Conference Paper

Tomographic rainbow schlieren measurements for underexpanded sonic jets from rectangular convergent nozzles

Author(s):
Ezoe, M.; Nakao, S.; Miyazato, Y.

Publication Date:
2018-10-05

Permanent Link:
https://doi.org/10.3929/ethz-b-000279217

Rights / License:
In Copyright - Non-Commercial Use Permitted
TOMOGRAPHIC RAINBOW SCHLIEREN MEASUREMENTS
FOR UNDEREXPANDED SONIC JETS
FROM RECTANGULAR CONVERGENT NOZZLES

M. Ezoe¹, S. Nakao¹, and Y. Miyazato¹, c

¹Department of Mechanical Systems Engineering, The University of Kitakyushu,
1-1 Hibikino, Wakamatsu-ku, Kitakyushu, Fukuoka, 808-0135, Japan
²Corresponding author: Tel.: +81936953219; Email:miyazato@kitakyu-u.ac.jp

KEYWORDS:
- Main subjects: density measurement, flow visualization
- Fluid: underexpanded free jet, shock-containing flow
- Visualization method(s): Rainbow Schlieren Deflectometry
- Other keywords: computed tomography

ABSTRACT: The underexpanded free jet issued from a rectangular convergent nozzle with an aspect ratio of 4.0 at the nozzle exit has been experimentally investigated using the rainbow schlieren deflectometry combined with the computed tomography. The nozzle operating pressure ratio is held constant at 4.0 to produce a flowfield including a Mach shock in the jet plume. Multidirectional rainbow schlieren pictures of the jet are acquired by rotating the nozzle about its longitudinal axis in equal angular intervals and the three-dimensional density field is reconstructed by the convolution back-projection algorithm. The three-dimensional compressible Reynolds-averaged Navier-Stokes (RANS) simulations with the Menter's SSTk-ω turbulent model also have been performed for a mutual comparison with the experimental results. The fine details of the jet structure are demonstrated with density contour plots on a jet cross-section, the streamwise and radial density profiles.

1 Introduction

Qualitative flow visualization including schlieren and shadowgraph techniques [1] is routinely used to evaluate the computed flowfield. Many researchers often compare simulated schlieren or shadowgraph images directly with those from experiments having the same flow conditions in order to validate the simulation. However, even though a computed schlieren picture is in good agreement with an experimental one, genuine scalar or vector fields cannot always be captured correctly by the computation because conventional schlieren images can only provide integrated information about the density gradient. Therefore, a comparison between experiment and simulation is only meaningful if the numerical results are spatially averaged along the view direction. In the present study, the three-dimensional density field of an underexpanded sonic jet issued from a rectangular convergent nozzle with an aspect ratio of 4 at the nozzle exit is captured for the first time by the rainbow schlieren deflectometry combined with the computed tomography, because rectangular supersonic jets attract special attentions due to their enhanced mixing for combustion applications and high-speed jet noise suppression when compared to those of comparable axisymmetric supersonic jets. Also, despite the pioneering work by a research group of Agrawal et al. [2], an application for supersonic jets of the rainbow schlieren deflectometry is lacking in the current literature. In addition to the experimental work, the three-dimensional underexpanded sonic jet is solved under the same conditions as those in the experiment using the RANS simulation with the SST k-ω turbulent model. A quantitative
comparison between experiment and simulation is carried out to investigate three-dimensional spatial variations of the shock-containing free jet from the rectangular nozzle.

2 Experimental apparatus
Experiments have been performed in a blow-down supersonic wind tunnel with the jet issued in the quiescent laboratory air. A schematic drawing of the experimental apparatus is shown in Fig. 1.

![Schematic drawing of experimental apparatus](image)

The air supplied by a compressor that pressurizes the ambient air up to 1 MPa is filtered, dried and stored in a high-pressure reservoir consisting of two tanks with a total capacity of 2 m³. The high-pressure dry air from the reservoir is stagnated in a plenum chamber and then discharged into the atmosphere through a convergent nozzle with a rectangular area of 5 mm × 20 mm at the nozzle exit. The total temperature in the plenum chamber is equal to the room temperature, and the plenum pressure is controlled and maintained constant during the testing by a valve. The jet issued from the nozzle is visualized by the rainbow schlieren deflectometry for a nozzle pressure ratio of 4.0. The rainbow schlieren system consists of rail-mounted optical components including a 50 μm diameter pinhole, two 100 mm diameter, 500 mm focal length achromatic lenses, a computer generated 35 mm wide slide with color gradation in a 1 mm wide strip, and a digital camera (Nikon D7100) with a 30 mm diameter focusing lens of 600 mm focal length. A continuous 250 W metal halide light source connected to a 50 μm diameter fiber optic cable provides the light input at the pinhole through a 16.56 mm focal length objective lens. The camera output in the RGB format is digitized by a personal computer with 24 bit color frame grabber. Multiple viewing rainbow schlieren pictures can be acquired over a range of nozzle angular angles from 0 deg to 180 deg by rotating the nozzle about its longitudinal axis (z axis) in equal angular intervals of 5 deg and the jet three-dimensional density field is reconstructed using the convolution back-projection algorithm.

3 Analytical method for reconstructing three-dimensional density field
The problem of obtaining reliable quantitative information about the flow structure of three-dimensional supersonic jets arises in many industrial fields. Although direct imaging such as the conventional schlieren cannot provide quantitative information about the structure of three dimensional
jets, the schlieren CT which combines the rainbow schlieren deflectometry with so-called computed tomography, often abbreviated as CT must be utilized. Computed tomography was originally developed for use in medical diagnostics with X-rays, but was later adapted for other applications in many scientific fields including physics, chemistry, astronomy, and geophysics, using in many cases other kinds of radiation and even elementary particles. Various CT reconstruction algorithms including the Fast Fourier transform (FFT) algorithm, the filter back projection (FBP) algorithm, and the convolution-back projection (CBP) algorithm have been proposed so far. Among them the present study adopts the CBP algorithm because it performs best when data are available at all viewing angles [2, 3]. A brief outline of computed tomography for density fields in the three-dimensional jet flows is given by Agrawal et al. [2] and Faris et al. [4].

Figure 2 shows a light ray traveling in a cross-section (x, y plane) of an asymmetric jet issued from a nozzle where the x, y, z rectangular Cartesian coordinate system is used and the z axis is perpendicular to the x, y plane which includes the vector along the optical axis direction of the schlieren system.

Also, the \( n \) and \( n_a \) in Fig. 2 indicate the refractive index in the jet cross-section and that of the surrounding air, respectively. The refractive index or density fields will now be investigated for a cross section of \( z = \text{constant} \). As shown in Fig. 2, let us consider the rotated coordinates (s, t) inclined at an angle \( \theta \) away from the fixed-original coordinates (x, y). Then, a light ray traveling in the s direction with an offset of \( t \) from the axis s is bent by the interaction with the jet flow and has an angle of inclination \( \varepsilon_{\theta}(t) \) with respect to its original path. As the incoming ray is initially parallel to the s axis, the deflection angle \( \varepsilon_{\theta}(t) \) after passing through the refractive index field is given by the line integral

\[
\varepsilon_{\theta}(t) = \int_{-\infty}^{\infty} \frac{\partial \eta(s,t)}{\partial t} ds
\]

along the s direction of the partial derivative \( \frac{\partial \eta}{\partial t} \) with respect to the t variable of the normalized refractive index difference

\[
\eta(s,t) \equiv \frac{n-n_a}{n_a}
\]

Fig. 2 Light ray travelling through asymmetric refractive index field.
for small ray deflections. The deflection angle \( \varepsilon_\theta(t) \) is taken for a range of various angles from \( \theta = 0 \) deg to 180 deg. The task of tomographic reconstruction in the present investigation is to find \( \eta(x,y) \) based upon a given knowledge of \( \varepsilon_\theta(t) \) and then density fields can be obtained through a well-known linear relation between refractive index and density, as described later.

After the one-dimensional Fourier transform of Eq. (1) with respect to the \( t \) variable and using the Leibniz rule for differentiation under the integral sign, it reduces to a relation,

\[
\int_{-\infty}^{\infty} \varepsilon_\theta(t) \exp(-i2\pi t\zeta) dt = 2\pi i \zeta \int_{-\infty}^{\infty} \eta(s,t) \exp(-i2\pi t\zeta) ds dt
\]  

(3)

The transformation from the \((s,t)\)-coordinate to \((x,y)\)-coordinate for the integral of the right-hand side of Eq. (3) yields the following form with \( u = -\zeta \sin \theta \) and \( v = \zeta \cos \theta \)

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \eta(s,t) \exp(-i2\pi t\zeta) ds dt = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \eta(x,y) \exp[-i2\pi(ux + vy)] dx dy
\]  

(4)

This equation expresses the two-dimensional Fourier transform of \( \eta(x,y) \). Hence, the inverse Fourier transform of Eq. (3) using the result of Eq. (4) with the convolution theorem leads to

\[
\eta(x,y) = \int_0^\pi \left[ \varepsilon_\theta(t) \ast k(t) \right]_{t=-\infty}^{t=\infty} \sin \theta \cos \theta d\theta
\]  

(5)

where \( k(t) \) is given by Agrawal et al. [2]

\[
k(t) = \frac{\sin^2 \left( \frac{\pi f_{\text{max}} t}{\pi} \right)}{\pi^2 t}
\]  

(6)

with the Nyquist frequency \( f_{\text{max}} \) and the symbol \( \ast \) denotes convolution between \( \varepsilon_\theta(t) \) and \( k(t) \).

When deflection data are sampled at a spacing of \( \Delta t \), only frequencies below the Nyquist frequency

\[
f_{\text{max}} = \frac{1}{2\Delta t}
\]  

(7)

are adequately sampled.

The ray deflection angle \( \varepsilon_\theta(t) \) is correlated with the focal length \( f_d \) of a decollimating lens and the ray transverse displacement \( d_\theta(t) \) at the cut-off plane of the schlieren system and given by

\[
\varepsilon_\theta(t) = \frac{d_\theta(t)}{f_d}
\]  

(8)

For air there is a simple linear relation between the refractive index \( n(x,y) \) and the gas density \( \rho(x,y) \):

\[
\rho(x,y) = \frac{n(x,y) - 1}{K}
\]  

(9)
The two-dimensional density fields obtained at any streamwise locations could be stacked together to form the three dimensional density field of the jet plume.

4 Numerical methods
For a mutual comparison with the experiment using the rainbow schlieren, numerical simulations have been performed using the commercial CFD software ANSYS Fluent Version 15.0. A schematic drawing of the computational domain for the present simulation is shown in Fig. 3.

![Fig. 3 Schematic of computational domain (dimensions in mm)](image)

Dry air flows with a specific heat ratio of $\gamma = 1.4$ through a convergent nozzle with a rectangular cross-section of 5 mm $\times$ 20 mm at the exit are numerically solved by the three dimensional compressible Reynolds-averaged Navier-Stokes (RANS) equations. The wall contours of the simulated nozzle are designed by the sinusoidal curve to provide uniform and parallel flows at the nozzle exit plane. The nozzle has a constant width of 20 mm from the inlet to exit. In the computations, the well-known Menter's SSTk-ω turbulent model which is blended model that exploits the advantages of both k-ω and k-ε models is utilized, because it has been extensively used in many mechanical, industrial and aerospace CFD applications.

The stagnation pressure and temperature far upstream of the nozzle, and back pressure and ambient temperature are set to $p_{os} = 404.0$ kPa, $T_{os} = 300$ K, $p_b = 101.0$ kPa, and $T_b = 300$ K, respectively. The nozzle pressure ratio NPR ($= p_{os} / p_b$) defined as the ratio of plenum to back pressures is equal to 4.0 to produce an underexpanded free jet for the simulated nozzle. The boundary conditions are the uniform input flow at the nozzle inlet boundary and the adiabatic no-slip on the nozzle wall, and the upper and downstream ends of the computational domain are taken as free boundaries. The computations are performed using unstructured tetrahedral grids which can accurately represent complicated geometries such as nozzle wall contours. To ensure statistical convergence, the computation is run over at least 200,000 time steps.
Maeda et al. [5] validated their simulations for a slightly underexpanded jet issued from a square supersonic nozzle using a vortex sheet model [6] and showed that there is very favorable agreement between simulation and theory. However, the analytical model can be only applied for the weak shocks, but it does not for the flow field including strong shock waves. There are no reliable experimental data to validate numerical simulations for shock containing free jets from rectangular nozzles. Therefore, the present simulations are validated by comparing the simulated results with the experimental data obtained by the rainbow schlieren deflectometry.

5 Results and Discussion
The three-dimensional structure of an underexpanded sonic jet issued from a rectangular convergent nozzle with an aspect ratio of 4 at the exit has been visualized using a rainbow schlieren system for NPR = 4.0. Schlieren pictures were taken using a shutter speed of 400 Hz with continuous schlieren light source. Also, the rainbow filter was placed at the cut-off plane in parallel with respect to the z axis and its orientation is illustrated above Fig. 4 with the location of the background hue represented as the dashed line on the filter.

Fig. 4 Rainbow schlieren pictures of rectangular underexpanded jet

Figures 4(a) and 4(b) show typical schlieren pictures of the rectangular jet of the minor and major sides, respectively, i.e., the view shown in Fig. 4(a) features the nozzle short dimension with the long
dimension being perpendicular to the plane of the schlieren picture and vice versa for the view shown in Fig. 4(b). Figure 4(a) is an archaetypal schlieren picture showing the shock-cell structure in the jet plume, however, it is gradually blurred toward the downstream due to the time-dependent unstable movement in the direction normal to the jet centerline for the shock-cells following the first shock-cell. Figure 4(b) exhibits a distinct shock wave in the first shock-cell and the shock consists of a leading oblique shock, a nearly normal Mach shock and a reflected shock with the three intersecting at the bifurcation point, and a similar shock can be dimly observed in the second shock cell. The density contour plots on the cross-section including the jet centerline with $x = 0$ and $y = 0$, are depicted in Figs. 5(a) and 6(a), respectively.

![Comparison between experiment and simulation for density contour plot on minor axis plane](image)

**Fig. 5** Comparison between experiment and simulation for density contour plot on minor axis plane

The contour levels with an interval of 0.1 kg/m$^3$ are shown at the top of each figure, and the spatial resolution in the experimental density map is around 13 µm. The schlieren pictures of Figs. 4(a) and 4(b) can only provide integrated information about the density gradient along the direction of the optical axis, however, Figs. 5(a) and 6(a) can clearly demonstrate the quantitative information of the density field in three-dimensional jet plume. Unlike the schlieren pictures of Fig. 4, the density contour plots shown in Figs. 5(a) and 6(a) illustrate the various flow features of the shock cell structures quantitatively, such as the shape and size of the expansion and compression regions, shock cell intervals, jet boundaries and so on. The spatial variations for the density fields within the rectangular shock-containing free jets are vividly demonstrated with detailed density representation. Figures 5(b) and 6(b) display numerical results corresponding to Figs. 5(a) and 6(a), respectively. Good overall agreement is reached between experiment and simulation for the sizes and streamwise locations.
of the compression and expansion regions on the major and minor axis planes except that the simulation shows no significant variation in the downstream direction on density contours near the jet boundaries ($x = \text{around} \pm 10 \text{ mm}$) in the major axis plane, while the experiment exhibits a wavy form with a peak at each jet boundary (upper or lower jet boundary) corresponding to the end location of each shock cell.

![Density Contour Plot](image)

*Fig. 6 Comparison between experiment and simulation for density contour plot on major axis plane*

Figures 6(a) and 6(b) show a Mach shock with a large diameter in the jet plume when compared to those for the minor axis plane view (Figs. 5(a) and 5(b)) and the oblique shock waves originating toward the Mach shock from the edges ($x = \text{around} \pm 10 \text{ mm}$) of the nozzle exit are identified.

A comparison between experiment and simulation for the density profile along the jet centerline is shown in Fig. 7. The theoretical density value estimated based upon the assumption of the one-dimensional isentropic flow from the inlet of the nozzle to the exit is shown as a leftward arrow on the vertical axis in Fig. 7. In addition, the fully expanded jet density $\rho_j$ was calculated from the definition that the flow of the nozzle inlet is isentropically expanded to the back pressure and the density level is shown as the solid line parallel to the abscissa. In this regard, it is necessary to keep in mind that the fully expanded jet pressure $p_j$ is identical with the back pressure $p_b$, but the $\rho_j$ is not equal to the ambient density $\rho_b$.

Let us focus on the experimental density profile in Fig. 7 for the moment. Since the static pressure at the nozzle exit plane is greater than the back pressure on the present nozzle operating condition ($p_{\text{os}}/p_b = 4.0$), Prandtl-Meyer expansion waves are produced from the nozzle lip. Therefore, the jet centerline density also gradually decreases with increasing streamwise distance by the expansion waves until
colliding with the Mach shock at $z = \text{around 10 mm}$, and then fall and rise in density is repeated in the flow downstream of the Mach shock. The values of the local maxima in the density profile decrease toward the downstream direction, while those of the local minima remain almost constant.

![Fig. 7 Comparison between experiment and simulation for centerline density profile](image)

The density increment by the Mach shock is in favourably good agreement with the theoretical density jump estimated using the Rankine-Hugoniot relations (shown as the two-way arrow in Fig. 7).

Next, we will discuss the simulated density profile in detail. The simulated density profile in Fig. 7 shows almost the same variation as the experimental profile and good quantitative agreement between them is reached on the density values at the local maxima and minima in the profile and their streamwise distances except for the density variation downstream of the second shock-cell. In addition, although densities at the local maxima for the experiment are always beyond $\rho_j$, those at the local minima are always below $\rho_j$. The density values at the local minima in the simulated density profile increase toward the downstream direction and the trend agrees with a finding of Panda et al. [7]. Comparisons between experiment and simulation for density profiles on jet major and minor axis planes at a streamwise distance of $z = 8 \text{ mm}$ are depicted in Figs. 8(a) and 8(b).

Both the figures indicate the density variation just upstream of the Mach shock. The density profile for the experiment in Fig. 8(a) indicates almost constant density values over a range of spanwise distances from $x = -5$ to 5 mm and it suggests uniform flow just upstream of the Mach shock. Additionally, the densities reach local maxima after crossing the oblique shock wave positioned at $z = \text{around 5 mm}$ before they decreasing to the ambient density. The simulation shows a similar trend as that in the experimental result and it agrees well with the experiment except that the density values at the local maxima in the simulated density profile are larger than those of the experiment. Also, it should be noted that the peak values (shown as the leftward arrow in Fig. 8(a)) on the simulated density profile are identical with the fully expanded jet density $\rho_j$. The locations nominally correspond to the inviscid jet boundaries [7].

As seen in Fig. 8(b), the density profiles for the experiment and simulation on the minor axis plane don’t show uniform density profile near the jet centerline like those on the minor axis plane. In addition, the densities at the local maxima in the density profile are lower than $\rho_j$ for both the experiment and simulation. The simulation is in reasonable agreement with the experiment in the
region of the negative $y$ axis, but the simulated densities are higher than experimental ones in the region of the positive $y$ axis.

![Comparison between experiment and simulation for lateral density profiles](image)

**Fig. 8 Comparison between experiment and simulation for lateral density profiles**

### 5 Concluding Remarks

The three-dimensional density field of an underexpanded sonic jet issued from a rectangular convergent nozzle with an aspect ratio of 4 at the nozzle exit plane was quantitatively obtained by the rainbow schlieren deflectometry combined with the computed tomography based on the convolution backprojection algorithm. Also, the free jet was solved under the same conditions as those in the experiment using the RANS equations with the SST $k-\omega$ turbulent model for a mutual comparison between experiment and simulation. As a result, the three-dimensional density fields obtained from both the rainbow schlieren and numerical simulation are in good quantitative agreement with each other. A distinct spatial variation of the jet structure was demonstrated. The present rainbow schlieren technique is found to be significantly effective to examine the fine structure of the shock-containing free jets and makes it possible to provide experimental data with high accuracy and reliability on the jet density field.

### 6 Acknowledgements

This work was funded by Grant-in-Aid for Scientific Research (C) No. 15K05804 of Japan Society for the Promotion of Science.
References


