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Author(s):
Gersbach, Hans; Hahn, Volker; Liu, Yulin

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H. Gersbach, V. Hahn and Y. Liu

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Macroprudential Policy in the New Keynesian World*

Hans Gersbach
CER-ETH
Center of Economic Research at ETH Zurich and CEPR
8092 Zurich, Switzerland
hgersbach@ethz.ch

Volker Hahn
Department of Economics
University of Konstanz
Box 143
78457 Konstanz, Germany
volker.hahn@uni-konstanz.de

Yulin Liu
CER-ETH
Center of Economic Research at ETH Zurich
8092 Zurich, Switzerland
liuyul@ethz.ch

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Abstract

We integrate banks and the coexistence of bank and bond financing into an otherwise standard New Keynesian framework. There are two policy-makers: a central banker, who can decide on short-term nominal interest rates, and a macroprudential policy-maker, who can vary aggregate capital requirements. The two policy instruments can be used to stabilize shocks, to moderate bank credit cycles, and to induce a more efficient allocation of resources across sectors. Moreover, we investigate the optimal combination of simple policy rules for interest rates and capital requirements. The optimal policy rules imply that the central bank should focus exclusively on price stability and the macroprudential policy-maker should react exclusively to changes in loan rate premia.

Keywords: central banks, banking regulation, capital requirements, optimal monetary policy.

JEL: E520, E580, G280

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1 Introduction

1.1 Motivation

Macrounancial and monetary policies are two policy areas with three objectives: banking stability, price stability, and output stability. How these policies should be conducted and organized is a major issue, and since the global financial meltdown, policy-makers and academics have been working on solutions.

This task is complicated by two problems. First, pursuing one objective may impede fulfilling the other objectives. This is well-known for potential inflation-output trade-offs. Moreover, it has been a long-standing issue whether and how changes in the monetary stance can strengthen or undermine financial stability. Similarly, macroprudential policies that are aimed at stabilizing output may have adverse consequences for financial stability. For instance, if aggregate capital requirements are relaxed in a downturn to counteract an output decline, the banks’ balance sheets may weaken, thereby exacerbating vulnerabilities in the financial system.

Second, policy-makers do not have enough instruments to attain the three key objectives: price, output, and financial stability. Typically, the central bank’s instrument is the short-term interest rate. Macroprudential policies have one additional macro-tool—varying bank capital requirements. Hence there are only two independent macro-instruments for attaining three objectives.

This paper integrates a banking sector into an otherwise standard New Keynesian framework in order to develop a microfounded model that enables us to study the interplay of monetary policy and macroprudential policy.\footnote{Several proposals on how monetary and macroprudential policies could be organized and conducted are reviewed in Section 1.3.} Specifically, we focus on an economy in which one portion of intermediate firms has to rely on bank financing to produce. Idiosyncratic shocks to the production of these firms may hamper their ability to repay loans and ultimately affect the solvency of banks. The optimal level of aggregate capital requirements for the banking system trades off the costs of higher capital requirements, which make bank financing more expensive and thereby lower output in the sector of the economy that relies on bank financing, and the benefits,
namely a higher stability of the banking sector. We also examine how the two policy instruments can and should be used to stabilize shocks to aggregate productivity and financial shocks. While interest rates have a long tradition as macroeconomic stabilization tools, policy-makers have started only recently to use changes in capital requirements for this purpose as well.²

More specifically, we are interested in these classic questions. First, how should monetary policy and macroprudential policy be used to stabilize shocks in the economy and to safeguard financial stability, price stability, and output stability? Second, should macroprudential policy-makers focus on price and output stability as well? Third, which policy-maker should focus on financial stability, and which indicator of financial stability should be used?

1.2 Approach and results

To integrate banks and different financing modes into the New Keynesian model, we start from five key observations.

1. While some firms mainly rely on financing through banks, other firms obtain funds via the capital market. The fraction of bank-financed firms varies widely across countries, but is non-negligible in general. It is particularly high in many continental European countries. Moreover, the relative significance of the two financing modes depends on the state of the economy (see e.g. De Fiore and Uhlig (2011) and Laeven and Valencia (2013) on these differences).

2. Firms that turn to banks and obtain bank loans are more risk-prone than firms that finance themselves through bond markets. These risks translate into risk premia on loan interest rates and default risks of banks.³

²Today it comes in the form of countercyclical capital requirements and has been included in national law in many countries. It is already used in some countries (see https://www.bis.org/bcbs/ccyb/, retrieved on 15th March 2017).

3. Bank deposits are protected by explicit or implicit guarantees, e.g., a deposit insurance system or the government’s commitment to bail out banks in the event of a crisis. This insurance is likely to create problems of moral hazard and may lead to excessive risk-taking by banks.

4. Defaults of banks impose additional costs on households, e.g., in the form of higher taxes that are required to finance bailouts.

5. Households face costs when they acquire and hold equity. For example, households may need time (or pay fees) to assess the return prospects of these risky assets. As a consequence, other assets like deposits or bonds are preferred to equity, unless the returns on equity compensate investors for these costs beyond a standard risk premium.

We proceed as follows: In a first step, we embed these features into a standard New Keynesian model and derive the relationships that must hold in equilibrium. With the help of these relationships, we illustrate the properties of the steady state for different levels of capital requirements. Typically, for low levels of aggregate capital requirements, the allocation of resources across sectors is inefficient: Due to banks’ limited liability and the fact that deposits are insured, banks take excessive risks and therefore too many resources are channeled through the banking sector, and too few through the capital market. If aggregate capital requirements are too high, however, this will excessively shift resources to safe firms and entail large equity management costs. Optimal macroprudential policies have to trade off these costs and benefits of higher capital requirements.

In a second step, we provide a first round of applications of our framework. Using a log-linearized version of our model, we begin with some examples of how the economy reacts to various shocks, where we consider exogenous disturbances to aggregate productivity, monetary policy, macroprudential policy as well as shocks to financial intermediation. We next derive a welfare formula by computing a second-order approximation to welfare around the steady state that is implied by unconditionally optimal policies. This provides a starting point for a wide variety of further possible policy-making investigations. In particular, we determine optimal rules for monetary policy as well as macroprudential policy rules from a fairly general class of rules. Our calibration
exercise reveals that the central bank should focus exclusively on price stability. The macroprudential policy-maker, however, should react vigorously to changes in loan rate premia.

1.3 Literature

We integrate banks and financial stability concerns into the New Keynesian framework, leaving all other essential parts of this framework intact. However, integrating the five features referred to above into the New Keynesian framework is a major undertaking.

Several papers already combine monetary and macroprudential policy-making. Angeloni and Faia (2013) show how bank capital requirements can mitigate the consequences of bank runs when monetary policy follows interest-rate rules. Christensen et al. (2011) link risky bank projects to the aggregate-credit-to-GDP ratio. Another branch of this literature has focused on optimal policies. Among others, De Paoli and Paustian (2013) and Collard et al. (2017) characterize Ramsey-optimal policies. The latter, in particular, show that prudential policies prevent excessive risk-taking by banks while monetary policy aims at smoothing business cycles. In contrast with them, we study the case where only a part of the firms rely on bank-financing. As a consequence, financial shocks and changes in macroprudential policy affect this sector to a larger extent, which has important implications for optimal policy rules.

Our contribution to this literature is as follows: First, we embed the five features referred to above into an otherwise standard New Keynesian framework. This allows for a comparison with standard results in monetary policy-making. Second, we investigate how such an economy responds to shocks affecting bank-financed or capital-market-financed firms and aggregate shocks. Third, we derive the unconditional optimal welfare formula and focus on institutional questions of how policy-making can be operationalized by Taylor-type rules for monetary policy and macroprudential policy. To our knowledge, this is the first paper that addresses optimal simple rules for both monetary and macroprudential policy makers.

\footnote{Loisel (2014) assesses the conclusions that can be drawn from the early literature on what monetary and macroprudential policy rules can achieve.}
Several frameworks for monetary and macroprudential policies have been proposed. Detailed outlines, rationales, and assessment can be found in Gersbach and Hahn (2011), Schoenmaker and Wierts (2016), Borio (2014), Claessens et al. (2013), and Jonsson and Moran (2014). Like these frameworks, our model shares the view that constraints on leverage and credit expansion are a key angle of macroprudential policies. Authors, however, differ as to how effective countercyclical policies can be and whether macroprudential policies should aim at smoothing credit cycles. We use macroprudential policies in the form of varying aggregate capital requirements for the banking system and investigate how such policies need to be conducted and organized.

1.4 Structure of the paper

The remainder of this paper proceeds as follows: In the next section we present the model. After establishing the equilibrium conditions in Section 3, we characterize the steady state of the economy and show the impulse responses to various types of shock in Section 4. In Section 5 we explore optimal policy rules. Section 6 concludes.

2 Model

2.1 Overview

There are six sectors in the model: households, banks, safe and risky intermediate firms, final firms, and the public sector made up of a fiscal agency, a central bank, and a macroprudential policy-maker. We start with the timeline of events in each period \( t = 0, 1, 2, \ldots \). Then we describe the agents’ optimization problems, the firms’ technologies, and the market structure in more detail.

The sequence of events is shown in Figure 1. Each period \( t \) is divided into two stages. At the beginning of the first stage, households own bonds \( B_t \), which they acquired in the previous period. In addition, aggregate shocks materialize. In particular, we will consider aggregate shocks to the productivity of intermediate firms, financial shocks, shocks to macroprudential policy and monetary policy. After observing the aggregate

\footnote{A more extended framework would also include microeconomic regulation and supervision of banks (see Gersbach and Hahn (2011) for such a framework).}
shocks, the monetary policy-maker (mon) chooses $I_t$, the nominal interest rate on bonds that mature in $t + 1$, where one unit of the bond represents a claim on one nominal unit at maturity.

The financial restriction for risky intermediate-goods firms implemented in our model is a variant of the working-capital requirement that has been adopted in other papers (e.g. in Jermann and Quadrini (2012)). Firms have to raise intra-period funds in order to finance wage payments to their workers before receiving the revenues from selling their products. In particular, safe firms issue real claims that are due in the second stage of period $t$ (safe firms’ bonds) in return for households’ labor. Risky intermediate firms cannot issue claims to households to hire labor because they have to be monitored closely. They rely on intra-period bank loans instead, where we assume that each bank serves exactly one risky firm.\textsuperscript{6} Hence risky firms take loans from banks and receive bank deposits at the same time. Risky firms use these deposits to hire labor from households. Deposits are riskless because they are insured by the government. The macroprudential policy-maker (mac) sets a capital requirement $\Gamma_t$ which banks have to fulfill in order to be allowed to operate. We also assume that households incur costs when acquiring and holding equity. This makes equity financing more costly for banks than debt financing.

At the beginning of the second stage, idiosyncratic shocks occur, affecting risky firms’ productivities. Subsequently, safe and risky intermediate firms choose their prices, taking the amount of labor hired in the first stage as given. While safe firms face Rotemberg price adjustment costs, risky firms live only for one period and can choose

\textsuperscript{6}Collard et al. (2017) make a similar assumption. The justification is that, under limited liability, a bank’s profit is highest if the risk is concentrated in a single loan.
the prices of their outputs freely. Safe firms can always repay their bonds, whereas some risky intermediate firms with adverse shock realizations cannot repay their bank loans in full. As a consequence, the corresponding bank may fail if its equity buffer is insufficient. These banks are bailed out by the fiscal agency, which uses a lump-sum tax on all households to guarantee that deposits are always repaid. All banks are dissolved, and the remaining funds are distributed to equity holders. Profits of intermediate firms accrue to households, and bonds \( B_t \) mature. Perfectly competitive final-good firms purchase the intermediate goods and use them to produce final goods. Households acquire new bonds \( B_{t+1} \) at a price \( 1/I_t \) as well as final goods. In the following, we describe the different agents in more detail.

### 2.2 Households

Each household has the instantaneous utility function:

\[
    u(c_t, n_t) = \frac{c_t^{1-\sigma} - 1}{1 - \sigma} - \frac{\psi n_t^{1+\varphi}}{1 + \varphi},
\]

where the relative risk aversion of consumption \( \sigma > 0 \), the inverse of the Frisch labor supply elasticity \( \varphi > 0 \), the relative utility weight of labor \( \psi > 0 \), and \( c_t \) and \( n_t \) denote consumption and total labor, respectively. It is sufficient to focus on the behavior of a representative household.

In the first stage of each period, the household provides labor to both safe and risky intermediate firms. Safe intermediate firms provide safe intra-period bonds \( \tilde{s}_t \) with gross return \( R_s^t \) to the household in exchange for the labor \( n_t^s \):\(^\text{7}\)

\[
    \tilde{s}_t = \tilde{w}_t n_t^s,
\]

where \( \tilde{w}_t \) represents the real market wage.

The household saves in bank deposits \( \tilde{d}_t \) with gross return \( R_d^t \) and bank equity \( \tilde{e}_t \). Thus, the amount of loans the bank can issue is

\[
    \tilde{l}_t = \tilde{d}_t + \tilde{e}_t.
\]

\(^{7}\)In line with other papers using working-capital constraints, we could normalize this safe intra-period interest rate to one.
Deposits are backed by the government and therefore riskless. Hence the following no-arbitrage condition holds
\[ R^d_t = R^s_t. \] (4)
As a consequence, we will not distinguish between \( R^d_t \) and \( R^s_t \) for the remainder of this paper.

The representative household acquires equal amounts of equity from each bank.\(^8\) Because the aggregate shocks are realized at the beginning of each period and already known when asset allocations are made, the equity portfolio held by the household is riskless. The gross return on the equity portfolio is denoted by \( R^e_t. \)\(^9\)

Using loans \( \tilde{l}_t \) granted by banks, risky intermediate firms hire labor \( n^r_t \) from the household
\[ \tilde{l}_t = \tilde{w}_t n^r_t. \] (5)

It will be useful to introduce \( s_t := R^s_t \tilde{s}_t, \) \( d_t := R^s_t \tilde{d}_t, \) \( l_t := R^s_t \tilde{l}_t, \) and \( e_t := R^s_t \tilde{e}_t. \)

Intuitively, \( s_t \) and \( d_t \) are the funds the household receives in the second stage from investing in safe bonds \( \tilde{s}_t \) and deposits \( \tilde{d}_t, \) respectively. \( l_t \) and \( e_t \) correspond to the hypothetical funds that one would receive in stage 2 if one invested \( \tilde{l}_t \) and \( \tilde{e}_t \) at rate \( R^s_t \) in stage 1.

We model the costs of equity financing by assuming that the household needs to spend resources to monitor and manage equities. For simplicity, we assume that the resources necessary for equity management are proportional to the dividend payments \( R^e_t \tilde{e}_t: \)
\[ m_t = \chi_t R^e_t \tilde{e}_t, \] (6)
where \( \chi_t \) is an exogenous positive random variable.

We introduce the premium on equity financing as \( \Delta^e_t := R^e_t / R^s_t. \) In the second stage, the household receives the gross returns on deposits, equities, and safe bonds, i.e. \( d_t, \Delta^e_t e_t, \) and \( s_t, \) respectively. In addition, safe and risky intermediate firms’ profits

\(^8\)We are effectively looking for symmetric bank equity allocations. Since banks are identical at this stage, this allocation is rationalized in the equilibrium.
\(^9\)The representative household holds all assets in the economy and is fully diversified. We could allow for heterogeneity of bank equity holding across households to further rationalize the costs of acquiring and holding risky bank equity.
$z_t^s$ and $z_t^r$ also go into the household’s pocket. On the expense side, the household consumes goods $c_t$ and pays lump-sum taxes $\tau_t^l$.

Using $w_t = R_t^s \tilde{w}_t$ and (2)-(6), we can write the total funds the household receives from safe bonds, deposits, and equity net of equity management costs as

$$s_t + d_t + \Delta_t^e e_t - \chi_t \Delta_t^e e_t = w_t n_t + (\Delta_t^e (1 - \chi_t) - 1) e_t,$$

where $n_t = n_t^s + n_t^r$ denotes total labor.

We are now in a position to state the household’s budget constraint in the second stage of period $t$ as

$$c_t + B_{t+1} \leq \frac{B_t}{p_t} + z_t^s + z_t^r - \tau_t^l + w_t n_t + (\Delta_t^e (1 - \chi_t) - 1) e_t.$$

The representative household maximizes the overall utility

$$\max_{\{c_t, B_{t+1}, n_t, e_t\} \in \mathbb{C}} \left\{ \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t^{1-\sigma} - 1}{1 - \sigma} - \psi n_t^{1+\varphi} \right) \right\} \text{s.t. (8),}$$

where $\beta$ is the discount factor with $0 < \beta < 1$. Let $\lambda_t$ be the current-value Lagrange multiplier associated with (8). Then we obtain the following first-order conditions of the household problem:

$$c_t : \quad \lambda_t = c_t^{-\sigma},$$

$$B_{t+1} : \quad \frac{\lambda_t}{\lambda_{t+1}} = E_t \left[ \frac{\beta \lambda_{t+1}}{p_{t+1}} \right],$$

$$n_t : \quad \psi n_t^\varphi = \lambda_t w_t,$$

$$e_t : \quad \Delta_t^e = -\frac{1}{1 - \chi_t},$$

where the last equation is a no-arbitrage condition, which involves that investing an additional unit of funds into equity delivers the same additional payoff—net of equity management costs—as investing the same amount in a deposit.
2.3 Final-good firms

There are infinitely many, perfectly competitive firms that purchase intermediate goods \( y_t(i) \) at prices \( p_t(i) \) and assemble them to a final good \( y_t \), which can be used for consumption:

\[
y_t = \left( \int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} \, di \right)^{\frac{\theta}{\theta-1}}, \tag{14}
\]

where \( \theta > 1 \) represents the elasticity of substitution between differentiated intermediate goods.

Hence each firm’s profit maximization problem can be formulated as

\[
\max_{\{y(i)\}_{i=0}^1} \left\{ p_t y_t - \int_0^1 p_t(i) y_t(i) \, di \right\} \quad s.t. \ (14).
\]

This problem leads to the following demand for intermediate good \( i \):

\[
y_t(i) = \left( \frac{p_t(i)}{p_t} \right)^{-\theta} y_t, \tag{15}
\]

and the price level is

\[
p_t := \left( \int_0^1 p_t(i)^{1-\theta} \, di \right)^{\frac{1}{1-\theta}}. \tag{16}
\]

2.4 Intermediate firms

In the first stage, monopolistically competitive intermediate firms attract loans from banks as well as households and hire labor. In the second stage, output materializes, prices are chosen, loans are repaid, and profits are transferred to households. The total number of intermediate firms is normalized to 1. A fixed proportion \( \nu \) constitutes the sector of safe firms. These firms are infinitely-lived and face quadratic price-adjustment costs. The remaining \( 1 - \nu \) intermediate firms, the sector of risky firms, live for one period and can choose their prices freely.
2.4.1 Safe firms

The safe firms’ production function is

$$y_s^t(i) = a_t n_s^t(i),$$

where $a_t$ is aggregate productivity, which is driven by an exogenous stochastic process.

In the first stage of each period $t$, safe firms take loans from (or issue bonds to) households at a gross real rate $R^s_t$ and use them to hire labor

$$s_t(i) = \bar{w}_t n^s_t(i).$$

Taking into account that the value of loans in stage 2 is $s_t(i) = R^s_t s_t(i)$, safe firm $i$’s real profit in the second stage of period $t$ can be written as

$$z^s_t(i) = p^s_t(i) y^s_t(i) - s_t(i) - \gamma^p \left( \frac{p^s_t(i)}{p^s_{t-1}(i)} - 1 \right)^2 y_t,$$

where $p^s_t(i)$ is the price for firm $i$’s output, and $\gamma^p > 0$ is the coefficient for price adjustment cost $\gamma^p \left( \frac{p^s_t(i)}{p^s_{t-1}(i)} - 1 \right)^2 y_t$.

Taking the wage as given, firms maximize the expected sum of discounted real profits

$$\max_{\{n_t^s(i), p_t^s(i)\}} \left( E_0 \sum_{t=0}^{\infty} Q_t z^s_t(i) \right),$$

subject to

$$a_t n^s_t(i) \geq \left( \frac{p^s_t(i)}{p_t} \right)^{-\theta} y_t,$$

where future profits are discounted by

$$Q_t = \frac{\beta^t \lambda_t}{\lambda_0} = \frac{\beta^t c^0_t}{c_t^0}.$$

The optimal behavior of safe firms is quite standard and is examined and described in Appendix A.
2.4.2 Risky firms

The risky firms’ production function is

\[ y_r(i) = (\phi(i))^\alpha Aa_nr(i), \]  

(22)

where \( \phi(i) \in [0, 1] \) with uniform distribution represents an idiosyncratic shock to firm \( i \)’s productivity. Parameter \( \alpha (\alpha > 0) \) affects the riskiness of production, where lower values involve less risk. Parameter \( A \) affects the relative productivity of risky firms compared to the safe firms.

Note that while the aggregate shock, \( a_t \), becomes commonly known at the beginning of each period \( t \), the idiosyncratic shock \( \phi(i) \) is realized at the beginning of the second stage of the corresponding period. Hence, in the first stage of each period it is unknown both to the risky firms and to the banks funding these firms.

Risky firms get loans from banks

\[ \tilde{l}_t(i) = \tilde{w}_tn_r(i) \]  

(23)

to finance the wage bill. We assume that like banks, risky intermediate firms live for one period. Consequently, they do not face price-adjustment cost. Thus, the risky firms’ real profit in period \( t \) can be written as

\[ z_r^*(i) = \frac{p_r^t(i)}{p_t}y_r^t(i) - R_r^t(i)\tilde{l}_t(i) \]  

(24)

subject to

\[ \phi(i)^\alpha Aa_nr(i) \geq \left( \frac{p_r^t(i)}{p_t} \right)^{-\theta} y_t. \]  

(25)

\( R_r^t(i) \) represents the gross return on loans paid by risky firms. If firms default, it is smaller than the market loan rate \( R_l^t \), i.e. \( R_r^t(i) < R_l^t \). Otherwise, \( R_r^t(i) = R_l^t \).

Aggregate profits, \( z^* = \int_0^1 \int_0^1 z_r^*(i) d\phi(i) di \), are paid out to households as dividends.

We analyze the optimal behavior of risky firms in Appendix B and obtain

**Proposition 1**

(i) A risky firm’s demand for bank loans is

\[ l_t(i) = \frac{(Aa_t)^{\theta-1}}{\tilde{w}_t^{\theta-1}(\Delta l)^\theta} L^* y_t, \]  

(26)
where $\Delta_t^l := \frac{R^l_t}{M_t}$ represents the loan rate premium\textsuperscript{10} and $L^*$ is the root of

$$
g(L) := (\theta - 1)L^{-\frac{1}{b}} + (1 + \alpha(\theta - 1))L^{-\frac{3}{\alpha(\theta - 1)}} - (\theta + \alpha(\theta - 1))$$

that satisfies $0 < L^* < 1$.

(ii) Defaulted firms are those with realized $\phi(i)$ below

$$
\phi^c = (L^*)^{\frac{1}{\alpha(\theta - 1)}}.
$$

We observe that the risky firms’ demand for loans is a decreasing function of the loan rate $\Delta_t^l$ and the real wage $w_t$. It increases with aggregate productivity $a_t$ and aggregate output $y_t$. Moreover, we note that the fraction of defaulting firms, $\phi^c$, is constant over the business cycle. However, we will see that the same is not true for the fraction of banks that default.

2.5 Banks

The banking system is characterized as follows: There is a continuum of banks.\textsuperscript{11} An individual bank lives for one period and is specialized in granting loans to a particular risky firm.\textsuperscript{12} Accordingly, banks do not hold a well-diversified loan portfolio. The banking system is competitive and operates as follows:

- Banks attract equity from households. A bank is founded if it receives a positive amount of equity.
- Banks attract deposits from households and decide on their capital structure.

\textsuperscript{10}The premium stems from two sources: default risk and higher costs of equity financing.

\textsuperscript{11}Since, ultimately, one risky firm will obtain a loan from one bank, we assume for measurement consistency that the measure of banks is at least $1 - \nu$.

\textsuperscript{12}Our assumption that banks serve only one firm is meant to capture the stylized fact that banks’ loan portfolios are not perfectly diversified. There are both theoretical as well as empirical arguments for why banks may choose non-diversified loan portfolios. First, empirical papers (Acharya et al. (2006), Berger et al. (2010) and Hayden et al. (2007)) find that banks with more specialized loan portfolios are more profitable. Second, in the context of our model where limited liability induces banks to take excessive risks, a bank with a diversified loan portfolio would offer a lower return on equity than a bank serving only one individual firm.
• The market for loans opens. Each firm $i$ in the pool of ex-ante identical risky firms demands a loan $l_t(i)$. If a bank satisfies capital requirements, it is allowed to operate and decides whether to offer its intermediation services, offering loans coupled with monitoring. Market clearing yields the loan rate $R^l_t$.

• The productivity of risky firms is affected by idiosyncratic shocks. If a risky firm cannot pay back the loan, banks will secure the liquidation value.

To examine the equilibrium in the loan market, we assume that perfect monitoring prevails, i.e. banks can enforce the terms of the contract to ensure that they either get the repayment of the loan or the liquidation value if the firm cannot pay back. Monitoring is costless for banks.

Once banks have received equity, the objective of a bank is to maximize returns on equity, taking into account limited liability, i.e. the fact that equity holders do not bear losses. In doing this, they decide on the capital structure, i.e. how many deposits they want to attract, whether they want to attract more equity, and whether they want to offer loans to risky firms. We assume that an individual bank can attract equity and deposits as long as it offers expected returns with which equity holders and depositors, respectively, are at least as well off as with other investment opportunities. Of course, given such individual choices, aggregate supply and demand for equity and deposits have to match in equilibrium.

Note that in our model, the maximization of the expected return on equity is equivalent to the maximization of the utility of shareholders. The reasons are as follows: First, as banks are perfectly competitive, an individual bank’s choice will not alter prices in the economy. Second, the bank’s decision to lend does not open up new insurance opportunities for households. As a consequence, all shareholders will agree that the bank should maximize its expected return on equity in order to contribute the maximal expected amount to the budget of shareholders.

We now consider a representative bank’s problem in more detail. Since loan and deposit markets are perfectly competitive, the bank demands an amount of deposits $d_t(i)$ at the prevailing deposit rate without worrying about whether this is consistent with market clearing for deposits and loans. Once the bank has chosen its capital structure $e_t(i)$, it decides whether to offer $e_t(i) + d_t(i)$ as loans to risky firms or to invest in safe firms’
bonds. As a tie-breaking rule, we assume that the bank will grant loans to risky firms if they generate at least the same expected return on equity as for other investment opportunities.

With these remarks, the problem of a representative bank can be formulated as in Appendix C. Three results that are crucial for our model are summarized in the next three propositions. First, we characterize the equilibrium capital structures.

**Proposition 2**

*Banks will always choose their capital structure to be equal to the aggregate capital requirement*

\[ e_t = \Gamma_t d_t. \]  

(29)

Similarly to the fraction of defaulting firms, we obtain the following proposition for the fraction of defaulting banks:

**Proposition 3**

*The fraction of defaulting banks is*

\[ \phi_t^\Gamma = \frac{\phi^c}{\left(\Delta_t^l(1 + \Gamma_t)\right)^{\frac{x}{\alpha(\theta - 1)}}} \in [0, \phi^c]. \]  

(30)

This proposition implies that the fraction of defaulting banks decreases with \( \Gamma_t \) and reaches 0 when banks are fully financed by equity, i.e. for \( \Gamma_t \rightarrow \infty \). The number of defaulting banks is also a decreasing function of \( \Delta_t^l \), i.e. the difference between the interest rates on loans and on deposits.

Accounting identities lead to the following relationship between loan rate \( \Delta_t^l \) and return on equity \( \Delta_t^e \):

**Proposition 4**

*
The market loan rate satisfies*

\[ \Delta_t^l = \frac{h^{-1}(\Gamma_t \Delta_t^e)}{1 + \Gamma_t}, \]  

(31)

where

\[ h(x) := \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} \left( \frac{1}{\frac{x}{\alpha(\theta - 1)}} - x \right) \phi^c + x - 1. \]  

(32)

\[ \text{See Gersbach et al. (2015a) on the uniqueness of bank capital structure in more general setups.} \]
As shown in Appendix C, function $h(\cdot)$ is a monotonically increasing function that satisfies $h(1) = 0$ and goes to infinity for large values of its argument. Hence, (31) establishes that the return on equity that the banks can generate, $\Gamma^{-1}h\left((1 + \Gamma t)\Delta^l_t\right)$, is an increasing function of the loan rate that banks charge risky intermediate firms. Moreover, we can conclude that for $\Gamma t \to 0$, $\Delta^l_t \to 1$. Therefore, if banks are fully financed by deposits, the rate on bank loans equals the rate on deposits. Positive values of $\Gamma_t$ result in values of $\Delta^l_t$ that are strictly larger than one.

### 2.6 The government

The sole function of the government is to use lump-sum taxes $\tau^l_t$ from households to bail out banks that have failed in the second stage. The government’s budget is balanced in every period. We assume a fraction $\mu$ of the bailout fees is dissipated when the government bails out the defaulting banks. Denoting the aggregate costs of these bailouts as $b_0$, the government’s budget constraint is

$$\tau^l_t + \frac{B_{t+1}}{p_{t+1}} = (1 + \mu)b_0 + \frac{B_t}{p_t}. \quad (33)$$

An expression for the bailout fees $b_0$ is presented in Appendix D.

We assume that monetary policy can be described by the following augmented Taylor rule

$$\frac{I_t}{I^*} = \left(\frac{\Pi_t}{\Pi^*}\right)^{\nu_{\text{mon}}} \left(\frac{\bar{y}_t}{\bar{y}^*}\right)^{\nu_{\text{y,mon}}} \left(\frac{\Delta^l_t}{\Delta^l^*}\right)^{\nu_{\text{\Delta,mon}}} e^{\xi^*_t}, \quad (34)$$

where variables with an asterisk denote steady-state values, $\xi_t$ stands for a monetary policy shock, and $(\nu_{\pi,\text{mon}}, \nu_{\text{y,mon}}, \nu_{\text{\Delta,mon}}, \nu_{\phi,\text{mon}})$ represent the Taylor-rule coefficients. A priori, we allow the Taylor rule to depend on inflation, the output gap, the aggregate volume of loans, the share of bank defaults, and the loan rate premium, where the gross rate of inflation is $\Pi_t := \frac{p_t}{p_{t-1}}$. While the first two variables are standard, the other three variables serve as indicators of financial stability, which the central bank may also take into account.\(^{14}\) We note that, for $\nu_{\pi,\text{mon}} = 1.5$, $\nu_{\text{y,mon}} = 0.5$, $\nu_{\phi,\text{mon}} = 0$, $\nu_{\text{\Delta,mon}} = 0$.

\(^{14}\)In a model with two sectors, one with flexible prices and one with sticky prices, Aoki (2001) shows that the central bank should target inflation in the sticky-price sector rather than aggregate inflation. In the present paper, we assume that the central bank focuses on a broad price index rather than sector-specific prices, which is in line with common practice among central banks. Exploring Taylor rules based on sector-specific inflation rates would be an interesting extension to our model.
and \( \nu_{\text{mon}} = 0 \), equation (34) simplifies to the standard Taylor rule:

\[
\frac{I_t}{I^*} = \left( \frac{\Pi_t}{\Pi^*} \right)^{1.5} \left( \frac{y_t}{y^*} \right)^{0.5} e^{\xi_t}.
\]  

(35)

We now turn to macroprudential policy-making. We assume the macroprudential policy-maker’s instrument is the capital requirement \( \Gamma_t \). Analogously to the central bank’s augmented Taylor rule (34), we write down a fairly general policy rule

\[
\frac{\Gamma_t}{\Gamma^*} = \left( \frac{\Pi_t}{\Pi^*} \right)^{\nu_{\text{mac}}} \left( \frac{y_t}{y^*} \right)^{\nu_{\text{mac}}} \left( \frac{\phi_t}{\phi^*} \right)^{\nu_{\text{mac}}} \left( \frac{\Delta_t}{\Delta^*} \right)^{\nu_{\text{mac}}} e^{\eta_t},
\]

(36)

where \((\nu_{\text{mac}}, \nu_{\text{mac}}^g, \nu_{\text{mac}}^d, \nu_{\text{mac}}^d, \nu_{\text{mac}}^\Delta)\) describe how vigorously the macroprudential policy-maker responds to the respective economic variables. Variable \( \eta_t \) represents a shock to macroprudential policy.

### 2.7 Market clearing

Finally, we state the market clearing conditions. Goods-market clearing implies that output equals the sum of consumption, the adjustment costs for prices, the equity management costs, and the dissipation when defaulting banks are bailed out,

\[
y_t = c_t + a_{adj}^p + m_t + \mu b o_t,
\]

(37)

where \( a_{adj}^p = \gamma^p \left( \frac{p_t(i)}{p_{t-1}(i)} - 1 \right)^2 y_t \).

Equilibrium on the labor market implies that the total supply of labor has to equal the demand from both safe and risky intermediate firms:

\[
n_t = n^s_t + n^r_t,
\]

(38)

where \( n^s_t = \int_0^\nu n^s_t(i)di \) represents total labor demand by safe firms, \( n^r_t = \int_0^\nu n^r_t(i)di \) total labor demand by risky firms.

The market for intra-period debt issued by safe firms is balanced if

\[
\tilde{s}_t = \int_0^\nu \tilde{s}_t(i)di.
\]

(39)

Finally, the following accounting identity must hold for banks:

\[
\tilde{l}_t = \tilde{d}_t + \tilde{e}_t,
\]

(40)

where \( \tilde{l}_t = \int_0^1 \tilde{l}_t(i)di \) represents total loans, \( \tilde{d}_t = \int_0^1 \tilde{d}_t(i)di \) total deposits, and \( \tilde{e}_t = \int_0^1 \tilde{e}_t(i)di \) total equity.
3 Equilibrium

In this section we summarize the equations describing the equilibrium. For this purpose, we observe that all safe firms are identical and thus set the same price $p_s^t := p_t^i(i)$. This enables us to introduce $q_t := p_t^s/p_t$, the ratio between the price of the intermediate goods produced by safe firms with respect to the aggregate price, and $\Pi_t^s := \frac{p_t^s}{p_{t-1}^s}$, the gross inflation for the intermediate goods produced by safe firms. With the help of this notation we now state the equations describing the evolution of endogenous variables for the paths of exogenous shocks \( \{a_t, \chi_t, \xi_t, \eta_t\}_{t=0}^{\infty} \).

As shown in Appendix B, a risky firm’s demand for bank loans is

$$l_t(i) = \left( \frac{Aa_t}{w_t} \right)^{\frac{\theta-1}{\theta}} \frac{L_s^t}{(\Delta l_t)^{\frac{\theta}{\theta-1}}} y_t^i, \quad (41)$$

where the loan is used to finance the wage bill, which implies

$$l_t(i) = w_t n_t^r(i). \quad (42)$$

The optimal price-setting of safe firms results in the following standard condition for price-setting in the presence of quadratic adjustment costs (see Appendix A):

$$\frac{y_t}{(q_t)^{\frac{\theta-1}{\theta}}} \left[ (1 - \theta) + \theta \frac{w_t}{q_t a_t} \right] - \gamma^p y_t \Pi_t^s (\Pi_t^s - 1)$$

$$+ \mathbb{E}_t \left[ \beta \frac{c_t^s}{c_{t+1}^s} \gamma^p y_{t+1} \Pi_{t+1}^s (\Pi_{t+1}^s - 1) \right] = 0. \quad (43)$$

Note that, in the absence of price adjustment costs, i.e. for $\gamma^p = 0$, each safe intermediate firm would charge a constant markup over real marginal costs, i.e. $q_t = p_t^s = \frac{\theta}{(\theta-1)} mc_t^s$, where real marginal costs are $mc_t^s = \frac{w_t}{a_t}$.

Inflation, $\Pi_t := \frac{p_t}{p_{t-1}}$ can be formulated as

$$\Pi_t = \frac{q_t^{-1} \Pi_t^s}{q_t}, \quad (44)$$

and, according to (30), the fraction of defaulting banks is

$$\phi_t^\Gamma = \frac{\phi^c}{(\Delta l_t(1 + \Gamma_t))^{\frac{\theta}{(\theta-1)}}}, \quad (45)$$
The demand function for the intermediate goods produced by safe firms results in the following equation (see (15) and (17)):

$$ a_t n_t^*(i) = q_t^{-\theta} y_t. \tag{46} $$

Aggregate output can be computed from the final-good firms’ production function and the production functions for intermediate firms (see (14), (17), and (22)):

$$ y_t = a_t \left( \nu n_t^*(i) \right)^{\theta - 1} + (1 - \nu) \frac{\theta}{\theta + \alpha(\theta - 1)} \left( A n_t^*(i) \right)^{\theta - 1} \right). \tag{47} $$

Combining (13) and (31) yields a relationship between the loan rate $\Delta^l_t$ and the equity management cost $\chi_t$

$$ \frac{1}{1 - \chi_t} = (\Gamma_t)^{-1} h \left( (1 + \Gamma_t) \Delta^l_t \right), \tag{48} $$

where $h(\cdot)$ is defined in (32).

Optimal bond holdings entail a standard consumption Euler Equation (see (10) and (11)):

$$ \frac{1}{I_t} c_t^{-\sigma} = E_t \left[ \beta c_{t+1}^{-\sigma} \frac{1}{\Pi_{t+1}} \right]. \tag{49} $$

Moreover, the marginal disutility from work has to equal the wage rate times the marginal utility of consumption (see (10) and (12)):

$$ \psi(\nu n_t^*(i) + (1 - \nu)n_t^*(i))^\sigma = c_t^{-\sigma} w_t. \tag{50} $$

Equilibrium on the goods market involves

$$ y_t = c_t + \frac{1}{2} \gamma^p \nu (\Pi_t^* - 1)^2 y_t + \frac{\chi_t}{1 - \chi_t} \frac{\Gamma_t}{1 + \Gamma_t} (1 - \nu) l_t(i) $$

$$ + \frac{\mu}{\theta + \alpha(\theta - 1)} \frac{\alpha(\theta - 1)}{\Pi_{t+1}} (L_t) \left( \Delta_t^l \right)^{\theta - 1}, \tag{51} $$

which is to be interpreted as output equals consumption, the costs for price adjustment and equity management, and the dissipations when the government bails out the defaulting banks. The expression for the bailout costs is derived in Appendix D.

To sum up, the equilibrium dynamics are described by the private-sector equilibrium conditions (41)-(51) and the policy rules for monetary and macroprudential policy, i.e. (34) and (36). It is then straightforward to determine the other variables not contained in this system of equations. We summarize the findings in the following proposition:
Proposition 5
For given shocks \(\{a_t, \chi_t, \xi_t, \eta_t\}_{t=0}^{\infty}\), the equilibrium \(\{n^*_t(i), n^*_t(i), w_t, l_t(i), \Delta^i_t, \phi^i_t, \Pi^*_t, \Pi_t, q_t, y_t, c_t, \Gamma_t, I_t\}_{t=0}^{\infty}\) is described by the system of Equations (34), (36), and (41)-(51).

4 Numerical Findings

4.1 Calibration

We follow Collard et al. (2017) in setting the discount factor to \(\beta = 0.993\), the inverse of labor supply elasticity to \(\varphi = 0.276\), the relative utility weight of labor to \(\psi = 3.409\), the relative risk aversion of consumption to \(\sigma = 1\), and the elasticity of substitution of intermediate goods to \(\theta = 7\) (which corresponds to a 17% markup). In addition, we set the steady-state value for the coefficient of equity management \(\chi_t\) to 0.0521, which results in an equity premium of 5.5% (U.S. data for 1900-2015).\(^{15}\) We set \(\gamma^p = 74.55\), which is the average of the values found in Table 1 in Ireland (2001).

We choose the fraction of safe firms \(\nu = 0.616\) such that the total revenue of the safe (bond-financed) firms is 1.5 times the total revenue of the risky (bank-financed) firms in steady state (see De Fiore and Uhlig (2011) and Gersbach et al. (2015b)). We select \(\alpha = 0.118\) such that the charge-off rate on loans

\[
\int_0^{\phi^c} \frac{R^1_t - R^i_t(i)}{R^1_t} d\phi(i) = \frac{\alpha (\theta - 1)}{\theta + \alpha (\theta - 1)} (L^*_t)^{\frac{\theta}{\theta + 1}}
\]

is equal to the empirical value 0.97%.\(^{16}\) The coefficient of bailout dissipation is set at \(\mu = 0.93\) capturing output losses and tax distortions, which amounts to quarterly output losses of 0.34%.\(^{17}\) We normalize the steady-state productivity of safe firms to \(a^* = 1\) and set \(A\) such that it satisfies

\[
\frac{\theta}{\theta + \alpha (\theta - 1)} A^{\frac{\theta}{\theta + 1}} = 1,
\]

which means that, loosely speaking, safe firms and risky firms are on average equally productive (see (47)).

\(^{15}\)See Damodaran (2016) for a review of the equity premiums across countries and over different periods.
\(^{16}\)See https://fred.stlouisfed.org/series/CORALACBN for the charge-off rate on loans for all commercial banks in the U.S. 1985—2016. Another approach is to choose \(\alpha\) such that the fraction of non-performing loans to total loans, i.e. \(\phi^c\), matches the ratio of non-performing loans to total loans for small enterprises in developed countries (see Beck et al. (2011)). Both approaches yield a very similar value for \(\alpha\).
\(^{17}\)We take the U.S. data from Laeven and Valencia (2012).
For our analysis of the dynamics of the economy in response to shocks, we will log-linearize the model around the unconditionally optimal (UO) steady state (see Damjanovic et al. (2008) and Damjanovic et al. (2015)), i.e. we determine the steady state associated with the UO policy, which maximizes the unconditional expectation of the household’s utility (1) subject to constraints (41)-(51). Given the other parameters, the corresponding numerical optimization problem results in the optimal capital requirement $\Gamma^* = 5.8\%$ and inflation rate $\Pi^* - 1 = 0.02\%$. Notably, the UO steady state features a mildly positive net rate of inflation. This is plausible as a mildly positive inflation rate alleviates the distortions arising from monopolistic competition to some extent. With this capital requirement and inflation rate, the corresponding fraction of defaulting banks is $\phi_{\Gamma^*} \approx 5.7\%$.

For the log-linearized economy, we will consider the following specification of shocks:

$$\hat{a}_t = \rho^a \hat{a}_{t-1} + \epsilon^a_t, \quad (52)$$
$$\hat{\chi}_t = \rho^x \hat{\chi}_{t-1} + \epsilon^\chi_t, \quad (53)$$
$$\hat{\xi}_t = \rho^\xi \hat{\xi}_{t-1} + \epsilon^\xi_t, \quad (54)$$
$$\hat{\eta}_t = \rho^\eta \hat{\eta}_{t-1} + \epsilon^\eta_t, \quad (55)$$

where variables with a “hat” stand for log deviations from the respective steady values. The $\rho$’s are weakly positive coefficients of persistence that are strictly smaller than one and the $\epsilon$’s are serially uncorrelated normally distributed error terms with zero mean.

In our analyses of optimal policy rules, it is clearly not restrictive to eliminate policy shocks, i.e. to set $\hat{\xi}_t = \hat{\eta}_t = 0$ in all periods. Hence it remains to select numerical values for the parameters in (52) and (53), i.e. $\rho^a$, $\rho^x$, as well as the standard deviations of $\epsilon^a_t$ and $\epsilon^\chi_t$. According to (13), the persistence $\rho^\chi$ of the financial shocks can be pinned down with the help of the empirical value of the persistence of the return on bank equity (see Goddard et al. (2011) for U.S. data). This procedure results in $\rho^\chi = 0.680$. In a similar vein, the standard deviation of the innovation to the financial shock process can be obtained from the standard deviation of the equity premium in the U.S. for 1900–2015, which is 0.196 (see Damodaran (2016)). As a consequence, we get that the standard deviation of $\epsilon^\chi_t$ is $\frac{1-\rho^\chi}{\chi} \sqrt{1-\rho^2} \cdot 0.196 = \frac{1}{1.680^2} \sqrt{1-0.680^2} \cdot 0.196 \approx 2.613$, where we have used (13). Collard et al. (2017) fit an
AR(1) process to match the evolution of de-trended log TFP. They find 0.966 for the persistence of deviations of TFP from its trend and 0.0068 for the standard deviation of the innovations. We follow them and choose the same values for the AR(1) process in (52).

4.2 Comparative statics

In this section we characterize and illustrate the properties of the steady state. While we keep inflation at the level implied by the UO steady state, we examine the consequences of different values for the capital requirement. Figure 2 shows the steady-state variables as functions of the aggregate capital requirement. We note that increases in

Figure 2: Steady-state variables as functions of the aggregate capital requirement.
capital requirements lead to a higher premium on bank loan rates because banks’ aver-
age costs of funding are higher as a result. These increases in loan rates make it more costly for risky firms to produce. As a result, output, labor and profits decrease in this sector. The decrease in the demand for labor by risky firms lowers wages, which entails that safe firms hire more workers. On balance, aggregate employment and output drop. However, higher capital requirements also have beneficial effects. They result in fewer defaults and hence less funds are necessary to bail out banks. The last panel shows that these costs and benefits of higher capital requirements lead to an optimal intermediate value of capital requirements. As has been mentioned before the UO steady state is characterized by a capital requirement of $\Gamma^* = 5.8\%$.

4.3 Impulse responses

In this section we examine how the economy reacts to shocks. More specifically, we consider shocks to monetary policy, macroprudential policy, productivity, and the costs of equity financing (see (52)-(55)). To illustrate the dynamic response of the economy to these shocks, we set the persistence of all shocks to 0.9 and the size of all shocks to 1 in this section. Monetary policy follows the standard Taylor rule (35), and the capital requirement does not depend on endogenous variables, i.e. follows Equation (36) with $(0, 0, 0, 0)$ policy rule coefficients.

Figure 3 shows the dynamics of the economy after an interest-rate shock (blue lines). High interest rates lead to lower output, consumption, declining wages, and lower total labor supply. Declining demand results in falling prices. However, due to price stickiness, safe firms are relatively slow to lower prices compared to risky firms. Thus the ratio between the price of the intermediate goods produced by safe firms with respect to the aggregate price, i.e., $\hat{q}_t$, hikes. With relatively high prices of their goods, safe firms face low demand, so less labor is hired by safe firms, and labor is shifted to risky firms. Due to price rigidity, the process of labor reallocation displays a hump shape. The loan rate premium does not change, as both the equity premium, which is only affected by $\hat{\chi}_t$, and the capital requirement $\hat{\Gamma}_t$ remain unchanged (see (48)). Equation (45) implies that the fraction of defaulting banks can be expressed as a function of the loan rate premium and the capital requirement. As both variables do
not change, $\phi^r = 0$ for a monetary-policy shock. Risky firms request and receive larger loans. As a consequence, slightly more resources are devoted to bailing out banks, although the fraction of defaulting banks is unaffected by the interest-rate shock.

It is instructive to study the dynamic effects of a shock to the macroprudential policy instrument next (red lines in Figure 3). In response to an increase in capital requirements, banks charge high loan rate premia to compensate the increased equity-financing cost. The high loan rate reduces total loan demand by risky firms. Thus, risky firms hire less labor, which leads to a decline in wages. As a consequence, safe firms hire more labor and therefore produce more goods, which leads to a decline in the relative prices of their goods, $\hat{q}_t$. Aggregate output drops moderately, as safe firms are overly invested and comparably few resources are channeled through the banking sector. It is also clear that the total funds that the government uses to bailout banks ($bo_t$) decline, because large equity buffers reduce the number of defaulting banks. An increase in

Figure 3: Impulse responses to a monetary-policy shock (blue lines, left axis) and a macroprudential-policy shock (red lines, right axis).
the macroprudential policy instrument is inflationary, as it makes bank-financing more expensive and thereby increases the costs of production for risky firms.

It is noteworthy that the two policy instruments have very different effects on the economy. As is common in models of monetary policy, changes in the interest rate always move both inflation and output in the same direction. By contrast, changes in the macroprudential policy instrument affect inflation and output differently. For example, stricter capital requirements increase inflation, while at the same time leading to lower output. These observations show that the macroprudential instrument is complementary to the monetary-policy instrument and that adding the macroprudential policy instrument to the policy-makers’ tool box might be welfare enhancing, as the joint use of both tools allows, for example, to influence output and inflation independently.

Figure 4: Impulse responses to a financial shock (blue lines, left axis) and a productivity shock (red lines, right axis).
We next examine the impact of a productivity shock (red lines in Figure 4). The increase in productivity yields higher output and consumption, which gradually decline to the steady level. Higher marginal productivity raises wages. Lower marginal costs lead to deflation. However, due to price rigidity, safe firms cannot lower the price as fast as risky firms. Thus the ratio between the price of the intermediate goods produced by safe firms with respect to the aggregate price, i.e., $\hat{q}$, displays a hump-shape hike. We observe a reallocation of resources between sectors: labor is shifted from safe firms to risky firms. Aggregate labor decreases.\textsuperscript{18} A high wage rate and more labor employed in risky firms imply larger bank loans granted to risky firms. Although the fraction of defaulted banks is unaffected by a shock to productivity, more rescue funds are needed as a consequence of the larger volume of loans.

Blue lines in Figure 4 show the evolution of the economy in response to a shock to equity management costs. As the costs of holding equity rise, investors will request a higher return on equity. This, in turn, implies that the loan rate premium will increase. Thus total loan demand declines, and labor is shifted from the risky sector to the safe sector. Furthermore, a high loan rate premium leads to a low fraction of defaulting banks and bailout fees. In addition, an increase in equity management costs leads to inflation, lower relative prices for intermediate goods produced by safe firms (safe firms adjust prices more slowly than risky firms) and declines in wage rates, aggregate labor, output (countercyclical equity premium), and consumption.

5 Optimal Policy Rules

The global financial crisis 2007-2009 has rekindled the debate on how to govern and coordinate monetary and macroprudential policies during financial crises and economically tranquil times. The last crisis has also cast considerable doubt on the consensus formed in the so-called Great Moderation that central banks should pursue inflation targeting, where monetary policy can be described by a Taylor rule which aims at stabilizing inflation and output. The crisis has shown that, despite stable inflation and

\textsuperscript{18}In a version of our model where all intermediate firms are safe, the income and substitution effect would cancel exactly. In our two-sector model, the reallocation of labor across sectors leads to a small deviation from this implication.
output for a long period, unsustainable sectoral booms and gradual buildups of financial risks—e.g. excessive leverage of banks—may lead to a financial meltdown with adverse macroeconomic consequences. Thus, a core issue is how to include financial stability measures—proxied by e.g. credit aggregates, non-performing loans or interest rates on bank loans—into macroeconomic policy-making.

To answer this question, one strand of the literature studied modified Taylor rules. Blanchard et al. (2013) and Woodford (2014) pointed out that monetary policy should incorporate multiple targets and multiple instruments.\textsuperscript{19} Woodford (2012) demonstrated that a temporary departure of monetary policy from the inflation and output target path due to financial stability concerns can be socially optimal.

Another strand of literature studied the optimal proxy or indicator for financial instability. Bernanke and Gertler (1999) and Cecchetti et al. (2002) used asset prices; Agnor et al. (2011) and Christiano et al. (2010) used credit aggregates; and Carlstrom et al. (2010), Angelini et al. (2014), Curdia and Woodford (2010), Quint and Rabanal (2014), and Ueda and Valencia (2014) used credit spreads and leverage. We contribute to both strands of literature by studying the interplay between monetary policy and macroprudential policy for the banking system and by investigating optimal policy rules for central banks and macroprudential policy-makers.

We derive the unconditionally optimal (UO) policies to obtain a welfare measure for different policy stances. Methodologically, we follow the approach of Damjanovic et al. (2015). As shown in Damjanovic et al. (2015), it is possible to derive a purely quadratic approximation to welfare around the unconditionally optimal steady state by using approximations to the social planner’s constraints up to the second order to eliminate all linear terms in the approximation of the household’s utility function up to the second order. This purely quadratic measure can be evaluated for constraints and policies that are correct up to the first order. As the computation of the welfare measure requires the computation of second-order derivatives of the constraints and the utility function, which is quite cumbersome, we perform the respective calculations with the help of a computer algebra system.\textsuperscript{20}

\textsuperscript{19}As documented in Goodhart et al. (1988), the original purpose of establishing central banks in certain countries was to prevent financial instability. Käfer (2014) reviewed the literature on Taylor rules augmented with a financial stability term.

\textsuperscript{20}Details are available upon request.
In the following, we first derive the UO policies and show the impulse responses of the economy under UO policies. Then, we investigate which simple monetary and macroprudential policy rules could replicate the results under UO policies.

### 5.1 Unconditional optimal policies

In this subsection, we analyze the UO policies, i.e. the stationary policies that maximize the unconditional expectation of the representative household’s utility (1) subject to the constraints (41)-(51). The respective impulse responses for productivity shocks and financial shocks are displayed in Figures 5 and 6 respectively.

**Figure 5:** Impulse responses to a productivity shock under UO policies (solid black lines) and under simple rules (green dashed lines).
According to Figure 5, which displays the response to aggregate productivity shocks under UO policies, the relative deviations of output, consumption, the real wage, bailout fees, and the total loan are exactly identical to \( \hat{a}_t \), i.e. the log deviation of productivity from its steady-state level. Moreover, we note that inflation, employment in both sectors, the relative price \( \hat{q}_t \) of intermediate goods across the sectors, and the capital requirement are completely unchanged. The following proposition shows that this a general feature of our model, provided that utility is logarithmic in consumption.

**Proposition 6**

Consider the non-linear economy characterized by (41)-(51). Suppose that \( \sigma = 1 \) and that there are no financial shocks, i.e. \( \chi_t = \chi^* \). Then the unconditionally optimal policy \( \{n^*_s(i), n^*_r(i), w_t, l_t(i), \Delta^*_t, \phi^*_t, \Pi^*_t, \Pi_t, \chi_t, y_t, c_t, \Gamma_t, I_t\}_{t=0}^\infty \) can be characterized as \( n^*_s(i) = n^s(i), n^*_r(i) = n^r(i), w_t = a_t w^*, l_t(i) = a_t l^*(i), \Delta^*_t = \Delta^s_t, \phi^*_t = \phi^s_t, \Pi^*_t = \Pi^s_t, \Pi_t = \Pi^*, q_t = q^*, y_t = a_t y^*, c_t = a_t c^*, \Gamma_t = \Gamma^*, I_t = \left( E_t \left[ \frac{a_t}{a_{t+1}} \right] \right)^{-1} I^* \).

The proposition, which is proved in Appendix F, implies that strict inflation targeting is optimal in the face of productivity shocks. This is related to the so-called “divine coincidence”, which states that it is possible to stabilize inflation and output at their socially optimal levels simultaneously in New Keynesian models. It is also noteworthy that employment remains fixed over time. This is a consequence of the fact that, for the utility function under consideration, the income and substitution effect triggered by changes in aggregate productivity directly offset one another.

Having discussed the response of the economy to productivity shocks under UO policies, we now turn to financial shocks in Figure 6. A positive financial shock \( \hat{\chi}_t \) increases the per-unit cost of bank capital. The UO policies therefore stipulate lower capital requirements in order to reduce aggregate bank capital. In line with (48), the loan rate premium \( \hat{\Delta}_t^l \) is an increasing function of both \( \hat{\chi}_t \) and capital requirement \( \hat{\Gamma}_t \) because higher \( \hat{\chi}_t \) as well as higher \( \hat{\Gamma}_t \) make bank financing more costly. It is therefore not clear a priori whether the loan rate premium increases or decreases in response to the increase in \( \hat{\chi}_t \) and the simultaneous decrease in \( \hat{\Gamma}_t \). Figure 6 reveals that the second effect dominates and therefore the loan rate premium drops in response to an increase in \( \hat{\chi}_t \). The reduction of the loan rate premium increases the total amount of loans provided by banks to the risky sector. As a consequence, more defaults of risky firms occur and
Figure 6: Impulse responses to a financial shock under UO policies (solid black lines) and under simple rules (green dashed lines).

the banking sector requires additional funds for bailouts. Since bank loans become cheaper, more labor is employed by risky firms and the prices of their intermediate goods decline.

Interestingly, the dynamics of the economy under UO policies are rather different from the ones under constant capital requirements. As has been shown in Section 4.3, constant capital requirements entail that loan rates increase in response to a higher $\chi_t$, as bank financing becomes more costly for risky firms. By contrast, the reduction of capital requirements in response to positive levels of $\chi_t$ leads to lower costs of bank financing for risky firms under UO policies. As a consequence, the total volume of loans,
aggregate labor, output and consumption move in exactly the opposite directions under UO policies than under constant capital requirements.

5.2 Optimal simple rules

In this subsection, we investigate which simple rules could replicate the outcomes of UO policies (the social loss under UO policies is $-0.01829$). To achieve the optimal simple rules, we first fix the capital requirement at its UO steady-state level and vary the coefficients on output and inflation in the Taylor rule, i.e. $v^y_{mon} \in [0, 10]$ and $v^\pi_{mon} \in [1.1, 10]$. Figure 7a shows that the social loss increases with $v^y_{mon}$ and decreases with $v^\pi_{mon}$. Extending the range of $v^y_{mon}$ to the negative territory, Figure 7b reveals that the social loss is minimized when the coefficient on output is 0. This experiment shows that given a fixed capital requirement, the monetary policy maker should stabilize inflation vigorously.

Henceforth we will therefore consider a fixed Taylor rule with coefficients $v^y_{mon} = 0$ and $v^\pi_{mon} = 10$. In the second step we investigate the social losses under different macro-prudential policy rules that respond to one of the following variables: the fraction of defaulting banks, output, inflation, the total volume of loans or the loan rate premium, respectively. We vary the coefficient for the respective single variable from -1000 to 1000, while fixing the other coefficients in (36) at zero. It turns out that the social loss is minimal when the macroprudential policymaker reacts to the loan rate premium with a coefficient of 256.40. Thus the optimal simple rules are

$$\hat{I}_t = 10\pi_t, \quad (56)$$

$$\hat{\Gamma}_t = 256.4\Delta^l_t. \quad (57)$$

It is straightforward to calculate that, in response to an increase in the loan rate premium $\Delta^l_t - 1$ by one basis point, the macroprudential policy-maker should increase the capital requirement $\Gamma_t$ by approximately 14 basis points.

Comparing green lines with black lines in Figures 5 and 6, we find that the optimal simple rules and the UO policies lead to very similar responses. We also note that the social loss under optimal simple rules ($-0.01826$) is almost indistinguishable from the social loss under UO policies ($-0.01829$). If the standard Taylor rule and the
fixed capital requirement examined above are replaced by the optimal simple rules, more than 99% of the theoretically possible welfare gains that could be attained by switching to the UO policies can be achieved.

It may be instructive to consider the intuition for why it is socially optimal to make the capital requirement a strongly increasing function of loan rates. We have already seen in Section 5.1 that the UO policies involve drops in loan rates in response to a positive shock $\chi_t$. How can optimal simple rules replicate this pattern? A positive shock to $\chi_t$ makes holding bank equity more costly and therefore raises \textit{ceteris paribus} the return on bank equity that households require. However, the desired decrease in loan rates for $\chi_t > 0$ tends to decrease the return on bank equity that banks can generate for a given level of equity. Hence, to raise the return on bank equity to the level required by investors, the drop in loan rates must be accompanied with a sharp decrease in bank-equity capital requirements. This explains why (57) is optimal.

For completeness, we note that social losses that are very low ($-0.01821$), though not as low as the ones achieved by (56) and (57), can be attained if the macroprudential policy tool responds exclusively to changes in the banks’ default probability $\phi_t^\Gamma$. For given monetary policy rule (56), the optimal coefficient is $\nu^\phi_{mac} = -1.73$. In this paper, we concentrate on bank loan premia as determinants of capital requirements, as this approach allows for slightly lower social losses. Moreover, information about loan rates is readily available to macroprudential policy makers. By contrast, the probability of bank default is more difficult to measure in practice.\footnote{For example, risk premia on bonds issued by commercial banks may reveal investors’ default expectations. However, they may also be affected by expectations of government bailout.}

To sum up, social welfare effectively attains its maximum when the monetary authority exclusively focuses on stabilizing inflation, and the macroprudential authority reacts to changes in the market loan rate premium. This observation provides some support for the separation of monetary policy and macroprudential policy in different policy-making bodies, since optimal policy requires both policies to focus on different economic variables.
6 Conclusion

We have integrated banks and the coexistence of banks and bond financing into an otherwise standard New Keynesian model. While interest rate policies stabilize shocks that affect aggregate variables, they are less suitable for stabilizing macroeconomic events driven by sectoral shocks. If they affect firms primarily financed by banks, such shocks are best dealt with by time-varying aggregate capital requirements. We found that monetary policy governed by a simple Taylor rule and a macroprudential policy rule that prescribes increases in capital requirements in response to increases in loan rate premia can largely replicate outcomes under UO policies. This behavior of the macroprudential policy maker ensures that capital requirements are high in periods where the social costs of bank capital are low; they are low in periods where bank capital is very costly.

While we have pursued a small number of applications, numerous extensions of the basic framework and further applications can and should be pursued. Regarding applications, a variety of alternative shocks could be investigated. For instance, markup shocks and demand shocks originating from preference shocks or government spending shocks are obvious candidates for an in-depth analysis of how monetary policy and aggregate bank-equity policies can jointly stabilize such shocks.
A Optimal Behavior of Safe Intermediate Firms

Stage 2: Price setting In line with (15), (19), and \( s_t(i) = \frac{w_t n_t(i)}{a_t} \), the real profits of a safe intermediate firm \( i \) in period \( t \) are given by

\[
 z^*_t(i) = \left( \left( \frac{p^*_t(i)}{p_t} - \frac{w_t}{a_t} \right) \left( \frac{p^*_t(i)}{p_t} \right)^{-\theta} y_t - \frac{\gamma_p^p}{2} \left( \frac{p^*_t(i)}{p^*_t(i) - 1} \right)^2 y_t \right).
\] (58)

Consequently, the present value of discounted profits can be written as

\[
 E_t \sum_{t=0}^{\infty} Q_t \left\{ \left( \frac{p^*_t(i)}{p_t} - \frac{w_t}{a_t} \right) \left( \frac{p^*_t(i)}{p_t} \right)^{-\theta} y_t - \frac{\gamma_p^p}{2} \left( \frac{p^*_t(i)}{p^*_t(i) - 1} \right)^2 y_t \right\}.
\] (59)

The first-order condition with respect to \( p^*_t(i) \) yields safe firms’ optimal price dynamics

\[
 0 = \left( \left( \frac{p^*_t(i)}{p_t} - \frac{w_t}{a_t} \right) \left( \frac{p^*_t(i)}{p_t} \right)^{-\theta} y_t \left[ (1 - \theta) + \theta \frac{w_t}{a_t} p_t \right] - \frac{\gamma_p^p y_t}{2} \left( \frac{p^*_t(i)}{p^*_t(i) - 1} \right)^2 \right)
\] (60)

B Optimal Behavior of Risky Intermediate Firms

Stage 2: Price setting In stage 2, after the realization of the idiosyncratic shock \( \phi(i) \), production is determined in line with (22). Each risky firm’s revenues are maximized by selecting the maximum price for which it can sell its output. Rewriting (15), we obtain the optimal price set by risky firms

\[
 p^r_t(i) = \left( \frac{y_t}{y^r_t(i)} \right)^{\frac{1}{\theta}} p_t,
\] (61)

It will be convenient to introduce the premium on bank financing:

\[
 \Delta^l_t := \frac{R^l_t}{R_t^l}.
\] (62)

Combining (24) and (61) yields the real profit, conditional on the firm’s being able to repay the loan:

\[
 z^r_t(i) = \frac{1}{\theta} y^r_t(i)^{\frac{\theta-1}{\theta}} - \Delta^l_t u_t(i) \geq 0.
\] (63)
Let \( \phi^c \) be the level of \( \phi(i) \) below which risky intermediate firms default and hence cannot fully repay the loan. We can solve the critical value \( \phi^c \) from \( l_t(i) = R_t^* l_t(i) \), \( w_t = R_t^* \bar{w}_t \), (22), (23) and (63):

\[
\phi^c = \left( \frac{1}{Aa_t \bar{n}_t^*(i)} \left( \frac{\Delta^1 l_t(i)}{y_t} \right)^{\theta - \frac{1}{\sigma}} \right)^{\frac{1}{\theta - 1}} = \left( \frac{w_t}{Aa_t} \left( \frac{l_t(i)}{y_t} \right)^{\theta - 1} \left( \Delta^1 l_t(i) \right)^{\theta - \frac{1}{\sigma}} \right)^{\frac{1}{\theta - 1}}. \tag{64}
\]

For risky intermediate firms with \( \phi(i) \in [0, \phi^c) \), i.e. firms that cannot repay the loan in full, profit is zero

\[
z^r_t(i) = 0, \tag{65}
\]

and all the revenue goes to the bank. Thus, provided that the firm defaults, the gross return on the bank loan is

\[
R^*_t(i) = \frac{y_t^1 y^r_t(i)^{\frac{\theta - 1}{\sigma}}}{l_t(i)} = R_t^* \in [0, R_t^*). \tag{66}
\]

**Stage 1: Attraction of loans**  Given the price (61) set in stage 2, firms determine the optimal amount of loan \( \tilde{l}_t(i) \).

The risky firms’ expected real profit is

\[
\int_0^1 z^r_t(i) d\phi(i) = \int_{\phi^c}^1 \left[ y_t^1 y^r_t(i)^{\frac{\theta - 1}{\sigma}} - \Delta^1 l_t(i) \right] d\phi(i), \tag{67}
\]

where we have taken into account the fact that profits are zero if \( \phi(i) < \phi^c \).

Equation (64) reveals that we have to restrict the choice of \( l_t(i) \) to values that involve \( \phi^c \leq 1 \), i.e.

\[
l_t(i) \leq \left( \frac{Aa_t}{w_t} \right)^{\theta - 1} \frac{y_t}{(\Delta^1)^{\theta}} =: l. \tag{68}
\]

Using (22), (23), (64), and (67), we can state the firm’s profit maximization problem in the following way:

\[
\max_{l_t(i)} \left( \frac{\theta}{\theta + \alpha(\theta - 1)} \right) l_t(i)^{\frac{\theta - 1}{\sigma}} + \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} l_t(i)^{\frac{1+\alpha(\theta - 1)}{\theta(\sigma - 1)}} - l_t(i) \right) \Delta^1_l \tag{69}
\]

s.t. \( l_t(i) \leq l. \)
Obviously, the condition $\phi^c \leq 1$ will be slack at the optimal choice of $l_t(i)$ because profits are zero when $\phi^c = 1$, which means that the firm defaults with probability one.

An optimal choice of $l_t(i)$ implies

$$
(\theta - 1) \left( \frac{\bar{I}}{l_t(i)} \right)^{\frac{1}{\sigma}} + (1 + \alpha(\theta - 1)) \left( \frac{l_t(i)}{\bar{I}} \right)^{\frac{1}{\sigma}} - (\theta + \alpha(\theta - 1)) = 0. $$

(70)

Consequently, the optimal value of $l_t(i)$ can be written as

$$
l_t(i) = L_t^* = \left( \frac{Aa_t}{w_l} \right)^{\theta - 1} \left( \frac{L^*}{\Delta_l} \right)^{\frac{1}{\sigma}} y_t,
$$

(71)

where $L^*$ is the root of

$$
g(L) := (\theta - 1)L^{\frac{1}{\sigma}} + (1 + \alpha(\theta - 1))L^\theta \left( \frac{1}{\sigma} \right)^{\frac{1}{\sigma}} - (\theta + \alpha(\theta - 1))
$$

that satisfies $0 < L^* < 1$.\footnote{For $\theta = 2$ and $\alpha = 1$, for example, the unique solution is $L = \frac{1}{4}(\sqrt{3} - 1)^2$.}

For arbitrary $\theta > 1$, the existence of such a solution can be readily established. Function $g(L)$ has at least one root on $(0, 1)$ because (i) $\lim_{L \to 0} g(L) = \infty$, (ii) $g(1) = 0$, and (iii) $g'(1) > 0$. The uniqueness of the solution follows from the additional observation that $g(L)$ has a single minimum on $(0, 1)$, which is straightforward to verify. To sum up, the risky firms’ expected profit is maximized when the real loan is given by $l_t(i) = L^*\bar{I}$, where $L^*$ is the solution to (72).

We observe that (71) also allows us to use a particularly simple expression for $\phi^c$. Inserting (71) into (64) results in

$$
\phi^c = (L^*)^{\frac{1}{\theta - 1}}.
$$

(73)

Hence $\phi^c$ will be constant in equilibrium and will not depend on the central bank’s policy rate $I_t$ or the capital requirement $\Gamma_t$.

Using (22), (23), and (71), we can write the output of risky firms as

$$
y_t^r(i) = (\phi(i))^\theta \left( \frac{Aa_t}{w_l \Delta_l} \right)^{\theta - 1} L^* y_t.
$$

(74)

The aggregate profits of all risky firms can be computed with the help of (67), (71), (72), (73), and (74) as

$$
z_t^r = (1 - \nu) \left( \frac{Aa_t}{w_l \Delta_l} \right)^{\theta - 1} \frac{L^*}{\theta - 1} \left( 1 - (L^*)^{\frac{1}{\theta - 1}} \right) y_t.
$$

(75)
C Optimal Behavior of Banks

We examine the problem of a representative bank in three steps.

Step 1: Loan provision for given capital structure

In the first step, we examine loan provision by a representative bank if it has a capital structure equal to the aggregate capital requirement and can provide a loan to one risky firm. For convenience, we denote the risky firm by \( i \) and use the same index \( i \) for the representative bank that lends to firm \( i \). On a bank’s balance sheet, the asset (loan to a risky firm) is equal to the sum of liabilities (deposits and equity). Thus we have

\[
\tilde{l}_t(i) = \tilde{d}_t(i) + \tilde{e}_t(i) = (1 + \Gamma_t) \tilde{d}_t(i).
\] (76)

We define \( R_t^\Gamma \) as the smallest gross return on bank loans \( R_t^r(i) \), such that the corresponding bank does not default. At this rate, the following condition must hold:

\[
\tilde{d}_t(i) R_t^r = \tilde{l}_t(i) R_t^\Gamma,
\] (77)

which means that the total repayment to depositors just equals the funds received from the risky firm. Combining (76) and (77) yields

\[
\Delta_t^\Gamma := \frac{R_t^\Gamma}{R_t^r} = \frac{1}{1 + \Gamma_t}.
\] (78)

As a next step, we compute \( \phi_t^\Gamma \), the value of \( \phi(i) \) below which the bank defaults. According to (22) and (66), \( \Delta_t^\Gamma \) has to satisfy

\[
\Delta_t^\Gamma = \frac{y_t^{\frac{1}{2}} \left[ (\phi_t^\Gamma)^\alpha A_n^r(i) \right]^{\frac{\alpha - 1}{\alpha}}}{l_t(i)}.
\] (79)

Equating (78) and (79) and solving for \( \phi_t^\Gamma \) results in

\[
\phi_t^\Gamma = \left( \frac{1}{A_n^r(i)} \left( \frac{l_t(i)}{(1 + \Gamma_t) y_t^{\frac{1}{2}}} \right)^{\frac{\alpha}{\alpha - 1}} \right)^{\frac{1}{\alpha}}.
\] (80)

Using (64) and (80) entails
\[ \phi^\Gamma = \frac{1}{(\Delta_t^i(1 + \Gamma_t))^{\frac{\theta}{\alpha(\theta - 1)}}} \phi^c = \left( \frac{L^*}{(\Delta_t^i)^\theta(1 + \Gamma_t)^\theta} \right)^{\frac{1}{\alpha(\theta - 1)}}. \]  

(81)

Note that \( \phi^\Gamma \) decreases with \( \Gamma_t \), which indicates that a high equity-to-debt ratio reduces the fraction of banks that fail. When \( \Gamma_t \to \infty \), i.e. banks are fully financed by equity, we obtain \( \phi^\Gamma \to 0 \), i.e. banks never default.

We note that the above equation implies
\[ \phi^\Gamma \leq \phi^c. \]  

(82)

Hence we have to distinguish between three ranges of \( \phi(i) \). For \( \phi(i) \geq \phi^c \), the firm can fully repay the loan and so the bank does not default. For an intermediate range of \( \phi(i) \), \( \phi(i) \in [\phi^\Gamma, \phi^c) \), the firm cannot fully repay the loan. However, the bank will not default because it can simply reduce dividends. For \( \phi(i) < \phi^\Gamma \), the repayment on the loan is not sufficient to repay depositors. In this case, the government has to bail out the bank and equity holders receive nothing.

The expected return on equity
\[ R_t^e = \int_0^{\phi^\Gamma} 0 \, d\phi(i) + \int_{\phi^\Gamma}^{\phi^c} R_t^e(i) \, d\phi(i) + \int_{\phi^c}^1 R_t^e \, d\phi(i), \]

where
\[ R_t^e(i) = \frac{R_t^r(i)\tilde{l}_t(i) - R_t^d\tilde{d}_t(i)}{\tilde{e}_t(i)} = \frac{(1 + \Gamma_t)R_t^r(i) - R_t^d}{\Gamma_t}, \]
\[ \tilde{R}_t^e = \frac{R_t^e(i)\tilde{l}_t(i) - R_t^d\tilde{d}_t(i)}{\tilde{e}_t(i)} = \frac{(1 + \Gamma_t)R_t^r - R_t^d}{\Gamma_t}. \]

We can rewrite the expected return on equity as
\[ R_t^e = \int_0^{\phi^c} \frac{(1 + \Gamma_t)R_t^r(i) - R_t^d}{\Gamma_t} \, d\phi(i) + \int_{\phi^c}^1 \frac{(1 + \Gamma_t)R_t^r - R_t^d}{\Gamma_t} \, d\phi(i). \]  

(83)

We observe that (66) can be combined with (22), (23), (71), and (73) to yield
\[ R_t^r(i) = \left( \frac{\phi(i)}{\phi^c} \right)^{\frac{\theta(\theta - 1)}{\alpha}} R_t^d. \]  

(84)

Inserting (84) into (83) yields the following relationship between \( \Delta_t^e \) and \( \delta_t^i \), where \( \delta_t^i := (1 + \Gamma_t)\Delta_t^i \):
\[ \Gamma_t\Delta_t^e = h(\delta_t^i) := \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} \left( \frac{1}{(\delta_t^i)^{\frac{\theta}{\alpha(\theta - 1)}}} - \delta_t^i \right) \phi^c + \delta_t^i - 1. \]  

(85)
Step 2: Uniqueness of loan rate

For $\Gamma_t \to 0$, which implies that banks would be entirely financed by deposits, (85) becomes

$$0 = \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} \left( \frac{1}{\Delta_l^t \theta^\alpha} - \Delta_l^t \right) \phi^c + \Delta_l^t - 1.$$  \hspace{1cm} (86)

In this case the solution is $\Delta_l^t = 1$. In the case of $\Gamma_t \to \infty$, i.e. with very strict capital requirements that lead to banks being financed entirely through equity, we obtain

$$\Delta_l^t = \frac{1}{1 - \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} \phi^e} \Delta_e^t > \Delta_e^t.$$  \hspace{1cm} (87)

For general values of $\Gamma_t$, (85) is more difficult to analyze. Recall that $(1 + \Gamma_t)\Delta_l^t \geq 1$ and therefore $\delta_l^t \geq 1$. It can be easily verified that $h(1) = 0$ and that $\lim_{\delta_l^t \to \infty} h(\delta_l^t) = \infty$. Moreover, $h'(\delta_l^t) > 0$, $\forall \delta_l^t \in [1, \infty)$. As a consequence, for all combinations of $\Gamma_t$ with $\Gamma_t \geq 0$ and $\Delta_e^t$ with $\Delta_e^t \geq 1$, there is a unique solution for $\Delta_l^t$ given by $\Gamma_t \Delta_e^t = h \left( (1 + \Gamma_t)\Delta_l^t \right)$ or $\Delta_l^t = \frac{1}{1 + \Gamma_t} h^{-1} (\Gamma_t \Delta_e^t)$. For fixed $\Gamma_t$, $\Delta_l^t$ is an increasing function of $\Delta_e^t$.

Step 3: Optimal capital structure

Finally, we show that it is optimal for banks to choose a capital structure that is equal to the aggregate capital requirement $\Gamma_t$ in each period. Suppose that except for one deviating bank, all banks choose $\Gamma_t$ in period $t$. Then the market loan rate $\Delta_l^t$ is given by (85) and illustrated in Graph 1 in Figure 2, since the deviating bank has no impact on equilibrium interest rates. Suppose that the deviating bank chooses a capital structure $\tilde{\Gamma}_t(i) > \Gamma_t$ and finances a loan to the risky firm $\tilde{l}_t(i) = \tilde{d}_t(i) + \tilde{c}_t(i)$. It is profitable for this bank to do so if the deviation strictly increases the return on equity. Hence, we have to verify whether for a given $\Delta_l^t$, $\Delta_e^t$ is increasing in the bank-specific capital structure that we denote by $\Gamma_t(i)$. Such a deviation cannot be profitable. For a given loan size and market loan rate $\Delta_l^t$, choosing $\Gamma_t(i) > \Gamma_t$ implies that in the case of default and bailout, expected transfers from the government are lower than for $\Gamma_t(i) = \Gamma_t$. The reason is that both the likelihood of default and the bailout transfer in the case of default are lower. Since bank revenues are unaffected by different choices of capital structure and return on equity is higher than deposit rates,
the preceding observation implies necessarily that the expected return on equity is lower with choice \( \Gamma_t(i) > \Gamma_t \) than with \( \Gamma_t(i) = \Gamma_t \). This is illustrated in Figure 8, which displays expected returns on equity for aggregate capital requirement ratios of 4%, 8%, and 12%, respectively. For instance, the solid black line represents the expected return on equity when the aggregate capital requirement ratio \( \Gamma_t \) is 4% (represented by the vertical dashed black line). The expected return on equity decreases with the equity-to-deposit ratio \( \Gamma_t(i) \). Thus the individual bank would select the lowest possible \( \Gamma_t(i) \), i.e. \( \Gamma_t(i) = \Gamma_t \). The realized return on equity is represented by the horizontal gray line at value \( \Delta^{e*} = \frac{1}{1 - \chi^*} = 1.055 \).

\[ \square \]

**D The Government**

The real bail-out fees amount to

\[ b_{0t} = (1 - \nu) \int_0^{\phi^*} (d_t(i) - R_t^*(i) \tilde{l}_t(i)) d\phi(i). \]

With the help of \( \phi^*/\phi^* = (\Delta^l)^{\theta/\theta - 1} (1 + \Gamma_t)^{\theta/\theta - 1}, \phi^* = (L^*)^{1/\theta - 1}, l_t(i) = w_t n_t^*(i), \) and (84), this expression can be stated as

\[ b_{0t} = \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} \frac{(1 - \nu)w_t n_t^*(i)(L^*)^{1/\theta - 1}}{(\Delta^l)^{\theta/\theta - 1} (1 + \Gamma_t)^{\theta/\alpha(\theta - 1)}}. \quad (88) \]

**E Log-linear Model**

**E.1 Log-linearized equilibrium conditions**

In this section we state log-linearized versions of the conditions (41)-(51). Observe that we log-linearize around a steady state that does not necessarily involve zero inflation. We use the symbol ` to denote log deviations from steady-state values and * for steady-state values. For the details, we refer to Appendix E.

Equation (41) can be approximated as

\[ \hat{l}_t(i) = (\theta - 1)\hat{a}_t - \hat{w}_t - \theta \hat{\Delta}_t + \hat{y}_t, \quad (89) \]
where
\[
\hat{l}_t(i) = \hat{w}_t + \hat{n}_t(i). \tag{90}
\]

The Phillips curve, Equation (43), has the following log-linearized approximation:
\[
0 = (1 - \beta)\gamma^p \Pi^* (\Pi^* - 1) (\hat{y}_t - \theta \hat{q}_t + \hat{w}_t - \hat{a}_t)
+ (q^*)^{1 - \theta} (\theta - 1) (\hat{w}_t - \hat{a}_t - \hat{q}_t)
+ \gamma^p (\Pi^*)^2 \mathbb{E}_t \left[ \beta (\sigma \hat{c}_t - \sigma \hat{c}_{t+1} + \hat{y}_{t+1} + 2 \pi^*_t + 1) - 2 \pi^*_t - \hat{y}_t \right]
+ \gamma^p \Pi^* \mathbb{E}_t \left[ \pi^*_t + \hat{y}_t - \beta (\sigma \hat{c}_t - \sigma \hat{c}_{t+1} + \hat{y}_{t+1} + \pi^*_t + 1) \right], \tag{91}
\]
where we write \( \pi^*_t = \hat{\Pi}^*_t \). Using \( \pi_t := \hat{\Pi}_t \) for the inflation rate, we obtain
\[
\pi_t = \hat{q}_{t-1} - \hat{q}_t + \pi^*_t. \tag{92}
\]

Moreover, \( \hat{\phi}^\Gamma_t \), the relative deviation of the fraction of defaulting banks from the steady-state value satisfies
\[
\hat{\phi}^\Gamma_t = -\frac{\theta}{\alpha(\theta - 1)} \hat{\Delta}_t - \frac{\theta}{\alpha(\theta - 1)} \frac{\Gamma^*}{1 + \Gamma^*} \hat{\Gamma}_t. \tag{93}
\]

Hence higher interest rates on loans and higher capital requirements are associated with fewer bank failures. While the relationship between capital requirements and bank failures is clear, the relationship between bank failures and loan rates is more subtle. It relies on the observations that higher interest rates on loans reduce the demand for loans and that the return on loans, given that a firm defaults, is the higher, the smaller the loan (see Eq (84)).

Equation (46) has the following log-linear approximation:
\[
\hat{a}_t + \hat{n}_t(i) = -\theta \hat{q}_t + \hat{y}_t. \tag{94}
\]

We obtain the following approximation to (47):
\[
\hat{y}_t = \hat{a}_t + \kappa_1 \hat{n}_t(i) + (1 - \kappa_1) \hat{n}_t(i), \tag{95}
\]
where \( \kappa_1 \in (0, 1) \) is given in Appendix E. According to (95), aggregate output increases with the quantities of labor employed in both sectors.
Equation (48) can be approximated as
\[ \hat{\Delta}_t^l = \tilde{\kappa}_2 \hat{\Gamma}_t + \chi^* \kappa^2 (1 - \chi^*) \hat{\chi}_t, \quad (96) \]
where \( \kappa^2 > 0 \) and \( \tilde{\kappa}_2 > 0 \) are given in Appendix E. This equation implies that the interest rate that banks charge on loans increases with capital requirements and equity management costs.

A log-linear approximation to the consumption Euler Equation, Equation (49), is
\[ \hat{c}_t = -\sigma^{-1} \left( \hat{I}_t - \mathbb{E}_t [\pi_{t+1}] \right) + \mathbb{E}_t [\hat{c}_{t+1}], \quad (97) \]

Equation (50) can be approximated as
\[ \hat{w}_t - \sigma \hat{c}_t = \varphi \kappa_3 \hat{n}_s^*(i) + \varphi (1 - \kappa_3) \hat{n}_r^*(i), \quad (98) \]
where \( \kappa_3 \in (0, 1) \) is given in Appendix E.

A log-linearized approximation to the resource constraint, Equation (51), is given by
\[ \hat{y}_t = \frac{1}{1 - \frac{\nu \gamma^p}{2}} \left( \frac{c^*}{y} \hat{c}_t + \nu \gamma^p (\Pi^* - 1) \pi^*_t + \left( 1 - \frac{c^*}{y} \right) \frac{1}{2} \gamma^p \nu (\Pi^* - 1)^2 \hat{\pi}_t(i) \right) + \frac{\kappa_4}{1 - \chi^*} \hat{\chi}_t - \kappa_5 \theta \hat{\Delta}_t^l + \frac{\kappa_4 - \kappa_5 (\theta + \alpha (\theta - 1)) \Gamma^*}{1 + \Gamma^*} \hat{\Gamma}_t, \quad (99) \]
where \( \kappa_4 \) and \( \kappa_5 \) are given in Appendix E.

The policy rules of the central bank and the macroprudential policy-maker can be approximated as:
\[ \hat{I}_t = \nu_{mon} \pi_t + \nu_{mon} \hat{y}_t + \nu_{mon} \hat{\pi}_t + \nu_{mon} \hat{\phi}_t + \hat{\xi}_t, \quad (100) \]
\[ \hat{\Gamma}_t = \nu_{mac} \pi_t + \nu_{mac} \hat{y}_t + \nu_{mac} \hat{\pi}_t + \nu_{mac} \hat{\phi}_t + \hat{\eta}_t. \quad (101) \]

After these steps, we are in a position to describe the equilibrium of the log-linearized economy as follows:

**Proposition 7**

For given shocks \( \{ \hat{a}_t, \hat{\chi}_t, \hat{\xi}_t, \hat{\eta}_t \}_{t=0}^\infty \), whose evolution is given by (52)-(55), the equilibrium \( \{ \hat{n}_s^*(i), \hat{n}_r^*(i), \hat{\phi}_t(i), \hat{\Gamma}_t(i), \pi_t, \pi^*_t, \hat{y}_t, \hat{c}_t, \hat{\Delta}_t^l, \hat{\xi}_t, \hat{\eta}_t \}_{t=0}^\infty \) of the log-linearized economy is described by the system of Equations (89)-(101).
E.2 Derivation

We now provide more details about how Equations (89)-(101) can be derived.

Equation (41)

\[ w_t n_t(i) = \left( \frac{Aa_t}{w_t} \right)^{\theta-1} \frac{1}{(\Delta_l^t)^{\theta}} L^* y_t. \]  \hfill (102)

Steady state:

\[ w^* n^*(i) = \left( \frac{Aa^*}{w^*} \right)^{\theta-1} \frac{1}{(\Delta^{*t})^{\theta}} L^* y^*. \]  \hfill (103)

Log-linearization:

\[ w^*(1 + \hat{w}_t)n^*(i)(1 + \hat{n}_t(i)) = \left( \frac{Aa^*(1 + \hat{a}_t)}{w^*(1 + \hat{w}_t)} \right)^{\theta-1} \frac{1}{(\Delta^{*t}(1 + \hat{\Delta}_l^t))^{\theta}} L^* y^*(1 + \hat{y}_t). \]  \hfill (104)

Using (103) to simplify the equation above yields

\[ \hat{w}_t + \hat{n}_t(i) = (\theta - 1)(\hat{a}_t - \hat{w}_t) - \theta \hat{\Delta}_l^t + \hat{y}_t. \]  \hfill (105)

Equation (43)

We can write (43) as

\[ 0 = q_t^{1-\theta} y_t \left[ (1 - \theta) + \frac{\theta w_t}{a_t q_t} \right] - \gamma^p y_t q_t \frac{q_t}{q_{t-1}} \Pi_t \left( \frac{q_t}{q_{t-1}} \Pi_t - 1 \right) \\
+ \beta E_t \left[ \frac{\sigma^t}{c^t+1} \gamma^p y_{t+1} q_{t+1} \Pi_{t+1} \left( \frac{q_{t+1}}{q_t} \Pi_{t+1} - 1 \right) \right]. \]  \hfill (106)

Steady state:

\[ (1 - \beta) \gamma^p \Pi^* (\Pi^* - 1) = (q^*)^{1-\theta} \left[ (1 - \theta) + \frac{\theta w^*}{a^* q^*} \right]. \]  \hfill (107)
Log-linear approximation around steady state:

\[
0 = (q^*)^{1-\theta} y^* \left[ (1 - \theta) + \frac{\theta w^*}{a^* q^*} \right] (\dot{y}_t - (\theta - 1)\dot{q}_t) \\
+ (q^*)^{1-\theta} y^* \frac{\theta w^*}{a^* q^*} (\dot{w}_t - \dot{a}_t - \dot{q}_t) \\
- \gamma^\rho y^* \Pi^* \left( \Pi^* - 1 \right) (\dot{q}_t - \dot{q}_{t-1} + \pi_t + \gamma \dot{y}_t) - \gamma^\rho y^* (\Pi^*)^2 (\dot{q}_t - \dot{q}_{t-1} + \pi_t) \\
+ \gamma^\rho \beta y^* \Pi^* (\Pi^* - 1) \mathbb{E}_t \left[ \sigma \dot{c}_t - \sigma \dot{c}_{t+1} + \dot{y}_{t+1} + \dot{q}_{t+1} - \dot{q}_t + \pi_{t+1} \right] \\
+ \gamma^\rho \beta y^* (\Pi^*)^2 \mathbb{E}_t \left[ \dot{q}_{t+1} - \dot{q}_t + \pi_{t+1} \right],
\]

(108)

where \( \pi_t = \dot{\Pi}_t \) is the relative deviation of inflation from its steady-state value. Note that \( \pi_t + \dot{q}_t - \dot{q}_{t-1} = \dot{p}_t^s - \dot{p}_{t-1}^s = \pi_t^s \) represents the relative deviation of the growth rate of the price of goods produced by safe firms from the corresponding steady-state inflation rate.

Combining (107) and (108) yields

\[
0 = (1 - \beta) \gamma^\rho \Pi^* (\Pi^* - 1) (\dot{y}_t - (\theta - 1)\dot{q}_t) \\
+ \left[ (1 - \beta) \gamma^\rho \Pi^* (\Pi^* - 1) + (q^*)^{1-\theta} (\theta - 1) \right] (\dot{w}_t - \dot{a}_t - \dot{q}_t) \\
- \gamma^\rho \Pi^* \left( \Pi^* - 1 \right) (\dot{q}_t - \dot{q}_{t-1} + \pi_t + \gamma \dot{y}_t) - \gamma^\rho (\Pi^*)^2 (\dot{q}_t - \dot{q}_{t-1} + \pi_t) \\
+ \gamma^\rho \beta \Pi^* (\Pi^* - 1) \mathbb{E}_t \left[ \sigma \dot{c}_t - \sigma \dot{c}_{t+1} + \dot{y}_{t+1} + \dot{q}_{t+1} - \dot{q}_t + \pi_{t+1} \right] \\
+ \gamma^\rho \beta (\Pi^*)^2 \mathbb{E}_t \left[ \dot{q}_{t+1} - \dot{q}_t + \pi_{t+1} \right],
\]

(109)

which can be re-arranged as

\[
0 = (1 - \beta) \gamma^\rho \Pi^* (\Pi^* - 1) (\dot{y}_t - \theta \dot{q}_t + \dot{w}_t - \dot{a}_t) \\
+ (q^*)^{1-\theta} (\theta - 1) (\dot{w}_t - \dot{a}_t - \dot{q}_t) \\
+ \gamma^\rho (\Pi^*)^2 \mathbb{E}_t \left[ \beta (\sigma \dot{c}_t - \sigma \dot{c}_{t+1} + \dot{y}_{t+1} + 2 \pi_{t+1}) - 2 \pi_t^s - \dot{y}_t \right] \\
+ \gamma^\rho \Pi^* \mathbb{E}_t \left[ \pi_t^s + \dot{y}_t - \beta (\sigma \dot{c}_t - \sigma \dot{c}_{t+1} + \dot{y}_{t+1} + \pi_{t+1}) \right],
\]

(110)
For a steady-state gross inflation rate of $\Pi^* = 1$, (110) simplifies to
\[
\pi_s^t = \frac{(\theta - 1)}{\gamma p(q^*)^{\theta - 1}} (\hat{w}_t - \hat{a}_t - \hat{q}_t) + \beta E_t[\pi_s^{t+1}].
\]  

**Equation (46)**
\[
a_t n_s^t(i) = q_t^\theta y_t.
\]  

Steady state:
\[
a^* n^{*s}(i) = (q^*)^{-\theta} y^*.
\]  

Log-linearization yields
\[
\hat{a}_t + \hat{n}_t^s(i) = -\theta \hat{q}_t + \hat{y}_t.
\]  

**Equation (47)**
\[
y_t = \left(\nu(n_s^t(i))^{\theta - 1} + (1 - \nu)\frac{\theta}{\theta + \alpha(\theta - 1)}(An_t^r(i))^{\theta - 1}\right)^{\frac{\theta}{\theta - 1}} a_t.
\]  

Steady state:
\[
y^* = \left(\nu(n^{*s}(i))^{\theta - 1} + (1 - \nu)\frac{\theta}{\theta + \alpha(\theta - 1)}(An^{*r}(i))^{\theta - 1}\right)^{\frac{\theta}{\theta - 1}} a^*.
\]  

Log-linearization:
\[
(1 + \hat{y}_t)^{\frac{\theta - 1}{\theta}} (y^*)^{\frac{\theta - 1}{\theta}} \\
= \left(\nu(1 + n_t^s(i))^{\theta - 1} (n^{*s}(i))^{\theta - 1}\right) \\
+ (1 - \nu)\frac{\theta}{\theta + \alpha(\theta - 1)}(1 + \hat{n}_t^r(i))^{\frac{\theta - 1}{\theta}}(An^{*r}(i))^{\frac{\theta - 1}{\theta}}(1 + \hat{a}_t)^{\frac{\theta - 1}{\theta}}(a^*)^{\frac{\theta - 1}{\theta}}.
\]  

An approximation that disregards all terms of order two and higher is
\[
\hat{y}_t(y^*)^{\frac{\theta - 1}{\theta}} = \left(\nu(n^{*s}(i))^{\theta - 1} \hat{n}_t^s(i) + (1 - \nu)\frac{\theta}{\theta + \alpha(\theta - 1)}(An^{*r}(i))^{\frac{\theta - 1}{\theta}} \hat{n}_t^r(i)\right) (a^*)^{\frac{\theta - 1}{\theta}} \\
+ \left(\nu(n^{*s}(i))^{\theta - 1} + (1 - \nu)\frac{\theta}{\theta + \alpha(\theta - 1)}(An^{*r}(i))^{\frac{\theta - 1}{\theta}}\right) \hat{a}_t(a^*)^{\frac{\theta - 1}{\theta}}.
\]  

Simplifying yields
\[
\hat{y}_t = \hat{a}_t + \kappa_1 \hat{n}_t^s(i) + (1 - \kappa_1)\hat{n}_t^r(i),
\]  

\[46\]
where \( \kappa_1 = \frac{\nu((\alpha^{*}(i)))^{\theta-1}}{\nu((\alpha^{*}(i)))^{\theta-1} + (1-\nu)((\alpha^{*}(i)))^{\theta-1}} \in (0,1) \).

**Equation (48)**

\[
\frac{\Gamma_t}{1-\chi_t} = h((1+\Gamma_t)\Delta l_t),
\]

where \((1+\Gamma_t)\Delta l_t = \delta_l^i\). With steady-state identity \(\delta^{i*} = (1+\Gamma^*)\Delta^{i*}\), the log-linearized version of \((1+\Gamma_t)\Delta l_t = \delta_l^i\) can be written as

\[
\delta_l^i = \frac{\Gamma^*\Gamma_t}{1+\Gamma^*} + \Delta l_t.
\]

**Equation (32):**

\[
\frac{\Gamma_t}{1-\chi_t} = h(\delta_l^i) = \frac{\alpha(\theta-1)}{\theta + \alpha(\theta-1)} \left( \frac{1}{(\delta_l^i)^{\alpha(\theta-1)}} - \delta_l^i \right) \phi^c + \delta_l^i - 1.
\]

with steady state:

\[
\frac{\Gamma^*}{1-\chi^*} = \frac{\alpha(\theta-1)}{\theta + \alpha(\theta-1)} \left( \frac{1}{(\delta^{i*})^{\alpha(\theta-1)}} - \delta^{i*} \right) \phi^c + \delta^{i*} - 1.
\]

Log-linearization:

\[
\frac{\Gamma^*(1+\hat{\Gamma}_t)}{1-\chi^*(1+\hat{\chi}_t)} = \frac{\alpha(\theta-1)}{\theta + \alpha(\theta-1)} \left[ \frac{1}{(1+\hat{\delta}_l^{i*})^{\alpha(\theta-1)}} - (1+\hat{\delta}_l^{i*})\delta^{i*} \right] (L^*(\theta))^{\frac{1}{\alpha(\theta-1)}} + (1+\hat{\delta}_l^{i*})\delta^{i*} - 1.
\]

Using the following equation:

\[
\frac{1}{(1+\hat{\delta}_l^{i*})^{\alpha(\theta-1)}(\delta^{i*})^{\alpha(\theta-1)}} - (1+\hat{\delta}_l^{i*})\delta^{i*} = \frac{1}{(\delta^{i*})^{\alpha(\theta-1)}} - \delta^{i*} - \frac{\theta}{\alpha(\theta-1)}(\delta^{i*})^{\frac{\theta}{\alpha(\theta-1)}} \hat{\delta}_l^{i*} - \delta^{i*}\hat{\delta}_l^{i*}
\]

yields

\[
\frac{\Gamma^*(1+\hat{\Gamma}_t)}{(1-\chi^*)(1-\frac{\chi^*}{1-\chi^*}\hat{\chi}_t)} = \frac{\alpha(\theta-1)}{\theta + \alpha(\theta-1)} \left( \frac{1}{(\delta^{i*})^{\alpha(\theta-1)}} - \delta^{i*} \right) (L^*(\theta))^{\frac{1}{\alpha(\theta-1)}} + \delta^{i*} - 1
\]

\[
- \frac{\alpha(\theta-1)}{\theta + \alpha(\theta-1)} \left( \frac{\theta}{\alpha(\theta-1)}(\delta^{i*})^{\frac{\theta}{\alpha(\theta-1)}} + \delta^{i*} \right) (L^*(\theta))^{\frac{1}{\alpha(\theta-1)}} \hat{\delta}_l^{i*} + \delta^{i*}\hat{\delta}_l^{i*}.
\]
Dividing by the steady-state equation yields

$$\hat{\Gamma}_t + \frac{\chi^*}{1 - \chi^*} \hat{\chi}_t = \kappa_2 \hat{\delta}_l^t,$$

where $$\kappa_2 = \frac{1 - \chi^*}{1 - \chi^*} \left( \delta^* - \frac{\alpha(\theta-1)}{\theta + \alpha(\theta-1)} \left( \frac{\theta}{\alpha(\theta-1)} \frac{1}{\delta^*} + \delta^* \right) L^*(\theta) \right).$$

With the help of (121), (127) can be restated as

$$\frac{\chi^*}{1 - \chi^*} \hat{\chi}_t = \left( \kappa_2 - 1 \right) \hat{\Gamma}_t - 1 \frac{1}{1 + \hat{\Gamma}_t} \hat{\chi}_t + \kappa_2 \hat{\Delta}_l^t,$$

(128)

Rewriting (128) yields

$$\hat{\Delta}_l^t = \tilde{\kappa}_2 \hat{\Gamma}_t + \frac{\chi^*}{\kappa_2 (1 - \chi^*)} \hat{\chi}_t,$$

(129)

where $$\kappa_2 > 0$$ and $$\tilde{\kappa}_2 = \frac{1 - (\kappa_2 - 1) \Gamma^*}{\kappa_2 (1 + \Gamma^*)}$$ > 0 in our calibration.

**Equation (49)**

$$\frac{1}{I_t p_t} c_t^{-}\sigma = \mathbb{E}_t \left[ \beta c_{t+1}^{-}\sigma \right]$$

(130)

is equivalent to

$$1 = I_t \beta \mathbb{E}_t \left[ \frac{c_t}{c_{t+1} \Pi_{t+1}} \right].$$

(131)

**Steady state:**

$$I^* = \frac{\Pi^*}{\beta}.$$  

(132)

**Log-linearization:**

$$\hat{c}_t = -\sigma^{-1} \left( \hat{I}_t - \mathbb{E}_t[\pi_{t+1}] \right) + \mathbb{E}_t[\hat{c}_{t+1}].$$

(133)

**Equation (50)**

$$\psi(\nu n(t) + (1 - \nu)n_t(i)) \sigma = c_t^{-}\sigma w_t.$$  

(134)

**Steady state:**

$$\psi(\nu n(t) + (1 - \nu)n_t(i)) \sigma = (c^*)^{-}\sigma w^*.$$  

(135)

$$\hat{w}_t - \sigma \hat{c}_t = \varphi \kappa_3 \tilde{n}_t(i) + \varphi (1 - \kappa_3) \dot{n}_t(i),$$

(136)

where $$\kappa_3 = \frac{\nu n^*(t)}{\nu n^*(t) + (1 - \nu)n^*(i)} \in (0, 1).$$

**Equation (51)**
\[ y_t = c_t + \frac{1}{2} \gamma p \nu \left( \Pi_t \frac{q_t}{q_{t-1}} - 1 \right)^2 y_t + \frac{\chi_t}{1 - \chi_t} \frac{\Gamma_t}{1 + \Gamma_t} (1 - \nu) w_t n_t^*(i) + \mu \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} (1 - \nu) w_t n_t^*(i) \frac{1}{(\Pi_t)^{\frac{\theta}{\theta - 1}}} \]  

\[ \frac{1}{1 - \chi^*} \frac{\Gamma^*}{1 + \Gamma^*} (1 - \nu) w^* n^{**}(i) (\hat{w}_t + \hat{n}_t^*(i)) + \frac{w^* n^{**}(i) \Pi^*}{y^*} \frac{1}{(\Delta^*)^{\frac{\theta}{\theta - 1}}} (\hat{w}_t + \hat{n}_t^*(i)) - \frac{\theta + \alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} \frac{\Gamma^*}{1 + \Gamma^*} \hat{\Gamma}_t. \]  

Steady state:

\[ g^* = c^* + \frac{1}{2} \gamma p \nu (\Pi^* - 1)^2 \hat{y}_t + \nu y^* \gamma^* \Pi^* (\Pi^* - 1) (\pi_t + \hat{q}_t - \hat{q}_{t-1}) \]

\[ + \frac{\chi^*}{1 - \chi^*} \frac{\Gamma^*}{1 + \Gamma^*} (1 - \nu) w^* n^{**}(i) (\hat{w}_t + \hat{n}_t^*(i)) + \frac{\hat{\chi}_t}{1 - \chi^*} + \frac{\hat{\chi}_t}{1 + \hat{\Gamma}_t} + \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} (1 - \nu) w^* n^{**}(i) \frac{1}{(\Pi^*)^{\frac{\theta}{\theta - 1}}} \]

Log-linearization:

\[ g^* \hat{y}_t = c^* \hat{c}_t + \nu y^* \gamma^* \Pi^* (\Pi^* - 1) (\pi_t + \hat{q}_t - \hat{q}_{t-1}) \]

\[ + \frac{\chi^*}{1 - \chi^*} \frac{\Gamma^*}{1 + \Gamma^*} (1 - \nu) w^* n^{**}(i) (\hat{w}_t + \hat{n}_t^*(i)) + \frac{\hat{\chi}_t}{1 - \chi^*} + \frac{\hat{\chi}_t}{1 + \hat{\Gamma}_t} + \mu \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} (1 - \nu) w^* n^{**}(i) \frac{1}{(\Pi^*)^{\frac{\theta}{\theta - 1}}} \]

Further simplification yields

\[ \hat{y}_t = \frac{1}{1 - \nu \gamma^* (\Pi^* - 1)^2} \left( \frac{c^*}{y^*} \frac{\hat{c}_t}{\hat{c}_t} + \nu \gamma^* \Pi^* (\Pi^* - 1) \pi_t^* + (1 - \frac{c^*}{y^*} - \frac{1}{2} \gamma^* \nu (\Pi^* - 1)^2) (\hat{w}_t + \hat{n}_t^*(i)) \right) \]

\[ + \frac{\kappa_4 \hat{\chi}_t}{1 - \chi^*} - \kappa_5 \theta \hat{\Delta}_t + \frac{\kappa_4 - \kappa_5 (\theta + \alpha(\theta - 1)) \Gamma^*}{1 + \Gamma^*} \hat{\Gamma}_t, \]

where \( \kappa_4 = \frac{(1 - \nu) \chi^* \Gamma^*}{1 - \chi^*} \frac{w^* n^{**}(i)}{y^*} \) and \( \kappa_5 = \frac{\mu}{\theta + \alpha(\theta - 1)} \frac{(1 - \nu) (L^*)^{\frac{\theta(\theta - 1)}{\theta - 1}} w^* n^{**}(i)}{y^* (\Delta^*)^{\frac{\theta}{\theta - 1}} (1 + \Gamma^*)^{\frac{\theta(\theta - 1)}{\theta - 1}}}. \)

\[ \square \]

## F Proof of Proposition 6

Damjanovic et al. (2008) derive necessary conditions for the unconditionally optimal policy. In our case, for \( x_t \in \{ n_t^*(i), n_t^*(i), w_t, l_t(i), \Delta_t^l, \phi_t^l, q_t, \Gamma_t, I_t \} \) the unconditionally optimal policy has to satisfy

\[ \frac{\partial u(c_t, \nu n_t^*(i) + (1 - \nu) n_t^*(i))}{\partial x_t} + \sum_{n=1}^{11} \lambda_{n,l} \frac{\partial}{\partial x_t} g_n(x_t, \Pi_{t+1}, \Pi_{t+1}^*, y_{t+1}, c_{t+1}) = 0, \]
where $g_n(x_t, \Pi_{t+1}, \Pi'_{t+1}, y_{t+1}, c_{t+1})$ $(n = 1, 2, ... 11)$ correspond to the left-hand sides net of the right-hand sides of the eleven conditions (41)-(51). Variables $\lambda_{n,t}$ $(n = 1, 2, ... 11)$ are the Lagrange multipliers associated with these constraints. For $x_t \in \{\Pi_t, \Pi'_t, y_t, c_t\}$, the first-order conditions are

$$\frac{\partial u(c_t, n_t)}{\partial x_t} + \sum_{n=1}^{11} \lambda_{n,t} \frac{\partial}{\partial x_t} g_n(x_t, \Pi_{t+1}, \Pi'_{t+1}, y_{t+1}, c_{t+1})$$

$$+ \sum_{n=1}^{11} \lambda_{n,t-1} \frac{\partial}{\partial x_t} g_n(x_{t-1}, \Pi_t, \Pi'_t, y_t, c_t) = 0. \quad (142)$$

It is comparably easy to see that $\lambda_{4,t} = \lambda_{5,t} = \lambda_{9,t} = 0$. Moreover, it is tedious but straightforward to show that the equations stated in the proposition combined with

$$\lambda_{n,t} = (a_t)^{-1} \lambda^*_n, \quad \text{for } n = 1, 2, 3, 6, 7, 11,$$

$$\lambda_{n,t} = \lambda^*_n, \quad \text{for } n = 8, 10,$$

solve conditions (141), (142), and (41)-(51).
References


Figure 7: Social loss as a function of the coefficients on inflation and output in the Taylor rule.
Figure 8: Expected returns on equity for different aggregate capital requirement ratios.
Working Papers of the Center of Economic Research at ETH Zurich

(PDF-files of the Working Papers can be downloaded at www.cer.ethz.ch/research/working-papers.html).

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