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RESEARCH ON THE EDGE POSITIONING TECHNOLOGY OF THE MARK CIRCULAR SUBPIXEL

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Fluid: high speed flows
Visualization method(s): Zernike moment
Other keywords: mark circular, image processing, edge positioning

ABSTRACT: Based on analysis of edge detection principle of Zernik moment subpixel image, while aiming at shortage of its insufficient phenomenon and poor edge positioning precision, introduce a subpixel edge detection method, which can specially target mark circular. While based on Canny operator detecting the pixel level edge, and combining with acquired mark circle geometric information, reduce calculation templates number to 1, thus advance algorithm operation speed. Meanwhile experimental result shows the method can improve both the precision and the anti-noise performance.

1 Introduction
As a basic feature of image, edge detection has always been an important aspect in image processing technology. While as a common positioning objects[1] in visual technology, circular target will turn to oval under perspective projection transformation, therefore rapid and accurate oval edge positioning plays an important role in computer vision and image processing. For traditional edge detection operators are usually at pixel level, sensitive to noise, with low accuracy, and in the field of computer vision measurement, detected edge point precision directly affects accuracy of measurement results, thus image edge sub-pixel detection algorithm has great practical significance.

Subpixel edge positioning method includes three types: quasi-legal, interpolation method and moment based method. For quasi-legal[2], the method fits gray value of image edge, thus make the edge point error minimum and get sub-pixel precision edge, improve positioning accuracy, and the disadvantage is time consuming. For interpolation method[3], according to rule of image gray distribution, and through methods like nearest neighbor interpolation, bilinear interpolation and cubic convolution interpolation to interpolate and obtain subpixel edges. It is characterized by short computation time and low noise immunity. For moment based method, which based on the first established ideal edge model, and using the invariant properties of moments to establish model parameter equation and get the exact parameters of the edge. Good anti noise performance is an outstanding advantage of all moment based methods. Lyvers et al.[4] put forward using the geometric moment to extract the subpixel edge of the image, which uses six image geometric moments to calculate the four step functions of the edge. When subpixel edge positioning is based on space moment, it is necessary to convolve the image with 6 moment templates. Ghosal et al.[5] used the orthogonal Zernike moment to carry out edge detection. The algorithm only needs three templates to calculate four parameters, with fast calculation speed, thus gets more attention and application. However, traditional Zernike moment needs repeat adjustment of threshold when judging the edge, which not only inefficient, but also easy to affect judgment accuracy.
Literature[6,7] put forward on the basis of rough location of Sobel operator, carry out related moment operations on obtained edge points, thus greatly reduce the number of pixels involved in template convolution operation and improve the arithmetic speed of the algorithm. Literature[8] expanded the template dimension and number of Zernike orthogonal moments to obtain higher positioning accuracy, yet both the algorithm complexity and operation time are increased.

In the paper, a subpixel edge location method based on ellipse target is proposed based on the gray distribution characteristics of ellipse targets. The method uses the principle of subpixel edge positioning based on moment, and through edge coarse positioning of Canny operator [9], reduce the number of involved calculation templates to one, therefore greatly reduce calculating time and guarantee accuracy.

2 Zernike moment subpixel edge detection

2.1 Zernike moment definition

The Zernike moment is an orthogonal function based on Zernike polynomials, and the set of orthogonal polynomials is a complete positive intersection on the unit circle. In the polar coordinate system (\(\rho, \theta\)), the Zernike moment of order \(m\) is defined as:

\[
V_{nm}(\rho, \theta) = R_{nm}(\rho)e^{jm\theta}
\]

\(n \geq 0\) in the formula, \(n - |m|\) is a positive even number, and the real value polynomial \(R_{nm}(\rho)\) is defined as:

\[
R_{nm}(\rho) = \sum_{s=0}^{(n-|m|)/2} \frac{(-1)^s(n-s)!\rho^{n-2s}}{s!(n+|m|-s)!(n-|m|-s)!}
\]

The \(m\)th order Zernike moment of image \(f(x, y)\) is defined as:

\[
A_{nm} = \frac{n+1}{\pi} \iint_{x^2+y^2 \leq 1} f(x, y) V_{nm}^*(\rho, \theta) dxdy
\]

In the formula, \(V_{nm}(\rho, \theta)\) is the integral kernel, \(m, n\) is all integers and \(n \geq 0\), \(n - |m|\) is even and \(n \geq |m|\).

2.2 Principle of Zernike moment subpixel edge detection

For a digital image of size \(N \times N\), the integral of the discrete 2d image \(f(x, y)\) is replaced by the summation form, and the Zernike moment is defined as:

\[
A_{nm} = \frac{n+1}{\pi(N-1)^2} \sum_{x=1}^{N} \sum_{y=1}^{N} f(x, y) \tilde{V}_{nm}(x, y)
\]

Based on the rotation invariance of Zernike moment, the image is rotated \(\phi\) angles, The relationship between Zernike moment \(A_{nm}\) before rotation and Zernike moment \(A'_{nm}\) after rotation is:

\[
A'_{nm} = A_{nm}e^{-j\phi}
\]

As formula (5) shows, the pattern of Zernike moment \(A_{nm}\) and Zernike moment \(A'_{nm}\) did not change before and after rotation, but the phase changed, which is the rotation invariant property of Zernike moment.
The Zernike moment is an integral operator, which is insensitive to noise in the case of noise. Its basic idea is to fit actual edge with ideal edge, and position the ideal edge position as actual marginal position. Figure 1 shows the parameterized model of ideal edge model.

As Figure 1 shows, the circle is unit circle, the line contained in the circle represents ideal edge. Gray values on both sides are \( h \) and \( h+k \), \( k \) means gray value difference. \( l \) means vertical distance from center to edge, \( l \in [-1,1] \), \( \phi \) is the angle between the line and X-axis, \( \phi \in [0, \pi] \). Rotate the edge clockwise around \( \phi \) angles until parallel Y axis, then:

\[
\int_{x^2+y^2\leq1} f'(x, y) y dy dx = 0
\]  

In the formula, \( f'(x, y) \) means image after rotation.

Only three different ranks of \( A_{nm} \) are used in derivation formula of subpixel edge detection, they are \( A_{00}, A_{11} \) and \( A_{20} \), and their corresponding integral kernel functions are \( V_{00} = 1, V_{11} = x + jy \) and \( V_{20} = 2x^2 + 2y^2 + 1 \). Relationship between \( A_{nm} \) before and after image rotation is: \( A'_{00} = A_{00}, A'_{11} = A_{11}e^{j\phi}, A'_{20} = A_{20} \)

Steps of using Zernike moment to detection edge are as the following:

1. Calculate \( A_{00}, A_{11}, A_{20} \) at Zernike moment.
2. Suppose that \( f(x, y) = 1 \) is the template, and the template for \( A_{nm} \) is \( M_{nm} \), then:

\[
M_{nm} = \int_{x^2+y^2\leq1} V_{nm}^*(\rho, \theta) d\rho d\theta
\]  

Let \( f(x, y) = 1 \) be the square area of row i and column j, \( C \) represents the unit circle area surrounded by \( x^2 + y^2 \leq 1 \), row i and column j coefficient of template \( M_{nm} \) is \( M_{nm-ij} \), The corresponding integral domain area is expressed as \( \Omega_{ij} = C \cap S_{ij} \), then :

\[
M_{nm-ij} = \int_{\Omega_{ij} = C \cap S_{ij}} V_{nm}^*(\rho, \theta) d\rho d\theta
\]  

The above equation can calculate \( A_{00}, \text{Im}[A_{11}], \text{Re}[A_{11}], A_{20} \) template coefficient separately.

3. The value of \( \phi \) can be calculated via \( \tan\phi = \frac{\text{Im}[A_{11}]}{\text{Re}[A_{11}]} \).

4. The value of \( A'_{00}, A'_{11}, A'_{20} \) can be calculated via \( A'_{00} = A_{00}, A'_{11} = A_{11}e^{j\phi}, A'_{20} = A_{20} \)

Edge parameters
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\[ l = \frac{A_{20}}{A_{41}} \]
\[ k = \frac{3A_{41}}{2(1-l^2)^{3/2}} \]
\[ h = \frac{A_{00} - \frac{k\pi}{2} + k\sin^{-1}(l) + kl\sqrt{1-l^2}}{\pi} \]  \hspace{1cm} (9)

(5) If \((x, y)\) is image edge point, the detection formula of the sub-pixel point of this point is derived.

\[
\begin{bmatrix}
    x_s \\
    y_s
\end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \frac{N}{2} \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix}
\]  \hspace{1cm} (10)

Where \(N\) is the size of Zernike moment’s \(N \times N\) template.

3 Pixel edge positioning based on marked circle.

3.1 Rough positioning of marked circle

Marked circle turns into an ellipse after perspective transformation. The most commonly used method of the elliptic geometric parameter is the fitting of the boundary by least square method. Yet premise is it is necessary to carry out basic operations on image like preprocessing, binarization and boundary extraction. Image preprocessing usually adopt smoothing filter\[10\], which aims to maintain edge characteristics when eliminating random noise.

Coarse orientation of target ellipse is based on rough fitting of two valued image contour. Basic idea: First preprocess image, then use OTSU method to obtain two-valued image. After that, use Canny operator to extract all contour of binary image. Finally, according to requirements, select desired target ellipse contour fitting to get initial parameters.

3.2 Calculation of edge model parameters

Fig. 2. Two dimensional edge model

Fig. 2 is a continuous 2-d edge model, \(h_1\) is background gray value, \(h_2\) is target gray value, \(l\) is normalized distance from actual boundary point to origin, \(\phi\) is Angle between edge normal and axis \(x\), \(S_1\) and \(S_2\) respectively represent area of background region in edge model and area of target area in edge model.

Zero order moment of continuous two-dimensional edge model is:

\[ M_{00} = \iint f(x, y)dx\,dy = S_1h_1 + S_2h_2 \]  \hspace{1cm} (11)

If the edge model is a unit circle, then:
According to preliminarily obtained geometric parameters of the target, edge Angle of $\phi$ can be quickly solved by using gradient direction of edge pixel as normal direction of the change point. Center of the edge pixel $(x_p, y_p)$ is located on the ellipse of $(x_0, y_0)$, and the edge Angle of the point is:

$$
\phi = \arctan \left( \frac{y - y_0}{x - x_0 + \exp} \right)
$$

(13)

$h_1$ and $h_2$ respectively refer to background gray value and target gray value. According to elliptic target distribution characteristics, it can be based on initial parameters, select two average gray values of contour line, one line inclusive of ellipse target, and the other is included by an elliptical target, as parameter values. As shown in figure 3:

![Background grayscale and target grayscale acquisition schematic.](image)

Where $l_0$ is initial contour of elliptical target, $R_1$ and $R_2$ are the short and half axes of target initial contour $l_0$. Both $l_1$ and $l_2$ are similar elliptic shapes of $l_0$. The average gray value of contour $l_1$ represents $h_2$ value, the average gray value of contour $l_2$ represents $h_1$ value, $l_1$ is 1/2 of $l_0$, $l_2$ is 3/2 of $l_0$.

<table>
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<th>Tab. 1. The z00 template for 7 × 7</th>
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<tr>
<td>0</td>
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</tbody>
</table>

For an ideal edge model, it needs to dispersed to image of $N \times N$, $N$ is an odd number greater than 1. Theoretically the greater the $N$, the higher the precision, yet big template increase calculation complexity, thus choose $N = 7$ can meet the requirements. The template $Z_{00}$ of 7×7 is shown in table 1. The $M_{00}$ value in formula (12) can be obtained by convolution of corresponding size image with the template $Z_{00}$.

Clockwise rotate continuous ideal edge model in figure 1 according to Angle $\phi$, and the top view of the continuous ideal edge model after rotation is shown in figure 4.
Where, $l$ represents distance from actual edge to origin, and $S_2$ shadow represents the above target area.

According to the geometric relationship:

$$S_2 = \beta - \frac{1}{2} \sin(2\beta)$$

$$l = \cos \beta$$

(14)

Under conditions of moment template $N = 7$ and $\beta \in (0, \pi)$, value range of $S_2$ is $[1.0708, 2.2249]$, so relationship function between $\beta - S_2$ in the region can be obtained according to Matlab simulation software:

$$\beta = 0.5096 \cdot S_2 + 0.7708$$

(15)

3.3 Implementation steps of edge positioning for marked circular sub pixel

According to above theoretical derivation, implementation steps of edge positioning for marked circular sub pixel are as follows:

(1) Image preprocessing, completion of Canny operator contour extraction and calculation of ellipse target geometry parameters.

(2) For each pixel edge point of elliptical target contour, obtain edge point of corresponding sub-pixel level when doing the following ①-⑤ steps:

① According to the geometrical parameters of ellipse, the Angle $\phi$ of the corresponding edge point is obtained by using the above equation (13).

② According to ellipse geometrical parameters, obtain target grayscale $h_1$ and background grayscale $h_2$ in grayscale image via method shown in figure 3.

③ Use convolution of zero order matrix template $7 \times 7$ and $7 \times 7$ square image centered on edge points on grayscale image, as Table 1 shows, to obtain $M_{00}$, the zero order matrix value of edge point.

④ Bring zero-order matrix value into equation (12) to obtain $S_2$ of target region in the edge model, and the edge parameter $l$ can be calculated according to formula (14) and (15).

⑤ According to equation (10), the real coordinate values of edge points can be calculated.

4 Experiment

According to above theories, with increase of torque, Zernike moment subpixel edge location accuracy gets higher, and considering algorithm efficiency and positioning accuracy, this experiment adopts template of $7 \times 7$. 
4.1 Simulation results
We use 3ds Max to simulate the view of camera to produce an ideal image containing elliptical target. Considering smooth action of camera lens, use $3 \times 3$ template mean smoothing operator to dispose. In order to test stability of the algorithm, add different SNR random noises to image. The signal-to-noise ratio is defined as: $SNR = 20 \log_{10} \frac{k}{\sigma} dB$

Where $k$ is contrast of the image, and $\sigma$ is a standard deviation of added random noise. Compare traditional Zernike moment method with simulated image of SNR=30,40,50,100db. Results in table 2 shows the method of this paper has good effect on edge positioning accuracy and anti-noise performance.

<table>
<thead>
<tr>
<th>Method of adoption</th>
<th>30dB RMSE</th>
<th>Max error</th>
<th>40dB RMSE</th>
<th>Max error</th>
<th>50dB RMSE</th>
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<td>4.65</td>
<td>4.03</td>
<td>4.77</td>
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</tbody>
</table>

4.2 Real experiment
Use camera to take a single target elliptical chart, as shown in figure 5(a). Respectively use Canny operator, traditional Zernike moment algorithm and the algorithm of this paper for edge positioning, the positioning effect is shown in fig.5 (b,c,d). Experimental results show that the method has higher smoothness and efficiency.

Fig. 5. Contrast of elliptical target edge positioning
In order to further verify stability of the algorithm in elliptical target edge of physical image, select a calibrated image with 99 elliptical targets at site, as shown in figure 6.

Fig. 6. Calibration plate image
Using traditional Zernike method and the method in this paper, the fitting error of all ellipse edges is shown in figure 7.

The above result shows the method of this paper is better than traditional moment method at fitting error, and it can significantly improve stability and precision.

5 Conclusion
This paper first analyzes traditional subpixel edge detection method based on Zernike moment, while aiming at perspective transformation of circular target ellipse target, propose an ellipse target subpixel edge detection method. In this method, the geometrical information of ellipse contour is obtained, and the edge parameter model is established, and the high-precision edge positioning can be obtained by using only one moment template. Experimental result shows this method not only speed up operation, but also improve accuracy and stability.

Reference: