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A Novel Seismic Structural Testing Protocol Based on Hybrid Simulation, Kriging and Active Learning: Methodology and Numerical Examples

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ABSTRACT

This paper presents a seismic structural testing protocol intended to maximize the relevance of benchmarks for model validation and calibration obtained via hybrid simulation experiments, by maximizing model discrepancy evaluated against an ab initio computational simulator. A probabilistic model of the ground motion excitation is used to parametrize the operational range of the emulated structure and Kriging surrogate modeling is extensively used to adaptively design hybrid simulation experiments. This paper presents both the methodology and a numerical verification.

1 INTRODUCTION

Computational Simulators (CS) aim to predict the response of real systems and are typically validated and calibrated against experimental benchmarks. In this context, Trucano et al. (2006) define validation as the procedure to quantify the confidence in the predictive capability of a CS through comparison of computational results with a set of benchmarks obtained from experiments. Validation entails the correctness of physics and for this reason is strictly related to engineering knowledge and judgment. In the same article, the authors define calibration as the procedure for optimizing a set of input parameters so that the resulting agreement of the CS with respect to an additional set of benchmarks is maximized. Finally, benchmarks are defined as pieces of information coming from experiments that serve as comparison for both calibration and validation. In this regard, it is convenient to introduce the concept of model discrepancy (or model inadequacy). Generally, some functional norm of the difference between benchmarks and CS predictions is used to measure model discrepancy, which was formally introduced and classified as a source of epistemic uncertainty by Kennedy and O'Hagan (2001). In general, benchmarks must be formulated accurately in the sense that experimental data has to be selected from the operational range e.g., loading frequency and magnitude, where the CS is required to produce most reliable predictions. For instance, in earthquake engineering, it is common practice to consider the variability of the ground motion excitation as aleatory (i.e. irreducible) and predominant with respect to all other sources of uncertainty e.g., material parameters or boundary conditions. Therefore, it is reasonable to state that ground motion variability fully determines the operational range for validation and calibration of CS of seismically excited structures.

From this idea, we developed a Seismic Structural Testing Protocol (SSTP) based on Hybrid Simulation (HS) that is intended to maximize the relevance of benchmarks for validation and calibration by maximizing model discrepancy measured against an ab initio CS. For the sake of example, Figure 1 shows the substructuring scheme for the HS of a two-story building with rigid floors, subjected to a ground motion excitation after McCrum and Williams (2016).
The Numerical Substructure (NS) comprises the mass and damping of each of the stories, and the Physical Substructure (PS) comprises the stiffness of the columns. Thus, the NS enters the equation of motion with inertia, damping, and external force terms while the PS provides the static restoring force of the frame. The resulting equation of motion reads as follows:

$$M^N \ddot{x} + C^N \dot{x} + R^P(x_{n+1}) = -M^N Ia_g(t)$$

where superscripts N and P indicate the NS and PS, respectively. $M^N$ and $C^N$ are the mass and the damping matrices characterizing the NS, and $R^P$ is the static restoring force measured on the PS as a function of displacement $x$. Finally, $a_g(t)$ represents the earthquake accelerogram. Numerical integration is performed on the equation of motion to solve for the acceleration, velocity and displacement of the structure at each incremental time step. Servo-controlled actuators apply displacements to the PS at each of the two story levels, and corresponding restoring forces are measured from these Degrees-of-Freedom (DoFs) using load cells. In principle, HS allows for reproducing the actual operational range of the emulated structure with a reduced effort compared to shake table experiments.

In the current practice of HS, earthquake accelerograms are selected from a collection of real records and scaled to produce some desirable response feature e.g., triggering of specific limit states, according to some $ab\ initio$ CS prediction. Here we propose to adopt a parametric ground motion model instead, which is adaptively sampled to maximize a pool of discrepancy norms between HS and CS responses. Therefore, benchmarks are more relevant for highlighting possible modeling errors and biases. In principle, CS and HS are both expensive to evaluate, hence Kriging metamodels (Santner et al., 2013) surrogate each discrepancy function and support the adaptive design of experiments, which is treated as a global optimization problem (Murphy, 2014). The key for using Kriging metamodels effectively lies in balancing the need to exploit the surrogate model (by sampling where Kriging predictions of discrepancy functions are maximized) with the need to improve its accuracy (by sampling where uncertainty of Kriging predictions may be high) (Jones et al., 1998). Surrogate modelling already showed promising results in enabling HS-based Uncertainty Propagation Analysis (UPA), Global Sensitivity Analysis (GSA) (Abbiati et al., 2015) and Reliability Analysis (RA) (Abbiati et al., 2017a) studies, correcting the bias of $ab\ initio$ CSs (Abbiati et al., 2017b) and optimization of experimental setups (Sauder et al., 2018).

First, the paper describes the SSTP. Then, it reports a numerical study that proves the validity of the proposed approach. Finally, conclusions and future perspectives are drawn.

2 DESCRIPTION OF THE SEISMIC STRUCTURAL TESTING PROTOCOL

In this study, HS is selected as testing method because of its capability of reproducing the actual history response of the tested component (i.e. the PS, given a realistic seismic excitation, McCrum and Williams, 2016) with a reduced effort compared to shake table tests. This feature makes HS ideal for exploring the expected operational range of the emulated structure. On the other hand, a ground motion model enables the parametrization of such operational range thus providing a support for the calibration.
of Kriging surrogates of a pool of discrepancy functions, which are evaluated against a refined \textit{ab initio} CS gathering all prior knowledge on the PS. Kriging surrogates (Santner et al., 2013) drives the adaptive design of HS experiments. In the following, Subsection 2.1 provides the basics of Kriging surrogate modeling, and Subsection 2.2 describes the proposed SSTP.

### 2.1 Basic of Kriging metamodeling

Kriging is a surrogate modeling technique that considers the computational model to be a realization of a Gaussian process (Santner et al., 2013):

\[
\tilde{M}(\mathbf{x}) = \mathbf{\beta}^T \mathbf{f}(\mathbf{x}) + \sigma^2 Z(\mathbf{x}, \omega) \tag{2}
\]

where \( \mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), \ldots, f_p(\mathbf{x})] \) is a set of regression functions, \( \mathbf{\beta} \) is a vector of coefficients, \( Z(\mathbf{x}, \omega) \) is a zero-mean, unit-variance, stationary Gaussian process, and, \( \sigma^2 \) is the variance of the process. \( Z(\mathbf{x}, \omega) \) is characterized by an autocorrelation function \( R(|\mathbf{x} - \mathbf{x}'|; \mathbf{\rho}) \), where \( \mathbf{\rho} \) is the vector of hyper-parameters of the autocorrelation function. The Kriging model is trained with a set of realizations \( \mathbf{X} = \{\mathbf{x}^{(i)}, i = 1, \ldots, i_{\text{max}}\} \) and the corresponding responses of the simulator \( \mathbf{y} = \{y^{(i)} = \mathcal{M}(\mathbf{x}^{(i)}), i = 1, \ldots, i_{\text{max}}\} \), which together form the so-called Experimental Design (ED) \( \{\mathbf{X}, \mathbf{y}\} \). Kriging parameters are obtained by generalized least-squared solution:

\[
\mathbf{\beta}(\mathbf{\rho}) = (\mathbf{F}^T \mathbf{R}^{-1} \mathbf{F})^{-1} \mathbf{F}^T \mathbf{R}^{-1} \mathbf{y} \tag{3}
\]

\[
\sigma^2(\mathbf{\rho}) = \frac{1}{N} (\mathbf{y} - \mathbf{F} \mathbf{\beta})^T \mathbf{R}^{-1} (\mathbf{y} - \mathbf{F} \mathbf{\beta}) \tag{4}
\]

where \( \mathbf{R}_{ik} = R(|\mathbf{x}^{(i)} - \mathbf{x}^{(k)}|; \mathbf{\rho}) \) is the correlation matrix and \( \mathbf{F}_{ii} = f_i(\mathbf{x}^{(i)}) \). In order to cope with control and measurement errors, which make HS experiments stochastic even when the PS is undamaged, a small nugget is added to the diagonal of the correlation matrix and all hyper-parameter values are estimated via maximum likelihood (Marelli and Sudret, 2014; Murphy, 2014). Having determined the Kriging parameters, the prediction value of the simulator at a test point \( \mathbf{x} \in \mathcal{D}_x \) is a Gaussian variable with the following mean value and variance:

\[
\mu_{\psi}(\mathbf{x}) = f(\mathbf{x})^T \mathbf{\beta} + \mathbf{r}(\mathbf{x})^T \mathbf{R}^{-1}(\mathbf{y} - \mathbf{F} \mathbf{\beta}) \tag{5}
\]

\[
\sigma^2_{\psi}(\mathbf{x}) = \sigma^2 \left( 1 - \mathbf{r}(\mathbf{x})^T \mathbf{R}^{-1} \mathbf{r}(\mathbf{x}) + \mathbf{u}(\mathbf{x})^T (\mathbf{F}^T \mathbf{R}^{-1} \mathbf{F})^{-1} \mathbf{u}(\mathbf{x}) \right) \tag{6}
\]

where \( \mathbf{r}(\mathbf{x}) = R(|\mathbf{x} - \mathbf{x}^{(i)}|; \mathbf{\rho}) \) and \( \mathbf{u}(\mathbf{x}) = \mathbf{F}^T \mathbf{R}^{-1} \mathbf{r}(\mathbf{x}) - f(\mathbf{x}) \). The trained Kriging model, produced by the process above supports the adaptive design of experiments described in the following section.

### 2.2 Adaptive design of experiments

In this study, \( \mathbf{x} \) represents the vector of the input parameters (i.e., the parameters defining the ground motion), \( \mathcal{M}_j(\mathbf{x}) \) is a function that measures the discrepancy between HS and CS on a generic response quantity of index \( j \) varying from 1 to \( j_{\text{max}} \) (e.g., displacement and restoring force peaks),

\[
\mathcal{M}_j(\mathbf{x}) = \left( \mathcal{R}_{\text{HS}, j}(\mathbf{x}) - \mathcal{R}_{\text{CS}, j}(\mathbf{x}) \right)^2 \tag{7}
\]

with \( \mathcal{R}_{\text{HS}, j}(\mathbf{x}) \) and \( \mathcal{R}_{\text{CS}, j}(\mathbf{x}) \) being respectively the response quantity of interest observed in a HS test and computed via CS. Both \( \mathcal{R}_{\text{HS}, j}(\mathbf{x}) \) and \( \mathcal{R}_{\text{CS}, j}(\mathbf{x}) \) combine the relevant structural simulator (HS or CS) and the parametric model used for simulating the ground motion. It is important to stress that a single simulator evaluation provides all response quantities of index \( j \). For a given set of samples \( \mathbf{x}^{(i)} \) of the input parameter vector, \( \mathbf{y}^{(i)} = \mathcal{M}_j(\mathbf{x}^{(i)}) = \left( \mathcal{R}_{\text{HS}, j}(\mathbf{x}^{(i)}) - \mathcal{R}_{\text{CS}, j}(\mathbf{x}^{(i)}) \right)^2 \) provide the complete ED \( \{\mathbf{X}, \mathbf{y}\} \) for the Kriging metamodel \( \mathcal{M}_j(\mathbf{x}) \). Accordingly, the expected improvement algorithm is used to explore the regions of maximum discrepancy for all discrepancy functions of index \( j \) so as to find,
\[ \hat{x}^* = \arg\max_{x, j} \mathcal{M}_j(x). \]  

As a result, the proposed SSTP performs HSs where the \textit{ab initio} CS is less predictive and experimental data has more relevance for highlighting possible modeling errors and biases. In the following, the proposed SSTP is introduced as a sequence of steps:

1) Generate a small initial realization \( X \) of the input parameter vector and evaluate the corresponding responses of both HS and CS to compute \( y_j(i) = \mathcal{M}_j(X(i)) \), which together form the initial ED \( \{X, y_j\}, \forall j \in \{1, ..., J_{\text{max}}\} \).

2) Train a Kriging metamodel \( \mathcal{M}_j(x) \) based on the relevant ED \( \{X, y_j\} \) as explained in Subsection 2.1, \( \forall j \in \{1, ..., J_{\text{max}}\} \).

3) Generate a large set \( S = \{x_1, ..., x_n\} \) from the ground motion model parameter space and compute the response of \( \mathcal{M}_j(x) \), i.e. \( \mu_{y_j}(x) \) and \( \sigma_{y_j}(x) \), \( \forall j \in \{1, ..., J_{\text{max}}\} \).

4) Enrich the ED by selecting the sample \( x^* \in S \) that, among all metamodels \( \mathcal{M}_j(x) \), maximize the expected value of the so-called “improvement random variable” (Jones et al. 1998),

\[
\{x^*, j^*\} = \arg\max_{x, j} E[I_j(x)] = \frac{\theta_j(x) - \mu_{y_j}(x)}{\sigma_{y_j}(x)} + \cdots
\]

where \( \theta_j(x) \) and \( \phi_j(x) \) are CDF and PDF of a standard Gaussian variable. For each metamodel \( \mathcal{M}_j(x) \), the expected improvement method locates samples by using a tradeoff between exploration of regions where Kriging variance predictors are higher and exploitation of regions function where Kriging average predictors are higher (Mockus et al., 1978, Jones et al. 1998).

5) Evaluate the corresponding responses of both CS and HS and add the pair \( \{x^*, y_j = \mathcal{M}_j(x^*)\} \) to each corresponding ED \( j \)-th.

6) Loop between Step #2 and #5 until \( \max_{x, j} E[I_j(x)] / \mu_{y_j}(x) < 0.01 \).

If all \( \mu_{y_j}(x) \) converge rapidly to true global maximum values, then the proposed SSTP represents a compelling and attractive method to minimize the number of HSs while increasing the relevance of benchmarks for validation and calibration of CSs.

The following section presents a numerical verification of the proposed SSTP. The two limit cases where three discrepancy functions are considered separately (\( J_{\text{max}} = 1 \)) and together (\( J_{\text{max}} = 3 \)) are examined and compared.

## Verification of the Seismic Structural Testing Protocol

### 3.1 The verification case study

The 2-Degrees-of-Freedom (DoFs) nonlinear HS of Figure 2, which highlights the partitioning of the domain into PS and NS, is selected to validate the proposed SSTP.
As can be appreciated from Figure 2, a nonlinear spring connecting the two masses is assumed to respond inelastically and represents the PS. Linear elastic springs $k_1$ and $k_2$ are constrained to the ground and belong to the NS while masses $m_1$ and $m_2$ are split between the two subdomains. In the real case, the PS spring would be tested in a laboratory, but in this study, its force-deformation response is modelled numerically using a Bouc-Wen model with stiffness deterioration (Ismail et al., 2009). The corresponding ODE set reads,

$$
\dot{r} = (A \cdot f(\Delta s) - (\beta \text{sign}(\Delta \dot{u} \cdot r) + \gamma)|r|^n)\Delta \dot{u}
$$

(10)

where $A$, $\beta$, $\gamma$ and $n$ are the parameters of the Bouc-Wen springs. On the other hand, $r$ represents the nonlinear restoring force while $\Delta u = u_2 - u_1$ and $\Delta \dot{u}$ are relative elongation and elongation rate, respectively. As can be appreciated from Eq. (10), an additional state variable $\Delta s$ is introduced to store the absolute elongation peak of the spring, which determines a reduction of the stiffness $A$ equal to $f(\Delta s)$,

$$
\begin{align*}
    f(\Delta s) &= 1, \\
    f(\Delta s) &= 1 - (1 - \alpha_r) \frac{\Delta s - \Delta u_y}{\Delta u_u - \Delta u_y}, \Delta u_y < \Delta s \leq \Delta u_u \\
    f(\Delta s) &= \alpha_r, \quad \Delta s > \Delta u_u
\end{align*}
$$

(11)

According to Eq. (11), no stiffness degradation occurs if the absolute elongation peak of the PS is less than $\Delta u_y$. A linear degradation occurs if the absolute elongation peak falls between $\Delta u_y$ and $\Delta u_u$, a constant residual stiffness multiplier $\alpha_r$ applies when the absolute elongation peak exceeds $\Delta u_u$. The reference values of the parameters of the 2-DoFs nonlinear HS read,

$$
\begin{align*}
    m_1 &= 8e3 \text{ kg}, m_2 = 9e3 \text{ kg}, k_1 = 4e4 \frac{N}{m}, k_2 = 1e5 \frac{N}{m}, \\
    A &= 4e5 \frac{N}{m}, \beta = 20, \gamma = 10, n = 1, \zeta = 0.05 \\
    \Delta u_y &= 0.010 m, \Delta u_u = 0.025 m, \alpha_r = 0.80
\end{align*}
$$

The undamped eigenfrequencies of the 2-DoFs system linearized about the initial (unreformed) configuration are 0.45 Hz and 1.62 Hz (i.e. the vibration mode periods are 2.20 s and 0.62 s). A uniform modal damping $\zeta = 0.05$ is assumed for the calculation of the damping matrix. A simper variant of Eq. (10), which corresponds to a classical Bouc-Wen spring - without stiffness degradation-, describes the PS restoring force of the CS. The corresponding ODE reads,

$$
\dot{r} = (A - (\beta \text{sign}(\Delta \dot{u} \cdot r) + \gamma)|r|^n)\Delta \dot{u}
$$

(12)

Same values are assumed for common parameters between HS and CS. In addition, in order to solve for the cumulate dissipated energy $E$, both HS and CS models include the following state equation,
\[ \dot{E} = r \cdot \Delta \dot{u} \]  

It is noteworthy that the task of building an *ab initio* CS model is often achieved via nonlinear Finite Element (FE) modelling and supports the design of the HS campaign e.g., dimensioning of actuation systems and reaction frames (Bursi et al., 2017).

In this study, only the effects of pulse like ground motions are investigated neglecting residual motions (Dabaghi and Der Kiureghian 2017, Broccardo and Der Kiureghian 2014). This corresponds to assume that most of the input seismic energy is carried by the pulse component. Accordingly, the near-fault pulse model of Dabaghi and Der Kiureghian (2017) is selected to parametrize the seismic excitation. In this respect, the following equations define the shape of the velocity pulse in time domain,

\[ v_{pu}(t) = \frac{1}{2} V_P \cos \left[ 2\pi \frac{(t - t_{max}) + \nu}{T_P} \right] - \frac{D_r}{\gamma T_P} \left\{ 1 + \cos \left[ 2\pi \frac{(t - t_{max})}{\gamma T_P} \right] \right\} \]  

\[ D_r = V_P T_P \sin(\nu + \gamma \pi) - \sin(\nu - \gamma \pi) \]  

\[ 4\pi(1 - \gamma^2) \]  

where \( V_P \) and \( T_P \) are velocity peak and duration of the pulse, \( \gamma \) is the number of oscillations within the pulse and \( \nu \) is the phase angle between time modulating function and pulse oscillations. The seismic accelerogram \( a_g(t) \) is found as time derivative of the velocity pulse.

Ranges of model parameters are calibrated based on a set of real records of moment magnitude \( M \in [5.4,7.2] \) and epicentral distance \( D \in [0,15] \) km downloaded from the NGA database (PEER, 2016). Pulses are extracted using the wavelet based algorithm proposed by Shahi and Baker (2014) and fitted following Dabaghi and Der Kiureghian (2017). The main advantage of this approach is that a discrepancy function can be trained on a reduced number of expensive-to-evaluate CS/HS responses sampled over the input parameter space of the ground motion model. A preliminary global sensitivity analysis highlighted a negligible sensitivity of the 2-DoFs system response to parameters \( \gamma \) and \( \nu \). Therefore, intervals of \( V_P \) and \( T_P \) define the input parameter space of the ground motion model, which is summarized in Table 1.

<table>
<thead>
<tr>
<th>Label</th>
<th>Unit</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>VP</td>
<td>[m/s]</td>
<td>0.177</td>
<td>0.607</td>
</tr>
<tr>
<td>TP</td>
<td>[s]</td>
<td>0.957</td>
<td>3.098</td>
</tr>
</tbody>
</table>

Constant values are assumed for \( \gamma = 2.41 \) and \( \nu = 2.56 \) [rad/s]. For the sake of example, Figure 3 depicts 1000 ground motion realizations as well as related response spectra assuming the same damping ratio of the 2-DoFs system, which is equal to 0.05.

**Figure 3**: Ground motion realization: a) accelerograms and b) response spectra (black dashed line highlight the 2-DoFs system periods). Grey curves correspond to 1000 random realization while red curves refer to parameters \( VP = 0.44 \) m/s; \( TP = 2.20 \) s.
Accordingly, Figure 4 compares the dynamic responses of HS and CS subjected to the accelerogram highlighted in red in Figure 3. A FE Matlab toolbox (MATLAB, 2018) was used to compute time history responses of both CS and HS models in this study.

![Figure 4: Comparison between CS and HS responses (VP = 0.44 m/s; TP = 2.20 s): a) hysteresis loop and b) dissipated energy history of the PS.](image)

It is important to stress that the proposed verification case study well represents the class of structural systems emulated via HS. In fact, two eigenmodes govern the dynamic response produced by the seismic loading, nonlinearities are confined to the PS and the overall duration of the simulation is less than 20 sec.

### 3.2 Numerical simulation of the experimental campaign

The proposed SSTP is verified considering the three following discrepancy functions,

$$
\mathcal{M}_{\Delta u}(x) = \left( R_{HS,\Delta u}(x) - R_{CS,\Delta u}(x) \right)^2
$$

$$
\mathcal{M}_r(x) = \left( R_{HS,r}(x) - R_{CS,r}(x) \right)^2
$$

$$
\mathcal{M}_E(x) = \left( R_{HS,E}(x) - R_{CS,E}(x) \right)^2
$$

where subscripts $\Delta u$, $r$ and $E$ indicate three scalar output response quantities related to the PS. In particular, subscript $\Delta u$ indicates the peak of the absolute elongation, subscript $r$ indicates the peak of the absolute restoring force and subscript $E$ indicates the total dissipated energy of the PS. In order to highlight the benefit of combining more discrepancy functions to drive the adaptive design of HS experiments, two different verification tests are conceived:

- Verification #1: the SSTP is applied to each single discrepancy function alone i.e., $f_{max} = 1$.
- Verification #2: the SSTP is applied combining all discrepancy functions i.e., $f_{max} = 3$.

According to Step #1 of the SSTP, an initial ED of four samples is generated by sampling the input parameter space of the ground motion model with a Sobol sequence (Sobol, 1967) and evaluating the corresponding responses of both HS and CS. Then, for both Verification #1 and #2, the ED is adaptively enriched by iterating the SSTP procedure from Step #2 to #5 until the stopping criterion is fulfilled. Kriging metamodels are estimated using the UQLab software framework developed by the Chair of Risk, Safety and Uncertainty Quantification in ETH Zürich (Marelli and Sudret, 2014). Figure 5 compares convergence plots of all discrepancy functions (16-18) obtained from Verifications #1 and #2.
As shown in Figure 5a for Verification #1, the SSTP succeeds in maximizing the discrepancy function \( M_{\Delta u}(\mathbf{x}) \) but fails in maximizing discrepancy functions \( M_r(\mathbf{x}) \) and \( M_E(\mathbf{x}) \), for which the stopping criterion is met too early. On the other hand, the SSTP succeeds in maximizing all discrepancy functions when they are evaluated altogether, as highlighted by Figure 5b, which refers to Verification #2. In addition, the convergence of the maximum value of \( M_{\Delta u}(\mathbf{x}) \) is achieved with 20 samples instead of 22.

Figures 6, 7 and 8 compare the Kriging surrogates of the three discrepancy functions at the last iteration of the SSTP to corresponding MC estimates for both Verification #1 and #2.

**Figure 5**: Convergence of discrepancy function maxima: a) Verification #1 \( (j_{\text{max}} = 1) \); b) Verification #2 \( (j_{\text{max}} = 3) \).

**Figure 6**: PS elongation \( \Delta u \): a) exact discrepancy function obtained via Monte Carlo; b) Kriging surrogate of the discrepancy function from Verification #1; c) Kriging surrogate of the discrepancy function from Verification #2.

**Figure 7**: PS restoring force \( r \): a) exact discrepancy function obtained via Monte Carlo; b) Kriging surrogate of the discrepancy function from Verification #1; c) Kriging surrogate of the discrepancy function from Verification #2.
Figure 8: PS dissipated energy $E$: a) exact discrepancy function obtained via Monte Carlo; b) Kriging surrogate of the discrepancy function from Verification #1; c) Kriging surrogate of the discrepancy function from Verification #2.

As can be appreciated from Figures 6c, 7c and 8c, which refer to Verification #2, all Kriging surrogates show a good agreement with exact counterparts depicted in Figures 6a, 7a and 8a when the SSTP considers the entire pool of discrepancy functions. On the other hand, as can be observed from Figure 7b and 8b, the SSTP fails when discrepancy functions $M_r(x)$ and $M_E(x)$ are taken alone. It is interesting to note that, for this relatively simple 2-DoFs case study, the same sample of the ground motion parameter maximizes all discrepancy functions. For this specific point (VP = 0.58 m/s; TP = 2.36 s), Figure 9 compares hysteretic loops and dissipated energy histories of HS and CS.

Figure 9: Comparison between CS and HS responses (VP = 0.58 m/s; TP = 2.36 s): b) hysteresis loop and a) dissipated energy history of the PS.

As can be appreciated from Figure 9, response histories are quite different meaning that the proposed SSTP efficiently explores the input parameter space where the \textit{ab initio} CS is less predictive, that is, where the effect of stiffness degradation observed in HS is most pronounced.

4 CONCLUSIONS

Assuming that models provide a simplistic representation of the reality, we assert that validation and calibration of computational simulators against experimental benchmarks should cover the expected operational range of the physical system. With reference to seismic structural simulators, ground motion variability, which is the strongly dominant source of uncertainty, underpins such operational range as reflected by the common practice of fragility assessment. Along this line, we presented a seismic structural testing protocol that samples the response of a hybrid simulator within the parameter space of a probabilistic model of ground motion excitation. In particular, the proposed procedure adaptively designs hybrid simulation experiments where the \textit{ab initio} computational simulator is less predictive. Accordingly, the expected improvement algorithm is used to find the global maxima of the Kriging surrogates of a pool of discrepancy functions by sampling the ground motion model parameters. Promising results are obtained on a 2-DoFs verification case study subjected to a pulse-like seismic excitation with two free parameters. In particular, we showed the advantage of combining more measures of model discrepancy to drive the adaptive design of experiments, which always succeeds in finding all global maxima in about 20 experiments.
REFERENCES


