A highway design concept based on probabilistic operational reliability

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A Highway Design Concept based on Probabilistic Operational Reliability

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ABSTRACT

This paper proposes a design load concept that treats capacity and traffic flow as random variables. It will be shown that the traffic flow is normally distributed and standard deviation changes with the volume to capacity ratio. A new definition of capacity is given and estimated. Given the new capacities the costs of a breakdown are assessed based on the value of travel time savings. It is shown how the results can be integrated into a cost-benefit analysis.
MOTIVATION

Highway design is commonly based on the idea that a particular percentile (e.g. 99.7%) of the annual distribution of hourly volumes defines the economically relevant load. The question whether a design providing for a fixed percentile of the hourly volumes of a year is economic or not remains a point of discussion.

This paper provides initial ideas of how one might be able to address this issue and obtain a new, consistent design concept for road infrastructures. While the paper will focus on motorways, it aims to be general and applicable to any type of road facility.

Central to any design concept is the conceptual separation of traffic load and facility capacity. Existing approaches often do not explicitly separate these two effects. Generally speaking, the capacity has been identified as the maximum expected traffic flow that can be achieved repeatedly (Transportation Research Board, 2000: HCM). In this context, the single value of capacity gives no information about the frequency or probability that the flow could reach the expected value given a sufficient demand.

A modern design concept has to take this into account and be includable into a cost-benefit framework to assess the alternatives to improve an infrastructure in a proper way. It is therefore desirable to use a method that links the estimated demand with the resulting generalised costs for a given infrastructure design.

ADOPTING THE IDEA OF A SCENARIO (LOAD CONFIGURATION)

It is known that the hours with the highest traffic volumes produce the largest contribution to the total generalised costs of a facility (Brilon and Zurlinden, 2003). The scenario concept proposed below is adapted from hydraulic engineering where the costs of a certain breakdown event (e.g. flooding due to high volumes) are estimated and valued (DVWK, 1989). By combining the period of repetition with the expected costs a breakdown (queuing) will
produce at a certain flow, one can define a marginal cost function which is needed for a cost-
benefit analysis (see Figure 1).

In this context, a scenario is an event that will result in increased generalised costs and is
relatively easy to identify from an engineering perspective. An example for a scenario of a
one hour duration would be a certain traffic volume during the peak hours of a common
weekday that is expected for about 200 times a year. It is assumed that this scenario has a
considerable share in the total generalised costs. Another scenario could be a lower traffic
volume that prevails 500 times a year for one hour, resulting in lower generalised costs for the
single event but having a higher frequency. A benefit of the scenario concept emerges from
the increasing accuracy of a cost-benefit analysis with the precision and level of
disaggregation of the defined scenarios. For application purposes not all possible scenarios
have to be considered for a cost-benefit analysis. The scenario concept focuses on hours with
high frequencies and high traffic volumes, for which the costs start growing non-linearly and
have substantial spatial spill-over effects. Taking this into account, boundaries are defined to
restrict the minimum and maximum traffic flows considered. Traffic flows below the lower
limit produce (nearly) no congestion costs and the upper limit excludes extremely rare events.
The boundaries have to be properly defined that the excluded scenarios have a negligible
influence on the total costs. Externalities and safety costs are primarily assumed to vary
directly with the volume; however, defined scenarios can also cover these effects.

Figure 1: General design of a cost function based on the occurrence function and the
breakdown function (numbers are illustrations), Source: adapted from (DVWK, 1989).

Generally speaking, the total annual costs of the facility must be known. That means, for each
event the resulting costs need to be provided. Since the hourly volume distribution is known,
the frequency of each demand level is known as well and can be described by an occurrence function that maps the number of occurrences per year to traffic flows. In Figure 1 the occurrence function (1st quadrant) is embedded into a nomogram of the cost function of an infrastructure element. In this graph a breakdown describes a major increase in travel time, so the function of the breakdown costs (2nd quadrant) returns the expected generalised marginal costs for a given flow. These costs are estimated with the willingness to pay for a reduction in travel time (see e.g. Axhausen et al., 2004) and for a reduction in the variation of travel time, having a share in the total generalised costs (Chen et al., 2003). Combining the occurrence function and the function of breakdown costs leads to the marginal cost function (4th quadrant). It serves as the basis for a cost-benefit analysis since it maps the probability of occurrence to the resulting costs of the scenarios considered. With the risk of a scenario being the product of the occurrence probability and the generalised costs of the event, the expected annual marginal costs are computed by integrating (or summing up in the discrete case) over risk of all regarded scenarios.

The next sections discuss the elements of the design concept. First, new results on breakdown probabilities are presented to support the idea of capacity as a random variable which is matched to the idea that long intervals of a flow can be thought to be a set of shorter intervals with the mean of the long intervals but a predictable standard deviation and distribution. These two concepts are combined with the idea of a reserve capacity, which is then used to estimate the capacity distribution. The reserve capacity is the basis of an initial cost estimate associated with a particular load situation (scenario). Finally, the costs of breakdowns are outlined to enable a cost-benefit analysis using the presented design concept.
CAPACITY AS BREAKDOWN PROBABILITY

Traditionally, it is assumed that a breakdown occurs when the flow regime changes from the upper branch (undersaturated flow) to the lower one (oversaturated flow) of the fundamental diagram (Transportation Research Board, 2000: HCM). Alternatively, one could define a breakdown as an event in which a flow is deteriorating by a defined speed reduction; e. g. 15 km/h after the event or below a threshold of $\frac{2}{3}$ of the free flow speed, as used here. The probabilities of such breakdowns grow with traffic flow as demonstrated by van Toorenburg (1986), Minderhoud et al. (1996), Okamura et al. (2000), Matt and Elefteriadou (2001), or Brilon and Zurlinden (2003). Capacity defined by capacity violations and speed reduction is therefore not a fixed value but is better described as a random variable with a certain set of moments (mean, variance, skew etc.). The probability of the event is associated with the traffic volume before its occurrence.

When estimating the probability distribution of breakdowns due to traffic volume from count data, it is important that the counting station is located at the bottleneck of an infrastructure element. Doing this, one avoids measuring effects due to upstream or downstream congestion. The count data for the following analysis was provided by the Swiss Federal Roads Authority (ASTRA); 13 sites on motorways (Autobahn) were chosen. For each site more than 180,000 5-minute intervals or approximately two years of measurements are available, which provide a good basis to capture all possible traffic scenarios.

The breakdown probability is calculated by defining capacity as the 60-minute traffic flow (average of twelve 5-minute flows) before a breakdown occurs (i. e. speed drop below 80 km/h in the following 5-minute interval). The probability is calculated by dividing the number of intervals marked as “before breakdown” by the total number of intervals in the same volume to capacity class.
It should be mentioned that the breakdown probabilities alone do not allow conclusions about the capacity of a given road type, as the traffic flow properties also affect the probability of a breakdown. The traffic flow during an interval is never constant but follows a distribution. Depending on the shape of the distribution, the breakdown probability will change in a way that for example traffic flow with a low mean but a high variance may result in the same breakdown probability as a flow with a higher mean but a small variance.

**WITHIN-CLASS TRAFFIC FLOW VARIANCE**

The traffic flow measured for a 60-minute interval aggregates 5-minute traffic flow intervals which will oscillate around the 60-minute average. To estimate this variance (standard deviation) the 60-minute average traffic flow $q_{60,A,t}$ was calculated for a counting site A at time t as follows:

$$q_{60,A,t} = \frac{1}{12} \sum_{i=-11}^{0} q_{5,A,t+i \cdot 5\text{min}}, \text{ if all } q_{5,A,t+i \cdot 5\text{min}} \text{ are defined and contain valid data.}$$

For each hourly traffic flow $q_{60,A,t}$ twelve 5-minute intervals $q_{5,A,t}$ were identified. To get comparable values for different road types with different number of lanes the flow to capacity ratios $r_{60,A,t}$ are calculated with the capacity $C_A$ (based on VSS, 2006: SN 640 018a) at all counting stations:

$$r_{60,A,t} = \frac{q_{60,A,t}}{C_A} \quad \text{and} \quad r_{5,A,t} = \frac{q_{5,A,t}}{C_A}.$$  

The volume to capacity ratios $r_{60}$ are assigned to $n$ (or more) groups $G$ defined by ratio intervals:

$$G_i = \left\{ r_{60,A,t} \mid r_{60,A,t} \geq \frac{i-1}{n} \land r_{60,A,t} < \frac{i}{n} \right\}. $$
As volume-to-capacity ratios higher than 1.0 can be measured due to traffic volumes higher than norm capacities, it is likely that more than \( n \) groups will be built. In addition, a few groups might be empty, especially those with very low ratios, as a result of a lack of observations.

Within a group \( G_i \) of \( J_i \) elements the mean of the ratios \( r_{60,Gi} \) are calculated:

\[
r_{60,G_i} = \frac{1}{J_i} \sum_{j=1}^{J_i} r_{60,A_j} \quad \text{with} \quad r_{60,A_j} \in G_i \quad \text{and} \quad J_i = |G_i|.
\]

Knowing the 5-minute ratios which build the 60-minute mean ratios, the standard deviation for each group can be estimated as follows:

\[
sd(r_{5,G_i}) = \sqrt{\frac{J_i \sum_{j=1}^{J_i} r_{5,A_j}^2 - \left( \sum_{j=1}^{J_i} r_{5,A_j} \right)^2}{J_i(J_i-1)}} \quad \text{with} \quad r_{5,A_j} \in G_i \quad \text{and} \quad J_i = |G_i|;
\]

where \( r_{5,G_i} = \frac{1}{J_i} \sum_{j=1}^{J_i} r_{5,A_j} \approx \frac{1}{J_i} \sum_{j=1}^{J_i} r_{60,A_j} = r_{60,G_i} \) is a good approximation, as for large \( J_i \)

\[
r_{5,G_i} = r_{60,G_i} \left( \lim_{J_i \to \infty} (r_{5,G_i}) = r_{60,G_i} \right).
\]

In Bernard and Axhausen (2005) it was demonstrated that the distribution of the 5-minute volume to capacity ratios can be described by a normal distribution within their corresponding 60-minute ratios. The standard deviation of this distribution varies predictably with the hourly volume to capacity ratio \( r_{60} = q_{60}/C \). Figure 2 shows a graph of this relationship. It can be seen that the standard deviation of the 5-minute volume to capacity ratios increases for higher values of a given hourly volume to capacity ratio up to ca. 30% (level of service A, LOS A, based on VSS, 2006). The LOS B with hourly ratios of 29 to 53% is characterised by a high volatility, which can be seen in the width of the 95%-interval of the means of all sites. In the following LOS C (53 to 76%) the standard deviation of the 5-minute ratios is slightly increasing, reaching a summit of 97‰ at the area of 76 to 90% of \( r_{60} \). Higher hourly volume
to capacity ratios (LOS E, 94 to 100%) lead to a steep decline of the standard deviation of the 5-minute ratios. This effect is due to increasing congestions and less freedom of the drivers to travel at their desired speed. The graph also gives a systematic alternative for the definition of boundaries of LOS.

Figure 2 Standard deviation of 5-minute volume to capacity ratios \( \text{sd}(r_5) = \text{sd}(q_5/C) \) vs. hourly flow to capacity ratio \( r_{60} = q_{60}/C \) with 95\% interval (grey) covering 95\% of the mean values of all sites. Values \( r_{60} \) larger than 1.0 are extrapolated.

\[
\begin{align*}
\text{sd}(r_5) &= \text{sd}(q_5/C) \\
r_{60} &= q_{60}/C \\
A \text{ to } F &= \text{quality of service by } (\text{VSS}, 2006: \text{SN 640 018a})
\end{align*}
\]

RESERVE CAPACITY OF A ROAD SECTION

In the following, the random variable of the capacity of an infrastructure element will be denoted as \( C \) with the probability density function \( f_C(x) \) and the traffic flow as the random variable \( Q \) with probability density function \( f_Q(x) \).

An infrastructure element fails to work properly (i.e. a breakdown occurs) if the traffic flow \( q \) exceeds the current capacity \( c \) (\( q \) and \( c \) denote realisations of the random variables \( Q \) and \( C \)).

With the probability density function of the random variable \( C \) the probability \( P_b \) of \( C \) being smaller than an actual \( q \) can be written as:

\[
P_b = P(C \leq q) = F_C(q) = \int_{-\infty}^{q} f_C(x) \, dx .
\]

If \( q \) itself is not known, but the distribution of \( Q \) is, then the probability that \( Q \) exceeds \( C \) becomes:

\[
P_b = P(C \leq Q) = P(C - Q \leq 0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_C(x) f_Q(x) \, dx \, dx = \int_{-\infty}^{\infty} F_C(x) f_Q(x) \, dx = \int f_{R_b}(x) \, dx .
\]
Here, the capacity $C$ and the traffic flow $Q$ are defined such that both variables are statistically independent. In structural reliability theory this case is called the fundamental case (Gulvanessian et al., 2002). The integral for two probability density functions $f_C$ and $f_Q$ of any shape cannot be solved in general but, assuming that $C$ and $Q$ are normally distributed, an analytical solution can be found.

If the reserve capacity (safety margin) is defined as:

$$M = C - Q$$

the breakdown probability $P_b$ becomes:

$$P_b = P(C - Q \leq 0) = P(M \leq 0).$$

If $C$ and $Q$ are normally distributed then $M$ is also normally distributed with the mean $\mu_M$ and standard deviation $\sigma_M$ as follows:

$$\mu_M = \mu_C - \mu_Q \quad \text{and} \quad \sigma_M = \sqrt{\sigma_C^2 + \sigma_Q^2} \quad \text{(if statistically independent)}.$$

With the cumulative probability density function of the normal distribution $\Phi=N(0, 1)$:

$$\Phi_X(x) = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{1}{2}x^2\right) dx$$

$P_b$ can be written as:

$$P_b = \Phi\left(0 - \frac{\mu_M}{\sigma_M}\right) = \Phi(-\beta)$$

with the reliability index (coefficient of variation) $\beta = \mu_M / \sigma_M$. 
ESTIMATION OF CAPACITY AS RANDOM VARIABLE

As shown above, one can estimate the breakdown probability on the basis of the distribution of the prevailing traffic flow together with the distribution of the capacity. The capacity’s distribution will be computed using a model of the traffic flow and the breakdown probability. The measured breakdown probabilities for a given mean traffic flow $\mu_Q$ on a given road using the 60-minute traffic volume before a breakdown of at least 5 minutes are denoted as $P_m(\mu_Q)$. The standard deviation of the mean hourly traffic volume $\sigma_Q(\mu_Q)$ is given by the graph shown in Figure 2 and the analytical breakdown probability $P_a(\Phi(-\beta)$, which is dependent on $\mu_m$ and $\sigma_m$, where the two parameters are dependent on $\mu_Q$, $\sigma_Q$, $\mu_C$ and $\sigma_C$. Since the traffic flow parameters ($\mu_Q, \sigma_Q$) are given only $\mu_C$ and $\sigma_C$ remain as unknown values. The analytical breakdown probability $P_a$ of the traffic volume $\mu_Q$ can be written as:

$$P_m(\mu_Q) \approx P_a(\mu_Q) = \Phi\left(\frac{\mu_C - \mu_Q}{\sqrt{\sigma_C^2 + \sigma_Q(\mu_Q)^2}}\right).$$

The index $Q$ describes a traffic volume class of the size of $333\frac{1}{3}$ vehicles per hour, i.e. the traffic volumes are rounded to match the classes with a mean of $0$, $333\frac{1}{3}$, $666\frac{2}{3}$, etcetera. The parameters $\mu_C$ and $\sigma_C$ are estimated by minimising the sum of squares of the residuals $r_Q = P_m(\mu_Q) - P_a(\mu_Q)$ using a nonlinear least squares method. As for example the frequency of high traffic volumes is relatively low in comparison to lower traffic volumes the accuracy of the measured breakdown probabilities is lower for traffic volumes that are infrequent. This effect is compensated by weighting the residuals by the square root of the number of observations ($N_Q$) in the class $Q$, considered:

$$r_Q = \sqrt{N_Q} \left( P_m(\mu_Q) - \Phi\left(\frac{\mu_C - \mu_Q}{\sqrt{\sigma_C^2 + \sigma_Q(\mu_Q)^2}}\right) \right) \text{ with } \sum (Q) r_Q^2 \rightarrow MIN.$$
It is important to note that the estimated capacity cannot be compared directly with the known values from e.g. (TRB, 2000: HCM), (FGSV, 2001), or (VSS, 2006: SN 640 018a), as these values are based on different concepts and therefore have a different meaning. The classical capacity is defined to be the maximum traffic volume that can be reached repeatedly whereas a traffic volume equal to the mean of the capacity seen as a random variable will result in a breakdown probability of 50%. Comparing that Bovy (2001) suggests a probability of congestion from 2 to 5% as an economic optimum, makes clear that in the design process the mean value of capacity should not be compared to the demand directly.

With the method described above the mean capacity $C_i$ and the standard deviation of the capacity $sd(C_i)$ was estimated for each site $i$. To compare the sites with different numbers of lanes and various grades the values of the estimated capacities seen as random variable are divided by the classical capacity $C_{i,VSS}$ based on the method of VSS (2006) in SN 640 018a:

\[
\frac{C_i}{C_{i,VSS}} \quad \text{and} \quad \frac{sd(C_i)}{C_{i,VSS}}.
\]

The values of the classical capacity in VSS (2006) are calculated using the same methods as in TRB (2000): HCM and (FGSV, 2001) and can therefore be directly compared among each other. The basic values of VSS (2006): SN 640 018a for grades smaller than 2% on motorways with two lanes per direction and free flow speeds of 120 or 100 km/h are listed as 4000 veh/h for a percentage of heavy vehicles of less than 5%, 3800 assuming a percentage of 5 - 15%, and 3600 veh/h for higher proportions of heavy vehicles. The capacity of motorways with three lanes per direction (120 km/h free flow speed) are found to be 5800 veh/h ($\leq$ 5% heavy vehicles), 5450 veh/h (5 – 15% heavy vehicles), and 5100 veh/h ($> 15\%$ heavy vehicles) with grades smaller than 2%. These values have to be considered as illustrations and can be used if no other measurements are available. The data used in this paper was analysed site by site to get the exact values of the classical capacity. Computing the
relative values for all sites and each class of heavy vehicle percentage one obtains the values shown in Table 1. It can be seen that the mean values do not differ much from the median, which supports the assumption that the values of capacity (expected value and standard deviation of capacity) can directly be calculated if the classical capacity is known – which is the case in most planning projects. Knowing the relationship between capacity as random variable and classical capacity, the expected breakdown probability $P_b$ can be expressed by the volume to capacity ratio with hourly estimates of the traffic volume $Q_{60}$ and the classical capacity $C_{VSS}$ as shown in Figure 3.

| Table 1 Estimated relative capacity: mean and median of expected value and standard deviation of all sites. $C_{VSS}$ denotes the classical capacity based on VSS (2006) for low percentages of heavy vehicles. |

| Figure 3 Breakdown probability $P_b$ vs. given hourly volume to classical capacity ratio ($Q_{60}/C_{VSS}$) for three classes of percentages of heavy vehicles (0-5%, 5-15%, 15-25%). |

**EXPECTED COSTS OF BREAKDOWNS**

As a reference value of travel speed not influenced by breakdowns the mean speed $v_m$ is introduced which represents the expected speed of a vehicle given a traffic volume (or volume to capacity ratio). The average free speed $v_m$ must be distinguished from the free speed $v_0$ which describes the maximum (allowed) speed of a road. To estimate the speed $v_m$ the intervals of each site were selected that are not affected by a breakdown. These intervals include all weather and traffic conditions (except breakdowns) to get a representative mean for an average day of the year. As the data contains counting stations on roads with speed limits of 120 km/h and of 100 km/h these sites are evaluated separately. The speed $v_m$ was computed by applying the *BPR-function* of Transportation Research Board (2000) in the form:
\[ v_m = \frac{v_0}{1 + \alpha r_{60}^\beta}, \]

where \( v_m \) denotes the measured average free speed for the given hourly volume to capacity ratio \( (r_{60}=Q_{60}/C_{VSS}) \). The parameters \( v_0, \alpha, \) and \( \beta \) are estimated by a nonlinear least squares method. With the estimated parameters the graphs in Figure 4 were plotted using the BPR-function. In these graphs the squares indicate the measured mean free speed of the given volume to capacity ratio of all sites of the same speed limit, the solid lines follow the BPR-function of the estimated parameters, the dotted lines mark the 95 %-confidence interval of the estimate, and the dashed lines show 95 % interval of the measured speeds of all sites.

**Figure 4** Free mean speed \( v_m \) not affected by breakdowns for motorways in Switzerland with speed limits of 120 km/h (black) and 100 km/h (grey) for percentages of heavy vehicles of 0-5% (a), 5-15% (b), and 15-25% (c).

It is assumed that the speed \( v_m \) has to be considered as the expected mean speed from the point of view of each driver, since traffic volumes and variables like weather and light conditions are taken into account when planning a trip however, random breakdowns that are not due to tailbacks of bottlenecks are supposed not to be scheduled. I. e. in the case of a breakdown the travel time will increase as the travel speed will drop during the time of the breakdown. Instead of assessing the reduced speed during a breakdown (denoted as \( v_b \)) directly the factor \( f_{mb} (v_b=f_{mb} \cdot v_m) \) being the quotient of the breakdown speed and the mean speed, which is not affected by breakdowns. The graph in Figure 5 was created using the measured breakdown speeds for given volume to capacity ratios divided by the measured average speeds \( (v_m) \) for the three classes of heavy vehicle moves (0 - 5 %, 5 - 15 %, 15 - 25 %). The 95 %-confidence interval for proportions of heavy vehicles between 0 and 5 % has an average width of 0.13, 0.19 for 5 - 15 % heavy vehicles, and 0.32 for 15 - 25 % heavy vehicles. As very few observations for volume to capacity ratios higher than 0.8 were
available in combinations with high percentages of heavy vehicles (class: 15 - 25 %) the values for ratios higher than 0.8 were extrapolated (marked as dashed line in graph).

**Figure 5** Factor $f_{mb} = \frac{v_b}{v_m}$, ratio of travel speed during breakdowns $v_b$ and mean speed $v_m$ vs. hourly volume to capacity ratio $r_{60}$ for three classes of percentages of heavy vehicles (0-5%, 5-15%, 15-25%).

When a breakdown occurs the actual travel speed, assumed to be the speed $v_m$, drops to the lower breakdown speed, given by the factor $f_{mb}$. As breakdowns are no instantaneous events but are of a temporal extent the duration of a breakdown is measured from the time when the speed drop is detected until the point where two consecutive 5-minute intervals with average speeds higher than the critical speed are following. With the graph shown in Figure 6 the expected value of the duration of a breakdown $t_b$ for a given volume to capacity ratio can directly be read off the graph.

**Figure 6** Mean duration of breakdowns $t_b$ for a given hourly volume to capacity ratio $r_{60}$ with 95%-confidence interval (dotted line) and 95%-interval of measured durations of all sites (dashed line).

The absolute travel time resulting from free flow time and congestion times is obviously dependent on the distance travelled. For that reason the expected travel speed will be described to include mean travel speed and the expected proportion of congestion. With $T_m$ being the total time (e. g. over a year) of flow being not affected by breakdowns for a given volume to capacity ratio and $T_b$ the sum of all durations of breakdowns for the same traffic volume the expected value of travel time can be written as:

$$E(v) = \frac{T_m}{T_m + T_b} v_m + \frac{T_b}{T_m + T_b} v_b,$$

where $v_m$ denotes the average speed which is not affected by breakdowns and $v_b$ the reduced average speed during a breakdown. If the times $T_m$ and $T_b$ are rewritten as $T_m=n_m\Delta t$ and
\( T_b = n_b \cdot t_b \) and if \( n_m \) is the number of the observed 5-minute intervals (\( \Delta t = 5 \text{ min} \)) with a free traffic flow and \( n_b \) the number of observed breakdowns with the duration \( t_b \), then the breakdown probability \( P_b \) can be written as \( P_b = n_b / n_m \) which results in the expression for the expected travel speed \( E(v) \):

\[
E(v) = \frac{1}{1 + P_b \frac{t_b}{\Delta t}} \cdot v_m + \frac{P_b}{1 + P_b \frac{t_b}{\Delta t}} \cdot v_b ,
\]

or with \( f_{mb} = \frac{v_b}{v_m} \):

\[
E(v) = \frac{1 + P_b \frac{t_b}{\Delta t} \cdot f_{mb}}{1 + P_b \frac{t_b}{\Delta t}} \cdot v_m .
\]

It should be remarked that the expression \( \Delta t \) is not a parameter of the function but a constant value (here 5 minutes) of the interval length of the observations from which the breakdown probability \( P_b \) was estimated. Other interval lengths will lead to different functions of the breakdown functions as shown in Matt and Elefteriadou (2001). The expected travel speed \( E(v) \) including speed reductions due to breakdowns is plotted in Figure 7.

**Figure 7** Expected travel speed vs. hourly volume to capacity ratio \((r_{60} = Q_{60} / C_{VSS})\) for given percentages of heavy vehicles (0-5%, 5-15%, 15-25%) and speed limits of 120 and 100 km/h on Swiss motorways.

Setting the travelled distance to \( s \) and assuming the expected travel time to be \( E(t) \) then \( s \) can be written as \( s = E(v) \cdot E(t) \), which is equal to the expression \( s = v_m \cdot t_m \), where \( t_m \) is the theoretical travel time at average free speed (see Figure 4). If \( \Delta t_b \) is defined to be the additional travel time caused by breakdowns with \( \Delta t_b = E(t) - t_m \) the expression \( \Delta t_b / s \) can be written as follows:
A straightforward concept to assess the benefit of a measure uses the willingness to pay for a reduction in travel time. A reduction in travel time is multiplied with the value of travel time savings (VTTS or willingness to pay in monetary units per time unit, e.g. €/h) for each user of the infrastructure element to compute the total benefit. Extending the simple approach leads to the calculation of users’ costs by taking into account that a higher willingness to pay exists to reduce late arrivals (e.g. due to congestions) than to reduce travel time as shown in Vickrey (1969) and Small (1982) or more recent Polak (1996) and König (2004). Adapting this concept to the travel time estimations leads to the formulation of the generalised costs of travel time (COSTt):

\[
\text{COST}_t = \text{VTTS}_m \cdot t_m \left(1 + \frac{P_b t_b}{\Delta t f_{mb} + \Delta t m_b + \Delta t b_m} \right) - 1 = \frac{1 - f_{mb}}{P_b t_b + f_{mb}} \cdot \frac{1}{v_m}.
\]

where \(\text{VTTS}_m\) denotes the value of travel time, \(t_m = \frac{s}{v_m}\) the free flow time, \(\text{VTTS}_b\) the willingness to pay for the reduction of late arrivals and \(\Delta t_b\) the expected additional travel time by reason of random breakdowns.

Using the previous function for all travellers on the observed section of the motorway gives the total generalised costs which should be compared to the building or modification costs of the section in a cost-benefit analysis.

**STRUCTURE OF NEW DESIGN CONCEPT**

Comparing the expected travel speeds of Figure 7 to the average free speeds in Figure 4 the reduction of the effective travel speed is substantial and should therefore be of interest when a new policy is introduced or an infrastructure element is build or modified. With this approach the tools are given to integrate travel speed and random delays into a cost-benefit-analysis.
Another advantage of this concept, in contrast to existing design concepts for motorways, is the detailed description of effects that influence travel speed. Classical design concepts mainly focus on modifications of road capacity or the influence of different traffic demands. The presented methodology offers variables to cover for example the effect of a reduction of the capacity’s or demand’s variation to reduce the breakdown probability having the still the same mean capacity and demand. Furthermore, an increase of average free speed, a reduction of the speed drop on breakdowns, or a reduction of the durations of breakdowns can be assessed and the profit in expected travel speed can directly be calculated.

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LITERATURE


### TABLES

<table>
<thead>
<tr>
<th>Percentage of heavy vehicles</th>
<th>mean E(C)/C_{VSS}</th>
<th>mean sd(C)/C_{VSS}</th>
<th>median E(C)/C_{VSS}</th>
<th>median sd(C)/C_{VSS}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-5%</td>
<td>1.4316</td>
<td>0.2173</td>
<td>1.38</td>
<td>0.22</td>
</tr>
<tr>
<td>5-15%</td>
<td>1.3746</td>
<td>0.2035</td>
<td>1.35</td>
<td>0.20</td>
</tr>
<tr>
<td>15-25%</td>
<td>1.2749</td>
<td>0.1615</td>
<td>1.24</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Table 1: Estimated relative capacity: mean and median of expected value and standard deviation of all sites. $C_{VSS}$ denotes the classical capacity based on VSS (2006) for low percentages of heavy vehicles.
FIGURE CAPTIONS

Figure 1 General design of a cost function based on the occurrence function and the breakdown function (numbers are illustrations), Source: adapted from (DVWK, 1989).

Figure 2 Standard deviation of 5-minute volume to capacity ratios \(sd(r_5)=sd(q_5/C)\) vs. hourly flow to capacity ratio \(r_{60}=q_{60}/C\) with 95%-interval (grey) covering 95% of the mean values of all sites. Values \(r_{60}\) larger than 1.0 are extrapolated.

\begin{align*}
sd(r_5) &= sd(q_5/C) & \text{standard deviation of 5-minute volume to capacity ratio} \\
r_{60} &= q_{60}/C & \text{normalised by capacity given by (VSS, 2006: SN 640 018a)} \\
A \text{ to } F &= \text{quality of service by (VSS, 2006: SN 640 018a)}
\end{align*}

Figure 3 Breakdown probability \(P_b\) vs. given hourly volume to classical capacity ratio \((Q_{60}/C_{VSS})\) for three classes of percentages of heavy vehicles (0-5%, 5-15%, 15-25%).

Figure 4 Free mean speed \(v_m\) not effected by breakdowns for motorways in Switzerland with speed limits of 120 km/h (black) and 100 km/h (grey) for percentages of heavy vehicles of 0-5% (a), 5-15% (b), and 15-25% (c).

Figure 5 Factor \(f_{mb} = v_b/v_m\), ratio of travel speed during breakdowns \(v_b\) and mean speed \(v_m\) vs. hourly volume to capacity ratio \(r_{60}\) for three classes of percentages of heavy vehicles (0-5%, 5-15%, 15-25%).

Figure 6 Mean duration of breakdowns \(t_b\) for a given hourly volume to capacity ratio \(r_{60}\) with 95%-confidence interval (dotted line) and 95%-interval of measured durations of all sites (dashed line).

Figure 7 Expected travel speed vs. hourly volume to capacity ratio \(r_{60}=Q_{60}/C_{VSS}\) for given percentages of heavy vehicles (0-5%, 5-15%, 15-25%) and speed limits of 120 and 100 km/h on Swiss motorways.
FIGURES

Figure 1

Figure 2
Figure 3
b) 5-15% heavy vehicles  

c) 15-25% heavy vehicles

Figure 4

Figure 5
Figure 6

Figure 7