Conference Paper

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Publication Date:
2019-01

Permanent Link:
https://doi.org/10.3929/ethz-b-000323693

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Modeling and integrated control of macroscopic heterogeneous traffic flow in large scale urban network using coloured Petri net

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Submission Date: July 31, 2018

Word Count:

Text = 6161 words
1 Table @ 250 words = 250 words

Total = 6411 words
ABSTRACT

The evaluation of perimeter control using Macroscopic Fundamental Diagram (MFD) concept is strongly related with the quality of the corresponding dynamic models. For capturing the real characteristics of traffic dynamics, an enhanced accumulation-based traffic model is proposed in which transfer flow, boundary queues and travel delay are considered simultaneously using coloured Petri net. Taking the advantage of graphical structure, the gated links and junctions on the border of the protected network are modeled as so-called buffers. A simple case study demonstrates the implementation mechanism of this proposed model and its capability of evaluating network performance.

Moreover, a perimeter control framework integrated with route guidance is proposed for enhancing the ability of perimeter control on alleviating total travel delay out of the protected network. There are mainly three modules in this integrated framework. Firstly, perimeter control inputs are optimized by the method of model predictive control at regional level. Secondly, a set of internal flow controllers are adopted to homogenize traffic density among subregions and route guidance strategies are used by monitoring the number of queuing vehicles in buffers. Finally, the proposed traffic model is served as a plant for evaluating network performances objectively. The numerical results clearly demonstrate the effectiveness of our proposed integrated control framework in the case of uneven high demand.

Keywords: Traffic dynamics, MFD (Macroscopic Fundamental Diagram), Petri net, Perimeter control, Route guidance
INTRODUCTION

As urban traffic comprises a classical nonlinear complex system, both mathematical and physical models have been adopted to model it. Existing traffic models can be categorized into microscopic or macroscopic one. A microscopic model focuses on the complex movements of individual vehicles, which is considered a result of the characteristics of drivers and vehicles, interactions between drivers and vehicles, interactions between driver-vehicle elements and the road characteristics, external conditions and traffic regulations (Hoogendoorn et al., 2001). For managing congestion in large scale road network, we should model urban traffic at an aggregate level and shift the modeling emphasis from microscopic prediction to macroscopic monitoring and control (Daganzo, 2007). Both queuing theory and shockwave theory are adopted for traffic modeling on urban or freeway links, which are not suitable for network level analysis. A powerful empirical tool to reveal the relationship between space-mean flow and density (or vehicle accumulation) is the Macroscopic Fundamental Diagram (MFD). The existence of MFD has been verified using real data from different cities (Geroliminis et al., 2008; Ambühl et al., 2016; Beibei et al., 2016), and the factors that influence the shape of MFD have also been explored in the literature (Geroliminis, 2011; Leclercq et al., 2015; Xiong et al., 2016). Previous studies have demonstrated that this relationship (i.e. MFD) is not very sensitive to the detailed demand profile, and moreover, it can be used to predict traffic states and design management strategies.

As one of the most promising and rapid developing direction, traffic management strategies based on MFD concept such as perimeter control (Geroliminis et al., 2013), gating control (Daganzo, 2007; Keyvan-Ekbatani et al., 2015), congestion pricing (Zheng et al., 2016a; Zheng et al., 2016b), and route guidance (Yildirimoglu et al., 2015) have shed some insights on large-scale traffic management which benefit from the accumulation-based dynamic models. According to the main idea of perimeter control, a road network is always divided as a protected region (PR) and a side region (SR), and then these two regions can be further partitioned into some medium-size subregions exhibiting well-defined MFDs. A number of control strategies have been proposed for alleviating or postponing traffic congestion in PR by implementing traffic control on the border of the PN, such as feedback control (Keyvan-Ekbatani et al., 2012; Haddad et al., 2016), hierarchical MPC control (Aboudolas et al., 2013; Fu et al., 2017), etc. It is noted that the capacity constraint and availability of real infrastructures on the border is essential for potential practical application of perimeter control. Keyvan-Ekbatani et al. (2015) stated that the control benefits will be largely limited, if the gated links cannot provide sufficient space or signalized junctions to store the gated vehicles (queuing) with limited obstruction for traffic flows not bound for the PR. For alleviating the negative effect of the gated vehicles, Keyvan-Ekbatani et al. (2012) proposed a feedback control strategy at junctions located further upstream of the PR rather than the exact border. Moreover, a queue management strategy based on a continuous quadratic knapsack problem aiming at balancing relative queues at the gated links was studied by Keyvan-Ekbatani et al. (2016). Nevertheless, the further investigation on the capacity or constraint of infrastructures on the border of the PR is needed. The first work that addressed the importance of carefully managing the queues concentrated at the boundary of the PR due to restricting control is Haddad (2017). Note that the integration of perimeter control and route guidance is a promising direction for its potential practical implementation in reality.

Despite of this, there are still some challenges on the modeling of macroscopic traffic dynamics, which is the foundation for the design and simulation of traffic management strategies. The primary motivation of the paper is to develop an accumulation-based traffic model to consider transfer flow and time delay simultaneously and intuitionistically. To this end, Coloured Petri net (CPN) (Jensen, 1994) is adopted in our model for representing traffic state and dynamics of a small number of subregions by partitioning a heterogeneous road network, in order to take advantage of well-defined MFDs. In addition, the buffers between PR and SR are also modeled as the corresponding number of separated subregions. In our model, accumulated vehicles are represented by the number of tokens in each subregion, and their destinations (i.e. OD matrix) are labeled by the corresponding colours of tokens. Moreover, the directed connection (so called transition) between two adjacent subregions is taken as the possible route with time constraint. Therefore, transfer flow can be clearly stored in a CPN and time delay of an OD pair can be easily derived by accumulating travel times of transitions on a specific route. To the best of our knowledge, this is the first attempt to model transfer flow as well as time delay in an accumulation-based traffic model.

The second motivation is to establish a novel perimeter control framework integrated with route guidance for multi-region system. As the extension of study on queue balancing and management at the gating
positions (Ramezani et al., 2015; Keyvan-Ekbatani et al., 2016), the gated links and junctions upstream of the PR are firstly modeled as buffers with bounded storage capacity between regions, and then the route guidance strategy is designed on consideration of queuing length or waiting time in these buffers which is integrated with the two layers perimeter control (Fu et al., 2017). Moreover, this integrated framework can be entirely represented by the aforementioned CPN, which is also easily coded and utilized as a plant for traffic simulation.

7 Modeling of traffic flow using coloured Petri Net

A Petri net is used to map an event driven system into a set of places and transitions. The places usually represent the states of a dynamical system, and the tokens in a place can be removed or added by firing an enabled transition. In other words, the flow of tokens represents state changing. As a mathematical tool, it enables systems to be governed by a set of mathematical equations, hence providing an avenue for system analysis (Murata, 1989). Petri net can suitably describe urban traffic and general transportation systems which consists of continuous, discrete, deterministic, stochastic, distributed, and parallel states (Ng et al., 2013).

As is well known, the timing and division of traffic light phases is determined by traffic flows from different directions. With the increasing number of intersections or scale of road network, the traffic system tends to become extremely complicated to be modeled by a classic Petri net. Various high level Petri nets can be utilized to overcome the state explosion problem. For example, coloured Petri net was introduced in DiCesare et al. (1994), where specific colour was assigned to each vehicle entering the network. Dotoli and Fanti (2006) used coloured time Petri net to model signalized traffic system, in which the place and token represented link cell and vehicle, respectively, and the colour of a token was used to record the destination of the corresponding vehicle. According to the previous studies, we can conclude that the application of Petri net on modeling transportation system is a promising direction, and the introduction of high level Petri net is essential for representing traffic flow characteristics in details. For taking advantage of high level Petri net, the coloured Petri net is adopted to achieve a compact model structure.

Definition 1: A Coloured Petri net (CPN) is an eight-tuple \((\Sigma, P, T, Co, Pre, Post, H, M_0)\), where:

- \(\Sigma\) is a finite set of non-empty types, also called colour sets.
- \(P\) is a set of places with coloured tokens, \(T\) is a set of transitions, \(H\) is an inhibition function.
- \(Co\) is a colour function that assigns a finite and non-empty set of colours to each place and a finite and non-empty set of modes to each transition. The set of token colours of place \(p_i\) is denoted as

\[ Co(p_i) = \{a_{i,1}, a_{i,2}, \ldots, a_{i,u_i}\} \subseteq \Sigma, \quad \text{where} \quad u_i = |Co(p_i)| \quad \text{is the number of possible colours of tokens in place} \]

\(p_i\). The set of possible colour occurrences is denoted as \(Co(t_j) = \{b_{j,1}, b_{j,2}, \ldots, b_{j,u_j}\} \subseteq \Sigma\) with

\[ u_j = \{Co(t_j)\}. \]

- \(Pre: Co(P) \rightarrow N(Co(T))\) is defined as the set of directed arcs from places to transitions, and

\[ Pre(p_i, t_j)(h,k) \] is the weight of the arc from place \(p_i\), with respect to colour \(a_{i,h}\), to transition \(t_j\) with colour \(b_{j,k}\).

- \(Post: Co(T) \rightarrow N(Co(P))\) is defined as the set of directed arcs from transitions to places, and

\[ Post(p_i, t_j)(h,k) \] is the weight of the arc from transition \(t_j\), with colour \(b_{j,k}\) to places \(p_i\) with respect to colour \(a_{i,h}\).
\[ M_0 = [m_1, m_2, \ldots, m_p]^T \] is the initial marking. For each place \( p_i \in P \), the marking \( m_i \) of \( p_i \) is defined as a non-negative multiset over \( Co(p_i) \).

Compared with the ordinary Petri net, the CPN is extended by colouring both places and transitions. As a result, the generalization of Petri net is largely enhanced; the firing rule as well as the control law become more complicated.

**Definition 2:** A transition \( t_j \) in a CPN is enabled with colour \( b_{j,k} \) at a marking \( M \) if and only if:

\[
\forall p_i \in t_j, m_i(h) \geq Pre(p_i, t_j)(h,k) \\
\forall p_i \in t_j, \sum_{h=1}^{u_i} m_i(h) < H(p_j, t_j)
\]

The firing of an enabled transition \( p_i^* = t_j \in T \) with colour \( b_{j,k} \) results into a new marking \( M' \), its \( h \)-th component \( m'_i(h) \) can be calculated as following (Dotoli and Fanti, 2006).

\[
m'_i(h) = m_i(h) - Pre(p_i, t_j)(h,k) + Post(p_i, l_i)(h,k)
\]

Consider a common urban network with a single city center (e.g. the central business district region), traffic control strategies are widely adopted to prevent the city center from suffering severe congestion. The real efficiency of traditional signal timing for large scale network is largely limited for considering a small group of connected intersections. For integrating perimeter control with route guidance in an accumulation-based traffic model, the elements and distinguishing features of the traffic flow CPN (TFCPN) are to be presented.

**Definition 3:** A macroscopic traffic flow system can be represented by a CPN if the following conditions hold.

- The macroscopic relationship between traffic parameters (i.e. MFD) can be expressed by the CPN.
- The CPN is implementable if there is no deadlock in it (i.e. gridlock in the traffic network).
- The CPN is applicable for network performance evaluation by simulating different scenarios.

In order to model the large scale traffic flow system as a compact CPN, the heterogeneous urban road network is firstly divided into one protected region (i.e. the city center) and one peripheral region (out of the city center). For achieving well-defined (i.e. low-scatter) MFDs, each region is further partitioned into several homogeneous subregions with evenly distributed link densities. Fig. 1 illustrates a TFCPN model with two regions, eight subregions, and four buffers (stand for the gated links on the border of two regions). The explanation of eight places (i.e. \( PS = \{PS_1, PS_2, \ldots, PS_8\} \)) in the TFCPN, is listed in the second row of Table 1.

As the gating control is executed on the border of the protected region, we set four buffers \( PB = \{PB_1, PB_2, PB_3, PB_4\} \) for representing the gated links or junctions at the four entrances to the city center.

The importance of these buffers should be highlighted. As the capacity of gated infrastructure is modeled by setting the maximal number of vehicles in the buffers from the peripheral region to the protected region, the states (i.e. the number of tokens, the delay time of tokens) of these buffers can be also used as the indicators for activating traffic measures. For instance, if the average waiting time of vehicles in buffer \( PB_1 \) in Fig. 1 is longer than a certain value (e.g. 15 min), route guidance can be adopted to alleviate traffic congestion in \( PS_3 \).
According to Fig. 1, there are 28 transitions (i.e. $TS = \{TS_1, TS_2, ..., TS_{28}\}$) between adjacent subregions and eight transitions (i.e. $TB = \{TB_1, TB_2, ..., TB_8\}$) connected with four buffers. The traffic dynamics with respect to a given traffic demand (i.e. OD pairs) is determined by these transitions and their firing rules (i.e. the control law). The outflow and inflow of one subregion $p_i \in PB \cup PS$ is given as $Pre(p_i, t_j)$ and $Post(p_i, t_j)$ respectively, where $t_j \in TB \cup TS$.

The total number of tokens in each place $p_i$ represents the accumulation of vehicles in the subregion or buffer, and the tokens with different destinations will be coloured as $Co(p_i) = \{a_{i,1}, a_{i,2}, ..., a_{i,u_i}\}$, where $u_i = |PS|$ is the number of subregions (i.e. possible destinations). As the route choices or OD pairs are expressed by the complete combination of transitions between any two adjacent subregions in our model, the colour of a transition can be simplified.

Fig. 1 The structure of TFCPN model with two regions and eight subregions.
Table 1. Interpretation of related symbols of the TFCPN model in Fig. 1.

<table>
<thead>
<tr>
<th>Items</th>
<th>Meaning</th>
<th>Protected region</th>
<th>Peripheral region</th>
</tr>
</thead>
<tbody>
<tr>
<td>Places PS</td>
<td>Homogeneous subregions</td>
<td>PS₁, PS₂, PS₃, PS₄</td>
<td>PS₅, PS₆, PS₇, PS₈</td>
</tr>
<tr>
<td>Places PB</td>
<td>Buffers for waiting queues</td>
<td>--------</td>
<td>PB₁, PB₂, PB₃, PB₄</td>
</tr>
<tr>
<td>Inhibitors HS</td>
<td>Storage capacities of subregions</td>
<td>CS₁, CS₂, CS₃, CS₄</td>
<td>CS₁, CS₂, CS₃, CS₄</td>
</tr>
<tr>
<td>Inhibitors HB</td>
<td>Storage capacities of buffers</td>
<td>--------</td>
<td>CB₁, CB₂, CB₃, CB₄</td>
</tr>
<tr>
<td>Transition TS</td>
<td>Travel time between subregions</td>
<td>TS₁, TS₂, TS₃, TS₄</td>
<td>TS₅, TS₆, TS₇, TS₈</td>
</tr>
<tr>
<td>Transition TB</td>
<td>Travel time entering city center</td>
<td>TB₁, TB₂, TB₃, TB₄</td>
<td>TB₅, TB₆, TB₇, TB₈</td>
</tr>
</tbody>
</table>

The conditions of firing a transition $t_{si} \in TS$ are given by the formulations (1) and (2), while the transition $tb_j \in TB$ in our model is different with $ts_i \in TS$ or that of classical CPN model. In addition, the firing rules of transitions in the TFCPN should be redesigned considering the characteristics of traffic flow in reality. Denoting the marking of buffer places $PB$ as $m = [m(pb_1), m(pb_2), ..., m(pb_{PB})]^T$, we can now define the firing conditions of a transition connected with a buffer place.

**Definition 4:** A transition $tb_j \in TB$ with colour $b_j$ in a TFCPN is enabled at a marking $m$ if and only if:

\[ \forall \, pb_t \in *tb_j, m(pb_t) \geq Pre(pb_t, tb_j) \]  \hspace{2cm} (4)


\[ \forall \, pb_t \in *tb_j, \sum_{i=1}^{[PS]} m(pb_t)(h) < H(pb_t, tb_j) \]  \hspace{2cm} (5)

where $m(pb_t)(h)$ stands for the number of tokens with colour $h$ in place $pb_t \in PB$, or the $h$-th component of marking $m$ for place $pb_t$.

A new marking $m'$ is generated by firing an enabled transition $*pb_t = tb_j \in TB$ in TFCPN.

\[ m'(pb_t)(h) = m(pb_t)(h) + Post(pb_t, tb_j)(h) - Pre(pb_t, tb_h)(h) \]  \hspace{2cm} (6)

\[ Post(pb_t, tb_j)(h) = \min\{MFD(m(*tb_j)(h)), (CB_j - m(pb_t)(h)) + (CS_j - m(ps_j)(h))\} \]  \hspace{2cm} (7)

\[ Pre(pb_t, tb_j)(h) = \min\{Post(pb_t, tb_j)(h) + m(pb_t)(h), CS_j - m(ps_j)(h)\} \]  \hspace{2cm} (8)

where $tb_h = pb_t^*$. $Post(pb_t, tb_j)(h)$ stands for the weight of the arc from transition $tb_j$ with colour $b_j$ to place $pb_t$ with colour $a_h$ (i.e. the inflow of buffer $PB_j$ with destination $PS_h$ in Fig. 1); $Pre(pb_t, tb_h)(h)$ is the weight of the arc from place $pb_t$ with colour $a_h$ to transition $tb_h$ with colour $b_k$ (i.e. the outflow of buffer $PB_i$ with destination $PS_j$ in Fig. 1). In addition, $CB_i$ and $CS_j$ are the storage capacities of buffer place $pb_i$ and subregion place $ps_j$, respectively, as illustrated in Table 1. Note that $MFD(\cdot)$ is a set of empirical MFDs, which can be defined as functions for mapping the total number of vehicles (i.e. accumulations [veh]) of subregions into the corresponding throughput (i.e. outflow [veh/h]).

There are several powerful analysis tools for deadlock detection and control in Petri net theory (Murata, 1989), where the control logic of the traffic system (i.e. perimeter control, or gating control) can be expressed by a
discrete event system. In the case that the number of states of the traffic system is too large, the analysis method of reachability tree cannot be used for deadlock detection. Consequently, the further analysis of structure and dynamic properties should be proposed in the future.

Integrated perimeter control framework based on TFCPN

Based on the proposed TFCPN model, a novel perimeter control framework integrated with route guidance is developed. The advantages of the proposed framework are concluded as two aspects. On the one hand, both transfer flow (standing for the spatial evolution of traffic flow) and time delay (standing for the temporal evolution of traffic flow) are considered in the TFCPN model, which will lead to the realistic representation of macroscopic traffic flow and more objective evaluation of the following control strategies. On the other hand, route guidance for traffic flow in the peripheral subregions is integrated with the hierarchical perimeter control strategy, which benefits from the modeling of the gated links or junctions as a set of buffer places. Moreover, the implementation mechanism of the proposed integrated control framework and the specific algorithm of its simulation are presented at the end of this section.

General implementation algorithm of a TFCPN model

Algorithm 1: Simulation of a TFCPN model.

Inputs: Network topology parameters, OD matrix, capacity of subregions and buffers, control law of transitions.

Outputs: Accumulations and heterogeneities of regions and subregions, accumulations and waiting time in buffers, outflows of regions and subregions, etc.

1. Parameters initialization. Initialize OD matrix, departure functions, road network parameters, such as adjacency matrix, number of lanes, length of each link, storage capacities of buffers and subregions, etc.
2. Network partitioning. Divide the given road network into the protected region and the peripheral region according to the real situation of urban network. Partition the big regions into several homogeneous subregions considering the network topology, and then achieve the MFDs of all subregions using real traffic data.
3. TFCPN modeling. Define the places, tokens, transitions and the corresponding colours sets, and build up a TFCPN model to represent OD demands and traffic dynamics between different subregions.
4. Optimization of control strategy. Design and optimize perimeter control strategy at the first layer to meter traffic flow between two regions, and then calculate the inner control inputs between each adjacent-subregion pair in the protected or outside region. In addition, optimize the subregion-level route guidance strategies in the peripheral regions considering the queuing length or waiting time in buffers.
5. Simulation of TFCPN model. Network performances are calculated by running the TFCPN, such as dynamic accumulation-outflow relationships (i.e. MFDs), average vehicle accumulations, mean waiting time in each subregion or waiting buffer, average queues in the waiting buffers, etc. The efficiency of the optimization algorithms can also be evaluated by using these indexes, and the candidate control strategies can be improved by running the simulation iteratively.
6. Finalization of the algorithm. If the maximum number of iterations is reached, plant will end up with some outputs.
**Integrated control framework based on the proposed TFCPN model**

As TFCPN can be easily coded, it is convenient to combine a TFCPN model with optimization algorithms for solving the traffic control or route guidance problems. Fig. 2 illustrates a hierarchical control framework using a TFCPN model in Fig. 1 as a plant for optimizing perimeter controllers and route guidance strategies. The framework consists of three modules, such as two layer hierarchical control modules and one plant based on a TFCPN model. Specifically, the upper layer perimeter control aims to optimize control variable $U_{12}(t)$ and $U_{21}(t)$ for metering traffic flow between two regions by solving a single objective (i.e. total network delay) optimization problem. The method based on PSO (Particle Swarm Optimization) is adopted in this paper for its high efficiency on computation (Eberhart et al., 1995).

With respect to the lower layer, route guidance strategy is integrated with subregion flow (or boundary) controller by calculating and updating transfer flow on consideration of waiting queues in buffers, which may result in severe congestion at upstream links of peripheral subregions. There are three modules at the lower layer, namely subregion boundary controller (see step 1 in Algorithm 2), buffer route guidance (using buffer as indicators) (see step 2 and 3 in Algorithm 2), and buffer controller (equal to the sum of boundary controller and surplus capacity of buffer) as illustrated by step 4 in Algorithm 2.

According to Algorithm 2, traffic dynamics of a TFCPN are actually determined by both internal flow controller $u_{ij}$ and route split rate $r_{ij}^q$ by taking advantage of the utilization of buffer as performance indicator of peripheral subregion. As a result, alleviation of congestion upstream of the buffers is expected in the following simulation experiments. Note that traffic flow conservation equations (18) - (21) in Algorithm 3 are different from the previous ones (Ramezani et al, 2015; Fu et al., 2017), as the proposed novel equations take account of transfer flow and travel time simultaneously for capturing more realistic traffic dynamics.
Algorithm 2: Lower layer perimeter control and route guidance.

Inputs: Control variables $U_{12}$ and $U_{21}$, accumulations of subregions and buffers, MFDs of subregions.

Outputs: Control variable $u_{ij}$, route split rate $r_{ij}^q$, transfer flow $m_{ij}^h$, $m_{ij}^q$, and accumulation in buffer $n_b^h$.

1. Compute subregion flow controller $u_{ij}$ using optimization method in Fu et al. (2017) considering single objective. As the perimeter controllers between different regions (i.e. $u_{51}, u_{62}, u_{73}, u_{84}$) are realized by implementing buffer control in this paper, the control variables $u_{ij}$ ($i=5,6,7,8, j=1,2,3,4$) are equivalent with $u_{ih}$ ($h=1,2,3,4$).

2. Compute weighted travel time $\theta_{ij}^q(t)$ from subregion $i$ with final destination $j$ through the subregion sequence or route $q$.

$$\theta_{ij}^q(t) = \alpha \times \sum_{h \in q} TB_h(t) + (1-\alpha) \times \sum_{a \in q \neq h} TS_a(t)$$ (9)

where $TB_h$ and $TS_a$ are waiting time in buffer $h$ and total travel time in subregion $i$, respectively, and $\alpha$ is a constant with $\alpha \in [0,1]$, e.g. $\alpha = 0.65$.

3. Compute split rate $r_{ij}^q(t)$ for route guidance using logit model with consideration of both waiting time in buffer(s) and travel time in subregion(s) on the route $q$, and then determine route transfer flow $m_{ij}^q(t)$.

$$r_{ij}^q(t) = \frac{1}{1 + \sum_{\xi \in Q_j(t)} e^{\lambda(C_{ij}^q(t) - C_j^q(t))}}$$ (10)

where the generalized route cost $C_{ij}^q(t) = \theta_{ij}^q(t) + \xi_j(t)$, $\lambda$ is the scale factor; $\xi$ is an alternative route, and $Q_j(t)$ is the set of possible routes from subregion $i$ to $j$; $\theta_{ij}^q(t)$ denotes the cost of route $q$ at time $t$, and $\xi_j(t)$ is the perception error.

$$m_{ij}^q(t) = r_{ij}^q(t) \cdot n_{ij}(t) / n_i(t) \cdot g(n_i(t))$$ (11)

where $n_{ij}(t)$ is accumulation in subregion $i$ with destination to $j$, $n_i(t)$ is the total accumulation in subregion $i$, and $g(n_i(t))$ is trip completion flow of subregion $i$ at time $t$, which is determined by empirical MFD.

4. Estimate subregion transfer flow $m_{ij}^h$ for each buffer $h$ belonging to route $q$ using route transfer flow $m_{ij}^q(t)$ in Eq. (11), and then compute accumulated vehicles in buffer $h$ using both inflow and outflow of
buffer \( h \) at time \( t \).

**Case 1:** if \( G(n_{ih}(t)) \geq [u_{ih}(t)(m_{ih}^h(t) + \sum_{j=1, j\neq h}^8 m_{ij}^h(t))] + [cb^h - nb^h(t - 1)] \)

\[
inB^h(t) = [u_{ih}(t)(m_{ih}^h(t) + \sum_{j=1, j\neq h}^8 m_{ij}^h(t))] + [cb^h - nb^h(t - 1)]
\] (12)

\[
outB^h(t) = u_{ih}(t)(m_{ih}^h(t) + \sum_{j=1, j\neq h}^8 m_{ij}^h(t))
\] (13)

And then the accumulated vehicles in buffer \( h \) equal its capacity (i.e. the buffer is full).

\[
b^h(t) = nb^h(t - 1) + inB^h(t) - outB^h(t) = cb^h
\] (14)

**Case 2:** if \( G(n_{ih}(t)) < [u_{ih}(t)(m_{ih}^h(t) + \sum_{j=1, j\neq h}^8 m_{ij}^h(t))] + [cb^h - nb^h(t - 1)] \)

\[
inB^h(t) = G(n_{ih}(t))
\] (15)

\[
outB^h(t) = \min [u_{ih}(t)(m_{ih}^h(t) + \sum_{j=1, j\neq h}^8 m_{ij}^h(t)), G(n_{ih}(t)) + nb^h(t - 1)]
\] (16)

\[
b^h(t) = nb^h(t - 1) + inB^h(t) - outB^h(t)
\] (17)

---

**Algorithm 3:** Network performance evaluation on running TFCPN model.

**Inputs:** Traffic demands, network parameters, travel times, MFDs, control variables \( u_{ij} \).

**Outputs:** Accumulation of each subregion and region, other performance parameters.

1. Initialize simulation parameters, such as control step, prediction step, etc.
2. Calculate internal accumulation \( n_{ih}(t) \) in region \( i \) at time \( t \), and accumulation \( n_{ij}(t) \) in subregion \( i \) with destination to \( j \) at time \( t \) using the below equations.

\[
n_{ih}(t) = n_{ih}(t - 1) + d_{ih}(t - 1) + \sum_{\Delta t=1}^T \sum_{h'=1}^8 u_{ih} m_{ih}(t - \Delta t) - \sum_{\Delta t=1}^T m_{ih}(t - \Delta t)
\] (18)

\[
n_{ij}(t) = n_{ij}(t - 1) + d_{ij}(t - 1) + \sum_{\Delta t=1}^T \sum_{h'=1}^8 u_{ij} m_{ij}(t - \Delta t) - \sum_{\Delta t=1}^T \sum_{h'=1}^8 u_{ij} m_{ij}(t - \Delta t)
\] (19)

where \( d_{ij} \) is traffic demand, \( u_{hi} \) is flow control variable between subregion \( h \) and \( i \), \( m_{ij}^h \) is the transfer flow for accumulation in subregion \( i \) with final destination \( j \) through the next immediate subregion \( h \), \( m_{ij}^h \)
stands for accumulation in $i$ with destination $i$ (without going through another subregion), time delay $\Delta t$ is determined by travel time inside or between the specific subregions, and natural number $T$ is the quotient of travel time divided by control step $K_c$.

3. Calculate the vehicle accumulations between the protected and peripheral region as $N_{12}(t) = \sum_{i=1}^{4} \sum_{j=5}^{8} n_{ij}(t)$, $N_{21}(t) = \sum_{i=5}^{8} \sum_{j=1}^{4} n_{ij}(t)$, $N_{22}(t) = \sum_{i=5}^{8} \sum_{j=5}^{8} n_{ij}(t)$.

4. Compute the total accumulation in the protected and peripheral region, respectively.

\[
N_1(t) = N_{11}(t) + N_{12}(t) = \sum_{i=1}^{4} n_i(t) \tag{20}
\]

\[
N_2(t) = N_{21}(t) + N_{22}(t) = \sum_{i=5}^{8} n_i(t) \tag{21}
\]

where $n_i(t) = \min(\sum_{j=1}^{8} n_{ij}(t), n_{ij}^{j_{\text{jam}}})$.

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**Simulation experiments on the TFCPN-based integrated control framework**

**Experiment setting and MFDs estimation**

The studied road network in Fig. 3 is located on the border of Tianhe, Yuexiu, and Haizhu Districts. As the economy and business center of Guangzhou, this area attracts dense traffic demands on morning peak hours and releases the corresponding traffic flows on evening peak hours on working days. The alleviation of traffic congestion in the studied network is deemed necessary in order to maintain the operating efficiency of BRT on Zhongshan Road, and improve the service level in Guangzhou. For preventing traffic congestion in Central Business District (CBD), the network with 413 intersections is partitioned into two regions: the protected Tianhe CBD $R_1$ and the periphery area $R_2$. $R_1$ consists of subregions 1, 2, 3 and 4, and subregions 5, 6, 7 and 8 belong to region $R_2$. We firstly collect real data (traffic flow and link occupancy) from 62 inductive loop detectors (small circles in Fig. 3) equipped on arterial lanes, and then estimate MFDs of eight subregions.
Fig. 3 Road network of central Guangzhou

Fig. 4 illustrates the resulting MFDs using real data collected on Jan. 14, 2016, which are aggregated in five minutes based on the raw 30-s aggregated observations. It shows that the critical accumulation in subregion 5 is largest, and flow drop of subregion 2 interprets traffic congestion in Tianhe CBD. On the other hand, Fig. 4 (b) and (d) show that there is no obvious flow drop or congestion in subregion 4 and 7. The fitting functions of these MFDs are achieved for monitoring traffic states of the simulated network.

Fig. 4 Estimated MFDs for (a) subregion 2; (b) subregion 4; (c) subregion 5; (d) subregion 7 using real data.
For the following numerical experiments, we set initial accumulations for two regions as: $N_1(0) = 400$ [veh] and $N_2(0) = 970$ [veh]. The critical accumulations for regions and subregions are given as:

- $N_1^{Cri} = 9000$ [veh],
- $N_2^{Cri} = 9100$ [veh],
- $n_1^{Cri} = 3200$ [veh],
- $n_2^{Cri} = 2230$ [veh],
- $n_3^{Cri} = 1200$ [veh],
- $n_4^{Cri} = 2280$ [veh],
- $n_5^{Cri} = 3000$ [veh],
- $n_6^{Cri} = 1800$ [veh],
- $n_7^{Cri} = 1500$ [veh],
- $n_8^{Cri} = 3750$ [veh].

Other selected parameters are as follows: predict step $K_p = 20$, control step $K_c = 300$ [s], and control time $T_c = 27000$ [s].

Storage capacity of buffers is initially set as 600 [veh], which will be tested within certain range.

**Perimeter control and route guidance with high demand**

![Graph 1](image1)

![Graph 2](image2)

![Graph 3](image3)

**Fig. 5** (a) Given traffic demands; (a) control inputs of $U_{12}$ and $U_{21}$ for two regions; (b) heterogeneity indexes of eight subregions.

In this case study, both the inbound and outbound flows of Region 1 (Tianhe CBD) are controlled, while the internal flows in Region 1 and 2 are left as uncontrolled. As the result, the studied network can be
taken as a four-state-two-region system. For investigating the efficiency of the proposed control framework, relatively high demands (see D21 in Fig. 5(a)) to Tianhe CBD (Region 1) are generated to represent morning peak situation in reality. We also simulate a sharp demand drop around 48 steps (4 hours) after simulation. Dynamic control inputs in Fig. 5(b) shows that these control variables are highly nonlinear. However, whether this variance has negative impact on control stability or not is not included in this paper.

According to Ramezani et al. (2015), mean occupancy can be taken as an indicator of network congestion level, and standard deviation of link occupancies can be regarded as the heterogeneity index (i.e. \( het \)) of the network. The positive \( het \) suggests the accumulation of a subregion exceed the desired point, which means possible occurrence of traffic congestion in the corresponding subregion. On the contrary, the negative \( het \) allows more vehicles to enter in. In this paper, we simply use \( het_i = \frac{n_i(t)}{N_i(t)} - \frac{n_i^{jam}}{N_i^{jam}}, n_i \in N_i \) to represent the heterogeneity of subregion \( n_i \); and use variance \( HET_i = \text{var}(het_i) \) to replace heterogeneity index of regions \( N_i (I=1, 2) \). The heterogeneity indexes of subregions presented in Fig. 5(c), which shows the \( het \) of every subregion varies within a limited range (i.e. \([-0.2, 0.2]\)) before the first 70 steps. However, the \( hets \) of subregion 2 and 5 will exceed 0.6 within the last 10 steps. It’s reasonable that the \( het \) during off-loading time increases considering the above formulation of \( het \). Note that Fig. 5(c) also illustrates the dynamic evolution of traffic flow in each subregion. For example, vehicles in subregion 5 are relatively less than the so-called capacity ratio (to Region 1) before the first 30 steps, and then the accumulation of subregion 5 increases gradually which may leads to possible congestion.

**Comparison of different control strategies**

In this section, different control scenarios are implemented to investigate the effect of both perimeter flow control (PFC) and route guidance based on buffer (RG) using model prediction control (MPC) method.

Strategy 1: Constant perimeter control. The control variables U21 and U12 are given as 0.4 and 0.6, respectively.

Strategy 2: Dynamic perimeter control. A hierarchical control strategy is adopted to consider PFC between subregions from the same region.

Strategy 3: PFC + RG. Route guidance strategy is integrated with perimeter control (i.e. the proposed integrated control framework in Fig. 2) to reduce network travel time.

Several network performances are investigated in this section, which consist of AAV (accumulated arrived vehicles), ATT (accumulated travel time aggregated by vehicle), and NCT (network clearance time or simulation step).

The results in Fig. 6 show that the proposed control framework (Strategy 3) performs better than other strategies in the case of high demand. The network can be cleared within 93 simulation steps (or 7.75 hours), and there will be 7.8% improvement on ATT with comparison of strategy 2. The obvious reduction of ATT in Fig. 6(a) verifies the efficiency of strategy 3. The reduction of network clearance time in Fig. 6(b) is benefited from our proposed control framework considering route guidance. It can be conclude that the advantages of introducing buffers into our control framework consists of two aspects. The one is to strengthen perimeter control on the border of the protected CBD, and the other is to serve as performance indicator for activating route guidance in the peripheral region.
As $HET$ is defined as a heterogeneity index for a region (see the specific equation in section 5.2), we can evaluate traffic state by observing the dynamic $HET$ of two regions. Perimeter control without consideration of route guidance contribute to lower heterogeneity in the protected region 1 (see Fig. 7(a)). Meanwhile, it leads to larger $HET$ in the peripheral region 2 in Fig. 7(b). It’s clear that strategy 3 have negative impact on the heterogeneity of both regions in comparison with constant control (i.e. strategy 1), which is related with specific route guidance method in our simulation experiments.

**CONCLUSIONS**

A traffic flow model based on colored Petri net (so-called CPN) is proposed for capturing macroscopic characteristics of urban traffic system in this paper. Transfer flow and travel delay are simultaneously considered in dynamic equations of this model, which will enable us to incorporate spatial and temporal evolution of traffic states in MFD modeling and then to evaluate the real effect of management strategy objectively. To the best of our knowledge, it’s the first effort to model the gated links and junctions on the border of two regions as buffers in the proposed CPN. Moreover, a perimeter control framework integrated with route guidance is established to enhance the balancing ability on preventing the protected region from congestion as well as alleviating potential time delay out of the protected region. The results demonstrate that the proposed framework contributes to less network clearance time and shorter travel time in the case of high demand.

Note that the theoretical foundation of CPN should be further improved in the future. For instances, the structure soundness and deadlock detection analysis of CPN, modeling multimodal traffic system,
simulating more large-scale networks using real data, designing a generic simulation platform, etc. With respect to the possible application of our integrated control framework, a field test is also expected in cooperation with the local authorities.

Acknowledgements

This research was funded by the National Natural Science Foundation of China (No. 61573110); the authors from EPFL have been funded by the ERC (European Research Council) Starting Grant “METAFERW: Modelling and controlling traffic congestion and propagation in large-scale urban multi-modal networks” (Grant #338205).

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Author Contribution Statement

The authors confirm contribution to the paper as follows: study conception and design: Hui Fu, Saifei Chen; data collection: Kaiyu Chen; analysis and interpretation of results: Kaiyu Chen; Hui Fu; Nikolaos Geroliminis; Anastasios Kouvelas; draft manuscript preparation: Kaiyu Chen; Hui Fu. All authors reviewed the results and approved the final version of the manuscript.