Master Thesis

Application and Comparison of Financial Risk Measures in Earthquake Engineering

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APPLICATION AND COMPARISON OF FINANCIAL RISK MEASURES IN EARTHQUAKE ENGINEERING

Master Thesis
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ABSTRACT

The devastating effects of earthquakes in terms of casualties and monetary losses on individuals, companies and on the whole society highlight the need for effective and informative seismic risk measurement and management. Risk measures enable to measure risk related to an associated hazard and facilitate decision making defining the risk appetite of the decision maker. These measures have been investigated extensively in quantitative financial risk management especially after the 2007-2008 global financial crisis. The current study presents risk measures used in financial industry and connects them to measures employed in the context of performance based earthquake engineering. A framework is proposed, allowing for adequate and rapid estimation of seismic risk measures. In a first application, the focus is on efficient resource allocation in a decision-making process related to seismic upgrading. By quantifying the benefit of seismic upgrading using different risk measures, it is shown that already partial upgrading is an effective measure to reduce risk substantially up to a high confidence level. The second application emphasizes limitations connected to quantification of risk for older and code-conforming buildings under the US and European building codes, but also according to the performance based earthquake design. More specifically, the results of the study verify that quantile-based risk measures are not sub-additive for earthquake intensities with probability of exceedance higher than the 10% in 50 years earthquake scenario (Design Basis Earthquake) and thus could lead to underestimation of financial losses for events with higher frequency of occurrence.
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1 INTRODUCTION

Earthquakes are (still) unpredictable, their potential to cause casualties and severe financial losses makes them one of the most feared natural hazards. Thus, it has already been a concern for long time in civil engineering to quantify earthquake risk. Similarly, risk quantification or measurement has a long history in the financial industry and different approaches have been studied extensively, especially after the 2007-2008 global financial crisis. Risk measurement aims to estimate risk associated with a single position or a portfolio and is crucial to define a subset of “acceptable” risks. An “unacceptable” risk (i.e. a position or portfolio leading to unacceptably high losses) could be mitigated through a variety of ways including risk avoidance, risk transfer or risk reduction. Simply put, seismic risk can be avoided by choosing a safer area and (to some extent) transferred by purchasing earthquake insurance. Most importantly for structural engineers, it can be reduced significantly by designing a new building or upgrading an existing building to achieve a desired performance in case an earthquake occurs. The current study applies a set of different risk measures in two case studies, where the first one focuses on efficient resource allocation in a decision-making process related to seismic upgrading. The second application emphasizes limitations connected to quantification of risk for older and code-conforming buildings under the US and European building codes but also according to the performance based earthquake design. Both case studies are briefly explained below.

The significant fraction of buildings constructed prior to the existence of modern seismic code guidelines recall the importance of seismic retrofit. Unfortunately, a seismic upgrade is an investment without a direct visible output in terms of positive monetary profit. Considering the limited amount of economical capital available to a decision-maker and/or building owner it is vital to seek for an efficient allocation of resources. Historically, building codes have been developed with a primary focus on the design of new buildings and only recently there have been efforts to develop guidelines for the assessment of existing structures. Still the fundamental question of efficient resource allocation, namely to what extent a non-code confirming building should be retrofitted, remains open. Such a question cannot be answered in a general way, but instead depends on the willingness of the decision-maker to take risk. The role of an engineer as a consultant is to provide adequate information that facilitates the decision process.
For this purpose, the current work examines the impact of seismic upgrading in reducing different risk measures.

The second main application estimates risk measures for a portfolio of new, code-compliant buildings, situated in a virtual hazard environment and compares them to the risk measured individually for each building. The importance and motivation of this part is best described using the findings of a recent study by Porter [1]. In the stated reference, an area of approximately 10,000 km is subjected to a Magnitue 7.8 ShakeOut Scenario corresponding to a repeat of the 1906 San Francisco earthquake (recurrence interval in the order of 200 years). Porter assumed a portfolio of newly-designed buildings conforming to the 2012 International Building Code (IBC). Results demonstrated that although only a small fraction of the buildings would collapse (0.8% of the inventory) approximately half of the buildings would be significantly impaired leading to extensive financial losses. The latter is counter-intuitive since current building codes should both prevent building collapse and casualties but also should prevent such extensive damage in an urban scale. This part of the thesis aims to provide an explanation for the findings of the stated reference, using the concept of coherent and non-coherent risk measures, as defined in quantitative financial risk management.

The thesis is structured into three main parts. The first part provides a review on risk measures used in financial industry, connects them to approaches used to quantify seismic risk and concludes with the definition of a coherent risk measure. The second part explains the methodology applied to quantify seismic vulnerability and to derive associated risk measures. The impact of seismic upgrading on reducing different risk measures is examined on the example of an existing, pre-code building, situated in different cities in the US and Europe. Within the third main part, risk measures are applied on a building portfolio situated in a virtual hazard environment.

Note that the work presented in this thesis is the result of a close collaboration between the author and his advisors, Dr. P. Galanis and Dr. M. Broccardo. In parallel to the present document we have worked on two future publications addressing the main parts of this thesis, explained above. Thus, it is inevitable that certain fragments cannot be attributed to a single author anymore and will appear also in the planned publications. All this is in full agreement between the author and his advisors.
2 RISK MEASUREMENT

Risk measurement attempts to quantify the riskiness of a position, whether this is a single financial position or a portfolio of bonds or shares held by a single company. In similar terms, it attempts to measure seismic risk of a single building or a geographically distributed system. By definition, it is not possible to quantify risk in a deterministic way. Whereas in the past often experience-based judgment has been applied to estimate risk, most modern approaches are based on statistical quantities extracted out of probability distributions. Note, that condensing the distribution to a single value always reduces information about the underlying risk. Nevertheless, many tasks require a single measure of risk: In a financial context to determine the needed capital buffer against unexpected events [2], whether this is a financial crisis or a natural catastrophe. Moreover, single metrics are necessary to define an objective function in optimization problems or in decision-making.

This chapter defines first a selection of risk measures used in the financial industry and in earthquake engineering. In a second part, the concept of coherent risk measures is explained by providing examples related to seismic risk.

2.1 Definition of Risk Measures

The current study estimates risk in terms of monetary losses as a percentage of the present building property value. On a probability space $\left( \Omega, F, P \right)$ we consider a loss as a random variable $L : \Omega \rightarrow \mathbb{R}_{\geq 0}$.

A risk measure $\rho$ is a functional assigning a real number to any random variable $L$. In other words, a risk measure measures the “riskiness of $L$ “. As discussed in [2] risk measures have different applications in practice including the amount of capital that a decision maker should hold as a reserve against unexpected future losses. As a tool for limiting the amount of risk exposure and thus for decision making to define the risk appetite of a decision maker. Risk measures are heavily used in financial industry regulations (Basel or Solvency) but only lately have been used in the engineering industry through the PEER performance based design framework.
The most common risk measures used in financial mathematics are the following:

**Expected Loss (EL):** For loss $L$ with CDF $F_L$ the expected loss is defined as:

$$EL = \int_0^\infty (1 - F_L) dl$$

(1)

EL, is one of the most popular measures to characterize a distribution of probable outcomes. By its definition it is a measure for expected outcomes which is, strictly speaking, a contradiction to the main objective within risk measurement, namely to deal with unexpected events. Nevertheless, it provides valuable information and is widely applied in cost-benefit analysis, under the assumption of risk-neutral decision makers. A concept which is well-known also in earthquake engineering. Additionally, EL is extensively used in actuarial science to define the pure premium that should be charged by an insurance company to the policy holder.

**Value-at-Risk (VaR):** Given some confidence level $\alpha \in [0,1]$. VaR of a loss random variable or portfolio $L$ is defined as the smallest number $l$ such that the probability that $L > l$ is no larger than $1 - \alpha$. Formally:

$$\text{VaR}_\alpha = \inf \{ l \in \mathbb{R} : P(L > l) \leq 1 - \alpha \}$$

(2)

VaR is the most widely used risk measure in finance and insurance (Basel II and Solvency II). Simply put, $\text{VaR}_\alpha$ is the $\alpha$-quantile of the probability distribution $L$. As discussed in [2], one of the main criticisms regarding $\text{VaR}_\alpha$ is that it is not a “what-if” risk measure, but rather provides information about the severity of losses which occur with probability $\leq 1 - \alpha$. As will be discussed further in Chapter 2.2, $\text{VaR}_\alpha$ is not a coherent risk measure since it does not satisfy the subadditivity property.

**Expected Shortfall (ES):** For a loss $L$ with CDF $F_L$ the expected shortfall at confidence level $\alpha \in [0,1]$ is defined as:

$$ES_\alpha = \frac{1}{1 - \alpha} \int_0^1 q_u(F_L) \, du = \frac{1}{1 - \alpha} \int_\text{VaR}_u^1 \text{VaR}_u \, du$$

(3)

Alternatively, ES can be defined as:

$$ES_\alpha = \text{VaR}_\alpha + \frac{1}{1 - \alpha} \int_{\text{VaR}_\alpha}^1 (1 - F_L) \, du$$

(4)
\( ES_\alpha \) is the average over \( VaR_u \) for all \( u \geq \alpha \). \( ES_\alpha \) looks further into the tail of \( F_L \) and is a what if risk measure. It can be shown that \( ES_\alpha \) fulfills all properties a coherent measure should possess, according to [3]. However, it is difficult to estimate since a larger sample size is required for accurate estimation of the entire tail distribution. A graphical representation of the listed risk measures based on a generic exceedance curve \((1 - F_L)\) is given in Figure 1.

![Figure 1](image)

**Figure 1** Evaluation of risk measures based on a generic exceedance probability curve: a) Expected Loss EL; b) Value-at-Risk (VaR) and Expected Shortfall (ES) at confidence level \( \alpha = 0.98 \).

The following paragraph provides a brief, and by no means complete, review on risk measurement approaches used in earthquake engineering. Seismic design prescriptions traditionally implemented in building codes aim to reduce collapse probability, given an intensity measure with a specified rate of exceedance or return period (e.g. 475 years). A shortcoming of this approach is, that it does not implicitly address outcomes of moderate earthquakes leading to non-collapse states but might still induce high economical costs, which led to the development of performance-based earthquake engineering.

**Earthquake Hazard Frequency-Based Seismic Response Limits:** This is a quantile based risk measure. Contrary to VaR, this measure focuses on hazard quantiles and associates them with upper limits in terms of seismic building performance. Assuming Poisson distribution to define the probability of exceedance for certain time horizon, FEMA 356 provides specific seismic performance objectives for structures under consideration for four levels of earthquake hazard recurrence interval. The hazard event descriptions considered together with the associated recurrence intervals are listed in Table 1.
Loss-at-Frequency (LaF): This measure corresponds to an extension of seismic performance limits for discrete intensity earthquake levels (as discussed above) to the whole range of earthquake events that could cause building damage. Moehle and Deierlein in 2004 [5] have defined a probabilistic framework including inherent uncertainties in earthquake hazard, engineering performance and associated damage. The output of the framework corresponds to the decision variable frequency, which in the context of this study will be defined as the loss expressed as a percentage of the Present Property Building Value (PBPV). The mathematical expression for loss frequency \( v \) is cited below.

\[
v(L) = \int \int \int G_1(L|DM) dG_2(DM|EDP) dG_3(EDP|IM) d\lambda(IM)
\]

where \( L \) is the seismic loss, DM is the damage measure (e.g. damage states ranging from minor damage to complete damage), EDP is the engineering demand parameter (e.g. max interstory drift), while IM is the earthquake intensity measure (e.g. spectral acceleration at the fundamental building period).

Another risk measure used extensively within seismic due-diligence for building transactions and mortgage issuing by the financial industry is called probable maximum loss (PML). Historically, PML had numerous very different explicit and implicit definitions, leading to confusion among stakeholders in its application. For this reason, the American society of testing materials (ASTM) developed standards for seismic risk assessment [6] and [7], where the terms

<table>
<thead>
<tr>
<th>Event Description</th>
<th>Recurrence Interval</th>
<th>Probability of exceedance in x years</th>
<th>x years</th>
<th>Damage Limits (Max. Interstory Drift)</th>
<th>Damage State</th>
<th>Damage Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequent EQ</td>
<td>43</td>
<td>50%</td>
<td>30</td>
<td>0%</td>
<td>Fully Operational</td>
<td>Negligible Minor local yielding at few places. No observable fractures. Minor buckling or observable permanent distortion of members</td>
</tr>
<tr>
<td>Occasional</td>
<td>72</td>
<td>50%</td>
<td>50</td>
<td>0.50%</td>
<td>Operational</td>
<td>Hinges form. Local buckling of some beam elements. Severe joint distortion. Isolated connection failures. A few elements may experience fracture</td>
</tr>
<tr>
<td>Rare</td>
<td>475</td>
<td>10%</td>
<td>50</td>
<td>1.50%</td>
<td>Life Safe</td>
<td>Extensive distortion of beams and column panels. Many fractures in connections.</td>
</tr>
<tr>
<td>Very Rare</td>
<td>2475</td>
<td>2%</td>
<td>50</td>
<td>2.50%</td>
<td>Near Collapse</td>
<td></td>
</tr>
</tbody>
</table>

Table 1 List of Earthquake Events considered in Performance Based Design (Damage Limits are defined according to Vision 2000 [4])
probable loss and scenarios loss are recommended to use. Probable loss refers to the loss associated with a certain frequency of exceedance or return period, which is identical to loss-at-frequency defined above. For scenario loss, the stated documents recommend the terminology of scenario expected loss and scenario upper loss, which will be defined in the following.

**Scenario Expected Loss (SEL):** This measure corresponds to the expected loss due to a specific seismic scenario event associated with a certain return period $RP$, often chosen to be consistent with design base earthquake and set to 475 years. A comprehensive scenario-based loss assessment builds up on hazard disaggregation to define a certain earthquake event with corresponding magnitude and source to site distance. Within this specific framework, a scenario is defined, as the IM associated with a certain rate of exceedance $\lambda(IM) = 1/RP$ and the scenario expected loss yields to:

$$SEL_{RP} = \int_0^\infty [1 - F_{L|IM_{RP}}] dL,$$  \hspace{1cm} (6)

where $F_{L|IM_{RP}}$ refers to the CDF of loss random variable $L$ conditioned on an intensity measure IM with return period RP.

**Scenario Upper Loss (SUL):** This measure refers to the 90% quantile of the loss distribution conditioned on an event with a certain return period $RP$, and thus can be defined as:

$$SUL_{RP} = \inf\{l \in \mathbb{R} : P(L > l \mid IM_{RP}) \leq 1 - 0.9\}.$$  \hspace{1cm} (7)

### 2.2 Coherent Risk Measure

As discussed in detail by Artzner et al. in [3] a risk measure satisfying the following four axioms is called coherent. Below, the four properties are defined together with the importance for a risk measure to satisfy them from an earthquake engineering perspective.

**Property 1** Monotonicity: If $L_1 \leq L_2$ for each outcome, then $\rho(L_1) \leq \rho(L_2)$

If two buildings are constructed in the same hazard environment and building one $L_1$ is less fragile than the other $L_2$ for all earthquake scenarios, then the owner of building one is exposed to higher risk than the owner of building two.

**Property 2** Translation Invariance: For constant $c \in R$, $\rho(L + c) = \rho(L) + c$
If an additional loss or gain amount $c$ is added then risk exposure is altered accordingly.

**Property 3** Subadditivity: For all random losses $L_1, L_2 \ldots L_n$, $\rho(L = L_1 + L_2 + \cdots L_n) \leq \rho(L_1) + \rho(L_2) + \cdots + \rho(L_n)$

The latter property reflects the idea of diversification. For earthquake engineering applications, this corresponds to diversification due to portfolios constructed from buildings of different building properties (i.e. corresponding to different fragility curves) but also geographic diversification (i.e. earthquake intensity is not uniform across the considered area).

**Property 4** Positive Homogeneity: For constant $b \in N$, $\rho(b \cdot L_1) = b \cdot \rho(L_1)$

If $b$ identical buildings are considered, then risk exposure becomes $b$ times higher. The latter property is not always desired, since for high number of $b$, risk exposure is not always increasing linearly. An example of such a case is related to Business Interruption. If a catastrophic event affects a high number of buildings above a certain threshold, losses due to business interruption could increase disproportionally to the number of buildings affected.

The only risk measures discussed in the previous section that satisfy Properties 1-4 and thus are coherent are expected shortfall and expected losses. VaR, loss-at-frequency and in general quantile based measures satisfy properties 1, 2, 4 but do not satisfy property 3. The implications of lack of this property are explained in detail in Chapter 4.

All risk measures defined in Chapter 2.1, require an estimate for the distribution of seismic losses. It is common practice in actuarial science, engineering and catastrophe modelling to characterize a loss distribution in terms of an exceedance probability curve $(1 - F_L)$. However, the term exceedance probability on its own may be misleading, since there are different definitions employed by specialists in the mentioned fields. For instance, aggregate exceedance probability (AEP) refers to the exceedance probability of the cumulative losses within a certain time period, whereas occurrence exceedance probability (OEP) describes the exceedance probability of at least one event in the given time period. Based on open-access hazard information and quantification of seismic vulnerability, the following chapter presents a framework to estimate AEP and OEP of seismic losses and illustrates the derivation of risk measures defined above. The latter are then applied to estimate impact on decisions related to seismic upgrading of an existing structure.
3 APPLICATION OF RISK MEASURES ON A SINGLE BUILDING

The underlying methodology to evaluate exceedance probability curves for seismic losses is based on the work described in Galanis et al. [8] and extended to allow for a quantification of risk measures. The framework combines principal elements of the PEER performance based framework [5] and the standard actuarial “frequency-severity” method. Based on available seismic hazard information, earthquake vulnerability is quantified and connected to building damage. The latter is used to derive seismic losses in terms of occurrence and aggregate exceedance probability.

The first part of this chapter explains each module of the underlying framework and demonstrates the derivation of seismic loss exceedance curves. Then, evaluation of the risk measures defined above is illustrated based on an existing building, situated in the hazard environments of Zurich, Lisbon, Seattle, Reggio Calabria and Los Angeles. An existing building, as defined in the present study, corresponds to a building constructed prior to the existence of modern seismic code guidelines, lacking adequate seismic detailing and having low lateral strength. The last chapter addresses the benefit of seismic upgrading of the existing building in reducing different risk measures.

The employed methodology quantifies earthquake vulnerability based on three structural parameters that can be estimated using an elasto-plastic idealization of a pushover analysis force-displacement response, namely: the yielding acceleration $a_y = F_y/M_s$, the structural ductility capacity $\mu_C (= d_u/d_y)$, and the fundamental vibration period $T$. $F_y$ is the yielding base shear force estimated according to the pushover elasto-plastic response idealization, $M_s$ is the seismic mass of the structure, and $d_y$ and $d_u$, are, respectively, the yielding and the ultimate displacement determined from a non-linear static pushover analysis of the structure. To compare the impact of seismic retrofit for different hazard environments the existing building is assumed to be the same at all considered site locations, having a yield strength $a_y = 0.05g$, a fundamental period of vibration $T_1 = 0.6s$ and a ductility capacity $\mu_C = 3$. 

3.1 Seismic Hazard Information

Seismic hazard data, accessible via open databases, serves as primary input. For the sites located in Europe (Zurich, Lisbon and Reggio Calabria) the corresponding information is provided by the European Facility for Earthquake and Risk [9] for soils with an average shear wave velocity of $v_{s,30} = 800$ m/s. For sites located in the United States similar data is provided by the United States Geological Survey [10]. For all site locations, seismic hazard curves illustrating the probability of exceedance in 50 years of peak ground acceleration PGA and 5% damped elastic spectral acceleration $S_{ae}(T)$ are extracted. The geographical coordinates of the corresponding cities are provided in Table 2. As an illustration of the hazard environments considered, Figure 2 shows the uniform hazard spectra for probabilities of exceedance of 50% (frequent earthquake), 10% (Design Basis Earthquake) and 2% (Maximum Considered Earthquake) in 50 years.

| Geographical coordinates of site locations considered in the current study. |
|---------------------------------|---------|--------|--------|---------|
| Zurich                          | Lisbon  | Seattle| Reggio. C. | Los Angeles |
| Longitude                       | 8.58    | -9.14  | 122.35  | 15.68    | -118.25  |
| Latitude                        | 47.40   | 38.72  | 47.60   | 38.10    | 34.00    |

**Figure 2** Uniform hazard spectra for the considered site locations and for probabilities of exceedance of 50%, 10% and 2% in 50 years.

3.2 Vulnerability Assessment

To assess the damage inflicted by an earthquake, fragility curves are derived, illustrating the probability of exceeding a certain discrete damage grade, conditioned on an intensity measure. The current work identifies for each damage grade $D_{G_k}$ limit states as a function of structural ductility capacity $\mu_c$ as shown in Table 3 and based on proposals given in [11].
Assuming a lognormal distribution for the fragility curve, the probability of exceeding a certain damage grade takes the mathematical form of:

\[
P(DG_k \mid S_{ae}) = \Phi \left[ \frac{\ln(S_{ae}) - \ln(\hat{s}_{a,k})}{\beta_k} \right],
\]

where \( \Phi \) denotes the standard normal cumulative distribution function, \( \hat{s}_{a,k} \) represents the median capacity of damage grade \( DG_k \) in units of the considered IM; and \( \beta_k \) is the corresponding lognormal standard deviation. The latter incorporates uncertainties on the demand side \( \beta_D \) (record-to-record variability) and the total modelling dispersion \( \beta_M \) related to uncertainties in structural characteristics, building quality and completeness of the mathematical model [12]. To estimate the structural response the current study applies the software tool SPO2IDA, developed by Vamvatsikos and Cornell and described in [13], capable of efficiently predicting results of incremental dynamic analysis (IDA) of a given SDoF. In simple words, it allows to estimate the 16%, 50%, and 84%-fractiles of lateral strength reduction factor \( R \) as a function of the ductility \( \mu \) (or vice versa), which is illustrated in Figure 3a, applied to the existing building defined above. The median capacity of a certain damage grade is evaluated by multiplying the yielding acceleration capacity with the 50%-fractile of \( R \) given \( \mu_{lim,k} \) :

\[
\hat{s}_{a,k} = R_{50\%}(\mu_{lim,k}) \cdot a_y.
\]

The total measure of dispersion \( \beta_k \) is defined as

\[
\beta_k = \sqrt{\beta^2_{Dk} + \beta^2_{ Mk}},
\]

where \( \beta_{Dk} \) represents the record-to-record variability and is estimated as:

\[
\beta_{Dk} = \frac{1}{2} \ln \left[ \frac{R_{84\%}(\mu_{lim,k})}{R_{16\%}(\mu_{lim,k})} \right].
\]

The parameter \( \beta_{ Mk} \) refers to modelling uncertainty and is estimated based on values provided in [14]. The fragility curves obtained with eq. 8 are illustrated in Figure 3b, evaluated for the existing building.

---

1 For dispersion due to modelling uncertainties \( \beta_M \) following values are provided in [13], depending on lateral strength reduction factor \( R \) : \( \beta_M = 0.25 \) given \( R \leq 1 \) ; \( \beta_M = 0.25 \) given \( R = 2 \) ; \( \beta_M = 0.35 \) given \( R = 4 \) ; \( \beta_M = 0.5 \) given \( R = 6 \) ; \( \beta_M = 0.5 \) given \( R \geq 8 \). For values of \( R \) situated between the given points linear interpolation is applied.
Table 3 Damage grade thresholds in terms of ductility capacity $\mu_c$, as defined in [11].

<table>
<thead>
<tr>
<th>Damage Grade</th>
<th>Damage Description</th>
<th>$\mu_{\text{lim,k}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Slight</td>
<td>0.7</td>
</tr>
<tr>
<td>2</td>
<td>Moderate</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>Extensive</td>
<td>0.5(1+$\mu_c$)</td>
</tr>
<tr>
<td>s45$^2$</td>
<td>Heavy-Complete</td>
<td>$\mu_c$</td>
</tr>
</tbody>
</table>

Figure 3 For the existing building: a) Outcome of SPO2IDA, namely the relation between lateral strength reduction factor $R$ and ductility $\mu$; b) Fragility curves derived for the damage limit states stated in Table REF.

3.3 Loss Analysis

The starting point of quantifying seismic risk is an estimate of the distribution of the size of losses inflicted, given that a single seismic event with certain seismic intensity occurs. The size of losses in this study is expressed as a percentage of the Present Building Property Value (PBPV). It is assumed that the PBPV remains constant during the considered building service time horizon. The seismic loss is quantified as:

$$P(L \leq l|im) = \sum_{D_k} G_2(L \leq l|dG_k) \cdot dG_1(DG_k|im), \quad (10)$$

$^2$ The following relations to determine the proportions of DG4 and DG5 are suggested in [11]:

$$P(DG_{s45}|S_{ae}) = 0.09 \cdot \sinh \left(0.6 \left(\sum_{k=1}^{3} k \cdot P(DG_{S_k}|S_{ae}) + 4 \cdot P(DG_{s45}|S_{ae})\right)\right) \cdot P(DG_{s45}|S_{ae})$$
where \( i_m \) is the seismic intensity measure quantified in terms of the elastic spectral (pseudo-) acceleration at the fundamental building period \( T \); \( DG_k \) corresponds to the damage grade \( k \) ranging from slight damage to complete collapse; \( L \) is the seismic loss incurred to structural elements by a single earthquake event expressed as a percentage of PBPV (e.g. \( L=100\% \) corresponds to complete damage of the building property); \( G_1 \) is the damage Cumulative Distribution Function (CDF) estimated using the vulnerability curves presented in the previous section. The CDF \( G_2 \) is assumed to follow a beta distribution, Beta\((L|DG_k; \alpha, \beta)\), where random variable \( L \) has finite support \([0,1]\) and \( \alpha \) and \( \beta \) are:

\[
\alpha = \frac{1 - MDR}{CoV^2} - MDR \quad \text{and} \quad \beta = \frac{\alpha \cdot (1 - MDR)}{MDR}
\]

where MDR is the Mean Damage Ratio and CoV is the coefficient of Variation, both stated in Table 4. MDR is defined as a percentage of PBPV by analogy to [15]. The complementary CDF \( G_2 \) conditioned on \( DG_k \) is illustrated in Figure 4.

**Table 4** MDR and CoV for each damage grade.

<table>
<thead>
<tr>
<th></th>
<th>DG1</th>
<th>DG2</th>
<th>DG3</th>
<th>DG4</th>
<th>DG5</th>
</tr>
</thead>
<tbody>
<tr>
<td>MDR</td>
<td>3.5%</td>
<td>14.5%</td>
<td>30.5%</td>
<td>80.0%</td>
<td>95.0%</td>
</tr>
<tr>
<td>CoV</td>
<td>1.23</td>
<td>0.39</td>
<td>0.36</td>
<td>0.14</td>
<td>0.06</td>
</tr>
</tbody>
</table>

**Figure 4** Conditional complementary CDF \( G_2 \) conditioned on \( DG_k \) as a percentage of Present Building Property Value (PBPV).
The amount of loss inflicted by a single seismic event (of any earthquake intensity) is defined as a random variable \( S \), called the severity. Severity \( S \in \mathbb{R}_{\geq 0} \) is a portion of PBPV. The Complementary Cumulative Distribution Function (CCDF) of \( S \) can be estimated using the concept of conditional probability as:

\[
P(S > s) = CEP = \frac{\sum_{i=min(im)}^{max(im)} (1 - P(L \leq l|im)) \cdot \lambda_i}{\sum_{i=min(im)}^{max(im)} \lambda_i},
\]

where \( \lambda_i \) is the annual rate of occurrence of a seismic event with intensity \( i = im \). The rate of occurrence ranges between \( \lambda_{\text{min}(im)} \), which is the lower bound of earthquake intensity that can cause damage or loss to the evaluated building property, and \( \lambda_{\text{max}(im)} \), which is the upper bound of earthquake intensity that can occur in the site of interest.

As explained above, the term exceedance probability can be misleading, since there are several definitions used by specialists in engineering, actuarial science and catastrophe modelling. The aggregate exceedance probability AEP is the probability that the sum of all losses during a given period exceeds some amount. The occurrence exceedance probability OEP defines the probability that at least one loss exceeds the specified loss amount within a certain time \( t \). In the following, it is illustrated how the mentioned quantities are derived using the framework described above.

Aggregate exceedance probability refers to the CCDF of aggregated seismic losses ASL. The latter are defined as the sum of losses for a given time period \( t \): \( ASL_t = \sum_i S_i \), where \( S_i \) are the earthquake severities with CCDF defined above. If earthquake occurrence follows a Poisson distribution, ASL can be assumed to follow a compound Poisson distribution, namely \( ASL_t \sim \text{CompPoi}(\lambda, S) \), where \( \lambda \) corresponds to the total frequency of all seismic intensities \( im \) considered for a given time period \( t \). Then the probability distribution of ASL is defined as:

\[
ASL_t \sim \sum_{i=1} P(i) \times S_i^i
\]

where \( P(i) \) corresponds to the probability of \( i \) occurrences of earthquakes that cause a loss within the time period \( t \) and \( S_i^i \) is the severity distribution convolved \( i \) times with itself. Note
that this procedure implicitly assumes that every time a loss is incurred the structure is instantaneously upgraded to the original state without any deterioration of the building performance. It is further assumed that the uncertainties are “renewed” after each earthquake event [16]. Figure 5b shows aggregated exceedance probability of a loss for a time horizon of 50 years and for all site locations considered in the present study: Zurich, Lisbon, Seattle, Reggio Calabria and Los Angeles.

For calculation of occurrence exceedance probability (OEP) the decomposition property of Poisson distribution is employed. Accordingly, if the number of occurrences in a fixed time period follows a Poisson distribution, then in any other subset of events built from the original event set the number of occurrences will also follow Poisson distribution with Poisson parameter equal to the original Poisson parameter multiplied by the probability of an event belonging to the corresponding subset of events. Based on the severity $S$ defined above, OEP for a certain time period $t$ is evaluated using:

$$OEP(L, t) = P(L > l \text{ in } t \text{ years}) = 1 - e^{-\lambda \cdot P(S > s)},$$

where $\lambda$ is the total frequency of all seismic intensities considered for a given time horizon. Figure 5a illustrates the occurrence exceedance probability of a loss within a time period of 50 years and for all different hazard environments considered in the present study.

**Figure 5** Exceedance probability of a certain loss amount in 50 years for the existing building situated in the different hazard environments: a) Occurrence Exceedance Probability b) Aggregated Exceedance Probability.
3.4 Evaluation of Risk Measures

Based on the derived exceedance probability curves the next paragraph explains the evaluation of risk measures in the same order as they are defined in Chapter 2.1.

**Expected Loss (EL):** Expected loss estimated based on the OEP curve refers to the mean losses due to at least one event in the considered time horizon t and is denoted as EL(OEP,t). If derived using the AEP curve, it represents the mean cumulative (aggregated) seismic losses in the considered time horizon, denoted as EL(AEP,t).

**Value-at-Risk:** This measure refers to the quantile evaluated on an exceedance probability curve. Assuming a building service time horizon of 50 years, a conveniently used measure is a loss, which is only exceeded with probability of 10% in 50 years. Estimated on OEP, it describes the loss amount due to a single event, for which the probability of exceedance is lower or equal to 10% in 50 years and is denoted as VaR\(_{90\%}\) (OEP,50) or more generally VaR\(_\alpha\) (OEP,t). Whereas using the AEP-Curve, it describes the loss due to all seismic events, for which the probability of exceedance is lower or equal to 10% in 50 years, denoted as VaR\(_\alpha\) (AEP,t). Figure 6 illustrates both mentioned risk measures as a function of exceedance level for all hazard environments considered in the present study.

**Expected Shortfall:** This measure estimated based on an occurrence exceedance probability of 10% in 50 years describes the average expected losses induced by at least one event given that the loss exceeded VaR at the same confidence level \(\alpha\), denoted as ES\(_\alpha\) (OEP,t). Estimated for an aggregated exceedance probability of 10% in 50 years it refers to the average cumulative

---

**Figure 6** VaR for the existing building and a time horizon of 50 years situated in different hazard environments, estimated based on: a) Occurrence Exceedance Probability, b) Aggregate Exceedance Probability.
expected losses given that the cumulative losses exceeded VaR at the same exceedance level and is denoted as $\text{ES}_\alpha(\text{AEP}, t)$. Figure 7 illustrates expected shortfall, estimated based on OEP and AEP, as a function of exceedance level $(1 - \alpha)$.

**Figure 7** ES for the existing building and a time horizon of 50 years situated in different hazard environments, estimated based on: a) Occurrence Exceedance Probability, b) Aggregate Exceedance Probability.

**Loss-at-frequency:** Using the definition of severity $S$ and total frequencies of all seismic intensities occurring within a time horizon of one year $\lambda$, the mean annual frequency of exceedance of a certain loss amount $\nu(L)$ is estimated as:

$$
\nu(L) = \lambda \cdot P(S > s).
$$

The corresponding exceedance curves, for each site location, are shown in Figure 8a. The risk measure Loss-at-frequency LaF is evaluated at the frequency or return period of interest, which is illustrated on the example of a return period equal to 475 years.

**Scenario Expected and Upper Loss:** Both measures rely on the distribution of losses conditioned on a scenario event. The current study defines a scenario as a certain seismic intensity measure, with an associated mean annual frequency of exceedance $\lambda(im)$ and $RP = 1/\lambda(im)$. These measures are evaluated by inserting $P(L \leq l | IM_{RP})$, obtained with eq. 10, in the definition of the corresponding risk measures provided by eq. 6 and 7. Figure 8b illustrates the CCDF of losses conditioned on IM with associated return period of 475 years.
Figure 8 For the existing building situated in different hazard environments: a) Loss-at-frequency, with indicated return period of 475 years; b) CCDF of seismic losses conditioned on intensity measure with return period of 475 years, the area below the curve corresponds to the risk measure Scenario Expected Loss and the 90% quantile of the distribution refers to Scenario Upper Loss.

Table 5 provides quantitative estimates of the risk measures explained above as a percentage of present building property value. EL is evaluated on an annual basis, whereas the other measures are evaluated for a probability level of 10% in 50 years. A typical application of risk measures is to compare different underlying risks, which in this example are represented by the different cities and their hazard environment. Note that the risk measures cannot be compared directly in a quantitative way, since they are defined in different ways and illustrate different aspects of the risk. Instead a relative comparison is possible, by normalizing results of one risk measure, with the same measure evaluated for a specific hazard environment, meaning a normalization along each row in Table 5.

To illustrate, how risks may be perceived differently, depending on the risk measure considered, Figure 9a shows for each of the nine risk measures listed in Table 5, the result for each city normalized by the result evaluated for Los Angeles. For this purpose, assume a decision maker has the choice of buying the exact same building in Seattle, Reggio C. or Los Angeles, where seismic risk exposure is the only decision criteria and that he or she only decides based on the value of one risk measure out of the nine stated above. If this measure is the annual expected loss, the choice will be Seattle.
Conversely, if the decision maker focuses only on tail events captured by ES or VaR based on OEP of 10% in 50 years, it is difficult to identify an optimal choice, because for this specific building and the three cities under consideration, the risk measure basically indicates full collapse of the structure. For high confidence levels (or low exceedance levels) expected shortfall and Value-at-Risk fail to provide an accurate “resolution” due to the bounded nature of an occurrence based approach, e.g. the loss to a single event cannot exceed 100% of PBPV. This is important, since it indicates that the choice of confidence level may have strong influence on the preferred option chosen by the decision-maker. This issue will be further discussed in the next section, where we introduce seismic upgrading.

<table>
<thead>
<tr>
<th>Nr.</th>
<th>Risk Measure</th>
<th>Zurich</th>
<th>Lisbon</th>
<th>Seattle</th>
<th>Reggio</th>
<th>Los Angeles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>EL (OEP,1yr)</td>
<td>0.15%</td>
<td>0.70%</td>
<td>1.51%</td>
<td>1.79%</td>
<td>2.79%</td>
</tr>
<tr>
<td>2</td>
<td>EL (AEP,1yr)</td>
<td>0.15%</td>
<td>0.71%</td>
<td>1.52%</td>
<td>1.81%</td>
<td>2.85%</td>
</tr>
<tr>
<td>3</td>
<td>VaR_{α=0.9} (OEP,50yr)</td>
<td>17.3%</td>
<td>91.6%</td>
<td>98.2%</td>
<td>98.5%</td>
<td>99.4%</td>
</tr>
<tr>
<td>4</td>
<td>VaR_{α=0.9} (AEP,50yr)</td>
<td>19.1%</td>
<td>101.6%</td>
<td>183.1%</td>
<td>199.3%</td>
<td>279.8%</td>
</tr>
<tr>
<td>5</td>
<td>ES_{α=0.9} (OEP,50yr)</td>
<td>52.1%</td>
<td>96.8%</td>
<td>99.4%</td>
<td>99.5%</td>
<td>99.8%</td>
</tr>
<tr>
<td>6</td>
<td>ES_{α=0.9} (AEP,50yr)</td>
<td>56.3%</td>
<td>141.7%</td>
<td>232.9%</td>
<td>254.6%</td>
<td>345.3%</td>
</tr>
<tr>
<td>7</td>
<td>LaF (RP=475yr)</td>
<td>17.4%</td>
<td>91.6%</td>
<td>98.2%</td>
<td>98.5%</td>
<td>99.4%</td>
</tr>
<tr>
<td>8</td>
<td>SEL (RP=475yr)</td>
<td>19.5%</td>
<td>60.9%</td>
<td>87.2%</td>
<td>81.3%</td>
<td>87.3%</td>
</tr>
<tr>
<td>9</td>
<td>SUL (RP=475yr)</td>
<td>56.6%</td>
<td>98.4%</td>
<td>99.4%</td>
<td>99.3%</td>
<td>99.5%</td>
</tr>
</tbody>
</table>

| Figure 9 | For the existing building situated in different hazard environments: a) Risk Measures stated in Table 5, normalized to the corresponding measure evaluated for Los Angeles, b) Hazard Curve, illustrating the probability of exceeding a certain Sa(T) in 50 years.
Furthermore, Table 5 and Figure 9a clearly state a problem, arising from scenario-based loss assessment. Considering only the loss which is due to a specific event, may provide misleading information about the underlying risk. Suppose instead of using EL, VaR or ES the only information the decision maker has is SEL or SUL, he might think that buying a house in Seattle is riskier than buying the exact same house in Reggio C, although all other measures indicate the opposite. As confirmed by the hazard curves illustrated in Figure 9b, the intensity measure at this specific level is higher for Seattle, whereas for frequent (EL) and rare (VaR, ES and LaF) events the seismic hazard is more severe in Reggio Calabria.

The following section addresses the impact of seismic upgrading on reducing different risk measures.

3.5 Quantification of Retrofit Utility in terms of different Risk Measures

To assess the effect of seismic upgrading and more specifically of partial upgrading, the term degree of seismic upgrading (DSU) is defined, where the existing building defined above corresponds to DSU=0%. A full seismic upgrade, e.g. DSU=100%, corresponds to an upgrade of the structure to an extent, such that it possesses the same properties as a modern, code-compliant building with proper seismic detailing. Stemming from the assumption that the retrofitted buildings have the same geometry and yielding material characteristics, the yielding displacement is assumed to be the same for all buildings representing different DSU. The yielding acceleration capacity of the code-compliant building (DSU=100%) is evaluated by:

\[ a_y(100\%) = S_{ae,10\%50\text{Years}}(T_1) \cdot \Omega \cdot \gamma_1/q \]  

where \( S_{ae,10\%50\text{Years}}(T_1) \) corresponds to the elastic 5% damped spectral acceleration at the fundamental period of vibration\(^3\) \( T_1 \), \( \Omega = 1.5 \) represents the structural system over-strength factor, \( \gamma_1 \) is the soil amplification factor (=1 for soil type A [17]) and \( q = 3.0 \) is the behavior factor to account for the inelastic response of the structure. The structural ductility capacity \( \mu_C \)

---

\(^3\) The fundamental period of vibration of the code-compliant building \( T_1(100\%) \) is evaluated using following relationship, which assumes constant yield displacement: \( T_1(100\%) = T_1(0\%) \cdot \sqrt{a_y(0\%)/a_y(100\%)} \). Thus, solving equation 16 becomes an iterative procedure.
is assumed to six. As implicitly stated in eq.16, the code-compliant building depends on the considered hazard environment. The parameters for the existing and code-compliant buildings are formulated in Table 6. The capacity curves of all intermediate DSU are derived based on a proportional increase of ductility capacity $\mu$ and yielding acceleration capacity $a_y$, leading to the following definition of $a_y(X\%)$:

$$a_y(X\%) = a_y(0\%) + DSU \cdot \left( a_y(100\%) - a_y(0\%) \right).$$  \tag{17}

The same equation is applied to estimate ductility capacity of intermediate DSU, whereas evaluation of the corresponding fundamental vibration period is based on the assumption of constant yield displacement, as described above for the code-compliant building.

**Table 6** Structural characteristics of existing and code-compliant buildings for the hazard environments considered in the present study.

<table>
<thead>
<tr>
<th>Site Location</th>
<th>Existing Building (DSU 0%)</th>
<th>High-Code Building (DSU 100%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_y$ (g)</td>
<td>$\mu$</td>
</tr>
<tr>
<td>Zurich</td>
<td>0.05</td>
<td>3</td>
</tr>
<tr>
<td>Lisbon</td>
<td>0.05</td>
<td>3</td>
</tr>
<tr>
<td>Seattle</td>
<td>0.05</td>
<td>3</td>
</tr>
<tr>
<td>Reggio C.</td>
<td>0.05</td>
<td>3</td>
</tr>
<tr>
<td>Los Angeles</td>
<td>0.05</td>
<td>3</td>
</tr>
</tbody>
</table>

The framework explained in the precedent chapters allows to estimate a risk measure $\rho$ for each of the buildings corresponding to different DSU, denoted by $\rho(DSU)$. For a given risk measure $\rho$ the retrofit benefit $RB_\rho$ of a specific DSU is defined as:

$$RB_\rho(DSU) = \rho(DSU 0\%) - \rho(DSU),$$  \tag{18}

where $\rho(DSU 0\%)$ refers to the risk measure $\rho$ evaluated for the existing building (with DSU=0%). Normalizing this quantity by the maximal considered benefit, obtained by upgrading the existing building to the code-compliant building defined in Table 6, is defined as retrofit utility $RU_\rho$ of a specific DSU:

$$RU_\rho(DSU) = \frac{RB_\rho(DSU)}{RB_\rho(DSU 100\%)} = \frac{\rho(DSU 0\%) - \rho(DSU)}{\rho(DSU 0\%) - \rho(DSU 100\%)},$$  \tag{19}
which, indicates by how much a given risk measure can be reduced for a given degree of seismic upgrade, compared to the maximum considered upgrade. The risk attitude of a decision maker defines the relevant risk measure. A risk-neutral decision maker decides according to minimizing the expected loss, which could additionally include the cost of implementing seismic upgrade.

The majority of decision makers in practice (especially home-owners) are risk averse. A risk-averse decision maker would prefer between two strategies leading to the same expected losses the one corresponding to lower risk e.g. if losses would follow normal distribution lower risk would correspond to the distribution with lower standard deviation [18]. This contrasts with a risk neutral decision maker that would be indifferent of the two strategies as long as they yield the same expected loss. In terms of a loss distribution, risk-averse decision makers tend to assign more weight on tail events depending on the degree of risk aversion, where low probability but high loss outcomes are located. Since risk-averse decision-makers are not deciding only based on the expected value, retrofit utility needs to be evaluated for different risk measures including those focusing on the tail of the distribution.

For the reasons listed above retrofit benefit and utility are discussed in detail for expected loss and expected shortfall, where the latter represents a typical tail measure and thus is likely to better suit the need of risk-averse decision makers.

Expected Loss

The expected loss of a distribution assigns weight proportional to the frequency of an event, and thus more frequent and less severe events control expected loss estimation. For the expected loss estimation, the loss due to multiple smaller events should be considered for a more realistic (and conservative) estimation. Thus, the expected value of the cumulative (aggregate) seismic losses due to all events is of interest, which in the present study is denoted by EL(AEP). Note that, due to the normalization of the \( RU_{EL}(DSU) \) (as defined in eq. 19), the latter is independent of the time horizon considered. This, however, holds only true for the mean expected cumulative losses.

Figure 10 illustrates that for all hazard environments considered, the relationship between seismic upgrading and retrofit utility is not only nonlinear, but also strictly concave indicating a
higher-than-proportional return on a partial seismic upgrade. Considering the significant difference in terms of seismic hazard exposure, the curves are similar in shape and an upgrade of 30% allows for a reduction of 52% (Zurich) to 75% (Los Angeles) in expected loss compared to the maximum considered reduction, achievable with full seismic upgrade.

Figure 10 Retrofit Utility in terms of expected loss for the hazard environments considered in the present study: Zurich, Lisbon, Seattle, Reggio and Los Angeles.

In addition to different hazard environments, the sensitivity is tested for different fragility curves, damage state thresholds and damage-to-loss functions. As Appendix A.1 and B.1 illustrate the retrofit utility curve in terms of expected loss is insensitive to the used fragility curves and to the range of losses associated to certain damage grades. Finally, in Appendix C.1 the sensitivity has been tested for two additional parameter sets for the existing building (DSU 0%), one set with values lower than the building discussed here and one set with slightly higher values. In the former case, the shape of the utility curve becomes more concave, indicating an even higher return on partial seismic upgrade, whereas for a “stronger” initial building the opposite holds true.

Expected Shortfall

In contrast to expected loss, expected shortfall is a tail measure and therefore focuses on low probability but high loss events. At this stage, it should be noted that as discussed above the present study assumes renewable uncertainties and immediate repair after each event occurrence. The latter corresponds to a strong assumption that becomes less realistic for high loss events (e.g. complete damage of the building property).
Since the focus of tail risk measures like the expected shortfall, is mainly affected by high loss events, the authors evaluated the expected shortfall on the annual occurrence exceedance probability curve $\text{ES}_\alpha(\text{OEP}, 1\text{yr})$ (although it leads to lower loss estimates compared to considering the cumulative annual loss probability curve instead). The latter is illustrated in Figure 11 for a confidence level $\alpha$ equal to 98%, 99% and 99.9%, respectively. In simple terms the expected shortfall for a confidence level $\alpha$ in this study estimates the average expected losses inflicted by at least one event, given that the loss exceeded the loss amount with associated OEP of $(1-\alpha)\%$. For low annual exceedance probabilities, the latter is approximately equal to the annual frequency of exceedance. Thus, the three panels in Figure 11, can be interpreted as the expected average loss induced by at least one event given that the loss exceeded the loss with return period of 50, 100 and 1000 years.

**Figure 11** Retrofit utility in terms of expected shortfall evaluated on annual occurrence exceedance probability curve for all hazard environments considered, at a confidence level $\alpha$ of: 98%, 99% and 99.9%. The dotted black line illustrates a linear relationship between seismic upgrade and utility.

In Figure 11 the benefit of seismic upgrading corresponds to the slope of the curves illustrated. Thus, a concave shape corresponds to disproportional benefit in terms of the retrofit utility compared to the degree of seismic upgrading that gradually decreases as the degree of seismic upgrading increases (i.e. after approximately DSU=50% the increase of retrofit utility decreases gradually). Contrary, a convex shape corresponds to a gradual increase of retrofit utility the higher the degree of seismic upgrading (i.e. the highest increase of retrofit utility is received for DSU=100%).

The curves illustrated in Figure 11 lead to several observations: First, even if the interest is on average losses beyond the 50-yr loss, the retrofit utility curves follow a concave shape and thus
appeal for partial retrofit holds true, similar to the utility in terms of expected loss as explained above. Secondly, increasing the confidence level further and looking at average losses beyond the 100yr loss the curves start to change their shape towards a convex form, leading to the third main observation. The confidence level, above which the utility curve becomes convex depends significantly on the hazard environment. Finally, as a general trend it is observed that for confidence levels below 99%, curves seem to be clustered for all the considered hazard environments (i.e. the findings of the benefit of seismic upgrading seem to be independent of the hazard environment, if normalized as done for the retrofit utility defined in eq. 19. These observations call for additional interpretation provided in the next paragraph.

To gain additional insight, why utility curves for certain hazard environments start to change their shape at different confidence levels $\alpha$ it is worth to disaggregate the normalized utility measure into its underlying parameters, namely expected shortfall and retrofit benefit. Figure 12a illustrates ES(OEP,1yr) for the city of Seattle and Figure 12b shows the measure of retrofit benefit in terms of ES. The latter indicates a minimal exceedance level or maximum confidence level $\alpha_{\text{max}}$, up until which the benefit of a certain DSU is an increasing function of $1-\alpha$. For a DSU equal to 25%, $\alpha_{\text{max}}$ takes the value of 98.7%, which means it is located between Panel (a) and (b) in Figure 11 and therefore is closely connected to the limit above which the utility curve for Seattle starts to see significant changes in shape.

![Figure 12](image_url)

**Figure 12** For the hazard environment of Seattle, annual OEP and different degrees of seismic upgrade DSU: a) Expected Shortfall ES over exceedance level $(1 - \alpha)$; b) Retrofit Benefit RB in terms of ES over $(1 - \alpha)$. 
Moreover, Figure 12b illustrates that the maximum confidence level is nearly constant for all DSU illustrated, indicating that $\alpha_{\text{max}}$ for different DSU depend on the expected shortfall curve of the existing building (DSU 0%). Indeed, ES evaluated at $\alpha_{\text{max}}$ for the existing building refers to a point of curvature change in Figure 12a. Above the confidence level, at which expected shortfall predicts losses higher than approximately 80% of PBPV for the existing building, the appeal for partial benefit starts to vanish and only a high or full upgrade can provide any benefit. This serves as an explanation for the sensitivity on hazard environment stated in the paragraph above and illustrated in terms of $\alpha_{\text{max}}$ in Figure 13a. Since by assuming the existing building to be the same at all sites, the confidence level at which ES reaches the point of curvature change, depends on hazard environment. Figure 13b indicates that the expected shortfall predicted for the existing building at $\alpha_{\text{max}}$ is indeed nearly independent of the hazard environment.

![Figure 13](image)

**Figure 13** For the hazard environments considered in the present study: a) Minimal exceedance level or maximum confidence level $\alpha_{\text{max}}$ over DSU and b) Expected shortfall predicted for the existing building at the confidence level $\alpha_{\text{max}}$ illustrated in panel a.

In similar terms as described above for expected loss, the results illustrated in Figure 11 are tested using different fragility curves, damage state thresholds and damage-to-loss functions (Appendix A.2 and B.2). The curve shapes are only marginally influenced by any of these parameters. As expected, based on the findings above, an initial building with lower strength and/or ductility capacity (Appendix C.2), leads to a lower value for the maximal confidence level defined above. This is explained by the fact that for a “weaker” initial building the ex-
pected shortfall approaches collapse estimates at higher levels of exceedance and thus the retrofit utility curves start becoming convex at lower confidence levels. The following paragraph summarizes the main findings of Chapter 3 and addresses also limitations of the presented framework.

Existing buildings, constructed prior to the existence of modern seismic code guidelines expose their residents, tenants and owners to a high risk in case an earthquake occurs. However, the limited amount of economical capital available to decision-makers calls for an efficient allocation of resources. The present chapter is not proposing an optimal risk mitigation strategy, since such a choice depends largely on the risk attitude of the decision maker.

The results presented in this chapter indicate that for risk-neutral decision makers the relative return on a partial seismic upgrade is higher, compared to an upgrade to a fully code-compatible structure, independent of the considered hazard environments. The appeal for a partial upgrade holds true even for the tail measure of expected shortfall up to a certain confidence level. The latter depends on the hazard environment and on the structural characteristics of the existing building.

The presented framework allows for an efficient estimation of various risk measures and thus has potential to significantly facilitate decision-making processes by allowing engineers to provide adequate information to their clients. However, the results and the proposed framework rely on several simplifying assumptions, such as focusing on financial loss and disregarding casualty risk and employing a simplified SDoF to evaluate dynamic response of the building. Furthermore, the cost of seismic upgrade is not included in the present analysis, mainly because it is strongly influenced by the local construction market. Thus, a comparison of different hazard environments would not be possible.
3 Application of Risk Measures on a Single Building
4 APPLICATION OF RISK MEASURES ON A BUILDING PORTFOLIO

Advances in probabilistic seismic hazard analysis, structural dynamics and finite element methods together with increased computational power have allowed engineers to estimate seismic performance not only for individual buildings subjected to a limited number of earthquake ground motions (selected deterministically) but to perform a fully probabilistic seismic risk assessment for building portfolios and estimate financial impacts of catastrophic earthquake events for urban seismic prone regions. Most of these studies focus on an assessment of seismic risk posed to the existing building stock, not implicitly addressing performance of new structures, designed to fulfill modern seismic code prescriptions. But today’s new buildings become tomorrow’s existing stock and are worth examining [1].

As discussed in the introduction to this thesis, building code prescriptions and more specifically seismic code prescriptions represent a typical application of risk measures by separating a set of risks into a subset of “acceptable” and “unacceptable” risks. If an engineer applies the guideline correctly to design a new building, his client can be (to some extent) sure that the final structure belongs to the subset of acceptable risks, as defined by the guideline. To create resilient cities and urban areas, it is of utterly importance that these prescriptions can limit not only the risk to a single structure but also to a group of structures, a portfolio of buildings. Similar concerns are widely known in the financial industry, where a director board defines risk limits to each individual trader, e.g. VaR$_{99\%}$ must be lower than some specified amount. By defining such limits, the board has to be sure that these measures are capable and efficient to limit the risks on an aggregated company level, in the sense that summing up the individual risk leads to a conservative upper bound of the underlying risk posed to the company.

Using a virtual city as a testbed, the framework presented in the precedent chapter is modified to allow for an estimation of risk measures of aggregated portfolio losses. The first three sections of this part explain the geometry of the virtual region, the seismological conditions of the underlying hazard environment and the methodology to derive spatially aggregated losses. Subsequently, risk measures for the aggregated portfolio are compared to risk measures obtained for individual structures. Finally, the sensitivity of the results is tested employing different parameter sets, describing the assumed hazard environment and building portfolio characteristics.
4.1 Virtual City and Hazard Analysis

The virtual city is embedded in a seismic hazard environment consisting of two linear fault sources. The underlying geometry is illustrated in Figure 14a, together with a hazard map for the city, showing peak ground acceleration PGA with 10% probability of exceedance in 50 Years. The latter result is obtained using classic probabilistic seismic hazard analysis (PSHA), where the mean annual rate of exceedance of a certain intensity measure \( im \) on a given source \( n \) is obtained by:

\[
\lambda^{(n)}(im) = \lambda^{(n)}_{\text{min}} \int_{m_{\text{min}}}^{m_{\text{max}}} \int_{r_{\text{min}}}^{r_{\text{max}}} P(IM > im | M = m, R = r) f^{(n)}_{R|M}(r|m) f^{(n)}_{M}(m) dr dm,
\]

where \( \lambda^{(n)}_{\text{min}} \) describes the mean annual rate of exceeding a minimal magnitude \( m \), \( P(IM > im | M = m, R = r) \) describes the probability of exceedance of \( im \) given a certain magnitude \( m \) and source-to-site distance \( R \) and is typically derived by means of a ground motion prediction equation (GMPE), \( f^{(n)}_{R|M}(r|m) \) refers to the probability density function of source-to-site distance \( R \) given magnitude \( m \) and \( f^{(n)}_{M}(m) \) represents the pdf of the magnitude \( M \). The parameters and equations applied in the current study are explained in the following paragraphs, starting with magnitude recurrence relationship. Note that the following paragraphs are not intended to provide a full introduction to the broad field of PSHA, for such purposes the reader is referred to available literature as [19].

Figure 14 Geometry of the virtual city considered in the present study including the hazard map for peak ground acceleration (PGA) with an exceedance probability of 10% in 50 years.
Both faults are modelled as strike-slip faults and the moment magnitude scale is used. As a recurrence law, a bounded GB-law is applied, having a probability density function \( f_M(m) \) of:

\[
f_M(m) = \frac{\beta \exp[-\beta (m - m_{\text{min}})]}{1 - \exp[-\beta (m_{\text{max}} - m_{\text{min}})]}, \quad m_{\text{min}} \leq m \leq m_{\text{max}}
\]

(21)

where \( \beta \) describes the relative ratio of small and large magnitudes, \( m_{\text{min}} \) refers to the lower bound of magnitude considered in the present study and \( m_{\text{max}} \) represents the maximal magnitude, given by a full rupture of the fault. The latter is derived using the Wells & Coppersmith [20] relation connecting rupture size \( s \) and magnitude \( m \) for strike-slip faults:

\[
s = 10^{0.74m - 3.55}.
\]

(22)

The parameters used in the equations above are stated in Table 7 for both faults, together with the mean annual rate of exceeding the minimal magnitude \( \lambda_{\text{min}} = \lambda(M \geq m_{\text{min}}) \) and the distance \( R_{\text{min}} \) from a fault to the closest border of the virtual city.

Table 7 Geometrical and seismological parameters of the virtual fault system.

<table>
<thead>
<tr>
<th></th>
<th>( L )</th>
<th>( m_{\text{min}} )</th>
<th>( m_{\text{max}} )</th>
<th>( \beta )</th>
<th>( \lambda_{\text{min}} )</th>
<th>( R_{\text{min}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fault 1</td>
<td>20 km</td>
<td>4</td>
<td>6.5</td>
<td>2.3</td>
<td>0.2</td>
<td>10 km</td>
</tr>
<tr>
<td>Fault 2</td>
<td>45 km</td>
<td>4</td>
<td>7</td>
<td>2.3</td>
<td>0.1</td>
<td>10 km</td>
</tr>
</tbody>
</table>

As explained above, ground-motion-prediction equations (GMPE) are employed to find the probability distribution of the chosen IM conditional on a set of explanatory variables. They model ground motion intensities and related heterogeneity at a site \( p \) due to earthquake \( j \) as in the following equation:

\[
\log I M_{p, j} = \log I M_{p, j}(M, R, \Theta) + \eta_j + \epsilon_{p, j},
\]

(23)

where \( \log I M_{p, j}(M, R, \Theta) \) is the mean of the logarithms of IM conditioned on parameters such as magnitude \( M \), source-to-site distance \( R \), and others \( \Theta \); \( \eta_j \) denotes the inter-event residual which is a constant term for all sites in a given earthquake and represents a systematic deviation of the mean of the specific event and finally, \( \epsilon_{p, j} \) is the intra-event variability of ground motion. \( \epsilon_{p, j} \) and \( \eta_j \) are usually assumed to be independent random variables normally distributed with zero mean and standard deviation \( \sigma_{\text{intra}} \) and \( \sigma_{\text{inter}} \) respectively. Consequently \( \log I M_{p, j} \) is
modeled as a normal random variable with mean $\log IM_{pj}(M,R,\Theta)$ and standard deviation $\sigma_r^2 = \sigma_{inter}^2 + \sigma_{intra}^2$ [21]. Following GMPEs are implemented in the framework: (1) Bindi et al. (2011), described in [22], which is derived based on the Italian Accelerometric Archive (ITACA database) and (2) Boore and Atkinson (2008), described in [23], which is derived based on the NGA database. For both models the source-to-site distance $R$ is defined as the Joyner-Boore distance, referring to the closest distance from a site to the surface projection of a rupture. Based on basic geometrical relations and the rupture length (eq. 22), it is straightforward to derive the probability distribution of $R$ conditioned on magnitude $M$, $f_{R|M}(r|m)$.

In most structural engineering applications, e.g. the design of a new building or bridge, one is interested in the seismic hazard at the specific location of the project, which is derived by solving the integral in eq. 20. If the objective is on seismic risk assessment for a spatially distributed portfolio of assets, the situation becomes more complex. Consider a portfolio of $N_B$ buildings subjected to earthquake $j$, each value in a GMPE (eq. 23) is a vector with size $N_B$. It is well-known that observations of strong ground motion intensity measures resulting from the same earthquake $j$ tend to show correlations over distance [24], which means the intra-event residual $\epsilon_{pj}$ takes the form of a multivariate distribution, rather than univariate normal distribution as described above. This issue is explained further in the following section.

To perform hazard assessment for the portfolio of buildings considered in the virtual city, the joint probability density function for the logarithms of IMs at all locations is modeled with a multivariate normal distribution assuming that logarithms of IMs form a Gaussian random field. The covariance matrix of the spatially correlated intra-event residuals is assumed to be only a function of separation distance $h$ (under the hypothesis of second-order stationarity and isotropy of the Gaussian random field, i.e. that is the spatial heterogeneity is assumed to not depend on the direction considered). This type of correlation is normally referred to as spatial correlation and has been studied extensively in literature for numerous ground motion databases.
Considering spectral acceleration $S_{ae}(T)$ as the intensity measure of interest and to preserve consistency between the GMPE and the spatial correlation model, the proposal of [21] is applied to the Italian GMPE and [25] is applied for the GMPE, derived from the NGA database. Both models propose an exponential function form for the coefficient of correlation as:

$$\rho_h(T) = \exp\left[-\frac{3h}{b(T)} \right], \quad (24)$$

where $h$ refers to the separation distance (in km) between two sites and $b(T)$ is the range of spatial correlation (in km) after which, the correlation is considered as technically lost. The latter is defined as a function of vibration period $T$ and estimated as:

$$b(T) = a_1 + a_2 \cdot T,$$

where parameters $(a_1, a_2)$ are defined in [21] as $(8.6, 11.6)$ and in [25] as $(8.5, 17.2)$.

The spatial correlation models discussed above provide information on the correlation structure between residuals of spectral acceleration at the same spectral period $T$ at two different sites. This is appropriate if one considers a homogenous portfolio, where all buildings are assumed to possess similar structural characteristics, e.g. all buildings of a certain building typology. If the portfolio is heterogenous consisting of buildings, with considerable variation in their fundamental periods, the use of the models described above may not be sufficient. Because observations from ground motion records indicate, that the intra-event ground motion residual term for two different periods of spectral acceleration at the same site are cross-correlated, with the coefficient of correlation decreasing in accordance with an increase in the spacing between periods [26]. A model addressing this issue is referred to as a spatial cross-correlation model. It should be noted that this topic is still relatively new and only recently got more attraction in within the earthquake engineering community. For a comprehensive overview on existing modelling approaches the interested reader is referred to [26].

For the purposes of this study, the choice of a suitable correlation model should depend mainly on two issues: First, the correlation model applied has be consistent with the implemented GMPE, meaning both should be based on the same database. Secondly, whether to include cross-correlation depends on the heterogeneity of the building portfolio. The latter is described in the following paragraph.
4.2 Description of Building Portfolio

As illustrated in Figure 14, the virtual city spreads over an area of 10x10 km. This area is populated with \( N_B \) buildings arranged in an orthogonal grid, where the grid spacing is denoted as \( d_g \). Furthermore, it is assumed that all buildings can be described by the same equivalent SDoF system. The corresponding capacity curve is defined with the yielding acceleration capacity \( a_y \), the fundamental period of Vibration \( T \) and the ductility capacity \( \mu_C \), in similar terms as described in Chapter 3. Especially the latter assumption is a significant simplification of a real city and its built inventory, but allows to neglect spatial cross-correlation as discussed in Chapter 4.1. This is preferred for the following reason: The few cross-correlation models available in literature [26], follow different assumptions in their derivation. In the authors’ opinion, the research community still not found a consensus on a consistent modelling approach. Note however, that the framework code has been developed to incorporate cross-correlation at a later stage and therefore is applicable for heterogenous portfolios as well.

| Table 8 Geometrical and structural parameters of the base-portfolio of buildings. |
|---|---|---|
| **Number of Buildings** | N\( B \) | 36 |
| **Grid Spacing** | \( d_g \) [km] | 2 |
| **Avg. Inter-Building Distance** | \( d_{avg} \) [km] | |
| **Structural Characteristics** | \( a_y \) [g] | 0.13 |
| | \( \mu_C \) [-] | 6 |
| | \( T \) [sec] | 0.05 |

The structural properties, discussed above, are defined using a similar procedure as discussed in Chapter 3.5. Assumed is that the considered buildings have the same yield displacement \( d_y \), thus assuming similar geometry and yielding material characteristics as in the referred chapter. However, the focus of this chapter is not addressing the benefit of a certain retrofit, but to assess the effect of using non-coherent risk measures on a code-compliant building stock. Thus, the yield strength \( a_y \) and the period \( T \) is defined using eq.16, with the difference that the seismic demand is not provided by an online database, but rather has been derived by PSHA for the site located at the centroid the portfolio. The properties of the base portfolio are provided in Table 8, the prefix “base” is added, since these properties will be modified in a later stage for
sensitivity testing purposes. The parameter $d_{avg}$ refers to the average inter-building distance of the portfolio. This parameter will be of interest in Chapter 4.4, when we assess the influence of the portfolio spatial footprint on results.

The next chapter provides a brief overview of the implemented simulation procedure and explains the process to derive seismic portfolio losses and corresponding risk measures.

### 4.3 Evaluation of aggregated Portfolio Losses

To obtain spatially aggregated losses of the whole building portfolio, the correlation structure between intensity measures must be considered. Thus, the computational framework is based on simulating gaussian random fields, rather than convolution of conditional probability distributions as described in Chapter 3. However, the individual parts of the referred chapter remain the same and are also applicable to the present study. Figure 15 illustrates the sampling chain leading to one realization $l_p$ of portfolio loss random variable $L_p$. The individual steps will briefly be explained in the following paragraphs. For a more comprehensive introduction into computational aspects related to modelling seismic risk of spatially distributed systems the reader is referred to [27].

![Figure 15](image.png)

**Figure 15** Monte-Carlo based sampling approach illustrated for one realization. The superscript (i) refers to building $i$ and its location.
In a first step, a magnitude $m$ is sampled from the probability distribution of magnitudes $f_M$, based on the GB-law provided in eq. 21. Then, a fault $k$ is sampled weighted with corresponding mean annual frequencies of exceeding the minimal magnitude $\lambda_{\text{min}}$. Conditional on the rupture length $s$ (eq. 22) and the length of fault $k$, a rupture location is randomly sampled. For each building $i$ and conditional on rupture location, the Joyner-Boore distance $r_{JB}^{(i)}$ is evaluated based on geometrical relations and the mean intensity measure is calculated using a GMPE of interest. Next, inter-event and intra-event residuals are sampled, weighted with univariate and multivariate normal distribution, as explained in Chapter 4.1. The total intensity measure $s_{ae}^{(i)}$ is estimated by combining residuals and corresponding mean value (eq. 23). Based on the fragility curves $P(DG_k|IM)$, provided in eq. 8, a damage grade is sampled for each individual building. The loss for each building $l^{(i)}$ is obtained by sampling from Beta($L|DG_k; \alpha, \beta$), the damage-to-loss function introduced in Chapter 3. Summing up realizations of individual loss $l^{(i)}$ over all buildings within the portfolio, results in a single realization of the aggregated portfolio loss $l_P$.

This procedure is repeated $N_{\text{sim}}$ times (typically $10^5$ to $10^6$) leading to a sufficiently large sample size. The CCDF of portfolio severity $S_p$ can be approximated by:

$$\hat{P}(S_p > s) = \frac{1}{N_{\text{sim}}} \sum_{j=1}^{N_{\text{sim}}} I(l_{p,j} > s),$$

where $I$ is the indicator function and equals 1 if $l_{p}$ is larger than some threshold $s$ and 0 otherwise. In similar terms eq. 25 can be applied to estimate the CCDF of severity of each individual building. The latter approximates the severity, as it is defined in Chapter 3.

Having estimated the severities of each individual building and of the complete portfolio, the procedures to derive occurrence exceedance probability (OEP), aggregate exceedance probability (AEP) and mean annual frequency of losses $\nu(L)$ are equivalent to the derivations explained thoroughly in Chapter 3. Thus, it is possible to derive all risk measures defined in Chapter 2 and illustrated in Chapter 3.4, either based on the loss distribution of the single building or for the building portfolio. Figure 16 illustrates Value-at-Risk VaR$_\alpha$ and Expected Shortfall ES$_\alpha$, both estimated based on annual OEP, as a function of exceedance level $(1 - \alpha)$ for two single buildings.
For two buildings in the virtual city: a) Geometrical location and b) Value-at-Risk over exceedance level \((1-\alpha)\) in terms of present building property value (PBPV), evaluated based on the occurrence exceedance probability in one year.

Denoting \(L^{(i)}\) as the loss random variable of single building \(i\) and \(L_P\) as the aggregated loss random variable of the portfolio. The concern raised in the introduction to this chapter, whether summing up individual risk metrics lead to a conservative upper bound for the aggregated risk, can mathematically formulated as:

\[
\rho(L_P) = \rho(L^{(1)} + \cdots + L^{(i)} + \cdots + L^{(NB)}) \leq \rho(L^{(1)}) + \cdots + \rho(L^{(i)}) + \cdots + \rho(L^{(NB)}) \quad (26)
\]

The latter is identical to the subadditivity property, that a coherent risk measure should possess (as defined in Chapter 2.2). Meaning, if the inequality holds true, the risk measure is sub-additive and thus also presents a conservative upper bound. If eq. 26 is not fulfilled, the risk measure is called supper-additive. As already mentioned in Chapter 2, quantile-based risk measure in general do not fulfill the property of subadditivity. This, however, does not mean that they are non-conservative in all possible cases, but only that they are in some of them. The following paragraph aims to illustrate the latter and connects such cases to the underlying seismic hazard.
4.4 Effect of Subadditivity in estimating Portfolio Losses

As discussed in Chapter 2 only expected shortfall is sub additive. In the context of earthquake engineering lack of this property has significant implications, when portfolio losses need to be estimated based on risk measures defined for an individual building. If subadditivity is satisfied for a risk measure, based on the plots illustrated in Figure 16 for each building of the virtual region one could sum the risk measure values for a certain confidence level and estimate an upper bound for the portfolio losses for the area of interest. The latter property is of high-importance at urban level since lack of it could result in counter-intuitive performance where portfolio risk measure values could exceed the sum of individual ones. In simple terms, although based on Performance Based Earthquake Design, all buildings contained in the area above are designed to experience only slight damage for earthquake events below a certain threshold.

Figure 17a illustrates the discussed phenomenon for Value-at-Risk, whereas Figure 17b shows that expected shortfall indeed is a coherent risk measure. Summing ES values for all confidence levels of individual buildings always yields higher value than the ES value of portfolio losses. The latter property leads to conservative results without the need to simulate the entire building portfolio but allows us to estimate portfolio losses only based on individual values of ES of the considered buildings of the portfolio.

![Figure 17](image)

**Figure 17** Risk Measures for portfolio loss and sum of risk measures of individual buildings (SingleSum) in the virtual city as a function of exceedance level: a) Value-at-Risk ; b) Expected Shortfall, both in terms of present portfolio property value (PPPV) and evaluated based on annual occurrence exceedance probability curve.
The VaR-curve in Figure 17a calls for additional interpretation, since this is an unintended effect of using non-coherent risk measures and could lead to non-conservative estimation of portfolio losses approximated by summing risk measures values of individual buildings. As discussed before Loss-at-Frequency, like VaR, is a quantile based risk measure and thus also suffers from lack of subadditivity, which is confirmed in Figure 18a. The point where the portfolio curve and the SingleSum curve crosses is defined as CP and illustrated in Figure 18b, with associated loss indicated by the vertical dashed line. In the next paragraph, this curve is separated in two parts, to characterize earthquake events leading to non-conservative (superadditive) estimates and to conservative (subadditive) estimates.

Figure 18 a) Loss-at-Frequency in terms of present portfolio property value (PPPV) for the aggregated portfolio and as the sum of each individual building (SingleSum) and b) Mean annual frequency of exceedance of a certain portfolio loss and illustration of point CP, separating sub- and superadditive losses.

To gain additional insight in the underlying seismic intensities connected with non-conservative loss estimates, portfolio losses are separated in two parts as described below:

$$\text{Case 1: } 0 < L_p < L_p(\text{CP}) \quad | \quad \text{Case 2: } L_p \geq L_p(\text{CP}) , \tag{27}$$

where Case 1 refers to all portfolio losses, for which the SingleSum predicts non-conservative estimates and Case 2 addresses the portfolio losses for which the SingleSum is a conservative upper bound. Figure 19b illustrates the distribution of mean spectral acceleration conditional on Case 1 and 2, respectively. Note that the mean spectral acceleration refers to the mean intensity over all site locations for a specific event. To facilitate assessment Figure 19a shows
the probability of exceedance in 50 years for mean spectral acceleration with indicated thresholds, corresponding to frequent (50% in 30 years or 50/30), occasional (50/50), rare (10/50) and very rare (2/50) seismic events. These thresholds are transferred to panel (b) and indicate that for the employed virtual city and hazard environment, spectral accelerations of frequent and occasional events lead to portfolio losses, for which the sum of individual measures is not conservative.

![Figure 19](image)

**Figure 19** a) Probability of exceedance in 50 years of mean spectral acceleration at fundamental period of vibration and indicated probability levels (dashed lines); b) Distribution of mean spectral acceleration conditional on case 1 (non-conservative) and case 2 (conservative), respectively. Vertical dashed lines correspond to intensities at probability levels indicated in panel a.

Since building codes and earthquake assessments of tomorrow need not only to avoid collapse for very rare earthquake events but should create resilient communities that are also able to recover fast after more frequent earthquakes, it is important the risk measure values employed to quantify seismic risk to be sub additive and thus to define an upper bound when one wants to estimate the portfolio performance based on the performance of an individual building. In the following paragraph, the sensitivity of the stated results is tested for different underlying parameters, such as building properties, number of buildings and associated population density, as well as minimal building to fault distance.
4.5 Sensitivity Analysis

To assess the influence of geometrical and structural parameters describing the building portfolio and its hazard environment, four different sets, listed in Table 9, are considered for sensitivity testing and compared to the results of the base case discussed above. The first set assumes a portfolio consisting of buildings with higher strength and stiffness compared to the base case. The parameters of sub-additivity threshold $CP$ indicate that portfolio loss at $CP$ is not affected, whereas associated frequency (or Return Period) is influenced significantly, corresponding to a left shift of $CP$ illustrated in Figure 18a above.

Table 9 Five parameter sets with associated sub-additivity thresholds considered for sensitivity analysis.

<table>
<thead>
<tr>
<th></th>
<th>Base</th>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
<th>Set 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Buildings</td>
<td>$N_B$</td>
<td>36</td>
<td>36</td>
<td>441</td>
<td>36</td>
</tr>
<tr>
<td>Grid Spacing</td>
<td>$d_g$</td>
<td>2</td>
<td>2</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>Avg. Inter-Building Distance</td>
<td>$d_{avg}$</td>
<td>6.1</td>
<td>6.1</td>
<td>5.4</td>
<td>0.31</td>
</tr>
<tr>
<td>Structural Characteristics</td>
<td>$a_y$</td>
<td>0.13</td>
<td>0.21</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>$\mu_C$</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Min. Fault to Site Distance</td>
<td>$T$</td>
<td>0.38</td>
<td>0.29</td>
<td>0.38</td>
<td>0.38</td>
</tr>
<tr>
<td>Area Dimensions</td>
<td>$R_{min}$</td>
<td>10 x 10</td>
<td>10 x 10</td>
<td>10 x 10</td>
<td>1 x 1</td>
</tr>
<tr>
<td>Portfolio Loss at CP</td>
<td>$L_p(CP)$</td>
<td>0.05</td>
<td>0.04</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Return Period at CP</td>
<td>$RP(CP)$</td>
<td>190</td>
<td>290</td>
<td>190</td>
<td>200</td>
</tr>
</tbody>
</table>

The second and third set address the influence of portfolio density, more specifically in set 2 the number of buildings are increased from 36 to 441, keeping the area size of the virtual city constant, whereas in set 3 the latter is substantially reduced and the number of buildings are the same as in the base case. Results listed in Table 9 show that the sub-additivity threshold $CP$ is not significantly affected by the increase of density for the fault system and hazard environment considered in the present study. Note, however, increasing density (and therefore also the correlation among seismic intensities at building locations) lead to higher aggregated portfolio losses in the tail of the distribution. Set 4 aims to address influence of the hazard environment, by increasing the minimal source-to-site location from 10 to 20 km. Observe, that structural characteristics of the building reflect the change in hazard environment. Similar to Set 2,
the loss at CP is not strongly influenced, whereas associated return period is increased significantly. Thus, for a portfolio of buildings with higher strength or for a lower hazard environment, more severe (and less frequent) seismic intensities are required to produce losses higher than the sub-additivity threshold. Appendix D illustrates portfolio and SingleSum loss-at-frequency curves (as Figure 18a) and the distribution of spectral accelerations conditional to Case 1 and 2, as defined above and illustrated in Figure 19b for the base case.

Finally, it should be noted that all results presented in this chapter are derived using the same distribution of magnitudes (e.g. G-R law) and using one ground-motion prediction equation [22], with associated spatial correlation model [21]. Furthermore, only two linear seismic sources with fixed length are considered in the present study. The influence of more complex fault systems or area sources on the sub-additivity threshold are not examined.
5 CONCLUSION

Fairly recent catastrophic earthquakes in 2011 in New Zealand and Japan have led to total economic losses of $15-20 billion and $300-400 billion respectively. As discussed by Yoshi-kawa and Goda [28], such events impose increased financial stress on governments/municipalities as well as insurers underwriting earthquake insurance policies revealing limitations of conventional financial risk-management instruments/tools, such as public support and earthquake insurance. To mitigate seismic risk, financial risk transfer on its own is not sufficient. Only adequate seismic design or retrofit can limit damage physically at first place. In the current study, financial risk measures are applied in a context of seismic retrofit of existing buildings, as well as to explore possible limitations of current seismic codes addressing the design of new buildings.

Existing buildings, lacking adequate seismic detailing and having relatively low lateral strength expose their residents, tenants and owners to a high risk in case an earthquake occurs. Nevertheless, implementing seismic upgrade is often perceived as overly expensive by building owners and/or decision makers. One reason, amongst others, is that many engineers still believe that the purpose of seismic upgrading is to achieve conformance with the lateral force requirement of modern building codes for new buildings [8]. As the current study shows, already partial seismic upgrading can be an effective measure to reduce risk substantially, independent of the hazard environment. By applying the tail measure expected shortfall, a threshold in terms of confidence level is found, indicating an upper bound for the “effectiveness” of partial seismic upgrading. If the desired level of confidence is higher than this threshold, only a full or high seismic upgrade can reduce risk. The proposed framework allows engineers to provide adequate information to their clients in a preliminary design phase to facilitate decision-making.

Risks related to natural hazards are most efficiently prevented by well-defined land-use planning. Considering the uncertainty associated with location and size of future earthquakes, this is rather difficult, if not impossible to achieve. Another highly efficient measure, from a governmental perspective, is to provide guidelines for the design of new buildings, that can limit seismic risk posed to the overall society. Building codes of tomorrow need to provide guide-
lines for designing communities that would remain resilient after a catastrophic event. Although the main focus of building codes today is related to designing individual buildings that are collapse-resistant for a rare earthquake event (i.e. for an earthquake with probability of exceedance 2% in 50 years), codes should also quantify and limit losses for more frequent earthquake events which has been one of the aims of performance based earthquake design. To perform so though, as this study shows, it is important to re-think how seismic performance should be quantified in terms of monetary losses. The results of the study verify that quantile-based risk measures are not sub-additive for earthquake intensities with probability of exceedance higher than the 10% in 50 years earthquake scenario (Design Basis Earthquake) and thus could lead to underestimation of financial losses for events with higher frequency of occurrence.

The findings of this study have significant implications in the context of Performance Based Earthquake Design, where different levels of ground motion intensity are examined in relation to seismic performance to meet the decision maker’s expectations. To illustrate the aforementioned issue, a relatively simple virtual hazard environment and building portfolio serves as a testbed. It is not the purpose of the current work to assess the degree of non-conservatism for a real case. To what extent a quantile-based measure is non-conservative, depends on the system under consideration and might be different for a dense road network in an urban environment and for a portfolio of single residential units, distributed over a large, sparsely populated, rural area. Thus, this thesis hopefully opens the field and highlights the need for a lot of future studies.

Acknowledgment:

During the past months, I had the opportunity to work on this interesting project. I would like to thank my advisors, Dr. P. Galanis and Dr. M. Broccardo, as well as my supervisor Prof. Dr. Bozidar Stojadinovic, for all the useful inputs and assistance, they provided me and especially for sharing their ideas with me. In my opinion those people are outstanding examples, how critical thinking should be implemented in a technical environment like ETH, combining high expertise in their field while continuously challenging current best practices.
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APPENDIX

A  Sensitivity of retrofit utility curves – Regression Equation

Sensitivity of Retrofit Utility Curves considering different regression equations available in literature to estimate the structural response of a SDoF system:

**Ruiz-Garcia and Miranda:** The stated authors propose equations for the central tendency and the dispersion of the inelastic displacement ratio $C_R$, which are based on statistical results computed from dynamic response of elastoplastic SDoF systems having a wide range of periods of vibration and lateral strength when subjected to a relatively large suite of ground motions [29].

The mathematical expression to estimate the 50% fractile is given by:

$$C_R = \begin{cases} 
1, & R \leq 1 \\
1 + \frac{1}{\Theta_1 \cdot T^{\Theta_2}}, & R > 1 
\end{cases}$$  \hfill (28)

where $\Theta_1 = 79.12$ and $\Theta_2 = 1.98$ are parameter estimates obtained through non-linear regression analysis, provided by the authors and $R$ refers to the lateral strength reduction factor defined before. The lognormal standard deviation of is estimated as:

$$\sigma_{\ln C_R} = \begin{cases} 
0, & R \leq 1 \\
\frac{1}{\beta_1} + \frac{1}{\beta_2 \cdot (R + 1)} \cdot \beta_3 \cdot (1 - e^{-\beta_4 (R - 1)}) & R > 1 
\end{cases}$$  \hfill (29)

where $\beta_1 = 5.876$, $\beta_2 = 11.749$, $\beta_3 = 1.957$ and $\beta_4 = 0.739$ are estimates provided in [29].

These equations allow to evaluate expressions for $\mu_{x\%} = R \cdot C_{R,x\%}$, where $x$ corresponds to the 16,50 and 84% fractile of $\mu$ given $R$ (see Figure 3a). These fractile curves are subsequently evaluated for $R_{x\%}(\mu_{\text{lim},k})$, which is possible if fractile quantities are used, and would not be correct if mean factors are applied [30]. The procedure to evaluate the median capacity of each damage grade in units of IM and the total dispersion is identical to the procedure described above.

**Krawinkler-Nassar:** The stated authors provide following regression expression to estimate the lateral strength reduction factor $R$ given ductility demand $\mu$ [31]:

$$R = \begin{cases} 
\mu, & \mu \leq 1 \\
\left[\frac{\mu}{c\cdot(\mu - 1)} + 1\right]^{1/c} & R > 1 
\end{cases}$$  \hfill (30)
where $c = T/(1 + T) + 0.42/T$ for elastoplastic SDOF systems. Since no measure of dispersion is provided in the stated reference, a total dispersion of $\beta = 0.6$ has been applied.

A.1 Retrofit Utility in terms of EL(AEP)

![Graphs showing Retrofit Utility in terms of EL(AEP)](image)

**Figure A.1** Retrofit Utility in terms of expected cumulative losses for all hazard environments considered in the present study derived using regression models of SPO2IDA, Ruiz-Garcia and Miranda, and Krawinkler and Nassar.
A.2 Retrofit Utility in terms of EL(AEP)

![Graphs of Retrofit Utility in terms of EL(AEP) for different confidence levels](image)

**Figure A.2** Retrofit Utility in terms of $ES_{\alpha}(\text{OEP}, 1\text{yr})$ for a confidence level of $\alpha$ equal to 95%, 98%, 99% and 99.9%, respectively. Results are derived using three different regression models to estimate structural response, as defined above, and for all hazard environments considered in the present study. The dotted black line illustrates a linear relationship between seismic upgrade and utility.
APPENDIX B

Sensitivity of Retrofit Utility Curves considering different damage state thresholds and damage-to-loss functions available in literature:

**Kappos et al.** The stated author and his co-workers propose in [32] and [33] a different set of damage state thresholds compared to the ones used in the current study. Especially they separate damage grade 4 and 5 directly on the capacity curve of the SDoF system. Furthermore, they provide mean damage ratios and associated damage ranges for the 5 damage states indicated in Table B.1.

**Table B.1** Damage State Thresholds, Mean damage ratios and associated damage ranges after [32] and [33].

<table>
<thead>
<tr>
<th>Damage Grade</th>
<th>Damage Description</th>
<th>$S_{d,k}$</th>
<th>MDR</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Slight</td>
<td>$0.7d_y$</td>
<td>2%</td>
<td>0-4%</td>
</tr>
<tr>
<td>2</td>
<td>Moderate</td>
<td>$d_y$</td>
<td>12%</td>
<td>4-20%</td>
</tr>
<tr>
<td>3</td>
<td>Extensive</td>
<td>$2d_y$</td>
<td>30%</td>
<td>20-40%</td>
</tr>
<tr>
<td>4</td>
<td>Heavy</td>
<td>$0.7d_u$</td>
<td>55%</td>
<td>40-70%</td>
</tr>
<tr>
<td>5</td>
<td>Complete</td>
<td>$d_u$</td>
<td>85%</td>
<td>70-100%</td>
</tr>
</tbody>
</table>

**B.1 Retrofit Utility in terms of EL(AEP,1yr)**

**Figure B.1** Retrofit Utility in terms of expected cumulative losses for all hazard environments considered in the present study derived using damage state thresholds and damage-to-loss equations of Lagomarsino and Giovanazzi [11] and Kappos et al. [32].
B.2 Retrofit Utility in terms of ES(OEP,1yr)

Figure B.2 Retrofit Utility in terms of $ES_d(OEP,1yr)$ for a confidence level of $\alpha$ equal to 95%, 98%, 99% and 99.9%, respectively. Results are derived using two different sets of damage state thresholds and damage to loss functions, as defined above, and for all hazard environments considered in the present study. The dotted black line illustrates a linear relationship between seismic upgrade and utility.
APPENDIX C

Sensitivity of Retrofit Utility Curves considering a higher and lower set of structural characteristics for the existing building as indicated in Table C.1. The high-code building remains unchanged, since it depends only on hazard environment.

**Table C.1** Structural characteristics for the base, low and high set of existing building.

<table>
<thead>
<tr>
<th></th>
<th>Existing Building (DSU 0%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_y$ (g)</td>
</tr>
<tr>
<td>Base Set</td>
<td>0.05</td>
</tr>
<tr>
<td>Low Set</td>
<td>0.03</td>
</tr>
<tr>
<td>High Set</td>
<td>0.07</td>
</tr>
</tbody>
</table>

**C.1 Retrofit Utility in terms of EL(AEP,1yr)**

![Graphs of Retrofit Utility](Zurich.png, Lisbon.png, Seattle.png, Reggio.png, LosAngeles.png)

**Figure C.1** Retrofit Utility in terms of expected cumulative losses for all hazard environments considered in the present study based on three different sets of structural parameters of the existing building (DSU 0%).
C.2 Retrofit Utility in terms of ES(OEP,1yr)

<table>
<thead>
<tr>
<th>Case</th>
<th>$a_Y$</th>
<th>$\mu_C$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>0.05g</td>
<td>3</td>
<td>0.6s</td>
</tr>
<tr>
<td>Low Case</td>
<td>0.03g</td>
<td>2.5</td>
<td>0.6s</td>
</tr>
<tr>
<td>High Case</td>
<td>0.07g</td>
<td>3.5</td>
<td>0.6s</td>
</tr>
</tbody>
</table>

Figure C.2 Retrofit Utility in terms of $ES_{\alpha}(OEP,1yr)$ for a confidence level of $\alpha$ equal to 95%, 98%, 99% and 99.9%, respectively. Results are derived using three different sets of structural parameters for the existing building DSU=0% and for all hazard environments considered in the present study. The dotted black line illustrates a linear relationship between seismic upgrade and utility.
APPENDIX D

Sensitivity testing of subadditivity threshold for different sets of geometrical parameters and structural characteristics as indicated in Table 9 and illustrated again below.

<table>
<thead>
<tr>
<th></th>
<th>Base</th>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
<th>Set 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of Buildings</strong></td>
<td>$N_B$</td>
<td>36</td>
<td>36</td>
<td>441</td>
<td>36</td>
</tr>
<tr>
<td><strong>Grid Spacing</strong></td>
<td>$d_g$</td>
<td>[km]</td>
<td>2</td>
<td>2</td>
<td><strong>0.5</strong></td>
</tr>
<tr>
<td><strong>Avg. Inter-Building Distance</strong></td>
<td>$d_{avg}$</td>
<td>[km]</td>
<td>6.1</td>
<td>6.1</td>
<td><strong>5.4</strong></td>
</tr>
<tr>
<td><strong>Structural Characteristics</strong></td>
<td>$a_y$</td>
<td>[g]</td>
<td>0.13</td>
<td>0.21</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>$\mu_C$</td>
<td>[-]</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>$T$</td>
<td>[sec]</td>
<td>0.38</td>
<td><strong>0.29</strong></td>
<td>0.38</td>
</tr>
<tr>
<td><strong>Min. Fault to Site Distance</strong></td>
<td>$R_{min}$</td>
<td>[km]</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td><strong>Area Dimensions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>[km]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>10 x 10</td>
<td></td>
<td>10 x 10</td>
<td>10 x 10</td>
</tr>
<tr>
<td><strong>Portfolio Loss at CP</strong></td>
<td>$L_p$(CP)</td>
<td>[PPPV]</td>
<td>0.05</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td><strong>Return Period at CP</strong></td>
<td>$RP$(CP)</td>
<td>[years]</td>
<td>190</td>
<td>290</td>
<td>190</td>
</tr>
</tbody>
</table>

First the risk measure loss-at-frequency estimated on aggregated loss curve and as the sum of the individual risk metrics is illustrated on the next page. Subsequently the conditional probability of mean spectral acceleration, leading to super-additive (Case 1) and sub-additive (Case 2) portfolio losses is shown.
D.1 Loss-at-Frequency: Portfolio and SingleSum

**Figure D.1** Loss-at-Frequency estimated for the aggregated portfolio and as a sum of single risk metrics of the individual buildings in the considered virtual cities using different parameter sets as defined in Chapter 4.4.
D.2 Distribution of Magnitude conditional on Case 1 and 2

Figure D.2 Distribution of mean spectral acceleration conditional on case 1 (non-conservative) and case 2 (conservative) using different parameter sets as defined in Chapter 4.4.