Basic Research, Openness, and Convergence

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Abstract

We study a model where economic growth is fueled by public basic-research investment and the importation of leading technology from foreign countries. In each period, the government chooses the amount of basic research, balancing the cost and benefits of stimulating growth through both channels. We establish the existence of steady states and the long-run share of technologically advanced sectors in the economy. Then, we explore how different degrees of openness affect long-term incentives to invest in basic research. Our main insight is that higher openness tends to encourage more investment in basic research which, in turn, yields a larger share of leading sectors. If, however, there are prospects of importing large technology advancements, highly open countries will reduce basic research as such imports become particularly valuable.

Keywords: basic research, openness, economic growth.

JEL: O31, O38

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1 Introduction

Policy-makers in industrialized countries often face subtle tradeoffs when they try to foster economic growth. For instance, should a country foster domestic innovations by investing in basic research, or should it rely on potential market entry by foreign firms with superior technology? In this paper we study this question.

We develop a Schumpeterian growth model in which government investments that take the form of employing labor for basic research can foster the innovation success of domestic firms. Successful domestic firms are able to produce as monopolists at the technological frontier for a certain time and may drive foreign firms out of the market. In sectors where domestic firms fail to innovate, foreign firms may enter with leading technology. The likelihood of this event depends on the degree of openness.

In such a model, higher investment in basic research for a particular generation has three effects on the economy. First, basic research draws labor from the production sector, thereby making labor more costly and reducing consumption. Second, as basic research fosters innovation, it has a positive effect on the productivity and consumption level of the economy. And third, by increasing innovation success basic research also helps to prevent foreign entry, thereby raising innovation rents and income.

Our results are twofold. First, we establish the circumstances under which the economy converges to a steady state with a particular share of leading industries. In the long-run, typically economies either exhibit a constant share of technologically advanced sectors or they converge to polar cases with only leading sectors or none.

Second, we examine how changes in the degree of openness affect the optimal level of basic research in the steady states and whether changes in openness foster a country’s convergence towards the world’s technological frontier. We show that for small and intermediate steps of innovation, an increase in openness induces higher investments in basic research which, in turn, yields a higher share of leading sectors in the economy in the long-run. The reason is that the benefits of foreign entry arising from the import of leading technology are smaller than the costs of foreign entry in terms of the domestic firm’s profit losses. However, if innovation steps are large, implying that the technological progress induced by foreign entry is large, we observe the opposite relationship.

Our paper is related to theoretical literature that incorporates basic research into
R&D-driven growth models (e.g. Arnold 1997, Cozzi and Galli 2009, Gersbach et al. 2009). Most of these contributions focus on the optimal level of basic research in closed economies. There are two papers that also investigate the impact of openness. In a two-country model Park (1998) analyzes how cross-country knowledge spillovers affect the optimal level of public basic research, whereas the degree of openness determines how large the spillovers are. Our notion of openness differs, as we focus on market entry by foreign firms. Gersbach et al. (2010) study how openness affects the interplay between basic and applied research in a static model. They provide preliminary empirical evidence that there is only a weak positive correlation at best between the degree of openness and basic-research investments in industrialized countries. Our paper shows that, in the long-run, the correlation between the degree of openness and basic-research investments is positive for moderate technology advancements but may be reversed for large technology advances.

Our paper is structured as follows: In the next section we present the model. Section 3 is a discussion of the effects of basic research. In section 4, we explore the dynamics of the model, derive the steady states and characterize their properties, followed by an analysis on the impact of openness in section 5. Section 6 concludes.

2 The Model

We consider a dynamic model with discrete periods, \( t = 0, 1, 2, \ldots \). There is a continuum with measure \( \bar{L} \) of identical households, each living for one period, enjoying strictly increasing utility in consumption, inelastically supplying one unit of labor, and receiving an equal share of the final good and intermediate firms’ profits. There is no population growth. In each period, a government representing the current generation maximizes the well-being of its citizens by publicly providing basic research financed by an income tax.\(^1\) We first describe the production side of the economy in a typical period and then proceed to solve the government’s optimization problem. In general, we omit the time index \( t \), as long as there is no possibility of confusion.

\(^1\)Essentially, we have a non-overlapping generations model in which each generation elects a government to provide public goods (here basic research) to maximize its well-being. This is equivalent to maximizing the consumption of the current generation.
2.1 Final-good sector

In the final-good sector, a continuum of competitive firms produces the homogeneous consumption good $y$ according to

$$y = \int_{0}^{1} A(i)^{1-\alpha} x(i)^{\alpha} \, di.$$  

(1)

$x(i)$ stands for the amount of intermediate input of variety $i$, and $A(i)$ is this variety’s productivity factor. The parameter $\alpha$ determines the output elasticity of the intermediate goods or the level of technology. The price of the final consumption good is normalized to one. In the following, we operate with one representative final-good firm. The final-good producer maximizes profits $\pi_y$

$$\max_{\{x(i)\}_{i=0}} \left\{ \pi_y = y - \int_{0}^{1} p(i)x(i) \, di \right\},$$

which yields the inverse demand functions for intermediate goods $x(i)$:

$$p(i) = \alpha \left( \frac{A(i)}{x(i)} \right)^{1-\alpha},$$

where $p(i)$ is the price of good $x(i)$.

2.2 Intermediate-goods sectors

The intermediate goods $x(i)$ are produced by labor $L_x(i)$ only, using a linear technology:

$$x(i) = L_x(i).$$

(2)

Each variety $i$ is produced by an intermediate firm. Intermediate-goods firms act competitively in the labor market and hold a monopoly position in their intermediate sector. Profits are given by $p(i)x(i) - wx(i)$, where $w$ denotes the wage level. Accordingly, the monopolistic intermediate firm asks a price $p(i) = \frac{w}{\alpha}$ for its goods, leading to a labor demand of

$$L_x(i) = \left( \frac{\alpha^2}{w} \right)^{\frac{1}{1-\alpha}} A(i) = x(i)$$

(3)

and profits

$$\pi_x(i) = (1 - \alpha) \left( \frac{\alpha^{1+\alpha}}{w^{\alpha}} \right)^{\frac{1}{1-\alpha}} A(i).$$

(4)
2.3 Technological state, innovation, and foreign entry

We assume that there is a world technological frontier which in period \( t \) is given by \( \bar{A}_t \) and grows exogenously over time in accordance with

\[
\bar{A}_t = \gamma \bar{A}_{t-1},
\]

where \( \bar{A}_{t-1} \) denotes the technological frontier of the preceding period and \( \gamma > 1 \).\(^2\)

We assume that each intermediate sector comprises a single domestic firm. At the end of the preceding period, each domestic intermediate firm can be of two types:

**Type 1** The firm’s technology is on a par with the current technological frontier, \( A_{t-1}(i) = \bar{A}_{t-1} \).

**Type 2** The firm’s technology lags one step behind the current technological frontier, \( A_{t-1}(i) = \bar{A}_{t-2} \).

An intermediate firm innovates with a certain probability. An innovation by a type 1 firm increases the firm’s technology level by a factor \( \gamma \), thus enabling it to retain its position at the technological frontier. Additionally, an innovation by a laggard type 2 firm enables the firm to leapfrog to the technology frontier.

The government can foster the innovation opportunities of domestic firms by investing in basic research. We specify the probabilities of type 1 and type 2 firms innovating successfully as

\[
\rho_1(L_B) = \min \{\theta L_B, 1\}, \quad (5)
\]

\[
\rho_2(L_B) = \min \{\eta \theta L_B, 1\}, \quad (6)
\]

where \( \theta > 0 \) and \( 0 < \eta < 1 \) are parameters that capture the efficiency of basic research with respect to a type 1 firm and leapfrogging by a type 2 firm, respectively. \( L_B \) denotes the amount of labor in the basic-research sector financed by the government. Equations (5) and (6) specify that basic research constitutes a public good from which domestic intermediate firms can benefit.

In this paper we focus on the impact of basic research on innovation activities by firms and do not explicitly model applied research activities by firms. Hence, probabilities

\[^2\text{In this section we follow Aghion et al. (2006). Instead of private incentives to innovate we focus on public basic-research investments to foster innovation.}\]
\( \rho_1(L_B) \) and \( \rho_2(L_B) \) must be interpreted as the differential impact basic research has on the success of private firms’ innovations. For simplicity, we normalize the probability of success to zero if no basic-research capacities are provided by the government.\(^3\)

A type 2 firm that does not innovate successfully has the option of adopting the mature technology, i.e., the technological level that lags one step behind frontier technology. We assume that these adoption costs are positive but small\(^4\) so that a type 2 firm currently holding a monopoly position and unsuccessful in leapfrogging will find it profitable to adopt the next technology. Adopting the next technology will enable a type 2 firm to maintain its monopoly position unless a foreign firm with a superior technology enters, as we will see subsequently. We note that the costly adoption of the next technology by the current type 2 firm prevents potential competitors from investing in technology adoption, as the costs for this could not be recovered in the ensuing price competition in the specific intermediate sector.

The concept of openness introduced in our model is as follows: We assume that firms headquartered in foreign countries and owned by foreigners (henceforth foreign firms) incur costs in introducing a leading technology into the domestic market and operating in a foreign country. Those costs are heterogeneous and depend on the country’s degree of openness.\(^5\) The higher the country’s openness, the more likely it is that an innovating foreign firm will find it profitable to introduce the new technology into the domestic market. In this way, we specify openness as the probability \( \sigma \) (\( 0 < \sigma < 1 \)) that an innovating foreign firm will benefit from entering the domestic market.

An alternative view on openness is the following: The outside world that forms the technological frontier is divided into two parts. Firms in the first part incur small costs by entering the domestic country and operating in it. The probability that innovations in this part will push the technological frontier of a particular industry to the next level is given by \( \sigma \) (\( 0 < \sigma < 1 \)). Firms in the second part of the outside world

\(^3\)The qualitative results remain unchanged if we employ \( \rho_1 = \min \{ \theta L_B + \rho_A, 1 \} \) and \( \rho_2 = \min \{ \eta \theta L_B + \rho_A, 1 \} \) with \( \rho_A \) being the innovation chances of private firms if there is no basic research.

\(^4\)These costs will be neglected in the analysis. If they were substantial, basic research might have further beneficial effects, as it can lower the costs of domestic firms in adopting mature technologies.

\(^5\)For example, if the technology is developed abroad, the introduction of the new technology may be hampered by the legal and institutional framework in the domestic country. Further, the introduction of a new technology in a different country may necessitate the build-up or restructuring of production capacities, the employment of experts from the foreign country, and the like. The amount of costs incurred by these activities is likely to differ depending on the technology and the structure of the innovating foreign firm. Further, it is also very likely that these costs will be lower in countries that are very open to foreign trade and FDI.
have high entry costs and are deterred from entering the domestic market under any circumstances. The probability of the rise in the technological frontier in a specific industry being caused by innovations in this part of the world is \(1 - \sigma\).

Our model is compatible with both perspectives on openness. The consequences of either view of openness for the domestic industries are as follows:

- If the domestic firm produces at the technological frontier, no foreign entry will occur in the respective sector.\(^6\)

- If the domestic firm lags behind the technological frontier, a foreign firm will enter with probability \(\sigma\) and capture the entire market in this sector.

The following remark is in order: Firms headquartered in a foreign country have superior technology if they manage to enter the domestic market. This reflects a common finding in the empirical literature indicating that foreign direct investment by leading-edge companies is a powerful mechanism for raising productivity in host countries (e.g. Baily and Gersbach 1995, Keller and Yeaple 2003 or Alfaro et al. 2006). FDI contributes directly to higher levels of productivity by transferring the best production techniques to the host country and indirectly by putting pressure on the host country’s domestic producers to improve.\(^7\)

In sum, each intermediate sector is in one of three states at the beginning of a particular period:

**State 1** Type 1 firm holding a monopoly

**State 2** Type 2 firm holding a monopoly

**State f** Foreign firm with frontier technology holding a monopoly

We denote the fractions of the states in period \(t\) by \(s_{1,t}\) (state 1), \(s_{2,t}\) (state 2), and \(s_{f,t}\) (state f), where for all \(t\), \(s_{1,t}, s_{2,t}, s_{f,t} \geq 0\) and \(s_{1,t} + s_{2,t} + s_{f,t} = 1\).

The way the sector states evolve depends on domestic innovation and foreign entry. If the domestic firms in sectors of state 1 or 2 innovate successfully, they will remain in

\(^6\)The entry costs a foreign firm incurs will prevent its market entry, since profits upon entering would be negative.

\(^7\)The most prominent examples are the US transplants of automotive companies headquartered in Japan.
state 1 or move up to state 1, respectively. If those firms fail to innovate, a foreign firm will enter and take over those sectors with probability $\sigma$. Thus, with the complementary probability $(1 - \sigma)$ these sectors achieve state 2.

If a sector is in state $f$ at the beginning of a particular period, it is possible for the domestic laggard to leapfrog and regain the monopoly position by innovating successfully.\(^8\) In this case, the foreign firm will leave the competitive market on account of the costs for firms operating in a foreign country. If there is no domestic innovation in the sector, the sector will remain occupied by a foreign firm, given that an innovating foreign firm will find it optimal to introduce the new technology with probability $\sigma$. This may either be the firm that has already occupied the domestic sector if it is able to keep up with the technological frontier or a new foreign firm replacing the old.\(^9\) Hence, with the complementary probability $(1 - \sigma)$ the sector is handed back to the domestic laggard.

### 2.4 Summary: Sequences of events and sector dynamics

It is useful to summarize the model’s timing of events and sector dynamics. In each sector there is one domestic firm that conducts research with success probability $\rho_1(L_B)$ if it is currently operating at the technological frontier and $\rho_2(L_B)$ if it is lagging behind or the respective sector is occupied by a foreign firm. In each period the following sequence of events occurs:

1. Government chooses basic research
2. Domestic firms conduct R&D
3. Technological frontier increases to $\bar{A}_t = \gamma \bar{A}_{t-1}$
4. Domestic firms failing to invent an intermediate at frontier level $\bar{A}_t$ adopt a mature technology at the level $\bar{A}_{t-1}$

\(^8\)An alternative way of motivating how domestic firms can drive the foreign firm out of the market is to think of spin-offs. There are several empirical studies (e.g. Bania et al. (1993) and Zucker et al. (1998)) that provide evidence that basic research has a positive effect on the creation of new firms. Hence, the innovation probability of type 2 firms could also be understood as the spin-off probability of domestic high-tech firms.

\(^9\)For simplicity, we neglect the likely fact that the costs of introducing a new technology are smaller for a foreign firm that has already occupied the domestic market.
5. Foreign firms decide whether to enter (or keep on operating in) the domestic market

6. Production of consumption good

On the assumption that operating in the domestic market is costly for a foreign firm, Bertrand competition implies negative profits for the foreign firm if it possesses the same technological level as the domestic firm (which would realize zero profits). Hence, the foreign firm will only enter or stay in the market if its technological level is higher than that of the domestic firm.

In Appendix B we provide an alternative microfoundation of our set-up with multiple domestic firms and patent races.

The following scheme illustrates the sector dynamics within a period:

\[
\begin{align*}
\rho_1 & : s_{1,t} \\
(1 - \rho_1)\sigma & : s_{f,t} \\
(1 - \rho_1)(1 - \sigma) & : s_{2,t}
\end{align*}
\]

\[
\begin{align*}
\rho_2 & : s_{1,t} \\
(1 - \rho_2)\sigma & : s_{f,t} \\
(1 - \rho_2)(1 - \sigma) & : s_{2,t}
\end{align*}
\]

\[
\begin{align*}
\rho_2 & : s_{1,t} \\
(1 - \rho_2)\sigma & : s_{f,t} \\
(1 - \rho_2)(1 - \sigma) & : s_{2,t}
\end{align*}
\]

Accordingly, we obtain the following equations of motion for the country’s industry structure:

\[
s_{1,t} = s_{1,t-1} \min \{\theta L_B, 1\} + (1 - s_{1,t-1}) \min \{\eta \theta L_B, 1\} \tag{7}
\]

\[
s_{2,t} = (1 - \sigma) [(1 - \min \{\theta L_B, 1\}) s_{1,t-1} + (1 - \min \{\eta \theta L_B, 1\})(1 - s_{1,t-1})] \tag{8}
\]

\[
s_{f,t} = \sigma [(1 - \min \{\theta L_B, 1\}) s_{1,t-1} + (1 - \min \{\eta \theta L_B, 1\})(1 - s_{1,t-1})] \tag{9}
\]

2.5 Equilibrium

The economy comprises the market for the final consumption good with price unity, the labor market with wage rate \(w\), and a continuum of intermediate-good markets with price \(p(i) = \frac{w}{\alpha}\).
In the labor market, labor \( \bar{L} \) is supplied inelastically. Labor demand consists of the government’s demand for basic researchers and the demand for workers in intermediate-goods production. Hence the labor market clears when

\[
\bar{L} = L_B + \int_0^1 L_x(i) \, di. \tag{10}
\]

The demand for workers in intermediate-goods production depends on the sector’s technological level after innovation activities and foreign entry have occurred. This reflects our assumption that foreign intermediate firms bring leading technology with them from abroad but produce the intermediate goods within the country. Consequently, the total intermediates’ demand for production workers is given by

\[
\int_0^1 L_x(i) \, di = (s_{1,t} + s_{f,t}) L_{1,x} + s_{2,t} L_{2,x}. \tag{11}
\]

where \( L_{1,x} \) denotes the labor demand in a technologically leading sector and \( L_{2,x} \) the labor demand in a technologically lagging sector. Using equation (3), we can rewrite (11) as

\[
\int_0^1 L_x(i) \, di = \bar{A}_t \left( \frac{\alpha^2}{w} \right)^{\frac{1}{1-\alpha}} \chi(L_B), \tag{12}
\]

where

\[
\chi(L_B) = s_{1,t} + s_{f,t} + s_{2,t} \frac{1}{\gamma}. \tag{13}
\]

Note that according to the system dynamics given in (7)-(9), \( \chi(L_B) \) is a linear function of \( L_B \). Therefore it is convenient to define \( \chi(L_B) = \chi' L_B + \bar{\chi} \). Inserting (12) into (10) we obtain the equilibrium wage for a given level of basic research:

\[
w(L_B) = \alpha^2 \left[ \frac{\bar{A}_t \chi(L_B)}{L - L_B} \right]^{1-\alpha}. \tag{14}
\]

An increase in basic research has two effects on the wage level. First, a higher technological level increases the productivity of the respective intermediates and consequently enhances demand. This leads to a wage increase. Second, by reducing the supply of labor for production, a rise of \( L_B \) also increases the wage level. The following lemma formalizes the effect of basic research on the equilibrium wage:

**Lemma 1**

\[
\frac{dw(L_B)}{dL_B} > 0.
\]
Proof: See Appendix A.1.

From the equilibrium wage we obtain the equilibrium prices for intermediate goods, from which the equilibrium quantities and the firms’ profits follow. To simplify notation, we will henceforth use \( w \) to denote the equilibrium wage associated with a particular level of basic research.

2.6 Government

In each period, the government chooses the amount of basic-research labor \( L_B \) required to maximize aggregate consumption \( c \) by the current generation. The expenditures \( wL_B \) are financed by a tax \( \tau \in [0, 1] \) on household income. Households earn wages and obtain profits from final-good and domestic intermediate-goods production. Consequently, the budget constraint for the government reads

\[
wL_B = \tau \left( w\bar{L} + s_{1,t}\bar{\pi}_{1,x} + s_{2,t}\bar{\pi}_{2,x} + \bar{\pi}_y \right), \tag{15}\]

where \( \pi_{1,x} \) and \( \pi_{2,x} \) represent the profits of a technologically leading firm and that of a technologically lagging firm, respectively. \( \bar{\pi}_y \) denotes the profits of the final-good sector.\(^{10}\) Aggregate consumption \( c \) equals total income after taxes:

\[
c = (1 - \tau) \left( w\bar{L} + s_{1,t}\bar{\pi}_{1,x} + s_{2,t}\bar{\pi}_{2,x} + \pi_y \right). \tag{16}\]

The government’s problem can also be written as

\[
\max_{L_B} c = y - s_{f,t}\bar{\pi}_{1,x} = \bar{A}_t \left( \frac{\alpha^2}{w} \right)^{\frac{\alpha}{1-\alpha}} \left[ s_{1,t} + s_{2,t} \frac{1}{\gamma} + s_{f,t} (1 - \alpha (1 - \alpha)) \right]. \tag{17}\]

Let us define

\[
\zeta(L_B) = s_{1,t} + s_{2,t} \frac{1}{\gamma} + s_{f,t} (1 - \alpha (1 - \alpha)). \tag{18}\]

The government’s objective can now be written as \( c = \bar{A}_t \left( \frac{\alpha^2}{w} \right)^{\frac{\alpha}{1-\alpha}} \zeta(L_B) \). And as we know from the equations of motion (7)-(9) that \( \zeta(L_B) \) is a linear function of \( L_B \), we can define \( \zeta(L_B) = \zeta'L_B + \bar{\zeta} \). The following proposition gives the solution to the government’s optimization problem:

\(^{10}\)Note that the profits in the final-good sector amount to \( \pi_y = (1 - \alpha)y \).
Proposition 1

The unique solution to the government’s maximization problem is given by

\[ L_B = \min \left\{ L_B^+, \frac{1}{\theta} \right\} \text{, if } L_B^+ \in \mathbb{R} \text{ and } L_B^+ \geq 0 \]

where

\[ L_B^+ = 1 - \frac{\alpha}{2} \bar{L} - \frac{1 + \alpha}{2} \bar{\chi} + \sqrt{\left( \frac{1 - \alpha}{2} \bar{L} - \frac{1 + \alpha}{2} \bar{\chi} \right)^2 - (\alpha \hat{\zeta} - \hat{\chi}) \bar{L} - \alpha \hat{\zeta} \hat{\chi}}, \quad (19) \]

\[ \hat{\zeta} = \frac{\zeta}{\zeta'}, \text{ and } \hat{\chi} = \frac{\chi}{\chi'} . \]

Proof: See Appendix A.2.

Our model exhibits a distance to frontier effect, as a higher level of \( s_{1,t-1} \) tends to support higher investment in basic research \( L_B^+ \).\(^{11}\)

3 Effects of Basic Research

Before turning to comparative statics with respect to a country’s openness, we here introduce the different effects of basic-research investment on aggregate consumption.

To identify the effects, we will use the derivative of \( c \) with respect to \( L_B \):

\[ \frac{dc}{dL_B} = -\frac{\alpha}{1 - \alpha} \bar{A}_t \alpha^{2/(\mu - 1)} \left[ \frac{d\zeta}{dL_B} (L_B) \right] + \bar{A}_t \left[ \frac{\alpha}{w} \right]^{\alpha/(\mu - 1)} \left[ \frac{d s_{1,t}}{dL_B} + \frac{d s_{2,t}}{dL_B} \frac{1}{\gamma} + \frac{d s_{f,t}}{dL_B} (1 - \alpha (1 - \alpha)) \right]. \quad (20) \]

The first summand reflects the change in consumption due to the change in the equilibrium wage induced by marginally higher basic research investment. We refer to this effect as the wage effect. Using Lemma 1 we infer that this effect is negative. The second summand captures the effect of basic research on the country’s industry structure.

From the equations of motion we obtain

\[ \frac{d s_{1,t}}{dL_B} = s_{1,t-1} \theta (1 - \eta) + \theta \eta, \]

\[ \frac{d s_{2,t}}{dL_B} = - (1 - \sigma) \frac{d s_{1,t}}{dL_B}, \]

\[ \frac{d s_{f,t}}{dL_B} = - \sigma \frac{d s_{1,t}}{dL_B} . \]

\(^{11}\)See Gersbach et al. (2010) for a detailed analysis of this effect in a static model with applied and basic research.
This reveals that basic research increases the number of domestic sectors operating at the technology frontier and decreases both the number of lagging sectors and the sectors with a foreign technology leader. Inserting the changes in sector sizes, the second summand of (20) can be written as follows:

\[
\bar{A}_t \left( \frac{\alpha^2}{w} \right)^{\frac{\alpha}{1-\sigma}} [s_{1,t-1}\theta (1 - \eta) + \theta \eta] \left[ (1 - \sigma) \left( 1 - \frac{1}{\gamma} \right) + \sigma \alpha (1 - \alpha) \right].
\]  

(21)

The term \( (1 - \sigma) \left( 1 - \frac{1}{\gamma} \right) \) reflects the positive effect that marginal basic-research investment has on consumption caused by the higher technological level. This reflects the productivity effect of basic research. The other term \( \sigma \alpha (1 - \alpha) \) stands for the escape entry effect. It captures the rise in consumption arising from the fact that the marginal basic-research investment induces some sectors to be held by a domestic technology leader instead of a foreign technology leader. Having a domestic firm is advantageous over a foreign firm at the same technological level as profits are retained in the country.

We summarize our findings in the next proposition.

**Proposition 2**

A marginal change in basic research has the following three effects on aggregate consumption:

(i) Wage effect:

\[
WE = -\alpha \frac{1}{1-\alpha} \bar{A}_t \alpha^{\frac{\alpha}{1-\alpha}} w^{\frac{1}{1-\sigma}} \frac{dw}{dL} \zeta (L_B)
\]  

(22)

(ii) Escape Entry Effect:

\[
EE = \bar{A}_t \left( \frac{\alpha^2}{w} \right)^{\frac{\alpha}{1-\sigma}} [s_{1,t-1}\theta (1 - \eta) + \theta \eta] \sigma \alpha (1 - \alpha)
\]  

(23)

(iii) Productivity Effect:

\[
PE = \bar{A}_t \left( \frac{\alpha^2}{w} \right)^{\frac{\alpha}{1-\sigma}} [s_{1,t-1}\theta (1 - \eta) + \theta \eta] \sigma \alpha (1 - \alpha) \left( 1 - \frac{1}{\gamma} \right)
\]  

(24)

The Escape Entry Effect and the Productivity Effect have a positive influence on aggregate consumption, whereas the Wage Effect lowers consumption. For our analysis it is important to see how the level of openness \( \sigma \) affects the Escape Entry Effect and the Productivity Effect. As \( \sigma \) increases entry threat, the Escape Entry Effect increases with \( \sigma \). For the extreme case of \( \sigma = 0 \), implying a closed economy, the Escape Entry Effect
vanishes. By contrast, the Productivity Effect decreases with $\sigma$. The reason is that the more open the economy is, the more it will benefit from the high technology of foreign firms. Accordingly, fewer domestic innovations can contribute to technological progress. For the maximum value $\sigma = 1$ the economy will feature the frontier technology in every sector, irrespective of how much basic research is performed. So in this case, the Productivity Effect is zero.

4 Dynamics and Steady State

In this section we first characterize the economy’s sectoral dynamics and then derive the model’s steady state. From the equations of motion (7)-(9) we obtain

$$s_{2,t} = \frac{1 - \sigma}{\sigma} s_{f,t}, \ \forall t,$$

and

$$s_{2,t} = (1 - \sigma)(1 - s_{1,t}), \ \forall t,$$

$$s_{f,t} = \sigma(1 - s_{1,t}), \ \forall t.$$

In other words, the sectors of the economy without a domestic technology leader are split between domestic laggards and foreign technology leaders in accordance with the degree of openness. Consequently, given openness, the industrial structure of the economy in period $t$ is entirely pinned down by the share of sectors occupied by type 1 firms. In this way, the dynamics of the economy are fully determined by the following difference equation:

$$s_{1,t} = L_B(s_{1,t-1}) \psi(s_{1,t-1}), \quad (25)$$

where

$$\psi(s_{1,t-1}) = s_{1,t-1} \theta(1 - \eta) + \eta \theta.$$

As $L_B(s_{1,t-1})$ is a linear function, there are two distinct steady state patterns that may emerge. Either there is a unique and stable steady state, or there exists one unstable steady state and two stable ones. The pattern that occurs depends on the impact of basic research on the innovation success of private firms. A complete characterization of all possible steady-state patterns and associated stability properties is given in Appendix A.3. Here we focus on two particularly interesting cases from an economic viewpoint.
Proposition 3

(i) If \( L^+(s_1,t-1 = 0) > 0 \) and \( \theta L^+(s_1,t-1 = 1) < 1 \), then there exists a unique and stable steady state with \( 0 < s^*_1 < 1 \).

(ii) If \( L^+(s_1,t-1 = 0) < 0 \) and \( \theta L^+(s_1,t-1 = 1) > 1 \), then there exists one interior steady state that is not stable. The stable steady states are given by the two corner solutions \( s^*_1 = 0 \) and \( s^*_1 = 1 \).

Proof: See Appendix A.3.

The specific interior steady-state values of the share of state 1 sectors are given by

\[
 s^*_1 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A},
 \tag{26}
\]

where

\[
 A = (1 - \alpha)\bar{L}\theta(1 - \eta) - 1,
 B = \bar{L} \left[ \theta(1 - \eta) \left( \frac{\bar{x}}{\bar{\chi}} - \frac{\alpha}{\frac{\tilde{\chi}}{\tilde{\zeta}}} \right) + (1 - \alpha)\eta\theta \right] - (1 + \alpha)\frac{\bar{x}}{\bar{\chi}},
 C = \eta\theta \bar{L} \left( \frac{\bar{x}}{\bar{\chi}} - \frac{\bar{\zeta}}{\tilde{\zeta}} \right) - \alpha \frac{\bar{x}}{\bar{\chi}} \frac{\tilde{\chi}}{\tilde{\zeta}},
 \tilde{\zeta} = (1 - \sigma)(1 - \frac{1}{\gamma}) + \sigma\alpha(1 - \alpha),
 \tilde{\chi} = (1 - \sigma)(1 - \frac{1}{\gamma}).
\]

Note that the case with two stable steady states as described in Proposition 3 may constitute a backwardness-trap situation with respect to welfare levels. Countries that are technologically advanced, i.e. that possess a high number of advanced domestic sectors, will converge to the stable steady state that comprises only state 1 sectors. And countries that are less advanced than the steady state level of the unstable fixed point will converge to the steady state without any state 1 sectors. As a consequence, the output level in the less advanced country is substantially lower than that of the advanced country at least if the degree of openness is small. Given that the costs of basic research for a country in the \( s_1 = 1 \) steady state do not outweigh the output gains relative to a country in the \( s_1 = 0 \) steady state, the latter will find itself in a backwardness trap.
5 Impact of Openness and Other Parameters

In this section, we analyze how the steady-state value of basic research denoted by $L_B^*$ is affected by changes in the degree of openness and other parameters. Moreover, we discuss how openness affects the convergence of a country to the technological frontier. Interior steady states of basic research are given by inserting (26) back into (19). As the steady state level of basic research cannot be derived analytically, we rely on numerical simulations. As a basic scenario, we choose the following set of parameters: $\alpha = 0.5$, $\gamma = 1.3$, $\theta = 2.96$, $\eta = 0.8$, $\sigma = 0.5$, $L = 1$, $\bar{A} = 100$. The choice of $\alpha$ is at an intermediate level. $\gamma = 1.3$ implies that the world’s technological frontier grows at a rate of 30% in each period. In our model, basic-research investments are considered for each generation, so it is convenient to think of a period as comprising one or two decades, which generates plausible annual growth rates. For an initial scenario we choose the intermediate degree of openness given by $\sigma = 0.5$. $L = 1$ and $\bar{A} = 100$ represent normalizations.

Finally, $\theta$ and $\eta$ are chosen to obtain a steady state with a balanced sector allocation of $(s^*_1, s^*_2, s^*_f) \approx (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. Figure 1 plots equation (25) in the $(s_{1,t-1}, s_{1,t})$-space and illustrates the resulting steady state. The economy features a steady-state pattern as described in Proposition 3 (i).
5.1 Changes in openness

Basic scenario

We start our analysis with a detailed discussion of the effect of openness. Figure 2 depicts how changes in the degree of openness affect the steady-state level of basic research in the basic scenario. We observe that $L_B^s$ increases with openness and also that a minimal degree of openness is needed to induce a positive level of basic research. To understand this result, it is instructive to consider the three effects of basic research described in section 3 ($WE$, $EE$, $PE$). As we saw there, an increasing degree of openness makes the Escape Entry Effect stronger and lowers the Productivity Effect. Increasing Openness also reduces the negative Wage Effect. The reason is as follows: Basic research increases the labor demand of intermediate firms by fostering technological progress. A higher degree of openness mitigates this demand effect and thus lowers the Wage Effect as the import of leading technology reduces the impact of basic research on the economy’s technological level.

Summarizing, we can say that on the one hand, a rise in openness will increase the incentives to invest in basic research by increasing the positive Escape Entry Effect and decreasing the negative Wage Effect. On the other hand, it lowers the investment incentives by decreasing the positive Productivity Effect. In the basic scenario we obtain a positive relationship between $L_B^s$ and openness as $|\frac{dWE}{d\sigma} + \frac{dEE}{d\sigma}|$ dominates $|\frac{dPE}{d\sigma}|$. This result can be interpreted in the following way: The government will prefer
to prevent foreign entry and keep the intermediate profits in the country instead of benefiting from imported leading technology. As a result, a larger entry threat will induce the government to increase investments in basic research and a steady state with a higher level of basic research is reached.

Large technology advances

From equation (24) we observe that the innovation step $\gamma$ is a major determinant of the productivity effect. The faster the frontier technology grows, the more important is the Productivity Effect, as domestic innovations cause higher technological progress. The way openness affects the economy when we consider larger technology advances is demonstrated in Figure 3. It reveals that in this case we have a negative relationship between openness and steady-state basic research. The reason is that the Productivity Effect is much larger, whereas the levels of the Wage and the Escape Entry Effect are only moderately affected by the rise in $\gamma$. As a result, $|\frac{dPE}{d\sigma}|$ now dominates $|\frac{dWE}{d\sigma} + \frac{dEE}{d\sigma}|$ and causes the falling pattern of $L_B$. In this case, implementing the leading technology in the domestic country is more important than protecting the domestic intermediates’ profits. Hence, the entry of foreign firms is welcome, as they implement leading technologies. Put differently, to achieve the leading technology it is cheaper to allow foreign entry and forgo domestic profits than to draw labor from production to invest in basic research.
Table 1: Effect of parameter changes on $L^*_B$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\alpha$</th>
<th>$\gamma$</th>
<th>$\theta$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L^*_B$</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Note that for $\sigma = 1$ the basic scenario and the scenario with high technology advances feature the same steady-state level of basic research. Maximum openness implies that all sectors of the domestic economy feature the leading technology, independently of the level of basic research. So there is no Productivity Effect, and the size of $\gamma$ is immaterial for the steady-state level of basic research.

5.2 Changes in other parameters

The way the remaining parameters affect steady-state basic research is straightforward and is summarized in Table 1.\footnote{We do not examine changes with respect to $\bar{L}$. Such an analysis would require an assumption on scale effects, as it would be necessary to specify $\theta$ as a function of $\bar{L}$. The one extreme would be to assume $\theta(\bar{L})$ to be constant with strong scale effects. The other extreme would be the absence of scale effects by assuming $\theta(\bar{L})\bar{L}$ to be constant.} Increasing $\alpha$ reduces the importance of the technological level in production and thus results in lower basic-research investments. We have already indicated in the previous subsection that a rise in $\gamma$ increases the Productivity Effect and yields larger incentives to invest in basic research. Research productivity increases with $\theta$ as well as with $\eta$, which explains the positive relationship between those parameters and $L^*_B$. The parameter $\theta$ could, for example, reflect the country’s level of human capital. Our theory would then imply that countries with higher stocks of human capital invest more in basic research and have a larger share of sectors with technologically leading domestic firms. Note that in this case there are two effects with an impact on higher basic-research investments: (a) a direct effect due to the higher probability of success when $\theta$ is higher, and (b) an indirect effect as the incentive to invest in basic research also increases if the share of $s_1$ sectors increases because there are more sectors with success probability $\rho_1$ relative to the smaller $\rho_2$. So there is also the effect that c.p. optimal basic-research investments are higher, the closer the country is to the technological frontier (measured in terms of the share of leading sectors).\footnote{This replicates the findings of Gersbach et al. (2010) in a dynamic context.}
5.3 Convergence

What do our results imply for the effect of openness on the convergence of a country to the technological frontier? We can apply two notions of convergence:

(1) A country converges to the technological frontier if the share of intermediates with leading technology increases.
   (i.e., a country’s production level comes closer to the output of a technologically advanced country with leading technology in every sector)

(2) A country converges to the technological frontier if the share of intermediates with leading technology produced by domestic firms increases (i.e., the technological knowledge in the country increases)

With respect to the second notion of convergence focussing on the steady-state share of state-1 sectors, we can directly infer from the previous results that greater openness will foster convergence if the innovation steps are small and will lead to divergence if the innovation steps are large.

Applying the broader notion of convergence (1) focussing on production output rather than technological knowledge, we come to the same result as under notion (2) if innovation steps are small. Figure 2 shows that increased openness fosters basic-research investments via a strong Escape Entry Effect leading to a higher share of state-1 sectors and, in addition, the share of f-sectors with high technology increases as well. Hence the country’s output level approaches the highest achievable at the technological frontier.

If innovation steps are large, we obtain an ambiguous result. Here a larger degree of openness leads to a higher share of foreign firms with leading technology but a lower share of domestic firms operating at the frontier. In this scenario, we observe a crowding out effect with respect to domestic research. Whether or not convergence occurs if openness increases depends on the magnitude of the effects, i.e. whether the share of incoming foreign firms is larger than the loss of domestic technology leaders. In our example setting, illustrated by Figure 3, basic-research investments do not decline strongly in response to an increased $\sigma$. Hence, the total share of sectors operating with leading technology increases. In this case, the country converges to the technological frontier in terms of production output but diverges in terms of domestic technological knowledge.
6 Conclusion

We develop a model of growth that incorporates basic research and the entry of foreign high-tech firms, while the level of basic research is determined by a government maximizing consumption of the current generation. On that basis we derive the steady states of the economy and study how changes in the degree of openness affects the incentives to invest in basic research. Our main insight that a higher degree of openness tends to justify higher investment in basic research may be important for policy discussions in industrialized countries.
A Proofs

A.1 Proof of Lemma 1

Using (14) to determine the derivative \( \frac{dw(L_B)}{dL_B} \) we obtain

\[
\frac{dw(L_B)}{dL_B} = (1 - \alpha)\alpha^2 \left[ \frac{\hat{A} \chi(L_B)}{\bar{L} - L_B} \right]^{-\alpha} \hat{A}_t \frac{\chi'(\bar{L} - L_B) + \chi(L_B)}{(\bar{L} - L_B)^2}.
\]

Since \( \chi(L_B) = \chi' L_B + \bar{\chi} \), the numerator of the last fraction can be written as \( \chi' \bar{L} + \bar{\chi} = \chi(\bar{L}) \). From the definition of \( \chi(L_B) \) according to equation (13) we know that \( \chi(L_B) \) is positive for all values of \( L_B \). As a consequence, \( \frac{dw(L_B)}{dL_B} > 0 \).

A.2 Proof of Proposition 1

By inserting (14) in (17), the government’s objective function reads

\[
\bar{A}_t^{1-\alpha} \zeta(L_B) \left( \frac{\bar{L} - L_B}{\chi(L_B)} \right)^\alpha.
\]

It yields the following first order condition:

\[
\frac{\alpha \chi(\bar{L})}{\chi(L_B)(\bar{L} - L_B)} \zeta(L_B) = \zeta'.
\]

By expanding, this condition can be transformed to

\[
\bar{L}(\alpha \hat{\zeta} - \hat{\chi}) + \alpha \hat{\zeta} \hat{\chi} + L_B[(1 + \alpha)\hat{\chi} - (1 - \alpha)\bar{L}] + L_B^2 = 0.
\]

The solution to this quadratic equation is

\[
L_B = \frac{1 - \alpha}{2} \bar{L} - \frac{1 + \alpha}{2} \hat{\chi} \pm \sqrt{ \left( \frac{1 - \alpha}{2} \bar{L} - \frac{1 + \alpha}{2} \hat{\chi} \right)^2 - (\alpha \hat{\zeta} - \hat{\chi})\bar{L} - \alpha \hat{\zeta} \hat{\chi}.}
\]

In order to determine which of the two solutions maximizes period \( t \) consumption, consider the government’s objective function as given in (17):

\[
\hat{A}_t \left( \frac{\alpha^2}{w} \right)^{\frac{\gamma}{1-\gamma}} \left[ s_{1,t} + s_{2,t} \frac{1}{\gamma} + s_{f,t}(1 - \alpha(1 - \alpha)) \right].
\]

From Lemma 1 we know that the wage strictly increases with \( L_B \). Taking a close look at \( w(L_B) \) given in equation (14), we see that for \( L_B \to \bar{L} \), the wage becomes infinite. As the second factor in (17), \( \zeta(L_B) \), is bound from above for \( L_B \to \bar{L} \), consumption converges to 0 if the entire labor force is employed in the basic research.
sector. Moreover, we see that $c$ cannot become negative, so when $L_B \to \bar{L}$, it converges to 0 from above. As a next step, we show that the objective function is concave in $L_B$, which implies that the second derivative of (27) with respect to $L_B$ should be negative:

$$A_i^{1-a} \left[ -2\alpha \zeta'(\bar{L} - L_B) \frac{(\bar{L} - L_B)^{\alpha-1} \chi(L_B) + \chi'(\bar{L} - L_B)}{\chi(L_B)^2} \right. $$

$$- \alpha(1 - \alpha) \zeta(L_B) \left( \frac{\bar{L} - L_B}{\chi(L_B)} \right)^{\alpha-2} \left( \frac{\chi(L_B) + \chi'(\bar{L} - L_B)}{\chi(L_B)^2} \right)^2 \left. + 2\alpha \zeta(L_B) \left( \frac{\bar{L} - L_B}{\chi(L_B)} \right)^{\alpha-1} \frac{\chi'(\chi(L_B) + \chi'(\bar{L} - L_B))}{\chi(L_B)^3} \right] < 0.$$ 

The middle term is negative, so we can neglect it. The remaining part can be reduced to

$$-\zeta' \bar{x} + \chi' \zeta < 0,$$

which equals

$$-[s_1L-1 \theta(1 - \eta) + \eta \theta] \left( 1 - \sigma \right) \left( 1 - \frac{1}{\gamma} \right) + \sigma \alpha(1 - \alpha) \left( \sigma + (1 - \sigma) \frac{1}{\gamma} \right) + [s_1L-1 \theta(1 - \eta) + \eta \theta] (1 - \sigma) \left( 1 - \frac{1}{\gamma} \right) \left( 1 - \sigma \frac{1}{\gamma} + \sigma(1 - \alpha(1 - \alpha)) \right) < 0.$$ 

The inequality can further be reduced to the form

$$-[s_1L-1 \theta(1 - \eta) + \eta \theta] \sigma \alpha(1 - \alpha) < 0,$$

which obviously holds. Hence, we now know that (27) either falls monotonically in the interval $L_B \in \{0, \bar{L}\}$ or it features a single extremum, which must be a maximum. To show that

$$L_B^+ = \frac{1 - \alpha}{2} \bar{L} - \frac{1 + \alpha}{2} \bar{x} + \sqrt{\left( \frac{1 - \alpha}{2} \bar{L} - \frac{1 + \alpha}{2} \bar{x} \right)^2 - (\alpha \zeta - \bar{\zeta}) \bar{L} - \alpha \bar{\zeta} \bar{x}},$$

the larger of the two solutions given in (28), always constitutes the possible maximum, it is sufficient to show that $L_B^+ < \bar{L}$ holds. A number of simple mathematical manipulations transform this condition into

$$(\bar{L} + \bar{\chi})(\bar{L} + \bar{\zeta}) > 0.$$  

(29)

That condition (29) is satisfied is straightforward, as $\bar{\chi} > 0$ and $\bar{\zeta} > 0$ due to $\bar{x}, \bar{\zeta}, \chi', \zeta' > 0$.

Finally, $L_B^+$ constitutes a maximum in the interval $L_B \in \{0, \bar{L}\}$, given that $L_B^+ \geq 0$ and $L_B^+ \in \mathbb{R}$. The latter condition guarantees that the expression under the square root of $L_B^+$ is positive.
A.3 Steady-State Analysis

Using the unique solution with respect to basic research investment as given in Proposition 1, we obtain the steady-state amount of domestic high technology sectors, \( s_1 \), by solving

\[
s_1 = L_B(s_1) \left[ \frac{s_1 \theta(1 - \eta) + \eta \theta}{\psi(s_1)} \right].
\]

(30)

First, we rearrange (30) according to

\[
\theta L_B(s_1) = \frac{\theta s_1}{s_1 \theta(1 - \eta) + \eta \theta},
\]

in order to check that for all interior solutions \( 0 < s_1 < 1 \), the condition \( 0 < \theta L_B < 1 \) must hold. It is straightforward that this is the case, as the right-hand side increases with \( s_1 \) and reaches one when \( s_1 = 1 \) and zero when \( s_1 = 0 \). This implies that we can focus on \( L_B = L_B^+ \) as given in Proposition 1 to determine the interior solutions.

We redefine \( \zeta' \) as \( \psi(s_1) \tilde{\zeta} \), where \( \tilde{\zeta} = (1 - \sigma)(1 - \frac{1}{\alpha}) + \sigma \alpha (1 - \alpha) \), and similarly we can rewrite \( \chi' \) as \( \psi(s_1) \tilde{\chi} \), where \( \tilde{\chi} = (1 - \sigma)(1 - \frac{1}{\alpha}) \). Now, by inserting \( L_B^+ \) into (30) the equation transforms to

\[
s_1^2 \left[ (1 - \alpha) \bar{L} \theta(1 - \eta) - 1 \right] + s_1 \left( \bar{L} \left[ \theta(1 - \eta) \left( \frac{\bar{X}}{\bar{X}} - \alpha \frac{\bar{\zeta}}{\bar{\zeta}} \right) + (1 - \alpha)\eta \theta \right] - (1 + \alpha) \frac{\bar{X}}{\bar{X}} \right) 
+ \eta \bar{\theta} \bar{L} \left( \frac{\bar{X}}{\bar{X}} - \alpha \frac{\bar{\zeta}}{\bar{\zeta}} \right) - \alpha \frac{\bar{X}}{\bar{X}} \frac{\bar{\zeta}}{\bar{\zeta}} = 0.
\]

It is now obvious that there can be at most two steady states where \( L_B = L_B^+ \).

With respect to steady states at the corners \( s_1 = 0 \) and \( s_1 = 1 \), we can state the following: If \( L_B^+(s_1,t-1) = 0 \) \( \leq 0 \) there exists a steady state at \( s_1 = 0 \). From Proposition 1 we know that \( L_B = 0 \) if \( L_B^+(s_1,t-1) \leq 0 \). Thus, it is clear that condition (30) is satisfied under these circumstances. Similarly, if \( \theta L_B^+(s_1,t-1) = 1 \) \( \geq 1 \) there exists a steady state at \( s_1 = 1 \). Again, from Proposition 1 we know that \( L_B = \frac{1}{\theta} \) if \( \theta L_B^+(s_1,t-1) \geq 1 \). That (30) holds for \( L_B = \frac{1}{\theta} \) and \( s_1 = 1 \) is straightforward.

With the above considerations, we can now proceed to a complete steady-state analysis:

1. If \( L_B^+(s_1,t-1) > 0 \) and \( \theta L_B^+(s_1,t-1) = 1 \), then \( L_B(s_1) \psi(s_1) \) crosses the bisectoral line once only and from above at \( 0 < s_1 < 1 \). Thus, there exists a unique and stable steady state with \( 0 < s_1 < 1 \).
2. If \( L_B(s_{1,t-1} = 0) > 0 \) and \( \theta L_B^+(s_{1,t-1} = 1) = 1 \), then \( L_B(s_1)\psi(s_1) \) either crosses the bisectrix once only and from above at \( s_1 = 1 \) or it crosses the bisectrix twice, first from above at \( 0 < s_1 < 1 \), and second from below at \( s_1 = 1 \). In the first case, the corner solution \( s_1^* = 1 \) is the unique and stable steady state. In the second case, only the steady state with \( 0 < s_1^* < 1 \) is stable.

3. If \( L_B^+(s_{1,t-1} = 0) > 0 \) and \( \theta L_B^+(s_{1,t-1} = 1) > 1 \), then \( L_B(s_1)\psi(s_1) \) either crosses the bisectrix once only and from above at \( s_1 = 1 \) or it crosses the bisectrix three times, first from above at \( 0 < s_a^1 < 1 \), second from below at \( 0 < s_b^1 < 1 \), where \( s_a^1 < s_b^1 \), and third from above at \( s_1 = 1 \). In the first case, the corner solution \( s_1^* = 1 \) is the unique and stable steady state. In the second case, only \( s_a^1 \) of the two interior steady states is stable. A second stable steady state is given at the corner \( s_1^* = 1 \).

4. If \( L_B^+(s_{1,t-1} = 0) = 0 \) and \( \theta L_B^+(s_{1,t-1} = 1) < 1 \), then \( L_B(s_1)\psi(s_1) \) either crosses the bisectrix once only and from above at \( s_1 = 0 \) or it crosses the bisectoral line twice, first from below at \( s_1 = 0 \) and then from above at \( 0 < s_1 < 1 \). In the first case, the corner solution \( s_1^* = 0 \) is the unique and stable steady state. In the second case, only the steady state with \( 0 < s_1^* < 1 \) is stable.

5. If \( L_B^+(s_{1,t-1} = 0) = 0 \) and \( \theta L_B^+(s_{1,t-1} = 1) = 1 \), \( L_B(s_1)\psi(s_1) \) crosses the bisectrix twice, first at \( s_1 = 0 \) then at \( s_1 = 1 \). If \( L_B(s_1)\psi(s_1) > s_1 \) at \( s_1 \in (0,1) \), then it crosses the bisectrix from below (above) at \( s_1 = 0 \) and from above (below) at \( s_1 = 1 \). Hence, the unique and stable steady state is given by the corner solution \( s_1^* = 1 \) \((s_1^* = 0)\).

6. If \( L_B^+(s_{1,t-1} = 0) = 0 \) and \( \theta L_B^+(s_{1,t-1} = 1) > 1 \), then \( L_B(s_1)\psi(s_1) \) crosses the bisectoral line either twice, first at \( s_1 = 0 \) from below then at \( s_1 = 1 \) from above, or it crosses the bisectrix three times, first from above at \( s_1 = 0 \), second from below at \( 0 < s_1 < 1 \), and third from above at \( s_1 = 1 \). In the first case, only the corner steady state given by \( s_1^* = 1 \) is stable. In the second case, the interior steady state is not stable, while both corner solutions \( s_1^* = 0 \) and \( s_1^* = 1 \) are stable steady states.

7. If \( L_B^+(s_{1,t-1} = 0) < 0 \) and \( \theta L_B^+(s_{1,t-1} = 1) < 1 \), then \( L_B(s_1)\psi(s_1) \) either crosses the bisectrix once only and from above at \( s_1 = 0 \) or it crosses the bisectoral line three times, first from above at \( s_1 = 0 \), second from below at \( 0 < s_1^* < 1 \), and
third from above at $0 < s_1^b < 1$, whereas $s_1^a < s_1^b$. In the first case, the corner solution $s_1^a = 0$ is the unique and stable steady state. In the second case, only $s_1^b$ of the two interior steady states is stable. A second stable steady state is given at the corner $s_1^a = 0$.

8. If $L_B^+(s_{1,t-1} = 0) < 0$ and $\theta L_B^+(s_{1,t-1} = 1) = 1$, then $L_B(s_1)\psi(s_1)$ either crosses the bisectrix twice, first at $s_1 = 0$ from above then at $s_1 = 1$ from below, or it crosses the bisectrix three times, first from above at $s_1 = 0$, second from below at $0 < s_1 < 1$, and third from above at $s_1 = 1$. In the first case, only the corner steady state given by $s_1^a = 0$ is stable. In the second case, the interior steady state is not stable, while both corner solutions $s_1^a = 0$ and $s_1^a = 1$ are stable steady states.

9. If $L_B^-(s_{1,t-1} = 0) < 0$ and $\theta L_B^-(s_{1,t-1} = 1) > 1$, then $L_B(s_1)\psi(s_1)$ crosses the bisectrix three times, first at $s_1 = 0$ from above, second at $0 < s_1 < 1$ from below, and third at $s_1 = 1$ from above. Thus, the interior steady state is not stable, while both corner solutions $s_1^a = 0$ and $s_1^a = 1$ are stable steady states.

**B Foundation of the Model with Patent Races**

In this section, we provide another interpretation of the model’s micro-foundation using patent races.

In each sector, there is a finite number of domestic firms that can engage in innovation/patent races at the beginning of each period. There are two types of R&D projects that the firms may conduct:

1. high-risk research aiming at technological level $\bar{A}_t$

2. low-risk research aiming either at technological level $\bar{A}_{t-1}$ (e.g., adopting an existing intermediate from the previous world technology frontier) or at inventing around an existing patent of an intermediate at technological level $\bar{A}_{t-1}$.

Establishing a research project of type (2) incurs a small fixed cost $\varepsilon > 0$, caused e.g. by the necessity to first learn about the existing intermediates of this technology level. For simplicity, we assume that the risky research project will not incur costs. In both
types of R&D, the first firm to succeed obtains a patent valid for two periods (which is ‘de facto’ equivalent to a patent of longer validity).

With respect to research project (1), we assume that each firm possesses the same probability of innovation success depending on the level of basic research. Also, each firm that participates in the race possesses equal probability of being the first to be successful. Since there are no fixed costs for participating in the patent race, all firms will participate in the risky innovation project (1).\textsuperscript{14} The probability that one firm will succeed in creating an innovation at the new technological frontier is $\rho_1(L_B)$ if the respective sector has been in state 1 in the previous period and $\rho_2(L_B)$ if the sector was in state 2 or f before.\textsuperscript{15}

For the low-risk project (2), the probability that a firm will be successful is one, and the fastest firm obtains the patent. In each period, the following sequence of events occur:

1. Government chooses basic research

2. Domestic firms engage in risky research projects

3. Technological frontier increases to $\bar{A}_t = \gamma \bar{A}_{t-1}$

4. If no domestic firm was successful in the risky research project, domestic firms may decide to enter the patent race with low-risk research at a small cost $\varepsilon$

5. Foreign intermediate firms decide whether to enter (or keep on operating in) the domestic market at positive costs

6. Production of consumption good

Again, in the subgame-perfect equilibrium, the foreign firm will only enter (or keep operating) if it is able to offer an intermediate good at a higher technological level. Concerning the patent race with low-risk research, domestic firms will only participate if in period $t$ no domestic patent for an intermediate with technological level $A_{t-1}$ exists. Note that if a patent held by a foreign firm exists, it will be profitable to invent around it in anticipation of the foreign firm leaving the market if competition at the

\textsuperscript{14}Note that we could also assume that the risky project is costly for the firms. Then, however, there exists a positive level of $L_B$ for which $\rho_1(L_B)$ and $\rho_2(L_B)$ are zero because no firm will participate in the innovation race due to prospects of negative profits from participating.

\textsuperscript{15}This assumption reflects the familiarity of the domestic firms with previous frontier technology.
same technological level ensues. Of course, this is just one interpretation of our set-up. One can easily find others that add further realistic features, such as simultaneous patent races with respect to high- and low-risk research.

\footnote{With the current specification of the game, it may happen for \( \sigma \) very close to one that no firm will find it profitable to engage in the second patent race because no firm succeeded in the first one, even if no domestic patent at the \( A_{t-1} \)-level exists. This can be avoided by assuming that the incumbent (e.g. the type 2 firm that operated in the market in the previous period or the one that has been outcompeted by the foreign firm) can participate in the race without costs, while the ‘outsiders’ incur costs when participating.}
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