equilibrium

Let’s assume that we are on a balanced growth path (BGP) in the decentralized equilibrium so that \( \dot{K}_t = \dot{K}, K_t/Q_t = K/Q, \) and \( L_{Y_t} = L_Y \). In this case the LHS of (5) is zero. The control variable, \( L_{Y_t} \), must adjust accordingly to any changes so that the RHS remains zero.

From (5) we can identify two direct effects and one indirect, that affect labor allocation among manufacturing and research. The first direct effect is the standard effect of an increase in the research subsidy, \( \phi \): an increase in the subsidy rate lowers the marginal cost of research (see (3)), increasing the incentives to perform R&D and thus to hire scientific labor. It decreases the denominator of the first term on the RHS stimulating lower employment in manufacturing and thus higher employment in research. The effect of an increase in the productivity factor, \( (K/Q)^{1 - \omega} \), the productivity effect, works in the same direction. We now turn to the indirect effect which works in the opposite direction. An increase in \( \dot{K} \), the growth rate of knowledge spillovers, lowers the cost of research in the future. Thus firms which can benefit from that tend to postpone research activity and hire less scientific labor, the spillover effect. A large value of \( \omega \) mitigates these effects since firms rely mainly on their own knowledge stock. In the limiting case of \( \omega \to 1 \) both the productivity and the spillover effects cancel out as the externality vanishes.

The only policy parameter is the subsidy rate, \( \phi \). Hence the social planner can increase the optimal R&D support whenever scientific labor falls short of its optimal level in order to give the correct research incentives. Below follows the intuition on how the timing of knowledge diffusion affects the way that the optimal subsidy depends on \( \omega \).

Take first the typical case of instantaneous knowledge diffusion, i.e. \( K = Q \). In that case the productivity effect cancels out and the optimal policy response should address only the spillover effect which falls in \( \omega \): as the degree of compatibility of firm-specific innovation decreases, so does the externality. This is the main argument in the literature in favor of differentiated sector-specific subsidies that increase with the degree of compatibility. According to this argument, firms with higher technological compatibility should be granted greater support as in the case of clean R&D (Dechezleprêtre et al., 2014). Conversely, for very slow knowledge adoption rate (long time lags) it holds that \( K ≪ Q → K/Q ≪ 1 \). In this case, due to the concavity of the productivity factor in \( \omega \), we do not get the same monotonic behavior of the optimal policy. For slow knowledge diffusion and low values of \( \omega \), there are too few incentives to perform research:

\footnote{In fact the value of the firm can be re-written as \( V_j = \lambda_j q_j \). Differentiating w.r.t time gives \( \dot{V}_j = \dot{\lambda}_j q_j + \lambda_j q_j \). Combining this with (4), (2), and (1) gives \( \pi_j + \dot{V}_j = rV_j \), the standard no-arbitrage condition between investing in research and in a riskless bond.}
inventive activity benefits largely from the economy-wide stock of innovation, \( K \), which, however, expands at a very slow rate. In such a case a marginal increase in \( \omega \) increases the productivity effect less than it mitigates the spillover effect. Thus, the productivity effect suffers most, indicating an increasing optimal subsidy rate in \( \omega \) until research is sufficiently stimulated. For high values of \( \omega \) the opposite occurs: a marginal increase in \( \omega \) results in a proportionally larger marginal increase of the productivity factor resulting in a decreasing optimal subsidy as \( \omega \) grows.

The previous discussion for low speed of knowledge diffusion can be qualitatively summarized in Figure 1. This shows for exogenous \( K/Q \ll 1 \) and \( \dot{K} \), how both the productivity factor, \((K/Q)^{1-\omega}\), and the spillover factor \(-(1-\omega)\dot{K}\), in equation (5), depend on \( \omega \). For slow diffusion, there exists a threshold, \( \bar{\omega} \), where the two slopes are equal. For \( \omega < \bar{\omega} \) the slope of the productivity effect is smaller than that of the spillover effect. In this case the positive productivity effect should be promoted more relative to dampening the adverse spillover effect; this requires an increase in the optimal subsidy. The contrary occurs for \( \omega > \bar{\omega} \).

Figure 1: Qualitative representation of the productivity effect vs. spillover effect for slow knowledge diffusion \((K \ll Q)\). Point \( \bar{\omega} \) is where the two slopes coincide.

4. General equilibrium results

4.1. Adding Knowledge diffusion

So far we have not specified the exact process of knowledge diffusion. As stated earlier, each individual invention creates innovative knowledge spillovers, the sum of which makes up the general pool of knowledge, \( K \). In order to include a time lag between an invention and its effect on the aggregate pool of knowledge we shall consider the following distributed lag formulation, commonly used in the literature; see for example Grossman and Helpman (1991, Chapter 3), or Eaton and Kortum (1999):

\[
k_{jt} = \int_{-\infty}^{t} \kappa e^{-\kappa(t-\tau)} q_{jt} d\tau.
\]

The rate of adjustment, denoted by \( \kappa \), measures the speed of adoption by the wider economy of an innovation made in sector \( j \). This can be interpreted as the reciprocal of the mean time-lag using the same exponential
distribution function; \( \tilde{\tau} = \int_0^\infty k e^{-\kappa \tau} \tau d\tau = 1/\kappa. \) The limiting case of \( \kappa \to \infty \) corresponds to instantaneous knowledge diffusion with \( \lim_{\kappa \to \infty} k_j = q_j \) and \( \lim_{\kappa \to \infty} K = Q. \) Differentiating the previous expression using the Leibniz rule gives

\[
\dot{k}_{jt} = \kappa (q_{jt} - k_{jt}).
\]  

(6)

4.2. Dynamics

Section 3 gave a qualitative explanation of the dominant forces that drive the results in partial equilibrium. However, in (5) both the productivity effect and the spillover effect, we previously identified, depend on \( \omega \) and \( \kappa, \) and should be endogenously determined. Symmetry still holds so that equation (2) gives the growth rate of the aggregate quality level, and (6) the growth rate of innovative spillovers:

\[
\dot{\hat{Q}}_t = \eta \left( \frac{K_t}{Q_t} \right)^{1-\omega} L_{St},
\]  

(7)

\[
\dot{\hat{K}}_t = \kappa \left( \frac{Q_t}{K_t} - 1 \right).
\]  

(8)

We define \( \gamma \equiv K/Q \in (0, 1]. \) Using the labor market clearing condition, the growth rate of this ratio with (7) and (8) reads

\[
\dot{\gamma}_t = \kappa \left( \frac{1}{\gamma_t} - 1 \right) - \eta \gamma_t^{1-\omega} (L - L_{Yt}).
\]  

(9)

Furthermore (5) with the definition of \( \gamma \) gives the growth rate of labor allocated to manufacturing as

\[
\dot{L}_{Yt} = \frac{\eta}{1 - \phi} \gamma_t^{1-\omega} L_{Yt} - (1 - \omega) \kappa \left( \frac{1}{\gamma_t} - 1 \right) - \rho.
\]  

(10)

Equations (9) and (10) give the dynamics of the economy in the \( \{L_Y, \gamma\} \) space for an active R&D sector. Here we are interested in the effect of policy along the balanced growth path and abstract from a thorough analysis of the dynamic system.\(^6\) In fact, in a recent contribution Grossmann et al. (2013), using a Jones (1995) model, study the optimal dynamic policy response to R&D externalities without time lags and find that the error of neglecting the transitional dynamics when designing the optimal R&D subsidy is small.

4.3. Optimal subsidy

Balanced growth can only exist if \( \gamma_t = \gamma \) and \( L_{Yt} = L_Y, \) constant. In this case \( g = \dot{C}/C = \dot{Y}/Y = \dot{Q}/Q = \dot{K}/K. \) The optimal subsidy rate, \( \phi^*, \) is the one needed to be imposed on the equilibrium allocation \( \{\gamma, L_Y\} \) (second best), in order to equate this with the socially optimal allocation \( \{\gamma^*, L_Y^*\} \) (first best). Equation (8) gives the steady state growth rate as a function of \( \gamma, \) and the speed of knowledge diffusion, \( \kappa: \)

\[
g = g_{Tech} = \kappa \left( \frac{1}{\gamma} - 1 \right).
\]  

(11)

\(^6\)Using equations (9) and (10), the determinant \( \Delta \) of the Jacobian matrix of the autonomous dynamic system reads

\[
\Delta = -\frac{\eta}{1 - \phi} \dot{\gamma}^{(1-\omega)} \left[ R + (1 - \omega)(1 - \phi) \right] + (1 - \omega)\eta L \dot{\gamma}^{(1-\omega)} < 0,
\]

which is negative for any of the permissible values of the parameters of the model, for any steady-state value \( \dot{\gamma} \in (0, 1], \) indicating a global saddle path stability.
This equation, denoted by $g_{Tech}$, represents the knowledge diffusion technology and holds for both allocations; the first and the second best. By setting (10) equal to zero, solving with respect to $L_Y$ and substituting in (9), while taking (11) into account, one gets the equilibrium steady state growth rate as a function of $\gamma$ and the policy parameter, $\phi$. This equation is defined as $g_{Dec}$:

$$g = g_{Dec} = \frac{\eta L y^{1-\omega} - (1 - \phi) \rho}{1 + (1 - \omega)(1 - \phi)}$$

(12)

The intersection of (11) and (12) in the $[\gamma, g]$ space gives the equilibrium steady state allocation depending on $\kappa$, $\omega$ and the policy $\phi$. Furthermore, solving (12) for $\phi$, with $g$ given by (11), gives $\phi(\gamma)$. We are now in position to study the decentralized equilibrium with the help of two functions: $g(\gamma)$ and $\phi(\gamma)$. These are plotted in Figure 2. Equation (12), the $g_{Dec}$ line, is shown for two levels of the subsidy rate; zero subsidy and the optimal subsidy. An increase in the subsidy rate leaves the $g_{Tech}$ line unaffected but shifts and turns the $g_{Dec}$ upwards, resulting in an increase in $g$ and a decrease in $\gamma$: higher subsidies stimulate research which, for a constant speed of knowledge diffusion, $\kappa$, increases $Q$ relative to $K$, thus lowering their steady-state ratio, $\gamma$.

The social planner seeks to maximize the present discounted value of utility taking into account $C = Y(1 - \beta)$, with (7) and (8). The result of this maximization is the $g_{S P}$ line that gives the optimal growth rate as a function of $\gamma$ (see Appendix):

$$g^* = g_{S P} = \frac{1 + \frac{\phi}{\kappa} \gamma}{1 + [1 + (1 - \omega)] \frac{\phi}{\kappa} \gamma} (\eta L y^{1-\omega} - \rho).$$

(13)

The optimal $\{\gamma^*, g^*\}$ can be found at the intersection of $g_{S P}$ with $g_{Tech}$. The optimal subsidy rate, $\phi^*$, is the subsidy rate needed to turn and lift the $g_{Dec}$ line to the point that $g_{Dec}(\gamma^*) = g^*$. The above reasoning is illustrated in Figure 2. At this point it would be instructive to study the behavior of the economy for the limiting cases of the relevant parameters $\kappa$ and $\omega$ and to confirm the qualitative findings of the previous section. The limiting case of instantaneous knowledge diffusion, $\kappa \to \infty$, implies $\gamma \to 1$ and so we can analytically get the optimal policy response as $\phi^* = 1 - \frac{\rho}{\eta L (1-\omega)(\eta L - \rho)}$, showing that $\phi^*$ is a monotonically decreasing concave function of $\omega$. As the compatibility of firm-specific innovation reduces, so does the externality and thus the optimal subsidy. For $\kappa \to 0$ innovation peters out and so subsidizing research becomes a moot point.

Interestingly, as we expected from our qualitative discussion, when one considers the timing of the knowledge diffusion process, optimal policy does not follow a clear cut rule anymore (see Figure 3). For high speed of knowledge diffusion (e.g. $\kappa = 0.2$, i.e. $\bar{\tau} = 5$ years), the spillover effect dominates, and since diminishing compatibility mitigates this effect, optimal policy falls in $\omega$. For low $\kappa$ (e.g. $\kappa = 0.05$, i.e. $\bar{\tau} = 20$ years), the optimal subsidy is non-monotonic in $\omega$. Ceteris paribus, $\phi^*$ is increasing in $\kappa$: fast adoption of sector-specific research results speaks in favor of higher optimal subsidies.
4.4. Comparative Statics

Using equations (11), (12) and (13) we define the implicit functions

\[ f_{Tech}(g^*, \gamma^*, \phi^*, \kappa, \omega) = g_{Tech}^* - g^* = 0, \]
\[ f_{Dec}(g^*, \gamma^*, \phi^*, \kappa, \omega) = g_{Dec}^* - g^* = 0, \]
\[ f_{SP}(g^*, \gamma^*, \phi^*, \kappa, \omega) = g_{SP}^* - g^* = 0, \]

and study the comparative statics of the optimum allocation for extreme cases of the relevant parameters \( \kappa \) and \( \omega \). The procedure can be found in the Appendix while the results in Figure 4. Since the behavior of the optimal policy in \( \kappa \) and \( \omega \) has been already extensively discussed, we present below only a brief summary.

- \( d\gamma^*/dk > 0 \): higher speed of diffusion expands the aggregate pool of knowledge at a higher rate, thus increasing the \( K/Q \) ratio.
Figure 3: Optimal subsidy $\phi^*$ for different values of $\kappa$ and $\omega$.

- $\frac{d\gamma^*}{d\omega} < 0$: parameter $\omega$ is used throughout as a proxy for the compatibility of firm-specific technological improvements with the average technology level of the market. Accordingly, an increasing $\omega$ implies less usable inter-sectoral knowledge spillovers and a decreasing $K/Q$ ratio.

- $\frac{d\phi^*}{d\kappa} > 0$: As we explained in the previous paragraph, a firm whose results are faster appreciated should deserve higher support since it expands the aggregate pool of knowledge at a higher rate.

- $\frac{d\phi^*}{d\omega}$ ambiguous: for low $\kappa$ and $\omega$ the firm lacks incentives to innovate and the productivity effect, which increases convexly in $\omega$, suffers most. The optimal response is then increasing in $\omega$. For large values of $\omega$, the social planner should give priority to correcting for the spillover effect. The spillover effect decreases current research activities, since entrepreneurs are anticipating falling R&D cost in the future, and is linearly mitigated by $\omega$, indicating a decreasing optimal subsidy rate in $\omega$.

- $\frac{dg^*}{d\kappa} > 0$: see equation (13) with $\frac{d\gamma^*}{d\kappa} > 0$.

- $\frac{dg^*}{d\omega} > 0$: combine (11) with $\frac{d\gamma^*}{d\omega} < 0$. In general, increasing $\omega$ lowers spillovers, increases the appropriability and stimulates research activity and growth.

Parameters: $\rho = 0.05, \eta = 0.5, L = 1$. 
Figure 4: Comparative statics for different values of $\kappa$ and $\omega$.

\[
\begin{array}{cccc|cccc|cccc|cccc|cccc}
 & d\kappa & d\omega & d\kappa & d\omega & d\kappa & d\omega & d\kappa & d\omega & d\kappa & d\omega & d\kappa & d\omega \\
d\gamma^* & + & - & d\gamma^* & + & - & d\gamma^* & + & - & d\gamma^* & + & - \\
d\phi^* & + & + & d\phi^* & + & - & d\phi^* & + & - & d\phi^* & + & - \\
dg^* & + & + & dg^* & + & + & dg^* & + & + & dg^* & + & + \\
\end{array}
\]

(a) $\kappa = 0.05$, $\omega = 0.1$  (b) $\kappa = 1$, $\omega = 0.1$  
(c) $\kappa = 0.05$, $\omega = 0.9$  (d) $\kappa = 1$, $\omega = 0.9$

Parameters: $\rho = 0.05, \eta = 0.5, L = 1$.

5. Extension: endogenous speed of diffusion

It is reasonable to assume that nowadays, in the era of technology and social media, information spreads faster than it used to in earlier times. Comin and Hobijn (2010) find that the average lag in technology adoption falls with the invention date of technologies, while Comin and Mestieri (2014) report a time lag of $121 \pm 53$ years for steam and motor ships (invented in 1788), $12 \pm 6$ for open heart surgery (invented in 1968), but only $7 \pm 3$ years for the internet (invented in 1983).

There are several ways to access firm-specific information over a practice or technology: internet research, through peers, by reverse engineering, or by imitation from others who have already accessed this information at an earlier stage. All of these processes become increasingly effective as the technology frontier of the market advances. In our model, at every instance of time, $Q$ is the technological level of the representative firm while $K$ the attained technology frontier of the market. Since the spillovers of advancements in $Q$ expand $K$, variable $K$ still lags behind $Q$ so that $K/Q \leq 1$ as in section 4.2.

Subsequently, we can think now of the effective speed of diffusion, $\kappa$, to be comprised of two parts: a technology-specific part, as before, and another part modulated by the $K/Q$ ratio, i.e. $\kappa = a + b\frac{K}{Q}$, with $a, b > 0$; once firm-specific information starts diffusing into the market it becomes increasingly easy to do so as the market’s technology benefiting from spillovers advances. This is in essence the basic Bass diffusion model for forecasting the adoption of a product/innovation, widely used in the marketing science, modified here to include an ever expanding technological potential $Q$; see Bass (1969). Accordingly, equation (8) becomes:

\[
\hat{K}_t = \left(a + b\frac{K_t}{Q_t}\right) \left(\frac{Q_t}{K_t} - 1\right). \tag{14}
\]

With this specification equations (9) and (10) become, respectively:

\[
\hat{\gamma}_t = (a + b\gamma_t) \left(\frac{1}{\gamma_t} - 1\right) - \eta\gamma_t^{1-\omega}(L - L_{yt}), \tag{15}
\]

\[
\hat{L}_{yt} = \frac{\eta}{1 - \phi}\gamma_t^{1-\omega}L_{yt} - (1 - \omega)(a + b\gamma_t) \left(\frac{1}{\gamma_t} - 1\right) - \rho. \tag{16}
\]

It can be proven that the dynamic system in $\{\gamma, L_y\}$ is again globally saddle stable. Following the same procedure as before we calculate the optimal subsidy rate that depends on the model’s parameters:
parameters $a$ and $\omega$ remain technology / sector-specific, whereas $b$ is now market specific.\footnote{Using the original version of the Bass diffusion model that gives for a certain market potential the rate of technology adoption over time, Sultan et al. (1990) and Teng et al. (2002) estimate $a$ and $b$ for several technologies: while $a$ can vary greatly within $0.005 - 0.08$, with a mean of $0.03$, parameter $b$ falls mostly within the range of $0.1 - 0.5$, with a mean of $0.38$.}

Figure 5: Optimal subsidy $\phi^*$ using the Bass diffusion technology

![Graph showing optimal subsidy rates for different values of $a$ and $\omega$]

(a) $b = 0.1$
(b) $b = 0.5$

Parameters: $\rho = 0.05, \eta = 0.5, L = 1$.

Figure 5 presents the optimal subsidy rate with the new diffusion specification. There are several things to note. First, the resulting optimal subsidy is highly non-linear in $\omega$ for low values of $a$ and $b$ (left panel). Second, as before, ceteris paribus, the optimal subsidy increases with $a$; technologies that diffuse faster should be granted higher support. Third, a higher value for $b$ produces monotonic results even for low values of $a$. This is intuitive because even if spillovers start diffusing at a very low speed, the effective speed of adoption continuously increases at an increasing rate; comparing the results here with Figure 3, in effect there is an upward shift in regimes from a low to a high value of $\kappa$. Finally, we confirm the thesis on the importance of the inter-temporal dimension of spillovers for optimal R&D support policies.

6. Conclusion

Empirical evidence suggests that there is room for improvement in the public budget allocation to private research activities. Motivated by findings on the inter-sectoral and inter-temporal aspects of knowledge spillovers, we study the effect of the timing of knowledge diffusion on the optimal industrial policy promoting research. Knowledge dissemination does not occur simultaneously, as commonly postulated in the literature, but rather with a time lag. We also differentiate between inventive activity in the lab and applicable innovation. This distinction reflects their true use in the economy, but also highlights the fact that the latter typically lags behind the former.
It is shown that, for large time lags, the optimal subsidy evolves non-monotonically in the parameter that proxies the compatibility of a sector-specific invention with the practices of the wider economy. This result contradicts the consensus in the literature that optimal subsidies should fall as innovation becomes more sector-specific and gives more room for improvement of R&D promoting policies. According to our results, the optimal R&D support policy should be sector-specific and such that it takes into account both the inter-sectoral and the inter-temporal aspect of knowledge spillovers. We use the example of green innovation to argue that wide applicability of green innovations alone does not justify higher research subsidies as commonly postulated in the literature, but only when coupled with the positive effects of a cleaner environment. The results also highlight the importance of including the inter-temporal dimension of knowledge spillovers in frameworks with more realistic market dynamics of the Schumpeterian type when studying firm-heterogeneity, reallocation, and endogenous entry and exit. This is left for future research.

Appendix A. Decentralized equilibrium

From (3) using \( w = (1 - \beta)Q, Y = QL_Y \) and \( g = \dot{Y}/Y \), we get \( \dot{\lambda}_j = g - \dot{L}_Y - \omega \dot{\mu}_j - (1 - \omega)\dot{K}_j \). Dividing (4) by \( \lambda_j \), substituting \( r = g + \rho \) from the Keynes-Ramsey rule, \( w = (1 - \beta)Q \), and assuming symmetry (i.e. \( L_{Sj} = L_S \) and \( q_j = Q \)), the growth rate of non-scientific labor employment in the final good sector follows as in (5).

Appendix B. Social Optimum

The associated Hamiltonian of the social planner reads

\[
H_t = \ln(Q_tL_Yt) + \mu_t\eta Q_t^\omega K_t^{1-\omega}(L - L_Yt) + \nu_tK_t - K_t).
\]

The first order conditions with respect to \( L_Y, Q \) and \( K \) are respectively

\[
L_{Yt}^{-1} = \eta(\mu_tQ_t) \left( \frac{K_t}{Q_t} \right)^{1-\omega},
\]

\[
Q_t^{-1} + \mu_t\omega \left( \frac{K_t}{Q_t} \right)^{1-\omega}(L - L_Yt) + \nu_tK_t = \rho\mu_t - \dot{\mu}_t, \tag{B.3}
\]

\[
\mu_t(1 - \omega) \left( \frac{K_t}{Q_t} \right)^{-\omega}(L - L_Yt) - \nu_tK_t = \rho\nu_t - \dot{\nu}_t. \tag{B.4}
\]

Variables \( \mu \) and \( \nu \) are the shadow prices for the two dynamic equations. By substituting \( L_Y \) from (B.2) to the other two equations, after dividing (B.3) with \( \mu \) and (B.4) with \( \nu \), and using (7) and (8), we have a system of differential equations in the \( \{Q, K, \mu, \nu\} \)-space. We can then redefine the variables \( \chi = \mu Q, \xi = \nu K \) and \( \gamma = K/Q \) to get the following autonomous system of dynamic equations for the social planner (in growth rates)

\[
\dot{\chi}_t = \rho - \frac{1}{\chi_t} \left( 1 + (1 - \omega) + \kappa \frac{\xi_t}{\gamma_t} \right) + (1 - \omega)\eta Q_Y t^{1-\omega} \tag{B.5}
\]

\[8\]See for example Klette and Kortum (2004) and Acemoglu et al. (2013).
\[ \hat{\xi}_t = \rho + \kappa + \frac{1 - \omega}{\xi_t} (1 - \chi_t \eta L \gamma_{t-1} - \omega) + \kappa \left( \frac{1}{\gamma_t} - 1 \right), \quad (B.6) \]

\[ \hat{\gamma}_t = \kappa \left( \frac{1}{\gamma_t} - 1 \right) - \eta L \gamma_{t-1} - \omega + \chi_t. \quad (B.7) \]

Furthermore the transversality conditions \( \lim_{t \to \infty} \chi_t e^{-\rho t} = 0 \) and \( \lim_{t \to \infty} \xi_t e^{-\rho t} = 0 \) apply. In the steady state \( \hat{\gamma}_t = 0 \), so that \( \hat{Q} = \hat{K} = g \). It follows as a necessary condition from (B.5)-(B.7) that \( \hat{\xi}_t = 0 \) and \( \hat{\xi}_t = 0 \) as well. Solving (B.5) and (B.6) for \( \chi \) in the steady state and substituting the solution in (B.7) while noting that \( g = \kappa (1/\gamma - 1) \) from (11) gives equation (13) of the main text.

**Appendix B.1. Comparative statics**

Suppose that using (11), (12) and (13) the steady state level of the vector \( z \) of the endogenous variables \( g^*, \gamma^*, \phi^* \) in the first best allocation is given by the system of implicit functions \( f(z, a) = 0 \), with vector \( a \) containing the relevant parameters \( \kappa \) and \( \omega \), as defined in the main text. Then, to a first approximation, sufficiently small changes in the exogenous parameters \( da \) will result in changes in the endogenous variables \( dz \) according to

\[ dz = -J_z^{-1} J_a da = 0, \quad (B.8) \]

with \( J_z \) the 3x3 matrix of partial derivatives of the \( f \) functions w.r.t. the elements of the vector \( z \) and \( J_a \) the 3x2 matrix of partial derivatives of \( f \) w.r.t. the elements of \( a \), both evaluated at the optimal steady state values \( g^*, \gamma^*, \phi^* \) for the specific parameters \( \kappa \) and \( \omega \). We can then get the static effect of each of the exogenous parameters on each of the endogenous variables by having the relevant parameter varying and the other constant.

**References**


Dechezleprêtre, A., Martin, R., Mohnen, M., 2014. Knowledge spillovers from clean and dirty technologies. CEP discussion papers, Centre for Economic Performance, LSE.


Working Papers of the Center of Economic Research at ETH Zurich

(PDF-files of the Working Papers can be downloaded at www.cer.ethz.ch/research/working-papers.html).

17/267 Christos Karydas
   The inter-temporal dimension to knowledge spillovers: any non-environmental reason to support clean innovation?

17/266 Christos Karydas, Lin Zhang
   Green tax reform, endogenous innovation and the growth dividend

17/265 Daniel Harenberg, Stefano Marelli, Bruno Sudret, Viktor Winschel
   Uncertainty Quantification and Global Sensitivity Analysis for Economic Models

16/264 Marie-Catherine Riekhof
   The Insurance Premium in the Interest Rates of Interlinked Loans in a Small-scale Fishery

16/263 Julie Ing
   Adverse selection, commitment and exhaustible resource taxation

16/262 Jan Abrell, Sebastian Rausch, and Giacomo A. Schwarz
   Social Equity Concerns and Differentiated Environmental Taxes

16/261 D. Ilic, J.C. Mollet
   Voluntary Corporate Climate Initiatives and Regulatory Loom: Batten Down the Hatches

16/260 L. Bretschger
   Is the Environment Compatible with Growth? Adopting an Integrated Framework

16/259 V. Grossmann, A. Schaefer, T. Steger, and B. Fuchs
   Reversal of Migration Flows: A Fresh Look at the German Reunification

16/258 V. Britz, H. Gersbach, and H. Haller
   Deposit Insurance in General Equilibrium

16/257 A. Alberini, M. Bareit, M. Filippini, and A. Martinez-Cruz
   The Impact of Emissions-Based Taxes on the Retirement of Used and Inefficient Vehicles: The Case of Switzerland

16/256 H. Gersbach
   Co-voting Democracy

16/255 H. Gersbach and O. Tejada
   A Reform Dilemma in Polarized Democracies
<table>
<thead>
<tr>
<th>Paper Number</th>
<th>Authors</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>16/254</td>
<td>M.-C. Riekhof and J. Broecker</td>
<td>Does the Adverse Announcement Effect of Climate Policy Matter? - A Dynamic General Equilibrium Analysis</td>
</tr>
<tr>
<td>16/253</td>
<td>A. Martinez-Cruz</td>
<td>Handling excess zeros in count models for recreation demand analysis without apology</td>
</tr>
<tr>
<td>16/252</td>
<td>M.-C. Riekhof and F. Noack</td>
<td>Informal Credit Markets, Common-pool Resources and Education</td>
</tr>
<tr>
<td>16/251</td>
<td>M. Filippini, T. Geissmann, and W. Greene</td>
<td>Persistent and Transient Cost Efficiency - An Application to the Swiss Hydropower Sector</td>
</tr>
<tr>
<td>16/250</td>
<td>L. Bretschger and A. Schaefer</td>
<td>Dirty history versus clean expectations: Can energy policies provide momentum for growth?</td>
</tr>
<tr>
<td>16/249</td>
<td>J. Blasch, M. Filippini, and N. Kumar</td>
<td>Boundedly rational consumers, energy and investment literacy, and the display of information on household appliances</td>
</tr>
<tr>
<td>16/248</td>
<td>V. Britz</td>
<td>Destroying Surplus and Buying Time in Unanimity Bargaining</td>
</tr>
<tr>
<td>16/247</td>
<td>N. Boogen, S. Datta, and M. Filippini</td>
<td>Demand-side management by electric utilities in Switzerland: Analyzing its impact on residential electricity demand</td>
</tr>
<tr>
<td>16/246</td>
<td>L. Bretschger</td>
<td>Equity and the Convergence of Nationally Determined Climate Policies</td>
</tr>
<tr>
<td>16/245</td>
<td>A. Alberini and M. Bareit</td>
<td>The Effect of Registration Taxes on New Car Sales and Emissions: Evidence from Switzerland</td>
</tr>
<tr>
<td>16/244</td>
<td>J. Daubanes and J. C. Rochet</td>
<td>The Rise of NGO Activism</td>
</tr>
<tr>
<td>16/242</td>
<td>M. Glachant, J. Ing, and J.P. Nicolai</td>
<td>The incentives to North-South transfer of climate-mitigation technologies with trade in polluting goods</td>
</tr>
<tr>
<td>16/241</td>
<td>A. Schaefer</td>
<td>Survival to Adulthood and the Growth Drag of Pollution</td>
</tr>
</tbody>
</table>
16/240  K. Prettner and A. Schaefer
Higher education and the fall and rise of inequality

16/239  L. Bretschger and S. Valente
Productivity Gaps and Tax Policies Under Asymmetric Trade

16/238  J. Abrell and H. Weigt
Combining Energy Networks

16/237  J. Abrell and H. Weigt
Investments in a Combined Energy Network Model: Substitution between Natural Gas and Electricity?

16/236  R. van Nieuwkoop, K. Axhausen and T. Rutherford
A traffic equilibrium model with paid-parking search

16/235  E. Balistreri, D. Kaffine, and H. Yonezawa
Optimal environmental border adjustments under the General Agreement on Tariffs and Trade

16/234  C. Boehringer, N. Rivers, H. Yonezawa
Vertical fiscal externalities and the environment

16/233  J. Abrell and S. Rausch
Combining Price and Quantity Controls under Partitioned Environmental Regulation

16/232  L. Bretschger and A. Vinogradova
Preservation of Agricultural Soils with Endogenous Stochastic Degradation

16/231  F. Lechthaler and A. Vinogradova
The Climate Challenge for Agriculture and the Value of Climate Services: Application to Coffee-Farming in Peru

16/230  S. Rausch and G. Schwarz
Household heterogeneity, aggregation, and the distributional impacts of environmental taxes

16/229  J. Abrell and S. Rausch
Cross-Country Electricity Trade, Renewable Energy and European Transmission Infrastructure Policy

16/228  M. Filippini, B. Hirl, and G. Masiero
Rational habits in residential electricity demand

16/227  S. Rausch and H. Schwerin
Long-Run Energy Use and the Efficiency Paradox