Explaining road speeds with spatial lag and spatial error regression models

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Abstract

Spatial regression on structure and network variables is used as an alternative to time series assignment for predicting link speeds on a road network. GPS floating car measurements in the Canton Zurich are matched to a network model and explained by link type, time of day and spatial structure (population and employment density) using weighted least squares with and without spatial autocorrelation and spatial error terms. The weighted least squares corrects for heteroscedastic variance of the speeds by road type. Two different types of spatial neighborhoods were investigated for their suitability in correcting for the spatial correlation between links: one based on a nearest neighbor criterion using Euclidean distance, and a second based on network distance, defined by the number of intersections between links. The significant spatial correlation coefficients estimated using either type of neighborhoods indicate the presence of both correlated spatial error and autocorrelation of speeds. The best-fit models for the two types of neighborhoods have different coefficient estimates, and the neighborhood based on network distance provides higher log-likelihood and adjusted R-square. Speed predictions are made against a holdout sample for validation. The performance and sensitivity indicate that this is a promising approach for monitoring the road system.

Keywords

spatial lag, spatial autocorrelation, speed forecast, correlated speeds, linear regression, weighted least squares, road speeds, spatial structure variables, assignment

Preferred citation style

1 Short and long term monitoring of speeds

Many parties are interested in spatially detailed estimates of link speeds by time of day: the travellers relying on their navigation systems, the system manager optimizing the control of the system, and over the longer term, the system owners and planners, who monitor system performance, adjustment and expansion in the context of land use and regional development.

At this point, those interested have three possibilities: they can rely on spatially sparse detectors measuring speeds for individual links or lanes; they can employ static or dynamic assignment models to predict speeds on all links of the modelled network, and third, they can use floating car information. The first approach is too limited in its coverage to be useful for navigation or system-wide monitoring. The second is cumbersome, requires continuous updating of the model and has well-known problems with modelling the spatial and temporal dynamics of congestion. The problems are attenuated with dynamic assignment, which however is more difficult to implement. The third is very expensive, if continuous real-time data is required on all links, due to equipment and telecommunication costs. In addition, such a purely descriptive approach does not give any insight into the structural reasons for the speed patterns across time and space.

An alternative method presented here is to estimate link speeds directly using the underlying population structure variables and easily obtainable samples of floating car measurements on a sample of the road system. Traffic speed depends on the local speed limit, but also on spatially localized features of the road infrastructure and land use which directly (road curvature, stoplights, pavement quality) or indirectly (network density, motorway access points, schools, office parks or large shopping centers) influence speed via the driver behavior or traffic volumes. A model which combines these variables in its explanation of links speeds is attractive because it offers a structural explanation of the speed in a more direct way than assignment models, even if it is not able to capture all the details in competition for road space which are modelled by an assignment model. In a longer-term monitoring context, for example, it would be possible to correct for the effects of land use changes when evaluating observed speed changes.

The approach presented assumes the availability of high-resolution data on land use and a road network topology, both of which are easily obtainable today. It also assumes the presence of a corresponding dataset of link speed measurements. Such datasets are proliferating
due to private investment in dynamic navigation systems. If missing, such data can be collected quickly at reasonable cost, as reliable and affordable GPS-based measurement units and the required map matching software is readily available today (Marchal et al., 2006).

This paper presents the estimation and validation of spatial regression models of road speed on population structure and road network variables, with an assessment of predictive power. The discussion centers on the treatment of the spatial error structures through appropriate spatial regression models. A brief discussion of spatial analysis approach follows. The data is then described and the choice of the weighting approach explained. A weighted least square model is estimated first. A set of spatial autocorrelation models using different contiguity matrices are estimated so that several treatments for spatial autocorrelation can be compared in a depth not known in the rest of the literature. The best spatial models are described and compared to the WLS. The paper concludes with recommendations for further research and advice for the practical application of the approach.

The family of spatial regression models was popularized by Anselin (1988). Applications of spatial autoregressive models or spatial lag models are common in fields of geography where spillover effects are expected, like real estate and agricultural economics (for a review see Anselin, 2002 and LeSage and Pace, 2004).

The application of spatial econometric regression models to traffic flow or speeds is rare. Bolduc et al. (1992), for example, correct for the spatial autocorrelation of flows between origin and destination zones. However spatial analysis may prove to be an insightful addition to other previous regressions in the field where it has not been applied, for example travel behavior and density (Cervero and Kockelman, 1997), congestion and density (Dunphy and Fisher, 1996), road accidents and traffic flows (Dickerson et al., 2000), or pedestrian behavior in large cities (Desyllas et al., 2003).

2 Spatial analysis goal, method

A priori, one would expect structural indicators and traffic volumes to be spatially correlated. Spatial dependence is explicit for many activities that generate traffic because they are located (strategically or otherwise) according to competing (or complimentary) activities and in such a way as to optimize access to roadways. Furthermore, traffic on a section of the network is influenced by the traffic or signalization ahead, when the speed limit is not the limiting constraint. One would therefore expect that explaining link speeds with structural variables would
yield spatially correlated residuals. If not treated, this will result in biased and inconsistent parameter estimates that cannot reliably be used for inference (LeSage, 2000).

The ordinary or weighted least squares (OLS, WLS) model can be corrected for such spatial correlations by adding information about the neighborhood (see Anselin, 1988 or LeSage, 1998, whose terminology is used here). The spatial correlation is derived either from the regression residuals or the values of the independent variable in the contiguous (neighboring) observations in the dataset. A neighborhood weighting matrix \( W \) (n x n) is employed to introduce the information into the equations predicting each of the n locations. Each row sum of the \( W \)-matrix is normalized to one. In contrast to other applications, the definition of the neighborhood and the calculation of the weights is not obvious in the case of network quantities like road speeds, and will be discussed in detail.

The spatial lag, or spatial autoregressive model (SAR), is a linear regression of a dependent variable \( y \) on independent variables \( X \) that includes a term for the spatial dependence of the observations in \( X \). The procedure is analogous to detrending a correlated time series:

\[
y = \rho W y + \beta X + \epsilon
\]

with

\[
\epsilon \sim N(0, \sigma).
\]

The spatial error model (SEM) corrects for the spatial correlation of the error terms and is analogous to stationary correlated errors in time series data:

\[
y = \beta X + u
\]

with

\[
u = \lambda W u + \epsilon, \quad \epsilon \sim N(0, \sigma).
\]

The general spatial autoregressive model with a correlated error term (SAC) includes both the spatial lag term and the correlation of the error terms:

\[
y = \rho W y + \beta X + u
\]

with

\[
u = \lambda W u + \epsilon, \quad \epsilon \sim N(0, \sigma).
\]

The parameter \( \rho \) in the SAR and SAC models represents the additional influence of neighboring observed values on the dependent variable. In the SEM and SAC model, the parameter \( \lambda \) corrects for spatially correlated errors.
3 Dataset description

The regressions use a dataset of three weeks’ continuous daytime floating car measurements in the Canton Zurich, matched topologically to a network model (Marchal et al, 2006). The network model features directed links which are categorized by link type, according to infrastructure features and speed limit (Tiefbauamt Zurich, 2002). After matching, the link speed is the link length divided by the travel time over the link (exit time – entry time). The matching yields 52,000 speed observations on 3680 directed links. 50% of the links were measured at least eleven times, and the average number of measurements per link is thirteen.

The observations are averaged by time period for consistency with the Canton’s planning office. The four periods are weekday peak (6:30-8:30am and 4:30-6:30pm), weekday shoulder (8:30am-4:30pm and 6:30-8:30pm), weekday off-peak (8:30pm-6:00am), and Saturday. The observations used in the regression represent the average speed on a directed link during one time period, and n = 10,506 observations because not all links were measured in all time periods.

The road (link) types to which the floating car data is matched are defined by a local traffic planning consultant (Tiefbauamt Zurich, 2002): Motorways, Trunk Roads, Collector Roads, Distributor Roads, and Other Roads. The category Other Roads are represented by zone centroid connectors in the network models. This road type is problematic because it represents many kinds of road infrastructure from neighborhood roads with speed limit 50 km/h to cul-de-sacs and gravel paths in the forest. These stretches are located at the beginning and end of the floating car measurement legs and their signage and lighting conditions are varied. These conditions made it difficult for test drivers to navigate consistently and to drive normally on these roads. The data has not been cleaned of these effects, and large variances are the result. The data are cleaned of gross driver errors and personal stops (Marchal et al, 2006).

The dataset is partitioned into an estimation sample and a validation sample that is used to determine goodness-of-fit for forecasting and to quantify the predictive quality of the model. The choice of the validation sample is constrained by the requirement in spatial analysis that the neighborhood of links be contiguous. The two samples were chosen to represent the two urban centers in the region in order to include similar land uses, densities, and network characteristics. The estimation sample is the Zurich metropolitan region of 9297 observations. The validation sample is the Winterthur metropolitan region consisting of 1209 observations.
Discontinuities in GPS data are caused by tunnels and by urban and natural canyons.

### 3.1 Dependent variable: Speed data by time and link type

Higher speed links exhibit higher variance (Hackney et al., 2004), indicating a heteroscedastic dependent variable. Indeed the OLS residuals are also heteroscedastic and treatment by weighted least squares (WLS) is indicated. The procedure groups appropriate observations of the heteroscedastic variable and divides the OLS equation by the group-specific residual variances (Maddala, 2001). Here, the framework of the problem provides convenient groups based on road type. Heteroscedasticity is no longer detected after dividing the OLS equation by the residual variance according to road type. The WLS parameters have the same units as in the OLS and can be used in the same way to calculate link speed predictions.
3.2 Spatial explanatory variables

The analysis attempts to explain the spatial speed variation with variables detailing the spatial structure and the structure of road network that might indicate the heterogeneous intensity of traffic in space. All GIS work was performed using ArcGIS Spatial Analyst 8.3.

The variables available at hectare resolution are the population, employment opportunities, and employed persons. This data is produced by the Swiss Federal Office of Statistics with census data (Swiss Federal Office of Statistics, 2000). Employed persons and population are nearly perfectly correlated. Population, instead of the number of employed people, is included in the models because it is more likely to be a variable available to planners elsewhere. These densities were weighted with a kernel density function over radii $R$ of 1km, 3km, and 5km to capture the effect of increasing distances from a link (Figure 2). The kernel density estimators take the form:

$$
\lambda(s) = n^{-1}b^{-2}\sum\kappa\{(s - s_i) / b\}
$$

where $s_1, s_2, \ldots, s_n$ are the variable values in the n hectares within the region $R$, $b$ is the bandwidth of 70.72 meters and $\kappa(\cdot)$ is the Gaussian spatial probability density function. The hectare value closest to the endpoint of a link is used in the regression. This endpoint values were used because the average link length is 456m, spanning several hectares.

The length of road by type per hectare (road density) and the number of motorway access points (on/off ramps) per hectare were calculated with a high-resolution network model of the Canton that was matched to the hectare (Navteq, 2004). The road density, in units of meters per hectare, and the number of motorway access points per hectare, are indicators of the local routing alternatives and the number of intersections near a link which could influence speed on the link by way of flow volume, signalization, or flow continuity. They are also indicators of land use, but the correlation with the structural variables is sufficiently low as to not cause concern for the regression. The road densities are not kernel-weighted, corresponding to the assumption that their effect on speed is localized. The motorway access points are kernel weighted in the same manner as the structural variables.
4 WLS results

The OLS regression is estimated in SPSS with a stepwise estimation/validation procedure that adds and eliminates variables by seeking marginal improvements in the F statistic, retaining only coefficients significant at the 5% level. The method is robust against overfitting but is insensitive to correlated independent variables which would invalidate the standard errors of the estimates. Specific combinations of network and structure variables were chosen for the stepwise regression based on their qualitative meaning in explaining speeds, their correlation with speed, and a low correlation with each other. The logarithm of the structural variables fits the relationship better and correlates stronger with speeds. Finally, only the combinations of vari-
ables with the lowest Variance Inflation Factor (Maddala, 2001) were used, to minimize correlation of the variables with the regression residuals.

Often there is little difference in fit quality or parameter statistics across different combinations of structural variables. Among those with the best statistics, the model with the most plausible qualitative explanation was retained for the final form of the WLS.

The WLS is estimated, like the spatial models, using the econometrics library in Matlab (LeSage 2005). It uses dummy flags for road type and time of day to capture assumed independent effects on average speeds. The variables used and the estimated parameters are in Table 2. Variables were kept if they were significant at $\alpha = 5\%$ or if they served illustrative purposes for the effects of the spatial correlation treatments. The adjusted $R^2$ for 9297 observations and 34 variables is 0.4960.

The average speeds correspond to the relative hierarchy in the Canton’s road system, travel period, and the speed limits on the different road types. Speeds are highest on Saturdays and during morning and evening shoulder periods for all road types, while off-peak speeds on Collector Roads and Other Roads are also nearly as high. During peak periods, speeds on the Motorways and major Trunk Roads are strongly reduced, and slightly reduced on Other Roads.

The parameters of kernel density-smoothed spatial variables employment opportunities, population, and number of motorway access points are consistent with expectations: speeds go down with increasing activity densities. The radii of maximum effect are slightly different for the different road types. Motorway speeds are more strongly associated with job density at a wide radius of 5 km, and with motorway access density locally at a radius of 1 km (this is nearly the average distance between motorway on- and off-ramps). Speed on lower ranked roads is associated more with the local employment density (1km) and the population density in a 5 km radius.

The parameters for the road density have mixed signs. The presence of urban collector and urban distributor roads within the same hectare as a link are associated with lower speed, while trunk roads, motorways, and distributor roads are associated with increased speed. The effect of the latter two road types is 2-3 times higher in absolute value than the effects of the other road types. The interpretation is that the presence of higher-speed roads near a link, in absence of lower-speed roads, is an indicator of land use dedicated to traffic throughput to destinations not directly involved with the immediate hectare. Higher speeds result. The pres-
ence of lower-speed roads, as an indicator of land use requiring high accessibility to a local origin or destination, would be expected to be associated with lower speeds on the link.

5 Spatial and network-topological basis of the W matrices

The clear correlation of speed observations demonstrated by Bernard et al. (2006) supports the a priori discussion above that spatial correlations should be expected in the WLS residuals. This section focuses on finding the most relevant set of contiguous neighbors comparing two alternative approaches to defining distance and neighborhood. One can measure distance between a pair of links either along the shortest network path between them, or as Euclidean distance by the midpoints of the links. The first measure is spatially inhomogeneous and not symmetric, due to, for example, one-way streets or limited access roads. The explanatory hypothesis is that the flows along the path create the correlations. The second measure is symmetric. Here, the explanatory hypothesis is that the abutting land uses and their travel generate the correlations.

While it is quite useful to assume spatially symmetric error correlations for regressions of geographically fixed variables like land rents, there are good reasons to expect the residual correlations of a traffic speed regression to be stronger on networks than symmetrically distributed in space. First, spatially proximate road links might only connect with each other at a distant part of the network, so traffic loads on proximate links might not be related except by the type and intensity of local land use. This would weaken a spatial model’s ability to discern between spatial error and autocorrelation terms. One example is the oncoming traffic lane: Travel demand is strongly directional at peak periods, so opposite lanes may carry much different flows, in which case the correlation of the speed variances in opposing directions will not be strong. A second reason that the error correlation structure for traffic is not likely to be spatially symmetric is the temporal dependence of a traffic state: Events that occur upstream in the traffic flow happen in the future and cannot have relevance to events in the present. While upstream events may indeed be correlated to the speed on the link, it only makes causal sense to model correlation from links downstream in the flow.

The Euclidean (spatially symmetric) set of nearest neighbors is constructed for link $i$ by searching outward in all directions from the midpoint of $i$ for the midpoints of the $N$ nearest links, where their Euclidean distance is the measure of nearness. The method `nnw` in the Matlab spatial econometrics toolbox is used (LeSage, 2005).
The network neighborhood of link $i$ is the set of downstream links within a given network distance $D$, in this case defined as the number of road intersections (nodes with 3 or more edges, Balmer et al., 2005). The network is searched from $i$ in the direction of link flow, including all branches of links encountered, up to $D$ downstream intersections (Figure 3). The number of nearest neighbor links will vary according to how many links join at each intersection. The oncoming lane is only reachable by a U-turn and has a distance of at least one intersection.

The temporal dependence of the traffic state is accounted for by assuming that speeds and residuals are independent across the four time periods used. Thus, if links $i$ and $j$ are within distance $D$ on the network or within $N$ nearest neighbors in space, they are only considered neighbors if there is a speed observation for both $i$ and $j$ during the same time period. Finally, the matrix elements are equally weighted and standardized to a row sum of one.
6 Spatial analysis results

6.1 Procedure for choosing the type of spatial model

Spatial regressions are indicated if analysis shows that the least squares residuals are correlated across the contiguity matrix. Fit statistics (e.g. Moran’s I or Lagrange Multiplier Statistic for SAR models) are desirable indicators of residual spatial errors. But their calculation requires inversion of the n * n contiguity matrix. Four GB of computer memory were not sufficient to calculate fit statistics for this dataset. In this case, in order to identify spatially correlated residuals, it is necessary to estimate the full regressions and to compare the significance of the estimated correlation parameter and the log likelihoods. The regressions can be calculated using sparse W matrices which save computer memory (LeSage, 2005).

The SAR and SEM models explain the correlated spatial model variance differently. The SEM model assumes a common but unidentified spatial process which affects all of the variables associated by the W matrix. A significant parameter indicates missing spatial variables (Bivand 1998). Examples are areas where older architecture or topography forces roads to be narrower and more curvy, areas where fog is or ice was present (endemic to the study area in
November), or the specific composition and distribution of structural variables within a hectare that impact on travel speed differently, such as whether the employment opportunities are associated with a large shopping mall versus offices.

The SAR model should be investigated if a process can be assumed which would lead to autocorrelated dependent variables. In this case it is an attempt to explain directly the speed on a link as a function of the speed of downstream traffic or signalization, as effects spill over from one road segment to the next along the path of influence in the W matrix. The SAR model must still be tested for spatially correlated residuals and corrected if necessary (SAC). The determination of the best spatial model using both an autoregressive and a spatial error term is described in section 6.5.

### 6.2 Quality of fit of the spatial autocorrelation models

Fit and maximum likelihood estimation statistics of the WLS and of spatial regressions using the first 8 network and 16 Euclidean orders of contiguity matrices are shown in Table 1.

The speed ($v$) and WSL residual ($r$) correlation ($\rho_v$ and $\rho_r$) are calculated using neighboring pairs of the speeds or WLS residuals as follows, for all non-zero elements of the W matrix:

\[
\rho_v = \frac{\sum_{(i,j) \neq 0} (v_i - \overline{v})(v_j - \overline{v})}{M \sigma_i \sigma_j} \quad ; \quad M = \sum_{(i,j) \neq 0} 1, \quad \overline{v} = \frac{\sum_{(i,j) \neq 0} v_k}{M}, \quad \sigma_k = \frac{\sum_{(i,j) \neq 0} (v_k - \overline{v})^2}{M - 1} \quad \text{with } k=i,j.
\]

\[
\rho_r = \frac{\sum_{(i,j) \neq 0} (r_i - \overline{r})(r_j - \overline{r})}{M \sigma_i \sigma_j} \quad ; \quad M = \sum_{(i,j) \neq 0} 1, \quad \overline{r} = \frac{\sum_{(i,j) \neq 0} r_k}{M}, \quad \sigma_k = \frac{\sum_{(i,j) \neq 0} (r_k - \overline{r})^2}{M - 1} \quad \text{with } k=i,j.
\]
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<th>WLS residual correlation R</th>
<th>SAR Adj. R²</th>
<th>SAR Log Lik.</th>
<th>SEM Adj. R²</th>
<th>SEM Log Lik.</th>
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</table>
The adjusted $R^2$ as well as log likelihoods of all the spatial models are higher than for the WLS, indicating that the SEM and the SAR fit the data slightly better than WLS, but only in the third significant decimal. However the spatial coefficients, $\rho$ or $\lambda$, of all the spatial models are highly significant, meaning that the WLS results are biased and inconsistent due to the uncorrected spatial correlations.

Best models chosen for illustration purposes are based on the statistics in Table 1. Because the iterative solution to the spatial regression maximizes the log likelihood, the models with the highest log likelihood are chosen as best fits. Though the highest residual correlation occurs as expected between nearest neighbors (e.g. Tobler 1970), the best fits are usually achieved with more neighbors. Also, the network distance W matrices fit the data better than the Euclidean nearest neighborhoods.

6.3 Best fit spatial error model (SEM)

The coefficient $\lambda$ shifts explanatory power from structure variables to the neighborhood context of the link. The best-fit spatial error models results with either the five nearest Euclidean neighbors, or with on average 21 neighbors within a network distance of four intersections. This large number indicates that persistence in speed variations is stronger along the network paths than across space. $\lambda$ is 0.59 in the network neighborhood model and 0.33 in the Euclidean-distance based neighborhood model (Table 2), meaning that network neighbor correlations contribute nearly twice as much to speeds as spatial neighbor correlations.

As in the WLS results, the relative road hierarchy is clear in the values of the dummy parameters by time period. In the SEM model however, one would choose to exclude the density of Urban Distributor roads and Urban Collector roads from the regression on the basis of their t-statistics. The effect of employment opportunities on the speeds of Other Roads is also not significant in the SEM model. The dummies for motorway for all time periods are roughly 3-7 km/h higher than in the WLS. The trunk, distributor, and collector road types have lower constants, while the constants for Other Roads are on average the same as in the WLS. The parameters estimated for the effect of road densities on speeds, when significant, are smaller in the SEM than in the WLS. Speeds on motorways and distributor roads are reduced more strongly by high job density in the SEM than in the WLS, and the effect is much stronger in the models based on network neighborhoods. The effect of population density on the speed on Distributor Roads remains insignificant.
6.4 Best fit spatial autocorrelation model (SAR)

The SAR corrects for the spatial autocorrelation of the speeds. Though the autocorrelation parameters were significant for all contiguity matrices tested in Table 1, like the WLS, the residuals remain correlated (see 6.5). The parameter estimates may thus be incorrect and are not shown in Table 2 for this reason. The best fit is obtained by using the five nearest Euclidean neighbors or the on average six neighbors within a network distance of two intersections. The autoregressive parameters, $\rho$, are very similar whether the Euclidean or network W is used, though statistically distinct (0.27 and 0.29). Both SAR models result in qualitatively similar differences in the fitted parameters relative to the WLS, which are also reflected in the SAC results.

6.5 Best fit general spatial regression model (SAC)

There is reason to suspect significant unobserved spatial influences and therefore spatial correlation of errors even after correction for autocorrelated speeds. The general spatial regression requires the use of two contiguity matrices: one for spatial autoregression and one for correlated spatial errors. It is not certain that the best SAR model will result in the best SAC model with the addition of a spatial error correction term. Therefore, the log likelihoods were calculated for combinations of W matrices for between zero and fifty nearest Euclidean neighbors and for zero to eight intersections, resulting in between zero and 78 neighbors. This range of localized and diffuse effects is shown in Figure 4.
Figure 4  Contours of the log-likelihood surface of the SAC model for different W matrices: axes are the number of neighbors in the autoregressive versus spatial error matrix

The two approaches (SEM, SAR) compete to a certain extent in the explanation of the underlying processes. The highest values are found on ridges parallel to the main axes. The solutions on the diagonal where identical W matrices are used for the autocorrelation and spatial error correlation are numerical artifacts (LeSage 2000, Dubin 2004).
Using either network or Euclidean matrices improves the fit in the SAC model beyond the underlying SEM or SAR models. The best fit is obtained with either the 11 and 4 nearest Euclidean neighbors, or with nearest network neighbors within four and one intersections (21 and 2 neighbors, on average). The difference between the log likelihoods in the best models is small. The coefficients of both models are similar and highly significant.

Both formulations the SAC use more neighbors for autocorrelations and fewer for the spatial error correlation. The influence of autocorrelation is approximately double that of the spatial residual correlation. These models therefore emphasize the causality arising from linked traffic flows more than the explanations due to unobserved spatial influences.

The SAC parameter estimates are rather similar to the SAR estimates (Table 2). The link type and time dummies for WLS and SEM are very similar, but are much lower for SAC. The difference is made up by the contribution of the speed on neighboring links. All dummies are insignificant for Other Roads, indicating that speed on this road type is completely accounted for by the autocorrelations of speeds and the residuals. Both the magnitude and the significance of road densities tend to play a stronger role in the WLS than if autocorrelation or spatial correlation is corrected. The link-type-specific effects of population structure variables also tend to have higher magnitude in the WLS, though this is not a general rule.

While the SEM has the highest $R^2$, the SAC has higher log likelihood. Either model is a clear improvement on the WLS due to the significant estimate of the correlation parameters. Inferences made without accounting for spatial correlation would overemphasize the importance of structure variables and even ascribe significance to variables that have no explanatory power when uncorrelated from neighborhood effects.
Table 2  Estimated model parameters for the subset of link speeds in Zurich (N = 9297)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model:</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>WLS</td>
<td>SEM, Eucl.</td>
<td>SAC, Eucl.</td>
<td>SEM, Net.</td>
<td>SAC, Net.</td>
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<td>Highways * Saturday</td>
<td>149.86</td>
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<td>157.20</td>
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<tr>
<td>Highways * Peak Period</td>
<td>139.14</td>
<td>141.20</td>
<td>106.13</td>
<td>145.63</td>
<td>112.94</td>
<td></td>
</tr>
<tr>
<td>Highways * Shoulder</td>
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<td>113.39</td>
<td>153.92</td>
<td>119.76</td>
<td></td>
</tr>
<tr>
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<td>115.94</td>
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</tr>
<tr>
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<td>93.23</td>
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<tr>
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<td>104.85</td>
<td>78.60</td>
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</tr>
<tr>
<td>Collector roads * Shoulder</td>
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<td>107.31</td>
<td>80.05</td>
<td>103.57</td>
<td>77.96</td>
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<td>75.24</td>
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<td>71.41</td>
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<tr>
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<td>20.16***</td>
<td>69.17</td>
<td>29.59**</td>
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<tr>
<td>Other roads * Shoulder</td>
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<td>21.00***</td>
<td>68.39</td>
<td>27.26**</td>
<td></td>
</tr>
<tr>
<td>Other roads * Off Peak Period</td>
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<td>74.94</td>
<td>24.93***</td>
<td>78.00</td>
<td>29.59**</td>
<td></td>
</tr>
<tr>
<td>Highways * Highway access points, r=1km</td>
<td>-2.23</td>
<td>-1.63</td>
<td>-2.14</td>
<td>-1.62</td>
<td>-1.56</td>
<td></td>
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<tr>
<td>Highways * LN(Employment Oppor., r=5km)</td>
<td>-9.88</td>
<td>-11.11</td>
<td>-7.20</td>
<td>-13.60</td>
<td>-10.16</td>
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<tr>
<td>Trunk roads * LN(Employment Oppor., r=1km)</td>
<td>-7.14</td>
<td>-6.21</td>
<td>-5.47</td>
<td>-6.76</td>
<td>-5.46</td>
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</tr>
<tr>
<td>Trunk roads * LN(Population, r=5km)</td>
<td>-4.83</td>
<td>-6.03</td>
<td>-4.55</td>
<td>-4.50</td>
<td>-4.51</td>
<td></td>
</tr>
<tr>
<td>Collector roads * LN(Employment Oppor., r=1km)</td>
<td>-2.55</td>
<td>-2.49</td>
<td>-1.96*</td>
<td>-3.64</td>
<td>-3.18</td>
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<td>Collector roads * LN(Population, r=3km)</td>
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<td>-6.35</td>
<td>-5.45</td>
<td>-5.01</td>
<td>-4.40</td>
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<td>Distributor roads * LN(Employment Oppor., r=1km)</td>
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<td>-4.58</td>
<td>-2.96</td>
<td>-6.35</td>
<td>-3.51</td>
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<tr>
<td>Distributor roads * LN(Population, r=5km)</td>
<td>-7.26</td>
<td>-6.85</td>
<td>-4.69</td>
<td>-3.31***</td>
<td>-4.87</td>
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<td>-5.55*</td>
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<td>-5.26**</td>
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<td>Density highways [m/m²]</td>
<td>417.03</td>
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<td>218.68</td>
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<td>Density trunk roads [m/m²]</td>
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<td>72.39</td>
<td>45.17*</td>
<td>85.28</td>
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<td>Density distributor roads [m/m²]</td>
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<td>180.06</td>
<td>155.11</td>
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<td>Density urban collector roads [m/m²]</td>
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<td>-26.12***</td>
<td>-8.45***</td>
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<td>Density urban distributor roads [m/m²]</td>
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<td>-17.98***</td>
<td>-12.09***</td>
<td>-26.28**</td>
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<tr>
<td>ρ</td>
<td>-</td>
<td>-</td>
<td>0.261</td>
<td>0.302</td>
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<tr>
<td>λ</td>
<td>-</td>
<td>-</td>
<td>0.326</td>
<td>0.182</td>
<td>0.091</td>
<td>0.146</td>
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<tr>
<td>adjusted R²</td>
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<td>0.535</td>
<td>0.540</td>
<td>0.537</td>
<td>0.545</td>
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<tr>
<td>Log-Likelihood (x 10⁴)</td>
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<td>-3.2401</td>
<td>-2.6996</td>
<td>-3.2397</td>
<td>-2.6960</td>
<td></td>
</tr>
</tbody>
</table>

Probability of rejecting $H_0 = \begin{align*} & \ast 5\% \leq p < 10\%; \quad \ast\ast 10\% \leq p < 15\%; \quad \ast\ast\ast p \geq 15\%; \quad \text{others: } p < 5\% \end{align*}$
7 Validation

The model’s suitability for prediction uses tests of reasonableness in comparing its output with data not used for calibration or estimation (FHWA 1997). The model results are validated by predicting speeds for the roads in Winterthur using the Zurich parameter estimates for the network-based neighborhood models from Table 2 and comparing them to the withheld measurements.

The aggregate statistics for the predictions from the best WLS, SEM and SAC in Table 3 show above all high consistency with one another. In transferring the Zurich model to the Winterthur area, all three model formulations tend to predict speeds for the holdout sample that are too high ($\hat{\Delta v} = \text{mean residual}$), though none of the differences are statistically significant. The variance of the predicted speeds is also higher for the Winterthur sample. The SEM formulation has the lowest mean residual, while the SAC model seems to reproduce the extreme values the best. This is supported by a speed histogram (not shown). The standard error of prediction is the proper gauge of the model’s ability to predict speed on a given link (NIST, 2006). $SEP = \sqrt{\sigma^2 + SDR^2}$.

Table 3 Characterization of model results based on the best network neighborhood contiguity matrices (all units km/h)

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Model</th>
<th>$\hat{v}$</th>
<th>$\Delta \hat{v}$</th>
<th>$\sigma_{\hat{v}}$</th>
<th>SEP</th>
<th>SDR</th>
<th>$\hat{v}_{\text{min}}$</th>
<th>$\hat{v}_{\text{max}}$</th>
</tr>
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<td>0.0</td>
<td>22.3</td>
<td>30.3</td>
<td>20.6</td>
<td>23.2</td>
<td>124.0</td>
</tr>
<tr>
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<td>SEM</td>
<td>54.1</td>
<td>0.2</td>
<td>17.8</td>
<td>27.7</td>
<td>21.2</td>
<td>28.9</td>
<td>114.4</td>
</tr>
<tr>
<td></td>
<td>SAC</td>
<td>54.0</td>
<td>0.0</td>
<td>22.7</td>
<td>29.9</td>
<td>19.5</td>
<td>8.4</td>
<td>150.0</td>
</tr>
<tr>
<td>Holdout dataset:</td>
<td>WLS</td>
<td>66.1</td>
<td>3.1</td>
<td>28.0</td>
<td>34.7</td>
<td>20.5</td>
<td>23.4</td>
<td>125.6</td>
</tr>
<tr>
<td></td>
<td>SEM</td>
<td>64.4</td>
<td>1.4</td>
<td>23.0</td>
<td>31.2</td>
<td>21.0</td>
<td>30.3</td>
<td>117.8</td>
</tr>
<tr>
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<td>SAC</td>
<td>68.6</td>
<td>5.6</td>
<td>27.7</td>
<td>34.8</td>
<td>21.0</td>
<td>24.5</td>
<td>128.4</td>
</tr>
</tbody>
</table>

SEP: standard error of prediction; SDR: standard deviation of the residuals

The residuals of the WLS, SEM, and SAR predictions of speeds in Zurich and in Winterthur were analyzed for systematic bias with respect to categories of travel period, road type, combined travel period and road type, road densities (by type of road), and categorized values of the regional structure variables. The categories of time and road type are defined above; the other categories were chosen to have equal widths. While the absolute value of the mean of
the residuals in certain cases exceeds 10 km/h, the mean residuals are insignificantly different from zero in nearly all categories. Where bias is significant, it is due to very small samples in the category. Thus there is no systematic indication of circumstances in which certain models perform better or worse.

Figure 5 depicts the prediction results minus observations for the Winterthur area during the peak travel period to enable spatial comparison of the models. The mean speeds for these roads were shown in Figure 1. The majority of the links in the WLS and SEM model are within ±10 km/h of the observations, with larger differences occurring with the same sign in the same places. The SAC model departs from the other two to overestimate speeds on most links. There is no apparent directional dependence, nor indication that the overestimates correlate with the underlying built areas.

Finally, a comparison with an assignment model of the Canton shows that the agreement with measurement is much better in the regression models, with the added benefit of more realistic spatial and temporal variation.
8 Conclusions and outlook

This paper is the first to report an approach to estimate link speeds employing both structural variables and the network context, with correction for the spatial error and autocorrelation.
terms. The validation with a large hold-out sample showed that the carefully implemented approach produces an acceptable fit, as measured by the standard error of prediction and comparison with assignment results. Low transfer error means that application of the model across the whole cantonal network is plausible. The W matrices are unique to the dataset however, including missing links that were not measurable with GPS, and would have to be re-made for a network-wide prediction.

Estimating the range of spatial models reveals that there are substantial spatial correlations which need to be accounted for. A simple linear regression is not enough and is likely to bias the conclusions. Spatial autocorrelation and spatial error correlation models are to some extent substitutes in terms of improving model fit, but they assume different understanding of the underlying processes, which is reflected in the parameters. The joint SAC model shows that the SAR has residual spatial correlations which are corrected with rather smaller neighborhoods to obtain a better fit of the data. A network neighborhood explains speeds better than a spatially symmetric neighborhood.

While the time period and road type interactions did not reveal any surprises, the different values estimated for the different model formulations highlight the need to be careful in the interpretation of spatial regressions for policy making. The new results on the impacts of the structural variables show that one has to account for them to understand variation in local speeds.

9 Acknowledgments

The data was collected as part of the Canton Zurich’s program to monitor the quality of service offered by its road network. The support of the Canton’s Amt für Verkehr and of Thomas Niederöst is gratefully acknowledged. The assistance of James LeSage in the application of the Econometrics Toolbox for Matlab to this special case is also appreciated.

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