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Dynamic measurement and its relation to metrology, mathematical theory and signal processing: a review

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Abstract. Dynamic measurements are measurements of quantities with time-dependent values using measuring instruments that are mathematically modelled by dynamic measurement systems. Typical examples are quantities with rapidly changing values, such as pressure in a combustion engine or the forces acting in a crash test. The theory underlying such measurements is considerably old but spread across different disciplines like mathematics, statistics, signal processing, engineering and control theory. In order to further develop metrology for dynamic measurements, the different elements have to be brought together under a common and harmonized umbrella. This contribution takes a first step towards this aim by reviewing mathematical theory, signal processing and elements of metrology.

1. Introduction

The measurement of time-dependent quantities, i.e. dynamic measurement, is a topic of growing importance for metrology and industry. The history of dynamic measurement in science and technology dates back to the late 19th century, leading to a rich toolset of methods and measurement principles. In metrology, though, this field - despite a growing list of scientific publications and draft guidelines - is still rather immature compared to metrology for quantities with nominally fixed value. One reason appears to be that measurement analysis for time-dependent quantities requires merging different fields from mathematics, statistics and engineering, harmonizing the vocabulary and developing novel techniques for the corresponding analysis of dynamic measurement uncertainties [1]. Due to these challenges, in many cases measuring devices are utilized without a proper dynamic calibration or the analysis of the measured values is done in an ad-hoc manner, neglecting dynamic effects. To this end, in this contribution we aim to combine the knowledge from mathematical modelling, stochastic process theory, traditional GUM uncertainty treatment and a rigorous analysis of the structure of dynamic measurements. Our aim is to provide a first step towards a harmonized treatment of dynamic measurements and allows identifying necessary fields of future research. Therefore, we review the historical background, carry out an analysis of the measurement principles and structures, review calibration and dynamic error compensation.

2. Brief historical background on dynamic measurements in science

The pioneer work on dynamic measuring is the paper of the French physicist Cornu [2] published in 1887. In this work he studied the motion of a galvanometer and associated the value of the damping



factor with physical parameters of the device. Later on, the inventor of an oscilloscope - the French engineer Blondel [3] - in 1893 considered dynamic models for some of its modules. Also, the work of D.I. Mendeleev [4], published in 1897 focusing on accurate weighing on the unsteady laboratory balance should be referred to as one of the first academic papers. The work of A.N. Krylov [5], where a device for measuring the mine explosion pressure was examined, can be considered as a starting point for development of the theory of dynamic measurements, studying a time-dependent measuring error. Based on improved technology, the interest in dynamic measurements grew rapidly in the 1970s and 1980s. A special role in the development of the theory of dynamic measurements belongs to V.A. Granovkii, who in his book [6] presented a comprehensive theory on dynamic measurements.

The importance of dynamic measurements becomes apparent whenever the values indicated by the measuring instrument show a time-dependent deviation from the actual measured values. Most of the publications dedicated to the compensation of such errors are based on the fundamental work of A.N. Tikhonov on methods for solving ill-posed problems [7, 8]. To some extent, these results coincide with the result of the analogous problem of Wiener-Kolmogorov's filtration theory [9]. Other authors proposed a Bayesian approach, which under certain conditions corresponds to the Tikonov method, see, e.g., [10]. A large number of further algorithms have been proposed leading to a rich toolbox for reconstructing the measured values from the erroneous indication signal.

3. General structure of dynamic measurements

The structure of dynamic measurements can be derived from signal and system theory (SST) which, in principal, can also be employed for non-dynamic measurements. In this way, the below given generic structure also provides a means to harmonise dynamic and non-dynamic measurement theory. In the field of SST, the concepts for *observation*, *observer* and *observation process* are well established and provide a clear mathematical framework, distinguishing in particular *observation* from *measurement* [11].

The term *quantity* is used as a global term and in SST the formal model of a *real-world quantity* is called *signal* being itself an abstract item. A quantity with a time-dependent value is called a *dynamic quantity*. Mathematical modelling deals with the abstract relations between signals. Such a set of relations is defined as a *system*, which is a simplified and reduced model of a real-world *process*. The common *measurement process M* consists of a *sensor process S* and a *reconstruction or estimation process R*. In addition, the generic structure can contain a process *P* that generates the *signal* $y(t)$ from an *input quantity* $u(t)$, see Figure 1.

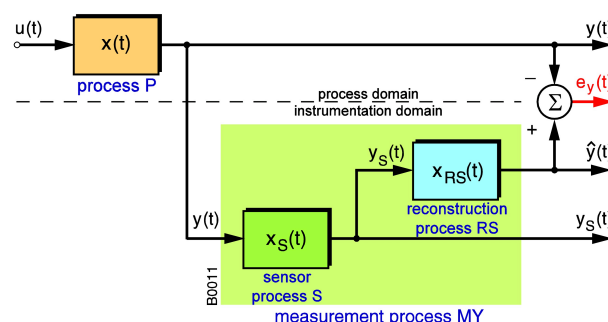


Figure 1 Generic structure of a measurement process with remaining errors $e_y(t)$ which cannot be compensated by the reconstruction process

Consequently, the measurement equation consists of the sensor equation plus the reconstruction equation. In the ideal case, the output of the measurement process, i.e. of the reconstruction, is numerically identical to the input signal to the sensor process. Note also that in this terminology, an *observation process O* may extend the measurement process with a data processing part in order to

estimate information $x(t)$ about the process P which is normally not accessible for a physical sensor process. The most famous observation procedures are the Luenberger observer and the Kalman filter.

4. Representations of linear time-invariant dynamic measuring systems

For *sensor processes* for which the associated *system* is linear in the input signals and has time-invariant parameters (LTI), the continuous mathematical model consists of a set of ordinary differential equations. This approach is widely used to represent the sensor process in the form of a physical-mathematical model. The *reconstruction process* is typically modelled using a discrete approach based on difference equations instead. This is used, for instance, for designing and implementing the reconstruction process in terms of digital signal processing, e.g. using digital filters [12, 13]. Nevertheless, mathematical theory for continuous models of the reconstruction process have been studied in the literature widely. There are methods to transform a continuous model to a discrete model and vice versa [14,15]. There are several equivalent characterizations for sensor processes modelled as LTI systems: transfer functions and its discrete correspondents; amplitude and phase frequency characteristics; impulse transient function – as impulse response; step transient function – as a step response. These representations are fully equivalent and there are mathematical methods to convert one characteristic to another. The best suited characterization to be used depends on the measurement method, measurement situation and, most importantly, the intended approach for the reconstruction process [1, 16].

5. Calibration of linear time-invariant systems

According to the VIM [17], the first step in a calibration in our generic setting from Figure 1 is the operation that establishes a (mathematical) relation between the quantity values which are the input to the *sensor process* and the corresponding indications, i.e. the output of the sensor process. For a dynamic *sensor system* this requires to take into account its dynamic properties, i.e. its frequency-dependent behaviour in the calibration. For instance, determination of the dynamic system's impulse response is carried out by exciting the sensor process with a known input pulse. Ideally, this pulse is as close to the Dirac delta function as possible to cover a wide frequency range. A typical example is the calibration of ultra-fast sampling oscilloscopes with frequency range in the Gigahertz spectrum and pulse durations in the pico-second range [18]. Similar to the impulse response calibration, the step response is obtained by exciting the sensor process with a known input step. For instance, in shocktube measurements for dynamic pressure calibration an ideal step pressure front can be assumed to calibrate a pressure transducer [19]. Another typical example is the calibration of thermometers by injection into a fluid of known temperature different to the ambient temperature [20]. The frequency response as the transfer function evaluated at complex frequency values can be determined, for instance, by exciting the sensor process with inputs of known frequency content and calculations in the frequency domain after a Fourier transform. This is done, for instance, for the dynamic calibration of hydrophones [21]. Determination of the transfer function is typically carried out by means of a parametric calibration and provides the most generic model approach. It requires either a physical-mathematical model or a black-box approach [22]. For instance, sensors for measuring time-dependent acceleration can be calibrated by means of sinusoidal excitation with known frequencies [23]. The result is a set of frequencies with corresponding amplitude and phase values of the frequency response. Using regression methods, the parameters of the transfer function model are then determined from these values [24]. Finally, the mathematical relation between the known sensor process input signal and the corresponding indications is then given either as a convolution integral model, ordinary differential equations system model, complex-valued Laplace domain model or complex-valued frequency domain model, depending on the chosen representation of the sensor process.

6. Estimation of time dependent measurands for LTI systems

In the second step of the calibration according to VIM the relation obtained in the first step is used to establish a relation to obtain a measurement result from an indication. Hence, the second step is the

design of a mathematical relation for the reconstruction process. Both steps together, thus, establish the mathematical relation of the measurement process.

When the sensor process causes time dependent errors the reconstruction process needs to be able to compensate these in order to reduce the remaining measurement errors. Consequently, the reconstruction process ideally is also modelled as a dynamic system. As for the sensor system several equivalent representations of the reconstruction system are possible: inverse filter, inverse frequency response, inverse transfer function and so on. The special situation in dynamic measurements is that in practice the ideal reconstruction process also depends on the frequency content of the measured signal, i.e. the measurand. That is, the second step of the calibration according to VIM has to take into account information about the input signals to the sensor process, i.e. the measurand. For example, for a given sensor process with a certain dynamic behaviour, a reconstruction process may provide a very high accuracy for a slowly varying signal $y(t)$, but a low accuracy for a rapidly varying signal. A typical example is a sensor process with finite bandwidth and a reconstruction process with a fixed low-pass characteristic. Consider, for instance, a sensor process with low-pass characteristics. The reconstruction process then has to have a high-pass characteristic up to a certain frequency, depending on the noise level *and* the frequency content of the measurand [12].

In practice, the reconstruction system is very sensitive to measurement noise. Mathematically speaking the design of the inverse to the sensor system is an ill-posed inverse problem which is very sensitive to small changes in its inputs. Basically, this results in a very large amplification of measurement noise and, thus, the ideal inverse system has to be extended to attenuate high-frequency noise [1]. Many approaches have been proposed for this task, see, e.g., [7, 8, 25, 26]. The generic challenge for metrology and the main difference to traditional approaches to uncertainty evaluation, though, is the systematic error that is caused by the regularization method itself that cannot be overcome. For example, for a measurand with high frequency content in a measurement with significant measurement noise a trade-off has to be made between large variations in the estimate of the measurand due to amplified noise and systematic errors in the estimate of the measurand due to noise attenuation. In order to evaluate this error, prior knowledge about the measurand's frequency content needs to be quantified. Such kind of information is not required for uncertainty evaluation for non-dynamic measurements.

7. Interpretation and determination of dynamic measurement error and uncertainty

The quality of a *measurement process* depends on the ability of the *reconstruction process* to correct the errors caused by the *sensor process*. For dynamic measurements, the value of the quantity of interest (measurand) is time dependent. Consequently, the error remaining after the reconstruction process and the measurement uncertainty associated with an estimate of the measurand are time dependent as well. For discrete-time signals the measurement uncertainty can usually be expressed and interpreted as for multivariate non-dynamic measurements [12, 27]. For continuous-time signals, though, one has to extend the GUM framework, for instance, to stochastic processes [28]. For the evaluation of uncertainties GUM-consistent approaches for several representations of the reconstruction process have been proposed in the literature. As mentioned above, though, one of the remaining challenges is the practical treatment of the regularization error.

For a dynamic measurement there are several ways to analyse the error remaining after the reconstruction process. In the case of no measurement noise, i.e. deterministic input and output signals, methods for analysing the measurement error are proposed as time functions for basic measuring signals [6, 25]. Others consider single numeric characters to express measurement error, for instance, in the form of the mean-square error for a given time interval [29] or based on the spectrum of the signal [30]. In both cases, the time information of the measurement error is compiled in a certain way to obtain a single number. This can be useful when, for instance, an algebraic (non-dynamic) reconstruction process is preferred. In this case, a dynamic error remains and has to be accounted for in the uncertainty budget [30]. Another case is that the reconstruction process includes a certain

regularization for which a known systematic dynamic error cannot be corrected for [31]. However, a generic method and guidance for the practical implementation are missing.

8. Conclusions and outlook

The analysis and characterization of dynamic measurements is a topic of growing importance in metrology and industry. A large amount of literature exists for the mathematical modelling of dynamic measurement processes, for the statistical analysis of time series, for the design of inverse filters and systems as well as for the regularization of ill-posed inverse problems. The huge challenge for the improvement of the still comparably immature field of dynamic metrology is to join these diverse fields and combine them with the existing framework for the evaluation of measurement uncertainties in metrology. Moreover, new methods need to be developed for the design of optimal reconstruction processes in the presence of uncertainties. This can be achieved, for instance, based on the theory of stochastic processes. Another area of future research is the optimization of measuring systems to improve their dynamic characteristics. This can be based, for instance, on modelling of the measurement process and dynamic calibration. In addition, new inputs from other fields such as automatic control will have to be considered in order to improve the design and analysis of dynamic reconstruction processes. This includes statistical approaches from time series analysis and stochastic process theory. For instance, Bayesian signal processing appears to be a promising field for dynamic metrology.

The importance of dynamic measurement analysis, in particular with simplified analysis approaches, is expected to grow rapidly in particular due to the emerging field of the industrial Internet of Things (IIoT), Industry 4.0 and Smart-X. Here measurement of dynamic quantities is the standard case and automated reliable data analysis methods which compensate for the time-dependent errors caused by the sensor process are required. For instance, so called “smart sensors” could, in the future, contain adaptive filtering with evaluation of measurement uncertainties resulting in a “smart traceability”. This would improve reliability of data analysis and automated decision-making in a huge way.

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