A short review of computational methods for uncertainty quantification in engineering

Other Conference Item

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A Short Review of Computational Methods for Uncertainty Quantification in Engineering

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Chair of Risk, Safety & Uncertainty Quantification

Common Challenges in Computationally-based Engineering Research
June 5th, 2013

Some common engineering structures

Cattenom nuclear power plant (France)

Cormet de Roselend dam (France)

Military satellite

Airbus A380

Bladed disk

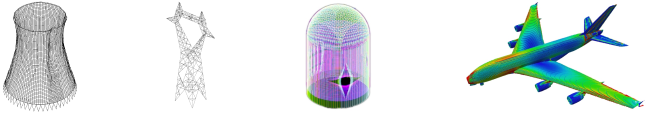
Computational models

Complex systems are designed using computational models that are based on:

- A mathematical description of the physics
- Numerical algorithms that solve the resulting set of (*e.g.* partial differential) equations, *e.g.* finite element models

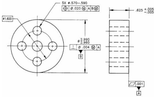
Computational models are used:

- Together with experimental data for calibration purposes
- To explore the design space ("virtual prototypes")
- To optimize the system w.r.t cost constraints
- To assess its robustness w.r.t uncertainty and its reliability

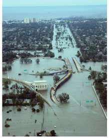


Sources of uncertainty

- Differences between the designed and the real system:
  - Dimensions (tolerances in manufacturing)
  - Material properties (*e.g.* variability of the stiffness or resistance)



- Unforecast exposures: exceptional service loads, natural hazards (earthquakes, floods), climate loads (hurricanes, snow storms, etc.)



Global framework for managing uncertainties

Step B

Quantification of sources of uncertainty

Step A

Model(s) of the system

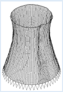
Assessment criteria

Step C

Uncertainty propagation

Random variables

Computational model



Moments

Probability of failure

Response PDF

Step C'

Sensitivity analysis

Global framework for managing uncertainties

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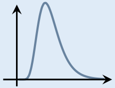
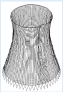


Computational model



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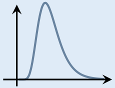

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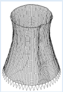

Moments

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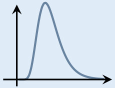
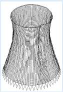


Computational model



Moments

Probability of failure

Response PDF

Step C'

Sensitivity analysis

Step A: computational model(s)

Vector of input parameters $\boldsymbol{x} \in \mathbb{R}^M$ — Computational model $\mathcal{M}$ — Model response $\boldsymbol{y} = \mathcal{M}(\boldsymbol{x}) \in \mathbb{R}^N$

- geometry
- material properties
- loading

- analytical formula
- finite element model
- comput. workflow

- displacements
- strains, stresses
- temperature, etc.

Step A: computational model(s)

Vector of input parameters $\boldsymbol{x} \in \mathbb{R}^M$

Computational model $\mathcal{M}$

Model response $\boldsymbol{y} = \mathcal{M}(\boldsymbol{x}) \in \mathbb{R}^N$

- geometry
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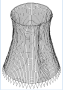
- displacements
- strains, stresses
- temperature, etc.

Step C: uncertainty propagation methods

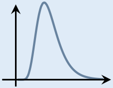
Computational model

Step A



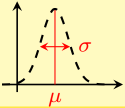
Probabilistic model

Step B

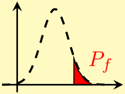


Probabilistic-computational model

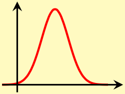
Central trend



Reliability analysis

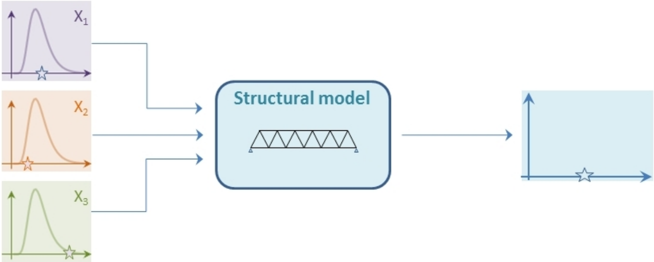


Response PDF



Monte Carlo simulation

- Monte Carlo simulation allows the engineer to assess the performance of a large number of virtual systems featuring different realizations of the input parameters.
- It uses a random number generator to compute a set of input parameters $\mathcal{X} = \{\boldsymbol{x}_i \, , \, i = 1, \, \dots \, , n\}$. The corresponding set of model responses $\mathcal{Y} = \{\mathcal{M}(\boldsymbol{x}_i) \, , \, i = 1, \, \dots \, , n\}$ is computed and post-processed.



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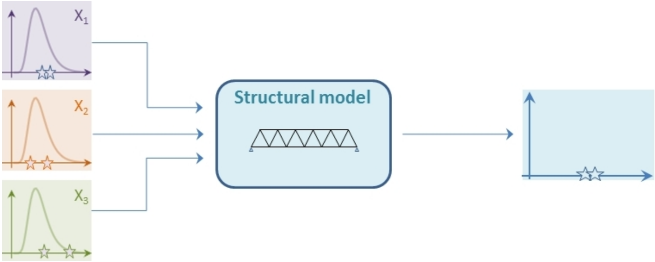

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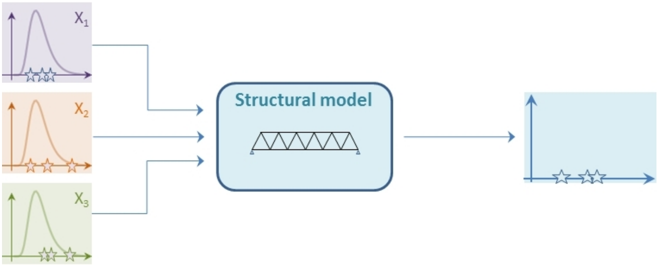

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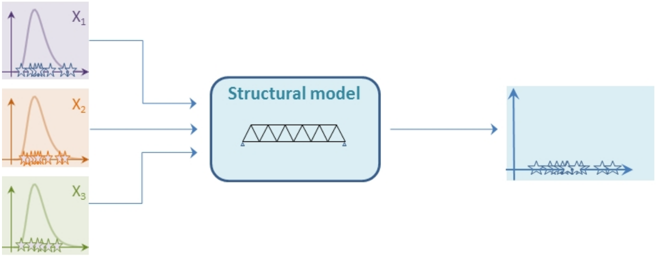
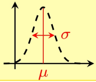


Step C: Central trend

Given a computer model $\mathcal{M}$ and a probabilistic model of its input parameters $\boldsymbol{X} \sim f_{\boldsymbol{X}}$, what is the expected value / scattering of the output $Y = \mathcal{M}(\boldsymbol{X})$?

Central trend



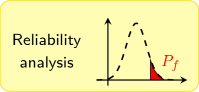
- mean value μ_Y
- standard deviation σ_Y
- higher order moments (skewness / kurtosis of the output distribution)

Methods

- Perturbation method
- Monte Carlo simulation
- Quadrature methods

Step C: Reliability analysis / rare event simulation

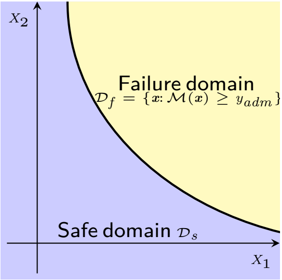
Given a computer model $\mathcal{M}$, a probabilistic model of its input parameters $\boldsymbol{X} \sim f_{\boldsymbol{X}}$ and a performance criterion (*e.g.* "$\mathcal{M}(\boldsymbol{X}) \leq y_{adm}$"), what is the probability of failure?



$$p_f = \mathbb{P}\left(\mathcal{M}(\boldsymbol{X}) \geq y_{adm}\right)$$

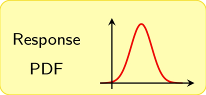
Methods

- ~~Monte Carlo simulation~~
- FORM/SORM methods: based on the assumption of a most probable failure point
- Advanced simulation methods: importance sampling, subset simulation
- Surrogate-based methods, *e.g.* using Kriging



Step C: Distribution analysis

Given a computer model $\mathcal{M}$ and a probabilistic model of its input parameters $\boldsymbol{X} \sim f_{\boldsymbol{X}}$, what are the characteristics of the output distribution of $Y = \mathcal{M}(\boldsymbol{X})$?



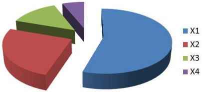
- range / shape (uni-/multi-modal?)
- quantiles (median, inter-quartile, 99%-quantile, etc.)

Methods

- Monte Carlo simulation + kernel smoothing (if large sample set available)
- Surrogate-based methods: polynomial chaos expansions, Kriging

Sensitivity analysis / Parametric study

What are the input parameters (or combinations thereof) that explain at best the variability of the output?



- Deconstruction of the model structure: detection of non linearities, interactions between input parameters, dummy variables
- Variance decomposition: Sobol' indices

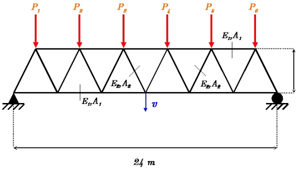
$$\mathrm{Var}\left[Y\right] = \sum_{i=1}^{M} D_i + \sum_{1 \le i < j \le M} D_{ij} + \cdots + D_{1\,2 \ldots M}$$

Methods

- Sobol' indices using Monte Carlo simulation (if large sample set available)
- Polynomial chaos expansions

Truss structure

Problem statement



Input: 10 independent random variables

- Bars properties (2 cross-sections, 2 Young's moduli)
- Loads (6 parameters)

Output: maximal deflection

Uncertainty quantification

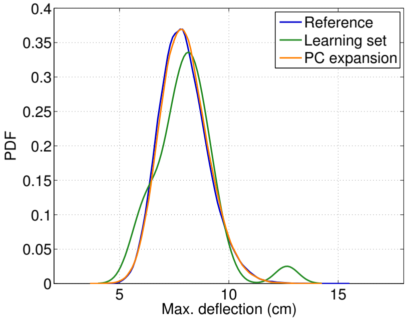
- Distribution of the maximal deflection?
- Mean value and standard deviation?
- Reliability analysis: Prob$\left[v \geq \frac{L}{200} = 12 \text{ cm}\right]$?

Blatman, G. Adaptive sparse polynomial chaos expansions for uncertainty propagation and sensitivity analysis, Université Blaise Pascal, Clermont-Ferrand, 2009.

Truss structure

Statistical moments

	Reference	Monte Carlo	Polynomial chaos
	100,000 runs	30 runs	
Mean (cm)	7.94	8.02 ± 0.49	7.98
Std. dev. (cm)	1.11	1.36 ± 0.10	1.10



Truss structure

Reliability analysis

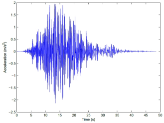
	Reference	Polynomial chaos
	100,000 runs	500 runs
10 cm	4.39e-02 $\pm$ 3.0%	4.30e-02 $\pm$ 0.9%
11 cm	8.61e-03 $\pm$ 6.7%	8.71e-03 $\pm$ 2.1%
12 cm	1.62e-03 $\pm$ 15.4%	1.51e-03 $\pm$ 5.1%
13 cm	2.20e-04 $\pm$ 41.8%	2.03e-04 $\pm$ 13.8%

Truss structure

Sensitivity analysis

Variable	Reference	QMC	Smolyak	Regression
A_1	0.388	0.366	0.372	0.367
E_1	0.367	0.373	0.372	0.367
P_3	0.075	0.077	0.077	0.080
P_4	0.079	0.077	0.077	0.080
P_5	0.035	0.046	0.037	0.039
P_2	0.031	0.039	0.037	0.039
A_2	0.014	0.014	0.013	0.012
E_2	0.010	0.013	0.013	0.012
P_6	0.005	0.014	0.005	0.005
P_1	0.004	0.005	0.005	0.005
# FE runs	5 500 000	10 000	231	66

Earthquake engineering: performance-based design



Question

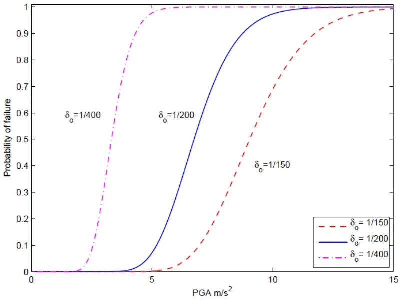
What is the probability of collapse of a building as a function of the “intensity” of a potentiel earthquake?

Uncertainties

- Properties of the structure (material strength, stiffness of the connections, etc.)
- Earthquake magnitude, duration, peak ground acceleration

Non linear transient finite element analysis of the structure for different synthetic earthquakes

Earthquake engineering: performance-based design



- The vulnerability is represented by a fragility curve (probability of attaining some state of damage conditionally on the PGA)
- Seismologists provide models for the PGA w.r.t. the local seismicity (occurrence / magnitude)
- Damage-related costs may be incorporated towards a global risk assessment

Performance-based earthquake engineering

Yang, T., Moehle, J., Stojadinovic, B. & Der Kiureghian, A. Seismic performance evaluation of facilities: methodology and implementation J. Struct. Eng. (ASCE), 2009, 135, 1146-1154.
Sudret, B., Mai, C.V, Computing seismic fragility curves using polynomial chaos expansions, ICOSSAR'2013, New York.

Conclusions

- Uncertainty quantification has become a hot topic in many (if not all) domains of applied science and engineering
- It is a transdisciplinary field which takes advantage from research progress in the mathematical- (statistics, PDEs), engineering- (civil, mechanical, chemical, etc.) and computer science communities
- Non intrusive approaches allow for applications in various fields of the same algorithms
- Generic analysis tools may be developed and disseminated towards the community

"The UQLab platform"

Thank you very much for your attention!