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Economic development on a finite planet with stochastic soil degradation

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ABSTRACT

World economic development is associated with growing food consumption. Agricultural land, however, suffers from over-exploitation and is subject to environmental shocks which are projected to become more severe due to climate change. We present a stochastic model of a dynamic economy where soil is an essential input and natural disasters are sizeable, multiple, and random. Expansion of economic activities raises effective soil units but contributes to an aggregate loss of soil-protective ecosystem services, which exacerbates soil degradation at the time of a shock. We provide closed-form analytical solutions and show that optimal development is characterized by a constant growth rate of stocks and consumption until an environmental shock arrives causing all variables to jump downwards. Optimal soil management consists of spending a constant fraction of output on preservation measures, which is an increasing function of the shocks hazard rate, degradation intensity of agricultural practices, and the damage intensity of environmental impact. We derive the optimal propensity to save and discuss the impact of human pressure and risk exposure on soil and output. We also discuss quantitative impacts of climate change on optimal soil management.

1. Introduction

1.1. Soils at risk

Rising food demand and growing risks of large-scale land degradation suggest bringing back to macroeconomics what is by now the “nearly forgotten resource” – soil.[1] It is estimated that by 2050 agricultural production will have to rise by 70% to meet the needs of growing world population (FAO, 2009).[2] However, already nowadays about one third of all soils are degraded and the global amount of productive land per person could in 2050 be only one-fourth of the level in 1960,

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[1] The expression “forgotten resource” is prominently used in FAO (2014). In France of the 18th century the physiocrats saw land as the primary generator of wealth. Classical economists such as Adam Smith and David Ricardo used it as one of three inputs into production. Ricardo (1817) starts his famous “Principles” by talking about “the produce of the earth.”

[2] Lanz et al. (2017) empirically estimate an endogenous growth model with finite land reserves to study the long-run evolution of global demand for food and find that agricultural production will double by the end of the century.

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if current management practices and policies remain unchanged.[3] A moderate interpretation of U.S. President Franklin D. Roosevelt's quote that "A nation that destroys its soils destroys itself" would read that important aspects of well-being like food security, human health, clean air, and clean water are at risk when quantity and quality of world's soils are not taken care of Wall and Six (2015). The challenge is amplified by changing weather patterns and climate variability, which force farmers to adopt ecosystem-unfriendly practices. This provides ample motivation to study the consequences of endogenous stochastic land degradation for long-run development.[4]

Soil cultivation raises availability and productivity of arable land but involves major risks and uncertainties. In fact, risk is an inherent element of all agricultural activities. The development of agriculture itself was a response to the risks of relying on hunting and gathering for food (Hardaker et al., 2004). The impact of risk has not disappeared, of course, but it has changed its character and has shifted to different areas. It is still inevitable, because food markets are closely interlinked with the rest of the economy and, in particular, because ecological systems and weather conditions are increasingly subject to perturbations. Many developing countries experience large variations in rainfall and are subject to frequent extremes of flooding or drought, both of which contribute to soil erosion and land degradation (UNEP, 2015). Drought and erosion are exacerbated by poor land management that responds inappropriately to climatic variations (UNCCD, 2009), which are predicted to increase in frequency and severity due to climate change, increasing land degradation even further (UNCCD, 2013).

Important negative effects on soil quality arise from the agricultural sector itself, creating potentially harmful changes in the ecological systems supporting the farms (Lichtenberg, 2002). For example, land use changes, irrigation, or deforestation may cause soil erosion and nutrient depletion. The use of pesticides, animal wastes, and soil siltation may contaminate surface and ground waters. Salinization of rivers may damage crop production in downstream areas, while irrigation and land clearing may lead to land loss to selenium and salt drawn up from subsoils. Degradation is often due to increased disruption of macroaggregates, reductions in microbial biomass, and loss of labile organic matter which are induced as negative externalities from aggregate economic activities. Under appropriate market conditions, farmers are able to respond to risks in an optimal manner. However, institutional deficiencies, such as incomplete land tenure systems as well as negative externalities from individual farming on aggregate soil protection, cause suboptimal development. Also, the lack of information about the impact of soil shocks and high individual rates of impatience may constitute management problems for soil.

Soil degradation can be addressed by different types of corrective measures. Labrière et al. (2015) empirically estimate the impact of contour planting, no-till farming and use of vegetative buffer strips on the reduction of soil erosion and find enormous potentials. They conclude that the government or natural resource managers can help decrease soil losses on a large scale. It has been stressed in the literature that such measures can be implemented by individual farmers, by farmer cooperations, or via public policies.[5]

It is often stated that human pressures on soils are reaching critical limits, reducing and sometimes eliminating essential soil functions (Islam and Weil, 2000). Human pressures may entail unsustainable land use and vulnerability to land degradation, including overcultivation, overgrazing, poor irrigation practices, deforestation, and polluting industrial activities (UNCCD, 2009).[6] Food production and security are global concerns but soil losses are especially acute in arid, semi-arid, mountainous, or tropical regions where the lack of protection may cause substantial deterioration in soil quality and reduction in yields. These types of environmental problems in production and food provision and their link to general economic development warrant a thorough investigation from a theoretical perspective.

1.2. Model and findings

We develop a dynamic model of an economy where investments increase the capital stock and the effective soil stock, defined as an index of soil quantity and quality. Investments in agricultural expansion have positive output effects but may also entail negative impacts on soil quality. For instance, clearing and fertilizing of soils are aimed at raising land productivity and profits but may, at the same time, harm the functioning of the ecosystem making it more vulnerable to degradation and natural disasters such as floods, droughts, storms, landslides, etc. The latter may typically have large economic consequences and at the same time are not easily predictable. We therefore model such calamities as random shocks. Our model refers to a planner internalizing such external effects in the economy. This is a generic approach to policy, so that we do not have to specify whether the political actors are public or private and, similarly, domestic or foreign.

The paper addresses several specific research questions. We first ask about the long-run development prospects of an economy which is subject to endogenous stochastic soil degradation. A second issue concerns optimal management practices and optimal public policy in the presence of random shocks. We specifically inquire about the optimal balance between economic expansion and protection of existing soils. Our model delivers closed-from analytical solutions for the optimal

[3] The scenario assumes world's population will reach 9.1 billion in 2050 (FAO, 2014).

[4] Another reason that soil has recently come roaring back is the land scarcity in agglomerations and associated income losses (Economist, 2015).

[5] Studying alternatives to governmental policies, Lopez (2002) deals with the endogenous evolution of rural environmental institutions which is particularly relevant for poor tropical areas where the agricultural and natural resource base is fragile.

[6] For example, there is a widespread concern that the clearing of Asia's upland mountain forests exacerbated the damages from floods in Bangladesh, Cambodia, China, India, Thailand, Vietnam and elsewhere in the Asian lowlands. Transboundary floods that affected India and Pakistan in 2014 resulted in losses of at least US$ 18 billion; the largest damage was the river basin flood in India causing 1281 fatalities and a loss of US$ 16 billion.

growth rate of consumption, the gross saving propensity, and optimal soil preservation measures. We extend the analysis to include human stress and increasing risk exposure.

Our main findings can be summarized as follows. First, we confirm that the damage-related parameters, such as the shock hazard rate, the extent of harmful agricultural practices, soil exposure, and damage intensity, have the expected negative effect on the optimal consumption growth rate. Productivity naturally fosters the optimal growth rate, while soil protection efficiency may have either a positive or a negative effect, which we explain in detail. Soil shocks are transmitted to the capital stock intensifying the economic downturn. Second, we show that the optimal soil preservation consists of devoting a constant fraction of output to soil protection. This fraction is an increasing function of the shock arrival rate, agricultural technology, proportion of harmful by-products, exposure component, and damage intensity. The role of each parameter is therefore identified precisely. Third, we show that the optimal propensity to save and invest crucially depends on the elasticity of intertemporal consumption substitution and therefore widely adopted logarithmic preferences, favored for their simplicity and tractability, may deliver misleading policy conclusions. Fourth, as long as agricultural labor force increases the marginal productivity of land, it also raises the optimal growth rate of consumption and the optimal fraction of output devoted to soil protection. Fifth, we show that higher population raises soil damages in case of a shock, reflecting the concern of human pressure on soil. Sixth, countries with a higher risk exposure turn out to experience a slower economic growth. Finally, we resort to the data on worldwide soil degradation to illustrate and discuss the consequences of increasing hazard rates of weather and ecosystem-unfriendly practices on optimal soil protection.

1.3. Contribution to the literature

Starting with the seminal contribution of Dasgupta and Heal (1979), growth possibilities in the presence of natural resource scarcity has been one of the main topics in resource and environmental economics. Within the endogenous growth literature it has been established that exhaustibility of natural resources may not pose a constraint on growth when innovations are resource-saving. On the other hand, exploitation of the resource and the associated pollution may pose environmental problems (Gradus and Smulders, 1993). In addition, resource scarcity may disrupt social processes that support innovation (Barbier, 1999). With the recent advances in endogenous growth theory, economic development under the restrictions of a finite planet has become again a topic in mainstream literature. Peretto and Valente (2015) call the sustainability of development in a limited habitat “one of the main future challenges” and investigate mechanisms linking resource availability, income, and population, thinking of the natural resource as being “land.” While their paper focuses on endogenous population and innovation dynamics, land input remains fixed. This is where our paper aims to make a contribution. Our model highlights that soil cultivation is able to increase land availability and/or land productivity even when the surface of the planet is finite. But we also stress that soil expansion comes at the cost of increasing the risk of land degradation triggered by environmental shocks. According to our setup, effective soil can be expanded by agricultural investment activities on the one hand but it is subject to stochastic and endogenous environmental shocks on the other. In this respect, our paper also builds on the literature on optimal environmental policies, where, for example, the negative impact of pollution on productivity or utility is stochastic and appropriate taxation measures are warranted (e.g. Bretschger and Vinogradova, 2017; Soretz, 2007).

Many contributions in the literature deal with the complex interplay between agriculture and ecosystem services. The early contribution of McConnell (1983) uses soil depth and soil loss for aggregate production to determine the difference between the private path of erosion from the socially optimal path. Dalea and Polasky (2007) analyze the environmental impacts of agricultural practices on a wide range of ecological services such as water nutrient cycling, soil retention, quality, pollination, carbon sequestration, and biodiversity conservation. They also show that ecosystem services have a positive impact on agricultural productivity. Heal and Small (2002) study agriculture as a producer and consumer of ecosystem services and stress that the quantity and quality of ecosystem services depend on the joint actions of many dispersed resource users.[7] Lanz et al. (2016b) stress that the concentration on a small number of highly productive crops has led to a significant loss of biodiversity which may have a negative feedback on agriculture. We build on these fundamental relationships in our framework by explicitly modeling the decrease of protective ecoservices as an externality of agricultural activity. Grepperud (1997) extends the previous studies by analyzing time-limited effects of optimal soil conservation measures and long-lasting effects on the soil base.[8] In the study of the tropical forest ecosystem in Bangladesh, Islam and Weil (2000) argue that “human population pressures upon land resources have increased the need to assess impacts of land use change on soil quality.” Our paper departs from the deterministic framework used in these papers, introducing random environmental shocks which have been classified as “production uncertainty” and qualified as a “quintessential feature of agricultural production” (Moschini and Hennessy, 2001, p. 90).

7 Pfaff (1996) confirms empirically that greater soil quality in the Brazilian Amazon is associated with more deforestation at the level of individual farms, which then diminishes protection of soils at the aggregate level. Ehui et al. (1990) study a two-sector model of agriculture and forestry to show the dynamic interactions between deforestation and agricultural productivity. They derive optimal rules when forest clearing raises growth of agricultural output but at the same time damages long-run productivity.

8 Cacho (2001) analyzes the impact of agroforestry on arable land prone to degradation in the presence of forest externalities. It is concluded that appropriate forestry is able to reduce land degradation and to contribute substantially to a sustainable use of soil.

Most studies of dynamic soil decisions under risk are empirical. An early exception is Hertzler (1991), who provides an overview of the analytical tools and various exemplary applications to the field. A paper closely related to our study is Shively (2001) which also considers a dynamic stochastic model of soil preservation but does not provide analytical solutions to the theoretical problem. The focus of Shively's analysis is on the role of farm size and liquidity constraints for the decision of subsistence-oriented households to make a one-time investment in protective installations reducing the risk of soil erosion. Thus, the dynamics of the investment process are not considered. His main conclusion is that public policy should aim at enhancing saving and insurance mechanisms for small farms which are most likely to face liquidity and subsistence constraints. Lanz et al. (2016a) introduce random shocks to total factor productivity in agriculture and show that uncertainty requires more land to be converted into agricultural use as a hedge against production shortages.

Our paper aims at making a theoretical contribution to the literature seeking to understand the interactions between growth and the environment. For this purpose, we combine the essential characteristics of soil dynamics from the agricultural literature with a workhorse endogenous growth model from the macroeconomics literature. The key strength of our approach is that it allows for closed-from solutions with respect to the main variables of interest, such as preservation measures, which are of particular importance from the policy perspective.

The remainder of the paper is organized as follows. Section 2 develops our baseline framework. In Section 3, we present the main results with respect to the optimal growth rate, soil protection, and saving propensity. Section 4 adds the development impacts of human stress and risk exposure and presents a quantitative application. Finally, Section 5 concludes.

2. The framework

2.1. General model

We analyze an economy which produces output Y_t using capital K_t, labor L_t, and soil Q_t as inputs; t is the time index. For aggregate production both the quantitative and the qualitative aspects of soil are important; accordingly, soil investments include both dimensions, e.g. land clearing to raise quantity and investment efforts to increase productivity. To obtain effective soil input, the units of arable land M_t are valued by their productivity which, following the agricultural literature, is captured by a quality index, Π_t.[9] Index Π_t aggregates basic biological, chemical, and physical soil characteristics such as infiltration rate, content of organic matter, structure, nutrient content, and soil depth. We define the stock of effective soil units at time t as $Q_t \equiv \Pi_t \cdot M_t$. We shall assume in this and the next section that labor is supplied inelastically ($L_t = L$). In Section 4 we discuss the implications of introducing a growing population.

Production process may involve soil management practices which are potentially harmful for effective soil in case of an environmental event, aggravating its consequences. Put differently, land clearing may weaken soil resilience or may result in by-products which lead to deterioration of soil-protecting ecosystem services. We postulate that a unit of output is accompanied by η units of harmful practices which cause damage to the ecosystem (Clarke, 1992). Exploitation of ecosystems leads to their weakening or even exhaustion, which in turn reduces their capacity to protect and preserve the quantity and/or the quality of agricultural land, making the soil vulnerable to degradation.[10]

The arrival of an environmental disaster follows a Poisson process with a constant intensity λ. When a shock occurs, an endogenously-determined amount $\Delta_t \in (0, Q_t)$ of the existing soil stock is degraded at time t. The size of the deterioration impact is directly and positively related to the magnitude of damages to the ecosystem, D_t. We shall also assume that damages increase in the amount of stocks exposed to disasters as, for example, in the case of volcanic eruptions, landslides, floods or droughts. They are equal to a fraction $\sigma \in [0, 1)$ of the total stock Q_t. The total degradation impact is then written as $\Delta(D_t, Q_t) = \delta D_t + \sigma Q_t$, where δ measures the degradation intensity of man-made damages and σ reflects the exposure to natural degradation independent of man-made activities.

The economy has a possibility to improve ecosystem services by adopting preservation and protection measures (e.g. contour hedgerows, antierosion ditches, grass strips, radical terraces). We assume that a share q_t of output is spent on protection, resulting in protection expenditure $E_t = q_t Y_t$. If the adverse effects of soil use occur at the firm level and are fully taken into account in individual decisions there are no negative externalities; optimal protection is obtained under free market conditions. However, when soil management is poor due to deficient institutions, e.g. incomplete land tenure or property rights systems, protection efforts become suboptimally low. Moreover, when suboptimal management practices of one firm affect other firms and households a case of negative externalities arises, calling for corrective policies. Prominent examples are salinization of rivers, individual management practices causing soil erosion and nutrient depletion in a larger region, and the adverse effects of soil management on public health. We will discuss the different cases in a separate section below.[11] At the aggregate level, total ecosystem protection, $\Omega(E_t)$, is an increasing function of total protection expenditure: $\Omega'(E_t) > 0$. The remaining output share $(1 - q_t)Y_t$ is split between current consumption, C_t, and investments in the accumulation of the two stocks K_t and Q_t, labeled by I_{Kt} and I_{Qt}, respectively. With the instantaneous utility given by $U(C_t)$, the planner's

[9] Broad assessment of soil quality indices is provided in Karlen et al. (1997) and Bastida et al. (2008), Smith et al. (2000) develop a dynamic soil quality model using an index that combines both soil-building and soil-reducing factors.

[10] Full exhaustion is a limiting case in our model which we do not explicitly analyze given our topic; for an extensive study of environmental damages which are irreversible and unknown see Ayong Le Kama et al. (2014).

[11] The different views on the optimality of markets vs. the existence of externalities are discussed in Eswaran et al. (2001).

program reads

$$\max_{C_t, q_t} \quad \mathbb{E}_0\left\{\int_0^\infty U(C_t)e^{-\rho t}dt\right\} \tag{1}$$

s.t.

$$dQ_t = [(1-q_t)Y_t(K_t, L, Q_t) - C_t - I_{Kt}]dt - (\delta D_t + \sigma Q_t)dz_t \tag{2}$$

$$dK_t = [(1-q_t)Y_t(K_t, L, Q_t) - C_t - I_{Qt}]dt \tag{3}$$

$$D_t = \eta Y_t - \omega E_t. \tag{4}$$

$$E_t = q_t Y_t(K_t, L, Q_t), \tag{5}$$

where $\mathbb{E}_0$ is the expectations operator, dz_t is an increment of the Poisson process with a constant arrival rate λ and ρ is the constant rate of time preference. We also require that the stocks and consumption are non-negative and $q_t \in (0, 1)$. In order to make further progress we shall introduce the following assumptions.

Damages We posit that the protection function exhibits constant returns to scale (e.g. a doubling of expenditure on protective measures will double the protection services) with the parameter ω denoting the effectiveness or efficiency of protection measures. The net damages to the ecosystem in period t, denoted by D_t, can be expressed as the difference between the impact of harmful practices and protection services, as stated in Eq. (4).[12]

Production We assume that output follows a Cobb-Douglas structure: $Y_t = F(K_t, L, Q_t) = AK_t^\alpha L^\beta Q_t^\gamma$, where A represents the level of total factor productivity and $\alpha, \beta, \gamma \in (0, 1)$. The function has the desirable property that all inputs are essential for production. To analyze soil degradation in the context of global economic integration we consider the case of a small open economy. A country's capital stock is held domestically as well as internationally, so that domestic return to capital is equalized with the world rate of interest, r, assumed given. The no-arbitrage condition on holdings of capital and soil units guarantees that the marginal productivities of accumulable inputs, net of depreciation, are equalized at each moment in time:[13] $F_K = F_Q - \tilde{\delta} = r$, with $\tilde{\delta} = \frac{\sigma}{\delta(\omega-\eta)} > 0$. The latter condition yields $\frac{Q_t}{K_t} = \frac{r\gamma}{\alpha(r+\tilde{\delta})} \equiv \theta > 0$. The no-arbitrage condition tells us that, in case of an environmental shock, capital use is reduced by the same proportion as soil, propagating the natural shock to all the stocks in the economy.[14] We believe, it is an important mechanism when analyzing growth, especially for less developed countries. We now define an integrated capital-soil stock $S_t \equiv K_t + \nu Q_t$, where $\nu > 0$ is a scaling parameter. Since $K_t = S_t/(1+\nu\theta)$ and $Q_t = \theta S_t/(1+\nu\theta)$ we can write the production function as $Y_t = \xi S_t^{\alpha+\gamma} L^\beta$ where $\xi = \frac{A\theta^\gamma}{(1+\nu\theta)^{\alpha+\gamma}}$. We follow endogenous growth theory by assuming constant returns to accumulable inputs, $\alpha + \gamma = 1$.[15] Output then becomes

$$Y_t = \xi S_t L^\beta \tag{6}$$

where $\xi = \frac{A\theta^{1-\alpha}}{1+\nu\theta}$.[16] The procedure allows to merge Eqs. (2) and (3) to a single equation of motion for stock S.[17]

Utility The economy's objective is to maximize the expected discounted value of utility over an infinite planning horizon with respect to consumption, C_t, and the share of output devoted to protection, q_t, subject to the stochastic process which describes the evolution of effective soil units over time. We assume the utility function takes a standard CRRA form

$$U(C_t) = \frac{C_t^{1-\varepsilon} - 1}{1-\varepsilon} \tag{7}$$

where ε is the coefficient of relative risk aversion.

[12] In our benchmark model it is assumed that the extent of ecosystem unfriendly practices is proportional to production with a constant proportionality factor η. It might, however, be the case that investment in protective measures may also reduce the extent of harmful practices, so that $\eta = \eta(q)$ with $\eta'(q) < 0$. We analyze this case in Appendix A.2.

[13] In our model soil is a natural accumulable input whose quantity depends on investment decisions and on economic activities affecting the damage size in case of a negative shock, while other renewable natural resources, like for example timber, follow a regeneration law which is deterministic and depends on the natural environment only. The no-arbitrage condition on capital markets prevents a divergence between resource prices and capital prices.

[14] Due to our assumption of a small open economy, adjustments of the capital stocks can be effectuated by international capital trade at given prices.

[15] The assumption is made for convenience and has no impact on the quality of the results.

[16] We are thus close to the class of the AK-models (Rebelo, 1991) with A being ξL^β in our case; the marginal product of soil, $\partial Y_t/\partial S_t$, depends on L which gives potential rise to a "scale effect" of growth which we will discuss below.

[17] As explained in the introduction there is no upper bound for S because soil cultivation increases land availability and/or land productivity which, however, comes at the cost of increasing the risks in the economy.

2.2. Solving the model

We denote by $V(S)$ the value function associated with the optimization problem described in (1)–(7) and the law of motion for the aggregate stock, S_t:

$$dS_t = [(1-q_t)Y_t - C_t]dt - (\delta D_t + \sigma S_t)dz_t.$$

The Hamilton–Jacobi–Bellman (HJB) equation may be written as

$$\rho V(S) = \max\left\{U(C) + V'(S)[(1-q)Y(S,L) - C] + \lambda\left[V(\tilde{S}) - V(S)\right]\right\}, \tag{8}$$

where $\tilde{S}$ is the aggregate stock after the occurrence of a shock: $\tilde{S} = S - \Delta(D, S)$. Time subscripts are omitted when there is no ambiguity. The set of optimality conditions is provided in the appendix. The first-order condition with respect to the choice of C, along with the differential of $V'(S)$ (using the change of variable formula sometimes referred to as Ito's Lemma for jump processes), can be used to derive an explicit solution for the law of motion of the consumption rate

$$\frac{dC}{C} = \frac{1}{\varepsilon}\left\{\xi L^\beta\left(1-\frac{\eta}{\omega}\right) + \frac{1-\sigma}{\omega\delta} - \rho - \lambda\right\}dt + \left(\frac{\tilde{C}}{C} - 1\right)dz, \tag{9}$$

where the consumption rate following a shock, $\tilde{C}$, is a constant fraction μ of the pre-jump rate

$$\tilde{C} = \mu C, \quad \mu \equiv (\lambda\omega\delta)^{\frac{1}{\varepsilon}} \in [0,1). \tag{10}$$

It follows that the last term on the RHS is negative and it represents the downward jump in consumption every time a shock occurs.

The first term on the RHS of (9) represents what we label the "trend" consumption growth rate. Specifically, while a shock has not arrived, consumption grows at the constant rate, defined as

$$g \equiv \frac{1}{\varepsilon}\left\{\xi L^\beta\left(1-\frac{\eta}{\omega}\right) + \frac{1-\sigma}{\omega\delta} - \rho - \lambda\right\}. \tag{11}$$

When a shock occurs, consumption jumps down to the new level, $\tilde{C}$, and then continues to grow at the rate g until the next shock.

It can be shown that the value function of the problem, satisfying the HJB equation and certain limiting conditions (see, e.g., Sennewald and Wälde, 2006), is of the form

$$V(S) = \frac{\psi^{-\varepsilon}S^{1-\varepsilon} - \frac{1}{\rho}}{1-\varepsilon}, \tag{12}$$

where ψ is a function of the parameters of the model

$$\psi \equiv \frac{1}{\varepsilon}\left\{\rho - (1-\varepsilon)\left[\frac{1-\sigma-(\lambda\omega\delta)^{\frac{1}{\varepsilon}}}{\omega\delta} + \xi L^\beta\left(1-\frac{\eta}{\omega}\right)\right] + \lambda\left[1-(\lambda\omega\delta)^{\frac{1-\varepsilon}{\varepsilon}}\right]\right\}.$$

Proposition 1. *The solution of the maximization problem (1)–(7) is characterized by*

(i) *optimal consumption is proportional to the effective capital-soil stock;*
(ii) *optimal protection expenditure is a constant fraction of output;*
(iii) *consumption, effective soil stock, and protection services increase at the same constant rate, given by (11), between two subsequent shocks.*

Proof. Provided in the appendix. □

3. Analysis of the solution

In this section, we provide a characterization of the optimal consumption growth and the optimal soil protection. In particular, we are interested in the effects of the key parameters such as exposure and damage intensities, protection efficiency, shock arrival probability, population size and technology.

3.1. Consumption growth

The trend growth rate of consumption is given by g, which we write as

$$g = \frac{1}{\varepsilon}\left\{\xi L^\beta\left(1-\frac{\eta}{\omega}\right) - \rho + \lambda\left(\frac{1}{\lambda\omega\delta} - 1 - \frac{\sigma}{\lambda\omega\delta}\right)\right\}. \tag{13}$$

The expression reveals that the consumption rate is increasing over time if the effective discount rate, which includes not only the pure rate of time preference ρ but also the Poisson intensity λ, is relatively low, formally $g > 0 \Leftrightarrow \xi L^\beta\left(1-\frac{\eta}{\omega}\right) +$

$\frac{1-\sigma}{\omega\delta} > \rho + \lambda$. Put differently, the emergence of natural shocks to soil, represented by λ, may be sufficient to cause negative growth rates in the economy.

The expression resembles the Keynes–Ramsey formula which is widely known in standard macroeconomics. The Keynes–Ramsey growth rate is typically equal to the difference between the real interest rate (usually the marginal productivity of physical capital) and the rate of pure time preference, multiplied by the elasticity of intertemporal consumption substitution (EICS). We note that in Eq. (13) the economy's implicit real interest rate is given by the first term inside the curly braces. It does not only include the marginal productivity of capital-soil input (ξL^{β}) but also the effect of harmful agricultural practices adjusted by the protection efficiency, i.e., the term η/ω. It follows that soil deterioration has an unambiguously negative growth effect. This adverse effect may be reduced by either increasing the protection efficiency, ω, or decreasing the proportion of harmful agricultural practices, η. We will discuss the role of labor for soil development separately in the section on "human stress" below.

The last term in Eq. (13), multiplying λ, represents the most important element of the growth rate – the effect of uncertainty, which includes the exposure and the jump components. The former is represented by the term $\frac{\sigma}{\lambda\omega\delta}$ and captures the slow-down effect due to the damage exposure of the proportion σ of the total stock. On the other hand, the jump component, represented by the term $\frac{1}{\lambda\omega\delta} - 1$, contributes to a *faster* consumption growth as compared to the standard Keynes–Ramsey formula. The term $\frac{1}{\lambda\omega\delta}$ is equal to the ratio of marginal utilities of post- to pre-jump consumption and therefore it is larger than unity (see also Eqs. (10) and (A.7)) implying that the term $\frac{1}{\lambda\omega\delta} - 1$ is positive. The optimal stochastic consumption path is therefore tilted counterclockwise, as compared to the consumption path in a deterministic Keynes–Ramsey model. The economy starts with a relatively low consumption rate at the beginning of the planning horizon, which implies the presence of the precautionary-saving motive, including saving for financing of protective measures. The result is analogous to what has been found in the literature on precautionary savings under uncertainty.[18] The peculiarity of the current setting is that the gross savings are endogenously split between two purposes: accumulation of effective capital-soil units and soil protection measures. It is clear that expansion of protective measures directly reduces soil degradation, while accumulation of effective units has a double-sided effect. On the one hand, a larger stock of effective soil units implies more output and thus more harmful by-products. On the other hand, having a larger stock of effective soil creates an "emergency buffer" in case a disaster strikes. In Section 3.3 we discuss in more detail the economy's optimal saving rate.

The responses of the consumption growth rate to changes in the fundamental parameters of the model are summarized in Proposition 2. It is important to distinguish between the effect of the expected frequency of disasters and the effect of the overall uncertainty. The former takes into account only the arrival rate λ. The latter includes both the arrival rate and the damage caused by a shock, as reflected in the last term in Eq. (13).

Proposition 2. *The optimal growth rate of consumption is*

(i) *a decreasing function of shock arrival rate (λ), proportion of harmful practices (η), and exposure intensity (σ);*
(ii) *an increasing function of the international interest rate (r) and labor force (L);*
(iii) *either an increasing or a decreasing function of soil protection efficiency (ω) and damage intensity (δ), depending on the parameter constellation.*

Proof. Provided in the appendix. □

The effect of the arrival rate (λ) on the optimal growth rate is directly proportional to the negative of the elasticity of intertemporal consumption substitution. Although there is not a general consensus on the magnitude of this elasticity, the empirically plausible range of values lies between 1 and 3. This suggests that if the frequency of disasters were to rise in the future due to a weakened ecosystem, the agrarian economy may experience an important growth slowdown. The intuition behind the effects of η and of the interest rate r is rather straightforward and has already been discussed.

An interesting result concerns the effect of the damage intensity δ, which is in general ambiguous. An increase in δ results in a fall in both stocks, with Q falling less than proportionately than K, so that the no-arbitrage condition is satisfied. The overall stock S therefore declines and its marginal productivity ξ increases. Thus the first term in (13) increases, while the last term declines in δ. If the exposure to non-man-made damages is zero, $\sigma = 0$, then g unambiguously declines in the damage intensity δ.

The effect of the protection efficiency ω is also ambiguous. On the one hand, a higher ω directly improves protection efficiency and contributes to soil preservation thus enhancing the growth rate through the first term in Eq. (13). On the other hand, a higher ω also means a lesser degradation of effective soil units and a smaller jump in consumption rate at the time of a disaster (the jump-smoothing effect). The ratio of post- to pre-shock marginal utilities of consumption is reduced (see Eq. (10)) and this contributes to a growth slowdown through the last term in Eq. (13). The overall effect of ω on g is positive when the marginal productivity of effective land units is relatively high (i.e. the level of technology (ξ) and/or agricultural labor force (L) is relatively high), or the extent of ecosystem-unfriendly practices (η) is relatively large or the degradation intensity (δ) is relatively high. This suggests that economies with a relatively high damage intensity of production and with a higher degree of vulnerability to shocks, such as numerous agriculture-dependent developing economies, may enjoy substantial gains in terms of their growth rates by adopting (more) efficient protective measures. At the same time, economies

[18] See, e.g., Wälde (1999).

with a relatively higher total factor productivity (such as advanced economies) may also experience an improvement in the growth rate of their agrarian sectors by enhancing their soil-protection technologies.

3.2. Optimal soil protection

We now address the question of how much of the current resources should be devoted to mitigation measures for soil preservation. When looking at world food supply and food security or when analyzing economies heavily relying on agricultural production this is a crucial decision problem. We have already shown in the previous section that it is optimal to allocate a constant fraction of output to protection activities. The solution for the optimal output share q^* is (see Eq. (A.6) in the Appendix):

$$q^* = \frac{\eta}{\omega} - \frac{1 - \sigma - (\lambda\omega\delta)^{\frac{1}{\varepsilon}}}{\xi L^{\beta}\omega\delta}. \tag{14}$$

If q^* were simply equal η/ω, then all the man-made damage (D_t) would be eliminated (by Eq. (4)). Given that $q^* < \eta/\omega$, it is not in general optimal for the economy to eliminate all the harmful by-products. If we ignore the exposure component of damages for the moment by setting $\sigma = 0$, we see that this policy is optimal only if the intertemporal substitution elasticity is zero (or coefficient of relative risk aversion is infinite). For finite ε, the optimal protection share falls short of 100% due to the "jump" effect. In fact, by bringing all the terms in (14) to the common denominator, we see that the optimal q^* depends on the difference between the marginal degradation caused by an extra unit of accumulated soil ($\xi L^{\beta}\eta\delta$) and the magnitude of the jump $(1 - (\lambda\omega\delta)^{\frac{1}{\varepsilon}})$ in the effective soil stock (and also consumption) when a shock occurs. The jump effect works to reduce q^*. The presence of the exposure component (σ) works in the opposite direction to increase q^*.

Expression (14) shows that an economy which is highly exposed and where damage intensity is sizeable has to devote a larger share of output to soil protection. When the arrival rate is lower or exposure is less pronounced, it becomes optimal to spend less effort on soil protection in equilibrium. The following proposition summarizes the effects of the fundamental parameters of the model on the optimal protection share.

Proposition 3. *The optimal fraction of output devoted to soil protection is:*

(i) *an increasing function of the event arrival rate (λ), technology (ξ), international interest rate (r), proportion of harmful practices (η), exposure component (σ), risk aversion (ε), and damage intensity (δ);*
(ii) *either a decreasing or an increasing function of protection efficiency (ω), depending on the parameter constellation.*

Proof. Provided in the appendix. □

The intuition behind the results in (i) is straightforward. A higher frequency of disasters (λ) requires more preservation measures in order to better protect land from degradation in the event of an adverse shock. If governments happen to misperceive the true arrival rate λ, the preservation policy would be sub-optimal. Specifically, if the perceived λ is lower than the true one, there is too little preservation. This might happen due to a regime switch from a low to a high shock frequency while the general expectations, if based on past experience, lag behind.

The world interest rate (r) – and hence the total factor productivity (ξ) – and the labor force (L) raise output and thus act in the same direction as η, damage intensity (δ) and exposure component (σ). The statement in (ii) warrants some further comments. The reason for the ambiguous sign in $\partial q^*/\partial\omega$ is that there are three effects which operate in different directions. They can be analyzed by examining the expression in (14). First, there is a direct effect of ω on the optimal protection share, operating through the first term on the RHS of (14): better protective technology requires a smaller expenditure on soil preservation, all else equal. Second, a better protection efficiency has a positive effect on the economy's growth rate (provided $\xi L^{\beta}\eta\delta > 1 - \sigma$), which in turn calls for a larger protection expenditure to compensate for an increase in soil exploitation. If $\xi L^{\beta}\eta\delta < 1 - \sigma$, the reverse is true. Finally, protection efficiency also affects the size of the downward jump in the consumption rate and in the soil stock when an adverse shock occurs (the last term in (14)). The direction of this latter effect depends on the intertemporal substitution elasticity, $1/\varepsilon$. When it is relatively high, above unity, the effect of ω on the downward jump is positive. Conversely, if EICS is below unity, the effect is negative. Overall, the first (direct) effect contributes to a decrease in the share of output devoted to soil protection; the second (growth) effect contributes to an increase or a decrease in protection; while the third (jump) effect can also be either positive or negative, depending on the intertemporal substitution elasticity.

Corollary 1. *If the intertemporal substitution elasticity is above (below) unity,*

(i) *the optimal protection share is convex (concave) in the arrival rate;*
(ii) *the response of the protection share to a change in the arrival rate is more (less) pronounced when protection technology is more (less) efficient and when damage intensity is relatively large (small).*

Proof. Provided in the appendix. □

The corollary implies that, when the frequency of disasters is already relatively high, a further increase in the frequency should be associated with a more (less) than proportional increase in protection measures if the intertemporal substitution elasticity is greater (smaller) than unity.

Corollary 2. *Assume that exposure to non-man-made damage is zero ($\sigma = 0$), then if the intertemporal substitution elasticity is*

(i) *below 3, then the optimal protection share is concave in the damage intensity;*
(ii) *below 2, then the response of the protection share to a change in the damage intensity is less pronounced when protection technology is more efficient. (These conditions are sufficient but not necessary.)*

Proof. Provided in the appendix. □

These results formally support the argument that it is optimal to increase soil protection when the magnitude of damages or the expected frequency of disasters increase. Our model predicts that the optimal increase in the protection share should be more (less) than proportional to an increase in the frequency of events if the intertemporal substitution elasticity is relatively high (low). The intuition is again straightforward. A higher elasticity of intertemporal consumption substitution implies that the economy is easily willing to forgo current consumption in exchange for more consumption in the future and thus an increase in the current protection expenditure is less burdensome. In the limiting case $\varepsilon = 1$ (logarithmic utility) and with $\sigma = 0$, q^* is linear in λ and monotone-increasing and concave in the damage intensity, $\frac{\partial^2 q^*}{\partial \delta^2} = -\frac{2}{\xi L^\beta \omega \delta^3} < 0$. It can be either monotone-decreasing and convex or monotone-increasing and concave in protection efficiency, depending on whether $\xi L^\beta \eta \delta \gtrless 1$.

3.3. Saving propensity

In addition to choosing the optimal soil preservation policy, the economy must also decide on another crucial variable, which is how much to invest in expansion of capital and effective soil units. The economy's gross savings are therefore endogenously split between soil preservation and capital-soil augmentation. The ratio of gross savings to output represents the economy's propensity to save (PTS), which we denote by s. We are particularly interested, from the macroeconomic perspective, in how the possibility of adverse shocks impacts on s. Moreover, knowing how shocks affect q^* and s allows us to deduce their impact on investment in soil stock accumulation. Using the result from Proposition 1(i) and setting $\sigma = 0$ for simplicity, we may express s as

$$s = 1 - \frac{\psi}{\xi L^\beta} = \frac{1}{\xi L^\beta \varepsilon}\left\{\xi L^\beta - \rho + (1-\varepsilon)\frac{1-(\lambda\omega\delta)^{\frac{1}{\varepsilon}} - \xi L^\beta \eta\delta}{\omega\delta} - \lambda\left[1-(\lambda\omega\delta)^{\frac{1-\varepsilon}{\varepsilon}}\right]\right\}. \tag{15}$$

Note that when log-utility is assumed ($\varepsilon \to 1$), the expression simplifies to $1 - \frac{\rho}{\xi L^\beta}$ and thus excludes the risk-related parameters altogether. This simplified preference structure implies that occurrence of random disasters would only cause a reallocation between investment in effective soil accumulation and preservation, but not between consumption and gross savings. When ε is different from unity, the effects of the key parameters characterizing adverse agricultural shocks on s are summarized in

Proposition 4. *If the EICS is above (below) unity, the optimal propensity to save is*

(i) *a decreasing (increasing) function of the disaster arrival rate (λ) and proportion of harmful agricultural practices (η);*
(ii) *an increasing (decreasing) function of the damage intensity (δ) and protection efficiency (ω).*

Proof. Provided in the appendix. □

The value of the intertemporal substitution elasticity appears to be crucial for resolving the ambiguity in the effects on the saving propensity. For instance, an increase in λ causes an unambiguous increase in the share of output devoted to soil preservation (q^*) but it leads to a decline in s if $1/\varepsilon > 1$ and to an increase in s if $1/\varepsilon < 1$. It follows that, when the elasticity is relatively high, the optimal response of the economy to an increase in disaster frequency is to increase both its soil preservation expenditure and current consumption – at the expense of investment in soil expansion. By contrast, when the elasticity is relatively low, an increase in the preservation share is accompanied by a *reduction* in both consumption and soil accumulation ($\frac{\partial s}{\partial \lambda} < \frac{\partial q^*}{\partial \lambda}$, see the exact expression for $\frac{\partial q^*}{\partial \lambda}$ in the proof of Proposition 2). The intuition behind these results lies in the understanding of the intertemporal consumption smoothing. If EICS is relatively low, this means that the economy is less willing to reallocate consumption possibilities over time. An increase in λ implies a higher chance of having a lower income in the future and thus a lower consumption. A low EICS dictates a preference for a smoother time-profile of consumption and thus it is optimal to lower the current consumption in response to an increase in λ as a lower consumption is also anticipated in the future. Therefore s increases, while current consumption and investment in soil accumulation fall. If EICS is relatively high, this means that the economy is more easily willing to reallocate consumption over time, so that the intertemporal smoothing is less important as compared to the overall lifetime consumption possibilities. As λ increases, indicating a more likely decline in consumption in the future, the economy's optimal response is to increase soil preservation measures and current consumption as well. This is akin to the "precautionary consumption" phenomenon (Müller-Fürstenberger and Schumacher, 2015). A similar reasoning can be applied to analyze the effects of an increase in harmful agricultural practices (η), damage intensity (δ) and protection efficiency (ω).

We note that when ε approaches unity, the derived impact of all the parameters – and most importantly those characterizing adverse events, λ, η, and δ – are at the lower bound of the empirically plausible impact range.

4. Development perspectives

4.1. Human stress

4.1.1. Population pressure

In the context of soil management, “human stress” denotes the hypothesis that an economy with growing population and expanding activities causes more intensive land use, faster soil degradation, and hence a sharper destruction of protecting ecosystem services. Reviewing the data and the literature, Eswaran et al. (2001, p. 7) conclude that a high population density in an area that is highly vulnerable to desertification poses “a very high risk for further land degradation.” In our model we have derived the joint development of a country's capital and soil stocks. Economic expansion involves both increasing capital and soil use causing higher soil damages when a shock occurs, see Eq. (4). Population size has also a major effect in our economy.[19] An increase in labor ($dL > 0$) raises the productivity of each capital-soil unit which results in a higher consumption growth. The result is an example of the so-called “scale effect” of growth which is a consequence of the constant returns to capital. It says that a larger population can exploit higher capital stock more efficiently which is favorable for growth. The empirical validation of the scale effect has been challenged in the literature, see Jones (1995). As a response to the critique and focusing on market structure, Peretto and Smulders (2002) show how an innovation-driven economy achieves endogenous growth without a scale effect. Specific to our contribution is that population size induces not only a growth effect but, at the same time, a negative damage effect. Indeed, following Eq. (4), the marginal damage as a side-product of soil investments grows with labor input (because labor is an argument in the production function). With the emergence of non-monotonic growth our model adds a new perspective on the scale effect and is not directly subject to the earlier critique about its consequences. The impact of population size on damage size addresses the often-stated concern about improper soil management in a situation of human stress.

According to our framework population size has a direct impact on optimal soil management. Recalling (14), which is reprinted here for convenience

$$q^* = \frac{\eta}{\omega} - \frac{1 - \sigma - (\lambda\omega\delta)^{\frac{1}{\varepsilon}}}{\xi L^{\beta}\omega\delta},$$

we see that larger population size calls for a higher output share q^* to be used for optimal soil protection. In case of poor institutions or negative externalities of soil management the government would have to invest more in soil protection the higher becomes the population size. This reflects the dual impact of population pressure prominently appearing in literature: it offers a potential for raising aggregate output but imposes a major concern for policy making, requiring a more stringent soil protection in the optimum. The high population growth in less developed countries highlights that the challenges for optimal land policies are big in those world regions. The demographic transition leading to low or zero population growth observed in middle-income and developed countries relaxes some of the growing pressure but does not mean that soil protection would have to decrease in time.

Population pressure also affects accumulation in the economy, i.e. the propensity of expanding capital and effective soil units. Specifically, population size has an ambiguous effect on the savings rate s:

$$\frac{\partial s}{\partial L} = \frac{\beta}{\varepsilon\xi L^{\beta+1}}\left\{\rho + \lambda + \frac{\varepsilon[1 - \sigma - (\lambda\omega\delta)^{\frac{1}{\varepsilon}}] - (1 - \sigma)}{\omega\delta}\right\} \gtrless 0.$$

With logarithmic preferences, however, the effect is unambiguously positive: $\frac{\partial s}{\partial L} = \frac{\beta\rho}{\xi L^{\beta+1}} > 0$.

4.1.2. Population dynamics

Although endogenizing population dynamics fully is beyond the scope of the present paper, in this subsection we examine the consequences of introducing an exogenously growing population over time. Suppose that population dynamics can be described by the following standard differential equation: $\hat{L}_t = n,$ i.e. population size increases exponentially at the rate n from an initial given size L_0. The objective of the social planner in this economy can then be written as

$$\max_{c,q} \ \mathbb{E}\left\{\int_0^{\infty} u(c_t)L_t e^{-\rho t}dt\right\} \text{ or } \max_{c,q} \ \mathbb{E}\left\{\int_0^{\infty} u(c_t)L_0 e^{(n-\rho)t}dt\right\},$$

where $c_t = C_t/L_t$ is per-capita consumption. Note the change in the discount factor from $-\rho$ to $n - \rho$.

Assume that output is produced with an AK-type technology: $Y_t = \xi S_t$. This is certainly a strong assumption in the context of agricultural production but it allows us to derive closed-form solutions and clearly see the role of population growth in the model. We shall subsequently discuss how these results would change if labor were an input in the production process. With our simplifying assumption we can rewrite the benchmark model in per-capita terms. The per capita output is given by $y_t = \xi S_t/L_t = \xi s_t$. Given the law of motion of the aggregate stock, S_t, the dynamics of s_t obeys (by the change of

[19] While population is exogenous in our setup, Peretto and Valente (2015) present a dynamic model with finite natural resources and endogenous population size in a deterministic framework.

variable formula):

$$ds_t = [(1-q)y_t - c_t - ns_t]dt + (\tilde{s}_t - s_t)dz_t,$$

where $\tilde{s} = \tilde{S}/L$ and $c_t = C_t/L_t$ is per capita consumption. The rest of the model remains as in Section 2. We relegate the detailed derivations to the appendix, while note here that in this setting the share of consumption in the per-capita soil stock is smaller than in the benchmark model due to the presence of population growth rate:

$$\psi = \frac{1}{\varepsilon}\left\{\rho - (1-\varepsilon)\left[\frac{1-\sigma-(\lambda\omega\delta)^{\frac{1}{\varepsilon}}}{\omega\delta} + \xi\left(1-\frac{\eta}{\omega}\right)\right] + \lambda\left[1-(\lambda\omega\delta)^{\frac{1-\varepsilon}{\varepsilon}}\right]\right\} - n.$$

The optimal protection measure, q, preserves its expression (Eq. (14)) with β set to zero, while the growth rate of per-capita consumption in-between two consecutive shocks is given by the familiar expression:

$$g = \frac{1}{\varepsilon}\left\{\left(1-\frac{\eta}{\omega}\right)\xi + \frac{1-\sigma}{\omega\delta} - (\rho+\lambda-n)\right\},$$

where the rate of time preference is adjusted by the shock hazard rate λ, as before, and now also by the population growth rate n. If λ serves to increase the effective discount rate – and thus reduce the consumption growth rate, n has the exactly opposite effect. If labor force were an input in the production production process, we would find that the growth rate of population would be reflected in the marginal productivity of the accumulable stock and thus appear in another term in the above equation. It would also affect the optimal propensity to consume, which would no longer be constant. On the one hand, growing population would have a positive effect on consumption growth by boosting the marginal productivity of S but, on the other hand, it would also cause more of ecosystem-unfriendly practices and thus more damages in case of a shock. The latter would call for more stringent protection policies, similarly to our results discussed in Section 4.1.1.

An interesting question is what would happen in our model if population size were fully endogenized. It is clear that such an analysis requires a different model setup and, in particular, a decentralized economy where households are allowed to make fertility decisions. A model of this type is elegantly presented in Peretto and Valente (2015). We note that, differently from our initial setup, their model includes in particular (i) a more sophisticated production sector and (ii) a constant endowment of a non-exhaustible resource. Since we do not study the effect of population size on innovation, we abstract from (i) and think of the simple production structure as in our Eq. (6). Our model adds in relation to point (ii), namely we endogenize the dynamics of resource, S. It is important at this stage to define how households perceive their impact on S. If they do not internalize the negative effect through damages to ecosystem, then they do not invest anything in protective measures and some policy is required to correct for the externality. In this case, and without appropriate policy intervention, it is clear that per-capita consumption will converge to zero in the long run. On the other hand, if households do internalize the externality, at least to some extent, their fertility decision will take into account the "human stress" effect on ecosystem and will result in a lower equilibrium fertility rate, as compared to the case without internalization, or even in a stabilization of population at a constant level. The latter case would correspond to our initial framework. In the former case, we conjecture, the endogenous fertility rate is closely related to the stochastic growth rate of S: assuming that a household size can adjust instantaneously, the population size increases at a constant rate in between shocks and drops when a shock occurs as households immediately respond to the scarcity of S.

4.2. Risk exposure

This section discusses the impact of event frequency on development, especially when countries are heterogeneous with respect to their exposure to environmental shocks. Recall that a country's development is driven by investments in soil and capital and is harmed by environmental shocks. If the country is hit by an adverse event, some amount of soil is destroyed, triggering capital exports which restore marginal return on capital K_t to the world market level. Employing the no-arbitrage condition for soil, $pF_Q + \dot{u} = r \cdot u$, one sees that with predetermined p, r, u in a small open economy any increase in F_Q due to soil loss has exactly to be compensated by a decrease in F_Q due to capital exports. The standard Keynes–Ramsey growth rate g equals the difference between the real interest rate and the rate of pure time preference, adjusted by the elasticity of intertemporal consumption substitution. Interest rates are identical for all the countries and the same for each country before and after the shock, so that the countries' growth rates are identical (assuming identical ε, ρ). But note that this is only valid for what we called the "trend growth rate" above, it does not take into account the possibility of jumps due to environmental shocks. If a country is especially vulnerable, it is hit more frequently than others. Then, also depending on the (endogenous) size of the shocks, average income growth will be lower than in a country which is less exposed to the shocks. We have thus derived a case where growth rates differ internationally even when countries are small, especially with respect to international capital markets.

To assess a country's average consumption growth rate including the possibility of jumps we calculate the *expected* consumption growth rate:

$$g^e = \frac{1}{\varepsilon}\left\{\xi L^\beta\left(1-\frac{\eta}{\omega}\right) - \frac{\sigma}{\omega\delta} - \rho + \lambda\left[\frac{1}{\lambda\omega\delta} + \varepsilon(\lambda\omega\delta)^{1/\varepsilon} - 1 - \varepsilon\right]\right\}. \tag{16}$$

It can be easily verified that the expected consumption growth rate is smaller than the trend growth rate: $g^e < g$. Moreover, the larger and the more frequent are the jumps, the lower becomes expected consumption growth and thus the relative income performance of a country (with our specification we have $\frac{\partial g^e}{\partial \lambda} < 0$).

To provide empirical evidence we may note that Africa is particularly vulnerable to land degradation and desertification (UNEP, 2015). Moreover, it is the poorest of the continents with a disproportionate share of low-income countries compared to other world regions and a very low level of economic development. The Economic Commission for Africa reported that in 2013 Africa's share of the world population was 13%, but its share of global GDP was only 1.6%. Comparing the different countries within the continent it is striking that many poor economies, in particular Burundi, DR Congo, R Congo, Gabon, Liberia, Sierra Leone and Zambia have a very high percentage of degraded land (UNEP, 2015, pp. 123–130), supporting our hypothesis of soil damages and land degradation negatively affecting economic development.[20] The result also makes clear that a growing arrival frequency in the future, e.g. induced by climate change, will deepen the income gap between the vulnerable countries of the South and the less vulnerable economies of the North, which acts against the sustainable development goals. Even if deficiencies in markets and policy of the South were absent, this would indicate a need for support of the South. Realistically assuming that market failures and weak institutions in less developed countries entail higher soil mismanagement, specific policy to correct for externalities and to strengthen institutions has to be implemented to improve welfare and development conditions in the South.[21]

4.3. Quantitative analysis

In this section we attempt to quantify our theoretical predictions, in spite of the fact that reliable numbers for soil degradation and its economic impact are rather difficult to obtain. Soil degradation has different dimensions, such as soil loss, sealing, contamination, acidification, salinization, compaction, and nutrient decline, which are not directly comparable. Moreover, the situation differs depending on climatic conditions, e.g. in arid regions vs. rain forests. Gibbs and Salmon (2015) report global estimates of total degraded area varying from less than 1 billion ha to over 6 billion ha, with equally wide disagreement in their spatial distribution. Eswaran et al. (2001) state average estimates for all degraded lands as percentage of total land for the continents: Africa 73%, Asia 71%, Europe 65%, and South America 73%. Lal (2001, p. 531) reports production losses due to soil erosion in Africa between 2% and 40% with a mean loss for the whole continent of 9% which could rise to 16.5% by 2020.

Soil erosion is by no means restricted to developing countries, it is also important in Europe and the U.S. The study of Kuhlman et al. (2010, p. 27) concludes that soil erosion by water and wind causes an average productivity loss per hectare in Europe of 0.16% per year. It is calculated that an effective soil erosion control programme in the EU would provide annual on-site benefits of 500 million Euros over a 20-year period equivalent to a present value of 3.25 billion euros. Comparing the figure to the costs of the programme supports our model hypothesis of external effects, motivating the need for public policy: the on-site benefits for reducing erosion on agricultural land turn out to be smaller than their costs, while adding the off-site benefits shows significants improvement in social welfare when implementing the programme. The need for public soil protection policy depends on the size of market failure in soil management. Den Biggelaar et al. (2003, p. 2) conclude: "Comparing the results of past and present erosion studies indicates that inappropriate soil management may amplify the effect of erosion on productivity by one or several orders of magnitude." An important case for externalities is the use of water. It is reported that the common water resources are rarely used efficiently and that "environmental externalities have arisen through excessive utilization of non-recharged aquifers while, in a number of cases, the excessive application of irrigation water has resulted in rising groundwater tables, soil salinization and sodification problems" FAO (2015, p. 402).

Even though the parameters of our model are not straightforward to calibrate under these preconditions we can provide some rough measures for the expected effects of accelerating climate change. We remain very cautious about choosing parameter values and limit the analysis to the impact of a few key factors on optimal soil protection. One such important factor is the frequency of natural disasters, which is predicted to increase over time due to rising global mean temperature (IPCC, 2014; Thomas and Lopez, 2015). For instance, the number of meteorological disasters has risen from about 25 per year to over a hundred during the period 1970–2014. Similarly, the number of hydrological events has risen from about 80 to almost 150 over the same time frame (Thomas and Lopez, 2015, Fig. 1). At the same time, exposure, defined as the presence of people, livelihoods, ecosystems, environmental services, resources, etc., has also increased due to societal change. IPCC (2012) documents that fatality rates and economic impact, e.g. losses as a proportion of gross domestic product (GDP), are higher in developing countries due to higher share of impoverished populations, weak infrastructure, lack of basic facilities, and limited government capacity. Residents of the developing economies are not only more vulnerable to natural disasters but at the same time they have less resources for management options and fewer coping strategies. Therefore, soil degradation and food security in general are more acute issues in those regions. When choosing parameters for calibration we shall restrict our attention mostly to those pertaining to the developing world.

An important general parameter is the coefficient of relative risk aversion ε. As a reference point and for the purpose of the comparison to other studies, we start with the logarithmic utility assumption ($\varepsilon = 1$). If soil is viewed as essential

[20] We do not include the possible feedback from poverty on soil mismanagement in the analysis because the empirical evidence is mixed in this respect.

[21] Borissov and Pakhnin (2016) compare private and public ownership of natural resources and show that the former usually results in a higher rate of growth but also in a higher inequality than a management by public decision making.

for food production, we may well assume a higher risk aversion because food security is crucial for survival of species and humans. Similarly, if we consider behavioral and policy aspects (reflecting opinions and objectives of policymakers) we will have to use a higher number; $\varepsilon = 3$ appears to be a frequent choice in the literature. In our setup, gross capital return is given by ξL^{β} which we set to 0.1. This might appear to be relatively high although may be appropriate in the context of the less developed countries we have primarily in mind. The statistics for natural disasters (floods and storms, including hurricanes) over the last two decades suggest an average hazard rate of $\lambda = 0.1$, i.e. a sizable event arrives around once in ten years. For the logarithmic utility function we can calculate the increase of optimal soil protection with rising event frequency, $\partial q^*/\partial\lambda$, from Proposition 3(i), which is given by $1/\xi L^{\beta} = 10$ in our case. This means that with increasing arrival frequency the optimal output share devoted to soil protection has to grow with a factor 10. For example, a 10% rise in the hazard rate from $\lambda = 0.1$ to $\lambda = 0.11$ implies an increase in the optimal output share devoted to soil protection of 0.1 (ten percentage points) which may represent a considerable burden on the budget of less developed countries. The examples for other event frequencies, e.g. pertaining to different regions, are readily calculated with this simple formula.

When $\varepsilon \neq 1$, additional effects enter into the picture which makes it necessary to calibrate other parameters of the model. For example, to compute $\partial q^*/\partial\lambda = (\lambda\omega\delta)^{\frac{1}{\varepsilon}-1}/\xi L^{\beta}\varepsilon$ we also need to calibrate $(\lambda\omega\delta)$. We know that in our model consumption drop at the time of a shock equals $1-(\lambda\omega\delta)^{1/\varepsilon}$. To get a ballpark value for the size of the consumption loss in the aftermath of a disaster we refer to UNEP (2015), where the loss of soil supporting ecosystem services is estimated to cost the 42 African countries about 278 million tons of cereals per year. In present value terms, the cost of inaction against nutrient depletion induced by soil erosion accounts for about 4.6 trillion PPP USD over the next 15 years (UNEP, 2015, p. 90). This is equivalent to about 286 billion PPP USD per year or about 12% of the average GDP of the 42 countries in 2010–2012, which corresponds to an approximate 8% GDP loss if a shock occurs every 10 years. We thus set the consumption drop to 8% so that, with $\lambda = 0.1$ and $\varepsilon = 3$, we obtain $\partial q^*/\partial\lambda = 3.94$. We know that a higher risk aversion requires a higher level of soil protection (Proposition 3) and also find now that even with a higher initial protection, optimal protection rate still has to grow by a factor 4 with an increase in arrival frequency. A 10% rise in the hazard rate from $\lambda = 0.1$ to $\lambda = 0.11$ implies an increase in the optimal output share devoted to soil protection of 0.04 which is still a considerable share of GDP. We conclude that increasing frequencies of soil degradation due to climate change pose a big challenge for food production and may impose a severe strain on the developing countries which are especially dependent on the agricultural sector. Efficient measures will have to cover both a direct support of protecting systems and the improvement of institutions to support private soil management practices.

In addition to disaster frequencies, ecosystem-exploitative practices have also risen over time – across the globe but mostly in the developing world (e.g., shrinking of the Amazonian rainforest). We therefore consider next the effects of η and ω on the optimal preservation measures. To do so, we use again the study of UNEP (2015) which relies on the well-established WOCAT (World Overview on Conservation Approaches and Technologies) database. For the 42 African countries, the study concludes that the loss of about 105 million hectares of croplands can be prevented provided that soil erosion is managed appropriately (UNEP, 2015, p. 11). The estimation of crop losses in terms of economic value involves an econometric estimation of the loss of ecosystem services and of the marginal physical product of different environmental variables as well as the use of market prices (UNEP, 2015, p. 58). With the already-mentioned cost of land degradation of about 286 billion PPP USD each year, the costs of inaction against land degradation in Africa turn out to be very high. Conversely, implementing better soil practices is found to be much less expensive. Specifically, expenditures for sustainable land management practices were calculated to cause an annual cost of about 9.4 billion USD or 1.15 % of the GDP (UNEP, 2015, p. 91).

If we assume that the UNEP study is sufficiently close to an optimal prescription for soil protection policy we can proxy the optimal output share for land conservation in our model by the 1.15%, so that we have $q^* = 0.0115$. With predicted benefits of sustainable land management being almost seven times the policy cost this value may also be seen as a lower bound for an optimal expenditure share. Assuming the exposure to natural degradation independent of man-made activities to be null for simplicity ($\sigma = 0$), we can determine the ratio of proportion of harmful practices to protection efficiency, η/ω, by using Eq. (14) for q^*. Adopting the other parameter values from above we obtain $\frac{\eta}{\omega} = 0.1142$. Given this information we are able to perform a further sensitivity analysis of our model results. Specifically, we can assess the impact of the extent of harmful practices, η, which may vary with sectoral change, trade exposure, and/or climate change. Invoking again Eq. (14), we calculate that a 10% increase in η *ceteris paribus* leads to an optimal output share for conservation measures of 2.3%. If harmful practices grow by a factor one half, the optimal expenditure share reaches 6.9% of output which is a relatively high number, especially for a less developed economy.

Protection efficiency, ω, may be raised by adopting more technologically-efficient equipment and techniques, as well as by applying fertilizers with high-nutrient content. We recall from our discussion of Proposition 3 that the impact of ω on q^* is ambiguous, because ω affects the leverage of the policy, the growth rate, and the downward jump in case of a shock. Using our calibrated parameter values we find that a 10% increase in protection efficiency leads to 0.1 percentage point decline in the optimal protection share, i.e. q^* falls to 1.05%.

In addition to the effects of rising hazard rates of climate shocks, ecosystem deterioration and changes in technological efficiency, one may be interested in the long-term projections of expenditure on soil preservation. Such projections can be useful for government budget planning when financial priorities may be shifting in light of changing climatic conditions and additional investment projects need to be undertaken. If we remain in the context of the African economies and, more

Table 1
Optimal spending on soil protection in Sub-Saharan Africa over next 30 years, in multiples of baseline year GNI. Baseline year is taken to be 2016 with corresponding GNI of 1566.7 bn USD (WorldBank, WDI).

$\lambda \backslash \varepsilon$	1	2	3	4
0.1	0.34	2.84	4.20	5.19
0.105	1.84	4.59	6.08	7.17
0.11	3.44	6.45	8.08	9.26

specifically, the most vulnerable region of Sub-Saharan Africa, we may ask ourselves how much this region should spend on agricultural policies aimed at soil protection over the next 30 years. Such a projection undoubtedly depends on the region's future growth rate. The World Bank data indicate that the average GDP growth rate in the region reached 1.2% and was −1.5% on a per capita basis. We shall assume for simplicity that the economies of this region will have a zero average growth rate in the next 30 years. We also posit that the protection spending derived from UNEP (2015) of 1.15% of GDP is optimal under current climate conditions. Depending on the assumptions about shock's hazard rates and coefficient of relative risk aversion, we obtain the following projections for expenditure on soil preservation measures:

Table 1 shows that optimal expenditure is sensitive to variations in the hazard rate and risk aversion. While with the current average hazard of $\lambda = 0.1$ and log-preferences ($\varepsilon = 1$) the total 30-year spending is only 34% of baseline year GNI (or 0.54 trillion USD), in the extreme case of $\varepsilon = 4$ and $\lambda = 0.11$ it rises to a multiple of 9 (or 14.5 trillion USD). This corresponds to the yearly output share devoted to soil protection of 30%. With a more moderate risk aversion of $\varepsilon = 2$ and less frequent climate events, $\lambda = 0.105$, the optimal output share for protection measures would reach about 15% or 7.2 trillion USD, which may still turn out to be an unsurmountable challenge for public management and associated financing. This number is somewhat lower than to the UNEP estimation of the cost of inaction against soil erosion in African countries.

As an alternative to the analysis of aggregate soil functions and overall economic linkages one may resort to the study of subsystems like soil organic carbon (SOC) which reveals some interesting characteristics of soil degradation. It has been shown that conventional agriculture leads to a loss of about fifty percent of SOC over a period of 20–30 years (Petersen and Hoyle, 2016). Conversely, increasing SOC by using alternative soil management practices is beneficial to soil functions, fertility, and buffering capacity. However, the benefits of alternative land policies strongly vary with soil quality. For example, in areas with low rainfall and sandy soils like in Western Australia, productivity improvement in agriculture is much less important compared to the sequestration value for climate policy when increasing SOC, see the numbers in Petersen and Hoyle (2016). As a consequence, the benefits of altered soil management heavily depend on the used carbon price an thus on the adopted climate policy, which is beyond the scope of the present paper. The analysis of other specific soil functions or of specific countries and regions reveals that the degree of heterogeneity is still considerable on a less aggregate level, especially when the economic valuation is a focus. Hence, different calculations and specific considerations would be needed for each of the many different cases. To concentrate on the main messages for the economics of soil degradation, we have restricted the analysis in this paper to the aggregate approach to soil degradation and protective policies, both in theory and the quantitative implications.

5. Conclusions

The present paper considers an economy which produces output employing three essential inputs: capital, soil and labor. Production process, accompanied by harmful agricultural practices, weakens the ecosystem and diminishes its protective services. The soils become vulnerable to random environmental shocks which lead to soil degradation. Such a scenario is observed in numerous developing countries with large agricultural sectors. In an attempt to raise yields and profits, farmers clear a slot to gain arable land but at the same time they lose the protective ecosystem services and make their land exposed to landslides, floods, droughts, winds and similar calamities. To ensure sustained yields, it thus becomes necessary to adopt soil preservation measures (e.g. installation of contour hedgerows to prevent landslides). Since the extent of soil degradation depends positively on the magnitude of harmful agricultural activities and negatively on the protection efforts, an optimal soil expansion and preservation policy can be designed to maximize the economy's expected lifetime welfare. In the present article we provide a clear-cut closed-form solution to this dynamic stochastic problem.

The optimal development of the economy is characterized by capital and soil stocks and the consumption rate which all grow at the same constant rate until a disaster occurs causing a downward jump in all variables. In the benchmark model we assume that shocks arrive at a constant Poisson rate. The percentage reduction in consumption, i.e. the size of the jump, is constant and depends on the arrival rate, the damage intensity, the protection efficiency and the intertemporal substitution elasticity (EICS). The optimal soil preservation strategy consists of devoting a constant fraction of output to protection measures. This fraction is an increasing function of the hazard rate, the damage intensity, the level of agricultural technology and agricultural labor force. It may be either increasing or decreasing in the protection efficiency due to three counteracting forces, the direct effect, the jump effect and the growth effect. The EICS appears to play a crucial role in determining how the economy's propensity to save responds to changes in the key parameters, including those characterizing adverse shocks. For a relatively high value of EICS (above unity), we find that an increase in disaster frequency leads to a decline in the saving propensity, implying that soil conservation measures and current consumption increase at the expense of capital-soil stock expansion. An increase in the damage intensity of shocks leads to an increase in both protection measures and saving propensity. Consequently, an increase in the disaster frequency and in the damage intensity, while both having a positive impact on preservation measures, have diverging effects on the propensity to save and thus on how consumption possibilities are spread over time. Population size has a positive effect on growth but intensifies negative soil shocks. Finally, a high soil risk exposure causes a lower economic growth, even when capital markets are fully integrated and the world interest rate is given.

Soil conservation has recently reemerged as an important issue in policy debate, especially for world food security and for developing economies where a significant share of population still relies on agriculture for subsistence. The present article provides a theoretical foundation for the analysis of optimal growth and for the formulation of management and policy prescriptions with respect to soil expansion and preservation nexus.

Appendix A. Appendix to Sections 2 and 3

A1. Optimization problem

The Hamilton–Jacobi–Bellman (HJB) equation may be written as

$$\rho V(S) = \max \left\{U(C) + V'(S)[(1-q)Y(S,L) - C] + \lambda\left[V(\tilde{S}) - V(S)\right]\right\}, \tag{A.1}$$

where $\tilde{S}$ is the aggregate stock after the occurrence of a shock: $\tilde{S} = S - \Delta(D, S)$. Time subscripts are omitted when there is no ambiguity. The first-order conditions consist of

$$C : U'(C) - V'(S) = 0, \tag{A.2}$$

$$q : -V'(S)Y + \lambda\delta\omega V'(\tilde{S})Y = 0, \tag{A.3}$$

$$S : \rho V'(S) = V''(S)[(1-q)Y - C] + V'(S)\xi L^{\beta}(1-q) + \lambda\left\{V'(\tilde{S})\left[1-\sigma-\delta(\eta-\omega q)\xi L^{\beta}\right] - V'(S)\right\}. \tag{A.4}$$

The optimality conditions are complemented by the transversality condition for S, the non-negativity constraints on C, S, and the requirement $q \in [0, 1)$. Calculation of the stochastic consumption growth rate follows similar steps as in, e.g., Sennewald and Wälde (2006).

A2. Proofs of propositions

Proof of Proposition 1. The result in (i) follows immediately from (A.2) and (12), so that

$$C^* = \psi S. \tag{A.5}$$

Statement (ii) follows from (A.3) and (12); by combining the two expressions we find that the optimal share of output devoted to protection services is given by:

$$q^* = \frac{\eta}{\omega} - \frac{1-\sigma-(\lambda\omega\delta)^{\frac{1}{\varepsilon}}}{\xi L^{\beta}\omega\delta}. \tag{A.6}$$

The non-negativity constraint on D requires that $q^* \leqslant \frac{\eta}{\omega}$ (see (4)). At the same time, q^* must be non-negative, so that both conditions lead to the inequality $0 \leqslant \frac{\eta}{\omega} - \frac{1-\sigma-(\lambda\omega\delta)^{\frac{1}{\varepsilon}}}{\xi L^{\beta}\omega\delta} \leqslant \frac{\eta}{\omega}$. After some rearrangements, we obtain

$$1-\sigma-\xi L^{\beta}\eta\delta \leqslant (\lambda\omega\delta)^{\frac{1}{\varepsilon}} \leqslant 1-\sigma, \tag{A.7}$$

which is the necessary restriction on the parameters of the model to ensure the existence of an interior solution.

To prove (iii), note that the stochastic time path of the soil stock can be found analytically by substituting the optimal controls (A.5) and (A.6) in (3) and solving the resulting stochastic differential equation

$$dS_t = [(1-q^*)\xi L^{\beta} - \psi]S_t dt - \left[\delta(\eta-\omega q^*)\xi L^{\beta} + \sigma\right]S_t dz_t.$$

The solution is given by

$$S_t = S_0 e^{[(1-q^*)\xi L^{\beta}-\psi]t + \ln\left[1-\sigma-\delta(\eta-\omega q^*)\xi L^{\beta}\right]z_t}.$$

We can verify that the term in the exponent involving the logarithm is well-defined since the argument of the logarithm is unambiguously positive and is equal to (using (A.6))

$$1-\sigma-\delta(\eta-\omega q^*)\xi L^{\beta} = (\lambda\omega\delta)^{\frac{1}{\varepsilon}} > 0.$$

Substituting the solution for q^* in $[(1-q^*)\xi L^\beta - \psi]$, we obtain the following stochastic path of the effective soil stock

$$S_t = S_0 e^{gt + \frac{1}{\varepsilon}\ln(\lambda\omega\delta)z_t}, \tag{A.8}$$

where the term q_t in the exponent is responsible for the discontinuous downward jump at the time of a shock. The jump is downward since $\ln(\lambda\omega\delta)$, which multiplies z_t, is negative. When $z_t = 0$, $S_t = S_0 e^{gt}$, i.e. the effective soil stock improves at the constant rate g, so that consumption and effective soil stock grow at the same rate as long as a shock has not arrived, in line with (A.5). Expenditure on protective measures, equal to a fraction q^* of output, evolves over time according to

$$E_t = q^*\xi S_t L^\beta = \frac{1}{\omega}\left[\eta\xi L^\beta\delta - (1-\sigma) + (\lambda\omega\delta)^{\frac{1}{\varepsilon}}\right] S_0 e^{gt + \frac{1}{\varepsilon}\ln(\lambda\omega\delta)z_t}$$

showing that it grows at the trend rate g while $q_t = 0$. □

Proof of Proposition 2. We shall make use of the following relationships:

$$\begin{aligned}
\frac{\partial\xi}{\partial A} &= \frac{\theta^{1-\alpha}}{1+\nu\theta} > 0,\\
\frac{\partial\xi}{\partial\theta} &= \frac{\xi(1-\alpha)}{\theta(1+\nu\theta)} > 0,\\
\frac{\partial\theta}{\partial r} &= \frac{\theta\sigma}{[\sigma + r\delta(\omega-\eta)]r} > 0,\\
\frac{\partial\theta}{\partial\delta} &= \frac{\theta\sigma}{[\sigma + r\delta(\omega-\eta)]\delta} > 0,\\
\frac{\partial\theta}{\partial\omega} &= \frac{\theta\sigma}{[\sigma + r\delta(\omega-\eta)](\omega-\eta)} > 0,\\
\frac{\partial\theta}{\partial\eta} &= -\frac{\theta\sigma}{[\sigma + r\delta(\omega-\eta)](\omega-\eta)} < 0,\\
\frac{\partial\theta}{\partial\sigma} &= -\frac{\theta}{\sigma + r\delta(\omega-\eta)} < 0.
\end{aligned}$$

By differentiating Eq. (13) we obtain:

$$\begin{aligned}
&\text{(i)}\quad \frac{\partial g}{\partial\lambda} = -\frac{1}{\varepsilon} < 0, \quad \frac{\partial g}{\partial\eta} = \frac{L^\beta}{\varepsilon}\left[\frac{\partial\xi}{\partial\theta}\frac{\partial\theta}{\partial\eta}\left(1-\frac{\eta}{\omega}\right) - \frac{\xi}{\omega}\right] < 0,\\
&\qquad \frac{\partial g}{\partial\sigma} = \frac{1}{\varepsilon}\left[\frac{\partial\xi}{\partial\theta}\frac{\partial\theta}{\partial\sigma}L^\beta\left(1-\frac{\eta}{\omega}\right) - \frac{1}{\omega\delta}\right] < 0,\\
&\text{(ii)}\quad \frac{\partial g}{\partial r} = \frac{L^\beta}{\varepsilon}\left(1-\frac{\eta}{\omega}\right)\frac{\partial\xi}{\partial\theta}\frac{\partial\theta}{\partial r} > 0, \quad \frac{\partial g}{\partial L} = \frac{1}{\varepsilon}\beta\xi L^\beta\left(1-\frac{\eta}{\omega}\right) > 0,\\
&\text{(iii)}\quad \frac{\partial g}{\partial\omega} = \frac{1}{\varepsilon\omega^2}\left(\xi L^\beta\eta - \frac{1-\sigma}{\delta}\right) \gtrless 0, \quad \frac{\partial g}{\partial\delta} = \frac{1}{\varepsilon}\left[\frac{\partial\xi}{\partial\theta}\frac{\partial\theta}{\partial\delta}L^\beta\left(1-\frac{\eta}{\omega}\right) - \frac{1-\sigma}{\omega\delta^2}\right] \gtrless 0.
\end{aligned}$$

□

Proof of Proposition 3. The results can be obtained from the following comparative statics, using Eq. (14)

$$\begin{aligned}
&\text{(i)}\quad \frac{\partial q^*}{\partial\lambda} = \frac{(\lambda\omega\delta)^{\frac{1}{\varepsilon}-1}}{\xi L^\beta\varepsilon} > 0, \quad \frac{\partial q^*}{\partial r} = \frac{1-\sigma-(\lambda\omega\delta)^{\frac{1}{\varepsilon}}}{\xi^2 L^\beta\omega\delta}\frac{\partial\xi}{\partial\theta}\frac{\partial\theta}{\partial r} > 0, \quad \frac{\partial q^*}{\partial L} = \beta\frac{1-\sigma-(\lambda\omega\delta)^{\frac{1}{\varepsilon}}}{\xi L^{\beta+1}\omega\delta} > 0,\\
&\qquad \frac{\partial q^*}{\partial\eta} = \frac{1}{\omega} + \frac{1-\sigma-(\lambda\omega\delta)^{\frac{1}{\varepsilon}}}{\xi^2 L^\beta\omega\delta}\frac{\partial\xi}{\partial\theta}\frac{\partial\theta}{\partial\eta} > 0, \quad \frac{\partial q^*}{\partial\sigma} = \frac{\xi + \left(1-\sigma-(\lambda\omega\delta)^{\frac{1}{\varepsilon}}\right)\frac{\partial\xi}{\partial\theta}\frac{\partial\theta}{\partial\sigma}}{\xi^2 L^\beta\omega\delta} > 0,\\
&\qquad \frac{\partial q^*}{\partial\delta} = \frac{\xi(\lambda\omega\delta)^{\frac{1}{\varepsilon}}\frac{1}{\varepsilon} + \left(1-\sigma-(\lambda\omega\delta)^{\frac{1}{\varepsilon}}\right)\left(\frac{\partial\xi}{\partial\theta}\frac{\partial\theta}{\partial\delta}\delta + \xi\right)}{\xi^2 L^\beta\omega\delta^2} > 0,\\
&\qquad \frac{\partial q^*}{\partial\varepsilon} = -\frac{(\lambda\omega\delta)^{\frac{1}{\varepsilon}}\ln(\lambda\omega\delta)}{\xi L^\beta\omega\delta\varepsilon^2} > 0,\\
&\text{(ii)}\quad \frac{\partial q^*}{\partial\omega} = -\frac{1}{\omega}\left\{\frac{\eta}{\omega} - \frac{\xi(\lambda\omega\delta)^{\frac{1}{\varepsilon}}\frac{1}{\varepsilon} + \left[1-\sigma-(\lambda\omega\delta)^{\frac{1}{\varepsilon}}\right]\left(\frac{\partial\xi}{\partial\theta}\frac{\partial\theta}{\partial\omega}\omega + \xi\right)}{\xi^2 L^\beta\omega\delta}\right\} \gtrless 0.
\end{aligned}$$

□

Proof of Corollary 1. Follows directly from

$$\text{(i)} \quad \frac{\partial^2 q^*}{\partial \lambda^2} = \left(\frac{1}{\varepsilon} - 1\right) \frac{\lambda^{\frac{1}{\varepsilon}-2}(\omega\delta)^{\frac{1}{\varepsilon}-1}}{\xi L^\beta \varepsilon} \gtrless 0 \Leftrightarrow \frac{1}{\varepsilon} \gtrless 1,$$

$$\text{(ii)} \quad \frac{\partial^2 q^*}{\partial \lambda \partial \omega} = \left(\frac{1}{\varepsilon} - 1\right) \frac{\omega^{\frac{1}{\varepsilon}-2}(\delta\lambda)^{\frac{1}{\varepsilon}-1}}{\xi L^\beta \varepsilon} \gtrless 0 \Leftrightarrow \frac{1}{\varepsilon} \gtrless 1,$$

$$\frac{\partial^2 q^*}{\partial \lambda \partial \delta} = \left(\frac{1}{\varepsilon} - 1\right) \frac{\delta^{\frac{1}{\varepsilon}-2}(\omega\lambda)^{\frac{1}{\varepsilon}-1}}{\xi L^\beta \varepsilon} \gtrless 0 \Leftrightarrow \frac{1}{\varepsilon} \gtrless 1.$$

□

Proof of Corollary 2. With $\sigma = 0$, the expression for the stocks ratio simplifies, so we have $\theta = (1-\alpha)/\alpha$ independent of other model parameters. The proof then follows directly from

$$\text{(i)} \quad \frac{\partial^2 q^*}{\partial \delta^2} = \frac{(\lambda\omega\delta)^{\frac{1}{\varepsilon}}(1-3\varepsilon) + 2\varepsilon^2\left[(\lambda\omega\delta)^{\frac{1}{\varepsilon}} - (1-\sigma)\right]}{\xi L^\beta \omega \delta^3} \gtrless 0,$$

$$\text{(ii)} \quad \frac{\partial^2 q^*}{\partial \delta \partial \omega} = \left(\frac{1-\varepsilon}{\varepsilon}\right)^2 \frac{(\lambda\omega\delta)^{\frac{1}{\varepsilon}}}{\xi L^\beta \omega^2 \delta^2} - \frac{1-\sigma}{\xi L^\beta \omega^2 \delta^2} \gtrless 0.$$

□

Proof of Proposition 4. Follows directly from:

$$\text{(i)} \quad \frac{\partial s}{\partial \lambda} = \frac{1}{\xi L^\beta \varepsilon}\left\{(\lambda\omega\delta)^{\frac{1}{\varepsilon}-1} - 1\right\} \gtrless 0 \Leftrightarrow \frac{1}{\varepsilon} \lessgtr 1,$$

$$\frac{\partial s}{\partial \eta} = -\frac{1-\varepsilon}{\varepsilon\omega} \gtrless 0 \Leftrightarrow \frac{1}{\varepsilon} \lessgtr 1,$$

$$\text{(ii)} \quad \frac{\partial s}{\partial \delta} = \frac{(1-\varepsilon)}{\xi L^\beta \varepsilon} \frac{(\lambda\omega\delta)^{\frac{1}{\varepsilon}}}{\omega\delta^2} \gtrless 0 \Leftrightarrow \frac{1}{\varepsilon} \gtrless 1,$$

$$\frac{\partial s}{\partial \omega} = \frac{(1-\varepsilon)}{\xi L^\beta \varepsilon} \frac{(\lambda\omega\delta)^{\frac{1}{\varepsilon}} + \xi L^\beta \eta\delta}{\omega^2\delta} \gtrless 0 \Leftrightarrow \frac{1}{\varepsilon} \gtrless 1.$$

□

A3. Endogenous polluting intensity

Let us assume that η is a decreasing function of q, i.e. the more we invest in protective measures, the less polluting our production process is. Suppose $\eta = \eta(q_t)$, with $\eta'(q_t) < 0$. Then we can write (4) as $D_t = [\eta(q_t) - \omega q_t]Y_t$. The optimality condition with respect to q is modified as follows:

$$-V'(S)Y - \lambda V'(\tilde{S})\delta Y(\eta'(q_t) - \omega) = 0,$$

which yields

$$1 - \tilde{\sigma} - \delta[\eta(q_t) - \omega q_t]\xi L^\beta = \left[\lambda\delta(\omega - \eta'(q_t))\right]^{1/\varepsilon}.$$

This equation can be solved for the optimal protection share q^*. For instance, assume the simplest linear relationship $\eta = \bar{\eta} - a q_t$, where a proxies efficiency of "greening" the current technology. Then we have

$$q^* = \frac{\bar{\eta}}{a+\omega} - \frac{1 - \tilde{\sigma} - [\lambda\delta(\omega + a)]^{1/\varepsilon}}{\delta\xi L^\beta(\omega + a)}.$$

Interestingly, q^* is non-monotonic in a, unless $\varepsilon = 1$ in which case $dq^*/da < 0$ unambiguously. In general, however, there is a threshold efficiency a^* such that if $a < a^*$, $dq^*/da > 0$, i.e. protective measures increase in a, and conversely, if $a > a^*$, $dq^*/da < 0$. Compared to our benchmark model with $a = 0$, the new q^* is larger (smaller) than the one derived in Eq. (14) if a is sufficiently low (high), i.e. below (above) a^*. In either case, the cleanliness of production process is higher (i.e. η is lower) than in our benchmark model, either because the optimal protection share is large (in the case of and in spite of somewhat lower a) or because the efficiency parameter a is large. Also note that zero "man-made" damage is optimal if and only if $q^* = \bar{\eta}/(\omega + a)$, which is not the case here

Appendix B. Population dynamics

Assume that output is independent of the population size: $Y_t = \xi S_t$, then we can write per capita output as $y_t = \xi S_t/L_t = \xi s_t$. Given the law of motion of the aggregate stock, S_t, the dynamics of s_t obeys (by the change of variable formula):

$$ds_t = [(1-q)y_t - c_t - ns_t]dt + (\tilde{s}_t - s_t)dz_t,$$

where $\tilde{s} = \tilde{S}/L$ and $c_t = C_t/L_t$ is per capita consumption. The objective of the planner is

$$\max_{c,q} \quad \mathbb{E}\left\{\int_0^\infty u(c_t)L_0 e^{(n-\rho)t}dt\right\}.$$

We may write the HJB equation in per capita terms as

$$(\rho - n)V(s) = \max\{u(c) + V_s[(1-q)y - c - ns] + \lambda[V(\tilde{s}) - V(s)]\}.$$

The first-order conditions are similar to those in the benchmark model:

$$c: \ u'(c) - V_s = 0,$$
$$q: \ -V_s y + \lambda\delta\omega\tilde{V}_s y = 0,$$
$$s: \ (\rho - n)V_s = V_{ss}[(1-q)y - c - ns] + V_s(1-q)\frac{\partial y}{\partial s} + \lambda\left(\tilde{V}_s\frac{\partial \tilde{s}}{\partial s} - V_s\right).$$

Using the same value function guess, as in the benchmark model, namely $V(s) = \frac{\psi s^{1-\varepsilon}}{1-\varepsilon}$, we obtain

$$c = \psi s, \quad \tilde{s}/s = (\lambda\delta\omega)^{1/\varepsilon} = \mu$$

Substituting the value function into the HJB equation and using the optimality conditions, we find a solution for ψ:

$$\psi = \frac{1}{\varepsilon}\left\{\rho - (1-\varepsilon)\left[\frac{1-\sigma-(\lambda\omega\delta)^{\frac{1}{\varepsilon}}}{\omega\delta} + \xi\left(1 - \frac{\eta}{\omega}\right)\right] + \lambda\left[1 - (\lambda\omega\delta)^{\frac{1-\varepsilon}{\varepsilon}}\right]\right\} - n.$$

Note that this expression is very similar to the one we obtained in the benchmark model, except for the last term which reflects the presence of the growing population. For $n > 0$, the optimal propensity to consume is unambiguously smaller than in the benchmark model.

The optimal protection measure, q^*, can be found by using the optimality condition with respect to q and the fact that

$$\tilde{s} = \frac{\tilde{S}}{L} = [1 - \sigma - \delta(\eta - \omega q)\xi]s.$$

We thus find that q^* preserves its expression (Eq. (14)) with β set to zero.

The growth rate of per-capita consumption in-between two consecutive shocks is found with the help of the optimality condition with respect to s and by computing the differential of V_s. It is therefore given by the already familiar expression:

$$g = \frac{1}{\varepsilon}\left\{\left(1 - \frac{\eta}{\omega}\right)\xi + \frac{1-\sigma}{\omega\delta} - (\rho + \lambda - n)\right\},$$

except that now the rate of time preference is adjusted not only by the shock hazard rate λ but also by the population growth rate n.

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