Mechanics and energetics of soil bioturbation by earthworms and growing plant roots

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Mechanics and Energetics of Soil Bioturbation by Earthworms and Growing Plant Roots

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Summary

Soil bioturbation by burrowing earthworms and by growing plant roots plays a key role in generating soil structure. The resulting biopores in soil become pathways for air and water flow, thereby enhancing deep drainage and extending the depth of oxic conditions into the soil profile. Decaying plant roots and other forms of particulate soil organic carbon serve as readily available nutrient sources for earthworms and stimulate soil biological activity. While there are ample reports of empirical evidence demonstrating the beneficial nature of soil bioturbation, the quantitative and mechanistic description of these important processes remains rudimentary. This becomes particularly important in order to determine future conditions that may critically reduce sub terrestrial activity by earthworms or plant roots (for example, soil compaction by intensified land use or changes in rainfall patterns and timing). For the research presented in this purpose, this PhD dissertation endeavors to develop a quantitative model that describes the mechanics of soil bioturbation by earthworms and plant roots.

The research begins by developing a biophysical-inspired mechanical model that considers steady cavity expansion into an elasto-plastic soil material. We model cone-like penetration by growing plant root tips or extending earthworm segments, followed by radial expansion, based on pressurized earthworm hydroskeleton or radial root growth. We consider the biomechanical limitations of the maximum earthworm and plant root radial pressures (200 kPa and 1000 kPa respectively) in order place limitations on suitable soil hydration conditions that would facilitate bioturbation activity. This research then considers soil mechanical properties correlated to different moisture contents and soil texture. The resulting mechanical penetration forces used to determine the amount of mechanical energy required for soil bioturbation for a range of saturation values. We use the model to estimate the influence of geometry (sem-apex angle and radius) on the energetic cost of soil bioturbation, which is converted to an equivalent amount of soil organic carbon required to sustain earthworm bioturbation activity.

Evaluation of the theoretical derivations is based on reported literature values, and own experiments. To test the forces and energetic components of the model, we developed an experimental protocol to test the simplified model. We designed miniature cone penetrometers enveloping the geometries of fine plant roots and large earthworms, and we mounted them to a force transducer and a stepper motor. Specifically, we used cones with 15 and 30 degree semi apex angle, and 1 and 2.5 mm radius at their bases. We also tested the influence of the penetrometer shaft (full vs recessed) to see the influence that it has on the measurement after the cone tip is inserted into the soil. We ran penetration tests in soils of known texture and water content and determined the
forces and stresses required to penetrate the conical tip into soil’s of known mechanical properties. The tests provided insight into the particularities of biophysical soil penetration vs rigid cone penetration, and yielded a striking duality between cone penetration stresses and forces.

The studies gave rise to two important biophysical differences that distinguish earthworms and growing roots: (1) their limiting radial pressures, and (2) their respective rates of penetration into the soil. We monitored the activity of both earthworms and plant roots using neutron radiography and custom rhizotrons (respectively). Earthworms instantaneous penetration rates are in the range of 100-500 $\mu$m s$^{-1}$, while plant roots axially extend at rates of 0.1-0.2 $\mu$m s$^{-1}$. The comparatively rapid rates of soil penetration by earthworms give rise to rate dependent forces caused by the plastic viscosity of unsaturated soils. For this purpose, we extended our modeling scheme to consider soil as an elasto-viscoplastic medium. Model predictions and experimental results show that earthworms penetrate the soil 1000 times faster than growing plant roots, and thus must exert 2-3 times the force that roots apply for similar geometry and soil conditions. This is equivalent to earthworms expending 2-3 times mechanical energy

Our modeling and experimental results showed that soil hydro-mechanical properties significantly limit earthworms to very narrow ranges of soil moisture conditions relative to that of plant roots. This is due to their relatively low hydroskeletal pressure limit (200 kPa). Given the centrality of this parameter, we developed a device that would allow us to directly measure the pressure limit in order to confirm this value. The device is a hallowed Plexiglas prism with a fitted tube that allows an earthworm to pass through the center. Once the earthworm is central to the device, we externally apply a pressure to the earthworm. We increase the pressure step wise until the earthworm is incapacitated, and subsequently release the earthworm. With this device, we not only determined the maximum earthworm pressure, but we also infer the dynamics of an earthworms peristaltic motion based on measured pressure fluctuations.

The study develops a mechanistic framework for quantifying soil bioturbation focusing on earthworm activity, but yielding new insights on mechanical and energetic constraints of growing plant roots. An earthworm’s mechanical activity is constrained by their hydroskeletal pressure, soil type and mechanical status, climate (rainfall and drying of a soil), and is sensitive to vegetation cover (providing fuel for subterranean activity). These constrains are intimately linked to the ecology of earthworms, seasonality and abundance, their presence or absence in an ecosystem, all of which feed directly into the structure and functionality and the hosting soil.
Riassunto

La bioturbazione del suolo da parte dei lombrichi e delle radici delle piante gioca un ruolo chiave nella generazione della struttura del terreno, a vantaggio di numerosi servizi ecosistemici. I biopori generati nel suolo da questa attività biologica fungono da vie preferenziali per il passaggio dell’aria e per il flusso d’acqua, migliorando di conseguenza il drenaggio e la disponibilità di ossigeno nelle profondità del terreno. Le radici delle piante, una volta in decomposizione, diven- tono a forme di carbonio organico particolato, stimolando ulteriormente l’attività biologica nel suolo. Nonostante l’abbondanza di prove empiriche che dimostrano la natura benefica della bioturbazione del suolo, la comprensione quantitativa dei processi meccanici ad essa sottesi ancora limitata. Lo studio di questi processi particolarmente importante per determinare quali condizioni future (fra cui, ad esempio, la compattazione del terreno a seguito del suo sfruttamento intensivo) possano ridurre in modo critico l’attività dei lombrichi e delle radici nel sottosuolo. La presente tesi di dottorato ha quindi lo scopo di sviluppare un modello quantitativo al fine di descrivere la meccanica della bioturbazione da parte dei lombrichi e delle radici delle piante.

La tesi inizia con lo sviluppo di un modello meccanico per l’espansione costante di una cavità in un materiale elasto-plastico (suolo). Si modellizza così la penetrazione da parte di una struttura conica che rappresenta la crescita delle punte delle radici delle piante o l’estensione dei segmenti del lombrico, seguita dall’espansione radiale basata sulla crescita radiale delle radici o sulla pressurizzazione dell’idroscheletro del lombrico. Le limitazioni biomeccaniche considerate consistono nella massima tensione radiale del lombrico e della radice: queste definiscono i limiti di idratazione del suolo che permettono l’attività di bioturbazione. Si considerano poi le proprietà biomeccaniche del suolo in termini di umidità e struttura. Le forze di penetrazione meccanica risultanti vengono quindi utilizzate per determinare la quantità di energia meccanica necessaria per la bioturbazione del suolo in un certo intervallo di valori di saturazione. Il modello viene utilizzato per stimare l’influenza della geometria (angolo apicale e raggio) sul costo energetico della bioturbazione, il quale viene convertito in unità di carbonio organico del suolo necessario ai fini dell’attività di bioturbazione.

Viene quindi sviluppato un protocollo sperimentale per testare il modello semplificato. I piccoli coni del penetrometro vengono progettati in modo da rappresentare la geometria delle radici sottili delle piante e quella dei lombrichi di grandi dimensioni, e vengono montati su un trasduttore di forze ed un motore passo-passo. Nello specifico, sono utilizzati coni con un mezzo angolo apicale di 15 e 30 gradi e con raggio di 1 e 2,5 mm alla base. L’influenza dell’asta del penetrometro (piena vs incassata) viene testata per valutare l’impatto che essa ha sulla misurazione
una volta che la punta del cono inserita nel terreno. I test di penetrazione sono eseguiti in terreni di consistenza e umidità note, e per ciascun set di proprietà meccaniche del suolo vengono determinate le forze e le sollecitazioni necessarie per far penetrare la punta conica nel terreno. I test forniscono informazioni sulle particolarità della penetrazione biofisica del suolo in rapporto alla penetrazione del cono rigido e rivelano una sorprendente dualità tra gli stress della penetrazione del cono e le forze.

Esistono due distinzioni biofisiche primarie fra i lombrichi che scavano nel suolo e le radici che crescono: le soglie limite di pressione radiale ed il rispettivo rate di penetrazione. Con l'uso, rispettivamente, della radiografia a neutroni e dell'uso di un apposito rizotrone stata quindi monitorata l'attività dei lombrichi e delle radici. I rate di penetrazione istantanea dei lombrichi sono risultati in un intervallo di 100-500 $\mu$m s$^{-1}$, mentre la cerscita delle radici delle piante avviene in un intervallo di valori corrispondente a 0.1-0.2 $\mu$m s$^{-1}$. L'elevato tasso di penetrazione dei lombrichi genera delle forze che dipendono dal tasso stesso, a causa della viscosità plastica tipica del suolo al di sotto della sua soglia di saturazione. A questo scopo, il modello viene migliorato considerando il suolo come un materiale elasto-viscoplastico. Viene dimostrato quindi che i lombrichi, che penetrano il suolo 1000 volte più velocemente delle radici, necessitano di forze 2-3 volte più elevate di queste ultime a parità di condizioni del suolo.

I presenti risultati modellistici e sperimentali mostrano che i lombrichi sono limitati in un intervallo di condizioni favorevoli di umidità del suolo più ristretto rispetto alle radici. Questo dovuto alla pressione relativamente bassa del loro idroscheletro (200 kPa). Data la centralità di questo parametro stato sviluppato un dispositivo che permette di misurare direttamente la pressione limite, al fine di confermare questo valore. Il dispositivo consiste in un prisma di Plexiglas forato contenente un tubo che permette al lombrico di attraversarlo. Una volta che il lombrico si trova al centro del dispositivo, gli viene applicata una pressione esterna. La pressione viene aumentata finché il lombrico non muoversi, e viene successivamente ridotta per permettergli di uscire. Con questo dispositivo stato possibile determinare non solo la pressione massima dei lombrichi, ma anche la dinamica del loro movimento peristaltico, la quale è stata ricavata dalle fluttuazioni della pressione misurate nel corso dello spostamento del lombrico.

Infine, i risultati di questo studio vengono discusso con esempi illustrativi allo scopo di predire i diversi intervalli di attività dei lombrichi e delle radici delle piante in contesti ambientali differenti. La presente tesi fornisce il quadro necessario per lo studio delle limitazioni dell'attività meccanica dei lombrichi e delle radici nei diversi biomi, suggerendo le possibili motivazioni fisiche che sottendono all'attività stagionale dei lombrichi, alla distribuzione della loro popolazione, e ai pattern migratori. Infine, vengono discusse possibili strategie per approfondire questo quadro nel contesto di suoli non omogenei.
Chapter 1

Introduction

Soil is one of the most critical living systems in the biosphere, comprised of rich biodiversity consequence of dynamic interactions between the atmosphere [1], hydrosphere [2], and nonlinearities that arise from this heterogeneous environment [3]. The complex arrangement of aggregated minerals and organic constituents comprise an important yet delicate trait known as soil structure [4] [5], which regulates many physical processes such as gas and water availability [6] [7] and transport [8] [9] [10] [11], chemical transport and reactions [12], mechanical rigidity and strength [13] [14] [15] [16] [17], and biological habitability [18] [19] [20]. This, in turn, impacts the efficiency of plant root water [21] [22] and solute uptake [23] [24] as well as mechanical impedance to root growth [25], which are reflected by crop yields [26]. Soil structure is characterized by aggregates and macropores resulting from mechanical and hydraulic stresses caused by climatic factors (e.g. wetting-drying [27] [28] or freezing-thawing cycles [29] [30]), anthropogenic practices (e.g. soil tillage and potential compaction on agricultural lands [31]), and biophysical processes (e.g. biopores created by soil flora and fauna [32] [33]). Agricultural practices (tillage, crop management, etc.) can have long term adverse impacts on soil structure. The rapid onset of compaction by a driving tire can take seconds to destroy soil structure and up to decades to recover [34]. Experimental studies have shown that biological activity can accelerate the soil structure recovery rates [35] [32]. In particular, earthworms and plant roots must move through soil in order to obtain resources. While increasing compaction can gradually decrease the mobility of these biological agents, both can work to break up soil and create biopores prior to becoming completely immobilized, thus ameliorating the negative impacts of compaction. Soil bioturbation by plant roots and earthworms results in a system of biopores that promote infiltration of water and gas exchange in soil [33]. Plant root biopores become biological hotspots for microbial activity [18] [36], which break down plant residues and act as a food source for earthworms. The tunnels left behind by earthworms also act as preferential growth paths for subsequent root growth. Biopores generated by earthworms and plant roots are known to be more structurally stable than abiotic soil structures due to wall compaction during the penetration process, excretion of biopolymers along the burrow wall (e.g. dried plant mucilage or earthworm...
mucus [37] [38]). One of the few quantitative studies has shown that earthworm sub terrestrial activity can displace as much as 100 kg of soil per square meter of ground area per year [39]. While this is comparatively less for plant roots (1 kg m\(^{-2}\) ground area per year), plant roots can exert radial pressures on the order of 2 MPa [40], which is an order of magnitude higher than earthworms, allowing roots to continue to grow in soils mechanically uninhabitable by earthworms, extending the duration of their activity window and efficacy to restore compacted soils. While the importance of soil structure recovery by plant roots and earthworms is evident, there have been few attempts to quantify the mechanical process of soil bioturbation. A mechanistic understanding of these processes could shed light on reasons why these biological agents are active during particular activity windows (seasonality, geography), the respective energetic costs for such biophysical activities, and possibly allude to more sustainable management practices for agricultural production.

**Modeling soil bioturbation by earthworms and plant roots**

The capability of earthworms and plant roots to move through soil results from their respective physiological traits. While the physical process of biopore generation by plant roots and earthworms can have similarities, the two biological agents have vastly different physiological features that permit sub terrestrial activity. These biological distinctions give rise to subtle differences in their environmental constraints. Plant roots growing into soil rely on synchronized processes occurring on multiple scales [41]. At the cellular level, plant cells are embedded in a stiff cell wall with a yield strength as high as 500 kPa [42] [43] [41], and these cells are able to extend and deform soil when the internal pressures exceed the cell wall strength and the soil strength. While plants can extend on a cellular level, the growth process relies on plant roots generating new cells at the root cap. The periodic reorientation of cellulose fibrils in the cell wall during an enzymatic loosening process [41] allows for a tip wise growth that minimizes frictional effects at the interface with the soil and allows plant roots to exert pressures in the range of 1-2 MPa, which is intense enough to fracture rigid chalk [40]. Roots growing into soils that are more mechanically impeding have been reported to thicken around the root cap [44], which could serve to extend fracture zones or distribute the stress concentration acting on the root tip. The relatively high stresses that plant roots can exert on the soil results from the multi scale growth processes that limit actuation speeds. Root growth rates are considered to have poroelastic limitations, interpreted to be fastest water driven actuation process within a plant. Therefore, typical root growth rates are on the order of 0.1-0.2 \(\mu\) m s\(^{-1}\) [45] [43].

Earthworms are comprised of a system of longitudinal and circumferential muscles that compensate for their lack of a rigid skeleton (known as a hydroskeleton [46] [47]. Their ability to crevice burrow [47] into soil relies on their ability to generate peristaltic motion through a synchronized alternation of locally contracting and stretching their muscle fibers. The cyclic for-
ward propulsion and subsequent anchoring are driven by peristalsis of the earthworms colonic fluid [48] [49] [50]. Radial contraction and relaxation of muscles results in a local expansion of the hydroskeleton, acting to anchor the earthworm in soil to prepare for forward propulsion, or to further expand small cavities. Radially contracted regions will axially elongate, advancing the segments forward to penetrate through the soil. Earthworms reliance on muscle fibers allows for relatively faster actuation speeds than plant roots, reportedly in the range of 100-500 $\mu$ m s$^{-1}$ [48]. However, lacking a rigid cell wall significantly limits the maximum pressures that earthworms can sustain. Previous measurements have estimated the earthworms pressure limits to be in the range of 60-230 MPa [51] [52]. Thus, this limits the allowable soil hydration conditions that earthworms can feasibly move through as consequence.

While the physiological differences are significant for underlining the different limiting conditions and energetic constraints for the two biological agents, the mechanical process of generating a biopore has physical similarities that are quantifiable. The tip of a growing plant root and burrowing earthworm into soil rely on locally penetrating into soil, displacing and deforming particles radially. Adopting the assumption that these processes are similar has given rise to some modeling attempts in the past.

Early examples of root growth models considered the pressure balance between the cell wall pressure and the soil pressure applied on the root tissue considering the analogy between the growing root and cone penetration [43]. This formulation was adopted to characterize earthworms analogous to extending plant roots [53]. Other models used for earthworms (developed for marine worms in sediment) quantified the burrowing process using crack propagation using linear elastic fracture mechanics [48]. While this had given certain quantitative insights into the bioturbation, fracture mechanics on wet soils or marine sediments seems unlikely given evidence of slow elasto-plastic deformations that occur during root growth and earthworm burrowing and considering the role of soil rheology. Looking towards a mechanistic model of soil bioturbation requires a mechanical description of soil penetration and cavity expansion [54]. A simplified mechanical description of the pressure required to penetrate into an elasto-plastic medium was observed to be proportional to the amount of pressure required to expand a cavity to a final diameter, and assuming a constant proportionality between the local cavity plastic deformation and the remote elastic field around the expanding cavity, the internal cavity pressure would converge to a limiting pressure at large enough deformations [54] [55]. This limit pressure is a function of the ratio between the soil shear strength and the shear modulus. A quantitative model that considers the mechanical process of soil penetration and expansion is described in Chapter 2.
Measuring forces, rates, and energy

Cone penetrometer tests (CPT) are often deployed to characterize mechanical and structural properties of a range of materials in different fields of study. The test utilizes a device consisting of a rod with a conical tip that is mounted onto a force transducer. The conical tip is pushed through a particular medium, and the resulting force measured over the displaced depth is used to infer the cone penetration resistance stress, typically by dividing the force by the active area of the cone. For standard civil and geotechnical engineering applications, the test is used to deduce the mechanical properties of soil in order to gauge the soils trafficability [56] or structural stability. The measurement extends to studies of snow mechanics, where CPT measurements are used to derive spatial gradients in snow stratigraphy in order to predict the potential for snow avalanche formation [57] [58] [59]. Similar to geotechnical applications, agricultural studies have also used CPT measurements to assess the suitability for crop growth [56].

Though an analogy could be drawn between plant root growth, earthworm burrowing, and cone penetration into soil, certain stipulations must be made. As reviewed in the work of Bengough and Mullins [60], there are several critical issues that have to be considered when comparing CPT to root growth, many of which also carry forward to earthworms burrowing. First is the notable fixed geometry of the cone penetrometers differs from plant roots, which grow tip wise followed by a radial expansion. Earthworms burrowing into soil also extend tip wise before subsequent radial expansion. Unlike the standard rough cone penetrometer, the local cell sloughing by plant roots and the production of mucilage significantly reduces the interfacial friction experienced by the plant root. Earthworms also utilize mucus to reduce the interfacial friction of the soil, and also generate an electrical negative action potential during peristaltic motion, which causes soil water to move towards the body surface creating a thin water film between the earthworm and the surrounding soil. Both earthworms and root tips navigate towards mechanical paths of least resistance, where CPT measurements are directed straight. Finally, earthworms and plant roots penetrate soil at much lower rates than cone penetrometer tests are typically conducted at. Furthermore, on the millimetric scale of earthworms and plant roots, there exist unresolved scale effects of soil penetration, causing larger penetration stresses local to the inserting tip [61].

In chapter 3, we developed miniature cone penetrometers enveloping the geometries of earthworms and plant roots and use these devices as analogs to the mechanical processes associated with soil bioturbation. We assess the distinctions between full vs recessed shaft in order to emulate the difference between tip wise penetration and isolate the influence of the shaft by the CPT. We tested blunt (30°) vs sharp (15°) semi cone angles to compare the influence of the standard (blunt) CPT measurement to the penetration of the bioturbation agents that displace soil radially [62] [63]. Finally, we outline the utility and limitations to our analytic penetration expansion model.
Biophysical limitations influencing soil penetration

The main biophysical characteristics that distinguish plant roots and earthworms are their limiting pressure thresholds and their soil penetration rates. Earthworm and plant root soil penetration rates have been monitored directly [64] [48] [45] [43], and their mechanical pressure limits have been inferred using different measurement techniques [40] [51] [52]. Assuming these biophysical parameters to be accurate, these have strict consequences when moving through partially saturated soils. When the absolute value of the principle stresses (in radial symmetric coordinates) exceeds the soil strength, multiscale deformations occur in series. Stable soil aggregates begin to breakdown and coalesce [27]. Simultaneously, soil clay platelets begin realigning and relaxing [65]. Water retained in the soil matrix begins to mobilize [66], which lubricates the rearranging soil particles, dissipating energy in the process. The coordinated scales of deformation manifests in a bulk flow of soil material [67], which is rate dependent. This deformation is quantified as soil elasto-viscoplastic deformation, where the soil viscosity must be distinguished from standard viscous flow of water during consolidation of saturated soils. Both the critical soil strength and the soil plastic viscosity are dependent on soil texture and water content, and influence the amount of energy required for earthworms and plant roots to penetrate soil. In Chapter 4, we extend our model to consider rate dependency by enhancing the standard yield criteria to consider viscoplastic soil deformation. We then discern the energetic costs of earthworms and plant roots to move below ground based on their biomechanical penetration rates. We confirm the values obtained in previous studies through imaging techniques such as neutron radiography and custom rhizotron devices. Lastly, we show limiting hydration conditions that would mechanically inhibit extended earthworm activity.

Measuring earthworms biomechanical pressure limitations

One of the most critical parameters used to outline limitations to earthworm activity is the maximum radial pressure that they can exert. Despite many estimates existing, most measurements rely on indirect measurements or inconsistent methods to infer pressure correctly. Early studies have tried to measure the pressure of the colonic fluid of anaesthetized earthworms [50]. This resulted in earthworm pressures on the order of 1.6 kPa, which is inconsistent with the regular burrowing activity of earthworms in soils exhibiting strengths one or two orders of magnitude higher. Subsequent tests were made by having earthworms burrow through soil cylinders of given compaction levels [51]. Small holes were drilled through the center to see if the earthworms were able to move through the center or even break the cylinders. However, the central borehole likely caused micro-fractures within the compacted soil cylinders, which would allow earthworms to break the soil cylinders at much lower stresses than the reported pre-compression stress. Attempts to later directly measure the earthworm limit pressures made use of force transducers and
mass scales [52] [47]. Force peaks recorded by a moving earthworm were used to determine the pressures that earthworms could exert, but the interpreted geometries reflected the forces with respect to the measurement system geometry rather than the pressures of the earthworms themselves. A pressure measurement system capable of directly measuring earthworm radial pressures during peristaltic motion is designed in Chapter 5.

**Objectives**

The objectives of this thesis are to: (1) Develop a physical model that quantifies the mechanics and energetics of soil bioturbation by plant roots and earthworms, (2) design experiments using miniature cone analogs to test the utility and limitations of our biophysical cavity-expansion penetration model, (3) quantify bioturbation rates by earthworms and plant roots through neutron imaging and rhizotron experiments, (4) extend our cavity-expansion penetration model to consider different penetration rates of burrowing earthworms and growing plant roots, and (5) design a measurement device that allows us to directly measure the limiting pressures of earthworms during peristaltic motion. The structure of this thesis follows the listed objectives. In chapter 2, a quantitative framework is developed describing the mechanical stresses acting on earthworms and plant roots during penetration and cavity expansion through elasto-plastic soils. Chapter 3 presents our designed custom miniature penetrometers as analogs to burrowing earthworms or growing plant roots with the aim of determining the limitations of our analytic model for compacted soils of different texture and hydration conditions. We then extend our soil bioturbation model in chapter 4 to consider elasto-viscoplastic soil deformations, and, with penetration rates obtained through direct imaging of burrowing earthworms and growing plant roots, we show the differences between their respective energetic costs based on soil rheological properties. In chapter 5, we design a device that directly measures the limiting pressures of an earthworm during peristaltic motion, inferring the biomechanical limitations to soil bioturbation by earthworms. These chapters are subsequently followed by a summary, conclusion, and an outlook on the remaining issues.

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Chapter 1


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Chapter 2

Soil Penetration by Earthworms and Plant Roots - Mechanical Energetics of Bioturbation of Compacted Soils

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Abstract

We quantify mechanical processes common to soil penetration by earthworms and growing plant roots, including the energetic requirements for soil plastic displacement. The basic mechanical model considers cavity expansion into a plastic wet soil involving wedging by root tips or earthworms via cone-like penetration followed by cavity expansion due to pressurized earthworm hydroskeleton or root radial growth. The mechanical stresses and resulting soil strains determine the mechanical energy required for bioturbation under different soil hydro-mechanical conditions for a realistic range of root/earthworm geometries. Modeling results suggest that higher soil water content and reduced clay content reduce the strain energy required for soil penetration. The critical earthworm or root pressure increases with decreased diameter of root or earthworm, however, results are insensitive to the cone apex (shape of the tip). The invested mechanical energy per unit length increase with increasing earthworm and plant root diameters, whereas mechanical energy per unit of displaced soil volume decreases with larger diameters. The study provides a quantitative framework for estimating energy requirements for soil penetration work done by earthworms and plant roots, and delineates intrinsic and external mechanical limits for biotur-
bation processes. Estimated energy requirements for earthworm biopore networks are linked to consumption of soil organic matter and suggest that earthworm populations are likely to consume a significant fraction of ecosystem net primary production to sustain their subterranean activities.

Introduction

The ability of earthworms to move and plant roots to grow through soil greatly affects their capacity to capture resources. Increasing soil compaction gradually decreases the speed and ability of biological growth [1] and movement in soil until a critical threshold is reached in some soils and movement is ceased. Prior to reaching this critical point, both earthworms and plant roots can work to break up soil, thereby ameliorating the negative impacts of soil compaction and improving their own biological habitat [2].

Bioturbation results in a network of channels that promote water infiltration and gas exchange in soil [3], and stimulate microbiological activity and preferential root growth along existing burrows. The stability of biopores is attributed to compaction at their walls [4], the excretion of biopolymers lining the burrows, and to hydrophobicity of mucilage excreted by plant roots [5] or mucus by earthworms [6].

The contributions of earthworms and plant roots to soil structural restoration after compaction [2; 7], and details of the mechanical processes involved have rarely been quantified. Capowiez and Belzunces [8] report that earthworms construct large tunnel networks with lengths ranging from 1 to 2 m per individual earthworm [8]. These burrows involve substantial amounts of displaced soil that may exceed 100 kg m$^{-2}$ ground area per year [7]. Comparatively, plant root growth contributes less to soil biopore construction; estimates of displaced soil mass associated with plant root growth are of the order of 1 kg m$^{-2}$ ground area per year in temperate regions [9].

Relatively few models for the mechanism of soil penetration by plant roots and earthworms have been proposed, such as the model of Greacen and Oh [10] for root growth that balances cell wall pressure and soil pressure applied on root tissue using the cone penetration analogy. This formulation became the standard root growth model ([11], [12], [13], [14]). Greacen et al. [4] have shown radiograph images of plant roots growing in a manner that compacts soil around the root circumference rather than at the forefront of the root tip. Hettiaratchi et al. [15] observed tissue thickening around the root cap that could promote soil weakening at the cap region resulting in fractures and thus lowering mechanical impedance for root growth. Misra et al. [12] have shown evidence that plant roots exert radial pressures notably larger than axial pressures with plant root radial pressures in excess of 2 MPa capable of cracking stiff rigid chalk [16; 12]. Analogously, dry compacted soil fracturing under radial pressures was measured by McKenzie and Dexter [17] in a procedure used to determine threshold radial pressures exerted by earthworms during penetration, showing that earthworms can exert radial pressures slightly
above 200 kPa.

The early studies by Dexter [18] on modeling root elongation as an analogue for earthworm penetration were only recently expanded by Dorgan et al. [19] and Murphy and Dorgan [20] that proposed models for earthworm burrowing in marine sediments. Dorgan et al. [19] used crack propagation models to describe peristalsis during burrowing by earthworms in soft marine sediments. This framework has been shown to provide certain quantitative insights, however, the reliance on fracture mechanics for locomotion in wet soils or soft marine sediments appears questionable, given evidence of slow elasto-plastic deformation during root growth and expansion rates in soft soils, or considering the role of soil rheological properties that mediate other mechanical processes [21]. The primary objective of the present study is to model gradual deformation processes linked with soil penetration by earthworms. The specific objectives are to:

1. Develop a mechanical model to quantify stresses and strains associated with the soil penetration by earthworms and their dependence on soil type, hydration status, earthworm characteristics and geometry

2. Convert mechanical stress-strains to energy equivalents for different soil conditions and earthworm geometries (assuming elasto-plastic soil)

3. Relate these mechanical energy estimates to observed earthworm activity and consumption of soil organic carbon to satisfy energy needs related to soil penetration

We first present theoretical considerations necessary to develop a physically based model for the mechanics and energetics of soil penetration by earthworms that is analogous to plant root penetration models. This is followed by derivation of a simplified analytic expression for minimum earthworm pressure and energy expenditure related to creating a cavity of a given length and diameter under prescribed soil mechanical conditions. Next, we use literature values to estimate soil mechanical conditions from soil water and clay contents and generate estimates of energetic costs and physical constraints on soil penetration in a range of soil conditions. Finally, mechanical and energetic predictions are compared with experimental and empirical evidence from the literature and ecological implications are discussed.

The modeling methodology employs a continuum mechanics approach, thus assuming that penetration occurs through a homogeneous soil medium. This assumption is most applicable in compacted soil, where mechanical constraints to soil penetration are also most severe. Under many natural conditions, soils structure is likely heterogeneous and soil mechanical parameters could be anisotropic. Under these conditions, earthworms and plant roots would likely follow paths of least resistance and greatly reduce energy expenditure for soil penetration.
Materials and Methods

The symbols used below are listed and described in Table 3.5.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>SI Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Semi-Apex Insertion Angle</td>
<td>rad</td>
</tr>
<tr>
<td>$\epsilon_r$</td>
<td>Radial Strain</td>
<td>$m \cdot m^{-1}$</td>
</tr>
<tr>
<td>$\epsilon_\theta$</td>
<td>Hoop Strain</td>
<td>$m \cdot m^{-1}$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Friction Angle</td>
<td>rad</td>
</tr>
<tr>
<td>$G$</td>
<td>Shear Modulus</td>
<td>Pa</td>
</tr>
<tr>
<td>$k$</td>
<td>Shape Parameter</td>
<td>--</td>
</tr>
<tr>
<td>$l$</td>
<td>Distance from Cone tip</td>
<td>m</td>
</tr>
<tr>
<td>$l_b$</td>
<td>Burrow Length</td>
<td>m</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson’s Ratio</td>
<td>$m \cdot m^{-1}$</td>
</tr>
<tr>
<td>$P$</td>
<td>Cavity Pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$P_L$</td>
<td>Limit Cavity Pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$r$</td>
<td>Radial Depth of Observation</td>
<td>m</td>
</tr>
<tr>
<td>$R$</td>
<td>Elasto-plastic interface</td>
<td>m</td>
</tr>
<tr>
<td>$r_c$</td>
<td>Cavity Radius</td>
<td>m</td>
</tr>
<tr>
<td>$r_{c0}$</td>
<td>Initial Cavity Radius</td>
<td>m</td>
</tr>
<tr>
<td>$r_f$</td>
<td>Final Expanded Cavity Radius</td>
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</tr>
<tr>
<td>$\sigma_n$</td>
<td>Normal Stress</td>
<td>Pa</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>Radial Stress</td>
<td>Pa</td>
</tr>
<tr>
<td>$\sigma_\theta$</td>
<td>Hoop Stress</td>
<td>Pa</td>
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<td>$\sigma_z$</td>
<td>Axial stress</td>
<td>Pa</td>
</tr>
<tr>
<td>$s_u$</td>
<td>Soil Strength</td>
<td>Pa</td>
</tr>
<tr>
<td>$\theta_m$</td>
<td>Water Content</td>
<td>$kg \cdot kg^{-1}$</td>
</tr>
<tr>
<td>$\theta_{min}$</td>
<td>Residual water Content</td>
<td>$kg \cdot kg^{-1}$</td>
</tr>
<tr>
<td>$\theta_{max}$</td>
<td>Saturated water Content</td>
<td>$kg \cdot kg^{-1}$</td>
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<tr>
<td>$\Theta$</td>
<td>normalized water content (actual divided by saturated)</td>
<td>--</td>
</tr>
<tr>
<td>$u$</td>
<td>Radial Deformation</td>
<td>m</td>
</tr>
<tr>
<td>$U$</td>
<td>Strain Energy</td>
<td>J</td>
</tr>
<tr>
<td>$U_0$</td>
<td>Strain Energy Density</td>
<td>$J \cdot m^{-3}$</td>
</tr>
</tbody>
</table>

Modeling penetration-expansion in soil.

The focus of the present study is on modeling the mechanics of soil penetration and the formation of macropores primarily by earthworms, in analogy to similar mechanisms applicable to elongating plant roots. We begin by discussing soil penetration by roots and worms concurrently as the equations governing the steady state mechanics of penetration-expansion for the simplified geometries are the same for both. The mechanical model formulation was first explained by Misra et al. [12], where the authors described the mechanics of axial and radial growth stresses of plant roots. A similar formulation was applied to earthworms by Mckenzie and Dexter[17]. Dexter
Despite the equivalent formulation of their mechanical soil penetration processes, earthworms and plant roots penetrate soil quite differently. Quillin [22] suggests that earthworm locomotion involves repeated penetration-expansion cycles driven by peristalsis of pressurized colonic fluid. The radial relaxation and contraction of muscles in a wave propagating manner [19; 20] results in localized radial expansion of the earthworm hydrostatic skeleton used for expanding cavities and anchoring during axial penetration. Local radial contraction is also used to elongate earthworm hydrostatic skeleton for more efficient axial penetration of soil [22] as seen in the sequence in Fig. 2.1 (taken in soft agar). Another exclusive feature attributed to earthworms is their ability to ingest soil, however, Quillin [22] reported that earthworm burrows are generally created by displacement of soil rather than ingestion.

Figure 2.1 A sequence of images of the front segments of an earthworm moving through agar by a series of penetration-expansion steps. (a) Displays the axial penetration inducing an initial cavity. (b) Illustration of cavity expansion when collecting expanded segments. (c) Further penetration post anchoring processes.

Root penetration occurs by tissue growth rather than by peristalsis [10; 23; 24]. The resulting differences originate already at the cellular level, where earthworms utilize muscle fibers for deformation and actuation, thus enabling peristalsis. Plant cells, on the other hand, are surrounded by a stiff cell wall, having yield strengths ranging from 200 to 500 kPa [23; 24]. Root growth occurs when internal cellular pressure within the plant root exceeds the yield strength of cell wall and the soil penetration resistance. This process is facilitated by episodic reorientation of cellulose micro fibrils in cell walls during an enzymatic loosening process, and cell wall thickening
during a tightening process following extension [24]. The sequence of cutting, reorientation, and rebuilding can produce pressures in excess of 1600 kPa for extended periods (up to two days) [24].

The most energetically relevant distinction between soil penetration by plant roots and earthworms relates to their penetration rates and penetration pressures. Growth-based soil penetration rates by plant roots range from 0.006 to 0.025 m day$^{-1}$ [13; 25], which is about 20 times slower than earthworm penetration rates. Additionally, plant roots exert pressures of up to 3500 kPa [25; 13; 15; 26; 27], 100 times larger than maximal earthworm pressures [13]. Both plant roots and earthworms exhibit larger radial pressures relative to axial pressures during soil penetration, yet both extend predominantly in the axial direction. Misra et al. [12] reported axial pressures for plant roots in the range of 200 to 500 kPa relative to radial pressures exceeding 2400 kPa [13]. Keudel and Schrader [28] reported the upper limits of radial pressures exerted by endogeic earthworms in the range between 60 and 195 kPa, whereas axial pressures were limited to the range of 27 to 39 kPa. The capability of plant roots and earthworms to inhabit heavily compacted soils (Capowiez et al. [2], Dexter [18]) rely on flexible tissue, reduction of interfacial friction between their bodies and the soil (e.g., mucilage and mucus), and seeking out least mechanically impeding pathways [13; 10; 18].

Bengough and Mullins [13] reviewed mechanical resistances experienced by plant roots during cone penetration and reported 2 to 9-fold higher cone resistances compared to root resistances at similar conditions. These authors have suggested that reduced root-soil friction compared to metal-soil friction [10] could offer an explanation, but mentioned other factors including dynamic effects due to rapid cone insertion rates relative to root growth rates, and radially dominated soil deformation by roots versus axially dominated deformation by a cone penetrometer. Despite differences between penetration by plant roots and cone penetrometers, Atwell [11] stated that penetrometers provide information that correlates well with mechanical impedance experienced by plant roots. Previous models based on pressure balance of a plant root or earthworm rely heavily on cone penetration formulations. Greacen and Oh [10] utilized a formulation describing penetrometer resistance in order to compute the external pressures being applied to the plant root by the soil. More empirical methods of correlating the root growth resistance to the penetration measurements were used by Dexter [14]. Furthermore, Dexter [18] modeled soil penetration by earthworms analogously to plant roots, establishing an indirect link between earthworm penetration and cone penetration.

Motivated by Bishop et al. [29], who observed that the pressure required to produce holes in an elastic-plastic medium is proportional to the pressures required to expand a cavity to the same final diameter, we employed a common approach for these linked processes of cone penetration and cavity expansion to describe soil penetration by earthworms. The model utilizes a simple theoretical limit pressure for cavity expansion that translates to axial penetration resistance as mediated by the cone geometry. For simplicity, we consider a steady state continuum elastic-
plastic mechanical model. Bishop et al. [29] found that for a sufficiently small cone angle, the resisting pressure converges towards the theoretical limiting pressure for cylindrical cavity expansion. Considering that the apex insertion angle of an earthworm and plant root is more acute than that of standardized cone penetrometers, the cylindrical cavity expansion model may be appropriate for describing the process. Bishop et al. [29] and Carter et al. [30] established the theoretical foundations for large deformation cavity expansion. Assuming a constant proportionality between the plastic and elastic fields around an expanding cavity, the authors showed that cavity pressures at large deformations would converge to a limit pressure depending on the ratio between the soil strength and shear modulus. Consequently, the stress-strain relationship obeys an associated flow law based on a Mohr-Coulomb criterion for all strains within a given radius, and act as a linear-elastic solid material outside of this boundary.

Plastic deformation is a critical process during soil penetration by earthworms in saturated and unsaturated soils. Plastic soil deformation is defined as an irreversible process resulting from cavity pressures exceeding the soil’s yield threshold. Soil plasticity is crucial for the ability of earthworms to exist in soil, otherwise earthworms would constantly need to invest energy to overcome elastic rebound and would not be able to create lasting channels. Earthworms subjected to dry compacted conditions have been reported to struggle in predominantly elastic soils [17].

Cavity expansion in an elasto-plastic media

At equilibrium, the stress around a cylindrical cavity decays with the radius $r$ from the center of the cavity into the surrounding medium (soil) as:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (2.1)$$

where $\sigma_r$ is the radial stress, and $\sigma_\theta$ is the hoop (circumferential) stress (Fig. 2.2). The mechanical transition from elastic deformation to plastic deformation is expressed by the Mohr-Coulomb criterion, relating the difference between the radial and the hoop stresses to the summation of the stresses multiplied by a component of the internal friction and the undrained soil shear strength:

$$\sigma_r - \sigma_\theta = (\sigma_r + \sigma_\theta)\sin(\phi) + (2s_u)\cos(\phi) \quad (2.2)$$

where $\phi$ is the internal angle of friction, and $s_u$ is the undrained soil shear strength. Following Yu [31] and Durban and Fleck [32], we assume frictionless soils, i.e. $\phi = 0$.

Shames [33] provides strain-displacement relationships in axially symmetric polar coordinates. Given the magnitude of radial soil deformation $u$ as a continuous function of the radial distance from the center of the cavity ($r$), the radial strain $\epsilon_r$ is defined as the deformation gradi-
Figure 2.2 Concept of Elasto-Plastic cavity expansion. Cavity expansion is based on the assumption of a constant ratio between the initial cavity \((r_c)\) and a fixed Elasto-Plastic interface \((R)\) at a distance proportional to the internally applied cavity pressure \((P)\). The stress field propagating into the soil, \((\sigma_r, \sigma_\theta)\) depends on the distance from the center \((r)\).

\begin{align*}
\epsilon_r &= -\frac{\partial u}{\partial r} \\
\epsilon_\theta &= -\frac{u}{r}
\end{align*}

The hoop strain \(\epsilon_\theta\) is defined as the ratio between the increase in circumference \((2\pi r - 2\pi(r + u))\) over the original circumference \((2\pi r)\) [33, pg 536] :

\begin{align*}
\epsilon_\theta &= -\frac{u}{r} \\
\epsilon_r &= ((1 - \nu)\sigma_r - \nu\sigma_\theta)/(2G) \\
\epsilon_\theta &= ((1 - \nu)\sigma_\theta - \nu\sigma_r)/(2Gk)
\end{align*}

where \(G\) is the shear modulus of elasticity, \(\nu\) is the Poisson’s ratio of the soil, and \(k\) is a shape parameter [30], distinguishing between spherical \((k = 2)\) and cylindrical \((k = 1)\) cavities. Elder[34]
solved this problem for incompressible conditions ($\nu = 0.5$) considering a spherical cavity i.e. $k = 2$. This is equivalent to assuming that $\epsilon_r = -2\epsilon_\theta$. Adapting Elder’s assumptions and applying them to a cylindrical cavity, we obtain $\epsilon_r = -\epsilon_\theta \Rightarrow \frac{\partial u}{\partial r} = -\frac{u}{r} \Rightarrow u = \frac{C_1}{r}$ (where $C_1$ is an integration constant). Equation 2.5b simplifies to

$$- 2G \frac{u}{r} = -\frac{1}{2} (\sigma_r - \sigma_\theta)$$

(2.6)

Substituting $u = \frac{C_1}{r}$ in Eq. 3.6 and subsequently Eq. 3.6 into Eq. 3.1 with the boundary condition that $\lim_{r \to \infty} \sigma_r = 0$ yields:

$$\sigma_r = \frac{2G}{r^2} C_1$$

(2.7)

Substituting $C_1 = ur$ from above and solving for $u$ gives:

$$u = \frac{\sigma_r}{2G} r$$

(2.8)

In the plastic regime, the radial stress can be estimated by substituting Eq. 3.2 into Eq. 3.1. For a boundary condition of $\sigma_r(r_c) = P$, where $r_c$ represents the cavity radius, the radial stress $\sigma_r$ is defined as:

$$\sigma_r = P - 2s_u \ln\left(\frac{r}{r_c}\right)$$

(2.9)

At the elasto-plastic interface $r = R$, the radial stress yields the value of the undrained soil strength, $\sigma_r = s_u$, hence the deformation at the elasto-plastic interface is as follows:

$$u(r = R) = \frac{R}{2G} \left( P - 2s_u \ln\left(\frac{R}{r_c}\right) \right) = R \left( \frac{s_u}{2G} \right)$$

(2.10)

Assuming that the change in the cavity zone ($\pi(r_c^2 - r_0^2)$) equates to the change in the plastic region($\pi(R^2 - (R - u)^2)$), we substitute in for Eq. 3.8 ($u(R) = \frac{s_u}{2G} R$) and solve for $\frac{R}{r_c}$. The relationship explicitly links cavity radius with the plastic radial domain ($r_c \leq r < R$) (Fig. 2.2).

$$\left(\frac{R}{r_c}\right)^2 = \frac{G}{s_u} \left( 1 - \left(\frac{r_0}{r_c}\right)^2 \right)$$

(2.11)

Inserting the relationship of Eq. 3.11 back into Eq. 3.10 ($\frac{R}{r_c}$), the pressure required to expand a cavity in an elasto-plastic material is expressed as:

$$P = s_u \left( 1 + \ln\left(\frac{G}{s_u} \left( \frac{1 - \left(\frac{r_0}{r_c}\right)^2}{1 - \frac{s_u}{4G}} \right) \right) \right)$$

(2.12)

and under the assumption that the system is incompressible, and $\frac{G}{s_u} >> 1$, the result yields the
limit cavity pressure [30] as \( r_{c_0} \rightarrow 0 \).

\[
P_L = s_u (1 + \ln \left( \frac{G}{s_u} \right))
\]  

(2.13)

Since plant root and earthworm radii are significantly smaller than standardized cone penetrometer radius, we use data from Bishop et al. [29] in conjunction with the limit pressure as a boundary condition. Considering that the deformation is predominantly plastic, we again combine Eq. 3.1 and Eq. 3.2 and apply the boundary condition \( \sigma_r(r_b) = P_L \) to solve for the radial stress as a function of cavity size:

\[
\sigma_r(r) = P_L - 2s_u \ln (r/r_b)
\]  

(2.14)

where \( r_b \) (4.45 mm) is the radius used by Bishop et al.[29] to derive the expression for the cavity limit pressure \( P_L \) as described by Carter et al. [30] . This formulation facilitates calculation of the required change in cavity pressure for different radii.

**Modeling penetration resistance**

Dexter [18; 14], Bengough and Mullins [13] report that plant roots and earthworms penetrate soil in a similar manner as that of sharp penetrometers, deforming the soil cylindrically (see Fig. 2.3). To model soil penetration by earthworms, a cavity expansion based cone penetration model is employed.

Yu [31] and Durban and Fleck [32] developed a semi-analytic expression based on cavity expansion for rough and smooth penetration at different angles. This formulation considers the angular effects when neglecting friction. The penetration resistance stress can be expressed as [32; 31]:

\[
\sigma_z = s_u (\pi + 2\alpha + \sin^{-1}(m) + \frac{D}{2} + m \cot \alpha - \sqrt{1 - m^2} - 1) + \sigma_r
\]  

(2.15)

where \( \alpha \) is the semi-apex cone insertion angle, \( m \in [0, 1] \) is the gauge of roughness where \( m = 0 \) is lubricated, and \( m = 1 \) is rough, and

\[
D = \frac{\sin (\frac{\pi - \alpha}{2}) + m \sin (\pi - \alpha)}{\cos (\frac{\pi - \alpha}{2}) - \cos (\pi - \alpha)}
\]

The penetration resistance force can be determined by integrating the total penetration resistance stress \( \sigma_z \) along the length of the cone for the changing cross sectional area:

\[
F_z = \begin{cases} 
2\pi \int_0^{r_f} r \sigma_z dr, & 0 \leq l < l_{cone} \\
\pi r_f^2 \sigma_z, & l > l_{cone}
\end{cases}
\]  

(2.16)

where \( r_f \) is the cone base radius, \( l_{cone} \) is the cone length, and \( l \) is the penetration depth. For
Figure 2.3 Cylindrical cavity expansion sequentially determines steady state penetration of acute cones. The conical cross section applies a boundary pressure that opens a cavity to some final steady state cylindrical burrow.

\[ l \leq l_{cone}, \; l = r \cot(\alpha) \] (see Fig. 2.3). For sufficiently long penetration depth where \( l >> l_{cone} \), the second expression suffices to account for the effective penetration force.

### Strain energy for cavity expansion in plastic soil - ecological considerations

All energetic costs calculated throughout the text pertain to the mechanical energy required for soil penetration by earthworms (involving plastic deformation and displacement of the soil). The mechanical strain energy is defined as the amount of energy invested to induce deformation based on the following:

\[ U = \int_{z_0}^{z_f} F_z dz \] (2.17)

where \( U \) is the strain energy of the system, \( F_z \) is the penetration resistance force, and \( dz \) is the change in the penetration depth. By substituting Eq. 2.16 into Eq. 2.17, we obtain:

\[ U = \begin{cases} 
2\pi r_0^2 \sigma_z \xi, & 0 \leq l < l_{cone} \\
\pi r_f^2 \sigma_z l, & l > l_{cone}
\end{cases} \] (2.18)

where \( \sigma_z \) is the axial stress, \( \alpha \) is the semi-apex cone insertion angle, \( l \) is the axial depth of penetration, \( l_{cone} \) is the axial length of the cone tip, \( r \) is the cavity radius, \( r_f \) is the earthworm
base radius, and $\xi$ is a dummy variable used for integration, representative of radius. For $l \gg l_{cone}$, the first term (during insertion) only accounts for a marginal amount of the total strain energy, therefore the second term can be used to approximate the total strain energy related to a penetration depth $l$.

The maximum strain energy density is estimated by dividing the strain energy required to construct small earthworm burrows by their burrow volume, resulting in the following expression:

$$U_0 = \frac{U(r_{min})}{\pi r_{min}^2 l}$$  \hspace{1cm} (2.19)

where $r_{min}$ is taken as the smallest earthworm radius based on ecological parameters, $U(r_{min})$ is the strain energy required for a small cavity, $\pi r_{min}^2 l$ is the resulting cavity volume. This expression for the strain energy density is used to determine the minimum amount of soil organic carbon content that would meet the mechanical energy demand of earthworm motility.

### Soil mechanical and biophysical properties

The mechanical properties of soils, and mechanical constraints on bioturbation are functions of soil water content, porosity, clay minerals, and other parameters. We explored a range of these soil physical parameters to delineate mechanical stresses and energy costs related to soil penetration by earthworms. A range of data for soil shear strength and elasticity as functions of clay and water contents were obtained from the literature (Table 2.2). The soil modulus of elasticity was estimated from the clay mechanical properties [38]. To capture the dependence of shear modulus of clay on its water content (Fig. 2.4), these results were expressed in terms of water content

<table>
<thead>
<tr>
<th>Clay content (%)</th>
<th>$\theta_m [kg \cdot kg^{-1}]$</th>
<th>$G [kPa]$</th>
<th>$s_u [kPa]$</th>
</tr>
</thead>
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<tr>
<td>15-25</td>
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<td>2 - 20 [21; 35; 36]</td>
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<td>70[21]</td>
<td>0.6[21]</td>
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<tr>
<td>15-25</td>
<td>0.35</td>
<td>-</td>
<td>5[35]</td>
</tr>
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<td>15-25</td>
<td>0.40</td>
<td>15[21]</td>
<td>0.20 - 20 [21; 35; 36]</td>
</tr>
<tr>
<td>40-50</td>
<td>0.20</td>
<td>500[21]</td>
<td>-</td>
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<td>0.45</td>
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<td>40-50</td>
<td>0.70</td>
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<tr>
<td>100</td>
<td>-</td>
<td>5000 - 17,000[37]</td>
<td>50 - 100[37]</td>
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<td>0.25</td>
<td>3000[38]</td>
<td>-</td>
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<td>2000[38]</td>
<td>6[39]</td>
</tr>
<tr>
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<td>0.40</td>
<td>500[21]</td>
<td>4[21]</td>
</tr>
<tr>
<td>100</td>
<td>0.50</td>
<td>350[21]</td>
<td>1.5[21]</td>
</tr>
</tbody>
</table>
and cohesion [39], suggesting that the inferred shear moduli were derived from samples with relatively low water contents. Studies conducted on saturated clays show greatly reduced shear modulus and soil shear strength [21].

Earthworm physical and ecological parameters were used to determine the mechanical limitations and to estimate the strain energy requirements for soil penetration by earthworms. Observed values of earthworm pressure thresholds were obtained from Newell [40], Keudel and Schrader [28], and McKenzie and Dexter [17], providing insights into the range of soil hydrological conditions that permit earthworms to penetrate soil. The internal earthworm pressure values rarely exceed 230 kPa, with mean values in the range of 60 to 200 kPa[28; 40; 17]. Estimates of earthworm radii were obtained from Vandenbygaart et al. [41] and Ehlers [42] with values ranging between 1.0 and 5.5 mm [41], and an earthworm population mean radius of 2.5 mm [42]. Estimates of tunnel lengths were obtained from Capowiez and Belzunces [8]. To estimate annual mechanical energy requirements per unit soil area (or volume) for a typical earthworm community, knowledge of population density and annual penetration rates were needed. Capowiez and Belzunces [8] reported penetration rates for individual earthworms in the range of 0.1 to 0.2 m day$^{-1}$ for measurements over 200 hours. Earthworm population density values were obtained from Daniel [43] and Chan [44] measured at 200 mm soil depths, Fragoso and Lavelle [45], measured at 100 mm soil depths[44] (data are presented in Table 2.3).
Table 2.3  Earthworm physical parameters.  \( r_f \): worm radius; \( l_b \): tunnel length; \( \frac{dl_b}{dt} \): penetration rate; \( n \): population density; \( P \): pressure.

<table>
<thead>
<tr>
<th>Classification</th>
<th>( r_f [mm] )</th>
<th>( l_b [m] )</th>
<th>( \frac{dl_b}{dt} [m/day] )</th>
<th>( n [ind m^{-3}] )</th>
<th>( P [kPa] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meadows [43]</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>700 – 1550</td>
<td>-</td>
</tr>
<tr>
<td>Trop. Forest [45]</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>40 – 4000</td>
<td>-</td>
</tr>
<tr>
<td>Ag. Fields [44]</td>
<td>1 – 5.5 [42; 41]</td>
<td>-</td>
<td>-</td>
<td>300 – 700</td>
<td>-</td>
</tr>
<tr>
<td>Lab</td>
<td>-</td>
<td>1 – 1.6 [8]</td>
<td>0.12 – 0.20 [8]</td>
<td>-</td>
<td>3 – 230 [40; 17]</td>
</tr>
</tbody>
</table>

Table 2.4  Input parameters for the Mechanical cavity expansion simulation. \( \theta_{min} \): residual water content; \( \theta_{max} \): saturated water content; \( \Theta \): normalized water content; \( G \): Shear modulus; \( s_u \): soil strength. (Values marked with an asterisk (\( ^* \)) were extrapolated based on the trend lines presented in [21; 46].)

<table>
<thead>
<tr>
<th>Clay</th>
<th>( \theta_{min} )</th>
<th>( \theta_{max} )</th>
<th>( \Theta )</th>
<th>( G [kPa] )</th>
<th>( s_u [kPa] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[( % )]</td>
<td>[kg/kg]</td>
<td>[kg/kg]</td>
<td>[( % )]</td>
<td>[kPa]</td>
<td>[kPa]</td>
</tr>
<tr>
<td>16</td>
<td>0.045</td>
<td>0.4</td>
<td>1</td>
<td>15</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.44</td>
<td>200</td>
<td>2.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.1*</td>
<td>2386*</td>
<td>40*</td>
</tr>
<tr>
<td>50</td>
<td>0.1</td>
<td>0.85</td>
<td>1</td>
<td>10</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.47</td>
<td>150</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.15*</td>
<td>5668*</td>
<td>38*</td>
</tr>
</tbody>
</table>

The analysis of penetration expansion was based on models that provided information for determining the amount of mechanical energy required to expand a cavity to radius \( r_f \). The range of radii was based on the smallest and the largest earthworm radius in Table 2.3 (\( r_f = 1 \) to 5 mm). Standard mechanical soil properties and their connection to hydration status and clay content were derived from the relations in Ghezzehei and Or [21] (see Table 2.4). For better comparability between soil types, the hydration status was expressed as normalized water content, defined as

\[
\Theta = \frac{\theta - \theta_{\text{min}}}{\theta_{\text{max}} - \theta_{\text{min}}} \tag{2.20}
\]

where \( \theta_{\text{max}} \) is the highest reported (saturated if available) water content, \( \theta_{\text{min}} \) is the lowest reported (residual if available) water content, and \( \theta \) is the actual gravimetric water content. The range of reported water content values are tabulated in Table 2.4. To account for lubrication by biopolymers (e.g. earthworm mucus), the interface between the soil and earthworm was considered to be a frictionless and smooth boundary. The range of apex angles representing earthworm geometries ranged from \( \alpha = 1 \) to 45°.
From penetration mechanics to soil displacement energy estimations

Using strain energy per unit length derived from mechanical considerations, estimates of energy demands for bioturbation were derived for different ecological conditions. Using the values of $l_b$ as the length of earthworm channels from Table 2.3, we calculated mechanical energy costs of soil penetration (per length) and determined the amount of strain energy required for incremental soil penetration. In conjunction with data regarding earthworm penetration rates and earthworm population densities in various ecological systems (Table 2.3), we estimated the amount of mechanical energy transferred to the soil by a population of earthworms. For simplicity, we consolidated reported vertical distributions of earthworm radius to a single mean value of 2.5 mm, assuming that the value does not vary too much in the top half meter of soil [42]. The estimated energetic costs of soil penetration were translated to equivalent soil organic carbon consumption by earthworms, using a conversion coefficient of $0.0484 \text{ g}\text{carbon J}^{-1}$ [7].

Model evaluation - comparison to numerical and empirical data

Predictions by the analytical model of cavity expansion (Eq. 3.12) were first evaluated in comparison with numerical calculations for the same conditions, and then with experimental results. A plane strain steady state finite elements model (FEM) was constructed using COMSOL [47]. The finite element model simulated deformation driven cavity expansion in an incompressible elastic-perfectly plastic medium with boundary walls infinitely far from the expanding cavity. Both the finite element model and the analytic model were compared for 9-fold expansion from an initial radius.

Additional comparisons were performed with an explicit cone penetration model developed by Walker and Yu [48]. Their model uses an adaptive finite element remeshing algorithm in order to simulate the actual motion and geometry of soil penetration directly rather than with cavity expansion. The analytic solution expressed in Eq. 3.14 was used for a cone roughness of $m = 1$ and a semi apex angle of $\alpha = 30^\circ$. Both models assume $s_u = 10$ kPa and $G = 1000$ kPa (taken from Walker and Yu [48]).

The model was lastly tested against experimental penetration data of Kurup et al. [49], where two replicates of a silty clay (50% Kaolinite and 50% Edgar fine sand) were measured to have soil strengths and shear moduli of a) $s_u = 65$ kPa and $G = 567 \times s_u$, and b) $s_u = 40$ kPa and $G = 150 \times s_u$ under saturated conditions. We assumed a friction effect of $m = 1$, semi apex angle of $30^\circ$, and utilized the final radial stresses for cone radii of 5.64 and 6.36 mm associated with the cone designs of the miniature piezocone penetrometer and miniature quasi-static cone penetrometer respectively [49].
Comparison to crack propagation model

Results from the present model were also compared with data provided by Dorgan et al. [50], who assumed that earthworms penetrate sediments by crack propagation and estimated energy requirements for soil penetration using linear elastic fracture mechanics. Estimations of energy requirements for soil penetration by crack propagation were conducted using linear elastic fracture mechanics (LEFM) utilizing an energy formulation defined by Dorgan et al. [50]:

\[ U = K_{Ic}^2 \frac{(1 - \nu)}{2G} l r_f \]  \hspace{1cm} (2.21)

where \( K_{Ic} \) is the fracture toughness, \( \nu \) is the Poisson’s ratio, \( G \) is the shear modulus, \( l \) is considered to be the distance over which the crack grows, and \( r_f \) is considered to be the width of the crack. For simplicity, this study assumes that the width of the crack is the same size as the radius of the earthworm penetrating the soil, and the length that the crack grows is equal to the depth of a given earthworm tunnel. For comparison with our model, we assumed a 1 m long penetration depth with a radius of 1.2 mm [50].

![Fracture toughness vs water content.](image)

**Figure 2.5 Fracture toughness vs water content.** [51; 50; 52]. Continuous curve was plotted through the data points in order to approximate fracture toughness values at different water contents.

The fracture toughness changes as a function of soil water content. Values for fracture tough-
ness for lower water contents were collected from Hanson et al. [51] and Wang et al. [52]. For a soil with clay content of 15-25% and saturated conditions (water content of $\theta_m = 0.44 \text{ kg kg}^{-1}$), the mechanical shear modulus is equivalent to that of gelatine used in the study by Dorgan et al. [50] ($G = 1.4 \text{ kPa}$). The fracture toughness parameters were fit to a continuous curve plotted against water content in order to estimate mechanical energy investments for a wider range of water contents (seen in Fig. 2.5).

## Results

### Model evaluation - comparison to numerical and empirical data

The results depicted in Fig. 2.6 illustrate that the limit pressures predicted by the analytic model and the finite element COMSOL model converge for large final cavity radii compared to initial radii, given same soil strength and shear modulus. Note that $PP_L^{-1}$ is the ratio of cavity pressure to limit pressure, and the limit pressure is invariant with respect to cavity radius [30].

![Figure 2.6 Analytical cavity expansion model vs. Finite Element cavity expansion model. (a) Relative cavity pressure vs. relative cavity radius; (b) strain energy density scaled by limit pressure vs. relative cavity radius scaled by initial radius. For both models, changes in soil mechanical parameters only changed the magnitude of $P_L$. Both analytic and numerical models showed close magnitudes of $P_L$. The discrepancy between the strain energy density values was measured as: $\frac{\|U_0,\text{NUM} - U_0,\text{ANA}\|_\infty}{\|U_0,\text{NUM}\|_\infty} = 0.11$.](image-url)
The results show that the convergence to the limit pressure occurs at a larger radial deformation in the numerical solution than predicted by the analytic solution. However, the difference between strain energy densities for the analytic solution and the numerical solution were less than 10% at 9-fold deformation (Fig. 2.6 (b)).

The penetration stress versus radial strain was compared between the cavity expansion based cone penetration model and the geometrically explicit adaptive FEM cone penetration model developed by Walker and Yu [48]. The comparison suggests that the simplifying assumptions employed in the analytical model introduced a relative error of less than 20% (Fig. 2.7).

![Comparison of analytical cavity expansion based model with an adaptive finite element explicit penetration model (Walker and Yu [48]).](image)

Figure 2.7 Comparison of analytical cavity expansion based model with an adaptive finite element explicit penetration model (Walker and Yu [48]). Comparison is drawn between the relative penetration stress vs the radial strain. Penetration stress is scaled by the shear soil strength. The discrepancy between the pressure values over two orders of magnitude never exceeds \( \frac{\| P_{NUM} - P_{ANA} \|_\infty}{\| P_{NUM} \|_\infty} < 0.2 \).

Finally, the soil impedance predicted with the present penetration-expansion model was compared with experimental impedance results reported by Kurup et al. [49] down to a soil depth of 150 mm (seen in Fig. 2.8). The comparison of the simulated stresses and the experimental data
reveals an error between measured and predicted soil mechanical impedance of about 8-18% for the first test, and 20 to 35% for the second test. The discrepancy towards the tip could be related to dynamic effects that are not properly accounted for in the current steady state solution.

![Figure 2.8](image)

**Figure 2.8** Soil mechanical impedance to cone penetration for different soil mechanical properties and cone types. Two replicates of a silty clay were measured to have (a) $s_u = 65kPa$ and $G = 567 \times s_u$; (b) $s_u = 40kPa$ and $G = 150 \times s_u$. Experimental data correspond to two tests conducted with duplicate cones of the same geometry but subtle physical design differences[49]: miniature piezocone penetrometer (PCPT4 and PCPT6) and miniature quasi-static cone penetrometer (PCPT3 and PCPT5). Data points were obtained from [49] with the dashed lines denoting the positions when the cone was fully inserted.

**Simulated forces and stresses during penetration-expansion in soil**

The internal earthworm (or root) pressures required for cavity expansion or for penetration of new soil volumes were computed for various cavity radii, apex insertion angles and combinations of hydration and soil properties listed in Table 2.2. The results in Fig. 2.9 illustrate that the pressure required for cavity expansion increases for drier soil and reaches the intrinsic maximum pressure (that an earthworm can exert) at normalized water content of roughly 0.1 and 0.2 for soils with clay contents of 16% and 50% respectively.

The exact cone geometry has only a small influence on the penetration and cavity expansion pressures. Fig. 2.9b illustrates the maximum penetration resistance pressures for varying semi
apex angles for the same moisture contents. The simulations were conducted assuming a frictionless \((m = 0)\) interface between the penetrating object (earthworm or plant root) and the soil. Soil drying (reduced water content) increases cavity limiting pressure and with it the penetration resistance. The results in Fig. 2.10 were computed for soils with a clay content of 16%, but similar trends were obtained for soils with 50% clay content (slightly higher mechanical resistance values). In summary, the penetration pressures increase with increasing clay content, decrease with increasing water content, decrease (slightly) with increasing radius, and slightly increase with increasing semi-apex angle.

Cone penetration resistance in terms of stresses and penetration resistive forces for different earthworm and plant root radii are depicted in Fig. 2.11. Fig. 2.11 (a) shows that the penetration stress decreases (slightly) with increasing radius. In contrast, the penetration force increases with cone radius (Fig. 2.11 (b)). The interplay between penetration (resistive) force and stress will be elaborated in the discussion section, in the context of estimating energy costs of bioturbation.
Figure 2.10 Maximum penetration resistance stress vs. cone apex angles for different normalized water contents with a base radius of 2.5 mm at 16% clay content. Simulations were conducted for normalized water contents of 0.1, and 1.0 at a soil clay content of 16%. Soils with larger clay content display similar mechanical behavior at larger normalized water contents.

**Strain energy of cavity expansion in plastic soil - ecological considerations**

Estimates of bioturbation strain energy were computed for different clay contents and normalized water contents, and for radii in the range of 1 to 5 mm, as listed in Table 2.2. The strain energy density was estimated from strain energy values associated with the minimum earthworm radius of 1 mm. Fig. 2.12 depicts the change in strain energy density as a function of clay content and water content. Applying a conversion coefficient of 0.0484 g$_{carbon}$ J$^{-1}$ to translate energy requirement to soil organic carbon requirement [7], we estimate minimum soil organic carbon (SOC) contents required to support penetration by earthworms in soils with different clay and water contents (Fig. 2.12). The strain energy (and required SOC) decreases with increasing water content, and increases with increasing clay content.

To estimate the annual carbon consumption of earthworm communities, we use published examples of earthworm abundances in different ecosystems (Table 2.3). For a soil with 16% clay content and normalized water content of 0.4 (representing field capacity [53]), the strain energy density was calculated to be 24.1 kJ m$^{-3}$. For an average earthworm of 2.5 mm radius, the energy
required to create a 1 m long tunnel would amount to $24.1 \text{ kJ m}^{-3} \times \pi \times (2.5 \times 10^{-3} \text{ m})^2 \times 1 \text{ m} = 4.7 \times 10^{-4} \text{ kJ}$. Assuming a mean penetration rate of 0.1 m day$^{-1}$ (or 36.5 m year$^{-1}$) (Table 2.3), we estimate an annual mechanical work of $17.2 \text{ J year}^{-1}$ per earthworm, which is equivalent to the energy contained in 0.8g of SOC. Assuming a typical earthworm population density in a paddock of 300 individuals per m$^3$ soil in the top 0.2 m of soil (Table 2.3) and the same penetration rate of 0.1 m day$^{-1}$ for each individual, we estimate a minimum consumption of $8 \times 10^{-4}$ kg year$^{-1} \times 300 \text{ ind m}^{-3} \times 0.2 \text{ m} = 0.05 \text{ kg year}^{-1}$ soil organic carbon per m$^2$ surface area to cover the energy expenditure for soil penetration by the whole earthworm community.

Comparison to crack propagation model

A comparison of energy requirements for creating a 1 m long cavity with a radius of 1.2 mm by plastic penetration (our model) or fracture propagation (model by Dorgan et al. [50]) is presented in Fig. 2.13. For a normalized water content of 1 [-], the crack propagation model predicts an energy expenditure of $3.2 \times 10^{-3} \text{ J}$, while the elasto-plastic cavity expansion model presented in this study yields an energy expenditure of $3.1 \times 10^{-3} \text{ J}$. For normalized water contents in the range of 0.2 to 1 [-], the strain energy required for penetration based on the elasto-plastic penetration-cavity expansion model is much lower than for fracturing. For normalized water contents below 0.02[-], the strain energy required for plastic penetration exceeds that of the crack propagation.
Figure 2.12 Required soil organic carbon content and maximum strain energy density as a function of normalized water content for clay contents of 16% and 50%. Analysis is based on the steady state mechanical model to determine strain energy. For given soil organic carbon content, one can determine the range of normalized soil moistures under which the energetic demands of displacing a volume of soil are less than the energy stored in the soil organic carbon in the same volume (range to the right of the respective line in the figure). This neglects any rate dependent effects. Note that mechanical energy is mapped to soil organic carbon content using a factor of $1.2/24.8$ kg kJ$^{-1}$ [7], and the tick marks on the left vertical axis were spaced in order to align with those on the right vertical axis.

Discussion

The theoretical estimates and their general agreement with limited experimental data inspire confidence in using these approximations to estimate the energy requirements for creation of earthworm burrows or plant root channels. The resistance stresses for plant roots and earthworms are reduced with increasing radius (Fig. 2.11 (a)), hence the mechanical energy per unit of displaced soil volume decreases with increasing radius. This implies that larger tunnel radii would be more energy efficient if displacing soil material where the primary goals of earthworms and plant roots. However, if movement through soil in search for resources drives bioturbation, the mechanical energy required to traverse a certain distance in the subsurface may be ecologically more relevant for plant roots and earthworms. The results in Fig. 2.11 (b), depict an increase in penetration force with increasing cavity radii, and the mechanical energy per distance travelled in the soil increases with radius. Hence, while smaller earthworms or plant roots are less energy-efficient in terms of volumetric soil displacement, worms or roots with smaller radii are more energy-efficient for traversing the subsurface.

A decrease in strain energy for wetter soils is attributed to the reduction in soil strength and shear modulus with increasing water content. The strain energy requirements increase with
increasing soil clay contents that affect soil strength and shear modulus. There is a slight increase in penetration resistance with increasing semi-apex insertion angle. The axial component of forces acting normal to the face of an earthworm or plant root increases with increased apex angles resulting in an increase in penetration resistance.

The plastic earthworm penetration model presented in this study compared well with the linear elastic fracture mechanics (LEFM) model of Dorgan et al. [50] at high water contents, predicting a mechanical energy expenditure of $3.1 \times 10^{-3}$ J, whereas Dorgan et al. [50] estimates $3.2 \times 10^{-3}$ J for an earthworm to propagate a crack of the same length. Despite many differences in the modeling approaches and underlying assumptions, the mechanical energy computed for crack propagation and penetration-expansion are similar for water contents near saturation[50].

For intermediate water contents, the LEFM model typically results in a larger expenditure per unit of cavity length than the plastic penetration model, while for very low saturation degrees (below 0.02 [-]), fracture propagation appears to be less energy consuming than plastic penetration (Fig. 2.13). However, LEFM would imply that creation of permanent fractures at saturation degrees below 0.4 [-] would require cavity pressures in excess of 230 kPa, which is the physiological limit of earthworms. The results suggest that plastically deforming the soil is probably more
energy efficient for wet soils, while for dry soils (saturation degrees lower than 0.02 [-]), Fig. 2.13 suggests that crack propagation is energetically advantageous over penetration-cavity expansion. These preliminary results suggest that different modes of soil penetration may be favorable for different soil water contents (or soil mechanical properties).

Plant roots have been observed to expand radially upon encountering soils with high mechanical impedance, and in this way weakening the forefront [15], potentially inducing a brittle failure close to the root tip, allowing for axial elongation into an open crack. For the case of this soil, the water contents where linear elastic fracture mechanics seems more energetically feasible than elasto-plastic deformation is below the physiological limit where plant roots can actually uptake water (corresponding to a saturation degree of 0.2 [-]), that is the matric potential of the soil exceeds the osmotic pressures that plant roots can extract water from the root soil interface (root suction pressures reported to range from -2.5 to -3.5 MPa [54]). It has been shown that soil plastic deformation and crack propagation are not mutually exclusive phenomena (Yoshida and Hallett [55]), and we note that more systematic studies are required to delineate respective envelopes of applicability of fracture mechanics and plastic deformation models. Despite neglecting potential crack formation, the elastic-perfectly plastic model presented in this study still determines a minimum energy requirement necessary for bioturbation.

The model developed in the present study could be used to assess the amount of soil organic carbon (SOC) necessary to support earthworm activity in soil. Fig. 2.12 depicts estimates for the minimum amount of SOC required for an individual earthworm to satisfy the energy demand of soil penetration (for clay contents of 16% and 50%). Based on these model estimates, earthworm penetration in soil with normalized water contents below 0.25 can only be economical if SOC content exceeds 10 kg m$^{-3}$. This value seems realistic, as Crumsey et al. [56] also found 10 kg m$^{-3}$ SOC in their mesocosm experiments, which were maintained at field capacity during the experiments. In order to provide conservative estimations of energetic costs of cavity creation, our analysis presented in the results section was based on normalized water content at field capacity ($\Theta = 0.4$).

Fig. 2.12 implies that the energetic costs for constructing a 1 m long tunnel by an earthworm with 2.5 mm radius [42; 3] at a normalized water content of 0.4 and 16% clay content are equivalent to the energy contained in roughly $2.3 \times 10^{-5}$ kg soil organic carbon. Considering an average earthworm maintenance respiration rate at rest of $3.6 \times 10^{-6}$ kg day$^{-1}$ (of carbon) [57], this implies that the strain energy required to burrow 1 m distance is equivalent to 7 days of maintenance respiration at rest. Provided the upper limit of earthworm penetration rate is 0.2 m day$^{-1}$ (Table 2.3) this would suggest that soil penetration could account for up to half of an earthworm’s energy budget.

Scaling up of our energy estimates to the burrowing activity of a typical earthworm community in a paddock resulted in energy expenditures equivalent to as much as 0.05 kg year$^{-1}$ SOC
per m² surface area. Alban and Berry [58] witnessed invasive earthworms depleting SOC at a rate of 0.06 kg year⁻¹ per m², suggesting that their estimates are in close agreement with field observations. Considering that, Beer et al. [59] estimated the net primary productivity (NPP) in cropland to be in the order of 0.9 kg m⁻², the estimates above suggest that earthworm populations could consume as much as 5% of the annual NPP in crop lands. Taking the lowest normalized water content for which soil penetration would still be economically feasible with an SOC concentration of 10 kg m⁻³ (Θ = 0.25 in Fig. 2.12), a similar calculation would yield that the earthworm community energy expenditure for the same burrowing activity would be equivalent to as much as 0.15 kg year⁻¹ SOC per m², which would be equivalent to 15% of the annual NPP in crop lands. However, note that these estimates are based on constant soil water content, whereas natural hydration dynamics would affect the energetic costs of soil penetration as well as bioturbation activity and earthworm population sizes.

An important aspect not considered in this analysis is the role of soil ingestion by earthworms, an essential process for extracting the required soil organic carbon and contributing to many soil regulatory functions [60]. Ingested soil passing through the earthworm gut stimulates microbial activity, enhances aggregation by mucus secretion and litter collection, and gives rise to mutual interactions between microbes and earthworms. On the other hand, earthworms ingest a wide variety of organic matter including fungus and bacteria [60; 61]. Evidence suggests beneficial impact on plant root activities (despite anecdotal evidence for earthworms ingesting living roots [61]). The partitioning between ingestion and soil displacement is not well understood. Future studies will endeavour to elucidate the mechanical ramifications of ingestion, and, equally important, the role of rapid penetration by earthworms in comparison to slow growth by plant roots that would incur different energetic costs (for the same burrow geometry) due to effects of visco-elastic displacement processes [21] not addressed in this study.

Finally, the results presented in this study suggest that biotic soil processes perform considerable mechanical work on soil, implying that they are likely effective contributors to soil structural restoration after disturbance, e.g. by soil compaction. Abiotic processes such as wetting and drying induce shrinking and swelling [62], which also contributes to expanding cavities. Spherical soil aggregates undergo coalescence due to wetting at axial strain rates of 0.005 m m⁻¹ s⁻¹, which is equivalent to the rate of spherical radial expansion due to aggregate swelling [62]. Plant roots expand cavities at strain rates up to 0.08 m m⁻¹ s⁻¹ [23]. This would imply that macropore creation by biotic processes occurs at rates nearly 20 times faster than that by abiotic processes, thus earthworms and plant roots have the capacity to accelerate soil structural restoration by at least an order of magnitude compared to abiotic processes only. More experimental and theoretical work is needed to understand the interplay between abiotic and biotic processes during post-compaction soil structural restoration. In particular, investigation of the initiation of the process, i.e. the creation of the first channels or cracks, and their benefit for subsequent colonization by roots and earthworms is important, in order to enable quantitative prediction of the speed of recovery of compacted soils.
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Bibliography


Chapter 3

Experimental Evaluation of Earthworm and Plant Root Soil Penetration - Cavity Expansion Models Using Cone Penetrometer Analogs

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Abstract

Recent mechanical models of soil penetration by earthworms and plant roots based on penetration-cavity expansion were tested using cone penetration measurements at scales compatible with the sizes of earthworms and plant roots. Measurements using different cone radii (1.0 to 2.5 mm) and cone semi-apex angles (15 to 30°) were obtained for a range of soils and water contents at highly resolved penetration forces and constant insertion rates. The cone penetration measurements were interpreted using independently determined soil mechanical parameters and yielded good agreement with predictions from an analytical mechanical model. Experimental confirmation of penetration force predictions supports estimates of energy costs associated with soil bioturbation that vary with soil hydration status and mechanical characteristics. Effects of soil friction and axial compaction were assessed by comparing the results from conventional and recessed cones (to eliminate soilshaft friction). The study provides new insights into quantitative soil bioturbation processes and expands predictive capabilities of the mechanics and energetics of earthworm
activity and root zone dynamics related to soil structure development.

**Introduction**

Soil penetration and internal volume displacement by growing plant roots and by burrowing earthworms require significant amounts of mechanical energy. Bioturbation processes associated with life in the subsurface play an important role in the formation of soil structure and affect a range of soil physical, hydrological, and ecological function (Bottinelli et al., 2014; Brown et al., 2000; Watts et al., 1999)[1; 2; 3]. For heterotrophic organisms, the mechanical costs of soil penetration are largely constrained by available energy sources needed to support subterranean activities. For example, the amount of (particulate) soil organic C (SOC) required to support the mechanical bioturbation energy requirements of generic earthworm communities (Lavelle et al., 2007)[4] was estimated to represent up to 5% of the net primary productivity of crop-lands per year (Ruiz et al., 2015)[5]. These theoretical, mechanically based estimates are within the range of observed soil C depletion rates by earthworm populations (Alban and Berry, 1994)[6].

Despite the significance of the energy costs of soil bioturbation for subsurface C storage and for the maintenance of soil structure, the experimental determination of the energetics of soil penetration by earthworms and plant roots remains a challenge. Mechanical analyses of bioturbation have focused on determining radial and axial stresses exerted by earthworms and plant roots necessary to fracture stiff dry soil or chalk (McKenzie and Dexter, 1988; Misra et al., 1986)[7; 8]. Such measurements are difficult to perform and are not general for different soil and climatic conditions, biological community structure, and other factors. A surrogate measure based on cone penetrometer resistive forces was found to be in good agreement with estimated plant root resistance pressures in soils (Abdalla et al., 1969; Dexter, 1987; Greacen and Oh, 1972)[9; 10] and has been proposed to represent the geometrical and mechanical features resembling earthworm burrowing (Ruiz et al., 2015)[5]. In addition to geometric similarities, cone penetrometer measurements are useful for determining various soil mechanical properties due to their simplicity and ability to provide real-time data (Salgado and Prezzi, 2007; Yu, 1993; Yu and Mitchell, 1998)[11; 12; 13]. Penetrometer measurements have been used as qualitative indicators of soil structural properties such as soil strength and compaction level (Adamchuk et al., 2004)[14]. A few studies have interpreted cone penetration resistance stress as the soil strength itself (Urnger and Kaspar, 1994)[15]; others have used supplemental measurements by coupling inflatable probes to the penetrometer shaft to deduce soil mechanical properties from rebounding pressures (Houlsby and Withers, 1988)[16]. Recent approaches have used empirical correlations to deduce the soil bulk density and soil strength as functions of penetrometer pressure and soil matric potential (Whalley et al., 2007)[17]. These mechanical factors are known to influence the rates of root growth and earthworm burrowing activity in soil (Bengough and Mullins, 1990; McKenzie and Dexter, 1988; Misra et al., 1986)[18; 7; 8].
Some of the early experimental attempts to relate miniaturized cone penetration with plant root growth were reported by Stolzy and Barley (1968)[19], who utilized 1.5-mm-radius cones with a semi-angle of 30° to characterize the ratio between probe and root resistance. Their early findings suggested that plant roots experience < 75% of the penetration forces of a metal cone penetrometer. Bengough et al. (1997)[20] quantified the coefficient of friction for different cone angles, how it changes during rotation, and how it differs from plant roots. However, a rigorous theoretical understanding of soil penetration mechanics was typically derived from geotechnical applications that rely on large penetrometer geometries and thus are not directly compatible with the geometries of earthworms and plant roots.

An early mechanical model for cone penetration of soil was developed by Bishop et al. (1945)[21] based on the concept of an expanding cavity for quantifying pressures required for cone penetration. Cavity expansion models account for both elastic and plastic soil deformation. Yu (1993) and Durban and Fleck (1992) [12; 22] developed semi-empirical relationships that account for angular and interfacial friction effects during modeled penetration. Other modeling approaches to cone penetration were developed based on discrete element and other numerical models (Johnson, 2003)[23].

Ruiz et al. (2015)[5] recently proposed a penetration model that characterizes the radial stresses at the penetrating tip as a function of the cavity radius and used it to estimate the mechanical stresses and energetics related to soil penetration by earthworms and plant roots. A key novelty in this modeling methodology is the dependency of stresses on the cone radius at the scales of earthworms and plant roots. In the present study, we endeavored to test this modeling approach and establish quantitative links between penetration measurements and soil mechanical properties. The specific objectives were to: (i) quantify and explain the relationship between penetration stresses and forces for geometries compatible with earthworms and plant roots; (ii) test the rootearthworm scale penetration theory of Ruiz et al. (2015)[5]; (iii) compare current model results with previously used penetration models; (iv) determine how friction affects penetration measurements; and (v) investigate the potential for determining soil mechanical properties from penetrometer measurements.

An analytical model based on cavity expansion processes suitable for small cone radii was presented by Ruiz et al. (2015)[5]. We have developed a new, physically based coupling of the frictional influences. We compared our model with the cone penetration measurements and then with the classical limit pressure model (Carter et al., 1986)[24], a model that considers the influence of friction and penetration angles (Yu, 1993)[12], and a linear elastic fracture mechanics (LEFM) model that has been proposed to describe penetration expansion of marine worms (Dorgan et al., 2011)[25]. We then investigated the potential for using such measurements to obtain soil mechanical parameters in undisturbed samples and estimated the mechanical energy requirements for bioturbation. We put the findings of the present study into context with the existing literature to highlight the main implications, potential caveats, and need for further research.
Materials and Methods

Modeling Penetration Expansion in Elasto-Plastic Media

We begin with an equilibrium equation for stresses associated with cylindrical cavity expansion (Carter et al., 1986)[24]. The stress around a cylindrical cavity decays with the radius \( r \) away from the center of the cavity into the soil:

\[
\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{3.1}
\]

where \( \sigma_r \) [kPa] is the radial stress and \( \sigma_\theta \) [kPa] is the hoop (circumferential) stress. Soil plasticity is represented by the Mohr-Coulomb criterion, relating the difference between the radial and hoop stresses to the summation of stresses multiplied by a component of the internal friction and undrained soil strength:

\[
\sigma_r - \sigma_\theta = (\sigma_r + \sigma_\theta) \sin(\phi) + 2s_u \cos(\phi) \tag{3.2}
\]

where \( \phi \) [rad] is the internal angle of friction and \( s_u \) [kPa] is the undrained soil strength. Following Yu (1993)[12] and Durban and Fleck (1992)[22], we assume frictionless soils, i.e., \( \phi = 0 \), which results in the von Mises yield criterion (von Mises, 1913). We assume that the deformation during penetration is plastically dominated, thus we utilize the plastic radial stress distribution to approximate the stresses for smaller cavity sizes, with the stresses becoming singular at the small surface area of the cone tip (Fig. 3.1).

In the plastic regime, the radial stress can be estimated by substituting Eq. 3.2 into Eq. 3.1, yielding the following differential equation (Ruiz et al., 2015)[5]:

\[
\frac{\partial \sigma_r}{\partial r} = -\frac{2s_u}{r} \tag{3.3}
\]

We treat the experimental geometry used by Bishop et al. (1945)[21] as the plastic radial boundary condition due to the cone radius being small and compatible with the limit pressure theory. Thus \( \sigma_r(r_p) = P_L \), where \( r_p \) (4.5 mm) is the radius of the cone used by Bishop et al. (1945)[21] and \( P_L \) is the limit pressure, given as

\[
P_L = s_u(1 + \ln(\frac{G}{s_u})) \tag{3.4}
\]

where \( G \) [kPa] is the shear modulus of rigidity and \( s_u \) [kPa] is the undrained soil strength. Solving the differential Eq. 3.3 for these boundary conditions gives

\[
\sigma_r(r) = P_L - 2s_u \ln\left(\frac{r}{r_p}\right) \tag{3.5}
\]
Figure 3.1 Radial cavity expansive stresses mapped along the radius of the cone face: (a) the cavity expansion model defines a plastic region between an elasto-plastic interface \((R)\) and a cavity \((r_c)\); (b) the progressively increasing cylindrical cavities are used to properly map the radial stresses along the boundary of a cone partitioned into cylindrical elements.

To map the cylindrical radial stresses in the correct direction to the conical surface, the cone is represented as the summation of many incrementally partitioned cylinders of increasing radius (Fig. 3.1 b). Each cylindrical surface has an incremental surface area \(A_{ci}\):

\[
\Delta A_{ci} = 2\pi r_i \Delta z_i \quad (3.6)
\]

where \(\Delta z_i \,[m]\) is the incremental cylinder length and \(r_i \,[m]\) is the cylinder radius of the \(i^{th}\) increment. The cylindrical surface area on which the radial stresses act is then given by:

\[
A_c = \lim_{||\Delta z_i|| \to 0} 2\pi \sum_{i=1}^{n} r_i \Delta z_i = 2\pi \int_{0}^{r} r \, dz \quad (3.7)
\]

Given a cone apex semi-angle \(\alpha\), the slope of the cone surface can be expressed as

\[
\frac{dr}{dz} = \tan(\alpha) \Rightarrow dz = \cot(\alpha) \, dr \quad (3.8)
\]

Insertion into Eq. 3.7 gives the effective cylindrical surface area as a function of the cone apex semi-angle:

\[
A_c = 2\pi \cot(\alpha) \int_{0}^{r} \, dr = \pi r^2 \cot(\alpha) \quad (3.9)
\]

This enables linking radial forces \(F_r \,[N]\) and radial stresses \(\sigma_r \,[kPa]\) according to

\[
F_r = \pi r^2 \cot(\alpha) \sigma_r \quad (3.10)
\]
where $\alpha$ [rad] is the cone apex semi-angle and $r$ [m] is the radius of the submerged section of the cone. The axial force acting on the cone face is then

$$F_z = F_r \tan(\alpha)$$

(3.11)

The radial and axial forces describe frictionless steady-state cone penetration.

### Interfacial Friction during Penetration

To include the effects of interfacial friction between the cone and soil during insertion, an operational expression for forces acting normal to the cone face is needed. The cone normal force is expressed as

$$F_n = F_r \hat{r} + F_z \hat{z}$$

(3.12)

where $r$ [m] and $z$ [m] are the unit vectors in the radial and axial directions, respectively. The frictional forces act in the direction perpendicular to the normal force:

$$F_f = \mu(-F_z \hat{r} + F_r \hat{z})$$

(3.13)

where $\mu$ is the coefficient of friction between the soil and the cone. The sum of all the forces in the axial direction is then expressed as

$$F_{Z,M} = F_z (\mu \cot(\alpha) + 1)$$

(3.14)

This result resembles the classical formulation of Greacen and Oh (1972) [10], but it illustrates explicitly the frictional influence on the axial forces, while previous formulations expressed the penetration resistance as a function of stresses acting normal to the cone face (Bengough and Mullins, 1990) [18]. Although the classical formulation is mathematically sound, penetration resistance measurements are measured forces converted to stresses. However, for a correct conversion, it is necessary to divide by the cross-sectional area, and it is commonly ignored that the relevant cross-sectional area increases during insertion (Whiteley et al., 1981) [26]. The axial stress, or penetrometer resistance stress, $q_p$, can be expressed as

$$q_p = \frac{F_{Z,M}}{\pi r^2}$$

(3.15)

where the radius $r$ [m] is increasing as the cone moves into the soil.
Figure 3.2  Representation of an earthworm by a cone penetrometer: (a) geometry during penetration of an earthworm in agar (radius of 2 mm); (b) miniature cone penetrometer (radius 2.5 mm); and (c) cone penetration test into a soil sample held by an aluminum soil sample holder and overlaid with a load of 1 kg (same cone as in b).

Experimental Procedure: Cone Penetration and Uniaxial Soil Tests

Custom miniature stainless steel penetrometers were designed to represent realistic geometries of earthworms and plant roots (Fig. 3.2), with radii of 1 and 2.5 mm and apex semi-angles of 15 and
The cones were mounted on an Omega LC703-10 load cell that was driven into soil samples using a PERO spindle motor (originally designed as a shear frame, described by Michlmayr et al., 2013 [27]). Data were collected with a CR1000 datalogger (Campbell Scientific) at 10-Hz recording frequency. Penetration tests were conducted at a constant rate of 10 mm min\(^{-1}\). 

Two soil types were used for the cone penetration experiments, a silt loam (Uetliberg) and a loamy sand (Winzler), selected to span a range of soil parameters for model testing (Table 1). Soil and water were mixed to prescribed water contents and were left in sealed plastic containers in an overhead shaker overnight to homogenize the water distribution in the soil sample. Gravimetric soil water content ranged from 15 to 30% for Uetliberg soil and from 10 to 15% for Winzler soil.

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Clay</th>
<th>Silt</th>
<th>Sand</th>
<th>(\alpha_v)</th>
<th>(n_v)</th>
<th>(\theta_s)</th>
<th>(\theta_r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uetliberg</td>
<td>11</td>
<td>50</td>
<td>39</td>
<td>0.52</td>
<td>1.5</td>
<td>0.58</td>
<td>0.15</td>
</tr>
<tr>
<td>Winzler</td>
<td>9</td>
<td>9</td>
<td>82</td>
<td>6.67</td>
<td>1.64</td>
<td>0.48</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Before each penetration experiment, the soil was treated with a mortar to destroy aggregates and then packed into a cylinder of 25-mm inner radius and 50-mm height with a centered penetration hole of 2.5-mm radius (Fig. 3.2 c). The soil cylinder was locked into a 50-mm-long cubic block for better positioning of the cone tip. A plastic cover was placed over the sample to reduce evaporation, and a 1-kg block was placed on the sample to prevent seepage of soil material during penetration (the resulting confining stress was 5 kPa). Cone penetration tests were conducted on each sample using the four different cones (combinations of two radii and two apex semi-angles). After each experiment, the soil was removed from the cylinder, mortared, and repacked into the cylinder for a follow-up experiment (each sample was tested twice for each cone). In summary, each soil type and water content sample was subjected to eight penetration tests (two per cone) for cones of semi-angles 15° and 30° and base radii of 1.0 and 2.5 mm.

Each soil sample was subsequently mortared and repacked to conduct an unconfined compression test (Wille Geotechnik PR 10). The unconfined compression tests provided information on the soil modulus of elasticity and yield strength. Several load and unload cycles were performed to determine soil elastic properties (Berli et al., 2006)[28]. During these loading-unloading cycles, Young’s modulus \(E\) was calculated as

\[
E = \frac{\Delta \sigma}{\Delta \varepsilon}
\]

where \(\Delta \sigma\) [kPa] is the change in the vertical stress within the elastic unloading branch and \(\Delta \varepsilon\) [m m\(^{-1}\)] is the change in the vertical strain within the elastic regime (predetermined as the maximum stress in the previous load step). Load-unload cycles were conducted until the stress-strain curve flattened at some stress value \((\sigma_y)\), which was then used to estimate the soil shear strength.
Figure 3.3 The continuous-shaft cone (top) vs. recessed-shaft cone (bottom). Radius of the cone base is 2.5 mm, and the semi-angle in the figure is 15°. The recessed shaft radius is 1.9 mm. Cones with a 30° semi-angle were also tested.

\( s_u \) based on the von Mises law (Durban and Fleck, 1992 [22]):

\[
s_u = \frac{\sigma_y}{\sqrt{3}} \tag{3.17}
\]

The sample was again mortared and repacked for a final confined uniaxial compression test. The confined compression test was used to determine the Poissons ratio of the soil (the ratio of transverse to axial strains). The confined tests also used load-unload cycles to construct a mechanical regime where the soil behaves elastically. By confining the transverse strains and determining Youngs modulus from the unconfined test, Poissons ratio was determined based on the formulation given by Eggers et al. (2006)[29], using the stress—strain relationship:

\[
\sigma_{ii} = \frac{E}{(1 + \nu)(1 - 2\nu)} (\nu(\epsilon_{jj} + \epsilon_{kk}) + (1 - \nu)\epsilon_{ii}) \tag{3.18}
\]

where \( \sigma_{ii} \) is a normal stress acting in the \( i \)th direction, \( \epsilon_{ii} \) is a normal strain acting in the \( i \)th direction, \( \epsilon_{jj} \) is a normal strain acting in the \( j \)th direction, and \( \epsilon_{kk} \) is a normal strain acting in the \( k \)th direction. The confined compression test allows only normal strains in the axial direction (\( z \)), thus the relationship becomes

\[
\sigma_z = \frac{E}{(1 + \nu)(1 - 2\nu)} ((1 - \nu)\epsilon_z) \tag{3.19}
\]
Poisson’s ratio is then obtained from [29]:

\[

\nu = \frac{1}{4} \left( \frac{\Delta \epsilon_z}{\Delta \sigma_z} E \pm \sqrt{(1 - \frac{\Delta \epsilon_z}{\Delta \sigma_z} E)(9 - \frac{\Delta \epsilon_z}{\Delta \sigma_z} E)} \right) - 1
\]

(3.20)

where \( E \) [Pa] is Young's modulus determined from the unconfined experiment, \( \Delta \epsilon_z \) (m m\(^{-1}\)) is the change in the axial strain within the elastic regime, and \( \Delta \sigma_z \) [Pa] is the change in the axial stress within the elastic regime. Because none of the soil samples tested behaved as an auxetic material, only positive values for Poisson's ratio were considered. The cone penetration model considers radial cavity expansion and requires the soil shear modulus, which was deduced from Young’s modulus and Poisson’s ratio as

\[

G = \frac{E}{2(1 + \nu)}
\]

(3.21)

The soil mechanical parameters determined from uniaxial experiments were used as input for the penetration—expansion model (Table 3.2).

<table>
<thead>
<tr>
<th>Soil</th>
<th>Water content</th>
<th>Shear modulus</th>
<th>Poisson’s ratio</th>
<th>Soil strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>kg kg(^{-1})</td>
<td>kPa</td>
<td>m m(^{-1})</td>
<td>kPa</td>
</tr>
<tr>
<td>Uetliberg</td>
<td>15</td>
<td>4615</td>
<td>0.3</td>
<td>69.3</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>3401</td>
<td>0.47</td>
<td>86.6</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>833</td>
<td>0.49</td>
<td>28.9</td>
</tr>
<tr>
<td>Winzler</td>
<td>10</td>
<td>1333</td>
<td>0.49</td>
<td>14.4</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>1000</td>
<td>0.48</td>
<td>17.3</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>666</td>
<td>0.49</td>
<td>17.3</td>
</tr>
</tbody>
</table>

The soil mechanical parameters used in this study were determined for fixed saturation values and assuming that variations in saturation would affect the soil strength. At the clay platelets scale, higher saturation lubricates and increases mobility, as well as facilitating the reorientation of clay particles (Mitchell and Soga, 2005) [30]. At the sample scale, these changes are manifested by lower yield stress and shear strength (Ghezzehei and Or, 2001) [31]. In granular soils, increasing water content reduces the soil suction-induced stresses (Lu et al., 2010) [32] and lowers soil strength. Hallett et al. (2014) [33] have shown evidence for a simple relationship between pore water pressure and yield stress:

\[

\sigma_y = \frac{2\gamma}{r_{pore}}
\]

(3.22)

where \( \sigma_y \) is the soil yield stress, \( \gamma \) is the surface tension of water (0.072 N m\(^{-1}\)), and \( r_{pore} \) is the radial pore size in the soil.
Cavity expansion Model Evaluation and Intercomparison

The model of Ruiz et al. (2015)[5] was compared with classical limit pressure models (Carter et al., 1986)[24] and with penetration—expansion models that consider the influence of friction and penetration angles (Yu, 1993)[12]. Although fracture mechanical parameters were not directly measured in this study, the LEFM model parameters were determined using the material correlations of Wang et al. (2007)[34]:

\[ K_{IC,EP} = \beta \sigma_y \]  \hspace{1cm} (3.23)

where \( K_{IC,EP} \) is the elasto-plastic fracture toughness, \( \sigma_y \) is the yield stress, and \( \beta \) is a proportionality coefficient (assumed by Wang et al. (2007)[34] to be 0.355). Due to the soils plastic deformation, a relationship between elasto-plastic fracture mechanics and LEFM parameters was also considered (Yoshida and Hallett, 2008)[35]:

\[ K_{IC,LE} = \frac{3}{2} K_{IC,EP} \]  \hspace{1cm} (3.24)

where \( K_{IC,LE} \) is the linear elastic fracture toughness. These were used to compare our model with the LEFM model proposed for sediment penetration by marine worms (Dorgan et al., 2011)[25].

Influence of Shaft Friction

A common concern when comparing cone penetrometer results with plant root or earthworm penetration are the potential effects of friction. Bengough et al. (1997)[20] estimated the coefficient of friction between metal cone penetrometers and soil on the order of \( \mu = 0.55 \). Earthworms and plant roots are known to exude biopolymers for reducing friction (among other functions) such as earthworm mucus (Gray and Lissmann, 1938)[36] and plant root mucilage (Kroener et al., 2014)[37]. For plant roots, frictional effects are likely to be further reduced due to their tip-wise growth, where roots extend by producing new tissue behind the advancing tip while the main root shaft remains anchored within the soil and thus experiences no frictional effects.

Standard cone penetrometers gradually increase the soil contact area with their cone during initial penetration and then with their shaft as soil rebounds behind the tip. The result is enhancement of the frictional effects after the cone is fully inserted in proportion to the inserted shaft length. We assume that the normal force per area on the shaft is constant, i.e., that the normal forces acting on the shaft are linearly proportional to the increasing surface area past the cone base:

\[ F_{n,s} \propto A_{s,s} \]  \hspace{1cm} (3.25)
where $F_{n,s}$ is the normal force acting on the penetrometer shaft and $A_{s,s}$ is the cylindrical surface area of the shaft after the cone base, given as:

$$A_{s,s} = 2\pi r(z - l_c)$$  \hspace{1cm} (3.26)

where $r$ is the shaft radius, $l_c$ is the cone length, and $z$ is the length of the shaft section in contact with the soil. The normal force acting on the shaft depends on soil rebound after passage of the cone. Houlsby and Withers (1988)[16] measured and quantified the radial rebounding pressures by fitting an inflatable probe to the shaft of a penetrometer, which resulted in normal forces acting on the shaft:

$$F_{n,s} = 2s_u A_{s,s}$$  \hspace{1cm} (3.27)

where $2s_u$ is the expression for the plastic unloading pressure acting on the cylindrical surface [16]. The resulting frictional forces are:

$$\|F_{f,s}\| = \mu\|F_{n,s}\|$$  \hspace{1cm} (3.28)

where $\mu$ (= 0.55 [dimensionless]) is the interfacial friction coefficient between the soil and metal, and $F_{f,s}$ is the frictional force acting in the axial direction. To isolate this frictional effect, we designed cones with recessed shafts, otherwise identical to the original cones (Fig. 3.3). The shafts were recessed to a 1.9 mm radius.

**Characterizing Axial Soil Deformation**

To better characterize the influence that the cone semi-angle has on axial soil deformation, we performed a finite-elements simulation. We conducted a detailed finite-element analysis using COMSOL 4.3b (COMSOL, 2012)[38] to visualize the affected soil domain around the different cones. We considered axial-symmetric conditions and assumed a frictionless soil-cone interface. To mitigate the geometric complexities and changes in nodal contacts, only small perturbations (0.5 mm insertion) were considered. An adaptive mesh refinement algorithm was implemented to obtain better resolution (Tolooiyan and Gavin, 2011)[39].

**Inverse Determination of Soil Mechanical Properties from Cone Penetration Measurements**

Typical soil mechanical properties are often determined at scales much larger than the millimeter scales experienced by earthworms and plant roots penetrating soil. To determine mechanical parameters at a scale compatible with soil penetration by earthworms and roots, we investigated the possibility of determining the parameters inversely by fitting model predictions to measured pen-
etrometer data. We used cone penetration measurements for the 15 and 30° apex semi-angles and 2.5 mm cone base radius. To avoid shaft frictional effects, we considered penetration forces up to full cone insertion only. For each cone geometry, we applied a nonlinear least squares procedure between the data and the model as described by Wraith and Or (1998)[40] and implemented with Microsoft Excel. To fit the model, we adjust the soil strength $s_u$ and shear modulus $G$. Initial values for $G$ and $s_u$ were estimated as 1667 and 5.8 kPa, respectively. The inverse procedure was applied as follows:

1. We first defined a sequence of measured data $\{(x_i, y_i), i = 1, N\}$, where $y_i = F_{Z, Mi}$ is the measured penetration force, and associated depth (distance) $x_i = z_i$.

2. We converted the measured axial force ($y_i = F_{Z, Mi}$) to estimate the penetration-resistant stress ($y^*_i = q_p$ using Eq. 3.15) and penetration depths to penetrometer radius ($x^*_i = r_i$ using Eq. 3.8).

3. We defined a parameter space $\{a_i, i = 1, M\} = \{G, s_u\}$, where $G$ is the shear modulus and $s_u$ is the shear strength.

4. We defined a function of the independent variable and parameters $f(x^*_i, a_i) = \sigma_r(r_i, G, s_u)$.

5. Assuming a fixed interfacial friction value of $\mu = 0.55$ and a fixed semi-apex angle $\alpha$, we converted the theoretical radial stress $f(x^*_i, a_i) = \sigma_r$ to an axial penetration-resistant stress $f^*(x^*_i, a_i) = q_p$ (using Eqs. 3.10, 3.11, 3.14 and 3.15).

6. We then define a cost function as

$$\chi^2(a) = \sum_{i=1}^{N} (y^*_i - f^*(x_i, G, s_u))^2$$ (3.29)

The process adjusts the soil strength $s_u$ and $G$ to minimize $\xi$.

For cone designs with obtuse angles, in addition to cavity expansion stresses, a certain compression ahead of the cone may add a component not accounted for by the model to the insertion force. Consequently, we expect model predictions based on cavity expansion only to underestimate the actual (observed) forces required for soil penetration.

**Results**

**Comparison of Measured and Predicted Cone Penetration Forces and Stresses**

For cones with the same apex semi-angle but different base diameters, we expect resistive forces (and stresses) to coincide up to the point of full insertion of the smallest cone because the geome-
try of the uninserted part of the cone plays no role in the mechanical behavior of the inserted part. Penetration impedance (resistive force) is commonly reported in terms of either stress or force, with the conversion from force to stress based on dividing force by the cross-sectional area of the cone base (Whalley et al., 2005)[17]. For a given cone geometry, insertion forces were initially independent of the cone base radius until a certain (geometrically determined) insertion depth. This suggests that division of the forces by the cone base cross-sectional area would result in stresses that do not coincide for cones with the same apex semi-angles but different diameters. However, division of the measured penetration forces by the cross-sectional area of the inserted part of the cone resulted in stresses showing consistent overlap up to the point of full cone tip insertion (as expected). In other words, the proper conversion of insertion forces to applied stresses must consider the immediately submerged cross-section and not the base cross-section. This result illustrates a seemingly unintuitive trade-off between penetration forces and penetration stresses.

Figure 3.4 Measured and modeled (a) insertion (axial) forces and (b) interpreted insertion stresses for 1 and 2.5 mm base radius cones with a semi-angle of 15° penetrating silt loam at a water content of 0.15 kg kg\(^{-1}\). Panel (a) shows modeled forces reaching a plateau at the depth of full cone insertion, whereas measurements demonstrate effects of friction on the cone shaft. Dashed lines in (a) illustrate model results when considering frictional effects using Eq. 3.28. Solid orange curve in (b) illustrates model results using Eq. 3.5, where the radius continuously increases with insertion depth, whereas the dotted lines illustrate division by the fixed cross-sectional base area (variable and constant area denoted with VA and CA, respectively). Each set of data points represents a mean of four separate penetration experiments for the given conditions.

Comparison of Measured and Predicted Cone Penetration Forces

With increasing penetration depth and cavity radius, the penetration axial force gradually increases while the average stress decreases (Fig. 3.4). This method for model evaluation was subsequently conducted for the two different soil types (silt loam in Fig. 3.5 and loamy sand in
Figure 3.5 Penetration force vs. penetration depth for two apex semi-angles (15° and 30°), shaft diameters (1 and 2.5 mm), and three water contents ($\theta_{\text{m}}$) for silt loam (note that the cone shaft was not recessed as seen in Fig. 3.3 top). The cone semi-apex angle is located at the top right of the column, water contents are indicated in the top left corner of each panel, and radius is listed next to the results (lines for model predictions and symbols for measurements). The dashed lines indicate the onset of modeled shaft friction after full cone submergence (3.7 and 9.3 mm for a 15° semi-angle and 1.7 and 4.3 mm for a 30° semi-angle). Each data point (symbol) represents an average of four separate penetration experiments for the given conditions (48 experiments in total). Shaded regions denote standard deviations from the mean values.
Figure 3.6 Penetration force vs. penetration depth for different semi-apex angles, shaft diameters, and water contents for loamy sand. Cone semi-apex angle is located at the top right of the column, water contents ($\theta_m$) are indicated in the top left corner of each panel, and radius is listed next to the results (lines for model predictions and symbols for measurements). The dashed lines indicate the onset of modeled shaft friction after full cone submergence (3.7 and 9.3 mm for a 15° semi-angle and 1.7 and 4.3 mm for a 30° semi-angle). Each set of data points represents a mean of four separate penetration experiments for the given conditions (48 experiments in total). Shaded regions denote standard deviations from the mean values.

Fig. 3.6) under different saturation conditions, cone angles, and cone base radii. Up to full insertion of the cone, the modeled forces are in good agreement with the measured forces. Following
the full insertion of the cone, the modeled forces reach a plateau while the measured penetration forces change slope and keep increasing at a nearly constant rate (Fig. 3.5 and Fig. 3.6), presumably due to shaft friction. Model estimates that consider shaft friction (Eq. 3.28) were in good agreement with observations for the sharp (15°) cones, while the model underestimated forces for the blunt cones (30°).

Most measurements showed remarkable agreement with model predictions (also after full cone insertion when shaft friction was considered) in both soil types. Penetration measurements in the loamy sand expressed larger standard deviations than those in the silt loam (Fig. 3.6). Clear deviations between modeled and measured penetration forces were apparent only for the acute cones (15°) in the loamy sand at intermediate wetness (Fig. 3.6c) and for the blunt cones (30°) in the silt loam at the wettest condition (Fig. 3.5f), as well as in the loamy sand at the driest and wettest conditions (Fig. 3.6b and f, respectively). In these cases, penetration forces were underestimated by the model for both penetrometer diameters.

The influence of shaft friction after full cone insertion was captured in most of the acute (15°) cone experiments but not for the blunt cone (30°) design. Despite consideration of shaft friction, the model underestimated the increasing forces after full cone insertion. This indicates the influence of additional factors on penetration forces for blunt cones that were not considered in our model, which are discussed below.

Penetration-Cavity-Expansion Model Intercomparison

Cone penetration force predictions by the model of Ruiz et al. (2015) [5] represented by Eq. 3.14 were compared with estimates by alternative models for similar cone geometry and compatible soil mechanical parameters (Fig. 3.7). This model inter-comparison considered both wet (0.15 kg kg⁻¹) and dry (0.30 kg kg⁻¹) silt loam soil. The results suggest that the simple limit pressure theory predictions of Carter et al. (1986)[24] significantly underestimated cone penetration forces for both wetness conditions. The cone penetration model of Yu (1993) [12], which uses semi-empirical factors to account for penetration angle and interfacial friction, also underestimated measured insertion forces but came much closer to observations than the Carter et al. (1986)[24] model.

In contrast, predictions by the LEFM model (Dorgan et al., 2011; Ruiz et al., 2015)[25][5] overestimated the measured penetration forces very strongly in the wet soil and less pronounced in the dry soil (Fig. 3.7). The LEFM also predicted a linear increase in penetration force with penetration depth, while the data and all other models expressed an increasing slope with penetration depth. The models of Yu (1993) and Carter et al. (1986) [24] scaled in a similar way with changes in soil wetness as our experimental data. The model by Ruiz et al. (2015) [5] was the only one that accurately reproduced the observed penetration forces up to full cone insertion,
Comparison between Full and Recessed Penetrometer Shafts

Penetration-Cavity-Expansion Model Intercomparison

Experiments using the Uetliberg (silt loam) soil were conducted to determine the effects of the recessed cone shaft (Fig. 3.8) on the measured frictional interactions after full cone insertion. For the acute cone (15° semi-angle) with recessed shaft, the measurement results in Fig. 3.8a imply that the insertion force becomes nearly constant after full insertion of the cone tip. This result stands in stark contrast to measurements with a standard (continuous) shaft, which often exhibit a gradual increase in penetration force due to soil–shaft frictional interactions. Results for the 30° cone semi-angle depicted in Fig. 3.8b did not show a clear difference between the recessed-shaft cones and the standard cones. In both cases, penetration forces continued to rise well after the cone tip was fully inserted into the soil, possibly suggesting that radial strains due to soil rebound were larger than the extent of the recession (0.6 mm).
Figure 3.8 Penetration force vs. penetration depth for different semi-apex angles, using full-shaft (blue empty markers) and recessed-shaft (red filled markers) penetrometers in a saturated silt loam. Measurements were conducted for cone semi-angles of (a) 15° and (b) 30° for a final cone radius of 2.5 mm. Soil water content was 0.35 kg kg\(^{-1}\). Each set of data points represents the mean of three separate penetration experiments for the given conditions (12 experiments in total). Shaded regions denote standard deviations from the mean values.

**Characterizing Axial Soil Deformation at the Cone Tip**

Relative to observations, the analytic model performed best for the cones with an acute angle (15°), but predictions deteriorated for blunt cone angles (30°) (Fig. 3.8). To better understand the differences in stress distribution in the soil related to different cone apex semi-angles, we performed COMSOL finite-element analyses for two different cone shapes in an elastic-plastic soil (Fig. 3.9). The stress distribution ahead of the cone with an obtuse angle (30°) shown in Fig. 3.9a is nearly spherical, with the yield zone extending nearly 2 mm ahead of the cone tip. We interpreted the pattern as indicative of large axial compressive stresses relative to those exerted by the acute (15°) cone (Fig. 3.9b). For the 15° cone, the yield zone remained close to the cone face, with negligible yielding ahead of the cone tip, suggesting that most of the deformation lay on the outer boundary of the cone face rather than the cone forefront (in agreement with the cavity expansion model assumptions).

**Inverse Determination of Soil Mechanical Parameters from Cone Penetration**

Soil mechanical parameters inferred from inversion of the cone penetration measurements for the Uetliberg and Winzler soils (for two cone angles and a range of water contents) are summarized in Table 3.3 (eight replicates per water content). The cone-based soil mechanical parameters were
Figure 3.9 Finite-element simulation of yield zones during cone tip penetration into an elastic-plastic soil (COMSOL, 2012). Models were designed with an automatic mesh refinement algorithm to adjust for the similarity point at the cone tip. The base radius of both cones was 2.5 mm and their apex semi-angles were (a) 30° and (b) 15°. The simulated displacement is 0.5 mm for both cases. The yield zones (represented in dark red) represent affected regions where stresses in the soil exceed its yield stress, resulting in deformation and compaction by cone insertion. The mechanical parameters used for the calculations are those of the silt loam soil at 0.15 kg kg\(^{-1}\) water content presented in Table 3.1.

<table>
<thead>
<tr>
<th>Soil</th>
<th>Water content</th>
<th>Uniaxial</th>
<th>CPT((\alpha = 15^\circ))</th>
<th>CPT((\alpha = 15^\circ))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>(G)</td>
<td>(s_u)</td>
</tr>
<tr>
<td>Uetliberg</td>
<td>0.15</td>
<td>4615 ± 600</td>
<td>69.3 ± 10</td>
<td>3543</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>3401 ± 800</td>
<td>86.6 ± 9</td>
<td>3565</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>833 ± 300</td>
<td>28.9 ± 6</td>
<td>2119</td>
</tr>
<tr>
<td>Winzler</td>
<td>0.1</td>
<td>1333 ± 41</td>
<td>14.4 ± 7</td>
<td>2184</td>
</tr>
<tr>
<td></td>
<td>0.12</td>
<td>1000 ± 52</td>
<td>17.3 ± 4</td>
<td>2350</td>
</tr>
<tr>
<td></td>
<td>0.15</td>
<td>666 ± 38</td>
<td>17.3 ± 6</td>
<td>2078</td>
</tr>
</tbody>
</table>

Table 3.3 Soil mechanical parameters (soil strength \(s_u\) and shear modulus \(G\)) determined from uniaxial tests and from cone penetrometer inversion (CPT).

in a similar range to those obtained from uniaxial tests with some notable differences. The values of soil strength \((s_u)\) deduced from the acute penetrometer measurements were slightly lower than values measured in the uniaxial tests for the Uetliberg soil and higher for the Winzler soil. However, the general trends in soil parameters with water content were in reasonable agreement. The acute penetrometer-determined shear modulus values \((G)\) were similar to values determined from the uniaxial test for the Uetliberg soil and nearly double the values measured by uniaxial tests for the Winzler soil.
The soil mechanical parameters deduced from the obtuse cone (30°) were less consistent with uniaxial test values, especially for the Winzler soil. Comparison of insertion forces for an obtuse semi-angle of 30° have shown that the measured force was larger by 30% than that predicted from cavity expansion (for known soil parameters). We could apply a correction factor to the computed \( f^* \) to account for force components that do not contribute to cavity expansion (Carter et al., 1986)[24]. Such corrections would require more systematic studies to evaluate different cone angles and radii; hence, for purposes of this study, we limited the applicability of the inversion method to acute cones (15°). The inversion procedure serves as proof of concept pending additional tests to evaluate the consistency of trends and to establish correspondence between soil mechanical parameters derived from cone and uniaxial tests.

**Cone Resistance and Energy Requirements for Burrowing in Soil: Ecological Considerations**

The physical model for cone penetration forces can be used to estimate the mechanical energy required for soil penetration by a cone, and by analogy, for creation of a biopore by an earthworm or root growing into the soil. We have used measurements for the 2.5-mm radius cone (similar to the average earthworm radius [Ehlers, 1975][41]) and to compute the energy per unit length of a burrow for different water contents and soil types. In the results depicted in Fig. 10, we did not consider interfacial frictional effects and converted the energy requirement for burrowing to the equivalent SOC that would be consumed by hypothetical earthworms. The conversion factor of 0.0484 g C J\(^{-1}\) was proposed by Lavelle et al. (2007)[4] and used by Ruiz et al. (2015)[5] to estimate SOC requirements for soil penetration by earthworms. Lastly, the inferred energy calculations were limited to biophysical constraints of the maximum earthworm pressure. Keudel and Schrader (1999) reported that earthworms have a biophysical pressure threshold of 250 kPa. Considering an average radius of 2.5 mm, this would amount to mechanical energy (neglecting interfacial friction) for a 1 m length of \((250 \times 103 \text{ Pa}) \times \pi \times (2.5 \times 10^3 \text{ m})^2 \times 1 \text{ m} \approx 5 \text{ J}\), and converting this to grams of C would result in \(5 \text{ J} \times 0.0484 \text{ g C J}^{-1} \approx 0.24 \text{ g C}\) for a 1-m-length burrow (Fig.3.10).

A range of consumed SOC estimates for the mechanical activity of earthworm communities were derived for a range of observed ecological parameters (Table 3.4). Considering an earthworm population density in an agricultural field of 100 individuals m\(^{-2}\) (observed in the top 20 cm [Chan, 2001][43] and assuming that an individual earthworm is capable of burrowing 15 m yr\(^{-1}\) on average (0.125 m d\(^{-1}\) [Capowiez and Belzunces, 2001; Ligthart and Peek, 1997][44][45] during five to eight active months per year [Edwards and Bohlen, 1996][46]) and factoring, we estimate that an earthworm community burrowing through a soil with a saturation degree of 60% (where an individual requires \(0.2 \times 10^{-3} \text{ kg C m}^{-1} \text{ burrow}\)) would consume 0.3 kg C m\(^{-2}\) per year just to cover its mechanical energy expenditure \((0.2 \times 10^{-3} \text{ kg C m}^{-1} \times 15 \text{ m yr}^{-1} \times 100\).
Figure 3.10  The energy costs of soil penetration per unit length for a cone-shaped object of 2.5-mm radius and 15° apex semi-angle as a function of water saturation in two soil types. The right axis displays the amount of soil organic C that would supply the energy given on the left axis, using a constant conversion factor of 0.0484 g C J⁻¹ (Lavelle et al., 2007)[4]. The red, dash-dotted line refers to silt loam soil, while the blue solid line refers to the loamy sand soil. Symbols represent values derived from integrating the force vs. displacement curves in Fig. 5 and 6. The orange dashed line is an upper bound based on an earthworm maximal mechanical pressure of 250 kPa (Keudel and Schrader, 1999; Ruiz et al., 2015)[42][5].

individuals m⁻² 0.3 kg C m⁻² yr⁻¹). The typically larger earthworm population densities observed in temperate climates (Bastardie et al., 2005; Lavelle, 1988)[47][48] and considering drier soils (at field capacity) would raise this estimate by a factor of 10.

Table 3.4  Population consumption rates for a range of physical and ecological parameters. Individual values were converted from saturation degree using Fig. 3.10.

<table>
<thead>
<tr>
<th>Relative saturation</th>
<th>Penetration rate m yr⁻¹</th>
<th>Population density no. m⁻²</th>
<th>Soil organic C requirement kg m⁻² yr⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>36.5</td>
<td>60</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>100</td>
<td>0.7</td>
</tr>
<tr>
<td>0.8</td>
<td>36.5</td>
<td>60</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>100</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Discussion

We found general agreement between experimental cone—penetration forces and model predictions based on independently obtained soil mechanical parameters (in particular for sharp 15° cones). These results inspire confidence in our understanding of the mechanical (and energetic)
characteristics of bioturbation by earthworms and roots provided that their soil penetration mode is mechanically analogous (this remains an untested hypothesis). The simple and analytic cone penetration cavity expansion model performed better in the silt loam soil due to continuum-like behavior relative to the coarser loamy sand with large grains (Winzler). Soil sand particles may rearrange and jam during deformation, violating the continuum assumptions in the model and affecting the resulting force measurements with small cones. In this context, it is interesting to note that earthworms predominantly inhabit light and medium loam soils rather than sandy soils (Booth et al., 2000), which could be either due to more suitable soil mechanical properties or because finer textured soils contain more SOC (10 g clay ≈ 1 g organic C [Dexter et al., 2008 [49]). Earthworm communities are also sensitive to soil drying (Holmstrup, 2001)[50], hence the characteristics of drying in loamy soils would support a wider window of earthworm activity relative to rapidly drying sandy soils (Curry, 1998)[51].

Comparison of the model presented here with previously proposed models (Fig. 3.7 ) highlights the need to consider tip geometry for modeling cone penetration. Conventional cavity expansion models often disregard geometric effects of soil penetration, leading to underestimation of penetration stresses during cone tip insertion. For example, the model of (Carter et al., 1986) [24] considers only radially applied pressures that are required to expand a cylindrical cavity in the soil and not axial insertion forces modulated by the cone shape to radial cavity expansion forces. The cone penetration model of Yu (1993) [12] uses semi-empirical factors to account for the penetration angle and interfacial friction but assumes that penetration stresses are invariant with respect to the cone radius. Instead of considering the gradual decay of stresses during cone insertion (as seen in Fig. 3.4 b), the penetration stresses computed using the Yu (1993)[12] model remain constant, hence the resulting forces do not vary during cone tip penetration. Both alternative models strongly underestimated the penetration forces obtained in our experiments (by factors of four and two, respectively). For earthworm activity in soil, such underestimation would affect estimates of the energy requirement and related C consumption (or the mechanical energy requirement for root growth).

The LEFM model proposed for modeling sediment penetration by marine worms (Dorgan et al., 2007) [52] generally overestimated penetration forces, which was much more pronounced in wet than in dry soil (Fig. 3.7b). This bias could be attributed to the way mechanical parameters scale to the force-displacement behavior. Because fracture toughness is linearly related to yield stress, this would require squaring the yield stress values and modifying the way insertion forces are scaled relative to the cavity expansion model. The LEFM model predicts a linear increase in insertion force with depth (with cavity width in Dorgan et al., 2011, Eq. [2][25]. However, experiments clearly show that this assumption is incorrect for cavity expansion in soil, as the force-displacement behavior follows a nonlinear (nearly quadric) shape until full cone insertion, when the insertion force becomes constant (in the absence of soilshaft friction).

The experiments presented in this study offer new insights into the role of interfacial friction
and forefront compaction during cone penetration and their dependence on cone geometry. Sharp cones (15° semi-angle) with recessed shafts greatly reduced frictional shaft effects, suggesting that they could be used to assess mechanical soil properties in undisturbed soil. The results for the recessed-shaft, sharp cones were captured using a simple representation of cavity expansion via cone penetration models (Houlsby and Withers, 1988) [16]. In contrast, penetration forces continued to increase past full cone insertion for the recessed-shaft obtuse cones (30° semi-angle). The difference is attributed to different patterns in the forefront soil compaction during cone penetration and subsequent soil rebound (Durban and Fleck, 1992; Yu, 1993)[12] [22]. Sharp cones efficiently displace the soil radially (closer to an ideal cavity expansion analog), and by recessing the shaft, the effect of rebounding soil contact is reduced. For the obtuse cones, soil deformation is pronounced at the forefront of the cone, where the pushed soil accumulates at the forefront up to a critical point when soil particles move around the cone (Fig. 3.9a) and result in increased rebound effects. This was also evident in X-ray tomography images presented by Greacen and Sands (1980)[53]. The simplified analytical model (Ruiz et al., 2015)[5] considers only normal forces acting on the cone face, with simple plastic deformation in the far field. Details of cone angle influences on the soil deformation and non-ideal elastic-plastic rebound are not considered.

Based on these observations and the burrowing characteristics of earthworms, we assume that recessed cone penetration captures key aspects of earthworm soil penetration. Although earthworms move through the soil with their entire body (full shaft), the kinematics during peristalsis suggest that the earthworm body probably contributes very little to the frictional forces acting on an earthworm. The various groups of muscles in the earthworm hydro-skeleton enable them to locally vary their radius and propel themselves forward with a sharp tip angle, while using radial expansion behind the tip for anchoring. The parts of the worm body that move axially at any time typically contract relative to the immobile anchoring parts. Additionally, earthworms excrete a mucus coating that may reduce friction (Gray and Lissmann, 1938)[36]. The extent of force reduction is not yet characterized, and additional studies are needed to quantify such effects.

For fine-textured soil, using cone penetration measurements (especially recessed cones with acute semi-angles of 15°) offers an attractive method for inferring the soil mechanical properties of undisturbed soils in situ, which are not possible using conventional compression tests. We observed that the cone-inferred soil strength (Table 3.3) agreed well with uniaxial test results for the silt loam soil. Noticeably larger values of the shear modulus ($G$) were obtained for high water contents (0.3 kg kg$^{-1}$). It is possible that the cone penetrometer measures $G$ more accurately than the uniaxial test due to the nature of radial deformation. The inverse determination of soil mechanical parameters was less consistent when determining them with cones possessing obtuse semi-angles (30°). They often failed to correlate with the uniaxial test. As discussed above, the large axial compaction could play a role in misrepresenting soil mechanical parameters (based on the simple model).
Energy calculations related to cone penetration provide estimates of energy requirements for soil bioturbation by earthworms (provided that the model captures the salient features of the burrowing mechanics of earthworms). According to our calculations, an earthworm would require at least 0.2 g of SOC to construct a burrow of 1-m length in silt loam at 0.25 kg kg$^{-1}$ water content (Fig. 10). This value is an order of magnitude higher than our previous estimate (Ruiz et al., 2015), which was based on literature values of soil mechanical properties obtained from shear tests (Cokca et al., 2004; Ghezzehei and Or, 2001) [54][31] rather than compression tests. Soil strengths measured in the present study were substantially higher, leading to higher energy consumption for soil penetration and suggesting that earthworms would operate near their (hydro-skeletal) pressure limit (250 kPa) even at the relatively high water contents considered in this study. Considering up to 8 d for the construction of a 1-m-long burrow (Capowiez and Belzunces, 2001)[44] by an earthworm of 1 g fresh weight and consumption of 0.2 g SOC in the process would result in a respiration rate of 25 mg C d$^{-1}$, which falls in the same range as observed respiration rates (between 2.4 and 72 mg C d$^{-1}$ g$^{-1}$ earthworm fresh weight [Edwards and Bohlen, 1996; Speratti and Whalen, 2008]) [46][55]. These values greatly exceed earthworm respiration rates at rest, which range between 0.37 and 0.48 mg C d$^{-1}$ g$^{-1}$ fresh weight (Burges and Raw, 1967; Phillipson and Bolton, 1976)[56][57]. This implies that respiration related to mechanical work could account for $\geq 98\%$ of total earthworm respiration in some cases.

Finally, considering a population density of 100 individuals m$^{-2}$ (Chan, 2001) [43] and assuming that an individual burrows 15 m yr$^{-1}$ on average (0.125 m d$^{-1}$ [Capowiez and Belzunces, 2001] [44] during five to eight active months per year [Edwards and Bohlen, 1996] [46]), we estimate that the earthworm population would consume roughly 0.3 kg C m$^{-2}$ yr$^{-1}$ to cover their energy costs of burrowing in a soil at 60% saturation. Considering the top 0.3 m of a soil with 1.5% SOC and 1500 kg m$^{-3}$ bulk density (Batjes, 1996) [58], the range of estimates of the earthworm SOC consumption rate suggest that earthworms may deplete up to 4.5% of the total SOC per year just to cover their mechanical energy requirements. Such consumption rates would deplete the soil C pool within 22 yr if not replenished (e.g., by decaying plant tissue or manure application). Note that these rates exceed by an order of magnitude the annual C depletion rates of 0.05 kg C m$^{-2}$ reported for invasive earthworm populations under field conditions by Alban and Berry (1994) [6]. Recently, Crumsey et al. (2013) [59] reported C consumption rates of about 0.15 kg C m$^{-2}$ over 8 mo for a similar population density as in our calculation (100 individuals m$^{-2}$), which is closer to our updated estimates for mechanical energy requirements.

Soil hydration conditions and mechanical properties strongly affect estimates of energy and C requirements for soil burrowing. Additionally, ecological constraints, earthworm population size, and physiological behavior play important roles in deriving such estimates of C consumption. In the long term, annual average C consumption by earthworms cannot exceed the rate of C input to the soil, which is some fraction of the net primary productivity (NPP). With NPP in soils supporting earthworm activity in the range of 1 to 4 kg C m$^{-2}$ yr$^{-1}$, our estimates of mechanical energy requirements would suggest consumption of a significant proportion of SOC. However,
note that our estimates are based on constant and homogeneous conditions, whereas earthworms have the possibility to modulate their activity in space and time, with the potential to substantially reduce their mechanical energy needs for burrowing.

Summary and Conclusions

The main findings of this study are summarized in the following:

1. Experimental evaluation of an analytic model for penetration cavity expansion forces yielded close agreement between measurements and model results using cones with a size and geometry similar to earthworms and roots (millimeter scale), much better than three other, previously proposed models.

2. The simple model enables the use of cone penetration measurements for in situ estimation of soil mechanical parameters and energy costs related to root and earthworm soil bioturbation.

3. Energy requirements for typical soil earthworm communities translated to SOC consumption required to support mechanical activity (formation of new burrows) represent a non-negligible fraction of the SOC pool on the order of 3 to 10% yr$^{-1}$.

The present modeling framework probably oversimplifies the complex array of biophysical processes and constraints associated with plant root growth and earthworm ecology in soil. However, the model allows estimates of energy requirements and physical limits for the creation of different biopores in soil. This study provides a means for assessing the capacity of the SOC pool to support the levels of bioturbation necessary for soil structure generation and maintenance. However, the rapid burrowing rates of earthworms relative to the slow growth of plant roots may require a larger energy investment for the same burrow size due to rate-dependent deformation processes, which were not considered in this study. These aspects will be the focus of future studies.

Symbols Used in This Study

The symbols used below are listed and described in Table 3.5.
Table 3.5  Symbols used in this study.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>SI Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_r$</td>
<td>Radial Stress</td>
<td>Pa</td>
</tr>
<tr>
<td>$\sigma_\theta$</td>
<td>Hoop Stress</td>
<td>Pa</td>
</tr>
<tr>
<td>$r$</td>
<td>Radius</td>
<td>m</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Internal angle of friction</td>
<td>rad</td>
</tr>
<tr>
<td>$s_u$</td>
<td>shear soil strength</td>
<td>Pa</td>
</tr>
<tr>
<td>$G$</td>
<td>Shear modulus of rigidity</td>
<td>Pa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
<td>m · m$^{-1}$</td>
</tr>
<tr>
<td>$P_L$</td>
<td>Cylindrical limit pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$r_b$</td>
<td>Initial plastic radius</td>
<td>m</td>
</tr>
<tr>
<td>$A_c$</td>
<td>Effective cylinder area</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$z$</td>
<td>Penetration depth</td>
<td>m</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Semi-apex cone angle</td>
<td>rad</td>
</tr>
<tr>
<td>$\mu$</td>
<td>cone-soil interfacial friction coefficient</td>
<td>—</td>
</tr>
<tr>
<td>$F_r$</td>
<td>Radial force</td>
<td>N</td>
</tr>
<tr>
<td>$F_z$</td>
<td>Axial force</td>
<td>N</td>
</tr>
<tr>
<td>$F_{n}$</td>
<td>Operational cone face normal force</td>
<td>N</td>
</tr>
<tr>
<td>$F_f$</td>
<td>Frictional force</td>
<td>N</td>
</tr>
<tr>
<td>$F_{ZF,M}$</td>
<td>Measured axial force</td>
<td>N</td>
</tr>
<tr>
<td>$q_p$</td>
<td>Cone penetration resistance stress</td>
<td>Pa</td>
</tr>
<tr>
<td>$F_{n,s}$</td>
<td>normal force to the shaft</td>
<td>N</td>
</tr>
<tr>
<td>$A_{s,s}$</td>
<td>Surface area of the shaft</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$l_c$</td>
<td>Cone length</td>
<td>m</td>
</tr>
<tr>
<td>$F_{f,s}$</td>
<td>Shaft frictional force</td>
<td>N</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Proportionality coefficient</td>
<td>m$^{1/2}$</td>
</tr>
<tr>
<td>$K_{IC,EP}$</td>
<td>Elasto-plastic fracture toughness</td>
<td>Pa · m$^{1/2}$</td>
</tr>
<tr>
<td>$K_{IC,LE}$</td>
<td>Linear elastic fracture toughness</td>
<td>Pa · m$^{1/2}$</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s modulus of elasticity</td>
<td>Pa</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>von Mises yield stress</td>
<td>Pa</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>Axial stress under compression</td>
<td>Pa</td>
</tr>
<tr>
<td>$\epsilon_z$</td>
<td>Axial strain under compression</td>
<td>m · m$^{-1}$</td>
</tr>
</tbody>
</table>

Acknowledgments

We gratefully acknowledge funding from the Swiss National Science Foundation National Research Program on Soil as a Resource NRP68 (SNF, Grant no. 406840_143061 Biophysical processes controlling restoration of compacted soil structure Long-term soil structure observatory and mechanistic studies) and Dr. Thomas Keller (Agroscope) for the many helpful discussions.

Bibliography


Chapter 4

The Mechanics and Energetics of Soil Penetration by Earthworms and Plant Roots – *Higher Burrowing Rates Cost More*

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**Abstract**

We quantify mechanics and energetics of soil penetration by burrowing earthworms and growing plant roots considering different penetration rates and soil mechanical properties. The mechanical model considers cavity expansion by cone-like penetration into a visco-elastic soil material in which penetration rates affect the resulting forces hence the mechanical energy required. To test the predicted penetration rates effects on forces and energetics, we conducted rate-controlled cone penetration experiments across rates ranging from 1 to 200 $\mu m \ s^{-1}$ to determine the mechanical resistance forces for cone geometries similar to plant roots and earthworms. These measurements also enabled inverse estimation of soil rheological parameters that were in good agreement with literature values for similar soils and water contents. The results suggest that higher soil penetration rates typical to earthworms activity (about 200 $\mu m \ s^{-1}$) may significantly increase resistance forces and energy expenditure by up to three fold relative to slower penetration rates of plant roots (0.2 $\mu m \ s^{-1}$) for similar soil properties and geometries. Another important mechanical difference between earthworms and roots is the radial pressures that earthworms’ hydro-skeleton exerts (<230 kPa) whereas plant roots may exert radial pressures exceeding 1 MPa. These inher-
ent differences in burrowing rates and expansion pressures may significantly extend the range of conditions suitable for root growth in drier and compacted soil compared to earthworm activity. Results suggest that mechanical energy costs of soil bioturbation under agricultural intensification and drier climate could greatly increase energetic costs of these ecologically important soil structure forming bioprocesses.

**Introduction**

Earthworms and plant roots must penetrate the soil matrix in pursuit of nutrients and other resources. The displacement of soil by earthworms and plant roots requires significant amounts of mechanical energy (Ruiz et al., 2015)\[1\] provided by ingestion of particulate soil organic carbon (SOC) by earthworms (Lavelle, 1988)\[2\] or by photosynthesis by plants (Lal, 2004)\[3\]. Soil hydration and its mechanical state (e.g., compaction) play important roles in determining the energy costs for soil penetration, especially in agricultural soils where increasing weights of modern farm machinery (Materechera et al., 1992, Schäffer et al., 2007)\[4][5\] and projected intensification for food production are expected to worsen soil compaction.

Bioturbation activity during certain time windows that permit root growth and earthworm burrowing is considered beneficial for soil functioning by improving soil structure and ameliorating effects of soil compaction (Blanchart, 1992, Bottinelli et al., 2014, Capowiez et al., 2009)\[6][7][8\]. The resulting soil structure following bioturbation supports richer biodiversity amongst microbial communities (Ebrahimi and Or, 2016, Kuzyakov and Blagodatskaya, 2015)\[9][10\], and promotes preferential growth paths for plant roots (Dexter, 1986, Stewart et al., 1999)\[11][12\]. Plant root and earthworm biopores remain stable for extended periods of time, due to wall compaction (Blackwell et al., 1990, Greacen and Sands, 1980)\[13][14\], and stabilization by either plant mucilage (Czarnez et al., 2000)\[15\] or earthworm mucus (Gray and Lissmann, 1938)\[16\]. Estimates of the displaced soil volumes associated with earthworm and plant root activities rely heavily on averaged values over time and space and vary among biomes, geography and other factors shaping community behavior (Capowiez et al., 2009, Lavelle et al., 2007)\[8][17\]. Reported measurements of earthworm community burrows suggest that earthworms could displace over 100 kg m\(^{-2}\) of soil per year (Lavelle et al., 2007)\[17\]. Fine root production rates in natural ecosystems range between 0.3 to 1.0 kg carbon per m\(^2\) ground area per year, depending on the ecosystem (Yuan and Chen, 2012)\[18\]. Assuming a maximum dry biomass of 1000 kg per m\(^3\) root volume and a soil bulk density of 1500 kg m\(^{-3}\), this amounts to soil mass displacements of 0.45-1.5 kg m\(^{-2}\) consistent with other estimates in the literature (Beer et al., 2007, Nadelhoffer et al., 1985)\[19][20\].

Although individual biopores formed in soil may appear similar, earthworms and plant roots penetrate soil in a different manner. The respective soil penetration rates by earthworms and
plant roots vary by three orders of magnitude; earthworm’s average rates of $200 \, \mu m \, s^{-1}$ relative to elongating root tips at $0.2 \, \mu m \, s^{-1}$ (Dorgan, 2015, Gregory, 1987)[21][22]. Additionally, earthworms are limited by the maximal pressure exerted by their hydroskeleton, often in the range of 60 to 230 kPa (McKenzie and Dexter, 1988)[23]. In contrast, plant roots may exert pressures exceeding 1 MPa (Materechera et al., 1992)[4]. These differences become important in the context of resistance forces and energy costs of burrowing as soil dries or becomes compacted, and delineate the range of climatic conditions for earthworm functioning (relative to plant growth).

Some of the early models for root growth mechanics in soil were developed based on the pressure balance between the cell wall and the soil pressure applied on the root tissue, treating the plant root analogously to a cone penetrometer (Greacen and Oh, 1972)[24]. This model was later adopted for earthworm penetration by treating the soil burrowing process of an earthworm analogous to an elongating plant root (Dexter, 1986)[11]. Radiography images by Greacen and Sands (1980)[14] revealed that plant roots grow in a manner that compacts soil around the root circumference rather than the forefront of the root tip, suggesting dominance of the radial expansion as the new root thickens. These observations and other evidence motivated the development of models describing soil penetration by earthworms and plant roots considering a mechanical analogy with cone-penetration based on cavity expansion typically used in geotechnical engineering to describe resistance forces for cones penetrating elasto-plastic soils (Bishop et al., 1945, Carter et al., 1986, Yu and Mitchell, 1998)[25] [26] [27]. Recent studies (Ruiz et al., 2015, Ruiz et al., 2016) [1][28] have expanded this physically-based modelling approach that uses first principles and direct soil mechanical properties to replace traditional (largely) empirical or correlative models.

Cavity expansion-penetration models have been used to determine soil-hydro-mechanical thresholds that would ultimately limit burrowing by earthworms and plant roots under different soil hydration conditions (Ruiz et al., 2015)[1]. However, because earthworms and plant roots penetrate soil at vastly different rates, static mechanical models used in the past (Dexter, 1987, Dorgan et al., 2007, Ruiz et al., 2015)[29] [30] [1] may not adequately capture the true mechanical forces and energetic costs incurred by these two different biological agents. Expanding the model framework to consider penetration rates requires consideration of soil rheological parameters that vary greatly among soil types and with hydration conditions (Barré and Hallett, 2009, Ghezzehei and Or, 2001, Hallett et al., 2014, Or and Ghezzehei, 2002)[31] [32][33][34]. To quantify the consequences of differences in penetration rates and the ranges of resistive forces and pressures exerted by earthworms and plant roots, the study set the following specific objectives, to:

1. develop a physically based model to quantify the force-rate relationship associated with soil penetration by earthworms/plant roots and their dependence on soil type, hydration status, and earthworm/plant root biophysical characteristics

2. test model predictions using published data and inversely determined mechanical parame-
ters using penetrometer tests (geometries compatible with earthworms and plant roots) at different penetration rates

3. Compute mechanical energy requirements of earthworms and plant roots based on physiological parameters (rates and pressures)

4. Estimate individual soil mechanical energy demands in terms of soil organic carbon consumption by earthworms

We begin by reviewing ecological and physiological factors related to mechanical soil penetration by earthworms and plant roots. We then extend an analytical model of cavity expansion mechanics developed by Ruiz et al. (2015)[1] to consider penetration rate dependency based on soil rheological properties (varying with soil type and hydration status). Next, we estimate the required mechanical energy per unit burrow length under prescribed soil mechanical conditions and evaluate model prediction in experiments using miniature cone penetrometers to deduce rate dependency on resistance forces. We then estimate the energetic costs and physical constraints on soil penetration by plant roots and earthworms in a range of soil conditions. We outline the minimum and maximum energetic costs for earthworms to penetrate into soil at different rates, and determine an equivalent amount of soil organic carbon required for earthworms to create a burrow of a given length. Finally, we discuss the similarities and differences between earthworms and plant roots and the potential environmental implications in sustaining below-ground mechanical activity.

Materials and Methods

Relating cone penetration to earthworm burrowing and plant root growth

In establishing analogy between plant root growth and earthworm burrowing and relate these to the mechanics of cone penetration into soil, certain caveats and stipulations must be made. We follow the work of Bengough and Mullins (1990)[35] who reviewed cone penetration data in the context of root growth and pointed out several critical issues. Plant roots do not have fixed geometries, instead they grow by tip extension followed by radial expansion. The axial soil resistance felt by root tips is reduced by local sloughing of cells and mucilage production that reduce the influence of friction by a factor of 3-5 (Ruiz et al., 2016)[28]. Root tip nutation is considered beneficial for determining paths of least resistance and potentially further reducing soil resistive forces (Bengough and Mullins, 1990)[35]. Lastly, but importantly, roots grow at much slower rates than most penetration speeds of typical cone penetrometer tests and are thus expected to experience lower resistive forces as elaborated shortly.
Similar aspects apply to soil penetration by earthworms. Earthworms exude mucus (Gray and Lissmann, 1938)[16] that heavily reduces interfacial friction, and they also navigate to paths of favorable soil structure (loosely packed). Furthermore, earthworms can also displace soil through ingestion, which may offer certain benefits (Dexter, 1978)[36]. Note, however, that contrary to common belief, soil ingestion and eggestion is energetically expensive (Lavelle, 1988)[2] and is reserved for extracting soil organic carbon rather than serving as the main mode of burrowing. Finally, despite the faster motion of earthworms relative to growing plant roots, they remain much slower than typical cone penetration experiments.

The study focuses on quantifying the local penetration forces required by an earthworm or a plant root to penetrate and expand a cavity in a soil under given hydration conditions. We highlight the local mechanics of cone-like tip penetration into the soil, as this description is similar for earthworms and plant roots penetrating the soil, and offers a simple and mechanistic description of these soil structure generating processes. To better emulate the predominately radial soil displacement resulting from sloughing root cap during axial penetration (Vollsnes et al., 2010)[37], we consider a sharp cone angle (15° semi angle) that reduces effects of forward displacement and resemble the shapes of root tips and earthworms (Greacen and Sands, 1980)[14]. Finally, we control the rates of cone penetration to mimic rates applied by earthworms and plant roots.

**Earthworm and plant root biomechanical traits**

The governing equations describing the mechanics of penetration-expansion for simplified geometries are similar for earthworms and plant roots. However, the intrinsic physiological differences impose constraints reflected in different mechanical limitations and energy expenditures for forming similar burrows. Soil penetration by plant roots involves multi-level growth processes (Pritchard, 1994)[38]. At the cellular level, plant cells are encapsulated in stiff cell wall with yield strengths up to 500 kPa (Dumais and Forterre, 2012, Greacen and Oh, 1972, Pritchard, 1994)[39][24][38]. When the internal (cell level) root pressure exceeds this value, the root can extend and penetrate soil (Greacen and Oh, 1972, Pritchard, 1994)[24][38]. This is followed by a periodic reorientation of cellulose fibrils in the cell wall during an enzymatic loosening process (Pritchard, 1994)[38]. This sequential reorienting and rebuilding process allows roots to exert pressures as high as 1000 kPa (Bengough and Mullins, 1990)[35], capable of fracturing rigid chalk (Gabet and Mudd, 2010, Misra et al., 1986)[40][41]. For wetter soils, plant root growth rates are not limited by soil mechanical properties but rather ‘poroelastic’ limitations in the plant roots, which are interpreted to be the fastest stable actuation processes driven by water movement within the plant (Dumais and Forterre, 2012)[39]. Root growth extention rates typically range between 0.1-0.2 μm s⁻¹ (Doerner et al., 1996, Greacen and Oh, 1972)[42][24].

The biomechanics of soil penetration by earthworms relies on peristaltic synchronization of contracting and stretching muscle fibers (Quillin, 2000), that constitute the earthworms’ hy-
drostatic skeleton (Dorgan, 2010)[43]. Cyclic penetration-expansion steps are driven by peristalsis of the earthworms’ pressurized colonic fluid (Dorgan et al., 2007, Murphy and Dorgan, 2011)[30][44]. Radial muscular contraction and relaxation produces localized radial expansion of the hydrostatic skeleton. This expansion is used to further enlarge cavities as well as anchor the worm during axial penetration push (as could be viewed in numerous videos in suplemental materials). Local elongation of earthworm hydrostatic skeleton then acts to axially penetrate the soil. Measurements suggest that maximum pressures exerted by earthworms are in the range of 60-230 kPa (Keudel and Schrader, 1999, McKenzie and Dexter, 1988)[45][23]. Furthermore, reliance on muscle fibers facilitate motion allowing for faster actuation that allow for penetration rates in the order of 100-500 µm s$^{-1}$ (Dorgan, 2015, Dorgan et al., 2007)[21][30].

In this study, we monitored plant roots growing and earthworms burrowing in soil in order to visualize in-situ modes of movement and penetration rates for the purpose of validating our modeling approach. Maize roots were visualized in a rectangular soil columns (20 × 8 × 1 cm$^3$) with a Dino-Lite digital camera (AM4113T-GFBW Dino-Lite Premier; AnMo Electronics Corp, Taiwan). Similarly, earthworms (Lumbricus rubellus provided by Rubellis GmbH, Stein am Rhein) were visualized in rectangular soil columns (20 × 8 × 1 cm$^3$) at the Paul Scherrer Institute (PSI, Villigen, Switzerland) by means of neutron radiography (NEUTRA beamline). The objective here to establish feasibility of using cavity expansion-penetration models to quantify penetration activity.

**Elastic-viscoplastic expansion-penetration model**

Theoretical considerations suggest that faster penetration rates by earthworms would alter soil resistive forces (relative to slow penetration by roots) at an amount that depend on the rate and on soil rheological properties (Ghezzehei and Or, 2000)[46]. To consider the role of soil rheology, we modify the static expression for cylindrical cavity expansion that describes the radial decay in stress around a cylindrical cavity (originally considered within an elasto-plastic soil):

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0$$

(4.1)

where $r$ [m] is the distance from the center of the cavity, $\sigma_r$ [Pa] is the radial stress and $\sigma_\theta$ [Pa] is the hoop (circumferential) stress. When the absolute value of the difference between the principle stress components ($\sigma_r$ and $\sigma_\theta$) exceeds the soils undrained strength (Carter et al., 1986)[26], stable soil aggregates will begin to break down and coalesce (Ghezzehei and Or, 2000)[46], soil clay platelets will start to relax and realign (Mitchell et al., 2005)[47], and water retained in the soil matrix will mobilize (Randolph and Wroth, 1979)[48] that may further lubricate rearranging soil particles as well as dissipate energy. These synchronous multiscale deformations manifests in a rate dependent viscous flow of bulk soil material (Ghezzehei and Or, 2000)[46], which should be distinguished from the viscous flow of water during consolidation of saturated soils. The
deformation behavior for rate dependent cavity expansion is expressed by the Von-Mises criterion (Mises, 1913)[49], which extends the dimensionality of viscous flow of a Bingham (soil) material (Denoual and Diani, 2002, Ghezzehei and Or, 2001)[50][32]. Adopting the assumptions used in Bishop et al. (1945)[25] to derive their stress-strain relationship during permanent deformation, we adapt the expression to consider a rate dependent yield criteria (Denoual and Diani, 2002)[50] equating the difference in the principal stresses to the sum of the undrained soil strength (Carter et al., 1986)[26] and the viscoplastic strain rate:

$$\sigma_r - \sigma_\theta = 2s_u + \frac{4}{3}\eta_p \dot{\epsilon}_r$$  \hspace{1cm} (4.2)

where $\eta_p$ [Pa s] is the soil viscosity, $s_u$ [Pa] is the undrained soil strength, and $\dot{\epsilon}_r$ is the radial strain rate. The $\frac{4}{3}$ factor emerges from the cylindrical geometry (Bishop et al., 1945)[25]. Considering viscous rate dependent deformation augments the limit pressure solution. We define the true radial strain as in Bishop et al. (1945)[25]:

$$\epsilon_r = \ln\left(\frac{r}{r_0}\right)$$  \hspace{1cm} (4.3)

where $r$ [m] is the radius of interest and $r_0$ [m] is the initial radius. If we differentiate this with respect to time, we obtain an expression for radial strain rate (Scherer, 1986)[51]:

$$\dot{\epsilon}_r = \frac{\partial \epsilon}{\partial r} \frac{dr}{dt} = \frac{\dot{r}}{r}$$  \hspace{1cm} (4.4)

we substitute D.2 into D.1 to obtain the following expression:

$$\frac{\partial \sigma_r}{\partial r} = -\frac{2s_u}{r} - \frac{4}{3}\eta_p \frac{\dot{r}}{r^2}$$  \hspace{1cm} (4.5)

and by integration, we determine the radial stresses as a function of the radius and time:

$$\sigma_r(r, t) = P_L - 2s_u \ln\left(\frac{r}{r_c}\right) - \frac{4}{3}\eta_p \int \frac{\dot{r}}{r^2} dr$$  \hspace{1cm} (4.6)

where $r_c$ [m] is the minimal cavity size where the time independent cavity pressures converge to $P_L$ [Pa]. In order to solve for the integral over the strain rate, we evoke the chain rule in order to simplify the expression:

$$\frac{\dot{r}}{r^2} = \frac{d(-\frac{1}{r})}{dt}$$  \hspace{1cm} (4.7)

we can re-arrange linear operators (Lokenath and Mikusinski, 2005)[52] and solve for the integral:

$$\int \frac{\dot{r}}{r^2} dr = -d\left(\int \frac{1}{r} dr\right)/dt = -\frac{\dot{r}}{r}$$  \hspace{1cm} (4.8)
and so, substituting 4.8 into D.6, we obtain:

$$\sigma_r(r, t) = P_L - 2s_u \ln\left(\frac{r}{r_c}\right) + \frac{4}{3} \eta \frac{\dot{r}}{r}$$ (4.9)

To solve for the limit pressure, we assume that the change in the cavity zone is the same as the change in the plastic region of the elasto-plastic solution of (Ruiz et al., 2015)[1]:

$$\left(\frac{R}{r_c}\right)^2 \rightarrow \frac{G}{s_u}$$ (4.10)

where $G$ is the shear modulus of rigidity. Assuming that at the elasto-plastic boundary ($r=R$), the visco-elastic soil material flow is zero, thus we obtain the following expression:

$$\sigma_r(R) = P_L - 2s_u \ln\left(\frac{R}{r_c}\right) = s_u$$ (4.11)

that define the relationship for the limit pressure (Bishop et al., 1945)[25] (Carter et al., 1986)[26]:

$$P_L = s_u(1 + 2 \ln\left(\frac{R}{r_c}\right)) = s_u(1 + \ln\left(\frac{G}{s_u}\right))$$ (4.12)

Thus, the radial stress (eq. 4.11) can be expressed as

$$\sigma_r(r, t) = s_u(1 + \ln\left(\frac{G}{s_u}\right)) - 2s_u \ln\left(\frac{r}{r_c}\right) + \frac{4}{3} \eta \frac{\dot{r}}{r}$$ (4.13)
and the stresses at the cavity could be simplified to:

$$\sigma_r(r_c, t) = s_u(1+ln\left(\frac{G}{s_u}\right)) + \frac{4}{3} \eta_p \frac{\dot{r}_c}{r_c}$$  \hspace{1cm} (4.14)$$

where $r_c$ is assumed to be the cavity radius (Figure 4.1).

In order to express radial stresses in the correct form onto the conical surface, we represented
the cone as the sum of gradually increasing cylinders with the summation of their incremental
surface areas is described as (Ruiz et al., 2016)[28]:

$$A_c = 2\pi \int_0^{r_c} r dz = \pi r_c^2 \cot(\alpha)$$ \hspace{1cm} (4.15)$$

where $dz$ [m] is the incremental length and $r$ [m] radius at a given increment, $\alpha$ [rad] is the cone
apex semi-angle, and $r_c$ [m] is the final cavity radius. The relationship between radial forces and
radial stresses is:

$$F_r = \pi r_c^2 \cot(\alpha) \sigma_r$$ \hspace{1cm} (4.16)$$

where $\alpha$ [rad] is the semi apex cone insertion angle and $r_c$ [m] is the radius of the submerged
conical tip of either the earthworm or plant root. The axial force acting on the submerging cone
would be related to the radial force based on the geometry of the semi apex angle.

$$F_z = F_r \tan(\alpha)$$ \hspace{1cm} (4.17)$$

The previous forces denote frictionless cone penetration. To include the effects of interfacing
friction acting between the penetrating surface and soil during insertion, we use the formulation
from Ruiz et al. (2016)[28] to describe the full penetration forces measured.

$$F_{Z,m} = F_z (1 + \mu \cot(\alpha))$$ \hspace{1cm} (4.18)$$

where $\mu$ [-] is the interfacial friction coefficient. For steel cone penetrometers, Bengough et al.
(1997)[53] provide a factor ranging from 0.5-0.6.

**The force-rate relationship based on cone penetration experiments**

The penetration rate dependency in the visco-elastic soil was determined in rate-controlled labo-
atory cone penetration tests. The tests were conducted using custom stainless steel penetrometers
resembling typical geometries of earthworms (2.5 mm radius and 15° semi-apex angle) as seen
in Figure 4.2 (a-b). The cone was mounted to an Omega LC703-10 load cell (Figure 4.2 (c))
and driven into soil samples using a PERO spindle motor (Michlmayr et al., 2013, Ruiz et al.,
2016)[54][28]. Data were collected with a CR 1000 data logger (Campbell scientific Inc., Logan
Utah) at 1-10 Hz recording frequency. Penetration tests were conducted at penetration rates of 5,
Figure 4.2  Emulation of soil penetration of an earthworm by cone penetrometer analogue.  
(a) Geometry of earthworm during penetration in agar (radius 2 mm, 13° semi-angle); (b) Miniaturized cone penetrometer analogue with recessed cone (2.5 mm max radius, 1.9 mm recessed radius and 15 degree semi-angle); (c) Experimental set up with D. detailing the position of the displacement gauge, L. is the load cell, P. is the cone penetrometer, and S. is the soil sample; (d) example of an experimental test where the penetrometer is driven through a soil sample from the top of the cylinder.

17 and 170 $\mu$m s$^{-1}$. The maximum rate corresponds to typical earthworm burrowing rates, and the lowest rate was chosen sufficiently close to root growth rates.
Table 4.1  Soil textural characteristics used in this study. The first two soils were directly used in this study. The second two soils are taken from literature used for a comparison.

<table>
<thead>
<tr>
<th>Soil</th>
<th>Texture class</th>
<th>Clay [%]</th>
<th>Silt [%]</th>
<th>Sand [%]</th>
<th>Bulk density [kg m⁻³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uetliberg</td>
<td>Silt loam</td>
<td>11</td>
<td>50</td>
<td>39</td>
<td>1200</td>
</tr>
<tr>
<td>Agroscope</td>
<td>Silty clay loam</td>
<td>30</td>
<td>49</td>
<td>20</td>
<td>1500</td>
</tr>
<tr>
<td>Milville [1]</td>
<td>Silt loam</td>
<td>16</td>
<td>55</td>
<td>29</td>
<td>1650</td>
</tr>
<tr>
<td>Fraternidad [1][2]</td>
<td>Clay</td>
<td>52</td>
<td>31</td>
<td>18</td>
<td>-</td>
</tr>
</tbody>
</table>


Tests were conducted on a silt loam soil (Uetliberg) and a silty clay loam soil (Agroscope), at three values of gravimetric water content of 0.20, 0.22, and 0.25 kg kg⁻¹; and 0.27, 0.33, and 0.37 kg kg⁻¹ for the silt loam and the silty clay loam soils, respectively (first two soils listed in Table 1). The water content range was selected to span a typical range of water contents that permit earthworm activity. Soil and water were mixed to attain the prescribed water contents and left in sealed plastic containers in an overhead shaker over night. Soils were then packed into a cylinder of 25 mm inner radius and 50 mm height at prescribed bulk densities. Sample cylinders were wrapped in plastic to reduce seeping and evaporation during experimentation. Cylinders were placed in their side, and the penetrometer was driven into the the face of the cylinder (Figure 4.2 (d)). Three penetration tests were conducted on a given sample (54 tests done in total). Mean values were taken for each penetrometer rate test at a given water content, and error bounds were determined based on the standard deviation from the mean.

Soil rheological properties vary with water content

The mechanical properties of soils and the physiological constraints on bioturbation are strong functions of soil water content, porosity, clay minerals, and other parameters. We explored a range of these soil physical parameters to delineate mechanical stresses and energy costs to plant roots and earthworms penetrating soil at different rates (and water contents). We have used the experimentally-determined penetrometer force-rate relationships to estimate soil rheological parameters (by inverse modeling) and compared the resulting values with data from literature for similar soil types.

To isolate the soil viscosity term, we first take the measured penetrometer force ($F_{Z,m}$) values and convert these to radial stresses ($\sigma_r$) required for cavity expansion to a given radius ($r = r_f$) considering Eqs. 4.18-D.8. For the final cavity radius (cone radius) at full cone insertion, difference in resistive force (or radial stresses) between experiments conducted at different rates are
Figure 4.3  (a) Strain-rate vs stress for idealized rheological materials. The radial strain rate is characterized at the point of full conical insertion where the radius and radial expansion rate are well defined ($\dot{\epsilon}_r = \dot{r}/r$). Similarly, the radial stress ($\sigma_r$) can be inversely computed from the measured axial force ($F_{Z,m}$). Flow curves for an ideal Bingham viscoplastic body was modified to consider remote elastic response ($\sigma_r(\dot{\epsilon}_r \rightarrow 0) \approx s_u(1 + \ln(G/s_u))$ where $s_u$ is the shear soil strength and $G$ is the shear modulus of rigidity) and a subsequent viscous flow proportionate to the applied stress ($\eta_p$). The mechanical analogue is characterized by (b) an idealized dashpot-slider, where the required threshold needed for the onset of viscous flow is characterized by the limit pressure ($\sigma_r(\dot{\epsilon}_r \rightarrow 0) \approx s_u(1 + \ln(G/s_u))$).

attributed to changes in radial expansion rate:

$$\Delta \dot{r} = \frac{4}{3} \eta_p \frac{\Delta \sigma_r}{r_f}$$  \hspace{1cm} (4.19)

where $\Delta \dot{r}$ [m s$^{-1}$] is the difference in radial expansion rates determined from different fixed penetration rates. The soil viscosity can then be determined as

$$\eta_p = r_f \frac{3 \Delta \sigma_r}{4 \Delta \dot{r}}$$  \hspace{1cm} (4.20)

Considering multiple tests at different penetration rates (and thus expansion rates), the radial strain rate was approximated at the final cavity size ($r = r_f$) with equation (D.4), and a linear interpolation through the strain rate vs radial stress data was plotted assuming the following rela-
tionship for a fixed final cavity:

\[
\dot{\epsilon}_r(r_f) = \frac{\sigma_r(r_f) - s_u(1 + \ln \left( \frac{G_s}{s_u} \right))}{\frac{4}{3} \eta_p}
\]  

(4.21)

where the x-intercept is used to estimate the static radial stress (illustrated in Figure 4.3). Using these parameters, we then fit the soil strength and shear modulus to the penetrometer via method of least squares (Wraith and Or, 1998)[56]. Modeled penetration forces are then compared with measurements to illustrate relevant trends between penetration rates and forces. Penetrometer-determined soil mechanical parameters are then compared with reported values in literature for similar soils to assess compatibility with measurements by other methods (Ghezzehei and Or, 2001)[32].

To explore effects of soil water content, a functional relationship between soil water content and Bingham model parameters was established. The relationship used (Ghezzehei and Or, 2001)[32] fits a two parameter exponential equation relating soil yield stress and soil viscosity to soil matric potential. We use similar functional relationship as Ghezzehei and Or 2000 [46]; however, this study utilizes the steady state relationship of the soil water retention characteristics (Van Genuchten, 1980)[57] to relate these parameters to water content rather than to matric. The equations are as follows:

\[
\eta_p = a_\eta \exp(b_\eta \theta_m)
\]

(4.22)

\[
s_u = a_y \exp(b_y \theta_m) / (\sqrt{3})
\]

(4.23)

where \(a_\eta\) and \(a_y\) are the magnitudes of the soil viscosity and the shear soil strength, and \(b_\eta\) and \(b_y\) determines the rate of change with respect to water content.

**Intercomparison between static and dynamic model parameters**

Rheological parameters determined for a fixed water content were compared to the set of mechanical parameters required to quantify the same penetration force-insertion profiles for different penetration rates using the static expansion-penetration model (Ruiz et al., 2016)[28]. For a fixed water content, we assume that the soil viscosity is zero. For zero plastic viscosity, the model simplifies to a classical elasto-plastic model (Ruiz et al., 2016)[28]. With the classical elasto-plastic model, we adjust the soil strength \(s_u\) and shear modulus \(G\) to obtain a match with the axial forces to the penetration profiles for each fixed penetration rate. The static mechanical parameters obtained for individual tests are then compared to the rheological parameters (including soil viscosity \(\eta_p\)) defined for all of the tested penetration rates given a fixed water content.
Comparison of mechanical energy requirements for earthworms and roots

The mechanical energy required to carry out soil penetration by earthworms and by plant roots was calculated from their respective penetration rates and considering their maximum pressure thresholds. To better investigate rate dependency, we represent both earthworms and plant roots with maxima radius of 1 mm. While this is a simplifying assumption, previous studies have documented the influence of geometry on penetration stresses and energetic requirements. Smaller geometries require larger magnitudes of stress to penetrate soil, while the energetic cost to penetrate soil scales proportional to the square of the radius (Ruiz et al., 2015)[1].

We model earthworms penetrating soil at rates of 500 $\mu$m s$^{-1}$ whereas plant roots are modeled to extend axially at a rate of 0.1 $\mu$m s$^{-1}$. A mechanical pressure limit for earthworms penetrating into soil was considered as 230 kPa (Keudel and Schrader, 1999, McKenzie and Dexter, 1988)[45][23]. Similarly, plant roots were allowed to penetrate soils up to a maximum pressure of 1000 kPa. Biomechanical modeling constraints are summarized in Table 4.2. Mechanical energy is computed as:

$$U = \int_0^l F_{Z,m} dz$$  \hspace{1cm} (4.24)

where $l$ [m] is the network length, $F_{Z,m}$ [N] is the penetration force, determined as the penetrometer force at full cone insertion. This is further used to generalize the energetic cost per unit tunnel length:

$$U' = \frac{U}{l}$$  \hspace{1cm} (4.25)

Earthworms require a carbon source for their life functions, which is primarily soil organic carbon (SOC) for endogeic worms (Edwards and Bohlen, 1996, Speratti and Whalen, 2008)[58][59]. To estimate SOC used by earthworms per unit burrow length, we use a conversion factor between earthworm energy utilization and SOC of 0.0484 g C J$^{-1}$ (Lavelle et al., 2007)[17].

Results and Discussion

Visualization of earthworm and plant root biomechanical traits

Root growth rates obtained from our own rhizotron images (Figure 4.4(a-c)) are in the range of 0.1-0.2 $\mu$m s$^{-1}$. Similarly, earthworm soil peristaltic motion was captured using neutron radiography (Figure 4.4(d-f)). Time-lapsed neutron radiography images revealed that earthworms are capable of producing instantaneous in situ penetration rates in the order of 100-250 $\mu$m s$^{-1}$ (reported in Table 2). This evidence reveals that earthworms are capable of traversing soils up to three orders of magnitude faster than plant roots. Kinematic motion of soil penetration by earthworms and plant roots exhibit key penetration-expansion features that are represented with our
Figure 4.4 Key biomechanical penetration-expansion process for plant roots (a-c) and earthworms (d-f) within soil visualized with a custom rhizotron and neutron radiography respectively. (a) Initial penetration of maize plant root within loose loamy sand soil, (b) after 3600 seconds of penetration, the root locally compacts the soil during penetration and begins to expand radially, (c) after 7200 seconds of penetration, the root expands both axially and radially further compacting soil particles during penetration. A similar penetration process occurs during earthworm peristaltic motion. (d) Pressurized expansion during earthworm anchoring in preparation for axial penetration, (e) after 20 seconds, earthworm propels forward segments through the clay soil, (f) after 40 seconds, the earthworm anchors again in preparation for the next forward penetration.

current modeling framework. Furthermore, observed soil penetration rates are consistent with values cited (Table 2). Radial displacement of soil during axial penetration can be seen by both earthworms and plant roots, which is in agreement with similar observations made by Greacen and Sands (1980)[14], and Vollnes et al. (2010)[37]. These results give us further confidence our modeling strategy.

Table 4.2 Observed and reported biophysical constraints for earthworms and plant roots. Values are used to constrain maximum rates and pressures that earthworms and plant roots are not allowed to exceed.

<table>
<thead>
<tr>
<th>Classification</th>
<th>Penetration rate $[\mu m \ s^{-1}]$</th>
<th>Maximum pressure $[kPa]$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Earthworms</strong></td>
<td>$100-250$ (100-500)</td>
<td>$230_2$</td>
</tr>
<tr>
<td><strong>Plant roots</strong></td>
<td>$0.1-0.2(0.1-0.2)_3$</td>
<td>$1000_1$</td>
</tr>
</tbody>
</table>

Figure 4.5 Observed cone penetration rate-force relationships with penetration force vs. depth profiles for three different penetration rates (170, 17, and 5 µm s⁻¹, illustrated in red, blue, and green respectively). The cone maximal radius of 2.5 mm, semi-angle of 15 degrees and three separate gravimetric water contents (θᵣₘ) for two different soils (Uetliberg and Agroscope). Experiments represented in (a-c) were conducted on Uetliberg soil at water contents of 0.25, 0.22, and 0.20 kg kg⁻¹. Experiments represented in (d-f) were conducted on Agroscope soil at water contents of 0.37, 0.33, and 0.27 kg kg⁻¹ respectively. Each data point (symbols) represents an average of three separate penetration experiments (54 experiments in total). Solid curves represent model results for the respective penetration rates, as labelled in the figures. Note the increased force magnitude range for (e) and (f) emphasized in red.
Figure 4.6 Radial strain rate vs radial stress curves used to inversely estimate yield stress and soil viscosity for two different soils (Uetliberg and Agroscope). Values for radial stress are determined from mean penetration forces (illustrated in Figure 5) at the point where the cone base is fully submerged into the soil and converted by factoring out interfacial friction effects. Likewise, radial strain is determined at the point of full radial expansion given the instantaneous rate of expansion (inferred from the rate controlled penetration tests and penetrometer geometry). Experiments represented in (a-c) were conducted on Uetliberg soil at water contents of 0.25, 0.22, and 0.20 kg kg\(^{-1}\) respectively. Experiments represented in (d-f) were conducted on Agroscope soil at water contents of 0.37, 0.33, and 0.27 kg kg\(^{-1}\) respectively. Soil viscosity is determined by the inverse of the slope of the linear dashed curve. Soil shear strength is determined by the x-intercept of the linear curve. Orange region highlights stresses that earthworms are expected apply on the soil during penetration expansion, while the green region highlights the stresses that a plant root would likely apply to the soil.
Experimental cone penetration rate-force relationship

The penetration rate-force relationships for cones inserted at different penetration rates and soil types are depicted in Figure 4.5. For Uetliberg soil (Figure 4.5 (a-c)), the measured penetration forces for the 170 \( \mu \text{m s}^{-1} \) data set (red) were nearly double the values measured at the slower penetration rates (blue 17\( \mu \text{m s}^{-1} \) and green 5 \( \mu \text{m s}^{-1} \)) for all water contents. It is also worth noting that penetration forces corresponding to the two slower rates (blue and green data sets) were similar, suggesting that static cone penetration models with no rate dependency (Ruiz et al. 2016)[28] provide useful approximations for the forces associated with very slow penetration rates typical for plant roots. The rate dependency was more distinct for the Uetliberg soil relative to the Agroscope soil (Figure 4.5 (d-f)), suggesting less pronounced viscous behavior of the Agroscope soil. For high water contents (0.33 and 0.37 kg kg\(^{-1}\)) the Agroscope soil exhibited noticeable rate dependence, but not for the drier conditions (0.27 kg kg\(^{-1}\) water content).

The solid curves in Figure 5 represent cavity-expansion model representation (Eqs. D.8, 4.19, and 4.20) using the extracted soil rheological parameters (as explained below) plotted along with measured values. The faster penetration rates indeed increase the penetration force, nearly doubling the value of the low rates. The force curves in the Agroscope soil continue to increase past the point of full cone insertion despite the recessed shaft behind the cone. This suggests a sufficient elastic rebound to allow re-attaching of soil to the shaft. These restitution stresses acting on the shaft were described in Houlsby and Withers (1988)[60], and their magnitude was estimated in Ruiz et al. (2016)[28].

The fastest penetration profile plots for the the Uetliberg soil in Figure 4.5 a-c reveal an overshoot and lagged decay subsequent to the full submergence of the conical base. The decaying effect allude to an visco-elastic restitution force that is accentuated during the faster experiments. It is likely that this visco-elastic effect also occurs during the faster penetration experiments in the Agroscope soil (Figure 4.5 d-f), but it is not clear due to the re-attachment of the soil to the shaft.

Determination of soil rheological parameters

The cone penetrometer experiments at different rates were used to plot strain-rate vs stress relationship to deduce the soil viscosity (inverse of the slope) and the soil strength (Figure 4.6). For Uetliberg soil (Figure 4.6(a-c)), it appears that the soil viscosity is larger than Agroscope soil’s viscosity (Figure 4.6(d-f)) for the respective tested water contents. In contrast, soil strength determined for the Agroscope soil (intersection) was consistently greater than the Uetliberg soil. This result suggests that there exists a more pronounced rate effect when penetrating through the silt loam soil than the silty clay loam soil, which is consistent with the data from Ghezzehei and Or (2001)[32]. This notion is more explicit when observing the penetration profiles for the Uetliberg
Figure 4.7  Rheological parameter comparison between Uetliberg, Agroscope, Millville, and Fraternidad soils parameterized using a penetrometer and a rheometer respectively. (a) Soil viscosity vs water content, and (b) yield stress vs water content. Note that the Fraternidad is a swelling clay capable of retaining large quantities of water.

and Agroscope soils (Figure 4.5).

### Table 4.3  Measured and literature values of soil rheological parameters

<table>
<thead>
<tr>
<th>Soil</th>
<th>$a_\eta$ [Pa s]</th>
<th>$b_\eta$ [-]</th>
<th>$a_y$ [Pa]</th>
<th>$b_y$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uetliberg</td>
<td>$6 \times 10^7$</td>
<td>-24.04</td>
<td>$3 \times 10^6$</td>
<td>-26.2</td>
</tr>
<tr>
<td>Agroscope</td>
<td>$2 \times 10^6$</td>
<td>-7.22</td>
<td>$3 \times 10^6$</td>
<td>-15.92</td>
</tr>
<tr>
<td>Milville</td>
<td>$5 \times 10^7$</td>
<td>-24.16</td>
<td>$2 \times 10^6$</td>
<td>-25.63</td>
</tr>
<tr>
<td>Fraternidad</td>
<td>$1 \times 10^6$</td>
<td>-8.62</td>
<td>$2 \times 10^6$</td>
<td>-5.97</td>
</tr>
</tbody>
</table>


Furthermore, the rheological parameters determined by the miniature penetrometer was compared to rheological parameters obtained for similar soils using a rheometer (Figure 4.7) (Ghezzehei and Or, 2001)[32]. The soil rheological parameters obtained from inversion of penetrometer test exhibited similar trends as rheometer measurements as a function of soil water content. The magnitudes of parameter values and variations were comparable for the different soils. The parameters obtained with the penetrometer are noticeably larger, but not significantly so. Fitting the exponential relationship to the data (parameters in Table 4.3) allows for a generalization for hydro-mechanical behavior for a wider span of water contents. These will be used in the following section.
Intercomparison between static and dynamic model parametrization

Intercomparison of soil mechanical parameters obtained from rate dependent vs static model parametrization is summarized in Table 4. We note that, for a given soil and water content, the rate dependent model parametrization is capable of characterizing all of the experiments (differing only in penetration rates) with a single set of rheological parameters (Figure 5). Characterization of the experiments in Figure 5 with the static model requires a set parameters for each fixed penetration rate. Penetration measurements were carried out only at three fixed penetration rates for a given water content. Results in Table 4 demonstrate that the soil strength obtained from the static model varies by a factor of two depending on the penetration rate at which we measure it. Consideration of visco-elasticity allows us to determine a fixed soil strength, and the rate dependency is proportional to the soil viscosity.

Estimation of energy requirement for soil penetration by earthworms and plant roots

Figure 4.8 Earthworm and plant root stresses (left axis) and specific mechanical energy requirement per unit length (right axis) required to penetrate (a) Uetliberg and (b) Agroscope soils at different fixed water contents. The underlying soil rheological parameters are given in Table 3. Earthworms (red) and plant roots (green) are assumed to have 1 mm radius. Earthworm penetration rate was assumed to be at 500 $\mu$m s$^{-1}$, while plant roots were assumed to penetrate soils at 0.1 $\mu$m s$^{-1}$. The blue dashed line marks the soil strength-imposed mechanical limitation beyond which earthworms are unable to penetrate the soil ($P_{\text{worm}} = 230$ kPa). Red vertical line point to minimum soil moisture contents allowing soil penetration for earthworms assuming the given penetration rate.

The soil hydro-mechanical parameters deduced from the cone penetration measurements were used to quantify the mechanical stresses required for soil penetration by earthworms and
Table 4.4  Intercomparison of static and rate dependent parameters. The upper section consists of material properties obtained based on the current rate dependent model that considers soil viscosity, thus parameters are consistent for all experimentally tested penetration rates. The bottom section presents mechanical properties obtained with previous models that do not consider the effects of rates. A unique set of mechanical properties were determined for each tested penetration rate.

### Parameters determined with current (rate dependent) model

<table>
<thead>
<tr>
<th>Soil name</th>
<th>Water content [kg kg(^{-1})]</th>
<th>Soil viscosity [kPa s]</th>
<th>Shear Modulus [kPa]</th>
<th>Shear strength [kPa]</th>
</tr>
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<tbody>
<tr>
<td>Uetliberg</td>
<td>0.20</td>
<td>498</td>
<td>22.1</td>
<td>10.8</td>
</tr>
<tr>
<td></td>
<td>0.22</td>
<td>358</td>
<td>8.6</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>156</td>
<td>3.0</td>
<td>2.7</td>
</tr>
<tr>
<td>Agroscope</td>
<td>0.27</td>
<td>233.5</td>
<td>45</td>
<td>25.7</td>
</tr>
<tr>
<td></td>
<td>0.33</td>
<td>171</td>
<td>13.6</td>
<td>6.8</td>
</tr>
<tr>
<td></td>
<td>0.37</td>
<td>156</td>
<td>5.9</td>
<td>4.6</td>
</tr>
</tbody>
</table>

### Parameters determined with previous (static) model

<table>
<thead>
<tr>
<th>Soil name</th>
<th>Water content [kg kg(^{-1})]</th>
<th>Penetration rate [µm s(^{-1})]</th>
<th>Shear Modulus [kPa]</th>
<th>Shear strength [kPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uetliberg</td>
<td>0.20</td>
<td>5</td>
<td>22.1</td>
<td>10.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>17</td>
<td>24.4</td>
<td>11.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>170</td>
<td>44.3</td>
<td>21.6</td>
</tr>
<tr>
<td></td>
<td>0.22</td>
<td>5</td>
<td>8.6</td>
<td>4.0</td>
</tr>
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<td></td>
<td></td>
<td>17</td>
<td>9.4</td>
<td>4.4</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>20.5</td>
<td>9.5</td>
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<td></td>
<td></td>
<td>170</td>
<td>7.7</td>
<td>7</td>
</tr>
<tr>
<td>Agroscope</td>
<td>0.27</td>
<td>5</td>
<td>37.1</td>
<td>21.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>17</td>
<td>41.0</td>
<td>23.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>170</td>
<td>44.4</td>
<td>25.4</td>
</tr>
<tr>
<td></td>
<td>0.33</td>
<td>5</td>
<td>13.2</td>
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<td></td>
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<td>5.7</td>
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<td>8.6</td>
<td>6.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>170</td>
<td>12.6</td>
<td>9.7</td>
</tr>
</tbody>
</table>

plant roots (Figure 4.8). Considering the different penetration rates, the model predicts that earthworms would apply larger penetration stresses than plant roots for the entire range of water contents that permit their activity. The stresses for earthworm penetration into Uetliberg soil (Figure 4.8 (a)) were about 2 to 3 times higher than stresses for slowly growing plant roots for similar conditions (and geometry). The parameters for the Agroscope soil (Figure 4.8 (b)) show a stronger dependency on penetration rate under wet conditions, but this tendency diminishes to-
wards drier conditions. This behavior could be attributed to the relatively low soil viscosity of this soil. Earthworms penetrating soil at the slow rates of plant roots (0.1 $\mu$m s$^{-1}$) are also plotted to illustrate the influence of rate dependency. The required mechanical energy is inferred from the penetration stresses for earthworms and plant roots. Estimates suggest that the mechanical energy per unit burrow length invested by earthworms is nearly three times larger than that by plant roots growing into the same soil considering the same geometry (Figure 4.8). Under dry conditions, where soil strength and its mechanical properties prohibit earthworm motion, plant roots growing into the soil would require to expend higher mechanical energy than the maximum possible for an earthworm. In other words, the ecological range of plant growth into soil in broader than that for earthworm activity.

The estimated mechanical energy requirements for burrowing by earthworms were converted into equivalent SOC mass per unit length of burrow (Figure 4.9). The energetic demand for an individual earthworm penetrating soil at 500 $\mu$m s$^{-1}$ (cf. slower rate of 0.1 $\mu$m s$^{-1}$) varies for soils with different rheological properties. For Uetliberg soil (Figure 4.9 (a)), the energetic expenditure increased by a factor of 2-3 due to rate dependency (a factor that remains nearly constant across most of the water content range). For the Agroscope soil, the soil viscosity plays a large role at high water contents (a factor of 3 higher for faster rates); however, at low water contents, the differences in the energetic costs of soil penetration between faster and and slower penetration rates are reduced (Figure 4.9 (b)).
Summary

The study highlights the effects of different soil rheological properties and the consequences of different burrowing rates by earthworms and growing plant roots on the resistive forces experienced and the energy costs of soil bioturbation. The validity of using cavity-expansion penetration models to represent root growth and earthworm burrowing through soil remains to be tested directly, however, qualitative evidence based on direct imaging of these bioturbation processes support the modeling approach. The local initialization of peristaltic earthworm motion or root growth and expansion appear the follow the path of penetration cavity-expansion at different rates. Time lapsed images were used to estimate typical soil penetration rates by the two organisms, where earthworms burrow at rates between 100-250 µm s$^{-1}$, and plant roots extend at rates of 0.1-0.2 µm s$^{-1}$. These measured values were consistent with previously reported values in several studies (Doerner et al., 1996, Dorgan, 2015, Dorgan et al., 2007, Greacen and Oh, 1972)[42] [21] [30] [24].

The selection of the earthworm species *L. rubellus*, which is a predominantly epigeic species, had practical reasons. At the time of the experiments (summer), which could not be freely scheduled due to constraints on the beamline, the endogeic species went into dormancy and did not show any burrowing activity. It is conceivable that endogeic worms that are specialized to burrow in mineral soil would be less constrained by our experimental conditions in their burrowing activities and therefore it would be good to carry out similar experiments with endogeic species.

The information gleaned from imaging guided the extension of the previously developed analytical mechanical model for soil penetration-cavity expansion to explicitly consider penetration rate dependency. Experimental results using miniature cones penetrating wet soils at different rates confirm model predictions that mechanical resistance to cone penetration increases nonlinearly with increasing penetration rate. The cone penetration measurements were used to estimate soil rheological parameters (by inverting the model), and yielded values consistent with rheometer measurements in silt-loam soils (Figure 4.7). These results inspire confidence in the modeling approach, and offers a means for in-situ measurement of soil mechanical properties that are directly compatible with the biomechanical processes of interest (Ruiz et al., 2016, Tej and Singh, 2012, Whalley et al., 2007)[28][61][62].

Estimates of the penetration stresses associated with soil bioturbation suggest that earthworms are required to exert larger stresses than plant roots by a factor of 2-3 (Figure 8) for identical hydro-mechanical soil conditions that permit their activity. These estimates are attributed to three orders of magnitude faster penetration rates by earthworms relative to growing plant roots. Standard static cone penetration mechanical models (Bishop et al., 1945, Carter et al., 1986, Ruiz et al., 2015, Yu, 1993)[25] [26] [1][63] are expected to underestimate the mechanical stresses required for penetration by earthworms. Root extension at lower rates reduces stresses exerted on the soil thereby extending the root capacity to grow under drier conditions and harder soils that
would be prohibitive for earthworm activity. This activity range is further expanded by higher stresses exerted by growing roots relative to earthworms’ maximal hydro-skeletal pressure.

Estimates of the energetic costs per unit burrow length further distinguish the differences between earthworms and plant roots (Figure 4.4). The capacity of plant roots to grow into stronger soils distinguishes their contribution to soil structure generation (biopores) for a wider range of soils and climatic conditions (relative to earthworms). For a range of soil and climatic conditions, the activity of earthworms and other soil fauna promotes formation of soil structure, a critical component of soil ecological function. The energetic cost for earthworm activity is higher relative to plant roots operating at similar conditions and geometries. However, the picture becomes more complicated if one considers the entire range of drying and wetting and full life cycle of plant roots and earthworms. In addition, one needs to consider that plant root biopores will likely become attractive preferential paths for earthworms. The mechanically worked soil will be weaker allowing earthworms to move through more easily. They will store an abundance of particulate soil organic carbon, which delivers the energy for soil bioturbations by earthworms (Brown et al., 2000, Curry and Schmidt, 2007)[64][65].

The soil water content is paramount for permitting soil bioturbation by earthworm activity (Supporting information). However, the soil hydration condition impacts many different facets pertinent to restricting the earthworm’s physical activity in soil. Detangling the various sources of restriction is non-trivial. The model developed in this study alludes to a critical soil hydromechanical limit that would severely hinder earthworms burrowing activity due to their limited hydroskeletal pressures. However, literature also suggests that dry soils could dehydrate earthworms resulting in a reduced volume and possibly reduced ability to exert pressure on the soil (Kretzschmar and Bruchou, 1991)[66]. On the other hand, Kretzschmar (1991)[67] had witnessed in a separate study that the earthworms weight was not as significant as the influence of soil compaction and soil drying. In this study, we show that drying plays a similar role to that of compaction on the soil mechanical strength, but the current modeling framework is not sufficient to address the impact that drying will have on earthworms. Excessive drying could be fatal for earthworms, but the influence that drying has on the biomechanical strength of the earthworms hydrostatic skeleton is a topic that must be investigated in future studies.

An interesting result of this study in the context of soil carbon cycle, is the amount of soil organic carbon required to support burrowing activity by earthworms. For wet Uetliberg soil near field capacity (matric potential -33 kPa corresponding to water content 0.2 kg kg\(^{-1}\)), we estimate that earthworms require 0.005 - 0.02 g of SOC per m of burrowing, depending on their burrowing rate. For the upper limit of burrowing rate (0.5 mm s\(^{-1}\)), the soil organic carbon requirement could be as high as their maximum rate of ingestion (\(~10^{-5}\) g SOC per s) (Johnston et al., 2014)[68]. As soils dry below field capacity (considering for example, Uetliberg soil at water content 0.18 kg kg\(^{-1}\)), earthworms will require 0.01-0.035 g of SOC per meter of burrow, nearly doubling the energetic requirement relative to field capacity (and the carbon requirement for the activity).
Hence, as the soil dries below field capacity, we expect slowing down of earthworms burrowing rates (still consuming up to 0.035 g of SOC per meter of burrow) until they can no longer operate. Consequently, earthworms will choose to continue burrowing under such dry conditions at lower rates, or enter a state of diapause until conditions become more favourable (mechanically and energetically). This behavior is likely species-specific (Kretzschmar, 1991)[67]. The reduction in the soil penetration rates would reduce the rates of soil structure generation, an issue that is expected to be aggravated with future agricultural intensification (Materechera et al., 1992, Schäffer et al., 2007)[4][5].

The simple modeling framework provides biophysical estimates for the mechanics and energetics of earthworms and plant roots in soil. These estimates outline physical constraints that would limit earthworm mobility, illustrating activity windows for earthworms burrowing. Model results could also allude to SOC requirements for earthworms mechanical activity. While there are currently many uncertainties regarding the data, the model could potentially infer trends in below-ground activity and associated carbon consumption under changing climatic conditions and projected intensification of agricultural production, and the associated soil compaction. With similar information regarding root length densities, future estimates could be made to determine the mechanical cost of placing carbon below ground in the form of plant roots. These estimates could potentially be used to better constrain predictions of carbon sequestration in soil.

Conclusion

1. We developed an elasto-viscoplastic expansion-penetration model that links soil rheological properties to rate dependent resistive forces (and energy consumption) by earthworms and growing plant roots.

2. Cone penetration experiments confirmed the predicted increase in penetration resistive forces with increasing penetration rate.

3. These cone penetration measurements were used to estimate soil rheological parameters, which were in good agreement with literature values for similar soil types.

4. Due to their 1000-fold faster penetration speed and (wet) soil viscous behaviour, earthworms must withstand twice the stress to penetrate soil relative to plant roots.

5. As soil dries, increasing soil strength impedes earthworm activity long before inhibiting plant root growth. Typical critical water contents for earthworms in silt loam and silty clay loam soils are near field capacity.
Acknowledgments

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Supporting Information

List of Symbols

Visualizing plant root and earthworms biomechanical traits

Maize roots were visualized in a rectangular soil columns (20 x 8 x 1 cm3) with a Dino-Lite digital camera (AM4113T-GFBW Dino-Lite Premier; AnMo Electronics Corp, Taiwan) taking photos every 10 minutes. Soil columns were fixed at a 30° incline in order to take advantage of gravitropism. This forces plant roots to grow along the face of the column. Course Winzlerboden soil [28] (Ruiz et al., 2016) packed at a bulk density of 1500-1700 kg m^-3 was used to visualize particle displacement during soil penetration by plant roots. Images in this article were taken every 10 minutes over a 25 hour period. Imaging earthworms (L. rubellus) was more difficult, as they prefer darker moist conditions. In situ observations were made at the Paul Scherrer Institute (PSI, Villigen Switzerland) using the NEUTRA beamline in the SINQ facility. NEUTRA allowed us to generate images through use of neutron radiography, allowing visualization inside of the soil as earthworms were burrowing. Rectangular soil columns (20 x 8 x 1 cm3) were filled with Agroscope soil mixed at 0.2 kg kg^-1 water content and packed at a bulk density of 1000-1200 kg m^-3. Images used in this article were sampled at 20 Hz, with an exposure time of 12 s. The field of view was 15 x 15 cm2, with a nominal pixel size of 0.104 mm.
### Table 4.5  Table of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>SI Unit</th>
</tr>
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<tr>
<td>$\alpha$</td>
<td>Semi-Apex Insertion Angle</td>
<td>$rad$</td>
</tr>
<tr>
<td>$\epsilon_r$</td>
<td>Radial Strain</td>
<td>$m \cdot m^{-1}$</td>
</tr>
<tr>
<td>$\eta_p$</td>
<td>Soil Viscosity</td>
<td>$Pa \cdot s$</td>
</tr>
<tr>
<td>$F_r$</td>
<td>Radial Force</td>
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<td>Distance of burrow length</td>
<td>$m$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Interface friction</td>
<td>–</td>
</tr>
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<td>Cavity Pressure</td>
<td>$Pa$</td>
</tr>
<tr>
<td>$P_L$</td>
<td>Limit Cavity Pressure</td>
<td>$Pa$</td>
</tr>
<tr>
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<td>Radial Depth of Observation</td>
<td>$m$</td>
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<tr>
<td>$r_f$</td>
<td>Final Expanded Cavity Radius</td>
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<td>Hoop Stress</td>
<td>$Pa$</td>
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<td>$\sigma_z$</td>
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<td>$Pa$</td>
</tr>
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<tr>
<td>$z$</td>
<td>Axial Penetration Depth</td>
<td>$m$</td>
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#### Earthworm activity limited by soil water content

A preliminary test was carried out to infer the influence of soil moisture and particulate organic matter on earthworm activity. In situ observations of earthworms (*L. rubellus*) were made at the Paul Scherrer Institute (PSI, Villigen Switzerland) using the NEUTRA beamline in the SINQ facility. Rectangular soil columns ($25 \times 8 \times 1 \text{ cm}^3$) were filled with Agroscope soil. The bottom half was mixed at 0.25 kg kg$^{-1}$ water content and the top half was mixed at 0.17 kg kg$^{-1}$ water content. The column was packed at a bulk density of 1000-1200 kg m$^{-3}$. Four small earthworms were placed at entry tubes at the bottom of the column and four at the top. Two sources concentrated sources of particulate organic matter were placed in the center of the cell. Images used in this article were sampled at 20 Hz, with an exposure time of 12 s. The field of view was 15 $15 \text{ cm}^2$, with a nominal pixel size of 0.104 mm (Video S3 [69]). Results showed that all of the earthworms enter into the bottom of the column with one guided directly towards a source of organic matter (Video S3 [69]). None entered from the top. In the upper half, the two closest to the organic matter appear to be attempting to enter (Video S3 [69]). At the very top, the two make no effort to enter (Video S3 [69]).
Bibliography


Chapter 5

Biomechanical Limits to soil penetration by Earthworms- Direct Measurements of Hydroskeleton Pressures and Peristaltic Motions

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Abstract

Earthworm’s activity and the resulting burrow networks are important for soil structure generation that supports various physical and ecological soil processes. Earthworm burrowing activity and rates are quantified using biomechanical models for earthworm penetration and subsequent cavity expansion that consider soil hydration status and mechanical properties. Key parameters in these models are the maximal axial and radial pressures exerted by an earthworm’s hydroskeleton (presently estimated at 200 kPa [1] [2]. We designed and tested a special pressure chamber for direct measurement of pressures exerted by moving earthworms under different confining pressures to delineate the limits of earthworm activity in soils at different mechanical and hydration states. The measurement chamber consists of a stiff Plexiglas prism fitted with inner flexible silicon tubing that hosts the earthworm in which the gap around the tubing is pressurized using water and the earthworm response is monitored using a pressure transducer. Earthworm peristaltic motion and concurrent pressure fluctuations were recorded by video camera and the pressure transducer.
A new model that links the earthworm’s kinematics with measured pressure fluctuations was developed. Results for anecic and endogeic earthworms in the flexible tube exhibit maximal values of radial pressures of 130 kPa and 195 kPa, respectively. Mean earthworm peristaltic frequencies were used to model burrowing rates, which were in excellent agreement with current measurements and previous results. By considering differences between the measurement chamber’s linear elastic behavior and the elasto-viscoplastic nature of soils, the study delineates mechanical constraints to soil bioturbation by earthworms for different mechanical conditions including compaction.

**Introduction**

In certain climatic regions, earthworms are considered among the most important soil and ecosystem engineers [3] due to their role in modifying soil structure and affecting its carbon resources [4]. Earthworms burrow through the soil and create preferential pathways for aeration and water flow [3], burrows may accelerate ground water recharge and by some accounts, enhance soil water retention [4] and extend the depth of where oxic conditions support soil fauna and flora [5]. The resulting tunnel networks are relatively stable and may be used multiple times by the earthworms, or serve as pathways of reduced mechanical resistance for growing plant roots [6] [7] [8]. Plant roots may benefit in other ways from earthworm-generated biopores; these become hotspots for microbial activity and promote synergy with plant rhizosphere thus potentially enhancing crop yields by as much as 25% [9].

Earthworms employ different mechanical strategies for penetrating the soil. The deformation of soil around their bodies as they wedge in, has been observed as the most common mode of burrowing [10] [11]. In certain regions, earthworm burrowing (and soil ingestion) have been reported to displace 100 kg m$^{-2}$ of soil per year [3]. The energetic costs of such subterranean activity may consume substantial amount of soil organic carbon, estimated in the range of 10 to 100g C m$^{-2}$yr$^{-1}$ [12] [13], or 1 to 10% of the annual net primary productivity of a typical agricultural soil [Beer et al., 2010]. These biomechanical energetic costs vary considerably across biomes depending on the soil hydration status [11], carbon inputs, and soil compaction level. With agricultural intensification and increase in soil compaction [14] and potentially warmer climate [15], soil bioturbation by earthworms is likely to become more energetically costly and the time windows under which soil conditions permit earthworm burrowing activity are expected to shrink. Given the high sensitivity of earthworm community ecology to details of soil hydration and carbon resources (with several earthworm species already on the brink of extinction [4]), it is important to estimate the biophysical limits for earthworm functioning under present and future climate and land use practices.

Recent biomechanical models for earthworm activity consider cavity expansion into an elasto-
viscoplastic soil with properties that may vary with soil texture, properties and hydration conditions [11]. Such models could be used to estimate critical external environmental factors that mechanically limit earthworm activity. The development of such scenarios relies heavily on a key variable: the maximal earthworm hydroskeletal pressure. This biomechanical parameter is difficult to quantify accurately, past estimates have been limited to indirect measurements [16] [2] or based on proxies [1] [17]. For example, early studies have measured the colonic fluid pressures of anaesthetized earthworms [16]; clearly, the resulting low pressures would not support earthworms’ regular burrowing activities even in wet and loose soil. Subsequent estimates were based on an earthworms ability to burrow through compacted soil cylinders [2], but cracks and defects in the soil cylinders may have resulted in over estimations of earthworm pressures. Keudel and Schrader [1] and Quillin [17] attempted to directly measure earthworms’ radial pressures by means of mass scales and force transducers. The recorded passive force peaks were used to infer pressure values, an inference confounded by the device geometry rather than the geometry of the earthworms themselves. The uncertainty in this biomechanical parameter and its centrality in making ecological and soil structure inferences warrant its reevaluation using reliable and direct methods that can resolve geometrical issues and be linked directly to a biomechanical model for earthworm kinematics in soil. Thus, the specific objectives of this study were to:

1. Develop a measurement device that can measure an earthworm’s radial pressure threshold for different confining pressures and different earthworm species

2. Formulate a theoretical framework for the peristaltic mechanical motion and resulting pressures to enable interpretation earthworm pressure pulses

3. Apply the results and place these new earthworm biophysical traits in the broader context of soil bioturbation and structure generation

We briefly review the mechanics of earthworms burrowing into natural soils and extend the model to peristaltic motion and associated quantities. We then propose a new pressure chamber for direct measurement of earthworm pressure pulses and their interpretation. We report results from pressure measurements for different species of earthworms, detailing their static pressure limits and kinematics. Finally, we evaluate the peristaltic motion model and its implications for linear elastic (compaction) conditions and compare the deformations to elasto-viscoplastic (natural soil) conditions.
Theoretical considerations

The mechanics of soil bioturbation by earthworms – an overview

Visual evidence of earthworms burrowing in soil has shown that the process is dominated by mechanical displacement and not by soil ingestion [Ruiz et al., 2017] [11]. Initially, earthworms locally contract radial muscles and axially extend local segments tips into soil gaps and pores. Subsequently, the earthworms axially contract hydroskeleton segments to exert a localized radial expansion and pressure. Alternating between these modes results in a peristaltic motion that permits earthworm locomotion [17]. In soils, the radial expansion serves two purposes. First, it allows an earthworm to locally anchor segments of its body while penetrating into soil. Second, it allows the earthworm to radially pressurize a cavity and deform the soil potentially exploiting paths of least resistance and loosening soil in the forefront [18].

The mechanics of earthworms radially expanding cavities in soil have been reported in detail [Ruiz et al., 2015; Ruiz et al., 2017] [13] [11], we thus provide only a brief overview. To quantify the magnitude of radial pressure required by an earthworm to expand in wet elasto-viscoplastic soils, we first consider the force balance at equilibrium:

\[
\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0
\] (5.1)

where \( r [m] \) is the distance from the center of the cavity, \( \sigma_r [Pa] \) is the radial stress and \( \sigma_\theta [Pa] \) is the hoop (circumferential) stress. The deformation behavior is expressed by the Von-Mises criterion considering viscous deformation (Bingham model [19] [20]), relating the difference between the radial and hoop stresses to the summation of the undrained soil strength and the viscoplastic strain rate:

\[
\sigma_r - \sigma_\theta = 2s_u + \frac{4}{3} \eta \dot{\epsilon}_r
\] (5.2)

where \( \eta [Pa \ s] \) is the soil plastic viscosity, \( s_u [Pa] \) is the undrained soil strength, and \( \dot{\epsilon}_r [m \ m^{-1} s^{-1}] \) is the radial strain rate. The \( \frac{4}{3} \) factor emerges from the cylindrical geometry [Bishop et al., 1945]. Substitution of Eq. (5.2) into (5.1) yields the following expression:

\[
\frac{\partial \sigma_r}{\partial r} = -\frac{2s_u}{r} - \frac{4}{3} \eta \frac{\dot{\epsilon}_r}{r}
\] (5.3)

By integration, we determine the radial stresses as a function of the radius (and the strain rate):
\[ \sigma_r (r) = P_L - 2s_u \ln \left( \frac{r}{r_c} \right) - \frac{4}{3} \eta \int \frac{\dot{\varepsilon}_r}{r} \, dr \] (5.4)

where \( r_c \) [m] is the minimum cavity size where the static cavity pressure converges to \( P_L \) [Pa] is the time independent limit pressure. Under static conditions, the strain rate term in the integral vanishes. We solve for the limit pressure by equating the change in the cavity zone to the change in the plastic region local to the cavity:

\[ \left( \frac{R_p}{r_c} \right)^2 \to \frac{E}{2s_u(1+\nu)} \] (5.5)

Where \( E \) [Pa] is the soils elastic modulus, \( \nu \) [m m\(^{-1}\)] is the soils Poisson’s ratio, and \( R_p \) is the elasto-plastic interfacial radius. Under static conditions, the radial stress at the elastoplastic interface could be expressed as:

\[ \sigma_r (R) = P_L - 2s_u \ln \left( \frac{R_p}{r_c} \right) = s_u \] (5.6)

And thus the minimum radial pressure required to expand a cavity in soil could be inferred by the limit pressure:

\[ P_L = s_u \left( 1 + 2 \ln \left( \frac{R_p}{r_c} \right) \right) = s_u \left( 1 + \ln \left( \frac{E}{2s_u(1+\nu)} \right) \right) \] (5.7)

The resulting expression would be the minimum amount of pressure an earthworm would have to exert with its hydroskeleton in order to expand a cavity radially in soil. Although we may have values for mechanical properties of soil, it is not given that an earthworm could induce the necessary pressures to expand a cavity in soil.

**Earthworm hydrostatic pressures**

The experimental system that will be presented in detail next, consists of a flexible tubing that hosts an earthworm and records the dynamic pulses generated by the earthworm’s hydroskeleton during motion (under imposed confining pressure). To describe the processes we first consider the static force balance between the thin tubing and the earthworm (Fig. 5.1), the pressure relationship between the inner tubing and the outer tubing could be derived from the equilibrium equation (5.1); however, as the problem remains linear elastic with no volumetric load source (neglecting gravity), simplification to the equations allow the use of the Airy Stress function in polar coordinates[Shames, 1997] [21]:
\[ \nabla^4 \Phi = \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) \right) \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \Phi}{\partial r} \right) \right) = 0 \]  

(5.8)

where \( \Phi \text{ [Pa m}^2] \) is the Airy function, or the scalar potential of the stress is defined by the principle stresses as:

\[ \sigma_r = \left( \frac{1}{r} \left( \frac{\partial \Phi}{\partial r} \right) \right) \]

(5.9)

And

\[ \sigma_\theta = \left( \frac{\partial^2 \Phi}{\partial r^2} \right) \]

(5.10)

Integrating equation (5.8), we obtain the following expression:

\[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \Phi}{\partial r} \right) \right) = \frac{C_1}{r} \]

(5.11)

And integrating equation (5.11), we obtain:

\[ \frac{\partial}{\partial r} \left( \frac{\partial \Phi}{\partial r} \right) = rC_1 \ln (r) + C_2 r \]

(5.12)

Integrating equation (5.12) gives us the final expression needed to characterize the stresses in the tubing [Shames, 1997] [21]:

\[ \frac{\partial \Phi}{\partial r} = \frac{C_1}{2} r \left( \ln (r) - \frac{1}{2} \right) + C_2 r + \frac{C_3}{r} \]

(5.13)

Substituting (5.13) into (5.9) gives the expression for radial stress:

\[ \sigma_r = \frac{C_1}{2} \left( \ln (r) - \frac{1}{2} \right) + C_2 + \frac{C_3}{r^2} \]

(5.14)

Substituting (5.13) into (5.10) gives the expression for circumferential stress:

\[ \sigma_\theta = \frac{C_1}{2} \left( \ln (r) + \frac{1}{2} \right) + C_2 - \frac{C_3}{r^2} \]

(5.15)
Figure 5.1  Pressure balance at static equilibrium between the pressure chamber and an earthworm. There is minimal resistance by the inner tubing, thus the pressure externally applied will cause the tubing to collapse. The earthworms outer circumference will be enveloped by the soft inner membrane (a), thus the pressures determined by the pressure system are representative of the earthworms geometries. The relationship between the outer pressure and earthworm hydroskeleton (b) is quantified by the pressure balance required to open and maintain a cavity.

For linear elastic cavity expansion, $C_1$ is considered to be zero thus enabling determination of the radial displacement [Shames, 1997] [21]. We consider as boundary conditions $\sigma_r (r_o) = -P_o$ the confining pressure acting on the outer tubing of the enveloped earthworm (Fig. 5.1), and $\sigma_r (r_i) = -P_i$ as the pressure applied by the earthworm’s hydroskeleton. These allow derivation of the following expressions for the radial stress propagation across the inner tubing:

$$\sigma_r = \frac{r_i^2 r_o^2 (P_o - P_i)}{r_o^2 - r_i^2} \left(\frac{1}{r^2}\right) + \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2} (5.16)$$

And the associated circumferential stress:

$$\sigma_\theta = -\frac{r_i^2 r_o^2 (P_o - P_i)}{r_o^2 - r_i^2} \left(\frac{1}{r^2}\right) + \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2} (5.17)$$

where $r_i$ [m] and $r_o$ [m] are the earthworms hydrostatic radius, and outer radius is the combination of the earthworm radius and the thickness of the tubing (see Fig. 5.1b), and $P_i$[Pa] and $P_o$ [Pa] are the inner and outer pressures. For known material mechanical properties:

$$\epsilon_\theta = \frac{u}{r} = \frac{1}{E} (\sigma_\theta - \nu \sigma_r) (5.18)$$

where $u(r)$ [m] is the deformation at a given radius $r$ [m] away from the inner tube radius, $E$
[Pa] is the elastic modulus of the tubing, \( \nu [m \text{ m}^{-1}] \) is the Poisson’s ratio of the tubing, we relate the inner tube pressure with the external confining pressure:

\[
P_i = \frac{E u (r_i) (r_o^2 - r_i^2) + 2r_i r_o^2 P_o}{(1 + \nu) r_i r_o^2 + (1 - \nu) r_i^2}
\]  

(5.19)

Neglecting any deformation of the inner membrane on the surface of the earthworm, the resulting expression is:

\[
P_i = \frac{2r_i r_o^2 P_o}{(1 + \nu) r_i r_o^2 + (1 - \nu) r_i^2}
\]  

(5.20)

Similar membranes were mechanically tested for stiffness and compressibility. These results showed that similar tubing would have a Poisson’s ratio \( \nu \approx 0.49 \) subject to hydrostatic pressures exceeding 25 kPa [Berli et al., 2006] [22]. If we assume that the membrane thickness has a negligible effect on the pressure measurement [22], \( r_o \to r_i \), then the internal pressure converges to the external pressure \( P_i \to P_0 \) at hydrostatic equilibrium.

The kinematics of earthworm’s peristaltic motion

Unlike the steady insertion of a conical tip into a soil where the pressures and cavity expansion are relatively steady, an earthworm advances by a series of peristaltic motions and associated pulses of pressures. To describe the process and enable interpretation of dynamic measurements of an earthworm held in a system under prescribed confining pressure (see Materials and Methods and supplementary materials S2), we develop a model for translating dynamic volume changes and pressure pulses to maximal pressures exerted by the earthworm. The local expansions and contractions of earthworm segments (Fig. 5.2 a) are reflected by the pressure changes in the measurement chamber. Several models have been developed to describe earthworm locomotion [Steigenberger and Behn, 2012] [23], we simplify the resulting peristaltic motion as a propagating sinusoidal wave through a thin cylindrical tube (Fig. 5.2b-c).

We employ a dynamic description of local pressure and volume wave propagation through a cylindrical tube to represent the earthworm’s kinematics during peristaltic motion. We assume that an earthworm moving through a confining cylindrical tubing applies longitudinal volume and pressure changes on sections of the tubing at a speed \( \frac{d w}{dt} \) [m s\(^{-1}\)] (Fig. 5.2 a). These dynamic pulses are estimated by considering a simplified force balance on an element of the thin tubing (Fig. 5.2 c) and the longitudinal impact during the earthworm’s propulsion [24]:
Figure 5.2  Quantifying the earthworm peristaltic volume changes and pressure pulses within a confined cylindrical geometry (similar to the measurement chamber used in this study). An earthworm passing through a flexible cylindrical membrane transmits a sinusoidal wave that propagates at a speed dependent on the amplitude of its peristaltic pulse (radial expansion $u$) and the frequency of these peristaltic pulses ($\omega_n$). Considering a radial axial symmetric thin tube (b), the force balance on an element of the thin tubing (c) makes use of the longitudinal wave speed in order to determine the reactionary radial deformation $u$. Considering the amplitude and frequency of the earthworm’s peristaltic motion, unique volumetric displacements are defined in the chamber ($\Delta V$), which is related to the measured pressure changes ($\Delta P$).

\[
\begin{align*}
\left(\frac{1}{c_p^2}\right) \frac{\partial^2 w}{\partial t^2} &= \frac{\partial^2 w}{\partial z^2} + \frac{\nu}{R} \frac{\partial u}{\partial z} \\
\left(\frac{1}{c_p^2}\right) \frac{\partial^2 u}{\partial t^2} &= -\frac{u}{R^2} - \frac{\nu}{R} \frac{\partial w}{\partial z} \\
\frac{\partial w}{\partial t} (z, 0) &= -\frac{u_0}{\nu R} c_p \\
\frac{\partial w}{\partial t} (z, 0) &= 0 \\
w (z, 0) &= 0 \\
u (z, 0) &= 0 \\
\frac{\partial w}{\partial t} (\infty, t) &= 0
\end{align*}
\] (5.21)

where $R$ [m] is the earthworm nominal radius (when the earthworm is relaxed), $c_p$ [m s$^{-1}$] is the wave speed along the interface of the earthworm and the tubing, $\nu$ ($\sim 0.5$) [m m$^{-1}$] is the Poisson’s ratio of the tubing, and $u_0$ [m] is the amplitude of the radial displacement during peristaltic motion. The system is initially assumed to be radially and axially static. We assume that outside
of the measurement chamber, a peristaltic wave is axially propagated into the system (front to back). The velocity of the incoming longitudinal wave \( \frac{\partial w}{\partial t} (\infty, t) \) proportionally transmits a radial displacement amplitude \( u_0 \) based on the Poisson’s effect. The initial analysis is done in the Laplace domain to isolate the geometric variability on the deformations:

\[
\begin{align*}
\bar{w}(z,s) &= \int_0^\infty w e^{-st} dt \\
\bar{u}(z,s) &= \int_0^\infty u e^{-st} dt \\
\left( \frac{1}{\zeta_p^2} \right) \left( s^2 \bar{w} + \frac{u_0}{\nu R} c_p \right) &= \frac{d^2 \bar{w}}{dz^2} + \nu \frac{d \bar{w}}{R dz} \\
\left( \frac{1}{\zeta_p} \right) s^2 \bar{u} &= -\frac{\bar{w}}{R^2} - \nu \frac{d \bar{w}}{R dz}
\end{align*}
\]

(5.22)

where \( s \) \([s^{-1}]\) is the complex frequency. To solve \( \bar{w} \) we have to substitute in for the \( \bar{u} \) term in the equation. We first solve the last equation in for \( \bar{u} \):

\[
 \bar{u} = -\frac{\nu c_p^2}{R} \left( \frac{1}{s^2 + \frac{c_p^2}{R^2}} \right) \frac{d\bar{w}}{dz}
\]

(5.23)

Substituting this into the equation for \( \bar{w} \):

\[
\frac{d^2 \bar{w}}{dz^2} - s^2 \zeta^2 \bar{w} = \zeta^2 \frac{u_0}{\nu R} c_p
\]

(5.24)

where:

\[
\zeta^2 = \frac{s^2 + \frac{c_p^2}{R^2}}{c_p^2 \left( s^2 + \frac{c_p^2}{R^2} (1 - \nu^2) \right)} \approx c_p^{-2}
\]

(5.25)

The approximation is needed later on when we want to estimate the radial deformation in the tubing. Solving (5.24) considering the boundary conditions:

\[
\bar{w}(z,s) = \left( -\frac{u_0}{\nu R} c_p \right) \frac{1}{s^2} (1 - e^{-s\zeta z})
\]

(5.26)

where the axial gradient is:

\[
\frac{d\bar{w}}{dz}(z,s) = -\frac{u_0}{\nu R} c_p \frac{1}{s} e^{-s\zeta z}
\]

(5.27)

Substituting into equation (5.24):
\[
\tilde{u}(z, s) = \frac{\nu c_p}{R} \left( \frac{u_0}{\nu R c_p} \right) \left( \frac{1}{s^2 + \frac{c_p^2}{R^2}} \right) \zeta \frac{1}{s} e^{-s\zeta} z
\] (5.28)

Considering the approximation in (5.25), we re-write the expression as:

\[
\tilde{u}(z, s) = \frac{\nu c_p}{R} \left( \frac{u_0}{\nu R c_p} \right) \left( \frac{1}{s^2 + \frac{c_p^2}{R^2}} \right) \frac{1}{s} e^{-s\frac{z}{c_p}}
\] (5.29)

Performing the inverse Laplace transformation on (5.29), we obtain:

\[
u(z, t) = -u_0 H(t - \frac{z}{c_p}) \left( \cos \left( \frac{c_p t - z}{R} \right) - 1 \right)
\] (5.30)

where \(H(x)\) is the Heaviside function. To simplify the expression, the solution for the peristaltic kinematics will take the form:

\[
u(z, t) = -u_0 \left( \cos \left( 2\pi \omega_n t - \frac{z}{R} \right) - 1 \right)
\] (5.31)

where \(u_0\) is the magnitude of the radial displacement and

\[
\omega_n = \frac{c_p}{2\pi R}
\] (5.32)

Given an analytic expression for the deformation, we obtain the change in the system volume as

\[
\Delta V = \pi \int_0^L \nu(z, t) \, dz = \pi \int_0^L \left( \cos \left( 2\pi \omega_n t - \frac{z}{R} \right) - 1 \right)^2 \, dz
\] (5.33)

The resulting volume change is then used to determine the pressure fluctuations:

\[
\Delta P = -K \frac{\Delta V}{V_0}
\] (5.34)

where \(V_0\) is the initial system internal chamber volume (physically introduced in the materials and methods) and \(K\) is the bulk modulus of the pressure chamber (see A.1 for details). Alternatively, measured pressure changes in the system are used to infer the amount of local volumetric deformation traveling through the confined system.
Spectral analysis of burrowing kinematic peristaltic frequency

Measurements of earthworm’s pressure pulses provide insights into the earthworm burrowing kinematics. Considering a pressure pulse \( \Delta P \) as a deviations from the mean pressure value at a certain confining pressure (Fig. 5.3 a-b), the frequencies of the pressure fluctuations could be analyzed by performing a Fourier transformation on the transient pulses (implemented in Mat- 
lab [25]):

\[
\Delta \hat{P}(\omega) = \left| \int_{-\infty}^{\infty} \Delta P(t) e^{-2\pi i \omega t} dt \right|
\]  
(5.35)

Where \( \omega [s^{-1}] \) is the frequency, \( t [s] \) is the time, and \( \Delta P [Pa] \) is the dynamic pressure pulses. The amplitude frequency response allows us to obtain the operational frequency associated with the primary pressure amplitudes (Fig. 5.3 c):

\[
\omega_n = \{ \omega_i \mid \Delta \hat{P}(\omega_i) \geq \Delta \hat{P}(\omega_j) \forall i, j \in \mathbb{N} \}
\]  
(5.36)

These frequency pulses define the wave speed of a peristaltic pulse:

\[
c_p = 2\pi R \omega_n
\]  
(5.37)

where \( R \approx r_i [m] \) is the nominal (relaxed) earthworm radius. Frequencies obtained from this analysis are used in conjunction with the dynamic model of peristaltic motion in order to check the model validity. Frequencies are also used to extend our inference on the earthworm’s kinematic motion outside of the limited measurable window that the device provides.

Materials and Methods

The earthworm pressure chamber

We developed an experimental chamber for measuring earthworm’s pressures against prescribed confining pressure to assess the limits of their hydroskeleton pressures. The system consists of a Plexiglas prism with a hallowed cylindrical cavity fitted with a thin silicon tubing of 1.5 mm inner radius and 15mm length for holding an earthworm. Water is injected and transmit prescribed pressure to the thin membrane of the system (Fig. 5.4 b-c). The chamber was hydraulically
Figure 5.3  A sketch of measured earthworm kinematics from pressure fluctuations for different confining pressures. For each pressure step prior to the earthworm incapacitation (a) we expect pressure fluctuations $\Delta P$ about the applied pressure $P_o$ induced by the earthworm motion. By subtracting the mean value pressure at a given pressure step window isolates the pressure pulses (b). The amplitude and frequency from measured pressure pulses in the time domain are transformed to the frequency domain (c) to determine the frequencies of the peristaltic pulses during earthworm motion.

connected to a pressure transducer (PR-23S/80549.55 Keller Druckmesstechnik, Switzerland) for monitoring pressure fluctuations in the chamber as earthworms pass through the inner tube. Pressure was measured at sampling frequencies (as high as 100 Hz) using a CR 1000 data logger (Campbell Scientific, Logan USA). A digital microscopic camera (AM4113T-GFBW Dino-Lite Premier, AnMo Electronics Corp, Taiwan) was used to acquire images of the experiments during earthworm passage (pixel size of 180 $\mu$m$^2$). Fig. 5.4 d illustrates the pressure application closing the internal silicon tube and transmitting pressure directly to passing earthworms.

The experimental procedure is administered similar to a sphygmmomanometer blood pressure measurement. Chamber water pressure ($P_o$ [Pa]) is increased until the earthworm becomes stationary and the pressure fluctuations diminish ($\Delta P = 0$ [Pa]). We then relieve the pressure down until we can monitor pressure fluctuations again. We interpret the response as the passing earthworm internal pressure exceeds this level of confining external pressure [Ward and Langton, 2007] [26]. While the pressure application is analogous to the sphygmmomanometer measurement, there are several important differences in the interpretation of the measurements. Unlike the blood pressure measurement, the muscles of the earthworm are not relaxed and we thus measure the earthworm’s hydroskeletal pressures (as evidenced by the high pressure pulses) and its blood pressures. The fluctuations are clearly associated with physical peristalsis of the earthworm body and not internal blood pressure (as imaging of the motion reveals). Earthworms also have no calcified skeletal system (porous and compressible), thus justifying the assumption of incompressibility [Evans and Mendelssohn, 1942] [27]. The confining fluid in the system was water to directly transmit the
pressures onto the earthworm without [Evans and Mendelssohn, 1942] [27]. The inner tubing was lightly lubricated (Vaseline) to reduce frictional effects and damage to the earthworm skin. Using a thin rigid tube, the earthworms were first placed at the entrance of the device and the end of the rigid tubing was slightly warmed up (warm paper towel) to coax the earthworms into entering the measurement chamber.

**Figure 5.4 Images of the pressure chamber measurement device.** (a) Device is comprised of a large Plexiglas prism with a cylindrical extrusion through the center. The inner spacing is fit with soft silicon tubing. The spaces between the membrane and the Plexiglas are filled with water and hydraulically connected at an inlet and an outlet. Inner tubing (b) allows the passage of an earthworm. Pressure is externally applied (c) via a syringe pump and transmitted to the earthworm, compressing the volume in the membrane space and measured on the external port by the pressure sensor.
Earthworm pressure measurement protocol

Earthworms used in this study were selected based on ecotypes that contribute generating soil structure. *L. terrestris* (anecic) earthworms were purchased locally (Reptile-food.ch GmbH, Duebendorf, Switzerland). This species is known to create deep burrows that facilitate drainage processes and ground water recharge. However, these burrows are often re-used by the earthworms, thus the species is not inclined to frequently generate burrows.

*A. caliginosa* (endogeic) earthworms were ordered from a research institute in France (INRA). This ecotype frequently creates new burrows and avoids reusing other burrows [28]. Previous measurements on the maximum pressure threshold have stated that *A. caliginosa* exert some of the largest pressures of any earthworm currently measured.

Using the pressure chamber, we place the earthworm specimen into the center of the chamber between the flexible tubing (Fig. 5.4b). Using a peristaltic pump, we incrementally increase the pressure step wise until the earthworm drastically slows down (Fig. 5.4c). We hold the pressure for 30-60 seconds. If the earthworm continues to move through the chamber, we increase the pressure in the chamber. This is continued until the earthworm is incapacitated. Once the earthworm stops moving, we release the pressure and remove the earthworm from the chamber. Experiments are carried out on 10 earthworms of a given species (20 in total).

Results

Measuring earthworm’s maximum hydroskeletal pressures

We begin with an illustrative example of the measured pressures applied by an endogeic earthworm passing through the pressure chamber (Fig. 5.5). The images on the right were taken at different times and are associated with certain pressure values that show a typical behavior of earthworm’s hydroskeleton motion. The small pressure pulses are associated with longitudinal deformation as the earthworm attempts to move through the pressurized tube. The confining pressure is increased in steps until the pressure pulses cease, indicating that the imposed pressure exceeds the earthworms maximum hydroskeletal pressure. Subsequently, the pressure is released and the earthworm passes through the device (in most cases, we have not detected physical damage). Details regarding the pressure values for the earthworms tested in the system are presented in Table 5.1 along with literature values from different studies. The maximum pressure under which motion of an endogeic (*A. caliginosa*) earthworm was measurable was 195 kPa. This value is nearly double the mean of maximum pressures measured for all 10 endogeic earthworms tested (92 kPa), and it resides outside of the standard deviation for the mean values. The maximum value measured for the anecic species (*L. terrestris*) was 130 kPa, and the mean maximum value
for all of the measured anecic species was 77 kPa. This maximum value also falls outside of the standard deviation (51 kPa). Overall, the anecic species appears to exert pressures only slightly greater than half of that of the endogeic earthworms, and there are variations among individuals that may exert extreme pressure values (as discussed above).

Figure 5.5  (a) Direct measurements of earthworms’ hydroskeletal pressures during peristaltic motion. (b) Water pressure is applied to the chamber once the earthworm entered the chamber. We first apply a confining pressure of 65 kPa, then we stepped up the pressure to 72 kPa where the earthworm appeared to pause. (c) after the earthworm resumed motion, the pressure was increased 75 kPa. The earthworm induced peristaltic pressure pulses were about $\Delta P = 2$ kPa higher than the confining pressure (75 kPa). (d) The pressure was increased to 87.5 kPa and then relaxed down to about 82.5 kPa. The earthworm was unable to move and remained stationary, we thus reduced the pressure and (e) the earthworm was allowed to exit the tube (the tube diameter increases and the tail of the earthworm is visible).

**Measuring earthworm’s peristaltic frequency**

We report a Fourier analysis of the earthworm’s peristaltic motion as depicted in Fig. 5.6. The earthworm pressure pulses were initially small for the lower confining pressures and gradually
Table 5.1 Maximum radial pressures obtained in this study and reported in the literature. The literature radial pressures are for two endogeic earthworms *A. rosea* and *A. caliginosa* and an anecic earthworm *L. terrestris*. The mean values (with standard deviations in parenthesis) from this study were determined on a sample size of 10 earthworms per species. Measurement techniques are noted along with the previous studies that presented them.

<table>
<thead>
<tr>
<th></th>
<th>Max pressure [kPa]</th>
<th>Mean pressure [kPa]</th>
<th>Technique</th>
<th>Mean diameter [mm]</th>
<th>Mean length [mm]</th>
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</thead>
<tbody>
<tr>
<td><em>A. caliginosa</em></td>
<td>195</td>
<td>92(±52)</td>
<td>Pressure chamber</td>
<td>2(±1)</td>
<td>100</td>
</tr>
<tr>
<td><em>L. terrestris</em></td>
<td>130</td>
<td>77(±51)</td>
<td>Pressure chamber</td>
<td>4(±2)</td>
<td>180</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Max pressure [kPa]</th>
<th>Mean pressure [kPa]</th>
<th>Technique</th>
<th>Study</th>
</tr>
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<tr>
<td><em>A. rosea</em></td>
<td>-</td>
<td>230</td>
<td>Soil disc</td>
<td>Mckenzie and Dexter [2]</td>
</tr>
<tr>
<td></td>
<td>182</td>
<td>104</td>
<td>Mass scale</td>
<td>Keudel and Schrader [1]</td>
</tr>
<tr>
<td><em>A. caliginosa</em></td>
<td>295</td>
<td>170</td>
<td>Mass scale</td>
<td>Keudel and Schrader [1]</td>
</tr>
<tr>
<td><em>L. terrestris</em></td>
<td>110</td>
<td>32.5</td>
<td>Mass scale</td>
<td>Keudel and Schrader [1]</td>
</tr>
<tr>
<td></td>
<td>1.6</td>
<td>-</td>
<td>Coelomic pressure</td>
<td>Newel [16]</td>
</tr>
</tbody>
</table>

increase as the confining pressure increases. Part of the apparent increase in the observed pressure pulses could be attributed to increasing system bulk modulus under higher confining pressure values (A.1). The earthworm peristaltic motion frequencies do not vary significantly with pressure confinement. The average peristaltic frequency was determined for the different experiments is reported in Table 5.2 (a pulse every 16 to 50 s on average). These measurements were used to infer wave speeds through the entire earthworm’s body. The pressure chamber measures within a length of about 15 mm (Fig. 5.3 b-c), and we monitor a pulse every 16-50 s. It is unclear whether the pulses traverse the entire length of the earthworm, however, considering the lowest frequency (pulse ever 50 s) we estimate a peristaltic pulse moving at a speed of 0.3 mm s⁻¹. For the full length of an earthworm (100-180 mm long), the wave would travel the entire length at about 2-5 min. To reconcile these estimates with our measurements, an earthworm would have to generate several (4-8) contractions along its body during locomotion. Observations suggest that unconfined earthworm peristaltic motion of an *A. caliginosa* (100 mm length) moving on wet agar surface generates about 4 simultaneous contractions on average (Fig. 5.7).
Figure 5.6 Inferring operating frequencies of earthworm peristaltic motion from a direct pressure measurement over time. An example (a) of time series pressure data is used to illustrate how a moving window across the different pressure steps is used to generate a amplitude frequency response (b) for the different pressure steps (blue data points illustrate fluctuations around 35 kPa, orange represent the frequency response at the 40 kPa pressure step, yellow for 50 kPa, purple for 60 kPa, and green for 70 kPa). This was conducted for all the possible experiments (c) *A. caliginosa* and (d) *L. terrestris* in order to determine mean operating peristaltic frequencies for the respective species.

<table>
<thead>
<tr>
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<th>Mean frequency [Hz]</th>
<th>Standard Deviation [Hz]</th>
<th>Max frequency [Hz]</th>
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</thead>
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<tr>
<td><em>A. caliginosa</em></td>
<td>0.0406</td>
<td>0.0191</td>
<td>0.13</td>
</tr>
<tr>
<td><em>L. terrestris</em></td>
<td>0.0428</td>
<td>0.0285</td>
<td>0.23</td>
</tr>
</tbody>
</table>
Figure 5.7 Image of an *A. caliginosa* earthworm on wet agar during locomotion. The earthworm in the figure is 100 mm long and has an approximate diameter of 2 mm. Four sections are contracted or extended simultaneously during forward motion, illustrating the peristalsis. Blue arrows indicate wave directions.

**Inferring volumetric changes using peristaltic pulses**

We may use the estimates for the mean wave speed to compare simulated and measured peristaltic pulses during earthworm wiggling within the pressure chamber. The pressure pulse was represented using Eq (A.1.1) and was compared to experimental data (Fig. 5.8). The confining
pressure initially set to 80 kPa and decayed to 68 kPa over the duration of 200 s measurement (Fig. 5.9). During this period, an earthworm under confinement continues to move through the device, resulting in the measured pressure pulses. The model predictions of pressure fluctuation are plotted over a single wave period for different volumetric strain amplitudes (Fig. 5.9 (c)) based on the simulated pulse. Considering a radial amplitude of the pulse of \( u_0 = 0.6 \) mm, and the maximum frequency for \( A. \text{ caliginosa} \) \( \omega_n = 0.13 \) Hz in Eq.(5.31), the estimated volumetric change (\( \Delta V \)) using Eq. (5.32), considering an initial volume in the system of \( V_o = 3.45 \) mL and a bulk modulus at external pressure \( P_0 = 120 \) kPa (see A.1 for details). The resulting local volumetric strain is \( 7.5 \times 10^{-3} \) for the comparison depicted in Fig. 5.9 (d)).

**Measured earthworm maximal pressures in the context of soil behavior**

In this section we seek to link the pressure measurements in the linear-elastic experimental system to conditions in deformable (elasto-viscoplastic) wet soil. We apply simulation results and measured earthworm pressure pulses in the pressure chamber to the pressures and deformations in natural soils (see S3 for details). Key soil mechanical properties corresponding to the bulk modulus of the experimental pressure chamber can be found in Table 5.3. The pressure and their respective radial strains are illustrated in Fig. 5.10. For all of the observed (and simulated) earthworm pressure responses, the theoretical soil viscous deformation would be an order of magnitude higher than in the linear elastic pressure chamber. This illustrates an important mechanical difference between measurements in the elastic pressure chamber and behavior in natural soils.

**Table 5.3** Soil elasto-viscoplastic properties corresponding to equivalent chamber elastic properties. Soil mechanical properties correspond to silt loam soil under different hydration conditions and were determined using a rheometer [Ghezzehei and Or, 2001] [20]

<table>
<thead>
<tr>
<th>Water content ((\theta_m)) [kg kg(^{-1})]</th>
<th>Yield stress ((\sqrt{3} s_u)) [kPa]</th>
<th>Approximate shear modulus ((K/2)) [kPa]</th>
<th>Soil plastic viscosity ((\eta)) [kPa s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>15</td>
<td>800</td>
<td>900</td>
</tr>
<tr>
<td>0.21</td>
<td>10</td>
<td>370</td>
<td>300</td>
</tr>
<tr>
<td>0.25</td>
<td>5</td>
<td>185</td>
<td>200</td>
</tr>
</tbody>
</table>
Figure 5.8  Simulated pressure field during peristaltic motion using the wave speed estimate obtained from previous measurements. Pulse starting from the front of the tube (a) propagates to the far end of the tube. (a) to the center (b) after 7.5 s, moving towards the back end after a subsequent 7.5s (c). The volumetric changes are computed based on the integrated displacement field over the entire domain.

Discussion

The study focused on establishing a method for obtaining direct measurements of earthworm maximal pressures (and other kinematic variables) important for predicting the envelopes of earthworm activity in different soils and climates. We developed a device and a measurement protocol for directly measuring an earthworm’s hydroskeletal pressures under a range of con-
fining pressures (under linear-elastic conditions). A thin flexible inner tubing conforms to the surface of the earthworm’s body when the hydroskeleton applies pressure. The resulting direct pressures are obtained under more realistic earthworm geometry than previously reported [2] [17] [1]. The experimental system permits confining pressures and its intrinsic compressibility vary with the confining pressure (See A.1 Fig. A.1.1 and Fig. A.1.2)). The setup permits passage of the earthworm and thus inference of kinematic information associated with motion in confined cavities.

The experimental system enabled direct measurements of earthworm pressures (Fig. 5.5) and imaging of the motion. Tests of 10 endogeic and 10 anecic earthworms revealed lower maximal pressures than previously reported (Table 5.1). The measured values were within a similar order of magnitude, roughly half the value reported for endogeic earthworms by Keudel and Schrader [1] and Mckenzie and Dexter [2]. In contrast, we found that the mean confining pressure that
Figure 5.10  Modeling pressure pulses responses during peristaltic motion for a range of mechanical properties corresponding to the pressure chamber (linear elastic) and natural partially saturated soils (elasto-viscoplastic). The resulting pressure pulses response (a) was simulated for three sets of mechanical properties associated with pressure chamber bulk modulus and partially saturated soils. For the same pressure input, the resulting deformations (b) can be seen for both the linear elastic pressure chamber and the elasto-viscoplastic soil.

limits of anecic motion was slightly higher than previously measured (Table 5.1) [1]. While the mechanical limitations reported here may represent the muscular limitations for earthworm’s hydroskeleton, other means such as ingestion or lubrication with mucus [Gray and Lissmann, 1938] [10] may extend the range of burrowing in soil.

In ecological terms, the mechanical activity of endogeic earthworms is limited to water contents of 25% for typical silt loam soils (about 2 - 5% higher than the previous minimum for uncompacted silt loam and silty clay loam respectively reported in [Ruiz et al., 2017] [11]). The estimates for the soil organic carbon required for mechanical activity were based on the earthworm pressure limits [11]. Considering the updated maximum earthworm pressures reported in this study (about half the values previously estimated), we estimate that endogeic earthworm communities would not use more than 0.05 kg m$^{-2}$ year$^{-1}$ (less than 5% of the annual NPP in crop lands based on estimates in Beers et al. 2007 [29]).

The lower maximal pressure value would reduce the window of earthworm activity in soil relative to that under which plant roots may grow into the soil [11]. Despite the lower growth rate (0.1-0.2 $\mu$m s$^{-1}$) compared to earthworms (100-500 $\mu$m s$^{-1}$) [11], earthworms would become mechanically limited to burrowing in wet soils with water contents higher than 20% gravimetric for silt loam soils. In contrast, plant roots would not be mechanically hindered growing into soils on the order of 10% gravimetric water content [11]. The link between the earthworm’s biomechanical limitations and water content could provide potential insights into the origins of
seasonality in earthworm activity. For example, Nakamura [30] has shown that the total earthworm population density [ind m$^{-2}$] reduces by 66% between December to August (8 months). Thus earthworm activity peaks during the Autumn months (August to November, 4 months in the northern hemisphere) [31] [30], which are generally the wettest months in the year.

The measurements with the pressure chamber yielded information on the peristaltic motion and frequencies (Fig. 5.6). Considering changes in the system’s bulk modulus with increasing pressure confinement (See A.1), the earthworms do not vary the volume of contraction moving through the pressure system (Fig. 5.6) with different external pressures. The frequency of earthworm peristaltic motions operate at wave speeds (Eq (5.36)) of the order of 0.3 mm s$^{-1}$, these were consistent with instantaneous penetration rates of earthworms in soils of (0.1-0.5 mm s$^{-1}$) [11]. These wave velocities were incorporated in a newly developed kinematic model (Eq (5.36)) to reproduce the measured pressure pulses (Fig. 5.8) and dynamics in the chamber (Fig. 5.9).

Notwithstanding the new insights from the pressure chamber and capability to obtain direct measurements of earthworm pressures, there are several notable differences between the conditions within the experimental device and in natural soils (Fig. 5.10). The radial strains induced by the earthworms were very small, of the order of <10% volumetric change in the system. This stands in contrast with the large displacements induced in natural soils (∼100%). One main difference lies in the elastic nature of the measurement system with strong confinement and rebound. The elasticity of the pressure chamber is much more persistent than in natural soils, thus an earthworm is never at rest during the measurements. The earthworms must always oppose the external pressures to some degree (to avoid internal collapse). In contrast, the conditions in soil where incremental volume changes are permanent once the earthworm overcomes the soils’ yield stress [11]. There are still influences of elasticity in the soil, but this is largely remote and negligible by comparison to the experimental system.

The experimental system may emulate extremely compacted soil [14] [32]. Precompression stresses could have effects as deep as 30 cm below the soil surface [33], which is within the typical depth of endogeic earthworm activity. Considering compacted soils to behave more elastic, a surface normal stress of 300 kPa is likely to result in a mean normal stress at a depth of 30 cm of approximately 60 kPa [33]. Considering the main burrowing methods of earthworms by cavity expansion (with hydroskeleton pressures overcoming mean normal stresses), our measurements suggest that for such compacted soil conditions would hinder more than 35% of the earthworms measured in this study. For compaction of an elasto-plastic soil [33], the mean normal stress at the 30 cm depth could be as high as 100 kPa, thus hindering 70% of the earthworms measured in this study. These examples show the great sensitivity of earthworm activity to soil compaction and the feedback into soil structure generation that would be delayed (relative to uncompacted soil) thereby exacerbating the loss of productivity and adverse impacts of soil compaction [34].
Conclusion

We designed a novel device for directly measuring earthworms’ hydroskeletal pressures and kinematics under external confining pressures. A new simple kinematic model uniquely considered the earthworm’s geometric changes and resulting pressures fluctuations during peristaltic motion. The measured maximum earthworm pressures for endogeic earthworms were (on average) half of the maximum pressures that were previously reported. Pressures measured for anecic earthworms were slightly higher than previously reported, but still on the same order of magnitude. Using the pressure chamber, we were able to develop a method for inferring kinematic information from the pressure pulses and infer peristaltic motion from the earthworms’ frequencies. Translating these pulses to moving waves, we determined that the inferred rates from the frequency information were consistent with burrowing rates previously reported. The mechanics in the pressure chamber exhibit a more pronounced elastic behavior from natural soils, but earthworm pressure results could still be used to compare and contrast resulting deformations from the pressure chamber from natural soils. The persistence of elasticity in the device, however, likely provides conditions analogous to compacted soil conditions. Given the reduction in the maximum earthworm pressure, biophysical activity windows are likely narrower than previously anticipated. The new pressure results also reduce the upper estimate for soil organic carbon required for mechanical activity (upper limit around 5% the NPP). Lastly, we could expect that compacted conditions could significantly hinder earthworm activity in soil by 35-70% until abiotic processes reduce compaction effects.

Acknowledgments

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## Table 5.4  Table of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>SI Unit</th>
</tr>
</thead>
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<tr>
<td>$c_p$</td>
<td>Wave speed</td>
<td>$m \cdot s^{-1}$</td>
</tr>
<tr>
<td>$\epsilon_r$</td>
<td>Radial Strain</td>
<td>$m \cdot m^{-1}$</td>
</tr>
<tr>
<td>$\eta_p$</td>
<td>Soil Viscosity</td>
<td>$Pa \cdot s$</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s modulus of elasticity</td>
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</tr>
<tr>
<td>$G$</td>
<td>Shear modulus of rigidity</td>
<td>$Pa$</td>
</tr>
<tr>
<td>$K$</td>
<td>Bulk modulus</td>
<td>$Pa$</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of chamber tubing</td>
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</tr>
<tr>
<td>$\mu$</td>
<td>Interface friction</td>
<td>$-$</td>
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<td>Poisson’s ratio</td>
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<td>Pressure pulses</td>
<td>$Pa$</td>
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<td>External chamber pressure</td>
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<tr>
<td>$P_i$</td>
<td>Internal chamber pressure</td>
<td>$Pa$</td>
</tr>
<tr>
<td>$P_L$</td>
<td>Limit Cavity Pressure in soil</td>
<td>$Pa$</td>
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<tr>
<td>$\Phi$</td>
<td>Airy stress function</td>
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<tr>
<td>$w$</td>
<td>Longitudinal deflections</td>
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<td>$z$</td>
<td>Axial Penetration Depth</td>
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## Bibliography


Chapter 6

Summary and conclusions

The unifying theme across the studies reported in this dissertation is a novel modeling framework for the mechanics of earthworm and plant root bioturbation into soil. In addition to the theoretical framework, experimental studies were carried out to examine the range of axial and radial forces associated with penetration-cavity expansion biophysical modeling. This included direct observations of earthworm activity using neutron radiography. While earthworms and plant roots employ vastly different biological mechanisms to move through soil, the physical description of penetration-expansion could be applied for both. One particularly important finding regarded the mode of soil bioturbation by earthworms specifically. The neutron images revealed that the soil was dominantly radially displaced during burrowing action, which is contradictory to the claim that earthworms ingest soil as a main mode of burrowing. This is not to suggest that earthworms do not ingest large quantities of soil, but it does not appear to be during the formation of burrows.

The mechanical model describing the penetration expansion processes was also tested using custom built cone penetrometers with diameters of 2 mm (root tip) to 5 mm (large earthworm) and semi-apex angles of 15-30 degrees. They were used to steadily penetrate soils of different textural class and different water contents. When comparing penetration forces to penetration stresses for these small geometries, we observe a counter intuitive duality between forces and stresses. While penetration forces scale with the size of the penetrometer diameter, the penetration stresses scale inversely (stresses decreased with increasing penetrometer size) with the penetrometer sizes for this range of diameters. While there are numerous reasons why this might arise (stress concentrations [1] [2], hardening associated with local gradients [3] [4], stochasticity [5], viscous effects [6] [7]), this work developed models that accurately estimate this effect for fine textured soils for a sufficient penetration rate. Tests were also conducted at different penetration rates in order to infer the rate-dependent differences that would be experienced by a slow growing plant root (0.1-0.2 \( \mu \text{m s}^{-1} \)) compared to relatively rapid burrowing earthworm (100-500 \( \mu \text{s}^{-1} \)). Isolating the rate effect, we found that earthworms would be required to exert forces 2-3 times greater than plant roots of comparative geometry in order to move through soils under identical conditions.
An important environmental factor that determines the force required for soil penetration is the soil moisture content. For silt loam soils, decreasing the (gravimetric) water content from 0.3 kg kg$^{-1}$ to 0.2 kg kg$^{-1}$ increases the force required to penetrate the soil by an order of magnitude. For silty clay loam, decreasing the gravimetric water content from 0.3 kg kg$^{-1}$ to 0.2 kg kg$^{-1}$ would increase the amount of force required to penetrate the soil by a factor of 5. Plant roots do not appear to be severely hindered by these mechanical constraints, as it has been reported that plant roots can exert expansion pressures on the scale of 1000 kPa [8]. This is particularly consequential for earthworms, as previous reports state that they can only exert maximal radial pressures on the order of 230 kPa [9] on average.

The centrality of a biophysical parameter related to the earthworm maximal radial pressures in the soil bioturbation model motivated efforts to directly measure this parameter. We designed and constructed a novel experimental device that hosts a live earthworm and directly measures radial pressures of earthworms during peristaltic motion. The experimental chamber applies prescribed confining pressure around the outer radius of the earthworms body. Using a procedure similar to a sphygmometer blood pressure test, we were able to infer the maximum radial pressures that would incapacitate the earthworm. The measurements made in this study have shown maximum earthworm pressure made over all the earthworms were as high as 195 kPa; however, on average, earthworm pressures are likely to be on the order of 100 kPa, which is half the value that had been previously reported. This would thus further constrain the range of soil moisture contents that are suitable for earthworm activity.

Prior to complete immobilization of the earthworm hydroskeleton, measurements obtained by the pressure chamber capture peristaltic pressure pulses associated with earthworm self-propulsion. Spectral analysis reveal that earthworms under confinement apply peristaltic pulses at low frequency of 0.04 Hz that do not vary significantly with confining pressure. Accounting for the earthworm length and other observations, such pulses travel along the body of the earthworm at intervals of 20-50 sec on average. We were able to use these pressure pulses and pulse frequencies to describe the kinematics of the earthworms locomotion. In doing so, we developed a kinematic model describing peristalsis during earthworm locomotion. The model was developed based on the dynamics of a transverse wave propagation through fixed annular tube. Unlike previous models of earthworm locomotion, the current model resolves both temporal and spatial deformations both radially and longitudinally during wave propagation along the earthworms longitudinal axis.

Our modeling results also shed light on some of the energetic costs of soil bioturbation by earthworms and plant roots. For the specific heterotrophic nature of earthworms, the amount of mechanical energy required for earthworms to move through soil were translated to equivalent soil organic carbon consumption by earthworms using the conversion 0.0484 g carbon J$^{-1}$ [10]. To estimate the annual consumption of earthworm communities (considering an earthworm population density of 300 individuals per m$^2$ in the top 0.2 m of soil) and average daily burrowing activity of 0.1 m day$^{-1}$, we found that earthworms would be required to consume up to 0.05 kg
of soil organic carbon per m$^2$ per year in order to account for their mechanical activity, which is as much as 5% of the estimated NPP in farmlands [11]. Throughout the studies, these estimates have been refined based on updated results. However, if we neglect the potential artifacts that came from mortaring and compacting the soil in the first experimental study, we found that our initial estimates were ultimately consistent. To summarize the findings in this dissertation:

1. New biophysical model for earthworm and plant roots penetration-cavity expansion was developed that considers different soil parameters and penetration rates to predict forces and energy requirements.

2. We evaluated the model using analog cone penetration tests and highlighted key differences in force and energy requirements between earthworms and plant roots for similar soil conditions (faster penetration is energetically more expensive).

3. The results provide direct links with earthworm distribution, seasonal activity and other ecological aspects.

4. We devised new methods to directly measure radial forces exerted by earthworm in a chamber under confining pressures.

5. Direct observations of earthworm activity in soils using neutron radiography established that penetration-cavity expansion is the dominant mode of bioturbation.

6. The important role of soil ingestion was not studied here and remains an open subject.

7. The modeling framework and tests provide new tools for quantitative studies of earthworm function and prediction of changes in behavior under future climate and land use changes.

**Bibliography**


Chapter 7

Outlook

The framework developed in this dissertation highlight the mechanical nature of soil bioturbation processes by plant roots and earthworms penetrating and expanding in wet elasto-viscoplastic soil, which are sensitive to the size and penetration rates of the respective bioturbation agents. The study outlined governing mechanisms for soil bioturbation by individual earthworms and plant roots burrowing and growing into a continuum soil medium. For future studies, a focus should be placed more explicitly on synergies between root systems and earthworm populations in order to have a broader understanding of how they contribute to soil structure formation.

Earthworm ecology and windows of activity

For earthworms, soil moisture plays a critical role in hindering their mechanical activities. While never explicitly explored in the dissertation, the mechanical model could potentially address activity windows of earthworms and plant roots. For example, we could use biome specific annual soil moisture data from a global database (Figure 7.1) [1]. Using such data, we consider soil moisture conditions for rain forests, tropical semi arid regions, wood/grasslands, and desert climates. Making simplifications regarding the soil textural class (assuming silt loam), we could use functional relationships between the soil moisture and the different soil mechanical properties (as done in Ghezzehei and Or, 2000) [2], and use the mechanical soil bioturbation model developed in this dissertation to map the soil moisture to minimum pressure limits by earthworms to penetrate and expand in the soil. We could then use the information regarding the maximum earthworm and plant root pressures [3] [4] in order to outline inner annual activity windows that would facilitate or hinder their sub-terrestrial activities (Figure 7.2) (for technical details, see Appendix D).

In this example, we would be able to see if the mechanical activity of earthworms follows very similar trends to some of the seasonality reported in different studies. For example, Nakmura [5] [6] reported earthworms activity peaks in the fall for a duration of about 4 months.
Our model shows that there are 4.5 months where mechanical activity is even plausible for similar biomes. Similarly, there is higher durations of reported activity in rain forests (9 months [7]). Our model would predict 8.5 months of mechanically suitable conditions. The model presented in this dissertation might be able to give a mechanically based explanation for some of the earthworms habitual trends.

**Figure 7.1  Biome specific 2010 time series soil moisture data for representative (a) rainforest in South America (b) tropical semi-arid forest in India, (c) wood/grassland in Africa and (d) deserts in Australia.** The solid curves for the respective plots come from the European Center for Medium-Range Weather Forecasts (ECMWF) land data assimilation system. Measurements are based on a 0-7 cm depth. The bands represent remotely sensed data (Soil Moisture and Ocean Salinity: SMOS Level 3) for the respective sites at 3 cm depth

**Outlining habitable regions for bioturbation activity apriori**

Similar to outlining the activity windows suitable for soil bioturbation, the model presented in this dissertation could also be extended to determine suitable habitats for earthworms and plant roots in different biomes. Earthworms have been found in nearly every type of ecosystem [8], thus it is difficult to accurately indicate which physical constraints may cause a regions to be inhospitable for earthworm communities. Our modeling framework would be able to show that there exist
Figure 7.2 Minimum cavity expansion pressures required to burrow/grow through soil considering the inner annual moisture data for (a) rain forests, (b) tropic arid regions, (c) wood/grasslands, and (d) deserts. Using the maximum root radial pressures and maximum radial earthworm pressures, we outline the mechanically feasible activity windows. The green shaded regions are all of the times where plant roots are not mechanically limited, and, similarly, the shaded red regions are all of the time frames when earthworms are not mechanically impeded.

mechanically limiting conditions for earthworms living in extremely dry regions. For example, Figure 7.2 (d) illustrates that there desert biomes are mechanically limiting for earthworm activities. These trends are reflected in maps if earthworm distributions around the Australian continent (Figure 7.3 redrawn from Abbott, 1994 [9]). Similar observations could be made for other regions as well (for example Europe [10] or the US [11]).

Outlining hospitable regions for earthworm activity would also allow us to illustrate potentially feasible migration trends. For example, assuming a similar biome in Mojave and Chihuahuan deserts as illustrated in Figure 7.1 (d), our mechanical model could provide an explanation as to why native earthworms never crossed these regions in North America after the glaciation [11], which is also reflected in earthworm distribution in North America (Figure 7.4 redrawn from Hendrix and Bohlen, 2002 [12]).
Interaction between plant roots and earthworms as a dynamical system

Many studies show that there is a mutualistic relationship between earthworms and plant roots. Earthworm activity relies on a sufficient amount of available energy in the form of soil organic carbon, and the soil organic carbon is predominantly comprised of plant root deposition. In contrast, earthworm activity is more relatively more rapid than root growth rates, thus the root biomass must be a limiting factor for stable earthworm population densities. However, the soil hydro-mechanical properties vastly limit the activity windows of earthworms relative to plant roots, enabling plant roots to grow when earthworms are dormant. Modeling these ecological feedbacks [13] as a dynamical system [14] could provide insights into population constraints for earthworm activity, and the modeling framework in this dissertation could provide useful constraints that could refine the modeling accuracy and predictability.

Role of plant roots and earthworms for soil structure generation

The pore structures generated by plant roots growing and earthworms burrowing in soil impact many of the physical processes such as water flow, retention, and gas diffusivity. Considering their respective activity windows and rates of soil penetration, we could use this information to see how soil bioturbation would impact soils pore size distribution. Soil bioturbation could
Figure 7.4  Current approximate distribution of earthworms in North America redrawn from Hendrix and Bohlen, 2002 [12]. Red regions indicate distributions of active earthworms. A dominate distribution of earthworms appear east of the 100° longitude line and a narrow distribution along the pacific coastline. Regions between 100° to 120° longitude west and 20° to 40° latitude north are deserts, which appear to hinder earthworm passage.

be incorporated into older stochastic models for pore space evolution (Fokker-Plank Equation [15] [16]) in order to see the impact that soil bioturbation might have on shifting the mean pore size, augmenting the modality, and the rates at which the pore size distribution peaks rise and fall. This could be linked directly to the activity windows for different biomes in order to see how structure would evolve for different climatic conditions. Ultimately, this would provide information about how other soil physical functions, such as water retention and flow, evolve under the influence of soil bioturbation.

Measuring plant root forces directly

While the studies in this dissertation re-investigated the maximum pressures that can be exerted by an earthworm’s hydroskeleton, they did not directly measure plant root pressures and forces during growth and expansion. New technology enables the possibility of directly measuring root
growth mechanical pressures much more accurately than previously reported \cite{17} \cite{18}. This would provide more confident mechanical limitations when considering biophysical parameters for our bioturbation model.

### The role of soil ingestion by earthworms

The research presented in this dissertation revealed that neutron radiography could be used to monitor (to some extent) crevice burrowing by earthworms in soil. However, the role of soil ingestion by earthworms was not resolved. Soil ingestion plays a crucial role in soil bioturbation when considering the carbon cycle (storage and emissions) \cite{19}. Ingestion may also allow earthworms to burrow under less hospitable conditions \cite{20}. Given the importance of ingestion, future studies should investigate the frequency of soil ingestion by earthworms and the soil conditions that prompt earthworm ingestion.

### The effects of soil compaction on soil bioturbation by earthworms and plant roots

Soil compaction by heavy machinery causes a reduction in pore space and an increase in the soil bulk density. This results in reduced gas diffusivity and an increased soil precompression stress. From a mechanical perspective, increasing the precompression stress would result in individual earthworms and growing plant roots to require more energy in order to move through the soil. This also limits the amount that earthworms and plant roots that can penetrate the soil, thus reducing the earthworm population density and potentially the root system. Although plant roots can exert enough pressure to overcome potentially drastic increases in precompression stress, plant roots may not be able to maintain pressure under a persistent elastic rebound (one that could be more pronounced in a compacted soil). Furthermore, oxygen limitations may also be the dominantly hindering factor when considering growing plant roots in compacted soils. Future investigations could be made in order to better understand to what extent soil compaction may impede soil bioturbation activity by these agents, and which aspects actually hinder their bioturbation activities.

### Bibliography


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Appendix A

Supporting Information: Biomechanical Limits to Soil Penetration by Earthworms-Direct Measurements of Hydroskeleton Pressures and Peristaltic Motion

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A.1 Quantifying system compressibility

Prior to complete immobilization of the earthworm, their peristaltic motion in the chamber results in pressure pulses measured by the system. The volumetric movement of water from the main pressure chamber must be accounted for by a change in the system volume. The external connectors and plastic syringe deform to accommodate the displacement of water, acting to reduce the overall pressure changes during the passage of the earthworm. To estimate the pressure response to all the volume changes in the different chamber components, we define an effective bulk system modulus:

\[ K = -V_0 \frac{\Delta P}{\Delta V} \]  

(A.1.1)
where $K [\text{Pa}]$ is the effective system bulk modulus, $V_o [\text{m}^3]$ is the systems initial volume, $-\Delta V [\text{m}^3]$ is the change in the system volume, and $\Delta P [\text{Pa}]$ is the change in the pressure. Measuring the initial system volume as $V_o = 3.45 \text{ mL}$, the system bulk modulus was estimated by inflating the inner tubing by fixed volumetric increments and measuring the changes in externally measured pressures. The internal tubing is initially closed by externally applied pressure ($P_o = 30, 50, \text{ and } 100 \text{ kPa}$) and the bulk modulus is estimated for different values, providing a relationship between the bulk modulus as a function of the externally measured chamber pressure.

### A.1.1 Determining system compressibility

Estimations were made while the internal tubing was closed by external pressures of $P_o = 30, 50, \text{ and } 100 \text{ kPa}$ (Fig. A.1.1 a). Evidence in the experimental tests showed that the bulk modulus in Fig. A.1.1 a does not have a constant bulk modulus, but gradually transitions until the chamber pressures above 150 kPa. Given the experimental evidence, the system bulk modulus was plotted as a function of externally applied pressures (Fig. A.1.1 b), and a simple logistic function was used to emulate this as a smooth transition.

![Figure A.1.1 Estimating the chamber effective bulk modulus for compressibility.](image)

(a) the chamber bulk modulus was estimated by inflating the internal tubing of the chamber by controlled volumetric intervals and observing the changes in externally measured pressure for initial externally applied pressures of 30 kPa (blue circles), 50 kPa (orange squares), and 100 kPa (yellow diamonds). Given the transition in pressure changes and consistency in slopes at higher externally applied pressures, we estimate the bulk modulus (b) as a smooth function of the external chamber pressure based on the experimental data.

Using the system bulk modulus, we estimate the pressure changes in the chamber for given radial pulses (Fig. A.1.2 ). The maximum $\Delta P \leq 25 \text{ kPa}$; however, this reduces for lower pulse amplitudes. Furthermore, we show the critical external chamber pressures where the bulk modulus
changes most significantly (Fig. A.1.2). When the device is closed at external pressures lower than 30 kPa, the bulk modulus $K = 375$ kPa. This would imply that the largest peristaltic pulse would result in pressure changes $\Delta P \leq 10\ kPa$.

![Figure A.1.2 Predicting the magnitude of pressure changes from a peristaltic pulse. Volumetric strains are computed for changing deformation amplitudes $u_0$ in equation (??). The transition region is denoted by the contour lines with the respective estimated slopes, outlining expected pressure changes in the chamber given a volumetric strain resulting from a peristaltic pulse.](image)

**A.2 Pressure measurements placed in the context of natural soil conditions**

While the pressure chamber can effectively determine maximum earthworm pressures, the deformations are dominantly elastic. Unlike the chamber, soil deformation resulting from burrowing activity is largely inelastic. To estimate magnitude of deformation that would result in soils, we utilize elasto-viscoplastic soil properties [Ghezzehei and Or, 2001] [1] corresponding to the elastic compressibility of the pressure chamber. We consider the definition of the shear modulus:
\[ G = \frac{E}{2(1+\nu)} = \frac{3K (1-2\nu)}{2 (1+\nu)} \quad (A.2.1) \]

where \( E \, [\text{Pa}] \) is the Young’s modulus of elasticity, \( \nu \, [\text{m m}^{-1}] \) is the Poisson’s ratio, and \( K \, [\text{Pa}] \) is the bulk modulus. Considering a typical Poisson’s ratio in soils (unsaturated clays, sandy clays, and silts) as \( \nu \approx 0.3 \, [\text{m m}^{-1}] \) [Bowles, 1988] [2], we estimate the shear modulus in soil as:

\[ G \approx \frac{K}{2} \quad (A.2.2) \]

where \( K \, [\text{Pa}] \) corresponds to the bulk modulus of the pressure chamber for a given externally applied pressure. Soil water contents used to estimate the rheological properties correspond to the externally applied pressures in the pressure chamber. Resulting radial deformations are estimated for the linear elastic pressure chamber as:

\[ \epsilon_{r,LE} = \frac{P_i}{2G} \quad (A.2.3) \]

where \( P_i \, [\text{Pa}] \) would be the pressure that the earthworm would apply in the chamber. These deformations are compared to the elasto-viscoplastic deformations that would occur in soils:

\[ \epsilon_{r,EVP} = \frac{P_i}{2G} + \frac{3}{4} \int_0^t \frac{P_i - P_L}{\eta} dt \quad (A.2.4) \]

where \( P_L \, [\text{Pa}] \) (eq. (D.7)) is the limit pressure for cavity expansion, and \( \eta \, [\text{Pa} \, \text{s}] \) is the soil plastic viscosity.

### A.3 Modeling peristaltic kinematics under extreme compression

Considering the system compressibility, we infer the exact changes in volume dynamically transmitted during earthworm’s peristaltic motion. The local expansions and contractions of earthworm segments (Fig. A.3.1 a-c) are estimated by local expansions within the pressure chamber. In order to estimate the transmission of the peristaltic pulse through the closed chamber, we assume that the closed chamber flattens out and can be approximated by planar 2D domain acting symmetrically on two sides (Fig. A.3.1 c).

The wave form is amplitude assumed dominated in opening perpendicular to the longitudinal direction of traveling wave and the tubing diameter. We neglect changes in the x-axis. More
A.3. Modeling peristaltic kinematics under extreme compression

Figure A.3.1 Physically modeling peristaltic pulses in the pressure chamber under extreme pressures. An earthworm passing through the chamber (a) will pressurize its hydroskeletal segments in a peristaltic manner inflating portions of the inner tubing. This pressure pulse will result in an inflation of the inner tubing (b) that will run along the inside of the pressure chamber as the earthworm passes through. These peristaltic pulses (c) will run through the inner tubing longitudinally, locally expanding portions of the inner tubing.

Specifically, the peristaltic pulse in the pressure chamber system is estimated as a propagating wave satisfying the following conditions:

\[
\begin{align*}
\frac{\partial^2 u}{\partial t^2} &= c_p^2 \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\
\frac{\partial u}{\partial x} \bigg|_{x=0} &= u \bigg|_{x=D} = 0 \\
\frac{\partial u}{\partial y} \bigg|_{y=0} &= \frac{\partial u}{\partial y} \bigg|_{y=L} = 0 \\
u(0, x, y) &= f(x, y) \\
\frac{\partial u}{\partial t} (0, x, y) &= g(x, y)
\end{align*}
\] (A.3.1)
where \( u \) [m] is the amplitude of the displacement in the tubing, \( u_0 \) [m] is the maximum amplitude of the change, \( D \) [m] is the diameter of the tubing, \( L \) [m] is the length of the tubing and \( c_p \) [m s\(^{-1}\)] is the speed of the propagating wave in the system, defined by the earthworms instantaneous burrowing rate. The boundary conditions along the x-axis requires that the opening amplitude is zero at the edges, which approximates the relaxed tube shape, having its apex at the center of the tubing (Fig. A.3.1 c). We assume that the amplitudes travel dynamically along the longitudinal direction (y-axis) allowing for non-zero values at the boundaries (Fig. A.3.1 c). The general solution given the boundary conditions is:

\[
u = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left( A_{mn} \cos (\omega_{tmn} t) + B_{mn} \sin (\omega_{tmn} t) \right) \sin (\omega_{xn} x) \cos (\omega_{ym} y)
\]

where \( \omega_{tmn} = c_p \sqrt{\left( \frac{n\pi}{D} \right)^2 + \left( \frac{m\pi}{L} \right)^2} \), \( \omega_{xn} = \left( \frac{n\pi}{D} \right) \), and \( \omega_{ym} = \left( \frac{m\pi}{L} \right) \). Considering the inner product orthogonality of \( \sin (\omega_{xn} x) \cos (\omega_{ym} y) \), the constants can be solved for as:

\[
A_{mn} = \frac{4}{LD} \int_0^D \int_0^L f(x,y) \sin \left( \frac{n\pi}{D} x \right) \cos \left( \frac{m\pi}{L} y \right) dy dx
\]

And

\[
B_{mn} = \frac{4}{\omega_{tmn} LD} \int_0^D \int_0^L g(x,y) \sin \left( \frac{n\pi}{D} x \right) \cos \left( \frac{m\pi}{L} y \right) dy dx
\]

If we specify initial conditions that would facilitate the transmission of a peristaltic pulse through the pressure system:

\[
\begin{cases}
    f(x,y) &= u_0 \sin \left( \frac{\pi}{D} x \right) \cos (\omega_y y) \\
    g(x,y) &= -c_p u_0 \sqrt{\left( \frac{\pi}{D} \right)^2 + (\omega_y)^2} \sin \left( \frac{\pi}{D} x \right) \sin (\omega_y y)
\end{cases}
\]

Obtaining the coefficients:

\[
A_{mn} = \begin{cases}
    u_0, & n = 1, m = 1 \\
    0, & n \neq 1 \text{ or } m \neq 1
\end{cases}
\]
\[ B_{mn} = \begin{cases} 
-2u_{0}\frac{(-1)^{m+1}}{\pi m(m^2-1)}, & n = 1, m > 1 \\
0, & \forall n \neq 1 \text{ or } m = 1 
\end{cases} \quad (A.3.7) \]

Given an analytic expression for the deformation, we could obtain an analytic expression for the change in the system volume. We could characterize the change in volume as:

\[ \Delta V = 4 \int_0^D \int_0^{\pi r_0} u(t,x,y) \, dx \, dy \quad (A.3.8) \]

Considering the current example, the maximum magnitude of the integral would result in:

\[ \| \Delta V \| = 4u_0 \int_0^D \int_0^{L/2} \sin \left( \frac{\pi}{D} x \right) \cos \left( \frac{\pi}{L} y \right) \, dx \, dy = 4u_0 \frac{L}{\pi} \left( \frac{2D}{\pi} \right) \quad (A.3.9) \]

The resulting volume change is then used with eq. (??) in order to determine the \( \Delta P \) for a given pressure change. Alternatively, measured pressure changes in the system are used to infer the amount of volumetric deformation that occurred in the system. Thus, for a fixed hydrostatic pressure, the earthworm can only transmit a fixed volume through the system at a time. This provides us with a volumetric flow rate during a discrete pulse.

**Bibliography**


Appendix B

Continuum cavity expansion and discrete micromechanical models for inferring macroscopic snow mechanical properties from cone penetration data

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Abstract

Digital cone penetration measurements can be used to infer snow mechanical properties, for instance to study snow avalanche formation. The standard interpretation of these measurements is based on statistically inferred micromechanical interactions between snow microstructural elements and a well-calibrated penetrating cone. We propose an alternative continuum model to derive the modulus of elasticity and yield strength of snow based on the widely used cavity expansion model in soils. We compare results from these approaches based on laboratory cone penetration measurements in snow samples of different densities and structural sizes. Results suggest that the micromechanical model underestimates the snow elastic modulus for dense samples by two orders of magnitude. By comparison with the cavity expansion based model, some of
the discrepancy is attributed to low sensitivity of the micromechanical model to the snow elastic modulus. Reasons and implications of this discrepancy are discussed and possibilities to enhance both methodologies are proposed.

Introduction

Cone penetration tests (CPT) are often used to characterize the mechanical and structural properties of a wide range of media in several disciplines. For typical engineering applications, CPT are used to deduce mechanical properties of soil for trafficability [W Whalley et al., 2007][1] and structural stability. In agriculture, CPT measurements are used to assess suitability for root growth [W R Whalley et al., 2007][1]. Recent studies have used penetrometer measurements analogously with the biophysical process of earthworm and plant roots growing or burrowing into soil [Bengough and Mullins, 1990; Ruiz et al., 2015][2][3]. In snow science, CPT are used to derive snow properties, for instance to investigate temporal and spatial changes in snow stratigraphy with regards to snow avalanche formation [B. Reuter et al., 2015; Schneebeli et al., 1999; Schweizer et al., 2016; van Herwijnen et al., 2009] [4][5][6][7] and to derive fundamental structural properties[Proksch et al., 2015] [8].

The wide range of cone penetrometer applications relies on a variety of mechanical models for parameter estimation and other inferences from cone penetrometer measurements [Adamchuk et al., 2004; Unger and Kaspar, 1994; H Yu, 2006; H Yu and J Mitchell, 1998] [9][10][11][12]. A class of continuum cavity expansion-penetration based models has been used in the geotechnical literature [H-S Yu, 2000; H Yu and J Mitchell, 1998; H Yu and Carter, 2002; H S Yu, 1993; 2006; H S Yu and J K Mitchell, 1998] [13][12][14][15][11][12] due to its ease of implementation and capability of describing elastic and plastic material deformation during cone penetration [H Yu and J Mitchell, 1998]. This methodology relies on macroscopic mechanical properties that could be determined independently from standard mechanical tests. The force penetration measurements are used to inversely determine mechanical properties and compare estimates to conventional mechanical tests under similar conditions [Ruiz et al., 2016][16].

For snow applications, a micromechanical model has been developed to interpret snow mechanical properties from specialized (high speed and highly resolved) cone penetrometer measurements [Johnson and Schneebeli, 1999] [17]. Unlike conventional continuum models, the micromechanical model statistically characterizes snow microstructural parameters based on data from the high resolution penetrometer SnowMicroPen (SMP) [Schneebeli et al., 1999] [5]. The microstructural parameters are up-scaled to represent snow sample scale mechanical properties. The original micromechanical model presented by Johnson and Schneebeli [17] has since been updated to generalize the suggested peak counting method [Marshall and Johnson, 2009] [18] as well as reformulated in terms of a Poisson shot noise process to reduce assumptions regarding spacing of snow
microstructural elements [Löwe and Van Herwijnen, 2012] [19] (details in supporting information C1). The micromechanical method has the potential to quantify the highly variable behavior during penetration through a layered snowpack. Nevertheless, there have only been a few attempts to assess the validity of the macroscopic mechanical properties obtained with this method [Capelli et al., 2016; Benjamin Reuter et al., 2013; Sigrist, 2006] [20] [21] [22]. Although, in general a good correlation was found between macroscopic snow mechanical properties obtained from the micromechanical model and from other measurement techniques, the absolute values are often diverging. Indeed, the absolute value of the SMP derived elastic modulus is generally underestimated with differences up to two orders of magnitude depending on the measurement method. [Capelli et al., 2016; Benjamin Reuter et al., 2013; Sigrist, 2006] [20] [21] [22]

Despite the benefit of the micromechanical model, it requires many underlying assumptions with regards to the distribution of rupture forces, inference of microstructural properties based on statistical parameters, and the scaling of these to bulk mechanical properties. We therefore investigate the utility of a continuum-based description of snow penetration. In this technical note, we explore the advantages, similarities and differences between the microstructural model and the cavity-expansion model in order to assess relevant mechanical properties of snow. The specific goals of this study are:

1. Review the micromechanical model (MMM) in relation to macroscopic mechanical properties
2. Introduce a continuum cavity expansion-penetration model (CEM)
3. Determine envelopes of compatibility amongst modeling approaches
4. Elucidate conditions for which the two models yield different interpretations

We begin by introducing the current state of the art method for estimating essential microstructural parameters via the shot noise process [Löwe and Van Herwijnen, 2012] [19] . Next, we determine the relationship between the micromechanical parameters and macroscopic mechanical properties [Johnson and Schneebeli, 1999; Marshall and Johnson, 2009] [17] [18]. We then introduce a continuum cavity expansion-penetration model, which uses macroscopic mechanical properties and compare the macroscopic properties determined by Johnson and Schneebeli [Johnson and Schneebeli, 1999] [17] with those inversely obtained from the continuum cavity expansion-penetration model [Ruiz et al., 2016][16] and highlight instances of equivalence. Finally, we highlight scenarios where the macroscopic parameters determined in [Johnson and Schneebeli, 1999] [17] are incompatible with the results obtained from continuum modeling [Ruiz et al., 2016][16] and discuss reasons and implications of such discrepancies with regard to other mechanical tests that have been carried out on snow.
Theoretical Considerations

All symbols can be found in supporting information Table C.0.1. A more detailed discussion of the methodology development can be found in the supporting information (Appendix C) [Ashby et al., 1986; Fierz et al., 2009; Johnson, 2003] [23] [24] [25].

Statistical determination of microstructural parameters (MMM)

The SMP measures the ensemble penetration force ($F_p$) which is interpreted as a super-positioning of microstructural forces resulting from snow microstructural elements deflecting to the point of rupture. These elements are assumed to be spaced at a specific characteristic length (Figure B.1 a-c). The Poisson shot noise process is used to statistically infer the values of element rupture forces, rupture deflection, and characteristic element length [Löwe and Van Herwijnen, 2012] [19]. Under the assumption that the rupture forces are equal in magnitude, the magnitude of a microstructural rupture force is defined as:

$$f_p = \frac{3}{2} \frac{\text{Var}(F_p(z))}{\text{Mean}(F_p(z))}$$

where $z$ [m] is the depth of penetration, $\text{Mean}(F_p(z))$ [N] is the mean value of the ensemble force, and $\text{Var}(F_p(z))$ [N$^2$] is the variance of the measured ensemble force (Figure B.1 b) over a moving window of 1.25 mm. The deflection at rupture according to Löwe and Van Herwijnen [2012] [19] is estimated by the following expression:

$$\delta_{n,r} = -\frac{3}{2} \frac{\text{Cov}(F_p(z))}{\Delta \text{Cov}(F_p(z))} \Delta z$$

where $\text{Cov}(F_p(z))$ [N$^2$] is the force covariance, $\Delta \text{Cov}(F_p(z))$ [N$^2$] is the incremental change in the force covariance, and $\Delta z$ [m] is the resolution of the penetration depth (Figure B.1 d). The microstructural element size according to Löwe and Van Herwijnen [2012] [19] and Proksch et al. [2015] [8] is the distance between given events and is defined as:

$$L_n = \left( \frac{A_s \delta_{n,r} f_p}{2 \text{Mean}(F_p(z))} \right)^{\frac{1}{3}}$$

where $A_s$ [m$^2$] is the surface area of the cone.
Figure B.1 Statistically inferring microstructural parameters from cone penetration measurements. (a) Idealized representation of the interaction between the penetrometer cone and snow microstructural elements (adapted from Johnson and Schneebeli [17]). The macroscopic penetration force profile (b) determines the mean value and variance of the penetration forces ($F_P$) along a moving window. (c) $F_P$ is interpreted as the super positioning of microstructural rupture forces ($f_p$) (arrow pointing to the ensemble of the individual ruptures). From the individual failure events (d), the microstructural parameters are defined $f_p$ rupture force, $\delta_{n,r}$ as deflection at rupture, and $L_n$ as element size.

**Derivation of micro and macro yield stress and elastic modulus**

The micromechanical parameters can be used to infer macroscopic mechanical properties. We start by deriving an expression for the microstructural element stiffness

[Johnson and Schneebeli, 1999; Marshall and Johnson, 2009] [17] [18]:

$$k_n = \frac{f_p}{\left(\delta_{n,r}(1 + \mu \cot(\alpha))\sin(\alpha)\right)}$$

(B.4)
\( \mu \) [-] (assumed 0.25) is the interfacial friction between the penetrometer (steel) and the snow (ice), and \( \alpha \) (30\(^\circ\)) is the semi-apex angle of the cone penetrometer. The micromechanical elastic modulus is then defined as [Johnson and Schneebeli, 1999; Marshall and Johnson, 2009] [17] [18]:

\[
E_n = \frac{k_n}{L_n}
\]  
(B.5)

and the micromechanical yield stress is defined as [Johnson and Schneebeli, 1999] [17]

\[
\sigma_n = \frac{k_n \delta_{n,r}}{L_n^2}
\]  
(B.6)

The micro scale properties are macroscopically scaled by taking the product of intact contact probability (defined as \( \delta_{n,r} \) [Johnson and Schneebeli, 1999; Marshall and Johnson, 2009] [17] [18]) and the respective microscopic parameters. The macroscopic elastic modulus is then [Johnson and Schneebeli, 1999] [17]

\[
E_{mac} = \frac{\delta_{n,r} E_n}{L_n}
\]  
(B.7)

and the macroscopic yield stress [Marshall and Johnson, 2009] [18]

\[
\sigma_{mac} = \frac{k_n}{V_e}
\]  
(B.8)

where \( V_e = \frac{4}{3} \pi (\frac{L}{2})^3 \) is the structural element volume [Marshall and Johnson, 2009] [18]. Substituting (B.5) and (B.6) into (B.7) and (B.8), we see that the macroscopic yield stress is related to the macroscopic modulus of elasticity:

\[
\sigma_{mac} = \frac{6 \delta_{n,r} E_{mac}}{\pi L_n}
\]  
(B.9)

Furthermore, the ensemble penetration force can be described as a function of the macroscopic yield stress. By substituting (B.6), (B.4), and (B.3) into (B.8), we obtain the following expression for the ensemble penetration force as a function of macroscopic properties [Marshall and Johnson, 2009] [18]:

\[
F_p(z) = \pi \sigma_{mac}(1 + \mu \cot(\alpha)) A_c/12
\]  
(B.10)
where $A_c = A_s \sin(\alpha) \text{[m}^2\text{]}$ is the cross-sectional area of the cone. This formulation implies that mean penetration forces are explicitly related to the sample scale yield stress only and only implicitly related to sample elasticity (in accordance with the second operating assumption [Johnson and Schneebeli, 1999] [17] in supporting information C1).

### A continuum mechanical model for cone penetration forces

We propose an alternative method for estimating snow bulk mechanical properties. Independent estimates of soil macroscopic mechanical properties were obtained using a continuum based cavity expansion-penetration model (CEM) [Bishop et al., 1945; Carter et al., 1986; H Yu, 2006; H Yu and J Mitchell, 1998] [26] [27] [11] [12]. The model is derived from the static equilibrium equation that describes the decay of radial stress along the radius $r$ from the center of the cavity into the surrounding medium:

$$\frac{\partial \sigma_r}{\partial r} = -\frac{\sigma_r - \sigma_\theta}{r}$$  \hspace{1cm} (B.11)

where $\sigma_r$ [Pa] is the radial stress and $\sigma_\theta$[Pa] is the circumferential stress. In this study, we neglect the effects of overburden stresses, as the strength of snow is primarily due to cohesion of snow elements. The transition from elastic to plastic deformation is expressed with the Von-Mises yield criteria:

$$\sigma_r - \sigma_\theta = \frac{2 \sigma_{mac}}{\sqrt{3}} + \left( \frac{4}{3} \right) \eta \dot{\varepsilon}_r$$  \hspace{1cm} (B.12)

where the full expression $(\frac{2 \sigma_{mac}}{\sqrt{3}})$ [Pa] represents the materials macroscopic shear strength, $\eta$ represents the material viscosity under yield, and $\dot{\varepsilon}_r$ is the radial strain rate of the yielding material.

While snow is known to exhibit rate dependent mechanical behavior [Mellor, 1974] [28], for a first estimate we simplify the problem by only considering a quasi-static system ($\eta = 0$). Substituting (B.12) into (B.11) and integrating yields the following expression:

$$\sigma_r(r) = P_L - \frac{2 \sigma_{mac}}{\sqrt{3}} \ln\left( \frac{r}{r_c} \right)$$  \hspace{1cm} (B.13)

where $P_L$ [Pa] is the radial pressure at the cavity walls as the cavity expands indefinitely and $r_c$ [m] is the cavity radius. To solve for the limit pressure $P_L$, we assume that for large enough cavity radius $r_c$, there is a remote elastic zone beyond the radius $R$ that contains the elasto-plastic deformation. The ratio between the cavity zone and the plastic zone converges to the following
expression [Bishop et al., 1945; Carter et al., 1986] [26] [27] (Figure B.2):

\[
\left( \frac{R}{r_c} \right)^2 \rightarrow \frac{\sqrt{3}E_{mac}}{2(1+\nu)\sigma_{mac}}
\]  

\(\text{Figure B.2 Relationship between cone penetration and cavity expansion.}\) The penetrating cone (a) creates a radial stress that compresses a zone around the face of the cone. This is represented by the expansion of a cavity (b), where the zone surrounding the cavity \((r_c)\) is the plastic region, and it is bound by an elasto-plastic interface at a radius \(R\). The cavity expansion-penetration model (CEM) is used to interpret the penetration force signal (c) by changing the yield stress and elastic modulus to fit the characteristic penetration profile to the SMP signal using a least squares fit.

where \(R\) [m] is the radius of the elasto-plastic interface, \(r_c\) [m] is the cavity radius, \(\nu = 0.2\) [-] is the Poisson’s ratio [Mellor, 1974] [28], \(E_{mac}\) [Pa] is the sample scale elastic modulus, and \(\sigma_{mac}\) [Pa] is the sample scale yield stress. At the elasto-plastic interface, the radial stress equals the shear strength \((\sigma_r (R) = \frac{\sigma_{mac}\sqrt{3}}{\sqrt{3}})\), and thus the limit pressure is of the form:

\[
P_L = \frac{\sigma_{mac}}{\sqrt{3}} \left( 1 + \ln \left( \frac{\sqrt{3}E_{mac}}{2(1+\nu)\sigma_{mac}} \right) \right)
\]

\(\text{B.15}\)

The standard limit pressure theory of cavity expansion is independent of any length scale. However, scale plays a particularly important role when addressing expansion of small cavities (e.g. \(r_c < 4.5\) mm [Ruiz et al., 2016][16]). Johnson [2003] [25] characterized scale effects as statistical deviations from the mean penetration resistance stress. Ladjal and Wu [2011] [29] later derived the scale effect from a hardening parameter, which is a function of second order strain gradient. The gradient based approach was originally used by Vardoulakis and Aifantis [1991] [30] to approximate the rate dependent flow law of the yielding material. The scale effect can be managed without assuming higher order gradient terms when considering rate dependency [Ruiz et al., 2017][31]. For the rate independent penetration model considered in this study, we
approximate of the scale dependency with the radial stress equation [Ruiz et al., 2016] [16]. For radius \( r \) smaller than \( r_c = 4.5 \text{ mm} \), the radial stresses during penetration expansion can thus be expressed as:

\[
\sigma_r(r) = \frac{\sigma_{mac}}{\sqrt{3}} \left( 1 + \ln \left( \frac{\sqrt{3}E_{mac}}{2(1+\nu)\sigma_{mac}} \right) \right) - 2\ln \left( \frac{r}{r_c} \right) \tag{B.16}
\]

where \( r_c \) is the minimal cavity radius where the limit pressure is attained [Bishop et al., 1945; Ruiz et al., 2016] [26] [16], and \( r \) is the active cone radius. We determine the radial forces acting on the cone face [Ruiz et al., 2016][16]:

\[
F_r = 2\pi \cot(\alpha) \int_0^{r_{cone}} \sigma_r(r)rdr \tag{B.17}
\]

where \( \alpha \) is the semi-apex angle of penetration (30° in this study) and \( r_{cone} \) is the cone base radius (2.5 mm). We determine the frictionless axial force required for cone penetration

\[
F_z = F_r \tan(\alpha) \tag{B.18}
\]

and by factoring in the frictional effects, we determine the final measured axial force [Ruiz et al., 2016][16]

\[
F_P = F_z(1 + \mu \cot(\alpha)) \tag{B.19}
\]

**Materials and Methods**

**SMP data and signal processing**

We used SMP force signals obtained by van Herwijnen and Miller [van Herwijnen and Miller, 2013] [32], who performed laboratory experiments on homogeneous snow samples to a depth of 80 mm over time to determine the sintering rate of snow. Six snow samples were analyzed for the first ten sintering times [32] (Details in supporting information C2 and Table C.0.1).
Estimating mechanical properties with MMM

For the different snow types, the micromechanical parameters were determined with the Poisson shot noise process (Löwe and Van Herwijnen [2012] [19]) using a moving window of 1.25 mm size and averaging the value for the entire signal length (mean of values of all windows up to 80 mm) with no overlap (illustrated in Figure B.1 b). Microstructural parameters were used to determine the bulk mechanical properties \(E_{\text{mac}}\) and \(\sigma_{\text{mac}}\).

Estimating mechanical properties with CEM

For each experiment, the curve characteristic to an ideal penetration profile (Figure B.2 c) was fit to the entire data using a least squares algorithm and adjusting the macroscopic yield stress and elastic modulus [Ruiz et al., 2016; Wraith and Or, 1998][16] [33]. Initial values for the yield stress and elastic modulus \(E_{\text{mac}}\) and \(\sigma_{\text{mac}}\) where chosen based on average values from the range presented in Mellor 1974 [28] for a given density of snow (Details in supporting information Table C.0.1).

Comparison to snow mechanical properties in Mellor 1974

In the absence of independent and standard measurements of the snow mechanical properties, we opted for comparison of the mechanical properties deduced from the CEM and the MMM with the expected mechanical properties for different snow densities compiled by Mellor [28]. Values of macroscopic yield stress and macroscopic elastic modulus obtained from the CEM and the MMM were plotted alongside the ranges of elastic moduli and yield stress values as a function of snow density as summarized in Figures 2 and 17 of Mellor [28]. Snow properties from Mellor were estimated for snow at temperatures ranging from -6 to -25\(^\circ\) C [28].

Results

Comparing mechanical properties obtained using MMM and CEM

We assess the bulk snow macroscopic elastic modulus and yield stress for all samples deduced from the MMM and CEM models as depicted in Figure B.3. While both estimates for yield stress (Figure B.3 a) correlate well \((R^2 = 0.96)\), there is a bias between the two approximated values, with yield stress determined by the MMM being consistently an order of magnitude greater than the yield stress determined by the CEM. The comparison between the macroscopic elastic modulus
was not as well correlated for all of the density values (Figure B.3 b). Focusing on the low density samples, the estimates for the elastic modulus from the two models are similar to one another (graphically near the 1:1 line); however, the macroscopic elastic modulus for the dense snow is estimated many orders of magnitude higher for CEM than with the MMM.

Comparison to snow mechanical properties in Mellor 1974

![Figure B.3 Comparison between (a) bulk snow yield stress and (b) the bulk snow effective elastic modulus obtained from the micromechanical model (MMM) and the cavity expansion-penetration model (CEM) penetration model for the different snow types for the first cavity expansion estimates. Snow types are plotted in order of descending bulk densities (see Table C.2.1 for details). The 1:1 line is marked red. For each sample, 10 penetration tests were considered for different sintering times, thus each sample has 10 colored symbols.]

For the macroscopic yield stress (Figure B.4 (a)) the estimates from the MMM perform very consistently with the yield stress expected from literature for the higher density snow samples. For the lower density snow samples, the MMM is consistently out of range by nearly an order of magnitude from the expected strength values. The CEM predicts lower yield stress values for higher density snow samples, with few of the predicted values overlapping the lower bound of the expected magnitude. However, the CEM predicts values for the yield stress within the expected range for the lower density samples. Looking at the expected values for elastic modulus (Figure B.4 (b)), the MMM is several orders of magnitude less than the expected range for the higher density snow. For the low density samples the MMM appears to be reasonably within the expected range. However, for the higher density samples, the MMM is two orders of magnitude below the expected range of elastic properties. The CEM estimated elastic modulus values fall within the expected range for the samples considered in this study, and are still consistent with
the average penetration forces.

**Discussion and conclusion**

![Figure B.4](image)

Figure B.4 (a) Bulk snow compressive yield stress and (b) bulk effective elastic modulus obtained from the micromechanical model (MMM) and the cavity expansion-penetration model (CEM) penetration model as a function of snow density taken from Mellor 1974 [28] (orange shaded region in the plots). Snow types are plotted in order of descending bulk densities (see Table C.2.1 for details). Open circles were the results from the CEM and closed diamonds are from the fit of the MMM.

In this study we presented a new approach to derive the yield stress and the elastic modulus of snow samples from SMP data using a method based on the continuum cavity expansion model. The macroscopic yield stress (Figure B.3 a) estimated by the MMM were an order of magnitude larger than that of the CEM. The macroscopic interpretation of the force signal by the MMM neglects the zone of compression around the cone tip and thus the influence of a remote elastic zone plays no explicit role in the MMM estimation of mechanical properties. Thus, the burden of the penetration force has to be entirely compensated by the macroscopic yield stress (and a factor of friction). This may be a reasonable estimate for lower density snow samples, where the SMP could be pulling down a net of loose snow elements during penetration, thus failing in tension. However, for the higher density snow samples, there will be compaction around the cone [van Herwijnen, 2013] [34]. This is considered in the CEM, thus the penetration force is partitioned...
between the macroscopic yield stress and the remote influence of elasticity.

The CEM and MMM estimated different values for the macroscopic elastic modulus (Figure B.3 b). While the two models produced similar values for the low density elastic modulus, the estimates for the elastic modulus by the CEM were two orders of magnitude greater than the MMM for the high density snow samples. For the dense samples, the compression zone around the cone tip will have a non-negligible influence on the force measurement. The failure zone is remotely bounded by the elasticity of the snow. This remote rebound contributes to the force measurement; however, the effect is dissipated in the proximity of the cone. The MMM would thus interpret this dissipated local effect as the elastic modulus and underestimates the elastic modulus.

These parameters were plotted in context with expected values [28] of macroscopic snow elastic modulus and yield stress (Figure B.4). Observing the comparison to the yield stress (Figure B.4 (a)), there are no clear drastic deviations from the expected values. The MMM estimates yield stress values that are an order of magnitude greater than the expected range for the low snow density. However, it predicts the yield stresses well for the high density samples. The CEM is within the expected range of yield stress values for the low density snow, and below the lower bound for the higher density snow. The estimated influence of scale in this study resulted in compressive strength 30% lower than the values that would be obtained with a scale free model. Considering the span of the data in Figure B.4, scale dependency presented little consequence on the trends of the CEM predictions. Despite falling out of the range presented by Mellor [28], they are consistent with cohesion values obtained for similar snow densities [Shapiro et al., 1997] [35].

The elastic modulus values obtained by the MMM (Figure B.4 (b)) were in good agreement for the loose snow samples. However, the MMM predicted values for elastic modulus do not seem to vary with density. Previous studies have shown that the elastic modulus of snow vary as a function of density over several orders of magnitude [Mellor, 1974 [28]], thus the MMM predicted elastic modulus does not seem sufficient for characterizing the elasticity of dense snow. The values for elastic modulus estimated by the CEM for high density snow were consistent with other studies that used independent methods to measure the elastic modulus and compared the values with the penetration forces obtained with the SMP ([Capelli et al., 2016; Benjamin Reuter et al., 2013; Sigrist, 2006] [20] [21] [22]).

While the determination of snow microstructural parameters is sufficient (in particular for loose snow), the macroscopic mechanical parameterization has difficulties estimating the elastic modulus in dense snow. Since it has been observed that there is a considerable zone of compaction local to the cone surface during penetration, neglecting the remote influence of elasticity is insufficient for properly inferring the macroscopic elastic modulus. While there have been attempts to incorporate the compaction zone in a version of the MMM [Johnson, 2003] [25], there have been no rigorous attempts to upscale microstructural parameters to macro scale mechanical properties. Therefore, a new parameterization is required that can account for the non-negligible influence of
this compacted zone.

Alternatively, the continuum mechanical model (CEM) inferences snow bulk mechanical properties considering a remote elastic influence that dense snow would have during cone penetration, and also allows the model to predict the higher elasticity of snow while characterizing the penetration force profile (Details in supporting information S3). Furthermore, the CEM is capable of considering effects of different cone penetration rates, and potential of snow viscous-deformation [Mellor, 1974 [28]] by considering different rheological formulations [Ruiz et al., 2017][31], which also manages scale dependency. However, unlike the MMM, estimates of mechanical properties require an initial value. For a given snow density, we tested the upper and lower bounds of the expected mechanical properties ($E_{mac}$ and $\sigma_{mac}$ [28]). The estimated properties were in the same range as presented in Figure B.4, with a consistent under estimation of $\sigma_{mac}$ for dense snow samples. The CEM also fails to resolve changes in properties over individual layers of stratified snow cover, which is a key function implicit in the MMM.

The importance for inference of snow bulk mechanical properties warrants further investigation and independent validation. A false estimate of snow strength could present a problem if used for snow stability evaluation [Schweizer et al., 2008][36] [B. Reuter et al., 2015][4]. Independent mechanical tests should therefore be carried out to assess the reliability of the both approaches to predict bulk mechanical parameters.

In conclusion, in this study we presented a continuum cone-penetration model for inferring macroscopic snow mechanical properties. A comparison between the CEM and the standard MMM showed the CEM is capable of estimating high elastic moduli values of denser snow that the MMM is not capable of doing. Given the value and simplicity of cone penetration measurements, the study proposes the CEM as an alternative for practical applications, capable of providing estimates for layered snow and for different penetration rates (both not tested in this study). An interesting synergy could be gained by linking MMM microstructural inferences with the CEM to improve interpretation of bulk mechanical properties (by both) in order to constrain the CEM and expand the range of predictability by the MMM.

**Acknowledgement**

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Bibliography


Appendix C

Supporting Information: Continuum vs discrete CPT Models in snow

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Introduction

This supplementary material presents (1) a comprehensive review of the development and current state of the micro-mechanical model (MMM), (2) physical properties of snow used in the penetration experiments, and (3) illustration of the cavity expansion model (CEM) fit to data compared with the mean value of the MMM fit to snow penetration data taken at different densities.

C.1 Review of the micromechanical model

The SMP was developed as a method to rapidly measure snow stratigraphy in particular to detect thin snow layers which are often associated with snow avalanche release [Schneebeli and Johnson, 1998] [1]. Among the novel aspects of the SMP were a miniature cone radius (2.5 mm) coupled with high resolution force measurements (currently at 5 kHz [van Herwijnen and Miller, 2013] [2]) and constant penetration speed (20 mm s⁻¹). The coupled geometry and measuring
frequency ideally enables the SMP to detect the failure of micro structural snow elements. Interpretation of these highly resolved penetration measurements prompted the formulation of the micro-mechanical model. The development of a micro-mechanical model dates back to Ashby et al. [1986] [3], whereas its formal introduction to snow was by Johnson and Schneebeli [1999] [4]. The original model has three fundamental assumptions:

1. The penetration force consists of the rupturing of elastic micro structural elements and ice friction on the cone

2. The influence of compaction around the cone is negligible, thus neglecting remote elastic effects

3. The micro structural elements have a mean constant size

Previous attempts to consider this zone of influence in the micromechanical model still greatly underestimate the influential region and provided no clear means of upscaling micro-
C.2. Physical properties of snow used in the penetration experiments [van Herwijnen and Miller, 2013] [2]

Table C.2.1 Physical properties of the different snow types determined at final sintering times. Grain type: (decomposed and fragmented particles (DF), rounded grains (RG) and depth hoar (DH)) according to Fierz et al. [2009] [8], and initial values for macroscopic Engineering elastic modulus and yield stress for least squares algorithm. Initial values were determined based on the median value within the range for a given snow density taken from Mellor [1974] [9]. The sample labels (including the *) are the same as in van Herwijnen and Miller [2013] [2].

<table>
<thead>
<tr>
<th>Sample</th>
<th>Density $\rho_b$ [kg m$^{-3}$]</th>
<th>Grain type and size [mm]</th>
<th>Final sintering time [min]</th>
<th>Engineering elastic modulus $E_{mac}$ [Pa]</th>
<th>Macroscopic yield stress $\sigma_{mac}$ [Pa]</th>
<th>Plot symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>400</td>
<td>RG (0.25)</td>
<td>474</td>
<td>$1.8 \times 10^8$</td>
<td>$1.2 \times 10^5$</td>
<td>◆</td>
</tr>
<tr>
<td>P</td>
<td>380</td>
<td>RG (0.55)</td>
<td>405</td>
<td>$1.4 \times 10^8$</td>
<td>$6.5 \times 10^4$</td>
<td>□</td>
</tr>
<tr>
<td>N</td>
<td>377</td>
<td>DH (1)</td>
<td>320</td>
<td>$1.2 \times 10^8$</td>
<td>$6.2 \times 10^4$</td>
<td>◊</td>
</tr>
<tr>
<td>L</td>
<td>375</td>
<td>DH (1)</td>
<td>320</td>
<td>$1.1 \times 10^8$</td>
<td>$6.1 \times 10^4$</td>
<td>△</td>
</tr>
<tr>
<td>C*</td>
<td>152</td>
<td>DF (1)</td>
<td>346</td>
<td>$1.0 \times 10^9$</td>
<td>$1.1 \times 10^4$</td>
<td>▼</td>
</tr>
<tr>
<td>A*</td>
<td>104</td>
<td>DF (0.75)</td>
<td>321</td>
<td>$2.8 \times 10^9$</td>
<td>$3.0 \times 10^2$</td>
<td>◊</td>
</tr>
</tbody>
</table>

We used SMP force signals obtained by van Herwijnen and Miller [van Herwijnen and Miller, 2013] [2], who performed laboratory experiments on homogeneous snow samples to a depth of
80 mm over time to determine the sintering rate of snow. Six snow samples were analyzed for the first ten sintering times [2].

### C.3 Illustrative plot of the different models overlaying the snow penetration data

![Penetration force with depth for snow sample (a) R, (b) L, (c) C* and (d) A*](image)

The measured cone penetration-force values for the different snow samples are depicted in Figure C.3.1 along with the resulting penetration model predictions for estimated mean snow mechanical parameters from the two models. For all snow densities, the two penetration models (MMM and CEM) yield similar estimates for the mean penetration force profiles.
Bibliography


Appendix D

Climatic and Biophysical Constraints affect Activity Windows by Earthworms and Plant Roots

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Abstract

Burrowing earthworms and decaying plant roots result in formation of networks of soil biopores. These important structural elements affect soil hydrological and ecological functioning. Recent biophysical models quantify the stresses and energetic costs of root growth and earthworm burrowing and thus offer new insights into climatic and ecological windows of activity for these biological agents. Earthworms and roots experience different constraints in part due to the rapid penetration rates by earthworms relative to growing plant roots, and different limiting pressures (200 kPa for earthworms and ≥ 1 MPa for plant roots). These biophysical differences permit activity under different soil and wetness conditions, much narrow windows for earthworms than for plant roots. The modeling results delineate climatic regions and periods where earthworm activity becomes mechanically prohibitive while plant roots remain unhindered. Activity windows derived from the soil bioturbation model potentially allow us to explore the ecological ramifications of these constraints under different climatic scenarios and outline regions that hinder soil bioturbation by earthworm activity.
Introduction

The mechanical and ecological consequences of subterranean activity by earthworms and plant roots are key to sustaining favorable soil structure that promotes numerous ecosystem services[1]. Soil biopores forming by burrowing earthworms and growing plant roots serve as preferential pathways for water and aeration [2], they may enhance ground water recharge, and extend water retention and oxic conditions [3] [4]. Empirical evidence suggests that earthworm burrowing activity could remediate negative effects of soil compaction [5] and, in some cases, enhance crop yields by 25% [6]. Similarly, biopores generated by decaying plant roots enhance aeration and infiltration and provide soil organic carbon for soil biological activity [7]. Both types of biopores (generated by earthworms and plant roots) may act as hot spots for microbial activity and promote other aspects of soil structure such as formation of soil aggregates [8].

The rates and extent of soil bioturbation by earthworms and plant roots depends on their capacity to burrow and growth through the soil. The extending tips of plant roots axially penetrate into soil and subsequently expand radially [9]. Earthworms may employ different mechanical modes for burrowing, direct imaging has shown that active motion through the soil is dominated by crevice burrowing or penetration-cavity expansion [10] [11]. This mode of burrowing relies on earthworms locally extending frontal segments in order to penetrate the soil, followed by a subsequent local expansion of segments of their body to anchor and recollect extended segments [10] [11].

The biophysical modeling of earthworms and plant root penetration employs penetration-cavity expansion principles similar in some ways to cone penetration [11]. A class of soil penetration models based on cavity expansion dates back to early studies by Bishop et al. [12] and Carter et al. [13]. Elements of these models were adopted for quantifying root growth into soil(Dexter, 1987, Farrell & Greacen, 1966, Greacen & Oh, 1972) [14] [15] [16] and subsequently for earthworm burrowing [17]. An interesting recent expansion of these models, considers the rates of penetration into wet elasto-viscoplastic soil (Ghezzehei & Or, 2001) [18] that gives rise to differences between earthworms and plant roots [11].

Earthworms can be found in virtually all ecosystems [19], yet, dry climates and periods where soil is dryer than a certain threshold limit the geographic distribution and seasonal activity patterns of earthworms. Some studies have alluded to links between seasonal activity and temperature and soil moisture conditions [20], more recently, this limits were formalized as mechanical constraints [11]. Yet, a systematic study of the ecological limits to earthworm activity as determined by climatic and soil conditions is lacking. We hypothesize that the mechanical model for earthworm bioturbation by earthworms [11] could provide the basis for predicting earthworm activity windows and geographic distributions. Preliminary examples were presented in the outlook attributed to the potential of the current mechanical model for doing this. Here, we will briefly discuss the technical aspects as to how we generated the activity windows for earthworms and
Theoretical considerations

Penetration cavity-expansion mechanical model – an overview

The mechanics of earthworms radially expanding cavities in soil have been reported in detail [Ruiz et al., 2015; Ruiz et al., 2017] [17] [11], we thus provide only a brief overview. To quantify the magnitude of radial pressure required by an earthworm or plant root to expand in wet elasto-viscoplastic soils, we first consider the force balance at equilibrium:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0$$  \hspace{1cm} (D.1)

where \(r\) [m] is the distance from the center of the cavity, \(\sigma_r\) [Pa] is the radial stress and \(\sigma_\theta\) [Pa] is the hoop (circumferential) stress. The deformation behavior is expressed by the Von-Mises criterion considering viscous deformation (Bingham model [25] [18]), relating the difference between the radial and hoop stresses to the summation of the undrained soil strength and the viscoplastic strain rate:

$$\sigma_r - \sigma_\theta = 2s_u + \frac{4}{3} \eta \dot{\varepsilon}_r$$  \hspace{1cm} (D.2)

where \(\eta\) [Pa s] is the soil plastic viscosity, \(s_u\) [Pa] is the undrained soil strength, and \(\dot{\varepsilon}_r\) [m m\(^{-1}\)s\(^{-1}\)] is the radial strain rate. The \(\frac{4}{3}\) factor emerges from the cylindrical geometry[Bishop et al., 1945]. Substitution of Eq. (D.2) into (D.1) yields the following expression:

$$\frac{\partial \sigma_r}{\partial r} = -\frac{2s_u}{r} - \frac{4}{3} \eta \dot{\varepsilon}_r$$  \hspace{1cm} (D.3)

By integration, we determine the radial stresses as a function of the radius (and the strain rate):

$$\sigma_r (r) = P_L - 2s_u ln\left(\frac{r}{r_c}\right) - \frac{4}{3} \eta \int \frac{\dot{\varepsilon}_r}{r} dr$$  \hspace{1cm} (D.4)

where \(r_c\) [m] is the minimum cavity size where the static cavity pressure converges to \(P_L\) [Pa] is the time independent limit pressure. Under static conditions, the strain rate term in the integral vanishes. We solve for the limit pressure by equating the change in the cavity zone to the change in the plastic region local to the cavity:
\[ \left( \frac{R_p}{r_c} \right)^2 \rightarrow \frac{G}{s_u} \quad (D.5) \]

Where \( G \) [Pa] is the soil's shear modulus, and \( R_p \) is the elasto-plastic interfacial radius. Under static conditions, the radial stress at the elasto-plastic interface could be expressed as:

\[ \sigma_r (R) = P_L - 2s_u \ln \left( \frac{R_p}{r_c} \right) = s_u \quad (D.6) \]

And thus the minimum radial pressure required to expand a cavity in soil could be inferred by the limit pressure:

\[ P_L = s_u \left( 1 + 2 \ln \left( \frac{R_p}{r_c} \right) \right) = s_u \left( 1 + \ln \left( \frac{E}{2 s_u (1 + \nu)} \right) \right) \quad (D.7) \]

thus the full expression for the rate dependent radial expansion stress during cavity expansion is:

\[ \sigma_r (r_c, t) = s_u \left( 1 + \ln \left( \frac{G}{s_u} \right) \right) - \frac{4}{3} \eta \int \frac{\dot{\varepsilon}_r}{r} dr \quad (D.8) \]

where \( r_c \) is assumed to be the cavity radius, and \( t \) is time (resolving the integration is explained in detail in Chapter 2). The resulting expression would be the minimum amount of pressure an earthworm or plant root would have to exert in order to expand a cavity radially in soil.

**Functional relationship between soil hydration status and mechanical properties**

A functional relationship between soil water content and soil mechanical parameters was established in order to estimate the activity windows. The relationship used (adapted from Ghezzehei and Or, 2001)[18] fits a two parameter exponential equation relating soil shear modulus, shear strength and soil plastic viscosity to soil water content. The equations are as follows:

\[ \eta_p = a\eta \exp(b\theta_m) \quad (D.9) \]

\[ s_u = a_y \exp(b_y \theta_m) / (\sqrt{3}) \quad (D.10) \]

\[ G = a_G \exp(b_G \theta_m) \quad (D.11) \]
where \(a_\eta\), \(a_y\), and \(a_G\) are the magnitudes of the soil viscosity, shear soil strength, and shear modulus, while \(b_\eta\), \(b_y\), \(b_G\) determines their exponential change as a function of water content \(\theta_m\).

Making simplifications regarding the soil textural class (assuming silt loam), we could use functional relationships between the soil moisture and the different soil mechanical properties using mechanical properties obtained in Ghezzehei and Or, 2001)[18]. Functional coefficients are listed in Table D.0.1.

Table D.0.1  Functional relationship coefficients for Millville silt loam soil

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_\eta) [Pa s]</td>
<td>(5 \times 10^7)</td>
</tr>
<tr>
<td>(b_\eta) [-]</td>
<td>-24.16</td>
</tr>
<tr>
<td>(a_y) [Pa]</td>
<td>(2 \times 10^6)</td>
</tr>
<tr>
<td>(b_y) [-]</td>
<td>-25.63</td>
</tr>
<tr>
<td>(a_G) [Pa]</td>
<td>(2 \times 10^8)</td>
</tr>
<tr>
<td>(b_G) [-]</td>
<td>-27.5</td>
</tr>
</tbody>
</table>

Ghezzehei & Or, 2001

**Methods and materials**

**Biome specific inner annual moisture data**

For the example used in the outlook, we used biome specific inner annual soil moisture data from a global database (Figure 7.1) [21]. We considered soil moisture conditions for rain forests, tropical semi arid regions, wood/grasslands, and desert climates. For all the biomes, we used the data from the European Center for Medeium-range Weather Forecasts (ECMWF) land data assimilation system, as it was the least noisy of the data sets provided in the study (7 cm depth). However, we provide bands outlining the soil moisture measurements made by the Soil Moisture and Ocean Salinity: SMOS Level 3 remote sensed data (3 cm depth). Soil moisture data was fed into the functional relationships in order to extract soil mechanical properties (shear modulus, soil shear strength, and soil plastic viscosity). With the innerannual soil moisture data and our mechanical model, we could estimate the minimum radial stresses required to expand a cavity in soil. This cavity expansion pressure is plotted throughout the year.

**Biomechanical constraints for earthworms and plant roots**

We consider the biomechanical radial limit pressure of earthworms (200 kPa) [26][23] and expanding plant roots (\(\approx 1000\) kPa) [27] along with the inner annual radial stresses required to
expand a cavity in soil given moisture conditions. While the radial stress requirement is below the maximum pressure threshold of earthworms or plant roots, we highlight these periods as feasible periods for bioturbation activity (Figure 7.2).

**Expected results**

The biome specific activity windows for earthworms and plant roots illustrated in Figure 7.2 make it possible for us to explore ecological questions from a mechanical perspective. In particular, we expect that from the preliminary findings reported in our outlook, we expect to answer some of the following questions:

1. Determination of biomes and conditions that would support earthworms’ and plant roots’ mechanical activities
2. Infer how these conditions could be used to predict earthworm migration or invasion trends
3. Make predictions on how this activity will be altered considering changes in climate and land use
4. Determine potential synergies between earthworms and plant roots based on their different activity windows and bioturbation rates.

**Bibliography**


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\[
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\] 29
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5.9 **Measuring and modeling peristaltic pulses in the pressure chamber.** Images of the earthworm passing through the pressure chamber (a-b) illustrate the radial displacement during peristaltic motion. An external chamber pressure was increased to 120 kPa as an earthworm passed through the system (c). As the earthworm passed through the pressure chamber, irregular pressure oscillations were monitored over time (blue curve). Pressure fluctuations as a result of propagating waves on the inner tubing were modeled for a volumetric strain of $7.5 \times 10^{-3}$ (orange solid curve) mL mL$^{-1}$ using the upper limit wave propagation speed obtained from the FFT analysis (frequency of 0.13 Hz in Table 5.2). Zooming into the comparison between the measured and modeled pulse (d), the resulting pressure fluctuations from a volumetric strain of $7.5 \times 10^{-3}$ (mL mL$^{-1}$) are consistent with the measured peristaltic pulses in the system.

5.10 **Modeling pressure pulses responses during peristaltic motion for a range of mechanical properties corresponding to the pressure chamber (linear elastic) and natural partially saturated soils (elasto-viscoplastic).** The resulting pressure pulses response (a) was simulated for three sets of mechanical properties associated with pressure chamber bulk modulus and partially saturated soils. For the same pressure input, the resulting deformations (b) can be seen for both the linear elastic pressure chamber and the elasto-viscoplastic soil.

7.1 **Biome specific 2010 time series soil moisture data for representative (a) rainforest in South America (b) tropical semi-arid forest in India, (c) wood/grassland in Africa and (d) deserts in Australia.** The solid curves for the respective plots come from the European Center for Medium-Range Weather Forecasts (ECMWF) land data assimilation system. Measurements are based on a 0-7 cm depth. The bands represent remotely sensed data (Soil Moisture and Ocean Salinity: SMOS Level 3) for the respective sites at 3 cm depth.
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Italian  Intermediate
Spanish  Beginner
\[
\frac{\partial \sigma_r}{\partial r} - \frac{\sigma_r - \sigma_\theta}{r} = 0
\]

\[
\sigma_r(t, r_c) = 2 \frac{\sigma_y}{\sqrt{3}} + \frac{4}{3} \eta \dot{e}_r
\]