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On the Day-to-Day Variability of Time-Space Prism Vertex Location

by

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Abstract
Longitudinal and cross-sectional variations in the location of prism vertices are evaluated using 42-day diary data collected in Germany. Stochastic frontier models are applied to determine the time coordinate of a prism vertex. The models are further used to evaluate observed and unobserved heterogeneity in prism vertex location, and to assess the relationship between the variability in prism vertex location and the variability in the starting time of the first trip in the prism. Statistical analyses indicate that a little less than one-half of the total variation in prism vertex location is systematic, of which about one-third is within-person variation. Notably, systematic variation by the day of the week is found insignificant for prism vertex location. As for departure time, systematic variation accounts for less than 17% of the total variation. The individual-specific error component is significant in the stochastic frontier model, indicating that there is significant unobserved heterogeneity in prism vertex location. It, however, accounts for only less than 1% of the total variation. The individual-specific error component in the departure time model, on the other hand, accounts for over 20% of the total variation. The origin vertices of workers’ morning prisms are located with a much smaller variance and are more systematically varying than the departure times of the first trips in the prisms. Large degrees of variability and unobserved heterogeneity are introduced when trips are made under prism constraints. Large degrees of flexibility appear to be associated with trip making, suggesting the presence of room for behavioral modification with respect to the departure times of morning trips.
Introduction
Variability in travel behavior is a subject area that has been discussed intensively, albeit by a relatively few researchers (e.g., Hanson and Huff, 1982; Pas, 1988; Kitamura, 1988a; Keuleers et al., 2001; Schlich and Axhausen, 2003; Schlich et al. 2003). A “typical” daily pattern is one of the key concepts that have emerged while addressing the variability in daily travel patterns. Hanson and Huff note: “An assumption that pervades theoretical and empirical work on urban travel behaviour is that individuals’ daily travel patterns are largely habitual and that these habitual patterns are remarkably stable in the short run,” and “[t]he notion that behaviour is habitual, routine, or stereotyped implies that people exhibit a high level of repetition in their choice of modes, destinations, and activities” (Hanson and Huff, 1988, p. 368).

Seeking such typical patterns, however, is not a trivial task. To begin with, data sets suitable for such an expedition are extremely rare and far apart. Moreover, locating typical patterns by itself does not reveal very much about the variability in daily travel. In fact, the finding that “over the five week observation period, each person exhibited more than one typical daily pattern” (Hanson & Huff, 1988, p. 369) suggests the need for a more elaborate analytical structure. Huff and Hanson propose “to differentiate among three very different sources of variation in an individual’s travel behaviour from one day to the next: systematic or predictable variation; ephemeral or non-recurring aspects of travel; and long term, structural change” (Huff & Hanson, 1990, p. 230).

For example, weekly grocery shopping, which may be pursued on a fixed day of the week, would generate “systematic or predictable variation.” Visiting an appliance store to acquire a large freezer would contribute to “ephemeral or non-recurring aspects of travel,” while possessing the large capacity freezer would probably induce “long term, structural change” in grocery shopping behavior. This differentiation, however, tells little about the magnitude and patterns of variability in daily travel behavior. Some reflections on travel behavior itself seem to be in order.

As the approaches by Chapin (1978) and Hägerstrand (1970) exemplify, an individual’s behavior can be characterized from two perspectives: that the individual’s behavior is driven by needs and desires, and that behavior is governed by a set of constraints. It is not difficult at all to see that needs and desires for certain activities vary from day to day. For example, the need for grocery shopping does not arise when there is an adequate level of food stock at hand. In fact theoretical models of shopping trip frequency and trip chaining have been developed as inventory control models where the cost of travel, purchase price, and inventory cost are balanced (e.g., Bacon, 1971; Narula et al., 1983; Thill, 1985, 1986). Likewise a typical individual would not have the desire to go to the movie theater everyday. Observed “waiting times,” or elapsed times, between two successive engagements in activities of a given type would reveal the characteristics of the accumulation of needs and desires over time. Duration models have been applied to investigate the distributive nature of waiting times (e.g., Kim and Park, 1997; Schönfelder and Axhausen, 2001; Bhat et al., 2004).

Variability in constraints can also be intuitively understood when social commitments constitute constraints. A commitment to be employed for paid-work typically implies that one must report at his workplace by a certain time each workday, and stay there to work for a certain duration of time. Quite often the commitment produces very stable, or repetitive, constraints and therefore repetitive behavioral patterns. Other types of social commitments may produce quite random constraints, e.g., meeting a client to show product samples or
making a business trip to a remote city. There are also random events that modify the constraints, e.g., one must take the subway to work because his car is in the garage for repair.

Such constraints as the worker having to report at work by a certain time of day, together with the speed with which one is capable to travel, constitute Hägerstrand’s time-space prism, which defines the domain in the time-space dimensions that the individual can occupy. The time-space prism thus acts as constraints that define the possible extent of the action space of an individual. Researchers in the area of activity-based analysis of travel behavior (e.g., Jones et al., 1983; Damm, 1983; Kitamura 1988b; Jones et al, 1990; Axhausen and Gärling, 1992; Ettema and Timmermans, 1997) have adopted this concept to examine the properties of travel behavior (Burns, 1977; Jacobson, 1979; Kitamura et al., 1981; Landau et al., 1981; Damm, 1982) or to simulate activity and travel in time and space (Lenntorp, 1978; Axhausen, 1989, 1990; Kitamura et al., 1997, 2000; Arentze et al., 2001). These studies in general demonstrate the importance and usefulness of the notion of prisms and the constraints in space and time that they represent. Yet, to the best knowledge of the authors, no results, either theoretical or empirical, have been accumulated about the day-to-day variability of prism constraints. Consequently it is entirely unknown how much of the variability in daily travel behavior is due to the variability in prism constraints. This is, presumably to a large extent, due to the fact that prism constraints are quite often unobservable.

The application of stochastic frontier models has been proposed to estimate the location of prism vertices (Kitamura et al., 2000; Pendyala et al., 2002; Yamamoto et al., 2004). This study builds on these earlier studies and attempts to evaluate both longitudinal and cross-sectional variations in the location of prism vertices using 42-day diary data collected in Germany. The objectives of the study are to quantify the longitudinal and cross-sectional variations in prism vertex location, to evaluate the relative magnitudes of observed and unobserved heterogeneity in vertex location, and to assess the relationship between the variability in prism vertex location and the variability in the starting time of the first trip in the prism.

Longitudinal, or day-to-day, variations in vertex location are decomposed into: systematic variations due to day-to-day changes in factors that influence vertex location, and purely random variations. Cross-sectional variations across individuals are decomposed into: systematic variations due to differences in contributing factors (observed heterogeneity) and random variations unaccounted for by observed factors (unobserved heterogeneity). The variability in the departure time of the first trip of the day is also examined using the same data set and compared with the variability of vertex location. Variations in departure time are decomposed in a similar manner, and the fraction that is attributable to the variation in prism vertex location is identified.

Although this study is motivated principally by the desire to gain knowledge about the variation characteristics of prism constraints, its results have certain practical implications. For example, by knowing the association between prism constraints and the attributes of the individual, it is possible to determine who tend to be subjected to tighter, or looser, constraints. One would anticipate that those individuals who are subjected to looser constraints have larger degrees of flexibility in trip making, and are therefore more amenable to policy measures that call for behavioral change. These individuals would be the primary target of the measures.

The rest of this paper is organized as follows. The analytical approaches, especially the
application of stochastic frontier models and the decomposition of variations, are discussed in
the next section. In the following sections, the data set used is briefly described, and the
results of model estimation are presented for both stochastic frontier models of prism vertex
location and linear regression models of the departure time of the first trip in the prism. The
variations in prism vertex location and departure time are then decomposed to systematic and
random variations. These are each further decomposed into within-person and
between-person variations. This is followed by a concluding section which offers a summary
of results and implications.

**Analytical Approach**

Consider a worker who must report at work by \( t_t \) in the morning. Suppose this worker is
located at his home before leaving for work. Then his/her behavior before work is constrained
by a prism whose beginning point, or “origin vertex,” is located at the home location, \( x_h \), and
ending point, or “terminal vertex,” at the work location, \( x_w \). The coordinates of the terminal
vertex in the time-space coordinates system is thus \((x_w, t_t)\). Turning to the origin vertex, its
time coordinate, say \( t_0 \), is unobserved and unknown. The stochastic frontier model can be
applied to estimate \( t_0 \).

Let \( t_o \) be the beginning time of the first trip and \( t_t \) be the ending time of the last trip in the
prism. Then, at the origin vertex, \( t_0 \leq t_o \), and at the terminal vertex, \( t_t \leq t_t \). From these
inequalities,

\[
t_o = t_o + u_o, \quad t_t = t_t - u_t
\]

where \( u_o \) and \( u_t \) are non-negative random variables. The stochastic frontier model can be
applied to estimate unobserved \( t_o \) or \( t_t \) based on the above relations.

Let

\[
Y_i = \beta X_i + \epsilon_i = \beta X_i + v_i + u_i
\]

where \( i \) denotes the individual, \( Y_i \) is the observed dependent variable (in this case an observed
trip beginning or ending time), \( \beta \) a vector of coefficients, \( X_i \) a vector of explanatory variables,
\( v_i \) and \( u_i \) random error terms, \(-\infty < v_i < \infty, u_i \geq 0 \), and \( \text{Cov}(u_i, v_i) = 0 \). Comparing Eqs. (1) and
(2) indicates that \( \beta X_i + v_i \) can be viewed as the time coordinate of the origin vertex of a
prism with the random element, \( v_i \). This formulation ensures that the observed trip starting
time, \( Y_i \), will not be before \( \beta X_i + v_i \) because \( u_i \) is non-negative. A model for the terminal
vertex can be formulated similarly as \( Y_i = \beta X_i + v_i - u_i \).

In the econometric literature on stochastic frontier models, a normal distribution is often
applied to \( v_i \), and a truncated (half) normal distribution to \( u_i \). For details on the distributional
forms and model estimation, see Aigner et al. (1977) and Waldman (1982).

As described in the next section, the data set used in this study comprises repeated
measurements of daily travel from each survey respondent. Let \( T \) be the total number of days
for which measurements are available from the respondent (for simplicity of exposition, it is
assumed that \( T \) repeated measurements are available from every respondent), and \( Y_i \) be the
starting time of the first trip in the prism on day \( i \). Let the individual-specific error component,
\( a_i \) be introduced into the model of Eq. (2) as
\[
Y_{it} = \beta' X_{it} + \varepsilon_{it} = \beta' X_{it} + \alpha_i + v_{it} + u_{it}
\tag{3}
\]
where \( u_{it} \geq 0 \) as before, and \( \alpha_i \sim N(0, \sigma^2_{\alpha}) \), \( v_{it} \sim N(0, \sigma^2_v) \), and \( u_{it} \) is from a half-normal distribution with parameter \( \sigma_u \) for \( \forall i, t \). The probability density function of \( u_{it} \) is given as
\[
g_{u_{it}}(x) = \frac{2}{\sqrt{2\pi\sigma_u}} \exp\left[-\frac{x^2}{2\sigma_u^2}\right], \quad x \geq 0.
\tag{4}
\]
The error components are all assumed to be uncorrelated, i.e., \( \text{Cov}(v_{it}, v_{i't'}) = \text{Cov}(u_{it}, u_{i't'}) = \text{Cov}(\alpha_i, u_{i't}) = \text{Cov}(\alpha_i, v_{i't}) = 0 \) for \( \forall i, i', \forall t, t' \), except the case where \( i = i' \) and \( t = t' \).

This model is adopted in this study with the intent of accounting for unobserved heterogeneity in the time coordinate of the prism vertex. It should be noted, however, that this error component may be interpreted to represent heterogeneity in the deviation between the prism vertex location and the trip starting time, rather than heterogeneity in vertex location. Unfortunately it is not possible to determine which is a more plausible interpretation based on statistical analysis of the data at hand.

Let the predicted time coordinate of the prism vertex on day \( t \) for individual \( i \) be \( \hat{V}_{it} = \hat{\beta}' X_{it} \), where \( \hat{\beta} \) is the estimated coefficient vector, and let \( \overline{V}_i = \frac{1}{T} \sum_{t=1}^{T} \hat{V}_{it} \), and \( \overline{V} = \frac{1}{N} \sum_{i=1}^{N} \overline{V}_i \). Then, the total systematic variation in vertex location can be decomposed into within-person variation and between-person variation as
\[
\sum_i \sum_t (\hat{V}_{it} - \overline{V}_i)^2 = \sum_i \sum_t (\hat{V}_{it} - \overline{V})^2 + \sum_i T (\overline{V}_i - \overline{V})^2. \tag{5}
\]
The total random variation, on the other hand, can be represented as
\[
NT(\sigma^2_{\alpha} + \sigma^2_v). \tag{6}
\]
The total variation in prism vertex location, which is the sum of the two, is not observable in this case, but can be estimated as
\[
\sum_i \sum_t (\hat{V}_{it} - \overline{V})^2 + NT(\sigma^2_{\alpha} + \sigma^2_v). \tag{7}
\]

**The Data and Models**

The Mobidrive data set is used in the statistical analysis of this study. The Mobidrive project, funded by the German Ministry of Research and Education, involved a six-week travel diary survey conducted in Karlsruhe and Halle, German cities of about 300,000 inhabitants, in the fall of 1999. A total of 317 persons over 6 years of age from 139 households participated in the survey. Altogether, the respondents reported 52,273 trips on 14,360 person-days. The sample used in the analysis of this study comprises 161 workers in the data set. For details of
the Mobidrive project and data, see Axhausen et al. (2002) and Zimmerman et al. (2001).

The analysis of this study focuses on the time coordinate of the origin vertex of a worker’s morning prism on a weekday (Monday through Friday), and the departure time of the first trip in the prism. Stochastic frontier models are developed for prism vertices, and linear regression models for departure times. Individual-specific error components are incorporated into both types of models.

The dependent variable of the models, the departure time of the first trip in the prism, is expressed in minutes, with midnight being 0; thus 6:00 AM is represented as 360 and 9:30 AM as 570. Repeated measurements of daily travel are available for an average of 28.0 weekdays from each respondent of the survey, and all available measurements are used in the estimation. An individual-specific random error component is introduced into stochastic frontier models as in Eq. (3).

The set of explanatory variables in the models (see Table 1) contains variables that may appear endogenous at first. The duration of the commute trip (commute time) depends on the mode used and the departure time of the trip. The dummy variable indicating whether the first trip in the prism is a commute trip (first trip is commute trip) of course depends on the individual’s travel decision. These two variables thus represent consequences of the individual’s travel decision and the travel decision is governed by prism constraints. They are endogenous in this sense.

These variables are included in the models as explanatory variables based on the view that a worker will either plan ahead for the first trip of the day, or adopt a well programmed routine, because the worker’s first trip on a weekday is closely tied to how the day is started, which is most presumably planned or programmed beforehand for a weekday. Attributes of the commute trip will undoubtedly be brought into the planning exercise when he decides the earliest possible time he is willing to leave and the actual departure time. Likewise, when an established routine is adopted, the trip attributes must have been taken into consideration when the routine was being formulated. For example, the earliest possible time a worker is willing to leave home will depend on whether he is commuting to work on the day. And how early he is willing to leave will depend on how long it takes to commute. These two variables can thus be considered as explanatory variables. At the same time, it is also the case that these variables reflect consequences of travel decisions which depend on prism constraints. Regarding these variables as exogenous is a rather strong assumption, and the authors are unable to provide either empirical evidence substantiating this assumption or previous arguments in the literature supporting it. This must be kept in mind when interpreting the empirical results of this study.

This leads to the next question of what exactly it is that the stochastic frontier models depict. It would be evident from Eqs. (1) and (2) that $\beta'X_i + v_i$ (or $\beta'X_{it} + \alpha_i + v_{it}$ in Eq. (3)) represents the earliest possible time of departure, and as such it constitutes a prism vertex. Yet, it is not clear whether this point represents a “constraint” that is fixed by external forces. It has been noted earlier (Kitamura et al., 2000) that observed travel behavior is governed by subjective beliefs and perceptions about the constraints supposedly at work. Then, there may be no prism vertex at work in the sense of Hägerstrand. In fact it is rather unconvincing to argue that the origin vertex of a morning prism is fixed completely by external forces as a constraint in the strict sense of the word. For example, one could drive out of home at 3:00 AM if one wishes and makes arrangements for his sleep beforehand. In fact one does so when
he has to take the first flight in the morning for a business trip, or when he is going out for fishing on a Saturday. Yet, one would view it infeasible to leave home at 3:00 AM on a normal workday. The view adopted in this study is that the origin vertex of the morning prism is a threshold that is set by the individual, rather than an externally established constraint. In other words, it is conjectured that the prism vertex constitutes a set of “soft” constraints. It is not entirely imposed by external forces, but is at least partially self-imposed by the individual while considering the plan for the day. From this view, the prism vertex location is partially a product of the individual’s planning effort.¹

Estimation Results

Table 1 summarizes estimation results of stochastic frontier models along with those of least-squares models of departure time (the models presented in this paper are all estimated using LIMDEP Version 7.0 or 8.0, by Econometric Software, Inc.). The first two models presented in the table are stochastic frontier models of the origin vertex of a worker’s morning prism. The coefficient estimates of \( \text{first trip is commute trip} \)² in the stochastic frontier models indicate that the vertex location is moved earlier by about 75 min when the first trip is a commute trip. The table also indicates that male workers tend to have vertices that are about 20 min earlier than those of their female counterparts; higher income workers tend to have later vertices; and those who live in central city or in Karlsruhe tend to have later vertices.

With a t-statistic of 2.92, parameter \( \sigma_a \), which represents the standard deviation of the individual-specific error component, \( \alpha_i \), is highly significant. The likelihood ratio statistic is also highly significant, offering evidence for the presence of unobserved heterogeneity in prism vertex location. The coefficient estimates of the explanatory variables are quite stable between the two models, except for those of family with child(ren) and living in CBD. It can also be noticed that the model with the error component has larger estimated t-statistics for most of the explanatory variables.

Although estimation results are not presented in this paper, models with four dummy variables representing the days of the week have been estimated. The coefficient estimates of the day-of-the-week dummy variables turned out to be insignificant, both individually and collectively as a group, for models both with and without the individual-specific error component (\( \chi^2 \) of 3.22 with 4 degrees of freedom for the model without the error component, and 0.86 for the model with the error component). As far as the time coordinate of the origin vertex of a worker’s weekday morning prism is concerned, there is no statistically significant systematic variation by day of the week.

As a result, the models of the origin vertex contain only two explanatory variables that may vary from day to day: commute time and first trip is commute trip. In other words, the models contain only these two explanatory variables to which systematic day-to-day variations in

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¹ The above discussions imply that the origin vertex of the morning prism and the beginning time of the first trip of the day be distinguished in the analysis. An alternative view would be to treat the beginning time of the first trip as the origin vertex of the morning prism, which is not adopted in this study based on the view discussed above. Likewise, it may be argued that the work starting time cannot be automatically assumed as the terminal vertex of a morning prism because the work starting time has nowadays become quite flexible for many workers and does not constrain workers’ behaviors as it once did. This is not addressed in this present study; reexamining roles played by work starting (or ending) time remains as a future task.

² The first trip of the day was a commute trip for approximately 80% of the time in the data.
vertex location may be attributed. This is primarily because there are only a limited number of variables in the data set that may be associated with day-to-day variability in travel. For example, a parent may have to chauffeur his child to a piano lesson on a certain day of the week, generating a systematic variation, but such information is not included in most travel survey data including the one used in this study. The reader should keep in mind that some of the variations reported as random variations in this study may in fact be systematic variations. As a future extension, it is desired that variables associated with day-to-day variability, such as those representing social commitments or intra-household task allocation, be sought and quantified.

Table 1 also shows estimation results for three versions of least-squares models of departure time, one without any error component, one with an individual-specific component, and one with individual- and time-specific components, formulated as

\[ Y_{it} = \theta' X_{it} + \epsilon_{it}, \]  
\[ Y_{it} = \theta' X_{it} + \zeta_i + v_{it}, \text{ and} \]  
\[ Y_{it} = \theta' X_{it} + \zeta_i + \sigma_i + \zeta_{it}. \]

where \( Y_{it} \) is the departure time by individual \( i \) on day \( t \) and \( X_{it} \) is a vector of explanatory variables as before, \( \theta \) is a vector of coefficients, \( \zeta_i \) is an individual-specific error component, \( \sigma_i \) is a time-specific error component, and \( \epsilon_{it} \), \( v_{it} \) and \( \zeta_{it} \) are purely random error terms. All error components are assumed to be normally distributed, mutually independent, and serially uncorrelated.

The least-squares models of departure time presented in Table 1 are formulated with the same set of explanatory variables as in the stochastic frontier models of prism vertex location. Comparing the coefficient estimates of the two sets of models indicates that the coefficients of commute time and first trip is commute trip are larger in their absolute values in the least-squares models of departure time. It appears the attributes of commute trips are more directly related to departure time than to vertex location.

The model with the time-specific error component does not show any noticeable improvement in the model’s fit, nor is its inclusion well supported theoretically. As was the case for prism vertex location, variations associated with the day of the week are statistically insignificant. Based on these results, no time-specific component is considered in the subsequent modeling effort.

Another set of models of departure time is developed without restricting the explanatory variables to the ones in the models of prism vertex. Results are presented in Table 2. Both models with and without an individual-specific component are estimated. The last two models include the expected location of the prism vertex (\( \hat{V} \)) obtained from the stochastic frontier models, to infer the effect of prism location on departure time. The new explanatory variables introduced here are: size of party, main mode = car/motorcycle, main mode = bicycle/walk, and number of activities per day. Excluded from the models are household size, household income and parent.

\[^3\] On the other hand, it is conceivable that commute time, which represents the travel time as perceived and reported by the respondent, is likely to be more stable than the actual commute time. If this is in fact the case, then commute time as an explanatory variable under-represents systematic day-to-day variation.
The most significant variable is *first trip is commute trip*, which has coefficient estimates of −110 to −120 in the models without the expected vertex location; workers leave home earlier by about 2 hours when the first thing they do is to commute. One might anticipate that a worker would leave home earlier if he pursues some non-work activity first. The estimation result here indicates that is not the case. The coefficient estimates of *number of activities per day* indicate that workers with more out-of-home visits to make tend to leave earlier, by approximately 10 min per visit. *Size of party*, which represents the number of co-travelers (including the worker himself) of the first trip, has significant positive coefficient estimates; a worker who is making the first trip of the day with others tend to leave home later. The coefficients of *main mode* = car/motorcycle and *main mode* = bicycle/walk indicate that workers using these modes tend to leave later.

Since the stochastic frontier models of prism vertex location and the least-squares models of departure time share several explanatory variables, inclusion of expected vertex location ($\hat{V}$) reduces the coefficient estimates of those common variables, most noticeably *first trip is commute trip*. Yet, these common variables have the same respective effects on departure time in the model with $\hat{V}$ as in the model without $\hat{V}$.

The coefficient of *expected vertex location* is estimated at 0.689 for the model without an error component, and at 0.650 for the model with one. Taking the estimate from the model with the error component, it may be inferred that, *ceteris paribus*, a worker’s departure time would on average be 39 minutes earlier when the vertex is moved earlier by 1 hour.

**Decomposing the Variation**

The variations in prism vertex location and departure time are decomposed into systematic variations and random variations, each of which is further decomposed into within-person variations and between-person variations. The results are summarized in Table 3. The sums of squares of the frontier model with the individual-specific error component are evaluated as shown in the table. Note that, since the term, $u_{it}$, of this model is for the deviation between the time coordinate of the prism vertex and the departure time of the first trip, its variance is not included as part of the error sum of squares for vertex location.

Quite noticeable in Table 3 is the difference in total variance between the two; the total variance of departure time is more than 7 times larger than that of prism vertex. The departure times of the first trips of workers on weekdays vary from day to day and from worker to worker much more substantially than do the origin vertices of their morning prisms.

Of the estimated total sum of squares of the frontier model, 47.9% is the regression sum of squares (SSR), and 52.1% is the error sum of squares (SSE). Of the regression sum of squares, approximately one-third (34.5%) is attributed to within-person variation and about two-thirds to between-person variation. Prism vertex location does vary systematically from day to day depending on the commute duration and on whether the first trip in the prism is a commute trip, which in turn depend on various factors that influence the worker’s daily itinerary. Of the error sum of squares, only 0.7% is attributable to the individual-specific error component. Although the estimation result has indicated the significance of $\sigma_u$, the variance of the error component is small relative to that of $\sigma_v$, which represents white noise.

Accounting for only 16.5% of the total sum of squares, the fraction of the regression sum of square is much smaller in the linear regression model of departure time. This sum is almost
evenly split between within-person variation and between-person variation. On the other hand, the individual-specific error component accounts for over one-quarter of the error sum of squares. Unlike the case with the stochastic frontier model of prism vertex location, unobserved heterogeneity is more dominant in the model of departure time.

The mean variance corresponding to each sum of squares is summarized in Table 4 in similar format. It can be seen essentially the same conclusions as have been discussed above can be drawn based on the mean variances in the table. Summarizing the findings of this section,

- The total sum of squares of departures time is more than 7 times larger than that of prism vertex location.
- The variation of prism vertex location is more systematic than that of departure time; 47.9% of total variation is systematic for prism vertex, while the corresponding value is only 16.5% for departure time.
- Of the systematic variation of departure time, 50.1% is within-person; for prism vertex, this is 34.5%. Prism vertex has a much larger fraction of systematic variation, but a smaller fraction of it is within-person.
- The individual-specific error component accounts for a larger fraction of total variation for departure time than for prism vertex (25.9% vs. 0.7%). Unobserved heterogeneity is more dominant with departure time than with prism vertex location.

**Conclusion**
Six-week travel diary records contained in the Mobidrive data set are used in this study to examine longitudinal and cross-sectional variations in the location of prism vertices, by applying stochastic frontier models to locate the unobserved time coordinate of a prism vertex. At the same time, linear regression models are developed for the departure time of the first trip in the prism. An individual-specific error component is introduced into both types of models to account for unobserved heterogeneity.

The statistical analyses of the Mobidrive data have revealed that almost one-half of the estimated total variation in prism vertex location is systematic, and further that over one-third of this is within-person. Of the random variation in vertex location, unobserved heterogeneity, although statistically significant, has been found to account for only a small fraction of total variation. Most of the random variation in vertex location is white noise. For the departure time of the first trip in the prism, on the other hand, systematic variation is much smaller, unobserved heterogeneity is much larger, while white noise accounts for over 60% of total variation.

The results imply that the origin vertices of workers’ morning prisms are located with a much smaller variance and are more systematically varying than the departure times of the first trips in the prisms. Large degrees of variability and unobserved heterogeneity are introduced when trips are made under prism constraints. This suggests large degrees of flexibility associated with trip making. It appears as if there is room for behavioral modification with respect to the departure times of morning trips.

By showing that the time coordinate of a prism vertex does vary from day to day as well as from person to person, and by comparatively analyzing the variations in vertex location and departure time, this study has shed new light on the variability in daily travel behavior. The empirical findings of this study add to the body of knowledge on multi-day travel behavior. At the same time, they will find practical applications in the simulation of individuals’ travel
behavior under prism constraints, or in the assessment of possibilities for behavioral modification in response to policy measures. The study, however, is subject to several limitations as discussed earlier in this paper, including the limited range of explanatory variables that account for within-person, or day-to-day, variations. Addressing these limitations remains as future tasks.

References


Table 1. Stochastic Frontier Models of Prism Vertex and Least-squares Models of Departure Time

<table>
<thead>
<tr>
<th></th>
<th>Stochastic Frontier Models of Prism Vertex</th>
<th>Least-squares Models of Departure Time of the First Trip in Prism</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Y_a = \beta X_a + v_a + u_a$</td>
<td>$Y_a = \theta X_a + \epsilon_a$</td>
</tr>
<tr>
<td></td>
<td>$Y_u = \beta X_u + \alpha + v_u + u_u$</td>
<td>$Y_u = \theta X_u + \zeta + v_u$</td>
</tr>
<tr>
<td></td>
<td>$Y_v = \theta X_v + \alpha + \zeta + u_u$</td>
<td>$Y_v = \theta X_v + \alpha + \zeta + u_u$</td>
</tr>
<tr>
<td>Coef. t</td>
<td>Coef. t</td>
<td>Coef. t</td>
</tr>
<tr>
<td>Constant</td>
<td>396 48.17</td>
<td>610 59.75</td>
</tr>
<tr>
<td>Commute time (min.)</td>
<td>-0.31 -2.58</td>
<td>-0.54 -4.60</td>
</tr>
<tr>
<td>First trip is commute trip [D]</td>
<td>-75.6 -17.28</td>
<td>-124.3 -20.05</td>
</tr>
<tr>
<td>Household size</td>
<td>-5.26 -1.68</td>
<td>-5.65 -1.60</td>
</tr>
<tr>
<td>Household income (in 1,000DM)</td>
<td>6.53 4.79</td>
<td>3.66 2.54</td>
</tr>
<tr>
<td>Male [D]</td>
<td>-22.0 -5.05</td>
<td>-22.6 -4.32</td>
</tr>
<tr>
<td>Parent [D]</td>
<td>29.8 4.53</td>
<td>22.9 3.00</td>
</tr>
<tr>
<td>Married [D]</td>
<td>-29.2 -5.52</td>
<td>-36.0 -5.87</td>
</tr>
<tr>
<td>Family with child(ren) [D]</td>
<td>-28.5 -4.65</td>
<td>-41.4 -6.24</td>
</tr>
<tr>
<td>Living in CBD [D]</td>
<td>19.3 3.21</td>
<td>31.6 3.65</td>
</tr>
<tr>
<td>Living in Karlsruhe [D]</td>
<td>34.3 7.87</td>
<td>18.7 3.55</td>
</tr>
<tr>
<td>$\sigma_\alpha$</td>
<td>3.45</td>
<td>136.2</td>
</tr>
<tr>
<td>$\sigma_\theta$</td>
<td>188</td>
<td>120.2</td>
</tr>
<tr>
<td>$\sigma_\epsilon$</td>
<td>50.5</td>
<td>70.3</td>
</tr>
<tr>
<td>$\sigma_\zeta$</td>
<td>192</td>
<td>120.0</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>40.1</td>
<td>70.4</td>
</tr>
<tr>
<td>$L(0)$</td>
<td>-20326</td>
<td>-20860</td>
</tr>
<tr>
<td>$L(C)$</td>
<td>-20087</td>
<td>-20602</td>
</tr>
<tr>
<td>$L(\theta)$</td>
<td>-19943</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.146</td>
<td>0.145</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.144</td>
<td>0.145</td>
</tr>
</tbody>
</table>

Number of sample individuals = 116, number of cases = 3,253

[D]: 0-1 dummy variable
The least-squares models of departure time are estimated with multi-stage procedures and the $R^2$’s or variance estimates are not comparable across the models.
### Table 2. Linear Regression Models of Departure Time

<table>
<thead>
<tr>
<th>Model</th>
<th>$Y_n = \theta \cdot X_n + \epsilon_n$</th>
<th>$Y_n = \theta \cdot X_n + \zeta_n + \nu_n$</th>
<th>$Y_n = \theta \cdot X_n + \hat{Y} + \epsilon_n$</th>
<th>$Y_n = \theta \cdot X_n + \nu_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef.</td>
<td>t</td>
<td>Coef.</td>
<td>t</td>
</tr>
<tr>
<td>Constant</td>
<td>581.</td>
<td>47.65</td>
<td>561.</td>
<td>28.03</td>
</tr>
<tr>
<td>Commute time (min.)</td>
<td>-0.69</td>
<td>-5.77</td>
<td>-0.43</td>
<td>-3.31</td>
</tr>
<tr>
<td>Size of party</td>
<td>46.0</td>
<td>8.72</td>
<td>38.1</td>
<td>7.36</td>
</tr>
<tr>
<td>Main mode = car/motorcycle</td>
<td>12.3</td>
<td>1.90</td>
<td>39.5</td>
<td>4.19</td>
</tr>
<tr>
<td>Main mode = bicycle/walk</td>
<td>-0.25</td>
<td>-0.03</td>
<td>25.4</td>
<td>2.56</td>
</tr>
<tr>
<td>Number of activities per day</td>
<td>-9.74</td>
<td>-7.92</td>
<td>-9.10</td>
<td>-7.31</td>
</tr>
<tr>
<td>Male [D]</td>
<td>-22.6</td>
<td>-4.29</td>
<td>-27.6</td>
<td>-1.95</td>
</tr>
<tr>
<td>Married [D]</td>
<td>-25.5</td>
<td>-4.95</td>
<td>-26.9</td>
<td>-1.91</td>
</tr>
<tr>
<td>Family with child(ren) [D]</td>
<td>-36.8</td>
<td>-5.73</td>
<td>-33.6</td>
<td>-1.89</td>
</tr>
<tr>
<td>Living in Karlsruhe [D]</td>
<td>25.7</td>
<td>5.22</td>
<td>24.4</td>
<td>1.80</td>
</tr>
<tr>
<td>Living in CBD [D]</td>
<td>33.4</td>
<td>4.02</td>
<td>38.1</td>
<td>1.72</td>
</tr>
<tr>
<td>First trip is commute trip [D]</td>
<td>-118.9</td>
<td>-18.73</td>
<td>-110.8</td>
<td>-16.08</td>
</tr>
<tr>
<td>Expected vertex location ($\hat{V}$)</td>
<td>0.689</td>
<td>4.36</td>
<td>0.650</td>
<td>1.51</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>133.3</td>
<td>117.7</td>
<td>133.0</td>
<td>117.7</td>
</tr>
<tr>
<td>$\sigma_n^2$</td>
<td>69.5</td>
<td>69.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L(0)</td>
<td>-20860</td>
<td></td>
<td>-20860</td>
<td></td>
</tr>
<tr>
<td>L($\hat{\theta}$)</td>
<td>-20533</td>
<td></td>
<td>-20523</td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.183</td>
<td>0.176</td>
<td>0.187</td>
<td>0.181</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.180</td>
<td>0.184</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Number of sample individuals = 116, number of cases = 3,253

[D]: 0-1 dummy variable

The models are estimated with multi-stage procedures and the $R^2$'s or variance estimates are not comparable across the models.
Table 3. Analysis of Variance in Prism Vertex Location and Departure Time

<table>
<thead>
<tr>
<th></th>
<th>Frontier Model of Vertex Location</th>
<th>Regression Model of Departure Time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SS</td>
<td>%</td>
</tr>
<tr>
<td>Within-person</td>
<td>$\sum \sum (\bar{V}_i - \bar{\bar{V}}_i)^2$</td>
<td>1670917</td>
</tr>
<tr>
<td>Between-person</td>
<td>$\sum T(\bar{V}_i - \bar{\bar{V}}_i)^2$</td>
<td>3174864</td>
</tr>
<tr>
<td>Subtotal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Between-Person</td>
<td>$NT \sigma^2_a$</td>
<td>38719</td>
</tr>
<tr>
<td></td>
<td>$NT \sigma^2_v$</td>
<td>5230857</td>
</tr>
<tr>
<td>Subtotal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The sums of squares are evaluated for the stochastic frontier model with the individual-specific error component in Table 1, and the linear regression model with the error component without expected vertex location in Table 2. The $R^2$ statistic shown in Table 2 does not agree with the square sums in this table because the latter are based on recalculated predicted values of departure times.

Table 4. Variance Components of Prism Vertex Location and Departure Time

<table>
<thead>
<tr>
<th></th>
<th>Frontier Model of Vertex Location</th>
<th>Regression Model of Departure Time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Variance</td>
<td>%</td>
</tr>
<tr>
<td>Within-person</td>
<td>532.6</td>
<td>35.3</td>
</tr>
<tr>
<td>Between-Person</td>
<td>976.0</td>
<td>64.7</td>
</tr>
<tr>
<td>Total</td>
<td>1508.6</td>
<td>100.0</td>
</tr>
<tr>
<td>Error</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error Component</td>
<td>11.9</td>
<td>0.7</td>
</tr>
<tr>
<td>White Noise</td>
<td>1608.0</td>
<td>99.3</td>
</tr>
<tr>
<td>Total</td>
<td>1619.9</td>
<td>100.0</td>
</tr>
<tr>
<td>Total</td>
<td>3128.5</td>
<td>100.0</td>
</tr>
</tbody>
</table>