SAR Tomography as an Add-On to Persistent Scatterer Interferometry for Improved Deformation Coverage

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SAR TOMOGRAPHY AS AN ADD-ON TO PERSISTENT SCATTERER INTERFEROMETRY FOR IMPROVED DEFORMATION COVERAGE

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Coherent scatterers obtained with differential SAR tomographic inversion of an interferometric data stack comprising 50 TerraSAR-X stripmap acquisitions over the city of Barcelona. The color-coding represents the estimated LOS deformation velocity between -10 mm/yr (subsidence, in red) and 10 mm/yr (uplift, in blue).
In the loving memory of my late father, with prayers for my mother's health and long life.
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Persistent scatterer interferometry (PSI) is a synthetic aperture radar (SAR) signal processing technique for the measurement of land surface deformation. As a conceptual derivative of differential interferometry, it strives to extract the interferometric phase variation induced by the line-of-sight component of the deformation. PSI limits the interferometric analysis to the so-called persistent (or synonymously coherent) scatterers (PS). These are single dominant scatterers exhibiting point-like, quasi-deterministic behavior. On the one hand, it implies reduced susceptibility to temporal and geometric decorrelation, but on the other hand, it incurs the limitation that range-azimuth resolution cells containing multiple scatterers are rejected in the PSI processing, even if they are individually coherent. Side-looking geometry of SAR sensors results in frequent layovers, whereby multiple scatterers situated at different elevations fall in the same resolution cell. Consequently, deformation coverage with PSI processing may remain limited in layover-affected areas.

SAR tomography is a means to alleviate the aforementioned limitation. It allows 3-D reconstruction of the scene reflectivity – a feature that offers the potential to resolve the layover problem. The coherent scatterers that are interfering in the same cell can be separated along the elevation. Additionally, differential SAR tomographic methods allow a joint spatio-temporal inversion of the coherent scatterers in layover, i.e., the position along the elevation axis as well as the deformation velocity of the interfering scatterers are simultaneously estimated. Therefore, differential SAR tomography can be used as an add-on to PSI techniques to improve deformation coverage in layover-affected areas.

This dissertation provides a comprehensive assessment of the utility offered by SAR tomography as an add-on to PSI. Several aspects of a tomographic processing framework, such as different phase models for tomography, phase calibration of the interferometric stack, statistical detection of coherent scatterers, etc., need to be investigated. To this end, three core investigations have been performed. In each case, a prior PSI solution has been used as a starting point. It serves not only as a reference to compare with, but is also shown to be a natural precursor to tomographic processing.

An interferometric data stack comprising 50 TerraSAR-X stripmap-mode acquisitions over an urban zone in the city of Barcelona, Spain, has been used in the first investigation. The phase models for classical SAR tomography (3-D SAR), differential tomography with the assumption of linear deformation over time, and the one further extended to simultaneously model thermal expansion, are compared against each other with respect to their suitability in resolving layovers. The results confirm that modeling thermal expansion of the scatterers, in addition to linear deformation and elevation, is indeed critical for effective layover separations, especially in the case of high-rise buildings. The quality of the scatterers obtained with tomography has been evaluated in terms of the dispersion of the residual phase and compared against the quality of the PS identified in the prior PSI processing. The results show a trade-off between the quantity and the quality of the scatterers.
The second investigation focuses on the problem of phase calibration for a potential application of SAR tomography in mountainous regions. It is a case study that assesses a regression-kriging approach to functionally model height-dependent atmospheric phase variations and lateral phase trends, and consider the turbulent mixing effects in a stochastic sense. The study has been performed on a stack comprising 32 Cosmo-SkyMed acquisitions over Matter Valley in the Swiss Alps. Phase corrections with the kriging approach extend the deformation coverage to parts of a mountainside (in layover) where no PS were identified in the prior PSI processing. However, a very few double scatterers are detected on the whole.

The third investigation explores how to perform scatterer detection for tomography extending the same quality considerations as used in the prior PSI processing. The outcome of this work is a detection strategy whereby quality parameters (in terms of the statistics of the phase residue or ensemble coherence) are used to determine the thresholds for hypothesis testing. The detection strategy is tested on the same data stack as for the first investigation to detect single and double scatterers in an urban area. An empirical analysis of the probability of false alarm is also provided.

As a whole, this dissertation covers several aspects that collectively highlight how the synergistic use of PSI and tomography can lead to improved deformation coverage.
L’interferometria a scatteratori persistenti (PSI) è un metodo di misura degli spostamenti del suolo basato sul radar ad apertura sintetica (SAR). La tecnica PSI applica l’analisi interferometrica a bersagli (scatteratori) persistenti; ovvero che si comportano come bersagli puntiformi. Questa restrizione fa in modo che la loro fase sia meno suscettibile ad effetti di decorrelazione temporale e spaziale, con lo svantaggio che tutte le celle di risoluzione composte da più bersagli coerenti vengono scartate dall’analisi PSI. Questo può ridurre la copertura spaziale delle stime di deformazione ottenute con la tecnica PS. La presenza di diversi bersagli in una cella di risoluzione è comune nei sistemi ad apertura sintetica a causa dello scavalcamento (layover) che si verifica quando oggetti a diverse altezze si trovano alla stessa distanza dal radar; in questo caso essi cadranno nello stesso pixel, costituendo un bersaglio multiplo.

La Tomografia SAR permette di alleviare questo problema tramite la ricostruzione tridimensionale della riflettività della scena; separando in elevazione i diversi bersagli che a causa del layover si troverebbero nella stessa cella di risoluzione. Alcune tecniche tomografiche consentono inoltre una ricostruzione spazio-temporale della scena, dove la posizione dei bersagli è stimata in congiunzione al loro tasso di deformazione. Grazie a questi metodi la tomografia SAR può essere usata in combinazione con tecniche PS per migliorare la copertura geografica delle stime di spostamento del terreno nelle regioni soggette al layover.

Questa dissertazione vuole essere una valutazione olistica dell’utilità della tomografia SAR combinata con l’interferometria a scatteratori persistenti. A questo fine è necessario considerare diversi aspetti tra i quali diversi modelli di fase per la ricostruzione tomografica, la calibrazione in fase dello stack interferometrico e la rilevazione statistica degli scatteratori coerenti. Essi sono stati analizzati in tre studi separati, tutti basati su una soluzione PSI, utilizzata come soluzione iniziale e quale paragone per la qualità dei risultati.

Lo scopo del primo studio era confrontare la capacità di diversi modelli tomografici a risolvere layover. In esso vengono comparati il modello classico di tomografia SAR, che permette solamente una ricostruzione tridimensionale, con due modelli più sofisticati. Il primo di questi consente di stimare, in aggiunta alla posizione in elevazione dei bersagli, il loro tasso di deformazione lineare. Il secondo ne è un’estensione che integra un modello di deformazione termica. Il raffronto si basa su uno stack interferometrico di 50 immagini TerrSAR-X della città di Barcellona acquiste in modalità stripmap.

La seconda ricerca è concentrata sul problema della calibrazione di fase applicata alla tomografia SAR in aree montagne, in particolare ai problemi legati al ritardo di fase atmosferico. Il comportamento delle variazioni laterali e verticali del ritardo di fase dovuto a turbolenza e stratificazione è stato analizzato statisticamente, derivando un metodo basato sulla combinazione di regressione e Kriging per modellare queste variazioni di fase. Per questo studio ci si è serviti di uno stack interferometrico di 32 immagini
Cosmo-SkyMed della valle della Matter, nelle Alpi Svizzere. Applicando il metodo di correzione delle fasi basato sul Kriging, è stato possibile ottenere misure di deformazione sul fianco di una montagna dove la soluzione preliminare basata su PSI non ha identificato alcuno scatteratore persistente. In generale solo pochi bersagli doppi sono stati identificati nella scena.

Nell’ultima investigazione si esplorano metodi per identificare bersagli coerenti nella tomografia SAR utilizzando criteri di qualità compatibili con l’analisi PSI. Il risultato dello studio è una tecnica di identificazione statistica dove gli indicatori di qualità di fase impiegate nell’interferometria PS, quali la coerenza d’insieme o le statistiche delle fasi residue, vengono tradotte in soglie per un test di verifica d’ipotesi mirato a verificare la presenza di bersagli singoli o doppi in una cella di risoluzione. La probabilità di falso allarme di questo test è analizzata empiricamente sullo stesso stack interferometrico usato nel primo studio.

Nel suo insieme questa dissertazione tratta diversi aspetti della tomografia SAR e dell’interferometria PS, mostrando che la loro sinergia può contribuire al miglioramento della copertura geografica delle misure di spostamento del suolo.
This cumulative dissertation focuses on two synthetic aperture radar (SAR) signal processing techniques for the measurement of surface deformation, namely persistent scatterer interferometry (PSI) and SAR tomography. In particular, it explores the use of SAR tomography as an add-on to PSI to improve deformation coverage. To this end, this dissertation comprises three investigations, which are presented as individual self-contained chapters (2-4). These investigations are either published or recently submitted for publication in peer-reviewed journals. An introduction is provided in chapter 1. Sections 1.1 - 1.3 provide a general background and an overview of the two techniques, aimed for an audience that has a scientific or technical background, though not necessarily familiar with SAR interferometric and tomographic processing. The rationale of the dissertation is stated in section 1.4. The main findings and contributions of the investigations are summarized in chapter 5.

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INTRODUCTION

In the earth, there are signs for those who (seek truth to) believe.
And in your own selves! So, do you not perceive observe? 

Holy Quran, 51:20-21

Earth – the planet we call home – is characterized by a complex interplay of several natural as well as anthropogenic activities that altogether influence its habitability. Be it the melting glaciers raising the sea-level and in turn threatening coastal regions with inundation [1, 2]; or the land subsidence in the wake of excessive sub-surface resource extraction causing damage to man-made structures and increasing the risk of floods [3-5]; or mass-wasting in the alps damaging infrastructure [6], etc., – their environmental impact can be severe, and at times put our lives at risk! In order to take timely preventive measures as well as to plan remedial strategies, we require an understanding of how these processes unfold. In other words, we need remote sensing for earth observation!

Remote sensing refers to the set of methods employed to acquire data and process information about the geophysical characteristics of the planet’s surface and atmosphere, without physical contact or close proximity. Instead, it exploits the Electromagnetic (EM) response, whether as self-emission or as return of the incident radiation. Passive sensors rely only on the self-emission, while active sensors involve a transmission towards the target followed by reception of the scattered radiation. In each case, the received EM field vector acts as the carrier of information which is embedded in the field strength, frequency/bandwidth, phase and/or polarization. Appropriate signal processing techniques strive to retrieve this information, and thereby the characteristics of earth’s environment. In this way, the sensors combined with the processing techniques constitute the tools that allow geophysical measurements to be taken from a distance. These sensors, especially when airborne or spaceborne, generally provide a wider spatial coverage than is feasible with field work. Moreover, repeated passes of the sensors aid in capturing the dynamics of the geophysical processes as they unfold in space and time.

1.1 BACKGROUND

Land surface deformation is at times a precursor of impending slope failures, mass wasting and structural collapse. For example, slow but continued subsidence in an urban zone may eventually result
in critical damage to housing and infrastructure [7–9], or gradual movements along a steep slope may eventually converge into a landslide [10, 11]. At other times, surface deformations appear in the wake of sudden events, like an earthquake or volcanic activity [12, 13]. In both cases, it has several scientific, economic and safety implications. In fact, measuring the deformation field is the first step towards understanding the underlying geophysical processes, be it to ascertain the scientific mechanism involved, or to plan measures to prevent loss of life and minimize economic risks.

Therefore, an accurate measurement of surface deformation has been the objective of various studies. Traditionally, repeated leveling-based methods [14–16], and afterwards Global Navigation Satellite System (GNSS)-based approaches or a combination of both [17], have been in use to monitor and measure land deformation in concerned regions. Leveling and GNSS measurements can only be performed locally (for discrete locations). For an area-wide assessment of land deformation, nowadays we often resort to remote sensing with spaceborne sensors, particularly synthetic aperture radar (SAR).

**Why synthetic aperture radar (SAR)?**

SAR is an imaging sensor that is nowadays among the standard devices for earth observation from space. Since it is an active sensor, it offers day and night imaging. It operates in the microwave spectrum, which allows it to penetrate the atmosphere. The spatial resolution of currently operational spaceborne SAR sensors ranges between a few tens of meters to a few meters, while covering swaths of several kilometers. In this way, they potentially allow fine-scale monitoring over wide areas (such as an entire urban region) with the flexibility to choose the area globally.

SAR is a coherent system, i.e., it preserves the phase information from transmission to reception of the EM waves. Consequently, SAR images are complex-valued, comprising both amplitude and phase values. The amplitude is related to the strength of the backscattered EM field as received by the sensor. The observed phase comprises the phase delays due to the optical path length traversed during the two-way wave propagation as well as the phase response of the target. The optical path length depends on the geometric sensor-to-target distance and the refractivity of the atmosphere. The phase response of the target is the result of the interaction of the incident EM wave with the scatterer(s) in the range-azimuth resolution cell, depending on their geometric orientation and electrical properties. The phase is observed wrapped between 0 and $2\pi$ radians. The exact number of cycles covered during the propagation from the transmitter to the receiver is, however, not retained.

Repeated passes of spaceborne SAR sensors provide temporal sampling of the imaged scene. In case the target undergoes deformation between two passes (within the resolution cell) with a component in the line-of-sight (LOS) direction of the radar, there will be a subtle phase change relative to the phase observed in the reference pass. In principle, a measurement of the phase changes in successive passes allows estimation of the deformation parameters. However, discriminating these phase changes among other sources of phase variations is typically non-trivial.

The set of SAR signal processing techniques that generally deal with discerning the individual sources of phase variations and exploiting them for parameter estimation are collectively referred to as SAR interferometry.
1.1. Background

**Repeat-pass SAR interferometry: An overview**

An interferogram is a coherent combination of two complex-valued SAR images resulting in the phase difference, i.e. the *interferometric* phase, among the two images at each resolution cell/pixel. Prior to the computation of the interferogram, the images are expected to be coregistered.

In an ideal scenario where 1) the SAR sensor repeats its pass identically, 2) there is no change in the propagation conditions between the passes, 3) the target’s phase response has not changed beyond any deformation-induced deterministic phase changes, and 4) there are no other phase disturbances, the observed interferometric phase will simply correspond to the deformation that took place in the radar line of sight (LOS). However, these assumptions are hardly met:

1) There is often a distance between the antenna phase centers among successive passes, called (spatial) *baseline* [18, 19].

   Due to non-zero spatial baseline, the interferometric phase becomes sensitive to topographic changes in the elevation axis, i.e. perpendicular to LOS. The phase change is proportional to the orthogonal component of the baseline. A digital elevation map (DEM) can be generated\(^1\). Nevertheless, in the context of deformation monitoring, topography-induced phase changes are unwanted. An external DEM is often used to compensate it. However, due to DEM gaps, errors or the elevation differences that may have taken place since the DEM generation, there is often some *residual* topography that remains to be estimated and compensated for. The interferometric methods that attempt to remove topography to extract the deformation are broadly classified as differential SAR interferometry (DInSAR).

   Geometric decorrelation of the interferometric phase may also occur due to non-zero spatial baseline. The EM backscatter from the cell may sum up differently due to slightly different viewing angles in each acquisition [23].

2) The atmospheric refractivity is generally varying in space and time [24, 25].

   The difference in the 3-D field of refractivity at the two acquisition times results in a phase footprint that is referred to as the *atmospheric phase screen* (APS) [26–28]. These phases are a disturbance, and if left uncorrected, they appear as erroneous deformation in differential interferometric processing [29–32].

3) Temporal decorrelation in repeat-pass interferometry is unavoidable in practice, especially given that the orbit repeat times for spaceborne sensors are generally in the order of days.

   Temporal decorrelation is attributable to non-deterministic changes in the physical distribution of the elementary scatterers in the resolution cell over time. Man-made structures such as building walls, pylons, etc. are generally more robust to temporal decorrelation compared to natural targets (e.g. vegetation being disturbed by wind).

\(^1\)Single-pass interferometry is more suited to the generation of a DEM. The two SAR images are acquired with an appropriate baseline at the same time, so that temporal decorrelation can be avoided and the atmospheric phases can effectively cancel each other out. In this context, spaceborne SAR missions like the Shutter Radar Topography Mission (SRTM) [20] and TanDEM-X [21, 22] are noteworthy.
4) There are generally several sources of phase noise, such as thermal or system noise, coregistration errors, orbital inaccuracies, interpolation errors, etc. In addition, the interferometric phases need to be unwrapped, since the phases are measured modulo 2π [33].

Furthermore, in order to capture the dynamics of the geophysical process causing the deformation, repeated interferometric sampling is required. This requirement motivated the need to develop techniques to process stacks of several multi-baseline repeat-pass interferograms, while addressing the aforementioned limitations. For a comprehensive discussion on the potential applications and limitations of SAR interferometry, interested readers are referred to detailed works in [19, 24, 34].

This dissertation focuses on the synergistic use of two state-of-the-art techniques for deformation monitoring with repeat-pass SAR interferometric stacks, namely persistent scatterer interferometry and (differential) SAR tomography. These techniques complement each other; combined, they have the potential to offer improved deformation coverage.

The following sections briefly review these techniques, relating the key aspects relevant in the context of their synergistic use.

1.2 Persistent Scatterer Interferometry (PSI)

Persistent scatterer interferometry (PSI) [35–48] is state of the art in multi-baseline SAR interferometry at an operational level [49, 50]. It allows exploiting stacks of several interferograms acquired over the same area with very wide temporal span (often extending over years) to estimate LOS deformation, while carefully separating residual topography-induced phase variations and mitigating other phase disturbances such as the atmospheric phases. PSI effectively addresses the issues of temporal and geometric decorrelation by performing parameter estimation on the so-called coherent or persistent scatterers (PS) which are characterized by geometrically and temporally stable phase response\(^2\). PSI techniques typically involve both functional and statistical modeling of the individual phase contributions, where applicable. Moreover, a quality assessment of the estimated parameters is usually carried out [40, 51]. Indeed, PSI is an invaluable tool, offering centimeter to potentially millimeter scale precision [52, 53].

**Persistent scatterer (PS) characterization**

The backscattering from an ideal PS is as if it has originated from a point source. It is localized in space, except for any displacement (deformation) within the resolution cell that can be modeled deterministically. Its phase response is identical in each direction, therefore, it does not decorrelate geometrically. Additionally, its temporal backscattering characteristics are constant in time, hence it is not affected

\(^2\)Depending on the context, the term persistent scatterer (PS) may refer to the scatterer (the physical entity) that is exhibiting phase stable response, or the range-azimuth resolution cell/pixel that contains such a scatterer. The latter is generally more common.
by temporal decorrelation. These aspects are an idealization though. In practice, we refer a resolution cell/pixel as a PS when the overall backscattering response is ‘point-like’ or quasi-deterministic. The received complex signal, $y$ is typically modeled as a vector sum of the deterministic response and the random backscatter, as follows:

$$y = \gamma e^{j\phi} + n. \quad (1.1)$$

$\gamma$ is the reflectivity, and $\phi$ represents the phase of the point scatterer. $n$ represents the additive noise or clutter in the resolution cell which are considered to be random in nature. For a PS, the deterministic response is dominant, i.e. $\gamma \gg |n|.$

The origin of the random backscatter is attributable to the presence of arbitrarily large number of elementary scatterers in the resolution cell, given the wavelength of the EM waves used for SAR imaging ($\sim$3 cm for X-band) are typically very small compared to the dimensions of cell (in meters). It gives rise to the speckle phenomenon [54]. In the absence of any single scatterer dominating the others, the overall backscattering response comprises the contributions from elementary scatterers. Under the central limit theorem, the real and imaginary components of $n$ can then be modeled as realizations of a zero-mean, complex circular Gaussian random process. In contrast to persistent scattering, this scenario is commonly called distributed scattering.

**PS candidate selection**

The methods developed for PS candidate selection can be broadly classified as those a) which attempt to assess the temporal stability of the backscattering, and b) those which evaluate the spectral correlation characteristics. In the former category, an example is the dispersion-index, as proposed in [36]. It is the ratio of the mean to the standard deviation of the observed amplitude of the received signal, $|y|$, across the data stack. Another approach to assess temporal variability of backscattering is suggested in [42]. In the latter category, low spectral phase diversity as a proxy for point-like behavior is also stated in [42]. Another method in this category is the sublook coherence approach (SCA) [55]. It is based on computing the correlation coefficient between two sublooks obtained with range compression using non-overlapping halves of the range bandwidth. An ideal PS would exhibit a high correlation among the bands (low spectral diversity in other words), while the noise or clutter components in the backscatter would be more decorrelated [55, 56]. Further examples of spectral correlation based methods include the sublook entropy approach (SEA) [55], phase variance approach (PVA) [55, 57], and more recently, a generalized likelihood ratio test approach (GLRTA) [56]. A performance analysis of these methods is provided in [58].

In urban areas, man-made structures such as building walls, railway lines, metallic lamp posts, and windowpanes, etc. are good PS candidates (PSCs). The prevalence of PS candidates in mountainous areas, vegetated lands or other natural environments is comparatively lower. Distributed scattering is more common in these environments, though occasional gravel patches, bare rocks or similar impenetrable surfaces may still exhibit PS behavior.
The interferometric phase, $\phi_m$ observed for a PS candidate in the $m^{th}$ layer of a single-master\textsuperscript{3} interferometric stack is modeled as a sum of several phase contributions [24, 30, 40]:

$$\phi_m = \phi_{\text{def}}^m + \phi_{\text{geo}}^m + \phi_{\text{atm}}^m + \phi_{\text{noise}}^m + 2\pi p$$ (1.2)

where $\phi_{\text{def}}^m$ is the phase change due to the deformation, $\phi_{\text{geo}}^m$ is the contribution of topography, $\phi_{\text{atm}}^m$ represents the atmospheric phases, and $\phi_{\text{noise}}^m$ represents phase noise. The term $2\pi p$ is added to account for phase wrapping, where $p$ is an unknown integer. Several techniques have been proposed over time to separate these phase contributions, such as the pioneer permanent scatterers approach [35, 36], Stanford Method for Persistent Scatterers (StamPS) [37, 38], Coherence Pixel Technique (CPT) [39], and the Interferometric Point Target Analysis (IPTA) [40–42], to name a few among several others [43–48]. Avoiding the specifics of each techniques, some common aspects are reviewed as follows, while a comparative study is referred to [50].

The direct dependence of topography-induced phase variations on the spatial baselines (due to across-track, repeat-pass imaging geometry [19, 59]) allows us to model these variations functionally as follows:

$$\phi_{\text{geo}}^m \approx \frac{4\pi}{\lambda} \left[ \frac{s^2}{2(r_0 - b_{m}^{\perp})} - \frac{b_{m}^{\perp} s}{r_0 - b_{m}^{\parallel}} \right]$$ (1.3)

where $\lambda$ is the wavelength, $r_0$ is the range distance from the sensor to the target location for the reference acquisition, $s$ represents the elevation, i.e. the position of the target in the perpendicular to LOS axis. $b_{m}^{\perp}$ and $b_{m}^{\parallel}$ are the orthogonal and parallel components of the spatial baseline for the $m^{th}$ interferogram, respectively.

Conversely, the dependence of deformation on time (e.g., whether the kinematics of the motion are step-wise, linear, accelerating, etc. [60]) is not known a priori – in fact, extracting it from the data is generally intended. Several PSI techniques take the first-order approximation, i.e. assume the deformation to be a linear function in time:

$$\phi_{\text{def}}^m = \frac{4\pi}{\lambda} v t_m$$ (1.4)

where $t_m$ is the $m^{th}$ temporal baseline, and $v$ is the deformation velocity. In case the total temporal span of the data stack extends over years, several studies consider the possibility of a seasonal deformation in addition to linear deformation (displacement), as follows [30, 61–63]:

$$\phi_{\text{def}}^m = \frac{4\pi}{\lambda} \left[ v t_m + a \cos \left( \frac{2\pi}{\tilde{T}} (T_m - T_0) \right) \right]$$ (1.5)

where $a$ is the amplitude of the seasonal deformation, $\tilde{T}$ is the period (1 year typically), and $T_0$ is a time constant. Some studies have shown that the LOS component of the thermal expansion of structures in urban areas can also induce phase variations [52, 62, 64, 65]. In [66–68], these interferometric phases

\textsuperscript{3}Throughout the work presented in this dissertation, only stacks of interferograms referenced to a single master acquisition have been considered. Moreover, all interferometric phases are additionally considered to be relative to a reference point in the scene.
are modeled as linearly dependent on temperature changes, as follows:

\[ \varphi_{m,\text{exp}}^{\text{th}} = \eta T_m \]

where \( T_m \) is the temperature change (with respect to the temperature for the reference layer), and \( \eta \) is the phase-to-temperature sensitivity.

A large data stack is generally desired to assess the statistical properties of the different sources of phase variations [30]. In general, PSI processing techniques adopt some form of spatio-temporal filtering of the observed interferometric phases to mitigate the impact of atmospheric phases (a phase disturbance) on parameter estimation, or to explicitly isolate them from other phase contributions. The atmospheric phases are typically considered to be spatially correlated up to a certain extent, and uncorrelated in time [35, 36, 41, 42, 69]. Even so, when using ground-based interferometric systems, the temporal spacing between consecutive acquisitions can be in the order of a few minutes and therefore, some temporal correlation is expected [70, 71]. However, in case of multi-baseline repeat-pass acquisitions from a space-borne SAR sensor, the assumption of an absence of temporal correlation remains reasonable, since repeated passes of satellites are generally several days apart. Any temporal filtering should preserve non-linear deformation, if any, as stressed by several studies [27, 35, 41, 72]. Moreover, temporally phase coherent targets, i.e. PS, are indispensable to a reliable extraction of the different phase contribution, while restricting the propagation of decorrelation noise during interferometric processing (phase filtering, unwrapping, etc.).

**Quality assessment**

The quality of the estimated parameters (residual topography and deformation parameters) is determined in terms of the statistics of the residue of the phase model fit to the observed phases. The lower the dispersion of the residual phases, the better is the perceived quality of the estimation process, indicating higher precision in the measurements. A high dispersion of the residual phase can be caused by decorrelation (temporal, geometric or other forms), uncompensated atmospheric phases, model imperfections, etc. Apart from the dispersion, some PSI techniques assess the quality in terms of the ensemble coherence, computed using the estimated residual phases across the stack [30, 35].

### 1.3 SAR tomography

Concurrent to developments in interferometric techniques, SAR tomography [73–77] emerged as a means of computing 3-D reflectivity. As for multi-baseline interferometry, SAR tomography also uses a stack of several SAR images, generally acquired in repeated passes in the same orbit or flight track. But unlike interferometry which exploits only the phase information, tomography uses both amplitude and interferometric phases.

A single SAR image is a projection of the true 3-D complex reflectivity of the scene in the range-azimuth plane. The elevation (orthogonal to the range-azimuth plane) information is not explicitly retained in a single projection. A given range-azimuth resolution cell contains all the backscatter that originates at
that range bin for that particular azimuth echo. In other words, there is no sensitivity along the elevation axis.

SAR interferometry provides sensitivity along the elevation, cf. eq. (1.3), but in a limited sense. The spatial baselines allow estimation of the elevation only for a single scatterer in the resolution cell, or a single phase center in case of distributed scattering. In case of multiple scatterers or phase centers in the same cell, the estimated elevation is an aggregate (possibly erroneous). Polarimetric Interferometric SAR (PolInSAR) provides a value addition by making use of the polarization diversity in addition to the baseline diversity; it, however, cannot explicitly distinguish more than three different scattering mechanisms (in a polarimetric sense). In case multiple scattering mechanisms exist at multiple phase centers along the elevation, the result is again an aggregate.

SAR tomography is a means to alleviate these limitations by synthesizing an aperture in the elevation axis using multiple acquisitions taken from slightly different viewing angles. The reflectivity along the elevation axis can be estimated, and in this way, multiple scatterers or phase centers can be discriminated [78–80]. For a detailed mathematical exposition, interested readers are referred to [81–84].

LAYOVER

SAR is a side-looking radar; the incidence angle is off-nadir. Consequently, real-life structures appear geometrically distorted [54, 85]. In case the top and the base of the structure commute their real positions in the SAR image, the distortion is called a layover. It occurs when the local terrain is steeper than the incidence angle [86]4. In urban areas, step-wise height changes, as depicted in Fig. 1.1, often result in layover where two or more scatterers fall in the same range-azimuth resolution cell. These scatterers may individually exhibit PS behavior, nonetheless they are typically rejected in a PSI processing (since a PS is a single dominant scatterer). In mountainous areas, layovers are typically more widespread due to drastic changes in the topography. Since layovers are rejected in the interferometric processing, it was shown in [86, 88] that deformation assessment would remain limited in more than 40% of the surface area in an alpine terrain due to layover effects when imaged by the European Remote Sensing Satellite (ERS) with 23° incidence angle.

The ability of SAR tomography to resolve layovers of potentially phase stable scatterers is central to the theme of this dissertation.

FROM AN IMAGING TECHNIQUE TO A MEASUREMENT TOOL!

SAR tomography was originally demonstrated as a technique for 3-D imaging of penetrable media (at microwave frequencies), such a forest canopies [76, 89–92], and more recently for crops as well [93, 94]. The so-called tomograms are 2-D slices of the estimated 3-D reflectivity, commonly represented as an image in elevation-range coordinates for a given azimuth echo. It allows discrimination of the structural features in the medium, such as canopies from the tree trunks, when the separation exceeds

4Sometimes, this distortion is distinguished as an active or a passive layover. The active region is the facet of the object whose top and base actually commute their positions, while the passive layover comprise the facets (though in the same range bin) on which the active part is overlaid [85–87]. This distinction is not relevant in our context (where we focus on point-like scatterers, and do not reconstruct facets), except for correct geocoding of the individual scatterers in layover.
1.3. SAR Tomography

Figure 1.1: Imaging geometry for multi-baseline repeat-pass processing techniques, such as persistent scatterer interferometry (PSI) and SAR tomography. Persistent versus distributed scattering in different resolution cells is depicted. The possibility of more than one point-like scatterer superimposed in the same cell, due to layover, is also shown.

The tomographic resolution limit. In case of an urban area with several man-made structures, such as building walls which are impenetrable, it allows discriminating the point-like scatterers falling into the same range bin.

The interferometric phase model classically introduced for SAR tomography assumes that the scatterers have not undergone any displacement over time, cf. eq. (1.3). This limitation is circumvented by differential tomography [66, 95–99] which introduces extended phase models for point-like scatterers to incorporate motion parameters of the individual scatterers in addition to their elevation. It simultaneously exploits both the spatial and temporal baselines of the acquisitions. In this way, differential SAR tomography can be used as a technique for deformation assessment. Additionally, it can offer a gain in deformation sampling by detecting multiple scatterers interfering in the same range-azimuth resolution cell, in comparison with the typical interferometric techniques which require the presence of a single point-like in a given resolution cell [68, 99–102]. Ignoring any phase disturbance or noise, a
mathematical model for differential SAR tomography can be written as follows:

\[ y_m = \int_{s_i} y(s) e^{-j[\phi_{\text{geo}}^m(s) + \phi_{\text{def}}^m(s, t_m)]} ds \] (1.7)

where \( y_m \) is the received SAR signal, \( \gamma(s) \) represents the reflectivity profile along the elevation axis, and \( s_i \) is the elevation support. The model has to be inverted to estimate the unknown elevation and deformation parameters. The number of coherent scatterers interfering in the resolution cell needs to be inferred. Moreover, a performance analysis is needed to characterize the impact of noise on detection and parameter estimation. Different tomographic processing frameworks have been suggested over time to address these aspects, such as [92, 103, 104]. While avoiding the specifics, the general aspects are related as follows.

**Tomographic processing framework: General aspects**

**Phase calibration** Prior to tomographic inversion, the interferometric data stack requires a precise phase calibration, i.e., an estimation and compensation of the atmospheric phases in each interferogram. If left uncorrected, these phases act as a disturbance in focusing the scatterers in 3-D [104–108]. Any residual platform motion or orbital inaccuracies also need to be mitigated to calibrate the interferometric data stack [105, 109]. In this work, however, atmospheric phases are considered as the main source of miscalibration, given that only spaceborne data from sensors like TerraSAR-X and CosmoSkyMed, whose orbital information is generally quite accurate [110–112], are used for the investigations.

Since the origin of the atmospheric (tropospheric) phases is variable refractivity at acquisition times, different methods have been proposed to estimate the 3-D field of refractivity using auxiliary information from external sources. Among such methods is the use of output from numerical weather models, e.g., the Weather Research and Forecasting (WRF) model [113] and the global atmospheric reanalysis data from the ERA-Interim project [114] at the European Center for Medium-Range Weather Forecasts (ECMWF). The hindcast of meteorological parameters from these models allow us to generate synthetic phase delay maps to compensate for the atmospheric effects [115, 116]. However, the applicability of the weather models is often questionable since the resolution of the weather data is too coarse and they are generally not concurrent with the SAR acquisitions. Moreover, temporal interpolation is needed which can introduce uncertainties. GNSS measurements are another source of auxiliary information [27, 117–119]. GNSS-based methods are generally effective, but their application is limited as continuously operating GNSS stations are often absent or sparsely distributed for most areas around the world.

Data-driven methods for phase calibration are typically the only choice – the atmospheric phase corrections have to be estimated from the interferometric phases themselves. In order to avoid propagation of phase noise in this estimation process, the atmospheric phases need to be estimated for phase stable scatterers, i.e., PS. Therefore, a PSI analysis is inevitably needed. The atmospheric phases isolated at PS locations are low-pass filtered and spatially interpolated over the scene, for example with a kriging interpolator [30, 36]. In this way, the need to use PSI prior to tomography indicates a natural synergy between the two techniques.
Tomographic inversion For a given resolution cell from a coregistered and phase calibrated interferometric data stack, the received single-look complex (SLC) signal values can be considered as samples of the far-field of the scatterers (sources) along the elevation profile [81]. The far-field can in turn be considered as the Fourier transform of the source distribution [120, 121]. Hence, SAR tomographic inversion is analogous to a source inversion problem, and each scatterer (or phase center in case of distributed scattering) along the elevation profile represents a cisoid embedded in noise, while the received SLCs are samples of its spectrum. In this way, tomographic inversion is analogous to a spectral estimation problem [77, 82, 122, 123].

Fast Fourier transform (FFT)-based spectral estimation, as employed in the SPECAN (SPECtral ANalysis) algorithm [124], requires uniformly sampled baselines which are typically not available. A phase correction was suggested in [76]. Examples of other commonly used spectral estimators for tomographic inversion are the minimum-variance spectral estimator/CAPON [89, 125–127] and multiple signal classification (MUSIC) [128, 129]. These estimators require the data covariance matrix. Since the covariance matrix is not known a priori, spatial multi-looking is generally implemented to estimate it by assuming each look to be an independent realization of the data. In case of distributed scattering, whereby the underlying signal model is assumed to zero-mean complex Gaussian, the information lies with the second-order statistics; therefore, the multi-looking operation for covariance estimation is inevitable. On the other hand, for point-like scatterers, single-look inversion methods can be used, preserving the full spatial resolution. Standard beamforming (BF) [84, 106, 130] and singular value decomposition (SVD)-based inversion are among such methods [78, 123]. BF is a robust method, but is prone to spectral leakage and high sidelobes, while the tomographic resolution is limited to Rayleigh. SVD-based techniques can suppress the side-lobes and offer slightly better tomographic resolution.

Another choice for single-look inversion is the time-domain back-propagation (TDBP) [131, 132]. It retrieves the reflectivity at each 3-D point of interest on a reconstruction grid by back-propagating the SLCs samples. It can be considered as a generalization of standard beamforming or matched filtering [91]. It is particularly suited to the case of tomography with airborne SAR data acquired in non-linear flight tracks [132], and its performance is shown to exceed that of the SPECAN algorithm [131]. However, it is computationally expensive. Compressive sensing [133, 134] based tomographic inversion is also an attractive choice, nonetheless, it is also computationally expensive. It offers super-resolution which can be valuable in urban areas where two more scatterers along the elevation can happen to be closely spaced below the Rayleigh resolution limit [133, 135].

A comparative study of some of the aforementioned inversion methods is presented in [130, 136].

Parameter estimation & scatterer detection The estimates of the unknown parameters are typically associated with the peak(s) of the absolute or squared reflectivity retrieved with tomographic inversion [137]. For parametric inversion methods, such as MUSIC or non-linear least squares (NLS) [123, 129], an a priori knowledge on the number of scatterers, i.e. model order, is required. The use of information-theoretic criteria (ITC) for model order estimation has been suggested in [138]. Hypothesis testing based on generalized likelihood ratio test (GLRT) has been proposed in [84] and [139] for single and double scatterers, respectively. These strategies allow statistical performance analysis in terms of probabilities
of detection and false alarm. The constant false alarm rate (CFAR) property satisfied by these detectors allows keeping the false alarm at a fixed level with appropriate thresholds on the test statistic. These detectors are implicitly based on BF-based inversion, therefore, no super-resolution is achieved. A recent advancement in this contest is a GLRT-based detector proposed in [140] which allows detection of multiple scatterers spaced closer than the Rayleigh resolution limit, while maintaining the CFAR property.

1.4 RATIONALE

1.4.1 Motivation

The lack of deformation data in many areas around the world is known to be among the limitations in modeling kinematics of mass movements and other crustal processes [13, 37, 141, 142]. An improvement in the coverage of deformation can be achieved in two ways. Firstly, more satellite missions will help in capturing more interferometric SAR data around the world. Larger data stacks with more frequent temporal sampling will be helpful in capturing the dynamics of the deformation process. Sentinel-1 [143] constellation, the upcoming NISAR [144] mission and the proposed Tandem-L [145] mission are noteworthy in this context. The second aspect is to push for improved processing to extract the most from the existing and upcoming data stacks! It is this second aspect where this dissertation aims to make a contribution. And hence the question – can we improve upon the deformation coverage of a typical PSI approach?

Since the PS are single dominant scatterers, the radar resolution cells comprising multiple scatterers, as for the case of layovers, are rejected in the PSI processing. Layovers represent ‘interfering signals’, and theoretically, it is possible to separately detect the interfering scatterers provided they are individually coherent and not buried in noise [137, 139]. Spatio-temporal inversion with differential tomographic methods can potentially resolve the layover and simultaneously provide an estimate of the deformation velocity of the interfering scatterers. In this way, SAR tomography as an add-on to PSI can improve deformation coverage.

1.4.2 Research objectives and questions

The ability of SAR tomography to resolve layovers of coherent scatterers is well-known and has been suggested in several studies [100, 123, 137, 139]. Nonetheless, a critical assessment of the utility offered by SAR tomography, as an add-on to PSI, is needed. To this end, several relevant aspects of a tomographic framework (phase models for inversion, phase calibration, scatterer detection, etc.) need to be investigated.

This dissertation is based on three core investigations.

Investigation # 1: Single-look SAR tomography as an add-on to PSI for improved deformation analysis in urban areas

This investigation focuses on urban areas, where layovers occur frequently due to step-wise changes
in the height. One of the objectives is to empirically analyze the role of different phase models for tomographic inversion, particularly in terms of their suitability for layover resolution and simultaneous estimation of deformation parameters. It also seeks to assess the overall gain with the added use of tomography, quantitatively as well as qualitatively, in comparison with an established PSI technique. In the PSI context, quality assessment is commonly performed in terms of the statistics of the residual phase, while for tomography, the performance is usually assessed in terms of the probabilities of false alarm and detection. Another objective of this investigation is to conduct a comparison of the quality of the scatterers obtained with tomography against the quality of the PS identified within the PSI processing using a common quality metric. In short, this investigation focuses on the following questions.

1. How do different phase models for tomographic inversion impact layover separation?

2. Is the quality of the scatterers obtained with tomography comparable to the quality of the persistent scatterers obtained with a PSI approach? What is the corresponding gain in deformation sampling with the added use of tomography?

Investigation #2: A case study on the correction of atmospheric phases for SAR tomography in mountainous regions

This investigation focuses on mountainous regions, where layovers are generally more widespread due to drastic variations in topography. Often, the slopes where deformation measurements are intended happen to be in layover.

Nonetheless, SAR tomography has thus far been applied for deformation monitoring in urban areas where the relatively larger prevalence of point-like scatterers is conducive to its application. Moreover, the problem of phase calibration of the interferometric stack is less challenging in urban areas. The underlying topography is generally flat. It can be assumed with reasonable justification that the atmospheric phases are spatially smooth (low-frequency variation). The unwrapped atmospheric phases computed for the PS during the prior PSI processing can be spatially filtered in local neighborhood and extrapolated over the scene to estimate an APS for each layer in the stack (as in investigation #1). However, the same procedure is not directly applicable in mountainous areas, where the phase calibration is more involved due to strong spatial variations of the local atmospheric conditions and propagation paths through the troposphere. The drastic variations in topography incur two critical implications for tomography. Firstly, a height-dependent phase delay variation owing to vertical stratification of the atmosphere cannot be ignored. Secondly, depending on the acquisition geometry, the possibly large height difference among the scatterers in layover implies that the spatial location of the individual scatterers in map coordinates (after geocoding) can also be very different. Therefore, the atmospheric correction needed for one scatterer may be very different from the other, notwithstanding that they are in the same range-azimuth pixel. Hence, a single correction for a range-azimuth pixel, as for PSI, would likely not suffice.

Keeping in view these considerations, this investigation aims to push the boundaries of research and seek a potential new avenue for the application of SAR tomography in mountainous regions, by seeking to answer the following main questions:
1. Can a data-driven approach build upon the prior PSI solution to reliably correct for atmospheric phases for the application of SAR tomography in mountainous regions?

2. How can multiple corrections for potentially multiple scatterers in layover be implemented along-side the spatio-temporal inversion?

**Investigation # 3: Differential SAR tomography as an add-on to PSI: Detection of coherent scatterers in the presence of phase instabilities**

This investigations focuses on another key aspect of a tomographic processing framework, that is coherent scatterer detection. The main question explored in this investigation is as follows:

1. Can we perform scatterer detection in the tomographic framework based on the same quality criteria as used in the prior PSI processing?

The instabilities in phase are represented as additive noise in interferometry, but their impact in tomography manifests as a multiplicative noise. However, most of the statistical detectors proposed in the literature model the noise in the tomographic model as additive only. The pioneer work to consider multiplicative noise in tomographic modeling was presented in [105]. The main objective of this investigation is to build upon the foundation laid in this work, and devise a detection strategy that would allow extending the same quality considerations as used in the prior PSI processing (in terms of the dispersion of the residual phase) to the subsequent tomographic analysis. At the same time, it should allow performance characterization in terms of probabilities of false alarm and detection as well.

### 1.4.3 Structure of the dissertation

The three investigations conducted in the frame of this thesis are presented as chapters 2, 3 and 4, respectively. The material in each chapter has either been published in a peer-reviewed journal, or recently submitted for publication, as indicated on the title page of each chapter. The publications are self-contained, and can be read independently. Chapter 5 provides a synopsis of the main findings and contributions of each investigation.

**REFERENCES**


REFERENCES


2

SINGLE-LOOK SAR TOMOGRAPHY AS AN ADD-ON TO PSI FOR IMPROVED DEFORMATION ANALYSIS IN URBAN AREAS

This chapter has been published as: Siddique, M. A.; Wegmüller, U.; Hajnsek, I. & Frey, O. (2016), Single-look SAR tomography as an add-on to PSI for improved deformation analysis in urban areas, IEEE Transactions on Geoscience and Remote Sensing, 54(10), 6119-6137.

Author’s contributions

• Expanded the initial PSI solution.

• Co-developed the tomographic processing framework, and co-designed and performed the experiments; implemented a sequential GLRT for scatterer detection; and evaluated the quality of the detected scatterers and the gain in deformation sampling.

• Drafted the manuscript, and interpreted the results.

Co-authors’ contributions

• U. Wegmüller provided the initial PSI solution used in this work.

• O. Frey initiated using SAR tomography as an extension of IPTA-based PSI processing; co-developed the tomographic processing framework, and co-designed the experiments; developed several routines for data preprocessing and PSI/tomo integration; and reviewed the interpretation of the results.

• All co-authors reviewed the manuscript.

Data sources

TerraSAR-X SAR data used in this project was obtained courtesy of the German Aerospace Center DLR under proposal MTH1717. SRTM is copyright USGS.
ABSTRACT

Persistent scatterer interferometry (PSI) is in operational use for spaceborne SAR-based deformation analysis. A limitation inherently associated with PSI is that by definition a persistent scatterer (PS) is a single dominant scatterer. Therefore, pixels containing signal contributions from multiple scatterers, as in the case of a layover, are typically rejected in the PSI processing, which in turn limits deformation retrieval. SAR tomography has the ability to resolve layovers. This paper investigates the value-addition that can be achieved by operationally combining SAR tomography with a PSI approach towards the objective of improving deformation sampling in layover-affected urban areas. Different tomographic phase models are implemented and compared as regards their suitability in resolving layovers. Single-look beamforming-based tomographic inversion and a generalized likelihood ratio (GLRT)-based detection strategy are used to detect single and double scatterers. The quantity of the detected scatterers is weighed against their quality as defined in terms of the phase deviation between the SLC measurements and the tomographic model fit. The gain in deformation sampling that can be derived with tomography relative to a PSI-based analysis is quantitatively assessed, and alongside the quality of the scatterers obtained with tomography is compared with the quality of the PSs identified with a PSI approach. The experiments are performed on an interferometric stack of 50 TerraSAR-X stripmap images. The results obtained show that, although there is a trade-off between the quantity and the quality of the detected scatterers, the tested SAR tomography approach leads to an improvement in deformation sampling in layover-affected areas.

2.1 INTRODUCTION

Multi-temporal synthetic aperture radar (SAR) interferometric techniques are widely used to monitor surface deformation caused by various geophysical processes (natural as well as anthropogenic), such as tectonic and volcanic activities [1–3], mass movements on unstable slopes [4, 5], mining and groundwater pumping [6–9], etc. The dynamics of such processes are sampled with time series of interferometric SAR data. Differential interferometric SAR (DInSAR) [10, 11] methods exploit the temporal baselines among one or more interferograms to extract the phase components correlated with scatterer motion. The performance of these methods is limited by the presence of unwanted phase components, such as the atmospheric phase screen (APS), and phase degradation caused by temporal and geometric decorrelations. Persistent scatterer interferometry (PSI) [12–16] is an advanced DInSAR concept that circumvents these limitations by observing the phase histories of the so-called persistent scatterers (PSs) in a relatively large stack of interferograms. These are single dominant point-like scatterers marked by high temporal coherence across the entire stack. Since they are less susceptible to decorrelation phenomena, their unknown motion parameters can be more reliably estimated. A large quantity of PSs is generally desired to effectively retrieve information about the observed geophysical process.

In the context of long-term monitoring of deformation in urban areas, PSI has proven to be an invaluable tool, offering millimeter-scale precision [17, 18] for extended areas of observation. Man-made structures such as rooftops, railways, metallic lamp posts, window-panes etc., are good persistent scatterer candidates (PSCs). A PSI analysis of an urban area generally reveals a sizable number of PSs, espe-
cially with high resolution spotlight images [19]. However, an inherent limitation associated with PSI is that by definition a PS is a single dominant scatterer within a range-azimuth resolution cell. Therefore, pixels containing backscatter of comparable energy from multiple scatterers, which may individually exhibit point-like behavior, are rejected. This situation arises often in layovers. Urban areas typically have buildings of different heights, and layovers such as those between the ground and the facade of a nearby building, or the rooftop of one building and the facade of a higher building in proximity, occur ubiquitously. A local PSI analysis of such buildings may suffer from poor deformation sampling due to the rejection of such layovers. To overcome this limitation, a higher-order analysis [20] (i.e. considering the possibility of more than one scatterer in a range-azimuth pixel) is required.

SAR tomography [21–30] is a multi-baseline interferometric technique that allows higher-order modeling using both the phase and amplitude of the backscatter. It serves as a means to separate individual scatterers in layover, which motivates its use as an add-on to PSI. While the classical use of SAR tomography has been the retrieval of reflectivity profile along the elevation (perpendicular to the line of sight direction), the more advanced tomographic techniques simultaneously allow modeling the motion parameters of one or more scatterers in addition to their elevation. Notwithstanding that PSI and tomography may share the same phase models, while the former can retrieve the elevation (as residual topography) and motion parameters for a single scatterer only, the latter allows it for multiple scatterers in a given range-azimuth pixel. This paper investigates the value-addition that can be derived by the combined use of SAR tomographic techniques and PSI, especially towards the objective of extending deformation analysis to layover-affected areas.

2.1.1 Related work

The early developments in SAR tomography focused on exploiting the spatial baselines of the data stack to build a synthetic aperture in the elevation, thereby extending the conventional 2D SAR imaging to 3D. The scatterers were assumed to be stationary. The concept of differential tomography, as introduced in [31, 32], raised interest in using tomographic techniques for deformation analysis as well. Differential tomography is a means to simultaneously model scatterer elevation and deformation. It exploits both the spatial and temporal baselines at the same time. The deformation is assumed to be temporally linear. Results on real data have been presented in various studies [26, 33–38]. As reported in some PSI investigations with X-band data, such as [17, 19, 39, 40], scatterers in urban areas may additionally be subject to non-linear motion due to thermal dilation. The phase model needs to be extended to account for the phase variations associated with non-linear motion. In [41, 42], the extended phase model considers the non-linear motion as a temporally sinusoidal variation alongside the linear component. The investigations presented in [43–45], however, explicitly include local temperature values to model phase variations related to thermal dilation; the results are provided for single scatterers only.

Different tomographic inversion methods have been proposed over time. In case the underlying signal model is assumed to be zero-mean circular Gaussian (fully developed speckle), multi-looking is typically needed to estimate the sample covariance matrix and allow for a reasonable inversion with spectral estimators such as CAPON [37, 46] and multiple signal classification (MUSIC) [47, 48]. Multi-looking, however, reduces the spatial resolution in range/azimuth. The approach recently proposed in
is a multi-look inversion method that uses a rather small degree of spatially-adapted multi-looking (averaging less than < 50 pixels) to estimate the sample covariance matrix. The method uses the principal component analysis (PCA)-based interferometric stack filtering approach given in [50]. Layovers are resolved by associating the eigen-vectors of the estimated covariance matrix with the different scattering mechanisms in the given (multi-looked) pixel. With a slight compromise on spatial resolution, the method offers better coverage compared to single-look tomographic inversion. In [50], it is additionally reported that layover separation can be achieved without a priori phase calibration of the stack (as required for SAR tomography) since the approach does not require any assumption about the structure of the steering vector (for the given spatial and temporal baselines).

In case of quasi-deterministic scatterers with stable phase ( speckle-free, point-like behavior), single-look inversion methods can be used. Among such methods are the standard beamforming (BF)[26, 34, 51] and singular value decomposition (SVD)-based techniques [30, 52]. BF is a simple and robust method, but it is susceptible to spectral leakage and high sidelobes. The resolution in elevation is determined by the extent and distribution of the spatial baselines, and is at best limited to the Rayleigh resolution for the theoretical case of uniformly distributed baselines over the same extent. SVD-based techniques generally offer better sidelobe reduction and possibly a slight super-resolution, i.e. the resolution achieved can be slightly better than the Rayleigh limit. In the context of super-resolution, the single-look tomographic processing introduced in a compressive sensing (CS) signal reconstruction framework in [53, 54] has been a significant advance. It is, however, computationally expensive. In [42], the overall computational expense of the framework is reduced by restricting the application of the CS-based reconstruction only to those pixels that contain closely-spaced double scatterers classified a priori. The classification process uses information theoretic criteria (ITC), a trained SVM-classifier and estimates of scatterer elevation and deformation parameters obtained with periodogram maximization (based on BF with amplitude-normalized single-look complex (SLC) pixel values). An alternative single-look method offering super-resolution with relatively inexpensive computation has been recently proposed in [55]. It suggests interpolating the spatial baselines at virtual uniform locations, followed by tomographic inversion with MUSIC and CAPON beamforming (which also offer super-resolution in elevation).

Prior to tomographic inversion, a PSI analysis is typically needed to isolate the atmospheric phase screen (APS) [56] and thereby phase calibrate the interferometric data stack. Another imperative step during tomographic processing is the estimation of the number of point-like temporally coherent scatterers in a given range-azimuth pixel, i.e. model order selection. In [57], ITC-based model order selection methods have been discussed. In case of tomographic focusing with parametric spectral estimators, e.g. non-linear least squares (NLS) [30, 48], MUSIC etc., model order selection has to be applied a priori. With non-parametric estimators, such as BF, CAPON, or the SVD-based techniques, the number of scatterers superposed in a given pixel can be estimated a posteriori by observing the 'maxima' [58] in the retrieved reflectivity profile. For the identification of single dominant scatterers (i.e. persistent scatterers), a detector based on the multi-interferogram complex coherence (MICC) has been used in [59] in the context of PSI processing. The MICC-based detector uses only the phase residuals (deviations between the differential phases and the phase model fit) neglecting the amplitude information. It has
been shown in [34] that a generalized likelihood ratio test (GLRT)-based detection of single scatterers, which implicitly incorporates beamforming-based tomographic focusing and hence includes both the amplitude and phase information, improves performance both in terms of higher probability of detection for a given level of false alarm as well as higher accuracy of estimates. A ‘second order’ version of MICC has been used in [20] for the detection of both single and double scatterers. A sequential use of GLRT has been theoretically developed and applied in [38] to decide among the hypotheses that a given pixel contains a single or double scatterer, or neither.

2.1.2 Research gaps

Single-look SAR tomography can be combined with a PSI approach to improve deformation sampling in urban areas. Given the fact that PSI is nowadays operationally used for deformation analysis, a critical assessment of the utility offered by SAR tomography is required to justify its operational use as an add-on to PSI. Different aspects of a tomographic framework (signal modeling, inversion, scatter detection etc.) need to be reviewed in this context and its performance compared with that of an established PSI approach using a common quality metric. The relative gain in deformation sampling with the added use of tomography, when using a typical SAR interferometric data stack (in terms of baselines, and spatial resolution), remains to be both quantitatively and qualitatively assessed.

2.1.3 Contributions of this paper

To fill in the aforementioned research gaps, this paper provides the following main contributions:

1. Different phase models for tomography are analyzed in terms of their suitability for layover resolution and simultaneous estimation of deformation parameters.

2. The potential gain in deformation sampling using a tomographic approach in addition to a PSI approach is quantitatively discussed.

3. The quality of the scatterers obtained with tomography is compared with the quality of the persistent scatterers (PSs) identified with a PSI approach (using a mutually consistent measure of quality).

2.1.4 Outline

First, we perform a PSI analysis for the selected urban area using the Interferometric Point Target Analysis (IPTA) [13] framework. A set of PSs is iteratively identified. The solution obtained includes the estimates of the residual topography, linear deformation, phase-to-temperature sensitivity and the atmospheric phase screen (APS) for each PS. Next, we implement beamforming-based tomographic inversion on the (phase calibrated) interferometric stack with three different phase models. The inversion is applied on all pixels, including those rejected in the PSI processing which potentially include double scatterers. The performance of the individual phase models, as regards the ability to resolve layovers, is
evaluated. For the detection of single and double scatterers, we adopt a sequential GLRT strategy as proposed in [38]. Relative to the number of PSs identified separately in the PSI processing, we quantify the gain in deformation sampling owing to layover separations achieved with tomography. As a next step, we compare the quality of the scatterers obtained with tomography against the quality of the PSs identified in the PSI processing. Since a traditional PSI analysis uses only the phase information, we assess the quality of the PSs in terms of the residual phase i.e., the root-mean-square (RMS) phase deviation between the differential phases and the phase model fit. Along similar lines, we evaluate the quality of the scatterers obtained with tomography on the basis of RMS phase deviation between the SLC measurements and the tomographic model fit. In this way, the quality metric used is mutually consistent and, therefore, allows for a performance comparison.

2.1.5 Additional remarks

In case of single/persistent scatterers, it has been shown in [34] that a tomography/GLRT-based detection and estimation approach offers improved quality in comparison with the classical PSI/MICC-based approach. In this investigation we conduct an extended (empirical) quality comparison that includes the case of double scatterers. We intend to comprehensively cross-examine the quality of the double scatterers against single scatterers obtained with tomography/GLRT at different detection thresholds, and in turn relate it to the quality of the PSs identified separately with an IPTA-based PSI processing.

SAR tomographic analysis in urban areas has often been conducted with very high resolution (~1 m) images acquired in spotlight imaging mode. The higher the resolution, the more is the tendency of the individual resolution cells towards point-like scattering. Therefore, compared to stripmap images at ~3 m resolution, spotlight images are more favorable for a PSI/tomographic analysis. Today, interferometric stacks of high-resolution spotlight-mode SAR data are available only for a very limited number of urban areas. Keeping in view this limitation, we perform this investigation on an interferometric stack comprising of stripmap images to assess the potential of tomography for a data type which is widely available and which is, therefore, more relevant in an operational scenario.

The remainder of this paper is organized as follows. Section II describes the methods used in this investigation for PSI processing, tomographic inversion and scatterer detection. The characteristics of the interferometric data stack used in this investigation are given in Section III. Results are provided in section IV, and a detailed discussion follows in section V.

2.2 METHODS

2.2.1 PSI processing approach

Prior to a tomographic analysis of an interferometric data stack, a PSI solution is generally needed to extract and remove the atmospheric phase contributions, thereby phase calibrating the stack. As per the objectives of this work, we require a PSI solution to also serve as a reference against which we can compare the potential value-addition (in terms of quantity and quality of the deformation samples)
offered by the added use of single-look SAR tomography. In this context, we performed PSI processing using the Interferometric Point Target Analysis (IPTA) [13] toolbox which supports SAR interferometric time-series analysis. The important aspects of the PSI processing in our case are as follows:

A reference layer was chosen from the stack of single-look complex (SLC) SAR images, and all the layers in the stack were then geocoded using a multilooked intensity image of the reference [60, 61] and coregistered. An initial list of persistent scatterer candidates was prepared on the basis of high temporal stability of the backscattering and low spectral diversity. These two criteria act as a proxy to identify the pixels containing single point-like scatterers exhibiting long-term temporal coherence. As a next step, we generated point differential interferograms for the candidates, using the (single) reference scene. It was assumed that the unwrapped differential interferometric phase is a sum of the phase contributions from the residual topography, deformation and the atmospheric path length delays. Exploiting the spatial and temporal baselines, we applied a 2D least-squares regression to obtain an initial estimate of the residual topography, \( h \) and (average) linear deformation velocity, \( v \). We evaluated the quality of the estimates in terms of the root-mean-square (RMS) phase deviation, \( \sigma_{\text{ipta}} \) between the differential phases and the model fit. Low RMS phase deviation represents a better match with the phase model, and hence indicates better quality. To ease computational burden of the processing, we curtailed the initial list of candidates using an adaptive point density reduction strategy [62]. The density of the candidates was reduced only in those local regions where it was too high (by masking preferentially the candidates of relatively low quality in the local neighborhood).

The atmospheric phase screen (APS) behaves as a nuisance in the regression fitting. Assuming the APS to be spatially low-frequency and temporally uncorrelated, we estimated it by spatial filtration and unwrapping of the phase residue in the neighborhood of the candidates that satisfied a quality criterion i.e. \( \sigma_{\text{ipta}} \) below a certain pre-selected threshold. The estimated APS was then subtracted from the differential interferograms and the regression-based estimation was iterated. After several iterations, the APS was well isolated and we obtained iteratively-refined estimates of the residual topography and linear deformation velocity.

In case of urban areas, especially when sensing high-rise buildings, the residual phase possibly contains a phase component associated with non-linear deformation which must be differentiated from the atmospheric phase. We used another regression-based routine that models the non-linear deformation as thermal expansion, assuming that the phase variations due to thermal expansion are linearly dependent on the temperature changes [40]. The regression coefficient is the phase-to-temperature sensitivity, \( \kappa \) [40, 43, 44]. The sensitivity is a measure of the phase variations undergone by a scatterer per unit change in temperature; the higher the sensitivity, the more pronounced is the thermal expansion. In this work, we used a single temperature value for the entire scene, i.e. one temperature per acquisition, for each layer, optimized in a way that minimizes the residual phase in each layer [40].

Toward the end of the processing, the PSI solution obtained on the reduced candidate list was expanded to include all the candidates in the initial list, excluding those that failed to satisfy the quality criterion. For more details on various interferometric processing strategies using the IPTA toolbox, the interested readers are referred to earlier works [13, 40, 62–64]. Throughout the rest of the paper, the term ‘persistent scatterers (PSs)’ identified with the PSI processing refers to the final set of candidates from the last
2. SAR TOMOGRAPHY AS AN ADD-ON TO PSI FOR URBAN AREAS

Figure 2.1: Characteristics of the interferometric data stack used in this work. (a) Distribution of the temporal baselines and the spatial baselines (orthogonal component). (b) Two-dimensional point spread function (PSF) in elevation-deformation plane.

Figure 2.2: Left: TerraSAR-X averaged intensity image of the observed urban area (Diagonal Mar, Barcelona, Spain). Right: Google Earth snapshot of the area. The blue pins mark the location of an example layover-affected pixel (a double scatterer containing signal returns from both the facade of the high-rise building as well as the roof-top of the triangle-shaped Diagonal Mar shopping center).

iteration.

2.2.2 Tomographic phase models

For a given range-azimuth pixel exhibiting ideal point scattering from one or more targets, assuming no additive or multiplicative noise, the mathematical model for classical SAR tomography (3D SAR) can be written as [23, 27, 30, 43, 51]:

\[ y_n = \int_{\Delta s} \gamma(s) \exp[-j \varphi_n(s)] \, ds. \]  \hspace{1cm} (2.1)

\( y_n \) is the SLC pixel value in \( n \)th layer of the coregistered and phase calibrated interferometric stack, for \( n = 0, 1, \ldots N - 1 \). \( \gamma(s) \) is the target reflectivity profile along the elevation, \( s \). \( \Delta s \) represents the observed
2.2. METHODS

Figure 2.3: PSI solution obtained with the Interferometric Point Target Analysis (IPTA) framework. The colored dots are the PSs identified in the PSI processing. Top: Estimated height, relative to WGS-84 reference ellipsoid. Middle: Deformation velocity in the line-of-sight (LOS). Bottom: Phase-to-temperature sensitivity.

elevation extent. In case of 3D SAR, the scatterer(s) is assumed to be stationary and the interferometric phase is modeled as follows:

$$\varphi_n (s) = 2k \Delta r_n (s)$$  \hspace{1cm} (2.2)

where $\Delta r_n (s)$ is the sensor-to-target path-length difference for the interferometric pair:

$$\Delta r_n (s) = r_n (s) - r_0 (s) \approx \frac{s^2}{2 \left( r_0 - b_{n} \right)} - \frac{b_{n}^\perp s}{r_0 - b_{n}^\parallel}.$$  \hspace{1cm} (2.3)

The range distance from sensor $n$ to the scatterer at elevation $s$ is represented by $r_n$. The orthogonal and parallel components of the $n^{th}$ spatial baseline are $b_{n}^\perp$ and $b_{n}^\parallel$, respectively. The layer $n = 0$ is considered as the reference layer. The phase model for 3D SAR depends only on the sensor-to-target geometry. It suffices in case the scatterer(s) remains stationary along the entire time series. However, in case there is some motion, there would be additional phase variations which must be accounted for in the phase
model. Assuming that the motion is a temporally linear displacement in the line of sight (LOS) over the entire time series, differential tomography [31, 35, 38] extends the phase model as follows:

\[ \phi_n(s, \nu) = 2k [\Delta r_n(s) + \nu t_n] \]  

(2.4)

where \( \nu \) is the (average) linear deformation velocity of the scatterer, and \( t_n \) is the temporal baseline for the \( n^{th} \) layer. The SLC pixel value from (3.2) can now be recast (as shown in [26, 33, 41]) as:

\[ y_n = \int_{\Delta \nu} \int_{\Delta \nu} \gamma(s, \nu) \exp \left[-j\phi_n(s, \nu)\right] ds d\nu \]  

(2.5)

where \( \Delta \nu \) is the range of the expected linear deformation velocity. Different studies, such as [17, 19, 39, 40], have shown that scatterers in urban areas may additionally be subject to non-linear motion due to thermal expansion of the buildings. Assuming that the LOS phase change due to thermal expansion of building structures is linearly dependent on temperature changes, the tomographic phase model is further extended as follows:

\[ \phi_n(s, \nu, \kappa) = 2k \left[\Delta r_n(s) + \nu t_n + \frac{1}{2k} \kappa \tau_n\right] \]  

(2.6)

where \( \kappa \) is the unknown phase-to-temperature sensitivity exhibited by the scatterer (or rather by the underlying physical structure), and \( \tau_n \) is the temperature change (with respect to the temperature for the reference layer) for \( n^{th} \) layer. With this phase model, the SLC pixel value is given by the following multi-variate integral [45]:

\[ y_n = \int_{\Delta \kappa} \int_{\Delta \nu} \gamma(s, \nu, \kappa) \exp \left[-j\phi_n(s, \nu, \kappa)\right] ds d\nu d\kappa. \]  

(2.7)

\( \Delta \kappa \) is the range in which the phase-to-temperature sensitivity values of the scatterer(s) is expected. These values can be related to the linear expansion coefficients of the materials [40].

For the sake of brevity, we label the aforementioned phase models as follows: ‘P1’ for the classical tomography (eq. 3.2), ‘P2’ for differential tomography (eq. 3.4), and ‘P3’ for the extended phase model which simultaneously models thermal expansion besides scatterer elevation and linear deformation (eq. 2.6). A general mathematical model for SAR tomography can be defined as follows:

\[ y_n = \int_{\mathcal{P}} \gamma(p) \exp \left[-j\phi_n(p)\right] dp \]  

(2.8)

where \( p \) is the unknown parameter vector for the scatterer, and \( \mathcal{P} \) represents the observed extent of the scatterer parameters i.e. the parameter space. In case of the phase model P1, \( p = [s] \); for P2, \( p = [s, \nu] \) and for P3, \( p = [s, \nu, \kappa] \).
2.2.3 Single-look tomographic inversion: Parameter estimation & scatterer detection

The estimation of the scatterer reflectivity, \(\gamma(p)\) and the scatterer parameter vector, \(p\) is an inverse problem, i.e. the tomographic model in eq. 2.8 has to be inverted starting with the SLC values (measurements). Additionally, a detection strategy has to be applied to ascertain whether a given pixel contains any coherent scatterers.

In urban areas, layover scenarios occur frequently. A layover-affected pixel may contain backscatter contribution from two or more scatterers that may individually be temporally coherent. In this work, considering that a given pixel has a maximum of two coherent scatterers, we make the following hypotheses: \(H_0\) – the pixel does not represent a stable scatterer, i.e. the SLC values represent merely noise; \(H_1\) – the pixel is a single scatterer, or \(H_2\) – the pixel is a double scatterer. The hypotheses are defined mathematically as follows [38]:

\[
H_0 : y = \mathbf{n} \tag{2.9}
\]
\[
H_1 : y = \gamma_1 a(p_1) + \mathbf{n} \tag{2.10}
\]
\[
H_2 : y = \gamma_1 a(p_1) + \gamma_2 a(p_2) + \mathbf{n}. \tag{2.11}
\]

\(y\) is the SLC vector:

\[
y = \begin{bmatrix} y_0 & y_1 & \ldots & y_{N-1} \end{bmatrix}^T. \tag{2.12}
\]

\(\gamma\) represents scatterer reflectivity and \(a(p)\) is the steering vector; the subscripts 1 and 2 indicate the first and the second scatterer in the pixel, respectively, with the first having more energy compared to the second. The general structure of the steering vector as a function of the unknown parameter vector \(p\) is given by

\[
a(p) = \begin{bmatrix} 1 & e^{-j\varphi_1(p)} & \ldots & e^{-j\varphi_{N-1}(p)} \end{bmatrix}^T. \tag{2.13}
\]

\(\mathbf{n}\) is the random noise vector generally considered zero-mean circular Gaussian. The structure of the phase values \(\varphi_n(p)\), and the unknowns in the parameter vector \(p\), depend on the choice of the phase model (eq. 3.2, 3.4 and 2.6).

Different methods have been proposed over time for tomographic inversion and scatterer detection, as in [26, 30, 34, 38, 51–54]. In this work, we use the reflectivity of single-look beamforming as a merit function for the parameter estimation problem. It can be directly associated with generalized likelihood ratio (GLRT)-based scatterer detection strategies[34, 38]. The reflectivity estimated with beamforming as a function of the parameter vector, \(p\) is given by

\[
\hat{\gamma}(p) = \frac{1}{N} a^H(p)y. \tag{2.14}
\]

Beamforming is a non-parametric method. The presence of one or more scatterers manifests in the peaks of the estimated absolute or squared reflectivity. Assuming the presence of at least one scatterer (i.e. either hypothesis \(H_1\) or \(H_2\) is true), we use the estimated absolute reflectivity as the objective
Figure 2.4: Reflectivity profiles obtained with tomography for an example layover-affected pixel (as marked with blue pins in Fig. 2.2). Results are shown for three different phase models: P1 (3D SAR), P2 (differential tomography) and P3 (elevation, linear deformation and thermal expansion are simultaneously modeled). The elevation coordinate has been projected to vertical (relative) height.

function in the following optimization:

\[
\hat{p}_1 = \arg\max_{p \in P} (|\hat{\gamma}(p)|).
\] (2.15)

The parameters in \( P \) that globally maximize the merit function are the estimated parameters for the first scatterer, \( \hat{p}_1 \). It is implicitly assumed that the steering vector for the first scatterer is orthogonal to both the noise vector and the steering vector of a second scatterer, if any. The estimated energy of the first scatterer, \( \hat{E}_1 \) (normalized with the total energy of the SLC vector) is as follows:

\[
\hat{E}_1 = \frac{|\hat{\gamma}(\hat{p}_1)|^2}{\|y\|^2}.
\] (2.16)

In case a second scatterer exists in the same pixel i.e. hypothesis \( \mathcal{H}^2 \) is true, in principle we may look for a second peak of the estimated reflectivity. However, a side-lobe of the first scatterer can always be mistaken for a second scatterer and thus lead to a false detection. An alternative way suggested by [38] is to subtract the estimated backscatter of the first scatterer from the SLC vector, and search for a potential second scatterer by applying inversion on the difference, \( y_c \):

\[
y_c = y - \hat{\gamma}(\hat{p}_1)a(\hat{p}_1)
= \hat{P}_1^\perp y
\] (2.17)

where

\[
\hat{P}_1^\perp = I_N - \frac{a(\hat{p}_1)a^H(\hat{p}_1)}{N}.
\] (2.18)

\( I_N \) is the identity matrix of size \( N \). \( \hat{P}_1^\perp \) can be considered as the projector onto the orthogonal comple-
2.2. Methods

Figure 2.5: Reflectivity in the height-deformation plane obtained with phase model P2 (differential tomography), i.e. \(|\hat{\gamma}(p)|\) with \(p = [s, v]\) (eq. 3.4).

The estimated energy of the second scatterer, \(\hat{\mathbf{E}}_2, c\) obtained after canceling the first scatterer from the SLC vector, and normalized with the energy of the difference, \(\mathbf{y}_c\) is given by

\[
\hat{\mathbf{E}}_{2,c} = \frac{|\mathbf{u}_c^H \mathbf{y}_c|^2}{\|\mathbf{y}_c\|^2}
\]

where

\[
\mathbf{u}_c = \frac{\hat{\mathbf{P}}_2^H \mathbf{a}(\hat{\mathbf{p}}_2)}{\|\hat{\mathbf{P}}_2^H \mathbf{a}(\hat{\mathbf{p}}_2)\|}
\]

To distinguish between the three hypotheses (\(\mathcal{H}_0\), \(\mathcal{H}_1\), and \(\mathcal{H}_2\)), we use the sequential generalized likelihood ratio test with cancellation (SGLRTC), as proposed in [38]. SGLRTC compares the normalized energies of the scatterers, \(\hat{\mathbf{E}}_1\) and \(\hat{\mathbf{E}}_{2,c}\), against preselected thresholds, \(T_1\) and \(T_2\), respectively. The test is sequential, as it comprises of two consecutive steps. Firstly, it is decided whether the pixel is a double scatterer or not, i.e. \(\mathcal{H}_2\) or \(\bar{\mathcal{H}}_2\), respectively:

\[
\hat{\mathbf{E}}_{2,c} \overset{\mathcal{H}_2}{\geq} T_2.
\]

If it is decided that the pixel is not a double scatterer, then it is tested against hypothesis \(\mathcal{H}_1\) (the pixel...
2. SAR Tomography as an Add-on to PSI for Urban Areas

Figure 2.6: Example of a tomographic inversion for the layover-affected pixel (as marked in Fig. 2.2 with blue pins) using the phase model P3, i.e. the scatterer elevation, temporally linear deformation and thermal expansion are modeled simultaneously (eq. 2.6). (a) The estimated three-dimensional reflectivity $|\gamma(s, v, \kappa)|$. (b) and (c) Slices of the reflectivity shown in (a) corresponding to the first and the second scatterer in the pixel, respectively. (d) The height-deformation plane showing the second scatterer after the cancellation of the first using the SGLRTC\[38\].

contains a single scatterer) and $\mathcal{H}_0$, (no scatterer detected), respectively:

$$\hat{E}_1 \geq T_1.$$  \hspace{1cm} (2.23)

The lower the thresholds, the more relaxed are the requirements on the minimum normalized energies to be detected as coherent scatterers. If the thresholds are too low, the energy of the clutter/noise may exceed the thresholds leading to false alarms. Similarly, a very high threshold may lead to missed detection. The choice of the thresholds can be associated with the desired probabilities of detection and false alarm. As suggested in [38], we keep the thresholds equal ($T_1=T_2$) to jointly maximize the probabilities of detection for both the single and double scatterers for a given probability of false alarm. For further details, interested readers are referred to [38] for a comprehensive discussion and comparison.
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![Figure 2.7: The number of single and double scatterers detected in the Diagonal Mar area using tomographic inversion and SGLRTC [38], for different thresholds of detection, $T_1$ and $T_2$, keeping $T_1 = T_2$. The tomographic phase model used simultaneously models scatterer elevation, linear deformation and thermal expansion (i.e. P3, eq. 2.6). The number of persistent scatterers identified with the IPTA-based PSI processing are also given for reference.](image)

of SGLRTC with other detection strategies. Although the thresholds $T_1$ and $T_2$ can be set to attain the desired probabilities of detection and false alarm, it remains to be assessed whether the thresholds can be related to the quality of the parameter estimates. We consider that the quality of the estimates is represented by the goodness of fit of the tomographic model with actual measurements (SLC values). Using the estimated parameters, we compute the estimated SLC vector, $y_{est}$ (model fit) and observe phase deviation between the estimated and the actual SLC values in each layer. Low phase deviations represent a better fit of the tomographic model to the measurements. We compute the root-mean-square (RMS) phase deviation as a metric for the goodness of fit, and hence as a metric for the quality of the estimates. In mathematical terms, the estimated SLC vector, $y_{est}$ for the single and double scatterers detected with SGLRTC is given by,

$$y_{est} = \begin{cases} \hat{\gamma}_1 a(\hat{p}_1) & \text{single scatterer} \\ \hat{\gamma}_1 a(\hat{p}_1) + \hat{\gamma}_2 a(\hat{p}_2) & \text{double scatterer} \end{cases} \tag{2.24}$$

where $\hat{\gamma}_1$ and $\hat{\gamma}_2$ are the estimated reflectivities of the first and second scatterer, respectively:

$$\hat{\gamma}_1 = a^H(\hat{p}_1) y \tag{2.25}$$

$$\hat{\gamma}_2 = a^H(\hat{p}_2) y_c / \| \hat{p}_1^a a(\hat{p}_2) \|. \tag{2.26}$$

The phase deviation for the $n^{th}$ layer, $\varphi_n^{res}$ is computed as follows:

$$\varphi_n^{res} = AD \{ \angle y_n, \angle y_{n,est} \} \tag{2.27}$$

where $y_{n,est}$ is the $n^{th}$ element of the vector $y_{est}$, $\angle (\cdot)$ returns the phase of the complex argument between 0 and $2\pi$, and the operator $AD \{\cdot, \cdot\}$ returns the absolute angle deviation between its arguments as
follows:

\[ AD\{w_1, w_2\} = \begin{cases} 
  d & d \leq \pi \\
  2\pi - d & d > \pi 
\end{cases} \]  

(2.28)

for \( d = |w_1 - w_2| \). The RMS phase deviation, \( \sigma^{\text{tomo}} \), is given by,

\[ \sigma^{\text{tomo}} = \sqrt{\frac{1}{N-1} \sum_{n=1}^{N} (\phi^{\text{res}}_n)^2} \]  

(2.29)

The quality of the estimates can also be related to the energy of the residue, i.e. the squared norm of the model mismatch, \( \|y - y_{\text{est}}\|^2 \). Low residual energy would imply a better match. However, in this work, we use phase deviations as a measure of model mismatch in order to be consistent with the quality assessment in IPTA-based PSI processing (which ignores amplitudes and operates only on the phases of the complex SLC values in the regression-based parameter estimation).

### 2.3 Data stack

The interferometric data stack used in this work comprises of 50 TerraSAR-X stripmap images acquired in repeated passes over the city of Barcelona, Spain. The images are spread over a time span of approximately 5 years, from December 2007 till October 2012. The polarization mode of the images is VV. The slant range resolution, \( \delta_r \), is 1.2 m, corresponding to a chirp bandwidth of 150 MHz. The resolution in azimuth is 3.3 m. The images were oversampled by a factor of 2 to allow for a more accurate coregistration. The pixel spacing in range and azimuth is 0.455 m and 1.89 m, respectively.

The distribution of both the temporal and spatial baselines is highly non-uniform, as shown in Fig. 2.1(a). The total orthogonal spatial baseline i.e. the aperture in elevation, \( B^\perp \), is 503.2 m. The resolution in elevation, \( \delta_s \), had the spatial baselines been uniformly distributed, would be

\[ \delta_s = \frac{\lambda r_0}{2B^\perp} = 19.2 \text{ m} \]  

(2.30)

where \( \lambda = 3.1 \text{ cm} \) and \( r_0 = 622.8 \text{ km} \). The corresponding resolution in height would be 11.1 m, for an incidence angle of 35.3° for the reference layer. The resolution in deformation velocity, assuming uniform temporal baselines, is

\[ \delta_v = \frac{\lambda}{2T} = 3.26 \text{ mm yr}^{-1} \]  

(2.31)

where \( T = 1738 \text{ days} \) is the total temporal span. Due to the non-uniformity of the baselines, the point spread function function (PSF) is distorted, as shown in Fig. 2.1(b). For a given range bin, the range migration due to the different viewing angles in each pass should be very small compared to range resolution range, which sets an upper limit on the observed extent of elevation, \( \Delta s \) [30]:

\[ \Delta s \ll \frac{\delta_v r_0}{B^\perp} = 1485 \text{ m}. \]  

(2.32)

Although we would expect an ambiguity in tomographic inversion with uniform baselines at an eleva-
2.4 Results

This section presents the results obtained on the interferometric data stack introduced in the previous section. The urban area under study is Diagonal Mar, Barcelona. The average SAR backscatter image and the corresponding Google image of the area are shown in Fig. 2.2. There are various man-made structures in the scene which may exhibit temporally coherent scattering. The area includes both lay-over (high-rise buildings) as well as non-layover affected areas, and therefore it exemplifies a general urban scenario from an operational point of view. As the temporal span of the acquisitions in the data stack extends over many seasons, we expect to see some non-linear deformation (besides any possible...
Figure 2.9: Single scatterers obtained with tomographic inversion and SGLRTC, and projected on Google Earth 3D building models. The thresholds for detection, $T_1$ and $T_2$, are set at 0.4. The colors represent the estimated parameters. Top row: Relative height. Middle row: Average deformation velocity in LOS. Bottom row: Phase-to-temperature sensitivity.
Figure 2.10: Double scatterers obtained with tomographic inversion and SGLRTC, and projected on Google Earth 3D building models. The thresholds for detection, $T_1$ and $T_2$, are set at 0.4. The colors represent the estimated parameters. Top row: Relative height. Middle row: Average deformation velocity in LOS. Bottom row: Phase-to-temperature sensitivity.
2.4.1 PSI solution

We perform a PSI analysis of the data stack using the Interferometric Point Target Analysis (IPTA) framework. The final solution is obtained after several iterations of the phase regressions for the estimation of scatterer residual height, linear deformation and the phase-to-temperature sensitivity, and the separation of the atmospheric phase screen (APS) from the interferometric stack. In this work, we used a threshold of 1.1 rad for the RMS phase deviation, $\sigma^{ipta}$, i.e., the set of persistent scatterer candidates with $\sigma^{ipta}$ greater than 1.1 rad were rejected in the iterative regression-based parameter estimation. Fig. 2.3 shows the persistent scatterers (PSs) identified in the PSI processing and the estimated scatterer parameters (relative height, LOS deformation velocity and phase-to-temperature sensitivity).

2.4.2 Tomographic inversions & scatterer detection

Prior to tomographic inversion, the interferometric stack needs to be phase calibrated. The APS isolated in the PSI processing for the PSs is extrapolated to all the pixels in the scene, and subtracted from the differential interferograms to allow for single-look tomographic inversions on all pixels. The reference layer, spatial and temporal baselines, and the reference point used for the tomographic inversion are the same as in the PSI processing. The reference temperature values used for modeling thermal expansion (due to the thermal expansion of the building structures).
2.4. Results

The estimation of scatterer parameters (elevation, deformation velocity and phase-to-temperature sensitivity) with beamforming-based tomographic inversion is otherwise independent of the PSI solution. The observed extents of the parameters are as follows: $\Delta s = [-50, 300]$ m, $\Delta \nu = [-5, 5]$ mm/yr and $\Delta \kappa = [-1, 1]$ rad/K.

A layover-affected pixel

The blue pins in Fig. 2.2 mark the location of a layover-affected pixel which has not been identified as a PS in the PSI processing. It is expected that the pixel contains two scatterers, one situated on the facade of a tall building while the other on the roof-top of a nearby building of relatively short height. We apply tomographic inversion for this pixel with each of the three different phase models discussed in section II.B. The reflectivity profiles retrieved for each phase model are shown in Fig. 2.4. For the case of classical SAR tomography (phase model P1, eq. 3.2), the reflectivity profile shows the presence of a single scatterer around 20 m. Similarly, differential tomography (phase model P2, eq. 3.4) also reveals the presence of a single scatterer. In addition to height, differential tomography models temporally linear deformation. Fig. 2.5 shows the two-dimensional reflectivity in the height-deformation plane. Tomographic inversion with the extended phase model (which simultaneously models scatterer height, linear deformation and thermal expansion; P3, eq. 2.6), is also applied. The three-dimensional reflectivity space is shown in Fig. 2.6. It can now be seen that the pixel is in fact a double scatterer, containing two individual scatterers of nearly comparable energy. The layover is resolved. The estimated heights of the first and the second scatterers are nearly 20 m and 100 m, respectively. The estimated normalized energy for the first scatterer is 0.6, while the normalized energy estimated for the second scatterer (after the cancellation of the first with SGLRTC) is 0.4.
Figure 2.13: Histograms of the RMS phase deviation, $\sigma_{\text{tomo}}$ for (a) scatterers detected as single scatterers, and (b) scatterers detected as single scatterers in addition to the persistent scatterers identified with IPTA-based PSI processing, at different thresholds (shown in different colors). The deviation bars show the interquartile range.

DIAGONAL MAR

Tomographic inversion with the extended phase model P3 is applied on the entire Diagonal Mar area, for all the pixels, whether identified as PS or not during the PSI processing. Single and double scatterers are detected using the SGLRTC-based detection strategy. The number of scatterers detected with SGLRTC is directly influenced by the choice of the thresholds, $T_1$ and $T_2$. Fig. 2.7 shows the variation in the number of the detected scatterers against the thresholds (while keeping $T_1 = T_2$). The lower the thresholds, the more are the detected scatterers. However, for thresholds below 0.3, we observe a sudden jump in the number of double scatterers accompanied by a sharp decline in the gradient for single scatterers. For thresholds between 0.3 – 0.4, we have nearly twice as many scatterers obtained with tomography than the PSs identified with PSI. For thresholds greater than 0.6, only a very few double scatterers are detected and the total number of scatterers obtained with tomography drops below the number of PSs. At a moderate value of 0.4 for the thresholds, Fig. 2.8 shows the single scatterers, as well as the scatterers with higher elevation among the double scatterers (usually referred to as the upper layer), in radar co-ordinates. Fig. 2.9 and Fig. 2.10 show the single and double scatterers, respectively, projected on Google Earth 3D building models. The color coding represents the estimated parameters.

2.4.3 Relative gain in deformation sampling with layover separations

We intend to assess how effective is SAR tomography in increasing deformation sampling (by resolving layovers) in comparison to a PSI-based analysis. To this end, we define the relative gain in deformation sampling, $G$ as follows:

$$G = \left( \frac{2N_{d,u} + N_{d,psi}}{N_{psi}} \right) \times 100\%.$$  (2.33)

$N_{d,u}$ is the number of pixels that are uniquely detected as double scatterers, i.e. the pixels were not identified as PSs in the PSI processing; $N_{d,psi}$ is the number of those pixels that are detected as double...
scatterers but were also identified as PSs, and \( N_{\psi} = 36312 \) is the total number of the PSs identified in the scene. We would expect that a PSI analysis should reject all the pixels containing double scatterers. However, if one of the two scatterers tends to dominate the other, the pixel may still get selected as a PS. Fig. 2.11 shows the double scatterers and the corresponding gain at the different thresholds of detection. It can be seen that the relative gain drops sharply with increasing thresholds. The choice of the thresholds should consider both the gain and the quality of the scatterers, as argued in the following subsection.

2.4.4 Quality of the estimates

The quality of the estimates obtained with IPTA-based PSI processing and beamforming-based tomographic inversion is assessed in terms of the RMS phase deviations, \( \sigma^{\text{ipta}} \) and \( \sigma^{\text{tomo}} \), respectively. The lower the RMS phase deviation, the better is the perceived quality of the estimates.

**Persistent scatterers (PSs) identified with IPTA-based PSI processing**

Fig. 2.12 shows the joint and marginal distributions of the \( \sigma^{\text{tomo}} \) and \( \sigma^{\text{ipta}} \) for the pixels that have been identified as the PSs in IPTA-based PSI processing. A strong correlation is clearly visible. The average values of the marginal distributions for tomography and PSI are 0.56 and 0.57 rad, respectively. There are a very few pixels (< 0.01%) for which \( \sigma^{\text{tomo}} \) is higher than 1.1 rad, which was used as the upper limit in the IPTA-based PSI processing.

**Single scatterers obtained with tomography**

Fig. 2.13a shows the distributions of the RMS phase deviation, \( \sigma^{\text{tomo}} \) for the single scatterers obtained with tomography at different thresholds. It can be seen that progressively increasing the thresholds improves the overall quality, but alongside there is a drop in the number of the detected scatterers.

Fig. 2.13b shows the distribution of RMS phase deviation, \( \sigma^{\text{tomo}} \) for the single scatterers obtained with tomography in addition to the PSs identified with the PSI processing at various thresholds. The quality improves with increasing thresholds, but it can be seen that on average, these scatterers exhibit a lower quality than for the PSs. To ensure that at least 95% of these pixels should have \( \sigma^{\text{tomo}} \) below 1.1 rad, we need thresholds of 0.40 or higher.

**Double scatterers obtained with tomography**

Fig. 2.14 shows the distribution of the RMS phase deviation, \( \sigma^{\text{tomo}} \) for the double scatterers obtained with tomography at different detection thresholds. There is a trade-off between the quality of the scatterers and the relative gain in deformation sampling, \( G \) (shown in Fig. 2.11). For \( T_1 = T_2 = 0.40 \), around 99% of the detected double scatterers have \( \sigma^{\text{tomo}} \) less than 1.1 rad, with a gain of 9.8%, while the average \( \sigma^{\text{tomo}} \) is 0.66 rad.

For a pixel that is truly a double scatterer, we expect that modeling the second scatterer after the first should lead to an improvement in quality in terms of reduced phase deviation between the measurements and the model fit. To this end, we define the relative decrease in the RMS phase deviation, \( \Delta \sigma \) as
follows:

\[
\Delta \sigma = \frac{\sigma_{\text{tomo}|\mathcal{H}^1} - \sigma_{\text{tomo}|\mathcal{H}^2}}{\sigma_{\text{tomo}|\mathcal{H}^1}}
\]  

(2.34)

where \( \sigma_{\text{tomo}|\mathcal{H}^1} \) is the RMS phase deviation computed considering only one scatterer in the pixel, while \( \sigma_{\text{tomo}|\mathcal{H}^2} \) is the RMS phase deviation when both the scatterers are modeled (referred to eq. 2.24). Fig. 2.15 shows the distribution of \( \Delta \sigma \) for the pixels detected as double scatterers at different thresholds. It can be seen that \( \Delta \sigma \) tends to increase with increasing thresholds, implying that the quality improvement is higher at higher thresholds. Fig. 2.15 also shows that for a few double scatterers, \( \Delta \sigma \) is undesirably negative implying a worsening of the quality. With increasing thresholds, the percentage of such pixels decreases. For the double scatterers detected at \( T_1 = T_2 = 0.4 \), there is 19% (on average) improvement in the quality, while around 2% of the double scatterers witness a decrease in quality.

2.4.5 Quantity and quality of the scatterers on a single layover-affected building

Fig. 2.16 shows the relative gain in deformation sampling and the RMS phase deviation, \( \sigma_{\text{tomo}} \) for the single and double scatterers detected on a single layover-affected building (Hilton hotel, as marked in the red outline). Again, we observe a sharp decline in the gain, and an improvement in quality, with increasing thresholds of detection. It can also be seen (in comparison with Fig. 2.11) that for the same thresholds, the relative gain in deformation sampling is higher for the case of the single building than for the Diagonal Mar area in general. This is naturally expected since it is the layover-affected areas where PSI is likely to reject candidates and tomography likely to provide a relative gain by resolving the layovers. At \( T_1 = T_2 = 0.4 \), the relative gain is 31% for the single building as opposed to 9.8% for Diagonal Mar.

2.5 DISCUSSION

This section provides an itemized discussion of the results presented in the previous section.

2.5.1 PSI solution

The persistent scatterers (PSs) identified in the IPTA-based PSI processing, and the estimated residual height, linear deformation and the phase-to-temperature sensitivity for each PS, are shown in Fig. 2.3. The residual heights shown are relative to the WGS-84 ellipsoid. The estimated heights correspond fairly well with the actual heights of the buildings. The area is mostly stable, except for some regions which are undergoing deformation as much as 3-5 mm/yr (Forum area in the northeast). It can be seen that for most of the buildings, the estimated phase-to-temperature sensitivity increases with height, implying that the upper parts of the building tend to undergo more thermal expansion. This seems plausible since the base of the buildings is grounded and the upper parts are less constrained in terms of expansion. However, it may not be true for all buildings. The structure of the individual buildings would likely have a significant impact on their thermal expansion characteristics. In an earlier work [40], the estimated phase-to-temperature sensitivities were related to the linear expansion coefficient of the building material.
2.5. Discussion

Figure 2.14: Histograms of the RMS phase deviation, $\sigma_{tomo}$ for double scatterers obtained with tomography, at different detection thresholds (shown in different colors). The deviation bars show the interquartile range.

It can be seen in Fig. 2.3 that the deformation sampling achieved with the PSI processing is fairly reasonable. Even for the layovers (high-rise buildings), the PSI solution provides a decent coverage. In case of a layover-affected pixel, quite often only one of the superposing scatterers behaves coherently over time, and when it tends to dominate the other scatterer(s) in terms of energy, the pixel may still be well-identified as a PS. In case the energy of the superposing scatterers is comparable and they exhibit temporal phase stability, the pixel would be rejected in the PSI processing, but tomography would be able to individually detect them and thus provide an added value.

2.5.2 Tomographic inversion & scatterer detection

A layover-affected pixel

The results obtained with classical SAR tomography (P1, eq. 3.2) and differential tomography (P2, eq. 3.4) are reasonable in their own right, but incorrect as they lead to the false inference that the pixel contains a single dominant scatterer (the first scatterer). The layover is resolved only for the extended phase model (P3, eq. 2.6) when the scatterer elevation, linear deformation and thermal expansion are modeled simultaneously. It implies that modeling the thermal expansion of the scatterers, in addition to their elevation and linear deformation, is indeed critical for effective layover separation, especially in case of high-rise buildings. The estimated heights of the two individual scatterers correspond fairly accurately with the actual heights of the two buildings. In addition, we obtain estimates of the linear deformation velocity as well as the phase-to-temperature sensitivity for each of the two scatterers. Considering the pixel was originally rejected during the PSI analysis, the layover separation provides two additional deformation samples. In this way, SAR tomography provides a value-addition to the PSI analysis by improving deformation analysis in layover-affected areas.
2.5.3 Relative gain in deformation sampling with layover separations

It can be seen in Fig. 2.11 that among the total pixels detected as double scatterers with tomography (for thresholds greater than 0.4), nearly half of them have also been identified as PSs in the IPTA-based PSI processing. It clearly indicates that tomography has improved the deformation sampling not only
by resolving those layovers which were altogether rejected in the PSI processing, but also by detecting
the second scatterer for those where the first (dominant) was identified as a PS. The relative gain in
deformation sampling, $G$ (eq. 2.33) declines sharply as the thresholds are increased due to a sharp
decrease in the number of double scatterers. For thresholds between 0.3 – 0.4, the gain varies between
22 – 9.8%; at 0.6, the gain is already below 1.5%. In order to choose a suitable range for the thresholds,
such that they are neither too restrictive nor too loose, it is imperative to consider the corresponding
quality of the estimated parameters.

2.5.4 Quality of the estimates

Persistently scatterers (PSs) identified with IPTA-based PSI processing

The average $\sigma^{IPTA}$ for the entire set of pixels identified as PSs is 0.57 rad, which is well below the upper
limit of 1.1 rad set in the IPTA processing, implying the overall good quality of the estimates obtained
with PSI. For these pixels, the corresponding quality of the estimates computed with tomography is
comparable, as indicated by the strong correlation between the $\sigma^{IPTA}$ and $\sigma^{tomo}$ (shown in Fig. 2.12).

Single scatterers obtained with tomography

It can be seen in Fig. 2.13a that progressively increasing the thresholds tends to reject the detected
single scatterers of relatively lower quality, while those with higher quality seem mostly unaffected. In
this sense, the threshold $T_1$ can be considered as a proxy for the quality of single scatterers. In order
that the quality of these scatterers is comparable with the quality of the PSs, we need thresholds greater
than 0.5. However, as discussed earlier, for thresholds above 0.5 we detect very few double scatterers, and
the relative gain in deformation sampling is merely 4% or less. Similarly, if we only observe the single
scatterers in addition to PSs, as shown in Fig. 2.13b, to match their quality with those of the PSs would
require thresholds that are too restrictive to allow for the detection of a reasonable number of double
scatterers.

Double scatterers obtained with tomography

Fig. 2.14 shows the quality of the double scatterers detected at different thresholds. As for the case
of single scatterers, there is a trade-off between the quality and the quantity of the double scatterers.
However, it can be seen that the effect of progressively increasing the thresholds on the average quality
is not as pronounced as it has been for the case of single scatterers. With increasing thresholds, some
pixels of relatively good quality are also rejected. This is not unexpected, since the phase residue for a
double scatterer depends on the quality of both the first and the second scatterer. It could be that the
second scatterer suffers from phase noise leading to poor inversion; a high value of $T_2$ may reject it as a
double scatterer although it may still exhibit a relatively low $\sigma^{tomo}$ due to a possibly good inversion
of the first scatterer. At the same time, there is a pronounced relative increase in the quality by modeling
the second scatterer in addition to the first, as depicted by the increasing $\Delta \sigma$ with increasing thresholds
in Fig. 2.15. The pixels detected as double scatterers but with $\Delta \sigma < 0$ are possibly false alarms; the
percentage of these pixels tends to decrease with increasing thresholds.
2.5.5 Relative gain and quality of the scatterers for a single layover-affected building

In Fig. 2.16, we can clearly observe the trade-off between the quantity and the quality of the scatterers for the case of a single (layover-affected) building as well. In order that the quality of the single scatterers obtained with tomography is on a par with the quality of the PSs identified with IPTA, we require $T_1 = T_2 >= 0.55$, which allows in this case around 10% gain in deformation sampling relative to the PSI solution. For thresholds above 0.6, we have too few detections to compute the statistics. Keeping the thresholds at 0.4 would allow as much as 31% gain, but of course with a compromise in quality.
2.6 Conclusion

In this paper, we have performed a case study on the usefulness of single-look SAR tomography as an add-on to persistent scatterer interferometry (PSI) for improving deformation analysis in urban areas using an interferometric stack of 50 TerraSAR-X images of an urban locality in Barcelona, Spain. Stripmap-mode images have been used in the study, keeping in view their wider prevalence compared to higher resolution imaging modes which are normally operated for a limited number of sites. A PSI analysis is performed using the Interferometric Point Target Analysis (IPTA) framework, and is followed by tomographic inversion on the phase-calibrated stack employing a single-look beamforming-based merit function. The phase models for classical SAR tomography (3D SAR), differential tomography, and the one further extended to simultaneously model thermal expansion, are compared against each other with respect to their suitability in resolving layovers. The results obtained confirm that modeling thermal expansion of the scatterers, in addition to linear deformation and elevation, is indeed critical for effective layover separations, especially in the case of high-rise buildings.

The PSI solution obtained with the IPTA framework offers a fairly reasonable coverage. Even in layover areas, many pixels have been identified as PSs (where one among the superposing scatterers tends to dominate the others and exhibits long term coherence). At the same time, there is a significant number of layover-affected pixels along the facades of the high-rise buildings which have been either totally rejected in the PSI processing, or partially rejected in the sense that a second coherent scatterer is present but has not been individually identified as a PS (since a PS is by definition a single dominant scatterer). Tomographic inversion effectively resolves these layovers and thus prevents the aforementioned rejections. Therefore, while PSI suffices for a large-scale deformation analysis, SAR tomography proves useful in performing a small-scale analysis for local infrastructure by increasing deformation samples in layover-affected areas.

We have used a generalized likelihood ratio-based hypothesis testing (following the tomographic inversion) to detect single and double scatterers in the observed scene. The impact of the detection thresholds on both the quantity and the quality of the detected scatterers has also been studied, and the relative gain in deformation sampling with layover separations is computed at different thresholds. The quality of the detected scatterers is assessed in terms of the root-mean-square (RMS) phase deviation between the measurements (SLC values) and the tomographic model fit, which is consistent with the way the quality of the persistent scatterers is evaluated in the IPTA-based PSI processing. The results obtained highlight a trade-off between the quantity and the quality; therefore, the choice of the detection thresholds has to be determined as a compromise between the desired values of the relative gain in deformation sampling and the corresponding quality. In order that the quality of the overall detected scatterers is comparable to the quality of the PSs identified with the PSI processing, the detection process may become too restrictive to allow for a sufficient number of layover separations. With a moderate compromise, we obtain a relative gain in deformation sampling of 9.8% for the Diagonal Mar area, while the RMS phase deviation for 99% of the detected double scatterers and single scatterers (in addition to the PSs) is below 1.1 rad (which is set as the upper limit to allow for the acceptance of a PS candidate in the final PSI solution). For the double scatterers, the relative decrease in the RMS phase deviation by modeling a second scatterer in addition to the first is 19% on average. This improvement in quality
clearly indicates that even in case the first scatterer was identified as a PS with a PSI approach, SAR to-
mography additionally allows detecting the second scatterer and thereby offers an improvement in the 
quality.

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A CASE STUDY ON THE CORRECTION OF ATMOSPHERIC PHASES FOR SAR TOMOGRAPHY IN MOUNTAINOUS REGIONS


Author’s contributions

• Conceived and implemented the proposed phase calibration approach.
• Developed the tomographic processing framework.
• Drafted the manuscript, and interpreted the results.

Co-authors’ contributions

• T. Strozzi provided the PSI solution used in this work. He and O. Frey initiated using SAR tomography in alpine areas.
• O. Frey developed several routines for data preprocessing and PSI/tomo integration; contributed to the variogram analysis; and reviewed the interpretation of the results.
• All co-authors reviewed the manuscript.

Data sources

The Digital Elevation Model (DEM) used in this work is © swisstopo (JD100042). The data stack used in this work is a COSMO-SkyMed Product - ©ASI - Agenzia Spaziale Italiana - (2013). All Rights Reserved.
3. CORRECTION OF ATMOSPHERIC PHASES FOR SAR TOMOGRAPHY IN MOUNTAINOUS REGIONS

ABSTRACT

Synthetic aperture radar (SAR) tomography with repeat-pass acquisitions generally requires a priori phase calibration of the interferometric data stack by compensating for the atmosphere-induced phase delay variations. These variations act as a disturbance in tomographic focusing. In mountainous regions, the mitigation of these disturbances is particularly challenging due to strong spatial variations of the local atmospheric conditions and propagation paths through the troposphere. In this paper, we assess a data-driven approach to estimate these phase variations under a regression-kriging framework. The vertical stratification of the troposphere is modeled functionally while the impact of spatial turbulence is considered in a stochastic sense. The methodology entails an initial persistent scatterer interferometry (PSI) analysis. The atmospheric phases isolated for the persistent scatterers (PS) within the PSI processing are considered as samples of the 3-D distribution of the phase delay variations over the scene. These atmospheric phases are regressed against the spatial coordinates in map geometry at PS locations. In turn, kriging predictions are obtained at each location along the elevation profile where tomographic focusing is intended. A key point of this approach is that the requisite atmospheric corrections are incorporated within the tomographic focusing model. A case study has been performed on a data stack comprising 32 Cosmo-SkyMed stripmap images acquired over the Matter Valley in the Swiss Alps, in the summers of 2008-2013. The results show locally improved deformation sampling with tomographic methods compared to the initial PSI solution, primarily due to the improved phase calibration. In general, the work underscores the indispensability of height-dependent correction of atmospheric phases for SAR tomography.

3.1 INTRODUCTION

Synthetic aperture radar imaging involves two-way wave propagation through the atmosphere which acts as a refractive medium. The propagation is through both the troposphere and parts of the ionosphere for space-borne sensors in low earth orbit. Ionospheric effects on microwaves are frequency-dependent, and can often be ignored for high-frequencies such as the X-band [1, 2]. The optical path traversed by the waves in the troposphere is longer than the geometric path; therefore, additional phase delays are accumulated. The refractivity of the troposphere is mainly governed by temperature, pressure and water vapor [3, 4]. These factors typically vary in time and space – therefore, the refractivity changes over the scene as well as from one pass to the next, incurring variable phase delays which in general do not cancel out in interferogram formation leaving behind a phase footprint that we refer to as the atmospheric phase in this paper. These phases are typically a nuisance. If left uncorrected, the atmospheric phases appear as erroneous deformation in differential interferometric processing [5–8]. In case of SAR tomography [9–12], these phases act as a disturbance in focusing the scatterers in 3-D [13–17]. Therefore, prior to tomographic inversion, the interferometric data stack requires a precise phase calibration i.e. an estimation and compensation of the atmospheric phases in each interferogram.

The impact of the atmosphere is commonly distinguished in two aspects: firstly, the 3-D heterogeneities in refractivity caused by turbulent mixing; and secondly, a general decrease in refractivity with increasing altitude under normal atmospheric conditions, considering the lower troposphere to be a vertically
3.1. Introduction

stratified medium comprising thin layers that are horizontally homogeneous. In mountainous regions, both the turbulent mixing and vertical stratification contribute to variable phase delays in the interferograms, contrary to regions of flat topography where only turbulent mixing effects are relevant [3, 18]. The dominant part of the atmospheric phase is due to water vapor distribution [19–21], which is highly variable around the globe, and the more so in mountainous regions where the wind rises over or around the orography causing local overturning and condensation [22]. Therefore, the atmospheric phases in mountainous regions may exhibit strong spatial variations. In fact, correction of these phases remains a challenging problem.

3.1.1 Motivation for SAR tomography in mountainous regions

In case of urban areas, single-look SAR tomography has been proposed as an add-on to persistent scatterer interferometry (PSI) [23–27], with the objective of improving coverage of the PSI solution by separating the coherent scatterers in layover and simultaneously estimating their deformation parameters [28–31]. As such, layovers are generally more widespread in case of mountainous regions due to drastic variations in topography. And at the same time, often the slopes of interest in terms of critical mass movements happen to be in layover. Consequently, the coverage of PSI solution along such slopes remains limited [32, 33]. The use of SAR tomography as an add-on to PSI in mountainous regions—though already at a loss given the typically low prevalence of coherent backscattering in these regions—can yet be rewarding as it may extend the coverage to slopes in layover. With this motivation, we intend to investigate the possibility of using SAR tomography in an alpine region. However, an immediate concern is how to achieve the requisite phase calibration.

3.1.2 Related work from SAR interferometry

Several investigations have addressed atmospheric phases since the first reports [5, 19, 20, 34], albeit only in the context of SAR interferometry. The methods proposed to correct these phases can be broadly categorized into those which utilize auxiliary information from external sources to estimate the phase delays, and those which are purely data-driven. In the former category, the commonly used auxiliary information are the output of numerical weather models, such as the Weather Research and Forecasting (WRF) model [35] and the global atmospheric reanalysis data from the ERA-Interim project [36]. The hindcast of the relevant meteorological parameters from the weather models is used to estimate 3-D field of refractivity of the atmosphere at the time of SAR acquisitions. These estimates are subsequently used to generate synthetic phase delay maps to compensate for the atmospheric effects [37, 38]. The usefulness of the weather models is often questionable since the resolution of the weather data is too coarse and they are generally not concurrent with the SAR acquisitions [39]. Global Navigation Satellite Systems (GNSS) measurements can also be used as auxiliary information. In general, the wet zenith delays measured with GNSS measurements are converted to delays in slant range followed by spatial interpolation over the area imaged by SAR [40]. Several techniques for the assimilation of GPS measurements into interferometric processing have been proposed over time [40–43]. GNSS-based approach is
generally effective, but its application remains limited as permanent GNSS stations are often absent or sparsely distributed for most areas around the world.

In the absence of or unsuitability of external sources due to the aforementioned limitations, purely data-driven alternatives are commonplace. They work directly on the interferometric data, customarily adopting spatio-temporal filtering to isolate the phase variations typically signified by atmosphere relative to other contributors such as deformation, residual topography as well as those occurring due to the multiplicity of the scatterers (in the context of tomography). The phase variations variations are typically considered to be spatially correlated up to a certain extent and uncorrelated in time [23, 24, 44–46] (especially in case of repeat-pass spaceborne acquisitions). In general, a network of several interferograms is desired to assess statistical properties of the different sources of phase variations. Any filtering in time should preserve non-linear deformation, if any, as emphasized by several studies [43–45, 47]. Moreover, temporally phase coherent targets, i.e. persistent scatterers (PS) [23, 24, 48] are needed to reliably extract the different phase contributions and to restrict the propagation of decorrelation noise during interferometric processing (phase filtering, unwrapping, etc.). The atmospheric phases estimated for the PS are low pass filtered and spatially interpolated over the scene, for example with a kriging interpolator [6, 23].

A linear dependence of interferometric phases on topography is adopted in several studies to model a vertically stratified atmosphere [49–52]. The investigation in [53] suggests a power-law model as a generalization of the linear model. While the topography-dependent part of atmospheric phases is typically modeled functionally, the contribution of turbulent mixing is often dealt with in a stochastic sense. The spatial heterogeneity in the atmospheric signal is modeled as a random process. The second-order statistics of the process are estimated by transforming the spatial spectra to corresponding spatial structure or covariance functions [3, 54, 55]. Under Kolmogorov turbulence theory in isotropic conditions, the spectrum and the associated covariance function theoretically follow a power-law distribution [19, 40]. In practice, spectra are quite variable and may not always be structured according to the Kolmogorov theory [3, 56]. The power-law exponent varies in different scaling regimes, and needs to be empirically estimated [41, 57]. As the lag between the observation points increases, the variance among them increases. In this way, an empirical variance-covariance matrix can be built up for data quality description as well as to constrain underlying geophysical processes that require the covariance of the noise [56, 58, 59]. Moreover, it can be used to improve spatial interpolation of the atmospheric phases from the observed locations (whether individual GPS sites, weather stations or PS locations where we have already estimated the atmospheric phases) towards the unobserved locations in the scene [60–62]. In [62], a 2-D kriging interpolation in easting-northing with a linear semivariogram model is used. The results obtained are shown to be better than spline interpolation.

### 3.1.3 Research gaps

To date, all atmospheric phase correction approaches estimate a single correction for each pixel. When the topography-dependence is modeled, this correction is estimated at the specific height for the given pixel. In the context of tomography in mountainous regions, multiple coherent scatterers may lie in the same pixel while being separated in height by as much as a kilometer or more, as shown in Fig. 1. Due to
the difference in the vertical refractivity profiles of the troposphere at each epoch relative to the master acquisition, the atmospheric phase delays experienced by each scatterer can be very different. While a single correction may work for one of the scatterers, the uncorrected phase delays for the second or more scatterers would prevail as noise and may hamper tomographic focusing. Consequently, modeling vertical stratification of the troposphere within the same pixel is required. Moreover, due to the side-looking geometry of the SAR sensors, the large difference in height between the scatterers in layover results in comparably large lateral separation among the scatterers. Therefore, a method that corrects for atmospheric phases for scatterers at different height levels must also account for possible lateral variations. In turn, any spatial interpolation should weight over observation points from local physical neighborhoods of each scatterer.

3.1.4 Contributions & limitations of this paper

Deformation assessment using differential tomography has thus far been applied in urban regions or specific infrastructure where the underlying topography is generally flat [14, 31, 63–65]. This investigation is a first step towards facilitating its application in rugged mountainous terrain by providing the following specific contributions filling in the aforementioned research gaps:

1. A data-driven, regression-kriging framework is presented to model both vertical stratification and mixing effects.

2. For a given layover-affected pixel, multiple corrections are applied for potentially multiple scatterers at different positions along the elevation axis.

3. These corrections are incorporated within tomographic focusing at each 3-D point of interest prior to inversion and scatterer detection.

At the same time, this paper does not specifically address some other potential limitations on the performance of tomography in mountainous regions. These include the possibility of range migration of the target from a given range bin to the adjacent bin when the tomographic focusing is performed over very large elevation extents [31, 64]. The severity of the problem depends on the baseline distribution as well as on the magnitude of topography changes. Another potential limitation is incurred as an aftermath of topography-induced radiometric effects [66, 67]. The physical pixel-area in layover and foreshortening-affected pixels may be too large to allow point-like scattering, which in turn reduces the prevalence of coherent scatterers.

3.1.5 Outline

The methodology proposed in this work entails a prior PSI analysis. The Interferometric Point Target Analysis (IPTA) [24] toolbox is used to iteratively identify a set of PS (which are single dominant scatterers). The atmospheric phases for these PS are estimated within the PSI processing by isolating these components from other phase contributions (e.g. residual topography and deformation) with appropriate spatio-temporal filtering. The PS are geocoded using an external digital elevation model (DEM).
We consider that these phases represent samples of the 3-D distribution of the atmospheric phase delay variations over the entire scene in map geometry. The atmospheric phases are regressed against easting, northing and height at the PS locations. The statistics of the residue of the regression fit are subjected to a variogram analysis. The empirical semivariogram for each interferometric layer in the stack is fit with a parametric model, and the corresponding covariance function is computed. For each range-azimuth pixel, and for each discrete location along the elevation axis where tomographic inversion is intended, the 3-D location in terms of range-azimuth-elevation is projected into map coordinates and a best linear unbiased predictor under universal kriging [68] is used to predict the atmospheric phase for that specific location. Single-look beamforming is used for differential tomographic inversion to retrieve the 2D reflectivity (as a function of the elevation and deformation velocity). A generalized likelihood ratio test is used to detect single and double scatterers in the scene.

The remainder of this paper is organized as follows. Section II discusses the implication of height-dependence of the atmospheric phases for SAR tomography. Section III describes the methodology proposed in this investigation. The characteristics of the interferometric data stack used in the investigation are given in Section IV. Results are provided in section V, and a detailed discussion follows in section VI.

3.2 Modeling vertically stratified atmosphere for SAR tomography

The dimensions of a SAR range-azimuth resolution cell are nowadays on the order of a few meters or tens of meters. They are small compared to the scales at which lateral variations in atmospheric phases occur, i.e. several hundreds of meters to kilometers [3]. Therefore, within a given resolution cell, lateral variations are not expected. However, vertical stratification effects within the same resolution cell may not be ignorable especially in mountainous regions when the multiple scatterers superposing in the cell may be separated by a kilometer or so, as shown in Fig. 3.1. In this section, first we consider the tomographic model in the absence of atmospheric phases, followed by a discussion on the impact of vertical stratification and its mathematical modeling.

3.2.1 SAR tomography

In the absence of any additive or multiplicative noise, classical SAR tomography (3-D SAR) relates the single-look complex SAR signal, $y_m$ for a given range-azimuth resolution cell, with the unknown reflectivity profile $\alpha (s)$ according to the following mathematical model [64, 69–71]:

$$ y_m = \int_{\mathcal{J}_s} \alpha (s) \exp \left[ -j \varphi_m (s) \right] ds \quad (3.1) $$

where $s$ denotes the elevation axis, $\mathcal{J}_s$ is the observed elevation extent, and the interferometric phase, $\varphi_m (s)$ models the phase variations along the elevation due to sensor-to-target geometry as follows:

$$ \varphi_m (s) = 2k \Delta r_m (s) \quad (3.2) $$
Figure 3.1: An imaging scenario for spaceborne SAR tomography in mountainous regions with multi-baseline repeat pass acquisitions. The total backscatter in a given range-azimuth resolution cell ($\delta r$) may comprise of multiple contributions along the elevation axis.

where $k = 2\pi/\lambda$ is the central wavenumber and $\Delta r_m(s)$ is the path-length difference:

$$\Delta r_m(s) = r_m(s) - r_0(s) \approx \frac{s^2}{2 \left( r_0 - b^\perp_m \right)} - \frac{b^\perp_m s}{r_0 - b^\perp_m}.$$  \hfill (3.3)

The variable $r$ represents the range distance from the sensor to the target location, and the subscript $m$ refers to the interferometric pair in consideration. The interferograms are set up relative to a single master acquisition denoted with $m = 0$. The orthogonal and parallel components of the $m^{th}$ spatial baseline are $b^\perp_m$ and $b^\parallel_m$, respectively.

The interferometric phase model for 3-D SAR assumes that no scatterer in the resolution cell has undergone any displacement during the time span between the first and the last acquisition in the data stack. If, however, they has been any displacement, there would be additional phase variations which must be accounted for in the phase model. Assuming that the motion is a temporally linear displacement in the line of sight (LOS), differential tomography [28–30] (4D SAR) extends the phase model as follows:
\[ \varphi_m(s, v) = 2k[\Delta r_m(s) + vt_m] \] (3.4)

where \( v \) is the (average) linear deformation velocity of the scatterer, and \( t_m \) is the temporal baseline for the \( m \)th interferogram. The SAR signal can now be rewritten as [71–73]:

\[ y_m = \int_\mathcal{S}_v \alpha(s, v) \exp[-j\varphi_m(s, v)] \, ds \, dv \] (3.5)

where \( \mathcal{S}_v \) is the extent of the expected linear deformation velocity.

### 3.2.2 Modeling atmospheric phases due to stratified troposphere

The standard model for atmosphere considers the refractivity, \( N \) of the troposphere to be exponentially decaying with increasing height [4, 43, 52]:

\[ N(z) = N_{sl} \exp\left(-\frac{z}{H}\right) \] (3.6)

where \( z \) represents the height above sea level (a.s.l.), \( H \) is the decay parameter typically set at 7.35 km, and \( N_{sl} \) indicates the refractivity at sea-level. While \( H \) is typically assumed to be constant, the temporal variations of the refractivity from one SAR acquisition to the next can be associated with time-dependence of \( N_{sl} \) [43]. With these considerations, the interferometric phase delay due to refractivity change between the acquisitions forming the \( m \)th interferogram, for a scatterer at a reference height, \( h_{ref} \) in the valley (see Fig. 3.1), is given as [3]:

\[ \psi_{h_{ref}} = 10^{-6} \frac{2k}{\cos\theta_{inc}} \int_{h_{ref}}^{h_{sat}} \Delta N_{sl} \exp\left(-\frac{z}{H}\right) \, dz. \] (3.7)

\( \theta_{inc} \) is the angle of incidence and \( h_{sat} \) is the height of the satellite bearing the sensor. They are considered here to be the same for each pass. If there is a second scatterer at an arbitrary height \( h_s \), though in the same range-azimuth resolution cell, additional phase variations will be incurred relative to the reference height:

\[ \Phi_{h_s, h_{ref}} = 10^{-6} \frac{2k}{\cos\theta_{inc}} \left( \int_{h_s}^{h_{sat}} \Delta N_{sl} \exp\left(-\frac{z}{H}\right) \, dz - \int_{h_{ref}}^{h_{sat}} \Delta N_{sl} \exp\left(-\frac{z}{H}\right) \, dz \right). \] (3.8)

Using the first two terms of the Taylor series expansion of \( \exp\left(-\frac{z}{H}\right) \) at \( z = h_s \), and that \( \exp\left(-\frac{z}{H}\right) \to 0 \) as \( z \to h_{sat} \) (which is in agreement with the fact that the relative tropospheric delays converge to zero typically around 7-13 km[53]), the expression in eq. (3.8) is simplified as follows:

\[ \Phi_{h_s, h_{ref}} \approx -10^{-6} \frac{2k}{\cos\theta_{inc}} \Delta N_{sl} \exp\left(-\frac{z}{H}\right) \cdot (h_s - h_{ref}). \] (3.9)
3.3. Methodology

The atmospheric phase for the scatterer at \( h \) is:

\[
\psi_m^{h_{s}} = \psi_m^{h_{\text{ref}}} + \Phi_m^{h_{s},h_{\text{ref}}} = \hat{\beta}_0 + \hat{\beta}_1 \cdot \Delta h
\]  

(3.10)

where the constants \( \hat{\beta}_0 \) and \( \hat{\beta}_1 \), following eq. (3.7) and eq. (3.9) respectively, are given by

\[
\begin{align*}
\hat{\beta}_0 &= 10^{-6} \frac{2 k}{\cos \theta_{\text{inc}}} \Delta N_m^s H \exp(-h_{\text{ref}}/H) \\
\hat{\beta}_1 &= -10^{-6} \frac{2 k}{\cos \theta_{\text{inc}}} \Delta N_m^s \exp(-h_{\text{ref}}/H).
\end{align*}
\]  

(3.11)

\( \Delta h = h_s - h_{\text{ref}} \) is the height difference between the scatterers, and its projection in the elevation axis is \( \Delta s = \Delta h \sin^{-1} \theta_{\text{inc}} \).

The expressions in eq. (3.10) and (3.11) indicate that apart from dependence on the wavelength and incidence angle, the atmospheric phases depend both on the magnitude of the unknown refractivity change \( \Delta N_s \) as well as the difference in elevation \( \Delta s \) between the scatterers in layover. Even if we have an accurate estimate of the atmospheric phase at the reference height (e.g. with phases reliably measured over a corner reflector, or with a GPS station in immediate proximity, or with local spatial filtering over several PS located in the built-up valley) and compensate for it, the additional phase variations due to the second scatterer at the higher altitude will remain uncorrected.

To model atmospheric phases in differential tomography, the mathematical model in eq. (3.5) is rewritten as:

\[
y_{pm}^{pd} = \int \int \alpha(s,v) \exp[-j \psi_m(s)] \exp[-j \varphi_m(s,v)] ds dv
\]  

(3.12)

where \( y_{pm}^{pd} \) is the SAR signal observed in the presence of atmospheric phases. It is assumed here that the refractivity changes are uncorrelated with the temporal baselines. In fact the refractivity changes are random from one interferogram to the next. Therefore, uncompensated atmospheric phases will act as a random phase disturbance and hamper tomographic focusing. It is not within the scope of this paper to describe the effect of these disturbances in a statistical sense; interested readers are referred to [13].

3.3 Methodology

The methodology adopted in this work is a data-driven approach to correct for the atmospheric phases and to apply differential SAR tomography on a data stack of repeat pass single-reference interferograms over a mountainous region. Fig. 3.2 shows the overall methodology in a flowchart, while the details of the different processing steps involved are given in the following subsections.

3.3.1 PSI processing

PSI processing is performed using the Interferometric Point Target Analysis (IPTA) [24, 45, 74] framework. The preprocessing includes selection of a reference acquisition and coregistration of the data.
3. Correction of atmospheric phases for SAR tomography in mountainous regions

Figure 3.2: Flowchart of the processing methodology.

A multi-look intensity image of the reference acquisition is geocoded using an external digital elevation model (DEM). DEM heights are transformed into radar geometry, and used in the coregistration of all the acquisitions to the reference. The overall coregistration process includes a last refinement step using the fine offsets estimated between the acquisitions [45, 66, 67].

An initial list of PS candidates is prepared on the basis of high temporal stability of the backscattering and low spectral diversity. These two criteria are proxies to identify single dominant scatterers with point-like response. In alpine regions, such scatterers are mostly man-made structures in the valley or occasional bare rocky patches on the mountainsides and tops. A reference point is selected, and double-differenced point differential interferograms are computed. The observed interferometric phase in each layer of the stack is modeled as the sum of atmospheric phase, phase variations due to residual topography and deformation, and phase noise (decorrelation or miscalibration). To separate these phase contributions, an iterative least-squares bivariate regression is applied exploiting the spatial and temporal baselines to obtain estimates of residual topography and linear deformation velocity.
3.3. Methodology

The PS candidates for which the standard deviation of the residual phase is higher than a preselected threshold are rejected. We consider several acquisitions are available to justify the assumption that the atmospheric phases are uncorrelated with other phase contributions. The residue of the fit is spatially filtered over the retained PS candidates in small local neighborhoods and unwrapped. The unwrapped phase is fit with a linear model for height-dependence, as in eq. 3.10, using DEM heights. The model fit is subsequently tried over more candidates. The aforementioned processing steps are repeated in several iterations, until the solution has converged i.e. no substantial changes in the estimated parameters (atmospheric phases, residual topography, linear deformation) occur in further iterations, nor do more PS are identified with the same quality restrictions.

For more details on various interferometric processing strategies using the IPTA toolbox, the interested readers are referred to earlier works [24, 45, 74–76].

3.3.2 Regression-kriging

The PS identified in the interferometric processing are geocoded. We consider the atmospheric phases estimated for the PS as samples of the physical 3-D distribution of the atmospheric signal over the scene. Considering the possibility of lateral variations besides vertical stratification effects, we model the unwrapped atmospheric phases for a given interferometric layer from the stack with the following multiple linear regression model:

$$
\psi(x) = x^T \beta + \varepsilon(x)
$$

$$
= \begin{bmatrix}
1 & x_e & x_n & h
\end{bmatrix}
\begin{bmatrix}
\beta_0 \\
\beta_1 \\
\beta_2 \\
\beta_3
\end{bmatrix} + \varepsilon(x) 
$$

(3.13)

where $x \triangleq (x_e, x_n, h)$ represents a general 3-D location in map geometry in terms of easting, $x_e$, northing, $x_n$ and height, $h$. $\beta$ terms are the regression coefficients. $x = \mathbb{T} \{r, a, s\}$ where $\mathbb{T} \{\cdot\}$ is the geocoding transformation applied on a range-azimuth-elevation tuple, $(r, a, s)$. The linear dependence on map coordinates attempts to extract ‘trends’ in the atmospheric phases, lateral as well as vertical. Within the interferometric layer, they are treated in a deterministic sense. The non-linear heterogeneity owing to tropospheric mixing effects is reflected in the residue, $\varepsilon(x)$, which we treat as stochastic. We assume it to be a zero-mean, spatially correlated, second-order stationary process. The variogram of such a process is defined as [77]:

$$
\gamma(\ell) = \frac{1}{2} E \left[ (\varepsilon(x) - \varepsilon(x + \ell))^2 \right]
$$

(3.14)

where $\ell$ is the spatial lag between two locations. Under this assumption, the variance of the process is considered to be a constant, while the spatial correlation does not depend on the location itself but only on the lag between the locations. Moreover, in case of a second-order stationary processes, the variogram and the covariance function are equivalent [77]:

$$
\gamma(\ell) = C(0) + C(\ell)
$$

(3.15)
where $C(0) = \sigma^2$ is the variance of the process. We further assume isotropy i.e. direction independence of the semivariance of $\varepsilon(x)$. It implies that the spatial correlation depends only on the magnitude of the lag, $\ell = \|\ell\|$, which is the Euclidean distance between two locations in map geometry in our case.

**Sample Variogram Estimation**

The atmospheric phases computed for the PS are regressed on their 3-D map coordinates for each interferometric layer, and the estimated residuals, $\hat{\varepsilon}(x)$ are used to obtain the sample variogram as follows [68]:

$$\hat{\gamma}(\ell_j) = \frac{1}{2N(\ell)} \sum_{i=1}^{N(\ell)} \{\hat{\varepsilon}(x_i) - \hat{\varepsilon}(x_i + \hat{i} \ell)\}^2, \quad \forall \ell \in \ell_j$$  \hspace{1cm} (3.16)

where $\hat{\varepsilon}(x_i)$ is the residue for the $i$th PS, $\hat{\varepsilon}(x_i + \hat{i} \ell)$ is the residue for a PS that is located such that the radial distance between the location pair $\{(x_i), (x_i + \hat{i} \ell)\}$ is within the given distance interval, $\ell_j$ and $N(\ell)$ is the number of such paired comparisons. $\hat{i}$ represents a unit vector in any direction. The subscript $j$ indicates the index over the distance intervals used to compute the sample variogram.

**Spatial Prediction**

The regression-kriging a.k.a. universal kriging-based best linear unbiased predictor (BLUP) of the atmospheric phase at a 3-D location $x_0$ is [68]:

$$\hat{\Psi}(x_0) = x_0^T \hat{\beta} + v^T V^{-1} \left( \Psi - X \hat{\beta} \right)$$ \hspace{1cm} (3.17)
3.3. Methodology

Figure 3.4: Data characteristics. (Top) Distribution of spatial (orthogonal component) and temporal baselines. (Bottom) Two-dimensional point spread function in elevation–deformation plane, for the baseline distribution shown above.

where $X$ is the design matrix and $\Psi$ is the vector of the atmospheric phases at PS locations:

$$
X = \begin{bmatrix}
    x^T_1 \\
    x^T_2 \\
    \vdots \\
    x^T_{N_{ps}}
\end{bmatrix}, \quad 
\Psi = \begin{bmatrix}
    \psi(x_1) \\
    \psi(x_2) \\
    \vdots \\
    \psi(x_{N_{ps}})
\end{bmatrix}. \tag{3.18}
$$

$N_{ps}$ is the number of PS used for kriging. The regression coefficients can be estimated with generalized least squares [68]:

$$
\hat{\beta} = (X^TV^{-1}X)^{-1}X^TV^{-1}\Psi. \tag{3.19}
$$

A parametric variogram model is fit to the sample variogram to get a smooth function that is continuous for all non-zero lags. The model selected must also ensure that the spatial predictions are associated with non-negative variances [68, 77]. Examples of models commonly used in the field of Geostatistics are spherical, exponential, power law, Gaussian, Matérn, etc. [78]. Using eq. (3.15), the model fit is used to estimate the covariance function, which in turn allows the estimation of the data covariance matrix $V$ (using the pairwise lags among the PS), and the covariance vector $v$ (using the lags between the location $x_0$ and the locations of the PS).
The prediction error variance for the predictor in eq. 3.17 is [68]:

$$\sigma^2_{e}(x_0) = \sigma^2 - v^T V^{-1} v + \zeta (X^T V^{-1} X)^{-1} \zeta^T$$ (3.20)

where $\zeta = x_0^T - v^T V^{-1} X$. The second term in eq. 3.20, $v^T V^{-1} v$ is equal to $\sigma^2$ when $x_0$ is identical to an observed location, i.e. a PS location in $X$. The expression $(X^T V^{-1} X)^{-1}$ in the third term becomes zero when $x_0$ is an observed location, and it increases otherwise when $x_0$ gets more distant from observed locations in $X$. Therefore, the prediction of atmospheric phases becomes less precise at locations farther from the PS distribution available/used in the kriging setup.

### 3.3.3 Tomographic focusing

Considering we have $M$ acquisitions, the observed SAR signal vector for a given range-azimuth pixel $(r, a)$ is

$$y^{pd} = \begin{bmatrix} y_0^{pd} \\ y_1^{pd} \\ \vdots \\ y_{M-1}^{pd} \end{bmatrix}^T.$$ (3.21)

The observed elevation and deformation extents, $I_s$ and $I_v$, respectively, are discretized. Inverting the differential tomographic model in eq. (3.12), the 2D scatterer reflectivity at the discrete pair $(s_k, v_l)$ is focused using single-look beamforming (BF) as follows:

$$\hat{a}(s_k, v_l) = \frac{1}{M} a^H (s_k, v_l) y^{pd}.$$ (3.22)

The steering vector $a(s_k, v_l)$ is set up such that atmospheric phase correction is incorporated within the tomographic focusing at each discrete point of interest along the elevation axis, as shown below:

$$a(s_k, v_l) = \begin{bmatrix} \exp \left[-j \left\{ \varphi_0 (s_k, v_l) + \tilde{\psi}_0 (T \{r, a, s\}) \right\} \right] \\ \exp \left[-j \left\{ \varphi_1 (s_k, v_l) + \tilde{\psi}_1 (T \{r, a, s\}) \right\} \right] \\ \vdots \\ \exp \left[-j \left\{ \varphi_{M-1} (s_k, v_l) + \tilde{\psi}_{M-1} (T \{r, a, s\}) \right\} \right] \end{bmatrix}.$$ (3.23)

### 3.3.4 Scatterer parameter estimation and detection

For a given range-azimuth resolution cell, we make the following hypotheses. It contains no coherent scatterer – $\mathcal{H}^0$; it comprises a single dominant scatterer – $\mathcal{H}^1$, or it is a double scatterer – $\mathcal{H}^2$. The unknown scatterer parameters (elevation and deformation velocity) for a potential first and second scatterer are estimated with the following beamforming-based maximization [29, 31], respectively:

$$(\hat{s}_1, \hat{v}_1) = \arg \max_{(s_j, v_j) \in (\mathcal{H}, \mathcal{H}_s)} \left\{ \| \hat{a}(s_j, v_j) \| \right\},$$ (3.24)

$$(\hat{s}_2, \hat{v}_2) = \arg \max_{(s_j, v_j) \in (\mathcal{H}, \mathcal{H}_s)} \left( \frac{\| a^H (s_j, v_j) \hat{P}_1^{\perp} y^{pd} \|}{\| \hat{P}_1^{\perp} a(s_j, v_j) \|} \right).$$ (3.25)
\( \hat{P}_1^\perp \) is the projector onto the orthogonal complement of the subspace spanned by the steering vector of the first scatterer, \( a^H(\hat{s}_1, \hat{v}_1) \):

\[
\hat{P}_1^\perp = \mathbf{I}_M - \frac{a(\hat{s}_1, \hat{v}_1) a^H(\hat{s}_1, \hat{v}_1)}{M}.
\] (3.26)

To distinguish between the aforementioned hypotheses, we use the sequential generalized likelihood ratio test with cancellation (SGLRTC), as proposed by Pauciullo et al. in [29]. SGLRTC compares the normalized energies of the potential scatterers against preselected thresholds. For details on SGLRTC the reader is referred to [29] and to our earlier works in [31, 79, 80].

### 3.4 Data Stack

The interferometric data stack used in this work comprises 32 Cosmo-SkyMed stripmap SAR images. These images were acquired during 2008-2013 in repeated passes over Matter Valley in the Swiss Alps, as shown in Fig. 3.3a. As the region is mostly snow-covered during the winters, we only use acquisitions from the summers. The region is known to have many dormant as well as active landslides, rockslides and rockfalls [81, 82]. The topography varies from around 1200 m to 4000 m a.s.l., from the valleys to the mountaintops. There are several slopes with considerable parts free of vegetation or sparsely covered with patches of alpine grass. An average SAR intensity image is shown in Fig. 3.3b. Due to the drastic height variations, most of the valleys are completely or partly in layover of the nearby mountains.

The region of interest (ROI) selected for tomographic investigation is the area around Zermatt, as marked in red in the full scene shown in Fig. 2b. Compared to the other settlements in the full scene, the Zermatt village is more built-up and therefore, more PS candidates are expected here. A zoom-in of the ROI is shown in Fig. 2c, where the top image is the SAR intensity image and the bottom image is an optical perspective from Google Earth. The height difference between the valley floor and the mountain
Residue of the regression fit, \( e(x) \) (geocoded)

Figure 3.6: Residue of the multiple linear regression fit of the unwrapped atmospheric phases against easting, northing and height, cf. eq. 3.13. It is shown here for each interferometric layer in the data stack for the PS identified in the interferometric processing over the full scene, and has been geocoded. The title of the sub-figures indicate the date of the SAR acquisition in YYYYMMDD format. Interferogram network is set up with reference to the acquisition on 20100920.

top is on the order of a kilometer. We can see vegetation stretched over the slope in near range. As to the one in far range, which casts the layover, we can observe some bare rocks on the mountainside and a few ridges which may exhibit long term coherence.

The distribution of the temporal and spatial baselines, and the corresponding point spread function (PSF), are shown in Fig. 3.4. Due to the non-uniformity of the baselines, the PSF is distorted with several irregularly-spread side-lobes. The total orthogonal baseline, i.e. the tomographic synthetic aperture in elevation, \( B^\perp = 1844 \) m. The resolution in the elevation, \( \delta_s \), had the aperture extended over uniform samples of interferometric baselines, would be as follows [11, 83]:

\[
\delta_s = \frac{\lambda r_m}{2B^\perp} = 6.6 \text{ m.} \tag{3.27}
\]

where \( \lambda = 3.12 \) cm and \( r_m = 701 \) km. The resolution projected in height is 3.0 m, corresponding to the incidence angle of 27.3º for the reference layer. This meter-level resolution is conducive to fine-geocoding of the detected scatterers as well as for the detection of closely-spaced multiple scatterers in layover. At the same time, considering the layover of the mountainside and the valley, the extent of the elevation to be observed (\( \mathcal{I}_s \)) for tomographic inversion is on the order of several hundred meters. A resolution too fine may require exceedingly high sampling rates along the elevation to accommodate
Figure 3.7: Sub-figures 1-31: Kriging prediction standard error, \( \sigma_e \), at different heights for 500 random locations in Zermatt valley, for each interferometric layer in the stack. The black dots represent the mean values of \( \sigma_e \) for the random locations at each height bin, while the blue vertical lines indicate the range between the minimum and the maximum. The dates of the SAR acquisitions are mentioned in YYYYMMDD format. Interferograms are referenced to the acquisition on 20100920. Top-rightmost sub-figure: Geocoded PS distribution in the valley, as used for the design matrix in the kriging setup. The random locations are initialized within 300 m radius (marked in circle) of the reference point. The easting and northing coordinates are given relative to the reference point. The color coding represents height variation, from 1550 (blue) to 2900 (red).

The possibility of detecting double scatterers between the valley and the mountainside, or even single scatterers along the mountainside when they are dominant over the backscattering from the valley. As a consequence, the computational load is increased.

Another inopportune implication of a large tomographic aperture in mountainous regions is the possibility of range migration of the target from a given range bin to the next or previous. Due to the different viewing angles in each pass, the iso-range curves for each sensor at the given range bin are not identical. Coregistration of the stack is, in principle, tying these iso-range curves at a specific height. In our case, as we use a terrain-based coregistration with the help of an external DEM, this height can be considered to be close to the DEM height. As we apply tomographic focusing at locations significantly farther from this height, the difference between the iso-range curves may exceed the dimension of the range resolution. Consequently, the true SLC signal value may actually migrate to the neighboring range bin. With larger baselines, this limiting point would occur at shorter height changes. Considering the total
aperture as the worst-case of baseline diversity, the limit on the observed elevation extent to avoid this range migration is [31, 64]:

\[
\mathcal{S} \ll \frac{\delta_r R_0}{\mathcal{B}^\perp} = 663 \text{ m. (3.28)}
\]

where \( \delta_r = 1.5 \text{ m} \) is the range resolution. Projecting vertically, the limiting height difference is nearly 300 m. In other words, it may not be possible to detect potential double scatterers whose separation in height is approaching 300 m. To overcome this limitation, a simple strategy is to select a subset comprising smaller baselines; for example, restricting the maximum baselines within \( \pm 400 \text{ m} \) with the same reference acquisition provides an improvement by raising the limit to 822 m. The drawback is the reduction in the size of the stack (in this case to 17) which in turn reduces the achievable precision in the estimates obtained with tomography [65, 84]. Incidentally, tomographic focusing with time domain back-projection (TDBP) [85, 86] or other range-migration correction techniques can be used to obviate the limitation on the observed extent, but it falls outside the focus of this work.

### 3.5 Results

This section presents the results obtained with the application of the proposed methodology on the interferometric data stack introduced in the previous section.

#### 3.5.1 PSI processing & regression-kriging

The distribution of the PS identified in the interferometric processing with IPTA is shown in Fig. 3.5. The left sub-figure shows the distribution over the full scene, while the right sub-figure shows the PS
3.5. Results

Figure 3.9: Geocoded scatterers. Left: Single and double scatterers obtained with tomography. Regression kriging-based height-dependent atmospheric corrections have been applied. A very few double scatterers have been detected. Nearly all of them are situated within the built-up area in the valley floor. The color-coding represents the estimated height. Middle: PS identified in the prior PSI processing, shown with the same color-coding as in the left sub-figure. Right: Single scatterers detected with tomography on the mountainside (enclosed in white boundary in the left sub-figure) around 230 m above the valley floor.

found in the Zermatt valley. For each PS, the final solution obtained after several iterations comprises the estimates of the residual topography, deformation in the LOS and atmospheric phases. A linear model for height-dependence of atmospheric phase has been applied using an external DEM at 5 x 5 m posting. The quality of the PSI solution is assessed in terms of the standard deviation of the residual phase, $\sigma_{\text{res}}$ for each candidate PS. Only the candidates with $\sigma_{\text{res}} < 0.9$ rad are qualified as PS.

In this investigation, we are specifically interested in the atmospheric phases of the PS. After geocoding, the trends in the atmospheric phases along easting, northing and height are estimated and removed for each interferometric layer. The residue of the regression fit, as shown in Fig. 3.6, is subjected to a variogram analysis. The empirical semivariance is fit with a parametric model, and thereby a spatial covariance function is selected for each layer. The regression coefficients and the covariance function allow universal kriging prediction, cf. eq. 3.17. In order to assess the precision of the kriging predictions in the ROI, we perform kriging at a set of random locations. These random locations are generated as follows. 500 easting-northing pairs are randomly initialized within a lateral radius of 300 m around the reference point, at the fixed height of 1600 m (valley floor). To allow vertical sampling, the height interval of [1600, 2900] is divided into 15 bins. For each random easting-northing pair, we perform kriging predictions at each height bin. Fig. 3.7 shows the standard error, $\sigma_e$ for these predictions against increasing height, for each interferometric layer in the stack. The black dots represent the mean values of the standard errors, while the blue vertical bars indicate the minimum and the maximum error for the random locations at each height bin. The subset of the PS used for the kriging setup are shown in the rightmost sub-figure in the first row. They are color-coded in height. The circle indicates the region in which the random locations are initialized.

3.5.2 Tomography

Single and double scatterers obtained with BF-based differential tomography and SGLRTC are shown in Fig. 3.8 in radar coordinates. The height and deformation velocity of the scatterers have been simultaneously estimated. The thresholds of detection are set at 0.48 \cite{29, 31}. The detected scatterers are
3.6 DISCUSSION

This section provides an itemized discussion of the results presented in the previous section.

3.6.1 PSI processing & regression-kriging

The natural terrain in mountainous regions generally limits the prevalence of coherent scatterers. Moreover, the ruggedness of the topography results in frequent layovers which are also typically rejected in

geocoded and projected in Google Earth, as shown in Fig. 3.9. The PS found in the Zermatt valley are also projected for comparison. It can be seen that some single scatterers, as encircled in white, are detected around 230 m above the valley floor along the mountainside only in the tomography solution. A few double scatterers are also detected, but nearly all of them are situated within the built-up area in the valley. Fig. 3.10 shows the squared 2D reflectivity in the height-deformation plane for a layover-affected pixel in the valley. It potentially contains coherent backscatter from a structure in the valley floor, as well as from rocky edges on the mountainside. The reflectivity is retrieved in three cases: I – no atmospheric correction is applied, II – a single atmospheric correction is applied, estimated by filtering the atmospheric phases of neighboring PS in the valley floor, and III – height-dependent regression kriging-based atmospheric correction is applied as proposed in this work (eq. 3.17, 3.22 and 3.23). Multiple scatterers are detected in the third case, as marked in white.
3.6. Discussion

Consequently, the PS identified in the interferometric processing, as shown in Fig. 3.5, correspond mostly to bare rocks and man-made structures in layover-free areas. Since the atmospheric phases for these PS are subsequently used in the kriging setup, low quality PS may lead to noise propagation in the prediction of the atmospheric phase at unobserved locations. Therefore, a quality control is imperative. In this work, we used only those PS candidates whose residual phase standard deviation is below 0.9 rad. A more strict threshold can further reduce the possibility of noise propagation, but at the expense of reduced coverage.

The residue of the regression fit at geocoded PS locations is shown in Fig. 3.6. In some interferometric layers, the phases are more smooth than the others. It is not unexpected as refractivity changes between the reference acquisition and the other acquisitions in the stack depend on meteorological conditions which can be more variable at times. Nonetheless, spatial correlation up to a certain extent can be observed in each case, which encourages the use of kriging interpolation.

It can be seen in Fig. 3.5 (left) that the Zermatt valley floor is partly covered with the layover cast by the adjoining mountain. A few PS are found even in the layover, representing those pixels where one among the other scattering contributions is dominant. Nearly all of such PS are situated within the valley, and there is no coverage along the mountainside or the top. Since the PS distribution also represents the sampling of the 3-D atmosphere that is subsequently interpolated with kriging, lack of PS at high altitudes is inopportunity as it may lead to high prediction error variance. To investigate it, we randomly initialized 500 locations within a 300 m radius of the reference point, and performed kriging predictions at various heights for these locations. The kriging prediction errors, as shown in Fig. 3.7, are increasing with height in each interferometric layer. The top of the mountain within the ROI is around 2800 m. At this height bin, the on-average prediction error is varying between [0.4, 1.4] rad among the different interferometric layers. For the majority of the layers, it is more than 0.85 rad (median value), which can indeed be a limiting factor in detecting coherent scatterers at high altitudes [13]. Moreover, these figures only represent the prediction error due to the kriging setup and the spatial covariance functions. Any errors in the prior estimation of the atmospheric phase for the PS, or their propagation during the PSI processing, are not accounted for.

3.6.2 Tomography

Single and double scatterers, as shown in Fig. 3.8 and 3.9, are obtained by setting the detection thresholds such that there are no obvious false alarms over decorrelated areas (e.g. forest) or detections at impossible locations (mid-air, below ground) after geocoding. A higher coverage is observed in the layover region relative to the PSI solution, though the apparent gain in coverage remains to be assessed vis-à-vis quality of the detected scatterers relative to the quality of the PS.

A group of single scatterers are detected on the mountainside, around 230 m above the valley floor, where no PS were found with the PSI processing. These scatterers are found on vegetation-free patches of rock, as can be seen in Fig. 3.9 (right). The reason they have not been identified in the prior PSI processing may be that they are in layover. However, since they are single dominant scatterers, there is possibly another explanation. In our PSI processing, although a linear model for height-dependence
of atmospheric phase has been used, only a single correction is applied corresponding to the reference/DEM height of the pixel (as used for the model fit). In case of the aforementioned scatterers, this reference height corresponded to the valley – rather than the mountainside – which would have been appropriate had the dominant scattering originated from the valley. The scatterers would still be detected at the correct location in terms of residual topography correction with PSI processing, if only there was no substantial height-dependence of the atmospheric phase. In this case, the scatterers along the mountainside were 230 m higher above the reference, and height-dependence of the atmosphere was not negligible. These limitations prevented the detection of these scatterers with the PSI processing. In short, these limitations are overcome with the methodology proposed in this work: the tomographic inversion retrieves the correct residual topography relative to the reference height as the atmospheric phases are corrected simultaneously while focusing along the elevation axis.

Fig. 3.10 presents the case of another layover-affected pixel, which was not detected as a PS. When no atmospheric correction was applied, no coherent scatterer was detected. On applying a single correction estimated with spatial filtering of the atmospheric phases of the neighboring PS, a single scatterer is detected corresponding to a structure in the valley. When a height-dependent atmospheric correction is applied, we observe multiple scatterers appearing around the mountainside, around 200 m above the valley – the layover is resolved. These results also substantiate the applicability and usefulness of the proposed methodology.

Notwithstanding, we do not observe any substantial number of single, double or higher order scatterers at higher altitudes along the mountainside or the top. A very few double scatterers are detected, merely 274, and nearly all of them are situated within the built-up area in the valley floor. Firstly, there may be a lack of coherent point-like scattering due to large pixel-area or unavoidable temporal decorrelation. Another factor may be the increasing atmospheric phase predicition error with increasing height. SAR tomography requires high phase stability. Referring to the relevant theoretical formulations in [13], for a data stack comprising 32 acquisitions, in order to achieve a probability of detection better than 95% while keeping the probability of false alarm under $10^{-4}$, the precision of the overall phase correction (for the atmospheric disturbances as well as any other source of phase instability) needs to be better than 1.0 rad for a single scatterer and 0.85 rad for a double scatterer, on average. Given the fact that just the kriging predication error approaches 0.85 rad around the mountain top for most of the interferometric layers, the predicted atmospheric corrections are (presumably) not precise enough to detect coherent scatterers at high altitudes [13]. Further investigations are needed to understand how the errors in the prior estimation of atmospheric phases for the PS propagate into errors in the kriging predictions at unobserved (non-PS) locations.

The possibility of target range migration with increasing height can also be a reason for lack of scatterers at high altitudes. In order to alleviate its impact, we experimented with a subset of 17 acquisitions from the data stack such that the orthogonal component of the spatial baselines are within $\pm 400$ m. However, no improvement is observed in the results. Arguably, any potential improvement is offset by the increase in phase stability requirements due to the reduction in the size of the stack [13].
3.7 Conclusion & outlook

This paper has discussed the role of atmospheric phases as one of the key limiting factors in the applicability of SAR tomography in mountainous regions. Given the possibility that the scatterers in layover may be separated in height by several hundreds of meters, the need to perform height-dependent atmospheric corrections is imperative. This paper has also proposed a methodology to correct for the atmospheric phases and incorporate it within tomographic processing. It is a data-driven approach that entails a prior PSI processing. Atmospheric phases are predicted at unobserved (non-PS) locations using universal kriging with 3-D spatial map coordinates as the regressors. The phase corrections are applied at each 3-D point of interest while focusing along the elevation axis. The proposed methodology has been experimented on a data stack comprising 32 Cosmo-SkyMed acquisitions over Matter Valley in the Swiss Alps. Differential SAR tomography is applied in a valley partly covered by the layover of the adjoining mountain. Apart from the single scatterers detected on the man-made structures in the valley, some single scatterers are detected along the mountainside around 230 m above the valley floor, where no PS were found in the prior PSI processing. The case of another layover-affected pixel which comprises multiple scatterers is presented: it was rejected during the PSI processing, but the proposed tomographic processing resolved the layover and separated the individual scatterers. These results substantiate the usefulness of the kriging-based atmospheric corrections introduced in the paper.

Nonetheless, increase in coverage in terms of layover separations remains limited as only a few double scatterers have been detected, and nearly all of them belong to the layovers occurring in the built-up area in the valley floor. Apart from the fact that large pixel-area squeezed in a few range bins limits point-like scattering, the sparsity of the PS distribution at high altitudes has also been a limiting factor. The precision of the atmospheric phase corrections gets lower for locations farther from the distribution which in turn hinders scatterer detection.

This paper has also hinted at factors other than atmospheric disturbances that may limit the detection of single and double scatterers at high altitudes, such as the possibility of range migration due to large elevation extents that may need to be observed in rugged mountainous regions. Choosing a subset of spatial baselines such that the total orthogonal baseline is not too large to cause significant range migration may in turn increase the requirement on the precision of the phase calibration to achieve a certain probability of detection. Therefore, future investigations should explore strategies for the correction of range-cell migration while focusing along the elevation, especially when layover separation is attempted over large elevation extents.

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REFERENCES


4

SAR TOMOGRAPHY AS AN ADD-ON TO PSI: DETECTION OF COHERENT SCATTERERS IN THE PRESENCE OF PHASE INSTABILITIES

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Author’s contributions

• Conceived and performed the experiments.
• Developed the tomographic processing framework, and the proposed detection strategy
• Drafted the manuscript, and interpreted the results.

Co-authors’ contributions

• U. Wegmüller provided the initial PSI solution used in this work.
• O. Frey contributed several routines for data preprocessing and PSI/tomo integration.
• All co-authors reviewed the manuscript.

Data sources

TerraSAR-X SAR data used in this project was obtained courtesy of the German Aerospace Center DLR under proposal MTH1717. SRTM is copyright USGS.
4. DETECTION OF COHERENT SCATTERERS IN THE PRESENCE OF PHASE INSTABILITIES

**ABSTRACT**

The estimation of deformation parameters using persistent scatterer interferometry (PSI) is limited to single dominant coherent scatterers. As such, it rejects layovers wherein multiple scatterers are interfering in the same range-azimuth resolution cell. Differential SAR tomography can improve deformation sampling as it has the ability to resolve layovers by separating the interfering scatterers. In this way, both PSI and tomography inevitably require a means to detect coherent scatterers, i.e. to perform hypothesis testing to decide whether a given candidate scatterer is coherent. This paper reports the application of a detection strategy in the context of “tomography as an add-on to PSI”. As the performance of a detector is typically linked to the statistical description of the underlying mathematical model, we investigate how the statistics of the phase instabilities in the PSI analysis carry forward to the subsequent tomographic analysis. While phase instabilities in PSI are generally modeled as an additive noise term in the interferometric phase model, their impact in SAR tomography manifests as a multiplicative disturbance. The detection strategy proposed in this paper allows extending the same quality considerations as used in the prior PSI processing (in terms of the dispersion of the residual phase) to the subsequent tomographic analysis. In particular, the hypothesis testing for the detection of coherent scatterers is implemented such that the expected probability of false alarm is consistent between PSI and tomography. The investigation is supported with empirical analyses on an interferometric data stack comprising 50 TerraSAR-X acquisitions in stripmap mode, over the city of Barcelona, Spain, from 2007-2012.

4.1 INTRODUCTION

Persistent scatterer interferometry (PSI) [1–7] is nowadays an operational geodetic technique for the monitoring of surface deformation with spaceborne synthetic aperture radar (SAR) data stacks. These stacks typically comprise several repeat-pass SAR acquisitions, spanning from months to years. PSI techniques attempt to extract the interferometric phase components correlated with the scatterer motion. The quality of the deformation estimates is tied to the precision of the interferometric phases. Temporal and geometric decorrelation, as well as uncompensated platform motion and atmosphere-induced optical path delay variations, are among the factors that cause random instabilities in phase. A quality control is necessary during the processing as well as when reporting the final results.

The single dominant scatterers that exhibit long term phase stability are generally termed as persistent scatterers (PS). PSI processing approaches often use a classifier to identify a priori a set of PS candidates, e.g. the pioneer permanent scatterers™ [1] approach uses the dispersion index as a proxy for phase stability. The PSI approaches based on the interferometric point target analysis (IPTA) framework, as in [3, 8], employ spectral diversity [3, 9–11] as a proxy for phase stability in addition to the stability of the backscattering amplitude. Low dispersion index and low spectral diversity are indicative of good phase quality. The observed differential interferometric phases are fit to a phase model and the unknown parameters, such as the deformation velocity and the residual topography, are thereby estimated. The dispersion of the residue of the fit is a means to characterize the quality of the estimates. It is often used to compute the multi-interferogram complex coherence (MICC) [1, 12, 13] which can in turn be used as a test statistic to perform statistical detection i.e., to decide among the hypotheses whether a given
PS candidate is a phase coherent single scatterer or if it comprises noise only. The statistics of the noise impact the probability of false alarm in the detection process.

An inherent limitation associated with PSI techniques is the fact that a phase-only model cannot consider multiple coherent scatterers with different complex reflectivity interfering in the same range-azimuth resolution cell. The cumulative phase response in this case is mismatched to the interferometric phase model, which is essentially based on the assumption of a single scatterer. Consequently, it may lead to erroneous estimation of the deformation parameters. Therefore, PSI processing approaches typically reject the cells that contain backscattering contributions from multiple scatterers, as for the case of layovers.

The aforementioned limitation can be alleviated by SAR tomography [14–17] which exploits both the amplitude and the phase of the received signal, thereby permitting a higher order analysis [18]. It allows 3D reconstruction of the scene reflectivity — a feature that renders it possible to resolve the layover problem [19–22]. Additionally, differential SAR tomographic methods [23–25] allow a joint spatio-temporal inversion of the coherent scatterers in layover, i.e., the position along the elevation axis as well as the deformation velocity of the interfering scatterers are simultaneously estimated. Therefore, differential SAR tomography has been proposed as an add-on to PSI techniques to improve deformation sampling by resolving the scatterers in layover that are rejected in the PSI processing [26–28]. Inevitably, a detection strategy is again required to classify whether the detection of one or more scatterers in the same resolution cell is true or false. In this context, it is pertinent to carry forward the same quality concerns as used in the prior PSI analysis so that the combined use of PSI and tomography holds compatibility.

The prevailing detection mechanisms for SAR tomography, such as the generalized likelihood ratio tests in [13, 24, 29], consider an additive noise model for the received complex signal vector. The source of the noise is attributed to the clutter in the resolution cell. However, the instabilities in the observed interferometric phases, albeit considered additive in the phase-only model, naturally represent themselves as multiplicative noise in the tomographic signal model. Therefore, in order to carry the impact of the phase instabilities from an interferometric to tomographic analysis, the detection strategy employed for hypothesis testing needs to account for the phase instabilities as a multiplicative disturbance in tomographic inversion.

Keeping in view the aforementioned concerns, this paper describes a strategy for the detection of single and double scatterers with SAR tomography whereby the hypothesis testing is directly linked to the MICC-based test statistic for PS detection in the prior PSI processing. Section 4.2 presents the mathematical models typically used for SAR interferometry and tomography, as well as the associated detection mechanisms. Section III presents the processing methodology adopted in the paper. The data stack for empirical analysis is introduced in section IV. The results obtained are presented in section V, followed by a discussion in section VI. As a whole, this paper is a follow-up to the earlier works in [12, 27, 30].
4.2 Models

We consider the availability of a coregistered, single-reference interferometric SAR data stack comprising $M$ layers of repeat-pass interferograms. For a given range-azimuth resolution cell in an interferometric layer, we denote the received single-look complex (SLC) signal as $y_m = z_m \exp(-j\varphi_m)$, where $z_m = |y_m| \in \mathbb{R}$ and $\varphi_m$ is the observed interferometric phase. The subscript $m$, where $m \in \{0, 1, \ldots, M-1\}$, is used to indicate a specific layer in the interferometric stack. In the following text, an underlined symbol represents a quantity that has been modeled as stochastic, or when the distinction between observables versus observations is emphasized. Bold symbols represent vectors, or matrices when capitalized.

4.2.1 Interferometric phase model

The interferometric phase observable, $\varphi_m$, is generally modeled as a sum of several phase contributions [31, 32]:

$$\varphi_m = \varphi_m^{\text{disp}} + \varphi_m^{\text{geo}} + \varphi_m^{\text{th.exp}} + \varphi_m^{\text{atm}} + \varphi_m^{\text{decor}} + 2\pi p$$  \hspace{1cm} (4.1)

where $\varphi_m^{\text{disp}}$ is the phase change due to the displacement of the target within the resolution cell:

$$\varphi_m^{\text{disp}} = \frac{4\pi}{\lambda} vt_m.$$  \hspace{1cm} (4.2)

$\lambda$ is the wavelength, $v$ is the deformation velocity in the line of sight (LOS), and $t_m$ is the temporal baseline for the $m$th interferogram. $\varphi_m^{\text{geo}}$ is the phase variation due to sensor-to-target geometry. Neglecting higher order terms [16, 33],

$$\varphi_m^{\text{geo}} \approx -\frac{4\pi b_m^\perp s}{\lambda (r_0 - b_m^\parallel)}.$$  \hspace{1cm} (4.3)

where $b_m^\perp$ and $b_m^\parallel$ are the orthogonal and parallel components of the spatial baseline for the $m$th interferogram, respectively. $r_0$ is the range distance from the sensor to the target location for the reference acquisition. $s$ represents the elevation, i.e. the position of the target in the perpendicular to LOS axis. In case of thermal expansion, the additional phase variations are linearly modeled as follows [27, 34]:

$$\varphi_m^{\text{th.exp}} = \eta T_m$$  \hspace{1cm} (4.4)

where $T_m$ is the temperature change (with respect to the temperature for the reference layer), and $\eta$ is the phase-to-temperature sensitivity. The term $2\pi p$, where $p \in \mathbb{Z}$, is added to account for phase wrapping. The phase variations $\varphi_m^{\text{atm}}$ are due to the optical path length variations while propagation through the atmosphere. They are modeled as stochastic variables due to the temporally varying nature of atmospheric refractivity [35–38]. The phase decorrelation term, $\varphi_m^{\text{decor}}$, is, by definition, a random quantity, which is typically modeled as an additive noise. The parameters $s$, $v$ and $\eta$ are treated as deterministic unknowns in this work.

The interferometric phase model in eq. (4.1) is implicitly assuming the presence of a single coherent scatterer in the resolution cell. In case of multiple coherent scatterers in the same resolution cell, it is
not possible to write the interferometric phase, $\varphi_m$, as a sum of the aforementioned sources of phase variations, independently of the reflectivity of the individual scatterers.

### 4.2.2 PSI: Model of observation equations

While several approaches to parameter estimation with PSI have been proposed over time, as in [1–6], the functional model of interferometric phase observation equations common to these approaches is as follows [32]:

$$\varphi = Ap + w$$

where $\varphi$ is the $M \times 1$ vector of interferometric phase observables, $A$ is the design matrix, and $p$ is the vector of the aforementioned unknown parameters. $w$ is the $M \times 1$ vector of phase residuals which collectively represent the phase instabilities owing to decorrelation, uncompensated atmospheric phases and model imperfections. The residuals in each layer are assumed to be zero-mean and independent random variables: $E\{w\} = 0$; and $D\{ww^H\} = Q_{ww}$ is the variance-covariance matrix for the residuals. If it can be assumed there are no phase unwrapping issues, and the data stack can be phase calibrated by compensating for the atmospheric phase with external data – although both assumptions are simplistic – then the remaining unknowns are $s$, $v$ and $\eta$. The design matrix is then constituted by the coefficients of these parameters (from eq. (4.2-4.4)) [1, 32]. Under Gauss-Markov conditions, the best linear unbiased estimate of the parameter vector using weighted least squares is given as [32]:

$$\hat{p} = \left( A^T Q_{ww}^{-1} A \right)^{-1} A^T Q_{ww}^{-1} \varphi.$$  

The variance-covariance matrix of the estimated parameter vector, $Q_{\hat{p}\hat{p}} = D\{\hat{p}\}$, is as follows:

$$Q_{\hat{p}\hat{p}} = \left( A^T Q_{ww}^{-1} A \right)^{-1}.$$  

The quality of the estimates is, therefore, dependent on the dispersion of the residuals. The vector of the estimated phase residuals is as shown below:

$$\hat{w} = \varphi - A\hat{p}.$$  

### 4.2.3 PSI: Statistics for PS detection

For each PS candidate, we distinguish between the following two hypotheses:

$\mathcal{H}^0$ – the null hypothesis. The range-azimuth resolution cell does not contain any coherent scatterer and comprises merely clutter;

$\mathcal{H}^1$ – the alternative hypothesis. The cell contains a phase coherent single scatterer, i.e. a PS.

In the presence of a coherent scatterer whose phase response is well-matched to the model in eq. (4.1), the phase residuals are expected to have a low dispersion around the expected value of zero. Contrarily,
in the absence of a coherent scatterer, the observed phase and the residuals are expected to have a wider dispersion. With these considerations, we assume that the phase residuals generally follow a von Mises (circular\(^1\) normal) distribution. The probability density function (PDF) is given by [39]:

\[
g(w; \mu, \kappa) = \frac{1}{2\pi I_0(\kappa)} e^{\kappa \cos(w - \mu)} \tag{4.9}
\]

where the support of the distribution is any \(2\pi\) interval. The parameter \(\mu = E\{w\}\) represents the ‘preferred direction’, which we consider to be zero under both \(\mathcal{H}^0\) and \(\mathcal{H}^1\). The support is then the interval \([-\pi, \pi]\) and the distribution is symmetric about zero. The parameter \(\kappa \geq 0\) is a measure of ‘concentration’ of the distribution around the mean value, i.e. \(\kappa^{-1}\) behaves analogous to dispersion of a linear random variable. \(I_0(\kappa)\) is the modified Bessel function of the first kind and order zero. Under \(\mathcal{H}^1\), we consider the residuals to exhibit a higher concentration.

**Test statistic**

A commonly used statistic to test among the two hypotheses is the *ensemble coherence*, as defined below [5, 31]:

\[
\gamma \triangleq E\{\exp(jw)\} \tag{4.10}
\]

An unbiased estimator of the coherence, given \(M\) interferometric layers, is the multi-interferogram complex coherence (MICC) [12, 31]:

\[
\hat{\gamma} = \frac{1}{M} \sum_m \exp(j\hat{w}_m) \tag{4.11}
\]

\[
= \frac{1}{M} \left( \sum_m x_m + j \sum_m y_m \right) \tag{4.12}
\]

\[
= \frac{1}{M} (X + jY) \tag{4.13}
\]

\[
= \bar{X} + j\bar{Y} = \text{Re}^{j\bar{\mu}} \tag{4.14}
\]

where \(X\) and \(Y\) are the sum of cosine and sine terms in the expression, respectively, and the length of the resultant, \(R = \sqrt{X^2 + Y^2}\). The overscore indicates sample mean. Hereafter, we refer to MICC simply as the sample coherence\(^2\). The phase residuals, \(\hat{w}_m\) are assumed to be independent and identically distributed (i.i.d.) random variables.

**Statistics under \(\mathcal{H}^0\)**

The statistics of the sample coherence depend on the distribution of the phase residuals. With reference to eq. 4.11, the phase residuals can be considered as angles subtended by phasors of unit length.

\(^1\)The term *circular distribution* as used in this paper refers to a directional distribution with support on the circumference of unit circle. [39]

\(^2\)Given \(\hat{R}\) is well-defined (i.e., \(\hat{R} > 0\)), the sample mean direction \(\hat{\mu}\), computed with sample coherence for any random sample \((w_1, w_2, \ldots, w_m)\) from a von Mises population, is the maximum likelihood estimator of the preferred direction, \(\mu\) [40, 41]. This property is characteristic of von Mises populations on a circle, analogous to a similar property holding for Gaussian distribution on a real line whose location parameter is estimated with maximum likelihood by the sample mean [39, 42].
Under $H_0$, when the phasors have no preferred direction, we consider the limiting case of von Mises distribution when $\kappa \to 0$ [39]:

$$\lim_{\kappa \to 0} g \left( w; \mu = 0, H_0 \right) = U (w) = \frac{1}{2\pi}, \quad -\pi \leq w < \pi$$  \hspace{1cm} (4.15)

where $U (w)$ is the circular uniform distribution. In this case, $E \{ x \} = E \{ y \} = 0$; therefore, $E \{ \hat{\gamma} \} = 0$. The second order moments are $E \{ x^2; H_0 \} = E \{ y^2; H_0 \} = \frac{1}{2}$. The terms $x$ and $y$ are not independent (as $x^2 + y^2 \equiv 1$), but they are uncorrelated as $E \{ x \cdot y \} = 0$ [12, 39]. The variance of the addends in the eq. (4.12) is finite. Therefore, under the assumption of a large sample size, multivariate central limit theorem holds, and we consider the joint distribution of $(\bar{X}, \bar{Y})$ to be converging to a Gaussian distribution, $\mathcal{N}_2 (0, \Sigma)$ where

$$\Sigma = \begin{bmatrix} \frac{1}{2M} & 0 \\ 0 & \frac{1}{2M} \end{bmatrix}.$$

$R$ is then approximately distributed as Rayleigh, and its PDF is as follows [39]:

$$f_\gamma (r; H_0) \propto 2r \exp \left( -\frac{r^2}{M} \right)$$  \hspace{1cm} (4.16)

where $0 \leq r \leq M$. Referring to [12, 39], the probability of false alarm can be computed as the upper tail of the Rayleigh distribution, as follows:

$$\Pr \{ R > r_{th} | H_0 \} = \exp \left( -\frac{r_{th}^2}{M} \right).$$  \hspace{1cm} (4.17)

It can be equivalently expressed as

$$\Pr \{ \parallel \hat{\gamma} \parallel > T_\gamma | H_0 \} = \exp \left( -MT_\gamma^2 \right)$$  \hspace{1cm} (4.18)

where $T_\gamma$ is the detection threshold such that $0 \leq T_\gamma \leq 1$.

**Statistics under $H_1$**

In case of $H_1$, the probability distribution of $R$ is given by [39],

$$f (r) = \frac{I_0 (kr)}{I_0^M (k)} r \psi_M (r)$$  \hspace{1cm} (4.19)

where

$$\psi_M (r) = \int_0^\infty J_0 (rt) J_0^M (t) \, td t.$$  \hspace{1cm} (4.20)

$J_0$ is the Bessel function of the first kind and zero-order. A closed form expression for the PDF is not available. We again assume a large sample size and invoke the multivariate central limit theorem. It allows us to consider the joint distribution of $\bar{X}$ and $\bar{Y}$ to be asymptotically normal, and expectation
Determination of Coherent Scatters in the Presence of Phase Instabilities

Figure 4.1: Estimates of the coherence magnitude obtained with $10^5$ Monte Carlo iterations assuming the residual phases have a von Mises distribution with concentration parameter, $\kappa$. Each solid line indicates the estimates for a specific number of acquisitions, $M$ in the data stack. The vertical bars represent $\pm$ standard deviation from the mean. The dashed line shows the coherence magnitude under the assumption that the residual phases follow a linear Gaussian distribution, cf. eq. 4.25 (assuming $\sigma_{\varphi}^2 = \kappa^{-1}$).

and the variance of the sample coherence can be approximated as follows [43]:

$$E \{|\hat{\gamma}|\} \cong v_1$$

$$\text{var} \{|\hat{\gamma}|\} \cong \frac{1 - 2v_1^2 + v_2}{2M}$$

where $v_j = E\{\cos(j\varphi)\}$:

$$v_1 = \frac{I_1(\kappa)}{I_0(\kappa)}$$

$$v_2 = 1 - 2\left(\frac{I_1(\kappa)}{\kappa \cdot I_0(\kappa)}\right)$$

**Limiting case when $\kappa \to \infty$** For sufficiently large $\kappa$, the von Mises distribution for the phase residuals can be approximated by a linear normal distribution with $\sigma_{\varphi}^2 = \kappa^{-1}$ [39]. The coherence in this case is given by [12, 30]:

$$E \{|\hat{\gamma}|\} = E \{\hat{\gamma}\} = \exp\left(-\frac{\sigma_{\varphi}^2}{2}\right)$$

For a discussion on the details about the corresponding probability of detection, interested readers are referred to earlier works in the literature [12, 13].

Since exact closed-form expressions for the PDF of $|\hat{\gamma}|$ are not available, we resort to numerical methods to compare the estimate of the coherence magnitude for the general case of $\kappa > 0$ against the estimate in case of the aforementioned linear normal approximation. For each value of $\kappa$ between $[1, 10]$, we use
10^5 Monte Carlo iterations and compute the coherence magnitude. The results are shown in Fig. 4.1 for three different values of \( M \). The estimate under the normal approximation (eq. 4.25) is also shown. It can be seen that the normal approximation for the limiting case tends to overestimate the coherence magnitude. The overestimation decreases for increasing values of \( \kappa \). For \( \kappa > 3 \), the difference between the coherence estimate under the assumption of von Mises distribution and the normal approximation is less than 5% on average. With increasing number of acquisitions, the variance in the estimation of the coherence magnitude decreases (in agreement with eq. 4.22).

### 4.2.4 SAR tomography: Mathematical model

In the absence of noise, for a given range-azimuth resolution cell, the mathematical model for classical SAR tomography (3D SAR) can be written as [16, 19, 21, 26, 44]:

\[
y_m = \int_{\mathcal{S}} \alpha(s) \exp[-j \varphi_m^\text{geo}(s)] \, ds \tag{4.26}
\]

where \( \alpha \) is the complex reflectivity and \( \mathcal{S} \) is the support of \( s \). This model assumes there has been no displacement in the line of sight during the observation time period. Differential SAR tomography [23, 25] with extended phases models [25, 27, 45] allows modeling linear displacement as well as seasonal or temperature-induced motion:

\[
y_m = \iint_{\mathcal{S}, \mathcal{V}, \mathcal{\eta}} \alpha(s, v, \eta) \exp\{-j [\psi_m(s, v, \eta)]\} \, ds \, dv \, d\eta \tag{4.27}
\]

where \( \psi_m \) is the sum of the deterministically modeled phase components as a function of the unknown parameters, i.e.,

\[
\psi_m(s, v, \eta) = \varphi_m^\text{geo}(s) + \varphi_m^\text{disp}(v) + \varphi_m^\text{th.exp}(\eta). \tag{4.28}
\]

It is assumed that the phase terms (and hence the spatial and temporal baselines, and temperature changes) are mutually independent of each other. A general mathematical model for SAR tomography can be defined as follows [27, 46]:

\[
y_m = \int_{\mathcal{P}} \alpha(p) \exp[-j \psi_m(p)] \, dp \tag{4.29}
\]

where \( \mathcal{P} \) represents the support of the parameter vector (i.e. the parameter space), and \( p \in \mathcal{P} \). It is analogous to a multi-dimensional Fourier transform [47]. In case the resolution cell contains a single point source with dirac delta response, \( \alpha(p) = \tau_1 \delta(p-p_1) \), with \( \tau_1 \in \mathbb{C} \), eq. (4.29) reduces to the following:

\[
y_m = \tau_1 \exp[-j \psi_m(p_1)]. \tag{4.30}
\]
For the general case of $Q$ point sources in the presence of clutter, the tomographic model is further extended as follows:

$$y_m = \sum_{q=1}^{Q} \tau_q \exp \left[ -j \psi_m \left( \mathbf{p}_q \right) \right] + n_m$$

(4.31)

$$= d_m + n_m$$

(4.32)

$$= z_m \exp \left( -j \phi_m \right)$$

(4.33)

where $n_m$ represents additive noise which is typically modeled as zero-mean complex Gaussian (with symmetric variances for the real and imaginary parts). We assume the noise samples are i.i.d. across the stack, i.e., $D \{n\} = \sigma_n^2 I_M$, with $\sigma_n^2 > 0$. $d_m$ represents the coherent sum of the deterministic components in the signal vector.

### 4.2.5 SAR tomography: Model inversion & parameter estimation

We use single-look beamforming for the inversion of the general tomographic model to estimate the unknown scatterer reflectivity as a function of the parameter vector $\mathbf{p}$ for a given range-azimuth resolution cell as follows [13, 16]:

$$\hat{\mathbf{a}}(\mathbf{p}) = \frac{1}{M} \langle \mathbf{y}, \mathbf{a}(\mathbf{p}) \rangle$$

(4.34)

where $\langle .. \rangle$ represents the inner product, $\mathbf{a}(\mathbf{p})$ is the steering vector as a function of $\mathbf{p}$, and $\mathbf{y}$ is the vector comprising the SLC observations:

$$\mathbf{y} = \left[ \begin{array}{c} y_0 \\ y_1 \\ \vdots \\ y_{M-1} \end{array} \right]^T.$$  

(4.35)

The steering vector is structured as follows:

$$\mathbf{a}(\mathbf{p}) = \left[ e^{-j\psi_0(p)} \quad e^{-j\psi_1(p)} \quad \ldots \quad e^{-j\psi_{M-1}(p)} \right]^T.$$  

(4.36)

For the estimation of the unknown parameters, we use the estimated absolute reflectivity as the objective function in the following maximization:

$$\hat{\mathbf{p}}_1 = \arg \max_{\mathbf{p} \in \mathbb{P}} (|\hat{\mathbf{a}}(\mathbf{p})|).$$

(4.37)

As more than one coherent scatterer may be present in the same resolution cell, successive maxima after the global maximum may indicate the presence of more scatterers. Assuming a maximum of two scatterers, an estimate of the parameter vector for the second scatterer is obtained as follows:

$$\hat{\mathbf{p}}_2 = \arg \max_{\mathbf{p} \in \mathbb{P} \setminus \{\hat{\mathbf{p}}_1 \pm \delta \mathbf{p}\}} (|\hat{\mathbf{a}}(\mathbf{p})|).$$

(4.38)
where $\delta p$ indicates the Rayleigh resolution for the tomographic profile along each of the unknown parameter.

Eq. (4.34), together with eq. (4.33), imply that instability in the observed phase will cause errors in the reconstructed target reflectivity. As a consequence, errors will propagate in the estimation of the parameters using the aforementioned maximizations. Therefore, a scatterer detection strategy is needed to classify whether a given resolution cell contains one or more phase coherent scatterers, or is merely clutter.

### 4.2.6 SAR tomography: Statistics for scatterer detection

A commonly used test statistic for coherent scatterer detection in the context of tomography is the absolute value of the estimated reflectivity, $|\hat{\alpha}|$. The same hypotheses are carried forward as introduced in section 4.2.3, except for the change that now we consider them for multiple coherent scatterer candidates for each range-azimuth resolution cell.

**Statistics under $H^0$**

In case the received signal is merely clutter, the received signal vector $y = n$. Using eq. 4.34,

$$
E \{ |\hat{\alpha}(p)|; H^0 \} = \frac{1}{M} E \{ |\mathbf{n} \cdot \mathbf{a}(p)| \} = \frac{1}{M} E \{ |\mathbf{n}| \} = 0
$$

where the third equality follows from rotational invariance of the Gaussian distributed samples, and therefore, the inconsequential difference between $\hat{n}$ and $n$ will be dropped. Since $\varphi_m = \angle n_m$ under $H^0$, the observed interferometric phase (and the residual phase in this case) follows a uniform distribution [12]. Along similar lines as in 4.2.3, the joint distribution of the real and imaginary parts of $\hat{\alpha}$ is a zero-mean Gaussian with the following covariance matrix:

$$
\Sigma_{\alpha|\mathbf{z}_m} = \begin{bmatrix}
\frac{1}{2M^2} \sum_m z^2_m & 0 \\
0 & \frac{1}{2M^2} \sum_m z^2_m
\end{bmatrix}.
$$

The PDF of $|\hat{\alpha}|$ in this case is Rayleigh, and the right tail probability to compute the probability of false alarm is as follows:

$$
\Pr\{ |\hat{\alpha}| > T_{\alpha}| H^0, y \} = \exp \left( -\frac{M^2 T^2_{\alpha}}{\|y\|^2_2} \right)
$$

where $\|y\|^2_2 = \sum_m z^2_m$ is the squared L2-norm of the observed signal vector.
In general, the received signal contains clutter besides the possibility of backscattering contribution from point-like sources. We assume that, under \( \mathcal{H}_1 \), the deterministic backscatter from the point sources is dominant over the clutter, i.e. \( |d_m| \gg |n_m| \ \forall \ m \). This assumption allows us to consider that the observed phase owes primarily to the vector sum of the backscatter from point-like sources (and not the clutter). Using eq. (4.31) and eq. (4.34), the expression for the estimated reflectivity can then be stated as follows:

\[
\hat{\alpha}(p) = \frac{1}{M} \left( \langle d, a(p) \rangle + \langle n, a(p) \rangle \right) = \frac{1}{M} \sum_m |d_m| \exp \left\{ -j[\phi_m - \psi_m(p)] \right\} + n \]

\[
= \frac{1}{M} \sum_m |d_m| \exp \left\{ j \hat{w}_m(p) \right\} + n
\]

Using eq. (4.10) and eq. (4.41),

\[
E\{\hat{\alpha}(p); \mathcal{H}_1\} = \frac{1}{M} \sum_m |d_m| E\{\exp[j \hat{w}_m(p)]\} = \frac{1}{M} \sum_m |d_m| \gamma
\]
4.2. Models

From eq. (4.46), it is clear that the phase instability is disturbing tomographic reconstruction in a multiplicative sense. The ensemble coherence has a direct impact on the expected value of the retrieved reflectivity profile, and thereby on the hypothesis testing. Closed-form expression for the PDF of $|\hat{a}|$ are not available when the residuals are assumed to follow a von Mises distribution with $\kappa > 0$. A Rician approximation can be taken, as suggested in [30], when the residuals can be considered to be normally distributed (i.e., the limiting case when $\kappa \to \infty$). The probability of detection, $f_D$ for a fixed false alarm rate can then be studied as the area under the upper tail of the Rician distribution [48]. Nonetheless, we resort to Monte Carlo simulation to study the probability of detection numerically in terms of inverse coefficient of variation (iCV) as defined below for the text statistic $\hat{a}$:

\[
\text{iCV}^2 \triangleq \frac{\left| E\{\hat{a}(p); \mathcal{H}_1\} \right|^2}{\text{var}\{\hat{a}(p); \mathcal{H}_1\}}.
\]  

(4.48)

Considering $\gamma \approx 0$, and dropping the dependence on $p$ to simplify notation,

\[
\text{var}\{\hat{a}; \mathcal{H}_1\} = \frac{1}{M^2} \sum_{l=1}^{M} \sum_{k=1}^{M} |d_l||d_k| \cdot \text{cov}\{e^{j\hat{\omega}_l}, e^{-j\hat{\omega}_k}\}
\]  

(4.49)

Using the assumption that the residual phases are i.i.d. random variables, the covariance term in eq. 4.49 simplifies as follows:

\[
\text{cov}\{e^{j\hat{\omega}_l}, e^{-j\hat{\omega}_k}\} = (1 - |\gamma|^2) \cdot \delta [l - k]
\]  

(4.50)

The definition in eq. (4.48) has been referred to as the signal-to-noise ratio (SNR) in [30]. Although, in the field of signal processing iCV is often referred to as the SNR, we avoid referring it so. In our context, formally the denominator in eq. (4.48) is not representing the noise power, whether additive ($\sigma^2_n$) or multiplicative ($\sigma^2_w$), but rather the dispersion of the test statistic.
4. DETECTION OF COHERENT SCATTERERS IN THE PRESENCE OF PHASE INSTABILITIES

Figure 4.4: Numerical analysis of the inverse coefficient of variation (iCV) of the test statistic \( \hat{\alpha} \) when point scatterers are embedded in different clutter levels, for \( M = 50 \) acquisitions. (a) iCV against concentration of the phase residuals for different levels of signal-to-clutter ratio (SCR) and number of scatterers, \( Q \in \{1, 2\} \). (b) Probability of detection against iCV for fixed levels of false alarm, \( f_F \in \{10^{-2}, 10^{-3}, 10^{-4}\} \), and \( Q \in \{1, 2, 3\} \).

Using this result, the eq. (4.49) reduces to the following [30]:

\[
\text{var} \{ \hat{\alpha}; \mathcal{H}^1 \} = \frac{1}{M^2} \left| \gamma \right|^2 \sum_m |d_m|^2.
\]  

(4.51)

Therefore,

\[
iCV^2 = \frac{\left| \gamma \sum_m |d_m| \right|^2}{(1 - \left| \gamma \right|^2) \sum_m |d_m|^2} = \left( \frac{\left| \gamma \right|^2}{1 - \left| \gamma \right|^2} \right) \frac{\| y \|^2}{\| y \|^2_2}.
\]  

(4.52)

Since \( \| y \|^2_2 \leq \| y \|^1 \leq \sqrt{M} \| y \|^2_2 \) [49], we reach the following bounds on the iCV for a given level of coherence:

\[
\frac{\left| \gamma \right|^2}{1 - \left| \gamma \right|^2} \leq iCV^2 \leq M \cdot \frac{\left| \gamma \right|^2}{1 - \left| \gamma \right|^2}.
\]  

(4.53)

iCV is a function of the ensemble coherence as well as on the ratio of the L1 to L2 norm of the signal vector. While the coherence is in turn a function of the concentration of the phase residuals (as shown in Fig. 1), the L1-L2 ratio is influenced by the 1) number of acquisitions and 2) the number of point-like scatterers in the same resolution cell. Fig. 4.2 shows the variation of the empirically estimated iCV against the concentration parameter for different number of scatterers, for \( M = 50 \) acquisitions as example. \( 10^5 \) realizations of the phase residue are generated under a von Mises distribution for each value of \( \kappa \) selected between \((0, 20]\). The dashed lines in Fig. 4.2 highlight the upper and lower bounds on iCV. The upper bound is reached theoretically when \( \| y \|^1 = \sqrt{M} \| y \|^2_2 \). Therefore, the more the number of acquisitions, the higher is the achievable iCV. At a given concentration of phase residuals, the iCV decreases for increasing number of scatterers. The iCV estimates for \( Q = 1 \) converge at the upper bound. The impact of the number of scatterers on the iCV is further discussed in Appendix A.

Fig. 4.3a is a plot of the numerically estimated \( f_D \) against the iCV. The detection thresholds are set to
4.3. Methods

This section presents the overall methodology adopted for the interferometric and tomographic processing of a real interferometric data stack. The models discussed in the previous section form the basis of this methodology. The data undergoes several preprocessing steps. A reference scene is selected, and a multilooked intensity image of the reference scene is used to geocode and coregister all the acquisitions in the stack. An external digital elevation model (DEM) is used in the process [50, 51]. A suitable reference point is selected to compute double-differenced interferograms.

4.3.1 Interferometric processing with IPTA

We use the IPTA [3, 8] framework for the PSI processing, whereby parameter estimation and phase calibration of the data stack are performed side by side using an iterative approach to least squares regression. An initial list of PS candidates is prepared on the basis of high temporal stability of the backscattering and low spectral diversity. The phase model assumed is as given in eq. (4.1). Point differential interferograms are obtained by subtracting the topographic phase computed using the DEM. A multivariable regression is used for each candidate to obtain an initial estimate of $s$ and $v$, as well as the phase unwrapping integer, $p$. The quality of the estimates is assessed in terms of the root-mean-square (RMS)
phase deviation, $\sigma_w$ of the residual phase. At the initial stage, atmospheric phases in each interferometric layer have not be corrected, and the possible temperature-induced phase variations of candidates on structures experiencing thermal expansion have also not be accounted for. Therefore, the residual phase typically exhibits a high dispersion. The PS candidates for which $\sigma_w$ is higher than a pre-selected threshold, $\sigma_c$ are masked out. The residue of the remaining candidates is analyzed further. Assuming the atmospheric phase screen (APS) to be spatially low-frequency and temporally uncorrelated, we estimate it by spatial filtration and unwrapping of the phase residue in the neighborhood of the candidates that satisfied the quality criterion. The estimated APS is subtracted and point-differential interferograms are re-computed for the full list of PSCs, and this time the phases related to the initial estimates of residual height, linear deformation and the atmospheric phase are subtracted as well. The resulting point differential interferograms are unwrapped and the regression is iterated. It is expected that the quality of the candidates would improve since an estimate of the atmospheric phase has been subtracted prior to the regression. $\sigma_w$ is computed again for all the candidates, and compared against $\sigma_c$ to mask out those with relatively low quality. For the retained candidates, the newly estimated regression coefficients (residual height and deformation velocity) act as ‘corrections’ on the previous estimates. The new phase residue is added to the previous estimate of the atmospheric phase, re-filtered and unwrapped to give a new estimate of the atmospheric phase. The process is iterated several times. In this way, there is progressive improvement in the quality of the estimates in consecutive iterations. For more details on various time-series processing strategies using the IPTA framework, the interested readers are referred to earlier works [3, 8, 9, 52].

For the candidates that are potentially undergoing thermal expansion, another regression-based routine is used that models it assuming that the corresponding phase variations are linearly dependent on the temperature changes [53–55]. The estimated regression coefficient is the phase-to-temperature sensitivity, $\eta$. Details are referred to the earlier work in [34].

After several iterations, the APS is well isolated and we obtain iteratively-refined estimates of the parameter vector, $\hat{p}$ for the PS candidates that satisfy the quality criterion. Assuming that these PS are of sufficiently good quality that the limiting case of von Mises distribution for the phase residuals being approximated by a linear normal distribution is justified, we compute the sample coherence threshold corresponding to $\sigma_c$ using eq. 4.25 as follows:

$$T_{\gamma_c} = \exp\left(-\frac{\sigma_c^2}{2}\right).$$

In turn, the corresponding probability of false alarm (theoretical) is computed using eq. (4.18).

4.3.2 Single-look differential SAR tomography with extended phase model

Prior to tomographic inversion, the interferometric data stack requires a precise phase calibration. For the pixels containing PS, we already have an estimate of the atmospheric phases from the PSI processing. Given a sufficient distribution of the PS over the imaged scene or the region of interest, we interpolate these phases over the surrounding pixels that may or may not have been PS candidates. Single-look differential tomographic inversion is applied for each pixel. The extended phase model, given in eq.
4.4 Data

The interferometric data stack used in the work comprises 50 TerraSAR-X stripmap acquisitions over the city of Barcelona, Spain in repeated passes. The temporal span of the acquisitions extends from 2007 to 2012. The images have been oversampled by a factor of 2 to allow for more accurate coregistration. The resolution in range and azimuth is 1.2 m and 3.3 m, respectively. The orthogonal component of the total spatial baseline is 503 m, which provides resolution in elevation axis of $\sim$19 m. The distribution of the spatial and temporal baselines, as shown in Fig. 1a, is highly non-uniform. The corresponding 2D point spread function is shown in Fig. 1b. The footprint of the acquisitions in map coordinates is shown in Fig. 1c. Apart from a dense urban stretch, some part of the viewed scene extends over the Balaeric sea.

4.5 Results on Real Data

This section presents the results obtained on the real interferometric data stack introduced in the previous section.
Figure 4.6: PSI solution obtained with iterative least-squares regression-based processing using the interferometric point target analysis (IPTA) toolbox. The colored dots are the PSs identified in the PSI processing. (a) Estimated height, relative to the WGS-84 reference ellipsoid. (b) Deformation velocity in the line-of-sight. (c) Phase-to-temperature sensitivity. (d) Sample coherence, and histogram of the estimated concentration parameter (shown as inset).

4.5.1 Interferometric processing

An initial list of PS candidates was prepared on the basis of low spectral diversity and high stability of the backscattering amplitude that is characteristic of single dominant scatterers [3]. There was no candi-
Figure 4.7: Point cloud of single scatterers obtained with differential SAR tomography. The threshold was set at $\sigma_c = 1.1$ rad for the proposed detection scheme. The color coding represents the estimated height. Several false alarms can be seen over the water surface.

Figure 4.8: False alarm rate observed over the sea patch in the viewed scene at different detection thresholds. The colored solid lines represent the case of 3/2/1-D tomographic inversion. The detection is performed on the retrieved reflectivity, $\hat{\alpha}$ according to eq. 4.55. The dot-dashed lines shows the case of PSI whereby the detection is performed on the sample coherence, $\hat{\gamma}$ without fitting any phase model to the observed interferometric phases.

date in unexpected areas, such as the water surface or radar shadows. After several iterations of the least squares regression within the IPTA framework, as outlined in section 4.3.1, a subset of the initial candidates is retained such that $\sigma_w \leq \sigma_c = 1.1$ rad for each candidate. Fig. 4.6 shows these candidates from the last iteration. These are 936649 in number, and spread over an area of nearly 4 km$^2$. In sub-figures (a)-(c), the color coding represents the estimated parameters, namely residual height, deformation velocity in the LOS and phase-to-temperature sensitivity, respectively. The sample coherence for these candidates is as shown in sub-figure (d). The median value for the sample coherence is 0.87. Using the
4. Detection of coherent scatterers in the presence of phase instabilities

Figure 4.9: Point cloud of single scatterers obtained with differential SAR tomography. The threshold was set at $\sigma_c = 1.0$ rad for the proposed detection scheme. (Top) Estimated height, relative to the WGS-84 reference ellipsoid. Middle: Deformation velocity in the line-of-sight. (Bottom) Phase-to-temperature sensitivity.

estimated coherence, we estimate the concentration parameter for each PS, as follows [56]:

$$
\hat{\kappa} = 
\begin{cases} 
2\hat{\gamma}^2 + \frac{5}{6}\hat{\gamma}^6 & \hat{\gamma} < 0.53 \\
-0.4 + 1.39\hat{\gamma} + \frac{0.43}{1 - \hat{\gamma}} & 0.53 \leq \hat{\gamma} < 0.85 \\
\frac{1}{3\hat{\gamma} - 4\hat{\gamma}^2 + \hat{\gamma}^3} & \hat{\gamma} \geq 0.85 
\end{cases}
$$

(4.56)

A histogram of the estimated concentration parameters is shown as an inset in Fig. 4.6d. The peak is at $\hat{\kappa} = 2.8$, while the median value is 4.1.
4.5. Results on real data

Figure 4.10: Point cloud of double scatterers obtained with differential SAR tomography. The threshold was set at $\sigma_c = 1.0$ rad for the proposed detection scheme. (Top) Estimated height, relative to the WGS-84 reference ellipsoid. (Middle) Deformation velocity in the line-of-sight. (Bottom) Phase-to-temperature sensitivity. The left column shows the lower layer and the right column shows the upper layer of the double scatterers, respectively. The inset focuses on a commercial complex (Diagonal Mar). The red polygon encloses a single building, part of which is in layover with a nearby building of shorter height.

4.5.2 Tomographic processing & empirical analysis of false alarms

The APS isolated in the IPTA-based PSI processing is extrapolated over the scene and compensated for over the entire scene in each layer of the interferometric stack. In this way, each pixel is considered to be
phase calibrated so that tomographic inversion can be applied next. Given that the city of Barcelona has several high-rise buildings, the elevation extent, $\mathcal{S}_s$ is set as $[-60,300]$ m. The parameter space for the deformation parameters is as follows: $\mathcal{S}_v \in [-10,10]$ mm/yr and $\mathcal{S}_\eta \in [-1,1]$ rad/K. The discretization in each dimension is 1/2.5 times the Rayleigh resolution, followed by a local refinement of the estimated reflectivity around the two candidate peaks at one-tenth the resolution. Using eqs. 4.54 and 4.55, and keeping $\sigma_c = 1.1$ rad, we threshold the reflectivity of the two candidates to perform the detection process. The point cloud of single scatterers thus detected is shown in Fig. 4.7. Several false alarms can be seen over the water surface.

A significant portion of the viewed sea extends over the sea, which is auspicious as it can be used as a test bed to conduct an empirical analysis of the false alarm rate. We perform sample coherence-based detection, as well as tomographic inversion and detection, for the range-azimuth pixels over the sea and observe the variation of the false alarm rate. These pixels constitute 1.4 million independent resolution cells. The results are shown in Fig. 4.8. The solid lines in the figure represent different cases of tomographic inversion and detection: 1) 3-D inversion and detection on the reflectivity, $\alpha$ retrieved as a function of elevation ($s$), deformation ($v$) as well as thermal expansion ($\eta$) where the support in each dimension is as mentioned above, 2) 2-D inversion i.e., thermal expansion is not considered, 3) 1-D inversion, whereby the reflectivity is retrieved only along the elevation profile, 4) 1-D inversion with the elevation support reduced to $[-25,50]$ m, and 5) 1-D inversion without the maximizations to detect peaks in the reflectivity, i.e., no fitting is performed in the parameter space to estimate the unknown elevation and deformation parameters. The dot-dashed line represents the PSI case whereby the thresholds are applied on the sample coherence without any parameter fitting. The black curve with diamond symbols shows the probability of false alarm (theoretical) according to the eq. 4.18. The bottom x-axis in the figure shows the detection thresholds, $T_\gamma$ and $T_\alpha$ (normalized between 0 and 1 as per eq. 4.55), while the top x-axis shows the equivalent standard deviation of the residual phase according to eq. 4.54.

Fig. 4.9 shows the point cloud of single scatterers obtained with tomographic inversion and detection with $\sigma_c = 1.0$ rad. In comparison with Fig. 4.7, we can see a reduction in the false alarms. 3-D tomographic inversion has been applied, therefore, we have estimates of height, deformation velocity as well as the phase-to-temperature sensitivity. Fig. 4.10 shows the point cloud of double scatterers obtained with the same threshold. They are separated as lower and upper scatterers, according to the estimated height for each of the two scatterers in layover. The inset in the sub-figures shows a commercial complex, namely Diagonal Mar, in focus. The red polygon encloses a high-rise building, which is partly in layover with the roof of a nearby building.

### 4.6 DISCUSSION

This section provides an itemized discussion of the results presented in the previous section.

#### 4.6.1 Interferometric processing

The PSI solution, as shown in Fig. 4.6, provides a good coverage over the viewed scene, which is typical with high resolution X-band interferometric imagery over urban areas such as Barcelona city [57, 58].
The PS heights fit reasonably with actual 3-D structures, as shown for selected buildings in our earlier work in [26, 27]. The PSI solution reveals deformation along the shoreline, which was partly observed in [57] as well. Several PS on high-rise buildings show temperature-dependent phase variations, which can be attributed to thermal expansion of the structures [29, 34, 45, 59, 60]. The observed coherence is high, and the estimated concentration parameters are all non-zero. With reference to Fig. 4.1, the fact that the median value of $\hat{\kappa}$ is greater than 3 substantiates the typical assumption of linear Gaussian statistics for the PS (since the approximation of von Misses as linear Gaussian distribution is accurate to within 5% error on average).

Interestingly, we do not observe false alarms over the sea patch in the scene. This owes to the fact that we have used high stability of the backscattering amplitude and spectral diversity as pre-classifiers to set up the initial PS candidate list. These classifiers are proxies for temporally coherent, single dominant scattering; therefore, they already preclude PS candidates from appearing on the water surface. Hence, no PSI solution has been sought (no regression fitting) on the pixels over the sea patch. In the context of tomography, these pre-classifiers cannot be used since they would tend to reject double scatterers as well.

### 4.6.2 Tomographic processing & empirical analysis of false alarms

We applied tomographic processing over the entire scene, regardless of any surface classification. The point cloud shown in Fig. 4.7 is obtained using the same cut-off phase standard deviation, $\sigma_c = 1.1$ rad, as for the iterative least squares based PSI processing. Nevertheless, several false alarms are visible over the sea patch. A simple mask (based on SAR intensity with spatial constraints for example) could have allowed us to remove the sea patch from the processing, but we choose to show these false alarms to highlight that similar false alarms may arise (due to noise) within the urban stretch as well though they may remain unnoticed.

Fig. 4.8, which shows the results of a false alarm analysis exclusively conducted over this patch, reveals that the false alarm rates can typically be higher in practice in comparison with the theoretical probability of false alarm (as the area under the upper tail of Rayleigh distribution). The maximizations (eqs. 4.37 and 4.38) allow degrees of freedom to fit the data; when the noise is fit incorrectly with the data model, it may lead to false alarm. The false alarm rate can be seen to decrease from 3-D to 2-D inversion, as reducing the dimensionality reduces the degrees of freedom to fit the data. Similar reduction in false alarm is observed when moving from 2-D to 1-D inversion, or when we reduce the support of the elevation in case of 1-D inversion. These findings imply that in case some a priori information is available – e.g. if significant thermal expansion is not expected (as is usually the case for buildings of low height), or if the support of deformation velocity can be reduced on the basis of local leveling measurements, or if the support for height corrections can be reduced given a digital surface model is available – then a reduction in false alarm rate can be achieved in practice.

Fig. 4.8 also shows the case where no parameter fitting is performed (for both tomography as well as sample coherence based detection). These results closely match with the theoretical relationship, indicating that the statistics of the test statistic ($\hat{\gamma}$ and $\hat{\alpha}$) closely follow the Rayleigh. The area shaded in
gray indicates the region where the results are not reliable due to insufficient number of independent range-azimuth resolution cells over the water surface. Given we have only 1.4 million of such cells, and assuming the test statistics are normally distributed over the scene, we can estimate a probability of false alarm no less than $1.1 \times 10^{-3}$ with a relative absolute error of 5% for 95% of the time [48].

Fig. 4.9 and Fig. 4.10 show the single and double scatterers, respectively, detected with $\sigma_c = 1.0$ rad. As expected, we observe fewer false alarms, and at the same time fewer scatterers are detected. $2.14 \times 10^6$ single scatterers and $1.01 \times 10^4$ double scatters (lower + upper) have been detected. Double scatterers constitute $< 1\%$ of the total scatterers detected over the scene. The gain in deformation sampling due to double scatterer detections [27], relative to the PSI solution, are around $2\%$ for Diagonal Mar complex and $4\%$ for the selected building marked in red, respectively. If the threshold is relaxed to $\sigma_c = 1.1$ rad, the gain improves to $6.4\%$ for Diagonal Mar and $17\%$ for the individual building.

4.7 Conclusion

In the context of SAR tomography as an add-on to PSI to potentially improve deformation coverage, following the directions set in earlier works in [12, 27, 30], this paper reports the application of a detection strategy that allows extending the same quality considerations to tomography as used in the prior PSI processing. In interferometric processing, the quality is typically assessed on the basis of the residual phase, either in terms of the phase dispersion (phase standard deviation) or the ensemble coherence computed using the residue of the fit. In both cases, under the proposed detection strategy, the quality parameters can be used to set up the threshold for hypothesis testing of coherent scatter candidates following tomographic inversion. Moreover, the theoretical probability of false alarm remains the same between the PSI and tomography. We have also highlighted that while the instabilities in phase are typically modeled as additive noise, their impact on tomography is multiplicative in nature. The experiments performed in this work with simulated data consider both multiplicative noise as well as additive disturbances (clutter) in the tomographic model. We show that the inverse coefficient of variation is a suitable parameter to assess the probability of detection, irrespective of the origin of noise. We have also performed experiments on real data, primarily to assess the variation of the observed false alarm rates against the thresholds set according to the proposed detection strategy. An interferometric data stack comprising 50 Terra-SAR-X acquisitions over the city of Barcelona, Spain has been used. We apply single-look beamforming for 1/2/3-D tomographic inversion, depending on whether the phase model used considers only the scatterer height, or height plus deformation velocity, or additionally thermal expansion. The results show that higher dimensionality and larger support sizes in each dimension lead to higher false alarm rates due to larger parameter space that may incorrectly fit noise to the data model. These results also suggest that in case an a priori information can reduce the dimensionality and/or support sizes, it should be adopted by the user to reduce the false alarm rate in practice.
Appendix A

The equality in eq. 4.53 holds when all elements in the vector $y$ are identical. In our context, theoretically it occurs for the case of a single point scatterer in the given range-azimuth resolution cell [30]. Interestingly, it is the PSI case wherein the PS is defined to be a single point-like scatterer. It can be explained by considering the SLC values as samples of the Fourier spectrum of the target situated along the elevation axis [14]. In case of point (dirac delta) scattering, the absolute value of the spectrum is a constant, and therefore, all samples (magnitude of the SLCs) are identical. Conversely, for a target that is extended continuously along the elevation axis, albeit deterministically, its spectrum is delta-like. In other words, the target can be considered as closely spaced sequence of several point-like scatterers along the elevation axis, in the same range-azimuth resolution cell. That is the case when the vector $y$ tends to be 1-sparse, and in turn, the ratio $\|y\|_1/\|y\|_2$ approaches 1. A real example of such an extended scatterer can be a mountain slope with a nearly zero local incidence angle (considering it to be vegetation-free and exhibiting a stable response).

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References


CONCLUSION

This chapter summarizes the main findings and contributions of the different investigations conducted in the frame of this dissertation, and provides an outlook for future investigations.

5.1 MAIN FINDINGS & CONTRIBUTIONS

The main findings are structured chapter-wise, as follows.

5.1.1 Chapter 2 (SAR tomography for improved deformation analysis in urban areas)


This empirical investigation was performed on an interferometric data stack comprising 50 TerraSAR-X stripmap-mode acquisitions over an urban zone in the city of Barcelona, Spain. A PSI solution was obtained using the Interferometric Point Target Analysis (IPTA) [1, 2]. The data stack was phase calibrated by extrapolating the atmospheric phases computed for PS identified in the PSI processing, and compensating it. The main research questions for this investigation were as follows:

1. How do different phase models for tomographic inversion impact layover separation?

2. Is the quality of the scatterers obtained with tomography comparable to the quality of the persistent scatterers obtained with a PSI approach? What is the corresponding gain in deformation sampling with the added use of tomography?

To seek the answer for the first question, different phase models were implemented for tomographic inversion. The phase models for a) classical SAR tomography (3-D SAR), b) differential tomography, and c) the one further extended to simultaneously model thermal expansion, are compared against each other with respect to their suitability in resolving layovers. The phase model for a) considers only topography-induced (elevation) phase variations; for b), deformation is considered in addition to elevation, while
the kinematics of the deformation are assumed to be linear in time; and for c) the phase model is further extended to include a direct dependence of phase variations on temperature changes. The results obtained in the investigation confirm that Unmodeled thermal expansion-related phases were shown to cause missed detections. For most of the buildings, the sensitivity of phase variations against temperatures is found to grow with height, which is plausible as the base of the buildings is grounded and the upper parts are less constrained in terms of thermal expansion.

In this investigation, a beamforming (BF)-based sequential generalized likelihood ratio test (GLRT) was used to detect single and double scatterers. The quality of these scatterers was assessed in terms of the root-mean-square (RMS) phase deviation between the measurements (SLC values) and the tomographic model fit (i.e. the residual phase), which is consistent with the way the quality of the PS is evaluated in the IPTA-based PSI processing. To seek the answer for the second question, the impact of the detection thresholds on both the quantity and the quality of the detected scatterers was studied, and the relative gain in deformation sampling with layover separations was computed at different thresholds. The results obtained indicate a trade-off between the quantity and the quality. The selected test site was Diagonal Mar, Barcelona – a locality which represents a general semi-commercial zone in Barcelona city, with several high-rise buildings affected by layover. Therefore, prospectively, it is a site where several double scatterers might exist. The investigation concluded in order that the quality of the overall detected scatterers is comparable to the quality of the PS identified with the PSI processing, the detection process may become too restrictive to allow for a sufficient number of layover separations. With a slight compromise in threshold setting, a relative gain in deformation sampling of 9.8% was obtained while keeping the RMS phase deviation for 99% of the detected double scatterers and single scatterers (in addition to the PS) below 1.1 rad (which was set as the upper limit to allow for the acceptance of a PS candidate in the final PSI solution).

Another contribution of this investigation is that it serves to showcase the use of a single-look non-parametric tomographic inversion method for the processing of a real data stack. Since no multi-looking is performed, the full spatial resolution of the data is preserved. Though it is not a unique contribution in this aspect (as similar studies have been reported in recent past by other researchers, as in [3–5]), but given that quantity vis-a-vis quality has been additionally investigated, and the analysis has been performed on the more commonly available stripmap product, this investigation can serve as a reference to compare with in future investigations.

5.1.2 Chapter 3 (Correction of atmospheric phases for SAR tomography in mountainous regions)


This investigation sought to explore the application of SAR tomography in mountainous regions, where even a minor improvement in deformation coverage relative to a PSI solution in layover-affected areas can be particularly rewarding (considering the already low prevalence of PS candidates in comparison
with urban areas). This investigation is the first case study in this direction. It focused on the issue of atmospheric phases which is one of main factors that limit the performance of tomography in mountainous areas. The main questions raised in this context were as follows:

1. **Can a data-driven approach build upon the prior PSI solution to reliably correct for atmospheric phases for the application of SAR tomography in mountainous regions?**

2. **How can multiple corrections for potentially multiple scatterer in layovers be implemented alongside the spatio-temporal inversion?**

In an effort to answer the first question, a regression-kriging approach was implemented. It essentially performs a 3-D interpolation/predication of the atmospheric phases computed at PS locations in the prior IPTA-based PSI processing. Map coordinates (northing, easting and height) are used as the regressors. In this way, height-dependent phase variations as well as lateral phase trends are modeled functionally. Additionally, in order to consider the turbulent mixing effects, the second-order spatial statistics (in terms of the sample variograms and covariance functions) are estimated, and used for the best linear unbiased prediction (BLUP) of the atmospheric phases at the unknown (non-PS) locations where tomographic inversion is intended. For each range-azimuth resolution cell in a layover-affected area, the inversion is applied at locations along the (discretized) elevation axis. In response to question 2, a geocoding step is implemented at each discrete location, transforming its radar coordinates to map coordinates, and predict the atmospheric phase at that location with regression-kriging. In this way, for each of the multiple scatterers at different elevation positions – though in the same range bin in case of layovers – the predictions are computed individually. The geocoding step also ensure that in case of very large height differences among the scatterers in layover, they are first projected to their real locations in map geometry and thus the BLUP are obtained by weighting over the correct local neighborhoods.

The investigation was performed on a data stack comprising 32 Cosmo-SkyMed acquisitions over the Matter Valley in the Swiss Alps. The region of interest (ROI) for differential tomographic inversion was selected around Zermatt village, which was partly covered by the layover cast by an adjoining mountain. The reliability of the regression-kriging based atmospheric corrections was assessed in terms of the prediction error. These errors naturally grow with increasing distance from the distribution of the PS (known locations). Since the PS distribution obtained in the prior PSI solution was sparse at high altitudes, it was found that predictions errors grow rapidly with increasing height from the valley floor to the top (around 2800 m). In most interferometric layers, the prediction error exceeded 0.85 rad, which can indeed be a limiting factor in detecting double scatterers at high altitudes [6]. Increase in coverage in terms of layover separations remained limited as only a very few double scatterers were detected, and nearly all of them belonged to the layovers occurring in the built-up area in the valley floor. In addition to the single scatterers detected on the man-made structures in the valley, some single scatterers were detected along the mountainside around 230 m above the valley floor, where no PS were found in the prior PSI processing, thus substantiating the usefulness of the kriging-based atmospheric corrections.

This was a first investigation towards facilitating the use of SAR tomography in mountainous regions, with emphasis on correcting the atmospheric phases. Although there have been several limiting factors
(e.g., range-cell migration with increasing elevation change, large pixel-area squeezed in a few range bins, etc.), and only the phase calibration issue was addressed, yet it serves to hint at the relevance of these factors in a broader context.

5.1.3 Chapter 4 (Detection of coherent scatterers in the presence of phase instabilities)

This chapter has been submitted for publication as: Siddique, M. A.; Wegmuller, U.; Hajnsek, I. & Frey, O. (2018), SAR tomography as an add-on to PSI: Detection of coherent scatterers in the presence of phase instabilities, Remote Sensing, (in review).

This investigation was partly motivated by a shortcoming noticed in the first investigation. The tomographic processing framework in the first investigation (chapter 2) used a sequential GLRT for hypothesis testing. The quality of the detected scatterers was evaluated only after the scatterers were detected, and then in turn compared with the quality of the PS. In other words, the detection threshold for hypothesis testing had to be adjusted \textit{a posteriori} to achieve comparable quality. In short, it raised the following question:

1. \textit{Can we perform scatterer detection in the tomographic framework based on the same quality criteria as as used in the prior PSI processing?}

The efforts in this direction were built upon the directions set in earlier works in [6, 7]. A detection strategy was devised whereby the quality parameters (in terms of the RMS phase deviation or ensemble coherence) can be used to set up the threshold \textit{during} hypothesis testing of coherent scatter candidates following tomographic inversion. In this way, the same quality criteria that were used in the prior PSI processing can be extended to tomography. It also allows keeping the theoretical probability of false alarm at the same level between PSI and tomography.

In this investigation, experiments were performed with simulated data, as well as on the real interferometric data stack from the first investigation (chapter 2). In the light of the pioneering work in [6], this investigation appreciated that while the instabilities in phase are typically modeled as additive noise, they directly impact tomography in a multiplicative sense, and should be considered as such in the detection process. With simulated data, it was shown that the coefficient of variation is a suitable parameter to assess the probability of detection, irrespective of the origin of noise (additive or multiplicative) in the tomographic model. Single-look beamforming with different phase models for tomographic inversion, as introduced in chapter 2 (i.e., 1/2/3-D inversion, depending on whether the phase model used considers only the scatterer elevation, or elevation plus deformation velocity, or additionally thermal expansion), were applied. The results show that higher dimensionality and larger support sizes in each dimension (i.e. larger parameter space) increase the false alarm rates.

5.2 Outlook

The investigations conducted in this dissertation show that while PSI suffices for a large-scale deformation analysis, SAR tomography as an add-on to PSI is particularly useful for a small-scale analysis (e.g.
deformation assessments on local infrastructure/buildings in an urban zone) by increasing deformation coverage in layover-affected areas. To push towards increased double or higher order scatterer detection, larger baseline spans will offer finer tomographic resolution to discern closely-spaced scatterers. On the contrary, relatively small baselines may suffice for tomography in mountainous areas given that the separations among the scatterers in layover can be in the order of several hundreds of meters. The use of a priori 3-D surface elevation data (e.g. a digital surface model (DSM)), when available, should be used to reduce the support sizes in tomographic inversion. In turn, it would allow the user to reduce the false alarm rate in practice. For improved detections, future investigations should explore methods to further improve phase calibration (correction of atmospheric phases) of the interferometric data stacks. Future missions with faster repeat passes will help reduce temporal decorrelation, and thereby improve phase stability in general. Moreover, missions with more than two (or several) sensors in the constellation acquiring simultaneously, will surely increase the relevance of SAR tomography in coming days for resolving scatterers in layover and estimating their deformation parameters.

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Research
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Teaching
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**PUBLICATIONS**

**Published articles**


**Submitted articles**


**Published book-chapter**


**Published in conference proceedings**


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