MACROSCOPIC MODELING OF ON-STREET AND GARAGE PARKING: IMPACT ON TRAFFIC PERFORMANCE

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Abstract

The short-term interactions between on-street and garage parking policies and the associated parking pricing can be highly influential to the searching-for-parking traffic and the overall traffic performance in the network. In this paper, we develop a macroscopic on-street and garage parking decision model and integrate it into a traffic system with an on-street and garage parking search model over time.

We formulate an on-street and garage parking-state-based matrix that describes the system dynamics of urban traffic based on different parking-related states and the number of vehicles that transition through each state in a time slice. This macroscopic modelling approach is based on aggregated data at the network level over time. This leads to data collection savings and a reduction in computational costs compared to most of the existing parking/traffic models. This easy to implement methodology can be solved with a simple numerical solver.

All parking searchers face the decision to drive to a parking garage or to search for an on-street parking space in the network. This decision is affected by several parameters including the on-street and garage parking fees. Our model provides a preliminary idea for city councils regarding the short-term impacts of on-street and garage parking policies (e.g., converting on-street parking to garage parking spaces, availability of garage usage information to all drivers) and parking pricing policies on: searching-for-parking traffic (cruising), the congestion in the network (traffic performance), the total driven distance (environmental conditions), as well as the revenue created for the city by the hourly on-street and garage parking fee rates. This model can be used to analyze how on-street and garage parking policies can affect the traffic performance; and how the traffic performance can affect the decision to use on-street or garage parking. The proposed methodology is illustrated with a case study of an area within the city of Zurich, Switzerland.
1. Introduction

As the population in urban areas is increasing, more and more cars need to find parking spaces in city centers. These vehicles normally have the choice between on-street and garage parking. Both parking possibilities follow diverse policies, which can sometimes lead to rather complex interdependencies and significant changes on the performance of a transportation network. In this research, we develop a macroscopic on-street and garage parking model such that the influences of different on-street and garage parking policies on the traffic system can be studied and illustrated. Hereafter, off-street parking is referred to as garage parking. The macroscopic model is built on a traffic system with a parking search model over time. It is incorporated into the on-street parking framework from [10].

Compared to methodologies concentrating on long-term demand management strategies, our dynamic macroscopic modelling approach focuses on the short-term effects in the traffic network. With limited data collection efforts, our macroscopic on-street and garage parking decision model shows the influence of different on-street and garage parking pricing rates, on the average searching time/distance. We analyze the relationship between on-street and garage parking, but also their interdependency on cruising-for-parking traffic and traffic performance with respect to different parking fees. Different pricing strategies affect the drivers’ decision to park on-street or to drive towards a parking garage. In case the drivers decide for on-street parking they might need to circulate in the city to search for an available on-street parking space, which contributes to the problem of traffic congestion. In case the drivers decide for garage parking, there is no need to search-for-parking. These vehicles drive towards the closest parking garage and access it depending on its current availability. Insights from this paper will help city councils or private agencies to analyze the short-term impacts on the traffic system, for example, when changing the hourly on-street and garage parking fee rates on the network, or when converting on-street to garage parking spaces (as it has been the case in cities like Zurich, Switzerland).

The existing literature contains a number of empirical approaches to model the interaction between on-street and garage parking. These empirical methods usually focus on collecting data for both on-street and garage parking, e.g., [26] uses its responsive pricing scheme to leave between 20 and 40 percent of on-street parking spaces open on every block, and [23] introduces parking pricing to have open spaces available in public garages at all times. Other garage parking models are based on questionnaires, e.g., [4]; [6]; or they use dynamic information to predict real-time garage parking availability ([8]). [19] estimates the effect of on-street parking fees on drivers’ choice between on-street and garage parking. [15] analyzes how garage parking demand is affected by on-street parking regulations. Our macroscopic modelling approach allows us to estimate the average impact of on-street and garage parking on the traffic system without any physical devices nor large data collections efforts.

For modelling both on-street and garage parking and the associated parking fees, [1] and [16] illustrate how the actual full price of parking contains both the interaction between garage operators and the cruising costs for on-street parking. They develop a spatial competition model to eliminate cruising by allocating excessive cruising demand to garage parking and focus on social optimum suggestions concerning the relationship between curbside and garage fares. [20] models variable on-street and garage pricing in real-time for effective parking access and space utilization by using a dynamic Stackelberg leader-follower game theory approach. [24] develops a real-time pricing approach for a parking lot based on its occupancy rate as a system optimal parking flow minimization problem. They assume a user equilibrium travel behavior and only focus on garage parking without analyzing its interdependency with on-street parking in the network. [27] studies park-and-ride (P+R) networks with multiple origins and one destination and focus on an optimal parking pricing strategy. They only focus on setting optimal parking fees for P+R terminals and do not consider the interaction with on-street parking. [29] models multi-modal traffic with limited on-street and garage parking and dynamic pricing based on a congestion- and cruising-responsive feedback parking pricing scheme. The
The proposed framework is based on the Macroscopic Fundamental Diagram (MFD) reflecting the dynamics of parking flows in an urban network ([13]; [14]). [3] analyzes how much curbside to allocate to parking when the private sector provides garage parking. [2] analyzes parking in a spatially homogeneous downtown area where the drivers choose between on-street and garage parking. Cruising for parking contributes to congestion, such that the price of the initially cheaper on-street parking is increased until it equals the price of garage parking. Then increasing the on-street parking fee may generate an efficiency gain through the reduction of cruising. These models focus on social optimum and user equilibrium methodologies. Compared to these papers that provide a long-term demand management strategy capturing interactions between on-street and garage parking policies for large-scale network applications, our model focuses on the short-term effects.

In summary, the existing literature approaches on-street and garage parking models either with empirical data collection efforts or with methodologies concentrating on user equilibrium or social optimum solutions that focus on long-term demand management strategies. Our on-street and garage parking decision model follows a macroscopic approach and focuses on short-term effects. Without large data collection efforts, our macroscopic decision model provides valuable insights into different on-street and garage parking fee rates and their impacts on cruising-for-parking traffic and the overall traffic performance.

The paper is organized as follows. Section 2 presents the overall framework of the macroscopic on-street and garage parking decision model. Section 3 illustrates the concept and mathematical model of the on-street and garage parking-state-based matrix. Section 4 shows a case study of an area within the city of Zurich, Switzerland. Section 5 concludes this paper.

2. On-Street and Garage Parking Decision

Several cost factors influence the on-street/garage parking decision as seen in Fig. 1. These cost variables include variables that have an impact on either the on-street parking option (e.g., the on-street parking pricing), the garage parking option (e.g., the garage parking pricing) or on both parking options (e.g., the number of parking spaces of each kind, and the desired parking duration). Drivers with desired long parking durations are more likely to choose garage parking. All drivers are assumed to be rational during their parking decision and only compare the relevant parking costs between on-street and garage parking, i.e., all drivers are treated as risk-neutral.

![Decision Model for on-street and garage parking based on several cost factors.](image)

The parking decision is then modeled macroscopically using a logistic function based on the on-street and garage parking cost variables. The data inputs are presented in section 2.1. All mathematical details of this modelling approach are illustrated in section 2.2.
2.1. Data inputs for decision model

The decision between on-street and garage parking is dependent on the input variables shown in Tables 1 and 2. The model parameters and all variables that are required to define the traffic network are presented in Table 1. These variables can either be directly measured, or estimated based on simulation results and/or the macroscopic fundamental diagram.

Table 1. Independent variables for parking decision (inputs to the model): Traffic network and model parameters.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>Size (length) of the network.</td>
</tr>
<tr>
<td>$b$</td>
<td>Average length of a block in the network.</td>
</tr>
<tr>
<td>$T$</td>
<td>Length of the simulation’s time horizon.</td>
</tr>
<tr>
<td>$t$</td>
<td>Length of a time slice.</td>
</tr>
<tr>
<td>$K$</td>
<td>Total number of user groups for demand input of the network. Each user group has a different value of time (VOT).</td>
</tr>
<tr>
<td>$VOT^k$</td>
<td>VOT for user group $k \in K$.</td>
</tr>
<tr>
<td>$v$</td>
<td>Free flow speed, i.e., maximum speed in the network, including stopped time at intersections.</td>
</tr>
<tr>
<td>$w$</td>
<td>Walking speed in the network.</td>
</tr>
</tbody>
</table>

All data inputs are based on a compact urban area with a relatively homogeneous network. The total time horizon is divided into small time slices (e.g., 1 minute). All traffic and parking conditions can change over multiple time slices, but they are assumed to be steady within each time slice. As stated in [10], the urban network is abstracted as one ring road with cars driving in a single direction, which has been proven to be reasonable for small, homogeneous traffic networks ([11]).

It is assumed that all trips are exclusively made by car, i.e., the mode choice has been previously made. In addition, we assume that drivers do not cancel their trips while searching for parking. The VOT is assumed to be different for individual vehicles depending on their user group. Such user group can be dependent on the residents’ location, income, careers, working states, etc.

Table 2 shows all independent variables associated with on-street and garage parking. This includes parking duration and parking pricing specific input parameters. These variables can be estimated based on real measurements, historical on-street and garage parking and pricing data, or defined otherwise. All variables related to the travel demand and the distances driven can be estimated based on historical data, e.g., traffic data on main roads to enter the network, etc. The distance variables that are associated with a transition into the next state can be reasonably assumed based on the length of the network, and other data collected from drivers.

Given the homogeneous network, parking searchers are assumed to be homogeneously distributed within the overall driving traffic. This is reasonable, as we also assume that all on-street parking spaces (not only the available ones) are uniformly distributed on the network. Recall that we focus on small compact areas with standard parking policies (e.g., downtown areas or portions thereof), and we are only interested in whether there is on average at least one car that takes each available parking space. As a result, we do not need to record the location of individual cars and parking spots throughout the different time slices in the system, i.e., only average numbers of vehicles during a time slice and total/average searching times and distances are tracked.

Parking garages are also assumed to be uniformly distributed within the network, and without loss of generality, all associated garage parking capacities are assumed to be equal. The distribution of desired parking durations is considered as an input to this model. Some distributions describe the parking duration better than others, see [9]; [25].
In theory, however, any distribution can be used, e.g., poisson, negative binomial ([10]). It is assumed that during the period of one working day drivers do not repark their car after the on-street parking time limit has expired.

Table 2. Independent variables for parking decision (inputs to the model): On-street and garage parking parameters.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Total number of existing on-street parking spaces (for public use) in the area.</td>
</tr>
<tr>
<td>$R$</td>
<td>Number of parking garages in the network.</td>
</tr>
<tr>
<td>$t_d$</td>
<td>Total capacity of all parking garages, i.e., total number of all garage parking spaces.</td>
</tr>
<tr>
<td>$t_{op}$</td>
<td>Parking duration of vehicles (independently of on-street and garage parking).</td>
</tr>
<tr>
<td>$t_{gp}$</td>
<td>Parking duration of vehicles focusing on on-street (op) or garage parking (gp), $\xi \in {op, gp}$.</td>
</tr>
<tr>
<td>$\tau_\ell$</td>
<td>On-street (op) or garage parking (gp) time limit, $\xi \in {op, gp}$.</td>
</tr>
<tr>
<td>$p_t$</td>
<td>Hourly on-street (op) or garage parking (gp) fee rate, $\xi \in {op, gp}$.</td>
</tr>
<tr>
<td>$p_{dist}$</td>
<td>Price per kilometer driven on the network (i.e., external costs as petrol, wear and tear of vehicles).</td>
</tr>
<tr>
<td>$\beta^i$</td>
<td>Proportion of new arrivals during time slice $i$ that is not searching for parking.</td>
</tr>
<tr>
<td>$t_{\text{d}/i}$</td>
<td>Distance that must be driven by a vehicle from user group $k \in K$ before it starts to search for parking.</td>
</tr>
<tr>
<td>$t_{\text{p}/i}$</td>
<td>Distance that must be driven by a vehicle from user group $k \in K$ before it leaves the area without having parked.</td>
</tr>
<tr>
<td>$l_{\text{d}/i}$</td>
<td>Distance that must be driven by a vehicle from user group $k \in K$ before it leaves the area after it has parked on-street or in a garage.</td>
</tr>
</tbody>
</table>

2.2. Mathematical decision framework

Table 3 summarizes the intermediate modelling variables that are needed to model the on-street and garage parking decision. The model outputs provide, amongst others, the results of the interactions between on-street and garage parking and their influence on the urban traffic system.

Table 3. Intermediate model variables.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{\text{d}/i}^\ell$</td>
<td>Total cost of on-street (op) or garage parking (gp) in time slice $i$ for user group $k \in K$, $\xi \in {op, gp}$.</td>
</tr>
<tr>
<td>$q_{\text{d}/i}^k$</td>
<td>Choice of drivers for garage parking (gp) in time slice $i$ for user group $k \in K$.</td>
</tr>
<tr>
<td>$q_{\text{gp}/i}^k$</td>
<td>Proportion of drivers deciding for on-street (op) or garage parking (gp) in time slice $i$ for user group $k \in K$, $\xi \in {op, gp}$.</td>
</tr>
<tr>
<td>$ACT_i^\ell$</td>
<td>Average cruising time for on-street parking in time slice $i$.</td>
</tr>
<tr>
<td>$ADD$</td>
<td>Average driving distance to closest garage location.</td>
</tr>
<tr>
<td>$AWD_i^\ell$</td>
<td>Average walking distance from on-street (op) or garage parking (gp) parking to destination, $\xi \in {op, gp}$.</td>
</tr>
<tr>
<td>$v_i$</td>
<td>Average travel speed in time slice $i$, including stopped time at intersections.</td>
</tr>
<tr>
<td>$d_i$</td>
<td>Maximum driven distance per vehicle in time slice $i$.</td>
</tr>
<tr>
<td>$A_i^\ell$</td>
<td>Number of available on-street parking spaces at the beginning of time slice $i$.</td>
</tr>
<tr>
<td>$R_i^\ell$</td>
<td>Garage parking availability of all parking garages in time slice $i$.</td>
</tr>
<tr>
<td>$P_{tot}$</td>
<td>Total revenue resulting from hourly on-street and garage parking fee rates for the city.</td>
</tr>
</tbody>
</table>

We model the parking decision between on-street and garage parking macroscopically in Eq. (1)-(3). This will then be incorporated into the on-street and garage parking-state-based matrix from [10], [17] and [18] in section 3. We assume that the on-street parking time limit $\tau_{op}$ is smaller than the garage parking time limit $\tau_{gp}$, i.e., $\tau_{op} \leq \tau_{gp}$. All drivers with a desired parking duration of $t_d \leq \tau_{op}$ decide between on-street and garage parking, whereas drivers with $\tau_{op} < t_d \leq \tau_{gp}$ are restricted and can only park at garages. Notice that $t_d$ is taken out of a distribution and no individual vehicles are tracked. We assume that $t_d \leq \tau_{gp}$ for all drivers, since vehicles with $t_d > \tau_{gp}$ cannot find a parking place according to their desired parking duration anywhere within this network. The choice for garage parking over on-street parking is modeled in Eq. (1) using a logistic function based on $R$ (the total capacity of all parking
garages), $A$ (the total number of existing on-street parking spaces), $C_{op}^{ik}$ (the cost of on-street parking, modelled in section 2.2.1), and $C_{gp}^{ik}$ (the cost of garage parking, modelled in section 2.2.2). To make sure the parking choice takes the supply into consideration we add the weight parameters $\frac{R}{R+A}$ and $\frac{A}{R+A}$ to Eq. (1). These terms are not time-dependent since there is no real-time usage information available. This constraint is relaxed later (section 4.5) when real-time information is available.

$$
\delta_{gp}^{ik} = \frac{R \cdot C_{op}^{ik} - A \cdot C_{gp}^{ik}}{a \cdot \min\left[\frac{R}{R+A} \cdot C_{op}^{ik}, \frac{A}{R+A} \cdot C_{gp}^{ik}\right]} 
$$

(1)

Notice that the decision of some drivers is restricted by $\tau_{op}$ and $\tau_{gp}$. This is taken into account when calculating $\gamma_{gp}^{ik}$ in Eq. (2), which described the proportion of vehicles deciding for garage parking, and $\gamma_{op}^{ik}$ in Eq. (3) which describes the proportion of vehicles deciding to search for on-street parking.

$$
\gamma_{gp}^{ik} = \int_{0}^{\tau_{gp}} f(t_d) dt_d \cdot \delta_{gp}^{ik} + \int_{\tau_{op}}^{\tau_{gp}} f(t_d) dt_d 
$$

(2)

$$
\gamma_{op}^{ik} = 1 - \gamma_{gp}^{ik} 
$$

(3)

Term 1 in Eq. (2) represents the portion of vehicles with a parking duration $t_d \leq \tau_{op}$ that have the option to decide for on-street or garage parking. Term 2 represents the portion of vehicles with $\tau_{op} < t_d \leq \tau_{gp}$ that have to park in a garage because of their desired parking duration. Notice that both $\int_{0}^{\tau_{op}} f(t_d) dt_d$ and $\int_{\tau_{op}}^{\tau_{gp}} f(t_d) dt_d$ are assumed to be $k$-independent, i.e., the distribution of the parking durations is assumed to be independent of the drivers’ VOT.

2.2.1 Cost of on-street parking

In Eq. (4), we derive the cost of on-street parking, $C_{op}^{ik}$ for each user group $k \in K$ in time slice $i$.

$$
C_{op}^{ik} = p_{op}^{term 1} + p_{dist}^{term 2} \cdot v^{term 3} \cdot ACT^{term 3} + \frac{VOT^{term 3} \cdot ACT^{term 3} + VOT^{term 3} \cdot AWD_{op}^{term 4}}{W}
$$

(4)

Term 1 represents the hourly on-street parking fee rate which, in the remainder of this paper, is assumed to be constant. In theory, however, the on-street parking fee could also be modelled as a responsive parking pricing scheme ([18]) that takes the parking search phenomenon into consideration. Notice that the parking decision in Eq. (1) is assumed to be based on the parking fee rates per hour independently of the parking durations. Term 2 represents the average cruising distance for on-street parking (i.e., external costs as petrol, wear and tear of vehicles) converted to price units. Term 3 represents the average cruising time based on the drivers’ VOT expressed in price units for $k \in K$. The average cruising time $ACT^{term 3}$ is determined as in [18], and is based on a queueing diagram showing the cumulative number of vehicles going through each transition event as a function of time. Notice that the longer the drivers search for on-street parking, the higher the average cruising time $ACT^{term 3}$ is, and consequently also the $C_{op}^{ik}$. Therefore, it is more likely that the drivers might decide for garage parking in congested areas. Term 4 represents the cost of walking from the on-street parking to the destination expressed in price units for $k \in K$. Even though our
abstracted network was a ring, we may assume without loss of generality that the real network is a square grid, where
the average length of a block \( b \) in the network is known. The total length of the ring network, \( L \), is then equivalent
to joining all blocks of length \( b \) together. As on-street parking spaces are uniformly distributed throughout the
network, the walking costs can be determined using the average distance traveled (Eq. (5)) between two random points
in the square grid ([21]).

\[
AWD_{gp} = \frac{2}{3} \cdot b \cdot \left( -\frac{1}{2} + \frac{1}{4} + \frac{L}{2b} \right)
\]

Term 1 represents the side length of the square grid.

2.2.2 Cost of garage parking

The cost of garage parking, \( C^{i,k}_{gp} \) for each group \( k \in K \) in time slice \( i \), is based on multiple cost terms as shown
in Eq. (6).

\[
C^{i,k}_{gp} = \left[ \frac{p_{gp}}{\text{term 1}} + \frac{p_{dist} \cdot ADD}{\text{term 2}} \right] + \frac{VOT^k \cdot ADD}{\text{term 3}} + \frac{VOT^k \cdot AWD_{gp}}{\text{term 4}}
\]

Term 1 represents the hourly garage parking fee rate which, in the remainder of this paper, is assumed to be constant.
Terms 2 and 3 show the cost of driving from the actual vehicle’s garage parking decision location to the closest
garage for \( k \in K \). It contains the average distance to the closest garage parking expressed in distance price units (term
2) and the average time expressed in price units for \( k \in K \) (term 3). Both terms include the associated average
driving distance \( ADD \), determined in Eq. (7).

\[
ADD = \frac{L}{2 \cdot G}
\]

Remember that the actual garage locations are assumed to be uniformly distributed on the network and that we assume
that traffic on the abstracted ring moves in a single direction.

Fig. 2. Simple example of uniformly distributed garage parking to illustrate the average walking distance in Eq. (8).

Term 4 in Eq. (6) represents the cost of walking from the garage parking to the destination expressed in price units
for \( k \in K \). As the number of garages is limited, they are expected to require, on average, some walking distance. The
walking speed \( w \) is assumed to be a constant input. To estimate the area served by each parking garage we take the
surface of the square grid \[ b \cdot \left( -\frac{1}{2} + \frac{1}{4} + \frac{L}{2b} \right) \] and divide it by \( G \) (Fig. 2). Assuming destinations are uniformly distributed in the network we can compute the average walking distance as \( \frac{2}{3} \) of the radius of each of the areas served by a parking garage in Eq. (8).

\[
A W D_{gp} = \frac{2b}{3\sqrt{\pi} \cdot G} \left[ -\frac{1}{2} + \frac{1}{4} \frac{L}{2b} \right]
\]  

(8)

Note that we enhance \( C_{gp}^{Lk} \) in section 4.5 by including garage usage information to all drivers.

2.2.3 Total revenue

One component of the parking decision is paying an hourly fee for on-street or garage parking. However, the drivers pay the final parking fee from on-street or garage parking depending on how long they have parked. Eq. (9) expresses the total revenue obtained from all user groups \( K \) for the time horizon \( T \).

\[
P_{\text{tot}} = \sum_{i=1}^{T} \sum_{k=1}^{K} \frac{\nu_{opns}^{Lk} \cdot p_{op} \cdot t_{d,op} + \nu_{gpns}^{Lk} \cdot p_{gp} \cdot t_{d,gp}}{\text{term 1}}
\]  

(9)

Term 1 shows the revenue from on-street parking for user group \( k \in K \) during time slice \( i \). Term 2 illustrates the revenue from garage parking for user group \( k \in K \) during time slice \( i \). \( t_{d,op} \) and \( t_{d,gp} \) illustrate the average on-street/garage parking duration obtained from all user groups \( K \) for the time horizon \( T \). Notice that \( \nu_{opns}^{Lk} \) in term 1 and \( \nu_{gpns}^{Lk} \) in term 2 are both defined in section 3.1 (Table 5).

3. On-Street and Garage Parking-State-Based Matrix

The on-street and garage parking-state-based matrix describes the system dynamics of urban traffic based on multiple parking-related states as in [10]. The matrix is used to incorporate our parking decision model into a macroscopic traffic system framework that emulates the interactions over time between the on-street and garage parking systems. This section shows an overview of all on-street and garage parking-related traffic states (section 3.1), and the analytical formulations for the transition events between those states (section 3.2).

3.1 Parking-related traffic states

The parking-related traffic states build the foundation for the parking-state-based matrix. The matrix updates all parking-related traffic states based on the number of vehicles going through different transition events in each time slice. The matrix is updated iteratively over time until the whole period is analyzed, or a defined criterion is reached (e.g., all the cars leave the area). By integrating our on-street and garage parking decision model from section 2, the matrix allows us to illustrate the effects of different on-street and garage parking policies on the searching time and searching distance.

The total traffic demand entering the network is divided into two groups; through-traffic, and vehicles searching for parking. The first group of vehicles represents the proportion of traffic that is driving through this area but does not want to park or has a destination outside (i.e., through-traffic). Therefore, it only experiences two transition events as seen in Fig. 3(a). The second group of vehicles needs to decide between searching for on-street parking or driving towards a parking garage as seen in Fig. 3(b). During one single time slice a vehicle may experience at most one
transition event.

(a) Through-traffic.

(b) Searching for parking traffic.

Fig. 3. The transition events of urban traffic focusing on on-street and garage parking in-between different parking-related states.

All vehicles searching for parking in Fig. 3(b) have the option to decide for on-street or garage parking at their current location. This decision involves the on-street and garage parking decision model from section 2. The vehicles that have decided to search for on-street parking can change their mind and switch to garage parking later. As soon as the vehicles decide for garage parking they will drive towards the closest parking garage and access it based on availability. For these drivers the location of the parking garages is assumed to be known or a guidance to the garage location is available. Once the garage parking decision is made, we assume the drivers do not change their decision while driving to the garage location. If there are no available garage parking spaces, the vehicles cannot access the parking garage and might move to the searching-for-on-street-parking state. After the vehicles have accessed on-street or garage parking, they depart and move back to the non-searching state before they leave the area. All traffic states in Fig. 3 are summarized in Table 4. The initial conditions of all traffic state variables are model input variables that can be measured, assumed or simulated.

Table 4. All traffic state variables for the on-street and garage parking-state-based matrix per time slice.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Name</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{ks}^{\text{ns}}$</td>
<td>Non-searching</td>
<td>Number of vehicles in the state “non-searching” for user group $k \in K$ at the beginning of time slice $i$.</td>
</tr>
<tr>
<td>$N_{ks}^{\text{sp}}$</td>
<td>Searching for on-street parking</td>
<td>Number of vehicles in the state “searching for on-street parking” for user group $k \in K$ at the beginning of time slice $i$.</td>
</tr>
<tr>
<td>$N_{ks}^{\text{os}}$</td>
<td>On-street parking</td>
<td>Number of vehicles in the state “on-street parking” for user group $k \in K$ at the beginning of time slice $i$.</td>
</tr>
<tr>
<td>$N_{ks}^{\text{dgp}}$</td>
<td>Driving to garage parking</td>
<td>Number of vehicles in the state “driving to garage parking” for user group $k \in K$ at the beginning of time slice $i$.</td>
</tr>
<tr>
<td>$N_{ks}^{\text{gp}}$</td>
<td>Garage parking</td>
<td>Number of vehicles in the state “garage parking” for user group $k \in K$ at the beginning of time slice $i$.</td>
</tr>
</tbody>
</table>

These parking-related states are determined using the information on the transition events. We introduce the transition events in Table 5.
Table 5. All transition event variables for the on-street and garage parking-state-based matrix per time slice.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Name</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_{i,k}^{ns} )</td>
<td>Enter the area</td>
<td>Number of vehicles that enter the area and transition to “non-searching” for user group ( k \in K ) during time slice ( i ) (i.e., travel demand per VOT user group).</td>
</tr>
<tr>
<td>( n_{i,k}^{ns/dgp} )</td>
<td>Go to parking (Decision to park: Driving to garage parking)</td>
<td>Number of vehicles that transition from “non-searching” to “driving to garage parking” (depending on their parking decision) for user group ( k \in K ) during time slice ( i ).</td>
</tr>
<tr>
<td>( n_{i,k}^{ns/s} )</td>
<td>Go to parking (Decision to park: Searching for on-street parking)</td>
<td>Number of vehicles that transition from “non-searching” to “searching on-street parking” (depending on their parking decision) for user group ( k \in K ) during time slice ( i ).</td>
</tr>
<tr>
<td>( n_{i,k}^{s/dgp} )</td>
<td>Switch to garage parking</td>
<td>Number of vehicles that transition from “searching for on-street parking” to “driving to garage parking” for user group ( k \in K ) during time slice ( i ).</td>
</tr>
<tr>
<td>( n_{i,k}^{s/op} )</td>
<td>Find and access on-street parking</td>
<td>Number of vehicles that transition from “searching for on-street parking” to “on-street parking” for user group ( k \in K ) during time slice ( i ).</td>
</tr>
<tr>
<td>( n_{i,k}^{dgp/gp} )</td>
<td>Access garage parking</td>
<td>Number of vehicles that transition from “driving to garage parking” to “garage parking” for user group ( k \in K ) during time slice ( i ).</td>
</tr>
<tr>
<td>( n_{i,k}^{dgp/s} )</td>
<td>Not access garage parking</td>
<td>Number of vehicles that transition from “driving to garage parking” to “searching on-street parking” for user group ( k \in K ) during time slice ( i ).</td>
</tr>
<tr>
<td>( n_{i,k}^{gp/ns} )</td>
<td>Depart on-street parking</td>
<td>Number of vehicles that transition from “on-street parking” to “non-searching” for user group ( k \in K ) during time slice ( i ).</td>
</tr>
<tr>
<td>( n_{i,k}^{gp/gp} )</td>
<td>Depart garage parking</td>
<td>Number of vehicles that transition from “garage parking” to “non-searching” for user group ( k \in K ) during time slice ( i ).</td>
</tr>
<tr>
<td>( n_{i,k}^{ns/} )</td>
<td>Leave the area</td>
<td>Number of vehicles that leave the area and transition from “non-searching” for user group ( k \in K ) during time slice ( i ).</td>
</tr>
</tbody>
</table>

Eq. (10) to (14) update the number of “non-searching”, “searching for on-street parking”, “on-street parking”, “driving to garage parking”, and “garage parking” vehicles, respectively. Notice that all equations need to be determined for every user group \( k \in K \), where \( K \) is the total number of user groups for the demand input of the network.

\[
N_{ns}^{i+1} = \sum_{k=1}^{K} N_{ns}^{i+1,k}, \text{ where } N_{ns}^{i+1,k} = N_{ns}^{i,k} + n_{i,k}^{ns} + n_{i,k}^{op/ns} + n_{i,k}^{dgp/ns} - n_{ns}^{i,k} - n_{ns}^{dgp} - n_{ns}^{i} \
\]

(10)

\[
N_{s}^{i+1} = \sum_{k=1}^{K} N_{s}^{i+1,k}, \text{ where } N_{s}^{i+1,k} = N_{s}^{i,k} + n_{i,k}^{ns/s} + n_{i,k}^{dgp/s} - n_{s}^{i,k} - n_{s}^{dgp/s} \
\]

(11)

\[
N_{op}^{i+1} = \sum_{k=1}^{K} N_{op}^{i+1,k}, \text{ where } N_{op}^{i+1,k} = N_{op}^{i,k} + n_{s}^{i,k} - n_{op}^{i,k} \
\]

(12)

\[
N_{dgp}^{i+1} = \sum_{k=1}^{K} N_{dgp}^{i+1,k}, \text{ where } N_{dgp}^{i+1,k} = N_{dgp}^{i,k} + n_{i,k}^{ns/dgp} + n_{i,k}^{s/dgp} - n_{dgp}^{i,k} - n_{dgp}^{i} \
\]

(13)

\[
N_{gp}^{i+1} = \sum_{k=1}^{K} N_{gp}^{i+1,k}, \text{ where } N_{gp}^{i+1,k} = N_{gp}^{i,k} + n_{i,k}^{dgp/gp} - n_{gp}^{i,k} \
\]

(14)

Eq. (10) updates the number of “non-searching” vehicles for each \( k \in K \) before aggregating them to \( N_{ns}^{i+1} \). Vehicles entering the area (i.e., \( n_{i,k}^{ns} \)), and vehicles that depart from on-street or garage parking (i.e., \( n_{i,k}^{op/ns} \) and \( n_{i,k}^{gp/ns} \)) join
this state; vehicles that start searching or drive to garage parking (i.e., $n_{ns/dgp}^{l_{k}}$) and vehicles leaving the
area (i.e., $n_{ns}^{l_{k}}$) quit this state. Eq. (11) updates the number of “searching” vehicles for each $k \in K$ and aggregates
them after to $N_{s}^{i+1}$. Vehicles starting to search for on-street parking (i.e., $n_{ns}^{l_{k}}$) and vehicles not able to access garage
parking (i.e., $n_{dgp/s}^{l_{k}}$) join this state; vehicles accessing on-street parking (i.e., $n_{s/op}^{l_{k}}$) and vehicles driving to garage
parking (i.e., $n_{s/dgp}^{l_{k}}$) leave this state. Eq. (12) updates the number of “on-street parking” vehicles for each $k \in K$
and then aggregates them to $N_{op}^{i+1}$. Vehicles accessing an on-street parking space (i.e., $n_{op}^{l_{k}}$) join this state; vehicles
departing from on-street parking (i.e., $n_{op/ns}^{l_{k}}$) leave this state. Eq. (13) updates the number of vehicles that “drive to
garage parking” for each user group $k \in K$ and then aggregates them to $N_{dgp}^{i+1}$. Vehicles that drive to a parking garage
during time slice $i$ (i.e., $n_{ns/dgp}^{l_{k}}$ and $n_{dgp/dgp}^{l_{k}}$) join this state; vehicles that actually access garage parking (i.e.,
$n_{dgp/dgp}^{l_{k}}$) and vehicles that cannot access garage parking (i.e., $n_{dgp/dgp}^{l_{k}}$) quit this state. Eq. (14) updates the number of
“garage parking” vehicles for each $k \in K$ before aggregating them to $N_{dgp}^{i+1}$. Vehicles that access a garage during
time slice $i$ (i.e., $n_{dgp/dgp}^{l_{k}}$) join this state; vehicles that depart garage parking (i.e., $n_{dgp/dgp}^{l_{k}}$) quit this state. The total
number of vehicles driving in the network at the beginning of time slice $i$ is $N_{ns}^{i} + N_{s}^{i} + N_{dgp}^{i}$.
The total number of vehicles parked at the beginning of time slice $i$ is $N_{op}^{i} + N_{gp}^{i}$.

3.2 Transition events

We model the transition events introduced in Table 5 in the subsections 3.2.1 to 3.2.9 below.

3.2.1. Enter the area

The traffic demand $n_{ns}^{l_{k}}$ is an input to the model. A portion $\beta^{i}$ of all vehicles entering the area is considered as
through-traffic, i.e., these vehicles will drive through the area without needing to park.

3.2.2. Go to parking (Decision to park)

We assume that the vehicles from user group $k \in K$ make their parking decision (searching for on-street parking or
driving to garage parking) after driving a distance $l_{ns}^{k}$ since they enter the area. $l_{ns}^{k}$ can be fixed or taken out of
any given probability density function. The vehicles have the option to drive to garage parking as modelled in Eq.
(15), or search for an on-street parking space as shown in Eq. (16). Both $n_{ns/dgp}^{l_{k}}$ and $n_{ns/s}^{l_{k}}$ may consist of vehicles
from user group $k \in K$ entering the network in any former time slice $l' \in [1, t - 1]$.

$$n_{ns/dgp}^{l_{k}} = \left[ \sum_{l=1}^{i-1} (1 - \beta^{i}) \cdot n_{ns}^{l_{k}} \cdot y_{ns}^{l_{k}} \right]_{\text{term 1}}, \quad y_{gp}^{l_{k}} \cdot y_{ns/dgp}^{l_{k}} \text{ term 3}$$

(15)

$$n_{ns/s}^{l_{k}} = \left[ \sum_{l=1}^{i-1} (1 - \beta^{i}) \cdot n_{ns}^{l_{k}} \cdot y_{ns/s}^{l_{k}} \right]_{\text{term 1}}, \quad y_{op}^{l_{k}} \cdot y_{ns/s}^{l_{k}} \text{ term 4}$$

(16)

where

$$y_{ns}^{l_{k}} = \begin{cases} 1, & \text{if } t_{ns}^{l_{k}} \leq \sum_{j=i-1}^{j=i} d_{j} \text{ and } \sum_{j=i}^{j=i-1} d_{j} \leq l_{ns}^{k} + d_{i-1} \\ 0, & \text{otherwise} \end{cases}$$

(17)

Term 1 in Eq. (15) and Eq. (16) shows the portion of the total demand $n_{ns}^{l_{k}}$ that needs to park, i.e., all vehicles
excluding through traffic. The proportion of through-traffic, \( \beta^i \), is assumed to be independent of the individual user group \( k \in K \). Term 2 indicates whether these vehicles can decide for parking in time slice \( i \) or need to continue driving until they cover a distance \( l_{ns}^k \) (Eq. (17)). Term 3 in Eq. (15) expresses the proportion of drivers deciding to drive towards a parking garage (from Eq. (2)) in time slice \( i \) depending on user group \( k \). Term 4 in Eq. (16) expresses the proportion of drivers deciding to search for an on-street parking space (from Eq. (3)) in time slice \( i \) depending on user group \( k \).

3.2.3. Switch to garage parking

In this subsection, the transition event \( n_{s/dgp}^{i,k} \) is modelled in Eq. (18) to determine the number of vehicles switching to garage parking after being in the searching-for-on-street-parking state for at least one time slice. This represents the drivers that change their mind regarding where to park.

\[
 n_{s/dgp}^{i,k} = \left[ N_s^{i,k} - n_{s/op}^i \cdot \frac{N_s^{i,k}}{N_s^i} \right] \cdot \min\{\frac{(N_s^i)^{-\alpha}}{1}, 1\} \tag{18}
\]

Term 1 represents all searching vehicles of user group \( k \) that have not parked on-street in this time slice \( i \). Further details on the computation of \( n_{s/op}^i \) can be found in [10] based on probability theory. Term 2 shows the proportion of searching vehicles deciding to drive towards a parking garage (Eq. (1)). Notice that the same vehicles have to go over the same decision at multiple time slices in the transition events “Go to parking” and “Switch to garage parking” (potentially revising their previous decision). Term 3 represents a penalty term that prevents drivers flipping between \( n_{s/dgp}^{i,k} \) and \( n_{dgp/s}^{i,k} \). It is dependent on \( N_s^i \) since the likelihood of flipping is high when there are a lot of searching vehicles on the network. The level of the penalty for the simulation is characterized by \( \alpha \), \( \alpha > 1 \). It can be shown in a sensitivity analysis that as long as \( \alpha > 1 \), changes to its value only have a marginal influence on the average searching time/distance, the average time/distance of drivers driving to garage parking, and on the revenue collected by on-street and garage parking fees in the network, but the details are omitted in this paper for brevity. In the remainder of this study, we assume a square root dependency and set \( \alpha = 2 \).

3.2.4. Find and access on-street parking

The vehicles from user group \( k \) searching for on-street parking that find and access a parking space is determined in Eq. (19).

\[
 n_{s/op}^{i,k} = n_{s/op}^i \cdot \frac{N_s^{i,k}}{N_s^i} \tag{19}
\]

Notice that all drivers decide to access the first available on-street parking space in the network, as all parking spaces have the same price. As previously stated, details on \( n_{s/op}^i \) can be found in [10].

3.2.5. Access garage parking

The transition event \( n_{dgp/gp}^{i,k} \) in Eq. (20) describes the process of accessing a parking garage. After the vehicles have decided to use garage parking, they drive towards the parking garage where they realize whether it is possible for them to access it depending on the garage parking availability.
\[ n_{dgp/ap}^{i,k} = \frac{N_{dgp}^{i,k}}{N_{dgp}^1} \cdot \min \left\{ \frac{1}{\text{term 1}} \sum_{k=1}^{K} \sum_{l' t=1}^{i-1} \left( n_{ns/dgp}^{i,k} + n_{s/dgp}^{i,k} \right) \cdot \gamma_{ADD}^{i,k} \cdot R^j \right\} \]  

where

\[ \gamma_{ADD}^{i,k} = \begin{cases} 
1, & \text{if } ADD - t_{ns}^k \leq \sum_{j=i}^{j=i-1} d^j \text{ and } \sum_{j=i}^{j=i-1} d^j \leq ADD - t_{ns}^k + d^{i-1} \\
0, & \text{otherwise} 
\end{cases} \]

Term 1 in Eq. (20) represents the portion of vehicles trying to access garage parking that belong to user group \( k \).

Term 2 shows the sum of all vehicles (from subsection 3.2.2 and 3.2.3) that have decided to use garage parking in any former time slice \( i' \in [1, i - 1] \). Term 3 (computed in Eq. (21)) indicates whether these vehicles have arrived at the garage after reaching \( ADD - t_{ns}^k \) (subsection 2.2.2). Note that the drivers driving to garage parking are assumed to drive directly towards their garage as soon as they enter the area. Thus, the distance \( l_{ns}^k \) is deducted from \( ADD \). Two conditions must be satisfied: the vehicles have driven enough distance to arrive at a parking garage after having decided for it, and they have not accessed a garage in a former time slice. Finally, the number of vehicles that can actually access garage parking is the minimum of the available garage parking spaces and the number of vehicles that want to park.

### 3.2.6. Not access garage parking

This transition event includes all vehicles that do not access garage parking due to limited availability. In this situation, some of these vehicles \( n_{dgp/s}^{i,k} \) in Eq. (22) return back to searching-for-on-street-parking state. However, depending on \( A \) and \( R \) some drivers might prefer to stay in the “drive to garage parking” state as a result of a low total number of existing on-street parking spaces compared to the total existing garage capacity.

\[ n_{dgp/s}^{i,k} = \frac{N_{dgp}^{i,k}}{N_{dgp}^1} \cdot \max \left\{ \frac{1}{\text{term 1}} \sum_{k=1}^{K} \sum_{l' t=1}^{i-1} \left( n_{ns/dgp}^{i,k} + n_{s/dgp}^{i,k} \right) \cdot \gamma_{ADD}^{i,k} - R^j \right\} \cdot \frac{A}{R + A} \]  

Terms 1, 2 and 3 are already determined as in Eq. (20). In case the garage parking availability limit is reached and the vehicles that would like to enter a parking garage, i.e., \( \sum_{k=1}^{K} \sum_{l' t=1}^{i-1} \left( n_{ns/dgp}^{i,k} + n_{s/dgp}^{i,k} \right) \cdot \gamma_{ADD}^{i,k} \), surpass \( R^j \), the remaining vehicles need to return to searching-for-parking state; otherwise all vehicles can successfully enter a garage. This portion of vehicles returning back to searching-for-on-street-parking state is reduced by term 4 that represents the drivers’ decision to stay in the “drive to garage parking” state due to a low \( A \) in comparison to \( R + A \). This term is not time-dependent since there is no real-time usage information available. This constraint is relaxed later (section 4.5) when real-time information is available. Notice that for more realistic applications, the capacity of garage parking will not be an active constraint. It is included here, however, for the sake of completeness.

### 3.2.7. Depart on-street parking

The number of vehicles that depart from on-street parking is based on the distribution of on-street parking durations \( f(t_{dgp}) \) and on the number of vehicles having accessed on-street parking, \( n_{s/op}^{i,k} \), in a former time slice \( i' \in [1, i - 1] \). The probability that these vehicles depart from on-street parking in time slice \( i \) equals to the probability
of the on-street parking duration being between \((i - i') \cdot t\) and \((i + 1 - i') \cdot t\), i.e.,
\[
\int_{(i - i') \cdot t}^{(i + 1 - i') \cdot t} f(t_{d, op}) \, dt_{d, op}
\]
The transition event is formulated as \(n_{op/ns}^{i,k}\) in Eq. (23) as in [10].
\[
n_{op/ns}^{i,k} = \sum_{i'=1}^{i-1} n_{s/op}^{i',k} \int_{(i - i') \cdot t}^{(i + 1 - i') \cdot t} f(t_{d, op}) \, dt_{d, op}
\]
(23)
The on-street parking availability is updated in Eq. (24) after vehicles access or depart from on-street parking. \(A^i\) cannot surpass the total number of existing on-street parking spaces, i.e., \(A^i \leq A\) for all time slices \(i\).
\[
A^{i+1} = A^i + \sum_{k=1}^{K} n_{op/ns}^{i,k} - \sum_{k=1}^{K} n_{s/op}^{i,k}
\]
(24)

3.2.8. Depart garage parking

The transition event \(n_{gp/ns}^{i,k}\) in Eq. (25) is modeled analogously to \(n_{op/ns}^{i,k}\). As we know the number of vehicles having decided to use garage parking in all former time slices, we can find \(n_{gp/ns}^{i,k}\) based on the distribution of garage parking durations \(f(t_{d, gp})\).
\[
n_{gp/ns}^{i,k} = \sum_{i'=1}^{i-1} n_{d,gp/gp}^{i',k} \int_{(i - i') \cdot t}^{(i + 1 - i') \cdot t} f(t_{d, gp}) \, dt_{d, gp}
\]
(25)
After vehicles access or depart from garage parking, the availability is updated in Eq. (26). \(R^i\) cannot surpass the total capacity, i.e., \(R^i \leq R\) for all time slices \(i\).
\[
R^{i+1} = R^i + \sum_{k=1}^{K} n_{gp/ns}^{i,k} - \sum_{k=1}^{K} n_{d,gp/gp}^{i,k}
\]
(26)

3.2.9. Leave the area

The vehicles leave the area after having driven for a given distance \(l_f^k\) or \(l_p^k\) depending on whether they have parked or not. Notice that the distances \(l_f^k\) and \(l_p^k\) analogously to \(l_{ns}^k\) can be fixed or taken out of any given probability density function. Vehicles leaving the area are modelled as \(n_{ns}^{i,k}\) in Eq. (27) and include through-traffic vehicles, \(\beta^i\). \(n_{ns}^{i,k}\) and vehicles from the transition events \(n_{op/ns}^{i,k}\) and \(n_{gp/ns}^{i,k}\).
\[
n_{ns}^{i,k} = \sum_{i'=1}^{i-1} \left( \beta^i \cdot n_{ns}^{i',k} \gamma_f^{i',k} + \left( n_{op/ns}^{i',k} + n_{gp/ns}^{i',k} \right) \gamma_p^{i',k} \right)
\]
(27)
where
Further details can be found in [10].

4. Applications

In this section, a case study of an area within the city of Zurich, Switzerland, is provided to illustrate the influences of on-street and garage parking on the traffic system. We use real data obtained by [11]. The results are obtained with the aid of a simple numerical solver such as Matlab. We discuss the findings regarding on-street and garage parking pricing, the related parking decision and the impacts on the average searching time/distance. We analyze the short-term effects of including garage usage information to all drivers, as well as the influences of converting on-street to garage parking spaces on the traffic system.

4.1. Case study of an area within the city of Zurich, Switzerland

Our study area (0.28 km²) in Fig. 4(a) is located around the shopping area Jelmoli in the city center of Zurich ([11]).

Fig. 4. Case study area and parking demand per minute computed as a moving average over 10 min (Source: [11]).

There is a significant amount of retail space and offices from the financial sector in this area. The total length of all roads in the area is \( L = 7.7 \) km with an associated area radius of 0.3 km and \( b = 76 \) m. Most streets in this area have two lanes (one per direction or two one-way lanes). There are \( A = 207 \) on-street parking spaces and \( G = 2 \) parking garages (Jelmoli and Talgarten garage) with a total capacity of \( R = 332 \) spaces. The on-street parking price is on average \( p_{op} = 1.5 \) CHF/hour and the garage parking price is on average \( p_{gp} = 3 \) CHF/hour ([11]). We consider time slices of 1 min during a working day, i.e., \( t = 1 \) min for a time horizon of \( T = 1440 \) min. The macroscopic fundamental diagram of the city of Zurich was used for the traffic properties (i.e., \( v = 12.5 \) km/h), based on ([12]; [22]).

The parking demand (Fig. 4(b)), parking durations, and initial conditions are extracted from an agent-based model in MATSim ([28]), that is based on previous measurements. There is a total travel demand of 2687 trips spread between

\[
\gamma^{i',k} = \begin{cases} 
1, & \text{if } l^k \leq \sum_{j=i'}^{j=i-1} d^j \text{ and } \sum_{j=i'}^{j=i-1} d^j \leq T + d^{i-1} \\
0, & \text{otherwise}
\end{cases}
\]

\[
\gamma^{i',k}_{p/i} = \begin{cases} 
1, & \text{if } l^k_{p/i} \leq \sum_{j=i'}^{j=i-1} d^j \text{ and } \sum_{j=i'}^{j=i-1} d^j \leq T_{p/i} + d^{i-1} \\
0, & \text{otherwise}
\end{cases}
\]
four different user groups (892/ 956/ 838/ 956 trips) in the network associated to different VOTs (VOT\(^1\) = 29.9 CHF/h; VOT\(^2\) = 25.4 CHF/h; VOT\(^3\) = 25.8 CHF/h; VOT\(^4\) = 17.2 CHF/h). All VOT values are based on the estimated mean values for the VOT for car journeys in Switzerland ([5]). Based on the parking demand and parking usage 23% (618 trips) of the daily demand (i.e., \(\beta = 0.23\), ∀i) does not search for parking and can be considered as through-traffic, while 77% (2069 trips) of the daily traffic searches for parking ([11]). At the beginning of every working day 183 vehicles are already in the area, where \(N_{op}^0 = 70\) are parked on-street and \(N_{gp}^0 = 113\) are in a garage. All other initial conditions are considered as zero, i.e., \(N_{ex}^0 = N_s^0 = N_{dgp}^0 = 0\). Taking the network properties into account, the travel distances \(l_{ai}^k\), \(l_f^j\) and \(l_{gp}^k\) are all uniformly distributed between 0.1 and 0.7 km for all \(k \in \{1, \ldots, 4\}\).

The parking durations of vehicles are differentiated by their parking destination. Fig. 5(a) displays the distribution of on-street parking durations and Fig. 5(b) the distribution of garage parking durations. The histogram in Fig. 5(a) is comparable to a gamma distribution with a shape parameter of \(a_1 = 3.5\) and a scale parameter of \(a_2 = 28.5\). The histogram in Fig. 5(b) represents the two types of drivers, those that have to park in a garage because \(t_d > \tau_{op}\) and those that chose to do it as \(t_d \leq \tau_{op}\). It is modelled using the gamma distribution with a shape parameter of \(a_1 = 2.1\) and a scale parameter of \(a_2 = 137.4\). Depending on the frequency a bi-modal gamma distribution might be suitable for other case studies. All on-street parking spaces have a parking time limit of \(\tau_{op} = 180\) min and the garages have no limit within 24 hours, i.e., \(\tau_{gp} = 1440\) min ([11]). The price per distance driven is assumed as \(P_{dist} = 0.3\) CHF/km and the walking speed is set to \(w = 5\) km/h ([7]).

### 4.2. Validation

In this section, we validate the garage parking occupancy rate using empirical data collected by the city of Zurich. The real garage occupancy data in Fig. 6 is generated through a local monitoring system (PLS Zurich) based on 15-minute intervals between the 1st and the 22nd of April, 2016. Only data from Tuesdays, Wednesday, and Thursdays from the Jelmoli and Talgarten garages are included in the study to represent a working day demand. Compared to [11] the garage parking occupancy obtained in this paper is already close to 100% after the 9.5th hour. This happens because the garage parking duration used here (gamma distribution with mean \(\mu = 293\) min in Fig. 5(b)) is on average longer than that used in [11] (gamma distribution with mean \(\mu = 230.2\) min) due to our differentiation of parking durations based on parking destinations. Hence, the turnover-rate of the garage parking spaces is reduced, and the 100% garage occupancy is reached at an earlier hour of the day.
Fig. 6. Comparison between the empirical garage and the estimated garage parking occupancy
(empirical data was collected and averaged over 12 working days from three weeks during 1st – 22nd April, 2016).

The curve reflecting the estimated garage parking occupancy rate shows a rather similar pattern to that of the real data. The approximation is more accurate compared to the validation in [11], where no differentiation between on-street and garage parking is modelled. The mean absolute error (MAE) of our estimation is 0.046, less than in [11].

4.3. Model results

In this section, we present some valuable insights with respect to on-street and garage parking. Table 6 illustrates the average/total time and driven distance for the vehicles in the states “Searching for on-street parking”, “Drive to garage parking” and “Non-searching” during a typical working day.

<table>
<thead>
<tr>
<th>State</th>
<th>Average time per vehicle (min/veh)</th>
<th>Total time (min)</th>
<th>Average driven distance (km/veh)</th>
<th>Total driven distance (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Searching for on-street parking state</td>
<td>3.72</td>
<td>4323</td>
<td>0.77</td>
<td>901</td>
</tr>
<tr>
<td>Driving to garage parking state</td>
<td>3.31</td>
<td>7458</td>
<td>0.69</td>
<td>1554</td>
</tr>
<tr>
<td>Non-searching state</td>
<td>4.46</td>
<td>10047</td>
<td>0.93</td>
<td>2093</td>
</tr>
<tr>
<td>Total</td>
<td>9.69</td>
<td>21827</td>
<td>2.02</td>
<td>4547</td>
</tr>
</tbody>
</table>

On average each vehicle spends 9.7 minutes in the network (excluding the time spent parked). Not surprisingly, vehicles spend on average longer in the “Searching for on-street parking”-state (3.7 minutes) than in the “Drive to garage parking”-state (3.3 minutes). A similar behavior can be detected when looking at the average driven distance in the network (Table 6). What is interesting, however, is that the absolute difference in average travel time between the two parking options is less than a minute. This happens because of two reasons. First, the area itself is rather small. Second, based on our decision framework in Eq. (2) on average only 48.8% of the parking vehicles are able to make a decision between on-street and garage parking. The remaining 51.2% must drive towards a parking garage, given that the on-street parking duration limit is set to $\tau_{op} = 180$ min.

Following the parking demand (Fig. 4(b)), the number of vehicles searching for on-street parking increases drastically between the 9th and the 13th hour, and the number of available on-street parking spaces goes down (Fig. 7(a)). After the 9.5th hour the number of available garage parking spaces gets close to zero (Fig. 7(b)). The vehicles that cannot access garage parking then return back to the searching-for-on-street-parking state. This leads to more searching vehicles and less available on-street parking spaces at an earlier hour compared to [11] (Fig. 7(a)). The number of vehicles driving to garage parking behaves analogously to the parking demand (Fig. 4(b)) and increases between the
5th and the 20th hour (Fig. 7(b)). Given the distribution of garage parking durations and the resulting turnover, the number of available garage parking spaces decreases drastically between the 9.5th and the 14th hour (see also Fig. 6):

Fig. 7. On-street and garage parking demand and supply over a typical working day.

Once there are no available on-street parking spaces anymore (Fig. 7(a)), the average cruising time increases (Fig. 8). This leads to an increase in the costs associated with cruising-for-on-street-parking.

Fig. 8. Average cruising time for on-street parking over a typical working day.

Fig. 9. Traffic composition and garage parking related transition events as a moving average over 10 min.

Fig. 9(a) shows the share of vehicles searching for on-street parking, driving to garage parking, or non-searching over time. This traffic composition is related only to the vehicles circulating on the network, and not those that are parked. Between the 10th and the 13th hour the network has the highest percentage of vehicles searching for on-street parking.
Fig. 9(b) shows the number of vehicles \( n_{ns/dgp}^{lk}, n_{s/dgp}^{lk} \) and \( n_{dgp/s}^{lk} \) summed over all user groups \( k \in K \) over a typical working day. \( n_{ns/dgp}^{lk} \) behaves analogously to the parking demand (Fig. 4(b)). It increases between the 5th and the 20th hour. \( n_{s/dgp}^{lk} \) is negligibly small. \( n_{dgp/s}^{lk} \) increases from approximately the 9.5th hour since the garage parking occupancy rate is close to 100% (Fig. 6). Thus, not enough available garage parking spaces are left (Fig. 7(b)) and vehicles are not able to access the parking garages.

### 4.4. Impacts of on-street and garage parking pricing

We now use our model to capitalize on the interactions between on-street and garage parking pricing to improve traffic performance in the short-term (i.e., minimize the average searching time and distance). This might be accomplished by increasing the attractiveness of garage parking such that less vehicles insist on searching for an on-street parking space, or vice versa, once the garages become full. Remember that this is only possible for drivers who actually have a choice and not for drivers who can only use a garage (Eq. (2)) due to the on-street parking time limit restrictions.

What is the ideal ratio between on-street and garage parking fees to attract drivers such that they avoid cruising for on-street parking? We study the impacts of a limited on-street and garage capacity in combination with different on-street and garage parking pricing parameters, i.e., due to the limited number of garage parking spaces and different related pricing schemes congestion might occur and affect the traffic performance in the network.

Remember that the hourly on-street and garage parking fee rates, \( p_{op} \) and \( p_{gp} \), are part of the decision related cost variables for on-street and garage parking. Based on these cost variables the drivers decide for on-street or garage parking, affecting the average travel time in each parking-related state as illustrated in Fig. 10. Increasing the ratio \( \frac{p_{op}}{p_{gp}} \) leads to a higher cost variable \( C_{op}^{lk} \) (section 2.2.1) and the drivers are more likely to drive to garage parking.

Thus, the average time for vehicles driving to garage parking increases, while the average searching time decreases (Fig. 10). Both times are equal for \( \frac{p_{op}}{p_{gp}} = 1.75 \). At the same time, the average vehicle time for both searching and driving to garage parking vehicles increases in the network after some initial drop (green dotted line in Fig. 10). The results for the average distance driven follow a similar pattern as Fig. 10, but the details are omitted in this paper for brevity. The financial short-term benefits for the city, i.e., the total revenue from both on-street and garage parking pricing leads to maximal results when both \( p_{op} \) and \( p_{gp} \) increase. The total revenue increases slightly faster when \( p_{gp} \) increases, since more than 60% of the parking spaces in the network are garage parking spaces.

![Fig. 10. The impact of on-street and garage parking pricing schemas on average time searching/driving to garage parking.](image)

Different on-street and garage parking fee rates can lead not only to more vehicle time/distance on the network, a worse traffic performance and worse environmental conditions, but also to various financial revenue outputs. Based
on these results cities can find reasonable hourly on-street and garage parking fees such that the average time driving to garage parking/searching for on-street parking are not negatively affected and additionally, acceptable financial revenues are obtained. Our methodology provides the tools to do a cost benefit analysis and to study the trade-off between revenues and the average travel time of vehicles trying to park.

4.5. Availability of garage usage information to all drivers

In reality, the actual garage parking availability also influences the drivers’ decision to park on-street or to drive towards a parking garage. This garage usage information can be made available to the drivers by providing real-time smartphone applications or garage information signs in the traffic network.

In this section, we include the availability of garage parking information into our on-street and garage parking model and assume that the garage parking availability is known to all drivers at all times. Since this garage parking availability has an influence on the driver’s parking decision, we replace all $R$ by $R^i$ in Eq. (1) and Eq. (22). Table 7 illustrates the average time and driven distance for the scenario with available garage usage information for all drivers in the network during a typical working day.

Table 7. Average time and driven distance in the network with available garage usage information during a typical working day. Value within parenthesis represents the percentage change with respect to the scenario without available garage usage information (Table 6).

<table>
<thead>
<tr>
<th>State</th>
<th>Average time per vehicle (min/veh)</th>
<th>Average driven distance (km/veh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Searching for on-street parking state</td>
<td>2.93 (-21.2 %)</td>
<td>0.61 (-20.8 %)</td>
</tr>
<tr>
<td>Driving to garage parking state</td>
<td>2.39 (-27.8 %)</td>
<td>0.5 (-27.5 %)</td>
</tr>
</tbody>
</table>

This additional information helps to reduce the average searching time by 21.2 % and the average time driving to garage parking by 27.8 % compared to the scenario without garage information available to all drivers (section 4.3).

The average driven distance in the network reduces similarly (Table 7). Allowing drivers to make their on-street or garage parking decision based on real-time occupancy data leads to a better traffic performance on the network, and on average, a faster journey for drivers searching for parking.

The parking choice for garage over on-street parking decreases drastically for drivers with available garage usage information between the 9.5th and the 14th hour compared to drivers who have no garage information available (Fig. 11). Since the increase in the average cruising time (Fig. 8) has an impact on the drivers’ decision, more drivers without any available garage usage information drive to garage parking between the 9.5th and the 13th hour. Due to the lack of garage information this parking choice is made even if the garage occupancy rate is low. Note that this parking choice only affects the portion of the parking demand that can make a decision between on-street and garage parking due to the on-street parking duration limit. By including the garage usage information into the decision framework the drivers react towards the garage occupancy rate. The garage occupancy rate (Fig. 6) is then reflected in Fig. 11 and the parking choice for garage parking increases from the 14th hour analogously to the decrease of the garage occupancy rate in Fig. 6.
Fig. 11. Parking choice for garage over on-street parking over a typical working day. The parking choice is illustrated for the scenarios without (section 4.3) and with (section 4.5) available garage usage information for all drivers. This choice is only possible for drivers with desired parking duration $t_d \leq \tau_{op}$.

4.6. Impacts of converting on-street parking to garage parking spaces

It has been one of the policies in Zurich, Switzerland to convert on-street to garage parking spaces. In this section, we evaluate the effects of this policy on the traffic performance and on the city’s revenue. We assume a new parking garage is built and the number of parking garages increases to $G = 3$. Then the total garage capacity starts at $R = 332$ (as in section 4.3) and increases dependent on the number of converted on-street parking spaces. Note that the initial conditions for $N_{op}^0$ and $N_{gp}^0$ are adapted accordingly.

The outputs in Fig. 12(a)-(b) show the impacts of the on-street parking conversion on the average searching/driving to garage parking time and the parking fee revenue. The impacts on the total time and the average/total driven distance follow a similar pattern, but the details are omitted in this paper for brevity.

![Diagram showing impacts of converting on-street parking to garage parking spaces](image)

(a) Impact on average time.  
(b) Impact on on-street and garage parking fee revenue.

Fig. 12. The influence of converting on-street to garage parking on the average/total time searching and driving to garage parking, and on the parking fee revenue in the network.

The more on-street parking spaces are converted to garage parking spaces, the less drivers drive to on-street parking. This reflects the drivers’ parking decision that is dependent on the total number of existing on-street and garage parking spaces in the network. It leads to a decreasing average searching time and an increasing average time driving to garage parking in the short-term assuming that drivers are used to their old on-street parking habits (Fig. 12(a)). When converting on-street parking, for simplicity, we assume that the distribution for the garage parking durations...
becomes the same for all levels (based on the combination of Fig. 5(a) and Fig. 5(b)) as in [11]. Fig. 12(b) shows the
impact of the on-street parking conversion on the total revenue created by on-street/garage parking. While a decreasing
number of on-street parking spaces leads to a decreasing total on-street parking revenue, it leads to an increasing total
revenue. A conversion of on-street parking to garage parking spaces might lead to a higher average time driving to
garage parking and a lower average searching time in the short-term with an increase in the total parking revenue for
the city.

5. Conclusions

In this study, we develop a dynamic macroscopic on-street and garage parking model such that the short-term
influences of different on-street and garage parking policies on the traffic system can be studied and illustrated. The
macroscopic model is built on a traffic system with a parking search model over time. It is incorporated into the on-
street parking framework from [10]. We validate this model based on real data for a case study of an area within the
city of Zurich, Switzerland.

The main contributions of this paper are three-fold.

First, we model garage parking macroscopically, including the parking searchers’ decision between driving to a
parking garage or searching for an on-street parking space in the network. This includes the influences on the
searching-for-parking traffic (cruising), the congestion in the network (traffic performance), the total driven distance
(environmental conditions), and the revenue created by on-street and garage parking fees for the city.

Second, we analyze the relationship between on-street and garage parking, but also their interdependency on cruising-
for-parking traffic and traffic performance with respect to different parking fees. Different hourly on-street and garage
parking fee rates can lead not only to more vehicle time/distance in the network, but also to various financial revenue
outputs. Thus, this analysis can be used for city councils or private agencies to find reasonable hourly on-street and
garage parking fees such that the average vehicle time/distance is not negatively affected and additionally, acceptable
financial revenues are obtained. Our methodology provides the tools to do a cost benefit analysis and to study the
trade-off between the revenue and the average travel time. In the long-term, drivers might avoid paying high on-street
or garage parking fees and quit their journeys. This could affect the demand, but long-term effects are out-of-scope of
this paper.

Third, our model allows us to analyze parking policies in city center areas, e.g., the short-term effects of converting
on-street to garage parking spaces on the traffic system can be simulated and recommendations for city councils can
be made. In the city of Zurich, a conversion of on-street parking to garage parking spaces might lead to a higher
average time driving to garage parking and a lower average searching time in the short-term with an increase in the
total parking revenue. Additionally, the influences of the availability of garage usage information to all drivers can be
analyzed. This might lead to a better traffic performance on the network and an on average faster and shorter journey
for each driver searching for parking.

The general framework provides an easy to implement methodology to macroscopically model on-street and garage
parking. All methods are based on very limited data inputs, including travel demand, VOT, number of garages with
their capacity, the traffic network, and initial parking specifications. Only aggregated data at the network level over
time is required such that there is no need for individual on-street and garage parking data. This macroscopic approach
saves on data collection efforts and reduces the computational costs significantly compared to existing literature.
Additionally, there is no requirement of complex simulation software and the model can be easily solved with a simple
numerical solver.
Overall, the usage of the model is far beyond the illustration in the case study of an area within the city of Zurich. In reality, vehicles often prefer parking possibilities in a central street or area of the network, while discarding other parking opportunities elsewhere. A further consideration is tiered parking pricing that can be included into the model. Certain cities have tiered pricing for both on-street and garage parking such that the driver may pay a low rate for the first hours, and then the rate jumps up significantly to increase turnover and promote higher parking availability. We can also study the usage of a responsive parking pricing scheme. In addition, we can include a traffic demand split with a fixed (low subsidized) parking fee for all on-street and/or garage parking spaces. All remaining portions of demand could be treated responsively, reflecting the external costs for parking. This approach can be motivated by, e.g., the subsidy by a company or a city for their residents.

In summary, the model can be used to efficiently analyze the influence of different on-street and garage parking policies on the traffic system for a smaller geographic scale network, despite its simplicity in data requirements. Based on scarce aggregated data, this model can be used to analyze how on-street and garage parking policies can affect the traffic performance; and how the traffic performance can affect the decision to use on-street or garage parking.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interests

The authors declare that there is no conflict of interest regarding the publication of this paper.

Funding Statement

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.
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